Physics

1. (b)

2. (b) We know, work done $W = F \cdot d$

Given, force, $F = 2\hat{i} - \hat{j} - \hat{k}$,

and position vector, $d = 3\hat{i} + 2\hat{j} - 5\hat{k}$

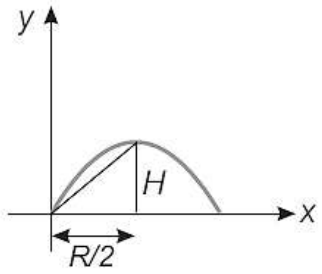
Using vector identify $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

Hence, $w = F \cdot d = (2\hat{i} - \hat{j} - \hat{k}) \cdot (3\hat{i} + 2\hat{j} - 5\hat{k})$

$= 6 - 2 + 5 = 9$ units

3. (c) We know, average velocity $= \frac{\text{displacement}}{\text{time}}$

$v_{av} = \frac{\sqrt{\mu^2 + R^2/4}}{T/2}$ ... (i)



where, $H =$ maximum height $= \frac{v^2 \sin^2 \theta}{2g}$.... (ii)

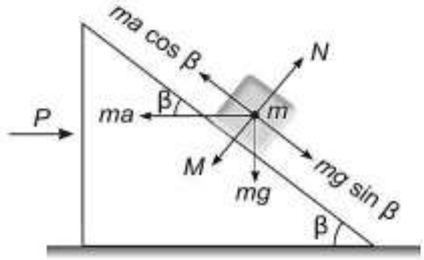
Range $R = \frac{v^2 \sin 2\theta}{g}$ ..... (iii)

Time of flight $T = \frac{2v \sin \theta}{g}$

Putting the values of Eqs. (ii), (iii) and (iv) in Eq. (i) we have

$v_{av} = \frac{v}{2}\sqrt{1 + 3\cos^2 \theta}$

4. (a) The free body diagram of the given situation is



(given)

Force = Mass × Acceleration

$$\therefore \quad P = (M + m)a$$

$$ma \cos \beta = mg \sin \beta$$

$$a = g\frac{\sin \beta}{\cos \beta}$$

$$= g \tan \beta$$

$$P = (M + m) g \tan \beta$$

5. (c) Energy of balls at rest, $K_1 = mgh_1$ and $K_2 = mgh_2$ percentage loss in KE $= \frac{K_1 - K_2}{K_1} \times 100$

$$\frac{25}{100} = \left(\frac{12 - h^2}{12}\right)$$

$$\Rightarrow \frac{25 \times 12}{100} = 12 - h_2$$

$$\Rightarrow h_2 = 12 - 3 = 9 \text{ m}$$

6. (a) Velocity of centre of mass is

$$\mathbf{v}_{CM} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2}$$

Given, $m_1 = 4$ kg, $m_2 = 5$ kg, $v_1 = 5\hat{\mathbf{j}}$ m/s

$v_2 = 3\hat{i}$ m/s

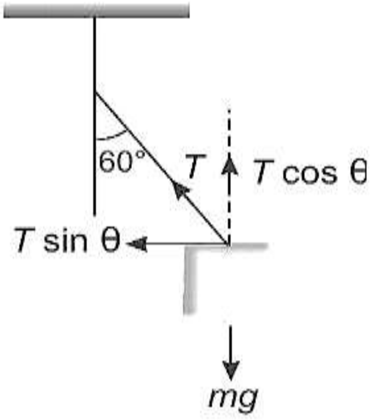
$\therefore\ \mathbf{v}_{CM} = \dfrac{4 \times 5\hat{j} + 5 \times 3\hat{i}}{5+4}$

$= \dfrac{20\hat{i}}{9} + \dfrac{15}{9}\hat{j}$

Hence, magnitude $|\mathbf{v}_{CM}| = \sqrt{\left(\frac{20}{9}\right)^2 + \left(\frac{15}{9}\right)^2}$

$= \frac{25}{9}$ m/s

7. **(b)** From law of conservation of momentum, we known,



$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$u_1 = 0, u_2 = 150$ m/s, $m_1 = 2.9$ kg and $m_2 = 0.1$ kg

So, $2.9 \times 150 = (2.9 + 0.1)v$

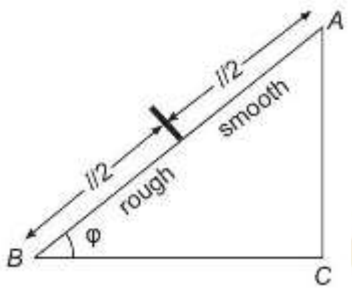
$\Rightarrow \quad \dfrac{2.9 \times 150}{3} = v$

$\Rightarrow \quad v = 145$ m/s

Also, $T\sin\theta = \dfrac{mv^2}{r}$

Putting the values and solving, we get T = 135 N.

8. **(a)** For upper half



From equation,

$v^2 = u^2 + 2as$

we have, $u = 0$ (from rest), $s = l/2$

$v^2 = 0 + 2(g\sin\varphi) \cdot \dfrac{l}{2}$

For lower half,

$v = 0$ and $a = g(\sin\varphi - \mu\cos\varphi)$,

$\Rightarrow \quad 0 = u^2 + 2g(\sin\varphi - \mu\cos\varphi) \cdot \dfrac{l}{2}$

$\Rightarrow \quad -gl\sin\varphi = gl(\sin\varphi - \mu\cos\varphi)$

$\Rightarrow \quad \mu\cos\varphi = 2\sin\varphi \Rightarrow \mu = 2\tan\varphi$

9. **(d)** Given, $I = 4$ kg $-$ m^2, $\tau = 8$ N $-$ m and $t = 20$ s

$$\tau = /\alpha$$
$$\Rightarrow \alpha = \frac{\tau}{l} = \frac{8}{4} = 2$$
$$\theta = \frac{1}{2}\alpha t^2$$
$$\Rightarrow \theta = \frac{1}{2} \times 2 \times 20 \times 20 = 400$$
$$\omega = \tau\theta = 8 \times 400 = 3200 \text{ J}$$

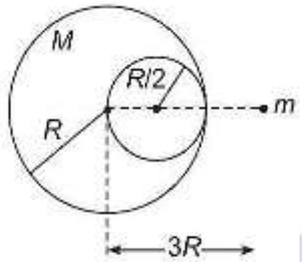
10. (b) We know the KE of a rotational circular disc

$$\text{KE} = \frac{1}{2}/\omega^2 \text{ and } I = \frac{1}{2}MR^2$$

Hence, the resultant loss rotational KE will be the addition of both energy loss is $= \frac{1}{4}/\omega^2$

11. (a) Gravitational force due to solid sphere is

$$F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2}$$



where, M and m are mass of solid sphere and particle respectively. Gravitational force on particle due to sphere with cavity

$$F_2 = \frac{GMm}{9R^2} - \frac{G\left(\frac{M}{8}\right)m}{(5R/2)^2}$$
$$= \frac{GMm}{R^2}\left[\frac{1}{9} - \frac{4}{8 \times 25}\right]$$
$$= \frac{GMm}{R^2}\left[\frac{41}{50 \times 9}\right]$$
$$\frac{F_1}{F_2} = \frac{50}{41}$$

12. (b) We knows, maximum velocity $V_{max} = A\omega = A\sqrt{\frac{K}{m}}$

Given, $K_1, m_1 = m, K_2, m_2 = 2\text{ m}$

$$(V_{max})_A = (V_{max})_B$$
$$A_A\sqrt{\frac{K_1}{M}} = A_B\sqrt{\frac{K_2}{2m}}$$
$$\Rightarrow \frac{A_A}{A_B} = \sqrt{\frac{K_2}{2K_1}}$$

13. (a) Given, $\sigma = 0.32, F = 20\text{ N}$

$$A = 0.01\text{ cm}^2 = 0.01 \times 10^{-3}\text{ m}$$

and $Y = 1 - 1 \times 10^{11}\text{ N/m}^2$

We know that

$$\frac{\Delta/}{l} = \frac{F}{AY} = \frac{20}{0.01 \times 10^{-3} \times 1.1 \times 10^{11}} = 18.1 \times 10^{-7}$$

and we also known

$$\sigma = \frac{-\Delta r/r}{\Delta //}$$

$-\dfrac{\Delta r}{r} = 0.32 \times 18.1 \times 10^{-7} = 5.79 \times 10^{-7}$

Hence, decrease in cross reactional area of wire is

$$\Delta A = 2\frac{\Delta r}{r} \times A = 2 \times 5.79 \times 10^{-7} \times 0.01 \times 10^{-3}$$
$$= 0.158 \times 10^{-10}\ m^2$$
$$= 1.26 \times 10^{-6}\ cm^2$$

14. (a) We knows height of water rise in a capillary tube

$$h = \frac{2T\cos\theta}{rdg}$$

$h_1 = \dfrac{2T_1 \cos\theta_1}{rdg},\ h_2 = \dfrac{2T_2 \cos\theta_2}{rdg}$

Given, $h_1 = h, T_1 = T, \theta_1 = 0$

$\therefore\ h = \frac{2T}{rdg} \ldots.$ (i)

Given, $T_2 = \sqrt{2}T, \theta = 45°, \cos 45° = \frac{1}{\sqrt{2}}$

$\therefore\ h_2 = \dfrac{2\sqrt{2}T \times \frac{1}{\sqrt{2}}}{rdg} \ldots.$ (ii)

From Eqs. (i) and (ii), we observe

$h_2 = h.$

Hence, same mass of liquid rises into the capillary as before 5×10^{-3} kg.

15. (c) Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

When, the two drops of same radius r coalesce then radius of new drop is R.

$\therefore\quad \dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi r^3 + \dfrac{4}{3}\pi r^3$

$\Rightarrow\quad R = 2^{1/3} \cdot r$

Critical velocity $\propto r^2$

$\therefore\quad \dfrac{v}{v_1} = \dfrac{r^2}{2^{2/3} \cdot r^2}$

$\Rightarrow\quad v_1 = \sqrt[3]{4} \cdot v$

16. (b) Due to volume expansion of both mercury and flask, the change in volume of mercury relative to flask is given by

$$\Delta V = V_0[Y_L - \gamma_g]\Delta\theta$$
$$= V[Y_m - 3\alpha_g]\Delta\theta$$

Given, $\gamma_m = 182 \times 10^{-6}/°C, \alpha_g = 10 \times 10^{-6}/°C$

$\Delta\theta = 100°C,\ V = 1\ L$

$\therefore\ \Delta V = 1[(182 \times 10^{-6} - 3 \times 10 \times 10^{-6})] \times 100$

$\Delta V = 15.2CC$

17. (c) In given condition

$$\frac{Y + 160}{-50 + 160} = \frac{340 - 273}{373 - 273}$$
$$\frac{Y + 160}{110} = \frac{67}{100}$$
$$Y + 160 = \frac{67 \times 110}{100}$$
$$Y = 73.7 - 160$$
$$Y = -86.3°Y$$

18. (a) We knows

$dQ = du + dw$

and we also known $du = 0$ for cyclio process so that

$dQ = dw$

Here, in given condition the work done during is a basic process

$w_{2-3} = P_2(v_3 - v_2)$

$w_{4-1} = p_1(v_1 - v_4)$

Total work done $= p_2(v_3 - v_2) + p_1(v_1 - v_4)$

From gas equation $pV = nRT = \frac{3\times T}{2}$

Hence, total work done

$$= \frac{3R}{2}(400 + 2500 - 700 - 1100)$$
$$= \frac{3}{2}R(2900 - 1800)$$
$$= \frac{3}{2}R(1100) = \frac{3300R}{2}$$
$$= 1650R$$

19. (c) $\frac{T_2}{T_1} = 1 - \eta = 1 - \frac{40}{100} = \frac{3}{5}$

$$\Rightarrow \quad T_1 = \frac{5}{3}T_2$$
$$\Rightarrow \quad T_1 = \frac{5}{3}\times 300 = 500\text{ K}$$

New efficiency $\eta' = 60\%$

$$\frac{T_2}{T_1'} = 1 - \eta' = 1 - \frac{60}{100} = \frac{2}{5}$$
$$\Rightarrow \quad T_1' = \frac{5}{2}\times 300 = 750\text{ K}$$

Increase in temperature $= 750 - 500 = 250$ K

20. (d) We knows from Stefan's law,

$$E = eA\sigma T^4$$
$$\text{Here,} \quad E_1 = e_1A\sigma T_1^4$$
$$E_2 = e_2A\sigma T_2^4$$
$$\text{so,} \quad E_1 = E_2$$
$$\therefore \quad e_1T_1^4 = e_2T_2^4$$
$$\Rightarrow T_2 = \frac{e_1}{e_2}T_1^{4\ 1/4} = \frac{1}{81}\times(5802)^4$$
$$\Rightarrow \quad T_B = 1934\text{ K}$$

From Wein's law, $\lambda_{AT_A} = \lambda_B T_B$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A}$$
$$\Rightarrow \frac{\lambda_B - \lambda_A}{\lambda_B} = \frac{T_B - T_B}{T_A}$$
$$\Rightarrow \frac{1}{\lambda_B} = \frac{5802 - 1934}{5802} = \frac{3968}{5802}$$
$$\Rightarrow \lambda_B = \frac{3}{2}\mu\text{ m}$$

21. (c) We knows frequency of a closed end an column

$$n_1 = \frac{v}{4/1}$$

We knows frequency of a open end an column

$$n_2 = \frac{v}{2I_2}$$

Given, $I_1 = 32$ cm, $I_2 = 66$ cm

and $n_1 - n_2 = 8$ heat/s

So, $n_1 = \frac{v}{4\times 32} = \frac{v}{128}$

and $n_2 = \frac{v}{2\times 66} = \frac{v}{132}$

In given condition,

$$\frac{v}{128}-\frac{v}{132}=8$$
$$v=8448\times 4$$
$$v=33792$$

Hence, $$n_1=\frac{33792}{128}$$
$$n_1=264\text{ Hz}$$
and $$n_2=\frac{33792}{132}$$
$$n_2=256\text{ Hz}$$

22. (b) We know that,

$$n'=n\left[\frac{v}{v-v_s\cos\theta}\right]$$

Hence,

$$n'=640\left[\frac{340}{340-\frac{100}{3}\times\frac{3}{5}}\right]$$
$$n'=640\left[\frac{340}{340-\frac{100}{5}}\right]$$
$$n'=640\times\frac{340}{320}=2\times 340$$
$$=680\text{ Hz}$$

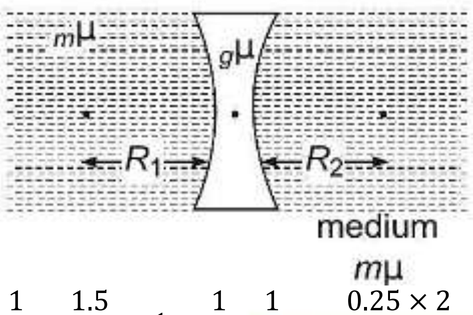
23. (a) From lens maker's formula

$$\frac{1}{f}=\left({}_g^m\mu-1\right)\frac{1}{R_1}-\frac{1}{R_2}$$

Now, $${}_g^m\mu=\frac{{}_g\mu}{{}_m}=\frac{1.5}{1.75}$$

For concave lens as shown in the figure, in this case

$R_1=-R$ and $R_2=+R$



$$\frac{1}{f}=\frac{1.5}{1.75}-1\;-\frac{1}{R}-\frac{1}{R}=+\frac{0.25\times 2}{1.75R}$$
$$\Rightarrow\ f=+3.5R$$

The positive sign shows that the lens behaves as a convergent lens.

24. (b) For eye piece

$$V_e=-25\text{ cm},\ f_e=5\text{ cm}$$
$$\Rightarrow\ \frac{1}{-25}-\frac{1}{u_e}=\frac{1}{5}$$
$$\Rightarrow\ u_e=-\frac{25}{6}\text{ cm}$$
$$v_0=L-|u_e|=10.5-\frac{25}{6}=\frac{38}{6}\text{ cm}$$

For objective

$$\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$$
$$\frac{1}{38/6}-\frac{1}{u_0}=\frac{1}{1.9}$$
$$\Rightarrow \quad \frac{1}{u_0}=\frac{6}{38}-\frac{1}{1.9}$$
$$\Rightarrow \quad u_0=2.7 \text{ cm}$$

25. **(b)** In Fresnel diffraction, no lenses are required for rendering light rays parallel and also the diffraction pattern may be dark or bright depending upon the number of half-period zones that superpose at the point. Hence, the intensity of light at a point a screen beyond or obstacle depends on the number of half-periods zones that superpose at the point.

26. **(b)** We know that the time period of a vibrating bar magnet

$$T=2\pi\sqrt{\frac{1}{MB_H}}$$

Given, $T_1=8\text{ s}, I_1=I, M=4\text{Am}^2$

$$8=2\pi\sqrt{\frac{1}{4\times B_H}} \quad \ldots\ldots \text{(i)}$$

Given, $T_2=6\text{ s}, I_2=9\times10^{-2}\text{ kg}-\text{m}^2, M=8\text{Am}^2$

$$6=2\pi\sqrt{\frac{9\times10^{-2}}{8\times B_H}} \quad ..\text{(ii)}$$

Dividing Eqs. (i) and (ii)

$$\frac{8}{6}=\sqrt{\frac{2/}{9\times10^{-2}}}$$

Squaring both sides and solving, we have

$I=8\times10^{-2}\text{ kg}-\text{m}^2$

27. **(c)** We knows, $B=\frac{\mu_0}{4\pi}\frac{M}{r^3}$

Hence the resultant horizontal magnetic induction point of the line joining their conters is

$$B=B_1+B_2+B_H$$
$$=\frac{10^{-7}\times1.2}{(10\times10^{-2})^3}+\frac{10^{-7}\times1}{(10\times10^{-2})^3}+3.6\times10^{-5}$$
$$=1.2\times10^{-4}+1\times10^{-4}+0.36\times10^{-4}$$
$$=2.56\times10^{-4}\text{ T}$$

28. **(a)** In deflection magnetometer, field due to magnet F and horizontal component B_H of earth's field are perpendicular to each other.

$\therefore$ Net field is $\sqrt{F^2+B_H^2}$

So the time period

$$T=2\pi\sqrt{\frac{1}{M\sqrt{F^2+B_H}}} \quad \ldots. \text{(i)}$$

When magnet is removed

$$T_0=2\pi\sqrt{\frac{1}{MB_H}} \quad \ldots\ldots \text{(ii)}$$

Also, $\frac{F}{B_H}=\tan\theta$

Dividing Eqs. (i) by (ii), we get

$$\frac{T}{T_0} = \sqrt{\frac{B_H}{\sqrt{F^2 + B_H^2}}}$$

$$= \sqrt{\frac{B_H}{\sqrt{B_H^2 + \tan^2\theta + B_H^2}}} = \sqrt{\frac{B_H}{B_H\sqrt{\sec^2\theta}}}$$

$$= \sqrt{\cos\theta}$$

$$T^2 = T_0^2 \cos\theta$$

29. (b) We knows capacitance $C = \frac{\varepsilon_0 A}{d}$

When plate is inserted

$$C' = \frac{\varepsilon_0 A}{d - \frac{d}{2}} = \frac{2\varepsilon_0 A}{d}$$

$$\frac{C'}{C} = \frac{2}{1}$$

30. (a) Applying junction law We have

$$I = I_1 + I_2$$

$$\frac{24 - V}{3} = \frac{10 - V}{2} + \frac{9 - V}{1}$$

$$\Rightarrow \frac{24 - V}{3} = \frac{28 - 3V}{2}$$

$$\Rightarrow 2(24 - V) = 3(28 - 3V)$$

$$\Rightarrow 48 - 2\text{ V} = 84 - 9\text{ V}$$

$$\Rightarrow 7V = 36$$

$$\Rightarrow V = 5.14\text{ V}$$

From Ohm's law

$$\Delta V = IR$$

$$\Delta V = 24 - 5.14 = 18.86, R = 3\Omega$$

$$\therefore \quad I = \frac{18.86}{3} \approx 6\text{ A}$$

31. (b) Here in given condition, we have

$$\frac{bx}{\frac{b + x}{2}} = \frac{0.625}{0.375}$$

$$\frac{bx}{(b + x)2} = \frac{25}{15}$$

$$\frac{5b}{(b + 5)2} = \frac{5}{3}$$

$$\frac{b}{2b + 10} = \frac{1}{3}$$

$$3b - 2b = 10$$

$$b = 10\Omega$$

32. (a) The heat developed per unit time in given condition is proportional to ΔT and l.

33. (d) We knows in LCR circuits

$$f = \frac{1}{2\pi\sqrt{LC}}$$

and $f \propto \frac{1}{\sqrt{L}}$

Here, in given condition

$$\frac{f_1}{f_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{L}{\sqrt{3}L}}$$

$$\Rightarrow f_1 = \frac{1}{3}^{1/4} f_2$$

34. **(d)** $i_g = \frac{S}{S+G} i$

$$\frac{1}{40} = \frac{10}{10+x}$$

$\Rightarrow \quad 10 + x = 400$

$\Rightarrow \quad x = 390\Omega$

35. **(b)** From equation of photoelectric effect, we have

$$E_1 = \frac{hc}{\lambda_1} - W \quad \ldots..(i)$$

$$E_2 = \frac{hc}{\lambda_2} - W \quad \ldots.(ii)$$

where, W is work function.

$$E_1 + W = \frac{hc}{\lambda_1} \ldots\ldots(iii)$$

$$E_2 + W = \frac{hc}{\lambda_2} \ldots..(iv)$$

From Eq. (iv)

$$W = \frac{hc}{\lambda_2} - E_2$$

∴ Putting this value in Eq. (iii), we have

$$E_1 + \frac{hc}{\lambda_1} - E_2 - \frac{hc}{\lambda_1}$$

$$\Rightarrow E_1 - E_2 = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow E_1 - E_2 = hc\left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right)$$

$$\Rightarrow hc = \frac{(E_1 - E_2)\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)}$$

36. **(c)** Maximum KE $= \frac{hc}{\lambda} - \phi_0$

Given, $\lambda = 3000Å = 3000 \times 10^{-10}$ m, $h = 6.6 \times 10^{-34}$ J − s, $c = 3 \times 10^8$ m/s, $\varphi = 2eV$

Maximum KE

$$= \frac{6 \times 10^{-34} \times 3 \times 10^8}{3000 \times 10^{-10}} \times \frac{1}{1.6 \times 10^{-12}} - 2$$

$$= 4.13 - 2 = 2.13eV$$

In Joules : $2.13 \times 1.6 \times 10^{-19} = 3.41 \times 10^{-19}$ J

37. **(b)** The relation between radius (R) and atomic number (A) is

$$\frac{R_1}{R_2} = \frac{A_1}{A_2}^{1/3}$$

Given, $R_1 = 6$ fermi, $A_1 = 125, A_2 = 27$

$$\frac{6}{R_2} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3}$$

$$R_2 = \frac{6 \times 3}{5} = \frac{18}{5} = 3.6 \text{ fermi}$$

$$= 3.6 \times 10^{-15} \text{ m}$$

38. **(b)** In 1 s , energy generated is 3.7×10^7 J

In 144×10^4 s, energy generated is

$= 3.7 \times 10^7 \times 144 \times 10^4$ J

Also, energy released in one fission is

$= 185 \text{meV}$

$= 185 \times 10^6 \times 1.6 \times 10^{-19}$ J

Number of fission $= \frac{3.7\times10^7\times144\times10^4}{185\times10^6\times1.6\times10^{-19}}$

$= 1.8 \times 10^{24}$ of U^{235} atoms.

Mass contained in 1.8×10^{24} atoms of U^{235} is

$$= \frac{235 \times 1.8 \times 10^{24}}{6.023 \times 10^{23}} = 702.3 \text{ g} = 0.70 \text{ kg}$$

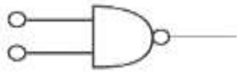
39. **(c)** Current gain $\beta = \frac{\Delta i_c}{\Delta b_b}$

$\Delta_c = (4 - 0.2)\text{mA} = 3.8 \times 10^{-3}$ A

$\Delta_b = (140 - 45)\mu\text{A} = 95 \times 10^{-6}$ A

$$\therefore \ \beta = \frac{3.8 \times 10^{-3}}{95 \times 10^{-5}} = 40$$

40. **(d)** The logic symbol of NAND gate is



Chemistry

41. **(b)** Number of spherical/radial nodes in any orbital

$= n - 1 - 1$

For s = orbitals, I = 0.

$\therefore$ Number of radial nodes in 3s-orbital

$= 3 - 0 - 1 = 2$

For p - orbitals, I = 1

$\therefore$ Number of radial nodes in 2p-orbitals

$= 2 - 1 - 1 = 0$

42. **(c)** The quantum or wave mechanical model of atom is based upon the dual nature of electron, i.e., the electron is not only a particle but has a wave character. The wave character of electron has partical significance since its wavelength is easily observed in electromagnetic spectrum.

43. **(c)** For 4th period, n = 4.

Orbitals being filled = 4s, 3d, 4p

Number of elements in the period = 2,10,6 = 18

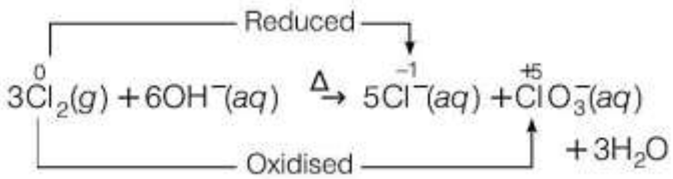
44. **(c)**

45. **(b)** (a) Hybridised orbitals show only head on overlapping and thus form only σ bonds. They never form π bonds.
(c) Head on overlapping is stronger than lateral or sideways overlapping. Therefore, the strength of bonds follows the order

$$\underbrace{\pi_{p\text{-}p}}_{\text{lateral overlapping}} < \underbrace{\sigma_{s\text{-}s} < \sigma_{s\text{-}p} < \sigma_{p\text{-}p}}_{\text{head on overlapping of same shell}}$$

(d) s-orbitals are spherically symmetrical and thus show only head on overlapping and form only σ bonds.

46. **(a)** A reaction in which the same species is simultaneously oxidised as well as reduced is called a disproportionation reaction.



47. **(d)** KE $= \frac{3}{2}$RT for 1 mole of the gas.

$\because$ 4 g of H_2 gas has 2 moles of H_2

$\therefore$ It has 2 times KE as compared to 1 mole of gas

But 8 g of O_2 gas has $1/4$ moles of O_2;

$\therefore$ It has one fourth part of the KE as compared to 1 mole of gas.

Hence, ratio of KE of H_2 and O_2

$KE_{H_2} : KE_{O_2} = 2 : \frac{1}{4} = 8 : 1$

48. (a) Two solutions are isotonic if they have same molar concentrations of the particles.

(a) 0.15 M NaCl and $0.1 M Na_2SO_4$ NaCl is an electrolyte which dissociates to give 2 ions, thus concentration of ions in the solution 0.30 M . Similarly for Na_2SO_4 (3 ions), the concentration of ions in the solution = 0.30M. Hence, both are isotonic.

49. (d) According to Raoult's law

$$\frac{p^\circ - p_s}{p^\circ} = \frac{n_2}{n_1 + n_2}$$

where, p° = vapour pressure of pure water at 100°C

= 760mmHg

p_s = vapour pressure of solution at 100°C

$$n_2 = \text{ moles of solute } = \frac{W_2}{M_2} = \frac{18}{180} = 0.1 \text{ mol}$$

$$n_1 = \text{ moles of solvent } = \frac{W_1}{M_1} = \frac{180}{18} = 10 \text{ mol}$$

By putting these values in the formula

$$\frac{p^\circ - p_s}{p^\circ} = \frac{0.1}{10 + 0.1}$$

or $10.1(p^\circ - p_s) = 0.1 p^\circ$

or $10 p^\circ = 10.1 p_s$

or $p_s = \frac{10 \times 760}{10.1} = 752.4$ mmHg.

50. (b) During the electrolysis of an aqueous solution of copper sulphate using copper electrodes, both Cu^{2+} and H^+ ions move towards cathode, but the discharge potential of Cu^{2+} ions is lower than that of H^+ ions, therefore Cu^{2+} ions are discharged in

preference to H^+ ions and copper is deposited on the cathode.

$Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$ (at cathode)

51. (c) In a fully charged lead accumulator or lead storage battery, sulphuric acid has a specific gravity (i.e., density that varies from 1.260 to 1.285. But during discharge (i.e., when the battery is in use). H_2SO_4 is used up.

$$Pb(s) + PbO_2(s) + 4H^+(aq) + 2SO_4^{2-}(aq) \rightarrow 2PbSO_4(I) + 2H_2O$$

As a result, the specific gravity of H_2SO_4 falls when it falls below 1.230 the battery needs recharging.

52. (a) In a close packed structure (hcp or ccp)

(i) Number of octahedral voids = Number of particles present in the close packing

(ii) Number of tetrahedral voids = 2 × Number of octahedral voids.

53. (c) Integrated rate equation for first order reaction

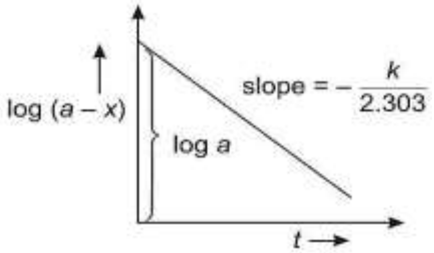
$$k = \frac{2.303}{t} \log \frac{a}{(a-x)}$$

$$\text{or } \frac{k}{2.303} t = \log \frac{a}{(a-x)}$$

$$= \log a - \log(a-x)$$

$$\text{or } \log(a-x) = -\frac{k}{2.303} t + \log a$$

Thus, if log $(a-x)$ values are plotted against time ' t ' the graph obtained should be a straight line.



54. (b)

55. (b) As a mixture of NH_4OH and NH_4Cl acts as a basic buffer, so its pH must be basic, (i.e., greater than 7), hence the answer must be 2nd. It can also be find by calculations :

$$pOH = pK_b + \log\frac{[\text{ salt }]}{[\text{ base }]}$$
$$= 4.8 + \log\frac{0.2M}{0.02M}$$
$$= 4.8 \times \log 10$$
$$pOH = 4.8 + 1 = 5.8$$
$$\therefore\ pH = 14 - pOH = 14 - 5.8 = 8.2$$

56. (d) Evaporation of water in an open vessel is a process which takes place by itself, by absorption of heat from the surroundings, because the gaseous water molecules are more random than the liquid water molecules.

$H_2O(l) \rightarrow H_2O(g);\ \Delta H = +40.8\ kJ\ mol^{-1}$

In other words, the process is spontaneous because it is accompanied by increase of entropy, which is further a measure of randomness or disorder of the system.

57. (a) The examples of colloidal systems are sols (solids in liquids), gels (liquids in solids), emulsions (liquids in liquids) and foams (gases in liquids) whereas aerosols are the colloidal system in which dispersed phase is liquid and dispersion medium is gas.

58. (a) (a) Heavy water is injurous to human beings, plants and animals since it slows down the rates of reactions occuring in them.

(b) Heavy water reacts with aluminium carbide forming deuteromethane.

$$\underset{\text{aluminium carbide}}{Al_4C_3} + 12D_2O \longrightarrow 4Al(OD)_3 + \underset{\text{deuteromethane}}{CD_4}$$

(c) In interstitial hydrates or deuterates, water molecules are present in interstitial sites or voids in the crystal lattice. e.g., $BaCl_2 \cdot 2H_2O$ and similarly, $BaCl_2 \cdot 2D_2O$ are interstitial compounds.

59. (b) For titration of a basic solution of Na_2CO_3 and $NaHCO_3$ against HCl , if phenolphthalein is used as indicator, the end point is indicated only for half neutralization of Na_2CO_3, i.e., (up to $NaHCO_3$).

$Na_2CO_3 + HCl \rightarrow NaHCO_3 + NaCl$

The remaining solution then contains the unreacted $NaHCO_3$ from this reaction plus the unreacted $NaHCO_3$ originally in the solution. At the phenolphthalein end point, there is no reaction between HCl and $NaHCO_3$.

From the equations

$$\text{Mol of HCl consumed} = \text{mol of } Na_2CO_3$$
$$20\ mL \text{ of } 0.1M = 20\ mL \text{ of } 0.1M$$

$\therefore$ The concentration of Na_2CO_3 in solution $X = 0.1M$.

Note that for a quantity of Na_2CO_3, exactly half volume of the HCl is used at the phenolphthalein end point and the second half volume of the HCl is required for complete neutralization of Na_2CO_3 at methyl orange end point.

$NaHCO_3 + HCl \rightarrow NaCl + CO_2 + H_2O$

$\therefore$ Volume of HCl required to neutralize

Na_2CO_3 in original sample $= 2 \times 20\ mL$

$= 40\ mL$

If methyl orange is used, the end point is indicated when all the alkali is neutralized.

$NaHCO_3 + HCl \rightarrow NaCl + CO_2 + H_2O$

As 40 mL of 0.1 M HCl is consumed in complete neutralization of Na_2CO_3 at methyl orange end point, so the volume of HCl used to neutralized $NaHCO_3$ from the original sample would be Remaining HCl $= 60 - 40 = 20$ mL of 0.1 M

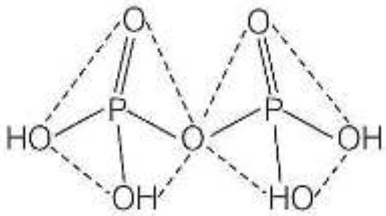
As per equation = 1 mol of $NaHCO_3$ = 1 mol of HCl $\therefore$ 0.1 mol of $NaHCO_3$ = 0.1 mol of HCl ,

60. (a) Diborane reacts with HCl in the presence of $AlCl_3$ catalyst and liberates H_2 gas.

$$\underset{\text{diborane}}{B_2H_6} + HCl \xrightarrow{AlCl_3} B_2H_5Cl + H_2 \uparrow$$

61. (c) If all the four corners, i.e., all the four oxygen atoms of each tetrahedra (SiO_4) are shared with other tetrahedra, three-dimensional network structure is obtained. i.e., different forms of silica such as quartz, tridymite and crystobalite.

62. (c) In pyrophosphoric acid ($H_4P_2O_7$), the phosphorus is bonded in tetrahedral manner with four sp^3 bonds. It has +5 oxidation state and it has four $P - OH$ bonds, two $P = O$ bonds and one $P - O - P$ linkage.



pyrophosphoric acid ($H_4P_2O_7$)

$P_4O_{10} + 6H_2O \rightarrow \underset{\text{orthophosphoric acid}}{4H_3PO_4}$

63. (d) Sodium thiosulphate is oxidised by moist Cl_2 or chlorine water and precipitates sulphur,

$\underset{\text{sodium thiosulphate}}{Na_2S_2O_3} + Cl_2 + H_2O \rightarrow Na_2SO_4 + 2HCl + \underset{\text{sulphur}}{S}$

64. (b) In Whytlaw-Gray's method for preparation of fluorine, the copper diaphragm is used to prevent the mixing of H_2 and F_2 liberated at cathode and anode respectively. Reactions in the electrolytic cell

$KHF_2 \rightarrow KF + HF$

$\Downarrow$

$K^+ + F^-$

At cathode

$K^+ + e^- \rightarrow K$

$K + HF \rightarrow KF + H$

$2H \rightarrow H_2$

At anode

$F^- \rightarrow F + e^-$

$2F \rightarrow F_2$

65. (d) Liquid xenon is used in bubble chamber for the detection of γ-photon and neutral mesons.

66. (a) When a compound absorbs a certain wavelength (i.e., 490 nm − 500 nm) from the visible light which corresponds to the blue-green light in the visible spectrum, the colour or light transmitted by it does not contain the colour of absorbed radiation and thus shows the complementary colour, i.e., red.

67. (d) In the extraction of silver from cyanide process (also Mac-Arthur Forrest process), the siliver compound dissolve in NaCN solution forming a soluble complex salt, in presence of air, then silver is precipitated from this complex salt by the addition of Zn , because Zn being more reactive displaces the Ag.

$Ag_2S + 4NaCN \rightleftharpoons 2Na[Ag(CN)_2] + Na_2S \downarrow$

$Na_2S + 2O_2 \rightarrow Na_2SO_4$

$2Na[Ag(CN)_2] + Zn \rightarrow Na_2[Zn(CN)_4] + 2Ag \downarrow$

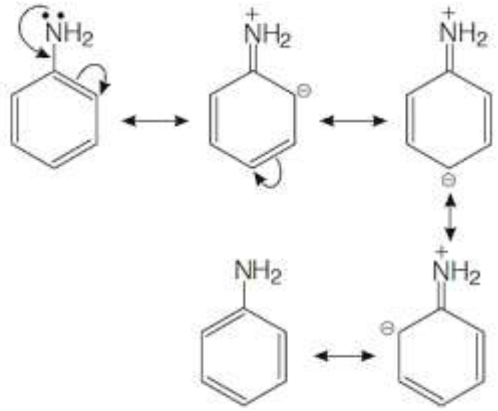
68. (b) The most serious effect of the delpetion of ozone layer is that the UV rays coming from the sun can pass through the stratosphere and reach the surface of the earth. IT has been found that with increase in the exposure to UV rays, the chance for occurance of skin cancer, increases, also exposure of eye to UV rays damages the cornea and lens of the eye and may cause cataract and even blindness.

69. (d) Ferric salts (such as $FeCl_3$) form Prussian blue (blue ppt or colouration) with potassium ferrocyanide.

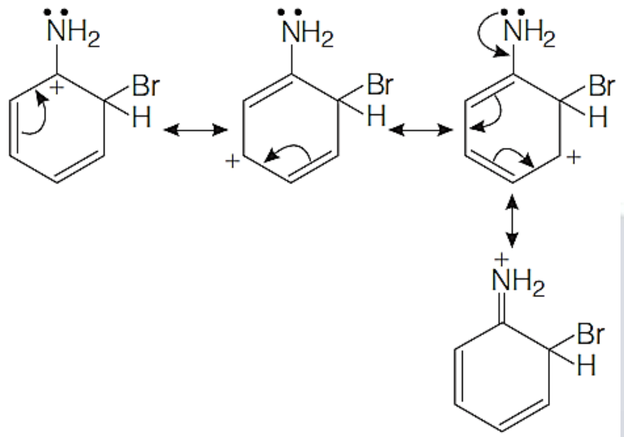
$4FeCl_3 + 3K_4[Fe(CN)_6] \longrightarrow \underset{\substack{\text{Prussian blue} \\ \text{(ferric ferrocyanide)}}}{Fe_4[Fe(CN)_6]_3} + 12KCl$

70. (c) Decomposition of a compound by application of heat is called pyrolysis and pyrolysis of higher alkanes into a mixture of lower alkanes, alkenes, etc. is also called cracking.

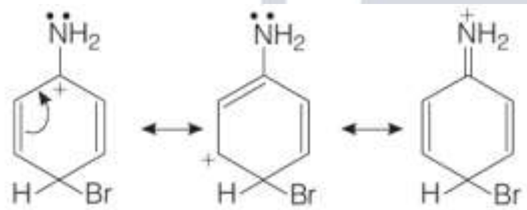
71. (a) The $-NH_2$ group of aniline is a very strong electron donor (+M effect), hence it activates the benzene ring thoroughly and the electrophilic aromatic substitutions on the benzene ring are very easy to take place at ortho and para-positions.



o- bromination



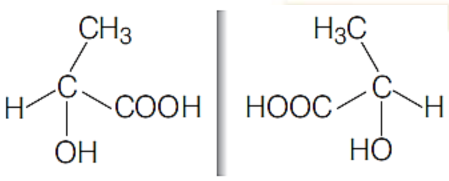
P - bromination



In addition to the usual resonating structures, that stabilizes the intermediate carbonium ion, the resonating strucutres formed by the interaction of lone pair electrons of nitrogen with the positively charged carbon of the ring also increase the stability of the carbonium ion formed during the atttack of Br^{+}ion (electrophile) on o - and p-positions.

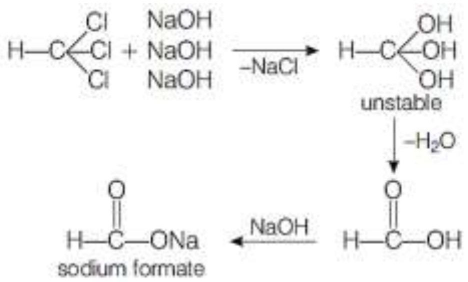
72. **(a)** The compound which is non-superimposable on its mirror image, is called optically active or chiral and its two non-superimposable mirror images are called enantiomers which have all physical and chemical properties same and also rotate the plane polarised light upto same extent but in opposite direction.

enantiomers of lactic acid

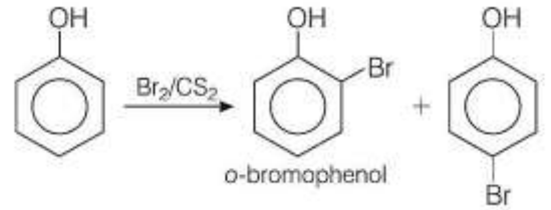


enantiomers of lactic acid

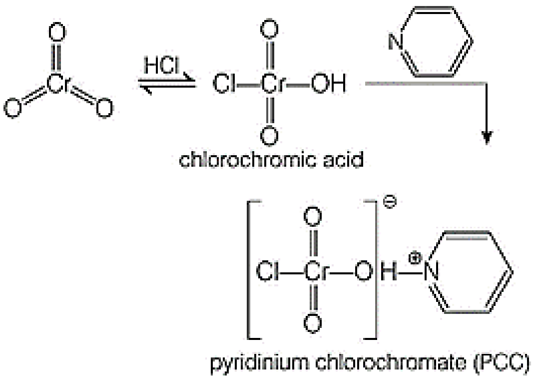
73. **(c)** On heating chloroform with concentrated aqueous or alcoholic NaOH , we get sodium formate,



74. (b) Phenol when treated with Br_2 in the presence of non-polar solvent CS_2, it gives only o - and p-bromophenols instead the trisubstituted products. Reason for the above observation is supression of phenoxide ion in non-polar solvent. Thus, we get only mono substituted products.



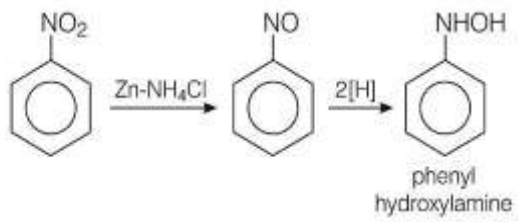
75. (d) Pyridinium chlorochromate (PCC) can be prepared by the dissolution of chromium trioxide in aqueous HCl . Addition of pyridine gives pridinium chlorochromate as orange cryotol.



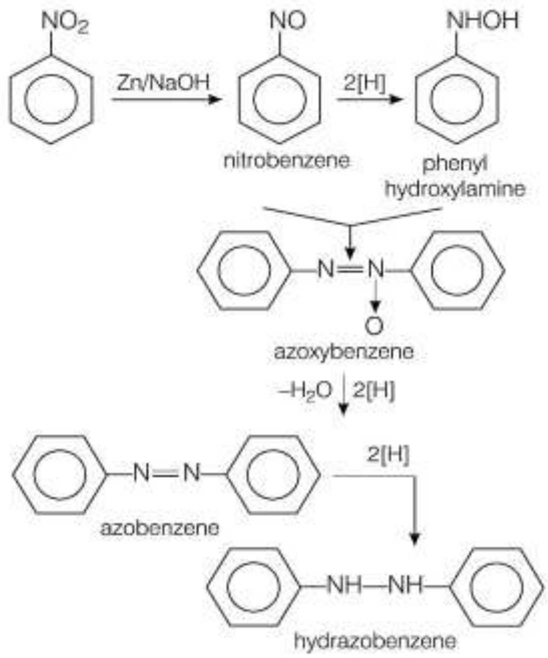
PCC offers the advantage of the selective oxidation of alcohols to aldehydes, whereas many other reagents are less selective.

76. (a) The more the value of pK_a, compound will be less acidic indicating the less value of K_a and smaller the value of pK_a, the compound will be more acidic, i.e., there will be more the value of K_a.

77. (c)(i) On reduction in neutral media, using Zn dust and NH_4Cl solution nitrobenzene gives phenyl hydroxylamine.

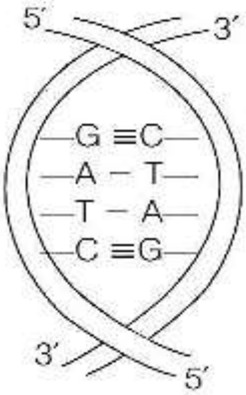


(ii) While in alkaline medium, using Zn – NaOH, mononuclear intermediate products (nitrobenzene and phenylhydroxyl amine) interact to each other to give dinuclear product. Final product using Zn – NaOH is hydrazobenzene, which in a formed viz the formation of azoxybenzene and azobenzene.



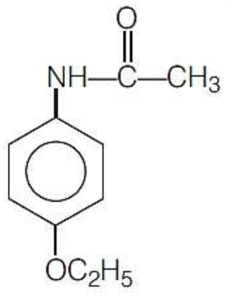
78. (b) The polymers which disintegrate by themselves during a certain period of time by enzymatic hydrolysis and to some extent by oxidation, are known as biodegradable polymers. Example → Poly- β-hydroxy-butyrate-co- β-hydroxyvalerate (PHBV), which is used in orthopaedic devices and in controlled drug release, and poly (glycollic acid) poly (lactic acid) or commonly known as dextron, which is used for stitching of wounds after operation.

79. (b) The only possible pairing in DNA are between G (guanine) and C (cytosine) through three H -bonds i.e., (C ≡ G) and between A (adenine) and T (thymine) through two H -bonds (i.e., A = T) as shown in figure.



The double α-helix structure of DNA.

80. (d) Phenacetin is a derivative of p-aminophenol and used as analgesic (pain killer). The main limitation of this drug is that it may act on red blood cells and thus may be harmful even in moderate dose.



phenacetin

Mathematics

81. (a) Given

$f(x) = (p - x^n)^{1/n}, p > 0$

$f[f(x)] = f[(p - x^n)^{1/n}]$

Now $= \{p - (p - x^n)^{1/n \times n}\}^{1/n}$

$= (x^n)^{1/n} = x$

82. (a) $x \in \mathbb{R} \mid \log[(1.6)^{1-x^2} - (0.625)^{6(1+x)}] \in \mathbb{R}\}$

Now, $(1.6)^{1-x^2} > (0.625)^{6(1+x)}$

$\Rightarrow \quad (1.6)^{1-x^2} > (0.625)^{6(1+x)}$

$= \left(\frac{8}{5}\right)^{1-x^2} > \left(\frac{8}{5}\right)^{-6(1+x)}$

$\therefore \quad 1 - x^2 > -6(1 + x)$

$\Rightarrow \quad x^2 - 6x - 7 < 0$

$\Rightarrow \quad (x - 7)(x + 1) < 0$

$\Rightarrow \quad x \in (-\infty, -1) \cup (7, \infty)$

Hence,

$x \in \mathbb{R} \middle| \log[(1.6)^{1-x^2} - (0.625)^{6(1+x)}] \middle| \in \mathbb{R}\}$

$= (-\infty, -1) \cup (7, \infty)$

83. (a)

84. (c) Given,

${}^nC_{r-1} = 330,\ {}^nC_r = 462$

and ${}^nC_{r+1} = 462$

Now, $\frac{{}^nC_{r+1}}{{}^nC_r} = 1$

$$\Rightarrow \frac{\frac{n!}{(r+1)!\,(n-r-1)!}}{\frac{n!}{r!\,(n-r)!}} = 1$$

$\Rightarrow$

$$\Rightarrow \frac{r!\,(n-r)(n-r-1)!}{(r+1)r!\,(n-r-1)!} = 1$$

$$\Rightarrow \frac{n-r}{r+1} = 1 \Rightarrow n - 2r = 1 \quad \ldots\ldots (i)$$

Again, $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{462}{330} = \frac{77}{55}$

$$\Rightarrow \frac{\frac{n!}{r!\,(n-r)!}}{\frac{n!}{(r-1)!\,(n-r+1)!}} = \frac{77}{55}$$

$$\Rightarrow \frac{(r-1)!\,(n-r+1)(n-r)!}{r(r-1)!\,(n-r)!} = \frac{77}{55}$$

$\Rightarrow \quad 55n - 132r + 55 = 0 \ldots\ldots (ii)$

From Eqs. (i) and (ii), we get

$22r = 110$

$\therefore \quad r = 5$

85. (d) M M M M M M M M M M

W W W W W W W W W W W

First, we arrange 10 men in a row at alternate position.

So, number of ways formula = 10 !

Now, 6 women can arrange in 11 positions

So, number of ways for women = ${}^{11}P_6$

Required number of ways = $10! \times {}^{11}P_6$

$= \frac{10!\,11!}{5!}$

86. (c) t_n = The number of triangles formed with n points in a plane, no three of which are collinear.

i.e., $t_n = {}^nC_3$

$\Rightarrow \quad t_{n+1} = {}^{n+1}C_3$

Now, $t_{n+1} - t_n = 36$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 36$

$$\Rightarrow \frac{(n+1)!}{(n-2)!\,3!} - \frac{n!}{(n-3)!\,3!} = 36$$

$$\Rightarrow \frac{(n+1)n(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 36$$

$\Rightarrow n(n-1)(n+1-n+2) = 36 \times 6$

$\Rightarrow 3n(n-1) = 36 \times 6$

$\Rightarrow n^2 - n - 72 = 0$

$\Rightarrow n^2 - 9n + 8n - 72 = 0$

$\Rightarrow n(n-9) + 8(n-9) = 0$

$\Rightarrow (n-9)(n+8) = 0$

$\therefore\ n = 9\ (\because n \neq -3)$

87. (b)

$$\left[\frac{(x+1)}{(x^{2/3} - x^{1/3} + 1)} - \frac{(x-1)}{(x-\sqrt{x})}\right]^{10}$$

$$= \left[\frac{\left(x^{1/3}\right)^3 + 1^3}{(x^{2/3} - x^{1/3} + 1)} - \frac{\{(\sqrt{x})^2 - 1\}}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$$= \left[\frac{\left(x^{1/3}+1\right)\left(x^{2/3} - x^{1/3} + 1\right)}{(x^{2/3} - x^{1/3} + 1)} - \frac{\{(\sqrt{x})^2 - 1\}}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$$= \left[\left(x^{1/3}+1\right) - \frac{(\sqrt{x}+1)}{\sqrt{x}}\right]^{10} = \left(x^{1/3} - x^{-1/2}\right)^{10}$$

The general term is

$$T_{r+1} = {}^{10}C_r\left(x^{1/3}\right)^{10-r}\left(-x^{-1/2}\right)^r$$

$$= {}^{10}C_r(-1)^r \cdot x^{\frac{10-r}{3} - \frac{r}{2}}$$

For the term independent of x, put

$$\frac{10-r}{3} - \frac{r}{2} = 0$$

$\Rightarrow \quad 20 - 2r - 3r = 0$

$\Rightarrow \quad 20 = 5r \Rightarrow r = 4$

$$\therefore \quad T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

88. (c) Given expression is

$$E = \frac{(1-2x)^{-1}(1-3x)^{-2}}{(1-4x)^{-3}}$$

$$= \frac{(1 + 2x + 2x^2 + \cdots)(1 + 6x + \cdots)}{(1 + 12x + \cdots)}$$

$$= \frac{(1 + 2x + 6x + \cdots)}{(1 + 12x)}$$

$= (1+8x)(1+12x)^{-1}$

(neglect higher powers)

$= (1+8x)(1-12x\ldots)$

$= (1 + 8x - 12x\ldots)$

$= (1-4x)$

($\because$ neglecting the higher term)

89. (d) Given,

$$\frac{1}{x^4+x^2+1}=-\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2-x+1}$$
$$\Rightarrow 1=(Ax+B)(x^2-x+1)+(Cx+D)(x^2+x+1)$$
$$\Rightarrow 1=(Ax^3+Ax^{2+}Ax+Bx^2-Bx+B)$$
$$+(Cx^3+Cx^2+Cx+Dx^2+Dx+D)$$
$$\Rightarrow 1=(A+C)x^3+(-A+B+C+D)x^2$$
$$+(A-B+C+D)x+(B+D)$$

On comparing, the coefficient of like powers on both sides, we get

$$A+C \quad =0 \ldots.(i)$$
$$-A+B+C+D \quad =0 \ldots(ii)$$
$$A-B+C+D \quad =0 \ldots..(iii)$$
$$B+D \quad =1 \ldots.(iv)$$

and

On adding Eqs. (ii) and (iii), we get

$$2(C+D) \quad =0$$
$$\Rightarrow \quad C+D=0$$

90. (b)

$$\frac{1}{2\cdot 3}+\frac{1}{4\cdot 5}+\frac{1}{6\cdot 7}+\frac{1}{8\cdot 9}+\cdots\infty$$
$$=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+\left(\frac{1}{8}-\frac{1}{9}\right)+\cdots\infty$$
$$=1-\log_e 2$$
$$=\quad \log_e e-\log_e 2$$
$$=\quad \log\left(\frac{e}{2}\right)$$

91. (d) Given equation is

$(5+\sqrt{2})x^2-bx+(8+2\sqrt{5})=0$

Let α and β be the roots of this equation.

$$\therefore \quad \alpha+\beta=\frac{b}{5+\sqrt{2}}$$
$$\text{and} \quad \alpha\beta=\frac{8+2\sqrt{3}}{5+\sqrt{2}}$$

Given that harmonic mean between the roots of the given equation is 4 .

$$\therefore \quad \frac{2\alpha\beta}{\alpha+\beta}=4$$
$$\Rightarrow \quad \frac{8+2\sqrt{5}}{5+\sqrt{2}}\times\frac{5+\sqrt{2}}{b}=2$$
$$\therefore \quad b=\frac{8+2\sqrt{5}}{2}$$
$$=4+\sqrt{5}$$

92. (b) Given, $x^2+5x+6\geq 0$ and $x^2+3x-4<0$

$\Rightarrow\ x^2+2x+3x+6\geq 0$

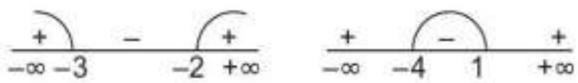
and $x^2+4x-x-4<0$

$\Rightarrow\ x(x+2)+3(x+2)\geq 0$

and $x(x+4)-1(x+4)<0$

$\Rightarrow\ (x+2)(x+3)\geq 0$

and $(x+4)(x-1)<0$



$\Rightarrow x\in(-\infty,-3]\cup[-2,\infty)$ and $x\in(-4,1)$

Common condition is

$x\in(-4,-3]\cup[-2,1)$

93. (c) Given, cubic equation is

$$x^3 - 42x^2 + 336x - 512 = 0$$

$$\Rightarrow \quad x^2(x-2) - 40x(x-2) + 256(x-2) = 0$$

$$\Rightarrow \quad (x-2)(x^2 - 40x + 256) = 0$$

$$\Rightarrow \quad (x-2)\{x^2 - 32x - 8x + 256\} = 0$$

$$\Rightarrow \quad (x-2)\{x(x-32) - 8(x-32)\} = 0$$

$$\Rightarrow \quad (x-2)(x-32)(x-8) = 0$$

$$\Rightarrow \quad (x-2)(x-8)(x-32) = 0$$

$$\Rightarrow \quad x = 2,8,32$$

Which represents a geometric progression in increasing order.

Common ratio $= \frac{8}{2} = 4:1$

94. (c)

95. (b) Given

$$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 64+10 & -40+20 \\ -16+8 & 10+16 \end{bmatrix}$$

$$= \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$$

$$4A = \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix}$$

$$-pl = \begin{bmatrix} -p & 0 \\ 0 & -p \end{bmatrix}$$

Since, the matrix A satisfies the equation

$$x^2 + 4x - p = 0, \text{ then}$$

$$A^2 + 4A - pl = 0$$

$$\Rightarrow$$

$$\begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} + \begin{bmatrix} -p & 0 \\ 0 & -p \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow$$

$$\Rightarrow \begin{bmatrix} 74-32-p & -20+20+0 \\ -8+8+0 & 26+16-p \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 42-p & 0 \\ 0 & 42-p \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing, we get

$42 - p = 0 \Rightarrow p = 42$

96. (d)

$$\text{Let } \Delta = \begin{vmatrix} 42-p=0 & \Rightarrow & p=42 \\ x+2 & x+3 & x+5 \\ x+4 & x+6 & x+9 \\ x+8 & x+11 & x+15 \end{vmatrix}$$

Apply operations $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} x+2 & x+3 & x+5 \\ 2 & 3 & 4 \\ 6 & 8 & 10 \end{vmatrix}$$

Again, apply operation $C_2 \to C_2 - C_1$,

$C_3 \to C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} x+2 & 1 & 3 \\ 2 & 1 & 2 \\ 6 & 2 & 4 \end{vmatrix}$$

Expand along R_1, we get

$\Delta = (x+2)(4-4) - 1(8-12) + 3(4-6)$
$= 0 + 4 + 3(-2) = 4 - 6 = -2$

97. (d) Given system of equation is

$3x + 2y + z = 6$
$3x + 4y + 3z = 14$
$6x + 10y + 8z = a$

Here, $A = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 4 & 3 \\ 6 & 10 & 8 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 14 \\ a \end{bmatrix}$

$C_{11} = (32-30) = 2, C_{12} = -(24-18) = -6,$
$C_{13} = (30-24) = 6$
$C_{21} = -(16-10) = -6, C_{22} = (24-6) = 18,$
$C_{23} = -(30-12) = -18$
$C_{31} = (6-4) = 2, C_{32} = -(9-3) = -6,$
$C_{33} = (12-6) = 6$

$$\operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}' = \begin{bmatrix} 2 & -6 & 6 \\ -6 & 18 & -18 \\ 2 & -6 & 6 \end{bmatrix}'$$

$$= \begin{bmatrix} 2 & -6 & 2 \\ -6 & 18 & -6 \\ 6 & -18 & 6 \end{bmatrix}$$

So, $(\operatorname{adj} A)B = \begin{bmatrix} 2 & -6 & 2 \\ -6 & 18 & -6 \\ 6 & -18 & 6 \end{bmatrix} \begin{bmatrix} 6 \\ 14 \\ a \end{bmatrix}$

$$= \begin{bmatrix} 12 - 84 + 2a \\ -36 + 252 - 6a \\ 36 - 252 + 6a \end{bmatrix} = \begin{bmatrix} -72 + 2a \\ 216 - 6a \\ -216 + 6a \end{bmatrix}$$

and

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 4 & 3 \\ 6 & 10 & 8 \end{vmatrix}$$

$= 3(32-30) - 2(24-18) + 1(30-24)$
$= 3(2) - 2(6) + 6 = 6 - 12 + 6 = 0$

We know that, if $|A| = 0$ and $(\operatorname{adj} A) \cdot B = 0$, then the system of equations is consistent and has an infinite number of solutions.

$(\operatorname{adj} A) \cdot B = 0$

$$\begin{bmatrix} -72 + 2a \\ 216 + 6a \\ -216 + 6a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

On comparing, we get

$2a - 72 = 0$
$\therefore \ a = 36$

98. (c)

99. (c)

$$\left(\frac{1+i}{1-i}\right)^4 + \left(\frac{1-i}{1+i}\right)^4$$
$$= \left\{\frac{(1+i)(1+i)}{(1-i)(1+i)}\right\}^4 + \left\{\frac{(1-i)(1-i)}{(1+i)(1-i)}\right\}^4$$
$$= \left\{\frac{(1+i)^2}{1-i^2}\right\}^4 + \left\{\frac{(1-i)^2}{1-i^2}\right\}^4$$
$$= \left\{\frac{1+i^2+2i}{1+1}\right\}^4 + \left\{\frac{1+i^2-2i}{1+1}\right\}^4$$
$$= \left\{\frac{1-1+2i}{2}\right\}^4 + \left\{\frac{1-1-2i}{2}\right\}^4$$
$$= (i)^4 + (-i)^4$$
$$= i^4 + i^4 = 2i^4 = 2(1) = 2$$

100. **(b)**

101. **(a)** $\frac{(1+i)x-i}{2+i} + \frac{(1+2i)y+i}{2-i} = 1$

$$\Rightarrow \frac{[(1+i)x-i](2-i)}{(4-i^2)} + \frac{[(1+2i)y+i](2+i)}{(4-i^2)} = 1$$
$$\Rightarrow \frac{2(1+i)x-2i-i(1+i)x+i^2}{4+1}$$
$$+\frac{2(1+2i)y+2i+i(1+2i)y+i^2}{(4+1)} = 1$$
$$\Rightarrow \frac{(2+2i-i-i^2)x-2i+i^2}{5}$$
$$+\frac{(4i+2+i+2i^2)y+2i+i^2}{5} = 1$$
$$\Rightarrow \frac{(2+i+1)x-2i-1}{5} + \frac{(5i+2-2)y+2i-1}{5} = 1$$
$$\Rightarrow (3+i)x-2i-1+(5i)y+2i-1 = 5$$
$$\Rightarrow (3+i)x+5iy = 7$$
$$\Rightarrow 3x+ix+5iy-7 = 0$$
$$\Rightarrow (3x-7)+(x+5y)i = 0+i0$$

On comparing, we get

$3x - 7 = 0$

$\Rightarrow x = \frac{7}{3}$ and $x + 5y = 0$

$\Rightarrow y = \frac{-7}{15}$

Hence, $(x, y) = \frac{7}{3}, \frac{-7}{15}$

102. **(d)** Given, $f(x) = \cos\left(\frac{x}{3}\right) + \sin\left(\frac{x}{2}\right)$

Period of $\cos x$ and $\sin x$ are 2π.

$\therefore$ Period of $f(x)$ $=$ Period of $\left[\cos\frac{x}{3} + \sin\frac{x}{2}\right]$

$=$ Period of $\cos\frac{x}{3} +$ Period of $\sin\frac{x}{2}$

$$= \frac{2\pi}{\left(\frac{1}{3}\right)} + \frac{2\pi}{\left(\frac{1}{2}\right)} = \frac{6\pi}{1} + \frac{4\pi}{1}$$
$$= \frac{\text{LCM of }(6\pi, 4\pi)}{\text{LCM of }(1.1)} = \frac{12\pi}{1} = 12\pi$$

103. **(d)** Given, $\sin\theta + \cos\theta = p \ldots$ (i)

and $\sin^3\theta + \cos^3\theta = q \ldots$ (ii)

$\Rightarrow (\sin\theta + \cos\theta)$
$(\sin^2\theta - \sin\theta \cdot \cos\theta + \cos^2\theta) = q$
$\Rightarrow p(1 - \sin\theta \cdot \cos\theta) = q$
[From Eq. (i) and $\sin^2\theta + \cos^2\theta = 1$]

$$\Rightarrow 1 - \sin\theta \cdot \cos\theta = \frac{q}{p}$$

$$\Rightarrow \sin\theta \cdot \cos\theta = 1 - \frac{q}{p} \ldots\ldots \text{(iii)}$$

On squaring both sides of Eq. (i), we get
$\sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta = p^2$
$\Rightarrow 1 + 2\left(1 - \frac{q}{p}\right) = p^2$ [from Eq. (iii)]
$\Rightarrow p + 2(p - q) = p^3$
$\Rightarrow 3p - 2q = p^3$
$\Rightarrow p^3 - 3p = -2q$
$\Rightarrow p(p^2 - 3) = -2q$

104. **(a)** Given, $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$$\Rightarrow \tan(\pi\cos\theta) = \tan\left\{\frac{\pi}{2} - \pi\sin\theta\right\}$$

$$\Rightarrow \pi\cos\theta = \frac{\pi}{2} - \pi\sin\theta$$

$$\Rightarrow \sin\theta + \cos\theta = \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos\theta \cdot \cos\frac{\pi}{4} + \sin\theta \cdot \sin\frac{\pi}{4} = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$$

105. **(d)** Given system of equation is
$x + y = \frac{2\pi}{3}$ .... (i)

and $\cos x + \cos y = \frac{3}{2}$, where x, y are real.

$$\Rightarrow 2\cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$$

$$\Rightarrow \cos\left(\frac{1}{2} \cdot \frac{2\pi}{3}\right) \cdot \cos\left(\frac{x-y}{2}\right) = \frac{3}{4}$$

[from Eq. (i)]

$$\Rightarrow \cos\left(\frac{\pi}{3}\right) \cdot \cos\left(\frac{x-y}{2}\right) = \frac{3}{4}$$

$$\Rightarrow \frac{1}{2}\cos\left(\frac{x-y}{2}\right) = \frac{3}{4}$$

$$\Rightarrow \cos\left(\frac{x-y}{2}\right) = \frac{3}{2} \ldots\ldots \text{(ii)}$$

Now, we have

$$\cos(x - y) = 2\cos^2\left(\frac{x-y}{2}\right) - 1$$

$$= 2 \times \frac{9}{4} - 1 = \frac{9}{2} - 1 = \frac{7}{2} \text{ [from Eq. (ii)]}$$

$$\text{and } \cos(x - y) = 1 - 2\sin^2\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \sin\left(\frac{x-y}{2}\right) < 0$$

So, system of equation have empty set of solution.

106. **(c)**

$$\cos^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\cos^{-1}x$$

$$=\cos^{-1}\left[\frac{5}{13}\cdot\frac{3}{5}-\sqrt{1-\frac{25}{169}}\cdot\sqrt{1-\frac{9}{25}}\right]=\cos^{-1}x$$

$$\Rightarrow\ \cos^{-1}\left[\frac{3}{13}-\frac{12}{13}\cdot\frac{4}{5}\right]=\cos^{-1}x$$

$$\Rightarrow\ \cos^{-1}\left[\frac{15-48}{65}\right]=\cos^{-1}x$$

$$\therefore\ x=\frac{-33}{65}$$

107. **(d)**

$$\tanh^{-1}\left(\frac{1}{2}\right)+\coth^{-1}(2)$$

$$\Rightarrow\tanh^{-1}\left(\frac{1}{2}\right)+\tanh^{-1}\left(\frac{1}{2}\right)[\coth^{-1}x=\tanh^{-1}(1/x)]$$

$$\Rightarrow 2\tanh^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow 2\cdot\frac{1}{2}\log\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)\left[\tanh^{-1}x=\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)\right]$$

$$=\log\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right)=\log 3$$

108. **(a)** In $\triangle$ ABC,

$$r_1r_2+r_2r_3+r_3r_1$$

$$=\frac{\Delta}{s-a}\cdot\frac{\Delta}{s-b}+\frac{\Delta}{s-b}\cdot\frac{\Delta}{s-c}+\frac{\Delta}{s-c}\cdot\frac{\Delta}{s-a}$$

$$=\frac{\Delta_2}{(s-a)(s-b)(s-c)}[s-c+s-a+s-b]$$

$$=\frac{\Delta^2 3s-(a+b+c)}{(s-a)(s-b)(s-c)}\ (\because 2s=a+b+c)$$

$$=\frac{\Delta^2(3s-2s)}{(s-a)(s-b)(s-c)}=\frac{\Delta^2\cdot s}{\frac{\Delta^2}{s}}$$

$$\because\frac{\Delta^2}{s}=(s-a)(s-b)(s-c)$$

$$=\frac{s^2\cdot\Delta^2}{\Delta^2}=\frac{\Delta^2}{\frac{\Delta}{s}{}^2}=\frac{\Delta^2}{r^2}\quad\because r=\frac{\Delta}{s}$$

109. **(c)** In $\triangle$ ABC,

$$\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$$

Let $\angle C=60°$, then

$$\cos C=\frac{\pi}{3}$$

$\Rightarrow\ a^2+b^2-c^2=ab$

$\Rightarrow b^2+bc+a^2+ac=ab+ac+bc+c^2$

$\Rightarrow\ b(b+c)+a(a+c)=(a+c)(b+c)$

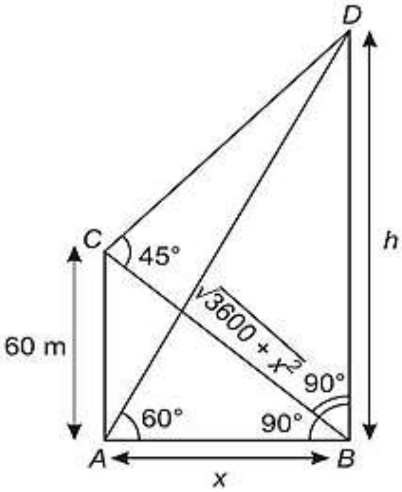
Divide by $(a+c)(b+c)$ and add 2 on both sides, we get

$$1+\frac{b}{a+c}+1+\frac{a}{b+c}=3$$

$$\Rightarrow \frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$$

So, $\angle C$ should be $60°$.

110. **(a)** In$\triangle$ ABD,



$$\tan 60° = \frac{h}{x} = \sqrt{3}$$

$\Rightarrow h = x\sqrt{3}$...... (i)

In $\triangle$ BAC,

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC = \sqrt{x^2+(60)^2}$$

$$= \sqrt{x^2+3600}$$

In$\triangle$ CBD, $\tan 45° = \dfrac{h}{\sqrt{3600+x^2}}$

$$\Rightarrow h = \sqrt{3600+x^2}$$

$h^2 - x^2 = 3600$ ….. (ii)

From Eqs. (i) and (ii), we get

$3x^2 - x^2 = 3600$

$\Rightarrow 2x^2 = 3600 \Rightarrow x^2 = 1800$

From Eqs. (i) and (ii), we get

$$\Rightarrow h^2 - 1800 = 3600$$

$$\Rightarrow h^2 = 5400$$

$$h = 30\sqrt{6} = 30\sqrt{2}\cdot\sqrt{3}$$

$$= 30\times\frac{2\cdot\sqrt{3}}{\sqrt{2}} = 60\sqrt{\frac{3}{2}}$$

111. **(c)** Let $a = 2i+3j+4k = OA$

$b = 3i+4j+2k = OB$

and $c = 4i+2j+3k = OC$

$AB = OB - OA = i + j - 2k$

$BC = OC - OB = i - 2j + k$

and $CA = OA - OC = -2i + j + k$

Now, $AB = \sqrt{1+1+4} = \sqrt{6}$

$BC = \sqrt{1+4+1} = \sqrt{6}$

and $CA = \sqrt{4+1+1} = \sqrt{6}$

Since, the length of all three sides are equal. So, the triangle is an equilateral triangle.

112. **(b)** Let the coordinates of four points P, Q, R and S be $(3,-4,5)$, $(0,0,4)$, $(-4,5,1)$ and $(-3,4,3)$ respectively.

Now, equation of line PQ is

$$\frac{x-3}{0-3}=\frac{y+4}{0+4}=\frac{z-5}{4-5}$$

$$\Rightarrow \frac{x-3}{-3}=\frac{y+4}{4}=\frac{z-5}{-1}=r_1 \quad \text{(say)....(i)}$$

Equation of line RS is

$$\frac{x+4}{-3+4}=\frac{y-5}{4-5}=\frac{z-1}{3-1}$$

$$\Rightarrow \frac{x+4}{1}=\frac{y-5}{-1}=\frac{z-1}{2}=r_2 \quad \text{(say)} \quad \ldots \text{(ii)}$$

Let $(-3r_1+3, 4r_1-4, -r_1+5)$ and $(r_2-4, -r_2+5, 2r_2+1)$ be the points on line (i) and (ii), respectively. Since, both lines intersect at a common point, then

$$-3r_1+3=r_2-4$$

$$\Rightarrow 3r_1+r_2=7 \quad \ldots\ldots \text{(iii)}$$

and $-r_2+5=4r_1-4$

$$\Rightarrow 4r_1+r_2=9 \ldots\ldots \text{(iv)}$$

On subtracting Eq. (iv) from Eq. (iii), we get

$r_1=2$

On putting the value of r_1 in Eq. (iii), we get

$3(2)+r_2=7 \Rightarrow r_2=1$

So, required point of intersection is $(-3,4,3)$ i.e.,

$-3i+4j+3k$

113. **(c)** Given,

$a\neq 0, b\neq 0, c\neq 0$ and $a\times b=0, b\times c=0$

$a\times b=0 \Rightarrow$ Vector a and b are parallel.

$\Rightarrow$ a and **c** are also parallel.

$\Rightarrow a\times c=0$

114. **(d)** The given lines are $r=a_1+\lambda b_1, r=a_2+\mu b_2$ where,

$a_1=3i+5j+7k, b_1=i+2j+k$

$a_2=-i-j-k, b_2=7i-6j+k$

$$|b_1\times b_2|=\begin{vmatrix} i & j & k\\ 1 & 2 & 1\\ 7 & -6 & 1\end{vmatrix}$$

$\Rightarrow |i(2+6)-j(1-7)+k(-6-14)|$

$\Rightarrow |8i+6j-20k|$

$\Rightarrow \sqrt{64+36+400}=\sqrt{500}=10\sqrt{5}$

Now, $[(a_2-a_1)b_1b_2]=(a_2-a_1)\cdot(b_1\times b_2)$

$=(-4i-6j-8k)\cdot(8i+6j-20k)$

$=-32-36+160$

$=160-68=92$

Shortest distance

$$=\frac{[(a_2-a_1)\cdot(b_1\times b_2)]}{|b_1\times b_2|}$$

$$=\frac{92}{10\sqrt{5}}=\frac{46}{5\sqrt{5}}$$

115. **(c)** Let the unit vector be

$r=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$

and $\mathbf{a}=\mathbf{i}+\mathbf{j}+3\mathbf{k}, \mathbf{b}=\mathbf{i}+3\mathbf{j}+\mathbf{k}$

and $\mathbf{c}=\mathbf{i}+\mathbf{j}+\mathbf{k}$

Given, $[r, a\ b\,]=0$, i.e., coplanar.

$$\Rightarrow \begin{vmatrix} x & y & z \\ 1 & 1 & 3 \\ 1 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad x(1-9) - y(1-3) + z(3-1) = 0$

$\Rightarrow \quad -8x + 2y + 2x = 0$

$\Rightarrow \quad -4x + y + z = 0 \ \ldots.(i)$

and $\mathbf{r} \cdot \mathbf{c} = 0$, i.e., perpendicular

$\Rightarrow \quad (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0$

$\Rightarrow \quad x + y + z = 0 \ \ldots\ldots(ii)$

On solving Eqs. (i) and (ii), we get

$5y + 5z = 0$

$\Rightarrow y = -z \quad \ldots.(iii)$

$\because$ r is a unit vector.

$\therefore \quad |\mathbf{r}| = 1 = \sqrt{x^2 + y^2 + z^2}$

$\Rightarrow \quad x^2 + y^2 + z^2 = 1$

$\Rightarrow \quad x^2 + 2y^2 = 1$ [from Eq. (iii)] ...(iv)

Put $y = -z$ in Eq. (i), we get

$-4x = 0 \Rightarrow x = 0$

From Eq. (iv), we get

$2y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{2}}$

Required vector is

$$\begin{aligned} r &= x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \\ &= 0i + \frac{1}{\sqrt{2}}\mathbf{j} \pm \frac{1}{\sqrt{2}}k \\ &= \frac{\mathbf{j} - k}{\sqrt{2}} \text{ or } \frac{-\mathbf{j} + k}{\sqrt{2}} \end{aligned}$$

116. **(c)** Since, a, b and $a \times b$ are non-coplanar, hence

$y = ax + tbx + (a \times b)z$

For some scalars x, t and z,

Now,

$b = a \times y$

$\Rightarrow \ b = (a \times a)x + (a \times b)t + a \times (a \times b)z$

$= 0x + (a \times b)t + [(a \cdot b)a - (a \cdot a)b]z$

$[\because a \times a = 0, a \cdot b = 0]$

$\Rightarrow \ b = (a \times b)t - (a \cdot a)b \cdot z$

$\Rightarrow \ t = 0$ and $z = \frac{-1}{|a|^2}$

Also, $c = a \cdot y = a \cdot ax + a \cdot bt + a \cdot (a \times b)z$

$= |a|^2 x + 0t + 0z$

$= |a|^2 x \ [\because a \cdot b = 0, (aab) = 0]$

$\Rightarrow \quad x = \frac{c}{|a|^2}$

$\therefore \quad y = ax + bt + (a \times b)z$

$y = \frac{ac}{|a|^2} + b0 + (a \times b)\frac{-1}{|a|^2}$

$\Rightarrow \quad y = \frac{1}{|a|^2}[ac - (a \times b)]$

117. **(c)** Case I When smaller of the two numbers is 1.

Then, total number of cases

$= 1 \times {}^7C_1 = 7$

Case II When smaller of two numbers is 2. Then, total number of cases

$= 1 \times {}^6C_1 = 6$

Case III When smaller of two numbers is 3. Then, total number of cases

$= 1 \times {}^5C_1 = 5$

Total favourable cases $= 7 + 6 + 5 = 18$

Total case $= {}^8C_2 = 28$

$\therefore$ Required probability $= \frac{18}{28} = \frac{9}{14}$

118. **(d)** Total samle points, $n(S) = 6 \times 6 = 36$

Favourable events

$= [(6,4), (6,5), (6,6), (5,5), (5,6), (4,6)]$

Total favourable events, $n(E) = 6$

Required probability

$$= \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

119. **(a)** The probability that the toss results is a tail $= \frac{(n+1)}{2(2n+1)}$

$\therefore 1 - \frac{(n+1)}{2(2n+1)}$ is the probability that the toss result is a head.

$$\therefore \quad 1 - \frac{n+1}{2(2n+1)} = \frac{31}{42}$$

$$\Rightarrow \quad \frac{4n+2-n-1}{4n+2} = \frac{31}{42}$$

$$\Rightarrow \quad \frac{3n+1}{4n+2} = \frac{31}{42}$$

$$\Rightarrow \quad 126n + 42 = 124n + 62$$

$$\Rightarrow \quad 2n = 20$$

$$\Rightarrow \quad n = 10$$

120. **(b)**

121. **(a)** In poission distribution

$$P(X = x) = \frac{\lambda^2 e^{-\lambda}}{x!} \quad \dots (i)$$

$$P(X = 1) = \frac{\lambda e^{-\lambda}}{1!} = \lambda e^{-\lambda}$$

$$P(X = 2) = \frac{\lambda^2 e^{-\lambda}}{2!} = \frac{\lambda^2 e^{-\lambda}}{2}$$

$$P(X = 1) = 2P(x = 2)$$

$$\Rightarrow \lambda e^{-\lambda} = 2 \times \frac{\lambda^2 e^{-\lambda}}{2}$$

$$\Rightarrow \lambda(\lambda - 1) = 0$$

$$\lambda \neq 0$$

$$\lambda = 1$$

Hence, $P(X = 3) = \frac{(1)^3 e^{-1}}{3!} = \frac{e^{-1}}{6}$ $(\because \lambda = 1)$

122. **(c)** Under the translation of origin to $(1,2)$ the point $(7,5)$ undergoes to $(7-1, 5-2) \equiv (6,3)$

Under the translation through 2 units along the negative direction of the new x-axis, the point $(6,3)$ undergoes to $(6-2, 3) \equiv (4,3)$

Under the rotation throw an angle $\frac{\pi}{4}$ about the origin of new system in the clockwise direction, the final position of point $(7,5)$

$$= \left(4\cos\frac{\pi}{4} + 3\sin\frac{\pi}{4}, -4\sin\frac{\pi}{4} + 3\cos\frac{\pi}{4}\right)$$

$$= \left(\frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}}, -\frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}}\right) = \left(\frac{7}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

123. **(a)** Given equations of straight lines are

$x\sec\theta - y\,\mathrm{cosec}\theta = a \quad \dots (i)$

$x\cos\theta + y\sin\theta = a\cos 2\theta \quad \dots (ii)$

Also,
p = Perpendicular distance from the origin to the line (i)

$$P=\frac{|0-0-a|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}=\frac{a\sin\theta\cdot\cos\theta}{\sqrt{1}}$$

$$=a\sin\theta\cdot\cos\theta=\frac{a}{2}\sin 2\theta$$

$$\Rightarrow\ 2p=a\sin 2\theta$$

and q = perpendicular distance from the origin to the line (ii)

$$q=\frac{|0+0-a\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}=\frac{a\cos 2\theta}{\sqrt{1}}=a\cos 2\theta$$

Now,

$$4p^2+q^2=a^2\sin^2 2\theta+a^2\cos^2 2\theta$$

$$=a^2(\sin^2 2\theta+\cos^2 2\theta)$$

$$a^2\,(1)=a^2$$

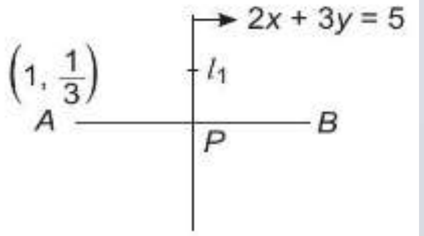
124. **(a)** Let $I_1\equiv 2x+3y=5\ldots\ldots$ (i)

Since, line $AB\perp l_1$

$\therefore$ Slope of l_1 is

$$m_1 \text{ say } =\frac{-2}{3}$$

$\therefore$ Slope of $AB=\frac{-1}{(-2/3)}=\frac{3}{2}$



Equation of line AB is

$$\left(y-\frac{1}{3}\right)=\frac{3}{2}(x-1)$$

$$\Rightarrow\quad 3x-2y=\frac{7}{3}\quad\ldots.(i)$$

Equation of line I_1 is

$2x+3y=5\quad\ldots.$ (ii)

From Eqs. (i) and (ii), we get

$13x\quad=17$

$$\Rightarrow\quad x=\frac{17}{13}$$

From Eq. (i), we get

$$3y=5-\frac{34}{13}\Rightarrow y=\frac{65-34}{13\times 3}=\frac{31}{39}$$

So, mid-point $P\rightarrow\left(\frac{17}{13},\frac{31}{39}\right)$

Coordinate of point B

$$=\left(\frac{17}{13}\times 2-1,\frac{31}{39}\times 2-\frac{1}{3}\right)$$

$$=\left(\frac{34-13}{13},\frac{62-13}{39}\right)=\left(\frac{21}{13},\frac{49}{39}\right)$$

125. **(a)** Since, the points (1,2) and (3,4) lie on the same side of the line $3x-5y+a=0$

$\therefore 3(1)-5(2)+a\geq 0$ or ≤ 0

$\Rightarrow\ a-7\geq 0$ or ≤ 0

$\Rightarrow\ a\geq 7$ or $a\leq 7$

and $3(3)-5(4)+a\geq 0$ or ≤ 0

$\Rightarrow\ a-11\geq 0$ or ≤ 0

$\Rightarrow\ a\geq 11$ or $a\leq 11$

So, common condition is [7, 11].

126. **(a)** Let m_1 and m_2 be the slopes of lines.

Then, $m_1 + m_2 =$ arithmetic mean $= \frac{13}{2}$ ... (i)

and $m_1 m_2 =$ geometric mean $= \sqrt{36} = 6.$ ... (ii)

Now, equation of the pair of lines passing through the origin is

$(y - m_1 x)(y - m_2 x) = 0$

$\Rightarrow y^2 - (m_1 + m_2)xy + m_1 m_2 x^2 = 0$

Using Eq. (i) and (ii), we get

$$y^2 - \frac{13}{2}xy + 6x^2 = 0$$

$\Rightarrow$

$$12x^2 - 13xy + 2y^2 = 0$$

127. **(a)** Comparing the given equation

$x^2 - 5xy + py^2 + 3x - 8y + 2 = 0$ .... (i)

with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ .... (ii)

we get

$$a = 1, h = \frac{-5}{2}, b = p, g = \frac{3}{2}, f = -4 \text{ and } c = 2$$

Eq. (i) represents a pair of straight lines, if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

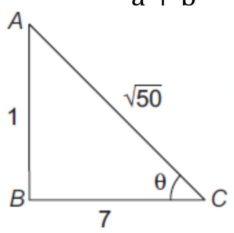
$$\Rightarrow \quad 1 \times p \times 2 + 2 \times (-4) \times \frac{3}{2} \times \left(\frac{-5}{2}\right) - 1 \times (-4)^2$$

$$-p \times \left(\frac{3}{2}\right)^2 - 2 \times \left(\frac{-5}{2}\right)^2 = 0$$

$$\Rightarrow \quad p = 6 \;\therefore p = 6$$

$\therefore$ Required angle,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b} = \frac{2\sqrt{\left(\frac{-5}{2}\right)^2 - 1 \times 6}}{1 + 6}$$



$$\Rightarrow \quad \tan\theta = \frac{1}{7} \Rightarrow \theta = \tan^{-1}\frac{1}{7}$$

$$\therefore \quad \sin\theta = \frac{1}{\sqrt{50}}$$

128. **(c)** We know that the point of intersection of the pair of straight line is

$$\left(\sqrt{\frac{f^2 - bc}{h^2 - ab}}, \sqrt{\frac{g^2 - ac}{h^2 - ab}}\right)$$

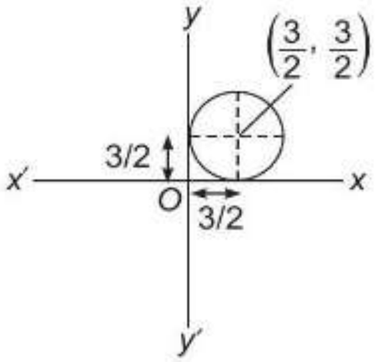
Required distance

$$= \left[\sqrt{\left(\sqrt{\frac{f^2 - bc}{h^2 - ab}} - 0\right)^2 + \left(\sqrt{\frac{g^2 - ac}{h^2 - ab}} - 0\right)^2}\right]^2$$

$$= \frac{f^2 - bc}{h^2 - ab} + \frac{g^2 - ac}{h^2 - ab}$$
$$= \frac{f^2 + g^2 - c(a+b)}{h^2 - ab}$$
$$= \frac{c(a+b) - f^2 - g^2}{ab - h^2}$$

129. **(a)** Given circle is

$$4x^2 + 4y^2 - 12x - 12y + 9 = 0$$
$$\Rightarrow \quad x^2 + y^2 - 3x - 3y + \frac{9}{4} = 0$$
$$\Rightarrow \quad (x^2 - 3x) + (y^2 - 3y) = -\frac{9}{4}$$



$$\Rightarrow \left(x - \frac{3}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$
$$\Rightarrow \left(x - \frac{3}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$$

Hence, centre $= \left(\frac{3}{2}, \frac{3}{2}\right)$ and radius $= \frac{3}{2}$

So, the given circle touches both the axes.

130. **(b)** Given equation of circle is

$C = x^2 + y^2 - 16x - 12y + 64 = 0 \ldots$ (i)

(i) Equation of polar at $(-5,1)$ w.r.t. to C is

$$x(-5) + y(1) - 8(x-5) - 6(y+1) + 64 = 0$$
$$\Rightarrow \quad -5x + y - 8x + 40 - 6y - 6 + 64 = 0$$
$$\Rightarrow \quad -13x - 5y + 98 = 0$$
$$\Rightarrow \quad 13x + 5y = 98$$

(ii) On differentiating Eq. (i) w.r.t. to x, we get

$$2x + 2y\frac{dy}{dx} - 16 - 12\frac{dy}{dx} + 0 = 0$$
$$\Rightarrow \quad (2y - 12)\frac{dy}{dx} = (16 - 2x)$$
$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{8 - x}{y - 6}\right)$$

At$(8,0)$ $\left(\frac{dy}{dx}\right)_{(8,0)} = \frac{8-8}{0-6} = 0$

$\therefore$ Equation of tangent at $(8,0)$ is $(y - 0) = 0(x - 8)$

$\Rightarrow \; y = 0$

(iii) Slope of normal is

$$\frac{dy}{dx} = \frac{6 - y}{8 - x}$$
$$\left(\frac{dy}{dx}\right)_{(2,6)} = \frac{6-6}{8-2} = 0$$

Equation of normal is

$(y-6) \quad = 0(x-2)$
$\Rightarrow \quad y \quad = 6$
(iv) Equation of the diameter of circle through (8,12) is
$x = 8$
131. **(c)** Given equations of circle is
$x^2 + y^2 = 16$
and $x^2 + y^2 + 2x + 2y = 0$
According to the questions,
Length of the tangent from (h, k) to the circle
$x^2 + y^2 = 16$
$= 2 \times$ Length of the tangent from (h, k) to the circle $x^2 + y^2 + 2x + 2y = 0$
$\Rightarrow \sqrt{h^2 + k^2 - 16} = 2 \times \sqrt{h^2 + k^2 + 2h + 2k}$
On squaring both sides, we get
$h^2 + k^2 - 16 = 4 \times (h^2 + k^2 + 2h + 2k)$
$\Rightarrow 3h^2 + 3k^2 + 8h + 8k + 16 = 0$
132. **(b)** Let the equation of circle whose centre $(a, 0)$ and radius (r) is
$(x-a)^2 + (y-0)^2 = r^2$
$\Rightarrow S_1 \equiv x^2 + a^2 - 2ax + y^2 - r^2 = 0$
and the equation of circle whose centre $(b, 0)$ and radius R is
$(x-b)^2 + (y-0)^2 = R^2$
$\Rightarrow S_2 \equiv x^2 + b^2 - 2bx + y^2 - R^2 = 0$
$\therefore$ Equation of radical axis is
$S_1 - S_2 = 0$
$\Rightarrow \quad a^2 - b^2 + 2bx - 2ax + R^2 - r^2 = 0$
$\Rightarrow \quad R^2 = r^2 - a^2 + b^2 - 2bx + 2ax$
Since, radical axis is y-axis.
Therefore, putting $x = 0$ in Eq. (i), we get
$R^2 = r^2 - a^2 + b^2 - 0 + 0$
$\Rightarrow \quad R = (r^2 + b^2 - a^2)^{1/2}$
133. **(d)** The common chord of the given circle is
$S_1 - S_2 = 0$
$\Rightarrow \quad (x^2 + y^2 + 4x - 6y + c)$
$-(x^2 + y^2 - 6x + 4y - 12) = 0$
$\Rightarrow \quad 10x - 10y + c + 12 = 0 \ldots.(i)$
Since, circle $x^2 + y^2 + 4x + 6y + c = 0$ bisects the circumference of the circle.
$x^2 + y^2 - 6x + 4y - 12 = 0$
Therefore, Eq. (i) passes through the centre of second circle i.e., (3, -2)
$\therefore \quad 10(3) - 10(-2) + c + 12 = 0$
$\Rightarrow \quad 30 + 20 + c + 12 = 0$
$\Rightarrow \quad c = -62$
134. **(c)** Given, equation of parabola is
$y^2 \quad = 8x \ldots.(i)$
$\therefore \quad a \quad = 2$
Let $(h, 4)$ be the coordinate of mid-point of chord. Then, equation of chord is
$y - 4 = m(x - h) \ldots.(ii)$
If line (ii) passes through the point $P(2t_1^2, 4t_1)$ and $Q(2t_2^2, 4t_2)$ on parabola Eq. (i), then
$y(t_1 + t_2) - 2x - 4t_1t_2 = 0 \ldots.(iii)$
having slope
$m = \frac{2}{t_1 + t_2} \ldots.(iv)$
Since, $(h, 4)$ is the mid-point of PQ. Therefore,
$2 \times 4 \quad = 4(t_1 + t_2)$
$\Rightarrow \quad t_1 \quad + t_2 = 2$
Hence, slope of chord PQ is

$m = \frac{2}{2} = 1$ [using Eq. (iv)]

135. **(a)** Equation of chord at $(2, -4)$ is

$$T = S'$$

$\Rightarrow \quad 2x + 4y(-4) - (x + 2) + 10(y - 4)$

$= (2)^2 + 4(-4)^2 + 2(2) + 20(-4)$

$\Rightarrow \quad 2x - 16y - x - 2 + 10y - 40$

$= 4 + 64 - 4 - 80$

$\Rightarrow \quad x - 6y = 42 - 16 \Rightarrow 26$

136. **(b)** Given, equation of ellipse is

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

and equation of hyperbola is

$\frac{x^2}{4} - \frac{y^2}{b^2} = 1$

eccentricity of ellipse

$b^2 = a^2(1 - e^2)$

$\Rightarrow \quad 16 = 25(1 - e^2)$

$\Rightarrow \quad e^2 = 1 - \frac{16}{25} = \frac{9}{25}$

$\Rightarrow \quad e = \pm\frac{3}{5}$

Focii of the ellipse $= (\pm ae, 0) = (\pm 3, 0)$ which coincide with focii of the hyperbola. Let e_1 be the eccentricity of the hyperbola.

$\therefore \quad \pm ae_1 = \pm 3$

$\Rightarrow \quad e_1 = \frac{3}{2}$

Now, $\quad b^2 = a^2(e_1^2 - 1)$

$\Rightarrow \quad b^2 = 4\frac{9}{4} - 1 = 4 \times \frac{5}{4}$

$\Rightarrow \quad b^2 = 5$

137. **(b)** Given that, $x = 9$ is a chord of contact of hyperbola.

$x^2 - y^2 = 9$ .... (i)

put $x = 9$, $\quad 81 - y^2 = 9$

$\Rightarrow \quad y^2 = 72$

$\Rightarrow y = 6\sqrt{2}$ or $-6\sqrt{2}$

$\therefore \quad$ Points are $(9, 6\sqrt{2})$ and $(9, -6\sqrt{2})$

Now, differentiating Eq. (i) w.r.t. x, we get

$$2x - 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

at $(9, 6\sqrt{2})$ $\left(\frac{dy}{dx}\right)_{(9,6\sqrt{2})} = \frac{9}{6\sqrt{2}} = \frac{3}{2\sqrt{2}}$

and at $(9 - 6\sqrt{2})$ $\left(\frac{dy}{dx}\right)_{(9,-6\sqrt{2})} = -\frac{3}{2\sqrt{2}}$

$\therefore$ Equation of tangent at $(9, 6\sqrt{2})$ is

$$(y - 6\sqrt{2}) = \frac{3}{2\sqrt{2}}(x - 9)$$

$\Rightarrow \quad 2\sqrt{2}y - 24 = 3x - 27$

$\Rightarrow \quad 3x - 2\sqrt{2}y - 3 = 0$

and equation of tangent at $(9, -6\sqrt{2})$ is

$$(y+6\sqrt{2})=\frac{-3}{2\sqrt{2}}(x-9)$$

$\Rightarrow\quad 2\sqrt{2}y+24=-3x+27$

$\Rightarrow\quad 3x+2\sqrt{2}y-3=0$

138. **(d)** Given points $(1,\pi),(1,0^\circ)$ and $1,\frac{\pi}{2}$ are in polar form.

Now, change in cartesian form,

$(1,\pi)\rightarrow(1\cdot\cos\pi,1\cdot\sin\pi)\rightarrow(-1,0)$

$(1,0^\circ)\rightarrow(1\cdot\cos 0^\circ,1\cdot\sin 0^\circ)\rightarrow(1,0)$

and $1,\frac{\pi}{2}\rightarrow 1\cdot\cos\frac{\pi}{2},1\cdot\sin\frac{\pi}{2}\rightarrow(0,1)$

Now, equation of the line passing through (1,0) and (0,1) is

$$(y-0)=\frac{1-0}{0-1}(x-1)$$

$$\Rightarrow\quad y=-x+1$$

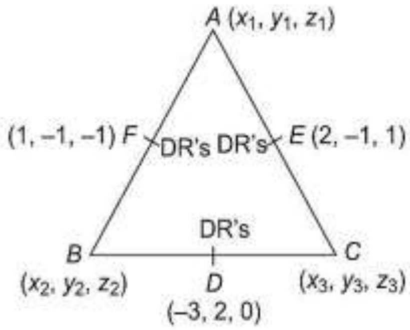
$\Rightarrow\quad x+y-1=0\ \dots(i)$

So, the perpendicular distance from the point $(-1,0)$ to the line (i) is

$$=\frac{|-1+0-1|}{\sqrt{1+1}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

139. **(b)**

140. **(a)** Direction ratios of AB and AC are $<1,-1,-1>$ and $<2,-1,1>$, respectively.



$$\therefore\left.\begin{aligned}x_2-x_1&=1\\y_2-y_1&=-1\\z_2-z_1&=-1\end{aligned}\right\}\dots\dots(i)$$

and

$$\left.\begin{aligned}x_1-x_3&=2\\y_1-y_3&=-1\\z_1-z_3&=1\end{aligned}\right\}\dots\dots(ii)$$

From Eqs. (i) and (ii),

$$\left.\begin{aligned}x_2-x_3&=3 &&\Rightarrow x_3-x_2=-3\\y_2-y_3&=-2 &&\Rightarrow y_3-y_2=2\\z_2-z_3&=0 &&\Rightarrow z_3-z_2=0\end{aligned}\right\}\dots(iii)$$

Let $<a,b,c>$ be the direction ratio of the normal to the plane ABC.

Then, $\quad a-b-c=0$

and $\quad 2a-b+c=0$

By cross-multiplication method,

$$\frac{a}{-1-1}=\frac{b}{-2-1}=\frac{c}{-1+2}$$

$$\frac{a}{-2}=\frac{b}{-3}=\frac{c}{1}$$

$$\Rightarrow\frac{a}{2}=\frac{b}{3}=\frac{c}{-1}$$

So, the direction ratio of the normal to the plane ABC is $\langle 2,3,-1\rangle$.

141. **(c)** A plane passing through the point $(-1,2,3)$, then its equation is

$$a(x+1)+b(y-2)+c(z-3)=0\quad\dots(i)$$

where $< a, b, c >$ are direction ratios of normal to the plane ABC.
So, the normal makes equal angles with coordinate axes

i.e., $(a, b, c) = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Now, from Eq. (i),

$$\frac{1}{\sqrt{3}}(x+1) + \frac{1}{\sqrt{3}}(y-2) + \frac{1}{\sqrt{3}}(z-3) = 0$$
$$\Rightarrow x + y + z - 4 = 0$$

142. **(c)** The foot of the propendicular from the origin to the plane lies on a sphere.

143. **(b)** $\because\ f(x+y) = f(x)f(y), \forall x, y \in R \ldots$ (i)

Put $x = y = 1$, we get
$f(2) = f(1) \cdot f(1) = 9 \quad [\because f(2) = 9]$
$\Rightarrow \ f(1)^2 = 9 \Rightarrow f(1) = 3$
Now, put $x = 2$ and $y = 1$ in Eq. (i), we get
$f(3) = f(2) \cdot f(1) = 3^2 \cdot 3 = 3^3$
Now, put $x = 3$ and $y = 1$ in Eq. (i), we get
$f(4) = f(3) \cdot f(1) = 3^3 \cdot 3 = 3^4$
Again, put $x = 4$ and $y = 2$ in Eq. (i), we get
$f(6) = f(4) \cdot f(2) = 3^4 \cdot 3^2 = 3^6$

144. **(b)**

145. **(b)** $f(x) = \frac{1}{1+\frac{1}{x}} = \frac{x}{1+x}$

and $g(x) = \frac{1}{1+\frac{1}{f(x)}} = \frac{1}{1+\frac{1+x}{x}} = \frac{x}{2x+1}$

$$\therefore\ g'(x) = \frac{(2x+1) \cdot 1 - x(2)}{(2x+1)^2} = \frac{1}{(2x+1)^2}$$

Now, $g'(2) = \frac{1}{(4+1)^2} = \frac{1}{25}$

146. **(c)** We have $\sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}} = 2$

$\Rightarrow\ y + x = 2\sqrt{x} \cdot \sqrt{y}$
On squaring both sides, we get
$x^2 + y^2 + 2xy = 4xy$
$\Rightarrow\ x^2 + y^2 - 2xy = 0$
$\Rightarrow\ (x-y)^2 = 0 \Rightarrow y = x$
$\therefore\ \frac{dy}{dx} = 1$

147. **(d)**

$$\frac{d}{dx}[(x+1)(x^2+1)(x^4+1)(x^8+1)]$$
$$=\frac{(15x^p-16x^q+1)}{(x-1)^2} \quad \text{... (i)}$$

LHS $$=\frac{d}{dx}\left[\frac{(x^2-1)(x^2+1)(x^4+1)(x^8+1)}{(x-1)}\right]$$
$$=\frac{d}{dx}\left[\frac{(x^4-1)(x^4+1)(x^8+1)}{(x-1)}\right]$$
$$=\frac{d}{dx}\left[\frac{(x^8-1)(x^8+1)}{(x-1)}\right]$$
$$=\frac{d}{dx}\left[\frac{(x^{16}-1)}{(x-1)}\right]$$
$$=\frac{(x-1)(16x^{15})-(x^{16}-1)}{(x-1)^2}$$
$$=\frac{16x^{16}-16x^{15}-x^{16}+1}{(x-1)^2}$$
$$=\frac{15x^{16}-16x^{15}+1}{(x-1)^2}$$

On comparing LHS = RHS, we get

$p=16$ and $q=15$

$(p,q)=(16,15)$

148. **(b)** $\cos^{-1}\left(\frac{y}{b}\right)=2\log\left(\frac{x}{2}\right), x>0$

On differentiating w.r.t. x we get

$$-\frac{1}{\sqrt{1-\frac{y^2}{b^2}}}\cdot\frac{1}{b}\frac{dy}{dx}=2\cdot\frac{1}{\left(\frac{x}{2}\right)}\cdot\frac{1}{2}$$
$$\Rightarrow \quad -\frac{1}{\sqrt{b^2-y^2}}\cdot\frac{dy}{dx}=\frac{2}{x}$$
$$\Rightarrow \quad x\frac{dy}{dx}=-2\sqrt{b^2-y^2} \quad \text{.... (i)}$$

Again, differentiating w.r.t. x we get,

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-2\cdot\frac{1}{2}(b^2-y^2)^{-1/2}(-2y)\frac{dy}{dx}$$
$$\Rightarrow \quad x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{2y}{\sqrt{b^2-y^2}}\cdot\frac{dy}{dx}$$
$$\Rightarrow \quad x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{2y}{-\frac{x}{2}\cdot\frac{dy}{dx}}\cdot\frac{dy}{dx} \quad [\text{ from Eq. (i) }]$$
$$\Rightarrow \quad x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-4y$$

149. **(d)** Given, $pV^{1/4}=a$, where a is constant.

$$\Rightarrow p=\frac{a}{V^{1/4}}$$

∴ Decreased volume

$$=V^{1/4}-\frac{V^{1/4}}{200}=\frac{199}{200}V^{1/4}$$

Increased pressure $=\frac{a}{\frac{199}{200}V^{1/4}}=\frac{200a}{199V^{1/4}}$

∴ Percentage increase in pressure

$$=\frac{\frac{200a}{199V^{1/4}}-\frac{a}{V^{1/4}}}{\frac{a}{V^{1/4}}}\times 100$$

$$=\frac{200}{199}-1\times 100$$

$$=\frac{100}{199}\approx\frac{1}{2}(\text{ approximate })$$

150. **(d)**

151. **(b)** Given, equation is $\frac{2}{f}=\frac{1}{v}-\frac{1}{u}\ldots$ (i)

Differentiating the given equation, we have

$$-\frac{2}{f^2}df=-\frac{1}{v^2}dv+\left(-\frac{1}{u^2}\right)du$$

$$=-p\left(\frac{1}{v}-\frac{1}{u}\right)\left(\frac{1}{v}+\frac{1}{u}\right)\quad\left(\because\frac{dv}{v}=\frac{du}{u}=p\right)$$

$$=\frac{-2p}{f}\left(\frac{1}{v}+\frac{1}{u}\right)\quad \text{using Eq. (i)]}$$

$$\therefore\ \frac{df}{f}=p\left(\frac{1}{v}+\frac{1}{u}\right)$$

152. **(d)** Given, $u=\log(x^3+y^3+z^3-3xyz)$

$$u_x=\frac{du}{dx}=\frac{3x^2-3yz}{(x^3+y^3+z^3-3xyz)}$$

$$u_y=\frac{du}{dy}=\frac{3y^2-3xz}{x^3+y^3+z^3-3xyz}$$

and $u_z=\frac{du}{dz}=\frac{3z^2-3xy}{x^3+y^3+z^3-3xyz}$

$$u_x+u_y+u_z$$

$$=\frac{3(x^2+y^2+z^2-xy-yz-zx)}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}$$

$$\Rightarrow\ (x+y+z)(u_x+u_y+u_z)=3$$

153. **(d)**

$$I=\int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx$$

$$=\int e^x\left\{\frac{2}{1+\cos 2x}+\frac{\sin 2x}{1+\cos 2x}\right\}dx$$

$$=\int e^x\left\{\frac{2}{2\cos^2 x}+\frac{2\sin x\cos x}{2\cos^2 x}\right\}dx$$

$$=\int e^x(\sec^2 x+\tan x)dx$$

$$=\int e^x\sec^2 x dx+\int e^x\tan x dx$$

$$=e^x\tan x-\int e^x\tan x dx+\int e^x\tan x dx$$

$$=e^x\tan x+C$$

154. **(a)**

155. **(b)**

156. **(d)** We have,

$$\int_0^b \frac{dx}{1+x^2} = \int_b^\infty \frac{dx}{1+x^2}$$

$\Rightarrow$ $[\tan^{-1} x]_0^b = [\tan^{-1} x]_b^\infty$

$\Rightarrow$ $\tan^{-1}(b) - \tan^{-1}(0) = \tan^{-1}(\infty) - \tan^{-1}(b)$

$\Rightarrow$ $2\tan^{-1}(b) - 0 = \frac{\pi}{2}$

$\Rightarrow$ $\tan^{-1}(b) = \frac{\pi}{4} \Rightarrow b = \tan\left(\frac{\pi}{4}\right)$

$\therefore$ $b = 1$

157. **(c)** Given, differential equation is,

$dx(1 + y + x^2y) + (x + x^3)dy = 0$

$\Rightarrow$ $\frac{dy}{dx} = -\left[\frac{1 + y + x^2y}{x + x^3}\right]$

$\Rightarrow$ $\frac{dy}{dx} = -\left\{\frac{1}{x(1+x^2)} + \frac{(1+x^2)y}{x(1+x^2)}\right\}$

$\Rightarrow$ $\frac{dy}{dx} - \frac{y}{x} = -\frac{1}{x(1+x^2)}$

Here, $p = -\frac{1}{2}, Q = -\frac{1}{x(1+x^2)}$

Integrating factor $\equiv e^{\int pdx}$

$= e^{\int\left(-\frac{1}{x}\right)dx}$

$= e^{-\log x}$

$= e^{\log\left(\frac{1}{x}\right)}$

$= \frac{1}{x}$

158. **(c)**

159. **(c)**

160. **(b)** Given, differential equation is

$\frac{dy}{dx} - 2y\tan 2x = e^x \sec 2x$

Here, $P = 2\tan 2x, Q = e^x \sec 2x$

$\therefore$ IF $= e^{\int -2\tan 2x dx}$

$= e^{\frac{-2\log\sec 2x}{2}}$

$= e^{-\log\sec 2x}$

$= \frac{1}{\sec 2x}$

$\therefore$ Required solution is

$$\frac{y}{\sec 2x} = \int e^x \cdot \frac{1}{\sec 2x} \cdot \sec 2x dx + C$$

$\Rightarrow$ $y\cos 2x = \int e^x \cdot 1 dx + C$

$\Rightarrow$ $y\cos 2x = e^x + C$

where, C is the constant of integration.