Physics

1. **(c)** Frequency of first mode of vibration

$n_1 = \frac{3v}{4l}$ ...... (i)

(closed pipe)

and frequency of first mode of vibration

$n_2 = \frac{v}{2l}$ .... (ii)

(open pipe)

According to the question,

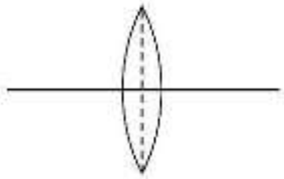
$$n_1 - n_2 = \frac{3v}{4l} - \frac{v}{2l} = 55$$

So $\frac{3v - 2v}{4l} = 55$

$\Rightarrow \frac{v}{4l} = 55$ Hz

2. **(c)** The given, $R_1 = R, R_2 = -R$

$f = F$



When convex lens is complete

Lens Maker's formula

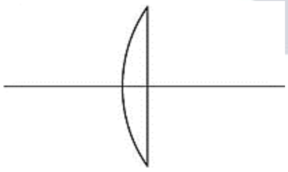
$$\frac{1}{F} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$$

$$f = \frac{R}{2(\mu - 1)}$$

$R = 2f(\mu - 1)$ .... (i)

Now, it is divided vertically into two identical plano convex lens



Plano convex lens

$$\frac{1}{f'} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

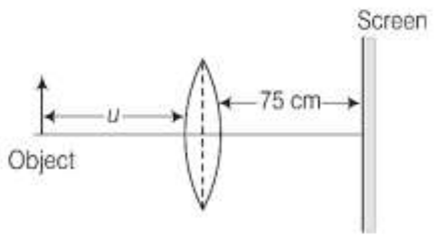
$$\frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right] \quad [\because R_1 = R, R_2 = \infty]$$

$$f_1 = \frac{R}{(\mu - 1)}$$

$$f_1 = \frac{2f(\mu - 1)}{(\mu - 1)}$$

$$f_1 = 2f$$

3. **(c)** According to the first condition,



$f = 25$ cm, $v = 75$ cm

$u = ?$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

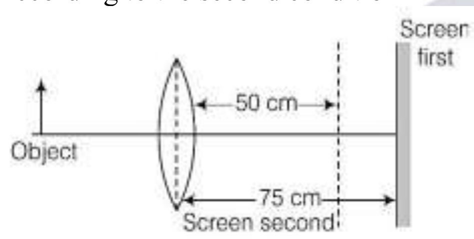
$$\frac{1}{25} = \frac{1}{75} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{75} - \frac{1}{25}$$

$$\frac{1}{u} = \frac{1-3}{75}$$

$$u = -\frac{75}{2} = -37.5 \text{ cm}$$

According to the second condition



$v_1 = 50$ cm, $f = 25$ cm, $u_1 = ?$

$$\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\frac{1}{25} = \frac{1}{50} - \frac{1}{u_1}$$

$$\frac{1}{u_1} = \frac{1}{50} - \frac{1}{25}$$

$$\Rightarrow \frac{1}{u_1} = \frac{1-2}{50}$$

$$\Rightarrow u_1 = -50 \text{ cm}$$

So, the screen is sharp again is

$$\Delta u = u_1 - u$$
$$= 50 - 37.5$$
$$= 12.5 \text{ cm}$$

4. (b) By Young's double slit interference experiment

$$\beta = \frac{\lambda D}{d}$$

The given $\beta_1 = 1.2$ mm, $\frac{d_2}{d_1} = 1\frac{1}{2} = 1.5$

So, $\beta \propto \frac{1}{d}$

$$\frac{\beta_1}{\beta_2} = \frac{1/d_1}{1/d_2}$$

$$\frac{\beta_1}{\beta_2} = \frac{d_2}{d_1} = 1.5 \Rightarrow \frac{1.2}{\beta_2} = 1.5$$

$$\beta_2 = \frac{1.2}{1.5} = \frac{4}{5} = 0.8 \text{ mm}$$

5. **(c)** By Coulomb's law,
When charge Q is divided into two charges q and Q − q

$$F=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot(Q-q)}{r^2}$$

The value of q

$$F_{max}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{Q}{2}\left(Q-\frac{Q}{2}\right)}{r^2}$$

(Putting $q=\frac{Q}{2}$ for the maximum force)

$$=\frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{Q}{2}\left(\frac{Q}{2}\right)}{r^2}$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{Q^2}{4}}{r^2}$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4r^2}$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot\frac{\left(\frac{Q}{2}\right)^2}{r^2}$$

6. **(a)** We know that, the surface charge densities is

$$\sigma=\frac{Q}{4\pi(r^2+R^2)}\quad \ldots\ldots (i)$$

The electric potential at the centre

$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma\times 4\pi r^2}{r}+\frac{\sigma\times 4\pi R^2}{R}\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\times\sigma\times 4\pi\left[\frac{r^2}{r}+\frac{R^2}{R}\right]$$

$$=\frac{\sigma}{\varepsilon_0}(r+R)$$

and $V=\frac{Q}{\varepsilon_0\cdot 4\pi(r^2+R^2)}\cdot(r+R)$

$$\Rightarrow\ V=\frac{Q}{4\pi\varepsilon_0}\cdot\frac{(r+R)}{(r^2+R^2)}$$

7. **(a)** The given,

$\rho_B=2\rho_A, R_A=R_B=R$

Let, $r_A=r$

$r_B=2r$

According to formula,

$$R_A=\rho_A\cdot\frac{I_A}{\pi r_A^2}\ \ldots (i)$$

$$R_B=\rho_B\cdot\frac{I_B}{\pi r_B^2}\ \ldots\ldots (ii)$$

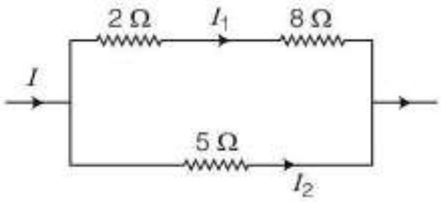
$$R_A=R_B$$

$$\rho_A\cdot\frac{I_A}{\pi r_A^2}=\rho_B\cdot\frac{I_B}{\pi r_B^2}$$

$$\frac{I_B}{I_A} = \frac{r_B^2}{r_A^2} \cdot \frac{\rho_A}{\rho_B}$$
$$= \frac{(2r)^2}{r^2} \cdot \frac{\rho_A}{2\rho_B}$$
$$= \frac{4r^2}{r^2} \cdot \frac{\rho_A}{2\rho_A}$$
$$= \frac{4}{2} = 2$$

8. **(a)** The given $P = 50$ J/s

$P = Vi \Rightarrow P = i^2R$



$$\Rightarrow i_2^2 = \frac{P}{R} = \frac{50}{5} = 10\text{Amp}^2$$
$$V = i_2R_{(5\Omega)}$$
$$= \sqrt{10} \times 5 = \sqrt{250}\ \text{V}$$

and 2Ω and 8Ω are in series.

So, the required resistance

$$= 2\Omega + 8\Omega = 10\Omega$$
$$i_1 = \frac{V}{R_{(10\Omega)}}$$
$$i_1 = \frac{\sqrt{250}}{10}\ \text{A}$$

The heat generated/second in 2Ω

$$P = Vi_1$$
$$= i_1^2 \times R$$
$$= \left(\frac{\sqrt{250}}{10}\right)^2 \times 2$$
$$= \frac{250}{100} \times 2 = \frac{25}{10} \times 2 = \frac{25}{5} = 5\ \text{J/s}$$

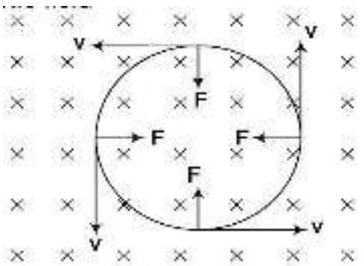
9. **(d)** The magnetic field a circular coil (for n number of loops)

$$B = \frac{\mu_0 ni}{2r}$$

When,

$$n_1 = 1, n_2 = 2$$
$$2\pi r_1 = n \times 2\pi r_2$$
$$\Rightarrow \frac{r_1}{r_2} = \frac{n}{1}$$
$$\text{So } \frac{B_1}{B_2} = \frac{\mu_0 n_1 i/2r_1}{\mu_0 n_2 i/2r_2}$$
$$\frac{B}{B_2} = \frac{n_1}{n_2} \cdot \frac{r_1}{r_2}$$
$$\frac{B}{B_2} = \frac{1}{n} \cdot \frac{1}{n}$$
$$\Rightarrow B_2 = n^2B$$

10. **(d)** By motion of charged particles in uniform magnetic field



$r = \frac{mv}{qB}$

So, $r \propto v$

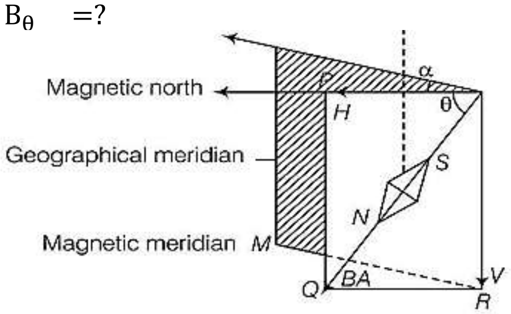
It travels in a circular path with a radius directly proportional to its velocity.

11. (d) Given,

$B_H = 0.8 \times 10^{-4}$ T

$\theta = 60°$

$B_\theta = ?$



We know that,

$B_H = B_e \cos\theta$

$0.8 \times 10^{-4} = B_e \cos 60°$

$B_e = \frac{0.8 \times 10^{-4}}{\frac{1}{2}}$

$= 1.6 \times 10^{-4}$ T

12. (b) The given,

$L_1 = 108\text{mH}, N_1 = 600$ turns, $N_2 = 500$ turns and $L_2 = ?$

By self-inductance of a plane coil

$L_1 = \frac{\mu_0 \pi N_1^2 a_1}{2}$ .... (i)

$L_2 = \frac{\mu_0 \pi N_2^2 a_2}{2}$ .... (ii)

From the Eqs. (i) and (ii), we get

$\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2$ $(\because a_1 = a_2)$

$\frac{108}{L_2} = \left(\frac{600}{500}\right)^2$

$L_2 = \frac{108 \times 25}{36}$

$L_2 = 3 \times 25 = 75\text{mH}$

13. (d) The given

$C = 50\mu\text{ F} = 50 \times 10^{-6}$ F

V = 220 Sin 50 t .... (i)

But we know that

$V = V_0 \sin\omega t$ ..... (ii)

Comparing both equations

$V_0 = 220\text{ V}, \omega = 50\text{rad/s}$

The capacitive reactance of the circuit is

$$X_C = \frac{1}{\omega C}$$
$$= \frac{1}{50 \times 50 \times 10^{-6}} = 400\Omega$$

The peak and the rms values of current in the circuit are

$$i_0 = \frac{V_0}{X_C} = \frac{220}{400} = \frac{11}{20}$$

and $i_{rms} = \frac{i_0}{\sqrt{2}}$

$$= \frac{11/20}{\sqrt{2}} = \frac{0.55}{\sqrt{2}}\ \text{A}$$

14. (c) The given

$\mathbf{E} = \mathbf{i}30\cos(kz - 5 \times 10^8 t)$

We know that,

$E = iV\cos(kz - \omega t)$

Comparing the both equations,

$\omega = 5 \times 10^8\,\text{rad/s}$

But we know that also

$$K = \frac{2\pi}{\lambda} \text{ and } \omega = 2\pi v$$

where, λ is wavelength and v is frequency of the wave

$$\therefore \quad \frac{\omega}{k} = \frac{2\pi v}{2\pi/\lambda} = v\lambda = C$$

$$\Rightarrow \quad C = \frac{\omega}{k}$$

$$\text{or} \quad k = \frac{\omega}{C} = \frac{5 \times 10^8}{3 \times 10^8}$$

$$\Rightarrow \quad k = \frac{5}{3} = 1.66\,\text{rad/m}$$

15. (b) We know that, de-Broglie wavelength of photon $E = hv$

But we know that also

$$E = \frac{1}{2}mv^2 \quad \text{(for proton)}$$

$$\therefore \quad 2E = \frac{P^2}{m}$$

$$\because \quad P = mv$$

$$\frac{P^2}{m} = mv^2$$

$$P = \sqrt{2mE} \quad \ldots\ldots(i)$$

According to the question

$\lambda_1 = \frac{h}{P}$ ..... (ii)

(de-Broglie of matter waves)

and $\lambda_2 = \frac{hc}{E}$ .... (iii)

From the Eqs. (ii) and (iii), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{h/P}{hc/E} = \frac{E}{Pc}$$

$$= \frac{E}{\sqrt{2mE} \cdot c} = \frac{\sqrt{E}}{\sqrt{2m} \cdot c}$$

$$\text{So,} \quad \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{E}}{\sqrt{2m} \cdot c}$$

$$\frac{\lambda_1}{\lambda_2} \propto \sqrt{E}$$

$$\Rightarrow \quad \lambda_1 : \lambda_2 = E^{1/2}$$

16. **(a)** The total energy of nth orbit

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2} \cdot \frac{1}{n^2}$$

Obviously $E_n \propto m$

$$\therefore \quad \frac{E_\mu}{E_e} = \frac{m_\mu}{m_e}$$

$$\Rightarrow \quad E_\mu = \frac{m_\mu}{m_e} \times E_e$$

Ground state energy of a proton in hydrogen atom,

$$E_\mu = -13.6 \times \frac{207 m_e}{m_e} \text{eV}$$

$$= -13.6 \times 207 \text{eV}$$

($\because m_\mu = 207 m_e$, where m_e is the mass of electron) We know that

$$r = \frac{\varepsilon_0 h^2 n^2}{207 \pi m_e e^2}$$

For ground state $(n - 1)$ for proton, we have

$$r_\mu = \frac{\varepsilon_0 h^2}{207 \pi m_e \cdot e^2}$$

But $\frac{\varepsilon_0 h^2}{\pi m_e \cdot e^2}$ = ground state radius of hydrogen atom

$$\frac{\varepsilon_0 h^2}{\pi m_e \cdot e^2} = r_H$$

$$\therefore \quad r_\mu = \frac{r_H}{207}$$

17. **(d)** By Rutherford's experiment

$R_1 = R_0 A_1^{1/3}$ .... (i)

$R_2 = R_0 A_2^{1/3}$ ... (ii)

Given

$R_1 = 1.5$ fermi

$A_1 = 125$

$A_2 = 64$

$R_2 = ?$

Now, $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$

[from the Eqs. (i) and (ii)]

$$\frac{1.5}{R_2} = \left(\frac{125}{64}\right)^{1/3}$$

$$\Rightarrow \frac{1.5}{R_2} = \frac{5}{4}$$

$$\Rightarrow \quad R_2 = \frac{1.5 \times 4}{5} = 0.3 \times 4$$

$$= 1.2 \text{ fermi}$$

18. **(d)** Given $n_1 = 1.6 \times 10^{16}/m^3$

$n_e = 4.8 \times 10^{20}/m^3$

$n_h = ?$

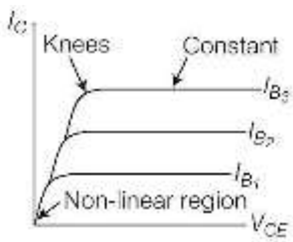
The concentration of holes in the semiconductor

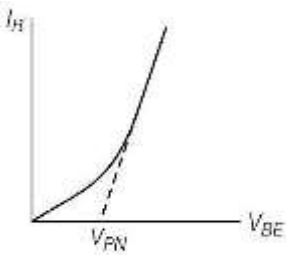
$$n_1^2 = n_e \times n_h$$

$$(1.6 \times 10^{16})^2 = 4.8 \times 10^{20} \times n_h$$

$$n_h = \frac{2.56 \times 10^{32}}{4.8 \times 10^{20}} = 5.3 \times 10^{11}/m^3$$

19. **(b)** The potential difference between base and emitter changes in I_C with changes in V_{CE} (when I_B = constant)





20. (b) Height of transmitting antenna $d \approx \sqrt{2R_e h_T}$

Given

$R_e = 6.4 \times 10^6$ m

$h_T = 128$ m

$d = ?$

The maximum distance

$$d = \sqrt{2 \times 6.4 \times 10^6 \times 128}$$

$$d = \sqrt{128 \times 10^5 \times 128}$$

$$d = \frac{128 \times 10^2 \sqrt{10}}{\sqrt{10}} \times \sqrt{10}$$

$$d = \frac{128 \times 10^3}{\sqrt{10}} \text{ m} = \frac{128}{\sqrt{10}} \text{ km}$$

21. (b) If angular acceleration is constant, we have

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

The given $\theta = 15°$

$\omega_0 = 0$

For the first conditions (time = tsec)

$$15° = 0 + \frac{1}{2}\alpha t^2$$

$$\Rightarrow 15° = \frac{1}{2}\alpha t^2 \text{ ... (i)}$$

For the second conditions (time = 3tsec)

$$\theta_1 = \frac{1}{2}\alpha(3t)^2 = \frac{1}{2}(\alpha)9t^2 \text{ .... (ii)}$$

So,

$$\Delta\theta = \theta_1 - \frac{1}{2}\alpha t^2$$

$$\Delta\theta = 9 \times \frac{1}{2}\alpha t^2 - \frac{1}{2}\alpha t^2$$

$$= 8\frac{1}{2}\alpha t^2$$

$$= 8 \times 15° = 120° \text{ (form eq (i))}$$

22. (d) The moment of inertia

$$I = \frac{3}{2} mR^2$$

[$\because$ Linear density $\rho = \frac{m}{l}$

$$m = I\rho$$
$$2\pi R = I$$
$$R = \frac{1}{2\pi}$$
$$I = \frac{3}{2} \times I\rho \times \left(\frac{1}{2\pi}\right)^2$$
$$= \frac{3}{2 \times 4} \cdot \frac{I^3\rho}{\pi^2}$$
$$= \frac{3I^3\rho}{8\pi^2}$$

23. **(d)** The kinetic energy

$$KE = \frac{1}{2}m\omega^2\left[a^2 - \left(\frac{a}{N}\right)^2\right] \quad \ldots. \quad \text{(i)}$$

The potential energy

$$PE = \frac{1}{2}m\omega^2\frac{a^2}{N^2} \quad \ldots. \text{(ii)}$$

From the Eqs. (i) and (ii), we get

$$\frac{KE}{PE} = \frac{\frac{1}{2}m\omega^2\left[a^2 - \left(\frac{a}{N}\right)^2\right]}{\frac{1}{2}m\omega^2\frac{a^2}{N^2}}$$
$$= \frac{a^2 - \frac{a^2}{N^2}}{\frac{a^2}{N^2}} = \frac{\frac{a^2}{N^2}(N^2-1)}{\frac{a^2}{N^2}} = N^2 - 1$$

24. **(c)** We know that,

Period of revolution

Let, T be the time of one revolution of the satellite.

Then,

$$T = \frac{2\pi r}{v_0} = \frac{2\pi(R_p + h)}{v_0}$$
$$T = \frac{2\pi(R_p + h)}{[GM_e/(R_p + h)]^{1/2}}$$
$$v_0 = \sqrt{\frac{GM_p}{(R_p + h)}}$$
$$T = 2\pi\sqrt{\frac{(R_p + h)^3}{GM_p}} \ldots.. \text{(i)}$$

Again, $GM_e = gR_e^2$

$$\therefore\ T = 2\pi\sqrt{\frac{(R_p+h)^3}{gR_p^2}} \ldots. \text{(ii)}$$

If the planet is supposed to be a sphere of mean density ρ, then the mass of the planet is given by

$$M_p = \text{volume} \times \text{density}$$
$$= \frac{4}{3}\pi r_p^3 \cdot \rho$$
$$T = \sqrt{\frac{3\pi(R_p + h)^3}{G\rho R_p^3}}$$

So,

If a satellite is orbiting very close to the planet's surface ($h \ll R_p$), then putting $h = 0$ in eq. (iii), we have

$$T = \sqrt{\frac{3\pi R_p^3}{G\rho R_p^3}}$$

$$\Rightarrow T = \sqrt{\frac{3\pi}{G\rho}}$$

25. (a) We know that,
Elastic potential energy in a stretched wire

$$U = \frac{1}{2} \text{ Young's modulus } \times (\text{ strain })^2$$

$$U = \frac{1}{2} \times \frac{F}{A} \times \frac{1}{l} \times \frac{Fl}{Y \cdot A}$$

$$U = \frac{1}{2} \frac{F^2}{A^2} \cdot \frac{1}{Y}$$

$$\Rightarrow U \propto \frac{1}{r^4}$$

So, $\frac{U_1}{U_2} = \frac{1/r_1^4}{1/r_2^4} = \left(\frac{r_2}{r_1}\right)^4$

$$\left[\because \frac{d_1}{d_2} = \frac{r_1}{r_2} = \frac{1}{2}\right]$$

$$\frac{U_1}{U_2} = \left(\frac{2}{1}\right)^2 = \frac{16}{1}$$

$U_1 : U_2 = 16 : 1$

26. (d) The energy of n small drop - the energy of the bigger drop = Energy loss by bigger drop

$$n \times 4\pi r^2 \times T - 4\pi r^2 \times T = 3 \times 4\pi R^2 \times T$$

$$n \times 4\pi r^2 \times T = 12\pi R^2 \times T + 4\pi R^2 \times T$$

$$n = \frac{16\pi R^2 \times T}{4\pi r^2 \times T}$$

$$n = 4\frac{R^2}{r^2}$$

27. (b) By calorimetry,
Heat lost = Heat gained

$$m \times L + m \times C \times (T_1 - T_2) = (m_1 + m_2) \times C \times (T_2 - T_3)$$

$$m \times 540 + m \times 1 \times (100 - 90) = (1000\text{ g} + 200\text{ g}) \times 1 \times (90 - 9)$$

$$= m \times 540 + 10\text{ m} = 1200 \times 81$$

$$550\text{ m} = 1200 \times 81$$

$$m = \frac{1200 \times 81}{550} = \frac{24 \times 81}{11}\text{ g}$$

$$= \frac{1944}{11}\text{ g} = 176\text{ g} = 0.176\text{ kg}$$

or $m = 0.18$ kg

28. (d) By Stefan's law If the absolute temperature of a black body is T, then the energy E emitted per second per unit surface area of the body

$$E \propto T^4$$

$$E = \sigma \cdot AT^4$$

Given,

$\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$

$T = 727 + 273 = 1000$ K

$A = 200 \times 10^{-6}$ m

$$E = 5.67 \times 10^{-8} \times (200 \times 10^{-6}) \times (1000)^4$$

$$= 5.67 \times 2 \times 10^{-12} \times 10^{12}$$

$$= 11.34\ J/s$$

29. (c) Work done by an ideal gas is adiabatic expansion

$$dU = n\frac{R}{\gamma-1}dT$$

$$dU = n\frac{R}{\gamma-1}(T_2 - T_1)$$

Given $T_1 = 273$ K

$T_2 = 673$ K

$R = 8.3$ J/mol − K

$n = 5$

$\gamma = 1.4$

$$\therefore dU = 5 \times \frac{8.3}{1.4-1}(673-273)$$

$$= \frac{41.5}{0.4} \times 400 \text{ J}$$

$$= 415 \times 100 \text{ J}$$

$$= 41.50 \text{ kJ}$$

30. (d) As, $dU = nC_v dT$

$$= \frac{R}{\gamma-1}(T_2 - T_1) = \frac{p\Delta V}{\gamma-1}$$

$(\because R\Delta T = pdV)$

$$= \frac{pV}{\gamma-1}$$

31. (a) By third's law of motion

$v^2 = u^2 + 2as$

$$v^2 - u^2 = -2\left(\frac{F}{m}\right)\cdot s$$

$$-u^2 = -2\frac{F}{m}\cdot s$$

$$u^2 = \frac{2F\cdot s}{m}$$

$$m = \frac{2F\cdot s}{u^2} \Rightarrow m \propto s$$

or

Given $s_1 = x$

$s_2 = ?$

$m_1 = m$

$m_2 = m \times 25\% + m$

Now, $\frac{m_1}{m_2} = \frac{s_1}{s_2}$

$$\frac{m}{m \times \frac{25}{100} + m} = \frac{x}{s_2}$$

$$s_2 = \frac{x \times m\left(\frac{100+25}{100}\right)}{m}$$

$$s_2 = \frac{125}{100}x$$

$$s_2 = 1.25x$$

32. (d) At highest point

$$mu\cos\theta = -\frac{m}{2}u\cos\theta + \frac{m}{2}v$$

$$mu\cos\theta + \frac{m}{2}u\cos\theta = \frac{m}{2}v$$

$$\frac{2mu\cos\theta + mu\cos\theta}{2} = \frac{m}{2}v$$

$$mv = 3mu\cos\theta$$

$$v = 3u\cos\theta \quad \ldots(i)$$

The kinetic energy

$$E_1 = \frac{1}{2} \times \frac{m}{2} u^2 \cos^2\theta$$

$$E_1 = \frac{1}{4} mu^2 \cos^2\theta \quad \ldots.. \text{(ii)}$$

Similarly,

$$E_2 = \frac{1}{2} \times \frac{m}{2} \times 9u^2 \cos^2\theta$$

$$E_2 = \frac{9}{4} mu^2 \cos^2\theta \ldots \text{(iii)}$$

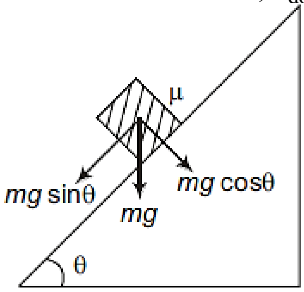
The relation between the E_1 and E_2

$$E_2 = 9 \times E_1 \text{(from the Eq. (ii))}$$

$$E_2 = 9E_1$$

33. **(c)** For upward motion, $F_{up} = mg(\sin\theta + \mu\cos\theta) \ldots \text{(i)}$

For downward motion, $F_{down} = mg(\sin\theta - \mu\cos\theta) \ldots. \text{(ii)}$



According to the question,

$$F_{up} = 2F_{down}$$

$$mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$$

$$\sin\theta + \mu\cos\theta = 2\sin\theta - 2\mu\cos\theta$$

$$3\mu\cos\theta = \sin\theta$$

$$\mu = \frac{1}{3}\tan\theta$$

$$\mu = \frac{1}{3} \times \tan 60^\circ$$

$$\Rightarrow \mu = \frac{1}{3}\sqrt{3} = \frac{1}{\sqrt{3}}$$

34. **(b)** By Newton's law

$F = T\sin\theta \ldots \text{(i)}$

$Mg = T\cos\theta \ldots. \text{(ii)}$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{F}{Mg} = \frac{T\sin\theta}{T\cos\theta}$$

$F = Mg\tan\theta$ ($\because \theta = 60^\circ$ given)

$F = Mg\tan 60^\circ$

$F = \sqrt{3}Mg$

35. **(c)** We know that,

Range of projectile

$$R = \frac{u^2 \sin 2\theta}{g}$$

As range is maximum, $\theta = 45^\circ$

$$R_{max} = \frac{u^2 \sin 2\times45}{g} = \frac{u^2}{g} \quad \ldots. \text{(i)}$$

Flight time of projectile,

$$T = \frac{2u\sin 45^\circ}{g}$$

$$= \frac{2u}{\sqrt{2} \cdot g} = \frac{\sqrt{2} \cdot u}{g}$$

or $u = \frac{Tg}{\sqrt{2}}$

Putting these value of u in Eq. (i), we get

$$R_{max} = \frac{1}{g}\left(\frac{Tg}{\sqrt{2}}\right)^2$$

$$R_{max} = \frac{1}{2}gT^2$$

36. (d) The given equation,

$y = ax - bx^2$ ... (i)

and we know that equation of trajectory is

$y = (\tan\theta)x - \frac{1}{2}\frac{g}{u^2\cos^2\theta} \cdot x^2$ .... (ii)

Compare both equations, we get

$$a = \tan\theta, b = \frac{1}{2} \cdot \frac{g}{u^2\cos^2\theta}$$

The maximum height

$$\frac{a^2}{b} = \frac{\tan^2\theta}{g} \times 2u^2\cos^2\theta$$

$$= \frac{\sin^2\theta}{g\cos^2\theta} \cdot 2u^2\cos^2\theta$$

$$= \frac{2u^2\sin^2\theta}{g} = 4\left(\frac{u^2\sin^2\theta}{2g}\right)$$

$$H_{max} = \frac{a^2}{4b}$$

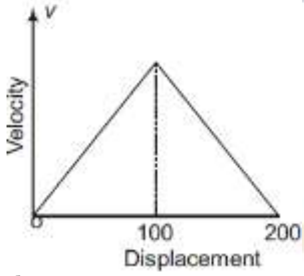
$a = \tan\theta$

$\theta = \tan^{-1}(a)$

So, required solution is $\frac{a^2}{4b}, \tan^{-1}(a)$

37. (c) The given graph

V= kx ...... (i)



$$\frac{dv}{dt} = K \cdot v$$

(from the Eq. (i))

$$\frac{dv}{dt} = K \cdot (Kx) = K^2x$$

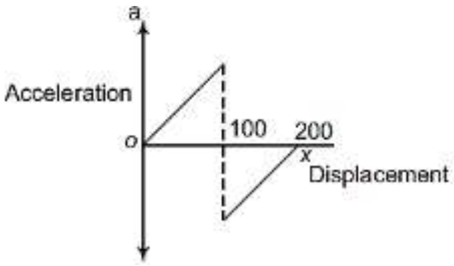
$$a = K^2x$$

and $v = -Kx + v_0$

$$\frac{dv}{dt} = -K \cdot v = -K(-Kx + v_0)$$

$$a = K^2x - Kv_0 \quad .... (ii)$$

So, according to the Eq. (ii).



38. (b) A according to question, $t_1 = \frac{2.5}{5} = \frac{1}{2} h = 30$ min

and $t_2 = \frac{2.5}{7.5} = \frac{1}{3}$ h $= 20$ min

So, the average speed

$$= \frac{\text{Total distance (in metre)}}{\text{Total time (in second)}}$$

$$= \frac{(2.5 + 2.5) \times 1000}{(30 + 20) \times 60}$$

$$= \frac{5 \times 1000}{50 \times 60} = \frac{50}{30} = \frac{5}{3} \text{ m/s}$$

39. (b) Let, $M = C^a h^b G^c$

$ML^0T^0 = [LT^{-1}]^a[ML^2\ T^{-1}]^b[M^{-1}L^3\ T^{-2}]^a$ ... (i)

where, $h = \frac{\text{Energy}}{\text{Frequency}}$

$$= \frac{[ML^2\ T^{-2}]}{[T^{-1}]} = [ML^2\ T^{-1}]$$

$$C = \frac{\text{Metre}}{\text{Second}} = [LT^{-1}]$$

$$G = \frac{\text{Force } \times (\text{ distance })^2}{(\text{ mass })^2}$$

$$= \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}\ L^3\ T^{-2}]$$

Comparing the coefficients M, L, T, of both sides we get

$$b - c \quad = 1 \ldots\ldots (ii)$$
$$a + 2b + 3c \quad = 0 \ldots\ldots (iii)$$
$$-(a + b + 2c) \quad = 0 \ldots\ldots (iv)$$

Solve the Eqs. (ii), (iii) and (iv), we get

$$a = \frac{1}{2}, b = \frac{1}{2}, c = -\frac{1}{2}$$

So,

$M = h^{1/2}C^{1/2}G^{-1/2}$

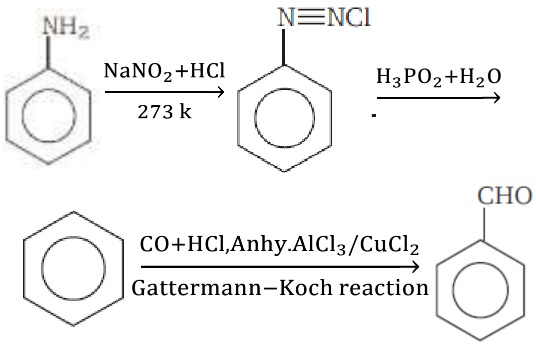
40. (c)

I	II
Fundamental forces in nature	Relative strength
(A) Strong nuclear force	(f) 1
(B) Weak nuclear force	(h) 10^{-13}
(C) Electromagnetic force	(e) 10^{-2}
(D) Gravitational force	(i) 10^{-39}

So, A-f, B-h, C-e, D-i

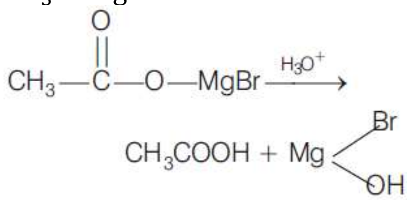
Chemistry

41. (c)

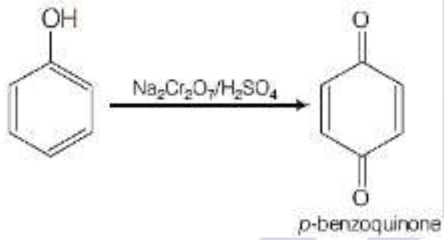


42. (b) When Grignard's reagent is treated with CO_2, a magnesium complex is formed, which on further hydrolysis yields monocarboxylic acid.

$CH_3 - Mg - Br + \xrightarrow{\text{Dry ether}}$



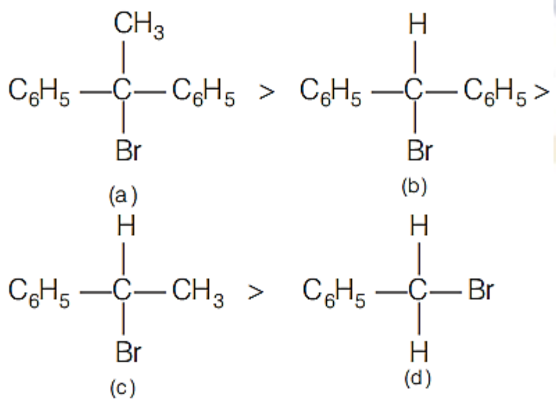
43. (b) In the presence of $Na_2Cr_2O_7$, and H_2SO_4, phenol is oxidised into p-benzoquinone.



44. (d) When an alkyl aryl ether is reacted with halogen acid (HX), the products are always phenol and an alkyl halide.

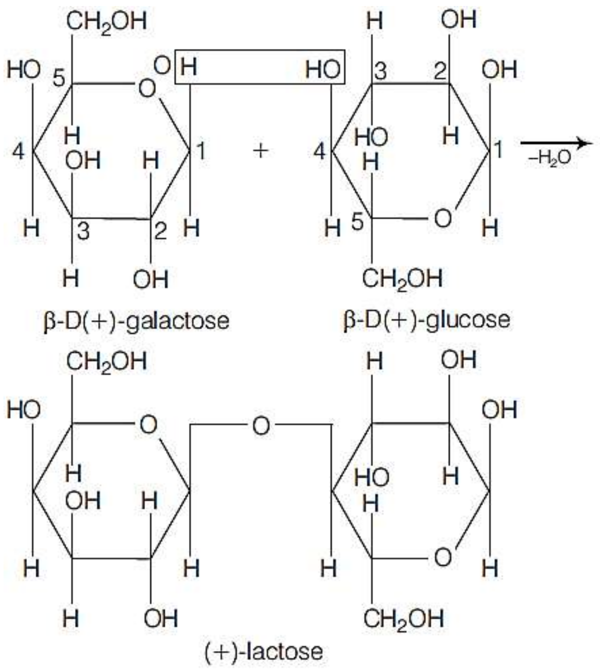
$$C_6H_5 - O - CH_2 - CH_3 + HI \xrightarrow{100^{o}C} C_6H_5OH + CH_3 - CH_2 - I$$

45. (a) The order of reactivity of halides towards S_N1 reaction is benzyl > allyl $> 3^{\circ} > 2^{\circ} > 1^{\circ} >$ Me Therefore, the orrde of hydrolysis by S_N1 mechanism is

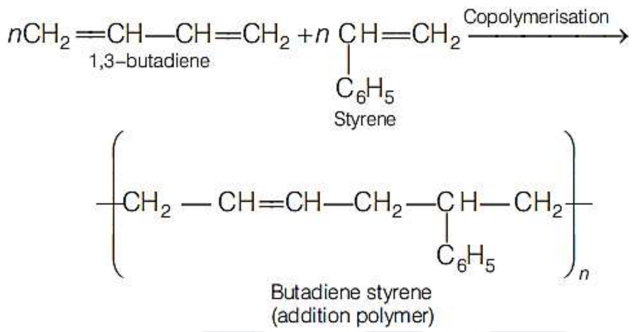


46. (c) An agonist is a chemical that binds to a receptor and activates the receptor to produce a biological response. It is a substance which mimic the natural chemical messengers.

47. (b) Lactose is a disaccharide of $\beta - D(+)$-glucose and $\beta - D(+) -$ galactose. These monosaccharides are joined together with $\beta - 1,4$-glycosidic linkage. It is a reducing sugar.



48. (a) When two or more different monomer units are allowed to polymerise, a copolymer is formed which contains multiple units of each polymer used, in the same polymeric chain.



49. (b)

50. (b)

$_{23}V$ $= [Ar]3d^3 4s^2$

V^{3+} $= [Ar]3d^2 4s^0 \Rightarrow$ 2 unpaired electrons

$_{26}Fe$ $= [Ar]3d^6 4s^2$

Fe^{3+} $= [Ar]3d^5 4s^0 \Rightarrow$ 5 unpaired electrons

$_{28}Ni$ $= [Ar]3d^8 4s^2$

Ni^{2+} $= [Ar]3d^8 4s^0 \Rightarrow$ 2 unpaired electrons

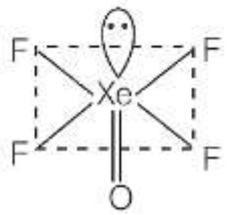
$_{25}Mn$ $= [Ar]3d^5 4s^2$

Mn^{2+} $= [Ar]3d^5 \Rightarrow$ 5 unpaired electrons

$_{24}Cr$ $= [Ar]3d^5 4s^1$

Cr^{3+} $= [Ar]3d^3 4s^1 \Rightarrow$ 3 unpaired electrons

51. (c) $XeOF_4$ has square pyramidal structure



52. (a) The charring of sugar is its dehydration process. When sugar is treated with conc. H_2SO_4, water molecules gets escaped and sugar charcoal is formed. Due to its further reaction with acid, smell of SO_2 is obtained.

$$\underset{\text{Sugar}}{C_{12}H_{22}O_{11}} + [H_2SO_4] \longrightarrow \underset{\text{Sugar charcoal}}{12C} + [H_2O + H_2SO_4]$$

$$C + 2H_2SO_4 \longrightarrow 2SO_2 + CO_2 + 2H_2O$$

53. (d) During extraction of iron from haematite ore, roasted ore, along with coke and limestone is heated in blast furnance. Here coke behaves as reducing agent and limestone is worked as flux. In this process, in reducing zone ($400 - 900^\circ C$), spongy iron is obtained. In slag zone (900-1200), limestone reacts with impurity (e.g., silica) to form slag. In the combustion and fusion zone ($1200 - 1600^\circ C$), all the charge melts and pig iron is obtained which purified further.

54. (d) The order of increase of energy can be calculated from $(n + l)$ rule. If two orbitals have same value of $(n + l)$, the orbital with lower value of n will be filled first.

(i) For $n = 4, I = 1, (n + l) = 4 + 1 = 5$
(ii) For $n = 4, I = 0, (n + l) = 4 + 0 = 4$
(iii) For $n = 3, I = 2, (n + l) = 3 + 2 = 5$
(iv) For $n = 3, I = 1, (n + I) = 3 + 1 = 4$

Therefore, correct order is
(iv) < (ii) < (iii) < (i).

55. (a) For 4d orbital, $n = 4, I = 2$
Number of angular node $= 1 = 2$
Number of radial node $= n - I - 1 = 4 - 2 - 1 = 1$

56. (b) $[AlCl(H_2O)_5]^{2+}$
Let oxidation state of Al is x.
$x + (-1) + 5(0) = +2,$
$x = +3$
Covalency of $Cl = 1$, covalency of $H_2O = 5$
$\therefore$ Total covalency $= 1 + 5 = 6$

57. (a) As we move from left to right in a period, effective nuclear charge increases. Due to this atomic radius decreases. Thus, correct order of atomic radii of 3rd period elements is
$Na > Mg > Al > Si > S$

58. (c) $InSCl_2$, number of lone pairs on $S = \frac{6-2(1)}{2} = 2$
$\therefore$ Total number of pairs of electrons
$= 2$ bonding $+2$ lone pairs $= 4$ pairs
$\therefore$ Total number of electrons in valency shell
$= 4 \times 2 = 8$

59. (a) The average distance between the centre of nuclei of the two bonded atoms in a molecule is known as bond length. It depends upon the size of atoms, hybridisation, steric effect, resonance etc. Usually bond length of polar bond is smaller as compared to a non-polar bond. Therefore, $C - C$ bond length is longest among given options.

60. (c) From Graham's law of diffusion

$$\frac{r_X}{r_Y} = \frac{1}{5} \ldots\ldots (i)$$

$$\frac{r_Y}{r_Z} = \frac{1}{6} \ldots\ldots (ii)$$

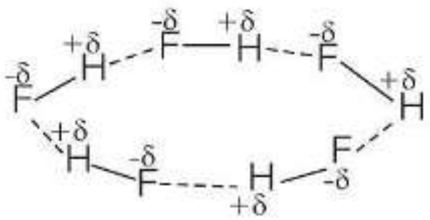
On multiplying Eqs. (i) and (ii),

$$\frac{r_X}{r_Z} = \frac{1}{5 \times 6} = \frac{1}{30}$$

$$\therefore \quad \frac{r_Z}{r_X} = \frac{30}{1} =$$

or $\quad r_Z : r_X = 30 : 1$

61. (b) Hydrogen bond is simply dipole-dipole attraction within oppositely partially charged ends. In HF molecule, electronegativity difference is remarkable, due to which it behaves as a dipole. In the gaseous state several HF molecules polymerises through H -bonding.



62. (b)

$$\underset{\text{n-factor}=3}{MnO_4^-} + I^- \rightleftharpoons MnO_2 + \underset{\text{n-factor}=1}{I_2}$$

Number of milli equivalents of
$MnO_4^- = 0.02 \times 3 \times 250 = 15$
Number of milli equivalents of
$I_2 = 0.1 \times 1 \times 250 = 25$
Thus, here MnO_4^- is limiting reagent.
$\therefore$ Number of milli equivalents of I_2 formed = Number of milli equivalent of $MnO_4^- = 15$
or number of equivalent of I_2 formed
$= \frac{15}{1000} = 0.015$
$\therefore$ Number of moles of I_2 formed $= \frac{0.015}{2}$
$= 0.0075$

63. (a) 40 g of oxygen is combined with = 60 g of metal
$\therefore$ 8 g of oxygen will be combined with $= \frac{60 \times 8}{40}$
= 12 g of metal
$\therefore$ Equivalent weight of metal = 12
Now, Atomic weight
= Equivalent weight $\times$ valency
$= 12 \times 2 = 24$

64. (b) From Gibbs-Helmholtz reaction,
$\Delta G = \Delta H - T\Delta S$
$0 = -20.5 \times 10^3 - T \times (-50.0)$
$\therefore T = \frac{-20.5 \times 10^3}{-50.0} = 410\text{ K}$

65. (a)
$$A + B \rightleftharpoons C + D$$
At $t = 0$, 1 1 0 0
At equil. $\underset{=0.6}{(1 - 0.4)}\underset{(1-0.4)}{(1 - 0.4)}0.4\,0.4$

$K_C = \frac{[C][D]}{[A][B]}$
$= \frac{0.4 \times 0.4}{0.6 \times 0.6} = 0.44$

66. (c)
$NaOH \rightleftharpoons \underset{CM}{Na^+} + \underset{CM}{OH^-}$
$Ni(OH)_2 \rightleftharpoons \underset{x}{Ni^{2+}} + \underset{x}{2OH^-}$
$\therefore\ K_{sp} = [Ni^{2+}][OH^-]^2$
$= x(x + C)^2$

$= x(x^2 + 2Cx + C^2)$

or $K_{sp} = xC^2$ (neglecting higher powers of x)

$$x = \frac{K_{sp}}{C^2} = \frac{1.9 \times 10^{-15}}{(1)^2}$$

$= 1.9 \times 10^{-15} M$

67. (d) The temporary hardness of water is due to the presence of bicarbonates of Ca and Mg . This may be removed by Clark's process. In this process, addition of calculated quantity of lime causes precipitation of $CaCO_3$ as

$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3 \downarrow + 2H_2O$

and $Mg(HCO_3)_2 + 2Ca(OH)_2 \longrightarrow 2CaCO_3 \downarrow$

$+ Mg(OH)_2 + 2H_2O$

68. (d) In KO_2, oxygen exists as O_2^- ion. Its molecular orbital electronic configuration is as follows

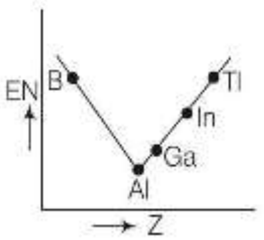
$O_2^- = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2, \sigma 2P_z^2$

$\pi 2p_x^2 \approx \pi^2 p_y^2, \overset{*}{\pi} 2p^2{}_x \approx \overset{*}{\pi} 2p^{-1}{}_y$

Due to presence of 1 unpaired electron in anti- bonding molecular orbital, it is paramagnetic inbehaviour.

69. (d) For group 13 elements, electronegativity decreases from B to Al. The decrease in electronegativity from B to Al is due to increased size of Al. The remaining three elements are more electro negative than Al and their electroonegativity increases from Ga to TI. Thus, the correct order of electronegativity of group 13 elements are as follows.

Element	B	Al	Ga	In	TI
EN (at Pauling scale)	2.0	1.5	1.6	1.7	1.8



70. (d) C, Si and Ge do not react with water while Sn reacts with steam and forms SnO_2 and H_2.

$Sn(s) + 2H_2O(g) \xrightarrow{\Delta} SnO_2(g) + 2H_2(g)$

71. (a) Since, chlorofluorocarbon compounds, i.e., freons (CFCs) are non-reactive, non-inflamable, non-toxic organic molecules, these are widely used in air conditioners, refrigerators, in electronic industry for cleaning computer parts. Due to very long life time ($CF_2Cl_2 - 12{:}139$ years, $CFCl_3 - 11 : 77$ years), these compounds ultimately react the stratosphere where they get broken down by powerful UV radiations and release chlorine free radical. The chlorine free radicals react with ozone and cause its depletion.

$CF_2Cl_2 \xrightarrow{hv} Ci + \dot{C}_2Cl$

$CFCl_3 \xrightarrow{hv} Ci + \dot{C}_2Cl$

Hence, $x + y = Ci + \dot{C}F_3Cl$

72. (c) Ethyne (C_2H_2)

Number of σ-bonds at C -atom = 2

Number of lone pair of electrons at C -atom = 0

∴ Total = 2 + 0 = 2

∴ Hybridisation is sp.

∴ C_2H_2 is linear

Methane (CH_4)

Number of σ bonds at C -atom = 4

Number of-lone pair of electrons at C-atom = 0

∴ Total = 4 + 0 = 4

∴ Hybridisation is sp^3.

∴ CH_4 is tetrahedral.

73. (c)

$$\underset{\text{Sodium propanoate}}{CH_3CH_2-CO_2^-Na^+} + NaOH \xrightarrow[\text{Decarboxylation}]{\text{Cao}}$$

$$\underset{\text{Ethane}}{CH_3-CH_3} + Na_2CO_3$$

74. (d) Aromatic compounds, e.g., C_6H_6 give sooty flame on combustion.

75. (a) The group 13 elements have 1 electron short to Ge, thus giving rise to electron deficient bond or a hole which is responsible for electrical conductivity. Thus, group 13 elements (like Ga) on dopping with Ge, make it p-type semiconductor. Here p-type specifies that conduction is through positive holes in them.

76. (a) For isotonic solutions

$$\frac{w_1}{V_1m_1} = \frac{w_2}{V_2m_2}$$

$$\frac{w_1}{V_1} = 1,\ \frac{w_2}{V_2} = 5$$

$$m_1 = ?,\ m_2 = 342$$

$$\therefore \frac{1}{m_1} = \frac{5}{342}$$

$$\text{or, } m_1 = \frac{342}{5}$$

$$\text{or, } m_1 = 68.4 = X$$

77. (c) From Raoult's law for very dilute solution.

$$\frac{p^\circ - p_s}{p^\circ} = \frac{w}{m} \times \frac{M}{W}$$

$$\frac{24 - p_s}{24} = 0.1 \times \frac{18}{180}$$

$$24 - p_s = 0.24$$

$$\therefore p_s = 24 - 0.24 = 23.76 \text{ mmHg}$$

78. (b) $\Lambda^\circ_m NH_4OH = \Lambda^\circ_m(NH_4Cl + KOH) - \Lambda^\circ_m(KCl)$

$= 152.8 + 272.6 - 149.8$

$= 275.6 \text{ S cm}^2 \text{ mol}^{-1}$

Degree of dissociation $(\alpha) = \frac{\Lambda_m}{\Lambda^\circ_m} = \frac{25.1}{275.6}$

$= 0.091$

$\therefore$ % degree of dissociation = 9.1%

79. (c) Since, concentration of the reactant decreases from 0.6 M to 0.3 M (i.e., halved) in 15 minutes, therefore half time for this reaction be 15 min .

i.e., $t_{1/2} = 15$ min

Now for a first order reaction

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{15} \text{ min}^{-1}$$

Again, $k = \frac{2.303}{t}\log\frac{[A]_0}{[A]}$

$$\therefore \frac{0.693}{15} = \frac{2.303}{t}\log\frac{0.1}{0.025}$$

$$t = \frac{2.303 \times 15}{0.693} \times \log 4$$

$$= \frac{2.303 \times 15}{0.693} \times 0.6020 = 30 \text{ min}$$

80. (a)

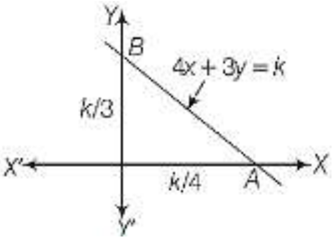
Mathematics

81. (b) Given equation of line is

$3x - 4y = 6$

Perpendicular to above line is

$4x + 3y = k$



$\therefore$ Area of $\triangle$ OAB

$= \frac{1}{2} \times OA \times OB$

$\Rightarrow \quad 6 = \frac{1}{2} \times \frac{k}{4} \times \frac{k}{3}$

$\Rightarrow \quad 6 = \frac{k^2}{24}$

$\Rightarrow \quad k^2 = 144$

$\Rightarrow \quad k = \pm 12$

$\therefore$ Required equation of line is

$4x + 3y = \pm 12$

82. (d) The coordinate of mid-point of PQ is

$M\left(\frac{-\frac{7}{5}+1}{2}, \frac{-\frac{6}{5}+2}{2}\right)$ i.e., $\left(-\frac{1}{5}, \frac{2}{5}\right)$

Now, slope of PQ is

$m = \frac{2+\frac{6}{5}}{1+\frac{7}{5}} = \frac{16}{12} = \frac{4}{3}$

Since, the required line is perpendicular to PQ and passing through $M\left(-\frac{1}{5}, \frac{2}{5}\right)$

$\therefore$ Equation of line is

$$y - \frac{2}{5} = -\frac{1}{4/3}\left(x + \frac{1}{5}\right)$$

$$\frac{5y-2}{5} = -\frac{3}{4}\frac{(5x+1)}{5}$$

$$20y - 8 = -15x - 3$$

$$\Rightarrow 20y + 15x = 5$$

or $4y + 3x = 1$

83. (a) Given points $A(k, 2k), B(3k, 3k)$ and $C(3,1)$ are collinear.

$\therefore$ Slope of AB = Slope of BC

$\therefore \quad \frac{2k-2k}{3k-k} = \frac{1-3k}{3-3k}$

$\Rightarrow \quad \frac{k}{2k} = \frac{1-3k}{3-3k} \Rightarrow 3 - 3k = 2(1-3k)$

$\Rightarrow \quad 1 = -3k \Rightarrow k = -\frac{1}{3}$

$\therefore$ Given points become $A\left(-\frac{1}{3}, -\frac{2}{3}\right), B(-1,-1)$ and $C(3,1)$.

$\therefore$ Equation of line passing through B and C is

$y + 1 = \frac{1+1}{3+1}(x+1)$

$\Rightarrow y + 1 = \frac{2}{4}(x+1)$

$\Rightarrow 2(y+1) = (x+1)$

$\Rightarrow 2y - x + 1 = 0$

Now, the distance from $(0,0)$ to the above line is

$$d=\frac{|2(0)-0+1|}{\sqrt{2^2+1^2}}\Rightarrow=\frac{1}{\sqrt{4+1}}=\frac{1}{\sqrt{5}}$$

84. (d) Given equations of line are

$x^2-3xy+y^2=0$

and $x+y+1=0$

Let, m_1 and m_2 be the slope of the line

$\therefore\ x^2-3xy+y^2=2$

$$\therefore\ m_1+m_2=-\frac{\text{Coefficient of } xy}{\text{Coefficient of } y^2}$$

$=+\frac{3}{1}=3\ \ldots\ldots(i)$

and

$$m_1m_2=\frac{\text{Coefficient of } x^2}{\text{Coefficient of } y^2}$$

Now,

$$m_1-m_2=\sqrt{(m_1+m_2)^2-4m_1m_2}$$
$$=\sqrt{(3)^2-4\times 1}$$
$$=\sqrt{9-4}=\sqrt{5}$$

$$\Rightarrow\ m_1-m_2=\sqrt{5}\ \ldots..(iii)$$

On solving Eqs. (i) and (iii), we get

and

$$m_1=\frac{3+\sqrt{5}}{2}$$
$$m_2=\frac{3-\sqrt{5}}{2}$$

$\therefore$ Equation of lines will be

and $y=\frac{3+\sqrt{5}}{2}x\ \ \ldots.(iv)$

$y=\frac{3-\sqrt{5}}{2}x\ \ \ldots.(v)$

and third line is given

$x+y+1=0\ \ \ldots(iv)$

$\therefore$ The points of intersection of these lines are

$$A(0,0),B\left(-\frac{2}{5+\sqrt{5}},\frac{3+\sqrt{5}}{5+\sqrt{5}}\right)\text{, and}$$
$$C\left(-\frac{2}{5-\sqrt{5}},\frac{3-\sqrt{5}}{5-\sqrt{5}}\right).$$

$\therefore$ Area of triangle

$$=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ -\frac{2}{5+\sqrt{5}} & \frac{3+\sqrt{5}}{5+\sqrt{5}} & 1\\ -\frac{2}{5-\sqrt{5}} & \frac{3-\sqrt{5}}{5-\sqrt{5}} & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[0+0+1\left(\frac{-2(3-\sqrt{5})}{5^2-(\sqrt{5})^2}+\frac{2(3+\sqrt{5})}{5^2-(\sqrt{5})^2}\right]\right.$$

$$=\frac{1}{2}\left[\frac{-6+2\sqrt{5}+6+2\sqrt{5}}{25-5}\right]$$

$$=\frac{1}{2}\left[\frac{4\sqrt{5}}{20}\right]=\frac{1}{2\sqrt{5}}$$

85. (b) Let given line be

$x^2+\alpha y^2+2\beta y-a^2=0$

Here, $a=1, b=\alpha, h=0, g=0, f=\beta, c=-a^2$
Condition for perpendicular line $a+b=0$
$\therefore\ 1+\alpha=0 \Rightarrow \alpha=-1$
Condition of pair of lines

$$abc+2fgh-af^2-bg^2-ch^2=0$$

$\therefore\quad 1\times\alpha\times(-a^2)+0-1(\beta)^2-0-(-a^2)(0)=0$

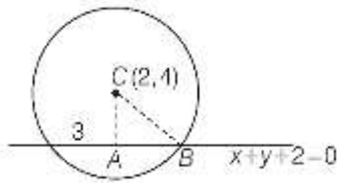
$\Rightarrow\quad -a^2\alpha-\beta^2=0$

$\Rightarrow\quad \beta^2=-\alpha a^2$

$\Rightarrow\quad \beta^2=-(-1)a^2\ (\because \alpha=-1)$

$\Rightarrow\quad \beta^2=a^2 \Rightarrow \beta=a$

86. (a) Let r be the radius of the circle.



Now, perpendicular distance

$AC=\dfrac{|2+4+2|}{\sqrt{1^2+1^2}}=\dfrac{8}{\sqrt{2}}$

$=4\sqrt{2}$

In right angled $\triangle$ CAB,

$r^2=(AC)^2+(AB)^2$

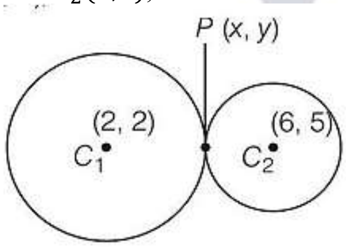
$=(4\sqrt{2})^2+(3)^2=32+9$

$\Rightarrow r^2=41 \Rightarrow r=\sqrt{41}$

87. (c) Centres and radii of given circles are

$C_1(2,2), r_1=\sqrt{2^2+2^2-7}=1$

and $C_2(6,5)$,



$r_2 \quad =\sqrt{5^2+5^2-45}$

$=\sqrt{36+25-45}=4$

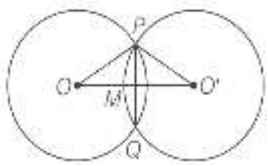
Let P be the point at which the circle touch.
Using internal ratio formula,

$P(x,y)=\left(\dfrac{1\times 6+4\times 2}{1+4},\dfrac{1\times 5+4\times 2}{1+4}\right)$

$=\left(\dfrac{6+8}{5},\dfrac{5+8}{5}\right)=\left(\dfrac{14}{5},\dfrac{13}{5}\right)$

88. (a) Condition for conjugate line is

$r^2(I_1+mm_1)=nn_1$



89. (d) Given equation of circles are

and

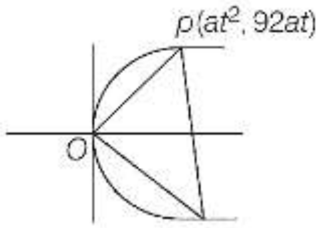
$x^2+y^2-4y=0$

and $x^2+y^2-8x-4y+11=0$

∴ Equation of chord
$x^2 + y^2 - 4y - (x^2 + y^2 - 8x - 4y + 11) = 0$
$\Rightarrow\ 8x - 11 = 0$
Centre and radius of first circle are $O(0,2)$
and $OP = r = 2$.
Now, perpendicular distance from $O(0,2)$ to the line $8x - 11$ is
$$d = OM = \frac{|8 \times 0 - 11|}{\sqrt{8^2}} = \frac{11}{8}$$
In $\triangle\, OMP$,
$$PM = \sqrt{OP^2 - OM^2}$$
$$= \sqrt{2^2 - \left(\frac{11}{8}\right)^2}$$
$$= \sqrt{4 - \frac{121}{64}} = \sqrt{\frac{256 - 121}{64}}$$
$$= \frac{\sqrt{135}}{8}$$
∴ Length of chord $PQ = 2PM = 2 \times \frac{\sqrt{135}}{8}$
$= \frac{\sqrt{135}}{4}$ cm

90. (c) Let the equation of circle be
$x^2 + y^2 + 2gx + 2fy + c = 0$
where, centre $(-g, -f)$
The centre of given circle $x^2 + y^2 - 20x + 4 = 0$ is $(10,0)$,
Condition of two circles cut.
$\therefore\ 2(g_1g_2 + f_1f_2) = c_1 + c_2$
$2(-g \times 10 + 0 \times (-f) = c + 4$
$2(-10g) = c + 4$
Also, circle touch the line $x = 2$.
∴ The perpendicular distance from centre to the circle is equal to radius of the circle.
$$\therefore\ \frac{|-g-2|}{\sqrt{1}} = \sqrt{g^2 + f^2 - c}$$
$\Rightarrow\ (g + 2) = \sqrt{g^2 + f^2 - c}$
$\Rightarrow\ g^2 + 4 + 4g = g^2 + f^2 - c$
$\Rightarrow\ f^2 - 4g - c - 4 = 0$
$\Rightarrow f^2 - 4g + 4 + 20g - 4 = 0$
$\Rightarrow\ f^2 + 16g = 0$
Hence, the locus of $(-g, -f)$ is
$y^2 - 16x = 0 \Rightarrow y^2 = 16x$

91. (b) The perpendicular of the normal to the parabola $y^2 = 4ax$ at P is



$y + tx - 2at + at^3$
Suppose, it meets the parabola at Q. If O be the vertex of the parabola, then the combined equation of OP and OQ is a homogeneous equation of second degree.

$$y^2 = 4ax\left(\frac{y+tx}{2at+at^3}\right)$$
$$\Rightarrow y^2(2at+at^3) = 4ax(y+tx)$$
$$\Rightarrow 4atx^2 + 4axy - (2at+at^3)y^2 = 0$$
Since, OP and OQ are at right angles, then
Coefficient of x^2 + Coefficient of $y^2 = 0$
$\therefore \quad 4at - 2at - at^3 = 0$
$\Rightarrow \quad t^2 = 2 \Rightarrow t = \sqrt{2}$

92. (c) Given equation of circle is
$x^2 + y^2 = (2)^2$
$\therefore$ The equation of tangent to the circle is
$y = mx \pm 2\sqrt{1+m^2} \left(\because y = mx \pm r\sqrt{1+m^2}\right)$
Also, equation of parabola is
$y^2 = 32x$
$\therefore$ Focus of parabola is (8,0).
Since, the line passes through focus (8,0).
$$\therefore \; 0 = 8m \pm 2\sqrt{1+m^2}$$
$$\Rightarrow \; -4m = \pm\sqrt{1+m^2}$$
$$\Rightarrow \; 16m^2 = 1 + m^2$$
$$\Rightarrow \; 15m^2 = 1$$
$$\Rightarrow \; m^2 = \frac{1}{15}$$
$$\Rightarrow \; m = \pm\frac{1}{\sqrt{15}}$$

93. (d) Given circle is a director circle to the given ellipse.
$\therefore$ Angle between tangelt is $\theta = \frac{\pi}{2}$

94. (b)

95. (c) Let the equation of hyperbola be
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \ldots$ (i)
Given equation of ellipse is
$$\frac{x^2}{(13)^2} + \frac{y^2}{(5)^2} = 1$$
Here, $a = 13, b = 5$
$$\therefore \; e = \sqrt{1 - \frac{b^2}{a^2}}$$
$$= \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$
$\therefore$ Focus $(\pm ae, 0) = \left(\pm 13 \times \frac{12}{13}, 0\right)$
$= (\pm 12, 0)$
Since, Eq. (i) passes through $(\pm 12, 0)$.
$$\therefore \frac{144}{a^2} - \frac{0}{b^2} = 1$$
$\Rightarrow a^2 = 144$
$\Rightarrow a = \pm 12$
Now eccentricity of hyperbola

$$e' = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{b^2}{144}}$$

According to the equation,

$$ee' = 1$$

$$\frac{12}{13} \times \sqrt{1 + \frac{b^2}{144}} = 1$$

$$\Rightarrow \Rightarrow \sqrt{1 + \frac{b^2}{144}} = \frac{13}{12}$$

$$\Rightarrow 1 + \frac{b^2}{144} = \frac{169}{144}$$

$$\Rightarrow \frac{b^2}{144} = \frac{169}{144} - 1$$

$$\Rightarrow \frac{b^2}{144} = \frac{25}{144}$$

$$\Rightarrow b^2 = 25$$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{144} - \frac{y^2}{25} = 1$$

96. (a) Using internally ratio formula,

$(-2,3,5)$

$$= \left(\frac{1 \times x + 3 \times 1}{1+3}, \frac{1 \times y + 3 \times 3}{1+3}, \frac{1 \times z + 3 \times 4}{1+3}\right)$$

$$\Rightarrow (2,3,5) = \left(\frac{x+3}{4}, \frac{y+9}{4}, \frac{z+12}{4}\right)$$

$\Rightarrow x + 3 = -8, y + 9 = 12, z = 12 = 20$

$\Rightarrow x = -11, y = +3, z = 8$

$\therefore$ Required point $= B(-11,3,8)$

97. (d) Given direction cosines of two lines are

$1 + m + n = 0$

and $l^2 - 5m^2 + n^2 = 0$

Also, $l^2 + m^2 + n^2 = 1$

$$(I_1, m_1, n_1) = \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

and

$$(I_2, m_2, n_2) = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$$

$$\therefore \cos\theta = |l_1 I_2 + m_1 m_2 + n_1 n_2|$$

$$= \left|-\frac{2}{\sqrt{6}} \times \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}} \times \frac{1}{\sqrt{6}} - \frac{2}{\sqrt{6}} \times \frac{1}{\sqrt{6}}\right|$$

$$\Rightarrow \cos\theta = \left|-\frac{2}{6} + \frac{1}{6} - \frac{2}{6}\right| = \left|-\frac{3}{6}\right|$$

$$= \frac{1}{2} \Rightarrow \theta = 60^\circ = \frac{\pi}{3}$$

98. (c) Given points are $A(3,4,5), B(4,6,3), C(-1,2,4)$ and $D(1,0,5)$.

Now DR's of DC $= (-2,2,-1)$

DR's of AB $= (1,2,-2)$

Let, θ be the angle between AB and DC.

$$\therefore\ \cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$
$$=\frac{-2\times1+2\times2-1\times(-2)}{\sqrt{(-2)^2+(2)^2+(-1)^2}\sqrt{(1)^2+(2)^2+(-2)^2}}$$
$$=\frac{-2+4+2}{\sqrt{4+4+1}\sqrt{1+4+4}}=\frac{4}{3\times3}=\frac{4}{9}$$

99. (c) $\lim_{x\to0}\frac{\sqrt{1+x^2}-\sqrt{1-x+x^2}}{3^x-1}$
$$=\lim_{x\to0}\frac{\sqrt{1+x^2}-\sqrt{1-x+x^2}}{3^x-1}$$
$$\times\frac{\sqrt{1+x^2}+\sqrt{1-x+x^2}}{\sqrt{1+x^2}+\sqrt{1-x+x^2}}$$
$$=\lim_{x\to0}\frac{(1+x^2)-(1-x+x^2)}{3^x-1\left(\sqrt{1-x^2}+\sqrt{1-x+x^2}\right)}$$
$$=\lim_{x\to0}\frac{x}{(3^x-1)}\times\frac{1}{\left(\sqrt{1-x^2}+\sqrt{1-x+x^2}\right)}$$
$$=\lim_{x\to0}\frac{1}{\frac{3^x-1}{x}}\times\frac{1}{\sqrt{1-x^2}+\sqrt{1-x+x^2}}$$
$$=\log_e\frac{1}{3}\times\frac{1}{\sqrt{1-0}+\sqrt{1-0+0}}$$
$$\left[\because \log\frac{(a^x-1)}{x}=1\right]$$
$$=\frac{1}{2\log_e3}=\frac{1}{\log_e9}$$

100. **(b)** Given, $f:[-2,2]\to R$
$$f(x)=\begin{cases}\frac{\sqrt{1+cx}-\sqrt{1-cx}}{x}, & -2\le x<0\\ \frac{x+3}{x+1}, & 0\le x\le 2\end{cases}$$
Now, LHL $=\lim_{x\to0^-}f(x)$
$$=\lim_{h\to0}\frac{\sqrt{1-ch}-\sqrt{1+ch}}{-h}$$
$$\times\frac{\sqrt{1-ch}+\sqrt{1+ch}}{\sqrt{1-ch}+\sqrt{1+ch}}$$
$$=\lim_{h\to0}\frac{(1-ch)-(1+ch)}{-h(\sqrt{1-0}+\sqrt{1+0})}$$
$$=\lim_{h\to0}\frac{-2ch}{-h(1+1)}=c$$
and
$$\text{RHL}=\lim_{x\to0^+}f(x)$$
$$=\lim_{h\to0}f(0+h)=\lim_{h\to0}\frac{0+h+3}{0+h+1}$$
$$=\lim_{h\to0}\frac{h+3}{h+1}=\frac{0+3}{0+1}=3$$
Since, f is continuous at $x=0$.

$\therefore$ LHL = RHL $\Rightarrow c=3$

101. **(d)** Given, $f(x)=x\tan^{-1}x$ then

Then, $\lim_{x\to1}\frac{f(x)-f(1)}{x-1}\left(\frac{0}{0}\text{ form }\right)$

$$= \lim_{x\to 1} \frac{f'(x) - 0}{1}$$

(using L'Hospital rule)

$$= \lim_{x\to 1} \left(\frac{x}{1+x^2} + \tan^{-1} x\right)$$

$$= \frac{1}{1+1^2} + \tan^{-1} = \frac{1}{2} + \frac{\pi}{4}$$

$$= \frac{2+\pi}{4} \text{ or } \frac{\pi+2}{4}$$

102. **(d)** Given, $y = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$

Put $ax = \tan\theta$

$$\therefore\ y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1} ax$$

$$\therefore\ y = \frac{1}{2}\tan^{-1} ax$$

On differentiating w.r.t. x, we get

$y' = \frac{1}{2(1+a^2x^2)}$ ..... (i)

Again, differentiating both side we get

$$y'' = -\frac{1}{2}\frac{(a^2 2x)}{(1+a^2x^2)^2}$$

$$\Rightarrow (1+a^2x^2)y'' = -\frac{a^2x}{1+a^2x^2}$$

$$= -a^2x(2y') \ [\because \text{ from Eq. (i) }]$$

$$\Rightarrow (1+a^2x^2)y'' + 2a^2xy' = 0$$

103. **(d)** Given, $f(x) = \frac{x}{1+x}$

and

$g(x) = f(f(x))$

$$\therefore\ g(x) = f\left(\frac{x}{x+1}\right)$$

$$= \frac{\frac{x}{1+x}}{1+\frac{x}{x+1}}$$

$$\Rightarrow\ g(x) = \frac{x}{2x+1}$$

On differentiating both sides w.r.t. x, we get

$$g'(x) = \frac{(2x+1)1 - x(2)}{(2x+1)^2}$$

$$= \frac{1}{(2x+1)^2}$$

104. **(a)** We know, two curves $\frac{x^2}{a_1^2}+\frac{y^2}{b_1^2}=1$ and $\frac{x^2}{a_2^2}+\frac{y^2}{b_2^2}=1$ cut orthogonally.

Then, $a_1^2-a_2^2=b_1^2-b_2^2$

Here, equation of curves are

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{25}+\frac{y^2}{16}=1$

∴ Condition of orthogonally,

$a^2-25=b^2-16$

$\Rightarrow \quad a^2-b^2=25-16$

$\Rightarrow \quad a^2-b^2=9$

105. **(d)** Given curve is

$f(x)=ax^3+bx^2+cx+d$

On differentiating w.r.t. x, we get

$f'(x)=3ax^2+2bx+c$

For extremum, $f'(x)=0$

$\therefore\ 3ax^2+2bx+c=0$

Since, it has no extremum value

$\therefore \quad b^2-4ac<0$

$\Rightarrow \quad (2b)^2-4\times 3a\times c<0$

$\Rightarrow \quad 4b^2-12ac<0$

$\Rightarrow \quad b^2-3ac<0$

$\Rightarrow \quad b^2<3ac$

106. **(d)** Given, $\Delta r=\pm\frac{0.04}{2}=0.02$

Volume of sphere

$V=\frac{4}{3}\pi r^3$

On differentiating w.r.t. r, we get

$\frac{dU}{dr}=\frac{4}{3}\pi\times 3r^2=4\pi r^2$

$\therefore\ \Delta V=\frac{dU}{dr}\Delta r=4\pi r^2\Delta r$

∴ Relative per cent error

$\frac{\Delta V}{V}\times 100=\frac{4\pi r^2\Delta r}{\frac{4}{3}\pi r^3}\times 100$

$=\frac{3\Delta r}{r}\times 100$

$=\frac{3\times(\pm 0.02)}{10}\times 100$

$=\pm\frac{6}{10}=\pm 0.6$

107. **(c)** Given, $f(x)=\sqrt{x-2}, x\in[2,6]$

We know that, by Lagrange's mean value theorem, their exist $c\in(2,6)$ such that

$f(c)=\frac{f(b)-f(a)}{b-a}$

$=\frac{1}{2\sqrt{c-2}}=\frac{f(6)-f(2)}{6-2}$

$\Rightarrow \frac{1}{2\sqrt{c-2}}=\frac{\sqrt{6-2}-\sqrt{2-2}}{4}$

$\Rightarrow \frac{1}{\sqrt{c-2}}=\frac{\sqrt{4}-0}{2}$

$\Rightarrow \frac{1}{\sqrt{c-2}}=1\Rightarrow\sqrt{c-2}=1$

$\Rightarrow \quad c-2=1\Rightarrow c=3$

108. **(b)** Given, $\int \frac{dx}{\sqrt{\sin^3 x\cos x}} = g(x) + c$

Now,

$$\int \frac{dx}{\sqrt{\sin^4 x\cot x}} = \int \frac{dx}{\sin^2 x\sqrt{\cot x}}$$

$$= \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cot x}} dx$$

Put $\cot x = t$

$\Rightarrow -\operatorname{cosec}^2 x dx = dt$

$$\therefore \int -\frac{1}{\sqrt{t}} dt = -\frac{t^{1/2}}{1/2} + c$$

$$= -2\sqrt{\cot x} + c$$

$$= -\frac{2}{\sqrt{\tan x}} + c$$

$$\therefore\ g(x) = -\frac{2}{\sqrt{\tan x}}$$

109. **(b)** Given,

$$\int \frac{dx}{(1+\sqrt{x})\left(\sqrt{x^2 - x^2}\right)} = \frac{A\sqrt{x}}{\sqrt{1-x}} + \frac{B}{\sqrt{1-x}} + c$$

On differentiating both sides, we get

$$\frac{1}{(1+\sqrt{x})\left(\sqrt{x-x^2}\right)}$$

$$= \frac{\sqrt{1-x}\left(A\frac{1}{2\sqrt{x}}\right) - (A\sqrt{x}+B)\frac{1}{2\sqrt{2-x}}(-1)}{(\sqrt{1-x})^2}$$

$$\Rightarrow \frac{1}{(1+\sqrt{x})\left(\sqrt{x-x^2}\right)} = \frac{(1-x)A + Ax + B\sqrt{x}}{(1-x)2\sqrt{x}\sqrt{(1-x)}}$$

$$\Rightarrow \frac{2(1-x)}{(1+\sqrt{x})} = A + B\sqrt{x}$$

$\Rightarrow\ 2 - 2\sqrt{x} = A + B\sqrt{x}$

$\Rightarrow\ A = 2, B = 2$

$\therefore\ A - B = 0$

110. **(d)** Given, $I_n = \int \tan^n xdx$

$$= \int \tan^{n-2} x \cdot \tan^2 xdx$$

$$= \int \tan^{n-2}(\sec^2 x - 1)dx$$

$$= \int \tan^{n-2} \sec^2 xdx - \int \tan^{n-2} dx$$

$$= \frac{\tan^{n-1}}{n-1} - I_{n-2}$$

$$\left[\begin{array}{l} \because\ \text{put } \tan x = t \Rightarrow \sec^2 xdx = dt \\ \therefore \int \tan^{n-2} dt = \frac{t^{n-1}}{n-1} = \frac{\tan x^{n-1}}{n-1} \end{array}\right]$$

But it is given

$$I_n = \frac{1}{a}\tan^{n-1} - bI_{n-2}$$

$$\therefore\ \frac{1}{a} = \frac{1}{n-1} \text{ and } b = 1$$

$\Rightarrow\ a = n-1, b = 1$

$\therefore\ (a, b) = (n-1, 1)$

111. **(c)**

112. **(b)** $\lim_{n\to\infty}\left(\frac{1^4}{1^5+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+\cdots+\frac{n^4}{n^5+n^5}\right)$

$$=\sum_{r=0}^{r=n}\frac{r^4}{r^5+n^5}=\int_0^1\frac{x^4}{1+x^5}dx$$

Put $1+x^5=t\Rightarrow 5x^4dx=dt$

$$=\int_1^2\frac{1}{5t}dt=\frac{1}{5}[\log t]_1^2$$

$$=\frac{1}{5}[\log 2-\log 1]=\frac{\log 2}{5}$$

113. **(d)** Let $I=\int_0^{\pi/6}\cos^4 3\theta\sin^2 6\theta d\theta$

Put $3\theta=t\Rightarrow d\theta=\frac{dt}{3}$

$$\therefore\ I=\frac{1}{3}\int_0^{\pi/2}\cos^4 t\sin^2 2tdt$$

$$=\frac{1}{3}\int_0^{\pi/2}\cos^4 t\times(2\sin t\cos t)^2dt$$

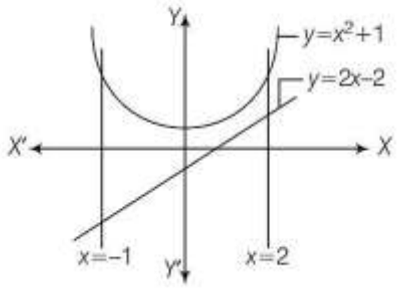
$$=\frac{4}{3}\int_0^{\pi}\cos^6 t\sin^2 tdt$$

$$=\frac{4}{3}\left[\frac{5\cdot 3\cdot 1}{8\cdot 6\cdot 4\cdot 2}\times\frac{\pi}{2}\right]$$

[use gamma rule of integration]

$$=\frac{5\pi}{192}$$

114. **(d)** Given curve is $y=x^2+1\Rightarrow x^2=y-1$ and line $y=2x-2$



The intersection point of curve and line is

$x^2=2x-2-1$

$\Rightarrow\ x^2-2x+3=0$

Now, $b^2-4ac=4-12<0$

Hence, there is no point of intersection

$$\therefore\ \text{Required area}=\int_{-1}^2(y_2-y_1)dx$$

$$=\int_{-1}^2[(x^2+1)-(2x-2)]dx$$

$$=\left[\frac{x^3}{3}+x\right]_{-1}^2-[x^2-2x]_{-1}^2$$

$$=\left[\frac{8}{3}+2-\left(-\frac{1}{3}-1\right)\right]-[4-4-(1+2)]$$

$$=\frac{14}{3}+\frac{4}{3}-[-3]$$

$$=6+3=9$$

115. **(c)** Equation of family of parabola having vertex $(0,-1)$ and axis along y-axis is

$x^2=4a(y+1)\ldots..$ (i)

On differentiating both sides w.r.t. x, we get

$2x = 4ay' \Rightarrow a = \frac{x}{2y'}$

On putting $a = \frac{1}{2y'}$ in Eq. (i), we get

$$x^2 = 4 \times \frac{x}{2y'}(y+1)$$

$$\Rightarrow x = \frac{2}{y'}(y+1)$$

$$\Rightarrow xy' - 2y - 2 = 0$$

116. **(d)** Given differential equation is

$$x\frac{dy}{dx} = y + xe^{y/x}$$

$$\frac{dy}{dx} = \frac{y}{x} + e^{y/x}$$

It is a homogeneous differential equation.

$$\therefore \text{ Put } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = \frac{vx}{x} + e^{vx/x}$$

$$\Rightarrow v + x\frac{dv}{dx} = v + e^{v}$$

$$\Rightarrow x\frac{dv}{dx} = e^{v}$$

$$\Rightarrow e^{-v}dv = \frac{1}{x}dx$$

On integrating both sides, we get

$$-e^{-v} = \log x + c$$

$$-e^{-y/x} = \log x + c$$

Given, $y(1) = 0$

$$\therefore \quad e^{-0/1} = \log 1 + c$$

$$-1 = 0 + c \Rightarrow c = -1$$

$$\therefore \quad -e^{-y/x} = \log x - 1]$$

$$\Rightarrow \quad 1 = \log x + e^{-y/x}$$

117. **(a)** Given differential equation can be rewritten as

$$\cos y\frac{dy}{dx} + x\sin y - 1 = 0$$

$$\Rightarrow \frac{dx}{dy} + (\tan y)x = \sec y$$

It is a linear differential equation of the form

$$\frac{dx}{dy} + Px = Q$$

$$\therefore \text{ IF} = e^{\int Pdy} = e^{\int \tan y dy}$$

$$= e^{\log \sec y} = \sec y$$

$\therefore$ Solution is

$$x\sec y = \int \sec^2 y dy$$

$$\Rightarrow x\sec y = \tan y + C$$

118. **(d)** Let $y = f(x) = \frac{2+x}{2-x}$

$$\Rightarrow 2y - xy = 2 + x$$

$$\Rightarrow x(1+y) = 2y - 2$$

$$\Rightarrow x = \frac{2(y-1)}{1+y}, y \neq -1$$

$\therefore$ Domain of y is $R-\{-1\}$.
Hence, range of f(x) is $R-\{-1\}$.

119. **(a)** Given, $f(x)=\begin{cases} x & \text{for } x\in Q \\ 1-x & \text{for } x\notin Q\end{cases}$ is defined for

$f:[0,1]\rightarrow[0,1]$
If x is rational, then
$f(x)=x$
$\therefore \quad f(f(x))=f(x)=x$
If x is irrational, then
$f(x)=1-x$
$\therefore\ f\circ f(x)=f(1-x)=1-(1-x)$
$=x$
$\therefore$ $f\circ f(x)=x$ is possible for all values of domain [0,1].

120. **(b)**

121. **(c)** Given,
$a^2+b^2+c^2+d^2=1 \quad \ldots..(i)$
and $A=\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib\end{bmatrix}$
Now $|A|=(a+ib)(a-ib)$
-(c +id) (-c +id)
$=a^2-(ib)^2-[(id)^2-(c)^2]$
$=a^2+b^2-[-d^2-c^2]$
$=a^2+b^2+d^2+c^2$
$=1$ (form eq (i)

$$A^{-1}=\frac{1}{|A|}\begin{bmatrix} a-ib & -(c+id) \\ -(-c+id) & a+ib\end{bmatrix}$$
$$=\frac{1}{1}\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib\end{bmatrix}$$
$$=\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib\end{bmatrix}$$

122. **(b)** Given, $A=\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & \alpha\end{bmatrix}$

Applying $R_2\rightarrow R_2-2R_1$, $R_3\rightarrow R_3-3R_1$ and $R_4\rightarrow R_4-2R_3$
$$=\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & \alpha-6\end{bmatrix}$$
Applying $R_4\rightarrow R_4+R_3$
$$=\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & \alpha-3\end{bmatrix}$$
Applying $R_4\rightarrow R_4-R_2$
$$=\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & \alpha-5\end{bmatrix}$$
Since, the matrix A is of rank 3 .
$\therefore\ \alpha-5=0\Rightarrow\alpha=5$

123. **(d)** Given $A=\begin{bmatrix} k & k\alpha & \alpha \\ 0 & \alpha & k\alpha \\ 0 & 0 & k\end{bmatrix}$

$\therefore\ |A| = \alpha k^2$

Now $|A^2| = |A|^2\ [\because |A^n| = |A|^n]$

$= (\alpha k^2)^2$

$= \alpha^2 k^4$

According to given condition,

$|A^2| = k^2$

$\therefore\ \alpha^4 k^4 = k^2$

$\Rightarrow\ \alpha^4 = \frac{1}{k^2}$

$\Rightarrow\ |\alpha| = \frac{1}{k}$

124. **(a)** Given, $z^3 + \bar{z} = 0$

Let $z = x + iy$

$\therefore\ (x+iy)^3 + x - iy = 0$

$\Rightarrow x^3 + (iy)^3 + 3x^2 iy + 3x(iy)^2 + x - iy = 0$

$\Rightarrow x^3 - y^3 i + 3x^2 yi - 3xy^2 + x - iy = 0$

$\Rightarrow (x^3 - 3xy^2 + x) + (-y^3 + 3x^2 y - y)i = 0$

On equating real and imaginary parts, we get

$x^3 - 3xy^2 + x = 0$

or $-y^3 + 3x^2 y - y = 0$

$\Rightarrow\ x(x^2 - 3y^2 + 1) = 0$

or $-y(y^2 - 3x^2 + 1) = 0$

$\Rightarrow\ x = 0$ and $x^2 - 3y^2 + 1 = 0$

or $y = 0$ and $y^2 - 3x^2 + 1 = 0$

Now,

$x^3 - 3y^2 + 1 = y^2 - 3x^2 + 1$

$\Rightarrow\ 4x^2 = 4y^2$

$\Rightarrow\ x = \pm y$

$y^2 - 3y^2 + 1 = 0$

$\Rightarrow\ 2y^2 = 1 \Rightarrow y = \pm\frac{1}{2}$

$\Rightarrow\ x = \pm\frac{1}{2}$

Hence, their solutions are (0,0) and $\left(\pm\frac{1}{2}, \pm\frac{1}{2}\right)$

Hence, number of solutions is 5 .

125. **(c)** Given, $(1+i)^n = (1-i)^n$

$\Rightarrow\ \frac{(1+i)^n}{(1-i)^n} = 1$

$\Rightarrow\ \left[\frac{(1+i)\times(1+i)}{(1-i)\times(1+i)}\right]^n = 1$

$\Rightarrow\ \left[\frac{1^2 + i^2 + 2i}{1^2 - i^2}\right]^n = 1$

$\Rightarrow\ \left[\frac{1 - 1 + 2i}{1+1}\right]^n = 1$

$\Rightarrow\ \left(\frac{2i}{2}\right)^n = 1$

$\Rightarrow\ (i)^n = 1$

$\therefore\ n = 4$

126. **(a)** Given, $x = p + q,\ y + p\omega + q\omega^2$ and $z = p\omega^2 + q\omega$

$\therefore xyz = (p+q)(p\omega + q\omega^2)(p\omega^2 + q\omega)$
$= (p+q)(p^2\omega^3 + pq\omega^2 + pq\omega^4 + q^2\omega^3)$
$= (p+q)(p^2 + pq\omega^2 + pq\omega + q^2)$
$(\because \omega^3 = 1)$
$= (p+q)(p^2 + pq(\omega + \omega^2) + q^2)$
$= (p+q)(p^2 - pq + q^2) = p^3 + q^3$
$(\because 1 + \omega + \omega^2 = -1)$

127. **(d)** Given, $z_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$

$\therefore z_1 z_2 z_3 \ldots = e^{i\frac{\pi}{2}} \cdot e^{i\frac{\pi}{2^2}} \cdot e^{i\frac{\pi}{2^3}} \cdots$
$= e^{i\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3} + \cdots\right)}$
$= e^{i\left(\frac{\pi/2}{1}\right)} = e^{i(\pi)}$
$= \cos\pi + i\sin\pi = -1 + i(0)$
$= -1$

128. **(d)** Given, x_1 and x_2 are the roots of the equation

$$x^2 - px + c = 0$$

$\therefore \quad x_1 + x_2 = k$
and $\quad x_1 x_2 = -c$
$\therefore$ The distance between $A(x_1, 0)$ and $B(x_2, 0)$ is
$AB = \sqrt{(x_2 - x_1)^2 + 0^2} = |x_2 - x_1|$
$= \sqrt{(x_1 + x_2)^2 - 4x_1x_2}$
$= \sqrt{k^2 - 4 \times 1 \times c}$
$= \sqrt{k^2 - 4c}$

129. **(b)**

130. **(c)** Given, p and q are distinct prime.
Let $p = 3$ and $q = 2$
$\therefore$ Given equation becomes

$$x^2 - 3x + 2 = 0$$
$$\Rightarrow \quad x - 2x - x + 2 = 0$$
$$\Rightarrow \quad x(x-2) - 1(x-2) = 0$$
$$\Rightarrow \quad (x-1)(x-2) = 0$$
$$\Rightarrow \quad x = 1,2$$

131. **(a)** Let, α, β and γ are the roots of
$x^2 - 2x^2 + 10x - 8 = 0$
$\alpha + \beta + \gamma = 2$
$\alpha\beta + \beta\gamma + \gamma\alpha = 10$
and $\alpha\beta\gamma = 8$
Now, $\alpha^2 + \beta^2 + \gamma^2$
$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$
$= (2)^2 - 2(10) = -16$
$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$
$= (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha + \beta + \gamma)(\alpha\beta\gamma)$
$= (10)^2 - 2(2)(8)$
$= 100 - 32 = 68$
and $\alpha^2\beta^2\gamma^2 = (8)^2 = 64$
$\therefore$ Required cubic equation is
$x^3 - (\alpha^2 + \beta^2 + \gamma^2)x^2 + (\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2)x$
$-\alpha^2\beta^2\gamma^2 = 0$
$\therefore x^3 - (-16)x^2 + (68)x - 64 = 0$
$\Rightarrow x^2 + 16x^2 + 68x - 64 = 0$

132. **(c)** Total number of points in a plane is 30 .
Out of them, 8 points are collinear.
$\therefore$ Total number of straight lines formed

$$= {}^{30}C_2 - {}^{8}C_2 + 1$$
$$= \frac{30 \times 29}{2} - \frac{8 \times 7}{2 \times 1} + 1$$
$$= 435 - 28 + 1 = 408$$

133. **(b)** We know, ${}^{11}C_n$ is maximum when $n = 5$.

$$\therefore\ {}^{11}C_n = \frac{11!}{n!\,(11-n)!}$$

$\therefore$ $n!\,(11-n)!$ is minimum when $n = 5$.

134. **(d)** Now, $(a + bx)^{-3} = a^{-3}\left(1 + \frac{bx}{a}\right)^{-3}$

$$= \frac{1}{a^3}\left(1 - {}^{3}C_1\left(\frac{bx}{a}\right) + \cdots\right)$$
$$\Rightarrow \frac{1}{a^3} - {}^{3}C_1 \frac{1}{a^3}\left(\frac{bx}{a}\right) + \cdots \text{(given)}$$
$$= \frac{1}{27} + \frac{1}{3}x$$

On equating constant and x, we get

$$\frac{1}{a^3} = \frac{1}{27}$$

and $-\ {}^{3}C_1 \frac{1}{a^3}\left(\frac{b}{a}\right) = \frac{1}{3}$

$\Rightarrow\ a = 3$

and $-\ {}^{3}C_1\left(\frac{1}{3^4}\right) \times b = \frac{1}{3}$

$$\Rightarrow\ -3 \times \frac{1}{3^4} b = \frac{1}{3}$$
$$\Rightarrow\ -b = 3^2 \Rightarrow b = -9$$
$$\therefore\ (a, b) = (3, -9)$$

135. **(a)** General term in the expansion of $\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)^{18}$ is

$$T_{r+1} = {}^{18}C_r(\sqrt{x})^{18-r}\left(-\frac{2}{\sqrt{x}}\right)^r$$
$$= {}^{18}C_r(x)^{\frac{18-r}{2}}(-2)^r(x)^{-\frac{r}{2}}$$
$$= {}^{18}C_r(-2)^r x^{9-r}$$

For independent of x,

put $9 - r = 0$

$\Rightarrow\ r = 9$

$\therefore\ T_{10} = {}^{18}C_9(-2)^9$

$= -\ {}^{18}C_9 2^9$

136. **(c)** Given, $\frac{2x^3+x^2-5}{x^4-25} = \frac{Ax+B}{x^2-5} + \frac{Cx+1}{x^2+5}$

$$\therefore 2x^3 + x^2 - 5 = (Ax + B)(2x^2 + 5) + (Cx + 1)(x^2 - 5)$$
$$\Rightarrow 2x^3 + x^2 - 5 = Ax^3 + 5Ax + Bx^2 + 5B + Cx^3 - 5Cx + x^2 - 5$$
$$\Rightarrow 2x^3 + x^2 - 5 = x^3(A + C) + x^2(B + 1) + x(5A - 5C) + 5B - 5$$

On equating the coefficients of x^3, x^2, x and constant, we get

$A + C = 2, B + 1 = 1, 5A - 5C = 0$

and $5B - 5 = -5$

Now, $B + 1 = 1 \Rightarrow B = 0$

$5A - 5C = 0 \Rightarrow A = C$

and $A + C = 2 \Rightarrow C + C = 2$

$\Rightarrow\ 2C = 2 \Rightarrow C = 1$
$\therefore\ A = C = 1$
$\therefore\ (A, B, C) = (1,0,1)$

137. **(d)** Given, $\cos x = \tan y, \cot y = \tan z$
and $\cot z = \tan x$

$\therefore\ \cos x = \tan y$

$\Rightarrow\ \cos x = \dfrac{1}{\tan z}$

$\Rightarrow\ \cos x = \cot z$

$\Rightarrow \cos x = \tan x$

$\Rightarrow \cos x = \dfrac{\sin x}{\cos x}$

$\Rightarrow \cos^2 x = \sin x$

$\Rightarrow 1 - \sin^2 x = \sin x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\therefore\ \sin x = \dfrac{-1 \pm \sqrt{1 - 4 \times (-1)}}{2 \times 1}$

$= \dfrac{-1 \pm \sqrt{5}}{2}$

$\therefore \sin x = \dfrac{\sqrt{5} - 1}{2}\ \left(\because \dfrac{-1 - \sqrt{5}}{2} < -1\right)$

138. **(d)** $\tan 81^\circ - \tan 63^\circ - \tan 27^\circ + \tan 9^\circ$

$= (\tan 81^\circ + \tan 9^\circ) - [\tan(63^\circ) + \tan(27^\circ)]$

$= \left[\dfrac{\sin 81^\circ}{\cos 81^\circ} + \dfrac{\sin 9^\circ}{\cos 9^\circ}\right] - \left[\dfrac{\sin 63^\circ}{\cos 63^\circ} + \dfrac{\sin 27^\circ}{\cos 27^\circ}\right]$

$= \left[\dfrac{\sin 81^\circ \cos 9^\circ + \sin 9^\circ \cos 81^\circ}{\cos 9^\circ \cos 81^\circ}\right]$

$- \left[\dfrac{\sin 63^\circ \cos 27^\circ + \sin 27^\circ \cos 63^\circ}{\cos 27^\circ \cos 63^\circ}\right]$

$= \dfrac{[\sin(81^\circ + 9^\circ)]}{\cos 9^\circ \cos(90^\circ - 90^\circ)}$

$- \dfrac{[\sin(63^\circ + 27^\circ)]}{\cos 27^\circ \cos(90^\circ - 27^\circ)}$

$= \dfrac{\sin 90^\circ}{\cos 9^\circ \sin 9^\circ} - \dfrac{\sin 90^\circ}{\cos 27^\circ \sin 27^\circ}$

$= \dfrac{1}{\sin 9^\circ \cos 9^\circ} \times \dfrac{2}{2} - \dfrac{1}{\sin 27^\circ \cos 27^\circ} \times \dfrac{2}{2}$

$= \dfrac{2}{\sin 18^\circ} - \dfrac{2}{\sin 54^\circ}$

$= 2\left[\dfrac{\sin 54^\circ - \sin 18^\circ}{\sin 18^\circ \sin 54^\circ}\right]$

$= 2\left[\dfrac{2\cos 36^\circ \sin 18^\circ}{\sin 18^\circ \cos 36^\circ}\right]$

$= 4$

139. **(d)** Given, $\cos x + \cos y = \frac{3}{2}$

$\Rightarrow\ 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right) = \dfrac{3}{2}$

and

$\sin x + \sin y = \dfrac{3}{4}$

$$\Rightarrow \; 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{4}$$

$$\therefore \; \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{3/4}{3/2}$$

$$\Rightarrow \; \tan\left(\frac{x+y}{2}\right) = \frac{1}{2}$$

$$\therefore \sin(x+y) = \frac{2\tan\left(\frac{x+y}{2}\right)}{1+\tan^2\left(\frac{x+y}{2}\right)}$$

$$= \frac{2\times\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2} = \frac{4}{4+1} = \frac{4}{5}$$

140. **(c)** We know,

$$\cos x\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right) = \frac{1}{4}\cos 3\theta$$

But it is given

$$\cos x\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right) = \frac{1}{4}$$

$$\therefore \cos 3\theta = 1$$

$$\Rightarrow \; 3\theta = 2\pi, 4\pi$$

$$\Rightarrow \; \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$\therefore$ Sum of solutions

$$= \frac{2\pi}{3} + \frac{4\pi}{3} = \frac{6\pi}{3}$$

$$= 2\pi$$

141. **(b)** Given, $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$$\Rightarrow \; \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right) = \pi$$

$$\Rightarrow \; \frac{x+y+z-xyz}{1-xy-yz-zx} = \tan\pi = 0$$

$$\Rightarrow \; x+y+z = xyz$$

142. **(c)** $\operatorname{sech}^{-1}x = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

and $\operatorname{cosech}^{-1}x = \log_e\left(\frac{1+\sqrt{1+x^2}}{x}\right)$

$$\therefore \operatorname{sech}^{-1}\left(\frac{1}{2}\right) - \operatorname{cosech}\left(\frac{3}{4}\right)$$

$$= \log_e\left(\frac{1+\sqrt{1-\left(\frac{1}{2}\right)^2}}{\frac{1}{2}}\right) - \log_e\left(\frac{1+\sqrt{1+\frac{9}{16}}}{\frac{3}{4}}\right)$$

$$= \log_e\left(\frac{1+\sqrt{\frac{3}{2}}}{\frac{1}{2}}\right) - \log_e\left(\frac{1+\frac{5}{4}}{\frac{3}{4}}\right)$$

$$= \log_e(2+\sqrt{3}) - \log_e\left(\frac{9}{3}\right)$$

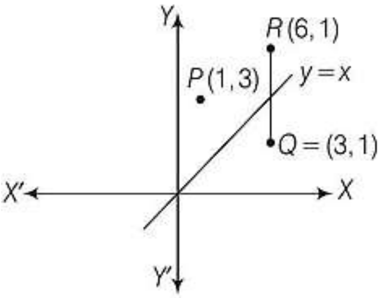
$$= \log_e(2+\sqrt{3}) - \log_e(3)$$

$$= \log_e\frac{(2+\sqrt{3})}{3}$$

143. **(d)**

$$\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$$
$$= \frac{2s(2s-2a)(2s-2b)(2s-2c)}{4b^2c^2}$$
$(\because a+b+c=2s)$
$$= \frac{16[s(s-a)(s-b)(s-c)]}{4b^2c^2}$$
$$= \frac{4\Delta^2}{b^2c^2} = \left(\frac{2\Delta}{bc}\right)^2 = \sin^2 A \left[\because \sin A = \frac{2\Delta}{bc}\right]$$

144. **(a)** The reflection of the point $P(1,3)$ about the line $y = x$ is $Q(3,1)$.



After translation through a distance 3 units along the positive direction of X-axis at the point whose coordinate are $R(6,1)$. After rotation through an angle of $\frac{\pi}{6}$ about the origin in the clockwise direction, then R goes to whose coordinates are

$$R'\left(\frac{6\sqrt{3}+1}{2}, \frac{\sqrt{3}-6}{2}\right)$$

145. **(c)** Given, vertices of a triangle are

$A(a\cos\theta, a\sin\theta), B(b\sin\theta, b\cos\theta)$ and $C(1,0)$

$\therefore$ Let the locus of centroid be (x, y).

$$\therefore (x,y) = \left(\frac{a\cos\theta + b\sin\theta + 1}{3}, \frac{b\cos\theta + 0}{3}\right)$$
$$\Rightarrow x = \frac{a\cos\theta + b\sin\theta + 1}{3}$$
and $y = \frac{a\sin\theta - b\cos\theta}{3}$
$$\Rightarrow a\cos\theta + b\sin\theta = x - 1$$
and $a\sin\theta - b\cos\theta = 3y$
$$\Rightarrow a^2\cos^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = (3x-1)^2$$
and $a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = 9y^2$

On adding, we get
$$a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta) = (3x-1)^2 + 9y^2$$
$$\Rightarrow a^2 + b^2 = (3x-1)^2 + 9y^2$$
which is the required locus of a point.

146. **(c)** Given, mean of binomial variable, $np = 8$ and variance of binomial variable, $npq = 4$

$$\therefore q = \frac{1}{2}$$
and
$$p = 1 - q$$
$$= 1 - \frac{1}{2} = \frac{1}{2}$$
and $n\left(\frac{1}{2}\right) = 8 \Rightarrow n = 16$

$\therefore P(X<3)=P(X=0)+P(X=1)+P(X=2)$

$= {}^{16}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{16-0} + {}^{16}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{16-1}$

$+ {}^{16}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{16-2}$

$= 1\left(\frac{1}{2}\right)^{16} + 16\left(\frac{1}{2}\right)^{16} + 120\left(\frac{1}{2}\right)^{16} = \frac{137}{2^{16}}$

147. **(b)** Given distribution is

X	1	2	3	4	5
P(X = x)	k	2k	3k	2k	k

$\therefore$ Variance $=\Sigma x_i^2 p-(\Sigma x_i p)^2$

$=(1k+8k+27k+32k+25k)$

$-(k+4k+9k+8k+5k)^2$

$=(93k)-(27k)^2=\left(93\times\frac{1}{9}\right)-\left(27\times\frac{1}{9}\right)^2$

$\left(\because \Sigma p=1, \therefore k=\frac{1}{9}\right)$

$=\frac{93}{9}-9=\frac{93-81}{9}=\frac{12}{9}=\frac{4}{3}$

148. **(a)** Required probability

= Probability of passing two test

+ Probability of passing all three test

= P (passing I and II tests and fail in third test)

+P (passing Ist test, fail in II test and passing in IIIrd test)

P (fail in I test, passing in IInd and IIIrd tests)

+P (passing in all three tests)

$= npq + pq\times\frac{p}{2} + q\frac{p}{2}\times p + ppp$

$= p^2(1-p) + p(1-p)\frac{p}{2} + (1-p)\frac{p^2}{2} + p^3$

$= p^2\left(1-p+\frac{1}{2}-\frac{p}{2}+\frac{1}{2}-\frac{p}{2}+p\right) = p^2(2-p)$

149. **(c)** Sum of the dice is 7 .

$S=-\{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}$

$\therefore n(S)=6$

Let, E = Event of getting atleast three 3 or a die

$\therefore n(E)=2$

$\therefore$ Required probability $=\frac{n(E)}{n(S)}=\frac{2}{6}=\frac{1}{3}$

150. **(d)** Given, $P(B)=\frac{3}{2}P(A)$

and $P(C)=\frac{1}{2}P(B)$

Since, A, B and C are exclusive events.

$\therefore\ P(A) + P(B) + P(C) = 1$

$\therefore\ P(A) + \frac{3}{2}P(A) + \frac{1}{2} \times \frac{3}{2}P(A) = 1$

$\Rightarrow\ P(A)\left(1 + \frac{3}{2} + \frac{3}{4}\right) = 1$

$\Rightarrow\ \frac{13}{4}P(A) = 1$

$\Rightarrow\ P(A) = \frac{4}{13}$

$\therefore\ P(C) = \frac{1}{2} \times \frac{3}{2}P(A) = \frac{3}{4} \times \frac{4}{13} = \frac{3}{13}$

Also, A, B and C are mutually exclusive.

$\therefore P(A \cap B) = P(B \cap C) = P(C \cap A) = 0$

$\therefore P(A \cup C) = P(A) + P(B) - 0$

$= \frac{4}{13} + \frac{3}{13} = \frac{7}{13}$

151. **(d)** Given, $\sum_{i=1}^{n} x_i^2 = 400$

and $\sum_{i=1}^{n} x_i = 80$

We know, $\frac{\Sigma(x_i)^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \geq 0$

$\Rightarrow\ \frac{400}{n} - \left(\frac{80}{n}\right)^2 \geq 0$

$\Rightarrow\ 400n \geq 6400$

$\Rightarrow\ n \geq 16$

$\therefore\ n = 16$

152. **(c)** Let four observations are be x_1, x_2, x_3 and x_4

Given, mean $(\bar{x}) = 3$

and $\Sigma x_i^2 = 48$

$\therefore SD = \sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{48}{4} - (3)^2} = \sqrt{12 - 9} = \sqrt{3}$

153. **(d)** Given skew lines are

$r = (\hat{i} + 2\hat{j} + 3\hat{k}) + t(\hat{i} + 3\hat{j} + 2\hat{k})$

and $r = (4\hat{i} + 5\hat{j} + 6\hat{k}) + t(2\hat{i} + 3\hat{j} + \hat{k})$

Here, $a_1 = \hat{i} + 2\hat{j} + 3\hat{j}$

$b_1 = \hat{i} + 3\hat{j} + 2\hat{k}$

$a_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$b_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

and

Now $a_2 - a_1 = 4\hat{i} + 5\hat{j} + 6\hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= 3\hat{i} + 3\hat{j} + 3\hat{k}$

and $b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$= \hat{i}(3 - 6) - \hat{j}(1 - 4) + \hat{k}(3 - 6)$

$= -3\hat{i} + 3\hat{j} - 3\hat{k}$

$\therefore$ The shortest distance between skew lines

$$= \frac{|(a_2 - a_1)\cdot(b_1 \times b_2)|}{|b_1 \times b_2|}$$

$$= \frac{|(3\hat{i} + 3\hat{j} + 3\hat{k})\cdot(-3\hat{i} + 3\hat{j} - 3\hat{k})|}{\sqrt{(-3)^2 + (3)^2 + (-3)^2}}$$

$$= \frac{|-9+9-9|}{\sqrt{9+9+9}} = \frac{9}{3\sqrt{3}} = \sqrt{3}$$

154. **(c)** Given, $\mathbf{a} = x\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$, $\mathbf{b} = x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{c} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$$\text{Now, } \mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & 2 & 0 \\ 0 & y & 3 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(6-0) - \hat{\mathbf{j}}(3x-0) + \hat{\mathbf{k}}(xy-0)$$

$$= 6\hat{\mathbf{i}} - 3x\hat{\mathbf{j}} + xy\hat{\mathbf{k}}$$

$$\Rightarrow 6\hat{\mathbf{i}} - 3x\hat{\mathbf{j}} + xy\hat{\mathbf{k}} = 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

On equating the coefficients of $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$, we get

$z = 3, x = 1$ and $xy = 1$

$\therefore xy = 1 \Rightarrow y = 1$

$\therefore\ a = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}, b = \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $c = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 6$

$$\therefore [a\quad c] = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 1 & 1 & 6 \end{vmatrix}$$

$$= 1(6-3) - 2(0-3) + 0 = 3 + 6 = 9$$

155. **(c)** Given, $|a| = 2, |b| = 3$ and $|c| = 4$

$$\therefore |a-b|^2 + |b-c|^2 + |c-a|^2$$

$$= a^2 + b^2 - 2a\cdot b + b^2 + c^2 - 2b\cdot c$$

$$+ c^2 + a^2 - 2a\cdot c$$

$$= 2(a^2 + b^2 + c^2) - 2(a\cdot b + b\cdot c + c\cdot a)$$

$$\Rightarrow 2(a\cdot b + b\cdot c + c\cdot a)$$

$$= 2(a^2 + b^2 + c^2)$$

$$-(|a-b|^2 + |b-c|^2 + |c-a|^2)$$

We know, $(a+b+c)^2 \geq 0$

$$\therefore a^2 + b^2 + c^2 + 2(a\cdot b + b\cdot c + c\cdot a) \geq 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(a^2 + b^2 + c^2)$$

$$-(|a-b|^2 + |b-c|^2 + |c-a|^2) \geq 0$$

$$\Rightarrow |a-b|^2 + |b-c|^2 + |c-a|^2$$

$$\leq 3(a^2 + b^2 + c^2)$$

$$\Rightarrow |a-b|^2 + |b-c|^2 + |c-a|^2$$

$$\leq 3(2^2 + 3^2 + 4^2)$$

$$\Rightarrow |a-b|^2 + |b-c|^2 + |c-a|^2$$

$$\leq 3(4 + 9 + 16)$$

$$\Rightarrow |a-b|^2 + |b-c|^2 + |c-a|^2 \leq 87$$

156. **(a)** Given lines are

$$r = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

and

$$r = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + \mu(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

Here DR's of given lines are (1,4,3) and (1,2,−3).

$\therefore$ Angle between these lines is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$= \frac{1 \times 1 + 4 \times 2 + 3 \times (-3)}{\sqrt{1^2 + 4^2 + 3^2}\sqrt{1^2 + 2^2 + (-3)^2}}$$
$$= \frac{1 + 8 - 9}{\sqrt{1 + 16 + 9}\sqrt{1 + 4 + 9}} = 0$$
$$\Rightarrow \theta = \frac{\pi}{2}$$

157. **(c)** Given, $\mathbf{d} = \frac{1}{x}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

and $\mathbf{d} = \frac{1}{y}(\mathbf{b} + \mathbf{c} + \mathbf{d})$

$\therefore\ a + b + c - xd = 0$

and $b + c + d - yd = 0$

$\Rightarrow\ a + b + c + d = 0$

$\therefore \dfrac{1}{xy}(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}) = \dfrac{1}{xy}(0) = 0$

158. **(a)** Given, $a + 3b$ is collinear with c.

$\therefore \quad a + 3b = \lambda c$

or $\quad a + 3b - \lambda c = 0 \ldots\ldots (i)$

And $3b + 2c$ is collinear with a.

$\therefore \quad 3b + 2c \quad = \mu a$

$3b + 2c - \mu a = 0 \quad \ldots. (ii)$

From Eqs. (i) and (ii), we get

$a + 3b - \lambda c = 3b + 2c - \mu a$

On equating c, we get

$\lambda = -2$

On putting $\lambda = -2$ in Eq. (i), we get

$a + 3b + 2c = 0$

159. **(d)**

160. **(d)** Given, the ratio of angles of a triangle is $1:1:4$. Let angles of a triangle are A, B and C.

$\therefore\ A:B:C = 1:1:4$

Let $A = x, B = x$ and $C = 4x$

$\because \quad A + B + C = 180°$

$\therefore \quad x + x + 4x = 180°$

$\Rightarrow \quad 6x = 180° \Rightarrow x = 30°$

$\therefore A = 30°, B = 30°$ and $C = 120°$

Hence, largest angle is $120°$. So largest side of a triangle is c .

$\therefore$ Perimeter of triangle: Largest side of a triangle

$= (a + b + c) : c$

$= (2R\sin 30° + 2R\sin 30° + 2R\sin 120°)$

$: 2R\sin 120°$

$[\because a = 2R\sin A, b = 2R\sin B$ and $C = 2R\sin C]$

$$= 2R\left[\frac{1}{2} + \frac{1}{2} + \frac{\sqrt{3}}{2}\right] : 2R \times \frac{\sqrt{3}}{2}$$
$$= \left(1 + \frac{\sqrt{3}}{2}\right) : \frac{\sqrt{3}}{2} = 2 + \sqrt{3} : \sqrt{3}$$