Physics

1. **(b)** Moment of inertia of a solid cylinder of mass M, length 2R and radius R about an axis passing through the centre of mass and perpendicular to the axis is

$$I_1 = M\left(\frac{L^2}{12} + \frac{R^2}{4}\right)$$

$$I_1 = M\left[\frac{4R^2}{12} + \frac{R^2}{4}\right] = M\left[\frac{7R^2}{12}\right]$$

Moment of inertia passing through one end of the cylinder and perpendicular to the axis of the cylinder

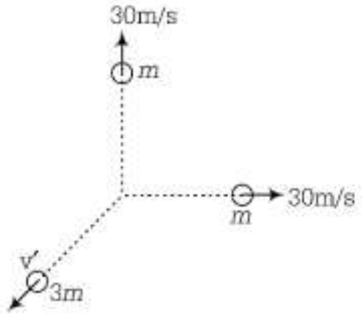
$$I_2 = M\left(\frac{L^2}{3} + \frac{R^2}{4}\right)$$

$$I_2 = M\left[\frac{4R^2}{3} + \frac{R^2}{4}\right]$$

$$= M\left[\frac{19R^2}{12}\right]$$

$\therefore\ I_2 - I_1 = MR^2$

2. **(a)** Initial momentum before explosion is zero. Final momentum after explosion.



$$\sqrt{2}\frac{m}{4} \times 30 - \frac{3m}{4}v'$$

From conservation of momentum

Initial momentum = Final momentum

$$\Rightarrow \quad \sqrt{2}\frac{m}{4} \times 30 - \frac{3m}{4}v' = 0$$

$$\Rightarrow \quad v' = 10\sqrt{2}\ \text{m/s}$$

3. **(a)** Equation of particle executing SHM

$$y = 8\cos\left[100t + \frac{\pi}{4}\right]\ \text{cm}$$

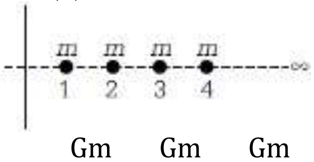
Here, $\omega = 100\text{rad/s}, a = 8\ \text{cm}$

Maximum kinetic energy $= \frac{1}{2}m\omega^2a^2$

$$= \frac{1}{2} \times 4 \times 100 \times 100 \times (8 \times 10^{-2})^2$$

$$= 128\ \text{J}$$

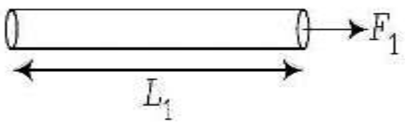
4. **(b)** Gravitational field at the origin due to mass at 1, 2, 4,8,16 ...



$$= \frac{Gm}{(1)^2} + \frac{Gm}{(2)^2} + \frac{Gm}{(4)^2} + \frac{Gm}{(8)^2} \ldots..$$

$$= Gm\left[1 + \frac{1}{4} + \frac{1}{16} + \cdots \ldots\right] = \frac{4Gm}{3}$$

5. **(c)** Change in length of wire of length L is directly proportional to the external force applied



$$F_1 \propto (L_1 - L)$$
$$\Rightarrow \quad F_1 = K(L_1 - L) \quad \ldots\ldots(i)$$
Similarly, $F_2 = K(L_2 - L) \ldots\ldots(ii)$

Dividing Eq. (i) by Eq. (ii), we get
$$\frac{F_1}{F_2} = \frac{L_1 - L}{L_2 - L}$$
$$\Rightarrow F_1L_2 - F_1L = F_2L_1 - F_2L$$
$$\Rightarrow \ L = \frac{F_1L_2 - F_2L_1}{F_1 - F_2}$$

6. (b) Volume of small spherical drops $= \frac{4}{3}\pi r^3$

Volume of larger spherical drops $= \frac{4}{3}\pi R^3$

n small spherical drops combine to form large drops.
$$\therefore \quad n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$
$$\Rightarrow \quad R = n^{1/3}r$$
Here,
$$n = 1000, r = \frac{1}{2} \times 10^{-8} \text{ m}$$
Amount of energy liberated

= Change in area × surface tension
$$= 4\pi R^2\left(n^{1/3} - 1\right) \times 0.075$$
$$= 6.75\pi \times 10^{-15} \text{ J}$$

7. (c) Let after getting equilibrium stage final temperature is T.

Then, according to loss of calorimetry

Heat loss = Heat gain
$$\Rightarrow (250)(1)(90 - T) = 20(1)(T - 5)$$
$$\Rightarrow \ T = 83.7^\circ\text{C}$$

8. (c) Let temperature of copper and iron junction is θ
$$\therefore \text{ Temperature of junction } (\theta) = \frac{K_1l_2\theta_1 + K_2l_1\theta_2}{K_1l_2 + K_2l_1}$$
$$= \frac{386.4 \times 125 \times 100 + 48.46 \times 75 \times 0}{386.4 \times 125 + 48.46 \times 75}$$
$$= \frac{48{,}300 \times 100}{48{,}300 + 3634.5} = \frac{48{,}300 \times 100}{51934.5}$$
$$= 0.93 \times 100 = 93^\circ\text{C}$$

9. (b) Heat required to convert 1 g of water at 100°C completely into steam at 100°C
$$dQ = mL$$
$$= 1 \times 540 \text{ calorie}$$
$$= 540 \times 4.2 = 2268 \text{ J}$$
Work done in expanding volume
$$dW = pdV = 10^5(V_2 - V_1)$$
$$= 10^5(1650 - 1000) \times 10^6$$
$$= 650 \times 10^{-1} = 65$$
Increase in internal energy
$$dU = dQ - dW = 2268 - 65 = 2203 \text{ J}$$

10. (b) RMS velocity of molecules at NTP
$$v_{rms} = \sqrt{\frac{3RT}{M}}$$
RMS velocity of oxygen molecules at NTP

$$(v_1) = \sqrt{\frac{3RT}{M_1}} = 0.5 \text{ km/s}$$

RMS velocity of hydrogen molecules at NTP

$$(v_2) = \sqrt{\frac{3RT}{M_2}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \quad \frac{0.5}{v_2} = \sqrt{\frac{2}{32}}$$

$$\Rightarrow \quad \frac{0.5}{v_2} = \frac{1}{4}$$

$$\Rightarrow \quad v_2 = 2 \text{ km/s}$$

11. **(c)** Ratio of fundamental frequency $(n_1 : n_2 : n_3) = 1 : 2 : 3$

$$I_1 : I_2 : I_3 = \frac{1}{n_1} : \frac{1}{n_2} : \frac{1}{n_3}$$

$$= \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$$

$$I_1 = \frac{6 \times 99}{11} = 54$$

$$I_2 = \frac{3 \times 99}{11} = 27$$

$$I_3 = \frac{2 \times 99}{11} = 18$$

12. **(b)** Combined focal length $\left(\frac{1}{F}\right) = \frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3}$

$$\Rightarrow \quad \frac{1}{F} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$\Rightarrow \quad F = 1 \text{ cm}$$

Magnification of the lens combination in normal adjustment is

$$M = 1 + \frac{D}{F} = 1 + \frac{25}{1} = 26$$

13. **(a)** Focal length of lens when placed in air

$$F_{air} = (\mu_{lens} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots. \text{(i)}$$

Focal length of lens when placed in water

$$F_{air} = \left(\frac{\mu_{lens}}{\mu_{liq}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots \text{(ii)}$$

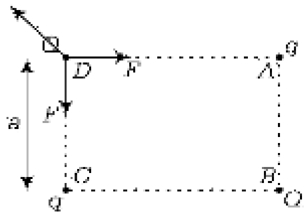
Dividing Eq. (i) by Eq. (ii)

$$\frac{F_{liq}}{F_{air}} = \frac{(1.45 - 1)}{\left(\frac{1.45}{1.3} - 1\right)} = \frac{0.45}{0.15} \times 1.3 = 3.9$$

14. **(b)** Width of central maxima $= \frac{2\lambda D}{a}$

$$= \frac{2 \times 6330 \times 10^{-10} \times 2}{2 \times 10^{-3}} = 1.27 \text{ mm}$$

15. **(a)** Net force on D due to A and B $= \frac{K\sqrt{2}Qq}{a^2}$



Force on D due to B $= \frac{KQ^2}{2a^2}$

For net electric field on D to be zero

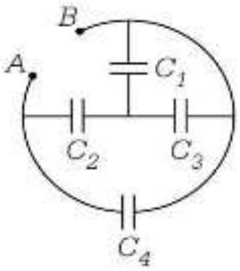
$$K\left[\frac{Q^2}{2a^2} + \frac{\sqrt{2}Qq}{a^2}\right] = 0$$

$$\Rightarrow \frac{Q}{2} + \sqrt{2}q = 0$$

$$Q = -2\sqrt{2}q$$

∴ q should be negative.

16. (b) Effective capacitance between the points A and B



C_1 and C_3 parallel

∴ $C_1 + C_3 = 18\text{pF}$

18 pF is in series with C_2

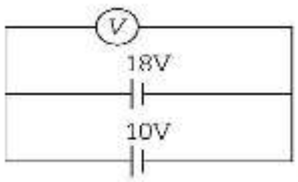
∴ $\frac{18 \times 9}{18 + 9} = 6\text{pF}$

6 pF is in parallel with 9 pF

∴ $6 + 9 = 15\text{pF}$

17. (b) Net emf of the combination

$= 18 - 10 = 8$



Net resistance of the circuit $= 3 + 1 = 4$

Current in the circuit $i = \frac{8}{4} = 2$ A

∴ Potential difference $(V) = E - ir = 18 - 6$

$= 12$ V

18. (d) If wire of aluminium and a wire of germanium are cooled to a temperature of 77 K the resistance of Al decreases and resistance of Ge increases.

19. (c) A voltmeter of 250 mV range having resistance of 10Ω

∴ $V_g = GI_g$

$\Rightarrow 250 \times 10^{-3} = 10 \times I_g$

$\Rightarrow I_g = 25 \times 10^{-3}$ A

Shunt required to convert this voltmeter into ammeter.

$$S = \frac{GI_g}{I - I_g}$$

$$= \frac{10 \times (25 \times 10^{-3})}{250 \times 10^{-3} - 25 \times 10^{-3}} = \frac{10}{9} = 1\Omega$$

20. (d) Dipole moments due to current carrying circular wire $M = iA$

Here, $\frac{M_1}{M_2} = \frac{i\pi r^2}{ia^2} = \pi\left[\frac{\left(\frac{1}{2\pi}\right)^2}{\left(\frac{1}{4}\right)^2}\right]$

$$= \pi\frac{16}{4\pi^2} = \frac{4}{\pi}$$

21. (a) Time period of magnet in magnetic field

$$T \propto \frac{1}{\sqrt{B}}$$

If magnetic field is doubled,

then $\quad T' \propto \frac{1}{\sqrt{2B}}$

$\Rightarrow \quad T' \propto \frac{T}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\ s$

[oscillation per second $= \frac{30}{60}$]

22. (d)

(A) Newton's law of motion is used in rocket propulsion

(B) Bernoulli's principle of fluid dynamics is used in aeroplane.

(C) Total internal reflection of light is used in optical fibres.

(D) magnetic confinement of plasma is used in fusion test reactor.

23. (a)

$$F = \frac{X}{\text{Linear density}}$$

Dimension of $F = [MLT^{-2}]$

Dimension of linear density $= [ML^{-1}]$

$\therefore\ X = F$ (linear density)

$$= [MLT^{-2}][ML^{-1}] = [M^2\ L^0\ T^{-2}]$$

24. (b) Displacement of a particle moving in a straight line $(x) = At^3 + Bt^2 + Ct + D$

$\therefore$ Velocity, $v = \frac{dx}{dt} = 3At^2 + 2Bt + C$

$v_{initial} = C$

and $a = \frac{d^2x}{dt^2} = 6At + 2B$

$a_{initial} = 2B$

$\therefore$ Ratio between initial acceleration and initial velocity is $\frac{2B}{C}$.

25. (b) If a body falls freely from rest at A. Then, time taken to travel from A to B (time taken)

$$t_1 = \sqrt{\frac{2h}{8}}$$

Time taken to travel from B to C

(time taken) $t_2 = \sqrt{\frac{4h}{8}} - \sqrt{\frac{2h}{8}}$

$\therefore\ \frac{t_2}{t_1} = \sqrt{2} - 1$

26. (a) Sum of two forces

$F_1 + F_2 = 25$ ..... (i)

Resultant of these forces

$(10)^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \quad \dots \text{(ii)}$

Since, resultant is normal to the smaller force

$\therefore\; F_1 + F_2\cos\theta = 0 \quad \dots\dots \text{(iii)}$

From Eqs. (i), (ii) and (iii), we get

$F_1 = 14.5\ \text{N}$

$F_2 = 10.5\ \text{N}$

27. (b) Maximum vertical height attained by body thrown with velocity v_1

$h_1 = \dfrac{v_1^2}{2g}$

Another body of mass 2m is projected with a velocity v_2 at an angle θ.

$$\therefore\; \text{Height attained } (h_2) = \frac{v_2^2\sin^2\theta}{2g}$$

$$\therefore\quad \frac{h_1}{h_2} = \frac{v_1^2}{v_2^2}\sin^2\theta$$

But $\quad t_1 = 2t_2$

$$\therefore\quad \frac{v_2}{g} = 2\left[\frac{v_2\sin\theta}{g}\right]$$

$$\Rightarrow\quad v_1 = 2v_2\sin\theta$$

$$\therefore\quad \frac{h_1}{h_2} = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$$

28. (b) From conservation of energy

$M \times v + m(0) = (M + m)v'$

$$v' = \frac{M \times 20}{(M + m)}$$

Average resistance of the wall to the penetration of the nail is

$$F = (M + m)\frac{(0^2 + v'^2)}{2 \times 10^{-2}}$$

$$\because v^2 = u^2 + 2as = (M + m) \times \frac{M^2 \times (20)^2}{(M + m)^2} \times \frac{1}{2 \times 10^{-2}}$$

$$= \frac{2M^2}{(M + m)} \times 10^4$$

29. (c) Body is acted upon by a force given by equation $F = (3t^2 - 30)\text{N}$

$\therefore\; m(v - u) = \int F dt$

$= \dfrac{3t^3}{3} - 30t = t^3 - 30t$

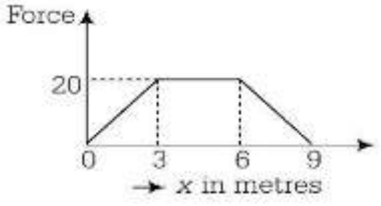
$= 5^3 - 30(5) = 125 - 150 = -25$

$\therefore\; m[v - 10] = -25$

$\Rightarrow$ Final velocity $v = 7.5\ \text{m/s}$

30. (a) Work done by the force of gravity is directly proportional to the distance travel in nth second. $W \propto S_n$ $\therefore$ Ratio of work done by the force is $1:3:5$.

31. (c) Work done in a moving body is get converted into kinetic energy.



$$\therefore\quad \int F dx = \frac{1}{2}m(v^2 - 0^2)$$

$$\Rightarrow\; \frac{1}{2}(9 + 3)(20) = \frac{1}{2}(2.4)v^2$$

$$\Rightarrow\; v = 10\ \text{m/s}$$

32. (d) Modulation index $(\mu)=\frac{E_m}{E_C}$

$\Rightarrow \frac{3}{4}=\frac{E_m}{12}$

$\Rightarrow E_m=9$

33. (d) n_e and n_h are electron and hole concentrations in an extrinsic semiconductor and n_i is electron concentration in an intrinsic semiconductor, then

$n_e n_h=n_i^2$

34. (a) In half wave rectifier frequency remain unchanged.

∴ Fundamental frequency of the output is 50 Hz .

35. (b) The effective half-life of the nucleus

$$\lambda=\lambda_1+\lambda_2$$

$$\frac{0.69}{T}=\frac{0.69}{T_1}+\frac{0.69}{T_2}$$

$$T=\frac{T_1T_2}{T_1+T_2}$$

$$=\frac{5\times10^3\times10^5}{5\times10^3[1+20]}=\frac{5\times10^5}{105}$$

$$=4762\text{yr}$$

36. (a) The wavelengths of a spectral lines of Lyman series are smaller than the wavelength of the second spectral line of Balmer series therefore statement A is false. Whereas, statement B is true.

37. (d) de-Broglie wavelength

$$\lambda=\frac{h}{\sqrt{2mE}}$$

$$E=\frac{1}{2m}\left(\frac{h}{\lambda}\right)^2$$

$$=\frac{1}{2\times9.1\times10^{-31}}\left(\frac{6.625\times10^{-34}}{5.5\times10^{-7}}\right)^2$$

$$=\frac{1}{18.2\times10^{-31}}\times1.45\times10^{-54}$$

$$=8\times10^{-25}\ \text{J}$$

38. (d) Point where ' B ' to be calculated is not mentioned.

On surface $\int B\cdot dl=\mu_0\left(\varepsilon_0\frac{d\phi}{dt}\right)$

$\Rightarrow B(2\pi r)=\frac{1}{C^2}\frac{d\phi}{dt}$

$\Rightarrow\ =2\times10^{-3}\ T$

39. (a) Current leads the voltage by 45°

∴ $\phi=45^\circ$

Since, $\tan\phi=\frac{X_C-X_L}{R}$

$\Rightarrow 1=\frac{X_C-X_L}{R}$

$\Rightarrow \frac{1}{\omega C}-\omega L=R$

$\Rightarrow \frac{1}{\omega C}=R+\omega L$

$\Rightarrow C=\frac{1}{\omega[R+\omega L]}$

$\Rightarrow C=\frac{1}{2\pi f[2\pi fL+R]}$

40. (b) A small square loop of wire of sidel is placed inside a large square loop, then mutual induction of the system depends upon.

$M \propto \frac{l^2}{L}$

Chemistry

41. (c)

42. (d)

$Ca_3(PO_4)_2 \rightleftharpoons 3Ca^{2+} + 2PO_4^{3-}$

$K_{sp} = [Ca^{2+}]^3[PO_4^{3-}]^2 = [3x]^3[2x]^2 = [27x^3][4x^2]$

$= 108x^5$

43. (a) Clark's method is not used to remove permanent hardness of water. This method is used to remove temporary hardness of water.

$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow CaCO_3 \downarrow + 2H_2O$

$Mg(HCO_3)_2 + 2Ca(OH)_2 \longrightarrow$

$2CaCO \downarrow + Mg(OH)_2 \downarrow + 2H_2O$

If excess of lime is added, water will again become hard because of adsorption of CO_2 from the atmosphere.

$Ca(OH_2 + 2CO_2 \longrightarrow Ca(HCO_3)_2$

44. (d) White metal is an alloy of Li and Pb .

45. (b)

46. (a) H_2CO_3 is a weak dibasic acid and dissociates in following two steps :

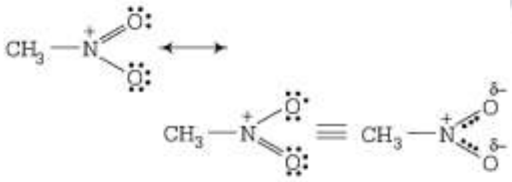
$H_2CO_3(aq) + H_2O(1) \rightleftharpoons HCO_3^-(aq) + H_3O^+(aq)$

$HCO_3^-(aq) + H_2O(l) \rightleftharpoons CO_3^{2-}(aq) + H_3O^+(aq)$

H_2CO_3/HCO_3^- buffer system helps to maintain pH of blood between 7.26 to 7.42 .

47. (b) The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water is called biochemical oxygen demand (BOD). The amount of BOD in the water is a measure of the amount of organic material in water in terms of how much oxygen will be required to break it down biologically. Clean water have BOD value less than 5 ppm whereas highly polluted water such as municipal sewage has BOD value more than 100 ppm.

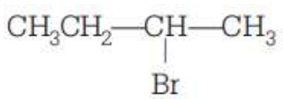
48. (d) Two bonds $N = O$ and $N - O$ in H_3CNO_2 are of same bond length due to resonance effect.



Resonance hybrid

49. (d) According to Markownikoff's rule, the addition of reagents such as HX, H_2O and HOX with unsymmetrical alkanes occurs in such a way that the negative part of the adding molecule goes to that carbon atom of the double bond which carries lesser number of hydrogen atoms.

$CH_3CH_2CH = CH_2 + HBr \xrightarrow[\text{Absence of peroxides}]{\text{Dark}}$

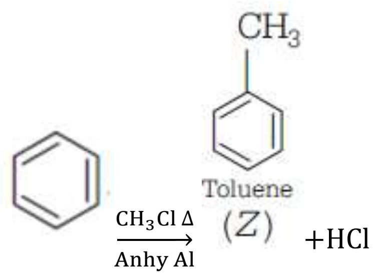


2-bromobutane (major product)

$+ CH_3CH_3 - CH_2 - CH_2 - Br$

1-bromobutane (minor product)

50. (b)



51. (c) Silicon carbide (SiC) is a covalent solid. It is black solid compound insoluble in water and soluble in molten alkali.

52. (b) Molarity

$$= \frac{\text{Weight of solute}}{\text{Molecular weight of solute}} \times \frac{1000}{\text{volume (in mL)}}$$

$$0.2 = \frac{\text{Weight of solute}}{106} \times \frac{1000}{250}$$

$$\therefore \ \text{Weight of solute} = \frac{0.2 \times 106 \times 250}{1000} = 5.3\text{gm}$$

53. (a) $\Delta T_b = K_b \cdot m$

$T_b - T_b^\circ = K_b \cdot m$

where, $K_b =$ molal elevation constant

$m =$ molality

$(100.052 - 100.00) = (0.52)(m)$

$0.052 = 0.52 \times m$

$m = 0.1$

54. (b) During recharging, the cell is operated like an electrolytic cell. The electrode reactions are the reverse of those that occur during discharge.

At cathod

$PbSO_4(s) + 2e^- \longrightarrow Pb(s) + SO_4^{2-}(aq)$

At anode $PbSO_4(s) + 2H_2O(1) \longrightarrow$

$PbO_2(s) + SO_4^{2-}(aq) + 4H^+(aq) + 2e^-$

Net reaction

$2PbSO_4(s) + 2H_2O(l) \longrightarrow$

$Pb(s) + PbO_2(s) + 4H^+(aq) + 2SO_4^{2-}(aq)$

55. (d) For the reaction,

$5Br^-(aq) + BrO_3^-(aq) + 6H^+(aq) \longrightarrow$

$3Br_2(aq) + 3H_2O(1)$

$$\text{Rate of reaction} = -\frac{1}{5}\frac{d[Br^-]}{dt} = -\frac{d[BrO_3^-]}{dt}$$

$$= -\frac{1}{6}\frac{d[H^+]}{dt} = +\frac{1}{3}\frac{d[Br_2]}{dt}$$

$$\therefore \ -\frac{1}{5}\frac{d[Br^-]}{dt} = -\frac{d[BrO_3{}^-]}{dt}$$

$$\Rightarrow \ -\frac{0.05}{5} = \frac{d[BrO_3]}{dt} \Rightarrow -\frac{d[BrO_3]}{dt} = 0.01$$

56. (d) Animal leather is colloidal in nature and has positively charged particles. When it is soaked in tannin, a negatively charged colloid, it results in mutual coagulation and gets harden. Thus, leather get hardened after tanning.

57. (d) German silver (nickel silver) is an alloy of copper, zinc and nickel often in the proportions 5: 2: 2. It is used in cheap jewellery and cutlery.

58. (d) The catalytic oxidation of SO_2 with O_2 to give SO_3 is the key step in the manufacturing of sulphuric acid by contact process.

$2SO_2(g) + O_2(g) \overset{V_2O_5}{\rightleftharpoons} 2SO_3(g);$

$\Delta_r H^\circ = -196 \text{ kJ mol}^{-1}$

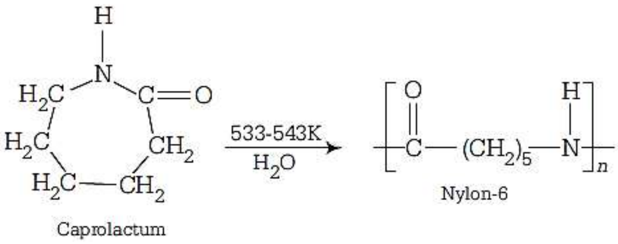
59. (c) The balanced equation is

$NH_3 + 3Cl_2 \longrightarrow NCl_3 + 3HCl$
Thus, the ratio of NH_3 to $3Cl_2$ is $1:3$.

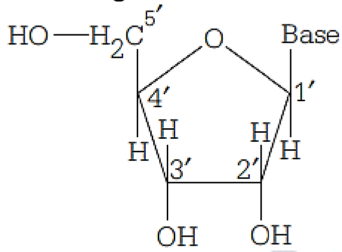
60. (a) Generally, paramagnetism is shown by lanthanoid ions. The paramagnetism arises due to presence of unpaired electrons in f-orbital. $Lu^{3+}(4f^{14})$ does not have unpaired electrons. Hence, it does not exhibit paramagnetism.

61. (b) Order of field strength of legands is as follows:
$I^- < Br^- < SCN^- < Cl^- < F^- < OH^- < C_2O_4^{2-}$
$< H_2O < NCS^- < EDTA^{4-} < NH_3 < en < CN^- < CO$

62. (c) Nylon-6 or perlon is a condensation homopolymer.

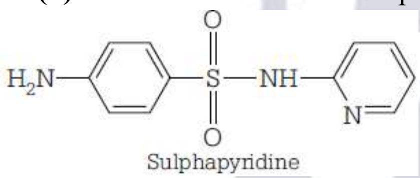


63. (a) Nucleoside consists of two basic components of nucleic acids, i.e. a pentose sugar and nitrogenous base. In this, unit base is attached at 1' position of sugar and sugar carbons are numbered as $1', 2', 3'$ etc., in order to distinguish these from the base.

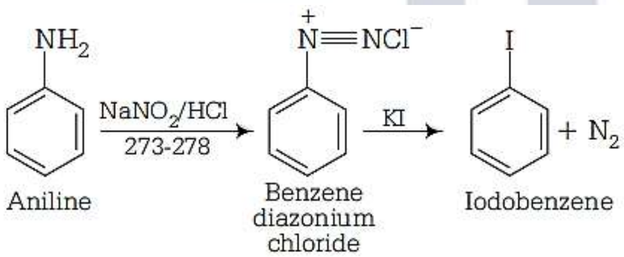


Structure of nucleoside

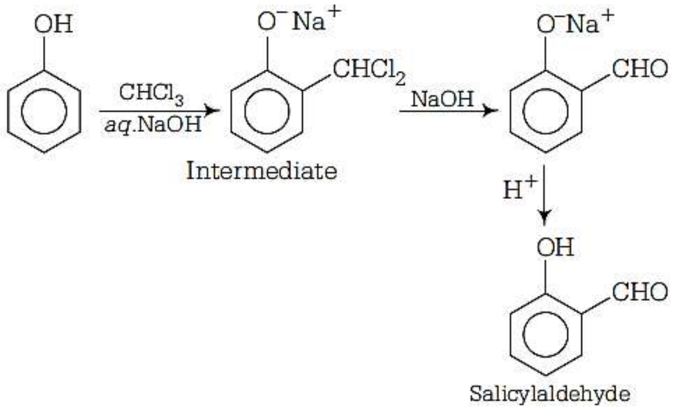
64. (b) The correct structure of sulpha pyridine is as follows:



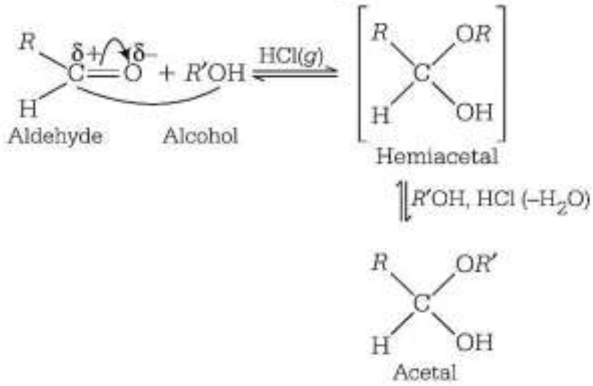
65. (c)



66. (d) In Reimer-Tiemann reaction, on treating phenol with chloroform in the presence of sodium hydroxide, a -CHO group is introduced at ortho position of benzene ring.



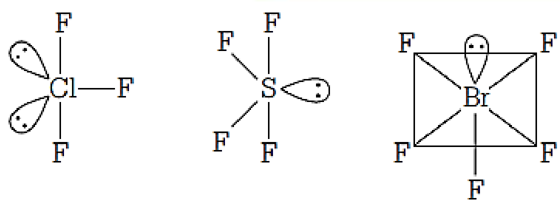
Hence, substituted benzyl chloride is intermediate in Reimer-Tiemann reaction.
67. (d) Acetal are organic compounds formed by addition of alcohol molecules to aldehyde molecules.



68. (b)
69. (b)
70. (a) Number of radial nodes $= n - 1 - 1$
For 3p-orbital, $n = 3, 1 = 1$
$\therefore$ Number of radial nodes $= 3 - 2 - 1 = 0$
71. (a)

Name	Frequency (in Hz)
X-rays	10^{17} to 10^{21}
Radiowaves	10^{5} to 10^{8}
UV rays	10^{15} to 10^{16}
IR rays	10^{12} to 10^{13}

72. (a) When an electron is added to a gaseous atom in its ground state to convert it into a negative ion, the enthalpy change accompanying the process is called electron gain enthalpy $(\Delta e_g H)$.
It is a direct measure of the ease with which an atom attracts an electron to form anion.
$X(g) + e^- \longrightarrow X^-(g); \Delta e_g H$
73. (b)
${}_{26}Fe = 1s^2, 2s^2 2p^6, 3s^2 3p^6, 4s^2, 3d^6$
${}_{26}Fe \longrightarrow {}_{26}Fe^{2+}$
Hence, atomic number of iron (Fe) is 26 and Fe^{2+} have 4 unpaired electrons in 3d-orbital.
74. (d)



ClF_3
3- bond pairs
2- lone pairs

SF_3
4- bond pairs
1- lone pair

BrF_5
5- bond pairs
1- lone pair

75. (a)
$N_2(14) = 1s^2, \overset{*}{\sigma}1s^2, \sigma 2s^2, \overset{*}{\sigma}2s^2, \pi 2p_x^2 \approx 2p_y^2, \sigma 2p_z^2$
Bond order $= \frac{N_b - N_a}{2} = \frac{10-4}{2} = 3$
76. (b)

List I	List II

(A) Viscosity	(V)	$kgm^{-1}\ s^{-1}$
(B) Ideal gas behaviour	(III)	Compressibility factor
(C) Liquefaction of gases	(I)	Critical temperature
(D) Charles' law	(II)	Isobars

77. (c) Most probable speed, $v_{mp} = \sqrt{\frac{2RT}{M}}$

$= \sqrt{\frac{2RT}{32}}$ (for oxygen, $M = 32$)

$= \sqrt{\frac{RT}{16}}$

78. (b) According to significant figure convention, during adding, the number of decimal places in the answer should not exceed the number of decimal places in either the numbers.

12.11
18.0
11.012

31.122

18.0 has only one digit after the decimal point, thus the result should be reported only upto one digit after the decimal point which is 31.1 .

79. (c)

Element	% amount	At.wt.	No. of moles	Simple molar ratio
C	52.14	12	$\frac{52.14}{12} = 4.34$	$\frac{4.34}{2.17} = 2$
H	13.13	1	$\frac{13.13}{1} = 13.13$	$\frac{13.13}{2.17} = 6$
O	34.73	16	$\frac{34.73}{16} = 2.17$	$\frac{2.17}{2.17} = 1$

$\therefore$ Empirical formula $= C_2H_6O$
Empirical formula mass $= 2(12) + 6(1) + 1(16)$
$= 46$
Molecular mass $= 46$

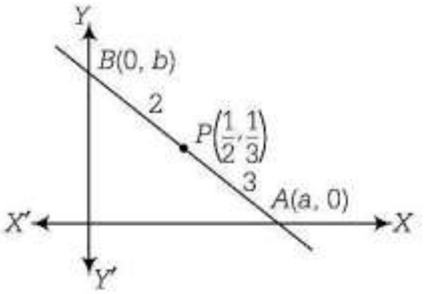
$\because n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}} = \frac{46}{46} = 1$

$\therefore$ Molecular formula = (empirical formal)n
$= (C_2H_6O)_1$
$= C_2H_6O$

80. (b) The variable depending only upon the initial and final states of a system but not on the path followed are called state functions, e.g. internal energy, entropy and free energy. Work done is a path function.

Mathematics

81. (c) Let A and B be the points on the coordinate axes, where the line intersect.



Since, P divides the line segment BA in the ratio 2: 3.
Then, the coordinates of point P will be
$\left(\frac{2a+0}{2+3}, \frac{0+3b}{2+3}\right)$ i.e. $\left(\frac{2a}{5}, \frac{3b}{5}\right)$
But coordinates of P are $\left(\frac{1}{2}, \frac{1}{3}\right)$.
$\therefore \frac{2a}{5} = \frac{1}{2}$ and $\frac{3b}{5} = \frac{1}{3} \Rightarrow a = \frac{5}{4}$
and $b = \frac{5}{9}$
So, equation of line will be

$$\frac{x}{\frac{5}{4}} + \frac{y}{\frac{5}{9}} = 1$$

$$\Rightarrow 4x + 9y = 5$$

82. **(c)** Slopes of lines $4x - y + 7 = 0$ and $kx - 5y - 9 = 0$ are 4 and $\frac{k}{5}$, respectively.

Let $m_1 = 4$ and $m_2 = \frac{k}{5}$

Then, angle between two lines whose slopes are m_1 and m_2, is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\therefore \tan 45^\circ = \left|\frac{4 - \frac{k}{5}}{1 + 4 \cdot \frac{k}{5}}\right|$$

$$\Rightarrow 1 = \left|\frac{20-k}{5+4k}\right|$$

$$\Rightarrow \frac{20-k}{5+4k} = \pm 1$$

$$\Rightarrow \frac{20-k}{5+4k} = 1 \text{ or } \frac{20-k}{5+4k} = -1$$

$$\Rightarrow$$

$$\Rightarrow 25 - k = 5 + 4k \text{ or } 20 - k = -5 - 4k$$

$$\Rightarrow k = 3 \text{ or } -\frac{25}{3}$$

But $k > 0$
$\therefore k = 3$

83. **(a)** Let the intercept of the line on coordinate axes be a and b.

Then, $\Rightarrow b = -(1 + a)$

b = - (1 +a)

$\therefore$ Equation of line will be
$\frac{x}{a} - \frac{y}{1+a} = 1$ …. (i)
Since, above line passes through (4,3).

$\therefore \quad \frac{4}{a} - \frac{3}{1+a} = 1$

$\Rightarrow \quad 4 + 4a - 3a = a(1+a)$

$\Rightarrow \quad 4 + a = a + a^2$

$\Rightarrow \quad a^2 = 4$

$\Rightarrow \quad a = \pm 2$

On putting $a = 2$ or -2 in Eq. (i), we get

$\frac{x}{2} - \frac{y}{3} = 1$ or $\frac{x}{-2} - \frac{y}{-1} = 1$

$\Rightarrow 3x - 2y - 6 = 0$ or $x - 2y + 2 = 0$

Hence, combined equation of the straight lines is

$(3x - 2y - 6)(x - 2y + 2) = 0$

84. (c) Let the new coordinates of the point be (X, Y).

Since, the origin is shifted to $(-\sqrt{2}, \sqrt{2})$ and rotates anti-clockwise through an angle 45°.

So, $(h, k) = (-\sqrt{2}, \sqrt{2}), \theta = \frac{\pi}{4}$ and $(x, y) = (1, -1)$

$\therefore X = (x - h)\cos\theta + (y - k)\sin\theta$

$= (1 + \sqrt{2})\frac{1}{\sqrt{2}} + (-1 - \sqrt{2})\frac{1}{\sqrt{2}} = 0$

and $Y = -(x - h)\sin\theta + (y - k)\cos\theta$

$= -(1 + \sqrt{2})\frac{1}{\sqrt{2}} + (-1 - \sqrt{2})\frac{1}{\sqrt{2}} = -2 - \sqrt{2}$

Hence, the new coordinates are $(0, -2 - \sqrt{2})$.

85. (d) Given lines are $3x + 4y + 5 = 0$ and $9x + 12y + 7 = 0$. Both lines are parallel. Hence, locus of the point P, which is equidistant from both lines, is a straight line which is an angular bisector.

86. (c) It is given that probability of showing head is p, then probability of showing tail will be $(1 - p)$. Also, probability of showing 50 and 51 heads in tossing a coin 100 times is same.

$\therefore P(X = 50) = P(X = 51)$

$\Rightarrow {}^{100}C_{50}(p)^{50}(1 - p)^{50} = {}^{100}C_{51}(p)^{51}(1 - p)^{49}$

$\Rightarrow \frac{100!}{50!\,50!}(1 - p) = \frac{100!}{51!\,49!}p$

$\Rightarrow \frac{1 - p}{50} = \frac{p}{51}$

$\Rightarrow 51 - 51p = 50p$

$\Rightarrow p = \frac{51}{101}$

87. (b) We have, $n = 6$.

It is given that,

$4P(X = 4) = P(X = 2)$

$\Rightarrow 4 \cdot {}^6C_4 p^4 q^2 = {}^6C_2 p^2 q^4$

$[\because P(X = r) = {}^nC_r p^r q^{n-r}]$

$\Rightarrow 4 \times \frac{6 \times 5}{2 \times 1} p^2 = \frac{6 \times 5}{2 \times 1} q^2$

$\Rightarrow 4p^2 = q^2$

$\Rightarrow 2p = q$

$\Rightarrow 2p = 1 - p \ [\because q = 1 - p]$

$\Rightarrow p = \frac{1}{3}$

88. (a) Let E_1, E_2 and A be the events defined as follows :

E_1 = Selected student is a woman

E_2 = Selected student is a man

A = Student is taller than 1.8 m

We have,

$P(E_1) = \frac{60}{100}, P(E_2) = \frac{40}{100}$

and $P(A/E_1) = \frac{1}{100}, P(A/E_2) = \frac{4}{100}$

By Baye's rule, we have

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{60}{100} \times \frac{1}{100}}{\frac{60}{100} \times \frac{1}{100} + \frac{40}{100} \times \frac{4}{100}} = \frac{3}{11}$$

89. (a) We have,

$P(A/B) = 0.6, P(B/A) = 0.3, P(A) = 0.1$

We know that,

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow 0.3 = \frac{P(B \cap A)}{0.1}$$

$\Rightarrow P(B \cap A) = 0.03$

or $P(A \cap B) = 0.03$

Also, $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow 0.6 = \frac{0.03}{P(B)}$$

$\Rightarrow P(B) = 0.05$

$\therefore P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - 0.1 - 0.05 + 0.03 = 0.88$

90. (b) We have,

$P(A \cup B) = \frac{5}{6}, P(\bar{A}) = \frac{1}{4}, P(B) = \frac{1}{3}$

$\therefore P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = \frac{3}{4} + \frac{1}{3} - P(A \cap B)$

$\Rightarrow P(A \cap B) = \frac{1}{4}$

and $P(A) \cdot P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

$\therefore P(A \cap B) = P(A) \cdot P(B)$

Hence, A and B are independent events.

91. (c) We have,

$\bar{X}_A = \bar{X}_B$ and $CV_A = 4, CV_B = 2$

Let σ_A and σ_B be the standard deviations of teams A and B respectively.

Then, $CV = \frac{\sigma}{\bar{X}} \times 100$ or $\sigma = \frac{CV \times \bar{X}}{100}$

$\therefore \sigma_A = \frac{4\bar{X}_A}{100}$ and $\sigma_B = \frac{2\bar{X}_B}{100}$

$\Rightarrow \bar{X}_A = 25\sigma_A$ and $\bar{X}_B = 50\sigma_B$

But $\bar{X}_A = \bar{X}_B$

$\therefore 25\sigma_A = 50\sigma_B$

$\Rightarrow \sigma_A = 2\sigma_B$

92. (c) Since, i is repeated i times for $i = 1,2,3, \ldots, n$. Then, mean of given data will be

$$\bar{X}=\frac{1\times1+2\times2+3\times3+\cdots+n\times n}{1+2+3+\cdots+n}=\frac{\Sigma n^2}{\Sigma n}$$
$$=\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}=\frac{2n+1}{3}$$

93. (d) We have,

$a=2\hat{i}-3\hat{j}+5\hat{k}$

$b=3\hat{i}-4\hat{j}+5\hat{k}$ and $c=5\hat{i}-3\hat{j}-2\hat{k}$

Now, $a+b=5\hat{i}-7\hat{j}+10\hat{k}$

and $b+c=8\hat{i}-7\hat{j}+3\hat{k}$ and $c+a=7\hat{i}-6\hat{j}+3\hat{k}$

∴ Volume of the parallelopiped with coterminous edges $a+b, b+c, c+a=[a+bb+cc+a]$

$$=\begin{vmatrix}5 & -7 & 10\\ 8 & -7 & 3\\ 7 & -6 & 3\end{vmatrix}$$
$$=5(-21+18)+7(24-21)+10(-48+49)$$
$$=-15+21+10=16$$

94. (d) Equations of given lines in vector form can be written as

$L_1: \mathbf{r}=3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-2\hat{\mathbf{k}}+\lambda(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$

and $L_2: \mathbf{r}=\hat{\mathbf{i}}-7\hat{\mathbf{j}}-2\hat{\mathbf{k}}+\mu(\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

Here,

$\mathbf{a}_1=3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-2\hat{\mathbf{k}}, \mathbf{a}_2=\hat{\mathbf{i}}-7\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

$\mathbf{b}_1=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}, \mathbf{b}_2=\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

Now, $\mathbf{a}_2-\mathbf{a}_1=-2\hat{\mathbf{i}}-11\hat{\mathbf{j}}+0\hat{\mathbf{k}}$

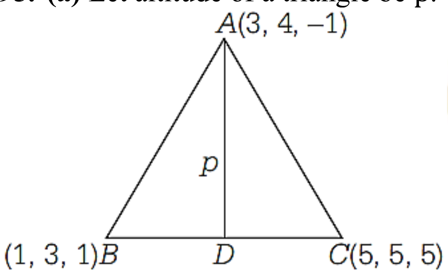
Also, $\mathbf{b}_1\times\mathbf{b}_2=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -1 & 2 & 1\\ 1 & 3 & 2\end{vmatrix}=\hat{\mathbf{i}}+3\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

Now, $|\mathbf{b}_1\times\mathbf{b}_2|=\sqrt{1+9+25}=\sqrt{35}$

∴ Distance between the above two skew-lines

$$=\left|\frac{(\mathbf{a}_2-\mathbf{a}_1)\cdot(\mathbf{b}_1\times\mathbf{b}_2)}{|\mathbf{b}_1\times\mathbf{b}_2|}\right|$$
$$=\frac{(-2\hat{\mathbf{i}}+11\hat{\mathbf{j}}+0\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+3\hat{\mathbf{j}}-5\hat{\mathbf{k}})}{\sqrt{35}}$$
$$=\left|\frac{-2-33+0}{\sqrt{35}}\right|=\sqrt{35}$$

95. (a) Let altitude of a triangle be p.



Now, AB $=-2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

AC $=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+6\hat{\mathbf{k}}$

BC $=4\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

We know that,

Area of $\triangle ABC=\frac{1}{2}|\mathbf{AB}\times\mathbf{AC}|=\frac{1}{2}|BC|\times p \ldots\ldots \text{(i)}$

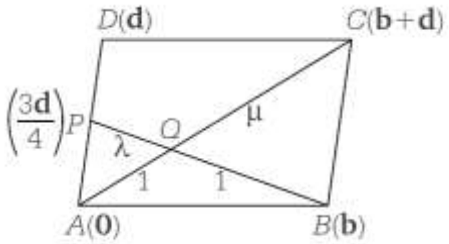
Now, $\mathbf{AB}\times\mathbf{AC}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -2 & -1 & 2\\ 2 & 1 & 6\end{vmatrix}=-8\hat{\mathbf{i}}+16\hat{\mathbf{j}}+0\hat{\mathbf{k}}$

From Eq. (i),

$$\frac{1}{2}\sqrt{(-8)^2+(16)^2+(0)^2}=\frac{1}{2}\sqrt{4^2+2^2+4^2}\times p$$

$$\Rightarrow\ p=\frac{\sqrt{320}}{6}=\frac{8\sqrt{5}}{6}=\frac{4\sqrt{5}}{3}$$

96. (a) Since, P divides AD in the ratio $3:1$, so P will be $\frac{3d}{4}$.



Let Q divides BP in $1:\lambda$ and AC in $1:\mu$.

$$\therefore\ \frac{\mathrm{b}+\mathrm{d}}{1+\mu}=\frac{\frac{3\mathrm{d}}{4}+\lambda\mathrm{b}}{1+\lambda}$$

$$\Rightarrow \mathrm{b}(1+\lambda)+(1+\lambda)\mathrm{d}=\frac{3}{4}(1+\mu)\mathrm{d}+\lambda(1+\mu)\mathrm{d}$$

On equating the vectors b and d, we get

$$1+\lambda=\frac{3}{4}(1+\mu)\text{ and }1+\lambda=\lambda(1+\mu)$$

$$\Rightarrow\ \lambda=\frac{3}{4}\text{ and }\mu=\frac{4}{3}$$

$$\therefore\ AQ:QC=1:\mu$$

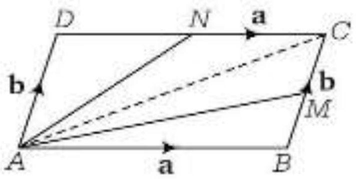
$$=1:\frac{4}{3}=3:4$$

97. (c) Since, M and N are the mid-points of BC and CD.

$$\therefore\qquad \mathrm{BM}=\frac{1}{2}\mathrm{b}$$

$$\text{and}\qquad \mathrm{DN}=\frac{1}{2}\mathrm{a}$$

Now, in $\triangle$ ABM,



$$\mathrm{AM}=\mathrm{AB}+\mathrm{BM}$$

$$=\mathrm{a}+\frac{1}{2}\mathrm{b}$$

and in $\triangle$ ADN,

$$\mathrm{AN}=\mathrm{AD}+\mathrm{DN}$$

$$=\mathrm{b}+\frac{1}{2}\mathrm{a}$$

$$\therefore\ \mathrm{AM}+\mathrm{AN}=\mathrm{a}+\frac{1}{2}\mathrm{b}+\mathrm{b}+\frac{1}{2}\mathrm{a}$$

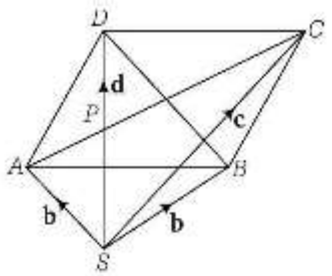
$$=\frac{3}{2}\mathrm{a}+\frac{3}{2}\mathrm{b}=\frac{3}{2}(\mathrm{a}+\mathrm{b})$$

$$\Rightarrow\ \mathrm{AM}+\mathrm{AN}=\frac{3}{2}\mathrm{AC}\ [\because \mathrm{AC}=\mathrm{a}+\mathrm{b}]$$

98. (b) Taking S as origin, let the position vectors of A, B, C and D be a, b, c and d respectively.

In $\triangle$ SAC, P is the mid-point of AC.

$$\therefore\ \mathrm{SA}+\mathrm{SC}=2\mathrm{SP}\ \ldots.\ \text{(i)}$$



In $\triangle$ SBD, P is the mid-point of BC.

$\therefore$ $SB + SD = 2SP$ .... (ii)

On adding Eqs. (i) and (ii), we get

$SA + SB + SC + SD = 4SP$

But it is given, $SA + SB + SC + SD = \lambda SP$

On comparing, we get $\lambda = 4$

99. **(a)** We have,

$$r_1 = 2r_2 = 3r_3$$

$$\Rightarrow \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c} = \lambda$$

$$\Rightarrow s-a = \frac{\Delta}{\lambda}, s-b = \frac{2\Delta}{\lambda}, s-c = \frac{3\Delta}{\lambda}$$

$$\Rightarrow s-a+s-b+s-c = \frac{\Delta}{\lambda} + \frac{2\Delta}{\lambda} + \frac{3\Delta}{\lambda} = \frac{6\Delta}{\lambda}$$

$$\Rightarrow s = \frac{6\Delta}{\lambda}$$

Now, $s-a = \frac{\Delta}{\lambda} \Rightarrow \frac{6\Delta}{\lambda} - a = \frac{\Delta}{\lambda} \Rightarrow a = \frac{5\Delta}{\lambda}$

$$s-b = \frac{2\Delta}{\lambda} \Rightarrow \frac{6\Delta}{\lambda} - b = \frac{2\Delta}{\lambda} \Rightarrow b = \frac{4\Delta}{\lambda}$$

$$s-c = \frac{3\Delta}{\lambda} \Rightarrow \frac{6\Delta}{\lambda} - c = \frac{3\Delta}{\lambda} \Rightarrow c = \frac{3\Delta}{\lambda}$$

$$\therefore\ b:c = \frac{4\Delta}{\lambda}:\frac{3\Delta}{\lambda}$$

$$\Rightarrow\ b:c = 4:3$$

100. **(b)** We have,

$$\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2}$$

$$= \frac{s^2}{\Delta^2} + \frac{(s-a)^2}{\Delta^2} + \frac{(s-b)^2}{\Delta^2} + \frac{(s-c)^2}{\Delta^2}$$

$$= \frac{s^2 + (s-a)^2 + (s-b)^2 + (s-c)^2}{\Delta^2}$$

$$= \frac{4s^2 - 2s(a+b+c) + a^2 + b^2 + c^2}{\Delta^2}$$

$$= \frac{4s^2 - 2s(2s) + a^2 + b^2 + c^2}{\Delta^2}$$

$$= \frac{4s^2 - 4s^2 + a^2 + b^2 + c^2}{\Delta^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$$

101. **(d)** Since, A, B, C are in an AP , so $A = 90°, B = 60°$ and $C = 30°$.

Now, $\cos B=\dfrac{a^2+c^2-b^2}{2ac}$

$\Rightarrow\ \cos 60^\circ=\dfrac{1}{2}=\dfrac{a^2+c^2-b^2}{2ac}$

$\Rightarrow a^2+c^2-b^2=ac$

$\Rightarrow\ c^2-ac+a^2-b^2=0$

$\Rightarrow\ c=\dfrac{a\pm\sqrt{a^2-4(a^2-b^2)}}{2}$

$=\dfrac{a\pm\sqrt{4b^2-3a^2}}{2}$

102. **(d)** We have,

$\cosh 2x=199$

$\Rightarrow\ \dfrac{1+\tanh^2 x}{1-\tanh^2 x}=199$

$\Rightarrow\ 1+\tanh^2 x=199-199\tanh^2 x$

$\Rightarrow\ 200\tanh^2 x=198$

$\Rightarrow\ \tanh^2 x=\dfrac{198}{200}\Rightarrow \tanh^2 x=\dfrac{99}{100}$

$\Rightarrow\ \tanh x=\dfrac{3\sqrt{11}}{10}\Rightarrow \operatorname{coth}x=\dfrac{10}{3\sqrt{11}}$

103. **(a)** We have,

$\cos\left(\cot^{-1}\left(\frac{1}{2}\right)\right)=\cot(\cos^{-1}x)$

$\Rightarrow\ \cos\cos^{-1}\dfrac{1}{\sqrt{5}}=\cot\cot^{-1}\dfrac{x}{\sqrt{1-x^2}}$

$\Rightarrow\ \dfrac{1}{\sqrt{5}}=\dfrac{x}{\sqrt{1-x^2}}$

$\Rightarrow\ \sqrt{1-x^2}=\sqrt{5}x$

On squaring both sides, we get

$1-x^2\ =5x^2\Rightarrow 1=6x^2$

$\Rightarrow\qquad x=\pm\dfrac{1}{\sqrt{6}}$

$\therefore\qquad x=\dfrac{1}{\sqrt{6}}$ [neglecting -ve sign]

104. **(c)** We have,

$\Rightarrow\qquad \sec x\cos 5x+1=0$

$\Rightarrow\qquad \cos 5x+\cos x=0$

$\Rightarrow\qquad 2\cos 3x\cos 2x=0$

$\Rightarrow\qquad \cos 3x=0 \text{ or } \cos 2x=0$

$\Rightarrow\qquad 3x=\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{5\pi}{2},\dfrac{7\pi}{2},\dfrac{9\pi}{2},\dfrac{11\pi}{2}$

or $\qquad 2x=\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{5\pi}{2},\dfrac{7\pi}{2}$

$\Rightarrow\quad x=\dfrac{\pi}{6},\dfrac{\pi}{2},\dfrac{5\pi}{6},\dfrac{7\pi}{6},\dfrac{9\pi}{6},\dfrac{11\pi}{6}$ or $\dfrac{\pi}{4},\dfrac{3\pi}{4},\dfrac{5\pi}{4},\dfrac{7\pi}{4}$

Hence, total number of solutions is 10 .

105. **(b)** We have, $\angle C=\frac{\pi}{3}$

Now, $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \cos\dfrac{\pi}{3} = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow a^2 + b^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 = ab + c^2$

$\Rightarrow a^2 + b^2 + ac + bc = ab + c^2 + ac + bc$

[adding both sides by $ac + bc$]

$\Rightarrow a(c + a) + b(b + c) = (b + c)(c + a)$

$\Rightarrow \dfrac{a}{b + c} + \dfrac{b}{c + a} = 1$

$\Rightarrow \dfrac{a}{b + c} + 1 + \dfrac{b}{c + a} + 1 = 3$

[adding 2 on both sides]

$\Rightarrow \dfrac{a + b + c}{b + c} + \dfrac{a + b + c}{c + a} = 3$

$\Rightarrow \dfrac{1}{b + c} + \dfrac{1}{c + a} = \dfrac{3}{a + b + c}$

$\Rightarrow \dfrac{3}{a + b + c} - \dfrac{1}{a + c} = \dfrac{1}{b + c}$

106. **(b)** We have,

$A = \sin^2\theta + \cos^4\theta$

$= 1 - \cos^2\theta + \cos^4\theta$

$= 1 - \cos^2\theta(1 - \cos^2\theta)$

$= 1 - \cos^2\theta\sin^2\theta$

$= 1 - \dfrac{1}{4}\sin^2 2\theta$

We know that,

$0 \le \sin^2 2\theta \le 1$

$\Rightarrow \dfrac{-1}{4} \le \dfrac{-1}{4}\sin^2 2\theta \le 0$

$\Rightarrow 1 - \dfrac{1}{4} \le 1 - \dfrac{1}{4}\sin^2 2\theta \le 1 \Rightarrow \dfrac{3}{4} \le A \le 1$

$\therefore A \in \left[\dfrac{3}{4}, 1\right]$

107. **(d)** Real function $f(x)$ defined on $[-1,1]$.

$\therefore \quad f(5x + 4) = g(x)$ defined for

$-1 \le 5x + 4 \le 1$

$\Rightarrow \quad -5 \le 5x \le -3$ [adding -4 in each term]

$\Rightarrow \quad -1 \le x \le \dfrac{-3}{5}$ [dividing each term by 5]

$\Rightarrow \quad x \in \left[-1, \dfrac{-3}{5}\right]$

108. **(c)** We have, $f: N \to R$ and $f(1) = -1$

Also, $f(n + 1) = 3f(n) + 2$ for $n \ge 1$

Now, put $n = 1$, we get

$f(2) = 3f(1) + 2$

$= 3(-1) + 2 = -1$

Put $n = 2$, we get

$f(3) = 3f(2) + 2 = 3(-1) + 2 = -1$

So, $f(n) = -1$ for all values of n.

Hence, $f(n)$ is a constant function.

109. **(c)** Let $f(n) = n^4 - 2n^3 - n^2 + 2n - 26$

$= n^3(n-2) - n(n-2) - 26$
$= (n-2)(n^3 - n) - 26$
$= (n-2)n(n^2-1) - 26$
$= (n-2)(n-1)n(n+1) - 26$
$= 24k - 48 + 22$
[$\because$ product of four consecutive natural numbers is divisible by 24]
$= 24[k-2] + 22$
Hence, remainder is 22 .

110. **(a)**

$$A(x) = \begin{vmatrix} 1 & 2 & 3 \\ x+1 & 2x+1 & 3x+1 \\ x^2+1 & 2x^2+1 & 3x^2+1 \end{vmatrix}$$

Apply $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$, we get

$$A(x) = \begin{vmatrix} 1 & 1 & 1 \\ x+1 & x & x \\ x^2+1 & x^2 & x^2 \end{vmatrix}$$
$$= 0 \ [\because C_2 \text{ and } C_3 \text{ are identical }]$$

$$\therefore \int_0^1 A(x)dx = \int_0^1 0dx = 0$$

111. **(a)** We have,

$$\begin{vmatrix} x^2+x+1 & x+1 & 2x-3 \\ 3x^2-1 & x+2 & x-1 \\ x^2+5x+1 & 2x+3 & x+4 \end{vmatrix}$$
$$= ax^4 + bx^3 + cx^2 + dx + e$$

Put $x = 0$ on both sides, we get

$$\begin{vmatrix} 1 & 1 & -3 \\ -1 & 2 & -1 \\ 1 & 3 & 4 \end{vmatrix} = e$$

$$\Rightarrow \quad e = 1(8+3) - 1(-4+1) - 3(-3-2)$$
$$= 11 + 3 + 15 = 29$$

112. **(d)** We have,

$$D = \begin{vmatrix} 4 & 1 & 2 \\ 1 & -5 & 3 \\ 9 & -3 & 7 \end{vmatrix}$$
$$= 4(-35+9) - 1(7-27) + 2(-3+45)$$
$$= -104 + 20 + 84 = 0$$
$$D_1 = \begin{vmatrix} 5 & 1 & 2 \\ 10 & -5 & 3 \\ 20 & -3 & 7 \end{vmatrix}$$
$$= 5(-35+9) - 1(70-60) + 2(-30+100)$$
$$= -130 - 10 + 140 = 0$$
$$D_2 = \begin{vmatrix} 4 & 5 & 2 \\ 1 & 10 & 3 \\ 9 & 20 & 7 \end{vmatrix}$$
$$= 4(70-60) - 5(7-27) + 2(20-90)$$
$$= 40 + 100 - 140 = 0$$
$$\text{and } D_3 = \begin{vmatrix} 4 & 1 & 5 \\ 1 & -5 & 10 \\ 9 & -3 & 20 \end{vmatrix}$$
$$= 4(-100+30) - 1(20-90) + 5(-3+45)$$
$$= -280 + 70 + 210 = 0$$

So, the given system of equations has infinite number of solutions.

113. **(d)** We have, $\alpha = \omega + 2\omega^2 - 3$

$\Rightarrow \quad \alpha = \omega + \omega^2 + \omega^2 - 3$
$\Rightarrow \quad \alpha = -1 + \omega^2 - 3$
$\Rightarrow \quad \alpha = \omega^2 - 4$
$\Rightarrow \quad \omega^2 = \alpha + 4$
$\Rightarrow \quad (\omega^2)^3 = (\alpha + 4)^3$
$\Rightarrow \quad \omega^6 = \alpha^3 + 12\alpha^2 + 48\alpha + 64$
$\Rightarrow \quad 1 = \alpha^3 + 12\alpha^2 + 48\alpha + 64$
$\Rightarrow \quad \alpha^3 + 12\alpha^2 + 48\alpha + 3 = -60$

114. **(a)** We have,

$\Rightarrow x + x^2 = 0$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-4}}{2} \Rightarrow x = \frac{-1 \pm \sqrt{3}i}{2}$$

$\Rightarrow x = \omega, \omega^2$

It is given that, α, β are the roots of the equation $1 + x + x^2 = 0$

$\therefore \alpha = \omega, \beta = \omega^2$

Now, $\alpha^4 + \beta^4 + \alpha^{-4}\beta^{-4} = \omega^4 + \omega^8 + \omega^{-4}\omega^{-8}$

$= \omega + \omega^2 + \omega^{-12}$

$= -1 + 1 = 0 \left[\because \omega^{-12} = \frac{1}{\omega^{12}} = 1\right]$

115. **(c)** Since, α, β are the roots of the equation

$x^2 - 4x + 8 = 0$

$$\alpha\,\beta = \frac{4 \pm \sqrt{16-32}}{2} = \frac{4 \pm 4i}{2} = 2 \pm 2i$$

$$\Rightarrow \alpha, \beta = 2\sqrt{2}\left(\frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}\right)$$

$$= 2\sqrt{2}\left(\cos\frac{\pi}{4} \pm i\sin\frac{\pi}{4}\right)$$

Now, $$\alpha^{2n} = (2\sqrt{2})^{2n}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{2n}, n \in N$$

$$= (2\sqrt{2})^{2n}\left(\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}\right)$$

Similarly, $\beta^{2n} = (2\sqrt{2})^{2n}\left(\cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2}\right)$

$\therefore \alpha^{2n} + \beta^{2n} = (2\sqrt{2})^{2n} \cdot 2\cos\frac{n\pi}{2}$

$= \left(2^{3/2}\right)^{2n} \cdot 2\cos\frac{n\pi}{2}$

$\Rightarrow \alpha^{2n} + \beta^{2n} = 2^{3n+1}\cos\frac{n\pi}{2}$

116. **(a)** Since, α and β are non-real cube roots of 2 .

$\therefore \alpha = 2^{1/3}\omega$ and $\beta = 2^{1/3}\omega^2$

Now, $\alpha^6 + \beta^6 = \left(2^{1/3}\omega\right)^6 + \left(2^{1/3}\omega^2\right)^6$

$= 2^2\omega^6 + 2^2\omega^{12}$

$= 4(\omega^3)^2 + 4(\omega^3)^4$

$= 4 + 4 = 8 \quad [\because \omega^3 = 1]$

117. **(c)** We have,

$\alpha^2 + 1 = 6\alpha$ and $\beta^2 + 1 = 6\beta$

Since, α, β are the roots of the equation

$x^2 - 6x + 1 = 0$

$\therefore \quad x = \frac{\alpha}{\alpha + 1}$

$\Rightarrow \quad \alpha = \frac{x}{1 - x}$

Hence, required quadratic equation whose roots are $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$, is $\left(\frac{x}{1-x}\right)^2 - 6\left(\frac{x}{1-x}\right) + 1 = 0$

$\Rightarrow x^2 - 6x(1-x) + 1(1-x)^2 = 0$

$\Rightarrow x^2 - 6x + 6x^2 + 1 + x^2 - 2x = 0$

$\Rightarrow 8x^2 - 8x + 1 = 0$

118. **(c)** We have,

$|x|^2 - 5|x| + 4 < 0$

Put $|x| = y$

$$\begin{aligned}
\therefore\quad y^2 - 5y + 4 &< 0 \\
\Rightarrow\quad y^2 - 4y - y + 4 &< 0 \\
\Rightarrow\quad y(y-4) - 1(y-4) &< 0 \\
\Rightarrow\quad (y-4)(y-1) &< 0 \\
\Rightarrow\quad y &\in (1,4) \\
\Rightarrow\quad 1 < |x| &< 4 \\
\therefore\quad x \in (-4,-1) &\cup (1,4)
\end{aligned}$$

119. **(c)** Since, α, β, γ are the roots of the equation $x^3 + x + 10 = 0$

$\therefore\ \alpha + \beta + \gamma = 0$

Now, $\alpha_1 = \frac{\alpha+\beta}{\gamma^2} = \frac{-\gamma}{\gamma^2} = \frac{-1}{\gamma}$

Similarly, $\beta_1 = \frac{-1}{\alpha}$ and $\gamma_1 = \frac{-1}{\beta}$

$\therefore\ \alpha_1, \beta_1, \gamma_1$ are the roots of equation $f\left(\frac{-1}{x}\right) = 0$.

Now, $f\left(\frac{-1}{x}\right)$ will be $\left(-\frac{1}{x}\right)^3 + \left(-\frac{1}{x}\right) + 10 = 0$

$\Rightarrow\ 10x^3 - x^2 - 1 = 0$

$\Rightarrow\ 10\alpha_1^3 - \alpha_1^2 - 1 = 0$

$\Rightarrow\ \alpha_1^3 = \frac{1}{10}\alpha_1^2 + \frac{1}{10}$

$\Rightarrow\ \Sigma\alpha_1^3 = \frac{1}{10}\Sigma\alpha_1^2 + \Sigma\frac{1}{10}$

$\Rightarrow \Sigma\alpha_1^3 - \frac{1}{10}\Sigma\alpha_1^2 = \frac{3}{10}$

$\therefore\ (\alpha_1^3 + \beta_1^3 + \gamma_1^3) - \frac{1}{10}(\alpha_1^2 + \beta_1^2 + \gamma_1^2) = \frac{3}{10}$

120. **(a)** Since, α, β, γ are the roots of $x^3 + x^2 + x + 2 = 0$.

$$\therefore\quad \alpha + \beta + \gamma = -1$$

$$\text{Now,}\quad \frac{\alpha+\beta-2\gamma}{\gamma} = \frac{-1-\gamma-2\gamma}{\gamma} = \frac{-1-3\gamma}{\gamma}$$

$$\therefore\quad x = \frac{-1-3\gamma}{\gamma} \Rightarrow \gamma = \frac{-1}{x+3}$$

Now, the equation whose root is $\frac{-1}{x+3}$, is

$\left(\frac{-1}{x+3}\right)^3 + \left(\frac{-1}{x+3}\right)^2 + \left(\frac{-1}{x+3}\right) + 2 = 0$

$\Rightarrow -1 + (x+3) - (x+3)^2 + 2(x+3)^3 = 0$

$\Rightarrow -1 + x + 3 - x^2 - 9 - 6x + 2x^3 + 54$
$+ 18x^2 + 54x = 0$

$\Rightarrow\ 2x^2 + 17x^2 + 49\,x + 47 = 0$

$\therefore \left(\frac{\alpha+\beta-2\gamma}{\gamma}\right)\left(\frac{\beta+\gamma-2\alpha}{\alpha}\right)\left(\frac{\gamma+\alpha-2\beta}{\beta}\right) = \frac{-47}{2}$

121. **(b)** $\sum_{r=0}^{10} {}^{40-r}C_5 = {}^{30}C_5 + {}^{31}C_5 + {}^{32}C_5 + {}^{33}C_5$
$+\ldots\ldots+\ {}^{40}C_5$

$\because \quad {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\Rightarrow \quad {}^{n}C_{r-1} = {}^{n+1}C_r - {}^{n}C_r$

$\therefore \quad {}^{30}C_5 = {}^{31}C_6 - {}^{30}C_6$

$\quad {}^{31}C_5 = {}^{32}C_6 - {}^{31}C_6$

${}^{32}C_5 = {}^{33}C_6 - {}^{32}C_6$

$\vdots \qquad \vdots \qquad \vdots$

${}^{40}C_5 = {}^{41}C_6 - {}^{40}C_6$

On adding, we get

$$\sum_{r=0}^{10} {}^{40-r}C_5 = {}^{41}C_6 - {}^{30}C_6$$

122. **(c)** The number of diagonals of a regular polygon having n sides is given by $\frac{n(n-3)}{2}$.

$$\therefore \frac{n(n-3)}{2} = 35$$

$\Rightarrow n^2 - 3n - 70 = 0$

$\Rightarrow n^2 - 10n + 7n - 70 = 0$

$\Rightarrow n(n-10) + 7(n-10) = 0$

$\Rightarrow (n-10)(n+7) = 0$

$\Rightarrow n = 10, -7$

$\therefore n = 10$

123. **(c)**

$$\because x = 1 + \frac{3}{1!} \times \frac{1}{6} + \frac{3 \times 7}{2!}\left(\frac{1}{6}\right)^2 + \frac{3 \times 7 \times 11}{3!}\left(\frac{1}{6}\right)^3 + \cdots \text{ ... (i)}$$

$(1-\alpha)^{-p/q}$

$$= 1 + \left(\frac{p}{q}\right)\frac{1}{1!}(\alpha) + \frac{\frac{p}{q}\left(\frac{p}{q}+1\right)}{2!}\alpha^2 + \frac{\frac{p}{q}\left(\frac{p}{q}+1\right)\left(\frac{p}{q}+2\right)}{3!}\alpha^3 + \cdots$$

$$= 1 + \frac{p}{1!}\left(\frac{\alpha}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{\alpha}{q}\right)^2 + \frac{p(p+q)(p+2q)}{3!}\left(\frac{\alpha}{q}\right)^3 + \cdots \text{ ... (ii)}$$

On comparing Eqs. (i) and (ii), we get

$p = 3, p + q = 7, p + 2q = 11$

And $\frac{\alpha}{q} = \frac{1}{6} \Rightarrow q = 4$

And $\quad \alpha = \frac{4}{6} = \frac{2}{3}$

So, let $x = (1-\alpha)^{-p/q} = \left(1 - \frac{2}{3}\right)^{-3/4}$

$$= \left(\frac{1}{3}\right)^{-3/4} = (3)^{3/4}$$

$\therefore x^4 = 3^3 = 27$

124. **(d)** We have,

$$\frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-1/5}}{(2-3x)^4}$$

$$=\frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-1/5}}{2^4\left(1-\frac{3}{2}x\right)^4}$$

$$=\frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-1/5}\left(1-\frac{3}{2}x\right)^{-4}}{16}$$

$$=\frac{1}{16}(1-2x)(1+3x)(1+6x)(\text{neglecting higher powers})$$

$$=\frac{1}{16}(1+x)(1+6x)$$

$$=\frac{1}{16}(1+7x)$$

125. **(c)** We have,

$$\frac{1}{x^2-5x+6}=\frac{1}{(x-3)(x-2)}$$

$$=\frac{1}{x-3}-\frac{1}{x-2}$$

$$=\frac{1}{-3}\left(\frac{1}{1-\frac{x}{3}}\right)-\frac{1}{-2}\left(\frac{1}{1-\frac{x}{2}}\right)$$

$$=\frac{1}{2}\left(1-\frac{x}{2}\right)^{-1}-\frac{1}{3}\left(1-\frac{x}{3}\right)^{-1}$$

$$\therefore \text{ Coefficient of } x^n=\frac{1}{2}\left(\frac{1}{2}\right)^n-\frac{1}{3}\left(\frac{1}{3}\right)^n$$

$$=\frac{1}{2^{n+1}}-\frac{1}{3^{n+1}}$$

126. **(c)** We have,

$$3\cos A+2=0$$

$$\Rightarrow \cos A=\frac{-2}{3}$$

$$\therefore \sin A=\sqrt{1-\cos^2 A}=\sqrt{1-\frac{4}{9}}=\frac{\sqrt{5}}{3}$$

and $\tan A=\dfrac{\sin A}{\cos A}=\dfrac{\sqrt{5}/3}{-2/3}=\dfrac{-\sqrt{5}}{2}$

Since, sin A and tan A are the roots of required equation.
Hence, equation can be written as

$$x^2-(\sin A+\tan A)x+\sin A\tan A=0$$

$$\Rightarrow x^2-\left(\frac{\sqrt{5}}{3}-\frac{\sqrt{5}}{2}\right)x+\left(\frac{\sqrt{5}}{3}\right)\left(\frac{-\sqrt{5}}{2}\right)=0$$

$$\Rightarrow x^2+\frac{\sqrt{5}}{6}x-\frac{5}{6}=0$$

$$\Rightarrow 6x^2+\sqrt{5}x-5=0$$

127. **(b)**
(P)

$$(x^3+1)\frac{dy}{dx}+x^2y=3x^2$$

$$\Rightarrow \frac{dy}{dx}+\frac{x^2}{x^3+1}y=\frac{3x^2}{x^3+1}$$

$\therefore$ Integrating factor $=e^{\int \frac{x^2}{x^3+1}dx}$

$=e^{\frac{1}{3}\log(x^3+1)}=(x^3+1)^{1/3}$

(Q) $x^2\frac{dy}{dx}+3xy=x^6$

$\Rightarrow \quad \frac{dy}{dx}+\frac{3x}{x^2}y=\frac{x^6}{x^2}$

$\Rightarrow \quad \frac{dy}{dx}+\frac{3}{x}y=x^4$

(R) $(x^3+1)^2\frac{dy}{dx}+6x^2(x^3+1)y=x^2$

$$\Rightarrow \frac{dy}{dx}+\frac{6x^2(x^3+1)}{(x^3+1)^2}y=\frac{x^2}{(x^3+1)^2}$$

$$\Rightarrow \frac{dy}{dx}+\frac{6x^2}{x^3+1}y=\frac{x^2}{(x^3+1)^2}$$

$\Rightarrow$ Integrating factor $=e^{\int \frac{6x^2}{x^3+1}dx}$

$=e^{2\log(x^3+1)}=(x^3+1)^2$

(S)

$$(x^2+1)\frac{dy}{dx}+4xy=\ln x$$

$$\Rightarrow \frac{dy}{dx}+\frac{4x}{x^2+1}y=\frac{\ln x}{x^2+1}$$

$\therefore$ Integrating factor $=e^{\int \frac{4x}{x^2+1}dx}$

$=e^{2\log(x^2+1)}$

$=(x^2+1)^2$

128. **(d)** Given equation is

$xy'=2xe^{-y/x}+y$

$\frac{dy}{dx}=2e^{-y/x}+\frac{y}{x}$ …. (i)

The above equation is homogeneous differential equation.

Put $y=vx$

$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

On putting these values in Eq. (i), we get

$$v+x\frac{dv}{dx}=2e^{-v}+v$$

$\Rightarrow \quad x\frac{dv}{dx}=2e^{-v}$

$\Rightarrow \quad \int e^v dv=\int \frac{2}{x}dx$

$\Rightarrow \quad e^v=2\log x+\log C$

$\Rightarrow \quad e^{y/x}=2\log x+\log C$

$\Rightarrow \quad e^{y/x}=2\log|Cx|$

129. **(d)** We have,

$y=ax+\frac{1}{a}$ …… (i)

$\Rightarrow \frac{dy}{dx}=a$

On putting $a = \frac{dy}{dx}$ in Eq. (i), we get

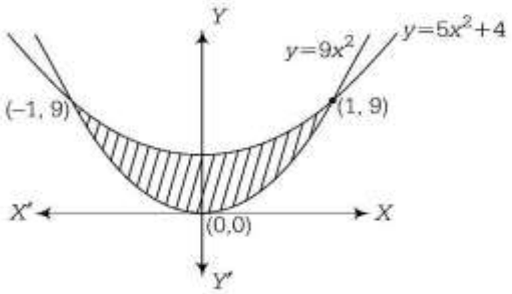
$$y = x\frac{dy}{dx} + \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$\Rightarrow y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + 1$$

which is a differential equation of degree 2.

130. **(d)** Intersecting point of $y = 9x^2$ and $y = 5x^2 + 4$ is given by

$9x^2 = 5x^2 + 4$



$\Rightarrow \quad 4x^2 = 4$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad x = \pm 1$

$\therefore \quad y = 9(\pm 1)^2 = 9$

Hence, intersecting points are $(\pm 1, 9)$.

$\therefore$ Required area $= 2\int_0^1 [(5x^2 + 4) - (9x^2)]dx$

$$= 2\int_0^1 (4 - 4x^2)dx$$

$$= 2\left[4x - \frac{4x^3}{3}\right]_0^1$$

$$= 2\left(4 - \frac{4}{3}\right) - (0 - 0) = 2 \times \frac{8}{3}$$

$= \frac{16}{3}$ sq units

131. **(c)**

$$\text{Let } I = \int_0^{\pi/2} \frac{16x\sin x\cos x}{\sin^4 x + \cos^4 x}dx \quad \ldots.(i)$$

$$= \int_0^{\pi/2} \frac{16\left(\frac{\pi}{2} - x\right)\sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)}dx$$

$$= \int_0^{\pi/2} \frac{(8\pi - 16x)\sin x\cos x}{\sin^4 x + \cos^4 x}dx \quad \ldots\ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = 8\pi\int_0^{\pi/2} \frac{\sin x\cos x}{\sin^4 x + \cos^4 x}dx$$

$$= 4\pi\int_0^{\pi/2} \frac{2\sin x\cos x}{\sin^4 x + \cos^4 x}dx$$

$$\therefore I = 2\pi\int_0^{\pi/2} \frac{2\tan x\sec^2 x}{\tan^4 x + 1}dx$$

$$= 2\pi\int_0^{\pi/2} d\{\tan^{-1}(\tan^2 x)\}$$

$$\therefore I = \pi^2$$

132. **(a)**

Let $I = \int_0^1 \sqrt{\frac{1-x}{1+x}}dx$

$= \int_0^1 \sqrt{\frac{1-x}{1+x}} \times \sqrt{\frac{1-x}{1-x}}dx$

$= \int_0^1 \frac{1-x}{\sqrt{1-x^2}}dx$

$= \int_0^1 \frac{1}{\sqrt{1-x^2}}dx - \int_0^1 \frac{x}{\sqrt{1-x^2}}dx$

$= [\sin^{-1}x]_0^1 + \left[\sqrt{1-x^2}\right]_0^1$

$= (\sin^{-1}1 - \sin^{-1}0) + (\sqrt{1-1} - \sqrt{1-0})$

$= \frac{\pi}{2} - 1$

133. **(a)** Let $I = \int \frac{x+5}{x^2+4x+5}dx$

Put $x + 5 = \lambda(2x + 4) + \mu$

On comparing both sides, we get

$1 = 2\lambda$ and $5 = 4\lambda + \mu$

$\Rightarrow \lambda = \frac{1}{2}$ and $\mu = 3$

$\therefore\ I = \frac{1}{2}\int \frac{2x+4}{x^2+4x+5}dx + 3\int \frac{dx}{x^2+4x+5}$

$= \frac{1}{2}\log(x^2+4x+5) + 3\int \frac{dx}{(x+2)^2+(1)^2}$

$= \frac{1}{2}\log(x^2+4x+5) + 3\tan^{-1}(x+2) + C$

$\therefore\ a = \frac{1}{2}, b = 3, k = 2$

134. **(b)** Let $I = \int \sqrt{e^x - 4}dx$

Put $e^x - 4 = t^2$

$\Rightarrow e^x dx = 2tdt$

$\Rightarrow\ dx = \frac{2t}{t^2+4}dt$

$\therefore\ I = \int t \cdot \frac{2t}{t^2+4}dt$

$= 2\int \frac{t^2}{t^2+4}dt = 2\int \frac{t^2+4-4}{t^2+4}dt$

$= 2\left[\int 1 - \frac{4}{t^2+(2)^2}dt\right]$

$= 2\left[t - 4 \cdot \frac{1}{2}\tan^{-1}\frac{t}{2}\right] + C$

$= 2t - 4\tan^{-1}\frac{t}{2} + C$

$= 2\sqrt{e^x-4} - 4\tan^{-1}\left(\frac{\sqrt{e^x-4}}{2}\right) + C$

135. **(d)** Let $I = \int e^{-x}\tan^{-1}(e^x)dx$

Put $e^x = t$

$\Rightarrow\ e^x dx = dt$

$\Rightarrow\ dx = \frac{1}{e^x}dt \Rightarrow dx = \frac{1}{t}dt$

$\therefore I = \int \frac{\tan^{-1} t}{t^2} dt$

$= -\frac{1}{t}\tan^{-1} t - \int \frac{-1}{t} \cdot \frac{1}{1+t^2} dt$

using integration by parts]

$= \frac{-1}{t}\tan^{-1} t + \int \left(\frac{1}{t} - \frac{t}{1+t^2}\right) dt$

$= \frac{-1}{t}\tan^{-1} t + \log t - \frac{1}{2}\log(1+t^2) + C$

$= -e^{-x}\tan^{-1}(e^x) + \log(e^x) - \frac{1}{2}\log(1+e^{2x}) + C$

$\therefore f(x) = -e^{-x}\tan^{-1}(e^x) + x = x - e^{-x}\tan^{-1}(e^x)$

136. **(d)** Let $I = \int \sqrt{\frac{2+x}{2-x}} dx$

$= \int \sqrt{\frac{2+x}{2-x}} \times \sqrt{\frac{2+x}{2+x}} dx = \int \frac{2+x}{\sqrt{4-x^2}} dx$

$= 2\int \frac{1}{\sqrt{(2)^2 - x^2}} dx + \int \frac{x}{\sqrt{4-x^2}} dx$

$= 2\sin^{-1}\frac{x}{2} - \sqrt{4-x^2} + C$

137. **(a)** Here,

$PQ = \sqrt{(t+1-t)^2 + (t^3 - 6t - 6 - t^3 + 16t + 3)^2}$

$= \sqrt{1 + (10t-3)^2}$

$\therefore PQ^2 = 1 + (10t-3)^2 \geq 1$

Hence, minimum value of PQ is 1.

138. **(d)** We have,

$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \end{cases}$

$\text{LHL} = \lim_{x \to 1^-} x$

$= \lim_{h \to 0}(1-h) = 1$

$\text{RHL} = \lim_{x \to 1^+}(2-x) = \lim_{h \to 0^+}\{2-(1+h)\} = 1$

and

$f(1) = 2 - 1 = 1$

$\therefore \text{LHL} = \text{RHL} = f(1)$

Hence, $f(x)$ is continuous.

Also, $f'(1^-) = 1$ and $f'(1^+) = -1$

$\therefore f'(1^-) \neq f'(1^+)$

Hence, $f(x)$ is not differentiable at $x = 1$.

So, Rolle's theorem is not applicable as $f(x)$ is not differentiable at $x = 1$

139. **(c)** Let A be the area and x be the side of an equilateral triangle.

$\therefore A = \frac{\sqrt{3}}{4}x^2$

on differentiating both sides w.r.t.x, we get

$\Rightarrow \frac{dA}{dx} = \frac{\sqrt{3}}{4}(2x)$

$\Rightarrow dA = \frac{\sqrt{3}}{2}x \cdot dx$ or $\Delta A = \frac{\sqrt{3}}{2}x\Delta x$

$\therefore$ Percentage error in area $= \frac{\Delta A}{A} \times 100$

$$= \frac{\frac{\sqrt{3}}{2} x\Delta x \times 100}{\frac{\sqrt{3}}{4} x^2} = \frac{2\Delta x}{x} \times 100$$

$$= \frac{2 \times 0.05}{10} \times 100 = 1\% \; [\because \Delta x = 0.05, \text{ given }]$$

140. **(a)** The slope of line $y = -4x + b$ is $m = -4$.
Also, slope of tangent of the curve $y = \frac{1}{x}$ is given by $\frac{dy}{dx} = \frac{-1}{x^2}$.
Since, given line is tangent to the curve.

$$\therefore \quad \frac{-1}{x^2} = -4 \Rightarrow x^2 = \frac{1}{4}$$

$$\Rightarrow \quad x = \pm\frac{1}{2} \text{ or } y = \pm 2$$

Put these values in $y = -4x + b$, we get $b = \pm 4$.

141. **(b)** We have,

$$x = \frac{1 - \sqrt{y}}{1 + \sqrt{y}}$$

$$\Rightarrow \quad x(1 + \sqrt{y}) = 1 - \sqrt{y}$$

$$\Rightarrow \quad x + x\sqrt{y} = 1 - \sqrt{y}$$

$$\Rightarrow \quad x + (x+1)\sqrt{y} = 1$$

On differentiating both sides w.r.t. x, we get

$$1 + \frac{(x+1)}{2\sqrt{y}} y_1 + \sqrt{y = 0}$$

$$\Rightarrow \quad (x+1)y_1 + 2y = -2\sqrt{y}$$

Again differentiating both sides w.r.t. X, we get

$$(x+1)y_2 + y_1 + 2y_1 = \frac{-2}{2\sqrt{y}} y_1$$

$$\Rightarrow \quad (x+1)y_2 + 3y_1 + \frac{1}{\sqrt{y}} y_1 = 0$$

$$\Rightarrow \quad (x+1)y_2 + \left(\frac{3\sqrt{y} + 1}{\sqrt{y}}\right) y_1 = 0$$

142. **(b)** We have,

$$x^2 + y^2 = t + \frac{2}{t} \text{ and } x^4 + y^4 = t^2 + \frac{4}{t^2}$$

$$\text{Now, } x^2 + y^2 = t + \frac{2}{t}$$

On squaring both sides, we get

$$x^4 + y^4 + 2x^2y^2 = t^2 + \frac{4}{t^2} + 4$$

$$\Rightarrow \quad x^4 + y^4 + 2x^2y^2 = x^4 + y^4 + 4$$

$$\Rightarrow \quad 2x^2y^2 = 4 \Rightarrow x^2y^2 = 2$$

$$\Rightarrow \quad y^2 = \frac{2}{x^2}$$

Now, differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = \frac{-4}{x^3} \Rightarrow x^3 y \frac{dy}{dx} = -2$$

143. **(d)** We have,

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) + \tan^{-1}\left(\frac{4x - 4x^3}{1 - 6x^2 + x^4}\right)$$

Put $x = \tan\theta$, we get

$$y=\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)+\tan^{-1}\left(\frac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}\right)$$
$$=\tan^{-1}\tan 3\theta+\tan^{-1}\tan 4\theta$$
$$=3\theta+4\theta$$
$$\Rightarrow\ y=7\theta$$
$$\therefore\ y=7\tan^{-1}x$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{7}{1+x^2}$$

144. **(a)** Since, $f(x)$ is continuous at $x=0$.

$$\therefore\ f(0)=\lim_{x\to0}f(x)$$
$$=\lim_{x\to0}(x+1)^{\cot x}=\lim_{x\to0}(1+x)^{\cot x}$$
$$=e^{\lim_{x\to0}x\cdot\cot x}=e^{\lim_{x\to0}\frac{x}{\tan x}}=e^1=e$$

145. **(a)** $\lim_{x\to0}\left[\tan\left(x+\frac{\pi}{4}\right)\right]^{1/x}$

$$=\lim_{x\to0}\left[1+\tan\left(x+\frac{\pi}{4}\right)-1\right]^{1/x}$$
$$=e^{\lim_{x\to0}\left[\tan\left(x+\frac{\pi}{4}\right)-1\right]\frac{1}{x}}$$
$$=e^{\lim_{x\to0}\left[\frac{\tan x+1}{1-\tan x}-1\right]\frac{1}{x}}$$
$$=e^{\lim_{x\to0}\frac{\tan x+1-1+\tan x}{x(1-\tan x)}}$$
$$=e^{\lim_{x\to0}\frac{2\tan x}{x(1-\tan x)}}=e^2$$

146. **(b)** Let the equation of the plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Since, plane meets the coordinate axes at P, Q, R respectively. So, the coordinates of the points P, Q, R are (a, 0,0), (0, b, 0), (0,0, c) respectively.

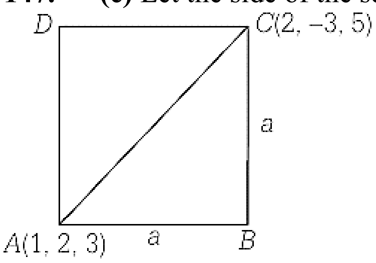
Also, centroid of $\triangle$ PQR is $\left(1,\frac{1}{2},\frac{1}{3}\right)$.

$$\therefore\quad \frac{a+0+0}{3}=1\Rightarrow a=3$$
$$\frac{0+b+0}{3}=\frac{1}{2}\Rightarrow b=\frac{3}{2}$$
$$\text{and}\quad \frac{0+0+c}{3}=\frac{1}{3}\Rightarrow c=1$$

$\therefore$ Equation of plane is $\frac{x}{3}+\frac{y}{3/2}+\frac{z}{1}=1$

$\Rightarrow\ x+2y+3z=3$

147. **(c)** Let the side of the square be a.



Then, $AC=\sqrt{(2-1)^2+(-3-2)^2+(5-3)^2}$

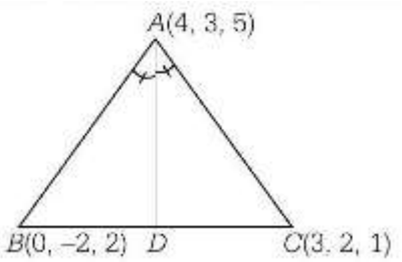
$=\sqrt{1+25+4}=\sqrt{30}$

Now, $AB^2+BC^2=AC^2$

$\Rightarrow a^2+a^2=30\Rightarrow 2a^2=30$

$\Rightarrow a^2=15\Rightarrow a=\sqrt{15}$ units

148. **(a)** We know that angle bisector of $\angle BAC$ meets BC at D which divides BC in the ratio AB: AC.



i.e. $\frac{BD}{CD} = \frac{AB}{AC}$

Now, $AB = \sqrt{(0-4)^2 + (-2-3)^2 + (2-5)^2}$

$= \sqrt{16+25+9} = \sqrt{18} = 5\sqrt{2}$

$AC = \sqrt{(3-4)^2 + (2-3)^2 + (1-5)^2}$

$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

$\therefore \frac{BD}{CD} = \frac{AB}{AC} = \frac{5\sqrt{2}}{3\sqrt{2}}$

$\Rightarrow \frac{BD}{CD} = \frac{5}{3}$

Hence, D divides BC in the ratio 5: 3.

So, coordinates of D

$= \left(\frac{5\times3+3\times0}{5+3}, \frac{5\times2+3(-2)}{5+3}, \frac{5\times1+3\times2}{5+3}\right)$

$= \left(\frac{15}{8}, \frac{4}{8}, \frac{11}{8}\right)$

149. **(c)** We know that, the product of lengths of perpendicular from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is $\frac{a^2b^2}{a^2+b^2}$.

Given, equation of hyperbola is

$x^2 - y^2 = 16$

$\Rightarrow \frac{x^2}{(4)^2} - \frac{y^2}{(4)^2} = 1$

Here, $a = 4$ and $b = 4$

Now, $\frac{a^2b^2}{a^2+b^2} = \frac{16\times16}{16+16} = \frac{16\times16}{2\times16} = 8$

150. **(d)** We have,

$\frac{(x+y-3)^2}{9} + \frac{(x-y+1)^2}{16} = 1$

To determine the centre of ellipse,

put $x + y - 3 = 0$

$\Rightarrow x + y = 3$ ..... (i)

and $x - y + 1 = 0$

$\Rightarrow x - y = -1$ ... (ii)

On adding Eqs. (i) and (ii), we get

$x = 1$ and $y = 2$

Hence, the centre of the ellipse is (1,2).

151. **(b)** Given equation of the ellipse is

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

Here, $a = 5$ and $b = 4$

(P) Equation of the directrix corresponding to the focus $(-3,0)$ is

$x = \frac{-a}{e}$

Now, $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

$\therefore\ x=-\frac{5}{3/5}$

$\Rightarrow\ 3x=-25\Rightarrow 3x+25=0$

(Q) Equation of tangent at the vertex (0,4) is

$y=b\Rightarrow y=4$

(R) Equation of latusrectum through (3,0) is

$x=ae$

$\Rightarrow\ x=5\times\frac{3}{5}\Rightarrow x=3$

152. **(a)** Let P be a point on parabola $y^2=8x$ whose coordinates are $(2t^2,4t)$ and A is the point (1,0).

Let (x,y) be the mid-point of $P(2t^2,4t)$ and $A(1,0)$.

$\therefore\quad x=\frac{2t^2+1}{2}\ \ldots\ldots(i)$

and $\quad y=\frac{4t+0}{2}=2t\ \ldots.(ii)$

On eliminating t, we get

$x=\frac{y^2+2}{4}$

$\Rightarrow\ 4x=y^2+2$

$\Rightarrow\ y^2=4\left(x-\frac{1}{2}\right)$

Hence, the locus of the mid-point of the line segment AP is $y^2=4\left(x-\frac{1}{2}\right)$.

153. **(a)** Let $P(x,y)$ be any point on the parabola. Since, the distance of P from the focus $S(1,-1)$ is equal to its distance from the directrix.

$\therefore\ PS=PQ$ or $PS^2=PQ^2$

where, Q is the point on directrix.

$\Rightarrow\ (x-1)^2+(y+1)^2=\left(\frac{x+y+3}{\sqrt{2}}\right)^2$

$\Rightarrow 2(x^2+y^2-2x+2y+2)=(x+y+3)^2$

$\Rightarrow\ 2x^2+2y^2-4x+4y+4$

$=x^2+y^2+9+2xy+6y+6x$

$\Rightarrow\ x^2+y^2-10x-2y-2xy-5=0$

154. **(c)** The equation of the circle passing through the points of intersection of the given circles, is

$(x^2+y^2-8x-6y+21)$

$+\lambda(x^2+y^2-2x-15)=0$ (i)

If this circle passes through point (1, 2), then

$(1+4-8-12+21)+\lambda(1+4-2-15)=0$

$\Rightarrow\ 6\lambda=12$

$\Rightarrow\ \lambda=\frac{1}{2}$

On substituting $\lambda=\frac{1}{2}$ in Eq. (i), we get the equation of the required circle as

$x^2+y^2-8x-6y+21+\frac{x^2}{2}+\frac{y^2}{2}-x-\frac{15}{2}=0$

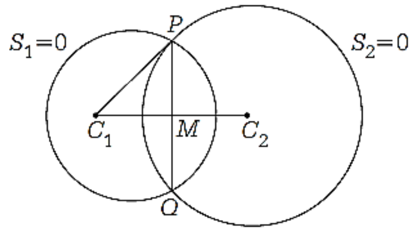
$\Rightarrow\ 2x^2+2y^2-16x-12y+42+x^2$

$+y^2-2x-15=0$

$\Rightarrow 3x^2+3y^2-18x-12y+27=0$

$x^2+y^2-6x-4y+9=0$

155. **(a)** The equation of the common chord PQ of the circles



$S_1: x^2+y^2-2ax=0$
$S_2: x^2+y^2-2by=0$
is $S_1-S_2=0$
$\Rightarrow$ $2by-2ax=0$
$\Rightarrow$ $ax-by=0$
The centre of S_1 is $(a,0)$ and radius is a.
The length of the perpendicular from $(a,0)$ to $ax-by=0$, is

$$C_1M=\left|\frac{a^2}{\sqrt{a^2+b^2}}\right|$$

Now,

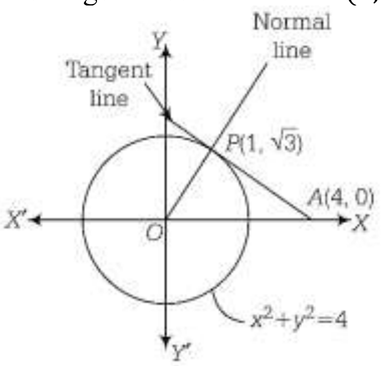
$$PQ=2PM=2\sqrt{C_1P^2-C_1M^2}$$
$$=2\sqrt{a^2-\frac{a^4}{a^2+b^2}}=\frac{2ab}{\sqrt{a^2+b^2}}$$

156. **(a)** Equation of polar of point (4,2) w.r.t. circle $x^2+y^2-5x+8y+6=0$ is $T=0$.

$$\therefore 4x+2y-\frac{5}{2}(x+4)+4(y+2)+6=0$$

Since, points (4,2) and $(k,-3)$ are conjugate points w.r.t. the given circle.
So, $(k,-3)$ satisfied line (i).
$\therefore 3k-36+8=0$
$\Rightarrow k=\frac{28}{3}$

157. **(c)** The equations of the tangent and normal to the circle $x^2+y^2=4$ at $P(1,\sqrt{3})$ are $x+\sqrt{3}y=4$ and $y=\sqrt{3}x$
The tangent meets X-axis at A(4,0).



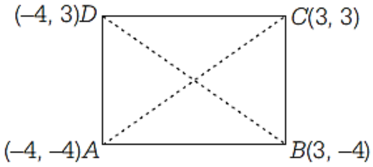
$\therefore$ Area of $\triangle$ OAP $=\frac{1}{2}\times 4\times\sqrt{3}=2\sqrt{3}$ sq units

158. **(d)** Given equation of the circle is
$x^2+y^2+4x-6y-a=0$
We know that, power of point P(1,6) w.r.t. the circle
$x^2+y^2+4x-6y-a$ is -16
$\therefore$ $1+36+4-36-a=-16$
$\Rightarrow$ $5-a=-16$
$\Rightarrow$ $a=21$

159. **(d)**

160. **(c)** Given equation $xy+4x-3y-12=0$ can be written as

$$x(y+4)-3(y+4)=0$$
$$\Rightarrow (y+4)(x-3)=0$$
Also, the equation $xy-3x+4y-12=0$ can be written as
$$x(y-3)+4(y-3)=0$$
$$\Rightarrow (y-3)(x+4)=0$$
Hence, equations of the sides of the square are
$x=3, x=-4, y=3, y=-4$
So, the coordinates of the vertex are $A(-4,-4)$, $B(3,-4)$, $C(3,3)$ and $D(-4,3)$.



$\therefore$ Equation of AC: $y+4=\frac{3+4}{3+4}(x+4)$
$$\Rightarrow y+4=x+4 \Rightarrow y=x$$
$$\Rightarrow x-y=0$$
$\therefore$ Equation BD: $y+4=\frac{3+4}{-4-3}(x-3)$
$$\Rightarrow y+4=-(x-3)$$
$$\Rightarrow y+4=-x+3$$
$$\Rightarrow x+y+1=0$$
Hence, combined equation of diagonals is
$$(x-y)(x+y+1)=0$$
$$\Rightarrow x^2+xy+x-xy-y^2-y=0$$
$$\Rightarrow x^2-y^2+x-y=0$$