Physics

1. **(b)** Electron microscope is based on the principle of wave nature of electron.
2. **(a)** From dimension logic, the quantities of same dimensions can be added or subtracted, so $[Bx] = [Dt]$ similarly,

$$\Rightarrow \left[\frac{x}{t}\right] = \left[\frac{D}{B}\right] \Rightarrow \left[\frac{D}{B}\right] = \frac{[L]}{[T]}$$

$\Rightarrow$ Dimension of $\frac{D}{B} = [LT^{-1}]$

$[L^{-1}]$ is the dimension of velocity.

3. **(a)** Given Acceleration of car$(\alpha) = 2\ m/s^2$

Distance covered by car in 20 seconds $(s) = 100\ m$

We know that,

$$s = \frac{1}{2} \cdot \frac{\alpha\beta}{\alpha+\beta} t^2$$

$$100 = \frac{1}{2} \times \frac{2\beta}{(2+\beta)} \times 20 \times 20$$

$$\frac{100}{400} = \frac{\beta}{(2+\beta)}$$

$$\Rightarrow \frac{1}{4} = \frac{\beta}{2+\beta}$$

$$\Rightarrow 4\beta = 2 + \beta$$

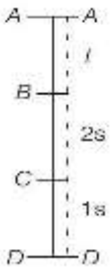
$$\Rightarrow 3\beta = 2 \Rightarrow \text{ retardation } \beta = \frac{2}{3}\ m/s^2$$

$\therefore$ Maximum velocity $(v_{max}) = \frac{\alpha\beta t}{\alpha+\beta}$

$$= \frac{2 \times \frac{2}{3} \times 20}{2 + \frac{2}{3}} = \frac{\frac{80}{3}}{\frac{8}{3}} = \frac{80}{3} \times \frac{3}{8}$$

$$= 10\ m/s$$

4. **(b)**



Let velocity of the particle at point B be v.

Now, $BC = 2v + \frac{1}{2} g \times (2)^2$

or, $BC = 2v + 2g \ \ldots\ldots (i)$

Similarly, $2BC = 3v + \frac{1}{2} \times g \times (3)^2$

$\Rightarrow 2BC = 3v + \frac{9g}{2} \ \ldots. (ii)$

From Eq. (i),

$$2(2v + 2g) = 3v + \frac{9g}{2}$$

$$\Rightarrow 4v + 4g = 3v + \frac{9g}{2}$$

$$\Rightarrow 4v - 3v = \frac{9g}{2} - 4g$$

$v = \frac{g}{2}$ ...... (iii)

From point A to B,

$$v = u + at$$

$$\Rightarrow \frac{g}{2} = 0 + g \times t$$

$$\Rightarrow t = \frac{1}{2} \text{ second}$$

$$\Rightarrow t = 0.5 \text{ s}$$

5. (c) Starting point of particle $= \hat{\imath} + 3\hat{k}$

Ending point of particle $= -3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$

force (F) $= \hat{\imath} + 5\hat{k}$

∴ Displacement of the particle

$(s) = (-3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) - (\hat{\imath} + 3\hat{k})$

$= -4\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$

We know that,

Work done, $W = F \cdot s$

or, $W = (\hat{\imath} + 5\hat{k}) \cdot (-4\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$

or, $W = -4 + 10$ or, $W = 6$ J

6. (c) Given, velocity of particle $= 2\sqrt{gh}$

Angle $(\theta) = 60°$

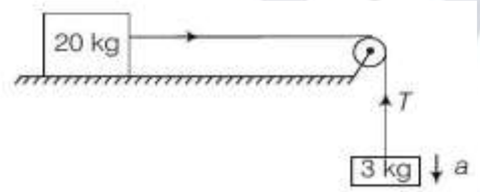
Distance covered by particle $= 2h$

Now, $2h = 2\sqrt{gh}\cos 60° t$

or, $\sqrt{h} = \sqrt{g} \times \frac{1}{2} t$

or, $t = 2\sqrt{\frac{h}{g}}$

7. (b)



Given, tension in string $(T) = 27$ N

Mass(m) $= 3$ kg

Let acceleration of the block be a.

$$3g - T = ma$$

$$\Rightarrow 10 \times 3 - 27 = 3a$$

$$\Rightarrow 30 - 27 = 3a$$

$$\Rightarrow a = \frac{3}{3} = 1 \text{ m/s}^2$$

For the body, we have,

$$27 - \mu 20g = 20a$$

$$\Rightarrow 27 - \mu \times 20 \times 10 = 20 \times 1$$

$$\Rightarrow 27 - 200\mu = 20$$

$$\Rightarrow 200\mu = 27 - 20$$

∴ Coefficient of kinetic friction,

$\mu = \frac{7}{200} = 0.035$

∴ Coefficient of kinetic friction is 0.035

8. (a)

According to the question, force, (F) is applied on m_2 mass.
So, tension in the string,
$T = m_1 \times a$
or, $T = m_1 \times \frac{F}{(m_1+m_2)}$ $\left[\because a = \frac{F}{m_1+m_2}\right]$
or, $T = \left(\frac{m_1}{m_1+m_2}\right)F$

9. (b) Given, mass of first body $(m_1) = 3$ kg
Velocity of first body $(v_1) = (2\hat{i} + 3\hat{j} + 3\hat{k})$m/s
Mass of second body $(m_2) = 4$ kg
Velocity of second body $(V_2) = (3\hat{i} + 2\hat{j} - 3\hat{k})$m/s
According to law of conservation of linear momentum,

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$\Rightarrow \quad (2\hat{i} + 3\hat{j} + 3\hat{k})3 + (3\hat{i} + 2\hat{j} - 3\hat{k})4 = (4 + 3)v$

$\Rightarrow \quad 6\hat{i} + 9\hat{j} + 9\hat{k} + 12\hat{i} + 8\hat{j} - 12\hat{k} = 7v$

$\Rightarrow \quad 18\hat{i} + 17\hat{j} - 3\hat{k} = 7v$

$\Rightarrow \quad v = \frac{1}{7}(18\hat{i} + 17\hat{j} - 3\hat{k})$

10. (a) According to the question, the bob is moving in a vertical circle, so the velocity at lowest point $v = \sqrt{5rg}$
Now, applying conservation of mechanical energy, we get

$\frac{1}{2}m \times 5rg = \frac{1}{2}mv^2 + mgr$

or, $\frac{5}{2}rg = \frac{v^2}{2} + rg$ or, $\frac{3}{2}rg = \frac{v^2}{2}$

or, $v^2 = 3rg$ .... (i)

Now, Horizontal force on the bob, $Fx = F_c = \frac{mv^2}{r}$

$= \frac{m \times 3rg}{r} = 3mg$ [from Eq. (i)]

Vertical force on the bob, $F_y = mg$

$\therefore \quad F_{net} = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(3mg)^2 + (mg)^2} = \sqrt{10}mg$

11. (c) In two particles system, if the first particle is pushed towards the centre of mass through a distanced. Then to keep the centre of mass at the same position, the second particle should be moved by the distance $= \left(\frac{m_1}{m_2}\right)$ d.

12. (a)

13. (a) Diameter of tube $= 2\sqrt{\frac{\pi}{5}}$m

$\therefore$ Radius of tube $(r) = \sqrt{\frac{\pi}{5}}$ m

Mass of solution (u) = 9 kg
We know that,

$\because$ Mass(u) = d.v

$\therefore$ $u = d\pi r^2 L$

or, $9 = 900 \times \pi \times \left(\sqrt{\frac{\pi}{5}}\right)^2 L$

or,

$9 = 900 \times \frac{\pi^2}{5} \times L$

Now, $L = \frac{5}{100\pi^2} \ldots\ldots \text{(i)}$

$T = 2\pi\sqrt{\frac{L}{2g}}$

From Eq. (i),

$T = 2\pi\sqrt{\frac{5}{200\pi^2 \times 10}}$

or, $T = 2 \times \sqrt{\frac{1}{400}} = \frac{2}{20}$

or, $T = 0.1$ s

14. (a) Mass of first body $(m_1) = 100$ kg

Mass of second body $(m_2) = 8100$ kg

Distance between two bodies = 1 m

Distance of null point from 100 kg

$x = \frac{d}{\sqrt{\frac{m_2}{m_1}} + 1}$

or $x = \frac{1}{\sqrt{\frac{8100}{100}} + 1}$

$x = \frac{1}{9+1} = \frac{1}{10}$

$= 0.1$ m

The gravitational potential at that point is given by

$= -G\left(\frac{m_1}{0.1} + \frac{m_2}{0.9}\right)$

$= -6.67 \times 10^{-11}\left(\frac{100}{10^{-1}} + \frac{8100}{9 \times 10^{-1}}\right)$

$= -6.67 \times 10^{-11}(1000 + 9000)$

$= -6.67 \times 10^{-7}$ J/kg

15. (d) According to the question,

In the first condition,

$W_1 = \frac{1}{2}kx^2 \quad \ldots\text{(i)}$

In the second condition,

$W_2 = \frac{1}{2}k(x+y)^2 \quad \ldots\text{(ii)}$

Work for x to y expansion

$$= \frac{1}{2}k(x+y)^2 - \frac{1}{2}kx^2$$
$$= \frac{1}{2}k[x^2 + y^2 + 2xy - x^2]$$
$$= \frac{1}{2}k[y^2 + 2xy]$$
$$= \frac{1}{2}ky[y + 2x]$$

16. (a) Given, radius of first soap bubble $(R_1) = 2.0$ cm
Radius of second soap bubble $(R_2) = 1.0$ cm We know that,
Radius of another bubble having same pressure difference,

$$R = \frac{R_1 R_2}{R_1 + R_2}$$
$$\text{or, } R = \frac{2.0 \times 1.0 \times 10^{-4}}{(2.0 + 1.0) \times 10^{-2}}$$
$$\text{or, } R = \frac{2 \times 10^{-4}}{3 \times 10^{-2}}$$
$$\text{or, } R = 6.67 \times 10^{-3} \text{ m}$$

17. (d) Given, area of slab of stone $(A) = 3600 \text{ cm}^2$
$= 3600 \times 10^{-4} \text{ m}^2$
Thickness of stone slab $= 10 \text{ cm} = 10 \times 10^{-2}$ m
Temperature difference $(\Delta\theta) = 100°\text{C}$
We know that,

$$\frac{Q}{t} = \frac{KA\Delta\theta}{l}$$
$$\Rightarrow \frac{4.8 \times 3.36 \times 10^5}{60 \times 60} = \frac{K \times 3600 \times 10^{-4} \times 100}{10 \times 10^{-2}}$$
$$\Rightarrow K = \frac{4.8 \times 3.36 \times 10^5 \times 10 \times 10^{-2}}{3600 \times 10^{-4} \times 100 \times 60 \times 60}$$
$$\Rightarrow K = \frac{161280}{129600}$$
$$\Rightarrow K = 1.24 \text{Js}^{-1} \text{ m}^{-1} \text{ K}^{-1}$$

18. (a) Given, Temperature of black body $(T) = 727°\text{C}$
$= 727 + 273 = 1000$ K
Cross-sectional area of black body $(A) = 1 \text{ m}^2$
Stefan's constant $(\sigma) = 5.7 \times 10^{-8} \text{ W/m}^2/\text{K}^4$
We know that,
Heat radiated per second

$$E = \sigma A T^4$$
$$= 5.7 \times 10^{-8} \times 1 \times (1000)^4$$
$$\text{or, } E = 57000 \text{ J}$$

∴ Heat radiated from this surface in one minute,
$E = 57000 \times 60$
or, $E = 3.42 \times 10^6$ or 34.2×10^5 J

19. (d) Given, In the first condition, work done $= W_1$
In the second condition, work done $= W_2$
Now, according to the question,
In the first condition,

$$W_1 = p(2V - V)$$
$$\text{or, } W_1 = pV \quad \dots\dots \text{(i)}$$

In the second condition,

$$W_2 = nRT\log_e\left(\frac{V_2}{V_1}\right)$$

or, $W_2 = pV\log_e\left(\frac{2V}{V}\right) \quad (pV = nRT)$

From Eq. (i),

$W_2 = W_1\log_e(2)$

20. **(c)** Given,

Mass of first uranium isotope = 238 units

Mass of second uranium isotope = 235 units

We know that, $v_{rms} = \sqrt{\frac{\gamma RT}{M}}$

$\Rightarrow \quad v_{rms} \propto \frac{1}{\sqrt{M}}$

So, the percentage ratio of difference in rms velocity of isotopes is given by

$$\frac{v_{rms_2} - v_{rms_1}}{v_{rms_2}} \times 100$$

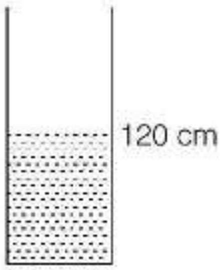
$$= \frac{\sqrt{M_1} - \sqrt{M_2}}{\sqrt{M_1}} \times 100$$

$$= \frac{\sqrt{238} - \sqrt{235}}{\sqrt{238}} \times 100$$

$$= \frac{15.427 - 15.329}{15.427} \times 100 = \frac{0.097}{15.427} \times 100$$

$$= 6.34 \times 10^{-3} \times 100 = 0.634 \approx 0.64$$

21. **(d)** The frequency of source, $f_s = 340$ Hz



Velocity of sound = 340 m/s

Length of the tube = 120 cm

The minimum height of the tube for resonance, $I_1 = \frac{v}{4f}$

$$= \frac{340}{4 \times 340} \text{ m}$$

$$= \frac{1}{4} = 0.25 \text{ m} = 25 \text{ cm}$$

From second resonance condition for closed pipe,

$h = I_2 - 3I_1$

Here, $I_2 = 120$ cm

$\Rightarrow h = 120 - 3 \times 25$

$= 120 - 75$

$= 45 \text{ cm} = 0.45 \text{ m}$

So, for second resonance, 0.45 m should be the height of water level. It will be the minimum height of water in the tube.

22. **(c)** When the convex lens of focal length f made of crown glass immersed in liquid of refractive index $\mu_l(\mu_l > \mu_c)$, here μ_c is the refractive index of crown glass, thus from lens maker's formula, we will get focal length of the lens as negative ($\because \mu_e > u_c$). Therefore, the convex lens will behave like a concave lens. So, it will be a divergent lens.

23. **(b)** The linear magnification of lens in terms of focal length f and u.

$m = \frac{f}{f+u}$

For first lens,

$u_1 = \left(\frac{1-m_1}{m_1}\right) f_1$

For second lens,

$u_2 = \left(\frac{1-m_2}{m_2}\right) f_2$

According to the question,

$$u_1 = u_2$$

$$\therefore \frac{(m_1-1)}{m_1} f_1 = \frac{(m_2-1)}{m_2} f_2$$

$$\frac{f_1}{f_2} = \frac{m_1}{m_2} \cdot \frac{(m_2-1)}{(m_1-1)}$$

$$= \frac{m_1(m_2-1)}{m_2(m_1-1)}$$

24. (b) The limit of resolution of a telescope is

$\theta = \frac{1.22\lambda}{d}$

where, λ = wavelength of light rays

d = aperture of objective lens of telescope.

Here, $\lambda = 6400 Å = 6400 \times 10^{-10}$ m

$d = 200$ cm $= 2$ m

Substitution of values gives

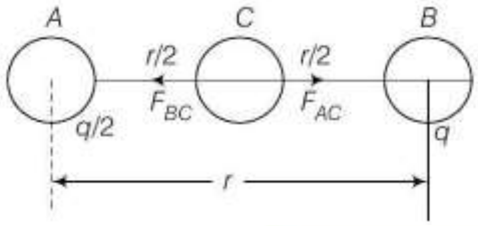
$$\theta = \frac{1.22 \times 6400 \times 10^{-10}}{2}$$

$$= \frac{1.22 \times 64 \times 10^{-8}}{2}$$

$$= 1.22 \times 32 \times 10^{-8}$$

$$= 39 \times 10^{-8} \text{rad}$$

25. (b) Spheres A and B have the same charge of same nature. When uncharged sphere C is touched with sphere A, then charge is transferred to C and this charge is equally divided between two spheres.



Let initially the charge on spheres A and B is q

Initially, the repelling force between charged spheres A and B $= 3 \times 10^{-5}$ N

When, sphere C, is placed between charged spheres A and B, then the force between A and C,

$F_{AC} = \frac{k\frac{q}{2} \times q/2}{(r/2)^2}$

Similarly, between B and C

$F_{BC} = \frac{kq \times q/2}{(r/2)^2}$

The magnitude of net force on C, $F' = F_{BC} - F_{AC}$

Also

$F = \frac{kq \times q}{r^2} = 3 \times 10^{-5}$ N

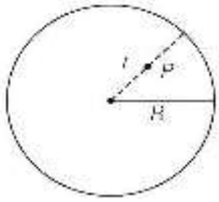
So, $\frac{F}{F_{BC}-F_{AC}} = \frac{(kq^2/r^2)}{k\left[\frac{q\times q/2}{(r/2)^2}-\frac{q/2\times q/2}{(r/2)^2}\right]}$

Here, $F = 3\times10^{-5}$ N, $F_{BC} - F_{AC} = F'$

$$\frac{3\times10^{-5}}{F'} = \frac{1}{2-1} = \frac{1}{1} = 1$$

$\therefore$ $F = 3\times10^{-5}$ N

26. (b) Let the volume charge density be ρ. The electric field inside the charged sphere,



$E = \frac{Kqr}{R^3}$

where, R = Radius of charged sphere

r = Distance of the point (P) inside the sphere

As volume charge density is ρ, then in terms of charge density

$$E = \frac{\rho r}{3\varepsilon_0}$$

$$\Rightarrow \rho = \frac{3E\varepsilon_0}{r}$$

Now, electric field, $E = -\frac{dV}{dr}$

Here, $V = Ar^2 + B$

$$\therefore E = -\frac{dV}{dr} = -2Ar + 0 = -2Ar$$

$$\therefore \rho = \frac{3E\varepsilon_0}{r}$$

$$= \frac{3(-2Ar\cdot\varepsilon_0)}{r} = -6A\varepsilon_0$$

27. (d) Let the resistances be R_1, R_2 and R_3, respectively.

According to the question, equivalent resistance in parallel = 1Ω

In parallel combination, the equivalent resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Here, $R_{eq} = 1$

$$\Rightarrow \frac{1}{1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Let $R_3 = 2R_2$

If we check the option given in the question

$$\frac{1}{1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{2R_2} = \frac{1}{R_1} + \frac{3}{2R_2}$$

If $R_1 = 1\Omega, R_2 = 2\Omega$

$$\frac{1}{1} = \left(\frac{1}{1} + \frac{3}{2\times2}\right) = \frac{1}{1} + \frac{3}{4} = \frac{7}{4}$$

will not hold the condition.

Similarly, if we check the other options only

$R_1 = 2\Omega, R_2 = 3\Omega$

$$\frac{1}{1} = \frac{1}{2} + \frac{1}{3} + \frac{1}{R_3}$$

$$= \frac{5}{6} + \frac{1}{R_3} \Rightarrow 1 - \frac{5}{6} = \frac{1}{R_3}$$

or, $\frac{1}{6} = \frac{1}{R_3}$

or, $R_3 = 6\Omega$

So, it will satisfy the equation,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or, $\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{6} = \frac{1}{1}$

or, $R_{eq} = 1\Omega$

$R_3 = 6\Omega$ will be highest resistance .

28. (d) Power, $P = \frac{V^2}{R}$

$$R = \frac{V^2}{P}$$

Here, $P = 320$ watts
$V = 220$ volts

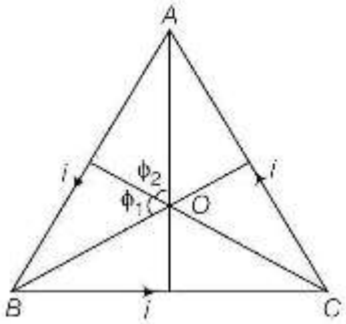
When the resistors are connected in series, then power developed across each resistor,

$$P' = \frac{(V')^2}{R} = \frac{\left(\frac{110}{2}\right)^2}{R} = \frac{110 \times 110}{2 \times 2} \times \frac{P}{V^2}$$

$$= \frac{110 \times 110}{2 \times 2} \times \frac{320}{220 \times 220}$$

$$= \frac{1}{4} \times \frac{1}{4} \times 320 = 20 \text{ watts.}$$

29. (b) Let the centroid of the triangle is O, magnetic field due to side AB



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{i}{r} (\sin\phi_1 + \sin\phi_2)$$

where, r is perpendicular distance of AB from centroid O. The magnetic field due to each side is same in magnitude and direction, so the resultant field

$B = 3B_1$

$$= 3\frac{\mu_0}{4\pi} \cdot \frac{i}{r} (\sin\phi_1 + \sin\phi_2)$$

(Here, $\mu_0 = 4\pi \times 10^{-7}$ H/m)

$i = 1$ A

In equilateral triangle, $\phi_1 = \phi_2 = 60°$

$\therefore \quad B = 3\dfrac{\mu_0}{4\pi}\cdot\dfrac{1}{r}(\sin 60^\circ + \sin 60^\circ)$

Also, $\quad r = 4.5\times 10^{-2}$ m

$$\Rightarrow \quad B = \frac{3\mu_0}{4\pi}\cdot\frac{1}{4.5\times 10^{-2}}\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)T,$$

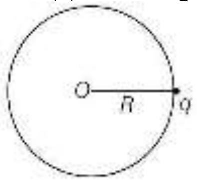
$$= \frac{3\sqrt{3}\cdot\mu_0}{4\pi}\cdot\frac{1}{4.5\times 10^{-2}}\ T$$

$$= \frac{3\sqrt{3}\times 10^{-7}}{4.5\times 10^{-2}}\ T\ \left[\because \frac{\mu_0}{4\pi} = 10^{-7}\ N/A^2\right]$$

$$= \frac{3\times 1.73}{4.5}\times 10^{-5}\ T$$

$$= 1.15\times 10^{-5}\ T \approx 2\times 10^{-5}\ T$$

30. (a) The magnetic moment of charge, $\mu = niA$



and angular momentum of charge,

$L = m\cdot v\cdot R$

Here, $n = 1$

Current,

$i = \dfrac{q}{T}$

Time-period, $T = \dfrac{2\pi R}{v}$

Equivalent current, $i = \dfrac{q}{2\pi R/v} = \dfrac{q\cdot v}{2\pi R}$

Area of loop or circle, $A = \pi R^2$

$\therefore \quad \mu = 1\times\dfrac{qv}{2\pi R}\times\pi R^2 = \dfrac{qvR}{2}$

$\Rightarrow \quad L = l\omega = mR^2\times\dfrac{v}{R} = mvR$

I = Moment of inertia of loop

$\therefore \dfrac{\mu}{L} = \dfrac{niA}{mvR} = \dfrac{qvR/2}{mvR} = \dfrac{q}{2m}$

31. (a) Let the mass of the first magnet = m

$\therefore$ Mass of second magnet = 2m

The time-period of oscillation of the magnet

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

where, I = MI of magnet about the axis passing through its centre

B_H = Earth's magnetic field

M = Magnetic moment of magnet

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{I_1}{M_1}\times\frac{M_2}{I_2}}$$

Let length of the magnet = L

MI of the magnets about the axis passing through one of the ends is $\frac{mL^2}{3}$

$\therefore\ I_1 = mL^2/3$ and $I_2 = \dfrac{(2m)(2L)^2}{3}$

$$\frac{T_1}{T_2} = \sqrt{\frac{mL^2/3}{M_1} \times \frac{M_2}{\frac{2m \times (2L)^2}{3}}}$$

It is given that maximum torques experienced by the magnets are same

$\therefore\quad M \cdot B_H = M_2 B_H \Rightarrow M_1 = M_2 [\because M_1 = M_2 \text{ (given) }]$

$\Rightarrow\ \dfrac{T_1}{T_2} = \dfrac{1}{\sqrt{8}} = \dfrac{1}{2\sqrt{2}}$

32. (b) Mutual inductance, $M = 0.005\text{H}$

$I = I_0 \sin \omega t$

$I_0 = 10\text{ A}$

$\omega = 100\pi \text{rad/s}$

The emf induced in second coil,

$$e = M\frac{dl}{dt} = M\frac{dl}{dt} = 0.005 \times \frac{dl_0 \sin \omega t}{dt}$$

$$= 0.005 \times I_0 \omega \cos \omega t = 0.005 \times I_0 \omega \cos \frac{2\pi}{t} t$$

$\Rightarrow e_{max} = 0.005 \times 10 \times 100\pi \times 1 = 5\pi \text{volt}$

33. (c) Capacitance, $C = \frac{10^{-3}}{2\pi}\text{F}$

Inductance, $L = \left(\frac{100}{\pi}\right)\text{mH} = \frac{100}{\pi} \times 10^{-3}\text{H}$

Resistance, $R = 10\Omega$

Voltage of source $= 220$ volt

Frequency of source, $f = 50$ Hz

Phase angle ϕ is given by

$\tan \phi = \dfrac{X_L - X_C}{R}$

where, $X_L =$ inductive impedance $= \omega L$

$= 2\pi f L\ (\because \omega = 2\pi f)$

$\therefore\ X_L = 2\pi \times 50 \times \dfrac{100}{\pi} \times 10^{-3}$

$= 10\Omega$

and $X_C =$ Capactive impedance

$$= \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1 \times 2\pi}{2\pi \times 50 \times 10^{-3}}\Omega = \frac{100}{5} = 20\Omega$$

$\therefore$ Phase angle,

$\tan \phi = \dfrac{X_C - X_L}{R} = \dfrac{20 - 10}{10} = 1$ or $\phi = 45^\circ$

34. (b)

(A) $\oint E \cdot d\mathbf{A} = \frac{Q}{E_0}$

It is Gauss law of electrostatic fields.

(B) $\oint B \cdot d\mathbf{A} = 0$

Because single pole of a magnet does not exist. So, it is Gauss law for magnetic fields. Option (b) is correct.

35. (b) The de-Broglie wavelength associated with a charged particle is given by

$\lambda = \dfrac{h}{\sqrt{mqV}}$

where, $h =$ planck's constant

$m =$ mass of charged particle

$q =$ charge on the particle

$V =$ potential at which the charged particle is accelerated.

$\therefore\ \lambda_1 = \frac{h}{\sqrt{m_1 q V_1}}, \lambda_2 = \frac{h}{\sqrt{m_2 q V_2}}$

Here,

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2 q V_2}{m_1 q V_1}} = \sqrt{\frac{V_2}{V_1}}$$

$$= V_2^{\frac{1}{2}} \cdot V_1^{\frac{1}{2}}$$

36. **(c)** From hydrogen spectrum, when electron transist from n_2 orbit to n_1 orbit, the emitted wavelength is given by the formula

$$\frac{1}{\lambda_L} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here, R = Rydberg constant.

For Lyman series, $n_1 = 1, n_2 = 2$

For First Lyman line,

$\frac{1}{\lambda_L} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \Rightarrow \frac{1}{\lambda_L} = \frac{3}{4} \cdot RZ^2 \quad \ldots. \text{(i)}$

For Balmer series, $n_1 = 2, n_2 = 3,4 \ldots \ldots$

For first balmer line,

$\frac{1}{\lambda_B} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = RZ^2 \cdot \frac{5}{36} \quad \ldots\ldots \text{(ii)}$

Dividing Eqs. (i) by (ii), we get

$$\frac{1/\lambda_L}{1/\lambda_B} =] \frac{3/4 \cdot RZ^2}{5/36 \cdot RZ^2} = \frac{3}{4} \times \frac{36}{5} = \frac{27}{5}$$

$$\therefore \frac{\lambda_B}{\lambda_L} = \frac{27}{5}$$

Here, $\lambda_L = 1215.4\text{Å}$

$$\lambda_B = \frac{27}{5} \times 1215.4\text{Å}$$

$$= 6563.16\text{Å} \approx 6563\text{Å}$$

37. **(a)** Decay constant, $\lambda = 7.9 \times 10^{-10}$ / second

Activity, $A = 55.3 \times 10^{11} \frac{\text{disintegration}}{\text{second}}$

Let the number of nuclei at that instant be N.

We know activity, $A = \lambda N$

$\therefore\ N = \frac{A}{\lambda} = \frac{55.3 \times 10^{11}}{7.9 \times 10^{-10}} = 7.0 \times 10^{21}$

38. **(a)** Change in current through junction diode,

$\Delta'/= 1.2\text{ mA} = 1.2 \times 10^{-3}\text{ A}$

Change in forward bias voltage, $\Delta V = 0.6$ V

Dynamic resistance,

$R_{\text{dyn.}} = \frac{\Delta V}{\Delta I} = \frac{0.6}{1.2 \times 10^{-3}} = 500\Omega$

39. **(a)** Electron concentration, $n_e = 2 \times 10^8\text{ m}^{-3}$

Hole concentration, $n_h = 2 \times 10^8\text{ m}^{-3}$

After doping with a impurity, the new electron concentration, $n_e' = 4 \times 10^{10}\text{ m}^{-3}$

New hole concentration, $n_h' = ?$

We know that, $n_e \times n_h = n_i^2$

and $n_e' \times n_h' = n_i^2$

$\therefore\quad n_e \times n_h = n_e' \times n_h'$

$$n_h' = \frac{n_e \times n_h}{n_e'} = \frac{2 \times 10^8 \times 2 \times 10^8}{4 \times 10^{10}}/\text{m}^3 = 10^6/\text{m}^3$$

$\Rightarrow\quad n_h' = 10^6\text{ m}^{-3}$

40. **(c)** Given, message signal peak voltage = 20 V

$\therefore\ E_m = 20$ V

Carrier voltage, $E_C = 30$ volt
Thus, modulation index,
$$m_a = \frac{E_m}{E_c} = \frac{20}{30} = \frac{2}{3} = 0.67$$

Chemistry

41. (a) The extra stability of half-filled and fully filled electronic configuration can be explained in terms of symmetry and exchange energy. All the orbitals of the same subshell are either completely filled or half-filled have more symmetrical distribution of electrons. Consequently, their shielding of one another is relatively small and the electrons are more strongly attracted by the nucleus. This leads to more stability of the atom.

42. (a) As the element copper has atomic number $= 29$ so, electronic configuration
$= 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^1 = [Ar] 3d^{10} 4s^1$

43. (d) Isoelectronic species are elements or ions that have the same, or equal number of electrons. Hence, $O^{2-}, F^-, Na^+, Mg^{2+}, Ne, N^{3-}, Al^{3+}$ are isoelectronic species. But Mg^+ is not isoelectronic with respect to other species.

44. (a) The atomic number of the element Uus is 117. Ununseptium (Uus) is the second heaviest known element and penultimate element of the 7 th period of the periodic table. Electronic configuration of Uus $=$ $[Rn] 5f^{14} 6d^{10} 7s^2 7p^5$

45. (c) Geometry of molecules according to VSEPR theory,
(A) PCl_3 Trigonal pyramidal
(B) BF_3 Trigonal planar
(C) CIF_3 T-shape
(D) XeF_4 Square planar

46. (d) Covalent character increases with increase in size of anion or decrease with size of cation. As, here cation (K^+)in all the species is common, hence the covalent character will increase in the order as: $KF < KCl < KI$.

47. (a) At same temperature, kinetic energy of both the gases will be same. In short
$\because \quad KE \propto \sqrt{T} \quad$ (for all gases)
$\therefore \quad KE_{(CH_4)} = KE_{(O_2)} = X$

48. (d) At lower temperatures, the molecules have less energy. Hence, the molecular speeds are lower, and distribution of molecules has smaller range. But as the temperature increases, distribution range flattens. Hence, correct order is: $T_2 > T_3 > T_1$

49. (a) The reaction proceeds as :
$N_2 + 3H_2 \rightleftharpoons 2NH_3$
28 g 6 g 34 g
$\because$ 6 gH_2 reacts with 28 g N_2.
$\therefore$ 10 gH_2 reacts with $\frac{28}{6} \times 10 = 46.67$ g N_2,
i.e. H_2 is limiting reagent and N_2 is in excess.
Now,
$\because$ 6 gH_2 produces 34 gNH_3.
$\therefore$ 10 gH_2 produces $\left(\frac{34}{6} \times 10\right)$ gNH_3.
Therefore, we can calculate number of moles of NH_3 as
$$n_{NH_3} = \frac{\text{Produced weight}}{\text{Molecular weight}}$$
$$= \frac{\left(\frac{34}{6} \times 10\right)}{17} = \frac{34}{6} \times \frac{10}{17} = \frac{10}{3} = 3.33$$

50. (b) In the reaction (in acidic medium),
$KMnO_4 + 8H^+ + 5e^- \longrightarrow K^+ + Mn^{2+} + 4H_2O$
$KMnO_4$ acts as an oxidising agent in acidic medium.
$KMnO_4$ gains 5 electrons from reductant.
$\therefore$ Equivalent weight of $KMnO_4 = \frac{158}{5}$
$= 31.6$ (in g/equivalent)

51. (d) Given,
$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g); \Delta_r H^\circ = -92$ kJ

Chemical reaction for molar enthalpy of formation of NH_3,

$$\frac{1}{2}N_2(g)+\frac{3}{2}H_2(g)\longrightarrow NH_3(g)$$

$(\because H_f^\circ \text{ for } N_2 \text{ and } H_2=0)$

Therefore, $\Delta_f H^\circ=\dfrac{-\Delta_r H^\circ}{2}=\dfrac{-92}{2}$

$=-46\text{ kJ mol}^{-1}$

52. **(b)** Correct option is (b) as (a), (c) and (d) are false.

(a) K_c depends on temperature.

(c) At equilibrium, both the rates are equal.

(d) $K_c=\dfrac{[Ni(CO)_4]}{[CO]^4}$

53. **(c)** Ammonium chloride is a salt of weak base (NH_3) and strong acid (HCl) hence, it is acidic in nature.

$NH_3+HCl\longrightarrow NH_4Cl$

pH of $NH_4Cl<7$

54. **(c)** Manganese exhibits oxidation states of $+2,+3,+4,+6$ and +7 .

So, in acidic medium on reaction of MnO_4^- with H_2O_2 it changes from 7 to 2.

55. **(d)** Beryllium having smallest atomic size in the group 2 has tightly held outermost electrons that do not get excited significantly during flame test.

56. **(b)** More the element has metallic character, more basic is its oxide. As the metallic character increases down the group hence, thallium (TI) has highest metallic character (among the given option) and forms the basic oxide. Al and Ga form amphoteric oxide and B (Boron) forms acidic oxide.

57. **(c)** Tha gas produced by passage of air over hot coke is called producer gas. The reaction proceeds as:

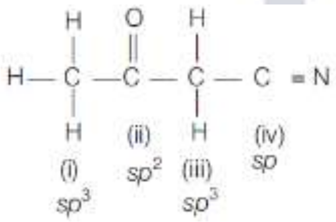
$$\underset{\text{Coke}}{2C}+\underbrace{O_2+N_2}_{\text{(From air)}}\xrightarrow{\Delta}\underbrace{2CO+N_2}_{\text{Producer gas}}$$

Other carbonaceous (carbon containing) fuels also produce producer gas of different compositions.

58. **(d)** In environmental chemistry, the medium which is affected by a pollutant is called receptors or targets.

Receptors, are the biotic or abiotic components, which affected adversely by the pollution

59. **(a)** In the compound,



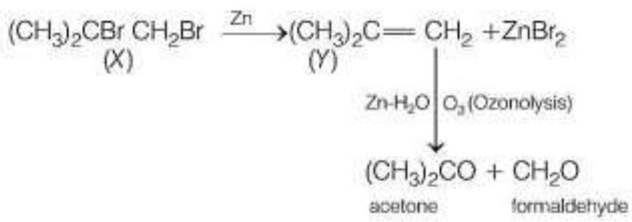
The C -atoms that is bonded to four different atoms involve four orbitals (1 of s and 3 of p - orbitals) during hybridisation. Hence, they have sp^3-hybridisation as in cases of (i) and (iii).

The C- atom bonded to three different atoms involves three orbitals (1 of s and 2 of p-orbitals). Hence, the sp^2-hybridisation in case of (ii). The C-atom bonded to two different atoms involves two orbitals (1 of s and 1 of p-orbitals). Hence, the sp - hybridisation in case of (iv).

60. **(b)** $H_3CC\equiv CH\xrightarrow{2HBr}H_3CCBr_2CH_3$

The reaction follows Markovnikov's addition.

61. **(c)** In this reaction,



62. **(d)** Packing efficiency for simple cubic

$$\frac{\frac{4\pi r^3}{3}}{a^3} \times 100 = \frac{\frac{4\pi r^3}{3}}{8r^3} \times 100 = 52.4\%$$

For body centred $= \dfrac{2 \times \frac{4}{3}\pi r^3}{\frac{64r^3}{3\sqrt{3}}} \times 100 = 68\%$

For cubic closed packing $= \frac{4\times4\pi r^3/3}{32r^3/\sqrt{2}} \times 100 = 74\%$

Hence, sc < bcc < ccp.

63. (c) Given, depression in freezing point of a dilute solution is 0.025 K .

$i = 2.0$

van't Hoff factor

$$(i) = \frac{\text{Observed colligative property (experimental)}}{\text{Calculated colligative property}}$$

$$2.0 = \frac{0.025}{\text{Calculated colligative property}}$$

Calculated depression in freezing point $= 0.0125$ K

64. (b) Given, $K_b = 0.52 \text{ kg mol}^{-1}$ K

Boiling point of water $= 100^\circ C$

$$m = \frac{\Delta T_b}{K_b}$$

$$0.1 = \frac{\Delta T_b}{0.52}$$

$\Delta T_b = 0.052$

Boiling point of solution $(T_b) = 100 + \Delta T_b$

$= 100 + 0.052 = 100.052$

65. (b) Molar conductivity is given as $\Lambda_m = \Lambda_m^\circ - A\sqrt{C}$

Thus, as the $\sqrt{C}$ increases , Λ_m decreases.

66. (c) $5Br^-(aq) + 6H^+(aq) + BrO_3^-(aq) \longrightarrow 3Br_2(aq)$

$+3H_2O(l)$

Since, we' ve $-\frac{1}{5}\frac{\Delta[Br^-]}{\Delta t} = -\frac{\Delta[Bro_3{}^-]}{\Delta t} = +\frac{1}{3}\frac{\Delta[Br_2]}{\Delta t}$

$$\therefore \frac{\Delta BrO^-3}{\Delta t} = \frac{1}{3}\frac{\Delta[Br_2]}{\Delta t}$$

$\Rightarrow$

$$\frac{\Delta[Br_2]}{\Delta t} = 3\frac{\Delta[BrO_3^-]}{\Delta t} = 3 \times 0.01 = 0.03$$

67. (a) Milk is an emulsion of fat and water. An emulsion is a dispersion of tiny droplets in another liquid. Thus, it is a liquid-liquid colloidal system.

68. (c) Copper matte contains Cu_2 S and FeS . On roasting the copper ore, i.e. copper pyrites ($CuFeS_2$), the impurities S and Fe present in it are converted into cuprous sulphide and ferrous sulphide. $2CuFeS_2 + O_2 \longrightarrow Cu_2S + 2FeS + SO_2$

69. (d) Zn reacts with dilute nitric acid to form 'laughing gas'.

$4Zn + 10HNO_3 \longrightarrow 4Zn(NO_3)_2 + N_2O + 5H_2O$

70. (a) XeF_4 is obtained by heating a mixture of xenon and fluorine in the molar ratio of 1: 5 at 873 K and 7 bar pressure in an enclosed nickel vessel for a few hours. The reaction proceeds as

$$Xe(g) + 2F_2(g) \underset{7 \text{ bar}}{\overset{873 \text{ K}}{\rightarrow}} XeF_4$$

The extra fluorine taken, increases the production.

71. (b) We know that, magnetic moment (μ) is related to number of unpaired electron ' n ', as $\mu_{eff} = \sqrt{n(n+2)}$BM

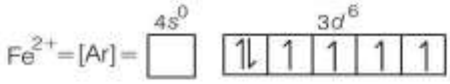
$\mu_{eff} = \sqrt{24}$BM

This corresponds, n = 4 as $\mu_{eff} = \sqrt{4(4+2)} = \sqrt{24}$

$= 2.\sqrt{2} \cdot \sqrt{3}$
$= 2 \times 1.414 \times 1.732$
$= 4.89 \text{Bm}$

(Trick = Number of unpaired e^- are equal to value of variable number of BM.)

Out of given options, Fe^{2+} has 4 unpaired electrons as



72. (c) All the other three exhibit geometrical isomerism but tetrahedral complex does not.

73. (d) The polydispersity index (PDI) of a polymer is the ratio of $\bar{M}_w$ and $\bar{M}_n$.

$$PDI = \frac{\bar{M}_w}{\bar{M}_n}$$

It is directly related to the standard deviation of polymer molecular weight distribution. Polydispersity index can be used as a measure of the width of the molecular mass distribution.

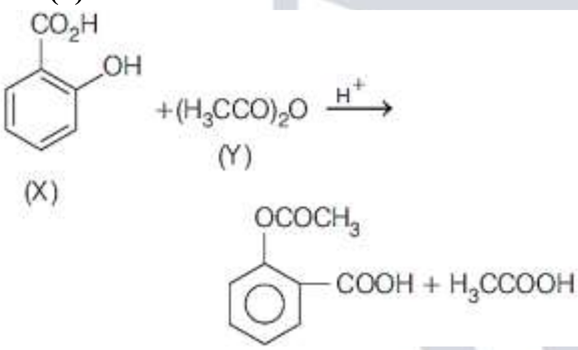
74. (b) Insulin is a peptide hormone secreted from pancreas. It regulates the blood glucose level metabolism within the limit by signalling liver, muscles and fat cells to take up extra glucose from blood.

75. (a) Chloroxylenol is an example of antiseptic. Dettol is a mixture of chloroxylenol and α terpineol. Chloroxylenol is responsible for the antiseptic and disinfectant properties of dettol.

76. (a) $H_3CCH_2CH_2CH_2Cl$

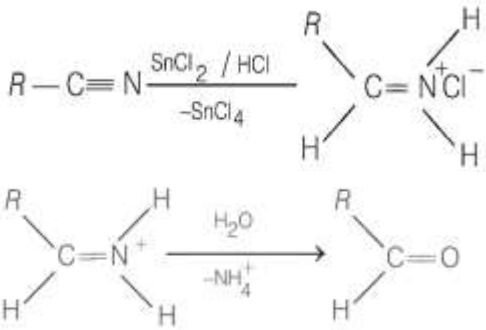
For isomeric alkyl halides, the boiling points decreases with branching. Branching of the chain makes the molecule more compact and therefore, the surface area decreases. Hence, the intermolecular attractive forces which depend upon the surface area, also become small in magnitude due to branching. Consequently, the boiling points of the branched chain having same molecular formula are less than the straight chain isomers. Higher the molecular weight, higher is the boiling point, but branching reduces boiling point.

77. (b)

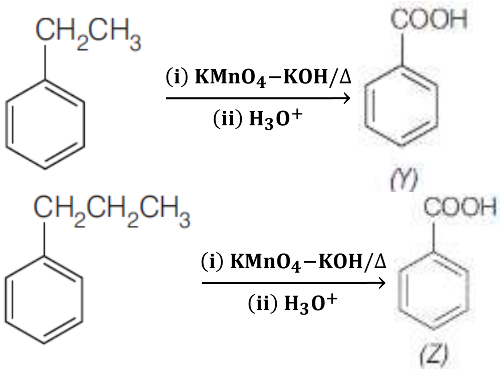


Acetyl salicylic acid or
2-acetoxy benzoic acid
(Aspirin)

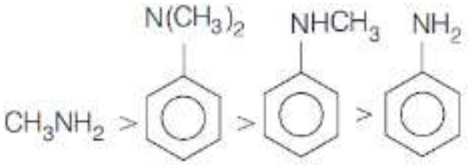
78. (c) Specifically, when reduction is carried out with $SnCl_2/HCl$. It is called Stephen's reaction. The generalised reaction looks like



79. (d) $KMnO_4$ (alk.) oxidise alkyl group (R) directly attached to benzene ring, irrespective of length of alkyl group (if has at least one $\alpha - H$).



80. **(a)** The species having more lone pair of electrons for donation are strong base. In case of CH_3NH_2, lone pair of electrons on N -atom is available for donation. Hence, it is most basic. Order of basic nature of the given compounds is



Mathematics

81. **(c)** Given, $f(x) = x^2 - 2x + 4$

Now, $f(x-1) = f(x+1)$

$\Rightarrow\ (x-1)^2 - 2(x-1) + 4 = (x+1)^2 - 2(x+1) + 4$

$\Rightarrow x^2 - 2x + 1 - 2x + 2 + 4$

$= x^2 + 2x + 1 - 2x - 2 + 4$

$\Rightarrow x^2 - 4x + 7 = x^2 + 3$

$\Rightarrow\ -4x = -4$

$\Rightarrow\ x = 1$

$\therefore\ x = \{1\}$

82. **(d)** Let $f(x) = ax + b$

Given,

$f(f(x)) = x + f(x)$

$\Rightarrow\ a(ax+b) + b = x + (ax+b)$

$\Rightarrow\ a^2x + ab + b = x + ax + b$

$\Rightarrow\ x(a^2 - a - 1) + ab = 0$

$\therefore\ a^2 - a - 1 = 0$ and $ab = 0$

$\Rightarrow\ a \neq 0, b = 0$

$$\therefore\ a = \frac{1 \pm \sqrt{5}}{2}$$

$$\text{Now, } f(x) = \left(\frac{1 \pm \sqrt{5}}{2}\right)x + 0 = \left(\frac{1 \pm \sqrt{5}}{2}\right)x$$

$$f(x) = \left(\frac{1 + \sqrt{5}}{2}\right) \times, \left(\frac{1 - \sqrt{5}}{2}\right)x$$

$\therefore$ It has two linear equation.

83. **(b)** We have, $7^n = (1+6)^n$

$= {}^nC_0 + {}^nC_1 6^1 + {}^nC_2 6^2 + {}^nC_3 6^3 + \cdots + {}^nC_n 6^n$

$= 1 + 6n + 6^2[{}^nC_2 + {}^nC_3 6 + \cdots + {}^nC_n 6^{\,n-2}]$

$= 1 + 6n + 36\lambda$ [where, ${}^nC_2 + \cdots + {}^nC_n 6^{n-2} = \lambda$]

$\Rightarrow\ 7^n - 6n = 36\lambda + 1$

$\Rightarrow\ 7^n - 6n - 50 = 36\lambda - 49$

$\Rightarrow\ 7^n - 6n - 50 = 36\lambda - 72 + 23$

$\Rightarrow\ 7^n - 6n - 50 = 36(\lambda - 2) + 23$

$\Rightarrow\ 7^n - 6n - 50 = 36\mu + 23$[where $\lambda - 2 = \mu$]

$\therefore$ When $7^n - 6n - 50$ is divided by 36 , then remainder will be equal to 23 .

84. (b) We have,

$ax + by + cz = 2$

$bx + cy + az = 2$

$cx + ay + bz = 2$

Now, $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\Delta = (a+b+c)\begin{bmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{bmatrix}$$

$\Delta = 0$ [$\because$ it is given that $a + b + c = 0$]

Therefore, the system of equations is inconsistent.

85. (b) Since, A and B are symmetric matrix, then

$A' = A$ and $B' = B$.

Now, let $P = AB - BA$

$\therefore\ P' = (AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$ [$\because (AB)' = B'A'$]

$= BA - AB$ [$\because A' = A, B' = B$]

$= -(AB - BA)$

$= -P$

Hence, P is a skew-symmetric matrix as $P' = -P$.

86. (b) Since,

$$= \begin{vmatrix} x+1 & x & x \\ 2x+1 & x & -2x \\ 3x+1 & -2x & x \end{vmatrix}$$

$$= x^2\begin{vmatrix} x+1 & 1 & 1 \\ 2x+1 & 1 & -2 \\ 3x+1 & -2 & 1 \end{vmatrix} \quad [\text{apply } C_2 \to C_2 - C_3]$$

$$= x^2\begin{vmatrix} x+1 & 0 & 1 \\ 2x+1 & 3 & -2 \\ 3x+1 & -3 & 1 \end{vmatrix} \quad [\text{ apply } R_2 \to R_2 + R_3]$$

$$= x^2\begin{vmatrix} x+1 & 0 & 1 \\ 5x+2 & 0 & -1 \\ 3x+1 & -3 & 1 \end{vmatrix} [\text{ expand along } C_2]$$

$= x^2(3)\{-x - 1 - 5x - 2\}$

$= 3x^2(-6x - 3) = -18x^3 - 9x^2$

$$\int_0^1 A(x)dx \quad = \int_0^1 (-18x^3 - 9x^2)dx$$

$$= \left[\frac{-18x^4}{4} - \frac{9x^3}{3}\right]_0^1$$

$$= \left[\frac{-18}{4}x^4 - \frac{9}{3}x^3\right]_0^1$$

$$= -\frac{18}{4} - \frac{9}{3}$$
$$= -\frac{18}{4} - 3$$
$$= \frac{-18-12}{4}$$
$$= -\frac{30}{4} = -\frac{15}{2}$$

87. (b) We have,

$$\bar{z}^{\frac{1}{3}} = a + ib$$
$$\Rightarrow \bar{z} = (a+ib)^3$$
$$\Rightarrow x - iy = (a+ib)^3$$
$$[\because \bar{z} = x - iy]$$
$$\Rightarrow x - iy = a^3 + i^3b^3 + 3a^2(ib) + 3a(i^2b^2)$$
$$\Rightarrow x - iy = a^3 - ib^3 + 3a^2bi - 3ab^2$$
$$\Rightarrow x - iy = (a^3 - 3ab^2) + i(3a^2b - b^3)$$
$$\Rightarrow x = a^3 - 3ab^2 \text{ and } y = -3a^2b + b^3$$
$$\Rightarrow \frac{x}{a} = a^2 - 3b^2 \text{ and } \frac{y}{b} = -3a^2 + b^2$$
$$\text{Now, } \frac{x}{a} + \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$$
$$= -2a^2 - 2b^2$$
$$\Rightarrow \frac{x}{a} + \frac{y}{b} = -2(a^2+b^2)$$
$$\therefore \frac{1}{a^2+b^2}\left(\frac{x}{a} + \frac{y}{b}\right) = -2$$

88. (c) Let $z = x + iy$

Now, we have $|z| + |z-1| = 3$

$$\Rightarrow |x+iy| + |x+iy-1| = 3$$
$$\Rightarrow \sqrt{x^2+y^2} + \sqrt{(x-1)^2+y^2} = 3$$
$$\Rightarrow \sqrt{(x-1)^2+y^2} = 3 - \sqrt{x^2+y^2}$$
$$\Rightarrow (x-1)^2 + y^2 = 9 + x^2 + y^2 - 6\sqrt{x^2+y^2}$$
$$\Rightarrow x^2 - 2x + 1 + y^2 = 9 + x^2 + y^2 - 6\sqrt{x^2+y^2}$$
$$\Rightarrow -2x + 1 = 9 - 6\sqrt{x^2+y^2}$$
$$\Rightarrow 6\sqrt{x^2+y^2} = 2x + 8$$
$$\Rightarrow 3\sqrt{x^2+y^2} = x + 4$$
$$\Rightarrow 9(x^2+y^2) = (x+4)^2$$
$$\therefore 9x^2 + 9y^2 = x^2 + 8x + 16$$
$$\Rightarrow 8x^2 + 9y^2 - 8x = 16$$
$$\Rightarrow 8(x^2 - x) + 9y^2 = 16$$
$$\Rightarrow 8\left[\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 9y^2 = 16$$
$$\Rightarrow 8\left(x - \frac{1}{2}\right)^2 - 2 + 9y^2 = 16$$
$$\Rightarrow 8\left(x - \frac{1}{2}\right)^2 + 9y^2 = 18$$
$$\therefore \frac{\left(x - \frac{1}{2}\right)^2}{\frac{9}{4}} + \frac{y^2}{2} = 1$$

Which is an equation of ellipse.

89. **(c)** Any circle with centre at origin and radius r has the equation

$|z| = r$

$\Rightarrow |1+i||1+2i||1+3i| \dots |1+10i| = r$

$\Rightarrow \sqrt{1^2+1^2} \cdot \sqrt{1^2+2^2} \cdot \sqrt{1^2+3^2} \dots \sqrt{1^2+10^2} = r$

$\Rightarrow \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{10} \dots \sqrt{101} = r$

$\therefore r^2 = 2 \times 5 \times 10 \times \dots \times 101$

90. **(b)**

$\because |(z-1)-(z-5)| \le |(z-1)| + |(z-5)|$

$\therefore \quad |z-1-z+5| \le |z-1| + |z-5|$

$\Rightarrow \quad |4| \le |z-1| + |z-5|$

$\Rightarrow \quad |z-1| + |z-5| \ge 4$

91. **(c)** We have,

$|x|^2 - 5|x| + 6 = 0$

Let $|x| = y$

$\Rightarrow \quad y^2 - 5y + 6 = 0$

$\Rightarrow \quad (y-2)(y-3) = 0$

$\Rightarrow \quad y = 2,3$

$\Rightarrow \quad |x| = 2$ or $|x| = 3$

$\Rightarrow \quad x = \pm 2$ or ± 3

$\therefore$ Number of real roots are 4 .

92. **(b)** We have, $x^2 - x + 1 = 0$

$\Rightarrow x = \omega, \omega^2$

Then, $\alpha = \omega$ and $\beta = \omega^2$

Now, $\alpha^{2015} = \omega^{2015} = \omega^{3\times 671+2} = \omega^2$

ω^2 and $\beta^{2015} = (\omega^2)^{2015} = \omega^{4030}$

$= \omega^{3\times 1343} + 1 = \omega$

$\therefore \alpha^{2015} + \beta^{2015} = \omega^2 + \omega = -1$

and $\alpha^{2015} \cdot \beta^{2015} = \omega^2 \cdot \omega = \omega^3 = 1$

$\therefore$ Equation whose roots are α^{2015} and β^{2015} will be

$x^2 - (-1)x + 1 = 0$

$\Rightarrow x^2 + x + 1 = 0$

93. **(d)** Given, the roots of

$x^3 - 5x + 4 = 0$ are α, β and γ

$\therefore \quad \alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = 5$ and $\alpha\beta\gamma = -4$

Since, $\alpha + \beta + \gamma = 0$

$\therefore \quad \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$

$= 3 \times (-4) = -12$

Hence, $(\alpha^3 + \beta^3 + \gamma^3)^2 = (-12)^2 = 144$

94. **(c)**

95. **(c)** Total number of ways $= 8^4$

$\therefore$ Required number of ways of 4 letter words that can be formed by the word 'EQUATION' when at least one letter repeated is

$= 8^4 - {}^8P_4 = 4096 - 1680$

$= 2416$

96. **(d)** $\because 7! = 2^4 3^2 5^1 7^1$

$\therefore$ Number of divisors

$= 5 \times 3 \times 2 \times 2$

$= 60$

97. **(a)** Let $S = 1 + \frac{2}{3}\left(\frac{1}{8}\right) + \frac{2\times 5}{3\times 6}\left(\frac{1}{8}\right)^2 + \frac{2\times 5\times 8}{3\times 6\times 9}\left(\frac{1}{8}\right)^3 + \cdots$

$$= 1 + \frac{\frac{2}{3}}{1}\left(\frac{1}{8}\right) + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{2!}\left(\frac{1}{8}\right)^2 + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)\left(\frac{8}{3}\right)}{3!}\left(\frac{1}{8}\right)^3 + \cdots$$

$$= \left(1 - \frac{1}{8}\right)^{-\frac{2}{3}} \left[\because (1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \cdots\right]$$

$$= \left(\frac{7}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{7}\right)^{\frac{2}{3}} = \left(\frac{64}{49}\right)^{\frac{1}{3}} = \frac{(64)^{\frac{1}{3}}}{\sqrt[3]{49}} = \frac{4}{\sqrt[3]{49}}$$

98. **(b)** Consider the given expression,

$$(-1)C_0^2 + 2C_1^2 + 5C_2^2 + \cdots + (3n-1)C_n^2$$

$$= \sum_{r=0}^{n} (3r-1)C_r^2$$

$$= \sum_{r=0}^{n} 3r \cdot C_r^2 - \sum_{r=0}^{n} C_r^2$$

$$= 3\sum_{r=0}^{n} (r \cdot C_r^2) - {}^{2n}C_n$$

$$[\because C_0^2 + C_1^2 + C_2^2 + \cdots + C_n^2 = {}^{2n}C_n]$$

$$= 3\sum_{r=0}^{n} (r \cdot C_r^2) - {}^{2n}C_n$$

$$= 3\sum_{r=1}^{n} (r \cdot {}^nC_r \cdot {}^nC_r) - {}^{2n}C_n$$

$$= 3\sum_{r=1}^{n} (n \cdot {}^{n-1}C_{r-1} \cdot {}^nC_r) - {}^{2n}C_n$$

$$= 3n\sum_{r=0}^{n} ({}^{n-1}C_{r-1}\, {}^nC_{n-r}) - {}^{2n}C_n$$

$$[\because r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}]$$

$= 3n$ [coefficient of x^{n-1} in the expansion of $(1+x)^{n-1}(1+x)^n] - {}^{2n}C_n$

$= 3n$ [coefficient of x^{n-1} in the expansion of $(1+x)^{2n-1}] - {}^{2n}C_n$

$$= 3n\, {}^{2n-1}C_{n-1} - {}^{2n}C_n$$

$$= 3n\frac{(2n-1)!}{(n-1)!\,(n)!} - \frac{(2n)!}{(n!)(n!)}$$

$$= \frac{3n \cdot (2n)(2n-1)!}{(2n)(n-1)!\,(n!)} - \frac{(2n)!}{n!\,n!} = \frac{3n}{2}\frac{(2n)!}{n!\,n!} - \frac{(2n)!}{n!\,n!}$$

$$= \frac{3n}{2} \cdot {}^{2n}C_n - {}^{2n}C_n = {}^{2n}C_n\left(\frac{3n}{2} - 1\right)$$

$$= \left(\frac{3n-2}{2}\right) \cdot {}^{2n}C_n$$

99. **(a)** Given,

$$\frac{x+1}{x^4(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+2}$$

$$\Rightarrow x + 1 = Ax^3(x+2) + Bx^2(x+2) + Cx(x+2) + D(x+2) + Ex^4$$

At $x = -1$,

$$-1 + 1 = -A + B - C + D + E$$

$$\Rightarrow 0 = B + D + E - A - C$$

$$\therefore B + D + E = A + C$$

100. **(b)** We have,

$$\cos^3\theta+\cos^3\left(\frac{2\pi}{3}+\theta\right)+\cos^3\left(\frac{4\pi}{3}+\theta\right)=a\cos 3\theta$$

$$\Rightarrow \frac{\cos 3\theta+3\cos\theta}{4}+\frac{\cos 3\left(\frac{2\pi}{3}+\theta\right)+3\cos\left(\frac{2\pi}{3}+\theta\right)}{4}$$

$$+\frac{\cos 3\left(\frac{4\pi}{3}+\theta\right)+3\cos\left(\frac{4\pi}{3}+\theta\right)}{4}=a\cos 3\theta$$

$$\Rightarrow \cos 3\theta+3\cos\theta+\cos(2\pi+3\theta)+3\cos\left(\frac{2\pi}{3}+\theta\right)$$

$$+\cos(4\pi+3\theta)+3\cos\left(\frac{4\pi}{3}+\theta\right)=4a\cos 3\theta$$

$$\Rightarrow \cos 3\theta+3\cos\theta+\cos 3\theta+\cos 3\theta+3\left[\cos\left(\frac{2\pi}{3}+\theta\right)\right.$$

$$\left.+\cos\left(\frac{4\pi}{3}+\theta\right)\right]=4a\cos 3\theta$$

$$\Rightarrow 3\cos 3\theta+3\cos\theta+3\left[2\cos(\pi+\theta)\cos\frac{\pi}{3}\right]$$

$$=4a\cos 3\theta$$

$$\Rightarrow 3\cos 3\theta+3\cos\theta+3\times 2\times(-\cos\theta)\times\frac{1}{2}$$

$$=4a\cos 3\theta$$

$$\Rightarrow 3\cos 3\theta+3\cos\theta-3\cos\theta=4a\cos 3\theta$$

$$\Rightarrow 3\cos 3\theta=4a\cos 3\theta\Rightarrow a=\frac{3}{4}$$

101. **(c)** Given, $\frac{\cos 13^\circ-\sin 13^\circ}{\cos 13^\circ+\sin 13^\circ}+\frac{1}{\cot 148^\circ}$

$$=\frac{1-\tan 13^\circ}{1+\tan 13^\circ}+\tan 148^\circ$$

$$=\frac{\tan 45^\circ-\tan 13^\circ}{1+\tan 45^\circ\tan 13^\circ}+\tan 148^\circ$$

$$=\tan(45^\circ-13^\circ)+\tan(180^\circ-32^\circ)$$

$$=\tan 32^\circ-\tan 32^\circ=0$$

102. **(d)** Given, $\cos x+\cos y+\cos\alpha=0$

$\therefore$ $\cos x+\cos y=-\cos\alpha$

$$\Rightarrow\ 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=-\cos\alpha\ \ldots..(i)$$

And $\sin x+\sin y+\sin\alpha=0$

$\therefore\ \sin x+\sin y=-\sin\alpha$

$$\Rightarrow\ 2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)=-\sin\alpha\ \ldots\ldots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{\cos\left(\frac{x+y}{2}\right)}{\sin\left(\frac{x+y}{2}\right)}=\frac{\cos\alpha}{\sin\alpha}$$

$$\therefore\ \cot\frac{(x+y)}{2}=\cot\alpha$$

103. **(b)** $f(x)=\cos^2 x+\cos^2 2x+\cos^2 3x=1$

$\Rightarrow \cos^2 x + (2\cos^2 x - 1)^2 + (4\cos^3 x - 3\cos x)^2 = 1$
$\Rightarrow \cos^2 x + 4\cos^4 x + 1 - 4\cos^2 x + 16\cos^6 x$
$+9\cos^2 x - 24\cos^4 x = 1$
$\Rightarrow 6\cos^2 x - 20\cos^4 x + 16\cos^6 x = 0$
$\Rightarrow 2\cos^2 x(3 - 10\cos^2 x + 8\cos^4 x) = 0$
$\Rightarrow \cos^2 x = 0 \text{ or } 8\cos^4 x - 10\cos^2 x + 3 = 0$
$\Rightarrow \cos^2 x = 0 \text{ or } (4\cos^2 x - 3)(2\cos^2 x - 1) = 0$

$$\Rightarrow \cos^2 x = 0 \text{ or } \cos x = \frac{3}{4} \text{ or } \cos^2 x = \frac{1}{2}$$

$$\Rightarrow \cos^2 x = 0 \text{ or } \cos x = \frac{\sqrt{3}}{2} \text{ or } \cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos^2 x = (2n+1)\frac{\pi}{2},$$

$$x = 2n\pi \pm \frac{\pi}{6}$$

$$\text{or } x = 2n\pi + \frac{\pi}{4}$$

$$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{\pi}{6}, \frac{7\pi}{4}, \frac{11\pi}{6}$$

Hence, there are six values of x for which $x \in [0, 2\pi]$.

104. **(a)** Given, $\sin(\cot^{-1} x) = \cos[\tan^{-1}(1+x)]$ ...(i)

And we know that, $\cot^{-1}\theta = \sin^{-1}\left(\frac{1}{\sqrt{1+\theta^2}}\right)$

And $\tan^{-1}\theta = \cos^{-1}\left(\frac{1}{\sqrt{1+\theta^2}}\right)$

∴ From Eq. (i), we get

$$\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+(1+x)^2}}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+(1+x)^2}}$$

$$\Rightarrow 1 + x^2 + 2x + 1 = x^2 + 1$$

$$\therefore x = -\frac{1}{2}$$

105. **(c)** $\because \operatorname{sech}^{-1}(x) = \log\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

$$\therefore \operatorname{sech}^{-1}(\cos\theta) = \log\left(\frac{1+\sqrt{1-\cos^2\theta}}{\cos\theta}\right)$$

$$= \log\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

$$= \log(\tan\theta + \sec\theta)$$

$$= \log\left[\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$$

106. **(c)** Given, $\angle A = 90°, \angle B \neq \angle C$

$\because \dfrac{b^2+c^2}{b^2-c^2}\sin(B-C)$

$= \dfrac{k^2\sin^2 B + k^2\sin^2 C}{k^2\sin^2 B - k^2\sin^2 C}\sin(B-C)$

$\left[\because \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k\right]$

$= \dfrac{\sin^2 B + \sin^2 C}{\sin^2 B - \sin^2 C}\sin(B-C)$

$= \dfrac{\sin^2 B + \sin^2(90° - B)}{\sin(B+C)\sin(B-C)}\sin(B-C)\ [B + C = 90°]$

$= \dfrac{\sin^2 B + \cos^2 B}{\sin(B+C)} = \dfrac{1}{\sin 90°} = 1$

107. **(a)** We have,

$$8R^2 = a^2 + b^2 + c^2$$

$\Rightarrow 8R^2 = 4R^2(\sin^2 A + \sin^2 B + \sin^2 C)$

$\left[\because R = \dfrac{a}{2\sin A} = \dfrac{b}{2\sin B} = \dfrac{c}{2\sin C},\right.$ where R is circumradius $\left.\right]$

$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$

$\Rightarrow 1 - \cos^2 A + 1 - \cos^2 B + \sin^2 C = 2$

$\Rightarrow (\cos^2 A - \sin^2 C) + \cos^2 B = 0$

$\Rightarrow \cos(A+C)\cos(A-C) + \cos^2 B = 0$

$\Rightarrow -\cos B\{\cos(A-C) - \cos B\} = 0$

$\Rightarrow -\cos B\{\cos(A-C) + \cos(A+C)\} = 0$

$\Rightarrow -2\cos A\cos B\cos C = 0$

$\Rightarrow \cos A = 0 \text{ or } \cos B = 0 \text{ or } \cos C = 0$

$\Rightarrow A = \dfrac{\pi}{2} \text{ or } B = \dfrac{\pi}{2} \text{ or } C = \dfrac{\pi}{2}$

Hence, $\triangle$ ABC is a right- angled triangle.

108. **(d)** We have,

$2R + r = r_2$

$\Rightarrow 2R = r_2 - r$

$\Rightarrow$

$2R = 4R\sin\dfrac{B}{2}\cos\dfrac{A}{2}\cos\dfrac{C}{2} - 4R\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

$\Rightarrow 2R = 4R\sin\dfrac{B}{2}\cos\left(\dfrac{A+C}{2}\right)$

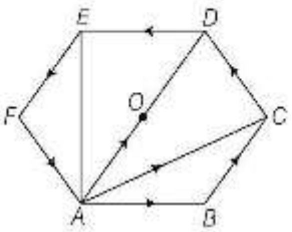
$\Rightarrow 1 = 2\sin^2\dfrac{B}{2} \Rightarrow \sin^2\dfrac{B}{2} = \dfrac{1}{2}$

$\Rightarrow \sin\dfrac{B}{2} = \dfrac{1}{\sqrt{2}} = \sin\dfrac{\pi}{4}$

$\Rightarrow \dfrac{B}{2} = \dfrac{\pi}{4}$

$\therefore B = \dfrac{\pi}{2}$

109. **(d)**



$\because \quad AB = ED, AF = CD$

$\therefore \quad AB + AC + AD + AE + AF$

$= AE + ED + AC + CD + AD$

$AD + AD + AD$

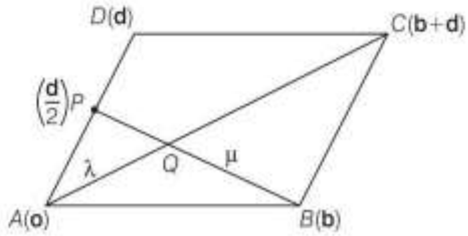
$= 3AD$

$\because$ O is the centre of regular hexagon.

$\therefore \ AD = 2AO$

$\Rightarrow \ 3AD = 3 \times 2AO = 6AO$

110. **(a)**



Let Q divides AC and BP in the ratio λ: 1 and μ: 1 respectively.

Now, point

$$Q = \frac{\lambda(b+d) + 1(0)}{\lambda + 1} = \frac{\lambda(b+d)}{\lambda+1} = \frac{\lambda}{\lambda+1} b + \frac{\lambda}{\lambda+1} d \ldots \text{(i)}$$

and

$$Q = \frac{\mu \frac{d}{2} + b}{\mu + 1} = \frac{1}{\mu+1} b + \frac{\mu}{2(\mu+1)} d \ldots . \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \frac{\lambda}{\lambda+1} = \frac{1}{\mu+1} \text{ and } \frac{\lambda}{\lambda+1} = \frac{\mu}{2(\mu+1)}$$

$$\Rightarrow \frac{1}{\mu+1} = \frac{\mu}{2(\mu+1)}$$

$$\Rightarrow \mu = 2$$

$$\therefore \frac{\lambda}{\lambda+1} = \frac{1}{2+1} = \frac{1}{3}$$

$$\Rightarrow 3\lambda = \lambda + 1$$

$$\Rightarrow 2\lambda = 1$$

$$\Rightarrow \lambda = \frac{1}{2}$$

Hence, required ratio $= AQ: QC = \lambda: 1$

$$= \frac{1}{2}: 1 = 1: 2$$

111. **(b)** Let x, y and z be the three scalars such that

$x(2\hat{i} - 3\hat{j} + \hat{k}) + y(\hat{i} - 2\hat{j} + 3\hat{k}) + z(3\hat{i} + \hat{j} - 2\hat{k}) = 0$

$\Rightarrow (2x + y + 3z)\hat{i} + (-3x - 2y + z)\hat{j} +$

$(x + 3y - 2z)\hat{k} = 0$

$\Rightarrow \ 2x + y + 3z = 0, -3x - 2y + z = 0$

and $\ x + 3y - 2z = 0$

This is a homogeneous system of equations. The determinant of coefficient matrix is

$$\begin{vmatrix} 2 & 1 & 3 \\ -3 & -2 & 1 \\ 1 & 3 & -2 \end{vmatrix} \neq 0$$

So, the system of equations has only the trivial solution given by $x = y = z = 0$.

Hence, the given vectors are linearly independent

112. **(c)** Given that,

$|a| = 1, |b| = 2, |c| = 3$

$\because$ b and c are perpendicular.

$\therefore \ b \cdot c = 0 \quad \ldots..\text{(i)}$

And projection of b on a $= \frac{a \cdot b}{|a|}$
Projection of c on a $= \frac{a \cdot c}{|a|}$
$\because$ Both projections are same.
$\therefore \frac{a \cdot b}{|a|} = \frac{a \cdot c}{|a|} \quad a.b = a\,c \ldots$ (ii)
Then, $|a - b + c|^2 = |a|^2 + |b|^2 + |c|^2 - 2a \cdot b$
$-2b \cdot c + 2c \cdot a$
$= (1)^2 + (2)^2 + (3)^2 - 2a \cdot b - 0 + 2a \cdot c$
$= 1 + 4 + 9 - 2a \cdot b + 2a \cdot b = 14$
[from Eqs. (i) and (ii)]
$\therefore |a - b + c| = \sqrt{14}$

113. **(c)** Given, $a + b + \sqrt{3}c = 0$
$\Rightarrow \quad a + b = -\sqrt{3}c \ldots\ldots$ (i)
$\Rightarrow \quad (a + b) \cdot (a + b) = (-\sqrt{3}c)(-\sqrt{3}c)$
$\Rightarrow \quad |a|^2 + 2a \cdot b + |b|^2 = 3|c|^2$
Since, $|a|, |b|$ and $|c|$ are unit vectors.
$\therefore \quad (1)^2 + 2(1)(1)\cos\theta + 1 = 3(1)$
$\Rightarrow \quad 1 + 2\cos\theta + 1 = 3$
$\Rightarrow \quad 2\cos\theta = 3 - 2$
$\Rightarrow \quad 2\cos\theta = 1$
$\Rightarrow \quad \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$
$\Rightarrow \quad \theta = \frac{\pi}{3}$

114. **(b)** Given,
$a \cdot b = 0$
$a \cdot c = 0$
$\therefore a \| b \times c$
and
$|b \times c| = |b| \cdot |c| \sin\left(\frac{2\pi}{3}\right)$
$[\because |a| = 2, |b| = 3$ and $|c| = 4]$
$= 3 \times 4 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$
Hence, $c \cdot (a \times b) = a \cdot (b \times c) [\because a \cdot b = |a||b|\cos\theta]$
$= |a||b \times c| = 2 \times 6\sqrt{3}$
$= 12\sqrt{3}$

115. **(b)** Given sequence 148,146,144, ... and average is 125 .
$\therefore S_n = \frac{n}{2}[2 \times 148 + (n - 1)(-2)] \; [\because d = -2]$
$= \frac{n}{2}[296 - 2n + 2]$
$= n[148 - n + 1] = n[149 - n]$
Hence, $125 = \frac{n[149 - n]}{n}$
$\Rightarrow 125 = 149 - n$
$\therefore n = 149 - 125 = 24$

116. **(c)** Given series,

$a, a+d, a+2d, \ldots, a+2nd$

Here, $a+2nd = a+(m-1)d$

$\Rightarrow 2nd = (m-1)d$

$\Rightarrow 2n = m-1$

$\Rightarrow 2n+1 = m$

$\therefore$ Standard deviation $= \sqrt{\frac{(2n+1)^2-1}{12} d^2}$

$= \sqrt{\frac{(2n+1-1)(2n+1+1)}{12} d^2}$

$= \sqrt{\frac{(2n+2)2n}{12} d^2} = \sqrt{\frac{(n+1)n}{3}} d$

117. **(a)** We have, $P(A) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \quad \ldots.(i)$

and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1}{2} \ldots\ldots(ii)$

$\Rightarrow \quad P(A \cap B) = \frac{1}{2} \times P(A) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Substitute the value of $P(A \cap B)$ in Eq. (i), we get

$P(B) = \frac{1}{8} \times 4 = \frac{1}{2}$

(I) $P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(A \cup B)'}{P(\bar{B})} = \frac{1-P(A \cup B)}{1-P(B)}$

$= \frac{1-[P(A)+P(B)-P(A \cap B)]}{1-P(B)}$

$= \frac{1-\left(\frac{1}{4}+\frac{1}{2}-\frac{1}{8}\right)}{1-\frac{1}{2}} = \frac{1-\frac{5}{8}}{\frac{1}{2}} = \frac{3}{4}$

(II) $P(A \cap B) = \frac{1}{8}$ [$\because$ A and B are not mutually exclusive]

(III) $P\left(\frac{A}{B}\right) + P\left(\frac{A}{\bar{B}}\right)$

$= \frac{P(A \cap B)}{P(B)} + \frac{P(A \cap \bar{B})}{P(\bar{B})}$

$= \frac{P(A \cap B)}{P(B)} + \frac{P(A)-P(A \cap B)}{1-P(B)}$

$= \frac{1}{4} + \frac{\frac{1}{4}-\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Hence, only (I) is correct.

118. **(a)** Number is divisible by 4 , if last two digits are 12, 24,32 and 52 .

Remaining 3 place will be filled by 3! ways.

$\therefore$ Favourable outcomes $= 3! \times 4$

Hence, required probability $= \frac{3! \times 4}{5!}$

$= \frac{3! \times 4}{5 \times 4 \times 3!} = \frac{1}{5}$

119. **(b)** Total outcomes $= 36$

Favorable outcomes $=$ (sum greater than 7) $= 15$

$\therefore$ Required probability $= \frac{15}{36} = \frac{5}{12}$

120. **(d)** Total number of outcomes $= 2^4 = 16$

Let X be the number of girls.
Now, probability (at least two girls)
$= 1 - \{P(X = 0) + P(X = 1)\}$
$P(X = 0)$, i.e. no girl or all boys.
Favourable outcome = {BBBB} = 1

$\therefore\ P(X = 0) = \frac{1}{16}$

and $P(X = 1)$, i.e. one girl.
Favourable outcomes
$= \{GBBB, BGBB, BBGB, BBBG\} = 4$

$\therefore P(X = 1) = \frac{4}{16}$

Now, required probability
$= 1 - \{P(X = 0) + P(X = 1)\}$
$= 1 - \left\{\frac{1}{16} + \frac{4}{16}\right\} = 1 - \frac{5}{16} = \frac{11}{16}$

121. **(d)** Let p denotes the ship departed at A reached B safely.

Then, $p = \frac{9}{10} = 0.9$

$\therefore\ q = 1 - p = 1 - \frac{9}{10} = \frac{1}{10} = 0.1$

Let X denotes the number of ships that departed at A and reached B safely. Now,
$n = 5$ and $p = 0.9$
$\therefore$ Required probability $= P(X \geq 4)$
$= P(X = 4) + P(X = 5)$
$= {}^5C_4(0.9)^4(0.1)^1 + {}^5C_5(0.9)^5(0.1)$
$= 5(0.9)^4(0.1)^1 + 1(0.9)^5(0.1)$
$= (0.9)^4(0.5 + 0.9)$
$= 1.4(0.9)^4$

122. **(a)** Given, $A(5, -4)$ and $B(7,6)$ are the points in a plane.

Now, $AP = \sqrt{(x-5)^2 + [(y-(-4)^2]}$

And $PB = \sqrt{(x-7)^2 + (y-6)^2}$

According to the question,
$AP : PB = 2 : 3$

$\Rightarrow \frac{\sqrt{(x-5)^2 + (y+4)^2}}{\sqrt{(x-7)^2 + (y-6)^2}} = \frac{2}{3}$

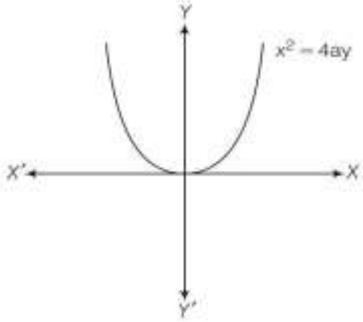
On squaring both sides, we get

$\Rightarrow \frac{(x-5)^2 + (y+4)^2}{(x-7)^2 + (y-6)} = \frac{4}{9}$

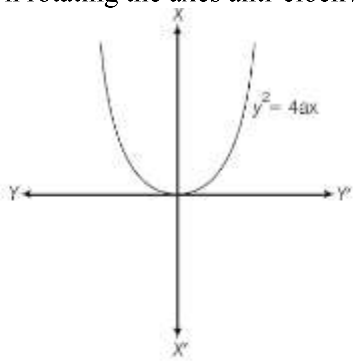
$\Rightarrow 9(x^2 + y^2 - 10x + 8y + 41)$
$= 4(x^2 + y^2 - 14x - 12y + 85)$
$\therefore\ 5x^2 + 5y^2 - 34x + 120y + 29 = 0$

Which represents the equation of an circle.

123. **(a)** Given, equation of parabola
$x^2 = 4ay$
Then,



On rotating the axes anti-clockwise through angle 90°, X-axis becomes Y-axis and Y-axis becomes X-axis.



Therefore, the equation of new parabola after rotation is $y^2 = 4ax$.

124. **(d)** Given,

$$y = kx + 1 \;\ldots\ldots\text{(i)}$$

and $3x + 4y = 9 \;\ldots\ldots\text{(ii)}$

Put the value of y by Eq. (i) in Eq (ii), we get

$$3x + 4(kx + 1) = 9$$
$$3x + 4kx + 4 = 9$$
$$\Rightarrow x(3 + 4k) + 4 - 9 = 0$$
$$\Rightarrow x(3 + 4k) - 5 = 0$$
$$\Rightarrow sx(3 + 4k) = 5$$
$$\Rightarrow x = \frac{5}{3 + 4k}$$
$$x = \pm 1, \pm 5$$

[$\because$ xis an integer]

$$\therefore\; 3 + 4k = \pm 1 \text{ or } 3 + 4k = \pm 5$$
$$k = \frac{-1}{2}, -1, -2, \frac{1}{2}$$

$\therefore$ we can choose value of k only integer number.

$\therefore\; k = -1, -2$

Hence, combined equation is

$$\{y = (-1)x + 1\}\{y = (-2)x + 1\} = 0$$
$$\Rightarrow (y = -x + 1)(y = -2x + 1) = 0$$
$$\Rightarrow (x + y - 1)(2x + y - 1) = 0$$

125. **(a)** Given lines,

$y - 3kx + 4 = 0 \quad \ldots\text{(i)}$

and $(2k - 1)x - (8k - 1)y - 6 = 0 \ldots\ldots\text{(ii)}$

Slope of Eq. (i), $m_1 = \frac{-(-3k)}{1} = 3k$

and slope of Eq. (ii) $m_2 = \frac{-(2k-1)}{-(8k-1)} = \frac{2k-1}{8k-1}$

Since, lines are perpendicular.

Since, lines are perpendicular.

$\therefore \quad m_1 m_2 = -1$

$\Rightarrow \quad 3k \times \frac{2k-1}{8k-1} = -1$

$\Rightarrow \quad 3k \times (2k-1) = -(8k-1)$

$\Rightarrow \quad 6k^2 - 3k = -8k + 1$

$\Rightarrow \quad 6k^2 - 3k + 8k - 1 = 0$

$\Rightarrow \quad 6k^2 + 5k - 1 = 0$

$\Rightarrow \quad 6k^2 + 6k - k - 1 = 0$

$\Rightarrow \quad 6k(k+1) - 1(k+1) = 0$

$\Rightarrow \quad (6k-1)(k+1) = 0$

$\therefore \quad k = \frac{1}{6}, -1$

126. **(b)** Given that,

$x_1 = 3, y_1 = 3$ and $x_2 = 7, y_2 = 6$

$\therefore$ Equation of straight line,

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$\Rightarrow \quad (y-3) = \frac{6-3}{7-3}(x-3)$

$\Rightarrow \quad (y-3) = \frac{3}{4}(x-3)$

$\Rightarrow \quad 4y - 12 = 3x - 9$

$\Rightarrow \quad 3x - 4y = -3$

For finding the points of intersection, put $x = 0$

then $\quad 3 \times 0 - 4y = -3$

$\Rightarrow \quad -4y = -3$

$\Rightarrow \quad y = \frac{3}{4}$

and when $y = 0$

$\Rightarrow 3x = -3$

$\Rightarrow x = -1$

Hence, the points are $A\left(0, \frac{3}{4}\right)$ and $B(-1, 0)$.

$\therefore$ Length of AB $= \sqrt{(0+1)^2 + \left(\frac{3}{4} - 0\right)^2}$

$= \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

127. **(b)** We know that, the equations of the line passing through the origin and perpendicular to the lines given by $3x^2 - 8xy + 5y^2 = 0$ is $5x^2 + 8xy + 3y^2 = 0$

Now, shifting the origin at (1,1) the equations of the required lines are

$5(x-1)^2 + 8(x-1)(y-1) + 3(y-1)^2 = 0$

$\Rightarrow 5(x^2 + 1 - 2x) + 8(xy - x - y + 1) + 3(y^2 + 1 - 2y) = 0$

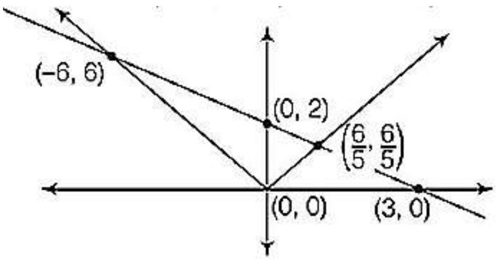
$\Rightarrow 5x^2 + 5 - 10x + 8xy - 8x - 8y + 8 + 3y^2$
$+3 - 6y = 0$

$5x^2 + 8xy + 3y^2 - 18x - 14y + 16 = 0$

128. **(c)** The sides of a triangle are given by

$(x^2 - y^2)(2x + 3y - 6) = 0$

or $\; x - y = 0, x + y = 0, 2x + 3y - 6 = 0$



We observe that the point $(0, \alpha)$ moves on $x = 0$. Thus, the value of α lies between $0 < \alpha < 2$.

129. **(c)** Let (a, b) be the point where line $4x - 3y + 7 = 0$ touches the circle $x^2 + y^2 - 6x + 4y - 12 = 0$.

Now, equation of the tangent at (a, b) to the given circle is

$$xa + yb - 3(x + a) + 2(y + b) - 12 = 0$$

$$\Rightarrow (a - 3)x + (b + 2)y - (3a - 2b + 12) = 0$$

If it represents the given line $4x - 3y + 7 = 0$

Then, $\frac{a-3}{4} = \frac{b+2}{-3} = \frac{-(3a-2b+12)}{7} = 1$

(say)

$\Rightarrow \quad a = 4l + 3, b = -3l - 2$

and $\quad -(3a - 2b + 12) = 7I$

$\Rightarrow \quad -3a + 2b - 12 = 7I$

$\Rightarrow \quad -3(4I + 3) + 2(-3/-2) - 12 = 7I$

$\Rightarrow \quad I = -1$

$\therefore \quad a = -1$ and $b = 1$

130. **(d)** Given equation of circle,

$x^2 + y^2 - 6x + 8y - 144 = 0$

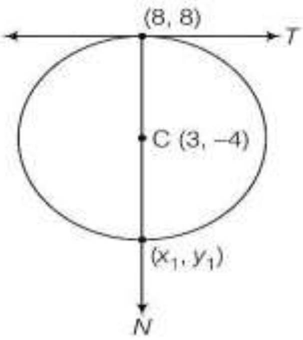
Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \Rightarrow g = -3$

$2f = 8 \Rightarrow f = 4$

Centre $(-g, -f) = (3, -4)$

We know that, normal to a circle passes through its centre.



Let (x_1, y_1) be the point at which normal meets, so

$$\frac{x_1 + 8}{2} = 3 \text{ and } \frac{y_1 + 8}{2} = -4$$

$\Rightarrow \quad x_1 + 8 = 6$ and $y_1 + 8 = -8$

$\Rightarrow \quad x_1 = -2$ and $y_1 = -16$

$\therefore$ Required point is $(-2, -16)$,

131. **(d)** The polar of point $P(2k, k - 4)$ with respect to the circle,

$x^2 + y^2 - 4x - 6y + 1 = 0$ is

$2kx + y(k - 4) + (-2)(x + 2k)$

$+ (-3)\{y + (k - 4)\} + 1 = 0$

$k(2x + y - 7) - 7y - 2x + 13 = 0$ ... ... (i)

Hence, (3,1) satisfies Eq. (i).

132. **(d)** The equation of the circle are

$x^2 + y^2 - 2\lambda x - 2y - 7 = 0$..... (i)

and $3(x^2 + y^2) - 8x + 29y = 0$

$\Rightarrow x^2 + y^2 - \frac{8}{3}x + \frac{29}{3}y = 0$ ...... (ii)

On comparing Eq. (i) and Eq. (ii) with
$x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$g_1 = -\lambda, f_1 = -1, c_1 = -7$
And $g_2 = -\frac{4}{3}, f_2 = \frac{29}{6}, c_2 = 0$
Since, circles are orthogonal, then
$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$
$$\Rightarrow \quad 2(-\lambda)\left(\frac{-4}{3}\right) + 2(-1)\left(\frac{29}{6}\right) = -7 + 0$$
$$\Rightarrow \quad \frac{8}{3}\lambda - \frac{29}{3} = -7 \Rightarrow \frac{8}{3}\lambda = -7 + \frac{29}{3}$$
$$\Rightarrow \quad \frac{8}{3}\lambda = \frac{-21+29}{3} \Rightarrow \lambda = \frac{8}{3} \times \frac{3}{8}$$
$$\therefore \quad \lambda = 1$$

133. **(d)** Given,
$$C_1 = x^2 + y^2 - 1 = 0 \ldots\ldots \text{(i)}$$
$$C_2 = x^2 + y^2 - 2x - 3 = 0 \ldots\ldots \text{(ii)}$$
$$C_3 = x^2 + y^2 - 2y - 3 = 0 \ldots\ldots \text{(iii)}$$
On solving Eqs. (i) and (ii), we get
$$2x + 2 = 0$$
$$\Rightarrow \quad x = -1$$
On solving Eqs. (i) and (iii), we get
$$2y + 2 = 0$$
$$\Rightarrow \quad y = -1$$
Hence, the radical centre of the given circles is $(-1, -1)$.

134. **(c)** Given, $y^2 = x$
$$\therefore \ 4a = 1 \Rightarrow a = \frac{1}{4}$$
Any normal to the parabola is
$y = mx - 2am - am^3$ ….. (i)
If it passes through $(C, 0)$, then
$$0 = mC - \frac{1}{2}m - \frac{m^3}{4} \quad \left\{a = \frac{1}{4}\right\}$$
$$\Rightarrow \ m = 0 \text{ and } \frac{m^2}{4} = C - \frac{1}{2}$$
$$\therefore \ C > \frac{1}{2}$$

135. **(c)** Given equation of parabolas
$y^2 = 5x$ ….. (i)
and $x^2 = 5y$ …. (ii)
Now, $x^2 = 5y$
$$\Rightarrow \ y = \frac{x^2}{5}$$
On substituting $y = \frac{x^2}{5}$ in Eq. (i), we get
$$\left(\frac{x^2}{5}\right)^2 = 5x$$
$$\Rightarrow \ \frac{x^4}{25} = 5x$$
$$\Rightarrow \quad x^4 = 125x$$
$$\Rightarrow \quad x^4 - 125x = 0$$
$$\Rightarrow \quad x(x^3 - 125) = 0$$
On solving Eqs. (i) and (ii), we get
$$x = 0 \text{ or } x = 5$$
$$\therefore \quad y = 0 \text{ or } y = 5$$

Hence, the point of intersection (0,0) and (5,5) lie on, line $x - y = 0$.

136. **(d)** Given equation of ellipse

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{16} = 1$$

Let $(x-3) = X$ and $(y-2) = Y$, then

$$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$

Here, $a = 5$ and $b = 4$.

Also, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Now, equation of the major axis $Y = 0$

$\Rightarrow y - 2 = 0$

$\therefore y = 2$

So, (i) → (q)

Equation of a directrix

$$x = \frac{a}{e} = \frac{5}{3} \times 5 = \frac{25}{3}$$

$$\Rightarrow x - 3 = \frac{25}{3}$$

$$\Rightarrow x = \frac{25}{3} + 3 = \frac{34}{3}$$

$\Rightarrow 3x = 34$

So, (ii) → (p)

Equation of a latusrectum,

$X = \pm ae$

$\Rightarrow x - 3 = \pm ae$

$$\Rightarrow x - 3 = \pm 5 \times \frac{3}{5}$$

$\Rightarrow x = \pm 3 + 3$

$\therefore x = 6,0$

So, (iii) → (s)

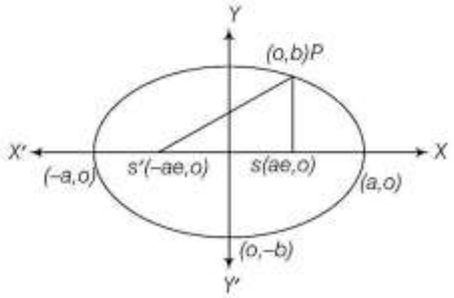
137. **(a)** Given equation of ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Here, $a = 5$ and $b = 4$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}}$$

$$= \sqrt{\frac{9}{25}} = \frac{3}{5}$$



Also, $PS + PS' = 2a = 2 \times 5 = 10$

$\Rightarrow PS' = 10 - 8\ [\because SP = 8]$

$\Rightarrow PS' = 2$

Now, $SS' = 2ae = 2 \times 5 \times \frac{3}{5} = 6$

From options,

$SS' = 4 + S'P$

138. **(a)** The normal at point $A(2\sec\theta, 3\tan\theta)$ is

$$2x\cos\theta + 3y\cot\theta = 4 + 9$$

$2x\cos\theta + 3y\cot\theta = 13$ .... (i)

And the normal at point $B(2\sec\phi, 3\tan\phi)$ is

$$2x\cos\theta + 3y\cot\theta = 4 + 9$$

$\Rightarrow 2x\cos\phi + 3y\cot\phi = 13$ .... (ii)

Multiplying Eq. (i) by $\cos\phi$ and Eq. (ii) by $\cos\theta$, then subtracting Eq. (ii) from Eq. (i), we get

$$3y\cot\theta\cos\phi - 3y\cot\phi\cos\theta = 13(\cos\phi - \cos\theta)$$

$$\Rightarrow 3y\left\{\cot\left(\frac{\pi}{2} - \phi\right)\cos\phi - \cot\phi\cos\left(\frac{\pi}{2} - \phi\right)\right\}$$

$$= 13\left\{\cos\phi - \cos\left(\frac{\pi}{2} - \phi\right)\right\}$$

$$\Rightarrow 3y\{\tan\phi\cos\phi - \cot\phi\sin\phi\} = 13(\cos\phi - \sin\phi)$$

$$\Rightarrow 3y(\sin\phi - \cos\phi) = 13(\cos\phi - \sin\phi)$$

$$\Rightarrow 3y = \frac{-13(\sin\phi - \cos\phi)}{\sin\phi - \cos\phi}$$

$$\therefore y = -\frac{13}{3}$$

Hence, the value of β is $-\frac{13}{3}$.

139. **(d)**

140. **(b)** Given,

$$I_1 = -\frac{2}{\sqrt{21}}, m_1 = \frac{C}{\sqrt{21}}, n_1 = \frac{1}{\sqrt{21}}$$

and $$I_2 = \frac{3}{\sqrt{54}}, m_2 = \frac{3}{\sqrt{54}}, n_2 = \frac{-6}{\sqrt{54}}$$

$\because$ $\cos\theta = I_1 I_2 + m_1 m_2 + n_1 n_2$

$\therefore$

$$\cos\frac{\pi}{2} = -\frac{2}{\sqrt{21}} \times \frac{3}{\sqrt{54}} + \frac{C}{\sqrt{21}} \times \frac{3}{\sqrt{54}} + \frac{1}{\sqrt{21}} \times -\frac{6}{\sqrt{54}}$$

$$\Rightarrow \quad 0 = \frac{-6}{\sqrt{21}\sqrt{54}} + \frac{3C}{\sqrt{21}\sqrt{54}} - \frac{6}{\sqrt{21}\sqrt{54}}$$

$$\Rightarrow \quad \frac{12}{\sqrt{21}\sqrt{54}} = \frac{3C}{\sqrt{21}\sqrt{54}}$$

$\Rightarrow$ $3C = 12$

$\therefore$ $C = 4$

141. **(c)** Let, $(x_1, y_1, z_1) = (5,2,6)$

and $a = 1, b = 1, c = 1$ and $d = -9$

$$\therefore \frac{x-5}{1} = \frac{y-2}{1} = \frac{z-6}{1} = \frac{-2(5+2+6-9)}{1+1+1}$$

Now, $x - 5 = \frac{-2\times4}{3} \Rightarrow x = \frac{-8}{3} + 5 = \frac{7}{3}$

and $y - 2 = \frac{-8}{3}$

$$\Rightarrow y = \frac{-8}{3} + 2 = -\frac{2}{3}$$

and $$z - 6 = \frac{-8}{3}$$

$$\Rightarrow \quad z = \frac{-8}{3} + 6 = \frac{10}{3}$$

$\therefore$ Image of the point $(5,2,6)$ is $\left(\frac{7}{3}, -\frac{2}{3}, \frac{10}{3}\right)$.

142. **(d)** Given, $\lim_{x\to\infty}\left[\frac{x^2+x+3}{x^2-x+2}\right]^x$

$$=\lim_{x\to\infty}\left[1-1+\frac{x^2+x+3}{x^2-x+2}\right]^x$$

$$=\lim_{x\to\infty}\left[1+\frac{x^2+x+3-x^2+x-2}{x^2-x+2}\right]^x$$

$$=\lim_{x\to\infty}\left[1+\frac{2x+1}{x^2-x+2}\right]^x$$

$$=e^{\lim_{x\to\infty}\frac{2x+1}{x^2-x+2}\cdot(x)}$$

$$\left[\because \lim_{x\to\infty}[1+f(x)]^{g(x)}=e^{\lim_{x\to\infty}f(x)\cdot g(x)}\right]$$

$$=e^{\lim_{x\to\infty}\frac{2x^2+x}{x^2-x+2}}=e^2$$

143. **(d)** At $x=0$, LHL $=\lim_{x\to 0^-}(1+|\sin x|)^{\frac{p}{\sin x}}$

$$=\lim_{x\to 0^-}(1-\sin x)^{\frac{p}{\sin x}}$$

$$\left[\because |\sin x|=-\sin x,\text{ for }\frac{-\pi}{6}<x<0\right]$$

$$=\lim_{h\to 0}(1+\sin h)^{\frac{-p}{\sin h}}$$

$$=e^{\lim_{h\to 0}(\sin h)\left(\frac{-p}{\sin h}\right)}=e^{-p}$$

$$\text{RHL }=\lim_{x\to 0^+}e^{\frac{\sin 2x}{\sin 3x}}$$

$$=e^{\lim_{x\to 0^+}\frac{\sin 2x}{\sin 3x}}=e^{\lim_{h\to 0}\frac{\sin 2h}{\sin 3h}}$$

$$=e^{\lim_{h\to 0}\frac{\sin 2h}{2h}\cdot\frac{1}{\frac{\sin 2h}{3h}}\cdot\frac{2}{3}}=e^{2/3}$$

$f(0)=q$

Since, $f(x)$ is continuous at $x=0$.

$\therefore$ LHL $=$ RHL $=f(0)$

$\Rightarrow\ e^{-P}=e^{2/3}=q$

$\Rightarrow\ p=\frac{-2}{3},q=e^{2/3}$

144. **(b)** Given,

$$y=\tan^{-1}\left[\frac{5\cos x-12\sin x}{12\cos x+5\sin x}\right]$$

$$=\tan^{-1}\left[\frac{\frac{5}{12}-\tan x}{1+\frac{5}{12}\tan x}\right]$$

[divide by $12\cos x$ in denominator and numerator]

$$=\tan^{-1}\left(\frac{5}{12}\right)-\tan^{-1}(\tan x)$$

$$y=\tan^{-1}\left(\frac{5}{12}\right)-x$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left\{\tan^{-1}\left(\frac{5}{12}\right)\right\}-\frac{d}{dx}(x)=0-1$$

$$\therefore\ \frac{dy}{dx}=-1$$

145. **(c)** Let $y=\tan^{-1}\left[\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right]$

$$=\tan^{-1}\left[\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$$

$$\left[\because \sin^2 x + \cos^2 x = 1; \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\right.$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

and $(a-b)^2 = a^2 + b^2 - 2ab$

$$= \tan^{-1}\left(\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right) = \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$\Rightarrow y = \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}$$

146. **(c)** Given,

$y = a\cos(\sin 2x) + b\sin(\sin 2x)$

On differentiating both sides w.r.t. x, we get

$$y' = -a\sin(\sin 2x)\cos 2x \cdot 2 + b\cos(\sin 2x) \cdot (\cos 2x) \cdot 2$$

$$y' = 2\cos 2x\{-a\sin(\sin 2x) + b\cos(\sin 2x)\}$$

Again on differentiating, we get

$$y'' = -4\sin 2x[-a\sin(\sin 2x) + b\cos(\sin 2x)] + 2\cos 2x[-a\cos(\sin 2x) \cdot \cos 2x \cdot 2 - b\sin(\sin 2x)\cos 2x \cdot 2]$$

$$= -4\sin 2x \cdot \frac{y'}{2\cos 2x} - 4(\cos^2 2x)y$$

$$= -2(\tan 2x)y' - 4(\cos^2 2x)y$$

$$\therefore y'' + 2(\tan 2x)y' = -4(\cos^2 2x)y$$

147. **(b)** Given, curves are

$x = a\cos^3 t$ ..... (i)

and $y = a\sin^3 t$ ...... (ii)

On differentiating Eq. (i) and Eq. (ii) w.r.t. t, we get

$$\frac{dy}{dt} = 3a\sin^2 t\cos t$$

and $\dfrac{dx}{dt} = -3a\cos^2 t\sin t$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t}$$

$$= \frac{-\sin t}{\cos t} = m_1$$

Equation of tangent at point $(a\cos^3 t, a\sin^3 t)$ is

$$y - a\sin^3 t = \frac{-\sin t}{\cos t}(x - a\cos^3 t)$$

$\Rightarrow y\cos t - a\cos t\sin^3 t = -x\sin t + a\cos^3 t\sin t$

$\Rightarrow x\sin t + y\cos t = a\cos t\sin t[\sin^2 t + \cos^2 t]$

$\Rightarrow x\sin t + y\cos t = a\cos t\sin t$

For finding the points cut off by the coordinate axes put $y = 0$ in Eq. (iii), we get

$\Rightarrow x\sin t = a\cos t\sin t$

$\Rightarrow x = a\cos t$

and put $x = 0$ in Eq. (iii), we get

$\Rightarrow y\cos t = a\cos t\sin t$

$\Rightarrow y = a\sin t$

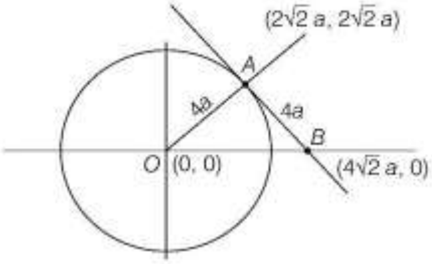
Hence, points are $A(a\cos t, 0)$ and $B(0, a\sin t)$,

$$\therefore \text{ Length of AB} = \sqrt{(a\cos t)^2 + (a\sin f)^2}$$

$$= \sqrt{a^2(\cos^2 t + \sin^2 t)} = \sqrt{a^2} = a$$

148. **(d)** Given curve is

$x^2 + y^2 = 16a^2$



On differentiating Eq. (i) w.r.t. x, we get

$2x + 2yy' = 0$

$y' = \frac{-x}{y}$

$\therefore\ (y')_{(2\sqrt{2}a, 2\sqrt{2}a)} = -1 = m_1$

and $m_2 = -\frac{1}{y'} = \frac{-1}{(-1)} = 1$

Equation of tangent is

$y - 2\sqrt{2}a = -1(x - 2\sqrt{2}a)$

$\Rightarrow\ y - 2\sqrt{2}a = -x + 2\sqrt{2}a$

$\Rightarrow\ x + y = 2\sqrt{2}a + 2\sqrt{2}a$

$\Rightarrow\ x + y = 4\sqrt{2}a$

And equation of normal

$y - 2\sqrt{2}a = 1(x - 2\sqrt{2}a)$

$\Rightarrow\ y - 2\sqrt{2}a = x - 2\sqrt{2}a$

$\Rightarrow\ x = y$

$\therefore$ Required area $=$ Area of $\triangle OAB$

$= \frac{1}{2} \times OA \times AB = \frac{1}{2} \times 4a \times 4a = 8a^2$

149. **(c)** Given,

$f(x) = \frac{1}{2}[|\sin x| + \sin x], 0 < x \leq 2\pi$

Case I, when, $0 < x \leq \pi$

$f(x) = \frac{1}{2}[\sin x + \sin x] = \sin x$

$f'(x) = \cos x$

$\cos x > 0$, for $0 < x < \frac{\pi}{2}$ (increasing)

$\cos x < 0$, for $\frac{\pi}{2} < x < \pi$(decreasing)

Case II When $\pi < x \leq 2\pi$

$f(x) = \frac{1}{2}[-\sin x + \sin x] = 0$

150. **(c)** Given, $f(x) = 9mx - 1 + \frac{1}{x} \geq 0$

Now, $9mx^2 - x + 1 \geq 0$

For $f(x) \geq 0$, D should be less than 0 .

$D = 1 - 36m$

$1 - 36m \leq 0$

$\therefore\ m \geq \frac{1}{36}$

151. **(a)** Let

$$I=\int \frac{x^2+1}{x^4+7x^2+1}dx$$

$$=\int \frac{1+\frac{1}{x^2}}{x^2+7+\frac{1}{x^2}}dx=\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+7}dx$$

$$=\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2+7}=\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+9}$$

Again, let $x-\frac{1}{x}=t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx=dt$$

$$\therefore I=\int \frac{dt}{t^2+(3)^2}=\frac{1}{3}\tan^{-1}\frac{t}{3}+C$$

$$=\frac{1}{3}\tan^{-1}\left(\frac{x-\frac{1}{x}}{3}\right)+C=\frac{1}{3}\tan^{-1}\left(\frac{x^2-1}{3x}\right)+C$$

152. **(c)** Let $I=\int \frac{x^3}{\sqrt{1+x^2}}dx$

$$=\int x^2\cdot\frac{x}{\sqrt{1+x^2}}dx$$

By using integration by parts

$$=x^2\int \frac{x}{\sqrt{1+x^2}}dx-\int \left\{2x\int \frac{x}{\sqrt{1+x^2}}dx\right\}dx$$

$$=x^2\sqrt{1+x^2}-\int 2x\sqrt{1+x^2}dx$$

$$=x^2\sqrt{1+x^2}-\frac{(1+x^2)^{3/2}}{\frac{3}{2}}+C$$

$$=x^2\sqrt{1+x^2}-\frac{2}{3}(1+x^2)^{3/2}+C$$

153. **(c)** Let $I=\int \frac{dx}{\cos(x+4)\cos(x+2)}$

$$=\frac{1}{\sin 2}\int \frac{\sin 2}{\cos(x+4)\cos(x+2)}dx$$

$$=\frac{1}{\sin 2}\int \frac{\sin[(x+4)-(x+2)]}{\cos(x+4)\cos(x+2)}dx=\frac{1}{\sin 2}$$

$$\int \frac{[\sin(x+4)\cos(x+2)-\cos(x+4)\sin(x+2)]}{\cos(x+4)\cos(x+2)}dx$$

$$=\frac{1}{\sin 2}\int \left[\frac{\sin(x+4)}{\cos(x+4)}-\frac{\sin(x+2)}{\cos(x+2)}\right]dx$$

$$=\frac{1}{\sin 2}\int [\tan(x+4)-\tan(x+2)]dx$$

$$=\frac{1}{\sin 2}\int \tan(x+4)dx-\frac{1}{\sin 2}\int \tan(x+2)dx$$

$$=\frac{1}{\sin 2}[\log\sec(x+4)-\log\sec(x+2)]+C$$

$$=\frac{1}{\sin 2}\log\left|\frac{\sec(x+4)}{\sec(x+2)}\right|+C$$

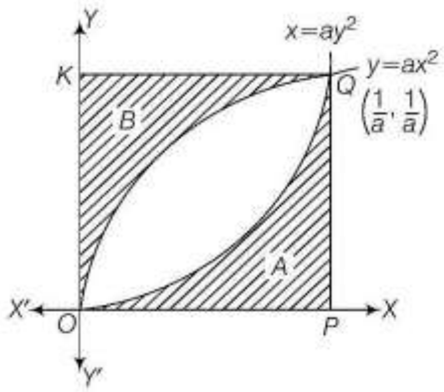
154. **(d)** Let $I=\int \frac{2x+2}{\sqrt{x^2-4x-5}}dx$

$$2x+2 = A\frac{d}{dx}(x^2-4x-5)+B$$
$$\Rightarrow \quad 2x+2 = A(2x-4)+B$$
$$\Rightarrow \quad 2x+2 = 2Ax-4A+B$$
Compare the coefficients of both sides, we get
$$2A = 2 \Rightarrow A = 1$$
and $\quad 2 = -4A+B$
$$\Rightarrow \quad 2 = -4+B$$
$$\Rightarrow \quad B = 6$$
$$\therefore \quad I = \int \frac{(2x-4)+6}{\sqrt{x^2-4x-5}}dx$$
$$\int \frac{2x-4}{\sqrt{x^2-4x-5}}dx + 6\int \frac{dx}{\sqrt{x^2-4x-5}}$$
Again, let $x^2-4x-5 = t$
and $(2x-4)dx = dt$
$$= \int \frac{dt}{\sqrt{t}} + 6\int \frac{dx}{\sqrt{(x-2)^2-9}}$$
$$= 2\sqrt{x^2-4x-5} + 6\log\left|(x-2)+\sqrt{x^2-4x-5}\right| + C$$

155. **(d)**

156. **(a)** $I = \int_0^{\pi/4} \sqrt{\tan x} + \sqrt{\cot x}dx$
$$= \int_0^{\pi/4} \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$$
$$= \int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{\sin x\cos x}}dx$$
$$= \int_0^{\pi/4} \frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{2\sin x\cos x}}dx$$
$$= \sqrt{2}\int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}}dx$$
Let $\sin x - \cos x = t$
$$\Rightarrow (\cos x + \sin x)dx = dt$$
$$\therefore I = \sqrt{2}\int_0^{\pi/4} \frac{dt}{\sqrt{1-t^2}}$$
$$= \sqrt{2}[\sin^{-1} t]_0^{\pi/4}$$
$$= \sqrt{2}[\sin^{-1}(\sin x - \cos x)]_0^{\pi/4}$$
$$= \sqrt{2}\left[\sin^{-1}\left(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right) - \sin^{-1}(\sin 0° - \cos 0°)\right]$$
$$= \sqrt{2}[\sin^{-1} 0 - \sin^{-1}(-1)]$$
$$= \sqrt{2}\left[0 + \frac{\pi}{2}\right] = \sqrt{2}\frac{\pi}{2} = \frac{\pi}{\sqrt{2}}$$

157. **(b)** Given curves are
$y = ax^2$ ….. (i)
and $x = ay^2$…… (ii)
Put the value of y by Eq. (ii) in Eq. (i), we get



$$\frac{\sqrt{x}}{\sqrt{a}}=ax^2 \Rightarrow \frac{\sqrt{x}}{\sqrt{a}}-ax^2 \quad =0$$

$$\Rightarrow \quad \sqrt{x}\left(\frac{1}{\sqrt{a}}-ax^{3/2}\right) \quad =0$$

$$\Rightarrow \quad \sqrt{x}\left(1-a^{3/2}x^{3/2}\right) \quad =0 \Rightarrow x=0,\frac{1}{a}$$

When, $x=0, y=0$

and $x=\frac{1}{a}, y \quad =\frac{1}{a}$

Since, shaded part lies in first quadrant, hence points of intersection of curves $y=ax^2$ and $x=ay^2$ are (0,0) and $\left(\frac{1}{a},\frac{1}{a}\right)$.

∴ Required area

$$\Rightarrow \quad 3=\int_0^{\frac{1}{a}}\frac{\sqrt{x}}{\sqrt{a}}dx-\int_0^{\frac{1}{a}}ax^2dx$$

$$\Rightarrow \quad 3=\frac{1}{\sqrt{a}}\int_0^{\frac{1}{a}}\sqrt{x}dx-a\int_0^{\frac{1}{a}}x^2dx$$

$$\Rightarrow \quad 3=\frac{1}{\sqrt{a}}\frac{2}{3}\left[x^{3/2}\right]_0^{\frac{1}{a}}-a\left[\frac{x^3}{3}\right]_0^{\frac{1}{a}}$$

$$\Rightarrow \quad 3=\frac{2}{3\sqrt{a}}\left[\left(\frac{1}{a}\right)^{3/2}\right]-\frac{a}{3}\left[\left(\frac{1}{a}\right)^3\right]$$

$$\Rightarrow \quad 3=\frac{2}{3\sqrt{a}}\times\frac{1}{a\sqrt{a}}-\frac{a}{3}\times\frac{1}{a^3}$$

$$\Rightarrow \quad 3=\frac{2}{3a^2}-\frac{1}{3a^2}$$

$$\Rightarrow \quad 3=\frac{2-1}{3a^2}\Rightarrow 9a^2=1$$

$$\Rightarrow \quad a^2=\frac{1}{9}\Rightarrow a=\frac{1}{3}$$

[since, shaded region lies in first quadrant] Hence, value of $a=\frac{1}{3}$.

158. **(b)** Given, family of curve $y=(\alpha+\beta x)e^{px}$ …. (i)

On differentiating Eq. (i) w.r.t. x, we get

$$\frac{dy}{dx}=\beta\cdot e^{px}+pe^{px}(\alpha+\beta x)$$

$Y=\beta.e^{px}+py$ …… (ii)

[by Eq. (i)]

On differentiating Eq. (ii) w.r.t. x, we get

$\Rightarrow \quad \frac{d^2y}{dx^2} = \beta \cdot pe^{px} + p\frac{dy}{dx}$

$\Rightarrow \quad y'' = \beta \cdot pe^{px} + py'$

$\Rightarrow \quad y'' = \frac{(y' - Py)}{e^{px}} \cdot pe^{px} + py'$

[by Eq. (ii), $\beta = \frac{(y'-py)}{e^{px}}$]

$\Rightarrow y'' = (y' - py) \cdot p + py'$

$\Rightarrow \quad y'' = py' - p^2y + py'$

$\Rightarrow \quad y'' + p^2y - 2py' = 0$

$\therefore y'' - 2py' + p^2y = 0$

159. **(a)**

160. **(a)** Given differential equation is $y' = \frac{1}{e^{-y}-x}$

$\frac{dy}{dx} = \frac{1}{e^{-y} - x}$

We can write this equation in the form

$\frac{dx}{dy} = e^{-y} - x$

$\frac{dx}{dy} + x = e^{-y} \ldots\ldots (i)$

Compare Eq. (i) by $\frac{dx}{dy} + Px = Q$, we get

$P = 1, Q = e^{-y}$

$\therefore \quad \text{If } = e^{\int Pdy} = e^{\int 1dy} = e^y$

Solution of given differential equation is

$x \cdot e^y = \int e^y \cdot e^{-y} dy + C$

$\Rightarrow \quad x \cdot e^y = \int 1 \cdot dy + C$

$\Rightarrow \quad xe^y = y + C$

$\therefore \quad x = e^{-y}(y + C)$