Physics

1. **(b)** Given,

Percentage error in length = 3%

Percentage error in force = 4%

We know that,

Pressure, $P = \frac{F}{A}$

Area of square $= (L)^2$

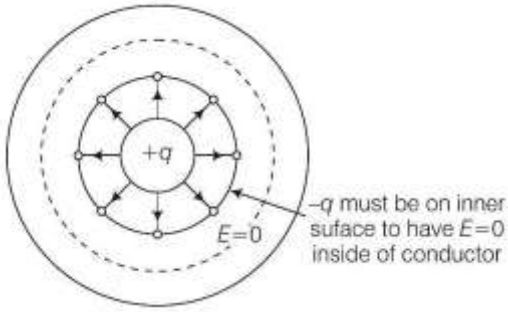
$$P = \frac{F}{L^2}$$

$$\frac{\Delta P}{P} = \frac{\Delta F}{F} + \frac{2\Delta L}{L}$$

$$\left(\frac{\Delta P}{P} \times 100\right) = \left(\frac{\Delta F}{F} \times 100\right) + 2\left(\frac{\Delta L}{L} \times 100\right)$$

$$= 4\% + 2(3\%) = 10\%$$

2. **(b)**



Total charge on shell = +Q

Here, total charge on shell is +Q. So, the (−Q) is on the inner surface of shell.

Hence, electric field inside the conductor = 0

According the Gauss's law,

$$\oint Eds = \frac{Q}{\varepsilon_0}$$

$$E\oint ds = \frac{Q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

Electric field, $E = \frac{Q}{4\pi r^2 \varepsilon_0}$

3. **(b)** Given,

Width of river (w) = 200 m

Velocity of river = 2 m/s

Velocity of man = 1 m/s

We know that,

$$\frac{w}{v_{man}} = \frac{d}{v_{strem}}$$

$$d = \frac{w \times v_{strem}}{v}$$

$$d = \frac{200 \times 2}{1}$$

$$d = 400 \text{ m}$$

4. **(b)** Given,

Number of turns in coil = n

We know that,

$\text{Current(i)} = \frac{e}{R'} = \frac{-\Delta\phi}{R'\Delta t}\ \left[\because e = \frac{-\Delta\phi}{\Delta t}\right]$

$R' = R + 4R$

$i = -\frac{(\phi_2 - \phi_1)}{(R + 4R)\Delta t}$

$i = \frac{(\phi_2 - \phi_1)}{5R(\Delta t)}$

Here, coil have n number of turns.

Hence, $i = -\frac{n(\phi_2 - \phi_1)}{5R(\Delta t)}$

Induced current, $i = -\frac{n(\phi_2 - \phi_1)}{5Rt}$

5. **(d)** Given,

Length of simple pendulum $(L) = 1\ \text{m}$

Acceleration $(a) = 2\ \text{m/s}^2$

Gravitation acceleration $(g) = 10\ \text{m/s}^2$

We know that,

$$T = 2\pi\sqrt{\frac{l}{g + a}} = 2\pi\sqrt{\frac{1}{10 + 2}}$$

$$= 2\pi\sqrt{\frac{1}{12}} = \pi\sqrt{\frac{4}{12}}$$

$I = \frac{\pi}{\sqrt{3}}\ S$

6. **(c)** Given,

Radius of metal ball $= r$

Velocity $= V$

Angle between direction of velocity and magnetic field $(B) = \alpha$

We know that,

$e = Blv\sin\alpha$

Here, $l = 2r$

$e = 2r|B||v|\sin\alpha$

7. **(d)** Resistance $= 4\Omega$

The equivalent emf of the combination

$E_{eq} = nE = 3 \times 20 = 60$

The cell have no internal resistance.

$\therefore$ Main current flowing through the load

$i = \frac{nE}{R} = \frac{60}{4} = 15\ \text{A}$

8. **(d)** We know that,

We have, $\lambda = \frac{h}{\sqrt{2mk}}$

$\therefore$ Kinetic energy (KE) $\propto \frac{1}{\lambda^2}$

Hence,

$\frac{KE_2}{KE_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^2$

Here, $\lambda_1 = 1\ \text{nm}$

$\lambda_2 = 0.5\ \text{nm}$

$\frac{KE_2}{KE_1} = \left(\frac{1}{0.5}\right)^2$

$KE_2 = 4KE_1$

Hence, the kinetic energy is increases three times.

$\Delta KE = 3KE_1$

9. (b) The net charge on upper plates both capacitor (2μ F and 3μ F) will be +80μC, so charge on upper plate of 3μC capacitor will be

$$q=\left(\frac{3}{3+2}\right)(80\mu C)$$

$$q=\frac{3}{5}\times 80=48\mu C$$

10. (d) Given,

Young's modulus of material $=2\times 10^{11}\ N/m^2$

Elastic limit or stress $=1\times 10^{8}\ N/m^2$

Length of the wire $=1$ m

We know that, $Y=\frac{\text{Stress}}{\text{Strain}}\Rightarrow Y=\frac{\text{Stress}}{\frac{\Delta L}{l}}$

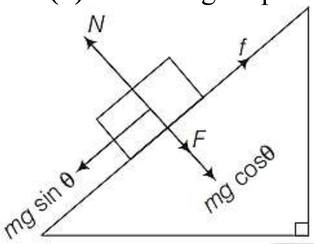
Here, $l=$ original length of the wire

$\Delta L=$ charge in the length of the wire

$$\Delta L=\frac{\text{Stress }\times l}{Y}=\frac{1\times 10^{8}\times 1}{2\times 10^{11}}$$

$$=0.5\times 10^{-3}=0.5\text{ mm}$$

11. (d) According to question, we can draw the following diagram.



$$mg\sin\theta=\mu(F+mg\cos\theta)$$

$$F=\frac{mg\sin\theta}{\mu}-mg\cos\theta$$

$$=mg\left[\frac{\sin\theta}{\mu}-\cos\theta\right]$$

Here,

$m=1$ kg, $g=10\ m/s^2, \theta=30°, \mu=0.2$

$$F=1\times 10\left[\frac{\sin 30}{0.2}-\cos 30°\right]$$

$$=10\left[\frac{1}{2\times 0.2}-\frac{\sqrt{3}}{2}\right]$$

$$=10\left[\frac{5}{2}-\frac{\sqrt{3}}{2}\right]=5[5-\sqrt{3}]$$

$$=5[5-1.732]=16.34\text{ N}$$

12. (a) Given, Coefficient of linear expansion of steel,

$\alpha=11\times 10^{-6}/°C$

We know that,

$$\Delta l=l\alpha\Delta t\Rightarrow \Delta t=\frac{\Delta l}{l\alpha}$$

Here, $\Delta l=6\times 10^{-5}$ m $\Rightarrow l=1$ m

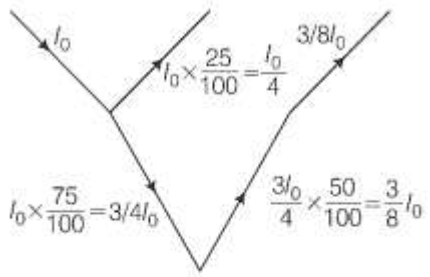
$$\Delta t=\frac{6\times 10^{-5}}{1\times 11\times 10^{-6}}$$

$$=5.45°C$$

So, the range of temperature in which the experiment can be hence performed him this metre scale will be 19°C to 31°C

13. (a) When the core is pulled out the solenoid, the change in current will remain same.

14. (a) According to question, we can draw the following diagram



$I_1=\frac{I_0}{4}\Rightarrow I_2=\frac{3}{8}I_0$

We know that,

$$\frac{I_{max}}{I_{min}}=\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}=\left(\frac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$$

15. (d) Given,

Total wall area (including the lid) (A) $= 1.0\ cm^2$

Thickness of wall (l) $= 3\ cm = 3\times10^{-2}\ m$

Average temperature outside the box $= 30^\circ C$

$$\Delta\theta = 30-0=30^\circ C$$

$L_{fusion\ (ice)} = 3\times10^5\ J/kg$

$K_{thermocol} = 0.03\ W/m\ K$

We know that,

$$\frac{Q}{t}=\frac{KA}{l}\Delta\theta$$

For one day, $t = 24\times60\times60\ s$

$$\frac{m\times L_{fusion\ (ice)}}{t}=\frac{KA}{l}\Delta Q$$

$$\frac{m\times3\times10^5}{24\times60\times60}=\frac{0.03\times1}{3\times10^{-2}}\times30$$

$$m=\frac{0.03\times1\times30\times24\times60\times60}{3\times10^{-2}\times3\times10^5}$$

$$m=\frac{77760}{9000}=8.64\ kg$$

16. (d) ${}^{239}_{94}Pu\xrightarrow{\alpha}{}^{235}_{92}U$

When α-particle is emitted that atomic mass is decreases by four and atomic number is decreases by two.

17. (d) Given, $E = 10\ V$

$\omega = 200 rad/s$

$R = 50\Omega$

$L = 400mH$

$C = 200\mu\ F$

We know that,

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$

$$=\sqrt{(50)^2+(80-25)^2}=\sqrt{(50)^2-(55)^2}$$

$$=\sqrt{2500+3025}$$

$$Z=\sqrt{5525}=74.3\Omega$$

$$I=\frac{E}{Z}=\frac{10}{74.3}=0.13459\ A$$

$$E_L=IX_L=0.13459\times80$$

$$=10.76\ V \text{ or } 10.8\ V$$

18. (a) Given,

$t_1 = 30°C$
$R_1 = 3.1\Omega$
$t_2 = 100°C$
$R_2 = 4.5\Omega$
We know that,
Temperature coefficient of resistance,

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1} = \frac{4.5 - 3.1}{(3.1)(100) - (4.5 \times 30)}$$
$$= \frac{1.4}{310 - 135} = \frac{1.4}{175} = 0.008°C^{-1}$$

19. (a) According to the question,
Velocity of an object half a second before maximum height
= Velocity of an object half a second after maximum height (return journey)

$$= 0 + gt = 0 + 9.8 \times \frac{1}{2} = 4.9 \text{ m/s}$$

20. (b) According to the question,
Energy given to hydrogen atom = Difference in energy level from $n_1 = 1$ to $n_2 = 3$ states

$$= 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 13.6\left(1 - \frac{1}{9}\right)$$
$$= 13.6 \times \frac{8}{9} = 12.08 \approx 12.1\text{eV}$$

21. (a) Given,
$x = a\cos t$,
$y = a\sin t$
and $z = t$
Path is circular in xy - plane,

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = \cos^2\theta + \sin^2\theta$$
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In one revolution, the particle move a distance of 1 unit along z-axis.

$$\frac{dz}{dt} = 1$$

Hence, path is helix.

22. (a) Given

$$\eta_1 = \frac{1}{6}$$

According to the question,

$$\eta_1 = \frac{T_1 - T_2}{T_1} \quad \frac{T_1 - T_2}{T_1} \quad \frac{1}{6} \text{ ..... (i)}$$
$$\eta_2 = \frac{T_1 - (T_2 - 62)}{T_1} \Rightarrow 2 \times \eta_1 = \frac{T_1 - T_2 + 62}{T_1}$$
$$2 \times \frac{1}{6} = \frac{T_1 - T_2 + 62}{T_1} \Rightarrow \frac{1}{3} = \frac{T_1 - T_2 + 62}{T_1} \text{ ..... (ii)}$$

From Eq. (i), we get

$$T_1 - T_2 = \frac{T_1}{6}$$

Substituting this value in Eq. (ii), we get

$$\frac{1}{3} = \frac{\frac{T_1}{6} + 62}{T_1} \Rightarrow \frac{1}{3} = \frac{T_1 + 372}{6T_1}$$
$$6T_1 = 3T_1 + 1116 \Rightarrow 3T_1 = 1116 \Rightarrow T_1 = 372 \text{ K}$$

From Eq. (i), we get

$$\frac{372 - T_2}{372} = \frac{1}{6}$$
$$6(372 - T_2) = 372$$
$$2232 - 6T_2 = 372$$
$$6T_2 = 2232 - 372$$
$$6T_2 = 1860$$
$$T_2 = \frac{1860}{6}$$
$$T_2 = 310\text{ K}$$

23. (a) Given,

Distance of light source (u) $= -1.5\text{ m} = -\frac{-3}{2}\text{ m}$

Radius of curvature of mirror (R) $= 1$ m

We know that,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \qquad \left(f = \frac{R}{2}\right)$$
$$\therefore \frac{2}{R} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{2}{1} = \frac{1}{v} + \frac{1}{\frac{-3}{2}}$$
$$\frac{2}{1} = \frac{1}{v} - \frac{2}{3} \Rightarrow \frac{1}{v} = 2 + \frac{2}{3}$$
$$\frac{1}{v} = \frac{6+2}{3} \Rightarrow \frac{1}{v} = \frac{8}{3} \Rightarrow v = \frac{3}{8} = 0.375\text{ m}$$

The image is virtual.

Magnification (m) $= \frac{-v}{u} = \frac{\frac{-3}{8}}{\frac{-3}{2}} = \frac{1}{4} = 0.25$

24. (c) Given,

Number of moles of air in room(n) $= 2000 = 2 \times 10^3$

Temperature difference (dT) $= 24 - 34 = -10°\text{C}$

We know that,

$$dQ = nC_VdT = 2 \times 10^3 \times \frac{R}{0.4}[-10]$$
$$= \frac{-2 \times 10^3 \times 8.314 \times 10}{0.4}$$
$$= \frac{2 \times 8.314 \times 10^5}{4} = \frac{16.628}{4} \times 10^5$$
$$= -4.2 \times 10^5\text{ J}$$

25. (d) Given,

Speed of ball (u) $= 20$ m/s

Angle (θ) $= 30°$

We know that,

$$H = \frac{u^2 \sin^2\theta}{2g}$$
$$H = \frac{(20)^2(\sin 30)^2}{2 \times 10}$$
$$\Rightarrow H = \frac{20 \times 20 \times \left(\frac{1}{2}\right)^2}{20}$$
$$\Rightarrow H = 20 \times \frac{1}{4}$$
$$H = 5\text{ m}$$

26. (c) Given,

Diameter of pipe (d) $= 5\text{ mm} = 5 \times 10^{-3}$ m

Density of gasoline (ρ) $= 720\text{ kg/m}^3$.

Viscocity of gasoline $(\eta) = 6 \times 10^{-3}$ Poise

We know that, $v_C = \frac{\eta}{\rho d} = \frac{6\times10^{-3}}{720\times5\times10^{-3}}$

$= \frac{6 \times 10^{-3}}{3600 \times 10^{-3}} = \frac{1}{600} = \frac{1}{6} \times 10^{-2}$

$= 1.66 \times 10^{-3}$ m/s

27. (c) Copper is a conductor and germanium is semiconductor. When both are cooled from room temperature to 80 K . Then, resistance of copper will decrease while that of germanium will increase.

28. (a) We know that,

$\lambda = \frac{\lambda_0}{\mu}$

According to the question,

$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2}$

$\lambda_2 = \lambda_1 \frac{\sin \alpha_2}{\sin \alpha_1}$

29. (b) Given,

Frequency of message signal$(f_m) = 1$KHz

$= 1 \times 10^3$ Hz

Peak voltage of message signal $(E_m) = 5$ V

Carrier frequency $(f_C) = 1$MHz $= 1 \times 10^6$ Hz

Peak voltage of carrier $(E_C) = 15$ V

We know that, The equation of amplitude modulated wave is given by

$$e(t) = E_C\left[1 + \frac{E_m}{E_C}\sin\omega_m t\right]\sin\omega_C t$$

$$= E_C\left[1 + \frac{E_m}{E_C}\sin(2\pi f_m)t\right]\sin 2\pi f_C t$$

$$= 15\left[1 + \frac{5}{15}\sin(2\pi \times 10^3)t\right]\sin 2\pi \times 10^6 t$$

$$= 15\left[1 + \frac{1}{3}\sin(2\pi 10^3 t)\right]\sin(2\pi 10^6 t)$$

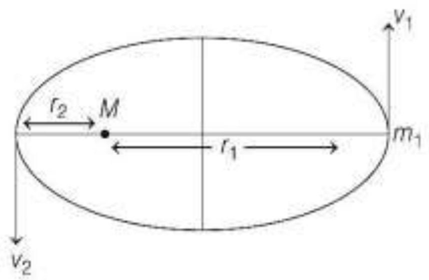
30. (d) SONAR technology is based on the reflection of ultrasonic waves.

31. (b) Given, Velocity of particle = v

Change in momentum = final momentum - initial momentum = $Mv - (-Mv) = 2Mv$

32. (a) When billiard ball collide elastically, they will exchange the kinetic energy, so they may move in straight path or at any angle.

33. (a)



According to the law of conservation of angular momentum,

$mv_1r_1 = mv_2r_2$

$V_2 = \frac{V_1r_1}{r_2}$ …… (i)

From the law of conservation of total mechanical energy.

$\frac{-GMm}{r_1} + \frac{1}{2}mv_1^2 = -\frac{GMm}{r_2} + \frac{1}{2}mv_2^2$ …. (ii)

From Eqs. (i) and (ii), we get

$$v_1 = \sqrt{\frac{2GMr_2}{(r_1 + r_2)r_1}}$$

Angular momentum,

$$L = mv_1r_1 = m\left(\sqrt{\frac{2GMr_2}{(r_1 + r_2)r_1}}\right) \times r_1$$

$$L = m\sqrt{\frac{2GMr_1r_2}{r_1 + r_2}}$$

34. (a) From the conservation of energy,

$mgh = \mu mgd$

$$d = \frac{h}{\mu}$$

35. (a) Given,

Number of turns in circular coil $= 100$

$Area(A) = 2 \times 10^{-2}\ m^2$

Magnetic field $(B) = 0.01\ T$

Frequency of rotation $(f) = 50\ Hz$

We know that,

$e = NBA\omega = NBA(2\pi f)$

$$= 100 \times 0.01 \times 2 \times 10^{-2} \times 2 \times \frac{22}{7} \times 50$$

$= 6.28\ V$

36. (c) The frequency of sound wave remains constant.

37. (c) The majority charge carriers in n-types semiconductor are electrons.

38. (d) Deceleration $\omega = -a\sqrt{v}$.

But $\omega = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = -a\sqrt{v}$

$$\frac{-dv}{\sqrt{v}} = a \cdot dt \Rightarrow \int_{v_i}^{0} \frac{dv}{\sqrt{v}} = \int adt$$

$$|2\sqrt{v}|_{v_i}^{0} = at \Rightarrow t = \frac{2}{a}\sqrt{v_iS}$$

Again,

$$\frac{-dv}{dt} = a\sqrt{v} \Rightarrow \frac{dv}{dx} \cdot \frac{dx}{dt} = -a\sqrt{v}$$

$$\frac{dv}{dx} \cdot v = -a\sqrt{v} \Rightarrow dv\sqrt{v} = -a \cdot dx$$

$$\int_{v_0}^{0} \sqrt{v}dv = -a\int_{0}^{s} ds$$

After solving, we get

$$s = \frac{2}{3a} \cdot v_0^{\frac{3}{2}}$$

39. (c) A current carrying conductor produces magnetic field in its neighbourhood.

40. (a) Given,

$F_1 = (\hat{i} + 2\hat{j} + 3\hat{k})N$

$F_2 = (4\hat{i} - 5\hat{j} - 2\hat{k})N$

$r_1 = 20\hat{i} + 15\hat{j}$ cm

$r_2 = 7\hat{k}$ cm

Total force on particle, $F = F_1 + F_2$

$= \hat{i} + 2\hat{j} + 3\hat{k} + 4\hat{i} - 5\hat{j} - 2\hat{k}$

$= (5\hat{i} - 3\hat{j} + \hat{k})N$

Displacement of particle, $s = r_2 - r_1$

$= 7\hat{k} - 20\hat{i} - 15\hat{j}$
$= (-20\hat{i} - 15\hat{j} + 7\hat{k})\text{cm}$

We know that,

$\text{Work}(\omega) = F \cdot s$
$= (5\hat{i} - 3\hat{j} + \hat{k})(-20\hat{i} - 15\hat{j} + 7\hat{k}) \times 10^{-2}$
$= (-100 + 45 + 7) \times 10^{-2}$
$= -0.48\ \text{J}$

Chemistry

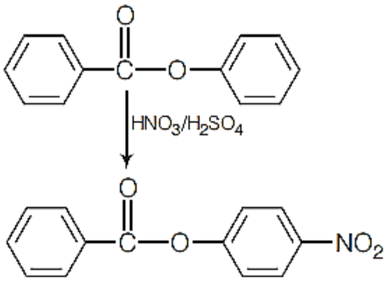
41. (d) Conditions that are correct for real solutions showing negative deviation from Raoult's law are as follows

$p_A < p_A{}^\circ x_A$ and $p_B < p_A{}^\circ x_B$

$\Delta_{mix} H = -\text{ve}$

Dissolution is exothermic heating decreases solubility $\Delta_{mixing} V = -\text{ve}$

42. (c)



It is an electrophilic aromatic substitution reaction. Because the COO^-acts as an electron withdrawing group to one phenyl group and a moderate electron donating group to the other phenyl group the nitration will occur on the phenyl group where $COO^-$is acting as an electron donating group.

The major product of the nitration will have the $-NO_2$ in the para position because the ortho position is partially blocked by the large COPh group.

43. (c) Electronic configuration of $_{59}Pr$ (praseodimium).

$[Xe]4f^3 5d^0 6s^2$

44. (c) Oxygen is common in all oxides of group-16, and electronegativity decreases as we down the group thus basic nature of oxide increase. Hence PoO is the most basic oxide.

45. (d) Thallium (Tl), due to inert pair effect shows stable lower oxidation state and forms stable compounds in low oxidation state.

46. (b) Al, Si belong to 3rd period of periodic table while N , and fluorine to second period.
The size of element decreases from left to right in a period hence the order of size is
$Al > Si > N > F$

47. (a) Calgon is a trade name of a complex salt, sodium hexametaphosphate $(NaPO_3)_6$. Calgon ionises to give a complex anion. The addition of calgon to hard water causes the calcium ions of hard water to displace sodium ions from the anion of calgon and form a complex with calgon.

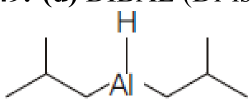
$(NaPO_3)_6$ or

$$Na_2(Na_4P_6O_{18}) \longrightarrow 2Na^+ + \underset{\text{(Complex anion)}}{Na_4P_6O_{18}^{2-}}$$

$$\underset{\substack{\text{Form}\\\text{hard water}}}{Ca^{2+}} + \underset{\text{Anion of calgon}}{Na_4P_6O_{18}^{2-}} \longrightarrow 2Na^+ + \underset{\substack{\text{Goes into}\\\text{solution}}}{CaNa_2P_6O_{18}^{2-}}$$

48. (b) Frequency of collisions increases in proportion to the square root of temperature.

49. (d) DIBAL (Di-isobutyl aluminium hydride)



DIBAL-H, BIBAH

DIBAL is a strong, bulky reducing agent. Its most useful for the reduction of esters to aldehydes.

50. (a) $MCl + e^- \longrightarrow M + Cl^-$cathode (reduction)

$Cl^- \longrightarrow \frac{1}{2}Cl_2 + e^-$anode (oxidation)

$MCl \rightarrow M + \frac{1}{2}Cl_2$

The K_C of the cell reaction is calculated from

Nernst equation $E_{cell} = E^{\circ}_{cell} - \frac{0.059}{n}\log K_C$

$-1.140 = -0.55 - \frac{0.059}{1}\log C$

$-0.59 = -0.059\log K_C$

$\log K_C = \frac{0.59}{0.059} = 10$

$\therefore\ K_C = 10^{10}$

K_{Sp} is for $M + \frac{1}{2}Cl_2 \longrightarrow MCl \longrightarrow M^+ + Cl^-$

$\therefore K_{Sp} = \frac{1}{K_C}$

$= \frac{1}{10^{10}} = 10^{-10}$

51. (a) Adsorption of H_2 gas on solid surface is exothermic and entropy decreases in the process. $\therefore \Delta S < 0$

52. (c) $4H^+ + 4e^- \rightleftharpoons 2H_2$

$E = E^{\circ} - \frac{0.059}{n}\log\frac{pH_2}{[H^+]^4}$

$-0.059 = 0 - \frac{0.059}{4}\log\frac{1}{[H^+]^4}$

$= [H^+] = 10^{-1} = 0.1M$

53. (a) The melting point order of IV A group elements is

$C > Si > Ge > Pb > Sn$

54. (d) The number of hydrogen bonds between adenine and thymine are two $(A = T)$ where as the number of hydrogen bond between guanine and cytosine are three $(G \equiv C)$.

55. (d) $CO_2(g) + H_2(g) \longrightarrow CO(g) + H_2O(g)$

$\Delta H_f{}^{\circ} = (\Delta H)_{products} - (\Delta H)_{reactants}$

Hence, $\Delta H_f{}^{\circ} = \left[\Delta H_{f_{CO}} + \Delta H_{f_{H_2O}}\right] - \left[\Delta H_{fCO_2}\right]$ ... (i)

Given, $\Delta H^{\circ}{}_{fCO_2(s)} = -393.5\ kJ\ mol^{-1}$

$\Delta H^{\circ}_{fCO} = -110.5\ kJ\ mol^{-1}$

$\Delta H^{\circ}_{fH_2O} = -241.8\ kJ\ mol^{-1}$

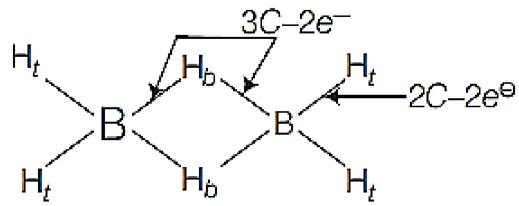
Put there value in equation (i)

$\Delta H_{f^{\circ}} = -110.5 - 241.8 + 393.5 = 41.2$

56. (d) Among hydrogen halides the strongest acids is HI. As the size of halogen increase, bond length increase hence bond energy decreases. So, the HI releases $H^{\oplus}$ ions easily and show high acidic nature.

57. (d) OsF_2 central atom sulphur contain 3 b.p and 1 l.p hence, OsF_2 is pyramidal. But it is given as OsF_2 $\therefore$

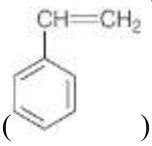
58. (a) Diborane molecular formula B_2H_6



Diborane has 4 two centered two electron $(2C - 2e^-)$bonds and 2 three centre two electron $(3C - 2e^-)$bonds.

59. (c) The monomers present in buna - S rubber are (i) 1, 3-butadiene

$(CH_2 = CH - CH = CH_2)$

(ii) styrene ()

60. (c) $2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$

Cuprous sulphide brings about reduction of cuprous oxide. This reaction takes place in bessemer converter during metallurgy of copper.

61. (c) The different energy levels of an atoms are given by the equation. $E_n = -13.6Z^2/n^2 eV$

Z is the atomic number for hydrogen $= 1$ $n = 1,2,3.$ and so on such that the ground state has $n = 1$ and the secondary energy level (first excited state) has $n = 2$

So, for first excited state, $n = 2$ $E = -13.6/2^2 = -13.6/4 = -3.4eV$

62. (b) (i), (ii), (iv)

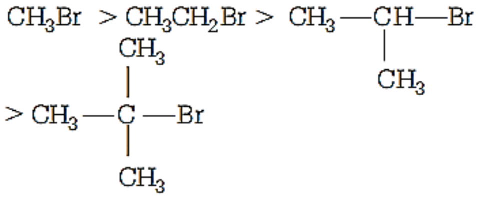
In zero order R′n

$t_{\frac{1}{2}} \propto \frac{[A]_0}{2K}$

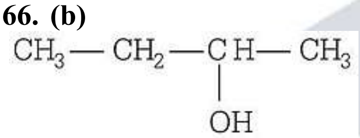
63. (d) As charge on cation increases size decrease and down the group size increases.

64. (a) When value of Δ_0 is greater than pairing energy, pairing of electron takes place hence low-spin complexes are formed

65. (a) In dry acetone reaction follows SN^2 mechanism Hence,



66. (b)



$\xrightarrow[(Dehydration)]{Few\ drops\ of\ acid} CH_3 - CH = CH - CH_3$

The above optically active compound undergo dehydration with few drops of acid and form stable alkene through the formation of stable carbocation.

67. (a) Given weight of solute 98 g , molar mass of solute 98 g mol^{-1} weight of solute = 2 g

$$\text{Molality} = \frac{\text{weight of } H_2SO_4}{\text{molecular weight of } H_2SO_4} \times \frac{1000}{\text{weight of water}}$$

$$= \frac{98}{98} \times \frac{1000}{2} = 500 \text{ m}$$

68. (d) Cyclohexylamine does not give azo dye test Because it is not the aromatic amine and azo dye test is given by aromatic amine only.

69. (a) Histamine is a potent vasodilator.

70. (c) $(\Delta H_f^\circ)_{water} = -286$ kJ/mol

71. (d) 3° alcohol reacts with Lucas reagent immediately.

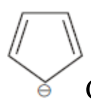
72. (b) $48KNO_3 + 5C_{12}H_{22}O_{11} \longrightarrow$

$24\ N_2 + 36CO_2 + 55H_2O + 24\ K_2CO_3$

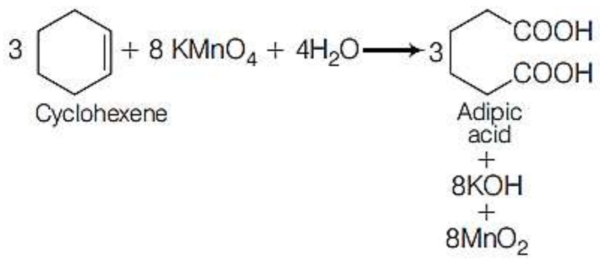
73. (b) Van Arkel method for refining Zr, Ti and Mond's process for Ni on vapour phase refining methods.

74. (c) Hydrogen and helium exhibit heating effect during Joule Thomson expansion due to low inversion temperature.

75. (a)

76. (b)  Cyclopentadienyl anion. It is cyclic, planar with $(4n + 2)\pi e^-$. Therefore, aromatic but structure is not similar to benzene.

77. (b)



78. **(c)** Number of spectral lines $= \frac{n(n-1)}{2}$ n is the orbital in which the electron exist.

79. **(b)** As the size of halogen atom increase, the bond length $X-X$ bond in X_2 molecule increase from F_2 to I_2 hence, the correct answer is (2).

80. **(b)** In a closed packed structure [CCP or HCP] if there are n spheres (atoms or ions) in packing; then;

Number of octahedral voids = n

Number of tetrahedral voids = 2n

In CCP;

Number of atoms = 4

So number of octahedral voids = 4

Number of tetrahedral voids = 8

Mathematics

81. **(d)** Given, $\tan 20^\circ = \lambda$

$$\therefore \frac{\tan 160^\circ - \tan 110^\circ}{1+(\tan 160^\circ)(\tan 110^\circ)}$$

$$= \frac{\tan(180^\circ - 20^\circ) - \tan(90^\circ + 20^\circ)}{1+\left(\tan(180^\circ - 20^\circ)\right)\left(\tan(90^\circ + 20^\circ)\right)}$$

$$= \frac{-\tan 20^\circ + \cot 20^\circ}{1+\tan 20^\circ \cot 20^\circ}$$

$[\because \tan(180^\circ - \theta) = -\tan\theta; \tan(90^\circ + \theta) = -\cot\theta]$

$$= \frac{-\lambda + 1/\lambda}{1+1}$$

$$= \frac{-\lambda^2+1}{2\lambda}$$

$$= \frac{1-\lambda^2}{2\lambda}$$

82. **(d)** We have,

$x^2 + y^2 - 6x + 4y = 12$

$\Rightarrow (x-3)^2 + (y+2)^2 = 25$

$\Rightarrow (x-3)^2 + (y+2)^2 = (5)^2$

Equation of tangent whose slope m is

$y + 2 = m(x-3) \pm 5\sqrt{m^2+1}$ ..... (i)

Now, this tangent is parallel to line $4x + 3y + 5 = 0$

$\therefore$ Slope of line is $-\frac{4}{3}$

Put the value of $m = -\frac{4}{3}$ is Eq. (i), we get

$$y + 2 = -\frac{4}{3}(x-3) \pm 5\sqrt{\left(\frac{-4}{3}\right)^2 + 1}$$

$$\Rightarrow y + 2 = \frac{-4}{3}(x-3) \pm 5\left(\frac{5}{3}\right)$$

$\Rightarrow 3y + 6 = -4x + 12 \pm 25$

$\Rightarrow 4x + 3y = 6 \pm 25$

$\Rightarrow 4x + 3y = 31$ or $4x + 3y = -19$

Hence, equation of tangent is
$4x+3y-31=0$ or $4x+3y+19=0$

83. **(b)** Given, mean $(\bar{x})=10$

$$\therefore \text{Mean}(\bar{x})=\frac{6+7+10+12+13+\alpha+12+16}{8}$$

$$\Rightarrow 10=\frac{76+\alpha}{8}$$

$$\Rightarrow \alpha=4|-16-10|+|7-10|+|10-10|+|12-10|$$

$$\text{MD}(\bar{x})=\frac{+|13-10|+|4-10|+|12-10|+|16-0|]}{8}$$

$$\text{MD}(\bar{x})=\frac{4+3+0+2+3+6+2+6}{8}$$

$$\text{MD}(\bar{x})=\frac{26}{8}=3.25$$

$\therefore$ Mean deviation about mean $=3.25$

84. **(a)** (I) $\int_{-1}^{1} x|x|dx=0$ [$\because x|x|$ is an odd function]

(II) Let $I=\int_0^{\pi/2}\left[1+\left(\log\frac{4+3\sin x}{4+3\cos x}\right)\right]dx$

$$I=\int_0^{\pi/2}\left(1+\left[\log\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]\right)dx$$

$$I=\int_0^{\pi/2}\left(1+\left(\log\frac{4+3\cos x}{4+3\sin x}\right)\right)dx$$

$$\Rightarrow 2I=\int_0^{\pi/2}\left(2+\log\frac{4+3\sin x}{4+3\cos x}\right.$$
$$\left.+\log\frac{4+3\cos x}{4+3\sin x}\right)dx$$

$$\Rightarrow 2I=\int_0^{\pi/2}\left(2+\log\frac{(4+3\sin x)(4+3\cos x)}{(4+3\cos x)(4+3\sin x)}\right)dx$$

$$\Rightarrow 2I=\int_0^{\pi/2}(2+\log 1)dx=\int_0^{\pi/2}2dx=\pi$$

$$\Rightarrow I=\frac{\pi}{2}$$

(III) $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

(IV) $$\int_{-a}^{a} f(x)dx=\int_0^a f(x)dx+\int_0^a f(-x)dx$$
$$=\int_0^a [f(x)+f(-x)]dx$$

85. **(d)** We have,
$f(x+y)=f(x)f(y)$
differentiate with respect to x, y as constant, we get
$f'(x+y)=f'(x)f(y)$
Put $x=0, y=3$, we get

$$f'(0+3)=f'(0)f(3)$$
$$\Rightarrow f'(3)=f'(0)\cdot f(3)$$

$\Rightarrow f'(3)=11\times 3=33$ [$\because f'(0)=11, f(3)=3$]

86. **(a)** We have,

$$I=\int_0^{\pi}\frac{x\,dx}{4\cos^2 x+9\sin^2 x}$$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)dx}{4\cos^2(\pi-x)+9\sin^2(\pi-x)}$$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)dx}{4\cos^2 x+9\sin^2 x}$$

$$\Rightarrow 2I=\int_0^{\pi}\frac{\pi dx}{4\cos^2 x+9\sin^2 x}$$

$$\Rightarrow 2I=\int_0^{\pi}\frac{\pi\sec^2 x dx}{4+9\tan^2 x}$$

$$\Rightarrow 2I=\int_0^{\pi/2}\frac{2\pi\sec^2 x dx}{4+9\tan^2 x}$$

$$\left.\because \int_0^{2a} f(x)dx-2\int_0^{a} f(x)dx \Rightarrow f(2a-x)=f(x)\right]$$

$$\Rightarrow I=\frac{\pi}{9}\int_0^{\pi/2}\frac{\sec^2 x dx}{\frac{4}{9}+\tan^2 x}$$

Put $\tan x=t\Rightarrow \sec^2 x dx=dt$

$$x=0, t=0, x=\frac{\pi}{2}, t=\infty$$

$$\Rightarrow I=\frac{\pi}{9}\int_0^{\infty}\frac{dt}{\left(\frac{2}{3}\right)^2+t^2}$$

$$\Rightarrow I=\frac{\pi}{9}\times\frac{3}{2}\left[\tan^{-1}\frac{3t}{2}\right]_0^{\infty}$$

$$\Rightarrow I=\frac{\pi}{9}\times\frac{3}{2}\times\frac{\pi}{2}=\frac{\pi^2}{12}$$

87. (c) We have,

X	P(X)	P_iX_i	$P_iX_i^2$
0	0.1	0	0
1	0.4	0.4	0.4
2	0.3	0.6	1.2
3	0.2	0.6	1.8
4	0	0	0
		$\Sigma P_iX_i=1.6$	$\Sigma P_iX_i^2=3.4$

$$\text{Variance}(\sigma^2)=\Sigma P_iX_i^2-(\Sigma P_iX_i)^2$$
$$=3.4-(1.6)^2=3.4-2.56=0.84$$

88. (b) We have,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 0 & 4 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & -1 \\ 2 & -1 & 8 \end{bmatrix}, A - A^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -0.5 \\ 1 & -0.5 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix}$$

$$A = B + C$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix} = -C^T$$

89. (a) We have,

$$|a \times \hat{i}|^2 + |a \times \hat{j}|^2 + |a \times \hat{k}|^2$$

$$(|a||\hat{i}|\sin\alpha)^2 + (|a||\hat{j}|\sin\beta)^2 + (|a||\hat{k}|\sin\gamma)^2$$

$$= \sin^2\alpha + \sin^2\beta + \sin^2\gamma \; [\because |a| = |\hat{i}| = |\hat{j}| = |\hat{k}| = 1]$$

$$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$$

$$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

$$= 3 - 1 = 2 \; [\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$$

90. (d) We have,

5 red balls, 3 black balls, 4 white balls

Total numbers of balls $= (5 + 3 + 4) = 12$

Three balls are drawn $= {}^{12}C_3 = 220$

Three balls are drawn of same colour

$= {}^5C_3 + {}^3C_3 + {}^4C_3 = 10 + 1 + 4 = 15$

Probability of are not same colour

$= 1 -$ Probability of same colour

$$= 1 - \frac{15}{220} = 1 - \frac{3}{44} = \frac{41}{44}$$

91. (d) We have,

$$S_1 = x^2 + y^2 - 4x - 6y + 5 = 0$$

$$S_2 = x^2 + y^2 - 2x - 4y - 1 = 0$$

$$S_3 = x^2 + y^2 - 6x - 2y = 0$$

$$S_1 - S_2 = 0 \Rightarrow 2x + 2y - 6 = 0 \Rightarrow x + y - 3 = 0 \ldots \text{(i)}$$

$$S_2 - S_3 = 0 \Rightarrow 4x - 2y - 1 = 0$$

Solving Eqs. (i) and (ii), we get

$$x = \frac{7}{6}, y = \frac{11}{6}$$

$\left(\frac{7}{6}, \frac{11}{6}\right)$ satisfies the equations $18x - 12y + 1 = 0$

92. (d) We have,

$\operatorname{cosec}\theta - \cot\theta = 2017 \quad \ldots\text{(i)}$

$\therefore \operatorname{cosec}\theta + \cot\theta = \frac{1}{2017} \quad \ldots\text{(ii)}$

$$\left[\begin{aligned} &\because \operatorname{cosec}^2\theta - \cot^2\theta = 1 \\ &\Rightarrow \operatorname{cosec}\theta - \cot\theta = \frac{1}{\operatorname{cosec}\theta + \cot\theta} \end{aligned}\right]$$

Adding Eqs. (i) and (ii), we get

$$2\operatorname{cosec}\theta = 2017 + \frac{1}{2017}$$

$$\Rightarrow \operatorname{cosec}\theta = \frac{1}{2}\left[2017 + \frac{1}{2017}\right] > 0$$

θ lie in Ist or IInd quadrant.

Subtracting Eq. (i) from Eq. (ii), we get

$$2\cot\theta = \frac{1}{2017} - 2017$$

$$\cot\theta = \frac{1}{2}\left(\frac{1}{2017} - 2017\right) < 0$$

$\therefore$ θ lie in IInd and IIIrd quadrant.

Hence, θ lies in IInd quadrant.

93. (a) We have,

$$\int e^{2x}f'(x)dx = g(x)$$

Let $I = \int (e^{2x}f(x) + e^{2x}f'(x))dx$

$$= f(x)\int e^{2x}dx - \int f'(x)\int e^{2x}dx)dx + \int e^{2x}f'(x)dx$$

$$= \frac{f(x)e^{2x}}{2} - \frac{1}{2}\int e^{2x}f'(x)dx + \int e^{2x}f'(x)dx$$

$$= \frac{e^{2x}}{2}f(x) - \frac{1}{2}\int e^{2x}f'(x)dx$$

$$= \frac{1}{2}\left[e^{2x}f(x) - \int e^{2x}f'(x)dx\right]$$

$$= \frac{1}{2}[e^{2x}f(x) - g(x)] + C$$

94. (c) Let $P(x, y), A = (5,3), B = (3,-2)$

$$\text{Area of } \triangle PAB = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 5 & 3 & 1 \\ 3 & -2 & 1 \end{vmatrix}$$

$$\Rightarrow 9 = \frac{1}{2}|x(3+2) - y(5-3) + (-10-9)|$$

$$\Rightarrow 18 = |5x - 2y - 19|$$

$$\Rightarrow 5x - 2y - 19 = |18|$$

$$\Rightarrow 5x - 2y - 19 = \pm 18$$

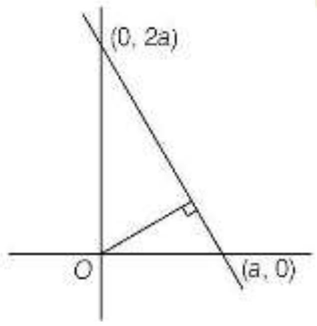
$$\Rightarrow 5x - 2y = 19 \pm 18$$

$$\Rightarrow 5x - 2y = 1 \text{ or } 5x - 2y = 37$$

which represents a pair of parallel lines.

95. (d) Let equation of line be

$$\frac{x}{a} + \frac{y}{2a} = 1$$



$$\Rightarrow 2x + y = 2a$$

Distance from origin is

$$\left|\frac{2a}{\sqrt{(2)^2 + (1)^2}}\right| = \left|\frac{2a}{\sqrt{5}}\right|$$

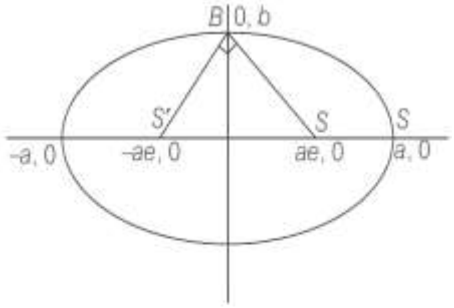
$$\Rightarrow \left|\frac{2a}{\sqrt{5}}\right| = 1$$

$$\Rightarrow \ 2a = \pm\sqrt{5}$$

Hence, equation of line is $2x + y = \pm\sqrt{5}$

96. (a) We have,

SBS is an isosceles right- angle triangle.



$\therefore \quad SS^2 = SB^2 + S\ \ B^2$

$\Rightarrow \quad (2ae)^2 = b^2 + (ae)^2 + b^2 + (ae)^2$

$\Rightarrow \quad 4(ae)^2 = 2(b^2 + (ae)^2)$

$\Rightarrow \quad (ae)^2 = b^2$

$\Rightarrow \quad e^2 = \dfrac{b^2}{a^2} \qquad \left[\because e = \sqrt{1 - \dfrac{b^2}{a^2}}\right]$

$\Rightarrow \quad 1 - e^2 = 1 - \dfrac{b^2}{a^2}$

$\Rightarrow \quad 1 - e^2 = e^2$

$\Rightarrow \quad 2e^2 = 1$

$\Rightarrow \quad e = \dfrac{1}{\sqrt{2}}$

97. (b) We have,

$$y^2 + 6y - 2x = -5$$

$$\Rightarrow \ y^2 + 6y = 2x - 5$$

$$\Rightarrow \ y^2 + 6y + 9 = 2x - 5 + 9$$

$$\Rightarrow (y + 3)^2 \ = 2x + 4$$

$$\Rightarrow \ (y + 3)^2 = 2(x + 2)$$

$\therefore \ 4a = 2 \Rightarrow a = \dfrac{1}{2}$

Vertex $= (-2, -3)$

Equations of directrix

$$x + 2 = -\frac{1}{2}$$

$$\Rightarrow \ x + \frac{5}{2} = 0$$

$$\Rightarrow \ 2x + 5 = 0$$

98. (c) We have,

$$\frac{x^2 + 5}{(x^2 + 1)(x - 2)} = \frac{A}{x - 2} + \frac{Bx + C}{x^2 + 1}$$

$\Rightarrow x^2 + 5 = A(x^2 + 1) + (Bx + C)(x - 2)$

$\Rightarrow x^2 + 5 = Ax^2 + A + Bx^2 - 2Bx + Cx - 2C$

Equating the coefficient of x^2, x and constant terms, we get

$1 = A + B, 0 = -2B + C, 5 = A - 2C$

Solving these equations, we get

$A=\frac{9}{5}, B=-\frac{4}{5}, C=-\frac{8}{5}$

$\therefore A+B+C=\frac{9-4-8}{5}$

$=\frac{-3}{5}$

99. **(b)** We have,

Conjugate of $(x+iy)(1-2i)$ is $1+i$.

i.e. $(x-iy)(1+2i)=1+i$

$\Rightarrow \quad x-iy=\frac{1+i}{1+2i}$

Taking conjugate both sides, we get

$x+iy=\frac{1-i}{1-2i}$

100. **(a)** Let $I=\int x^4e^{2x}dx$

$\Rightarrow I=x^4\int e^{2x}dx-\int \frac{dx^4}{dx}\cdot\int e^{2x}\cdot dx\cdot dx+C$

$=\frac{x^4\cdot e^{2x}}{2}-\frac{1}{2}\int 4x^3e^{2x}dx+C$

$\Rightarrow\frac{x^4e^{2x}}{2}-2\left[\frac{x^3e^{2x}}{2}-\int\frac{3x^2e^{2x}}{2}dx\right]+C$

$\Rightarrow\frac{x^4e^{2x}}{2}-x^3e^{2x}+3\left[\frac{x^2e^{2x}}{2}-\frac{1}{2}\int 2xe^{2x}dx\right]+C$

$\Rightarrow\frac{x^4e^{2x}}{2}-x^3e^{2x}+\frac{3}{2}x^2e^{2x}-3$

$\left[\frac{xe^{2x}}{2}-\int\frac{e^{2x}}{2}dx\right]+C$

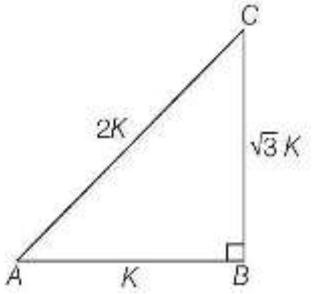
$\Rightarrow\frac{x^4e^{2x}}{2}-x^3e^{2x}+\frac{3}{2}x^2e^{2x}-\frac{3}{2}xe^{2x}+\frac{3e^{2x}}{4}+C$

$\Rightarrow\frac{e^{2x}}{4}[2x^4-4x^3+6x^2-6x+3]+C$

101. **(a)** We have,

Ratio of sides of triangle are $1:\sqrt{3}:2$.

Let the sides are $k, \sqrt{3}k, 2k$.



Since, this triangle is a right- angle triangle

$(2k)^2=(\sqrt{3}k)^2+(k)^2=(2k)^2$

$\therefore \sin A=\frac{\sqrt{3}k}{2k}=\frac{\sqrt{3}}{2}\Rightarrow A=60^\circ$

$\Rightarrow \sin C=\frac{k}{2k}=\frac{1}{2}\Rightarrow C=30^\circ$

$\Rightarrow B=90^\circ$

$\therefore$ Ratio of angles are $30^\circ:60^\circ:90^\circ$

$\Rightarrow 1:2:3$

102. **(a)** We have,

$$(x-1)^3 + 64 = 0$$
$$\Rightarrow \quad (x-1)^3 = -64$$
$$\Rightarrow \quad (x-1)^3 = (-4)^3$$
$$\Rightarrow \quad x - 1 = -4, -4w, -4w^2$$
$$\Rightarrow \quad x = -3, -4w + 1, -4w^2 + 1$$

Complex roots of the equations are $-4w + 1, -4w^2 + 1$

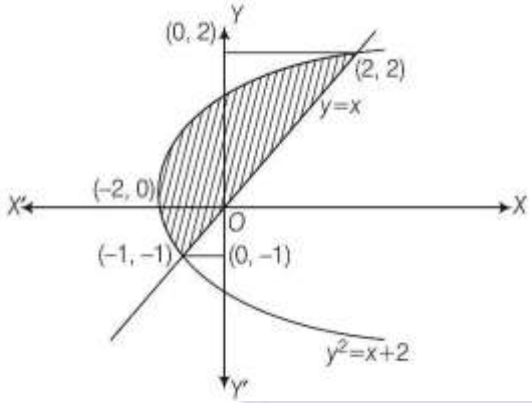
Sum of complex roots are

$$-4w + 1 - 4w^2 + 1 = -4(w + w^2) + 2$$
$$= -4(-1) + 2 = 4 + 2 = 6 \; [\because 1 + w + w^2 = 0]$$

103. **(c)** We have,

$x = y^2 - 2$ and $x = y$

Graph of $x = y^2 - 2$ and $x = y$ are



Solving, $x = y^2 - 2$ and $x = y$, we get $(-1,-1)$ and $(2,2)$.

Area of shaded region is

$$\int_{-1}^{2} y\,dy - \int_{-1}^{2} (y^2 - 2)\,dy$$
$$\Rightarrow \left[\frac{y^2}{2} - \frac{y^3}{3} + 2y\right]_{-1}^{2} = \left(\frac{4}{2} - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right)$$
$$= \frac{10}{3} + \frac{7}{6} = \frac{27}{6} = \frac{9}{2}$$

104. **(b)** We have,

$$a = x\hat{i} + y\hat{j} + z\hat{k}$$
$$(a \times \hat{i})(\hat{i} + \hat{j}) + (\hat{a} \times \hat{j})(\hat{j} + \hat{k}) + (a \times \hat{k})(\hat{k} + \hat{i})$$
$$= [a\hat{i}\hat{i}] + [a\hat{i}j] + [a\hat{j}\hat{j}] + [a\hat{j}\hat{k}] + [a\hat{k}\hat{k}] + [a\hat{k}\hat{i}]$$
$$= [a\hat{i}\hat{j}] + [a\hat{j}\hat{k}] + [a\hat{k}\hat{i}]$$
$$[\because [a\hat{i}\hat{i}] = [a\hat{j}\hat{j}] = [a\hat{k}\hat{k}] = 0]$$
$$= a \cdot (\hat{i} \times \hat{j}) + a \cdot (\hat{j} \times \hat{k}) + a \cdot (\hat{k} \times \hat{i})$$
$$= a \cdot \hat{k} + a \cdot \hat{i} + a \cdot \hat{j} = a \cdot (\hat{i} + \hat{j} + \hat{k})$$
$$= (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = x + y + z$$

105. **(b)** Let $z = x + iy$

$$\therefore \frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{2x+1+iy}{(-y+1)+ix}$$
$$= \frac{[(2x+1)+iy][(-y+1)-ix]}{[(-y+1)+ix][(-y+1)-ix]}$$
$$= \frac{(2x+1)(-y+1) - (2x+1)xi + y(-y+1)j + xy}{(-y+1)^2 + x^2}$$
$$= \frac{(2x+1)(-y+1) + xy + (-y^2 + y - 2x^2 - x)i}{(-y+1)^2 + x^2}$$

It is given that imaginary part of $\frac{2z+1}{iz+1} = -2$

$$\therefore \frac{-y^2+y-2x^2-x}{(-y+1)^2+x^2}=-2$$
$$\Rightarrow -y^2+y-2x^2-x=-2(-y+1)^2-2x^2$$
$$\Rightarrow -y^2+y-x=-2y^2-2+4y$$
$$\Rightarrow y^2-3y-x+2=0$$
which represent the equations of parabola.

106. **(d)** We have,

$$g(x)=(f(2f(x)+2))^2$$
$$g'(x)=2(f(2f(x)+2)\cdot f'(2f(x)+2)f'(x)\cdot 2$$
$$\Rightarrow g'(0)=2(f(2f(0)+2)\cdot f'(2f(0)+2)\cdot 2f'(0)$$
$$\Rightarrow g'(0)=2(f(2(-1)+2)\cdot f'(2(-1)+2\cdot 2(1)$$
$$[\because f(0)=-1, f'(0)=1]$$
$$\Rightarrow g'(0)=2[f(-2+2)\cdot f'(-2+2)]\cdot 2$$
$$\Rightarrow g'(0)=4f(0)\cdot f'(0)$$
$$\Rightarrow g'(0)=4\times(-1)(1)$$
$$=-4$$

107. **(d)** We know that, distance from point (x_1,y_1) to the line $ax+by+c=0$ is
$$\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$
$\therefore$ Distance from $(1,1)$ to $3x+4y+c=0$ is 7
$$\therefore \quad 7=\left|\frac{3(1)+4(1)+c}{\sqrt{3^2+4^2}}\right|$$
$$\Rightarrow \quad 7=\left|\frac{7+c}{5}\right|$$
$$\Rightarrow \quad 35=|7+c|$$
$$\Rightarrow \quad 7+c=\pm 35$$
$$\Rightarrow \quad c=-7\pm 35$$
$$c=28,-42$$

108. **(a)** We have,
$$\frac{dy}{dx}=\frac{x+y}{x-y}$$
Put $y=vx$
$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$
$$\therefore v+x\frac{dv}{dx}=\frac{x+vx}{x-vx}=\frac{1+v}{1-v}$$
$$\Rightarrow x\frac{dv}{dx}=\frac{1+v}{1-v}-v=\frac{1+v-v+v^2}{1-v}=\frac{1+v^2}{1-v}$$
$$\Rightarrow \frac{1-v}{1+v^2}dv=\frac{dx}{x}$$
$$\Rightarrow \int\frac{1}{1+v^2}dv-\frac{1}{2}\int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}$$
$$\Rightarrow \tan^{-1}v-\frac{1}{2}\log|1+v^2|=\log x+C$$
$$\Rightarrow \tan^{-1}\frac{y}{x}=\log x+\frac{1}{2}\log\left(1+\frac{y^2}{x^2}\right)+C$$
$$\Rightarrow \tan^{-1}\frac{y}{x}=\log\frac{x\left(\sqrt{x^2+y^2}\right)}{x}+C$$
$$\Rightarrow \tan^{-1}\frac{y}{x}=\log\sqrt{x^2+y^2}+C$$

109. **(a)** We have,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$
Let $x=a\cos\theta, y=b\sin\theta$
$$\therefore \frac{dx}{d\theta}=-a\sin\theta, \frac{dy}{d\theta}=b\cos\theta$$
$$\frac{dy}{dx}=-\frac{b}{a}\cot\theta$$
On differentiating w.r.t. θ, we get
$$\frac{d^2y}{dx^2}=-\frac{b}{a}(-\operatorname{cosec}^2\theta)\frac{d\theta}{dx}$$
$$\Rightarrow \frac{d^2y}{dx^2}=\frac{b\operatorname{cosec}^2\theta}{-a^2\sin\theta}$$
$$\Rightarrow \frac{d^2y}{dx^2}=-\frac{b}{a^2\sin^3\theta}$$
$$\Rightarrow \frac{d^2y}{dx}=\frac{-b^4}{a^2y^3}\ \left[\because \sin\theta=\frac{y}{b}\right]$$

110. **(a)** We have, $\lim_{y\to1}\left(\frac{1}{y^2-1}-\frac{2}{y^4-1}\right)$
$$=\lim_{y\to1}\left(\frac{y^2+1-2}{y^4-1}\right)$$
$$=\lim_{y\to1}\left(\frac{y^2-1}{(y^2-1)(y^2+1)}\right)$$
$$=\lim_{y\to1}\frac{1}{y^2+1}=\frac{1}{1+1}=\frac{1}{2}$$

111. **(c)** We have,
$$(y-3x^2)dx+xdy=0$$
$$\Rightarrow ydx-3x^2dx+xdy=0$$
$$\Rightarrow ydx+xdy=3x^2dx$$
$$\Rightarrow dxy=3x^2dx$$
On integrating both sides, we get
$$xy=x^3+C\Rightarrow y=x^2+\frac{C}{x}$$

112. **(b)** We have, $(1+x)^{42}$
$$T_{2r+1}={}^{42}C_{2r}X^{2r}$$
$$T_{r+1}={}^{42}C_rX^r$$
Coefficient of $(2r+1)^{\text{th}}$ and $(r+1)^{\text{th}}$ term are equal
$$\therefore {}^{42}C_{2r}={}^{42}C_r$$
$$\therefore 2r+r=42\ [\because {}^nC_x={}^nC_y\Rightarrow x+y=n]$$
$$\Rightarrow r=14$$

113. **(b)** Equation of plane passes through the points $(1,-1,6),(0,0,7)$ and perpendicular to the plane $x-2y+z=6$ is
$$\begin{vmatrix} x-1 & y+1 & z-6\\ 0-1 & 0+1 & 7-6\\ 1 & -2 & 1\end{vmatrix}=0$$
$$\Rightarrow \begin{vmatrix} x-1 & y+1 & z-6\\ -1 & 1 & 1\\ 1 & -2 & 1\end{vmatrix}=0$$
$$\Rightarrow (x-1)(1+2)-(y+1)(-1-1)$$
$$+(Z-6)(2-1)=0$$
$$\Rightarrow 3x-3+2y+2+z-6=0$$
$$\Rightarrow 3x+2y+z-7=0$$
This plane is passes through (1,1,2).

114. **(a)** The curve $y=ax^3+bx+4$ is passes through (2,14)

$\therefore \quad 14 = a(2)^3 + b(2) + 4$

$\Rightarrow \quad 14 = 8a + 2b + 4$

$5 = 4a + b$ …… (i)

Slope of tangent to the curve $y = ax^3 + bx + 4$

i.e. $\frac{dy}{dx} = 3ax^2 + b$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,14)} = 3a(2)^2 + b$

$21 = 12a + b$ ….. (ii)

$\left[\because \frac{dy}{dx} = 21\right]$

Solving Eqs. (i) and (ii), we get

$a = 2, b = -3$

115. **(c)** We have

x	P(x)	$P_i x_i$
1	a	a
2	a	2a
3	a	3a
4	b	4b
5	b	5b
6	0.3	1.8

$\Sigma P_i = a + a + a + b + b + 0.3$

$\Rightarrow \quad 1 = 3a + 2b + 0.3$

$\Rightarrow \quad 3a + 2b = 0.7$ ….. (i)

$\Sigma P_i x_i = a + 2a + 3a + 4b + 5b + 1.8$

$\Rightarrow \quad 4.2 = 6a + 9b + 1.8$

$\Rightarrow \quad 2a + 3b = 0.8$ ….. (ii)

Solving Eqs. (i) and (ii), we get

$a = 0.1, b = 0.2$

116. **(d)** We have,

f(x) is a quadratic equation.

$\therefore f(x) = ax^2 + bx + c$

$f(0) = c, f(1) = a + b + c$

$\Rightarrow f(0) + f(1) = 0$

$\Rightarrow c + a + b + c = 0$

$a + b + 2c = 0$ ….. (ii)

$f(-2) = a(-2)^2 + b(-2) + c$

$0 = 4a - 2b + c$

$4a - 2b + c = 0$ …. (ii)

From Eqs. (i) and (ii), we get

$$\frac{a}{5} = \frac{b}{7} = \frac{c}{-6}$$

$\Rightarrow \quad a = 5k, b = 7k, c = -6k$

$\therefore \quad f(x) = k(5x^2 + 7x - 6)$

$\Rightarrow \quad 5x^2 + 7x - 6 = 0$

$\Rightarrow \quad 5x^2 + 10x - 3x - 6 = 0$

$\Rightarrow \quad 5x(x + 2) - 3(x + 2) = 0$

$\Rightarrow \quad (x + 2)(5x - 3) = 0$

$x = -2, x = \frac{3}{5}$

Hence, $f\left(\frac{3}{5}\right) = 0$.

117. **(b)** We have,

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

On differentiating w.r.t. x, we get

$$\frac{nx^{n-1}}{a^n} + \frac{ny^{n-1}}{b^n}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b^n x^{n-1}}{a^n y^{n-1}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,b)} = \frac{-b^n a^{n-1}}{a^n b^{n-1}} = \frac{-b}{a}$$

Equations of tangent at (a, b) is

$$y - b = \frac{-b}{a}(x - a)$$

$$\Rightarrow ay - ab = -bx + ab$$

$$\Rightarrow bx + ay = 2ab$$

$$\Rightarrow \frac{bx}{ab} + \frac{ay}{ab} = \frac{2ab}{ab}$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$$

118. **(b)** We know that equation of normal of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 - b^2$$

$\therefore$ Equations of normal to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is

$$\frac{9x}{x_1} + \frac{4y}{y_1} = 9 + 4$$

$$\Rightarrow \frac{9x}{x_1} + \frac{4y}{y_1} = 13$$

Since, line $x + y = k$ is normal to the given hyperbola

$$\therefore \frac{\frac{9}{x_1}}{1} = \frac{\frac{4}{y_1}}{1} = \frac{13}{k}$$

$$\Rightarrow \frac{9}{x_1} = \frac{4}{y_1}$$

$$= \frac{13}{k}$$

$$\Rightarrow x_1 = \frac{9k}{13}$$

$$y_1 = \frac{4k}{13}$$

Since (x_1, y_1) lie on the hyperbola.

$$\therefore \frac{\left(\frac{9k}{13}\right)^2}{9} - \frac{\left(\frac{4k}{13}\right)^2}{4} = 1$$

$$\Rightarrow \frac{9k^2}{169} - \frac{4k^2}{169} = 1$$

$$\Rightarrow 5k^2 = 169$$

$$\Rightarrow k = \pm\frac{13}{\sqrt{5}}$$

119. **(b)** We have,

$$x^2 - 8x + 9 - \frac{8}{x} + \frac{1}{x^2} = 0$$

$\Rightarrow x^4 - 8x^3 + 9x^2 - 8x + 1 = 0$

Product of roots $= 1$

120. **(b)** Given, $\Delta = \begin{vmatrix} 1 & 5 & 6 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{vmatrix}$

$= 1(1-0) - 5(0-0) + 6(0-0)$

$= 1 - 0 + 0$

$\Rightarrow \Delta = 1$

If $\Delta = 1$ and $\Delta' = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 0 & 3 \\ 4 & 6 & 100 \end{vmatrix}$

$= 1(0-18) - 0(300-12) + 1(18-0)$

$= -18 - 0 + 18$

$\Delta' = 0$

Now, $(\Delta + \Delta')^2 - 3(\Delta + \Delta') + 2$

$= (1+0)^2 - 3(1+0) + 2$

$= 1 - 3 + 2$

$= -2 + 2 = 0$

121. **(a)** Total number of ways of choosing such terms is ${}^{10}C_5 \cdot {}^5C_1$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} \times 5$

$= 10 \times 3 \times 7 \times 6 = 1260$

122. **(c)** We know that point of intersection of two lines i.e. $ax_1 + by_1 + c_1 = 0$ and $ax_2 + by_2 + c_2 = 0$ is

$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$

$\therefore$ Point of intersection of two lines i.e. $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ is

$\left(\frac{-6 \times 5 - 2 \times -1}{5 \times 2 - 3 \times -6}, \frac{-1 \times 3 - 5 \times 5}{5 \times 2 - 3 \times -6}\right)$

$= \left(\frac{-30+2}{10+18}, \frac{-3-25}{10+18}\right)$

$= \left(\frac{-28}{28}, \frac{-28}{28}\right) = (-1, -1)$

and slope of line $3x - 5y + 11 = 0$ is $m = \frac{3}{5}$

Equation of the line passing through $(-1, -1)$ and perpendicular to $3x - 5y + 11 = 0$ is

$$\begin{aligned} (y+1) &= -\frac{5}{3}(x+1) \\ \Rightarrow 3(y+1) &= -5x - 5 \\ \Rightarrow 3y + 3 &= -5x - 5 \\ \Rightarrow 5x + 3y + 3 + 5 &= 0 \\ \Rightarrow 5x + 3y + 8 &= 0 \end{aligned}$$

123. **(d)** Let $S = \{2k \mid -9 \le k \le 10\} = \{-18, -16, -14, \ldots, 0, 2, 4, 6, \ldots 20\}$

Total number of possible outcomes $= 20$ Favourable outcomes are - 12, 0 and 12 So, number of favourable outcomes $= 3$

$\therefore$ Required probability $= \frac{3}{20}$

124. **(b)** $\int \frac{dx}{x(x^4+1)} = \int \frac{x^4+1-x^4}{x(x^4+1)} dx$

$= \int \frac{x^4+1}{x(x^4+1)} dx - \int \frac{x^4}{x(x^4+1)} dx$

$= \int \frac{1}{x} dx - \int \frac{x^3}{x^4+1} dx$

$= \log|x| - \int \frac{x^3}{x^4+1} dx + C$

Let $x^4 + 1 = t \Rightarrow x^3 dx = dt$

$\Rightarrow x^3 dx = \frac{1}{4}$

$$\therefore \int \frac{dx}{4(x^4+1)} = \log|x| - \frac{1}{4}\int \frac{dt}{t} + C$$

$$= \log|x| - \frac{1}{4}\log|t| + C$$

$$= \log|x| - \frac{1}{4}\log|x^4+1| + C$$

$$= \frac{1}{4}\log|x^4| - \frac{1}{4}\log|x^4+1| + C$$

$$= \frac{1}{4}[\log|x^4| - \log|x^4+1|] + C$$

$$= \frac{1}{4}\log\left|\frac{x^4}{x^4+1}\right| + C$$

125. **(b)** We have,

$$\sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\sqrt{\frac{2}{3}}$$

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}}{2}\sqrt{1-\left(\sqrt{\frac{2}{3}}\right)^2} + \sqrt{\frac{2}{3}}\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right]$$

As $0 < \frac{\sqrt{3}}{2} = 0.866, \sqrt{\frac{2}{3}} = 0.816 \leq 1$ and

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\sqrt{\frac{2}{3}}\right)^2 = 1.4167 > 1$$

[$\because \sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}$
$x\sqrt{1-y^2} + y\sqrt{1-x^2}$, if $0 < x, y \leq 1$ and
$x^2 + y^2 > 1$]

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}}{2}\sqrt{1-\frac{2}{3}} + \sqrt{\frac{2}{3}}\sqrt{1-\frac{3}{4}}\right]$$

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}}{2}\sqrt{\frac{3-2}{3}} + \sqrt{\frac{2}{3}}\sqrt{\frac{4-3}{4}}\right]$$

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{1}{2}\right]$$

$$= \pi - \sin^{-1}\left[\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}}\right]$$

126. **(c)** Given, α and β are the roots of $x^2 + 2x + c = 0$.

$\therefore$ Sum of roots, $\alpha + \beta = \frac{-\text{ coefficient of } x}{\text{coefficient of } x^2}$

$\alpha + \beta = \frac{-2}{1} = -2$ ……. (i)

and Product of roots, $\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$

$\Rightarrow \alpha\beta = \frac{c}{1} = c$ ….. (ii)

Since, $\alpha^3 + \beta^3 = 4$ [given]

$\Rightarrow (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)=4$
$[\because a^3+b^3=(a+b)(a^2+b^2-ab)]$
$\Rightarrow (\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]=4$
$[\because a^2+b^2=(a+b)^2-2\alpha\beta]$
$\Rightarrow (-2)[(-2)^2-3\times c]=4$ [From Eqs. (i) and (ii)]
$\Rightarrow (-2)[4-3c]=4$
$\Rightarrow 4-3c=\frac{-4}{2}$
$\Rightarrow 4-3c=-2$
$\Rightarrow -3c=-2-4$
$\Rightarrow -3c=-6$
$c=2$

127. **(b)** Given equations of circle is
$S\equiv x^2+y^2-13=0$
On differentiating it w.r.t x, we get
$2x+2y\frac{dy}{dx}=0$
$\Rightarrow \frac{dy}{dx}=\frac{-x}{y}$
Now, slope of tangent, $m=\left.\frac{dy}{dx}\right|_{at\ (2,3)}=\frac{-2}{3}$
$\Rightarrow m=\frac{-2}{3}$
$\therefore \left(m,-\frac{1}{m}\right)=\left(-\frac{2}{3},\frac{3}{2}\right)$
On substituting this point in LHS Eq. (i), we get
$LHS=\frac{4}{9}+\frac{9}{4}-13$
$=\frac{16+81-468}{36}<0$
$\therefore \left(m,-\frac{1}{m}\right)$ is an internal point with respect to the circle $S=0$.

128. **(c)**

		C		

If we fix C in the middle, then the rest 4 letters can be arranged in ${}^4P_4=4!$ ways $=24$ ways.

129. **(b)** Given equation of curves are
$x^2=8y$ ... ... (i)
and $xy=8$ .... (ii)
On substituting the value of y from Eq. (i) in Eq. (ii), we get
$x\left(\frac{x^2}{8}\right)=8$
$\Rightarrow x^3=64$
$\Rightarrow x=4$
On substituting $x=4$ in Eq. (ii), we get
$y=2$
Thus, the point of intersection of given curves is (4,2).
Now, let m_1 be the slope of tangent to the curve (i) at point (4,2) and m_2 be the slope of tangent to the curve (ii) at point (4, 2).
Then, $m_1=\frac{4}{4}=1$
$\left[\because x^2=8y\Rightarrow 2x=8\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{x}{4}\right]$
and $m_2=\frac{-2}{4}=-\frac{1}{2}$

$\left[\because xy = 8 \Rightarrow x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x}\right]$

Now, the angle θ between the curves is given by

$$\tan\theta = \left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$$

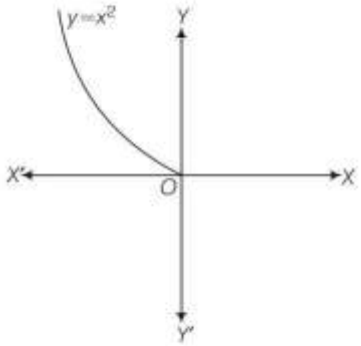
$$= \left(\frac{\frac{-1}{2} - 1}{1 - \frac{1}{2}}\right) = \left(\frac{\frac{-3}{2}}{\frac{1}{2}}\right)$$

$\Rightarrow \tan\theta = -3$

$\Rightarrow \theta = \tan^{-1}(-3)$

130. **(a)** We have a function $f: (-\infty, 0] \rightarrow [0, \infty)$ defined as $f(x) = x^2$.

The graph of above function is



Since, each line parallel to x-axis cuts the above curve at maximum one point, therefore f is one - one. Also from the graph it is clear that range $f = [0, \infty)$

∴ f is onto.

Thus, f is invertible.

Hence, $f^{-1}: [0, \infty) \rightarrow (-\infty, 0]$

$\Rightarrow \text{Domain}(f^{-1}) = [0, \infty)$ and Range of $(f^{-1}) = (-\infty, 0]$

131. **(c)** We have,

a, b, c are unit vector

$\therefore |a| = |b| = |c| = 1$ and $a + b + c = 0$, angle between a and b is $\frac{\pi}{3}$.

Now, $a + b + c = 0$

$\Rightarrow a \times (a + b + c) = 0$

$\Rightarrow a \times a + a \times b + a \times c = 0$

$\Rightarrow a \times b = c \times a$

$\Rightarrow |a \times b| = |c \times a|$

Similarly $|a \times b| = |b \times c|$

$\therefore |a \times b| + |b \times c| + |c \times a|$

$= 3|a \times b|$

$= 3|a||b|\sin(a, b)$

$= 3 \times 1 \times 1 \times \sin\frac{\pi}{3} = \frac{3\sqrt{3}}{2}$

132. **(b)** Given, $x = A\cos(nt + \alpha) \ldots\ldots(i)$

On differentiating x w.r.t. ' t ', we get

$$\frac{dx}{dt} = A\frac{d}{dt}\cos(nt+\alpha)$$
$$= -A\sin(nt+\alpha)\frac{d}{dt}(nt+\alpha)$$
$$= -A\sin(nt+\alpha)(n)$$
$$\Rightarrow \frac{dx}{dt} = -An\sin(nt+\alpha)$$
$$\Rightarrow \frac{dx}{dt} = -\frac{xn\sin(nt+\alpha)}{\cos(nt+\alpha)} \quad \text{[form eq (i)]}$$
$\frac{dx}{dt} = nx\tan(nt+\alpha)\ldots\ldots$ (ii)

Again, differentiating w.r.t ' t ', we get

$$\frac{d^2x}{dt^2} = -\left[nx\frac{d}{dt}\tan(nt+\alpha) + \tan(nt+\alpha)\frac{d}{dt}nx\right]$$
$$= -\left[nx\sec^2(nt+\alpha)\frac{d}{dt}(nt+\alpha) + \tan\right.$$
$$\left.(nt+\alpha)\frac{dx}{dt}\times n\right]$$
$$= -[nx\sec^2(nt+\alpha)n + n\tan(nt+\alpha)$$
$$\times -nx\tan(nt+\alpha)]$$
[from Eq. (ii)]
$$= -[n^2x\sec^2(nt+\alpha) - n^2x\tan^2(nt+\alpha)$$
$$= -n^2x[\sec^2(nt+\alpha) - \tan^2(nt+\alpha)]$$
$$= -n^2x \times 1 \; [\because \sec^2\theta - \tan^2\theta = 1]$$
$$\Rightarrow \frac{d^2x}{dt^2} = -n^2x$$
$$\Rightarrow \frac{d^2x}{dt^2} + n^2x = 0$$

133. **(a)** We have, $|a| = |b| = 1$ and α is the angle between a and b.

Clearly, $\cos\alpha = \frac{a\cdot b}{|a||b|}$

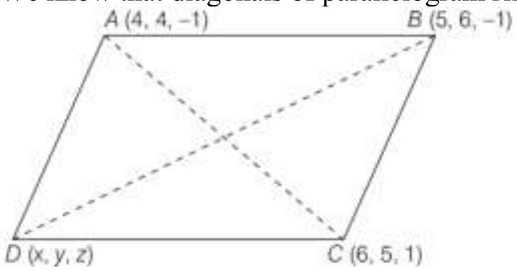
$\cos\alpha = a\,b$ ...... (i)

Now, let $a + b$ is a unit vector, then

$$|a+b| = 1$$
$$\Rightarrow |a+b|^2 = 1$$
$$\Rightarrow (a+b)\cdot(a+b) = 1$$
$$\Rightarrow a\cdot a + a\cdot b + b\cdot a + b\cdot b = 1$$
$$\Rightarrow |a|^2 + 2a\cdot b + |b|^2 = 1 \; [\because a\cdot b = b\cdot a]$$
$$\Rightarrow 1 + 2\cos\alpha + 1 = 1 \; \text{[using Eq. (i)]}$$
$$\Rightarrow 1 + 2\cos\alpha = 0$$
$$\Rightarrow \cos\alpha = -\frac{1}{2}$$

134. **(c)** Given, ABCD is a parallelogram with vertices $A(4,4,-1), B(5,6,-1), C(6,5,1)$ and $D(x,y,z)$.

We know that diagonals of parallelogram ABCD bisects each other.



$\therefore$ Mid-point of AC = Mid-Point of BD

$$\Rightarrow \left(\frac{4+6}{2}, \frac{4+5}{2}, \frac{-1+1}{2}\right) = \left(\frac{x+5}{2}, \frac{y+6}{2}, \frac{z-1}{2}\right)$$
$$\Rightarrow \left(\frac{10}{2}, \frac{9}{2}, 0\right) = \left(\frac{x+5}{2}, \frac{y+6}{2}, \frac{z-1}{2}\right)$$

On comparing both sides, we get

$$\frac{x+5}{2} = \frac{10}{2}, \frac{y+6}{2} = \frac{9}{2} \text{ and } \frac{z-1}{2} = 0$$
$$\Rightarrow x+5 = 10, y+6 = 9 \text{ and } z-1 = 0$$
$$\Rightarrow x = 10-5, y = 9-6 \text{ and } z = 1$$
$$\Rightarrow x = 5, y = 3 \text{ and } z = 1$$

Thus, $D(x, y, z) = D(5,3,1)$

135. **(c)** On comparing the given equations with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 2, b = 2\lambda, h = -5, g = \frac{5}{2}$,

$f = -8, c = -3$

Since, the given equations represents a pair of straight lines, therefore

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$
$$\Rightarrow \begin{vmatrix} 2 & -5 & 5/2 \\ -5 & 2\lambda & -8 \\ 5/2 & -8 & -3 \end{vmatrix} = 0$$
$$\Rightarrow 2(-6\lambda - 64) + 5(15+20) + \frac{5}{2}(40 - 5\lambda) = 0$$
$$\Rightarrow -12\lambda - 128 + 175 + 100 - \frac{25}{2}\lambda = 0$$

$\Rightarrow$

$$\Rightarrow -\frac{49}{2}\lambda = -147$$

$\Rightarrow$

$$\lambda = \frac{147 \times 2}{49} = 6$$

Now, the point of intersection of given lines is given by

$$\left(\frac{bg - fh}{h^2 - ab}, \frac{af - gh}{h^2 - ab}\right) = \left(\frac{30 - 40}{25 - 24}, \frac{-16 + \frac{25}{2}}{25 - 24}\right)$$
$$= \left(-10, \frac{-7}{2}\right)$$

136. **(c)** $A = \begin{pmatrix} x & x & x \\ x & x^2 & x \\ x & x & x+1 \end{pmatrix}$

Since, rank $A = 1$, therefore atleast one determinant of order 1 should be non-zero and all the determinants of order 2 and 3 should be zero.

If $x = 0$, then $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, which have non-zero determinant of order 1 only.

If $x = 1$, then $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$, which have a non-zero determinant of order 2, nearly $\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$

$\therefore$ x can take value 0 only.

137. **(d)** Given, $a = \hat{i} + \hat{j} + \hat{k}$,

$b = \hat{i} - \hat{j} + 2\hat{k}$

and

$c = x\hat{i} + (x-2)\hat{j} - \hat{k}$

Since, the given vectors are coplanar, therefore

$[a \quad b \quad c] = 0$

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$

$\Rightarrow 1(1-2x+4) - 1(-1-2x) + 1(x-2+x) = 0$

$\Rightarrow 5-2x+1+2x+2x-2 = 0$

$\Rightarrow \ 2x+4 = 0$

$\Rightarrow \ 2x = -4$

$\Rightarrow \ x = -2$

138. **(a)** On comparing the given equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 4, h = 4, b = 10, g = -4, f = -22$ and $c = 14$

For removal of first degree terms, shift the origin to

$\left(\frac{bg-fh}{h^2-ab}, \frac{af-gh}{h^2-ab}\right) = \left(\frac{-40+88}{16-40}, \frac{-88+16}{16-40}\right)$

$= \left(\frac{48}{-24}, \frac{-72}{-24}\right) = (-2,3)$

139. **(c)** Let S_1 be the circle with centre $(2,3)$ and radius a.

Let S_2 be the circle with centre $(5,6)$ and radius a.

Then $S_1 \equiv (x-2)^2 + (y-3)^2 - a^2 = 0$

and $S_2 = (x-5)^2 + (y-6)^2 - a^2 = 0$

Now, radical axis of these circle is given by $S_1 - S_2 = 0$

$(x-2)^2 + (y-3)^2 - a^2 - (x-5)^2 - (y-6)^2 + a^2 = 0$

$\Rightarrow (4-4x+9-6y) - (25-10x) - (36-12y) = 0$

$\Rightarrow \ 13-4x-6y-25+10x-36+12y = 0$

$\Rightarrow \ 6x+6y-48 = 0$

$\Rightarrow \ x+y = \frac{48}{6} = 8 \quad \ldots.(i)$

Since, the given circle cut orthogonally, therefore

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow 2(-2)(-5) + 2(-3)(-6) = ((-2)^2 + (-3)^2 - a^2)$
$+((-5)^2 + (-6)^2 - a^2)$

$[\because$ radius $= \sqrt{g^2+f^2-c} \ \therefore a^2 = g^2+f^2-c \Rightarrow$
$c = g^2+f^2-a^2]$

$\Rightarrow \quad 20+36 = (13-a^2) + (61-a^2)$

$\Rightarrow \quad 56 = 74 - 2a^2$

$\Rightarrow \quad 2a^2 = 74-56 = 18$

$\Rightarrow \quad a^2 = 9$

$\Rightarrow \quad a = 3[\because$ radius can't be negative]

Clearly, option (c) satisfy the Eq. (i)

140. **(b)** We have, $\tan\theta_1 = k\cot\theta_2$

$\Rightarrow \ \tan\theta_1\tan\theta_2 = k \quad \ldots$ (i)

Consider, $\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)} = \frac{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2}{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2}$

$= \frac{1-\tan\theta_1\tan\theta_2}{1+\tan\theta_1\tan\theta_2}$

$= \frac{1-k}{1+k}$

141. **(a)** We have, $a = 2\hat{i} + \hat{j} - 3\hat{k}$

$b = \hat{i} + 3\hat{j} + 2\hat{k}$

Clearly, c is parallel to $a \times b$

Here, $a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 3 & 2 \end{vmatrix} = \hat{i}(11) - \hat{j}(7) + \hat{k}(5)$

$= 11\hat{\imath} - 7\hat{\jmath} + 5\hat{k} = d$ (say)

Now, $\hat{d} = \frac{a \times b}{|a \times b|} = \frac{11\hat{\imath} - 7\hat{\jmath} + 5\hat{k}}{\sqrt{195}}$

Thus, $c = |c|\hat{d} = \frac{2}{\sqrt{195}}(11\hat{\imath} - 7\hat{\jmath} + 5\hat{k})$

Hence, volume of the parallelopiped $= [a \quad b \quad c]$

$$= \begin{vmatrix} 2 & 1 & -3 \\ 1 & 3 & 2 \\ \frac{22}{\sqrt{195}} & \frac{-14}{\sqrt{195}} & \frac{10}{\sqrt{195}} \end{vmatrix}$$

$$= \frac{2}{\sqrt{195}} \begin{vmatrix} 2 & 1 & -3 \\ 1 & 3 & 2 \\ 11 & -7 & 5 \end{vmatrix}$$

$$= \frac{2}{\sqrt{195}}[2(29) - 1(-17) - 3(-40)]$$

$$= \frac{2}{\sqrt{195}}[58 + 17 + 120]$$

$$= \frac{2}{\sqrt{195}} \times 195$$

$= 2\sqrt{195}$

142. **(a)** Given, $y = x^3 - 3x^2 + 5$ ….. (i)

On differentiating both sides w.r.t. ' x ', we get

$\frac{dy}{dx} = 3x^2 - 6x$ …. (ii)

For local maxima or local minima, put $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x = 0$

$\Rightarrow 3x(x - 2) = 0$

$\Rightarrow x = 0$ or $x = 2$

Now, differentiating Eq. (ii) w.r.t. ' x ' we get

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = -6 < 0$$

$\therefore$ $x = 0$ is a point of local maxima.

and $\left(\frac{d^2y}{dx^2}\right)_{x=2} = 6 \times 2 - 6 = 12 - 6 = 6 > 0$

$\therefore$ $x = 2$ is a point of local minima.

143. **(d)** In the expansion of $(1 + x)^n$, the general term i.e. $(r + 1)$ th term is

$T_{r+1} = {}^nC_r(1)^{n-r}X^r = {}^nC_rX^r$

$\therefore$ Coefficient of $(r + 1)$ th term is ${}^nC_r$

Similarly, coefficient of pth term $= {}^nC_{p-1}$

$\therefore$ $p = {}^nC_{p-1}$ (given)

p $p = \frac{n!}{(p-1)!(n-P+1)!}$……….(i)

and coefficient of $(p + 1)$ th term $= {}^nC_p$

$\therefore$ $q = {}^nC_p$ (given)….. (ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{p}{q}=\frac{\frac{n!}{(p-1)!\,(n-p+1)!}}{{}^{n}C_{p}}=\frac{\frac{n!}{(p-1)!\,(n-p+1)!}}{\frac{n!}{(p)!\,(n-p)!}}$$

$$=\frac{(p)!\,(n-p)!}{(n-p+1)!\,(p-1)!}=\frac{p(p-1)!\,(n-p)!}{(n-p+1)(n-p)!\,(p-1)!}$$

$$\Rightarrow \frac{p}{q}=\frac{p}{n-p+1}$$

$$\Rightarrow \frac{1}{q}=\frac{1}{n-p+1}$$

$$\Rightarrow n-p+1=q$$

$$\Rightarrow n+q=n+1$$

144. **(d)** Given,

$$f(x)=\begin{cases}\sin x, & \text{if } x\le 0\\ x^2+a^2, & \text{if } 0<x<1\\ bx+2, & \text{if } 1\le x\le 2\\ 0, & \text{if } x>2\end{cases} \text{ is continuous on IR,}$$

$\therefore \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)$

and $\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)$

$\Rightarrow \lim_{x\to 0^-}\sin x=\lim_{x\to 0^-} x^2+a^2$

and $\lim_{x\to 2^-} bx+2=\lim_{x\to 2^+} 0$

$\Rightarrow\ 0=0+a^2$ and $2b+2=0$

$\Rightarrow\ a=0$ and $b=-1$

Now, $a+b+ab=0+(-1)+0=-1$

145. **(d)** We have, $\cosh^{-1}x=2\log_e(\sqrt{2}+1)$

$$\Rightarrow \log_e\left(x+\sqrt{x^2-1}\right)=\log_e(\sqrt{2}+1)^2$$

$$\Rightarrow x+\sqrt{x^2-1}=(\sqrt{2}+1)^2$$

$$\Rightarrow x+\sqrt{x^2-1}=3+2\sqrt{2}=3+\sqrt{8}$$

On comparing rational and irrational part, we get

$x=3$

146. **(b)** Consider,

$$\sum_{k=1}^{n} k(k+2)=\sum_{k=1}^{n}(k^2+2k)=\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k$$

$$=\frac{n(n+1)(2n+1)}{6}+2\frac{n(n+1)}{2}$$

$$=n(n+1)\left[\frac{(2n+1)}{6}+1\right]=n(n+1)\left[\frac{2n+7}{6}\right]$$

$$=\frac{n(n+1)(2n+7)}{6}$$

147. **(b)** Given equations of ellipse can be rewritten as

$((5x)^2+2(5)(10)x+10^2)+$

$((2y)^2-2(2)(1)y+1^2)-10^2-1^2+100=0$

$$\Rightarrow (5x+10)^2+(2y-1)^2=1$$

$$\Rightarrow 25(x+2)^2+4\left(y-\frac{1}{2}\right)^2=1$$

$$\Rightarrow \frac{(x+2)^2}{(1/5)^2}+\frac{(y-1/2)^2}{(1/2)^2}=1, \text{ which is of the form}$$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a<b$

Here, $a = \frac{1}{5}, b = \frac{1}{2}$ and major axis of ellipse $x + 2 = 0$ i.e. $x = -2$.

Now, $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{25}} = \sqrt{\frac{21}{25}} = \frac{\sqrt{21}}{5}$

$\therefore$ foci are $\left(-2, \frac{1}{2} \pm be\right) = \left(-2, \frac{1}{2} \pm \frac{\sqrt{21}}{10}\right)$

$= \left(-2, \frac{5 \pm \sqrt{21}}{10}\right)$

148. **(c)**

149. **(a)** We have, $f(x) = -3x - 3$

(i) Let $f(x) = 3$, then we get

$3 = -3x - 3 \Rightarrow 6 = -3x \Rightarrow x = -2$

(ii) Let $f(x) = -6$, then we get

$-6 = -3x - 3 \Rightarrow -3 = -3x \Rightarrow x = 1$

(iii) Let $f(x) = -9$, then we get

$-9 = -3x - 3 \Rightarrow -6 = -3x \Rightarrow x = 2$

(iv) Let $f(x) = -18$, then we get

$-18 = -3x - 3 \Rightarrow -15 = -3x \Rightarrow x = 5$

Thus, domain of $f = \{-2,1,2,5\}$

Hence, -1 cannot be in the domain of f.

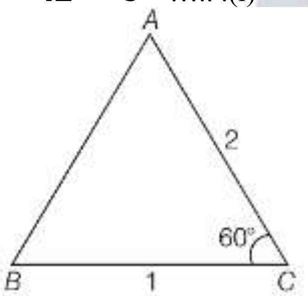
150. **(a)** Clearly $\Delta = \frac{1}{2} \cdot a \cdot b \sin c$

$\Rightarrow \quad \Delta = \frac{1}{2} \cdot 2 \cdot 1 \cdot \sin 60°$

$\Rightarrow \quad \Delta = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \Delta^2 = \frac{3}{4}$

$\Rightarrow \; 4\Delta^2 = 3 \quad$ ….. .(i)



By sine rule, we get

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \frac{\sin A}{1} = \frac{\sin B}{2} = \frac{\sin 60°}{c}$

$\Rightarrow \frac{\sin A}{\sin B} = \frac{1}{2} = \frac{\sin 60°}{c}$

Considering last two terms, we get

$\frac{1}{2} = \frac{\frac{\sqrt{3}}{2}}{c}$

$c = \sqrt{3} \quad$ …. (ii)

Thus, $4\Delta^2 + c^2 = 3 + 3 = 6$ [(Using Eqs. (i) and (iii)]

151. **(d)** We have, $|a| = 3, |b| = 4$ and $|a + b| = 5$

Since,

$|a+b|=5$

$\therefore\ |a+b|^2=25$

$\Rightarrow\ (a+b)\cdot(a+b)=25$

$\Rightarrow a\cdot a+a\cdot b+b+a+b\cdot b=25$

$\Rightarrow\ |a|^2+2a\cdot b+|b|^2=25\ [\because a\cdot b=b\cdot a]$

$\Rightarrow\ 9+2a\cdot b+16=25$

$\Rightarrow\ a\cdot b=0$

Now, consider $|a-b|^2=(a-b)(a-b)$

$=a\cdot a-a\cdot b-b\cdot a+b\cdot b$

$=|a|^2-2a\cdot b+|b|^2\ [\because a\cdot b=b\cdot a]$

$=9-0+16=25$

$\Rightarrow |a-b|=5$

152. **(a)** We have,

$$\int f(x)\cos x dx=\frac{1}{2}(f(x))^2+C$$

On differentiating w.r.t ' x ', we get

$$f(x)\cos x=\frac{1}{2}\times 2f(x)\cdot f'(x)$$

$\Rightarrow\quad f(x)\cos x=f(x)\cdot f'(x)$

$\Rightarrow\quad f'(x)=\cos x$

$\Rightarrow\quad f'(0)=\cos 0$

$\Rightarrow\ f'(0)=1\ [\because \cos 0=1]$

153. **(c)** We have,

α,β are the roots of the equations $ax^2+bx+c=0$

$$\therefore\ \alpha+\beta=-\frac{b}{a},\alpha\beta=\frac{c}{a}$$

Equations whose roots are $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$ are

$$x^2-\left(\frac{1-\alpha}{\alpha}+\frac{1-\beta}{\beta}\right)x+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1-\beta}{\beta}\right)=0$$

$$\Rightarrow\ x^2-\left(\frac{1}{\alpha}+\frac{1}{\beta}-2\right)x+\left(\frac{1-(\alpha+\beta)+\alpha\beta}{\alpha\beta}\right)=0$$

$$\Rightarrow x^2-\left(\frac{\alpha+\beta-2\alpha\beta}{\alpha\beta}\right)x+\left(\frac{1-(\alpha+\beta)+\alpha\beta}{\alpha\beta}\right)=0$$

$$\Rightarrow \alpha\beta x^2-(\alpha+\beta-2\alpha\beta)x+(1-(\alpha+\beta)+\alpha\beta)=0$$

$$\Rightarrow\ \frac{c}{a}x^2-\left(-\frac{b}{a}-\frac{2c}{a}\right)x+\left(1+\frac{b}{a}+\frac{c}{a}\right)=0$$

$$\Rightarrow\ cx^2+(b+2c)x+(a+b+c)=0$$

Comparing with $px^2+qx+r=0$, we get

$r=a+b+c$

154. **(a)** Parametric equations of given circle is $x=12\cos\theta, y=12\sin\theta$

$[\because$ Parametric equation of $x^2+y^2=r^2$

is $x=r\cos\theta, y=r\sin\theta]$

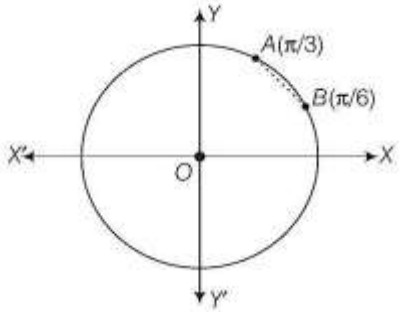
Now, coordinates of point A are given by

$$x=12\cos\frac{\pi}{3}, y=12\sin\frac{\pi}{3}$$

$$\Rightarrow\ x=12\cdot\frac{1}{2}, y=12\cdot\frac{\sqrt{3}}{2}$$

$$\Rightarrow\ x=6; y=6\sqrt{3}$$

i.e. $A\equiv(6,6\sqrt{3})$



and coordinates of point B are given by

$$x = 12\cos\frac{\pi}{6}, y = 12\sin\frac{\pi}{6}$$

$\Rightarrow \quad x = 12\cdot\frac{\sqrt{3}}{2}\cdot y = 12\cdot\frac{1}{2}$

$\Rightarrow \quad x = 6\sqrt{3}, y = 6$

i.e. $\quad B \equiv (6\sqrt{3}, 6)$

Clearly, length of chord

$$AB = \sqrt{(6\sqrt{3}-6)^2 + (6-6\sqrt{3})^2}$$

$$= \sqrt{2\times 6^2(\sqrt{3}-1)^2}$$

$$= 6\sqrt{2}(\sqrt{3}-1)$$

$$= 6(\sqrt{6}-\sqrt{2})$$

155. **(b)** Given equations of pair of straight lines, $xy - x - y + 1 = 0$ can be rewritten as

$x(y-1) - 1(y-1) = 0$

$\Rightarrow \quad (x-1)(y-1) = 0$

$\Rightarrow \quad x = 1$ and $y = 1$

Thus, the three concurrent lines are

$x = 1$ ….. (i)

$y = 1$ ….. (ii)

and $x + ay - 3 = 0$ ….. (iii)

Since, Eqs. (i) and (ii), intersect at only point, namely (1, 1), therefore this point also satisfy the Eq. (iii), we get

$1 + a - 3 = 0$

$\Rightarrow \quad a = 2$

Now, the pair of lines

$ax^2 - 13xy - 7y^2 + x + 23y - 6 = 0$ becomes

$2x^2 - 13xy - 7y^2 + x + 23y - 6 = 0$

The acute angle θ between these lines is given by

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{\left(-\frac{13}{2}\right)^2 + 14}}{-5}\right|$$

$$= \left|\frac{2\sqrt{\frac{169+56}{4}}}{-5}\right| = \left|\frac{2\sqrt{\frac{225}{4}}}{-5}\right| = \left|\frac{2\times\frac{15}{2}}{-5}\right| = |-3| = 3$$

$$\theta = \tan^{-1}(3) = \cos^{-1}\left(\frac{1}{\sqrt{1+3^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$$

156. **(b)** Consider the equations

$\cos 2\theta = \sin\theta$
$\Rightarrow 1 - 2\sin^2\theta = \sin\theta$
$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$
$\Rightarrow 2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$
$\Rightarrow 2\sin\theta(\sin\theta + 1) - 1(\sin\theta + 1) = 0$
$\Rightarrow (\sin\theta + 1)(2\sin\theta - 1) = 0$
$\Rightarrow \sin\theta = -1$ or $\sin\theta = \frac{1}{2}$
$\Rightarrow \theta = \frac{3\pi}{2}$ or $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $[\because \theta \in (0, 2\pi)]$

Thus, number of solutions of the given equation is 3 .

157. **(d)** Let $a = 13, b = 14$ and $c = 15$. Then

$S = \frac{a+b+c}{2} = \frac{13+14+15}{2} = 21$

and area of triangle,

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{21(21-13)(21-14)(21-15)}$
$= \sqrt{21 \times 8 \times 7 \times 6} = 7 \times 3 \times 4 = 84$

Now, as we know $R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$

$\therefore R = \frac{13 \times 14 \times 15}{4 \times 84}$ and

$r = \frac{84}{21}$

$\Rightarrow R = \frac{65}{8}$ and $r = 4$

So, $8R + r = 65 + 4 = 69$

158. **(a)** We know that variance of n number which are in A.P. whose first terms is ' a ' and common difference d is

$\sigma^2 = \frac{d^2(n^2-1)}{12}$

$\therefore \mathrm{Var}(\sigma_1) = \frac{2^2(n^2-1)}{12} = \frac{(n^2-1)}{3} = A$

$\mathrm{Var}(\sigma_2) = \frac{2^2(n^2-1)}{12} - \frac{n^2-1}{3} = B$

$\therefore A = B$

159. **(c)** Since, $x - y = -4k$ or $y = x + 4k$ is a tangent to the parabola $y^2 = 8x$, therefore

$4k = \frac{2}{1}$ [$\because$ Here $4a = 8 \Rightarrow a = 2$]

$\Rightarrow k = \frac{1}{2}$

Also point of contact p is $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = (2,4)$

Now, equation of normal to the $y^2 = 8x$ at $(2,4)$ is

$(y-4) = \frac{-4}{2(2)}(x-2)$

[$\because$ Equation of normal to the parabola $y^2 = 4ax$ at

(x_1, y_1) is $y - y_1 = \frac{-b}{2a}(x - x_1)$]

$\Rightarrow y - 4 = -1(x-2)$
$\Rightarrow y - 4 = -x + 2$
$\Rightarrow x + y = 6$
$\Rightarrow x + y - 6 = 0$

The perpendicular distance of normal from $(k, 2k)$

i.e $\left(\frac{1}{2}, 1\right)$

$$= \frac{\left|\frac{1}{2} + 1 - 6\right|}{\sqrt{1^2 + 1^2}} = \frac{\left|\frac{3}{2} - 6\right|}{\sqrt{2}} = \frac{9}{2\sqrt{2}}$$

160. **(d)** Given, $P(A) = 0.6, P(B) = 0.4$ and $P(A \cap B) = 0$, then $P($ neither A nor B$) = P(\bar{A} \cap \bar{B})$

$= P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A \cap B)$

$[\because P(A \cup B) = P(A) + P(B) - P(A \cap B)]$

$= 1 - 0.6 - 0.4 + 0 = 1 - 1 = 0$