Physics

1. **(d)**(A) Energy conservation is a universal law, so there is no violation possibility of this in nature. (False) (R) This is the statement of energy conservation law. (True)

2. **(a)** Given, $pV^{5/3} = k$

Using dimensional balance method

Dimension of $p = ML^{-1}T^{-2}$

Dimension of $V = L^3$

$\Rightarrow$ Dimension of $V^{5/3} = L^5$

As, dimension of LHS = Dimension of RHS

So, dimension of $k = [ML^{-1}T^{-2}] \times [L^5] = [ML^4T^{-2}]$

3. **(b)** Relation between average velocity and instantaneous velocity is

$$\langle v \rangle = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v(t)dt$$

In the given problem, $v(t) \propto y^{\beta}$ (independent of t)

So,

$< v > \propto v(t)$ or $(v) \propto y^{\beta}$

But according to question,

$< v > \propto y^{1/3}$

$$\beta = \frac{1}{3}$$

4. **(a)** Given,

$v = \alpha\sqrt{x}$ and at $x = 0, t = 0$

To find velocity and acceleration in terms of time, we need to find x as function of t.

$$\because \frac{dx}{dt} = \alpha\sqrt{x}$$

$$\therefore \frac{dx}{\sqrt{x}} = \alpha dt$$

On integrating both sides, we get

$$2\sqrt{x} = \alpha t + C$$

$$x = 0 \text{ at } t = 0 \Rightarrow C = 0$$

$$\therefore \sqrt{x}\frac{\alpha}{2}t$$

Hence, velocity, $v = \frac{\alpha^2}{2}t \quad (\because v = \alpha\sqrt{x})$

and acceleration, $a = \frac{dv}{dt} = \frac{\alpha^2}{2}$

5. **(a)** Given, $r = 3t^2\hat{i} + 2t\hat{j} + \hat{k}$

(i) Velocity vector, $v = \frac{dr}{dt} = 6t\hat{i} + 2\hat{j}$

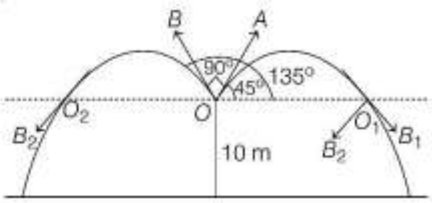
Magnitude of velocity vector,

$|v|_{t=2} = \sqrt{(12)^2 + (2)^2} = \sqrt{148}$

(ii) Acceleration vector, $a = \frac{dv}{dt} = 6\hat{i}$

Magnitude of acceleration $|a|_{t=2} = \sqrt{(6^2)} = 6$

6. **(b)**



In this projectile motion, both objects are projected at the angle difference of 90°. They will be again have same angle difference, when they are again at same height (0 f 10 m) from ground. Taking, plane O_2OO_1 as reference, time taken to reach at point O_1 by first object is

Time of flight, $T_1 = \frac{2u\sin\theta}{g}$

$$= \frac{2 \times 2\sqrt{2}\sin 45^\circ}{10} = 0.4 \text{ s}$$

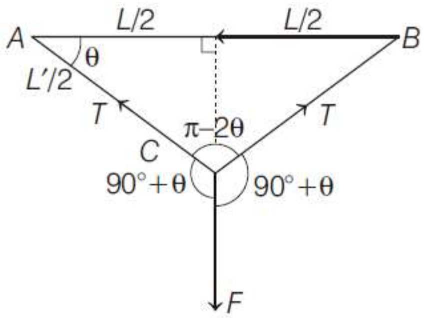
Time taken by second object to reach at point O_2 is

$$T_2 = \frac{2 \times 2\sqrt{2}\sin 135^\circ}{10} = 0.4 \text{ s}$$

$\because \quad T_1 = T_2$

So, velocities of both objects will be perpendicular to each other after 0.4 s .

7. **(c)** Extension x in AC is $L'/2 - L/2$



$$= \frac{L}{2\cos\theta} - \frac{L}{2} = \frac{L}{2}\left(\frac{1-\cos\theta}{\cos\theta}\right)$$

Given, $\frac{T}{x} = k$

$$\therefore \; T = \frac{Lk}{2}\left(\frac{1-\cos\theta}{\cos\theta}\right)$$

Using Lami's theorem, $\frac{F}{\sin(\pi-2\theta)} = \frac{T}{\sin(90^\circ+\theta)}$

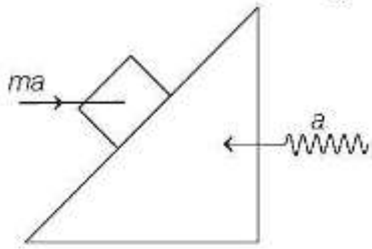
$$\frac{F}{\sin(\pi-2\theta)} = \frac{\frac{L}{2}k\left(\frac{1-\cos\theta}{\cos\theta}\right)}{\sin(90^\circ+\theta)}$$

$$\Rightarrow F = \frac{kL}{2}\frac{(1-\cos\theta)\sin 2\theta}{\cos^2\theta}$$

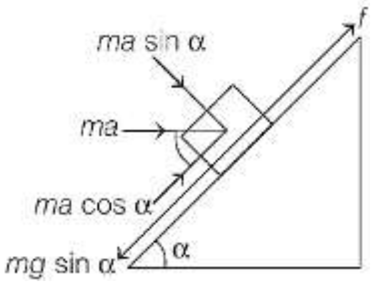
$$= \frac{kL}{2}\frac{(1-\cos\theta)2\sin\theta\cos\theta}{\cos^2\theta}$$

$$F = kL(1-\cos\theta)\cdot\tan\theta$$

8. **(b)** Let a = acceleration of wedge, then force on block due to acceleration of wedge = ma.



Then, from FBD we have, For equilibrium of block,



$$mg\sin\alpha = ma\cos\alpha + \mu(ma\cos\alpha + ma\sin\alpha)$$

$\Rightarrow g = a + \mu(g + a) \Rightarrow \frac{0.6}{1.4} g = 9$

$\Rightarrow a = \frac{30}{7}\ \text{ms}^{-2}$

9. **(a)** Given, $a_c \propto t^{\alpha}$

or $a_c = kt^{\alpha}$

Now, $a_c = \frac{v_t^2}{r}$

(where, v_t = tangential component of velocity)

$\Rightarrow \quad v_t = \sqrt{kr \cdot t^{\alpha}}$

and $\quad a_t = \frac{dv_t}{dt} = \sqrt{(kr)} \cdot \frac{\alpha}{2} \cdot t^{\frac{\alpha}{2}-1}$

Now, power developed is

$P = Fv = ma_t v_t$

$= \quad m\sqrt{kr} \cdot \frac{\alpha}{2} \cdot t^{\frac{\alpha}{2}-1}$

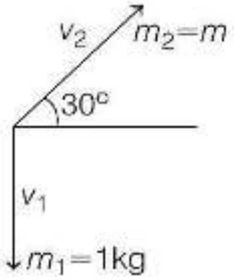
$\Rightarrow \quad P \propto t^{\alpha-1}$

10. **(d)**

$\overset{u_1}{\rightarrow} \quad u_2 = 0$

$m_1 = 1\text{kg} \quad m_2 = m$

Before collision



According to the law of conservation of momentum,

$1 \times u_1 = mv_2 \cos 30°$ (x axis)

and $u_1 = \frac{\sqrt{3}}{2} mv_2$

$\Rightarrow$

$1 \times v_1 = mv_2 \sin 30°$ (y axis)

$v_1 = \frac{1}{2} mv_2$

From kinetic energy conservation,

$\frac{1}{2} u_1^2 = \frac{1}{2} v_1^2 + \frac{1}{2} mv_2^2$

$\Rightarrow u_1^2 - v_1^2 = \frac{1}{2} mv_2^2$

On putting the values of u_1 and v_1, we get

$\frac{3}{4} m^2 v_2^2 - \frac{1}{4} m^2 v_2^2 = \frac{1}{2} mv_2^2$

$\Rightarrow \quad m = 2\ \text{kg}$

11. **(a)** Mass of bullet $= 35\ \text{g} = \frac{35}{1000}$ kg

Number of bullets per second $= \frac{200}{60} = \frac{10}{3} 5^{-1}$

Speed of bullet $= 750\ \text{m/s}$

$\therefore$ Force exerted by gun on bullets,

$F = \frac{35}{1000} \times \frac{10}{3} \times 750 = 35 \times 2.5 = 87.5\ \text{N}$

12. **(a)** By forces balancing perpendicular to inclined plane,

$N = mg\cos\theta$

By forces balancing parallel to inclined plane,

$$mg\sin\theta - f_r = m\frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{mg\sin\theta - f_r}{m} \Rightarrow v(t) = \int g\sin\theta dt - \frac{1}{m}\int f_r dt$$

Now, torque, $\tau = I\frac{d\omega}{dt} = \mathbf{r} \times \mathbf{F}_r$ and $I = mk^2$

$$\therefore \quad mk^2\frac{d\omega}{dt} = rF_r \text{ or } \frac{d\omega}{dt} = \frac{rF_r}{mk^2}$$

$$\Rightarrow \quad \omega(t) = \frac{R}{mk^2}\int f_r dt$$

or $\frac{1}{m}\int f_r dt = \frac{k^2}{R}\omega(t) = \frac{k^2}{R^2}v(t)$ (using $\omega = v/R$)

$$\Rightarrow \ v(t) = \int g\sin\theta dt - \frac{k^2}{R^2}v(t)$$

or $v(t) = \frac{1}{\left(1+\frac{k^2}{R^2}\right)}\int g\sin\theta dt$

So, acceleration,

$$a(t) = \frac{dv(t)}{dt} = \frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)}$$

13. (b) Along Y-axis, net force on the particle is

$F = -(kx\sin 45^\circ + kx\sin 45^\circ + 2kx\sin 135^\circ + 2kx\sin 135^\circ$

$$= -\frac{6}{\sqrt{2}}kx$$

$$\Rightarrow \ m\frac{d^2x}{dt^2} = -3\sqrt{2}kx \Rightarrow \frac{d^2x}{dt^2} + \frac{3\sqrt{2}kx}{m} = 0$$

This is the equation of SHM.

Angular frequency, $\omega^2 = 3\sqrt{2}\frac{k}{m}$

$$\Rightarrow \ f = \left(\frac{1}{2}\pi\right)\sqrt{\frac{3\sqrt{2}k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{3\sqrt{2}k}}$$

14. (b) Gravitation at height h from surface,

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Gravitation at depth d from surface,

$$g_d = g(1 - d/R)$$

For $\quad g_h = g_d$

$\Rightarrow \quad d = 2h$

So, ratio of height w.r.t. depth is

$$\frac{h}{d} = \frac{1}{2} = 0.5$$

15. (c) Given, $F = 400kN = 400 \times 10^3$ N

$L = 2$ m

$r = 50 \text{ mm} = 5 \times 10^{-2}$ m

$\Delta l = 0.5 \text{ mm} = 5 \times 10^{-4}$ m

Young's modulus, $Y = \frac{F\cdot L}{A\cdot\Delta l}$

$$= \frac{4 \times 10^5 \times 2 \times 10^4 \times 10^4}{\frac{22}{7} \times 25 \times 5} = \frac{8 \times 7}{22 \times 5 \times 25} \times 10^{13}$$

$$\approx 2 \times 10^{11} Nm^2$$

16. (d) Pressure at bottom

$$= g \times \int_{z=0}^{z=H} \frac{1}{\pi r^2} \times \pi r^2 \times dz \times \rho_0\left(2 - \frac{z^2}{H^2}\right)$$

$$= g \times \int_{z=0}^{z=H} \rho_0 \left(2 - \frac{z^2}{H^2}\right) dz = g \times \left[\rho\left(2z - \frac{z^3}{3H^2}\right)\right]_{z=0}^{z=H}$$

$$= g\rho_0 \left(2H - \frac{H^3}{3H^2}\right) = g\rho_0 \left(2H - \frac{H}{3}\right)$$

$$= g\rho_0 \left(\frac{5H}{3}\right) = \frac{5}{3}\rho_0 gh$$

17. (d) Using $pV = RT$ and $p = 3 - g\left(\frac{V^2}{{V_0}^2}\right)$, we get

$$T = \frac{3V}{R} - \frac{gV^3}{RV_0^2}$$

Differentiating w.r.t. volume, we get

$$\frac{dT}{dV} = \frac{3}{R} - \frac{g}{RV_0^2} \times 3V^2$$

For maximum T, $\frac{dT}{dV} = 0$

$$\Rightarrow \frac{3}{R} - \frac{g}{RV_0^2} \times 3V^2 = 0 \Rightarrow V = \frac{V_0}{\sqrt{g}}$$

Substituting in Eq. (i), we get

$$T_{max} = \frac{3V_0}{R\sqrt{g}} - \frac{g\left(V_0/\sqrt{g}\right)^3}{RV_0^2}$$

$= \frac{2V_0}{R\sqrt{g}}$.

18. (a) Using first law of thermodynamics,

$dU = dQ - dW$

Here, $dW = pdV = 0 (V = \text{constant})$

$\therefore$ Change in internal energy, $dU = dQ$

Using, $du \propto dt \quad (dQ = C_v dt)$

$pV = RT$

$$dT = \frac{V}{R} dp$$

$\Rightarrow dT \propto dp$

$\Rightarrow dU \propto dp$

Hence, internal energy will increase with increment in pressure.

19. (b) Efficiency of Carnot cycle is

$$\eta = 1 - \frac{T_2}{T_1}$$

$\Rightarrow 1 - \eta = \frac{T_2}{T_1} \Rightarrow \frac{T_1}{T_2} = \frac{1}{(1-\eta)}$ .....(i)

In second case,

$$1.5\eta = 1 - \frac{T_2}{T_1 + \Delta T}$$

or $\quad 1 - 1.5\eta = \frac{T_2}{T_1 + \Delta T}$

$$\Rightarrow \frac{T_1 + \Delta T}{T_2} = \frac{1}{(1 - 1.5\eta)} \text{ ... ... (ii)}$$

Using Eqs. (i) and (ii), we get

$$\frac{\Delta T}{T_2} = \frac{1}{1 - 1.5\eta} - \frac{1}{1 - \eta}$$

$$= \frac{1 - \eta - 1 + 1.5\eta}{(1 - \eta)(1 - 1.5\eta)}$$

$$\Rightarrow \Delta T = \frac{0.5\eta T_2}{(1 - \eta)(1 - 1.5\eta)}$$

20. (b) Using

$$\frac{(p_a+\rho gh)r_1^3}{T_1}=\frac{p_a r_2^3}{T_2}$$

$\because \quad T_1 = T_2$

$\therefore \quad r_2^3 = \frac{1}{p_a}(p_a+\rho gh)r_1^3$

Using all values in SI system,

$$r_2^3 = \frac{1}{10^5}[10^5+10^3 \times 9.8 \times 5]r_1^3$$

$$r_2^3 \approx 1.5 r_1^3$$

$$\Rightarrow \frac{4}{3}\pi r_2^3 \approx (1.5)\frac{4}{3}\pi r_1^3$$

$$V_2 = 1.5 \times V_1$$

$\Rightarrow V_2 = 1.5 \times 3 = 4.5\ mm^3$

21. (d) Let $kx - \omega t = \alpha$

$y_1 = a\sin\alpha, y_2 = a\sin(\phi - \alpha)$

Superimposed wave, $y = y_1 + y_2$

$\Rightarrow y = [a\sin\alpha + a\sin\phi\cos\alpha - a\cos\phi\sin\alpha]$

$\Rightarrow y = [a(1-\cos\phi)\sin\alpha + (a\cos\phi)\cos\alpha]$

Let $R\cos\theta = a(1-\cos\phi)$ and $R\sin\theta = a\sin\phi$.

Then, magnitude of R will be the magnitude of resultant wave,

$$R^2 = a^2[\sin^2\phi + (1-\cos\phi)^2]$$

$$= a^2[1+1-2\cos\phi]$$

$\Rightarrow R^2 = 4a^2\sin^2\frac{\phi}{2} \quad \Rightarrow R = 2a\sin\frac{\phi}{2}$

22. (b) We know that, lens formula, $\frac{1}{f}=\frac{1}{u}-\frac{1}{v}$

Magnification, $m = -\frac{v}{u} = -4$

$\Rightarrow \quad v = +4u$

$\therefore \quad \frac{1}{+8} = \frac{1}{-u} - \frac{1}{4u}$

($-u$, because of sign convention)

$\Rightarrow \frac{5}{4u} = -\frac{1}{8} \quad \Rightarrow u = \frac{-40}{4} = -10$ cm

23. (c) Image of lens will act as object for mirror.

Here,

$v_1 + u_2 = 40$ cm

Lens formula, $\frac{1}{f_1} = \frac{1}{u_1} - \frac{1}{v_1}$

Mirror formula, $\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$

For final image, $v_2 = \infty$

So, $\quad \frac{1}{f_2} = \frac{1}{u_2}$

$\Rightarrow \quad u_2 = 15$ cm

Now, $v_1 = 40 - 15 = 25$ cm

So, value of u_1 should be

$$\frac{1}{20} = \frac{1}{u_1} - \frac{1}{25} \quad \Rightarrow \frac{1}{u_1} = \frac{1}{20} + \frac{1}{25}$$

$\Rightarrow u_1 = 100/9$ cm $\Rightarrow u_1 = 11.11$ cm

$\simeq 12$ cm

24. (c) Position of bright fringe on screen is given by

$x = \frac{n\lambda D}{d}[x = 2.5\ mm]$

For coinciding case, $n_1\lambda_1 = n_2\lambda_2$

$\therefore n = 6$ for wavelength of 500 nm

and n = 5 for wavelength of 600 nm .
Using any one of those,

$$25 \times 10^{-3} = \frac{6 \times 500 \times 10^{-9} \times D}{3 \times 10^{-3}}$$

$$\Rightarrow \quad D = 2.5 \text{ m}$$

25. **(a)** Distance between point charge and observation point is
$x = r - r_0 = 6\hat{i} - 8\hat{j}$
Magnitude of $x = x = \sqrt{6^2 + 8^2}$
$\Rightarrow \; x = 10$ m
$\therefore$ Electric field, $E = \frac{Kq}{x^2}$

$$= \frac{9 \times 10^9 \times 50 \times 10^{-6}}{10 \times 10}$$

$$= 4.5 \text{kV/m}$$

26. **(c)** Work done,

$$W = U_{12} + U_{23} + U_{13}$$

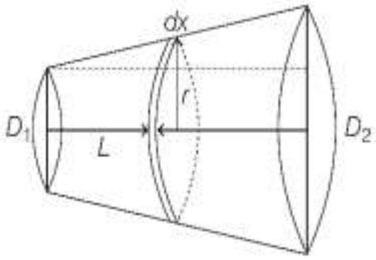
$$= \frac{kq_1q_2}{r_{12}} + \frac{kq_2kq_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$$

$$= \frac{1}{4\pi\varepsilon_0 a}[-2q^2 + 2q^2 - q^2]$$

$$= -\frac{q^2}{4\pi\varepsilon_0 a}$$

27. **(b)** Statement B is not true because
$R_{eff} = 20R$ (Let's assume R is resistance of one bulb) So, $i = \frac{V}{R_{eff}} = \frac{V}{20R}$ which will be minimum.

28. **(d)** Similarity of triangles,



$$\frac{r - a}{x} = \frac{b - a}{l}$$

$\Rightarrow$ (Here, $a = D_1/2, b = D_2/2$)

$$r = a + \left(\frac{b - a}{l}\right)x$$

Resistance of elemental disc is

$$dR = \frac{\rho dx}{\pi r^2} = \frac{\rho dx}{\pi\left[a + \left(\frac{b - a}{l}\right)x\right]^2}$$

Total resistance of tapered wire is

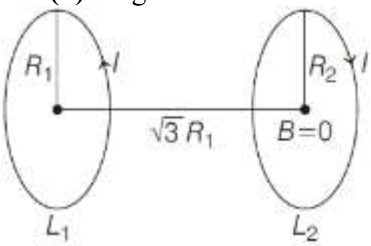
$$R = \int dR = \int_0^l \frac{\rho dx}{\pi\left[a + \left(\frac{b - a}{l}\right)x\right]^2}$$

$$\Rightarrow R = \frac{\rho}{\pi}\left[-\frac{1}{\left[a + \left(\frac{b - a}{l}\right)x\right.}\frac{l}{(b - a)}\right]_0^l$$

$$= \frac{\rho}{\pi} \times \frac{l}{b - a}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{\rho l}{\pi ab}$$

$$= \frac{4\rho l}{\pi D_1 D_2}$$

29. (d) Magnetic field at centre of L_2 will be zero when



$$B_{L_1} = B_{L_2} \text{ (magnitude)}$$

$$\frac{\mu_0 I R_1^2}{2(R_1^2 + x^2)^{3/2}} = \frac{\mu_0 I}{2R_2}$$

Here= $\sqrt{3}R_1$

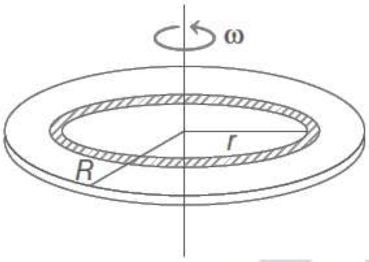
$$\therefore \quad \frac{R_1^2}{2(R_1^2 + 3R_1^2)^{3/2}} = \frac{1}{2R_2}$$

$$\Rightarrow \quad \frac{R_1^2}{8R_1^3} = \frac{1}{R_2}$$

$$\Rightarrow \quad R_2 = 8R_1$$

30. (a) Total charge on the disc, $Q = \sigma(\pi R^2)$

Period of motion, $T = 2\pi/\omega$



So, current in the ring, $dI = \frac{dQ}{T}$

$\because \quad Q = \sigma\pi R^2 \Rightarrow dQ = 2\sigma\pi r dr$

$\therefore \quad dI = \frac{2\sigma\pi r dr}{2\pi} \times \omega = \sigma\omega r dr$

Magnetic field due to ring is

$$dB = \frac{\mu_0 dI}{2r}$$

So, magnetic field due to disk is

$$B = \int dB = \int_0^R \frac{\mu_0}{2r} \cdot \sigma\omega r dr$$

On putting value of σ, we get

$$B = \frac{\mu_0\sigma_0\omega}{2}\int_0^R \left[1 + \left(\frac{r}{R}\right)^{\beta}\right] dr$$

$$= \frac{\mu_0\sigma_0\omega}{2}\left[R + \frac{R^{\beta+1}}{R^{\beta}(1+\beta)}\right] = \frac{\mu_0\sigma_0\omega}{2} \times \left(\frac{2+\beta}{1+\beta}\right) R$$

Now,

$$B = \frac{9}{10}\mu_0\sigma_0\omega R = \frac{\mu_0\sigma_0\omega R}{2} \times \frac{(2+\beta)}{(1+\beta)}$$

$$\Rightarrow \quad 5\beta + 10 = 9\beta + 9$$

$$\Rightarrow \quad \beta = \frac{1}{4}$$

31. (b) Magnetic field at axial point (d >> l)

$\mathbf{B}_1 = \frac{2\mu_0 M}{4\pi d^3}$ (from S to N)

$= \frac{2\mu_0 M}{4\pi D^3}$ (d = D)

Magnetic field at equatorial line (d >>)

$\mathbf{B}_2 = \frac{2\mu_0 M}{4\pi d^3}$ (from N to S)

Now, for equatorial point d = 2D

$\therefore$ $\mathbf{B}_2 = \frac{1}{8}\frac{\mu_0 M}{4\pi D^3}$(from N to S)

So, resultant magnetic field,

$\mathbf{B} = \mathbf{B}_1 - \mathbf{B}_2$(from S to N)

$$= \frac{\mu_0 M}{4\pi D^3}\left(2 - \frac{1}{8}\right)$$

$$\mathbf{B} = \frac{15}{8}\frac{\mu_0 M}{4\pi D^3} \text{ (in magnitude)}$$

On comparing with given formula, we get

$\alpha = 15/8$

32. **(a)** Emf

$$= \frac{d}{dt}\phi_B$$

$$= \left(\frac{d}{dt}A\right)\cdot B$$

$$= \left(\frac{\pi R^2}{2\pi/\omega}\right)B$$

$$= \frac{1}{2}BR^2\omega$$

33. **(b)** When switch S_1 is closed and S_2 is open, capacitor C will be fully charged. After charging, when S_1 is opened and S_2 is closed, this L − C circuit will act as tank circuit.

In tank circuit,

$$\frac{1}{2}CV^2 = \frac{1}{2}LI^2 \text{ (total energy)}$$

$$\Rightarrow \quad I^2 = \frac{C}{L}V^2$$

$$\Rightarrow \quad I = V\sqrt{C/L}$$

34. **(c)** Intensity of radiations, $I = \frac{\text{Power}}{\text{Area}} \times \text{Efficiency}$

$$= \frac{100}{4\times\pi\times(10)^3}\times\eta$$

Now, half of intensity is provided by electric field and another half by magnetic field.

So, $\frac{1}{2}I = \frac{1}{2}\varepsilon_0 E_{rms}^2 c = \frac{1}{2}\varepsilon_0\frac{E_{max}^2}{2}c$

$[E_{rms} = E_{max}/\sqrt{2}]$

$$\Rightarrow \quad \frac{100}{4\pi\times10^3}\eta = \frac{1}{2}\times 8.85\times10^{-12}\times\frac{4}{2}\times3\times10^8$$

$$\Rightarrow \quad \eta = \frac{8.85\times2\times3\times1256\times10^{-4}}{100}$$

$$\Rightarrow \quad \eta = \frac{6.66}{100}$$

or $\eta = 6.66\%$

35. **(d)** Photoelectric effect, $h\nu = h\nu_0 + eV$ (Einstein's equation)

$$\Rightarrow V = \frac{h}{e}(\nu - \nu_0)$$

Given, $V_1 = \frac{h}{e}(\nu_1 - \nu_0)$ and $3V_1 = \frac{h}{e}(2\nu_1 - \nu_0)$

$\Rightarrow \quad \frac{3V_1}{V_1} = \frac{2v_1 - v_0}{v_1 - v_0} \Rightarrow v_1 = 2v_0$

$\therefore \quad V_1 = \frac{h}{e}(2v_0 - v_0) = \frac{h}{e}v_0$

At frequency $4v_1, nV_1 = \frac{h}{e}(4v_1 - v_0)$

$= \frac{h}{e}(8v_0 - v_0) = 7 \times \frac{h}{e}v_0$

$\Rightarrow nV_1 = 7V_1 \Rightarrow n = 7$

36. (a) For electron, $pc = \sqrt{2KEm_0c^2}$

$\Rightarrow \quad pc = \sqrt{2 \times 9eV \times 0.58 \times 10^6}$

$\Rightarrow \quad pc = 3 \times 10^3 eV$

Now, de-Broglie wavelength,

$\lambda = \frac{hc}{pc} = \frac{4 \times 10^{-15} \times 3 \times 10^{10}}{3 \times 10^3} = 4 \times 10^{-8}$ cm

$(\because c = 3 \times 10^{10}$ cm/s$)$

37. (b) Decay equation, $N = N_0 \exp(-\lambda t)$

Given, $N_1 = N_0/3, N_2 = ?$

$t_1 = 20$ h $\quad t_2 = 20$ h

So, $N_1 = N_0 \exp(-\lambda t_1)$

$\Rightarrow \quad \frac{1}{3} = e(-20\lambda) \Rightarrow e^{20\lambda} = 3$

Now, $N_2 = N_0 \exp(-\lambda t_2) \Rightarrow \frac{N_0}{N_2} = e^{80\lambda}$

$\Rightarrow \quad \frac{N_0}{N_2} = \left(e^{20\lambda}\right)^4 = (3)^4$

$\Rightarrow \quad N_2 = \frac{1}{(3)^4}N_0 = \frac{N_0}{81}$

38. (b) $R_L = 10 \times 10^3 \Omega, \Delta I_c = 4 \times 10^{-3}$ A, $\Delta I_B = 20 \times 10^{-6}$ A, $V_{BE} = 40 \times 10^{-3}$ V

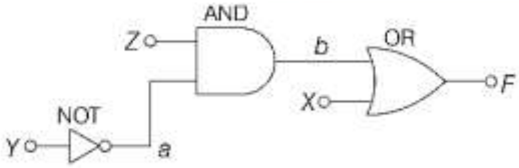
Power gain $= \beta^2 R_L/R_B$

$= \left(\frac{\Delta I_c}{\Delta I_B}\right)^2 \cdot \frac{R_L}{R_B} = \left(\frac{\Delta I_c}{\Delta I_B}\right)^2 \times \frac{R_L}{(V_{BE}/\Delta I_B)}$

$= \left(\frac{40 \times 10^{-3}}{20 \times 10^{-6}}\right)^2 \times \frac{10 \times 10^3}{40 \times 10^{-3}} \times 20 \times 10^{-6}$

$\Rightarrow A_P = 2 \times 10^{-7}$ W

39. (a)



Let's name intermediate states as a and b as shown in figure.

Now,

$a = \bar{Y}$

$b = Za = z\bar{y}$

$F = b + X = Z\bar{Y} + X$

40. (c) Modulation index,

$m = \frac{\text{Peak value of } m(t)}{A} = \frac{M}{A}$

$\therefore \quad M = mA$

$= 20 \times 60\% = 20 \times \frac{60}{100} = 12$ V

Chemistry

41. (b) For electron in 3rd orbit of H -atom.

Energy $= -\frac{2\pi^2 mk^2 e^4}{n^2 h^2}$

Substituting the standard values, we get

Energy

$$= -\frac{-2 \times \left(\frac{22}{7}\right)^2 \times 9.1 \times 10^{-31} \times (9 \times 10^9)^2 \times (1.6 \times 10^{-19})^4}{(6.6 \times 10^{-34})^2 \times (3)^2}$$

$$= \frac{-2 \times 3.14 \times 3.14 \times 9.1 \times 9 \times 9 \times 1.6 \times 1.6)^2 \times 10^{-31+18-76}}{9 \times 6.6 \times 6.6 \times 10^{-68}}$$

$$= -2.42 \times 10^{-19}\ \mathrm{J}$$

42. (c) Wavelength $= \frac{\text{Planck's constant}}{\text{Momentum}} = \frac{h}{m}$

But, momentum $= \sqrt{2Em}$

$$\therefore\ \lambda = \frac{h}{\sqrt{2Em}}$$

Energy (E) of a moving particle is nothing but its kinetic energy $\left(\frac{1}{2}mv^2\right)$.

$$\therefore\ \lambda = \frac{h}{\sqrt{2Em}} = \frac{h}{\sqrt{2 \times \frac{1}{2}mv^2 \times m}} \lambda = \frac{h}{mv}$$

Given, mass of the particle $= m$

$= 11.043 \times 10^{-26}$ kg

Velocity of particle $= v = 6.0 \times 10^7\ ms^{-1}$

Planck's constant $= h = 6.626 \times 10^{-34}\ J - s$

Substituting these values, we get

$$\lambda = \frac{6.626 \times 10^{-34} Js}{11.043 \times 10^{-26} \times 6.0 \times 10^7}$$

$$= \frac{6.626 \times 10^{-34}}{6.626 \times 10^{-18}} = 1.0 \times 10^{-16}\ m$$

43. (b) Formula of chlorine molecule is Cl_2.

Thus, the covalent radius of chlorine is half its bond length.

$\therefore$ Covalent radius $= \frac{1.98}{2} = 0.99$Å

44. (d) Covalency is the number of bonds formed by central metal atom with ligands. In $[AlCl(H_2O)_5]^{+2}$, Al central metal atom is bonded by coordination bond with one chlorine and 5 water molecules. So, covalency is 6 .

45. (b) As we move down the group electronegativity decreases resulting in decrease of electron density. This- results decrease in repulsive interactions between the bonded electron pairs hence, H − M − H bond angle reduces. The correct order of decreasing bond angle is

H — *M* — H	NH_3 >	PH_3 >	AsH_3 >	SbH_3
Bond angle	107°	92°	91°	90°

46. (d) When all the electrons in a molecule are paired it is diamagnetic and when unpaired electrons are present it is paramagnetic. Considering the filling of electrons in molecular orbitals the e.c. of molecules is

$Be_2(8) = (\sigma ls)^2, (\sigma^*1s)^2, (\sigma 2s)^2, (\sigma^*2s)^2$
$C_2(12) = (\sigma ls)^2, (\sigma^*1s)^2, (\sigma 2s)^2, (\sigma^*2s^2)$
$(\pi 2p_y)^2, (\pi 2p_z)^2$
$N_2(14) = (\sigma ls)^2, (\sigma^*1s)^2, (\sigma 2s)^2, (\sigma^*2s)^2$
$(\pi 2p_y)^2, (\pi 2p_z)^2, (\sigma 2p_x)^2$
$O_2(16) = (\sigma ls)^2, (\sigma^*1s)^2, (\sigma 2s)^2, (\sigma^*2s)^2$
$(\pi 2p_y)^2, (\sigma 2p_x)^2$
$(\pi 2p_z)^1 (\pi 2p_z)^1$
Unpaired electron is present only in O_2, so it is paramagnetic.

47. (d) The correct form of van der Waals' equation for n moles of the gas is

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

Where,

$p =$ pressure, $V =$ volume
$n =$ number of moles $R =$ gas constant
$T =$ temperature $a, b =$ constants

It can be written in a modified form as:

$$pV + \frac{an^2}{V} - \frac{abn^3}{V^2} - pnb = nRT$$

48. (d) Kinetic energy of gas $= \frac{3}{2} nRT$

Given,
$n = 1$ mol, $R = 8.314\ J\ mol^{-1}\ K^{-1}$,
$T = 27^\circ C = 300\ K$
$\therefore$ Kinetic energy of N_2 gas
$= \frac{3}{2} \times 1 \times 8.314 \times 300$
$= 3741.3\ J = 3.741\ kJ$

49. (b) The chemical reaction involved during the titration of $I_2(aq)$ by $S_2O_3^{2-}(aq)$ using starch indicator is
$I_3^- + 2\ S_2O_3^{2-} \longrightarrow S_4O_6^{2-} + 3I^-$
At the end point deep blue colour disappears due to the decomposition of the iodine-starch complex. Thus, the colour change at end point is from blue to colourless.

50. (b) Limestone $(CaCO_3)$ molar mass $= 100$ g/mol
10 g of 100% pure limestone $= 10$ g of $CaCO_3$ But 10 g of 90% pure limestone
$= 9$ g of $CaCO_3$
9 g of $CaCO_3 = 0.09$ mole of $CaCO_3$
Chemical reaction involved on heating $CaCO_3$ is given by the equation.

$$\underset{1\ mol}{CaCO_3} \xrightarrow{\Delta} CaO + \underset{1\ mol}{CO_2(g)}$$

Thus, 1 mole of $CaCO_3$ yields 1 mole of $CO_2(g)$ or 22.4 L of $CO_2(g)$.
Hence, 0.09 mole of $CaCO_3$ will yield
$= 0.09 \times 22.4$ L of CO_2
$= 2.016\ L^2$ of CO_2

51. (d) Free energy change is linked to equilibrium constant (K_p) by the formula.

$$\Delta G^\circ = -2.303 RT \log K_p$$

Value of $R = 8.314\ J\ mol^{-1}\ K^{-1}$
$T = 298\ K$
$K_p = 3 \times 10^{-29}$
Substituting these- value in equation and solving the value, ΔG° obtained is 163 kJ mol^{-1}.

52. (b) K_p is related to K_C by the formula.

$K_p = K_C(RT)^{\Delta n}$
For the equation $2\ A(g) \rightleftharpoons 2B(g) + C(g)$

$\Delta n = (2+1) - 2 = 1$
$R = 0.0821 \text{bar mol}^{-1} K^{-1}$
$T = 1069 K$
$K_C = 375 \times 10^{-6}$
Substituting the values in equation and solving we get $K_p = 3.3 \times 10^{-4}$

53. (d) The reaction equation of dissociation of CH_3COOH is
$CH_3COOH(aq) \longrightarrow CH_3COO^- + H^+$

$$K_a = \frac{[CH_3COO]^-[H]^+}{[CH_3COOH]} = 1.0 \times 10^{-5}$$

When $N = 1,\ K_a = 1.0 \times 10^{-5}$

$$\text{When } N = 0.1, K_a = \frac{1 \times 10^{-5}}{0.1 \times 10^{-2}}$$

$\Rightarrow\ K_a = 10^{-2}$

54. (d) The reaction equations for the given reactants are:
(a) $H_2O + H_2S \longrightarrow HS^- + H_3O^+$
(b) $3H_2O + N^{3-} \longrightarrow NH_3 + 3OH^-$
(c) $2H_2O + SiCl_4 \longrightarrow SiO_2 + 4HCl$
(d) $H_2O + F_2 \longrightarrow 2HF + \frac{1}{2}O_2$

$$\longrightarrow 2H^+ + 2F^- + \frac{1}{2}O_2$$

55. (d) On dissolving sodium (Na) metal in liquid ammonia (NH_3) blue colour solution is obtained due to formation of ammoniated electrons which absorbs energy in the visible region. Reaction involved is
$Na + (x+y)NH_3 \longrightarrow [Na(NH_3)_x^+] + [e(NH_3)_y]^-$

56. (a) (i) In orthoboric acid (H_3BO_3) is boron sp^2 hybridised. It forms three bonds with three hydroxyl group which lie on the same plane. Thus, in orthoboric acid, boron is in planar geometry, thus statement is correct.
(ii) BCl_3 is a non-polar molecule with a symmetrical distribution of electrons and flat geometry. It forms a polar non-covalent bond with NH_3 molecule due to presence of empty orbital in boron and lone pair of electrons over N -atom, which has tetrahedral geometry. Thus, statement is correct.
(iii) Borax is a salt of a strong base and a weak acid (H_3BO_3). Hence, it is basic in nature. Thus, this statement is incorrect.

57. (d) The approx yield of various methyl substituted chlorosilanes is as follows:
$CH_3SiCl_3 < 10 - 30\%$ yield
$(CH_3)_2SiCl_2 < 50\%$ yield
$(CH_3)_3SiCl < 10\%$ yield
$(CH_3)_4Si < 5\%$ yield
Thus, yield of $(CH_3)_4Si$ is minimum.

58. (a) When hydrocarbon is burnt in absence of give oxygen it is known as anaerobic digestion, during which vegetation is biochemically digested to give carbon dioxide and methane.

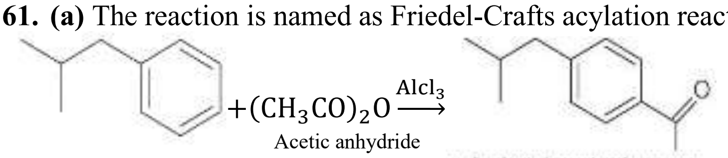
59. (c)

$$\underset{5}{CH_3}-\underset{4}{\overset{\overset{Cl}{|}}{CH}}-\underset{3}{CH_2}-\underset{2}{\overset{\overset{CH_2-CH_3}{|}}{CH}}-\underset{1}{CHO}$$

Numbering of carbon chain is done from the side of carbonyl functional group. Denoting the position of substituents, the correct name of organic compound is 4-chloro-2-ethylpentanal.

60. (b)

61. (a) The reaction is named as Friedel-Crafts acylation reaction.



Isobutyl benzene p-isobutyl acetophenone

The major product obtained is para since steric bulk of isobutyl group is para/ortho directing due to its weakly electron donating nature.

62. (a) Since, Z forms fcc lattice so per unit cell number of Z atoms = 4.

In fcc structure total number of octahedral voids = 4 = number of X atoms.
In fcc structure, total number of tetrahedral voids = 8
Thus, number of Y atoms $= \frac{1}{3} \times 8 = \frac{8}{3}$
Thus, formula of compound is $Z_4X_4Y_{8/3}$ or $X_4Y_{8/3}Z_4$ or $XY_{2/3}Z$ or $X_3Y_2Z_3$.

63. (a) Parts per million

$$= \frac{\text{Number of parts of component}}{\text{Total number of parts of all components of the solution}} \times 10^6$$

Here,
Number of parts of component = amount of dissolved oxygen $= 6 \times 10^{-3}$ g
Total number of parts of all components of the solution = amount of sea water = 1030 g
∴ Concentration of dissolved oxygen in ppm

$$= \frac{6\times10^{-3}}{1030} \times 10^6 = \frac{6}{1030} \times 10^3$$
$$= 0.005825 \times 10^3 \text{ppm}$$
$$= 5.8\text{ppm}$$

64. (a) Osmotic pressure $(\pi) = \text{nRT}$
where, i = vant-Hoff's factor = 1 for sucrose

$$\text{n} = \text{ number of moles } = \frac{\text{weight in gram}}{\text{molecular weight}}$$
$$= \frac{40}{342} \text{ for sucrose}$$

R = gas constant $= 8.314 \times 10^6$ cm^3 Pa K^{-1} mol^{-1}
T = temperature = 300 K
Substituting in formula of osmotic pressure, we get

$$\pi = 1 \times \frac{40}{342} \times 8.314 \times 10^6 \times 300$$
$$= 292\text{kPa}$$

65. (a) The cell is represented as:
$$\text{Pt }|H_2(1\text{ atm})|H^+||H^+|H_2(\text{latm})|\text{Pt}$$
$$E^\circ_{\text{cell}} = 0.0591\log\frac{[H^+]_{\text{cathode}}}{[H^+]_{\text{anode}}}$$
On substituting the value of E°_{cell} and solving
$$-0.17 = 0.0591\log\frac{[H^+]}{10^{-3}}$$
$$\frac{-0.17}{0.0591} = \log\frac{[H^+]}{10^{-3}}$$
$$\log[H^+] = -2.87 - 3 = -5.87$$
$$\text{pH} = -\log[H^+] = -(-5.87) = 5.87$$

66. (a) For a reaction working at two different temperature we have
$$\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{\Delta T}{T_1T_2}\right]$$
Substituting $k_1 = 0.002 \text{ s}^{-1}, k_2 = 0.06 \text{ s}^{-1}$
$T_1 = 500 \text{ K}, T_2 = 700 \text{ K}$
$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
$$\log\frac{0.002}{0.06} = \frac{E_a}{2.303 \times 8.314}\left[\frac{700-500}{700\times500}\right]$$
$$\log 0.030 = \frac{E_a}{19.147}\left[\frac{200}{350000}\right]$$
$$\log(3.0 \times 10^{-2}) = \frac{E_a}{19.147}\left[\frac{2}{3500}\right]$$
$$E_a = \log(3\times10^{-2}) \times \frac{19.147\times3500}{2}$$
$$E_a = 49.49 \text{ kJ mol}^{-1}$$

67. (b) The degree of absorption increases with the decrease in temperature. The correct order of temperature is $T_2 < T_3 < T_1$.

68. (b) Only following ores are sulphide ores.
Sphalerite = zinc sulfide
Fool's gold = iron disulfide
Chalcopyrites = copper iron sulphide
Rest are not sulphide ores
Calamine = zinc carbonate and silicate
Malachite = copper carbonate hydroxide
Siderite = iron carbonate
Zincite = zinc oxide
Kaolinite = basic aluminium silicate

69. (c) On completing the equations:

(i) $(NH_4)_2SO_4 + NaOH \longrightarrow NH_3 + H_2O + Na_2SO_4$

(ii) $NH_3 + 3Cl_2 \text{ (excess)} \longrightarrow N_2 + 3HCl + Cl_2$

(iii) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + H_2O$

(iv) $NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$

(v) $NH_4Cl(aq) + NaNO_2(aq) \longrightarrow N_2 + H_2O + NaCl$

N_2 is being liberated in equation b, c and e.

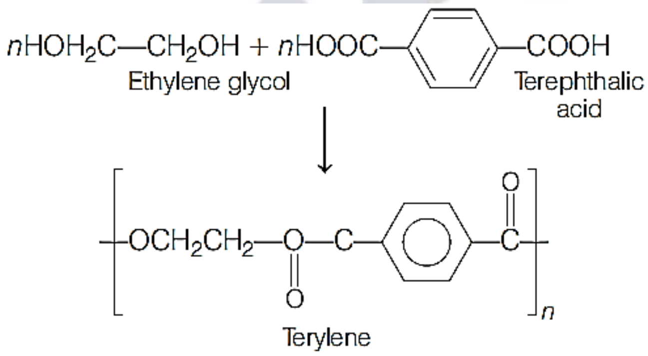
70. (c) $3Au + \underset{\text{(aqua-regia)}}{HNO_3} + 3\underset{HCl}{AuCl} + \underset{(x)}{NO} + H_2O$

$3Pt + \underset{\text{(aqua-regia)}}{4HNO_3} + 12HCl \underset{3PtCl_4}{\rightleftharpoons} + \underset{(Y)}{4NO} + 8H_2O$

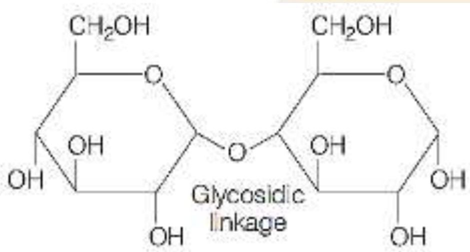
71. (a)

72. (b)

73. (a) Formation of terylene from ethylene glycol and terephthalic acid is a condensation polymerisation reaction which can be represented by the equation.

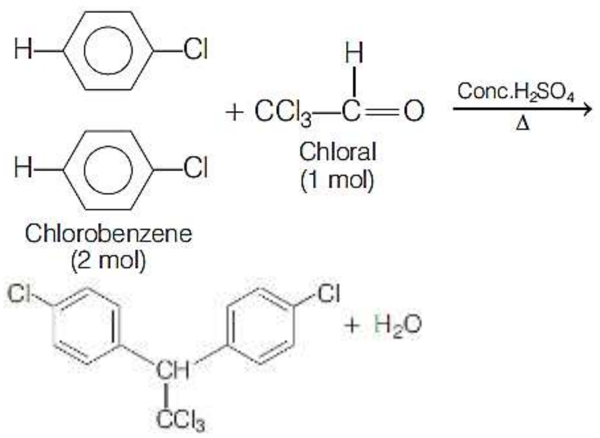


74. (c) Glycosidic linkage is a type of covalent bond that joins a carbohydrate molecule to another group. Out of the given compound maltose has glycosidic linkage as is clearly visible from its structure.



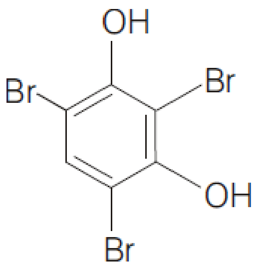
75. (b) Antioxidants are substances that prevent or slow the damage of cells caused by free radicals, butylated hydroxytoluene is an antioxidant. Antiseptic are antimicrobial substances that are applied to living tissue to reduce the possibility of infection. Bithionol is an antiseptic. Antibiotic are antimicrobial substances that act against bacteria. An example is chloramphenicol.

76. (c)

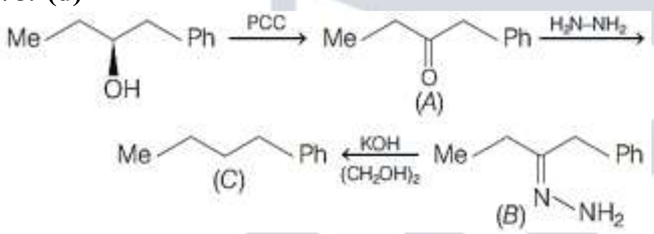


D.D.T. (dichlorodiphenyl trichloroethane)

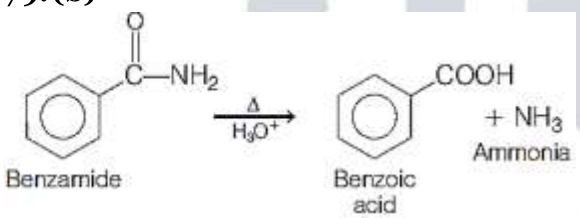
77. **(c)** 1, 3-dihydroxybenzene forms ortho-para brominated product on reacting with bromine. This happens because of electron donating nature of hydroxy (-OH) group attached at meta position (w.r.t each other), so, the major product is



78. **(d)**



79. **(b)**



80. **(d)** Carbylamine test is also known as Hofmann's isocyanide test, it is used for detection of primary amines. If primary amine is present it forms carbylamine (isocyanide) and gives foul smelling substance. Since, on reduction of tertiary amide with $LiAlH_4$ no primary amine is obtained as product, so carbyl amine test fails.

Mathematics

81. **(b)** $\because fog(x) = x$ and $f(x) = 2x + \cos x + \sin^2 x$

$\therefore$ g(x) is inverse function of f(x).

So, $1 + (2n - 1)\pi = 2x + \cos x + \sin^2 x$

$\Rightarrow 1 + (2n - 1)\pi = 2x + \cos x + 1 - \cos^2 x$

$\Rightarrow 2x + \cos x - \cos^2 x = (2n - 1)\pi$

For $n = 1, x = \frac{\pi}{2}$

$n = 2, x = \dfrac{3\pi}{2}$

$n = 3, x = \dfrac{7\pi}{2}$

$n = n = x = \frac{2\pi - 1}{2}\pi$

So, $\sum_{n=1}^{99} g(1+(2n-1)\pi)$
$= \frac{\pi}{2} + \frac{3\pi}{2} + \frac{7\pi}{2} + \cdots +$ upto 99 terms
$= \frac{\pi}{2}[1 + 3 + 7 + \cdots +$ upto 99 terms]
$= (99)^2 \frac{\pi}{2}$

82. **(c)** $\because f(x) = \frac{1+\sqrt{1+4\log_2 x}}{2} = y$ (Let)

$\Rightarrow \quad 1 + 4\log_2 x = (2y-1)^2$

$\Rightarrow \quad \log_2 x = \frac{(2y-1)^2 - 1}{4}$

$\Rightarrow \quad x = 2^{\frac{(2y-1)^2-1}{4}} \Rightarrow f^{-1}(x) = 2^{\left(\frac{(2x-1)^2-1}{4}\right)}$

$\therefore \quad f^{-1}(3) = 2^{\frac{(2\times3-1)^2-1}{4}} = 2^6 = 64$

83. **(b)** For all $n \in N$
α is greatest divisors of $n(n^2-1)$
$=$ value of $n(n^2-1)$
for $n = 2 = 2[2^2 - 1] = 6$
and β is greatest divisors of $2n(n^2+2)$
$=$ value of $2n(n^2+2)$ for $n = 1$
$= 2 \times 1(1+2) = 6$
So,
$\alpha\beta = 6 \times 6 = 36$

84. **(b)**

$$\because A = \begin{bmatrix} \frac{1}{6} & -\frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{bmatrix} = 6\begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

$A^p = A$, for every $p \in N$
So, $A^{2016l} = A^{2017m} = A^{2018n}$
$= A$, for every $l, m, n \in N$

So, $A^{2016l} + A^{2017m} + A^{2018n} = 3A = \frac{1}{\alpha}A$ (given)

$\Rightarrow \alpha = \frac{1}{3}$

85. **(c)** $\because$ A is 3×4 matrix, so rank of $A \le 3$.
$\because$ It is given that rank of $A = 3$

$$\therefore \begin{vmatrix} 1 & r & r^2 \\ r & r^2 & 1 \\ r^2 & 1 & r \end{vmatrix} \neq 0 \Rightarrow r^3 + r^3 + r^3 - r^6 - 1 - r^3 \neq 0$$

$\Rightarrow$

$\therefore$

$\therefore \; -(r^3-1)^2 \neq 0 \Rightarrow r \neq 1$
$r \in R - \{1\}$

86. **(d)** The given system of equations

$x + y + z = 9$
$2x + 5y + 7z = 52$
and $x + 7y + 11z = 77$

$$\because \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 1 & 7 & 11 \end{vmatrix} = 55 + 7 + 14 - 5 - 49 - 22 = 0$$

$$\Delta_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 77 & 7 & 11 \end{vmatrix} = 495 + 539 + 364 - 385$$

$-441 - 572 = 0$

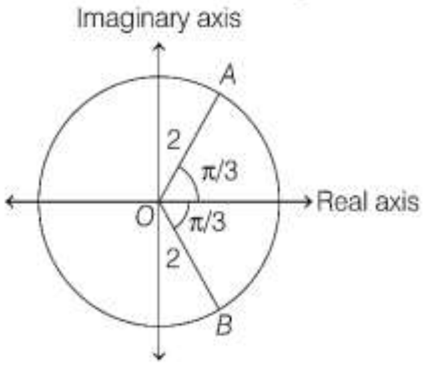
$$\Delta_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 1 & 77 & 11 \end{vmatrix} = 0$$

and $\Delta_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 1 & 7 & 77 \end{vmatrix} = 0$

$\because \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$\therefore$ System of equations have infinitely many solutions.

87. (c) Diagram of $|Z| \leq 2$ and $-\frac{\pi}{3} \leq \text{amp}Z \leq \frac{\pi}{3}$, is



Area of sectorial area OAB is

$= 2 \times \frac{2\pi}{3} = \frac{4\pi}{3}$ units

88. (d) $\because |Z_1 - Z_2| = |(-3 + 5i) - (-1 + 6i)|$
$= |-2 - i| = \sqrt{5}$
$|Z_2 - Z_3| = |(-1 + 6i) - (-2 + 8i)| = |1 - 2i| = \sqrt{5}$
$|Z_3 - Z_4| = |(-2 + 8i) - (-4 + 7i)| = |2 + i| = \sqrt{5}$
$|Z_4 - Z_1| = |(-4 + 7i) - (-3 + 5i)|$
$= |-1 + 2i| = \sqrt{5}$
$|Z_1 - Z_3| = |(-3 + 5i) - (-2 + 8i)|$
$= |-1 - 3i| = \sqrt{10}$
and $|Z_2 - Z_4| = |(-1 + 6i) - (-4 + 7i)|$
$= |3 - i| = \sqrt{10}$

$\because$ Length of sides are same and equal to $\sqrt{5}$ and length of diagonals are same and equal to $\sqrt{10}$.

$\therefore$ Points in the argand plane represents a square.

89. (a) $\because \sqrt{3} + i = -2\omega i$,

$\therefore \sqrt{3} - i = 2\omega^2 i$

where, ω is complex cube root of unity.

$\therefore \quad (\sqrt{3} + i)^n + (\sqrt{3} - i)^n$
$= (-2\omega i)^n + (2\omega^2 i)^n$

when $n = 8$

$(-2\omega i)^8+(2\omega^2 i)^8 \quad = 2^8[\omega^8+\omega^{16}]$

$= 2^8[\omega^2+\omega] \qquad [\because \omega^3=1]$

$= -2^8 = -256 \quad [\because \omega+\omega^2=-1]$

90. (b) $2\text{ cis}\frac{7\pi}{5} = 2\left(\cos\frac{7\pi}{5}+i\sin\frac{7\pi}{5}\right) = z^{\frac{1}{5}}$

$\Rightarrow\ z = 2^5(\cos 7\pi + i\sin 7\pi)$

$= 2^5(-1) = -32$

91. (a) $\because |x-1|+|x+1|\begin{cases}-2x, & x<-1\\ -2, & -1\le x\le 1\\ 2x, & x>1\end{cases}$

For, $x<-1$

$-2x<4$

$\Rightarrow \quad x>-2$

$\Rightarrow \quad x\in(-2,-1)\ \ldots\ldots(i)$

For, $x\in[-1,1]$

$-2<4$

$\Rightarrow \quad x\in[-1,1]\ \ldots\ldots(ii)$

and for $\quad x>1,$

$\Rightarrow \quad 2x<4$

$\Rightarrow \quad x<2$

$\Rightarrow \quad x\in(1,2)\ \ldots\ldots(iii)$

From Eqs. (i), (ii) and (iii),

$x\in(-2,2)$

92. (b) For the quadratic equation $x^2+x+a=0$, if roots exceed ' a ', then

$a^2+a+a>0$

$\Rightarrow \quad a\in(-\infty,-2)\cup(0,\infty)\ \ldots\ldots(i)$

and $\quad D=1-4a\ge 0$

$\Rightarrow \quad a\in\left(-\infty,\frac{1}{4}\right)\ \ldots.(ii)$

and $\quad -\frac{1}{2}>a$

$\Rightarrow \quad a<-\frac{1}{2}$

$\Rightarrow \quad a\in\left(-\infty,-\frac{1}{2}\right)\ \ldots.(iii)$

From Eqs. (i), (ii) and (iii),

$a<-2$

93. (b) Let $\sqrt{\frac{x}{1-x}}=y$

So, $\quad y+\frac{1}{y}=\frac{5}{2}$

$\Rightarrow \quad 2y^2-5y+2=0$

$\Rightarrow \quad y=2,\frac{1}{2}$

So, $\quad x=\frac{4}{5},\frac{1}{5}$

$\because \quad p>q$

$\therefore \quad p=\frac{4}{5}, q=\frac{1}{5}$

Now, for the equation $(p+q)x^4-pqx^2+\frac{p}{q}=0$

$$\Sigma\alpha = 0, \Sigma\alpha\beta = -\frac{pq}{p+q} \text{ and } \alpha\beta\gamma\delta = \frac{p}{q(p+q)}$$

$$\text{So, } (\Sigma\alpha)^2 - \Sigma\alpha\beta + \alpha\beta\gamma\delta = 0 + \frac{pq}{p+q} + \frac{p}{q(p+q)}$$

$$= \frac{p}{p+q}\left(q + \frac{1}{q}\right)$$

$$= \frac{\frac{4}{5}}{\frac{4}{5}+\frac{1}{5}}\left(\frac{1}{5}+5\right) = \frac{4}{5} \times \frac{26}{5} = \frac{104}{25}$$

94. (c) $\because x^5 - 5x^3 + 5x^2 - 1 = (x-1)^3(x^2+3x+1)$

So, $\alpha + \beta = -3$ and $\alpha\beta = 1$

so, $\alpha + \beta + \alpha\beta = -2$

95. (d) Number of possible 5 digit numbers is 120 Number of possible 4 digit numbers is 120 Number of possible 3 digit numbers start with 7 is 12

Number of possible 3 digit numbers start with 5 is 12

Then, according to order 3 digit numbers are 375,372,371,357,352,351,327

So, rank of number 327 is 271 .

96. (b) $\because 10800 = 2^4 3^3 5^2$

So, number of all even divisors ' a ' $= 4 \times 4 \times 3 = 48$ and number of all odd divisors ' b ' $= 4 \times 3 = 12$

So, $2a + 3b = (2 \times 48) + (3 \times 12) = 96 + 36 = 132$

97. (a) The general term in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{13}$

$$T_{(p+1)} = {}^{13}C_p (ax^2)^{13-p} \frac{1}{(bx)^p}$$

$$= {}^{13}C_p \frac{a^{13-p}}{b^p} x^{26-3p}$$

For $x^5, p = 7$

So, coefficient of x^5 in the expansion of

$\left(ax^2 + \frac{1}{bx}\right)^{13}$ is ${}^{13}C_7 \frac{a^6}{b^7}$ $\quad [\because p = 7]$

Similarly, the general term in the expansion of

$$\left(ax - \frac{1}{bx^2}\right)^{13}$$

$$T_{q+1} = {}^{13}C_q (ax)^{13-q}\left(-\frac{1}{bx^2}\right)^q$$

$$= {}^{13}C_q (-1)^q \frac{a^{13-q}}{b^q} x^{13-3q}$$

for $x^{-5}, q = 6$

So, coefficient of x^{-5} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{13}$ is ${}^{13}C_6 \frac{a^7}{b^6}$.

According to the question,

$${}^{13}C_7 \frac{a^6}{b^7} = {}^{13}C_6 \frac{a^7}{b^6}$$

$$\Rightarrow \quad ab = 1$$

98. (b) $\because$ Sum of all binomial coefficients is 2^n. and $200 < 2^n < 400 \Rightarrow n = 8$

$$\because \text{ General term } T_{r+1} = {}^nC_r \left(\frac{1}{x^{\frac{3}{4}}}\right)^{n-r} a^r \left(x^{\frac{5}{4}}\right)^r$$

$$= {}^nC_r a^r x^{-\frac{3}{4}n + \frac{3}{4}r + \frac{5}{4}r}$$

For independent of ' x '

$$-\frac{3}{4}n + 2r = 0 \quad [n = 8]$$

$\Rightarrow \qquad 2r = 6$

$\Rightarrow \qquad r = 3$

According to the question,

$${}^{n}C_r a^r = 448$$

$\Rightarrow \qquad {}^{8}C_3 a^3 = 448$

$\Rightarrow \qquad 56a^3 = 448$

$\Rightarrow \qquad a^3 = 8$

$\Rightarrow \qquad a = 2$

99. (c) $\frac{x^4+x^3+2x^2-2x+1}{x^3+x^2}$

$$= x + \frac{2x^2 - 2x + 1}{x^3 + x^2}$$

$$= x + \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$\Rightarrow 2x^2 - 2x + 1 = Ax(x+1) + B(x+1) + Cx^2$

By comparing the coefficients of x^2, x and constant terms.

$A + C = 2, A + B = -2$ and $B = 1$

$\Rightarrow A = -3, B = 1$ and $C = 5$

So, $A + B + C = 3$ and $P(x) = x$

So, $A + B + C = P(3)$

100. (b)

101. (c)

102. (a)

103. (c) Given equation

$\sin x - \sin 2x + \sin 3x = 2\cos^2 x - 2\cos x$

$\Rightarrow 2\sin 2x\cos x - \sin 2x = 2\cos x(\cos x - 1)$

$\Rightarrow 2\sin x\cos x(2\cos x - 1) = 2\cos x(\cos x - 1)$

Either, $\qquad \cos x = 0$

$\Rightarrow x = \frac{\pi}{2} \ldots\ldots (i)$

or $\sin x(2\cos x - 1) = \cos x - 1$

$\Rightarrow 2\sin x\cos x + 1 = \sin x + \cos x$

$\Rightarrow 4\sin^2 x\cos^2 x + 1 + 4\sin x\cos x$

$= 1 + 2\sin x\cos x$ [on squaring both sides]

$\Rightarrow 2\sin x\cos x(2\sin x\cos x + 1) = 0$

Either $\sin x = 0$ or $\cos x = 0$ or $\sin 2x = -1$

$\because \qquad x \in (0, \pi)$

$\therefore \qquad \sin x \neq 0$

So, $\qquad \sin 2x = -1$

$\Rightarrow \qquad 2x = \frac{3\pi}{2}$

$\Rightarrow \qquad x = \frac{3\pi}{4} \ldots\ldots\ldots (ii)$

From Eqs. (i) and (ii),

$x = \frac{\pi}{2}$ or $\frac{3\pi}{4}$

Only two solutions possible in $x \in (0, \pi)$.

104. (b) $2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-}\frac{1}{8}$

$$= 2\left[\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right] + \sec^{-1}\frac{5\sqrt{2}}{7}$$

$$= 2\tan^{-1}\frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{40}} + \sec^{-1}\frac{5\sqrt{2}}{7}$$

$$= 2\tan^{-1}\frac{13}{39} + \sec^{-1}\frac{5\sqrt{2}}{7}$$

$$= 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}\left[\because \sec^{-1}\frac{5\sqrt{2}}{7} = \tan^{-1}\frac{1}{7}\right]$$

$$= \tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{6}{8} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\left(\frac{3}{4}\times\frac{1}{7}\right)}$$

$$= \tan^{-1}\left(\frac{21+4}{28-3}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

105. **(d)** $\because \cosh^2 x - \sinh^2 x = 1$

$\Rightarrow \cosh^2 x - \cos^2\theta = 1 \; [\because \sinh x = \cos\theta]$

$\Rightarrow \cos^2\theta = \cosh^2 x - 1$

$= \frac{14}{9} - 1 = \frac{5}{9}$

So,

$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{5}{9} = \frac{4}{9}$

$\because \quad -\pi < \theta < -\frac{\pi}{2}$

$\therefore \quad \sin\theta = -\frac{2}{3}$

106. **(c)** $\because \tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2} = \frac{a-b}{a+b}\tan\frac{A+B}{2}$

$\therefore \quad \frac{a-b}{a+b} = \frac{1}{4}$

$\Rightarrow \quad \frac{5-b}{5+b} = \frac{1}{4}$

$\Rightarrow \quad 20 - 4b = 5 + b$

$\Rightarrow \quad 5b = 15 \Rightarrow b = 3$

$\therefore \sqrt{a^2 - b^2} = \sqrt{25-9} = \sqrt{16} = 4$

107. **(c)** According to sine law,

$\frac{\sin A}{\alpha+1} = \frac{\sin B}{\alpha-1}$

$\Rightarrow \quad \frac{2\sin B\cos B}{\alpha+1} = \frac{\sin B}{\alpha-1} \quad [A = 2B]$

$\Rightarrow \quad \cos B = \frac{\alpha+1}{2(\alpha-1)}$

$\Rightarrow \quad \frac{(\alpha+1)^2 + \alpha^2 - (\alpha-1)^2}{2\alpha(\alpha+1)} = \frac{\alpha+1}{2(\alpha-1)}$

$\Rightarrow \quad \frac{4\alpha + \alpha^2}{2\alpha(\alpha+1)} = \frac{\alpha+1}{2(\alpha-1)}$

$\Rightarrow \quad (4+\alpha)(\alpha-1) = (\alpha+1)^2$

$\Rightarrow \quad \alpha^2 + 3\alpha - 4 = \alpha^2 + 2\alpha + 1$

$\Rightarrow \quad \alpha = 5$

108. **(c)** $\because \; A = \frac{\pi}{2}$

$\therefore \; B + C = \frac{\pi}{2}$

and $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$$\Rightarrow \frac{r}{R} = 4\frac{1}{\sqrt{2}}\sin\frac{B}{2}\sin\left(\frac{\pi}{4}-\frac{B}{2}\right)$$

$$\Rightarrow \frac{5}{2} = 2\sqrt{2}\sin\frac{B}{2}\left[\frac{1}{\sqrt{2}}\cos\frac{B}{2}-\frac{1}{\sqrt{2}}\sin\frac{B}{2}\right]$$

$\frac{5}{4} = \sin\frac{B}{2}\cos\frac{B}{2} - \operatorname{Sin}^2\frac{B}{2}$....(i)

and $r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

$$\Rightarrow \frac{r_1}{R} = 4\times\frac{1}{\sqrt{2}}\cos\frac{B}{2}\cos\left(\frac{\pi}{4}-\frac{B}{2}\right)$$

$$\Rightarrow \frac{\lambda}{2} = 2\sqrt{2}\cos\frac{B}{2}\left[\frac{1}{\sqrt{2}}\cos\frac{B}{2}+\frac{1}{\sqrt{2}}\sin\frac{B}{2}\right]$$

$\frac{\lambda}{4} = \cos^2\frac{B}{2} + \cos\frac{B}{2}\sin\frac{B}{2}$.....(ii)

From Eqs. (i) and (ii),

$$\frac{\lambda}{4}-\frac{5}{4} = 1$$
$$\Rightarrow \lambda - 5 = 4$$
$$\Rightarrow \lambda = 9$$

So, the roots of quadratic equation

$x^2 - 4x + 3 = 0$ are 1,3

109. **(a)** $\because$ BA is parallel to $2\hat{i}-\hat{j}+2\hat{k}$, so

$$BA = \pm|BA|\frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{2^2+1^2+2^2}}$$
$$= \pm\frac{18}{3}(2\hat{i}-\hat{j}+2\hat{k})$$
$$= \pm(12\hat{i}-6\hat{j}+12\hat{k})$$
$$\therefore \quad OA - OB = \pm(12\hat{i}-6\hat{j}+12\hat{k})$$
$$\Rightarrow \quad OA = (3\hat{i}+\hat{j}-\hat{k}) \pm (12\hat{i}-6\hat{j}+12\hat{k})$$
$$= 15\hat{i}-5\hat{j}+11\hat{k} \text{ or } -9\hat{i}+7\hat{j}-13\hat{k}$$

110. **(c)** Let the required vector is

$a\hat{i}+b\hat{j}+c\hat{k}$

$\because$ The above vector is parallel to $2\hat{i}-2\hat{j}-4\hat{k}$, so

$\frac{a}{2} = \frac{b}{-2} = \frac{c}{-4} = k(\text{ Let })$ .... (i)

and coplanar with the vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$, so

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ a & b & c \end{vmatrix} = 0$$

$\Rightarrow \quad 1(c-b)+1(a) = 0$

$\Rightarrow \quad a-b+c = 0$ .....(iii)

Now, only option $\hat{i}-\hat{j}-2\hat{k}$ satisfy the relation Eqs. (i) and (ii).

111. **(a)** Equation of given line is

$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{2} = r$ (let)..... (i)

The point on the line is $P(2r+1, r+2, 2r-3)$

Now, equation of plane is

$$(r-(\hat{i}+\hat{j}+\hat{k}))\cdot[\{(\hat{i}+\hat{j}+\hat{k})-(\hat{i}-\hat{j}-\hat{k})\} \times (\hat{i}-2\hat{j})] = 0$$

$$\Rightarrow \quad (r-(\hat{i}+\hat{j}+\hat{k}))\cdot[(2\hat{j}+2\hat{k})\times(\hat{i}-2\hat{j})] = 0$$

$$\Rightarrow \quad (r-(\hat{i}+\hat{j}+\hat{k}))\cdot(4\hat{i}+2\hat{j}-2\hat{k}) = 0$$

$$\Rightarrow \quad 4x+2y-2z = 4 \quad \text{......(ii)}$$

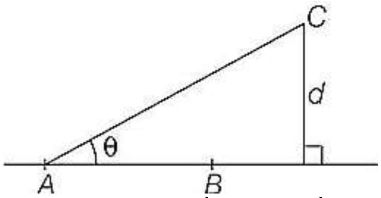
$\because$ Point P is point of intersection of line Eq. (i) and plane Eq. (ii).

$\therefore \quad 4(2r+1)+2(r+2)-2(2r-3)=4$

$\Rightarrow \quad r=-\frac{5}{3}$

So, point P is $\left(-\frac{7}{3}, \frac{1}{3}, -\frac{19}{3}\right)$ or $\frac{1}{3}(-7\hat{i}+\hat{j}-19\hat{k})$

112. **(c)** The given vectors $A(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}), B(2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}), C(\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$, so the perpendicular distance of point C from the line AB is



$$|AC| \sin\theta = \frac{|AB \times AC|}{|AB|} = d \text{ (let)}$$

$\because \quad AB=\hat{i}-3j+k, AC=-\hat{j}+\hat{k}$

$$\text{So, } d = \frac{\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 0 & -1 & 1\end{matrix}\right\|}{\sqrt{1+9+1}}$$

$$= \frac{|-2\hat{i}-\hat{j}-\hat{k}|}{\sqrt{11}} = \frac{\sqrt{4+1+1}}{\sqrt{11}} = \sqrt{\frac{6}{11}}$$

113. **(b)** Given vertices of a tetrahedron are

$A(4\hat{i}+5\hat{j}+\hat{k}), B(-\hat{j}+\hat{k})$

$C(3\hat{i}+9j+4k), D(-2\hat{i}+4\hat{j}+4\hat{k})$

$SO, AB=-4\hat{i}-6\hat{j}\ AC=-\hat{i}+4\hat{j}+3\hat{k}$

$AD=-6\hat{i}-\hat{j}+3\hat{k}$

Now, volume of the given tetrahedron is

$$v=\frac{1}{6}\left|\begin{vmatrix}-4 & -6 & 0 \\ -1 & 4 & 3 \\ -6 & -1 & 3\end{vmatrix}\right|$$

$$=\frac{1}{6}\times|[-4(12+3)+6(-3+18)+0]|$$

$$=\frac{1}{6}|[-60+90]|=\frac{30}{6}=5 \text{ cubic units}$$

114. **(a)**

115. **(c)** (a) $\because \bar{x}=\frac{x_1+x_2+x_3+\cdots+x_n}{n}$

$\Rightarrow \quad n\bar{x}=x_1+x_2+x_3+\cdots+x_n$

$\Rightarrow \quad (x_1-\bar{x})+(x_2-\bar{x})+\cdots+(x_n-\bar{x})=0$

$$\Rightarrow \quad \sum_{i=1}^{n}(x_i-\bar{x})=0$$

(b) $\because$ Variance $(\sigma^2)=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$ = Mean of the squares of the deviations from mean.

(c) $\because$ Mean deviation of observations is mean of the absolute deviations from any measure of central tendency.

(d) $\because$ Coefficient of variation is used to find the homogeneity of given two series.

$\therefore$ (a) $\rightarrow$ (iii), (b) $\rightarrow$ (v), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (ii)

116. **(d)** $\because$ The variance is remains unchanged if we add or subtract a positive number to each observation, but if any positive number ' k ' is multiplied to each observation, then variance becomes k^2 times. So, variance of the new data is $6^2\times7=36\times7=252$

117. **(a)** Since number of getting prime numbers {2,3,5} is 3 therefore number of getting prime numbers on both the dice is 3×3.

And getting a head and a tail on two coins $=1\times1\times2$

And total number of possibilities $= (6 \times 6) \times (2 \times 2)$

So, required probability $= \frac{3\times3\times1\times1\times2}{16\times6\times2\times2} = \frac{1}{8}$

118. **(b)** From ground floor to 7 -floor, a person can leave the lift cabin in six floor (beginning with the first floor). Since, 5 persons leaving the cabin at different floors, so number of way to do so is ${}^6P_5$. And total number of possible ways $= 6^5$

So, required probability $= \frac{{}^6P_5}{6^5}$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2}{6 \times 6 \times 6 \times 6 \times 6} = \frac{5}{54}$$

119. **(c)** Total number of items produced in a day $= 10000$.

Let E_1, E_2, E_3 be the events of selecting a item produced by Machines A, B, C respectively. Then,

$$P(E_1) = \frac{2500}{10000} = \frac{25}{100} = \frac{5}{20}$$

$$P(E_2) = \frac{3500}{10000} = \frac{35}{100} = \frac{7}{20}$$

$$P(E_3) = \frac{4000}{10000} = \frac{40}{100} = \frac{8}{20}$$

Let E be the event of selecting a defective item. Then,

$$P\left(\frac{E}{E_1}\right) = \frac{2}{100}, P\left(\frac{E}{E_2}\right) = \frac{3}{100}, P\left(\frac{E}{E_3}\right) = \frac{5}{100}$$

So, required probability $= P\left(\frac{E_3}{E}\right)$

$$= \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot \left(\frac{E}{E_3}\right)}$$

$$= \frac{\left(\frac{8}{20} \times \frac{5}{100}\right)}{\left(\frac{5}{20} \times \frac{2}{100}\right) + \left(\frac{7}{20} \times \frac{3}{100}\right) + \left(\frac{8}{20} \times \frac{5}{100}\right)}$$

$$= \frac{40}{10 + 21 + 40} = \frac{40}{71}$$

120. **(a)** Given that,

$$\frac{P(x=1)}{\frac{1}{2}} = \frac{P(x=2)}{\frac{1}{3}} = \frac{P(x=3)}{1} = \frac{P(x=4)}{\frac{1}{5}} = k$$

$$\because \ P(x=1) + P(x=2) + P(x=3) + P(x=4) = 1$$

$$\Rightarrow \ \frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$$

So,

$$P(x=1) = p_1 = \frac{30}{2 \times 61} = \frac{15}{61}$$

$$P(x=2) = p_2 = \frac{30}{3 \times 61} = \frac{10}{61}$$

$$P(x=3) = p_3 = \frac{30}{61}$$

and $P(x=4) = p_4 = \frac{30}{5\times61} = \frac{6}{61}$

$$\because \ \sigma^2 = \left(\sum_{i=1}^{4} p_i x_i^2\right) - \mu^2$$

$$\Rightarrow \ \sigma^2 + \mu^2 = \left(\frac{15}{61} \times 1^2\right) + \left(\frac{10}{61} \times 2^2\right)$$

$$+ \left(\frac{30}{61} \times 3^2\right) + \left(\frac{6}{61} \times 4^2\right)$$

$= \frac{15+40+270+96}{61} = \frac{421}{61}$

121. **(b)** The number of telephone calls per hour is a random variable with mean $= 5$.
The required probability is given by

$$P(r>2) = 1 - P(r \le 2) = 1 - \sum_{r=0}^{2} \frac{e^{-5}\cdot 5^r}{r!}$$

$$=1 - \left[\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!}\right]\frac{1}{e^5} = 1 - \frac{37}{2e^5} = \frac{2e^5 - 37}{2e^5}$$

122. **(d)** Let $D(h,k)$

$$\text{Now, } \triangle ABC = \frac{1}{2}\left|\begin{vmatrix} 3 & 4 & 1 \\ -3 & 6 & 1 \\ -5 & 1 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}|[3(6-1) + 4(-5+3) + 1(-3+30)]|$$

$$= \frac{1}{2}|[15 - 8 + 27]| = 17 \text{ and } \triangle ADC = \frac{1}{2}\left|\begin{vmatrix} 3 & 4 & 1 \\ h & k & 1 \\ -5 & 1 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}|[3(k-1) + 4(-5-h) + 1(h+5k)]|$$

$$= \frac{1}{2}|(3k - 3 - 20 - 4h + h + 5k)| = \frac{1}{2}|8k - 3h - 23|$$

$\because$

$\Rightarrow \triangle ABC = \triangle ADC$

$\Rightarrow 34 = |3h - 8k + 23|$

By taking locus of point (h,k), $|3x - 8y + 23| = 34$

$\Rightarrow 3x - 8y - 11 = 0$ or $3x - 8y + 57 = 0$

or $(3x - 8y - 11)(3x - 8y + 57) = 0$

123. **(d)** $\because \frac{3\sqrt{3}-1}{2\sqrt{2}} = \cos\alpha + 2\sin\alpha$

and $\frac{\sqrt{3}+3}{2\sqrt{2}} = 2\cos\alpha - \sin\alpha$

$$\therefore\ 5\cos\alpha = 5\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = 5\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$\Rightarrow\ \cos\alpha = \cos\frac{\pi}{12}\ \Rightarrow\ \alpha = \frac{\pi}{12}$$

124. **(b)** Since, $L(p,q) = \frac{ap+bq+c}{\sqrt{a^2+b^2}}, \forall p,q \in R$

Now, $L\left(\frac{2}{3},\frac{1}{3}\right) + L\left(\frac{1}{3},\frac{2}{3}\right) + L(2,2) = 0$

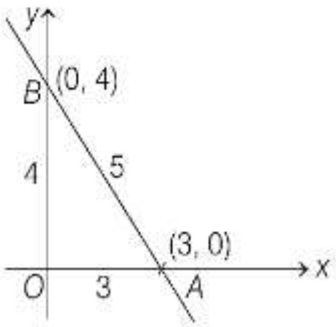
$\Rightarrow \frac{2}{3}a + \frac{1}{3}b + c + \frac{1}{3}a + \frac{2}{3}b + c + 2a + 2b + c = 0$

$\Rightarrow\ 3a + 3b + 3c = 0$ (given)

$\Rightarrow\ a + b + c = 0\ \ldots.$ (i)

So, line $ax + by + c = 0$ passes through the point $(1,1)$.

125. **(d)**



$\therefore$ Incentre of $\triangle$ OAB is $\left(\frac{(5\times0)+(4\times3)+(3\times0)}{5+4+3}, \frac{(5\times0)+(4\times0)+(3\times4)}{5+4+3}\right)$

$= \left(\frac{12}{12}, \frac{12}{12}\right) = (1,1)$

126. **(b)** Let the equation of required line is

$x + 2y + c = 0 \quad \ldots(i)$

According to the question,

$$\frac{2|C-3|}{\sqrt{5}} = \frac{|8-C|}{\sqrt{5}}$$

$\Rightarrow 2(C-3) = 8 - C \Rightarrow 3C = 14$

$\Rightarrow$

$C = \frac{14}{3}$

So, equation will be

$x + 2y + \frac{14}{3} = 0 \quad \ldots. (ii)$

Let the normal form is

$x\cos\alpha + y\sin\alpha = p \quad \ldots.. (iii)$

From Eqs. (ii) and (iii),

$$\frac{\cos\alpha}{-1} = \frac{\sin\alpha}{-2} = \frac{p}{\frac{14}{3}}$$

$\Rightarrow \alpha = \pi + \tan^{-1} 2$ and $p = \frac{14}{3\sqrt{5}} \Rightarrow p = \frac{14}{\sqrt{45}}$

So, required equation is

$x\cos\alpha + y\sin\alpha = \frac{14}{\sqrt{45}}, \alpha = \pi + \tan^{-1} 2$

127. **(a)** According question equation of lines

$(y-1) - \tan\theta(x-1) = 0$

and $(y-1) - \cot\theta(x-1) = 0$

So, combined equation is

$(y-1)^2 - (x-1)(y-1)[\tan\theta + \cot\theta] + (x-1)^2 = 0$

$\Rightarrow x^2 - \frac{2}{\sin 2\theta}xy + y^2 + \left(\frac{2}{\sin 2\theta} - 2\right)(x + y - 1) = 0$

On comparing with the equation

$x^2 - (a+2)xy + y^2 + a(x + y - 1) = 0$

$\Rightarrow a + 2 = \frac{2}{\sin 2\theta}$

$\Rightarrow \sin 2\theta = \frac{2}{a+2} \Rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{2}{a+2}\right)$

128. **(a)**

129. **(b)** The chord $y - mx - 1 = 0$ is tangent to the circle

$$S_1 \equiv x^2 + y^2 - 4x + 1 = 0$$

or $$(x-2)^2 + (y-0)^2 = 3$$

So, $$\frac{|2m+1|}{\sqrt{1+m^2}} = \sqrt{3}$$

$\Rightarrow$ $$4m^2 + 1 + 4m = 3 + 3m^2$$

$\Rightarrow$ $$(m+2)^2 = 6$$

$\Rightarrow$ $$m = -2 \pm \sqrt{6}$$

Now, equation of chord is $L \equiv y + (2 \pm \sqrt{6})x - 1 = 0$, Let possible point is (x_1, y_1) for which the chord $L = 0$ is chord of contact of circle $S \equiv x^2 + y^2 - 1 = 0$.

So, $xx_1 + yy_1 - 1 = 0$ and $y + (2 \pm \sqrt{6})x - 1 = 0$

represent same line, so $\frac{x_1}{2\pm\sqrt{6}} = \frac{y_1}{1} = \frac{-1}{-1}$

$\Rightarrow (x_1, y_1) \equiv (2 \pm \sqrt{6}, 1)$

130. **(b)** The given circle is

$$x^2 + y^2 - 6x - 2y + 1 = 0$$

$(x-3)^2 + (y-1)^2 = 9$ … (i)

and the line $y + c = 0$ is tangent to the circle at point $(a, 4)$, so

$$4 + c = 0$$

C = -4 ….. (ii)

and slope of line joining the point of contact and centre is infinite, so

So,

$$\frac{a-3}{4-1} = 0 \Rightarrow a = 3$$

So ac = -12

131. **(b)** For four common tangent, the distance between centres of two circles C_1C_2 should be more than the sum of radii $(r_1 + r_2)$.

$\therefore \sqrt{49+9} > \sqrt{49+9-33} + a$

$\Rightarrow \sqrt{58} > \sqrt{25} + a \Rightarrow a < \sqrt{58} - 5$

$\because a \in N$ so, $a = \{1,2\}$

$\therefore$ Two circles are possible only.

132. **(d)** Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ …. (i)

Since, circle (i) passes through point (1,0),

$1 + 2g + c = 0$……. (ii)

$\because$ Circle (i) cutting the circles

$x^2 + y^2 - 2x + 4y + 1 = 0$ and

$x^2 + y^2 + 6x - 2y + 1 = 0$, orthogonally, so

$2g(-1) + 2f(2) = c + 1$…. (iii)

and $2g(3) + 2f(-1) = c + 1$ …. (iv)

From Eqs. (ii), (iii) and (iv)

$f = 0$ and $g = 0$

So, $c = -1$

So, equation of required circle is $x^2 + y^2 = 1$

$\therefore$ Centre is (0,0).

133. **(a)** Let the equation of required circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ….. (i)

Since, circle (i) cuts the circles

$x^2 + y^2 - 2x + 6y = 0$

$x^2 + y^2 - 4x - 2y + 6 = 0$ and

$x^2 + y^2 - 12x + 2y + 3 = 0$ orthogonally, then

$-2g + 6f = c$ …. (ii)

$-4g - 2f = c + 6$ …… (iii)

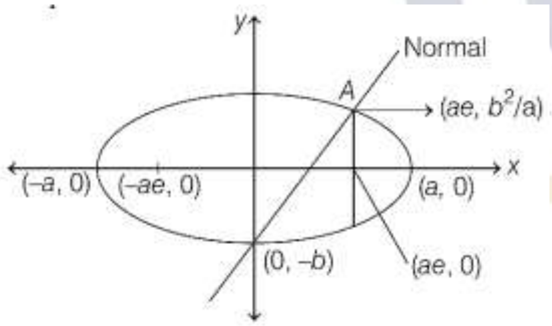
and $-12g + 2f = c + 3$ …. (iv)

From Eqs. (ii), (iii) and (iv), we get
$g=0, f=-\frac{3}{4}$ and $c=-\frac{9}{2}$
So, equation of required circle is
$x^2+y^2-\frac{3}{2}y-\frac{9}{2}=0$
Now, equation of tangent at point $(0,3)$ is
$3y-\frac{3}{4}(y+3)-\frac{9}{2}=0 \Rightarrow y=3$

134. **(c)** Let point of intersection of required tangents is (h,k), then equation of pair of tangents drawn through the point (h,k) to the parabola $y^2=8x$ is
$(y^2-8x)(k^2-8h)=[yk-4(x+h)]^2$ .... (i)
$\because$ Equation of tangent at the vertex of parabola is $x=0$, so let $M(0,y_1)$ and $N(0,y_2)$, so we will get y_1 and y_2 by putting $x=0$ in Eq. (i).
$y^2(k^2-8h)=(yk-4h)^2$
$\Rightarrow y^2k^2-8hy^2=y^2k^2+16h^2-8hky$
$\Rightarrow y^2-ky+2h=0$ having roots y_1 and y_2
$\therefore |y_1-y_2|=\frac{\sqrt{k^2-8h}}{1}=4(\text{ given })$
$\Rightarrow k^2-8h=16 \Rightarrow k^2=8(h+2)$
By taking locus of point (h,k) we are getting $y^2=8(x+2)$.

135. **(c)** Since, equation of normal to parabola $y^2=x$, having slope ' m ' is
$y=mx-2\left(\frac{1}{4}\right)m-\frac{1}{4}m^3$ ..... (i)
$\because$ Normal drawn through the point $(c,0)$.
$\therefore\ m^3+2m-4mc=0$
$\Rightarrow\ m[m^2+(2-4c)]=0$
Either $m=0\{\because$ normal is X-axis also $\}$
or $m^2+(2-4c)=0$
$\because$ Remaining two are perpendicular to each other.
$\therefore\ 2-4c=-1 \Rightarrow 4c=3 \Rightarrow c=\frac{3}{4}$

136. **(c)** Given, equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, let $(a>b)$ the an end point of a lat us rectum be $\left(ae,\frac{b^2}{a}\right)$, then the equation of the normal at this end point is



$$\frac{x-ae}{\frac{ae}{a^2}}=\frac{y-\frac{b^2}{a}}{\frac{b^2}{ab^2}}$$
It will pass through the end of the minor axis $(0,-b)$, if $-a^2=-ab-b^2$
$\Rightarrow\ 1=\frac{b}{a}+\left(\frac{b}{a}\right)^2 \Rightarrow 1-\left(\frac{b}{a}\right)^2=\frac{b}{a}\left[\because e=\sqrt{1-\left(\frac{b}{a}\right)^2}\right]$
$\Rightarrow\ e^2=\sqrt{1-e^2}$
$\Rightarrow\ e^4+e^2=1$

137. **(d)** Equation of tangent at point (x_1, y_1) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1 \ldots$ (i)

Intersecting point of tangent with the axes is

$A\left(\frac{a^2}{x_1}, 0\right)$ and $B\left(0, \frac{b^2}{y_1}\right)$.

Now, let middle point of point A and B is $P(h, k)$, then

$$h=\frac{a^2}{2x_1} \text{ and } k=\frac{b^2}{2y_1}$$

$$\Rightarrow x_1=\frac{a^2}{2h} \text{ and } y_1=\frac{b^2}{2k}$$

$\because (x_1, y_1)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

so $\frac{a^2}{h^2}+\frac{b^2}{k^2}=4$

On taking locus of point $P(h, k)$, we are getting $\frac{a^2}{x^2}+\frac{b^2}{y^2}=4$

138. **(d)** Let a function $f(x)=2x^3+10x-13$

$\because f'(x)=6x^2+10>0, \forall x \in R$

$\therefore f(x)$ is strictly increasing function.

Now, $f(1)=2+10-13=-1<0$

and $f(2)=16+20-13=23>0$

So, the one and only one root $e \in (1,2)$, where e is the eccentricity of the conic.

$\because 1<e<2$

$\therefore$ Conic is a hyperbola.

139. **(c)** Except the equilateral triangle, the centroid, orthocentre and circumcentre are collinear and centroid divides the line segment joining the orthocentre and the circumcentre in the ratio 2: 1. So, if $(-1,3,2)$ and $(5,3,2)$ are respectively the orthocentre and circumcentre of triangle, then coordinate of centroid is

$$\left(\frac{(-1\times 1)+(5\times 2)}{1+2}, \frac{(3\times 1)+(3\times 2)}{1+2}, \frac{(2\times 1)+(2\times 2)}{1+2}\right)=(3,3,2)$$

So, (A) is true and (R) is false.

140. **(a)** Given relations

$al+bm+cn=0 \quad \ldots\ldots$ (i)

and $mn+nl+lm=0 \quad \ldots\ldots$ (ii)

From relations Eqs. (i) and (ii),

$\left(\frac{al+bm}{-c}\right)(l+m)+lm=0$, [by eliminating ' n ']

$al^2+(a+b-c)lm+bm^2=0$

$$\Rightarrow a\left(\frac{l}{m}\right)^2+(a+b-c)\left(\frac{l}{m}\right)+b=0 \ldots\ldots \text{(iii)}$$

Let roots of Eq. (iii) is $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$,

$\therefore \frac{l_1 l_2}{m_1 m_2}=\frac{b}{a} \quad \ldots$ (iv)

Similarly, $\frac{l_1 l_2}{n_1 n_2}=\frac{c}{a} \quad \ldots$ (v)

So, $\frac{l_1 l_2}{\frac{1}{a}}=\frac{m_1 m_2}{\frac{1}{b}}=\frac{n_1 n_2}{\frac{1}{c}}$ [from Eqs. (iv) and (v)]

If lines whose direction cosines given by the relation Eqs. (i) and (ii) are perpendicular if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

141. **(b)** If the plane $ax+by+cz=1$, passes through $(1,2,3)$, then $a+2b+3c=1$.

142. **(b)** $\lim_{x\to-\infty}\frac{3|x|-x}{|x|-2x}-\lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x}$

$$= \lim_{x\to-\infty} \frac{-3x - x}{-x - 2x} - \lim_{x\to 0} \frac{\frac{\log(1+x^3)}{x^3} \times x^3}{\left(\frac{\sin x}{x}\right)^3 \times x^3}$$

$[\because |x| = -x, x < 0]$

$$= \frac{3+1}{1+2} - 1 = \frac{4}{3} - 1 = \frac{1}{3}$$

143. **(c)** Given, $f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & x < 2 \\ a + b, & x = 2 \\ \frac{x-2}{|x-2|} + b, & x > 2 \end{cases}$

$$= \begin{cases} -1 + a, & x < 2 \\ a + b, & x = 2 \\ 1 + b, & x > 2 \end{cases}$$

If $f(x)$ is continuous at $x = 2$, then

$-1 + a = a + b = 1 + b$

So, $a = 1, b = -1$

$\therefore a + b = 0$

144. **(c)** Given function,

$$f(x) = \begin{cases} \frac{x^2 \log(\cos x)}{\log(1+x^2)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\because$ LHL (at $x = 0$) $= \lim_{h\to 0} \frac{\log(\cos(0-h))}{\frac{\log(1+h^2)}{h^2}} = \frac{\log(1)}{1} = 0$

and RHL (at $x = 0$)

$$= \lim_{h\to 0} \frac{\log(\cos h)}{\frac{\log(1+h^2)}{h^2}} = \frac{\log(1)}{1} = 0$$

And $f(0) = 0$

$\therefore f(x)$ is a continuous at $x = 0$.

Now, LHD (at $(x = 0)$)

$$= \lim_{h\to 0} \frac{\frac{(0-h)^2 \log(\cos(0-h))}{\log(1+h^2)} - 0}{-h}$$

$$= \lim_{h\to 0} \frac{\frac{h^2 \log(\cos h)}{\log(1+h^2)}}{-h} = \lim_{h\to 0} h \times \frac{\log(\cos h)}{\frac{\log(1+h^2)}{-h^2}}$$

$$= 0 \times \frac{\log(1)}{-1} = 0$$

and RHD (at $x = 0$)

$$= \lim_{h\to 0} \frac{\frac{h^2 \log(\cos h)}{\log(1+h^2)}}{h}$$

$$= \lim_{h\to 0} \frac{h \log(\cos h)}{\frac{\log(1+h^2)}{h^2}} = 0 \times \frac{\log(1)}{1} = 0$$

$\therefore f(x)$ is differentiable at $x = 0$.

145. **(c)**(a) $\because y = |x| + |x - 2| = \begin{cases} 2x - 2, & x \geq 2 \\ 2, & x < 2 \end{cases}$

Now, LHD (at $x = 2$) $= \lim_{h\to 0} \frac{2-2}{-h}$

$= \lim_{h\to 0} \frac{0}{-h} = 0$ RHD (at $x = 2$)

$= \lim_{h\to 0} \frac{[[_2(2+h)] - 2}{h}$

$= \lim_{h\to 0} \frac{2h}{h} = 2$

So, $\frac{dy}{dx}$, at $x = 2$ does not exist.

(b) $\because f(x) = |\cos 2x| = \begin{cases} 0, & x = \frac{\pi}{4} \\ -\cos 2x, & x > \frac{\pi}{4} \end{cases}$

So $f'\left(\frac{\pi^+}{4}\right) = \lim_{h\to 0} \frac{-\cos\left[2\left(\frac{\pi}{4}+h\right)\right]-0}{h}$

$= \lim_{h\to 0} \frac{\sin 2h}{h} = 2$

(c) Since, $f(x) = \begin{cases} \sin\pi = 0, & 1 \le x < 2 \\ \sin 0 = 0, & 0 \le x < 1 \end{cases}$

So, $f'(1^-) = \lim_{h\to 0} \frac{0-0}{-h} = 0$

(d) Since, $f(x) = \begin{cases} \log\left(\frac{1}{2}\right), & x = \frac{1}{2} \\ \log(1-x), & 0 < x < 1 \end{cases}$

So, LHD(at $x = \frac{1}{2}$) $= \lim_{h\to 0} \frac{\log\left(1-\left(\frac{1}{2}-h\right)\right)-\log\left(\frac{1}{2}\right)}{-h}$

$= \lim_{h\to 0} \frac{\log\left(\frac{1}{2}+h\right) - \log\left(\frac{1}{2}\right)}{-h}$

$= \lim_{h\to 0} \frac{\log(1+2h)}{-h} = -2$

RHD at($x = \frac{1}{2}$) $= \lim_{h\to 0} \frac{\log\left(1-\left(\frac{1}{2}+h\right)\right)-\log\left(\frac{1}{2}\right)}{h}$

$= \lim_{h\to 0} \frac{\log\left(\frac{1}{2}-h\right) - \log\left(\frac{1}{2}\right)}{h}$

$= \lim_{h\to 0} \frac{\log(1-2h)}{h} = -2$

So, $f'\left(\frac{1}{2}\right) = -2$

$\therefore$ (a) → (iv), (b) → (i), (c) → (ii), (d) → (iii)

146. **(c)** Given, $y = \frac{(\sin^{-1}x)^2}{2}$

$\Rightarrow \quad 2y_1 = 2(\sin^{-1}x) \times \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \quad \sqrt{1-x^2}y_1 = \sin^{-1}x$

$\Rightarrow \quad \sqrt{1-x^2}y_2 - \frac{2x}{2\sqrt{1-x^2}}y_1 = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \quad (1-x^2)y_2 - xy_1 = 1$

147. **(c)** Given that relative errors $\frac{\Delta r}{r} = \frac{\Delta h}{h} = 0.02$

For cone, the volume is $V = \frac{1}{3}\pi r^2 h$

percentage error $\frac{\Delta V}{V} \times 100 = \left(2\frac{\Delta r}{r} + \frac{\Delta h}{h}\right) \times 100 = 6$

148. **(c)** Given curve is $x = a(1+\cos\theta), y = a\sin\theta \Rightarrow (x-a)^2 + y^2 = a^2$, is a circle.

The normal to the circle always passes through the centre of the circle. So, normal at a point to the given curve passes through centre $(a, 0)$.

149. **(d)** According to Lagrange's Mean value theorem

$$g'(c)=\frac{g(6)-g(0)}{6-0}=\frac{\frac{f(6)}{6+1}-\frac{f(0)}{0+1}}{6}$$

$$=\frac{-\frac{4}{7}-12}{6}=-\frac{88}{42}=-\frac{44}{21}$$

150. **(b)** For turning point $f'(x)=0$

$\Rightarrow \quad 6x^2-30x+36=0$

$\Rightarrow \quad (x-2)(x-3)=0$

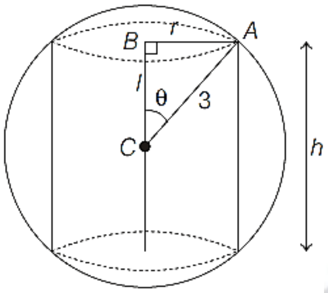
$\Rightarrow \quad x=2,3$

and $f(2)=20$ and $f(3)=19$

So, $(\alpha,\beta)=(2,20)$ and $(\gamma,\delta)=(3,19)$

So, $\alpha-\gamma-\beta+\delta=2-3-20+19=-2$

151. **(b)** Since volume of required cylinder $(V)=\pi r^2h$ ….(i)



$\because$

$\Rightarrow$

and $l^2+r^2=9$

$r^2=9-l^2$

$h=2l$

So, $V=\pi(9-l^2)h=\pi\left(9-\frac{h^2}{4}\right)h$

For maximum volume $\frac{dv}{dh}=0$

$\Rightarrow \pi\left(9-\frac{3h^2}{4}\right)=0$

$\Rightarrow \quad h=2\sqrt{3}$

152. **(b)** $I=\int\frac{dx}{(e^x+e^{-x})^2}=\int\frac{e^{2x}}{(e^{2x}+1)^2}dx$

Let $(e^{2x}+1)=t$

$\Rightarrow 2e^{2x}dx=dt$

So, $I=\frac{1}{2}\int\frac{dt}{t^2}=-\frac{1}{2t}+c$

$=-\frac{1}{2(e^{2x}+1)}+c$

153. **(d)** $I=\int_0^{\frac{\pi}{2}}\frac{dx}{1+(\tan x)^{\sqrt{2018}}}$

$$=\int_0^{\frac{\pi}{2}}\frac{(\cos x)^{\sqrt{2018}}}{(\cos x)^{\sqrt{2018}}+(\sin x)^{\sqrt{2018}}}dx\ \ldots(i)$$

By using property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$I=\int_0^{\frac{\pi}{2}}\frac{(\sin x)^{\sqrt{2018}}}{(\sin x)^{\sqrt{2018}}+(\cos x)^{\sqrt{2018}}}dx\ \ldots(ii)$$

By adding Eqs. (i) and (ii),

$$2I=\int_0^{\frac{\pi}{2}}\frac{(\sin x)^{\sqrt{2018}}+(\cos x)^{\sqrt{2018}}}{(\sin x)^{\sqrt{2018}}+(\cos x)^{\sqrt{2018}}}dx$$

$$\Rightarrow\ 2I=\int_0^{\pi/2}1dx=\frac{\pi}{2}$$

$$\Rightarrow\ I=\frac{\pi}{4}$$

154. **(c)** Let $\frac{x}{(x^2+1)(x-1)}=\frac{P}{x-1}+\frac{Qx+R}{x^2+1}$

$\Rightarrow\ x=P(x^2+1)+(Qx+R)(x-1)$

On comparing the coefficients of different terms

$P+Q=0\ldots.\ (i)$

$R-Q=1\ldots..\ (ii)$

and $P-R=0\ \ldots..\ (iii)$

From Eqs. (i), (ii) and (iii),

$P=R=\frac{1}{2}$ and $Q=-\frac{1}{2}$

So, $I=\int\frac{x}{(x^2+1)(x-1)}dx$

$$=\frac{1}{2}\int\frac{dx}{x-1}+\frac{1}{2}\int\frac{-x+1}{x^2+1}dx$$

$$=\frac{1}{2}\log|x-1|-\frac{1}{4}\int\frac{2xdx}{x^2+1}+\frac{1}{2}\int\frac{dx}{x^2+1}$$

$$=\frac{1}{2}\log|x-1|-\frac{1}{4}\log|x^2+1|+\frac{1}{2}\tan^{-1}(x)+d$$

So, $A=-\frac{1}{4},B=\frac{1}{2},C=\frac{1}{2}$

$$\therefore\ A+B+C=-\frac{1}{4}+\frac{1}{2}+\frac{1}{2}=\frac{3}{4}$$

155. **(b)** Let $I=\int_0^{\frac{\pi}{2}}\frac{\cos^3 x}{\sin x+\cos x}dx\ \ldots.\ (i)$

By using property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$I=\int_0^{\frac{\pi}{2}}\frac{\sin^3 x}{\cos x+\sin x}dx\ \ \ldots\ (ii)$

By adding Eqs. (i) and (ii),

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)}{\sin x+\cos x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}\left(1-\frac{1}{2}\sin 2x\right)dx$$

$$=\frac{\pi}{2}-\frac{1}{2}\left(\frac{-\cos 2x}{2}\right)_{20}^{\frac{\pi}{2}}=\frac{\pi}{2}+\frac{1}{4}(-1-1)$$

$$\Rightarrow\ 2I=\frac{\pi}{2}-\frac{1}{2}$$

$$\Rightarrow\ I=\frac{\pi-1}{4}$$

156. **(c)** $\int_0^3(2x+x^2)dx=\int_0^3 f(x)dx$

where, $f(x)=(2+x^2)$

Now, $\int_0^3 f(x)dx=\lim_{n\to\infty}\frac{1}{n}\Sigma_{r=1}^{3n}f\left(\frac{r}{n}\right)$

$$=\lim_{n\to\infty}\frac{1}{n}\Sigma_{r=1}^{3n}\left[2+\left(\frac{r}{n}\right)^2\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[(2\times 3n)+\frac{1^2+2^2+3^2+\cdots+(3n)^2}{n^2}\right]$$

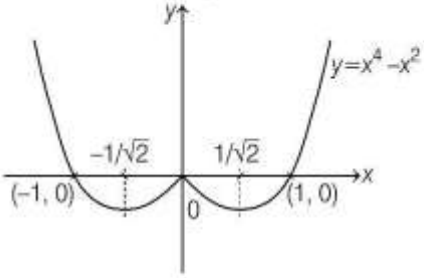
$$= \lim_{n\to\infty} \frac{1}{n}\left[6n + \frac{1^2+2^2+3^2+\cdots+9n^2}{n^2}\right]$$

157. **(d)** For points of minimum

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 4x^3 - 2x = 0$$

$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}$$



The required area $= 2\left|\int_0^{\frac{1}{\sqrt{2}}}(x^4-x^2)dx\right|$

$$= 2\left[\frac{x^5}{5} - \frac{x^3}{3}\right]_0^{1/\sqrt{2}}$$

$$= 2\left|\frac{1}{5\times 4\sqrt{2}} - \frac{1}{3\times 2\sqrt{2}}\right|$$

$$= 2\left|\frac{3-10}{60\sqrt{2}}\right| = \frac{14}{60\sqrt{2}} = \frac{7}{30\sqrt{2}}$$

158. **(d)** Equation of family of circles having centres on X-axis and passing through the origin is

$x^2 + y^2 + 2ax = 0$ …. (i)

On differentiation

$2x + 2y\frac{dy}{dx} + 2a = 0$ …. (ii)

From Eqs. (i) and (ii), on the elimination of 2a

$$x^2 + y^2 + \left(-2x - 2y\frac{dy}{dx}\right)x = 0$$

$$\Rightarrow \quad x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad y^2 - x^2 - 2xy\frac{dy}{dx} = 0$$

159. **(b)** Given differential equation

$$\frac{dy}{dx} = \frac{y^2}{x^2+xy}$$

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now, the differential equation will be

$$v + x\frac{dv}{dx} = \frac{v^2x^2}{x^2+vx^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2}{1+v} - v = \frac{v^2 - v - v^2}{1+v} = -\frac{v}{1+v}$$

$$\Rightarrow$$

$$\frac{1+v}{v}dv = -\frac{dx}{x}$$

$\Rightarrow \quad \log v + v = -\log x - \log c$

$\Rightarrow \quad \log\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right) = -\log(cx)$

$\log\left(\frac{y}{x} \times cx\right) = -\frac{y}{x} \Rightarrow cy = e^{-\frac{y}{x}} \Rightarrow e^{-\frac{y}{x}} = cy$

160. **(a)** According to the question,

$$\frac{dy}{dx} = x + xy$$

$$\Rightarrow \frac{dy}{1+y} = xdx$$

$\Rightarrow \quad \log|1+y| \ldots . \text{(i)}$

$\because$ The curve (i) passes through the point $(0,1)$

$\therefore$ $\log 2 = c$

$\therefore$ The equation of required curve is

$$\log(1+y) = \frac{x^2}{2} + \log 2$$

$$\Rightarrow \frac{1+y}{2} = e^{\frac{x^2}{2}} \Rightarrow y = 2e^{\frac{x^2}{2}} - 1$$