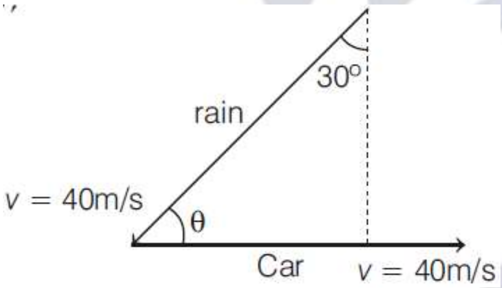
Physics

1. (a) The four fundamental forces of nature are as follows,
(i) Strong nuclear force which is between nuclear particles like, quarks. Its strength is very high and of the order of 1 . But it's range is very short.
(ii) Weak nuclear force which is also between nuclear particles, however its strength is low and of the order of 10^{-13}. This force is usually found in radioactive nuclei. It is also a short range force.
(iii) Electromagnetic force which is the force between the electrically charged particles. Its strength is of the order of 10^{-2}. But its range is very high, upto infinity.
(iv) Gravitational force which is the force of attraction between bodies due to gravity. Its strength is very low and of the order of 10^{-39}. But it's range is very high upto infinity.
So, the correct sequence of match is
A → (ii), B → (iv), C → (i) and
D → (iii).

2. (b, d) In statement (b), the percentage error in case of 1 m length is
$$\frac{\Delta l_1}{l_1} \times 100 = \frac{0.01}{1} \times 100 = 1\%$$
while in case of 0.5 m length, it is
$$\frac{\Delta l_2}{l_2} \times 100 = \frac{0.01}{0.5} \times 100 = 2\%$$
As percentage error in case of 0.5 m length is greater. So, the accuracy is less as compared to that of 1 m length. Hence this statements is incorrect. In statement (d), according to the result of rounding off, if the number to be rounded off is 5 , then the preceding digit remains unchanged if it is even or increased by l, if it is odd. So, the correct result is 2.44. Hence, this is also an incorrect statement.
However, In statement (a), as the second measured value i.e., 5.51 km is more closed to the result i.e., 5.678 km so, it is more precise. Hence, this statement is correct.
Similarly, in the statement (c), the significant digits in first number is 2 and in second number is also 2 as the trailing zero after non-zero number,

3. (a)

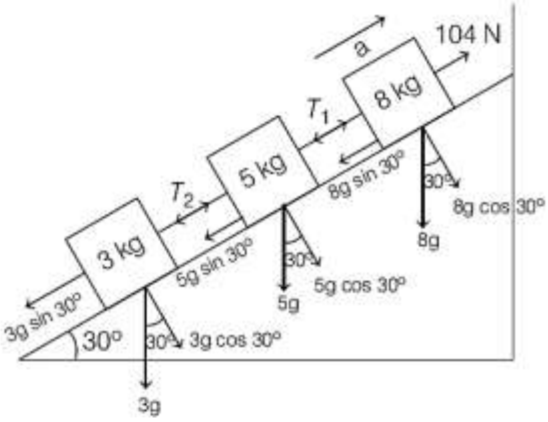
4. (b) The given situation can be shown in figure below,



As the rain makes an angle of 30° with the vertical. So, the angle made by it with the horizontal is
$\theta = 90° - 30° = 60°$
And as the car is moving in horizontal direction with the same speed as that of the rain. So, the rain appears to him falling at an angle of 60°.

5. (d)

6. (d) The given situation is shown in the figure below,



If the system moves upward with acceleration a m/s^2 on the application of 104 N , then the equation of motion for 8 kg block is

$$104 - T_1 - 8g\sin 30^\circ = 8a$$

$$\Rightarrow 104 - T_1 - 8 \times 10 \times \frac{1}{2} = 8a \; [\because g = 10 \text{ m/s}^2]$$

$64 - T_1 = 8a$ …… (i)

Equation of motion for 5 kg block is

$$T_1 - T_2 - 5g\sin 30^\circ = 5a$$

$T_1 - T_2 - 25 = 5a$ …….(ii)

Equation of motion for 3 kg block is

$T_2 - 3g\sin 30^\circ = 3a$

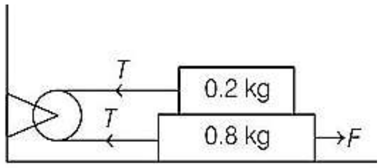
$T_2 - 15 = 3a$ ……. (iii)

Adding Eqs. (i), (ii) and (iii), we get

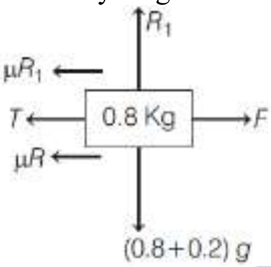
$24 = 16a \Rightarrow a = 1.5 \text{ m/s}^2$

Hence, acceleration a of block is 1.5 m/s^2.

7. (a) Given, acceleration of block, $a = 5 \text{ m/s}^2$ and coefficient of friction between two blocks and table, $\mu = 0.1$



Free body diagram for 0.8 kg block.



For 0.8 kg block, friction force due to 0.2 kg block is in opposite direction of applied force F.

∴ Equation of motion for 0.8 kg block,

$F - T - \mu R - \mu R_1 = 0.8a$

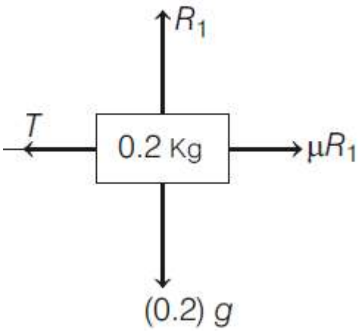
putting the given values, we get

$$F - T - 0.1(0.8 + 0.2)g - 0.1 \times 0.2g = 0.8a$$

$$F - T - 0.1 \times 10 - 0.1 \times 0.2 \times 10 = 0.8 \times 5$$

$F - T = 5.2$ ……. (i)

Free body diagram for 0.2 kg body,



For 0.2 kg block, friction force due to 0.8 kg block is in opposite direction of tension force.

So, $T - \mu R_1 = 0.2a$

$$T - 0.1 \times 0.2g = 0.2 \times 5$$

$$T - 0.02 \times 10 = 1.0$$

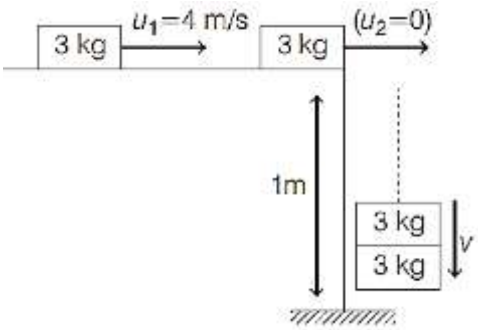
$T = 1.2$

From Eqs. (i) and (ii), we get

$$F - 1.2 = 5.2$$

$$F = 5.2 + 1.2 = 6.4 \text{ N}$$

8. (d) The given situation can be shown as below,



According to law of conservation of momentum, total momentum before collision = total momentum after collision.
$m_1u_1 + m_2u_2 = (m_1 + m_2)v$
Given, mass of boxes, $m_1 = m_2 = 3$ kg
speed of the moving box, $u_1 = 4$ m/s
and initial speed of second box, $u_2 = 0$
Putting the given values, we get

$$3 \times 4 + 3 \times 0 = (3 + 3)v$$

$$\Rightarrow \quad v = \frac{12}{6} = 2 \text{ m/s}$$

Thus, the two bodies move with the velocity of 2 m/s. Applying law of conservation of energy at height of 1 m and at the bottom of table.
$KE_1 + PE_1 = KE_2 + PE_2$
At the bottom just above the ground, the total energy is KE as height is approximately equals to zero.

$$\frac{1}{2}(m_1 + m_2)v^2 + (m_1 + m_2)gh = KE_2$$

$$\Rightarrow KE_2 = \frac{1}{2} \times 6 \times 4 + 6 \times 10 \times 1$$

$$= 12 + 60 = 72 \text{ J}$$

Hence, the kinetic energy just before the boxes strike the floor is 72 J .

9. **(a)** Given, velocity of the ball, $\mathbf{v} = (20\hat{\mathbf{i}} + 24\hat{\mathbf{j}})$m/s and mass of the ball, m = 2 kg From the equation of motion, when a ball is thrown,

$\mathbf{v} = \mathbf{u} - \mathbf{g}t$ ..... (i)
Substituting the given values in Eq. (i), we get
$\mathbf{v} = (20\hat{\mathbf{i}} + 24\hat{\mathbf{j}}) - (10\hat{\mathbf{j}})8$
$= 20\hat{\mathbf{i}} + 24\hat{\mathbf{j}} - 80\hat{\mathbf{j}} = 20\hat{\mathbf{i}} - 56\hat{\mathbf{j}}$...... (ii)

Fom the law of conservation of energy, change in potential energy of the ball = change in kinetic energy of the ball

$$\Rightarrow \Delta PE = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

$$= \frac{1}{2}m(u^2 - v^2)$$

$$= \frac{2}{2}(u \cdot u - v \cdot v)$$

$$= [(20\hat{i} + 24\hat{j}) \cdot (20\hat{i} + 24\hat{j}) - [(20\hat{i} - 56\hat{j}) \cdot (20\hat{i} - 56\hat{j})]$$

$$= (20)^2 + (24)^2 - (20)^2 - (56)^2$$

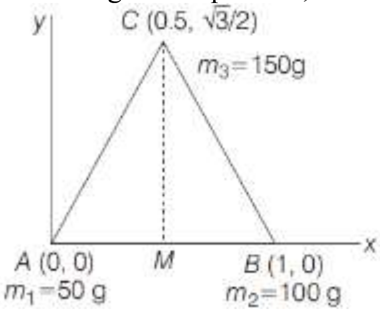
$$= (24)^2 - (56)^2 = 576 - 3136$$

$$= -2560 = -2.56 \text{ kJ}$$

Hence, change in potential energy of the ball after t = 85 s is -2.56 kJ .

10. **(d)** Given,
mass of ball A, $m_1 = 50$ g
mass of ball B, $m_2 = 100$ g
mass of ball C, $m_3 = 150$ g
and length of each side of triangle = 1 m

According to the question,



$\because y_3 = \sqrt{AC^2 - AM^2}$

$= \sqrt{1-(0.5)^2} = \sqrt{0.75} = \frac{\sqrt{3}}{2}$ $[\because AM = x_3 = 0.5]$

Coordinates of Centre of mass is (x, y), then

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150} = \frac{175}{300} = \frac{7}{12}$$

and $y = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$

$$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150} = \frac{75\sqrt{3}}{300} = \frac{\sqrt{3}}{4}$$

The coordinates (x, y) for the centre of the mass of this system of balls is $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{4}\text{m}\right)$. So, no option given is correct.

11. (d) Given, radii of ring, solid disc and solid sphere are identical, say R.

When a body of mass m starting from rest rolls down an inclined plane, then the velocity of body when it reaches at the ground is given by

$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

where, h = height of inclined plane,
R = radius of body,
and k = radius of gyration
For a ring, $k^2 = R^2$

$\therefore$ Speed of the ring, $v_R = \sqrt{\frac{2gh}{1+\frac{R^2}{R^2}}} = \sqrt{gh}$..... (i)

For a solid sphere, $k^2 = \frac{2R^2}{5}$

$\therefore$ Speed of the solid sphere, $v_S = \sqrt{\frac{2gh}{1+\frac{2}{5}}}$

$\Rightarrow \sqrt{\frac{10gh}{7}} = 1.19\sqrt{gh}$ .... (ii)

For a solid disc, $k^2 = \frac{R^2}{2}$

$\therefore$ Speed of the solid disc, $v_D = \sqrt{\frac{2gh}{1+\frac{1}{2}}}$

$\Rightarrow \sqrt{\frac{4gh}{3}} = 1.15\sqrt{gh}$

From Eqs. (i), (ii) and (iii), it is clear that $V_S > V_D > V_R$.

12. (c) We know that time period of a spring mass system,

$T_1 = 2\pi\sqrt{\frac{m}{k}}$
where, m = mass of body
and k = force constant of the spring
Time period of simple pendulum,

$T_2 = 2\pi\sqrt{\frac{l}{g}}$

According to the question, initially time period of a spring mass system, T_1 = time period of simple pendulum, T_2

$2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}}$

$\sqrt{\frac{m}{k}} = \sqrt{\frac{l}{10}}$

...(i) [$\because g = 10\ m/s^2$]

When both of them are put in an elevator going downwards with an acceleration $5\ m/s^2$, then no effect occurs on the time period of spring mass system.

i.e., $T_1' = T_1 = 2\pi\sqrt{\frac{m}{k}}$

But time period of simple pendulum changes in elevator and is given by

$T_2' = 2\pi\sqrt{\frac{l}{g_{eff}}}$, where $g_{eff} = g - a$ = effective acceleration of the pendulum when elevator is accelerating downwards with a m/s^2.

$$T_2' = 2\pi\sqrt{\frac{l}{g-5}} = 2\pi\sqrt{\frac{l}{5}}$$

$$\therefore \quad \frac{T_1'}{T_2'} = \frac{2\pi\sqrt{\frac{m}{k}}}{2\pi\sqrt{\frac{l}{5}}} = \frac{\sqrt{l/10}}{\sqrt{l/5}} \quad \text{[form eq (i)]}$$

$$\therefore \quad \frac{T_1'}{T_2'} = \frac{1}{\sqrt{2}}$$

Hence, the ratio of time period of the spring mass system to the time period of the pendulum is $\frac{1}{\sqrt{2}}$.

13. (c) We know that the escape velocity of a particle from the surface of a planet is given by

$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2gR^2}{R}}$ [$\because GM = gR^2$]

$= \sqrt{2gR}$

where, g = acceleration due to gravity,
and R = radius of the planet
Given, the velocity at equator, $v_E = V$
and acceleration due to gravity at equator,

$g_E = \frac{1}{3} g_P$

$g_E = 3g_E$ ….. (i)

The escape velocity of a particle at equator,

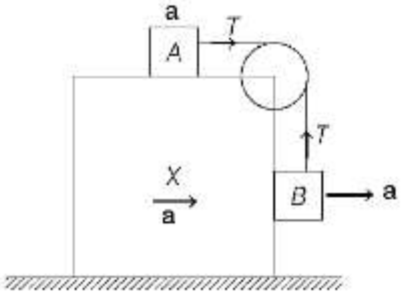
$v_E = \sqrt{2g_E R}$…… (ii)

and at poles,

$$v_P = \sqrt{2g_PR}$$
$$= \sqrt{2 \times 3g_ER} = \sqrt{3}\sqrt{2g_ER}$$
$$= \sqrt{3}v_E \quad \text{[From Eq. (i)]}$$
$$= \sqrt{3}v \quad \text{[From Eq. (ii)]}$$
$$[\because v_E = V]$$

Hence, $\sqrt{3}v$ is the escape velocity for a particle at the pole of this planet.

14. (a) Given, Mass of block A and B, $m_A = m_B = m$. Coefficient of friction between each block and surface, $\mu = 0.5$.



Block X moves with acceleration a such that block A and B remains stationary. Block B is stationary only when, Tension force on block B is equal to friction force on B.

i.e., $T = \mu R$

$T = \mu ma \quad \ldots\ldots(i)[\, R = ma]$

Block A is stationary only when friction force on block A is equal to the sum of tension force on block A and applied force on block A.

$$\mu R = T + ma$$
$$\mu mg = \mu ma + ma \quad \text{[From Eq. (i),]}$$
$$\mu g = \mu a + a$$
$$\mu g = a(\mu + 1)$$
$$a = \frac{\mu g}{\mu + 1} = \frac{0.5g}{0.5 + 1} = \frac{0.5g}{1.5} = \frac{g}{3}$$
$$\therefore\ a = \frac{g}{3}$$

Thus, g/3 should be its minimum acceleration such that blocks A and B remains stationary.

15. (c) Given, percentage change in volume $= 0.4\%$ Bulk modulus of water, $B = 2 \times 10^9$ Pa

The change in volume of water is given by

$$\Delta V = -\frac{p \cdot V}{B}$$

where, p = pressure applied on the sample of the matter.

Given,

$$\frac{\Delta V}{V} = -0.4\% = \frac{0.4}{100}$$

Pressure required, $p = B\left(-\frac{\Delta V}{V}\right)$

$$= \frac{0.4}{100} \times 2 \times 10^9 \text{ Pa} = \frac{0.4}{100} \times \frac{2 \times 10^9}{101325} \text{ atm}$$
$$= 79 \text{ atm} \approx 80 \text{ atm}$$

Hence, 80 atm pressure is needed to compress a sample of water by 0.4%.

16. (c) Given, mass of iron ball, $m = 100$ kg

density of iron ball, $\rho_{iron} = 8 \times 10^3\ kg/m^3$

density of sea water $\rho_{sea\ water} = 1000\ kg/m^3$

and acceleration due to gravity, $g = 10\ m/s^2$

When the ball is submerged in sea water, then upthrust force acting on the ball in upward direction. When the ball is completely submerged, it's effective weight gets decreased. This decrease in weight would be equal to the upthrust on the body according to Archimedes principle. Hence tension T in the massless cable is equal to the apparent weight on the ball.

i.e., T = apparent weight

$$T = W\left(1 - \frac{\rho_{\text{sea water}}}{\rho_{\text{iron}}}\right) = mg\left(1 - \frac{1000}{8 \times 10^3}\right)$$

$$= 100 \times 10\left(1 - \frac{1}{8}\right) = 1000 \times \frac{7}{8} = 875 \text{ N}$$

17. **(b)** Given, area of a circular copper coin increases by 0.4%.

i.e., $\frac{\Delta A}{A} = 0.004$

Increase in temperature, $\Delta T = 100^\circ\text{C}$

We know that, the coefficient of areal expansion β is given by

$$\beta = \left(\frac{\Delta A}{A}\right) \cdot \frac{1}{\Delta T} = \frac{0.004}{100} = 4 \times 10^{-5} / {}^\circ\text{C}$$

∴ Coefficient of linear expansion, of the coin is

$$\alpha = \frac{\beta}{2} = \frac{4 \times 10^{-5}}{2}\ {}^\circ\text{C} = 2 \times 10^{-5} / {}^\circ\text{C}$$

18. **(c)** Given, specific heat of water,

$W = 4200 \text{ J/Kg}^\circ\text{C} = 4.2 \times 10^3 \text{ J/Kg}^\circ\text{C}$

change in temperature of water,

$\Delta T = T_2 - T_1 = 100 - 25 = 75^\circ\text{C}$

mass of water, $m = 100 \text{ g} = 10^{-1} \text{ kg}$

and power of heater, $H = \frac{Q}{t} = 210 \text{ W}$

$\Rightarrow Q = 210t$ .... (i)

Where Q is amount of heat required to raise the temperature of water which is given by

$Q = mc\Delta T$

$210t = mc\Delta T$ [From Eqs.

$210t = 10^{-1} \times 4.2 \times 10^3 \times 75$

$$t = \frac{10^{-1} \times 4.2 \times 10^3 \times 75}{210} = 150 \text{ s}$$

Therefore, the time required to raise the temperature of water is 150 s .

19. **(d)** Given, initial temperature of 1 mole of N_2 gas

$T_0 = 300 \text{ K}$

initial pressure of gas, $p_1 = p$

and final pressure of gas, $p_2 = 10p$

By adiabatic process relation between p and T,

$$p_1^{1-\gamma} T_1^\gamma = p_2^{1-\gamma} \cdot T_2^\gamma$$

$$\left(\frac{p_1}{p_2}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^\gamma \Rightarrow \left(\frac{p}{10p}\right)^{1-\gamma} = \left(\frac{T_2}{300}\right)^\gamma$$

$$10^{\gamma-1} = \frac{T_2^\gamma}{300^\gamma} \Rightarrow T_2^\gamma = 10^{\gamma-1} \times 300^\gamma$$

$$\Rightarrow T_2 = 10^{\frac{\gamma-1}{\gamma}} \times 300 = 10^{1-\frac{1}{\gamma}} \times 300$$

$$= 10^{1-\frac{1}{7/5}} \times 300 \left[\because \text{ For diatomic, } \gamma = \frac{7}{5}\right]$$

$$= 10^{2/7} \times 300 = (100)^{1/7} \times 300$$

$$= 1.9 \times 300 \left[\because 100^{1/7} = 1.9\right]$$

$$= 570 \text{ K}$$

Therefore, the final gas temperature after compression is 570 K .

20. **(c)** For monatomic gas A,

atomic mass of molecules of gas A, $m_A = 4u$

and temperature of gas, $T_A = 27^\circ\text{C}$

For diatomic gas B,

atomic mass of molecules of gas B,

$m_B = 2 \times 20u = 40u$

Temperature of gas, $B = T_B$
Since, rms speed of the molecules of gas, A = rms speed of molecules of gas, B
$(v_{rms})_A = (v_{rms})_B$
$$\sqrt{\frac{3RT_A}{m_A}} = \sqrt{\frac{3RT_B}{m_B}}$$
$$\Rightarrow \frac{T_A}{m_A} = \frac{T_B}{m_B} \Rightarrow \frac{27}{4u} = \frac{T_B}{40u}$$
$\Rightarrow T_B = 270°C$

21. **(b)** Given, length of spring, $l = 16$ m
speed of wave, $v = 32$ m/s
and total number of nodes between the two fixed ends of the string = 9
∴ Total number of segments of vibrations between fixed ends, $p = 9 + 1 = 10$
∴ frequency of the wave, $f = \frac{p}{2L} \times v$
$$= \frac{10}{2 \times 16} \times 32 = 10 \text{ Hz}$$

22. **(a)** Here, let frequencies of horn A and B are n_A and n_B, respectively.
As given, $n_A - n_B = \frac{50}{10}$
$n_A - n_B = 5$ beats
When, the horn B is blown while moving towards the wall the apparent frequency,
$$n'_B = n_B \frac{v + v_0}{v - v_S}$$
Since, both the observer and source are moving with truck.
So, $v_s = v_0 = 10$ m/s
$$n_B' = n_B\left(\frac{330+10}{330-10}\right)$$
$$n_B' = n_B 1.0625$$
$$= n_B' - n_B = 0.0625 n_B$$
As given, $n'_B - n_B = 5$
$$0.0625 n_B = 5$$
$$n_B = 80 \text{ Hz}$$
∵ Given, that if frequency of A is decreased then beat frequency is increases.
So, $n_B > n_A$
∴ $n_A = n_B - 5 = 80 - 5$
⇒ $n_A = 75$ Hz

23. **(c)** Given, maximum intensity along y-axis,
$I_Y = I_0$
and minimum intensity along x-axis,
$$I_X = \frac{2I_0}{3}$$
The intensity transmitted, through a polaroid with pass axis 45° to y-axis (i.e., 45° to x-axis also) is given by Brewster's law,
$$I = I_x\cos^2 45° + I_y\cos^2 45°$$
$$= I_0\cos^2 45° + \frac{2I_0}{3}\cos^2 45°$$
$$= I_0 \cdot \frac{1}{2} + \frac{2I_0}{3} \cdot \frac{1}{2} = \frac{I_0}{2} + \frac{I_0}{3} = \frac{5I_0}{6}$$

24. **(b)** Let m^{th} bright fringe of wavelength λ_m and n^{th} bright fringe of wave length λ_n formed at a distance x_n from central maxima at same point P, i.e., they coincide, therefore, $x_m = x_n$.
$$\frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d}$$
$$\Rightarrow \frac{m}{n} = \frac{\lambda_n}{\lambda_m}$$
Putting the given values, we get

$$=\frac{600\times10^{-9}}{400\times10^{-9}}$$

[Given, $\lambda_m=400\times10^{-9}$ m, $\lambda_n=600\times10^{-9}$ m]

$$\therefore\ \frac{m}{n}=\frac{3}{2}$$

So, least integral values of m and n satisfying above requirement are m = 3 and n = 2.

25. (b) Given, mass of each spherical ball,

$m=20\text{ g}=2\times10^{-2}\text{ kg}, g=10\text{ m/s}^2$

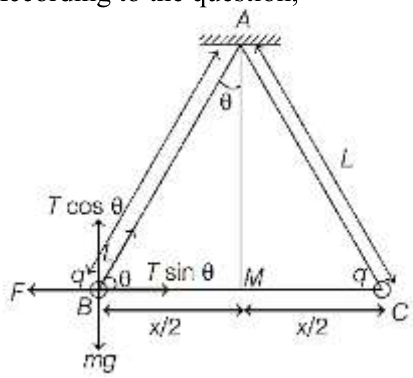
charge on each spherical ball,

$q_1=q_2=q=10^{-10}\text{C}$

and length of thread,

$L=300\text{ cm}=3\text{ m}$

According to the question,



Equilibrium at point B,

$T\cos\theta=mg$ ..... (i)

$T\sin\theta=F$

$T\sin\theta=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{x^2}$ ..... (ii)

(From Coulomb's law)

From Eqn. (i) and (ii), we get

$$\frac{T\sin\theta}{T\cos\theta}=\frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{x^2}}{mg}$$

$$\left[\text{Given, } 4\pi\varepsilon_0=\frac{1}{9\times10^9}C^2/N-m^2\right]$$

From △ ABM,

$$\tan\theta=\frac{\frac{x}{2}}{\sqrt{L^2-\left(\frac{x}{2}\right)^2}}=\frac{x}{\sqrt{4L^2-x^2}}=\frac{x}{2L}$$

($\because$ L >> x)

∴ From Eq. (iii), we get

$$mg\cdot\frac{x}{2L}=\frac{9\times10^9\times q^2}{x^2}$$

Putting the given values, we get

$$2\times10^{-2}\times10\times\frac{x}{2\times3}=\frac{9\times10^9\times10^{-20}}{x^2}$$

$$\Rightarrow\ x^3=\frac{27}{10^{10}}\text{ m}$$

$$\Rightarrow\ x=\frac{3}{10^3\times10^{1/3}}=\frac{3}{10^{1/3}}\text{ mm}$$

Therefore, magnitude of equilibrium separation distance of balls is $\frac{3}{10^{1/3}}$ mm.

26. **(c)** According to the question, the space between the two large parallel plates is filled with a material of uniform charge density ρ i.e., It is a homogeneous medium, therefore the potential at any point between these plates is calculated by Poisson's equation as given below,

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \qquad \ldots \text{(i)}$$

Since, we have to calculate the potential at any point along x-direction, hence the derivative of the potential along y and z directions is zero.

Hence, from Eq. (i), we get

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_0}$$

Integrate on both the sides, we get

$$\Rightarrow \int \frac{d^2V}{dx^2} dx = -\int \frac{\rho}{\varepsilon_0} dx \Rightarrow \frac{dV}{dx} = \frac{-\rho x}{\varepsilon_0} - A$$

Integrate on both sides again, we get

$$\Rightarrow \int \frac{dV}{dx} dx = \int \frac{-\rho x}{\varepsilon_0} dx - \int Adx$$

$$V = \frac{-\rho x^2}{2\varepsilon_0} - Ax - B$$

$$\therefore V = -\left(\frac{+\rho x^2}{2\varepsilon_0} + Ax + B\right]$$

27. **(c)** When the temperature of a metal is increased, the atoms in it starts to vibrate more rigorously. This leads to increase in number of collisions amongst the electrons. Thus, time between successive collision or relaxation time ' τ ' decreases. Since, the resistivity of a metal is given by

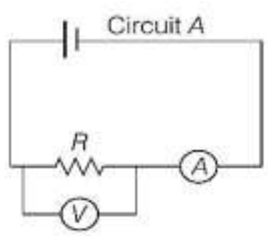
$$\rho = \frac{m}{ne^2\tau}$$

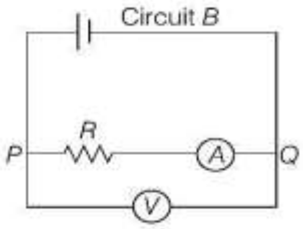
Where, m = mean of electron,
n = number of charge and
e = charge on electron

$$\Rightarrow \rho \alpha \frac{1}{\tau}$$

So, on τ decreases, resistivity ' s ' increases.

28. **(b)** In the given circuits, A represents an ammeter, which is used to measured the current passing through the resistor it is not a low resistive device, so that it does not affect the circuit. It is connected in series with resistor. Similarly, V represents a voltmeter, which is used to measure the potential drop across the resistor. It is usually a high resistive device, so that the negligible current passes through it. Thus, does not affect no current passing through the resistor. It is connected in parallel with to the resistor, This implies that, these devices are used measure. Current and voltage drop across a resistor, without actually affecting any of these quantities because of their addition in the circuit. So, the given circuits A and B as shown below,



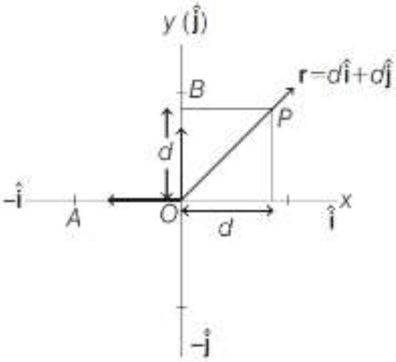


We can conclud that the circuit A has been correctly connected. Therefore, it can be used to measure R accurately, which can be calculated with the help of ammeter and voltmeter reading as,

$R = \frac{\text{voltage(V)}}{\text{current (I)}}$ [From ohm's law]

However, when the arrangement of ammeter and voltmeter is as per given in circuit B. The circuit then is usually used for measuring higher resistance as compare to circuit A.

29. **(b)** The given situation for two current carrying wires are shown in the figures



Position vector of charge particle,

$r = d(\hat{\imath} + \hat{\jmath}) = d\hat{\imath} + d\hat{\jmath}$

The magnetic field intensity due to the wire along $-\hat{\imath}$ direction,

$B_1 = \frac{\mu_0}{2\pi} \cdot \frac{I}{d}(-\hat{k})$

Similarly, magnetic field intensity due to wire along $\hat{\jmath}$ direction.

$B_2 = \frac{\mu_0}{2\pi} \cdot \frac{I}{d}(-\hat{k})$

∴ Resultant magnetic field at point P due to both the wires,

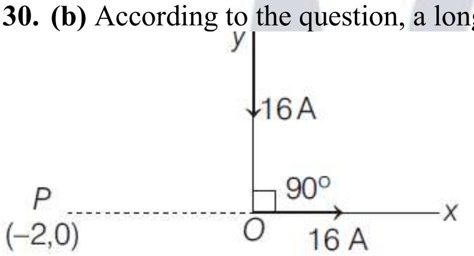
$B = B_1 + B_2 = 2 \cdot \frac{\mu_0}{2\pi} \cdot \frac{I}{d}(-\hat{k})$

$B = \frac{\mu_0}{\pi} \cdot \frac{I}{d}(-\hat{k})$

∴ Force on the charged particles,

$F = q(v \times B) = q\left[v\hat{\imath} \times \frac{\mu_0}{\pi}\frac{I}{d} \cdot (-\hat{k})\right] = \frac{\mu_0 Iqv}{\pi d} \cdot \hat{\jmath}$

30. **(b)** According to the question, a long straight wire is bent at 90° as shown in the figure,



Where, current flowing through the wire, $I = 16$ A

$\mathbf{r} = OP = (-2\hat{\mathbf{i}} + 0\hat{\mathbf{j}})$mm

Magnitude of the magnetic field at point P due to current carrying wire along x-axis is zero because pont P lies in the direction of conducting wire along x-axis.

∴ Magnitude of the magnetic field due to current carrying wire along y-axis at point P is given by

$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}$

Putting the given values in above relation, we get

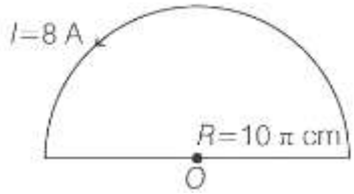
$= \frac{\mu_0}{4\pi} \times \frac{16}{2 \times 10^{-3}}$ [$\because r = 2 \text{ mm} = 2 \times 10^{-3}$ m]

$= 10^{-7} \times 8 \times 10^3 = 0.8 \times 10^{-3}$ T $= 0.8$mT

31. **(a)** Given, current flowing through wire, $I = 8$ A and radius of circular wire,

$R = 10\pi$ cm $= 10\pi \times 10^{-2}$ m

According to the question,



$\because$ Magnetic field produced by semi-circular current carrying thin wire at centre O.

$B = \frac{\mu_0 I}{4R}$

Putting the given values, we get

$$B = \frac{4\pi \times 10^{-7} \times 8}{4 \times 10\pi \times 10^{-2}}$$

$\Rightarrow B = 8 \times 10^{-6}$ T

$\therefore$ Magnitude of the force on thin wire per unit length.

$F = IB$

Putting the given values,

$= 8 \times 8 \times 10^{-6}$

$= 64 \times 10^{-6}$ N/m $= 64\mu$ N/m

32. **(c)** Given, resistance of a coil, $R = 10\Omega$ number of turns in the coil, $N = 180$ and diameter of the coil $= 4$ cm $= 4 \times 10^{-2}$ m Magnetic flux associated with the coil is maximum.

Hence, $\theta = 0°$

$\therefore$ Magnetic flux, $\phi = BA$

When magnetic field is suddenly removed, then some charge flows through the galvanometer.

Given, resistance of a galvanometer, $R_g = 618\Omega$ and charge flowing through the galvanometer, $q = 360\mu C = 360 \times 10^{-6}C$

As $q = I \cdot \Delta t$

$$\Rightarrow q = \frac{e}{R_{eq}}\Delta t = \frac{\Delta\phi}{\Delta t} \cdot \frac{\Delta t}{R_{eq}} \left(\because e = N\frac{\Delta\phi}{\Delta t}\right)$$

So, for N turns, $q = \frac{N}{R_{eq}}\Delta\phi$

Here, $R_{eq} = R + R_g$

Putting the given values, we get

$$360 \times 10^{-6} = \frac{180}{(618 + 10)} \cdot (BA - 0)$$

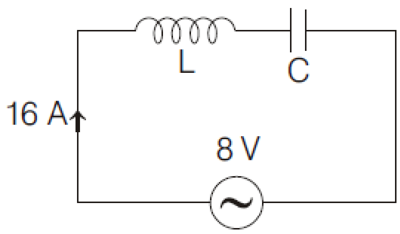
$$2 \times 10^{-6} \times 628 = B \cdot \pi \times (2 \times 10^{-2})^2 \; [\because A = \pi r^2]$$

$$\Rightarrow B = \frac{2 \times 10^{-6} \times 628}{3.14 \times 4 \times 10^{-4}} = 1 \text{ T}$$

Hence, the magnetic field is 1 T .

33. **(c)** Given, rms voltage of AC source $V_{rms} = 8$ V and the rms current in the circuit, $I_{rms} = 16$ A

According to the question,



$\therefore$ Impedance of the circuit,

$Z = \frac{V_{rms}}{I_{rms}} = \frac{8}{16}$

$Z = 0.5\Omega$

When the inductor coil is connected to a 6 V DC battery then $V'_{rms} = 6 \text{ V} = V_{DC}$

$\therefore$ Magnitude of steady current,

$I' = \frac{V'_{rms}}{Z} = \frac{6}{0.5} = 12$ A

34. **(a)** Given, frequency of EM wave,

$f = 3\text{MHz}$

$= 3 \times 10^6$ Hz

and permittivity of non-magnetic medium,

$\varepsilon = 16\varepsilon_0$

Wavelength of EM wave,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^6}$$

$\lambda = 100$ m

Velocity of electromagnetic (EM) into non-magnetic material,

$$v = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{16}} \left[\because \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 16\right]$$

$$= \frac{3}{4} \times 10^8 \text{ m/s}$$

$\therefore$ Wavelength, $\lambda' = \frac{v}{f} = \frac{\frac{3}{4}\times 10^8}{3\times 10^6} = 25$ m

$\therefore$ Change in wavelength $= \lambda' - \lambda$

$= 25 - 100 = -75$ m

35. **(d)** The de-Broglie wavelength λ of charged particle of charge q mass m and energy E is given by

$\lambda = \frac{h}{\sqrt{2mE}}$ .... (i)

For another particle mass $m' = 2$ m

charge, $q' = 2q$

and energy, $E' = 2E$

de-Broglie wavelength,

$$\lambda' = \frac{h}{\sqrt{2m'E'}}$$

$$= \frac{h}{\sqrt{2 \times 2m \times 2E}}$$

$$= \frac{1}{2} \cdot \frac{h}{\sqrt{2mE}} \lambda' = \frac{\lambda}{2}$$

36. **(d)** Given, kinetic energy of electron $= 5.5\text{eV}$. The energy required to excite a hydrogen atom from ground state $(n = 1)$ to first excited state $(n = 2)$ is

$\Delta E = E_2 - E_1$

$= -3.4 - (-13.6) = 10.2\text{eV}$

When an electron collides with hydrogen atom in the ground state with energy of 5.5 eV which is less than 10.2 eV , then it will not be able to excite hydrogen atom into first excited state. Therefore, electron will not give any energy to the hydrogen atom. Hence, the total kinetic energy of electron remains conserved. Hence, its collision with hydrogen atom is elastic.

37. **(a)** Given, initial amount of radioactive element, A = initial amount of radioactive element, B

i.e., $(N_0)_A = (N_0)_B$ ...... (i)

half- life of element A, $\left(t_{1/2}\right)_A = 10\text{yr}$

and half- life of element B, $\left(t_{1/2}\right)_B = 20\text{yr}$

After time t, remaining the amount of element A,

$N_A = \frac{1}{e}$ kg

and remaining amount of element B,

$N_B = 1$ kg

For element A, after time t, amount of element left undecayed is given as

$$N_A = (N_0)_A \left(\frac{1}{2}\right)^{\frac{t}{(t_{1/2})_A}}$$

$\frac{1}{e} = (N_0)_A \left(\frac{1}{2}\right)^{\frac{t}{10}}$ ..... (ii)

Similarly, for element B,

$N_B = (N_0)_B \left(\frac{1}{2}\right)^{\frac{t}{20}}$

$1 = (N_0)_A \left(\frac{1}{2}\right)^{\frac{t}{20}}$ ….. (iii)

Dividing Eqs. (ii) and (iii), and using Eq. (i), we get

$$\frac{1}{e} = \frac{\left(\frac{1}{2}\right)^{\frac{t}{10}}}{\left(\frac{1}{2}\right)^{t/20}} = \left(\frac{1}{2}\right)^{\frac{t}{20}}$$

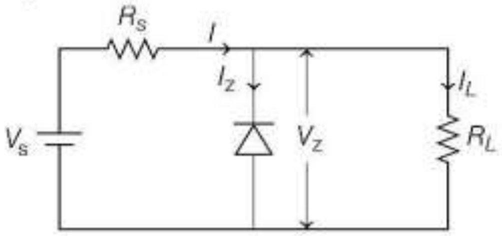
Taking log on the both sides, we get

$$\log\frac{1}{e} = \frac{t}{20}\log\frac{1}{2}$$

$$t = \frac{20}{\log 2}$$

$$= \frac{20}{0.7} = \frac{200}{7}\text{yr } [\because \log 2 = 0.7]$$

38. (c) According to the question, the circuit diagram of zener diode as a voltage regulator is shown below,



Given, voltage supply, $V_s = 16$ V, voltage across zener diode, $V_Z = 10$ V, current passes through the zener diede, $I_Z = 5I_L$ and load resistance $R_L = 2k\Omega$ The current through load resistance is

$$I_L = \frac{\text{voltage}(V_Z)}{\text{resistance}(R_L)}$$

$$= \frac{10}{2 \times 10^3}$$

The current through in the resistance in series

$$I = I_Z + I_L$$
$$= 5I_L + I_L$$
$$= 6I_L$$
$$= 6 \times 5 \times 10^{-3}$$
$$= 3 \times 10^{-2} \text{ A}$$

Also, $I = \frac{V_S - V_Z}{R_S}$

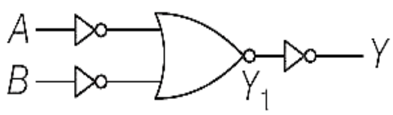
$\Rightarrow R_S = \frac{V_S - V_Z}{I}$ ……. (ii)

substituting the values in Eqs. (ii), we get

$$R_S = \frac{16 - 10}{3 \times 10^{-2}} = \frac{6}{3 \times 10^{-2}} = 200\Omega$$

Thus, the series resistance for zener diode is 200Ω.

39. (b) The truth table for the given logic circuit,



A	B	Y_1	Y
0	0	0	1
0	1	0	1

1	0	0	1
1	1	1	0

The truth table of the logic circuit is same as that of NAND gate.

40. (d) We know that, the modulation index is given by

$\mu = \frac{A_m}{A_c}$ ...(i)

where, A_m = maximum amplitude of message signal and A_c = maximum amplitude of carrier-signal.

Given, A_m and A_c are increased by 0.1% and 0.3%, then the modulation index becomes,

$$\mu_1 = \frac{A_m + 0.001A_m}{A_c + 0.003A_c}$$

$$= \frac{1001A_m}{1003A_c} = \frac{1001}{1003}\mu \text{ [From Eq. (i)]}$$

$$\Rightarrow \frac{\mu_1}{\mu} = \frac{1001}{1003}$$

The percentage change in modulation index,

$$= \frac{\mu_1 - \mu}{\mu} \times 100$$

$$= \frac{1001 - 1003}{1003} \times 100$$

$$= -\frac{2}{1003} \times 100 \simeq -0.2\%$$

Chemistry

41. (c) $E_n = -2.18 \times 10^{-18} \cdot \frac{Z^2}{n^2}$

where, E_n = energy of an electron in nth orbital.

Z = atomic number

n = number of orbital

For hydrogen, $Z = 1$

Therefore,

$$E_3 = -2.18 \times 10^{-18} \times \frac{1}{3^2}$$

$$= -\frac{2.18\times10^{-18}}{9}$$

$$= -0.242 \times 10^{-18}$$

and $$E_\infty = -2.18 \times 10^{-18} \times \frac{1}{\infty}$$

$$E_\infty = 0$$

Hence, $E_\infty = 0$ and $E_3 = -0.242 \times 10^{-18}$ J and option (c) is the correct answer.

42. (d)

(A) Nodes (III) $|\psi^2|$ is zero because $|\psi^2|$ represents the region where probability of electron finding is zero.

(B) Subsidiary Quantum Number

(I) is a three -dimensional shape of the orbital. It is also called azimuthal quantum number (l). The values of (l) gives three dimensional shapes of orbitals.

(C) White Light

(V) Continuous spectrum because when all the energy waves are so close to each other that they appear as in continuous pattern, the spectrum is called continuous spectrum.

(D) (II) Heisenberg's Uncertainty Principle

Significant only for motion of microscopic objects, because, according to this principle, it is impossible to determine simultaneously, the exact position and exact momentum (or velocity) of an electron. Hence, (d) is the correct option.

43. (a) Element with $Z = 120$ is an hypothetical element having IUPAC name = unbinilium. It has two electrons in its outermost subshell with configuration 2,8,18,32,32,18,8,2. Thus, it is expected to belong to group 2 of s-block, in periodic table.

44. (a) Common oxidation state of f-block elements is $(+)3$, but Eu and Tb can also show (+)2 and (+)4 oxidation states respectively because of $4f^7$ and $4f^9$ electrons in 4f-subshell respectively.

45. (b) According to molecular orbital theory, species with all paired electrons in their electronic configuration are diamagnetic in nature.

$He_2^+ \rightarrow$ Has total 3 electrons, thus it contains one unpaired electron and is paramagnetic in nature. $H_2 \rightarrow$ Has total 2 electrons, thus it contains all paired electrons and is diamagnetic in nature.

$H_2^+ \rightarrow$ Has only 1 electron, thus it is paramagnetic in nature.

$H_2^- \rightarrow$ Has total 3 electrons, thus it contains one unpaired electron and is paramagnetic in nature. He $\rightarrow$ Contains total two (2) electrons and has no unpaired electron, thus is diamagnetic in nature. Hence, among the given species, we have only two of them which are diamagnetic and thus, option (b) is the correct answer.

46. (d) More be the electronegativity difference between H -atom and the atom bonded with H -atom, more- stronger is the H-bonding. Since, electronegativity difference between H -atom and F -atom is maximum

47. (c) $\because$ Kinetic energy (KE) $= \frac{3}{2}RT$

where, R = gas constant

T = temperature

$\therefore\ RT = \frac{2}{3}(KE) \ldots\ldots (i)$

Most probable velocity (MP) $= \sqrt{\frac{2RT}{M}} \quad \ldots. (ii)$

where, M = molecular mass of oxygen = 32

Given $\Rightarrow$ KE = 3741.3 J

Substituting the value of RT from Eq. (i), we have,

$$MP = \sqrt{\frac{2 \times 2(KE)}{3 \times M}}$$

$$= \sqrt{\frac{2 \times 2 \times 3741.3}{3 \times 32} \times 1000}$$

$= \sqrt{155.88 \times 1000} = \sqrt{155887}\ J\ kg^{-1}$

Hence, option (c) is the correct answer.

48. (c) According to van der Waals' correction for pressure for real - gases, the pressure correction is given by $p + \frac{an^2}{V^2}$. Correction term for pressure of a real gas $= \frac{an^2}{V^2}$ and option (c) is the correct answer.

49. (b)

50. (d) The disproportionation reaction for MnO_4^{2-} occurs as follows :

$$3\overset{+6}{Mn}O_4^{2-} + 4H^+ \longrightarrow \overset{+4}{Mn}OO_2 + 2\overset{+7}{Mn}O_4^- + 2H_2O$$

Hence, MnO_4^{2-} is oxidised to MnO_4^- and reduced to MnO_2. Thus, oxidation state of Mn when MnO_4^{2-} undergoes disproportionation reaction under acidic medium are +7 and +4 .

51. (d) Given,

Initial temperature = 0°C

Final temperature = 30°C

Thus, $\Delta T = 30°C$

Mass of ice (m) = 10 g

Enthalpy of fusion (L) $= 333.5 Jg^{-1}$

C_p of water $= 4.18 Jg^{-1} K^{-1}$

$\because$ Heat required (Q) $= mL + mC_p\Delta T$

$$= 10 \times 333.5 + 10 \times 4.18 \times 30$$
$$= 3335.0 + 1254.00$$
$$= 4589\ J$$
$$= 4.59\ kJ$$

52. (a) For the reaction,

$0.5C(s) + 0.5CO_2(g) \rightleftharpoons CO(g)$

$0.5CO_2(g)$ can give CO(g).

If 50% conversion of $CO_2(g)$ takes place.
We have concentration of $CO_2(g) = [0.25]$
$\because K_p$ is calculated only for gaseous items,
thus

$$K_p = \frac{[\text{ Product }]_{(g)}}{[\text{ Reactant }]_{(g)}} = \frac{[1]}{[0.25]} = 4$$

53. (a) $\because$ Solubility product $\left(K_{sp}\right) = x^x \cdot y^y \cdot S^{x+y}$
where, x and y are number of Ca^{2+} and PO_4^{3-} ions in 1 L of pure water respectively.
i.e. $InCa_2(PO_4)_3$
Here, $x = 2$ and $y = 3$
Therefore, $K_{sp} = 2^2 \cdot 3^3 S^{2+3}$
$K_{sp} = 108S^5$.

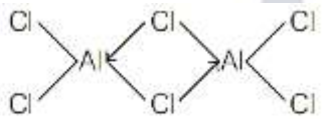
54. (a) The balanced equation between the reaction of $KMnO_4$ and H_2O_2 in acidic medium is as follows:
$2KMnO_4 + 5H_2O_2 + 3H_2SO_4 \longrightarrow 5O_2 + 2MnSO_4$
$+K_2SO_4 + 8H_2O$
Thus, 2 moles of $KMnO_4 \equiv 5$ moles of
$H_2O_2 \equiv 5$ moles of $O_2(g)$ at STP.
According to above relation:
34 g of H_2O_2 will give $= 11.2$ L of O_2 at STP.

$\therefore$ (34 × 5), i.e. 170 g of H_2O_2 will give $= \frac{11.2 \times 170}{34}$
$= 56$ L

$\because$ 1 L of H_2O_2 give 10 L of O_2 at STP.
$\therefore$ 1 mL of H_2O_2 give 10 mL of O_2 at STP. and 56 mL of H_2O_2 will give 560 mL of O_2 $\therefore$ Volume of $H_2O_2 = 56$ mL

55. (d) On hydrolysis of NaO_2, (a superoxide of sodium), it gives O_2, H_2O_2 and NaOH as products. i.e. $2NaO_2 +$
$2H_2O \longrightarrow 2NaOH + O_2 + H_2O_2$

56. (b) Structure of Al_2Cl_6 is



Therefore,
(i) Oxidation state (x) of 'Al' in Al_2Cl_6 :
$= 2x - 6 = 0$ or $x = (+)3$
(ii) Coordination number of Al in Al_2Cl_6 is four (4), it is bonded to 4 chlorine atoms in Al_2Cl_6.
(iii) Number of valence electrons around Al in
$Al_2Cl_6 = 2 \times 4 = 8$

57. (a) $\because$ Graphite is the most stable state of carbon and its ΔH_f° is considered as zero ($\because$ has more van der Waals' force) also ΔH_f° for $C_{60} > \Delta H_f^\circ$ for diamond.
ΔH_f° values of graphite, diamond and C_{60} are = 0, 1.9 and 38.1 kJ mol^{-1} and ΔH_f° values are in order : Diamond < Fullerene < Graphite

58. (d) Enamel of teeth is $Ca_{10}(PO_4)_6(OH)_2$, when it reacts with F^-ions, following reaction will take place.
$Ca_{10}(PO_4)_6(OH)_2 + 2F^- \longrightarrow Ca_{10}(PO_4)_6 F_2 + 2OH^-$
$Ca_{10}(PO_4)_6 F_2$ can be rewritten as :
$3[Ca_3(PO_4)_2]CaF_2$ or $3[Ca_3(PO_4)_2 \cdot CaF_2]$

59. (d)
(a) Kjeldahl method can be used for estimation of nitrogen in an organic sample.
(b) Dumas method can be used for estimation of nitrogen in an organic sample.
(c) Lassaigne method can be used for estimation of N, S, P and halogens in an organic sample.
(d) Carius method can be used to find out the percentage composition of halogen present in an organic compound using Carius tube.

60. (c)

61. **(b)** Conditions for species to be an aromatic are:
(i) The aromatic structures follows Huckel's rule, i.e. species have $(4n+2)\pi$-electrons in its structure.
where, $n =$ integer $0,1,2,3, \ldots$ so on.
(ii) Should be planar ring structure.
(A) It follows Huckel's rule and has planar structure thus, is an aromatic ($\because n=1, \pi e^{-}=6$).
(B) It does not follow Huckel's rule ($\because$ has 8π electrons with $n=2$).
(C) It follows Huckel's rule and has planar structure. Thus, is an aromatic ($\because n=1, \pi e^{-}=6$).
(D) It follows Huckel's rule and has planar structure as it has $n=3$ and 14π electrons.
(E) It does not follow Huckel's rule, thus is not an aromatic compound ($n=1, \pi e^{-}=4$).
(F) It also does not follow Huckel's rule. Thus, is not an aromatic ($n=0, \pi e^{-}=4$)

62. **(c)** In ferrimagnetic substances, the magnetic moments of the domains are alligned in parallel and antiparallel directions in unequal number. The net magnetic moment is small. Thus, these are weakly attracted by magnetic field than ferromagnetic substances and these also lose ferrimagnetism on heating and become paramagnetic. e.g. Fe_3O_4 and ferrites like $ZnFe_2O_4, MgFe_2O_4$ and $NiFe_2O_4$ etc.

$\uparrow\uparrow\downarrow\uparrow\ \uparrow\downarrow$
Ferrimagnetism

Among the given options, we have 3 compounds (i.e. $Fe_3O_4, MgFe_2O_4$ and $NiFe_2O_4$) that show ferrimagnetism.

63. **(b)** Given,
Moles of solute $(n) = 1$ mol
Mass of solvent $(w_A) = 50$ g
$\because$ Molality $(m) = \frac{n \times 1000}{w_A(\text{ in g})}$
$\therefore\ m = \frac{1000}{50} = 20 \text{ mol kg}^{-1}$

64. **(d)** Given,
Volume of solute $= 10$ mL
$\Delta T_f = 0 - (-0.413) = 0.413^{\circ}C$
w (solvent) $= 500$ g
$K_f(\text{ water }) = 1.86 \text{ K kg mol}^{-1} (= w_A)$
Molecular weight of (A) or $M_B = 60$ g
Let mass of solute (A) $= w_B$

$$\therefore \quad \Delta T_f = K_f \times \frac{w_B}{M_B} \times \frac{1000}{w_A}$$
$$\therefore \quad 0.413 = 1.86 \times \frac{w_B}{60} \times \frac{1000}{500}$$
$$w_B = \frac{0.413 \times 60 \times 500}{1.86 \times 1000}$$

Mass of solute $= 6.66$ g
Also,
$\because$ Density (d) of solution $= \frac{\text{Total mass}}{\text{Total volume}}$
$$= \frac{6.66+500}{500+10} = \frac{506.66}{510} = 0.993(\text{ g mL}^{-1})$$
Density (d) of solution $= 0.993 \text{ g mL}^{-1}$

65. **(c)** Given,
Charge used $= 19296C$
Molar mass of Cu $= 63.5$
$\because$ Cu in $CuSO_4$ has Cu^{2+} charge means:
$2 \times 96500C$ of charge give $= 63.5$ g of Cu
Thus, 19296 C of charge give $= \frac{63.5 \times 19296}{2 \times 96500}$
$= 6.35$ g of copper (Cu).

66. **(c)** For zero-order reaction, integrated rate equation is given as:
$$k = \frac{[R_0]-[R]}{t}$$
$k =$ rate constant

where, $[R_0]$ = initial concentration
and $[R]$ = concentration at time (t)
When, $t = t_{1/2}$

$$[R] = \frac{[R_0]}{2}$$

Thus,

$$K = \frac{[R_0] - \frac{1}{2}[R_0]}{t_{1/2}}$$

67. (b) Silver sols can be used as an eye-lotion because it can heal eye infections.

68. (a) In the given reaction, (A) and (B) are respectively : $[Au(CN)_2]^-$and $[Zn(CN)_4]^{2-}$ The whole reaction is as follows :

$$4Au + 8CN^-(aq) + 2H_2O(aq) \xrightarrow{O_2(g)} 4\underset{(A)}{[Au(CN)_2]^-} + H_2O$$

$$\xrightarrow{Zn(s)} 2\left[Zn\left(\underset{(B)}{CN)_4}\right]^{2-} + 2Au$$

69. (a) Statement (A) Sulphur vapour is paramagnetic is correct statement as sulphur vapour has formula S_2, i.e. similar to O_2 and O_2 has two unpaired electrons in its anti-bonding molecular orbitals. Statement (B) Reaction of HCl (dil.) with finely divided iron forms $FeCl_3$ and H_2 gas is a wrong statement as the whole reaction is as follows:

$$Fe + 2HCl(\text{ dil. }) \longrightarrow FeCl_2 + H_2(\text{ g})$$

The H_2(g) produced will prevent the formation of Fe^{3+} (i.e. $FeCl_3$).

70. (c, d) Noble gases have low boiling point and low melting point because of
(i) weak van der Waals' interactions and,
(ii) weak dispersion forces.

71. (a) (A) $Co^{2+} \longrightarrow$ (V) Pink colour
(B) $Fe^{2+} \longrightarrow$ (IV) Pale green colour
(C) $Ni^{2+} \longrightarrow$ (II) Dark green colour
(D) $Cu^{2+} \longrightarrow$ (III) Blue colour

72. (b) The crystal field splitting, i.e. Δ_0, depends upon the field produced by the ligand and charge on the metal ion. So, ligands can be arranged in a series in the order of increasing field strength. This series is known as spectrochemical series.

e.g., $I^\ominus < Br^\ominus < SCN^\ominus < Cl^\ominus < F^\ominus < OH^\ominus < C_2O_4^{2-}$
$< H_2O < NCS^\ominus < EDTA^{4-} < NH_3 < en < CN^\ominus < CO:$

Among the given options, ligand (NH_3) has highest magnetic field therefore, it has maximum value for Δ_0

73. (c) ∵ Number average molecular mass $(M\bar{n})$

$$= \frac{N_1M_1 + N_2M_2 + N_3M_3}{N_1 + N_2 + N_3}$$

where, N_1, N_2 and N_3 are number of molecules and M_1, M_2 and M_3 are respectively their molecular masses.
Given, $N_1 = 50$ and $M_1 = 5000$
$N_2 = 100$ and $M_2 = 10,000$
$N_3 = 50$ and $M_3 = 15,000$
Thus,

$$M\bar{n} = \frac{N_1M_1 + N_2M_2 + N_3M_3}{N_1 + N_2 + N_3}$$

$$= \frac{(50 \times 5000) + (100 \times 10,000) + (50 \times 15000)}{50 + 100 + 50}$$

$$= \frac{(25,0000) + (1,000,000) + (75,00,00)}{200}$$

$M\bar{n} = 10,000$

74. (d) The species which contain free - CHO group are reducing sugars.

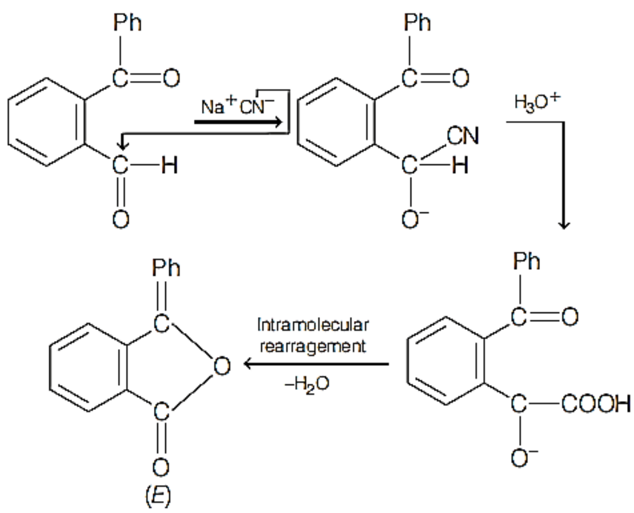
(A) Sucrose is a non-reducing sugar.
(B) Maltose is a reducing sugar.
(C) Lactose is a reducing sugar.
(D) Fructose is a reducing sugar.

75. (a) (A) Codeine, (D) morphine and (F) heroin are opiates or narcotics. The most common effect of medicinal opiates is pain relief.
(B) Thymine is a nucleoside present in DNA.
(C) Epinephrine is used under extreme stress. It affects on hormones which release stress producing chemicals.
(E) Thiamine is a vitamin.

76. (d) (A)$CH_3-CHBr-CH_2Br \xrightarrow{KOH/C_2H_5OH}$
(IV) Alkenyl bromide.
$\because$ KOH (alc.) will remove one H and one Br to give alkenyl bromide.
(B) $CH_3-CH_2-CH=CH_2 \underset{(C_6H_5CO)_2O_2\Delta}{\overset{HBr}{\rightarrow}}$ (I)
gives 1° alkyl bromide due to presence of peroxide and HBr. (Anti-markownikoff's addition)
(C) $CH_3-CH_2-CH_3 \overset{Br_2,hv}{\rightarrow}$ (II), gives 2° alkyl bromide because 2° carbocation is more stable.
(D) $CH_3-CH=CH_2 \overset{NBS}{\rightarrow}$ (III) gives allyl bromide due to presence of NBS.

77. (b)

78. (c) The given reaction proceed in a similar way as that of benzoin condensation. The complete reaction is given as below:



79. (b) Acidic strength increases with presence of EWG (i.e. groups show-I effect) and decreases with the presence of EDG (i.e. groups show +I or hyperconjugation effect). More be the number and more close be the EWG to the -COOH group, more is the acidic strength.
(A) $\overset{2}{C}H_3-\overset{1}{C}OOH$, has only one EDG, i.e. (by hyperconjugation) one CH_3 group is bonded with - COOH group.
(B) $CH_3-\overset{3}{C}H(Cl)-\overset{2}{C}H_2-\overset{1}{C}OOH$, has one EWG
(i.e. Cl) at $C-3$ position and CH_3, i.e. one EDG (by hyperconjugation) at C_3-position.
(C) $Cl-\overset{2}{C}H_2-\overset{1}{C}OOH$, has one EWG (i.e. Cl -atoms) at C_2 - position.
(D) $Cl_2-\overset{2}{C}H-\overset{1}{C}OOH$, has two EWG's (i.e. two Cl -atom) at C_2-position. Thus, order of acidic strength is (D) > (C) > (B) > (A) and option (b) is the correct answer.

80. (b)

Mathematics

81. (d) (A) $f: R \rightarrow R$
$f(x) = \cos(112x - 37)$

$f(x)$ is periodic function.
$\therefore$ It is not injective.
Range of $f(x)$ is $[-1,1]$
$\therefore$ It is also not surjective.
$\therefore\ A \to IV$
(B) $f: A \to B, A \in [-2,2]$ and $B \in [-4,4]$
$f(x) = x|x|$
$$f(x) = \begin{cases} -x^2 & -2 \le x < 0 \\ x^2 & 0 \le x \le 2 \end{cases}$$
Clearly $f(x)$ is bijective function.
$\therefore B \to III$
(C) $f: R \to R$
$f(x) = (x-2)(x-3)(x-5)$
$f(2) = f(3) = f(5) = 0$
$\therefore$ It is not injective.
Range of $f(x) = R$
$\therefore$ It is surjective.
$\therefore C \to II$
(D) $f: N \to N$
$f(x) = n + 1$
Clearly it is injective.
Range of $f(x) = \{2,3,4, \dots\}$
Range $\neq$ Codomain
$\therefore$ It is not surjective.
$\therefore D \to I$

82. (c) We have,
$$f(x) = \sqrt{\frac{x-[x]}{\log(x^2-x)}}$$
It is defined
$\log(x^2-x) > 0\ [\because x-[x] = [x] \ge 0, \forall x \in R]$
$\Rightarrow x^2 - x > 1 \Rightarrow x^2 - x - 1 > 0$
$$\because x^2 - x - 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{5}}{2}$$
$$\therefore\ x \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, \infty\right)$$
$\therefore$ Domain of $f(x) = R - \left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$

83. (a) We have,
$$\frac{x}{x+1} = x(1+x)^{-1} = x(1 - x + x^2 - x^3 + x^4 - \cdots)$$
$$= x - x^2 + x^3 - x^4 + x^5 - \cdots$$
$$= \sum_{n=0}^{\infty} (-1)^n x^{n+1}$$
$\therefore$ Assertion is true.
Reason $(R) = (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \cdots$
is also true (A) and (R) are true, (R) is correct explanation of (A).

84. (c) We have,

$$A=\begin{bmatrix}1&2&3\\1&3&5\\2&1&6\end{bmatrix}$$
$$|A|=1(18-5)-2(6-10)+3(1-6)$$
$$=13+8-15=6$$
$$\text{Adj}A=\begin{bmatrix}13&-9&1\\4&0&-2\\-5&3&1\end{bmatrix}$$
$$A^{-1}=\frac{\text{Adj}A}{|A|}=\frac{1}{6}\begin{bmatrix}13&-9&1\\4&0&-2\\-5&3&1\end{bmatrix}$$
Now, $\text{Adj}(\text{Adj}A)=|A|A$
$\text{Adj}(\text{Adj}A)=6A$
$(\text{Adj}(\text{Adj}A))^{-1}=(6A)^{-1}$
$$=\frac{1}{36}\begin{bmatrix}13&-9&1\\4&0&-2\\-5&3&1\end{bmatrix}$$
$$\therefore (\text{Adj}(\text{Adj}A))^{-1}=\frac{1}{36}\begin{bmatrix}13&-9&1\\4&0&-2\\-5&3&1\end{bmatrix}$$

85. **(a)** We have,
$$\begin{vmatrix}x^2+3x&x+1&x-3\\x-1&2-x&x+4\\x-3&x-3&3x\end{vmatrix}$$
$$=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4$$
Put $x=1$ both sides, we get
$$\begin{vmatrix}4&2&-2\\0&1&5\\-2&-2&3\end{vmatrix}=a_0+a_1+a_2+a_3+a_4$$
$$4(3+10)-2(10+2)=a_0+a_1+a_2+a_3+a_4$$
$$52-24=a_0+a_1+a_2+a_3+a_4$$
Put $x=-1$ both sides, we get
$$\begin{vmatrix}-2&0&-4\\-2&3&3\\-4&-4&-3\end{vmatrix}=a_0-a_1+a_2-a_3+a_4$$
$$-2(-9+12)-4(8+12)=a_0-a_1+a_2-a_3+a_4$$
$$-6-80=a_0-a_1+a_2-a_3+a_4$$
On adding Eqs. (i) and (ii), we get
$$2(a_0+a_2+a_4)=-58$$
On subtracting Eq. (ii) from Eq. (i), we get
$$2(a_1+a_3)=28+86=114$$
$$\Rightarrow a_1+a_3=57$$
$$\therefore (a_1+a_3)+2(a_0+a_2+a_4)=57-58=-1$$

86. **(c)** Given, $AX=D$ be a system of three linear non-homogeneous equation.
$|A|=0$
$\therefore$ Equation have not unique solution.
But $\text{rank}(A)=\text{rank}(AD)=\alpha$
$\therefore$ If $\alpha<3$, then equation has infinite number of solutions.

87. **(b)** We have,
$$x+iy=(1+i)^6-(1-i)^6$$
$$1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$
$$(1+i)^6=\left(\sqrt{2}^2\right)^6\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^6$$
$$=\left(\sqrt{2}^6\right)^6\left(\cos\frac{6\pi}{4}+i\sin\frac{6\pi}{4}\right)$$

Similarly,

$$(1-i)^6 = (\sqrt{2})^6\left(\cos\frac{6\pi}{4} - i\sin\frac{6\pi}{4}\right)$$

$\therefore$

$$x + iy = (\sqrt{2})^6\left[\cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4} - \cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4}\right]$$

$$x + iy = 2^3\left[2i\sin\frac{3\pi}{2}\right] = -16i$$

$\therefore\ x = 0, y = -16$

Hence, $x + y = 0 - 16 = -16$

88. (d) $\because i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \forall n \in$ Integer

So, $i^2 + i^3 + i^4 + \cdots + i^{4000}$

$= -i + [i + i^2 + i^3 + \cdots + i^{4000}] = -i$

89. (b) $\because a^2 + b^2 + c^2 - ab - bc - ca$

$= (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

and $a^3 + b^3 + c^3 - 3abc = (a + b + c)$

$(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

So, $(a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

$= a^3 + b^3 + c^3 - 3abc$

90. (d) Let the roots of $x^3 + bx + c = 0$ is

$\cos\alpha + i\sin\alpha, \cos\beta + i\sin\beta$ and $\cos\gamma + i\sin\gamma$

Sum of roots $= \cos\alpha + i\sin\alpha + \cos\beta$

$+ i\sin\beta + \cos\gamma + i\sin\gamma$

$\Rightarrow 0 = (\cos\alpha + \cos\beta + \cos\gamma) + i(\sin\alpha + \sin\beta + \sin\gamma)$

Hence, $\cos\alpha + \cos\beta + \cos\gamma = 0$

and $\sin\alpha + \sin\beta + \sin\gamma = 0$

$\Rightarrow 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = -\cos\gamma$ ….. (i)

and $2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = -\sin\gamma$ …. (ii)

(i) divided by (ii), we get

$$\cot\left(\frac{\alpha+\beta}{2}\right) = \cot\gamma$$

$\alpha + \beta = 2\gamma$

Similarly, $\beta + \gamma = 2\alpha$ and $\gamma + \alpha = 2\beta$

and $\alpha = \beta = \gamma$

$\therefore\ \cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow\ 3\cos\alpha = 0$

$\Rightarrow\ \cos\alpha = \cos\beta = \cos\gamma = 0$

Now, $b = \cos(\alpha + \beta) + \cos(\beta + \gamma) + \cos(\gamma + \alpha)$

$+ i[\sin(\alpha + \beta) + \sin(\beta + \gamma) + \sin(\gamma + \alpha)]$

$\therefore$ Real part of b

$= \cos(\alpha + \beta) + \cos(\beta + \gamma) + \cos(\alpha + \gamma)$

$= \cos 2\gamma + \cos 2\alpha + \cos 2\beta$

$= 2\cos^2\gamma - 1 + 2\cos^2\alpha - 1 + 2\cos^2\beta - 1$

$= 0 - 1 + 0 - 1 + 0 - 1 = -3$

91. (c) Let $y = \frac{ax^2 - 2x + 3}{2x - 3x^2 + a}$

$\Rightarrow\ 2xy - 3x^2y + ay = ax^2 - 2x + 3$

$\Rightarrow x^2(a + 3y) - 2x(y + 1) + 3 - ay = 0$

as $x \in R, D \geq 0$

$\therefore\ 4(y + 1)^2 - 4(a + 3y)(3 - ay) \geq 0$

$\Rightarrow\ (y^2+2y+1)-[3a-a^2y+9y-3ay^2)\geq 0$
$\Rightarrow\ y^2(3a+1)+(a^2-7)y+1-3a\geq 0$
as $y\in R, D\leq 0$
$\therefore\ (a^2-7)^2+4(3a+1)(3a-1)\leq 0$
$\Rightarrow\ a^4-14a^2+49+36a^2-4\leq 0$
$\Rightarrow\ a^4+22a^2+45\leq 0$
$\Rightarrow$

$[\because a^4+22a^2+45>0, \forall a\in R]$

$\Rightarrow\ a\in\phi$

92. (b)
93. (c) Let α,β,γ are roots of equation
$x^3-ax^2+ax-1\ =0\ \ldots\ldots(i)$
$\alpha+\beta+\gamma=a$
$\alpha\beta+\beta\gamma+\alpha\gamma\ =a$
$\alpha\beta\gamma\ =-1$
Cubic equation whose roots are $\alpha^2,\beta^2,\gamma^2$ is
$x^2-(\alpha^2+\beta^2+\gamma^2)x^2+(\alpha^2\beta^2+\ \beta^2\gamma^2+\alpha^2\gamma^2)x$
$-\alpha^2\beta^2\gamma^2=0\ \ldots(ii)$
Eqs. (i) and (ii) are identical.
$\therefore\ \frac{a}{\alpha^2+\beta^2+\gamma^2}=\frac{a}{\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2}=\frac{1}{\alpha^2\beta^2\gamma^2}$
$a=\alpha^2+\beta^2+\gamma^2\quad[\alpha\beta\gamma=\ -1]$
$a=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$
$a=a^2-2a\Rightarrow\ a^2=3a$
$\Rightarrow\ a=3$
$[\because$ a is non-zero real]
94. (b) We have,
$x^3+px^2+qx+r=0$
α,β,γ are roots of equation
$\alpha+\beta+\gamma=-p$
$\therefore\ \alpha\beta+\beta\gamma+\alpha\gamma=q$
$\alpha\beta\gamma=-r$
Now, $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$
$=(\alpha+\beta)(\beta\gamma+\alpha\beta+\alpha\gamma+\gamma^2)$
$=\alpha\beta\gamma+\alpha^2\beta+\alpha^2\gamma+\alpha\gamma^2+\beta^2\gamma+\alpha\beta^2+\alpha\beta\gamma+\beta\gamma^2$
$=2\alpha\beta\gamma+\alpha^2\beta+\alpha^2\gamma+\alpha\gamma^2+\beta^2\gamma+\alpha\beta^2+\beta\gamma^2$
$=(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\alpha\gamma)-\alpha\beta\gamma$
$=-p(q)-(-r)=r-pq$
95. (d) Number of ways of arranging m boys and m girls in a row such that no two boys sit together, i.e. $x=(m+1)!\,m\,!$
Number of ways of arranging m boys and m girls in a row such that boys and girls sit alternately, i.e. $y=m!\times m!\times 2!$
Number of ways of arranging m boys and m girls in a circular table such that boys and girls sit alternately, i.e. $z=(m-1)!\,m\,!$
$\therefore\ x:y:z=(m+1)!\,m!:m!\,m!\times 2:(m-1)!\,m!$
$=(m+1)m:2m:1$
96. (c) The numbers greater than 1000000 that can be formed using all digits 1,2,0,2,4,2 and 4 is
$\frac{7!}{3!\,2!}-\frac{6!}{3!\,2!}=360$
The odd numbers greater than 1000000 that can be formed by using 1,2,0,2,4,2 and 4 is

$$\frac{6!}{3!\,2!}-\frac{5!}{3!\,2!}=50$$

$\therefore$ Total even number $= 360 - 50 = 310$

97. **(c)** Let$(2+\sqrt{3})^5 = \mathrm{I}+\mathrm{F}$

[$\because$ I is integer, F is fraction]

$\because\ 2-\sqrt{3}<1 \Rightarrow (2-\sqrt{3})^5<1$

Let $(2-\sqrt{3})^5 = \mathrm{F}'$

$\therefore\ \mathrm{I}+\mathrm{F}+\mathrm{F}' = (2+\sqrt{3})^5+(2-\sqrt{3})^5$

$= 2(2^5 + {}^5C_2 2^3(3) + {}^5C_4 2\cdot 3^2)$

$\mathrm{I}+\mathrm{F}+\mathrm{F}' = 2(32+240+90)$

$\mathrm{I}+\mathrm{F}+\mathrm{F}' = 724$

$\mathrm{I} = 724-(\mathrm{F}+\mathrm{F}')$

$\mathrm{I} = 724-1 = 723\ [\because \mathrm{F}+\mathrm{F}'=1]$

98. **(d)** We have,

$$\left(3-\sqrt{\frac{17}{4}+3\sqrt{2}}\right)^{10} \Rightarrow T_6 = -\,{}^{10}C_5(3)^5\left(\frac{17}{4}+3\sqrt{2}\right)^{5/2}$$

$\therefore\ T_6 =$ negative irrational number

99. **(c)** We have,

$$\frac{1}{(x^2-3)^2} = \frac{A_1}{x-\sqrt{3}}+\frac{A_2}{(x-\sqrt{3})^2}+\frac{A_3}{(x+\sqrt{3})}+\frac{A_4}{(x+\sqrt{3})^2}$$

$$\Rightarrow 1 = A_1(x+\sqrt{3})(x^2-3)+A_2(x+\sqrt{3})^2 + A_3(x-\sqrt{3})(x^2-3)+A_4(x-\sqrt{3})^2$$

Put $x=-\sqrt{3}$

$\therefore\ A_4 = \frac{1}{12}$

Put $x=\sqrt{3} \Rightarrow A_2 = \frac{1}{12}$

Put $x=0$

$\therefore\ 1 = -3\sqrt{3}A_1+3A_2+3\sqrt{3}A_3+3A_4$

$1 = 3\sqrt{3}(A_3-A_1)+3(A_2+A_4)$

$1 = 3\sqrt{3}(A_3-A_1)+3\left(\frac{1}{12}+\frac{1}{12}\right)$

$3\sqrt{3}(A_3-A_1) = 1-\frac{1}{2} = \frac{1}{2}$

$3A_1+A_3 = \frac{-1}{6\sqrt{3}}$...... (i)

Put $x=2\sqrt{3}$

$3A_1+A_3 = \frac{-1}{6\sqrt{3}}$ .... (ii)

From Eqs. (i) and (ii), we get $A_1 = \frac{-1}{12\sqrt{3}}$ and

$A_3 = \frac{1}{12\sqrt{3}}$

Clearly, $\sum_{i=1}^{4} Ai \neq 1$

Hence only statement IV is false.

100. **(c)** We have,

$\cos(x + 8x + 27x + \cdots + n^3x)$

$= \cos\{(1^3 + 2^3 + 3^3 + \cdots + n^3)x\} = \cos\left(\left\{\frac{n(n+1)}{2}\right\}^2 x\right)$

Period $= \frac{2\pi \times 4}{n^2(n+1)^2} = \frac{8\pi}{n^2(n+1)^2}$

101. **(a)** We have,

$\sin^2(3°) + \sin^2(6°) + \sin^2(9°) + \ldots + \sin^2(84°) + \sin^2(87°) + \sin^2 90°$

$= \sin^2(3°) + \sin^2(87°) + \sin^2(6°) + \sin^2(84°) + + \cdots \sin^2 90°$

$= 1 + 1 + 1 + \cdots 15 \text{ times } + \left(\frac{1}{\sqrt{2}}\right)^2 = 15 + \frac{1}{2} = \frac{31}{2}$

102. **(d)** We have,

$\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} - \cos\frac{6\pi}{7}$

$\left(\cos\frac{\pi}{7} - \cos\frac{6\pi}{7}\right) - \left(\cos\frac{2\pi}{7} - \cos\frac{5\pi}{7}\right) + \left(\cos\frac{3\pi}{7} - \cos\frac{4\pi}{7}\right)$

$= \left(\cos\frac{\pi}{7} + \cos\frac{\pi}{7}\right) - \left(\cos\frac{2\pi}{7} + \cos\frac{2\pi}{7}\right) + \left(\cos\frac{3\pi}{7} + \cos\frac{3\pi}{7}\right)$

$= 2\left(\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7}\right)$

$= 2\left(\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7}\right)$

$= \frac{2\cos\left(\frac{\pi}{7} + \frac{2\pi}{7}\right)\sin\left(\frac{3\pi}{7}\right)}{\sin\frac{\pi}{7}}$

$= \frac{2\cos\frac{3\pi}{7}\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}} = \frac{\sin\frac{6\pi}{7}}{\sin\frac{\pi}{7}} = \frac{\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}} = 1$

103. **(c)** We have,

$7\cos x + 5\sin x = 2k + 1$

Maximum and minimum value of $7\cos x + 5\sin x$ is $\sqrt{49+25}, -\sqrt{49+25} \Rightarrow \sqrt{74}, -\sqrt{74}$

$\therefore -\sqrt{74} \le 2k + 1 \le \sqrt{74}$

$\because$ k is an integer

$\Rightarrow -8 \le 2k + 1 \le 8 \Rightarrow -9 \le 2k \le 7 \Rightarrow -\frac{9}{2} \le k \le \frac{7}{2}$

$\therefore k = \{-4, -3, -2, -1, 0, 1, 2, 3\}$

Total number of $k = 8$

104. **(c)** We have,

$\sec\left\{\tan^{-1}\left(\frac{y}{2}\right)\right\}$

Let $\tan^{-1}\frac{y}{2} = x$

$\therefore \frac{y}{2} = \tan x$

Squaring on both sides, we get

$\Rightarrow \frac{y^2}{4} = \tan^2 x$

$\Rightarrow \frac{y^2}{4} = \sec^2 x - 1 \Rightarrow \sec^2 x = 1 + \frac{y^2}{4}$

$\Rightarrow \sec x = \sqrt{\frac{4+y^2}{4}} \Rightarrow \sec\left(\tan^{-1}\frac{y}{2}\right) = \frac{\sqrt{4+y^2}}{2}$

105. **(b)** We have,

$\sqrt{2} + e^{\cosh^{-1} x} - e^{\sinh^{-1} x} = 0$

$\because \quad \cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$

and $\quad \sinh^{-1} x = \log\left(x + \sqrt{x^2 + 1}\right)$

$\therefore \quad e^{\cosh^{-1} x} = x + \sqrt{x^2 - 1}$

and $\quad e^{\sinh^{-1} x} = x + \sqrt{x^2 + 1}$

$\therefore \quad \sqrt{2} + x + \sqrt{x^2 - 1} - x - \sqrt{x^2 + 1} = 0$

$\Rightarrow \quad \sqrt{2} + \sqrt{x^2 - 1} - \sqrt{x^2 + 1} = 0$

$\Rightarrow \quad \sqrt{2} + \sqrt{x^2 - 1} = \sqrt{x^2 + 1}$

Squaring on both sides, we get

$\Rightarrow \quad 2 + x^2 - 1 + 2\sqrt{2}\sqrt{x^2 - 1} = x^2 + 1$

$\Rightarrow \quad \sqrt{x^2 - 1} = 0$

$\Rightarrow \quad x^2 - 1 = 0$

$\Rightarrow \quad x = \pm 1, x \neq -1$

Total number of roots $= 1$

106. **(b)** Given, Length of an arc $= 44$ cm

Radius $= 12$ cm

$\therefore \ \theta = \frac{l}{r} \Rightarrow \theta = \frac{44}{12} = \frac{11}{3}$ radian

θ (in degree) $= \frac{11}{3} \times \frac{180}{\pi} = \left(\frac{660}{\pi}\right)^\circ$

107. **(d)** We have,

$a : b : c = 2 : 3 : 4$

Let $a = 2k, b = 3k, c = 4k$

$\therefore \ R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$

$\therefore \ \frac{R}{r} = \frac{s \cdot abc}{4\Delta^2}$

$\Rightarrow \ \frac{R}{r} = \frac{s \cdot (2k)(3k)(4k)}{4s(s-2k)(s-3k)(s-4k)}$

$\Rightarrow \ \frac{R}{r} = \frac{6k^3}{\left(\frac{9k}{2} - 2k\right)\left(\frac{9k}{2} - 3k\right)\left(\frac{9k}{2} - 4k\right)}$

$\left[\because s = \frac{a+b+c}{2}\right]$

$\Rightarrow \ \frac{R}{r} = \frac{6 \cdot 2^3 \cdot k^3}{5k \cdot 3k \cdot k} \Rightarrow \frac{R}{r} = \frac{16}{5}$

$\therefore \ R : r = 16 : 5$

108. **(a)**

109. **(a)** We have,

$OA = a + b + c, OB = a - 2b + 3c$

$AB = (a - 2b + 3c) - (a + b + c)$

$= -3b + 2c$

Point P divide AB in the ratio $1 : 3$ internally,

$$\therefore\ OP = \frac{a - 2b + 3c + 3a + 3b + 3c}{1+3} = \frac{4a + b + 6c}{4}$$

Point Q divides AB in the ratio 1: 3 externally,

$$\therefore\ OQ = \frac{a - 2b + 3c - 3a - 3b - 3c}{1-3} = \frac{2a + 5b}{2}$$

$$PQ = \left(\frac{2a + 5b}{2}\right) - \left(\frac{4a + b + 6c}{4}\right) = \frac{9b - 6c}{4}$$

$$|AB| = \sqrt{9b^2 + 4c^2} \text{ and } |PQ| = \frac{1}{4}\sqrt{81b^2 + 36c^2}$$

$$|PQ| = \frac{1}{4}\sqrt{9(9b^2 + 4c^2)} \Rightarrow |PQ| = \frac{3}{4}\sqrt{9b^2 + 4c^2}$$

$$|PQ| = \frac{3}{4}|AB| \Rightarrow 4|PQ| = 3|AB|$$

110. **(c)** $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are three non-collinear point and $k\mathbf{a} + 2\mathbf{b} + 3\mathbf{c}$ is a point in the plane $\mathbf{a}, \mathbf{b}, \mathbf{c}$.

$\therefore\ k\mathbf{a} + 2\mathbf{b} + 3\mathbf{c} = 0$

$\Rightarrow\ k + 2 + 3 = 0 \Rightarrow\ k = -5$

111. **(d)** Given,

$a = 3\hat{j} + 4\hat{k}$ and $b = \hat{i} + \hat{j}$

$a = a_1 + a_2$

a_1 is parallel to b.

$\therefore\ a_1 = \lambda b = \lambda(\hat{i} + \hat{j})$

a_2 is perpendicular to b.

$\therefore\quad a_2 \cdot b = 0$

$(a - a_1) \cdot b = 0$

$(3\hat{j} + 4\hat{k} - \lambda(\hat{i} + \hat{j})) \cdot (\hat{i} + \hat{j}) = 0$

$(-\lambda\hat{i} + (3 - \lambda)\hat{j} + 4\hat{k}) \cdot (\hat{i} + \hat{j}) = 0$

$\Rightarrow\quad -\lambda + 3 - \lambda = 0 \Rightarrow \lambda = \dfrac{3}{2}$

$\therefore\quad a_1 = \dfrac{3}{2}(\hat{i} + \hat{j})$

112. **(c)** We have,

$r \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$

$r \cdot (3\hat{i} + 3\hat{j} - 5\hat{k}) = 3$

Direction ratio of line of intersection of two planes

is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -3 \\ 3 & 3 & -5 \end{vmatrix} = -\hat{i} + \hat{j}$

Direction ratio of line

$r = 3\hat{i} + 2\hat{j} + \hat{k} + t(5\hat{i} + 5\hat{j} - 7\hat{k})$

is $5\hat{i} + 5\hat{j} - 7\hat{k}$

$\therefore$

$$\theta = \cos^{-1}\frac{(-\hat{i} + \hat{j}) \cdot (5\hat{i} + 5\hat{j} - 7\hat{k})}{|-\hat{i} + \hat{j}||\hat{i} + 5\hat{j}|} = \cos^{-1} 0$$

$$\theta = 90^\circ = \frac{\pi}{2}$$

113. **(c)** Given,

$x = \hat{i} + \hat{j}, y = 3\hat{i} - 2\hat{k}$

$r \times x = y \times x$ and $r \times y = x \times y$

$\therefore\ (r - y) \times x = 0$

$r - y$ is parallel to x.

$\therefore \quad r = y + \lambda x = 3i - 2k + \lambda(i + \hat{j})$

$= (3+\lambda)\hat{\imath} + \lambda\hat{\jmath} - 2\hat{k}$

$|r| = \sqrt{21}$

$\therefore \quad \sqrt{(3+\lambda)^2 + \lambda^2 + 4} = \sqrt{21}$

$\Rightarrow \quad 9 + 6\lambda + 2\lambda^2 + 4 = 21$

$\Rightarrow \quad \lambda^2 + 3\lambda - 4 = 0$

$\Rightarrow \quad (\lambda+4)(\lambda-1) = 0 \Rightarrow \lambda = 1$

$\therefore \quad r = 4\hat{\imath} + \hat{\jmath} - 2\hat{k}$

114. **(a)** Given,

and$r = (-\hat{\imath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

$\mathbf{r} \cdot (10\hat{\mathbf{\imath}} + 2\hat{\mathbf{\jmath}} - 11\hat{\mathbf{k}}) = 3$

If angle between line and plane is θ, then

$$\sin\theta = \frac{|\, 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) \cdot (10\hat{\imath} + 2\hat{\jmath} - 11\hat{k}\,|}{|2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}||10\hat{\imath} + 2\hat{\jmath} - 11\hat{k}|}$$

$$\sin\theta = \frac{20 + 6 - 66}{\sqrt{4+9+36}\sqrt{100+4+121}}$$

$$\sin\theta = \frac{-40}{7 \times 15} \Rightarrow \sin\theta = \frac{8}{21} \Rightarrow \theta = \sin^{-1}\left(\frac{8}{21}\right)$$

115. **(a)** Given,

$$\sum_{i=1}^{15} x_i^2 = 3600, \sum_{i=1}^{15} x_i = 175$$

When 20 is replace by 40 , then

$$\sum_{i=1}^{15} x_i = 175 - 20 + 40 = 195$$

$$\sum_{i=1}^{15} x_i^2 = 3600 - (20)^2 + (40)^2 = 4800$$

$\therefore$ Corrected variance $= \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$$= \frac{4800}{15} - \left(\frac{195}{15}\right)^2 = 320 - 169 = 151$$

116. **(b)** Given,

Coefficient of variation $= 7.2$

Variance $\sigma^2 = 3.24$

Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

$$7.2 = \frac{\sqrt{3.24}}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{1.8}{7.2} \times 100 \Rightarrow \bar{x} = 25$$

Hence, mean $= 25$

117. **(d)** Five dice are thrown simultaneous $n = 5, p =$ probability of show same numbered face

$$\therefore p = \frac{1}{6}, q = 1 - \frac{1}{6} = \frac{5}{6}$$

$\therefore$ Required probability

$= 6(P(r \geq 3)) = 6[P(r=3) + P(r=4) + P(r=5)]$

$$= 6\left[{}^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + {}^5C_4\left(\frac{1}{6}\right)^4\frac{5}{6} + \left(\frac{1}{6}\right)^5\right]$$

$$= \frac{6}{6^5}(10 \times 25 + 25 + 1) = \frac{276}{6^4}$$

118. **(b)** Let A be the events sum appeared on two unbiased dice is 7 and B be the event sum appeared on two unbiased dice is either 7 or 11 .

$\therefore P(A)=\frac{6}{36}, P(B)=\frac{6}{36}+\frac{2}{36}=\frac{2}{9}$

$\therefore$ Required probability

$=P(A)+P(\bar{B}A)+P(\bar{B}\bar{B}A)+P(\bar{B}\bar{B}\bar{B}A)+\cdots$

$=\frac{1}{6}+\frac{7}{9}\times\frac{1}{6}+\left(\frac{7}{9}\right)^2\times\frac{1}{6}+\cdots$

$=\frac{1}{6}\left(\frac{1}{1-\frac{7}{9}}\right)=\frac{1}{6}\times\frac{9}{2}=\frac{3}{4}$

119. **(d)** Let A be the event that the first mango is good and B be the event that second is good.

$\therefore$ Required probability $=P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$

$P(A)=\frac{{}^6C_2}{{}^{10}C_2}+\frac{{}^6C_1\times{}^4C_1}{{}^{10}C_2}$

$P(A\cap B)=\frac{{}^6C_2}{{}^{10}C_2}$

$\therefore P\left(\frac{B}{A}\right)=\frac{{}^6C_2}{{}^6C_2+{}^6C_1\times{}^4C_1}=\frac{15}{15+24}=\frac{15}{39}=\frac{5}{13}$

120. **(d)** Two dice are rolled

X = Absolute difference of two number

X	P(X)	P_iX_i
0	6/36	0
1	10/36	10/36
2	8/36	16/36
3	6/36	18/36
4	4/36	16/36
5	2/36	10/36

$\mu(\text{ Mean })=\Sigma P_iX_i=\frac{0+10+16+18+16+10}{36}$

$=\frac{70}{36}=\frac{35}{18}$

121. **(c)** $n=10, p=\frac{1}{2}, q=\frac{1}{2}$

$\therefore$ Required probability $=P(r\geq 6)$

$=P(r=6)+P(r=7)+P(r=8)+P(r=9)+P(r=10)$

$={}^{10}C_6\left(\frac{1}{2}\right)^{10}+{}^{10}C_7\left(\frac{1}{2}\right)^{10}+{}^{10}C_8\left(\frac{1}{2}\right)^{10}+{}^{10}C_9\left(\frac{1}{2}\right)^{10}+\left(\frac{1}{2}\right)^{10}$

$=\frac{1}{2^{10}}({}^{10}C_6+{}^{10}C_7+{}^{10}C_8+{}^{10}C_9+1)$

$=\frac{1}{2^{10}}(210+120+45+10+1)=\frac{386}{2^{10}}=\frac{193}{2^9}$

122. **(b)** Given,

Area of triangle = 15sq unit

$A(1,-2), B(-5,3), C(x,y)$

$$\Delta = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & -2 & 1 \\ -5 & 3 & 1 \end{vmatrix}$$

$$15 = \frac{1}{2}|x(-2-3) - y(1+5) + 1(3-10)|$$

$$30 = |-5x - 6y - 7|$$

$$5x + 6y + 7 = \pm 30$$

$$= 5x + 6y + 37 = 0 \text{ or } 5x + 6y - 23 = 0$$

Hence, point lies on

$(5x + 6y + 37)(5x + 6y - 23) = 0$

123. **(a)** The given equation is

$x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ ..... (i)

Since, axes are rotated through an angle 30°.

$\therefore\ x = X\cos 30^\circ - Y\sin 30^\circ$

$$= \frac{1}{2}(\sqrt{3}X - Y)$$

and $y = X\sin 30^\circ + Y\cos 30^\circ = \frac{1}{2}(X + \sqrt{3}Y)$

On putting the value of x and y in Eq. (i), we get

$\Rightarrow \frac{1}{4}(\sqrt{3}X - Y)^2 + \frac{2\sqrt{3}}{4}(\sqrt{3}X - Y)(X + \sqrt{3}Y) - \frac{1}{4}(X + \sqrt{3}Y)^2 = 2a^2$

$\Rightarrow 3X^2 + Y^2 - 2\sqrt{3}Y + 2\sqrt{3}\left(\sqrt{3}X^2 + 2XY - \sqrt{3}Y^2\right)$

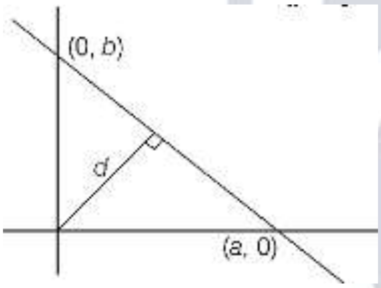
$-\left(X^2 + 3Y^2 + 2\sqrt{3}XY\right) = 8a^2$

$\Rightarrow 3X^2 + Y^2 - 2\sqrt{3}XY + 6X^2 + 4\sqrt{3}XY - 6Y^2 - X^2$

$-3Y^2 - 2\sqrt{3}XY = 8a^2$

$\Rightarrow 8X^2 - 8Y^2 = 8a^2 \Rightarrow X^2 - Y^2 = a^2$

124. **(c)** Let the equation of line is $\frac{x}{a} + \frac{y}{b} = 1$.



Distance from origin

$d = \left|\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right| \Rightarrow \frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2}$.... (i)

When line is rotated perpendicular distance from origin is same of line $\frac{x}{p} + \frac{y}{q} = 1$

i.e. $d = \left|\frac{1}{\sqrt{\frac{1}{p^2} + \frac{1}{q^2}}}\right| \Rightarrow \frac{1}{d^2} = \frac{1}{p^2} + \frac{1}{q^2}$..... (ii)

From, Eqs. (i) and (ii), we get

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$$

125. **(c)** Slope of BC $= \frac{4-2}{3-1} = 1$

Let the slope of AB $= m_1$

$\therefore\ \tan 30^\circ = \left|\dfrac{m_1 - 1}{1 + m_1}\right| \Rightarrow \pm\dfrac{1}{\sqrt{3}} = \dfrac{m_1 - 1}{1 + m_1}$

$m_1 + 1 = \pm\sqrt{3}(m_1 - 1)$

$m_1 + 1 = \sqrt{3}(m_1 - 1)$

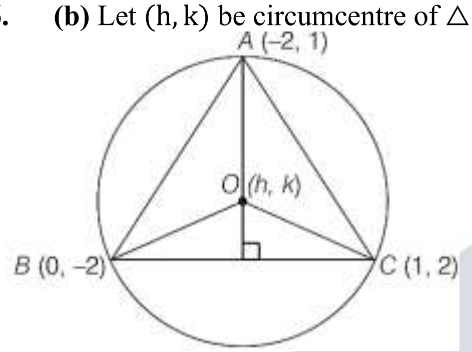
$m_1(\sqrt{3} - 1) = \sqrt{3} + 1$

$m_1 = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3}$

Similarly, we get $m_2 = 2 - \sqrt{3}$

$\therefore\ \dfrac{m_1}{m_2} = \dfrac{2 + \sqrt{3}}{2 - \sqrt{3}} = 7 + 4\sqrt{3}$

126. **(b)** Let (h, k) be circumcentre of $\triangle$ ABC and $A(-2,1), B(0,-2), C(1,2)$ are the vertices of $\triangle$ ABC.



$\therefore\quad OA = OB = OC$

$OA = OB$

$(h + 2)^2 + (k - 1)^2 = h^2 + (k + 2)^2$

$\Rightarrow\quad 4h - 6k + 1 = 0 \ldots\ldots (i)$

$OB = OC$

$h^2 + (k + 2)^2 = (h - 1)^2 + (k - 2)^2$

= $2h + 8k - 1 = 0 \ldots\ldots$ (ii)

On solving Eqs. (i) and (ii), we get

$h = -\dfrac{1}{22}, k = \dfrac{3}{22}$

Equation of BC

$y + 2 = \dfrac{2 + 2}{1 - 0}(x)$

$4x - y - 2 = 0$

Distance from circumcentre to line BC

$$d = \left|\frac{4\left(\frac{-1}{22}\right) - \frac{3}{22} - 2}{\sqrt{4^2 + 1^2}}\right|$$

$$d = \frac{51}{22\sqrt{17}} = \frac{3\sqrt{17}}{22}$$

127. **(c)** Equation of angle bisector of line positive coordinate axes is $y = x$

Putting the value of $y = x$ in

$ax^2 + 2hxy + by^2 = 0$, we get

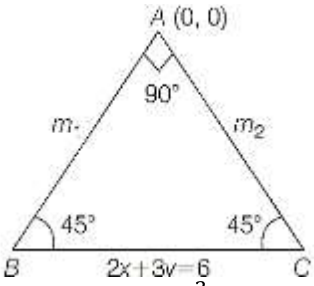
$ax^2 + 2hx^2 + bx^2 = 0$

$\Rightarrow\ a + 2h + b = 0$

$\Rightarrow\ a + b = -2h$

$\Rightarrow\ (a + b)^2 = 4h^2$

128. **(a)** Equation of line BC is $2x + 3y = 6$.



Slope of BC $= \frac{-2}{3}$

Slope of AB $= m_1 \tan 45^\circ = \left|\frac{m_1+\frac{2}{3}}{1-\frac{2}{3}m_1}\right|$

$1-\frac{2}{3}m_1 = m_1+\frac{2}{3} \Rightarrow m_1\left(1+\frac{2}{3}\right) = 1-\frac{2}{3}$

$\Rightarrow\ m_1 = \frac{1}{5}$

$\because$ Line AB and AC are perpendicular

$\therefore m_2 = -5$

Equation of line AB is $y = \frac{1}{5}x \Rightarrow 5y - x = 0$

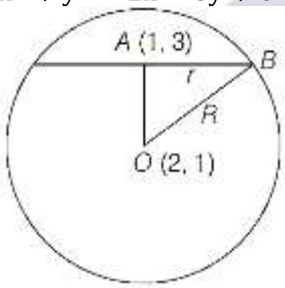
Equation of line AC is $y = -5x \Rightarrow y + 5x = 0$

Combining equation is $(5y - x)(y + 5x) = 0$

$\Rightarrow 5y^2 + 24xy - 5x^2 = 0 \Rightarrow 5x^2 - 24xy - 5y^2 = 0$

129. **(d)** Equation of circle

$x^2 + y^2 - 2x - 6y + 6 = 0$



Centre $(1,3), r = \sqrt{1+9-6} = 2$

$OA = \sqrt{(2-1)^2 + (1-3)^2} \Rightarrow OA = \sqrt{1+4} = \sqrt{5}$

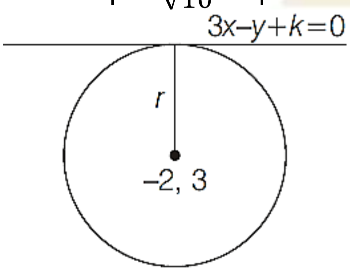
$\because AB = r = 2$ and $R^2 = r^2 + OA^2$

$\Rightarrow R^2 = 4 + 5 \Rightarrow\ R = 3$

130. **(c)** Line $3x - y + k = 0$ touches the circle $x^2 + y^2 + 4x - 6y + 3 = 0$ having

Centre $(-2,3), r = \sqrt{4+9-3} = \sqrt{10}$

$r = \left|\frac{-6-3+k}{\sqrt{10}}\right|$



$\because$

$10 = |-9+k| \Rightarrow \pm 10 = k - 9$

$\therefore\ k = -1, 19$

$k_1 = -1, k_2 = 19$

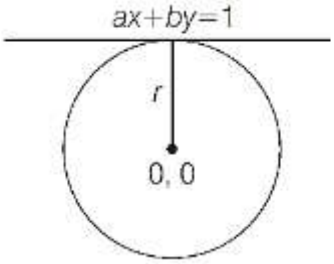
Equation of chord of contact of $(-1,19)$ to the circle

$-x + 19y + 2(x-1) - 3(y+19) + 3 = 0$

$\Rightarrow\quad x + 16y - 56 = 0$

131. **(a)** We have,

$ax+by=1$ is tangent of circle $x^2+y^2-r^2=0$



$\therefore\ r=\frac{1}{\sqrt{a^2+b^2}} \Rightarrow r^2=\frac{1}{a^2+b^2} \Rightarrow a^2+b^2=\frac{1}{r^2}$

$r=1,(a,b)$ lie on the circle

$\therefore (a,b)$ lie on the circle when $S_1=0$

132. **(a)** Let the equation of the circle be

$x^2+y^2+2gx+2fy+c=0$

Since, this circle is passes through $(2,0)$ and $(-2,0)$

$\therefore\ 4+4g+c\ =0 \ldots\ldots$ (i)

$4-4g+c\ =0 \ldots..$ (ii)

On solving Eqs. (i) and (ii), we get $g=0, c=-4$

$\therefore$ Equation of circle $x^2+y^2+2fy-4=0$

Now, $y=mx+c$ is tangent of circle

$\therefore\quad \left|\frac{-f+c}{\sqrt{1+m^2}}\right|=\sqrt{f^2+4}$

$\Rightarrow\quad (c-f)^2=(1+m^2)(f^2+4)$

$\Rightarrow\quad m^2f^2+2cf+4(1+m^2)-c^2=0$

Let f_1f_2 are roots of equation

$\therefore\ f_1f_2=\frac{4(1+m^2)-c^2}{m^2}$

Now, equation of this circle are

$x^2+y^2+2f_1y-4=0$ and $x^2+y^2+2f_2y-4=0$

Since, these are orthogonals.

$\therefore\ 2f_1f_2=c_1+c_2$

$\Rightarrow\ 2f_1f_2=-8$

$\Rightarrow\ f_1f_2=-4$

$\Rightarrow \frac{4(1+m^2)-c^2}{m^2}=-4$

$\Rightarrow\ c^2=4(1+2m^2)$

133. **(d)** Equation of given circles

$S_1=x^2+y^2-3x+y-10=0$

$S_2=x^2+y^2-x+2y-20=0$

$\because$ Equation of family of circles passes through intersection of circles S_1 and S_2 is

$S_1+\lambda S_2=0$

$x^2(1+\lambda)+y^2(1+\lambda)-x(3+\lambda)+y(1+2\lambda)$

$-10-20\lambda=0 \ldots$ (i)

$\because$ Equation of common chord of circles S_1 and S_2

$S_1-S_2=-2x-y+10=0$

$2x+y-10=0$ is diametre of circle $S_1+\lambda S_2=0$

Centre of circle $S_1+\lambda S_2=0$ is lie on

$2x+y-10=0$

$\because$ Centre $\left(\frac{3+\lambda}{2(1+\lambda)},-\frac{(1+2\lambda)}{2(1+\lambda)}\right)$

$\therefore\ 2\left(\frac{3+\lambda}{2(1+\lambda)}\right)-\left(\frac{1+2\lambda}{2(1+\lambda)}\right)-10=0$

$6+2\lambda-1-2\lambda-20-20\lambda=0$

$\lambda=\frac{-3}{4}$

Putting the value of λ in Eq. (i), we get

$$x^2\left(1-\frac{3}{4}\right)+y^2\left(1-\frac{3}{4}\right)-x\left(3-\frac{3}{4}\right)+y\left(1-\frac{6}{4}\right)$$

$$-10+\left(20\times\frac{3}{4}\right)=0$$

$$\Rightarrow x^2+y^2-9x-2y+20=0$$

134. **(c)** (I) $x=ly^2+my+n$

$\Rightarrow\ y^2+\frac{m}{l}y=\frac{x}{l}-\frac{n}{l}$

$\Rightarrow\ y^2+\frac{m}{l}y+\frac{m^2}{4l^2}=\frac{x}{l}-\frac{n}{l}+\frac{m^2}{4l^2}$

$\Rightarrow\ \left(y+\frac{m}{2l}\right)^2=\frac{1}{l}\left[x-\left(n-\frac{m^2}{4l}\right)\right]$

Vertex $=\left(n-\frac{m^2}{4l},\frac{-m}{2l}\right)$

Statement I is true.

(II) We have,

$$y=lx^2+mx+n\Rightarrow\frac{y}{l}-\frac{n}{l}=x^2+\frac{mx}{l}$$

$$\frac{y}{l}-\frac{n}{l}+\frac{m^2}{4l^2}=\left(x+\frac{m}{2l}\right)^2$$

$$\frac{1}{l}\left[y-\left(n-\frac{m^2}{4l}\right)\right]=\left(x+\frac{m}{2l}\right)^2$$

$$\text{Focus }=\left(-\frac{m}{2l},n-\frac{m^2}{4l}+\frac{1}{4l}\right)$$

Statement II is false.

Statement III is also true.

135. **(a)** We know that, length of normal of parabola $y^2=4ax$ at $(at^2,2at)=8a\sqrt{t^2+1}$ and length of focal chord $=a\left(t+\frac{1}{t}\right)^2$.

Point $P(3,6)$ lie on parabola $y^2=12x$

$\therefore$ Here $a=3,t=1$

Hence $l_1=8\times3\sqrt{1+1}=24\sqrt{2}$

$l_2=3(1+1)^2=12$

$$\frac{l_1}{l_2}=\frac{24\sqrt{2}}{12}=2\sqrt{2}$$

136. **(a)** Equation of tangent at $(3\sqrt{3}\cos\theta,\sin\theta)$ on the ellipse $\frac{x^2}{27}+\frac{y^2}{1}=1$ is

$$\frac{3\sqrt{3}x\cos\theta}{27}+\frac{y\sin\theta}{1}=1$$

$$\frac{x}{3\sqrt{3}}\cos\theta+\frac{y\sin\theta}{1}=1$$

Sum of intercepts of tangent

i.e. $L=3\sqrt{3}\sec\theta+\operatorname{cosec}\theta$

$\because\ \frac{dL}{d\theta}=3\sqrt{3}\sec\theta\tan\theta-\operatorname{cosec}\theta\cot\theta$

For maxima or minima $\frac{dL}{d\theta}=0$

$3\sqrt{3}\sec\theta\tan\theta-\operatorname{cosec}\theta\cot\theta=0$

$\tan^3\theta = \frac{1}{3\sqrt{3}} \Rightarrow \tan\theta = 1\sqrt{3} \Rightarrow \theta = \frac{\pi}{6}$

Minimum at $\theta = \frac{\pi}{6}$

137. **(b)** Let the equation of line perpendicular to X-axis cut the circle $x^2 + y^2 = 9$ at A is $(3\cos\alpha, 3\sin\alpha)$

Equation of tangent at A is $x\cos\alpha + y\sin\alpha = 3$

Slope $= -\cot\alpha$

Similarly, cut the ellipse $4x^2 + 9y^2 = 36$ at B is

$(3\cos\alpha, 2\sin\alpha)$

Equation of tangent at B

$2\cos\alpha + 3\sin\alpha = 6$

$$\text{Slope} = -\frac{2}{3}\cot\alpha$$

Angle between tangents is θ, then

$$\tan\theta = \frac{\left(-\frac{2}{3}+1\right)\cot\alpha}{1+\frac{2}{3}\cot^2\alpha} = \frac{\cot\alpha}{3+2\cot^2\alpha}$$

$$\text{Let } \tan\theta = z \Rightarrow z = \frac{\cot\alpha}{3+2\cot^2\alpha}$$

$$\because \frac{dz}{d\alpha} = -\left[\frac{(3+2\cot^2\alpha)(\text{cosec}^2\alpha) - \cot\alpha(4\cot\alpha\,\text{cosec}^2\alpha)}{(3+2\cot^2\alpha)}\right]$$

For maxima or minima $\frac{dz}{d\alpha} = 0$

$\therefore 3\text{cosec}^2\alpha + 2\cot^2\alpha\,\text{cosec}^2\alpha - 4\cot^2\alpha\,\text{cosec}^2\alpha = 0$

$$\Rightarrow \cos^2\alpha = \frac{3}{2} \Rightarrow \cot\alpha = \sqrt{\frac{3}{2}}$$

$$\because \tan\theta = \frac{\sqrt{\frac{3}{2}}}{3+2\times\frac{3}{2}} = \sqrt{\frac{3}{2}}\times\frac{1}{6} = \frac{1}{2\sqrt{6}}$$

$\therefore$ Greatest acute angle when, $\tan\theta = \frac{1}{2\sqrt{6}}$.

138. **(b)** Equation of ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

Foci $= (0, \pm 3)$

$$\Rightarrow \quad e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5} \text{ given } e_1 e_2 = 1$$

(where e_2 is eccentricity of hyperbola)

Let equation of hyperbola $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

its passes through $(0, \pm 3)$

$\therefore \quad b^2 = 9$

$$\Rightarrow \quad e_2 = \sqrt{1 + \frac{a^2}{b^2}}$$

$$\Rightarrow \quad e_2^2 = 1 + \frac{a^2}{b^2}$$

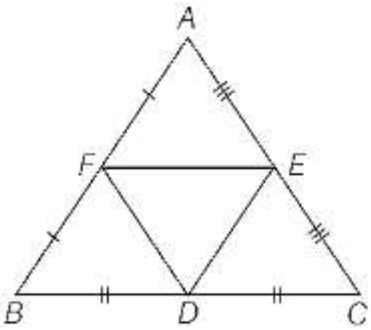
$$\Rightarrow \quad \frac{1}{e_1^2} = 1 + \frac{a^2}{9} \Rightarrow \frac{25}{9} = 1 + \frac{a^2}{9}$$

$\Rightarrow \quad a^2 = 16$

Hence, the equation of hyperbola is $\frac{y^2}{9} - \frac{x^2}{16} = 1$

139. **(b)** We have, $(1,0,3), (2,1,5), (-2,3,6)$ is mid point of the sides of triangle.

Centroid of $\triangle$ ABC is also the centroid of $\triangle$ DEF



$\therefore$ Centroid $= \left(\frac{1+2-2}{3}, \frac{0+1+3}{3}, \frac{3+5+6}{3}\right)$

$= \left(\frac{1}{3}, \frac{4}{3}, \frac{14}{3}\right)$

140. **(b)** Let the direction ratio of normal to the plane P is (a, b, c).

$\because$ equation of another given plane $x + y = 3$ having direction ratio of normal $= (1,1,0)$

$\therefore \cos\frac{\pi}{4} = \frac{a+b}{\sqrt{a^2+b^2+c^2} \times \sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{a+b}{\sqrt{a^2+b^2+c^2} \times \sqrt{2}}$

$\Rightarrow (a+b)^2 = a^2 + b^2 + c^2 \Rightarrow 2ab = c^2$

which satisfied by the option (b)

$\therefore a = 1, b = 1, c = \sqrt{2}$

141. **(a)** Let the equation of plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Distance from origin is 6 .

$\therefore 6 = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{36}$ ... ... (i)

Centroid of plane is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\therefore$ Let $x = \frac{a}{3} \Rightarrow a = 3x$

Similarly, $b = 3y, c = 3z$

On putting the value of a, b, c in Eq. (i), we get

$\frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{36} \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{4}$

142. **(b)** Let $L = \lim_{x\to\infty} \left(\frac{2x^2+3x+4}{x^2-3x+5}\right)^{\frac{3|x|+1}{2|x|-1}}$

$L = \lim_{x\to\infty} \left(\frac{2 + \frac{3}{x} + \frac{4}{x^2}}{1 - \frac{3}{x} + \frac{5}{x^2}}\right)^{\frac{3+\frac{1}{|x|}}{2-\frac{1}{|x|}}} = \left(\frac{2}{1}\right)^{3/2} = 2\sqrt{2}$

143. **(a)** We have,

$$f(x) = \begin{cases} \cos 2x, & -\infty < x < 0 \\ e^{3x}, & 0 \le x < 3 \\ x^2 - 4x + 3, & 3 \le x \le 6 \\ \frac{\log(15x - 89)}{x - 6}, & x > 6 \end{cases}$$

$\because \lim_{x\to 3^-} e^{3x} = e^9$ and $\lim_{x\to 3^+} x^2 - 4x + 3 = 9 - 12 + 3 = 0$

Clearly, $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$

$\therefore f(x)$ is discontinuous at $x = 3$.

$\therefore a = 3$,

Now, $\lim_{x\to 3} \frac{x^2-9}{x^3-5x^2+9x-9} = \lim_{x\to 3} \frac{(x-3)(x+3)}{(x-3)(x^2-2x+3)}$

$$= \frac{3+3}{(3)^2 - 2(3) + 3} = \frac{2}{2} = 1$$

144. **(d)** We have,

$$y = (x+1)(x^2+1)(x^4+1)(x^8+1)$$

$$\frac{dy}{dx} = (x^2+1)(x^4+1)(x^8+1) + 2x(x+1)(x^4+1)$$

$$(x^8+1) + 4x^3(x+1)(x^2+1)(x^8+1) + 8x^7(x+1)$$

$$(x^2+1)(x^4+1)$$

$$\left(\frac{dy}{dx}\right)_{x=-1} = (1+1)(1+1)(1+1) + 0 + 0 + 0 = 8$$

145. **(c)** We have,

$h(x) = (f(x))^2 + (g(x))^2$

So, $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$\Rightarrow h'(x) = 2f(x)g(x) + 2g(x)f''(x)$

$[\because f'(x) = g(x) \text{ and } f''(x) = g(x)]$

$\Rightarrow h'(x) = 2f(x)g(x) - 2g(x)f(x)\ [f''(x) = -f(x)]$

$\Rightarrow h'(x) = 0$

$\because\ h(x) =$ Constant function

$h(1) = 2$

$\therefore h(2) = 2,$

146. **(d)** We have

$$y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \cdots \infty}}}}$$

$$y = \sqrt{x + \sqrt{y + y}} \Rightarrow y^2 = x + \sqrt{2y}$$

$\Rightarrow y^2 - x = \sqrt{2y} \Rightarrow (y^2 - x)^2 = 2y$

$\Rightarrow y^4 - 2xy^2 + x^2 = 2y$

$\Rightarrow y^4 - 2xy^2 - 2y + x^2 = 0$

On differentiating w.r.t. to x, we get

$$4y^3\frac{dy}{dx} - 2y^2 - 4yx\frac{dy}{dx} - \frac{2dy}{dx} + 2x = 0$$

$$\frac{dy}{dx}(4y^3 - 4xy - 2) = -2x + 2y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$$

147. **(c)** We have,

$xy = 1$ ….. (i)

and $x^2 + 8y = 0$….. (ii)

On solving Eqs. (i) and (ii), we get

$$x = -2, y = -\frac{1}{2}$$

Now, $xy = 1$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x} \Rightarrow \left(\frac{dy}{dx}\right)_{\left(-2,\frac{-1}{2}\right)} = -\frac{1}{4} = m_1$$

and $x^2 + 8y = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{4}$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\left(-2,-\frac{1}{2}\right)} = \frac{1}{2} = m_2$$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \tan\theta = \left|\frac{-\frac{1}{4} - \frac{1}{2}}{1 - \frac{1}{8}}\right|$

$\therefore\ \tan\theta = \frac{6}{7}$

148. **(a)** We have,

$x = t^2 - 7t + 7, y = t^2 - 4t - 10$

Put $x = 1$

$1 = t^2 - 7t + 7$

$\Rightarrow t^2 - 7t + 6 = 0$

$\Rightarrow (t - 6)(t - 1) = 0 \Rightarrow t = 1,6$

Put $y = 2$

$2 = t^2 - 4t - 10 \Rightarrow t^2 - 4t - 12 = 0$

$(t - 6)(t + 2) = 0 \Rightarrow t = -2,6$

Hence, $t = 6$ satisfies both equations.

Now, $\frac{dx}{dt} = 2t - 7 \Rightarrow \left(\frac{dx}{dt}\right)_{t=6} = 5$

and $\frac{dy}{dt} = 2t - 4 \Rightarrow \left(\frac{dy}{dt}\right)_{t=6} = 8$

$\therefore\ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{8}{5}$

149. **(c)** We have,

$f(x) = 2x^3 - 3x^2 - x + 1$

Let $g(x) = \frac{x^4}{2} - x^3 - \frac{x^2}{2} + x$

$g(-1) = \frac{1}{2} + 1 - \frac{1}{2} - 1 = 0$

and $g(0) = 0$

$\therefore$ $f(x) = 0$ has roots lie in $[-1,0]$.

Similarly, $g(0) = g(1) = 0$

$\therefore$ $f(x) = 0$ has roots lie in $[0,1]$.

$g(2) = \frac{16}{2} - 8 - \frac{4}{2} + 2 = 8 - 8 - 2 + 2 = 0$

$g(1) = g(2) = 0$

But, $g(-2) \neq 0$

$\therefore$ $f(x) = 0$ has roots in every interval except I_4.

150. **(a)** We have,

$f(x) = \int_x^{x+1} e^{-t^2} dt \Rightarrow f'(x) = e^{-(x+1)^2} - e^{-x^2}$

$f'(x) = \frac{1}{e^{(x+1)^2}} - \frac{1}{e^{x^2}}$

Since, $f(x)$ is decreasing function.

$\therefore\ f'(x) < 0$

$\therefore\ \frac{1}{e^{(x+1)^2}} - \frac{1}{e^{x^2}} < 0 \Rightarrow e^{x^2} < e^{(x+1)^2}$

$x^2 < (x + 1)^2 \Rightarrow x^2 + 2x + 1 - x^2 > 0$

$2x + 1 > 0 \Rightarrow x > -\frac{1}{2}$

$\therefore\ x \in \left(\frac{-1}{2}, \infty\right)$

151. **(a)** We have,

$$\int \frac{\cos x + x}{1+\sin x}dx = f(x) + \int \frac{3\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}dx + c$$

$$\therefore f(x) = \int \left(\frac{\cos x + x}{1+\sin x} - \frac{3\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)dx$$

$$= \int \frac{\cos x}{1+\sin x} + \frac{x}{1+\sin x} - \frac{3\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}dx$$

$$= \int \frac{\cos x}{1+\sin x} + \frac{x}{1+\sin x}$$

$$- \frac{\left(3\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{1+\sin x}dx$$

$$= \int \left(\frac{\cos x}{1+\sin x} + \frac{x}{1+\sin x}\right.$$

$$\left.- \frac{\left(3\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} - \sin^2\frac{x}{2}\right)}{1+\sin x}\right)dx$$

$$= \int \frac{\cos x + x - 2\cos^2\frac{x}{2} - \cos x - \sin x}{1+\sin x}dx$$

$$= \int \frac{x - 1 - \cos x - \sin x}{1+\sin x}dx = \int \left(\frac{x - \cos x}{1+\sin x} - 1\right)dx$$

$$= \int x(\sec^2 x - \sec x \tan x) - \int \frac{\cos x}{1+\sin x}dx - x$$

$$= x(\tan x - \sec x) - \int (\tan x - \sec x)dx$$

$$- \int \frac{\cos x}{1+\sin x} - x$$

$$= x(\tan x - \sec x - 1) + \int \frac{1-\sin x}{\cos x}dx - \int \frac{1-\sin x}{\cos x}dx$$

$$f(x) = x\left(\frac{-1+\sin x}{\cos x} - 1\right) = -x\left(\frac{\cos x}{1+\sin x} + 1\right)$$

$$f(x) = -x\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} + 1\right)$$

$$f(x) = \frac{-2x\cos\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

$$= \frac{-2x}{1 + \tan\frac{x}{2}}$$

152. **(b)** We have, $I = \int \frac{\sqrt{\cos 2x}}{\sin x}dx$

Put $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

$$\because \quad I = \int \frac{\sqrt{1-\tan^2 x}}{\sec x \sin x}dx$$

Put $1 - \tan^2 x = t^2$

$\Rightarrow \; -2\tan x \sec^2 x\, dx = 2t\,dt$

$$I = -\int \frac{t^2}{(1-t^2)(2-t^2)}dt = -\int \frac{t^2}{(t^2-1)(t^2-2)}dt$$

$$I=-\int\left(\frac{2}{t^2-2}-\frac{1}{t^2-1}\right)dt=2\int\frac{-1}{t^2-2}dt+\int\frac{dt}{t^2-1}$$

$$I=\frac{-2}{2\sqrt{2}}\log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|+\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+c$$

$$I=\frac{-1}{\sqrt{2}}\log\left|\frac{\sqrt{1-\tan^2 x}-\sqrt{2}}{\sqrt{1-\tan^2 x}+\sqrt{2}}\right|+\frac{1}{2}\log\left|\frac{\sqrt{1-\tan^2 x}-1}{\sqrt{1-\tan^2 x}+1}\right|+c$$

$$I=\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+\sqrt{1-\tan^2 x}}{\sqrt{2}-\sqrt{1-\tan^2 x}}\right|-\frac{1}{2}\log\left|\frac{1+\sqrt{1-\tan^2 x}}{1-\sqrt{1-\tan^2 x}}\right|+c$$

153. **(a)** We have,

$$I=\int\frac{2x+3}{x(x+1)(x+2)(x+3)+1}dx$$

$$=-\frac{1}{px^2+qx+r}+c$$

$$I=\int\frac{2x+3}{(x^2+3x)(x^2+3x+2)+1}dx$$

Put $x^2+3x=t\Rightarrow(2x+3)dx=dt$

$$\therefore\quad I=\int\frac{dt}{t(t+2)+1}$$

$$I=\int\frac{dt}{(t+1)^2}=\frac{-1}{t+1}+c$$

$$\Rightarrow\quad I=\frac{-1}{x^2+3x+1}+c$$

$$\therefore\quad p=1,q=3,r=1$$

$$\Rightarrow\quad\frac{3p-q}{r}=\frac{3-3}{1}=0$$

154. **(c)** Let $I=\int(\log x)^2dx$

$$I=(\log x)^2\int dx-\int\frac{d(\log x)^2}{dx}\int dx\cdot dx+c$$

$$I=x(\log x)^2-\int 2\log x dx+c$$

$$I=x(\log x)^2-2(x\log x-x)+c$$

$$I=x(\log x)^2-2x\log x+2x+c$$

$$I=x(\log x)^2-2x(\log x-1)+c$$

155. **(a)** We have,

$$I=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+k^2}=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{k/n}{1+\left(\frac{k}{n}\right)^2}$$

$$\Rightarrow\ I=\int_0^1\frac{x}{1+x^2}dx=\frac{1}{2}[\log(1+x^2)]_0^1=\frac{1}{2}\log 2$$

156. **(c)** Let

$$I=\int_0^{10}5-\sqrt{10x-x^2})dx, I=\int_0^{10}\left(5-\sqrt{(5)^2-(x-5)^2}dx\right.$$

$$I=\left[5x-\frac{x-5}{2}\sqrt{10x-x^2}-\frac{25}{2}\sin^{-1}\frac{x-5}{5}\right]_0^{10}$$

$$I=\left(50-0-\frac{25}{2}\sin^{-1}1\right)-\left(0-0-\frac{25}{2}\sin^{-1}(-1)\right)$$

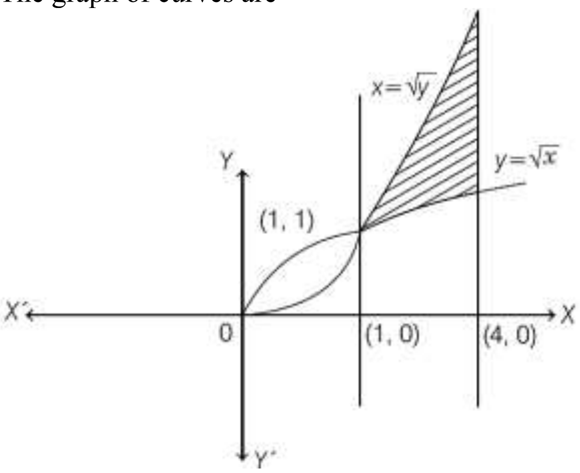
$$I=50-\frac{25}{2}\times\frac{\pi}{2}-\frac{25}{2}\times\frac{\pi}{2}$$

$$I=50-\frac{25\pi}{2}\Rightarrow I=\frac{1}{2}(100-25\pi)$$

157. **(b)** We have,

$y=\sqrt{x}, x=\sqrt{y}, x=1, x=4$

The graph of curves are



Area of shaded region

$$=\int_1^4 (x^2-\sqrt{x})dx=\left[\frac{x^3}{3}-\frac{2}{3}(x)^{3/2}\right]_1^4$$

$$=\left(\frac{64}{3}-\frac{16}{3}\right)-\left(\frac{1}{3}-\frac{2}{3}\right)$$

$$=\frac{64-16-1+2}{3}=\frac{49}{3}$$

158. **(c)** Family of circle of constant radius r is $(x-a)^2+(y-b)^2=r^2$

Let $x=a+r\cos\theta, y=b+r\sin\theta$

$$\frac{dx}{d\theta}=-r\sin\theta, \frac{dy}{d\theta}=r\cos\theta \Rightarrow \frac{dy}{dx}=-\cot\theta$$

$$\frac{d^2y}{dx^2}=\operatorname{cosec}^2\theta\frac{d\theta}{dx}=\frac{-\operatorname{cosec}^3\theta}{r}$$

$$\frac{d^2y}{dx^2}=\frac{-(1+\cot^2\theta)^{3/2}}{r}$$

$$ry''=-(1+(y')^2)^{3/2}$$

Squaring on both sides, we get

$r^2(y'')^2=(1+(y')^2)^3$

159. **(b)** We have,

$$(2x-3y+5)dx+(9y-6x-7)dy=0$$

$$\frac{dy}{dx}=\frac{2x-3y+5}{3(2x-3y)+7}$$

Put $2x-3y=z$

$$\Rightarrow \quad 2-\frac{3dy}{dx}=\frac{dz}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{3}\left(2-\frac{dz}{dx}\right)$$

$$\therefore \quad \frac{1}{3}\left(2-\frac{dz}{dx}\right)=\frac{z+5}{3z+7}$$

$$\Rightarrow \quad 2-\frac{dz}{dx}=\frac{3z+15}{3z+7}$$

$$\Rightarrow \quad \frac{dz}{dx}=2-\frac{3z+15}{3z+7}$$

$$\Rightarrow \quad \frac{dz}{dx}=\frac{3z-1}{3z+7}$$

$$\Rightarrow \frac{3z+7}{3z-1}dz = dx$$

$$\Rightarrow \int\left(\frac{3z-1}{3z-1}+\frac{8}{3z-1}\right)dz = dx$$

$$= z+\frac{8}{3}\log|3z-1| = x+c$$

$$= 3z+8\log(3z-1) = 3x+c$$

$$= 3x-9y+8\log(6x-9y-1) = c$$

160. **(a)** We have,

$$\sqrt{1-y^2}dx + xdy - \sin^{-1}ydy = 0$$

$$\frac{dx}{dy}+\frac{x}{\sqrt{1-y^2}} = \frac{\sin^{-1}y}{\sqrt{1-y^2}}$$

$$\text{IF} = e^{\int\frac{dy}{\sqrt{1-y^2}}} = e^{\sin^{-1}y}$$

Solution of given differential equation is

$$xe^{\sin^{-1}y} = \int e^{\sin^{-1}y}\frac{\sin^{-1}y}{\sqrt{1-y^2}}dy$$

$$xe^{\sin^{-1}y} = e^{\sin^{-1}y}(\sin^{-1}y-1)+c$$

$$x = \sin^{-1}y-1+ce^{-\sin^{-1}y}$$