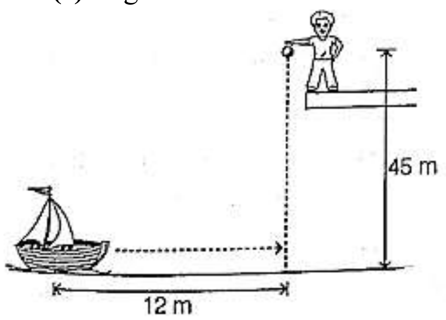
Physics

1. **(c)** Tokamak reactor is a fusion reactor in which controlled fusion is achieved by heating a dense mass by lasers and confining that heated matter or plasma by forming magnetic bottle.
2. **(d)** Dimensions on both sides of equation must be same. So, exponent term must be dimensionless and there must not be any other term except energy on right side.

$E = E_0\left(e^{-t/t_0}\right)$

3. **(c)** In given situation



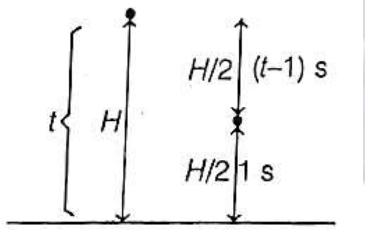
Time taken by ball to cover 45 m = Time taken by boat to cover 12 m

$$\Rightarrow \sqrt{\frac{2h}{g}} = \frac{\text{Distance (x)}}{\text{Speed (v)}} \Rightarrow \sqrt{\frac{2\times 45}{10}} = \frac{12}{s}$$

$$\Rightarrow v = 4\ m/s$$

4. **(b)** In last 1s, ball covers $H/2$ distance. Let total time for falling by H height is t second.

So, $H = \frac{1}{2}gt^2$



In $(t-1)$ second, ball falls by $\frac{H}{2}$ height.

So, $\frac{H}{2} = \frac{1}{2}g(t-1)^2$ .... (ii)

Now, dividing Eq. (i) by Eq. (ii), we get

$$\frac{H}{H/2} = \frac{\frac{1}{2}gt^2}{\frac{1}{2}g(t-1)^2} \Rightarrow \sqrt{2} = \frac{t}{t-1}$$

$$\sqrt{2}t - \sqrt{2} = t$$

$$t = \frac{\sqrt{2}}{\sqrt{2}-1} = \frac{\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= 2 + \sqrt{2} = 2 + 1.41$$

$$\Rightarrow t = 3.41\ s$$

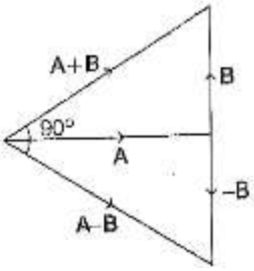
5. **(c)** Planet completes 2 rounds in 360 days;

$\Rightarrow$ $2 \times 2\pi$rad in 360 days

So, angular frequency $= \frac{\text{angle}}{\text{time}} = \frac{4\pi}{360}$

$= 3.5 \times 10^{-2}$rad/day

6. **(c)** Let forces are A and B. Then given that, $(A + B)$ and $(A - B)$ are perpendicular, which is shown in the following diagram.



$\Rightarrow \quad (\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}-\mathbf{B})=0$

$\Rightarrow \quad \mathbf{A}^2-\mathbf{B}^2=0$

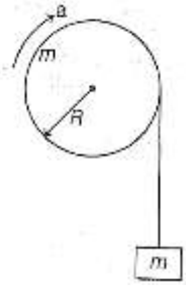
$\Rightarrow \quad \mathbf{A}^2=\mathbf{B}^2$

As square of a vector is equal to square of its, magnitude,

$\Rightarrow \quad |\mathbf{A}|^2=|\mathbf{B}|^2$

$\Rightarrow \quad |\mathbf{A}|=|\mathbf{B}|$

7. **(d)** The given situation is shown in the following figure



Initial torque on cylinder due to force of mass m is

$\tau = FR = mgR$

If α is angular acceleration of cylinder, then $\tau = I\alpha$ where, I = moment of inertia

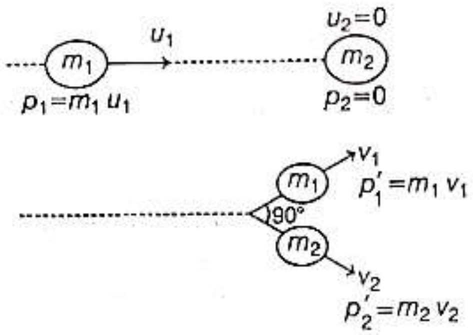
$= mR^2$

$$\alpha = \frac{\tau}{I} = \frac{mgR}{mR^2} \Rightarrow \alpha = \frac{g}{R}$$

As linear acceleration, $a = R\alpha$

$$= R\cdot\frac{g}{R} = g$$

8. **(a)** According to given situation,



Momentum conservation gives,

$p_1' + p_2' = p_1$ .....(i)

KE conservation gives,

$$\frac{p_1'^2}{m_1}+\frac{p_2'^2}{m_2}=\frac{p_1^2}{m_1}\left[\because k=\frac{p^2}{2m}\right] \ldots \text{(ii)}$$

Subtituting the value of p_1 from Eq. (i) into Eq. (ii), we have

$$\frac{p_1'^2}{m_1}+\frac{p_2'^2}{m_2}=\frac{1}{m_1}(p_1'+p_2')^2$$

$$\frac{p_1'^2}{m_1}+\frac{p_2'^2}{m_2}=\frac{p_1'^2}{m_1}+\frac{p_2'^2}{m_1}+\frac{2p_1'p_2'}{m_1}$$

As particles fly off at 90° after collision.

$\therefore \quad \mathbf{p}_1 \cdot \mathbf{p}_2 = 0$

$\Rightarrow \quad p_1 \cdot p_2 \cos\theta = 0$

$\Rightarrow \quad p_1 p_2 = 0$

$$\frac{p_2'^2}{m_2} = \frac{p_2'^2}{m_1} \text{ or } \frac{m_2}{m_1} = 1$$

9. **(b)** Loss of KE of block = Gain of PE of spring

$\Rightarrow \frac{1}{2}m(u^2 - v^2) = \frac{1}{2}kx^2 \quad \ldots\ldots(i)$

Here,

$$m = 100\text{ g} = \frac{100}{1000}\text{ kg} = 0.1\text{ kg}$$

$$u = 2\text{ ms}^{-1}$$

$$v = \frac{4}{2} = \frac{2}{2} = 1\text{ ms}^{-1}$$

$$x = 2\text{ cm}$$

$$= 2 \times 10^{-2}\text{ m}$$

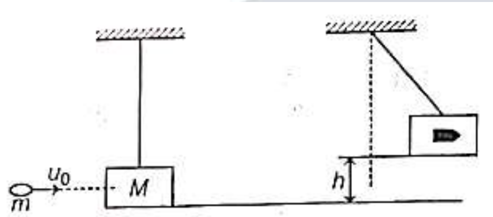
So, from Eq. (i),

$$\frac{1}{2} \times 0.1 \times (2^2 - 1^2) = \frac{1}{2} \times k \times (2 \times 10^{-2})^2$$

$$\Rightarrow \frac{3}{10 \times 4 \times 10^{-4}} = k$$

$$\Rightarrow k = \frac{3 \times 1000}{4} = 750\text{ N/m}$$

10. **(a)**



Bullet hits block and gets lodged into the block. After this, block + bullet system moves to rise upto height h.

Let, v = common velocity of block and bullet with which mass system begins to rise.

This is an example of perfectly inelastic collision, hence according to conservation of linear momentum,

$\Rightarrow \quad mv_0 = (m + M)v$

So, $\quad v = \dfrac{mv_0}{(m + M)}$

$\therefore$ Loss of KE

= Final KE of system - Initial KE of bullet

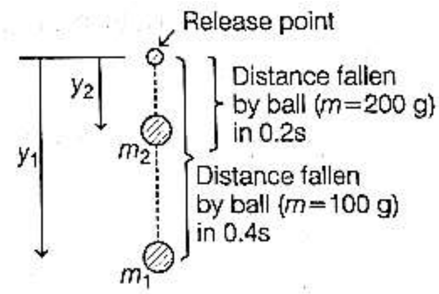
$$= \frac{1}{2}(m + M)v^2 - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}(m + M)\frac{m^2v_0^2}{(m + M)^2} - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}mv_0^2\left(\frac{m}{m + M} - 1\right)$$

$$= \frac{1}{2}mv_0^2\left(\frac{M}{m + M}\right)$$

11. **(a)** As second ball is dropped 0.2 s after first ball, we have following situation at $t = 0.4$ s.



Now, y_1 = distance travelled by first ball in 0.4 s

$= \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.4)^2 = 0.8$ m

and y_2 = distance travelled by second ball in 0.2 s

$= \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.2)^2 = 0.2$ m

Now, position of centre of mass of ball w.r.t. release point (taken origin) is

$$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$= \frac{100 \times 0.8 + 200 \times 0.2}{100 + 200}$$

$$= \frac{120}{300} = 0.4 \text{ m}$$

12. (c) According to given situation,

As there is no external force, velocity and hence KE of centre of mass remains same before and after collision.

Now, $v_{CM} = \frac{m_1v_1+m_2v_2}{m_1+m_2}$

$\Rightarrow v_{CM} = \frac{2M+1}{M+1}$

Kinetic energy of centre of mass is given 4/3 J.

$\therefore \frac{4}{3} = \frac{1}{2}(M+1) \cdot v_{CM}^2$

$\Rightarrow \frac{4}{3} = \frac{1}{2}(M+1)\left(\frac{2M+1}{M+1}\right)^2$

$\Rightarrow \frac{8}{3} = \frac{(2M+1)^2}{M+1}$

$\Rightarrow 8(M+1) = 3(2M+1)^2$

$\Rightarrow 12M^2 + 4M - 5 = 0$

$\Rightarrow 12M^2 + 10M - 6M - 5 = 0$

$\Rightarrow 2M(6M+5) - 1(6M+5) = 0$

$\Rightarrow (2M-1)(6M+5) = 0$

$\Rightarrow M = \frac{1}{2}$ or $M = \frac{-5}{6}$

So, mass, $M = 1/2 = 0.5$ kg

13. (b) Given, frequency of particle oscillating in SHM, f = 1 Hz

At t = 0, phase = 1rad

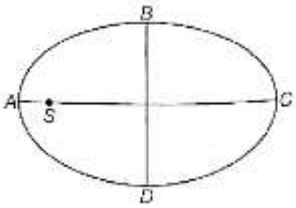
Time period $= \frac{1}{f} = \frac{1}{1} = 1$ s

So, in 1 s , phase changes by 2πrad.

or a phase change of 1 rad occurs in $\frac{1}{2\pi}$ s.

Hence, to make phase of particle zero, we have to shift time origin at $-\frac{1}{2\pi}$ s

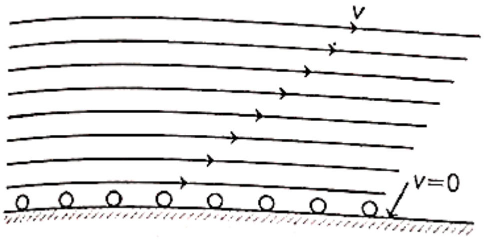
14. (a) Speed of planet is more, than when it is near to sun.



So, in arc DAB, it is moving with more speed than when it is in arc BCD.
$\therefore$ Time taken in travelling arc DAB is less than that for BCD.

15. **(b)** Gases have much larger intermolecular distances than those of solids and liquids .
So, compressibility of gases is very larger than solids or liquids. Except statement 2 all are correct.

16. **(b)** The given situation is shown below



Force of viscous drag is given by

$F = \eta A\left(\frac{dv}{dx}\right)$

where, dv/dx = velocity gradient.

In given case, $\frac{dv}{dx} = \frac{v}{H}$

So, shearing stress $= \frac{F}{A} = \eta\frac{dv}{dx} = \eta \cdot \frac{v}{H}$

17. **(b)** As both steel rod and metal scale expands at 25°C, we use

True value of length of rod at 25°C = Scale reading $\times \left(1 + \alpha(\theta' - \theta)\right)$ .... (i)

where, α = coefficient of linear expansion of scale ($= 20 \times 10^{-6} \rho^\circ C$)

θ' = temperature at which observation is taken

$(= 25^\circ C,)$

and θ = temperature at which metre scale reads correctly $(= 0^\circ C)$

Subtituting values in Eq. (i), we get Length of rod at 25°C

$= 1 \times [1 + 20 \times 10^{-6}(25 - 0)]$

$= 1 + 5 \times 10^{-4} = 1.0005$ m

Now, if l_2 = length of steel rod at 25°C and

l_1 = length of steel rod at 0°C, then we have

$l_2 = l_1(1 + \alpha\Delta\theta)$

Subtituting values, we have

$1.0005 \quad = l_1[1 + 12 \times 10^{-6} \times (25 - 0)]$

$\Rightarrow l_1 \quad = \frac{1.0005}{1.0003} = 1.000199$

≈ 1.0002 m

18. **(d)** Heat required to convert water at 100°C into steam at 100°C is

$Q = mL$

Where, m = mass = 100 kg and

L = latent heat

$= 22.6 \times 10^5$ J/kg

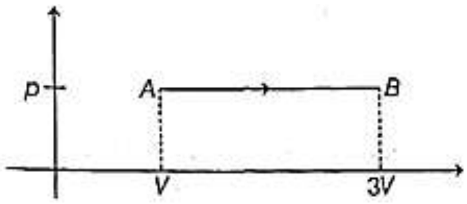
$Q \quad = 100 \times 22.6 \times 10^5$

$= 22.6 \times 10^7$ J

19. **(a)** Work done in a thermodynamic process

= Area under p − V graph

For process 1,



Work done, $W_1 = p\Delta V$
$= p(3V - V) = 2pV$
Let change of internal energy in process 1 (point A to point B) is U_1.
Then, by first law of thermodynamics, heat required is
Here, $\Delta Q_1 = \Delta U + \Delta W = U_1 + W_1$
$Q_1 = 5pV$(given
or $5pV = U_1 + 2pV$
Now for process 2 ,
$U_1 = 3pV$
As process 2 and process 1 lies between same isotherms.
Also, work done for process 2 is
$W_2 =$ Area under process 2

$$= \frac{1}{2} \times 2V \times \frac{1}{2}p + p \times 2V$$

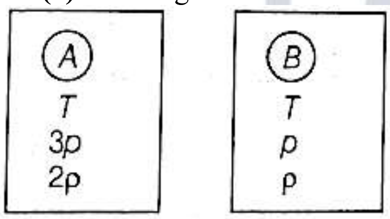
$$= \frac{5}{2}pV$$

Again, using first law of thermodynamis, heat required in process 2 is

$$\Delta Q_2 = \Delta U_2 + \Delta W_2$$

$$= 3pV + \frac{5}{2}pV = \frac{11}{2}pV$$

20. (a) We are given with following condition,



Let n_A and n_B are the number of moles of gases in vessel A and vessel B respectively, then by gas equation, we have $p_A V_A = n_A R T_A$ and $p_B V_B = n_B R T_B$ but given $T_A = T_B$

$$\Rightarrow \frac{p_A V_A}{n_A} = \frac{p_B V_B}{n_B}$$

$$\Rightarrow \frac{p_A V_A}{m_A/M_A} = \frac{p_B V_B}{m_B/M_B}$$

$$\left[\text{As, number of moles } = \frac{\text{mass of gas}}{\text{molar mass}}\right]$$

$$\Rightarrow \frac{p_A M_A}{\rho_A} = \frac{p_B M_B}{\rho_B}$$

$$\left[\because \frac{m}{V} = \rho = \text{density}\right]$$

Subtituting values, we have

$$\frac{3p \cdot M_A}{2\rho} = \frac{p \cdot M_B}{\rho}$$

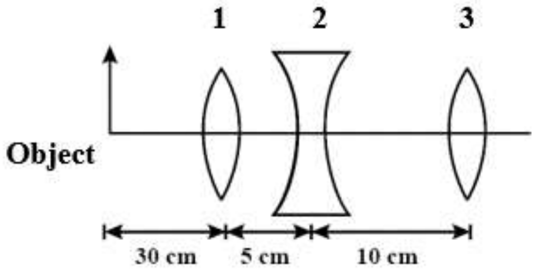
$$\Rightarrow \frac{M_A}{M_B} = \frac{2}{3}$$

21. (c) Resultant intensity when two plane waves interfere is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

Here, $I_1 = I_2 = I =$ intensity
and $\phi = 60^\circ =$ phase difference.
So, $I_P = I + I + 2\sqrt{I}\sqrt{I}\cos 60^\circ = 3I$

22.(d) Given, lens combination and position of object is as shown below



For first lens,
$u = -30$ cm, $f = 10$ cm
By lens formula,

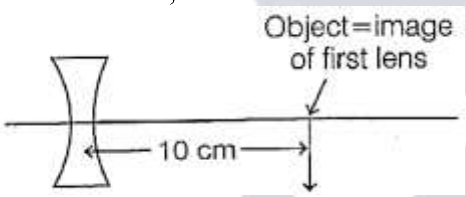
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{15} \text{ or } v = 15 \text{ cm}$$

Now, image of first lens acts like object for second lens.
For second lens,



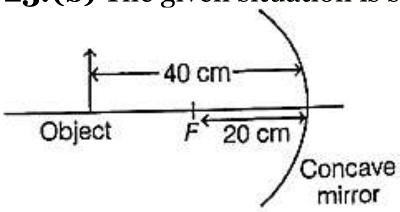
$u = +10$ cm, $f = -10$ cm
Again, using lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-10} + \frac{1}{10}$$

$$\Rightarrow \frac{1}{v} = 0 \text{ or } v = \infty$$

This means light rays becomes parallel after refraction from the second lens. When these parallel light rays passes through third lens, their image is formed at focus. Hence, distance of image from third lens is 30 cm .

23.(b) The given situation is shown below



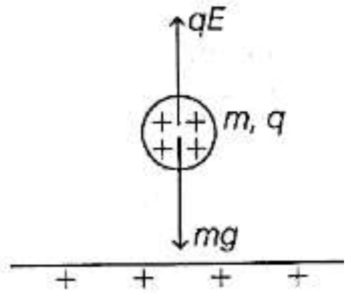
As, focal length, f= - 20 cm
Radius of curvature, R = 2f = -40 cm
This means object is at centre of curvature. So, its image is also formed at centre of curvature which is real, inverted and of same size.

24. (b) Intensity when one of wave is shifted by angle θ is
$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\theta$ .... (i)
Here, $I = 84$ mW, $I_1 = 64$ mW
and $I_2 = 4$ mW
Subtituting all values in Eq. (i), we have

$84 = 64 + 4 + 2\sqrt{64}\sqrt{4}\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

25. (a)



As particle hangs over the surface as shown above weight of particle = repulsive electrostatic force

$\Rightarrow mg = qE \Rightarrow mg = q\frac{\sigma}{\varepsilon_0}$

(As $E = \frac{\sigma}{\varepsilon_0}$, near a conducting surface)

Hence, $\sigma = \frac{\varepsilon_0 mg}{q}$

Here, $m = 2 \times 10^{-6}$ kg, $q = 5 \times 10^{-6}$C

$g = 10\ m/s^2$ and $\varepsilon_0 = 8.85 \times 10^{-12} C^2/Nm^2$

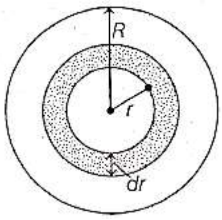
Hence, $\sigma = \frac{8.85 \times 10^{-12} \times 2 \times 10^{-6} \times 10}{5 \times 10^{-6}}$

$= 35.4 \times 10^{-12} C/m^2$

26. (b)

27. (b) Heat produced in a section of width dr at a distance r from centre is

$H = \int_0^R V \cdot t \cdot j \cdot 2\pi r dr$



Here, $V = 50$ V, $t = 100$ s

$j = 2 \times 10^{10} r^2, R = 2 \times 10^{-3}$ m

So, $H = \int_0^R 50 \times 100 \times 2 \times 10^{10} r^2 \times 2\pi r \times dr$

$= \int_0^R 2\pi \times 10^{14} \cdot r^3 \cdot dr$

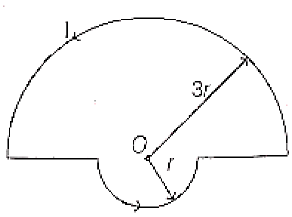
$= 2\pi \times 10^{14} \times \left[\frac{r^4}{4}\right]_0^R$

$= \frac{\pi}{2} \times 10^{14} \times (2 \times 10^{-3})^4$

$= 8 \times \pi \times 10^{14} \times 10^{-12} = 800\pi$ J

28. (a)

29. (b) Suppose, current I is flowing each of the circuit



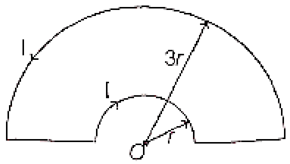
Case I

Net magnetic field at centre is

$B_1 = B_{\text{upper part}} + B_{\text{lower part}}$

$$= \left|\frac{\mu_0 I}{4(3r)} + \frac{\mu_0 I}{4(r)}\right|$$

$$= \left|\frac{4\mu_0 I}{12r}\right| = \left|\frac{\mu_0 I}{3r}\right| = \frac{\mu_0 I}{3r}$$



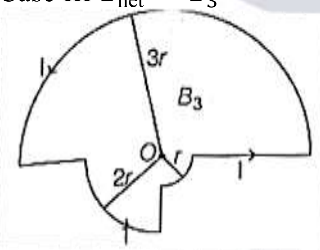
Case II

$B_{\text{net}} \quad = B_2 = \left|B_{\text{upper part}} - B_{\text{lower part}}\right|$

$$= \left|\frac{\mu_0 I}{4(3r)} - \frac{\mu_0 I}{4(r)}\right| = \left|\frac{-2\mu_0 I}{12r}\right|$$

$$= \left|\frac{-\mu_0 I}{6r}\right| = \mu_0 I/6r$$

Case III $B_{\text{net}} = B_3$



$$= \left|B_{\text{upper part}} + B_{\text{ower part}} + B_{\text{smallest part}}\right|$$

$$= \left|\frac{\mu_0 I}{4(3r)} + \frac{\mu_0 I}{8(2r)} + \frac{\mu_0 I}{8r}\right| = \frac{13}{48}\frac{\mu_0 I}{r}$$

Clearly, $B_1 > B_3 > B_2$

30. **(b)** As magnetic force is perpendicuar to velocity, so work done by force is zero. So, there is no change in magnitude of velocity and hence kinetic energy remains constant. But force produces a change in direction of velocity and hence momentum changes.

31. **(b)** Let magnetic dipole moment of given magnet is M. Then, its oscillation time period is

$$T = 2\pi\sqrt{\frac{I}{MB}} = 2\text{ s (given}\Bigg)$$

where, l = moment of inertia $= \frac{\omega l^2}{12}$

and M = magnetic dipole moment of magnet.

When magnet is cut into 3 parts, each part length is $I/3$ and mass of each part will be $\omega/3$.

So, moment of inertia of each part will be

$$I' = \frac{\omega' l^2}{12} = \frac{\frac{\omega}{3} \times \frac{l^2}{9}}{12} = \frac{1}{27} \times I$$

Also, magnetic dipole moment of each part will be

$$M' = \frac{M}{3}$$

Now, when these parts are placed one over other with like poles together, Moment of inertia of combination

$= I_1 = 3 \times \frac{1}{27} I = \frac{1}{9} I$

Also, dipole moment of combination is

$M_1 = 3 \times \frac{M}{3} = M$

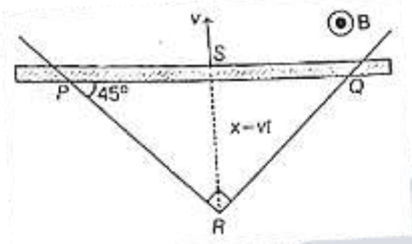
Hence, time period of oscillation of combination is

$$T = 2\pi \sqrt{\frac{I_1}{M_1 B}} = 2\pi \sqrt{\frac{I/9}{MB}}$$

$$= \frac{1}{3}\left(2\pi \sqrt{\frac{I}{MB}}\right)$$

$$= \frac{1}{3} \times 2 = \frac{2}{3} \text{ s}$$

32. (a) The given situation is shown below



For triangle PQR, $x = v \times t$

So, at $t = 4$ s,

$x = 5 \times 4 = 20$ m

Now in $\triangle$ PRS,

$$\frac{SR}{SP} = \tan 45^\circ$$

$\Rightarrow SR = SP \; (\because \tan 45^\circ = 1)$

or length PQ of rod is $2 \times SR = 2x = 40$ m

i.e., area swept in 4 s = area $\triangle$ PQR

$$= \frac{1}{2} \times \text{ base } \times \text{ height}$$

$$= \frac{1}{2} \times 2x \times x$$

$$= x^2 = (20)^2 = 400 \text{ m}^2$$

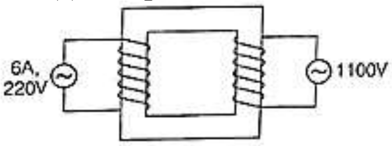
Area swept in 1 s

$$= \frac{dA}{dt} = \frac{400}{4} = 100 \text{ m/s}^2$$

So, magnitude of emf induced in triangular loop is

$$E = \frac{d\phi}{dt} = B\left(\frac{dA}{dt}\right) = 0.1 \times 100 = 10 \text{ V}$$

33. (c) The given situation is shown below



For given transformer, 40% of power is lost.

So, power output $= \frac{60}{100} \times$ power input

$$\Rightarrow 1100 \times I_s = \frac{60}{100} \times 220 \times 6$$

$$\Rightarrow I_s = \frac{6 \times 22 \times 6}{1100} = 0.72 \text{ A}$$

∴ None of the option is matching. But if we take output power = 40% of input power, then answer will be 0.48 A .

34. (b) Given, waves are

$$y_1 = a\sin(\omega t - kx)$$

$$y_2 = b\cos\left(\omega t - kx + \frac{\pi}{3}\right)$$

As $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

$$\Rightarrow\ y_2 = b\sin\left(\frac{\pi}{2} + \omega t - kx + \frac{\pi}{3}\right)$$

$$\Rightarrow\ y_2 = b\sin\left(\omega t - kx + \frac{5\pi}{6}\right)$$

So, phase difference of y_1 and y_2 is $\frac{5\pi}{6}$ rad.

35. (c) Maximum velocity of emitted photoelectrons is

$v_{max} = 1.6 \times 10^6\ ms^{-1}$

Maximum kinetic energy of emitted photoelectrons is

$$K_{max} = \frac{1}{2} m v_{max}^2$$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.6 \times 10^6)^2\ J$$

$$= \frac{91 \times 10^{-31} \times (1.6 \times 10^6)^2}{2 \times 1.6 \times 10^{-19}} eV$$

$$= 7.2 eV$$

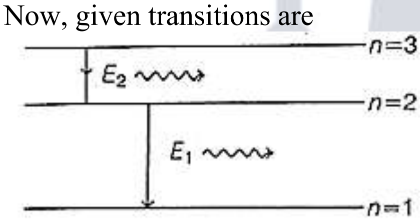
So, work function = incident energy maximum kinetic energy of electron

$= 10.5 - 7.2 = 3.3 eV$

36. (d) Energy value of nth state for H -electron is

$$E_n = \frac{E_1}{n^2}$$

Now, given transitions are



Energy radiated in transition $n = 3$ to $n = 2$ is

$$E_2 = \frac{E_1}{9} - \frac{E_1}{4} = \frac{-5E_1}{36}$$

Also, energy radiated in transition

$n = 2$ to $n = 1$ is

$$E_1 = \frac{E_1}{4} - \frac{E_1}{1} = \frac{-3E_1}{4}$$

Hence, ratio of $\frac{E_2}{E_1}$ is $\frac{5}{27}$.

37. (a) Given, mass of ${}^{29}_{14}Si = 28.976495u$

Now, mass of 14 protons

$= 14 \times$ mass of 1 proton

$= 14 \times 1.007276 = 14.101864u$

Also, mass of $29 - 14 = 15$ neutrons $= 15 \times$ mass of 1 neutron

$= 15 \times 1.008664 = 15.12996$

Total mass of constituents of Si atom

$= 14.101864 + 15.12996$

$= 29.231824$

So, mass defect = mass of constituents - mass of atom

$= 29.231824 - 28.976495$

$= 0.255329u$

Binding energy = Energy equivalent of mass defect

$= \Delta M \times 931.5 \text{MeV}$
$= 0.255329 \times 931.5 = 237.84 \text{MeV}$

38. (c) In an n-type semiconductor, majority carriers are electrons. Only statement 3 is incorrect.

39. (d) Copper is a metal and germanium is a semiconductor.
On cooling resistance of copper decreases as atomic vibrations decreases and causes a reduction in electronic collisions. Whereas on cooling germanium, number of electrons and holes decreases and hence its resistance increases.

40. (b) Modulation index,

$$m = \frac{\text{peak value message signal}}{\text{peak value of carrier signal}} = \frac{V_2}{V_1}$$

Lower side band (LSB) and upper side band (USB) frequencies are

$v_+ = v_1 + v_2 \quad \ldots. \text{(i)}$

And $v_- = v_1 - v_2 \ldots.. \text{(ii)}$

Adding Eq. (i) and Eq. (ii), we get

$$v_1 = \frac{v_+ + v_-}{2} \text{ and } v_2 = \frac{v_+ - v_-}{2}$$

Chemistry

41. (d) Series of spectral lines

(1) Lyman series ($n' = 1$)
• The series is named after its discoverer Theodore Lyman.
• Lyman series lies in the ultraviolet region.
(2) Balmer formula ($n' = 2$)
• The series is named after its discoverer Johann Balmer.
• It is technically visible part of the spectrum with wavelength longer than 400 nm and shorter than 700 nm .
(3) Paschen series ($n' = 3$)
• Named after the German physicist-Friedrich Paschen
• The paschen lines all lie in the infrared band.
(4) Brackett series ($n' = 4$)
• Named after the American physicist Frederick Sumner Brackett.
• The spectral lines of Brackett series lie infrared band.
(5) Pfund series ($n' = 5$)
• Discovered by August Herman Pfund.
• Lie in infrared region.
• Humphreys series ($n' = 6$)
• American physicist Curtis J. Humphreys.
The spectral lines of Humphreys series lie in infrared region.

42. (c) Quantisation is the process of constraining an input from a continuous or otherwise large set of values to a discrete set. So, a person can stand on any step of a stair case can be compared to the concept of quantisation.

43. (c) Element with atomic number 123.

Number	Root
0	nil
1	un
2	bi
3	tri
4	quad
5	pent
6	hex
7	sept
8	oct

9	enn

The roots are put together in the order of the digits, which make up the atomic number and terminated by 'ium' to spell out the name. The symbol of the element is composed of the initial letter of the numerical routs, which make up the name. Hence, symbol for atomic number 123 is Ubt and IUPAC name is unbitrium.

44. (c) Enthalpy of atomisation is the energy required to break off atoms, which are joined together. From top to bottom in a group, the atomic radii increases due to addition of extra electron shells. Atoms with larger radii attract each other more

10 weakly than atoms with smaller radii, so it is easier to separate the larger size atoms by the less input of energy and hence on moving down a group (s-block and p-block) have low atomisation enthalpy. 1uy. But in the case of d-block elements enthalpy of atomisation increases on moving down a group due to presence of lanthanide and actinides contractions

45. (b) Ionic character in a molecule increases with increasing electronegativety difference in bonded atoms.

$$\underbrace{LiF > K_2O > SO_2}_{\substack{\text{Polar}\\\text{(Ionic bond)}}} > \underbrace{N_2}_{\substack{\text{Non-polar}\\\text{(Covalent bond)}}}$$

46. (d) Bond order

$$= \frac{\text{bonding electrons} - \text{antibonding electrons}}{2}$$

Molecules	Bond order
He_2	$\frac{2-2}{2}=0$
He_2^+	$\frac{10-1}{2}=0.5$
O_2	$\frac{10-6}{2}=2$
O_2^+	$\frac{10-5}{2}=2.5$

47. (b) From gas equation, $pV = nRT$

Given,

$V = 12$ L

$n = 1$

$R = 0.082l$ L atm/mol/K

$T = 297\,^\circ C + 273 = 570$ K

$$p = \frac{nRT}{V} = \frac{1 \times 0.821 \times 570}{12}$$

$$= 38.99 \text{ atm} (1 \text{ atm} = 101325 \text{ Pa})$$

$$= 395066.18 \text{ Pa} = 395 \text{kPa}$$

48. (d) 4.4 g of a gas at STP($0^\circ C$) occupies a volume of 2.73 L .

At STP, 1 mole of any gas occupies a volume of 24.4 L .

4.4 g of a gas at STP occupies a volume of 2.73 L .

Hence, the number of moles of gas

$$= \frac{22.4}{2.73} = 0.1 \text{ mol}$$

0.1 mole of a gas = 4.4 g

1 mole of a gas $= 4.4 \times \frac{1}{0.1} = 44$ g

Thus, the molecular weight of gas is 44 g/mol. It is the molecular weight of CO_2gas.

49. (d) $N_2 + 3H_2 \longrightarrow 2NH_3$ or

$$N_2 + 5H_2 \longrightarrow xNH_3$$

$$\frac{3\ \text{molH}}{2} = \frac{5\ \text{molH}_2}{x\ \text{molNH}_3}$$

$3 \times x = 5 \times 2 = x = \frac{10}{3} = 3.3$ mol

50. (c)

$$\left(C_2H_4(g) + H_2(g) \overset{\text{Cataylst}}{\rightarrow} C_2H_6(g)\right.$$

$$\frac{1gC_2H_4}{1gC_2H_6} = \frac{x\ gC_2H_4}{50\ gC_2H_6}$$

$$\frac{28}{30} = \frac{x}{50}$$

$$\begin{bmatrix} 28 = \text{ molar mass of } C_2H_4 \\ 30 = \text{ molar mass of } C_2H_6 \end{bmatrix}$$

$$28 \times 50 = 30x \Rightarrow x = \frac{28 \times 50}{30}$$

$$= 46.67\ g$$

51. (a) For solids and liquids, volume changes are insignificate.

$\therefore \quad \Delta V = 0$

$\Delta H = \Delta U + p\Delta V \Rightarrow \Delta H = \Delta U$

Therefore, the difference between ΔH and ΔU is almost negligible for solids and liquids.

52. (a) Given, K_b (ammonium hydroxide)

$= 5.00 \times 10^{-10}$

$K_w = 10^{-14}$ at 298 K

$K_h = \frac{K_w}{K_b} = \frac{10^{-14}}{5\times10^{-10}}$

$= 2 \times 10^{-5}$

53. (b) $2A(g) \rightleftharpoons 2B(g) + C(g)$

$K_p = K_C \cdot (RT)^{\Delta n_g}$

Given

$$\left.\begin{aligned} K_C &= 4 \times 10^{-4} \\ T &= 1000\ K \\ \Delta n_g &= \text{moles of product} - \text{moles of reactant} \end{aligned}\right\}$$

$= 3 - 2 = 1$

$K_p \quad = 4 \times 10^{-4}(8.314 \times 800)^1$

$K_p \quad = 0.026$

54. (a) Hydrogen peroxide can easily break down, or decompose, into water and oxygen. There are no other molecules to react with, the parts will form water and oxygen gas as these are more stable than the original molecule H_2O_2.

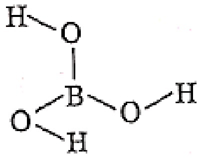
$2H_2O_2 \longrightarrow 2H_2O + O_2$

55. (c) The word slaking means the addition of water to calcium oxide powder (lime). The resulting product is calcium hydroxide. (milk of lime)

$CaO + H_2O \longrightarrow Ca(OH)_2$

The reaction is exothermic, so the mixture heats up. Slaking is a key step in the most widely used procedure for formation of precipitated calcium carbonate.

56. (a) Boric acid (H_3BO_3) also called hydrogen borate. Boric acid and orthoboric acid is a weak, monobasic Lewis acid of boron.

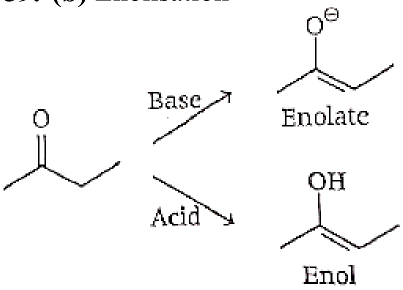


However, some of its behaviour towards some chemical reactions suggest, it to be tribasic acid in the Bronsted sense as well.

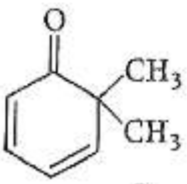
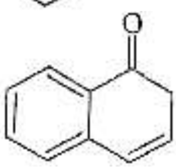
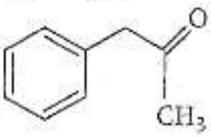
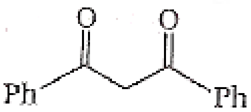
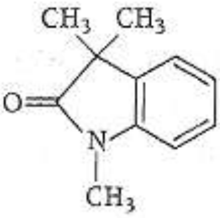
57. **(b)** In CO_2molecule, carbon atom has two effective pairs or two double bonds exist in it.
The molecule can be represented as $O = C = O$, In CO_2 molecule, the hybridisation of carbon is sp and oxygen sp^2.

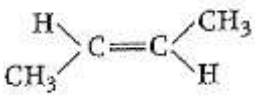
58. **(b)** Global warming stresses ecosystems through temperature rises, water shortages, increased fire threats, drought weed and pest invasion, intense storm damage and salt invasion. Just to name a few, temperature rises is responsible to melting of Himalayan glaciers.

59. **(b)** Enolisation



Organic esters, ketons, and aldehydes with an α-hydrogen.
In the case of ketones, the conversion is called a keto-enol tautomerism,

Molecule	Enolisation
(i) 	no
(ii) 	yes
(iii) 	yes
(iv) 	yes
(v) 	no

60. **(d)** $H_3C - C \equiv C - CH_3 \xrightarrow[Liq.NH_3]{Na}$ 

trans 2-butene
(Birch reduction)

61. **(b)**

62. (b) $LiCoO_2$ crystallises in a rhomobohedral structure. Let at initial 100 atoms are present of both Li and Co and after 50%Li is extracted only 50 atoms of Li is remaining. But total charge on the crystal is converted so cobalt have to increase its oxidation state. So,

$$100\text{Li} + 100\text{Co} = 400$$
$$50\text{Li} + 100\text{Co}^* = 400$$
$$100\text{Co}^* = 350$$
$$\text{Co}^* = 3.5$$

So, after extraction of 50%Li atom new oxidation state of Co to 3.5 .

Change in oxidation state = final oxidation state - initial oxidation state

$= 3.5 - 3$

Change in average oxidation state of Co $= 0.5\%$ of change in average oxidation state of

Co $= \frac{0.5}{3} \times 100 = 16.66\%$ (increases)

63. (c)

64. (d)

65. (b)

66. (c) Activation energy,

$$\log\frac{K_1}{K_2} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$
$$\log 2 = \frac{E_a}{2.303 \times 8.3}\left(\frac{10}{295 \times 305}\right)$$
$$0.3010 = \frac{E_a}{2.303 \times 8.3}\left(\frac{10}{295 \times 305}\right)$$
$$\frac{0.3031 \times 2.303 \times 8.3 \times 295 \times 305}{10} = E_a$$

$E_a = 51.76\ \text{kJ mol}^{-1}$ (nearest value to $49.8\ \text{kJ mol}^{-1}$)

67. (b) We know, $\frac{x}{m} = Kp^{1/n}$ or $\log\frac{x}{m}\log K + \frac{1}{n}\log p$

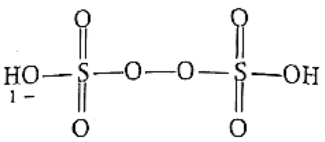
$\therefore$ Plot of $\log\frac{x}{m}$ vs $\log p$ is linear with slope $= \frac{1}{n}$ and intercept $= \log K$ Hence, $\frac{1}{n} = \tan\theta = \tan 45^\circ = 1$, i.e. $n = 1$ and $\log K = 0.3 = \text{antilog}(0.3) = 2$

At $p = 1$ atm, $\frac{x}{m} = Kp^{1/n} = 2 \times (1)^{1/1} = 2$

68. (c) Ellingham diagram is a graph is drawn between change in free energy and temperature. The slope of the curve is the entropy and the intercept represents the enthalpy.

69. (a) A common nitrate test known as the brown ring test can be performed by adding iron sulphate to a solution of a nitrate, then slowly adding concentrated sulphuric acid such that the acid forms a longer below the aqueous solution. A brown ring will form at the junction of the two layers, indicating the presence of the nitrate ion.

70. (b)



(Marshall's acid)

$$2x + 4(-1) + 4(-2) = 0$$
$$\Rightarrow \quad 2x - 4 - 8 = 0$$
$$2x = 12 \Rightarrow x = +6$$

71. (a) Paramagnetic properties are due to the realignment of the electron path caused by the external magnetic field.

Paramagnetic nature $\propto$ number of unpaired electron

Metalion	d -electrons	Number of unpair e^-
Cu^{2+}	↑↓ ↑↓ ↑↓ ↑ ↑	2
V^{2+}	↑ ↑ ↑ _ _	3

Cr^{2+}	1 1 1 1	4
Mn^{2+}	1 1 1 1 1	5

So increasing order of paramagnetic nature
$Cu^{2+} < V^{2+} < Cr^{2+} < Mn^{2+}$

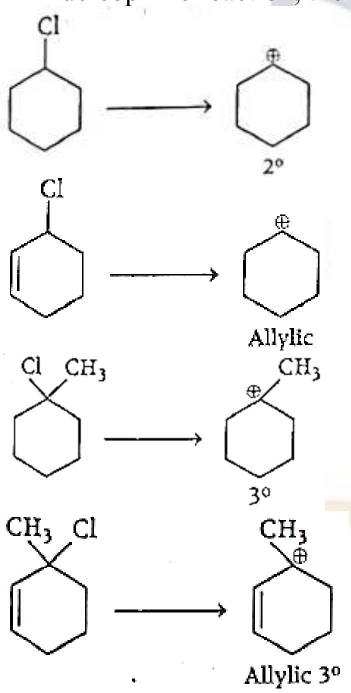
72. (c) Coordination complex
$= [Co(OH_2)_6]^{2+}$
- Symmetry of complex is octahedral.
- H_2O is weak ligand, so it form outer orbital complexes.
- Complex is paramagnetic in nature due to presence of unnaired electrons.

73. (a) Step-growth polymerisation It refers to a type of polymerisation mechanism in which bi-functional or multifunctional monomers react to form first dimers, then trimers, longer oligomers and eventually long chain polymers. Nylon-6,6 is type of polyamide or nylon. It and nylon-6, are the most common for textile and plastic industries. It is formed by step-growth polymerisation by polycondensation of hexamethylene diamine and adipic acid.
$n(HOOC - (CH_2)_4 - COOH)$

74. (b) The pancreas secretes insulin and glucogen. Both hormones work in balance to play a vital role in regulating blood sugar levels. If the level of one hormone is higher or lower than the ideal range, blood sugar levels may spike or droop.

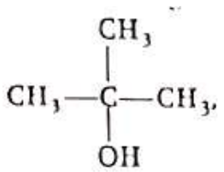
75. (d) A type of detergent in which the active part of the molecule is a positive ion (cation) is known as cationic detergents are usually quaternary ammonium salts and often also have bactericidal properties. Cationic-detergents contains a long- chain cations that is responsible for their surface-active properties. Cationic detergents also possess excellent germicidal properties and are utilised in surgery in dilute form, it is not cheap and widely used.

76. (d) Reactivity of alkyl halides towards S_N1 nucleophilic substitution reaction is $3° > 2° > 1°$ because in S_N 1 nucleophilic reaction, the first and the slow step is the formation of a carboaction.

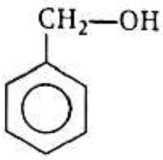


Reactivity order (decreasing order) allylic $3° > 2°$ allylic $> 3° > 2°$

77. (c) Lucas reagent react with alcohols used to differentiate between primary, secondary and tertiary alcohols. Tertiary alcohols react immediately with Lucas reagent as evidenced by turbidity owing to the low solubility of the organic chloride in the aqueous mixture.
(i) $\underset{\text{n-butanol}}{CH_3 - CH_2 - CH_2 - CH_2 - OH}$,
1° alcohol-not give reaction

(ii) 

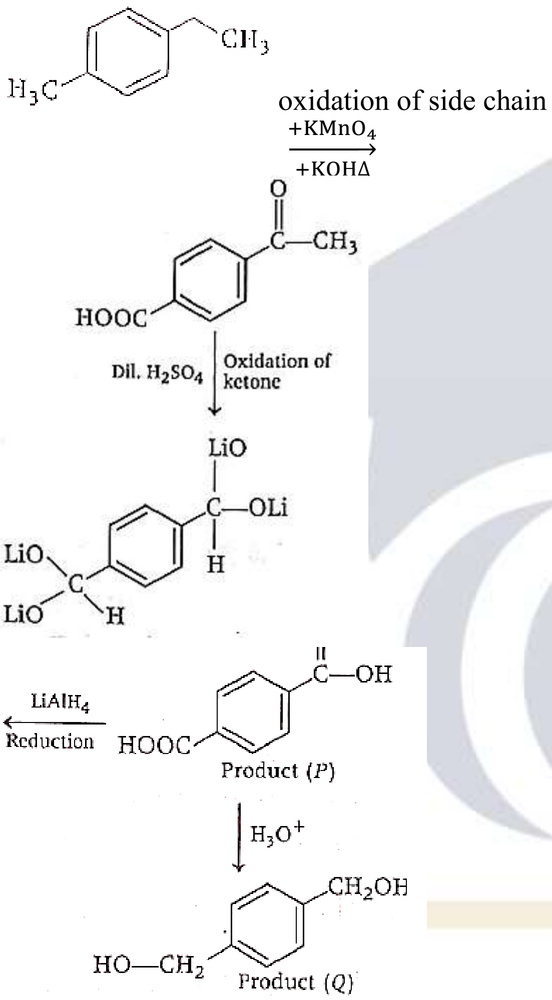
3° alcohol-white turbidity

(iii)  , 1° alcohol benzylic cation - white turbidity

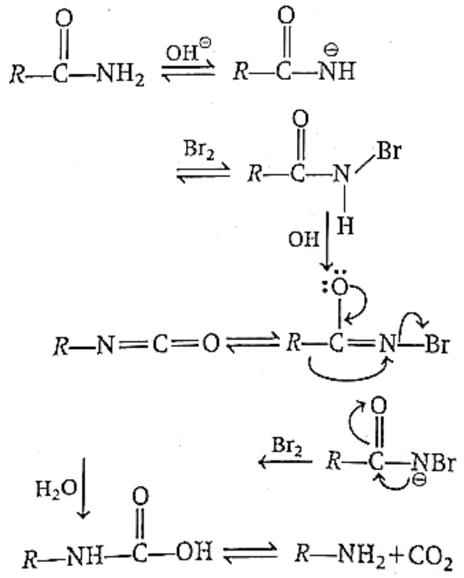
(iv) $CH_3 = CH - CH_2 - OH \cdot 1°$ alcohol allylic cation-white turbidity.

78. (b)

79. (a)



80. (c) Hofmann degradation reaction It is the organic reaction of a primary amide to a primary amine with one fewer carbon atom. The reaction is named after its discoverer August Wilhelm won Hofmann.



Mathematics

81. (a) (A) We know that,

$-1 \le \cos x \le 1$

$\Rightarrow \quad 0 \le \cos^2 x \le 1$

$\Rightarrow \quad 1 \le 1 + \cos^2 x \le 2$

$\therefore \quad [1 + \cos^2 x] = \{1, 2\}$

So, range of

$\sec^{-1}[1 + \cos^2 x] = \{\sec^{-1} 1, \sec^{-1} 2\}$

(A) → (V)

(B) We have,

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

$$f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2$$

Put $y = x + \frac{1}{x}$

$\therefore \; f(y) = y^2 - 2$

Now, AM ≥ GM

Case II When $x < 0$,

$$\frac{-|x| + \frac{1}{-|x|}}{2} \ge \sqrt{(-|x|) \times \frac{1}{-|x|}}$$

$$\Rightarrow \; -\left(|x| + \frac{1}{|x|}\right) \ge 2 \Rightarrow |x| + \frac{1}{|x|} \le -2$$

$$\therefore \; x + \frac{1}{x} \in \mathbf{R}$$

$R - (-2, 2)$

So, domain of $f(x)$ is $R - (-2, 2)$

(B) → (IV)

(C) We have

$f(x + y) = f(x) + f(y)$

Put $x = y = 0$, we get

$f(0) = f(0) + f(0)$

$\Rightarrow \quad f(0) = 0$

Now, put $y = -x$, we get

$$f(x-x) = f(x)+f(-x)$$
$$\Rightarrow f(0) = f(x)+f(-x)$$
$$\Rightarrow f(x)+f(-x) = 0 \ [\because f(0)=0]$$
$$\Rightarrow f(-x) = -f(x)$$

So, f(x) is a odd function.

(C) → I

D. We have

$$\sin^{-1}x-\cos^{-1}x+\sin^{-1}(1-x)=0$$
$$\Rightarrow \sin^{-1}(1-x)=\cos^{-1}x-\sin^{-1}x$$
$$\Rightarrow \sin^{-1}(1-x)=\frac{\pi}{2}-\sin^{-1}x-\sin^{-1}x$$
$$\Rightarrow \sin^{-1}(1-x)=\frac{\pi}{2}-2\sin^{-1}x$$
$$\Rightarrow 1-x=\sin\left(\frac{\pi}{2}-2\sin^{-1}x\right)$$
$$\Rightarrow 1-x=\cos(2\sin^{-1}x)$$
$$\Rightarrow 1-x=\cos\left(\sin^{-1}2x\sqrt{1-x^2}\right)$$
$$\Rightarrow 1-x=\cos\cos^{-1}(1-2x^2)$$
$$\Rightarrow 1-x=1-2x^2$$
$$\Rightarrow 2x^2-x=0 \Rightarrow x(2x-1)=0$$
$$\Rightarrow x=0,1/2$$
$$\therefore x\in\left\{0,\frac{1}{2}\right\}$$

(D) → II

82. (d) We have,

$$f(x) = \sec^{-1}(3x-4)+\tanh^{-1}\left(\frac{x+3}{5}\right)$$
$$= \sec^{-1}(3x-4)+\frac{1}{2}\ln\left(\frac{1+\frac{x+3}{5}}{1-\frac{x+3}{5}}\right)$$
$$= \sec^{-1}(3x-4)+\frac{1}{2}\ln\left(\frac{8+x}{2-x}\right)$$

Now, for $\sec^{-1}(3x-4)$

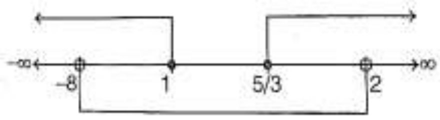
$$3x-4\le -1 \cup 3x-4\ge 1$$
$$\Rightarrow 3x\le 3\cup 3x\ge 5$$
$$\Rightarrow x\le 1\cup x\ge\frac{5}{3}$$
$$\therefore x\in(-\infty,1]\cup x\in\left[\frac{5}{3},\infty\right)$$

Again, for $\ln\left(\frac{8+x}{2-x}\right)$

$$\frac{8+x}{2-x}>0$$

$x\in(-8,2)$

$\therefore$ Domain of $f(x)=(-8,1]\cup\left[\frac{5}{3},2\right)$



83. (a) We have,

$m = (9n^2 + 54n + 80)(9n^2 + 45n + 54)$
$(9n^2 + 36n + 35)$
$= [(9n^2 + 30n + 24n + 80)]$
$[(9n^2 + 27n + 18n + 54)]$
$= [(3n(3n + 10) + 8(3n + 10)]$
$[9n(n + 3) + 18(n + 3)]$
$[3n(3n + 5) + 7(3n + 5)]$
$= (3n + 10)(3n + 8)(n + 3)(9n + 18)$
$(3n + 5)(3n + 7)$
$=$
$(3n + 5)(3n + 6)(3n + 7)(3n + 8)$
$(3n + 9)(3n + 10)$

Now, we know that $n(n + 1)(n + 2) \dots (n + r - 1)$ is divisible by r !
$\therefore$ m is divisible by 6! i.e, 720 .

84. (b) We have,

$$(\mathrm{adj}(A))^T = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -5 & -2 \\ -2 & 4 & 1 \end{bmatrix})$$

$$(\mathrm{adj}(A) = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -5 & -2 \\ -2 & 4 & 1 \end{bmatrix})$$

$$\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$$

$$(\mathrm{adj}(A)| = \begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix})$$

$= 1(-5 + 8) - 4(-2 - 4) - 2(+4 + 5)$
$= 3 + 24 - 18 = 9$

Now, we know that

$$|\mathrm{adj}\, A| = |A|^{n-1}$$

$\Rightarrow \quad |A|^{3-1} = 9$
$\Rightarrow \quad |A|^2 = 9$
$\Rightarrow \quad |A| = \pm 3$

85. (a) We have,

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$\therefore A^2 = A \cdot A$

$$= \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 - 6 + 0 & -9 + 9 - 4 & 12 - 12 + 4 \\ 6 - 6 + 0 & -6 + 9 - 4 & 8 - 12 + 4 \\ 0 - 2 + 0 & 0 + 3 - 1 & 0 - 4 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

Now,

Nov,

$|A^2| = 3(3 - 0) + 4(0) + 4(0 - 2) = 1$

Now, again cofactors of A^2 are given

by $= C_{11} = 3, C_{12} = 0, C_{13} = -2,$
$C_{21} = -4, C_{22} = -1,$
$C_{23} = 2, C_{31} = -4, C_{32} = 0, C_{33} = -3$
adj

$$(A^2) = \begin{bmatrix} 3 & 0 & -2 \\ -4 & -1 & 2 \\ -4 & 0 & -3 \end{bmatrix}^T = \begin{bmatrix} 3 & -4 & -4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$\therefore (A^2)^{-1} = \frac{1}{|A^2|} \text{ adj}$$

$$(A^2) = \frac{1}{1}\begin{bmatrix} 3 & -4 & -4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} = A^2$$

$$\therefore (A^2)^{-1} = A^2$$

86. (c) We have, $x + y + z = 3$
$x + 2y + 2z = 6$
$x + ay + 3z = b$
Above equations can be written as
$AX = B$
Where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & a & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 3 \\ 6 \\ b \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & a & 3 \end{vmatrix}$$

$$= 1[6 - 2a] - 1[3 - 2] + 1[a - 2]$$
$$= 6 - 2a - 1 + a - 2 = 3 - a$$

For unique solution
$|A| \neq 0 \Rightarrow 3 - a \neq 0$
$\Rightarrow a \neq 3$
$\therefore$ Given system have unique, solution when $a \neq 3$, b can have any value.

87. (a) We have,

$$\frac{1 + iz}{1 - iz} = \frac{1 + i\frac{(b + ic)}{1 + a}}{1 - i\left(\frac{b + ic}{1 + a}\right)}$$

$$= \frac{1 + a + ib - c}{1 + a - ib + c}$$

$$= \frac{(1 + a - c) + ib}{(1 + a + c) - ib}$$

$$= \frac{(1 + a - c) + ib}{(1 + a + c) - ib} \times \frac{(1 + a + c) + ib}{(1 + a + c) + ib}$$

$$= \frac{(1 + a - c)(1 + a + c) + (1 + a - c)bi + (1 + a + c)bi - b^2}{(1 + a + c)^2 + b^2}$$

$$= \frac{\begin{array}{l}(1+a+c+a+a^2\\ +ac-c-ac-c^2-b^2)\\ +ib(1+a-c+1+a+c)\end{array}}{(1+a+c)^2+b^2}$$

$$= \frac{1+2a+a^2-(1-a^2)+2(1+a)ib}{1+a^2+c^2+2a+2ac+2c+b^2}$$

$$= \frac{2a(1+a)+2(1+a)ib}{2+2a+2ac+2c}$$

$$= \frac{2(1+a)(a+ib)}{2(1+a)+2c(1+a)}$$

$$= \frac{2(1+a)(a+ib)}{2(1+a)(1+c)} = \frac{a+ib}{1+c}$$

88. **(d)** If $z = x + iy$, then $\bar{z} = x - iy$,

As, $\frac{\bar{z}-1}{z-i} = 2$

$\Rightarrow \left|\frac{\bar{z}-1}{z-i}\right| = 2$

$\Rightarrow (x-1)^2 + y^2 = 4[x^2 + (y-1)^2]$

$\Rightarrow 3x^2 + 3y^2 + 2x - 8y + 3 = 0$

$\Rightarrow x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + 1 = 0$

So, the locus of point $P(x, y)$ in the cartesian plane is a circle of radius $\frac{2}{3}\sqrt{2}$ and the centre $\left(-\frac{1}{3}, \frac{2}{3}\right)$.

89. **(d)** We have,

$(1-\omega+\omega^2)^6 + (1-\omega^2+\omega)^6$

$= (-\omega-\omega)^6 + (-\omega^2-\omega^2)^6 \quad [1+\omega+\omega^2 = 0$

$= (-2\omega)^6 + (-2\omega^2)^6$

$= 64\omega^6 + 64\omega^{12} = 64(\omega^3)^2 + 64(\omega^3)^4$

$= 64(1)^2 + 64(1)^4 \; [\because \omega^3 = 1]$

$= 64 + 64 = 128$

90. **(c)**

91. **(d)** The quadratic expression $ax^2 + bx + c$ and a will have the same sign for some value of $x \in R$ when $b^2 - 4ac > 0$

And the quadratic inequality

$3x^2 - 16x + 4 > -16$

$\Rightarrow 3x^2 - 16x + 20 > 0$ for some value of $x \in R$ as discriminant

$D = 16^2 - 4(3)(20) = 256 - 240 = 16 > 0$

Since, $3x^2 - 16x + 20 > 0$

$\Rightarrow 3x^2 - 6x - 10x + 20 > 0$

$\Rightarrow 3x(x-2) - 10(x-2) > 0$

$\Rightarrow (x-2)(3x-10) > 0$

$\Rightarrow x \in (-\infty, 2) \cup \left(\frac{10}{3}, \infty\right)$

$\therefore$ (A) is false but (R) is true.

92. **(c)** We have,

Equation $ax^2 + bx + c = 0$ has

imaginary roots

$\therefore\; b^2 - 4ac < 0 \Rightarrow b^2 < 4ac$

Now,

$$f(x) = 3a^2x^2 + 6abx + 2b^2$$
$$= 3a^2\left(x^2 + \frac{2b}{a}x\right) + 2b^2$$
$$= 3a^2\left[\left(x + \frac{b}{a}\right)^2 - \frac{b^2}{a^2}\right] + 2b^2$$
$$= 3a^2\left(x + \frac{b}{a}\right)^2 - 3b^2 + 2b^2$$
$$= 3a^2\left(x + \frac{b}{a}\right)^2 - b^2$$

$\therefore$ Minimum value of f(x) is

$> -4ac - b^2$ [$\because b^2 < 4ac$]

93. (b) We have, $x^3 + px + q = 0$

Since, α, β, γ are the roots of the given equation

$\therefore \quad S_1 = \Sigma\alpha = 0$

$S_2 = \Sigma\alpha\beta = p$

$S_3 = \alpha\beta\gamma = -q$

Now, $\Sigma\alpha^2\beta = S_1S_2 - 3S_3$

$= (0)(p) - 3(-q) = 3q$

Again, $x^3 + px + q = 0$

$\Rightarrow x^3 = -[px + q]$

$\Rightarrow x^4 = -[px^2 + qx]$

$\therefore \Sigma\alpha^4 = -[p\Sigma\alpha^2 + q\Sigma\alpha]$

$= -[p((\Sigma\alpha)^2 - 2(\Sigma\alpha\beta)) + q\Sigma\alpha]$

$= -[p(S_1^2 - 2S_2) + qS_1]$

$= -[p(0 - 2p) + q \times 0] = 2p^2$

$\therefore \Sigma\alpha^2\beta + \Sigma\alpha^4 = 3q + 2p^2$

Now,

$f(x) = 3p^2x^2 + p^2x + 3q$

$\therefore f(1) = 3p^2 + p^2 + 3q = 4p^2 + 3q$

$f(-1) = 3p^2 - p^2 + 3q = 2p^2 + 3q$

$f(0) = 3q$

$f(2) = 12p^2 + 2p^2 + 3q = 14p^2 + 3q$

$\therefore \Sigma\alpha^2\beta + \Sigma\alpha^4 = f(-1)$

94. (d) We have, $x^3 + ax^2 - bx + c = 0$

$\therefore \quad S_1 = \Sigma\alpha = -\frac{(a)}{1} = -a$

$S_2 = \Sigma\alpha\beta = \frac{(-b)}{1} = -b$

$S_3 = \alpha\beta\gamma = -\frac{(c)}{1} = -c$

Now, $\Sigma\beta^2(\gamma + \alpha) = S_1S_2 - 3S_3$

$= (-a)(-b) - 3(-c) = ab + 3c$

95. (a) Total number of words that can be formed with the letters of the word 'MATHEMATICS'

$= \frac{11!}{2!\,2!\,2!} = 4989600$

Number of words in which all the repeated letters are together

$= 8! \times \frac{2!}{2!} \times \frac{2!}{2!} \times \frac{2!}{2!} = 8! = 40320$

$\therefore$ Required words

$= 4989600 - 40320$

$= 4949280$

It is given that

$$982(X) = 4949280$$

$$\Rightarrow \quad X = \frac{4949280}{982} = 5040$$

96. (a) We have, 10 Red and 5 Yellow roses of different sizes.

Now, x = number of garlands that can be formed with all these flowers so that no two yellow roses come together

$$= \frac{1}{2}(10-1)! \times {}^{10}P_5$$

$$= \frac{1}{2} \times 9! \times \frac{10!}{5!}$$

$$= x = \frac{10!}{2}\left[\frac{9!}{5!}\right] \Rightarrow \frac{2x}{10!} = \frac{9!}{5!}$$

and y = number of garlands formed with all these flowers so that all the red roses coming together

$$= \frac{1}{2}(6-1)! \times 10! = \frac{1}{2} \times 5! \times 10!$$

$$\Rightarrow \quad \frac{2y}{10!} = 5!$$

$$\therefore \quad \frac{2(x-y)}{10!} = \frac{9!}{5!} - 5!$$

97. (b) We have,

$$y = \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \cdots \infty$$

$$y + 1 = 1 + \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \cdots \infty$$

$$= 1 + \left(\frac{-3}{2}\right)\left(\frac{-1}{2}\right) + \frac{\left(\frac{-3}{2}\right)\left(\frac{-3}{2}-1\right)\left(\frac{-1}{2}\right)^2}{2!}$$

$$+ \frac{\left(\frac{-3}{2}\right)\left(\frac{-3}{2}-1\right)\left(\frac{-3}{2}-2\right)\left(\frac{-1}{2}\right)^3}{3!} + \cdots$$

$$= \left(1 - \frac{1}{2}\right)^{-3/2} = \left(\frac{1}{2}\right)^{-3/2} = 2^{3/2} = 2\sqrt{2}$$

$$\therefore y + 1 = 2\sqrt{2}$$

$$\Rightarrow (y+1)^2 = 8$$

$$\Rightarrow y^2 + 2y + 1 = 8$$

$$\Rightarrow y^2 + 2y - 7 = 0$$

98. (d)

99. (d) We have,

$$\frac{x^2+1}{x^3+3x^2+3x+2} = \frac{A}{x+2} + \frac{B}{x^2+x+1} + \frac{C}{(x+2)(x^2+x+1)}$$

$$\Rightarrow x^2 + 1 = A(x^2+x+1) + B(x+2) + C$$

$$\Rightarrow x^2 + 1 = Ax^2 + Ax + A + Bx + 2B + C$$

$$\Rightarrow x^2 + 1 = Ax^2 + (A+B)x + A + 2B + C$$

On comparing, we get $A = 1$

$$A + B = 0$$

$$A + 2B + C = 1$$

$$\therefore A = 1, B = -1, C = 2$$

$$\therefore A - B + C = 1 - (-1) + 2 = 4$$

100. (b)

We have, $\sqrt{x}+\frac{1}{\sqrt{x}}=2\cos\theta$

$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=4\cos^2\theta$

$\Rightarrow x+\frac{1}{x}+2=4\cos^2\theta$

$\Rightarrow x+\frac{1}{x}=4\cos^2\theta-2$

$\Rightarrow \quad x+\frac{1}{x}=2(2\cos^2\theta-1)$

$\Rightarrow \quad x+\frac{1}{x}=2\cos 2\theta$

$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2=4\cos^2 2\theta$

$\Rightarrow \quad x^2+\frac{1}{x^2}+2=4\cos^2 2\theta$

$\Rightarrow \quad x^2+\frac{1}{x^2}=2\cos 4\theta$

$\Rightarrow \quad \left(x^2+\frac{1}{x^2}\right)^3=8\cos^3 4\theta$

$x^6+\frac{1}{x^6}+3\left(x^2+\frac{1}{x^2}\right)$

$=8\left[\frac{\cos 12\theta+3\cos 4\theta}{4}\right]$

$\Rightarrow \quad x^6+\frac{1}{x^6}+6\cos 4\theta=2\cos 12\theta+6\cos 4\theta$

$\Rightarrow \quad x^6+x^{-6}=2\cos 12\theta$

101. **(c)** We have, $\frac{99\pi}{2}\le\theta\le\frac{100\pi}{2}$

$\Rightarrow \theta\in$ IV quadrant

$\therefore\ |\sin\theta|=-\sin\theta$ and $|\cos\theta|=\cos\theta$

$\therefore$

$A=\sin|\sin\theta|=\sin(-\sin\theta)=-\sin^2\theta$

and

$B=\cos\theta|\cos\theta|=\cos\theta(\cos\theta)=\cos^2\theta$

$\therefore\ B-A=\cos^2\theta-(-\sin^2\theta)$

$=\cos^2\theta+\sin^2\theta=1$

102. **(b)** We have,

$\frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)}+\frac{\cos(\theta_3-\theta_4)}{\cos(\theta_3+\theta_4)}=0$

$\Rightarrow \frac{\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)}=-\frac{\cos(\theta_3-\theta_4)}{\cos(\theta_3+\theta_4)}$

$\Rightarrow \frac{\cos(\theta_1+\theta_2)-\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)+\cos(\theta_1-\theta_2)}$

$=\frac{-\cos(\theta_3-\theta_4)-\cos(\theta_3+\theta_4)}{-\cos(\theta_3-\theta_4)+\cos(\theta_3+\theta_4)}$

[using componendo and dividendo rule]

$\Rightarrow \frac{-2\sin\theta_1\sin\theta_2}{2\cos\theta_1\cos\theta_2}=\frac{-2\cos\theta_3\cos\theta_4}{-2\sin\theta_3\sin\theta_4}$

$\Rightarrow -\tan\theta_1\tan\theta_2=\cot\theta_3\cot\theta_4$

$\Rightarrow \cot\theta_1\cot\theta_2\cot\theta_3\cot\theta_4=-1$

103. **(b)** We have,

$$\sin^4 x - (K+3)\sin^2 x - K - 4 = 0$$

$$\Rightarrow \quad \sin^2 x = \frac{(K+3) \pm \sqrt{(K+3)^2 + 4(K+4)}}{2}$$

$$= \frac{(K+3) \pm \sqrt{K^2 + 6K + 9 + 4K + 16}}{2}$$

$$= \frac{(K+3) \pm \sqrt{K^2 + 10K + 25}}{2}$$

$$= \frac{(K+3) \pm \sqrt{(K+5)^2}}{2}$$

$$= \frac{K+3 \pm (K+5)}{2} = \frac{2K+8}{2}, -1$$

$$= K+4, -1$$

$$\Rightarrow \sin^2 x = K + 4 \; [\because \sin^2 x \geq 0]$$

Now, we know that

$$0 \leq \sin^2 x \leq 1$$

$$\Rightarrow 0 \leq K + 4 \leq 1 \Rightarrow -4 \leq K \leq -3$$

104. **(c)** (A) We have,

$$\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \sin^{-1}\left(\frac{1}{3}\right)$$

$$= \sin^{-1}\left[\frac{2\sqrt{2}}{3}\sqrt{1 - \frac{1}{9}} + \frac{1}{3}\sqrt{1 - \frac{8}{9}}\right]$$

$$= \sin^{-1}\left[\frac{2\sqrt{2}}{3} \times \frac{2\sqrt{2}}{3} + \frac{1}{3} \times \frac{1}{3}\right]$$

$$= \sin^{-1}\left[\frac{8}{9} + \frac{1}{9}\right] = \sin^{-1} 1 = \frac{\pi}{2}$$

$\therefore$ (A) $\rightarrow$ (V)

(B) We have,

$$\sin^{-1}\left(\frac{(-1)^n}{2}\right) = x$$

$$\Rightarrow \quad \sin x = \frac{(-1)^n}{2}$$

$$\Rightarrow \quad \sin x = \frac{-1}{2} \text{ or } \frac{1}{2}, x \in Z$$

Now, $\sin x = \frac{1}{2}$

$$\Rightarrow \; x = n\pi \pm (-1)^n \frac{\pi}{6} \text{ and } \sin x = -\frac{1}{2}$$

$$\Rightarrow \; x = n\pi \pm (-1)^n \left(\frac{-\pi}{6}\right)$$

$$\therefore \; x = n\pi \pm (-1)^n \frac{\pi}{6}$$

(B) $\rightarrow$ (I)

(C) We have,

$$\tan^{-1}\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) = \tan^{-1}(\sqrt{2} + 1)$$

$$= \frac{3\pi}{8}$$

(C) $\rightarrow$ (IV)

(D) We have,

$\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$

$\Rightarrow \quad y = \sqrt{y}$, where $y = \sin^{-1}|\sin x|$

$\Rightarrow \quad y^2 - y = 0$

$\Rightarrow \quad y(y-1) = 0 \Rightarrow y = 0,1$

Now, $\sin^{-1}|\sin x|$ is periodic with period $\pi \text{In}(0,\pi)$,

$\sin^{-1}|\sin x| = \begin{cases} 1, & x = 1 \\ \pi - 1, & \text{otherwise} \end{cases}$

$\therefore$ General solution, $x = K\pi \pm 1, K \in \mathbf{Z}$ (D) $\rightarrow$ (II).

105. **(a)** We have,

$$\sin h^{-1}(-2) + \operatorname{cosec} h^{-1}(-2) + \cot h^{-1}(-2)$$

$$= \ln\left(-2 + \sqrt{(-2)^2 + 1}\right)$$

$$+\ln\left(\sqrt{1 + \frac{1}{(-2)^2}} + \frac{1}{(-2)}\right)$$

$$+\frac{1}{2}\left[\ln\left[1 + \frac{1}{(-2)}\right] - \ln\left(1 - \frac{1}{(-2)}\right)\right]$$

$$= \ln(\sqrt{5} - 2) + \ln\left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right) + \frac{1}{2}\left[\ln\frac{1}{2} - \ln\frac{3}{2}\right]$$

$$= \ln(\sqrt{5} - 2) + \ln\frac{(\sqrt{5} - 1)}{2} + \frac{1}{2}\ln\frac{3}{2}\Bigg]$$

$$= \ln\left[(\sqrt{5} - 2) \times \frac{(\sqrt{5} - 1)}{2} \times \frac{1}{\sqrt{3}}\right]$$

$$= \ln\left[\frac{5 - \sqrt{5} - 2\sqrt{5} + 2}{2\sqrt{3}}\right] = \ln\left[\frac{7 - 3\sqrt{5}}{2\sqrt{3}}\right]$$

106. **(a)** We have, $a : b : c = 4 : 5 : 6$

$\Rightarrow \quad a = 4K, b = 5K, c = 6K$

Now, $\quad R = abc/4\Delta$ and $r = \Delta/s$

$$\Rightarrow \quad \frac{R}{r} = \frac{\left(\frac{abc}{4\Delta}\right)}{\left(\frac{\Delta}{s}\right)} = \frac{abcs}{4\Delta^2}$$

Now, $\Delta^2 = s(s-a)(s-b)(s-c)$

$$\text{and } s = \frac{a+b+c}{2} = \frac{4K + 5K + 6K}{2} = \frac{15K}{2}$$

$$\therefore \quad \frac{R}{r} = \frac{\left[abc\frac{(a+b+c)}{2}\right]}{[4s(s-a)(s-b)(s-c)]}$$

$$= \frac{abc}{8(s-a)(s-b)(s-c)}$$

$$= \frac{4K \times 5K \times 6K}{4\left(\frac{15K}{2} - 4K\right)\left(\frac{15K}{2} - 5K\right)\left(\frac{15K}{2} - 6K\right)}$$

$$= \frac{120K^3}{4 \times \frac{7K}{2} \times \frac{5K}{2} \times \frac{3K}{2}} = \frac{16}{7}$$

Required ratio $= 16 : 7$

107. **(d)**

We have, $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=15$

$\Rightarrow a\left[\frac{1+\cos C}{2}\right]+c\left[\frac{1+\cos A}{2}\right]=15$

$\Rightarrow \frac{a}{2}\left[1+\frac{a^2+b^2-c^2}{2ab}\right)$

$+\frac{c}{2}\left(1+\frac{b^2+c^2-a^2}{2bc}\right)=15$

$\Rightarrow \frac{a}{2}\left[\frac{(a+b)^2-c^2}{2ab}\right]$

$\Rightarrow \frac{c}{2}\left[\frac{(b+c)^2-a^2}{2bc}\right]=15$

$\Rightarrow \frac{1}{4b}[(a+b)^2-c^2]$

$+\frac{1}{4b}[(b+c)^2-a^2)]=15$

$\Rightarrow \frac{1}{4b}[a^2+b^2+2ab-c^2+b^2+c^2$

$+2bc-a^2]=15$

$\Rightarrow \frac{1}{4b}[2b^2+2ab+2bc]=15$

$=\frac{2b}{4b}[b+a+c]=15$

$\Rightarrow\ a+b+c=30$

Now, $\cot\frac{B}{2}=\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$

$=\sqrt{\frac{s^2(s-b)^2}{s(s-a)(s-b)(s-c)}}$

$=\frac{s(s-b)}{\Delta}=\frac{15(15-10)}{15\sqrt{3}}$

$\left[\because s=\frac{a+b+c}{2}=\frac{30}{2}=15,\right.$

$b=10,\Delta=15\sqrt{3}]$

$=\frac{5}{\sqrt{3}}$

108. **(a)** We have,

$d_1=2r_1, d_2=2r_2, d_3=2r_3$

Now, $d_1d_2+d_2d_3+d_3d_1=(2r_1)(2r_2)$

$+(2r_2)(2r_3)+(2r_3)(2r_1)$

$=4[r_1r_2+r_2r_3+r_3r_1]$

$=4\left[\frac{\Delta}{s-a}\times\frac{\Delta}{s-b}+\frac{\Delta}{s-b}\times\frac{\Delta}{s-c}\right.$

$\left.+\frac{\Delta}{s-c}\times\frac{\Delta}{s-a}\right]$

$=4\left[\frac{\Delta^2}{(s-a)(s-b)}+\frac{\Delta^2}{(s-b)(s-c)}\right.$

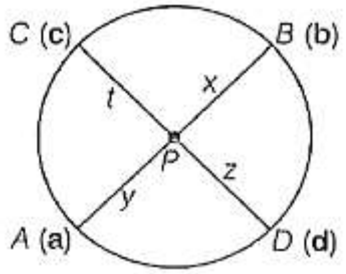
$\left.+\frac{\Delta^2}{(s-c)(s-a)}\right]$

$$= 4\Delta^2\left[\frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)}\right]$$
$$= 4\Delta^2\left[\frac{3s-(a+b+c)}{(s-a)(s-b)(s-c)}\right]$$
$$= 4\Delta^2\left[\frac{3/2(a+b+c)-(a+b+c)}{(s-a)(s-b)(s-c)}\right]$$
$$= \frac{4\Delta^2}{2}\left[\frac{a+b+c}{(s-a)(s-b)(s-c)}\right]$$
$$= \frac{2s(s-a)(s-b)(s-c)(a+b+c)}{(s-a)(s-b)(s-c)}$$
$$= (a+b+c)\cdot(a+b+c) = (a+b+c)^2$$

109. **(b)** Given, $A(a), B(b), C(c), D(d)$ are concyclic points

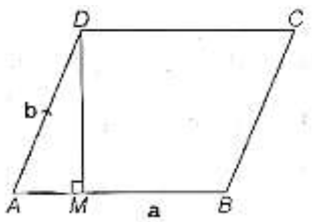
and $\begin{aligned} xa+yb+zc+td &= 0 \\ x+y+z+t &= 0 \end{aligned}$

$$\frac{xa+yb}{x+y} = \frac{(zc+td)}{x+t}$$



$$\frac{xa+yb}{x+y} = \frac{(zc+td)}{z+t}$$
$$\therefore\ P = \frac{xa+yb}{x+y} \text{ or } P = \frac{zc+td}{z+t}$$
$$PA = \frac{xa+yb}{x+y} - a = x(a-b)$$
$$PB = \frac{xa+yb}{x+y} - b = y(a-b)$$
$$PC = \frac{zc+td}{z+t} - c = t(d-c)$$
$$PD = \frac{zc+td}{z+t} - d = z(d-c)$$
$$\therefore PA \times PB = PC \times PD$$
$$|x(a-b)||y(a-b)| = |t(d-c)\,\|\,z(d-c)|$$
$$|xy||a-b|^2 = |zt||c-d|^2$$

110. **(b)**



Clearly, AM = projection of b on

$$a = \frac{a\cdot b}{|a|}$$
$$\therefore\ AM = \left\{\frac{a\cdot b}{|a|^2}\right\}a$$

In $\triangle$ AMD, we have

$$AD = AM + MD$$
$$\Rightarrow DM = AM - AD$$
$$\Rightarrow DM = \frac{(a \cdot b)a}{|a|^2} - b$$
Also, $DM = \frac{1}{|a|^2}\{(a \cdot b)a - |a|^2 b]\}$
$$= \frac{1}{|a|^2}\{(a \cdot b)a - (a \cdot a)b\}$$
$$= \frac{a \times (a \times b)}{|a|^2}$$

111. **(b)** We have,
$A(1,2,3), B(3,7,-2)$,
$C(6,7,7), D(-1,0,-1)$
Let E be the centroid of $\triangle$ ABD, then
$$E = \left(\frac{1+3-1}{3}, \frac{2+7+0}{3}, \frac{3-2-1}{3}\right)$$
$$= (1,3,0)$$
and F be the centroid of ΔACD Then
$$F = \left(\frac{1+6-1}{3}, \frac{2+7+0}{3}, \frac{3+7-1}{3}\right)$$
$$= (2,3,3)$$
$\therefore$ Equation of linc passing through

$\therefore$

$E(1,3,0)$ and $F(2,3,3)$ is
$$\mathbf{r} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 0\hat{\mathbf{k}} + t((2-1)\hat{\mathbf{i}}$$
$$+(3-3)\hat{\mathbf{j}} + (3-0)\hat{\mathbf{k}})$$
$$= \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + t(\hat{\mathbf{i}} + 3\hat{\mathbf{k}})$$
$$= (1+t)\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3t\hat{\mathbf{k}}$$

112. **(b)** We have,
a and b are orthogonal vectors
$\therefore\ a \cdot b = 0$
$$\Rightarrow -\tan\theta + \tan^2\theta$$
$$+\frac{3}{\sqrt{\sin\frac{\theta}{2}}}\left(-2\sqrt{\sin\frac{\theta}{2}}\right) = 0$$
$$\Rightarrow\ -\tan\theta + \tan^2\theta - 6 = 0$$
$$\Rightarrow\ \tan^2\theta - \tan\theta - 6 = 0$$
$$\Rightarrow (\tan\theta - 3)(\tan\theta + 2) = 0$$
$$\Rightarrow\ \text{at } \tan\theta = 3, -2$$
Now, c makes an obtuse angle with X-axis
$\therefore\ \sin 2\theta < 0$
$$\Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} < 0 \Rightarrow \tan\theta < 0$$
$$\therefore \tan\theta = -2$$
$$\Rightarrow \tan\theta = \tan(\tan^{-1}(-2))$$
$$\Rightarrow \theta = n\pi + \tan^{-1}(-2) = n\pi - \tan^{-1}2$$

113. **(a)** We have,

$$(a+b)\cdot p = (a+b)\cdot \frac{b\times c}{a\cdot(b\times c)}$$
$$= \frac{a\cdot(b\times c)+b\cdot(b\times c)}{a\cdot(b\times c)} = \frac{[abc]+[bbc]}{[abc]} \;...$$
$$= \frac{[a\cdot bc]}{[ab]} \qquad [\because [bbc]=0]$$
$$=1$$
Similarly, $(b+c)\cdot q = (c+a)\cdot r = 1$
$\therefore\ \alpha = 1+1+1 = 3$
Now, $(a+b)\cdot(b+c)\times(a+b+c)$
$$= \left[\begin{matrix} a+b \quad b+ca+b+c] \\ = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} [abc] \end{matrix}\right.$$
$$= \left[\begin{matrix} 1(1-1)-1(0-1)][abc] \\ = [a\ b\ c] \end{matrix}\right.$$
and $b\cdot(a\times c) = [b \quad a \quad c]] - [abc]$
$\therefore\quad \beta = \frac{[abc]}{-[abc]} = -1$
$\therefore\quad \alpha+\beta = 3-1 = 2$

114. **(b)** It is given that, $a+lb+l^2c=0$
$\therefore\ a\times b = l^2(b\times c)$
and $\mathbf{c}\times\mathbf{a} = l(\mathbf{b}\times\mathbf{c})$
and $\mathbf{a}\times\mathbf{b} = l(\mathbf{c}\times\mathbf{a})$
and it is also given that,
$$\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a} = 3(\mathbf{b}\times\mathbf{c})$$
$\Rightarrow\quad \mathbf{a}\times\mathbf{b}+\mathbf{c}\times\mathbf{a} = 2(\mathbf{b}\times\mathbf{c})$
$\Rightarrow\quad l^2(\mathbf{b}\times\mathbf{c})+l(\mathbf{b}\times\mathbf{c}) = 2(\mathbf{b}\times\mathbf{c})$
$\Rightarrow\quad l^2+l-2=0$
$\Rightarrow\quad l^2+2l-l-2=0$
$\Rightarrow\quad l(l+2)-1(l+2)=0$
$\Rightarrow\quad l=-2,1$
$\therefore$ Minimum value of $l=-2$

115. **(c)**

Cl	f_l	x_l	f_ix_l	$\lvert x_l-\bar{x}\rvert$	$f_l\lvert x_l-\bar{x}\rvert$
$0-4$	4	2	8	4	16
$4-8$	3	6	18	0	0
$8-12$	2	10	20	4	8
$12-16$	1	14	14	8	8
Total	$\Sigma f_i = 10$	$\Sigma f_ix_i = 60$			$\Sigma f_i\lvert x_i - x\rvert$ =32

Now,
$$\bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{60}{10} = 6$$

$\therefore$ Mean Deviation $= \frac{\Sigma f_i|x_i-\bar{x}|}{\Sigma f_i}$

$= \frac{32}{10} = 3.2$

116. **(c)** We have,

$$CV_A = \frac{\sigma_x}{x} \times 100 = \frac{2}{x} \times 100$$

$$\text{and } CV_B = \frac{\sigma_y}{\bar{y}} \times 100 = \frac{3}{\bar{y}} \times 100$$

Now, A is more consistent than B

$\therefore CV_A < CV_B$

$$\Rightarrow \frac{2}{\bar{x}} \times 100 < \frac{3}{\bar{y}} \times 100 \Rightarrow \frac{2}{\bar{x}} < \frac{3}{\bar{y}}$$

$$\Rightarrow \frac{\bar{x}}{\bar{y}} > \frac{2}{3}$$

117. **(c)** We have,

$x = P(\text{ getting sum at most } 7)$

$= P(\text{ sum } = 2) + P(\text{ sum } = 3)$

$+ P(\text{ sum } = 4)$

$+P(\text{ sum } = 5) + P(\text{ sum } = 6) + P(\text{Sum} = 7)$

$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36}$$

$$= \frac{21}{36}$$

And $y = P$ (getting sum $= 7$ at least once when pair of dice rolled n times) $= 1 - P$ (getting sum $= 7$ zero times)

$$= 1 - {}^nC_0 \left(\frac{6}{36}\right)^0 \left(\frac{30}{36}\right)^n$$

$$= 1 - \left(\frac{30}{36}\right)^n$$

Now $y > x$

$$1 - \left(\frac{5}{6}\right)^n > \frac{7}{12}$$

$$1 - \frac{7}{12} > \left(\frac{5}{6}\right)^n$$

$$\left(\frac{5}{6}\right)^n < \left(\frac{5}{12}\right) \Rightarrow \left(\frac{5}{6}\right)^n < \frac{5}{6} \times \frac{1}{2}$$

$$\Rightarrow \left(\frac{5}{6}\right)^{n-1} < \frac{1}{2}$$

$\therefore$ Minimum value of n is 5 .

118. **(c)** Let, E be the event that sum of the selected numbers is even and F be the event that both the numbers are odd.

$$\therefore (E) = {}^6C_2 + {}^7C_2$$
$$= \frac{6 \times 5}{2 \times 1} + \frac{7 \times 6}{2 \times 1}$$
$$= 15 + 21 = 36$$
$$n(F)\ {}^7C_2 = \frac{7 \times 6}{2 \times 1} = 21$$
$$\text{and } n(E \cap F) = {}^7C_2 = \frac{7 \times 6}{2 \times 1} = 21$$
$$\text{Required probability } = P(F/E)$$

$$= \frac{P(F \cap E)}{P(E)} = \frac{n(F \cap E)}{n(E)}$$
$$\frac{21}{36} = \frac{7}{12}$$

119. **(b)** We know that,

$\Sigma P(X) = 1$

$\Rightarrow$ $0.1 + 0.15 + 0.3 + 0.25 + k + k = 1$

$\Rightarrow$ $0.80 + 2k = 1$

$\Rightarrow$ $2k = 0.2$

$\Rightarrow$ $k = 0.1$

Now, mean, $\bar{X} = \Sigma XP(X)$

$= 1 \times 0.1 + 2 \times 0.15 + 3 \times 0.3 +$
$4 \times 0.25 + 5k + 6k$

$= 3.4$ [$k = 0.1$]

and $\Sigma X^2 P(X) = 1 \times 0.1 + 4 \times 0.15 + 9 \times 0.3$
$+16 \times 0.25 + 25k + 36k = 13.5$

$\therefore$ Variance $= \Sigma X^2 P(X) - (\Sigma XP(X))^2$

$= 13.5 - (3.4)^2$

$= 13.5 - 11.56$

$= 1.94$ or 1.93

120. **(d)** Required probability = 1 − P (Number of head and tail are equal)

$$= 1 - {}^{2n}C_n \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^n$$
$$= 1 - \frac{2n!}{n!\,n!}\left(\frac{1}{4}\right)^n = 1 - \frac{2n!}{(n!)^2} \cdot \frac{1}{4^n}$$

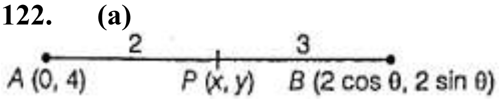
121. **(b)** In Poisson distribution

$$p(x) = \frac{e^{-\lambda}\lambda^r}{r!}$$
$$\Rightarrow p(2) = \frac{e^{-\lambda}(3725)^2}{2!} \quad [\because \lambda = 3725]$$
$$p(2) = e^{-\lambda}\left(\frac{13.875}{2}\right) = e^{-\lambda}(6.937)$$
$$p(3) = \frac{e^{-\lambda}(3725)^3}{3!} = e^{-\lambda}(8.614)$$
$$p(4) = \frac{e^{-\lambda}(3.725)^4}{4!} = e^{-\lambda}(8.022)$$
$$p(5) = \frac{e^{-\lambda}(3.725)^5}{5!} = e^{-\lambda}(5.976)$$

Clearly, p(3) has maximum value

$\therefore$ Value of x = 3

122. **(a)**



Let P be (x, y). Then

$$(x, y) = \left(\frac{4\cos\theta + 0}{5}, \frac{4\sin\theta + 12}{5}\right)$$

$$\therefore \quad x = \frac{4\cos\theta}{5} \text{ and } y = \frac{4\sin\theta + 12}{5}$$

$$\Rightarrow \quad x = \frac{4}{5}\cos\theta \text{ and } y - \frac{12}{5} = \frac{4}{5}\sin\theta$$

$\therefore$

$$x^2 + \left(y - \frac{12}{5}\right)^2 = \frac{16}{25}\cos^2\theta + \frac{16}{25}\sin^2\theta$$

$$\Rightarrow \quad x^2 + \left(y - \frac{12}{5}\right)^2 = \frac{16}{25}$$

which is equation of a circle.

123. **(c)** Here, $\tan\theta = 2$

$$\therefore \ \cos\theta = \frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$$

$$\therefore \ x = X\cos\theta - Y\sin\theta = \frac{X - 2Y}{\sqrt{5}}$$

$$\text{and } y = X\sin\theta + Y\cos\theta = \frac{2X + Y}{\sqrt{5}}$$

The equation $3x^2 - 4xy = r^2$ reduces to

$$3\left(\frac{X - 2Y}{\sqrt{5}}\right)^2 - 4\left(\frac{X - 2Y}{\sqrt{5}}\right)\left(\frac{2X + Y}{\sqrt{5}}\right) = r^2$$

$$\Rightarrow \ 3(X^2 - 4XY + 4Y^2) - 4$$

$$\Rightarrow \ (2X^2 - 2Y^2 - 3 \times 4) = 5r^2$$

$$\Rightarrow \ 20Y^2 - 5X^2 = 5r^2$$

$$\Rightarrow \ 4Y^2 - X^2 = r^2$$

124. **(d)** We have, $x + 2y + 1 = 0$

$$\Rightarrow \quad x + 2y = -1$$

$$\Rightarrow \quad -x - 2y = 1$$

$$\Rightarrow \quad \frac{-1}{\sqrt{5}}x - \frac{2}{\sqrt{5}}y = \frac{1}{\sqrt{5}}$$

$$\therefore \quad P = \frac{1}{\sqrt{5}}, \cos\theta = \frac{-1}{\sqrt{5}}, \sin\theta = \frac{-2}{\sqrt{5}}$$

$$\therefore \ \tan\theta = \frac{\sin\theta}{\cos\theta} = 2$$

Now, again

$$\Rightarrow \quad x + 2y + 1 = 0 \Rightarrow 2y = -x - 1$$

$$\Rightarrow \quad y = \frac{-1}{2}x - \frac{1}{2}$$

$$\therefore \quad m = \frac{-1}{2}, c = \frac{-1}{2}$$

$$\therefore \quad \tan^{-1}(\tan\theta + m + c)$$

$$= \tan^{-1}\left(2 - \frac{1}{2} - \frac{1}{2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

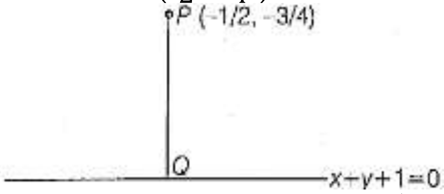
125. **(a)** We have, $ax + by + c = 0$ and $2a + 3b = 4c$

Putting $c = \frac{2a+3b}{4}$ in $ax + by + c = 0$,

we get

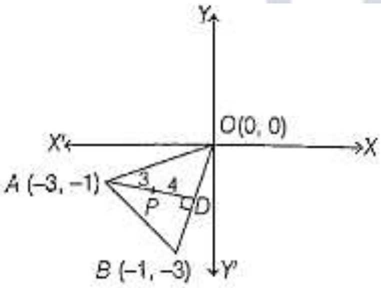
$$ax + by + \frac{2a + 3b}{4} = 0$$
$\Rightarrow \quad 4ax + 4by + 2a + 3b = 0$
$\Rightarrow \quad a(4x + 2) + b(4y + 3) = 0$
$\Rightarrow \quad (4x + 2) + \frac{b}{a}(4y + 3) = 0$

This is of the form $L_1 + \lambda L_2 = 0$
So, set of lines are
$4x + 2 = 0$ and $4y + 3 = 0$
These two lines intersect at $\left(\frac{-1}{2}, \frac{-3}{4}\right)$
$\therefore \; P(l, m) = \left(\frac{-1}{2}, \frac{-3}{4}\right)$

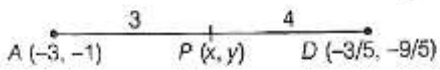


Equation of line passing through $\left(\frac{-1}{2}, \frac{-3}{4}\right)$ and perpendicular to $x + y + 1 = 0$ is
$$y + \frac{3}{4} = 1\left(x + \frac{1}{2}\right)$$
$\Rightarrow \; 4y = 4x - 1 \Rightarrow 4x - 4y - 1 = 0$
On solving $x + y + 1 = 0$
and $4x - 4y - 1 = 0$
we get $Q\left(-\frac{3}{8}, -\frac{5}{8}\right)$

126. (b)



Slope of $OB = \frac{-3-0}{-1-0} = 3$
$\therefore$ Slope of $AD = \frac{-1}{\text{Slope of } OB} = \frac{-1}{3}$
Now, equation of OB is given by
$y - 0 = 3(x - 0) \Rightarrow y = 3x \ldots.$(i)
and equation of AD is given by
$$y + 1 \quad = \frac{-1}{3}(x + 3)$$
$3y + 3 \quad = -x - 3$
$x + 3y \quad = -6 \ldots..$(ii)
On solving Eqs. (i) and (ii), we get
$D\left(\frac{-3}{5}, \frac{-9}{5}\right)$
Now, P divides AD in the ratio $3:4$



$$\therefore\ P(x,y) = \left(\frac{\frac{-9}{5}-12}{7}, \frac{\frac{-27}{5}-4}{7}\right)$$
$$= \left(\frac{-69}{35}, \frac{-47}{35}\right)$$

$\therefore$ Equation of line passing through $P\left(\frac{-69}{36}, \frac{-47}{35}\right)$ and parallel to OB is

$$y+\frac{47}{35} = 3\left(x+\frac{69}{35}\right)$$
$\Rightarrow\ 35y+47 = 3(35x+69)$
$\Rightarrow\ 35y+47 = 105x+207$
$\Rightarrow\ 105x-35y+160 = 0$
$\Rightarrow\ 21x-7y+32 = 0$

127. **(d)** Let the equation of line through the point $(1,0)$ having slope $m, (m \neq 0)$ is $y = mx - m$,
Now, the combined equation of pair of straight lines joining the point of intersections of given curve $2x^2+5y^2-7x = 0$ and line $y = mx - m$ to the origin is

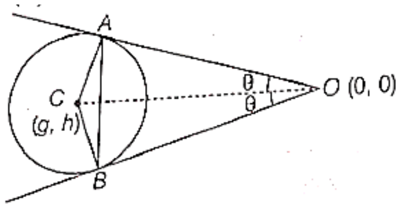
$$2x^2+5y^2-7x\left(\frac{mx-y}{m}\right) = 0$$
$\Rightarrow 2mx^2+5my^2-7x(mx-y) = 0$
$\Rightarrow\ -5mx^2+5my^2+7xy = 0 \ldots..(i)$

Since, the Eq. (i) represents the perpendicular lines, because sum of coefficients of x^2 terms and y^2 terms is zero.

128. **(d)** We have, $8x^2-14xy+5y^2 = 0$
$\Rightarrow\ 8x^2-4xy-10xy+5y^2 = 0$
$\Rightarrow\ 4x(2x-y)-5y(2x-y) = 0$
$\Rightarrow\ (2x-y)(4x-5y) = 0$
$\therefore$ Sides of triangles are
$2x-y = 0, 4x-5y = 0$
$x-2y+3 = 0$
On solving above equations, we get the vertices of triangle as $(0,0), (1,2)$ and $(5,4)$
Since (p,q) is the centroid of the triangle

$$\therefore\ (p,q) = \left(\frac{0+1+5}{3}, \frac{0+2+4}{3}\right)$$
$\Rightarrow\ (p,q) = (2,2)$
$\therefore\ p = q = 2$

129. **(d)**



We have, equation of circle is

$$x^2+y^2-2gx-2hy+h^2 = 0$$
$\therefore$ Radius of circle,
$AC = \sqrt{g^2+h^2-h^2} = g$
and Length of tangent,
$OA = \sqrt{0+0-0-0+h^2} = h$

Now, in $\triangle$ OAC

$$\tan\theta = \frac{AC}{OA} = \frac{g}{h}$$

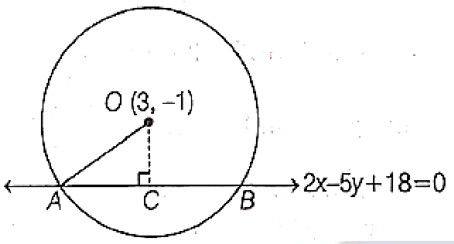
$$\therefore \sin\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\times\frac{g}{h}}{1+\frac{g^2}{h^2}} = \frac{2gh}{h^2+g^2}$$

$$\therefore \text{ Area of } \triangle OAB = \frac{1}{2} OA \times OB \times \sin 2\theta$$

$$= \frac{1}{2}\times h\times h\times\frac{2gh}{h^2+g^2}$$

$$= \frac{gh^3}{h^2+g^2}$$

130. **(b)**



Given, equation of circle is

$x^2+y^2-6x+2y-28=0$

$\therefore$ Centre of circle, $0=(3,-1)$

and radius of circle

$$OA = \sqrt{9+1+28} = \sqrt{38}$$

$$\text{Now, } OC = \left|\frac{3\times 2-(-1)\times 5+18}{\sqrt{2^2+(-5)^2}}\right|$$

$$= \left|\frac{6+5+18}{\sqrt{4+25}}\right|$$

$$= \frac{29}{\sqrt{29}} = \sqrt{29}$$

Now, in $\triangle$ OAC

$$AC = \sqrt{OA^2 - OC^2}$$

$$= \sqrt{38-29} = \sqrt{9} = 3$$

$$\therefore \lambda = AB = 2AC = 2\times 3 = 6$$

131. **(a)** We have,

$S_1: x^2+y^2+2x+8y-23=0$

$S_2: x^2+y^2-4x+10y+19=0$

Now, centres of both the circles are

$C_1(-1,-4)$

And $C_2(2,-5)$

$\therefore$ Pole of $C_1(-1,-4)$ with respect to S_2 is

$$-x-4y-2(x-1)+5(y-4)+19=0$$

$$\Rightarrow -x-4y-2x+2+5y-20+19=0$$

$$\Rightarrow -3x+y+1=0$$

$$\Rightarrow 3x-y-1=0 \;\ldots\ldots(i)$$

and Pole of $C_2(2,-5)$ with respect to S_1 is

2x-5y+(x+2)+4(y-5)-23=0

$$2x-5y+x+2+4y-20-23=0$$

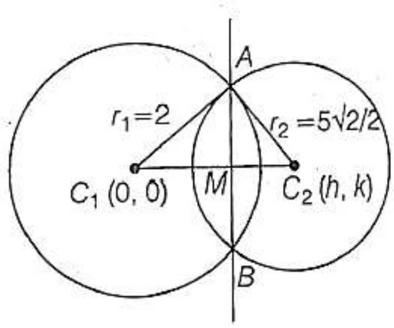
$$3x-y-41=0 \;\ldots.(ii)$$

Eqs. (i) and (ii) shows parallel lines and distance between them

$= \left|\frac{-41+1}{\sqrt{9+1}}\right| = \left|\frac{-40}{\sqrt{10}}\right|$

$= 4\sqrt{10}$ units

132. **(b)**



Let

$S_1: x^2+y^2-4=0$

$S_2: (x-h)^2+(y-k)^2 = \frac{25}{2}$

So, equation of common chord will be

$S_1 - S_2 = 0 \Rightarrow 2hx+2ky-h^2-k^2 = -\frac{17}{2}$

$\Rightarrow 2hx+2ky-h^2-k^2+\frac{17}{2} = 0$

Now, slope of common chord $= \frac{1}{4}$

$\therefore \frac{-2h}{2k} = \frac{1}{4} \Rightarrow k = -4h$ .....(i)

Now, for maximum length of chord

$C_1M = 0$

So, that AB become diameter of S_1

$\therefore \left|\frac{-h^2-k^2+\frac{17}{2}}{\sqrt{4h^2+4k^2}}\right| = 0$

$\Rightarrow h^2+k^2 = \frac{17}{2}$

On solving Eqs. (i) and (ii), we get

$(h,k) = \left(\frac{\sqrt{2}}{2}, \mp 2\sqrt{2}\right)$

133. **(a)** We have, equation of circles

$S_1: x^2+y^2-4x-6y+k=0$

and $S_2: x^2+y^2+8x-4y+11=0$

Let C_1, C_2 are centres of circles and r_1, r_2 are radius of circles S_1 and S_2 respectively.

$\therefore C_1 = (2,3), C_2 = (-4,2)$

$r_1 = \sqrt{4+9-K} = \sqrt{13-x},$

$r_2 = \sqrt{16+4-11} = 3$

Now, angle between circles is given by

$\cos\theta = \frac{r_1^2+r_2^2-c_1c_2^2}{2r_1r_2}$

$\cos\frac{\pi}{3} = \frac{13-k+9-\left(\sqrt{(2+4)^2+(3-2)^2}\right)^2}{2\times\sqrt{13-k}\times 3}$

$\Rightarrow \quad \frac{1}{2} = \frac{22-k-37}{6\sqrt{13-k}}$

$\Rightarrow \quad 3\sqrt{13-k} = -15-k$

$\Rightarrow \quad 9(13-k) = (15+k)^2$

$\Rightarrow \quad 117-9k = 225+k^2+30k$

$\Rightarrow \quad k^2+39k+108 = 0$

$\Rightarrow \quad (k+3)(k+36) = 0$

$\Rightarrow \quad k = -3, -36$

$\therefore \quad k = -3$

134. **(c)** We have, $y^2 = 4x$

$\Rightarrow \ 2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

At A(1,2)

$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{2}{2} = 1$

$\therefore$ Tangent is given by

$y-2 \quad = 1(x-1)$

$y-2 \quad = x-1$

$y \quad = x+1 \quad \ldots.(i)$

At $B(4,-4)$

$\left(\frac{dy}{dx}\right)_{(4,-4)} = \frac{2}{-4} = -\frac{1}{2}$

$\therefore$ Tangent is given by $y+4 = \frac{-1}{2}(x-4)$

$2y+8 \quad = -x+4$

$x+2y+4 \quad = 0 \quad \ldots..(ii)$

At $C(2,2\sqrt{2})$

$\left.\frac{dy}{dx}\right|_{(2,2\sqrt{2})} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\therefore$ Tangent is given by

$y-2\sqrt{2} = \frac{1}{\sqrt{2}}(x-2)$

$\Rightarrow \quad \sqrt{2}y-4 \quad = x-2$

$\Rightarrow \quad x-\sqrt{2}y+2 \quad = 0$

Let P, Q, R the vertices of triangle formed by tangents (i), (ii), (iii). On solving Eqs. (i), (ii) and (iii), we get $P(-2,-1), Q(-2\sqrt{2}, \sqrt{2}-2)$ and $R(\sqrt{2}, \sqrt{2}+1)$

Now, ar $(\triangle ABC) = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 4 & -4 & 1 \\ 2 & 2\sqrt{2} & 1 \end{vmatrix}$

$= \frac{1}{2}[1(-4-2\sqrt{2}) - 2(4-2) + 1(8\sqrt{2}+8)]$

$= \frac{1}{2}[-4-2\sqrt{2}-4+8\sqrt{2}+8] = 3\sqrt{2}$sq unit and ar ($\triangle$ PQR) $= \frac{1}{2}\begin{vmatrix} -2 & -1 & 1 \\ -2\sqrt{2} & \sqrt{2}-2 & 1 \\ \sqrt{2} & \sqrt{2}+1 & 1 \end{vmatrix}$

$= \frac{1}{2}[-2[\sqrt{2}-2-\sqrt{2}-1]+1[-2\sqrt{2}-\sqrt{2}]$

$+1[-4-2\sqrt{2}-2+2\sqrt{2}]]$

$= \frac{1}{2}[6-3\sqrt{2}-6]$

$= \frac{-3}{2}\sqrt{2}$

$= \frac{3}{2}\sqrt{2}$ sq unit

[area cannot be negative]

$\therefore\ \alpha = 3\sqrt{2}$ and $\beta = \frac{3}{2}\sqrt{2}$

$\therefore \alpha\beta = 3\sqrt{2} \times \frac{3}{2}\sqrt{2} = 9$

135. **(d)** We have, $x - 2y + k = 0$ is tangent to the parabola $y^2 - 4x - 4y + 8 = 0$, then

Now put $x = 2y - k$ in equation of parabola, we get

$\therefore\ y^2 - 4(2y - k) - 4y + 8 = 0$

$\Rightarrow\ y^2 - 8y + 4k - 4y + 8 = 0$

$\Rightarrow\ y^2 - 12y + 4k + 8 = 0$

Since line is tangent to parabola.

$\therefore\ D = 0$

$\Rightarrow\ (-12)^2 - 4(4k+8) = 0$

$\Rightarrow\ 144 - 16k - 32 = 0$

$\Rightarrow\ 16k = 112$

$\Rightarrow\ k = 7$

Now, we have

$y^2 - 4x - 4y + 8 = 0$

$\therefore\ 2y\frac{dy}{dx} - 4 - 4\frac{dy}{dx} = 0$

$\Rightarrow\ 2\frac{dy}{dx}(y-2) = 4$

$\Rightarrow\ \frac{dy}{dx} = \frac{2}{y-2}$

$\therefore$ Slope of tangent at $(l, k) = \frac{2}{k-2}$

$= \frac{2}{7-2}$ $[\because k = 7]$

$= \frac{2}{5}$

136. **(c)** Let the equation of ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$

Now, we have

Length of latus rectum = 4

$\Rightarrow\ \frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a \ldots\ldots \text{(i)}$

and distance between foci $= 4\sqrt{2}$

$\Rightarrow 2ae = 4\sqrt{2} \Rightarrow ae = 2\sqrt{2}$

$\Rightarrow a^2e^2 = 8$

$\Rightarrow a^2\left(1 - \frac{b^2}{a^2}\right) = 8$

$\Rightarrow a^2 - b^2 = 8$

$\Rightarrow a^2 - 2a - 8 = 0$ [From Eq. (i)]

$\Rightarrow (a-4)(a+2) = 0$

$\Rightarrow a = 4, -2$

$\Rightarrow a = 4\ [\because a \neq -2]$

$\therefore b^2 = 2a = 2 \times 4 = 8$

$\therefore$ Equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{8} = 1$$

$\Rightarrow x^2 + 2y^2 = 16$

137. **(c)** We have, Equation of hyperbola is

$$\Rightarrow \frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$$

$$\Rightarrow \frac{x^2}{\left(\frac{144}{25}\right)} - \frac{y^2}{\left(\frac{81}{25}\right)} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

$\therefore$ Foci $= (\pm ae, 0)$

$$= \left(\pm\frac{12}{5}\sqrt{1 + \left(\frac{9/5}{12/5}\right)^2}, 0\right)$$

$$= \left(\pm\frac{12}{5} \times 3, 0\right) = (\pm 3, 0)$$

Equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$$

$\therefore$ Foci $= (\pm ae, 0)$

$$= \left(\pm 4\sqrt{1 - \frac{b^2}{4^2}}, 0\right)$$

$\therefore$

$$= \left(\pm\frac{4\sqrt{16 - b^2}}{4}, 0\right)$$

$$= \left(\pm\sqrt{16 - b^2}, 0\right)$$

Since both have same foci

$\therefore \sqrt{16 - b^2} = 3 \Rightarrow 16 - b^2 = 9$

$\Rightarrow b^2 = 7$

138. **(c)** We have, Equation of hyperbola is

$9x^2 - 16y^2 = 144$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

We know that, the pole of the line $lx + my + n = 0$ with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\left(\frac{-a^2 l}{n}, \frac{b^2 m}{n}\right)$.

$\therefore$ Pole of $3x - 16y + 48 = 0$ with respect to the hyperbola $\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$ will be $\left(\frac{-(4)^2 \times 3}{48}, \frac{(3)^2(-16)}{48}\right)$

$\therefore \quad (\alpha, \beta) = (-1, -3)$

$\alpha = -1, \beta = -3$

$\therefore \quad \alpha - \beta = -1 - (-3) = 2$

139. **(c)** We have,

$A(2,3,-4), B(-3,3,-2), C(-1,4,2),$
$D(3,5,1)$

Now, $E =$ centroid of $\triangle ABC$

$$= \left(\frac{2-3-1}{3}, \frac{3+3+4}{3}, \frac{-4-2+2}{3}\right)$$

$$= \left(\frac{-2}{3}, \frac{10}{3}, \frac{-4}{3}\right)$$

$F =$ centroid of $\triangle ABD$

$$= \left(\frac{2-3+3}{3}, \frac{3+3+5}{3}, \frac{-4-2+1}{3}\right)$$

$$= \left(\frac{2}{3}, \frac{11}{3}, \frac{-5}{3}\right)$$

$G \quad =$ centroid of $\triangle ACD$

$$= \left(\frac{2-1+3}{3}, \frac{3+4+5}{3}, \frac{-4+2+1}{3}\right)$$

$$= \left(\frac{4}{3}, 4, \frac{-1}{3}\right)$$

Let centroid of $\triangle EFG$ is H

$$\therefore H = \left[\frac{\frac{-2}{3}+\frac{2}{3}+\frac{4}{3}}{3}, \frac{\frac{10}{3}+\frac{11}{3}+4}{3}, \frac{\frac{-4}{3}\frac{-5}{3}\frac{-1}{3}}{3}\right]$$

$$= \left(\frac{4}{9}, \frac{11}{3}, \frac{-10}{9}\right)$$

140. **(b)** According to the Cauchy's inequality

$(lm + mn + nl)^2 \le (l^2 + m^2 + n^2)^2$

$\Rightarrow \quad (lm + mn + n)^2 \le 1$

$\Rightarrow \quad \{\because l^2 + m^2 + n^2 = 0\}$

$\Rightarrow \quad -1 \le (lm + mn + nl) \le 1$

Therefore, the maximum value of $lm + mn + nl = 1$ and it is possible only when $l = m = n$, as $l = \cos\alpha, m = \cos\beta$ and $n = \cos\gamma$, so

$\alpha = \beta = \gamma$

141. **(a)** We have, combined equation of plane is plane is

$S \equiv 2x^2 - 6y^2 - 12z^2 + 18yz + 2zx$
$+ xy = 0$

Let the planes are

$a_1x + b_1y + c_1z + d_1 = 0$

and $a_2x + b_2y + c_2z + d_2 = 0$

$\therefore a_1a_2 = 2, b_1b_2 = -6, c_1c_2 = -12$

Now, one plane say

$a_1x + b_1y + c_1z + d_1$ is parallel to

$x + 2y - 2z = 5$

$\therefore a_1 = 1, b_1 = 2, c_1 = -2$

$\therefore a_2 = 2, b_2 = -3, c_2 = 6$

Let θ be acute angle between the planes

$\therefore$

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$$= \left|\frac{2-6-12}{\sqrt{1+4+4}\sqrt{4+9+36}}\right| = \frac{16}{21}$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{16}{21}\right)$$

142. **(a)** We know that;

$$3x < 6x < 9x \qquad [X\ E\ R^+]$$

$$f(3x) < f(6x) < f(9x)$$

[as f(x) is a increasing function]

$$\Rightarrow \frac{f(3x)}{f(3x)} < \frac{f(6x)}{f(3x)} < \frac{f(9x)}{f(3x)}$$

$$\Rightarrow [\because f(x) > 0 \forall x]$$

$$\Rightarrow \quad \Rightarrow \lim_{x\to\infty}\frac{f(3x)}{f(3x)} < \lim_{x\to\infty}\frac{f(6x)}{f(3x)} < \lim_{x\to\infty}\frac{f(9x)}{f(3x)}$$

$$\Rightarrow$$

$$\Rightarrow 1 < \lim_{x\to\infty}\frac{f(6x)}{f(3x)} < 1$$

$$\therefore \lim_{x\to\infty}\frac{f(6x)}{f(3x)} = 1$$

143. **(a)** We have,

$$f(x) = \begin{cases} 1, & 0 < x < \pi/2 \\ 1-\sin 1, & \frac{\pi}{2} < x < \pi \\ \frac{1-\sin 1}{\cos 1}, & \pi < x < \frac{3\pi}{2} \\ \frac{1}{\cos 1}, & \frac{3\pi}{2} < x < 2\pi \end{cases}$$

From above definition of f(x), we have f(x) is continuous on $\left(0, \frac{\pi}{2}\right)$.

144. **(b)** We have,

$$x = \frac{1-\sqrt[3]{y}}{1+\sqrt[3]{y}} \Rightarrow x + x\sqrt[3]{y} = 1 - \sqrt[3]{y}$$

$$\Rightarrow \quad \sqrt[3]{y}(x+1) = 1 - x$$

$$\Rightarrow \quad \sqrt[3]{y} = \frac{1-x}{1+x} \Rightarrow y = \frac{(1-x)^3}{(1+x)^3}$$

On differentiating both the sides, we get

$$\frac{dy}{dx} = \frac{3(1-x)^2(-1)(1+x)^3 - (1-x)^3 3(1+x)^2}{(1+x)^6}$$

$$= \frac{-3(1-x)^2(1+x)^2[(1+x)+1-x]}{(1+x)^6}$$

$$= \frac{-6(1-x)^2}{(1+x)^4}$$

$$\therefore \frac{dx}{dy} = \frac{-(1+x)^4}{6(1-x)^2}$$

145. **(d)** Let $f(x) = \log|x|$

$$= \begin{cases} \log(-x), & x < 0 \\ \log x, & x \geq 0 \end{cases}$$

$$\therefore f'(x) = \begin{cases} \frac{-1}{x}(-1), & x < 0 \\ \frac{1}{x}, & x \ge 0 \end{cases} = \begin{cases} \frac{1}{x}, & x < 0 \\ \frac{1}{x}, & x \ge 0 \end{cases}$$

$$\text{and } f'(x) = \begin{cases} \frac{-1}{x^2}, & x < 0 \\ \frac{-1}{x^2}, & x \ge 0 \end{cases}$$

$$\therefore f'(x) = \frac{-1}{x^2} = \frac{-1}{|x|^2}$$

So, A is false.

We know that, $|x| = -x$, when $x < 0$ So, R is true.

146. **(d)** We have, .

$$ax^2 + 2hxy + by^2 = 0 \ldots..(i)$$

$$\therefore \quad 2ax + 2h\left(1 \cdot y + x\frac{dy}{dx}\right) + 2by\frac{dy}{dx} = 0$$

$$\Rightarrow \quad ax + hy + hx\frac{dy}{dx} + by\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(ax+hy)}{(hx+by)}$$

$$\therefore \quad \frac{d^2y}{dx^2}$$

$$= \quad -\left[\frac{\left(a + h\frac{dy}{dx}\right)(hx+by) - (ax+hy)\left(h + b\frac{dy}{dx}\right)}{(hx+by)^2}\right]$$

$$= -\left[\frac{[a(hx+by) - h(ax+hy)] + \left[h(hx+by) - b(ax+hy)\frac{dy}{dx}\right.}{(hx+by)^2}\right]$$

$$= -\left[\frac{(ahx + aby - ahx - h^2y) + [h^2x + hby - abx - bhy]\frac{dy}{dx}}{(hx+by)^2}\right]$$

$$= -\left[\frac{(ab - h^2)y + (h^2 - ab)x\frac{dy}{dx}}{(hx+by)^2}\right]$$

$$= -\frac{(h^2 - ab)}{(hx+by)^2}\left[-y + x\left(-\left(\frac{ax+hy}{hx+by}\right)\right)\right]$$

$$= -\frac{(h^2 - ab)}{(hx+by)^3}[-hxy - by^2 - ax^2 - hxy]$$

$$= \frac{h^2 - ab}{(hx+by)^3}[ax^2 + 2hxy + by^2] = 0$$

147. **(c)** Let the point be (x_1, y_1).

$\therefore\ y_1^2 = x_1 \quad \ldots.(i)$

Now, we have

$$y^2 = x$$

$\therefore\ 2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

$\therefore$ Slope of normal at

$$(x_1, y_1) = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}$$

$$= \frac{-1}{\left(\frac{1}{2y_1}\right)} = -2y_1$$

It is given that, $-2y_1 = x_1$ …. (ii)

From Eqs. (i) and (ii), we have

$$y_1^2 = -2y_1 \Rightarrow \Rightarrow y_1^2 + 2y_1 = 0$$

$$\Rightarrow y_1(y_1 + 2) = 0 \Rightarrow y_1 = 0, -2$$

$$\therefore x_1 = 0, 4$$

Required points are $(0,0), (4,-2)$

148. **(b)** We have, $x^2 + ax - b = 0$ has roots α and β

$\therefore\ \alpha + \beta = -a$ and $\alpha\beta = -b$

Now, equation of curve is

$$\frac{x^n}{\alpha^n} + \frac{y^n}{\beta^n} = 2$$

$$\therefore\ \frac{n}{\alpha^n}x^{n-1} + \frac{n}{\beta^n}y^{n-1}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\beta^n}{\alpha^n}\frac{x^{n-1}}{y^{n-1}}$$

$\therefore$ Slope of tangent at (α, β) $= \left.\frac{dy}{dx}\right|_{(\alpha,\beta)}$

$$= \frac{-\beta^n}{\alpha^n}\frac{\alpha^{n-1}}{\beta^{n-1}} = \frac{-\beta}{\alpha}$$

So, equation of tangent at (α, β) is given by

$$y - \beta = \frac{-\beta}{\alpha}(x - \alpha)$$

$$\Rightarrow \alpha y - \alpha\beta = -\beta x + \alpha\beta$$

$$\Rightarrow \beta x + \alpha y = 2\alpha\beta$$

But equation of tangent is

$x\cos\theta + y\sin\theta = C$

$$\therefore\ \frac{\cos\theta}{\beta} = \frac{\sin\theta}{\alpha} = \frac{C}{2\alpha\beta}$$

$$\Rightarrow \cos\theta = \frac{C}{2\alpha} \text{ and } \sin\theta = \frac{C}{2\beta}$$

$$\therefore\ \cos^2\theta + \sin^2\theta = \frac{C^2}{4\alpha^2} + \frac{C^2}{4v\beta^2}$$

$$\Rightarrow \frac{C^2}{4}\left[\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right] = 1$$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{4}{C^2}$$

$$\Rightarrow \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{4}{C^2}$$

$$\Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{4}{C^2}$$

$$\Rightarrow \frac{a^2 + 2b}{b^2} = \frac{4}{c^2} \Rightarrow \left(\frac{a}{b}\right)^2 + \frac{2}{b} = \frac{4}{c^2}$$

149. **(c)** We know that,

$$f'(C) = \frac{f(b) - f(a)}{b - a}$$

Now,$\Rightarrow$ $f'(x) = x$

$\Rightarrow$ $f'(x) = 1$

Also,

$$f(b) = f(5) = 5$$
$$f(a) = f(2) = 2$$

$$\therefore \frac{f(b) - f(a)}{b - a} = \frac{5 - 2}{5 - 2} = 1$$

$\therefore$ $f'(C) = \frac{f(b)-f(a)}{b-a}$ is always true

For infinite value of $C \in (2,5)$.

150. **(a)** We have,

$$f(x) = x^3 - 4x^2 + 4x + 3$$

$\therefore$ $f'(x) = 3x^2 - 8x + 4$

Now, $f'(x) = 0 \Rightarrow 3x^2 - 8x + 4 = 0$

$\Rightarrow$ $3x^2 - 6x - 2x + 4 = 0$

$\Rightarrow$ $3x(x - 2) - 2(x - 2) = 0$

$\Rightarrow$ $(x - 2)(3x - 2) = 0 \Rightarrow x = 2, \frac{2}{3}$

Now,

$f(-1) = -1 - 4 - 4 + 3 = -6$

$$f(2/3) = \frac{8}{27} - 4 \times \frac{4}{9} + 4 \times \frac{2}{3} + 3$$
$$= \frac{8 - 48 + 72 + 81}{27} = \frac{113}{27}$$

$f(2) = 8 - 14 + 8 + 3 = 5$

$f(3) = 27 - 36 + 12 + 3 = 6$

$\therefore$ $f(x)$ has minimum value at $x = -1$

151. **(c)**

$$\text{Let } I = \int \frac{\tan^{-1} x}{x^3} dx$$
$$= \int \tan^{-1} x \cdot \frac{1}{x^3} dx$$
$$= \tan^{-1} x \left[\frac{-1}{2x^2}\right] - \int \frac{-1}{2x^2} \cdot \frac{1}{1 + x^2} dx$$
$$= -\frac{1}{2x^2} \tan^{-1} x + \frac{1}{2} \int \frac{1}{x^2(1 + x^2)} dx$$
$$= \frac{-1}{2x^2} \tan^{-1} x + \frac{1}{2} \int \left(\frac{1}{x^2} - \frac{1}{1 + x^2}\right) dx$$
$$= \frac{-1}{2x^2} \tan^{-1} x + \frac{1}{2}\left[\frac{-1}{x} - \tan^{-1} x\right] + C$$
$$= \frac{-1}{2x^2} \tan^{-1} x - \frac{1}{2x} - \frac{1}{2} \tan^{-1} x + C$$
$$= \frac{-1}{2} \tan^{-1} x \left[\frac{1}{x^2} + 1\right] - \frac{1}{2x} + C$$
$$= \frac{-1}{2x} - \left(\frac{1}{2x^2} + \frac{1}{2}\right) \tan^{-1} x + C$$

152. **(d)**

$$\text{Let } I=\int \frac{1-(\cot x)^{2019}}{\tan x+(\cot x)^{2020}} dx$$

$$=\int \frac{1-\frac{(\cos x)^{2019}}{(\sin x)^{2019}}}{\frac{\sin x}{\cos x}+\frac{(\cos x)^{2020}}{(\sin x)^{2020}}} dx$$

$$=\int \frac{\sin^{2019} x-\cos^{2019} x}{\sin^{2021} x+\cos^{2021} x} \cdot \sin x \cos x dx$$

Put $\sin^{2021} x+\cos^{2021} x=t$

$\Rightarrow [2021 \sin^{2020} x \cdot \cos x$

$+2021 \cos^{2020} \cdot(-\sin x)]$

$dx=dt$

$\Rightarrow \sin x \cos x[\sin^{2019} x-\cos^{2019} x]$

$$dx=\frac{1}{2021} dt$$

$$\therefore\ I=\frac{1}{2021} \int \frac{1}{t} dt=\frac{1}{2021} \log t+C$$

$$=\frac{1}{2021} \log[\sin^{2021} x+\cos^{2021} x]+C$$

$\therefore\ f(x)=\sin x, g(x)=\cos x, n=2021$

$$\therefore n\left[(f(x)^4+(g(x)^4)\right]_{x=\frac{\pi}{3}}$$

$$=2021\left[\sin^4 \frac{\pi}{3}+\cos^4 \frac{\pi}{3}\right]$$

$$=2021\left[\left(\frac{\sqrt{3}}{2}\right)^4+\left(\frac{1}{2}\right)^4\right]$$

$$=2021\left[\frac{9}{16}+\frac{1}{16}\right]$$

$$=\frac{2021 \times 10}{16}=\frac{10105}{8}$$

153. **(a)** Let

$$I=\int \frac{x^3+x^2-x-1}{(x^5+x^4+3x^3+3x^2+x+1)}dx$$

$$\tan^{-1}\left(\frac{x^2+1}{x}\right)$$

$$=\int \frac{x(x^2-1)+x^2-1}{[x(x^4+3x^2+1)+x^4+3x^2+1]}dx$$

$$\tan^{-1}\left(\frac{x^2+1}{x}\right)$$

$$=\int \frac{(x+1)(x^2-1)}{(x^4+3x^2+1)(x+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$=\int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$\text{Put } \tan^{-1}\left(\frac{x^2+1}{x}\right)=t$$

$$\frac{1}{1+\left(\frac{x^2+1}{x}\right)^2}\cdot\frac{2x(x)-(x^2+1)}{x^2}dx=dt$$

$$\Rightarrow \frac{x^2-1}{x^4+3x^2+1}dx=dt \Rightarrow I=\int \frac{1}{t}dt$$

$$=\log t+C$$

$$=\log \tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

$$\therefore A=1, f(x)=\tan^{-1}\left(\frac{x^2+1}{x}\right)$$

$$\therefore A-\tan f(2)=1-\tan\tan^{-1}\frac{5}{2}$$

$$=1-\frac{5}{2}=-\frac{3}{2}$$

154. **(a)** We have, $I_n=\int x^n \sin x dx$

$\therefore I_6 = \int x^6 \sin x dx$

$= x^6(-\cos x) - \int -\cos x \cdot (6x^5) dx$

$= -x^6 \cos x + 6\int x^5 \cos x dx$

$= -x^6 \cos x + 6\left[x^5 \sin x - \int \sin x \cdot (5x^4) dx\right]$

$= -x^6 \cos x + 6x^5 \sin x - 30\int x^4 \sin x dx$

$= -x^6 \cos x + 6x^5 \sin x - 30$

$\left[x^4(-\cos x) - \int -\cos x(4x^3) dx\right]$

$= -x^6 \cos x + 6x^5 \sin x + 30x^4 \cos x$

$-120\int x^3 \cos x dx$

$= -x^6 \cos x + 6x^5 \sin x + 30x^4 \cos x - 120$

$\left[x^3 \sin x - \int \sin x(3x^2) dx\right]$

$= -x^6 \cos x + 6x^5 \sin x + 30x^4 \cos x - 120$

$x^3 \sin x + 360\int x^2 \sin x dx$

$= -x^6 \cos x + 6x^5 \sin x + 30x^4 \cos x$

$-120x^3 \sin x + 360 I_2$

$\therefore I_6 - 360 I_2 = (-x^6 + 30x^4) \cos x$

$+ (6x^5 - 120x^3) \sin x$

$\therefore\ f(x) = -x^6 + 30x^4$

and $g(x) = 6x^5 - 120x^3$

$\therefore f(1) + g)1) = -1 + 30 + 6 - 120$

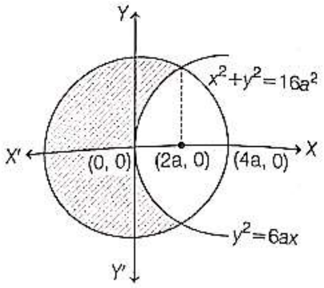
$\therefore$

$\therefore = -85$

155. **(d)**

156. **(d)**

157. **(a)**



We have, $x^2 + y^2 = 16a^2$ and $y^2 = 6ax$ On solving both equations, we have

$x^2 + 6ax = 16a^2$

$\Rightarrow\ x^2 + 6ax - 16a^2 = 0$

$\Rightarrow\ x^2 + 8ax - 2ax - 16a^2 = 0$

$\Rightarrow\ x(x + 8a) - 2a(x + 8a) = 0$

$\Rightarrow\ (x + 8a)(x - 2a) = 0$

$\Rightarrow\ x = -8a, 2a$

$\Rightarrow\ x = 2a\ [x \neq -8a$, as $y^2 = 6ax]$

$\therefore$ Required area = Area of circle -
Area of unshaded region

$$= \pi(4a)^2 - 2\left[\int_0^{2a} \sqrt{6ax}dx\right.$$
$$\left.+ \int_{2a}^{4a} \sqrt{16a^2 - x^2}dx\right]$$
$$= 16a^2\pi - 2\left[\sqrt{6a}\cdot\left[\frac{x^{3/2}}{3/2}\right]_0^{2a}\right.$$
$$\left.+ \left[\frac{x}{2}\sqrt{16a^2 - x^2} + \frac{16a^2}{2}\sin^{-1}\frac{x}{4a}\right]_{2a}^{4a}\right.$$
$$= 16a^2\pi - 2\left[\frac{2}{3}\sqrt{6a}(2\sqrt{2}a)^{3/2}\right.$$
$$\left.+ \frac{16a^2}{2}\cdot\frac{\pi}{2} - \frac{2a}{2} \times 2\sqrt{3}a - \frac{16a^2}{2^2} \times \frac{\pi}{6}\right]$$
$$= 16a^2\pi - \frac{16\sqrt{3}}{3}a^2 - 8a^2\pi + 4\sqrt{3}a^2 + \frac{8}{3}a^2\pi$$
$$= \frac{32\pi a^2}{3} - \frac{4a^2\sqrt{3}}{3}$$
$$= \frac{4a^2}{3} - (8\pi - \sqrt{3}) \text{ sq unit}$$
$=$

158. **(d)** We have,

$$\sqrt{1 + y^2}Cxe^{\tan^{-1}x} \quad \ldots\ldots(i)$$
$$\Rightarrow \frac{1}{2\sqrt{1 + y^2}}\cdot 2y\frac{dy}{dx} =$$
$$= Ce^{\tan^{-1}x} + \frac{Cxe^{\tan^{-1}x}}{1 + x^2}$$
$$\Rightarrow \frac{y}{\sqrt{1 + y^2}}\frac{dy}{dx} = Ce^{\tan^{-1}x\left[1+\frac{x}{1+x^2}\right]}$$
$$\Rightarrow \frac{y}{\sqrt{1 + y^2}}\frac{dy}{dx}$$
$$= \frac{\sqrt{1 + y^2}}{xe^{\tan^{-1}x}}\cdot e^{\tan^{-1}x}\left[\frac{1 + x^2 + x}{1 + x^2}\right]$$

[From Eq. (i)]

$$\Rightarrow \frac{y}{\sqrt{1 + y^2}}\frac{dy}{dx} = \frac{\sqrt{1 + y^2}(1 + x + x^2)}{x(1 + x^2)}$$
$$\Rightarrow xy(1 + x^2)dy$$
$$= (1 + y^2)(1 + x + x^2)dx$$
$$\Rightarrow xy(1 + x^2)dy$$
$$- (1 + y^2)(1 + x + x^2)dx = 0$$

which is required differential equation

159. **(b)** We have,

$$\frac{dy}{dx} = \frac{x + y - 3}{x + y - 7}$$

$$\Rightarrow \frac{dy}{dx}=\frac{x+y-5+2}{x+y-5-2}$$

Put $x+y-5=V$

$$\Rightarrow 1+\frac{dy}{dx}=\frac{dV}{dx}\Rightarrow\frac{dy}{dx}=\frac{dV}{dx}-1$$

$$\therefore \frac{dV}{dx}-1=\frac{V+2}{V-2}\Rightarrow\frac{dV}{dx}=\frac{V+2}{V-2}+1$$

$$\Rightarrow \frac{dV}{dx}=\frac{2V}{V-2}\Rightarrow\int\frac{V-2}{V}dV=2\int dx$$

$$\Rightarrow \int\left(1-\frac{2}{V}\right)dV=2\int dx$$

$$\Rightarrow V-2\log V=2x+K$$

$$\Rightarrow x+y-5-2\log(x+y-5)=2x-k$$

$$\Rightarrow y-x-5-K=\log(x+y-5)^2$$

$$\Rightarrow (x+y-5)^2=e^{y-x-5-K}$$

$$\Rightarrow (x+y-5)^2=e^{y-x}\cdot e^{-5-k}$$

$\Rightarrow (x+y-5)^2=Ce^{y-x}$, where $C=e^{-5-K}$

Which is required solution.

160. (a)

We have, $x\frac{dy}{dx}=x^2+3y$

$$\Rightarrow x\frac{dy}{dx}-3y=x^2\Rightarrow\frac{dy}{dx}-\frac{3}{x}y=x$$

$$\therefore \text{IF}=e^{\int\frac{-3}{x}dx}=e^{-3\log x}=\frac{1}{x^3}$$

$\therefore$ Solution is given by

$$y\cdot\frac{1}{x^3}=\int x\cdot\frac{1}{x^3}dx\Rightarrow\frac{y}{x^3}=\frac{-1}{x}+C$$

$$y=-x^2+Cx^3$$

Now, it is given $y(2)=4$

$\therefore\quad 4=-4+8C\Rightarrow C=1$

$\therefore\quad y=x^3-x^2$

$\therefore\quad y(4)=64-16=48$