Physics

1. (b) According to question we have to find incorrect option but

Option (a) According to Noether's theorem, law of conservation of energy is associated with symmetry of nature. As a result of which, the amount of conserved quantity of any isolated system doesn't change, until some in flow/out flow of physical quantities takes place.

Hence, option (a) is the correct.

Option (b) As we know that, the strength of fundamental force i.e. in order of

(i) strong nuclear force $= 1$

(ii) electromagnetic force $= \frac{1}{137}$

(iii) weak nuclear force $= 10^{-6}$

(iv) gravity $= 6 \times 10^{-39}$

$\therefore$ Strong nuclear force and electromagnetic force > weak nuclear force > gravitational force.

Hence, option (b) is the incorrect.

Option (c) As we know that, the law of conservation of mass, momentum, energy etc. can only be easily observed by experimental observations.

Hence, option (c) is the correct.

Option (d) Since there are two types of nuclear reaction i.e. nuclear fission & fusion and in both cases we find generation of energy due to breaking of heavy nuclei (U_{235}) or by fusion of two light nuclei i.e. (H_1^2 and H_1^3), respectively.

2. (b) Given,

By using Stefan's law, $P = \sigma A e T^4$

where,

$P =$ power,

$\sigma =$ Stefan's constant

where, $A =$ area,

$e =$ emissivity,

and $T =$ temperature.

$$\therefore [\sigma] = \left[\frac{P}{AeT^4}\right] = \left[\frac{ML^2\,T^{-2}}{L^2\,T^1\,K^4}\right]$$

$$= [ML^0\,T^{-3}\,K^{-4}] \ldots\ldots \text{(i)}$$

Now, By Wien's displacement law,

$[\lambda T] =$ constant

$$= [b] = [M^0\,L^1\,K^1] \ldots\ldots \text{(i)}$$

where, $\lambda =$ wavelength,

$T =$ temperature,

and $b =$ Stefan's constant.

Now, quadrating the power of Eq. (ii) and multiply by Eq. (i), we get

$$\therefore [\sigma b^4] = [ML^0\,T^{-3}\,K^{-4}][M^0\,L^1\,K^1]^4$$

$$= [ML^0\,T^{-3}\,K^{-4}][M^0\,L^4\,K^4]$$

$$= [M^1\,L^4\,T^{-3}]$$

3. (a) Given, equation of distance

$x(t) = \alpha t + \beta t^3 \ldots\ldots$ (i)

where, $\alpha = 2\ \mathrm{m/s}$ and $\beta = 0.01\ \mathrm{m/s^3}$.

Initial timc, $t_1 = 2$ s and final time,

$t_2 = 4$ s

As we know that,

Average velocity

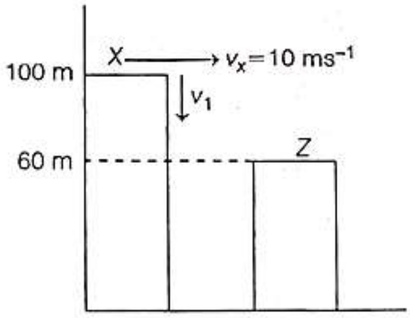
$$(v_{av}) = \frac{\text{Total displacement }(x)}{\text{Total time }(t)}$$

Since $x(t) = \alpha t + \beta t^3$

$\therefore$ Distance travelled by car in time interval $(t_2 - t_1)$ will be

$$x = \alpha(t_2 - t_1) + \beta(t_2^3 - t_1^3)$$
$$= 2(4-2) + \frac{1}{100}(4^3 - 2^3)$$
$$= 2(2) + \frac{1}{100}[64 - 8]$$
$$= 4 + \frac{1}{100}(56) = 4.56 \text{ m}$$
$$\therefore\ v_{av} = \frac{x}{t_2 - t_1} = \frac{4.56}{4-2}$$
$$= \frac{4.56}{2} = 2.28 \text{ m/s}$$

4. **(a)** Given, speed of car at X, $v_x = 10$ m/s.
Acceleration due to gravity, $g = 10\ \text{ms}^{-2}$ Let, mass of car be m.



Speed of car at X in downward direction (v_1) be $0\ \text{ms}^{-1}$,
Speed of car at Z in downward direction be v_2 and v be the speed of car at Z,
h_1 and h_2 are the position of X and Z from ground i.e., 100 m and 60 m , respectively.
Now, by using law of conservation of energy,

$$\frac{1}{2}mv_2^2 = mg(h_1 - h_2)$$
$$\Rightarrow\ v_2 = \sqrt{2g(h_1 - h_2)}$$
$$= \sqrt{2 \times 10(100 - 60)}$$
$$= \sqrt{2 \times 10 \times 40} = \sqrt{800}$$
$$= 20\sqrt{2}\ \text{ms}^{-1}$$
$$\therefore\ \mathbf{v} = 10\hat{\mathbf{i}} + 20\sqrt{2}\hat{\mathbf{j}}$$
$$|\mathbf{v}| = \sqrt{10^2 + (20\sqrt{2})^2}$$
$$= \sqrt{100 + 400 \times 2}$$
$$= \sqrt{900} = 30\ \text{ms}^{-1}$$

5. **(a)**

Given, $a = 3\hat{i} + 2\hat{j} + 5\hat{k}$
$b = 5\hat{i} + 3\hat{j} + \hat{k}$
Since, $a \cdot b = |a||b|\cos\theta$
$$\therefore\ \cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\Rightarrow \cos\theta = \frac{(3\hat{i}+2\hat{j}+5\hat{k})\cdot(5\hat{i}+3\hat{j}+\hat{k})}{\sqrt{3^2+2^2+5^2}\sqrt{5^2+3^2+1^2}}$$

$$\Rightarrow \cos\theta = \frac{15+6+5}{\sqrt{9+4+25}\sqrt{25+9+1}}$$

$$= \frac{26}{\sqrt{38}\sqrt{35}}$$

$$= \frac{26}{\sqrt{1330}}$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{26}{\sqrt{1330}}\right)$$

6. (a) Given, acceleration of particle,

$\mathbf{a} = (4\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\mathrm{ms}^{-2}$

Let at time, $t = 0$

Velocity, $u_x = 4\hat{\mathbf{i}}\ \mathrm{ms}^{-1}$

and $u_y = 0\hat{\mathbf{j}}\ \mathrm{ms}^{-1}$

Speed of body be v_x along X-axis and v_y along Y-axis after covering a displacement s_x of 6 m .

Now, as we know that,

$v^2 - u^2 = 2as$

and for displacement along X-axis

$$v_x^2 - u_x^2 = 2a_x s_x$$

$$\Rightarrow \quad v_x^2 - 16 = 2\times 4\times 6$$

$$\Rightarrow \quad v_x = \sqrt{48+16}$$

$$\Rightarrow \quad v_x = \sqrt{64} = 8\ \mathrm{ms}^{-1}$$

$$\therefore \quad \mathbf{v}_x = 8\hat{\mathbf{i}}\ \mathrm{ms}^{-1}$$

Now,

Since,

$v = u + at$

$$\therefore\ t = \frac{v_x - u_x}{a_x}$$

where, t is time taken.

$$\therefore \qquad t = \frac{8-4}{4} = 2\ \mathrm{s}$$

Again $\quad v_y = u_y + a_y t$

$$\therefore \qquad v_y = 0 + 4\times 1 = 4\ \mathrm{ms}^{-1}$$

$$\Rightarrow \qquad v_y = 4\hat{\mathbf{j}}\ \mathrm{ms}^{-1}$$

$$\therefore \qquad v = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = 8\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$$

Now, resultant vector

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{8^2+4^2}$$

$$= \sqrt{64+16}$$

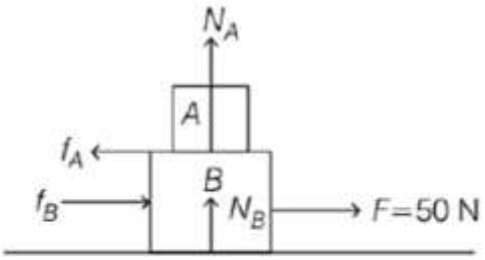
$$= \sqrt{80}$$

$$= 4\sqrt{5}\ \mathrm{ms}^{-1}$$

7. (a) Given, mass of block A and B are $(m_A), (m_B)$ be 3 kg and 7 kg ; respectively.

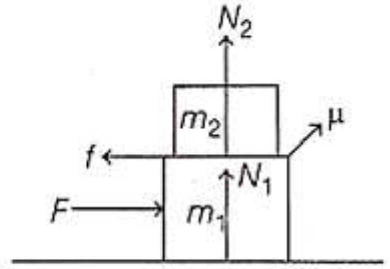
Coefficient of friction between block A and B, B and floor $(\mu_A), (\mu_B)$ be 0.4 and 0.55 respectively,

Applied force (F) on block B be 50 N. Let f_A, f_B be the friction forces between blocks A, B and B, floor respectively.



Now, from block B diagram,
$f_B = \mu_B N_B \ldots$ (i)
where, N_B be the normal reaction on block B
$= \mu_B(m_A + m_B)g$
$= 0.55(3 + 7)10 = 55$ N
Similarly, $f_A = \mu_A N_A$
where, N_A is normal reaction on block A.
$\therefore \quad f_A = \mu_A m_A g$
$f_A = 0.4 \times 3 \times 10 = 12$ N
Now, since, $f_B > f$
$\therefore$ the block B will not move.
Hence, $f_A = 0$ N

8. **(b)** Given, force F is the applied force on mass m_1.



As we know that,
$F = ma$
$\Rightarrow \ a = \frac{F}{m}$
where, a is acceleration of system. $m = m_1 + m_2$ i.e. mass of system
$\therefore \ a = \frac{F}{m_1 + m_2}$
and since, there should be no relative motion between m_1 and m_2
$\therefore \ f = \mu N_2 = m_2 a$
where, f = friction force and N_2 = normal reaction of mass m_2.
$\therefore \quad \mu m_2 g = m_2 \frac{F}{m_1 + m_2}$
$\Rightarrow \quad F = \mu(m_1 + m_2)g$

9. **(a)** Given, equation of potential energy,
$U(r) = \frac{1}{r^2} - \frac{1}{r}$
As we know that, attractive force,
$F = -\frac{dU}{dr}$
$\therefore F = -\frac{d}{dr}\left(\frac{1}{r^2} - \frac{1}{r}\right)$
$= -\frac{d}{dr}(r^{-2} - r^{-1}) = -(-2r^{-3} + r^{-2})$
$= (2r^{-3} - r^{-2})$
Now, for F_{max} i.e. maximum value of force

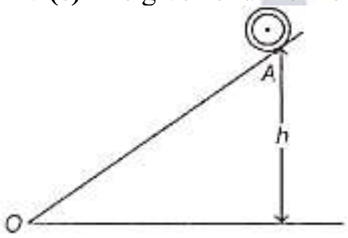
$$\frac{dF}{dr}=0$$
$$\therefore \frac{d}{dr}(2r^{-3}-r^{-2})=0$$
$$\Rightarrow -6r^{-4}+2r^{-3}=0$$
$$\Rightarrow 2r^{-3}=6r^{-4} \Rightarrow 1=3r^{-1}$$
$$\Rightarrow r=3$$
Substituting in Eq. (i), we get
$$F=\frac{2}{r^3}-\frac{1}{r^2}\bigg|_{r=3}$$
$$=\frac{2}{3^2}-\frac{1}{3^2}=\frac{2}{27}-\frac{1}{9}$$
$$=\frac{2}{27}-\frac{3}{27}=-\frac{1}{27}$$

10. **(b)** Given, let initial kinetic energy, momentum and mass be KE_1, p_1 and m_1 and final kinetic energy, momentum and mass be KE_2, p_2 and m_2

$KE_2 = 16KE_1 \ldots\ldots$ (i)
$$m_2=\frac{m_1}{2}\ldots\ldots(ii)$$
since, $KE=\frac{p^2}{2m}$
$$\therefore \frac{KE_1}{KE_2}=\left(\frac{p_1}{p_2}\right)^2\times\left(\frac{m_2}{m_1}\right)$$
$$\Rightarrow \frac{KE_1}{16KE_1}=\left(\frac{p_1}{p_2}\right)^2\times\left(\frac{m_1}{2m_1}\right)$$
$$\Rightarrow \frac{p_1}{p_2}=\sqrt{\frac{1}{8}} \Rightarrow p_2=2\sqrt{2}p_1$$

11. **(a)**

12. **(c)** The given situation is shown below



Mass of small disc, $m = 500$ g
Radius of disc, $R = 5$ cm
Initial kinetic energy of the disc will be zero i.c. $K_i = 0$
Final kinetic energy of the disc
$$K_f=\frac{1}{2}mv^2$$
where, v is velocity of disc at bottom. According to conservation of energy.
Total energy at point A = Total energy at point O
$\Rightarrow U_i + (K_{rot})_{initial} + K_i$

$= U_f + (K_{rot})_{final} + K_f$

$\Rightarrow mgh + 0 + 0 = 0 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$\Rightarrow mgh = \frac{1}{2} I\omega^2 + \frac{1}{2}mv^2$

Since, $\omega = \frac{v}{R}$

and $I = \frac{1}{2}mR^2$

$\therefore mgh = \frac{1}{2} \times \frac{1}{2} + mR^2 \times \frac{v}{R} + \frac{1}{2}mv^2$

$\therefore$

$\Rightarrow v = \sqrt{\frac{4}{3}gh}$

$\Rightarrow v \propto \sqrt{h}$

Hence, speed of centre of mass of disc at bottom depends on height of inclined plane.

13. (b) Given, mass of body, $m = 1$ kg Displacement equation of SHM,

$y = \left[6\sin\left(100t + \frac{\pi}{4}\right)\right]$ cm

Let v_y = velocity

Hence, $v_y = \frac{dy}{dt}$

$\Rightarrow v_y = \frac{d}{dt}\left[6\sin\left(100t + \frac{\pi}{4}\right)\right]$

$= 6 \times 100\cos\left(100t + \frac{\pi}{4}\right)$

$= 600\cos\left(100t + \frac{\pi}{4}\right) cms^{-1}$

$= 6\cos\left(100t + \frac{\pi}{4}\right) ms^{-1}$

Maximum velocity, $(v_y)_{max} = 6$ m/s

$\therefore$ Maximum kinetic energy,

$K_{max} = \frac{1}{2}m(v_y)^2_{max}$

$= \frac{1}{2} \times 1 \times 6^2$

$= \frac{1}{2} \times 1 \times 36$

$= 18$ J

14. (b) We know that, mass is the amount contained in any body, as it remains unaffected by change of its position.

15. (d) Given, length of rod, $l = 1$ m

Area of cross-section $= 0.5\ cm^2$

$= 0.5 \times 10^{-4}\ m^2$

Mass of rod, $m = 500$ kg

Young's modulus, $Y = 10^{11}$ Pa

Let Δl be the elongation.

As we know that,

$Y = \frac{F \cdot l}{A\Delta l}$

$\therefore \Delta l = \frac{Fl}{AY} = \frac{mgl}{AY}$

$\therefore \Delta l = \frac{500 \times 10 \times 1}{0.5 \times 10^{-4} \times 10^{11}}$

$= 1 \times 10^{-3}$ m

$= 1$ mm

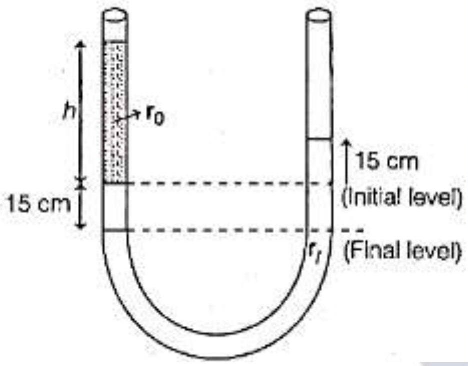
16. (a) Given, speed of water coming out, $v = 10\ ms^{-1}$
Radius of tube, $r = 1\ cm = 1 \times 10^{-2}$ m, Height of window, $H = 3$ m, Acceleration due to gravity, $g = 10\ ms^{-2}$ and density of water, $\rho = 1000\ kg/m^3$ As we know that,

$$\text{Power (P)} = \frac{\text{Work (W)}}{\text{Time(t)}}$$

and W = total energy $= mgh + \frac{1}{2}mv^2$

$$\therefore P = \frac{mgh + \frac{1}{2}mv^2}{t}$$
$$= \frac{m}{t}\left(gh + \frac{v^2}{2}\right)$$
$$= \frac{\rho A \cdot l}{t}\left(h + \frac{v^2}{2}\right)$$
$$= \rho Av\left(gh + \frac{v^2}{2}\right)$$
$$= 1000 \times \pi\left(\frac{1}{100}\right)^2 \times 10\left(10 \times 3 + \frac{100}{2}\right)$$
$$= 1000 \times \frac{10}{10000} \times \pi 80$$
$$= 80\pi\ W$$

17. (b) Given, density of liquid, $\rho_l = 1.2$ gcm^{-3}
density of oil $= \rho_0 = 0.9 gcm^{-3}$



As we know that,
At same level, pressure remains same.
$\therefore p_0 + \rho_0 g(h + 15) = p_0 + \rho_l g(15 + 15)$
where, p_0 is atmospheric pressure.

$$\therefore \quad \rho_0(h + 15) = \rho_l 30$$
$$\Rightarrow \quad 0.9(h + 15) = 1.2 \times 30$$
$$\Rightarrow \quad 3(h + 15) = 4 \times 30$$
$$\Rightarrow \quad 3h + 45 = 120$$
$$\Rightarrow \quad 3h = 75$$
$$\Rightarrow \quad h = 25\ cm$$

$\therefore$ Difference in level of oil and liquid
$= 25 - 15 = 10$ cm

18. (b) As we know that,

$$\alpha = \frac{\Delta l}{l_0 \Delta T}$$

where, α = coefficient of linear expansion,
$\Delta l =$ change in length
$l_0 =$ initial length
and ΔT = change in temperature.
As we know that,

$$\beta=\frac{\Delta A}{A_0\Delta T}=\frac{2\Delta l}{l_0\Delta T}$$

where, $\beta=$ real expansion / areal expansion,
$\Delta A=$ change in area
and $A_0=$ initial area.

$$\therefore \frac{\alpha}{\beta}=\frac{\frac{\Delta l}{l_0\Delta T}}{2\frac{\Delta l}{l_0\Delta T}}=\frac{1}{2}=0.5$$

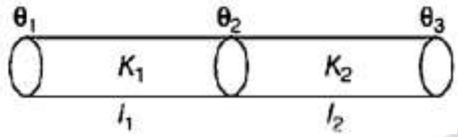
19. (a) The given, situation is shown in the figure.

As we know that, $\frac{Q}{t}$ (Heat flow rate)

$$=\frac{KA\Delta\theta}{l}=\text{ Heat current}$$

where, $\Delta\theta=$ change in temperature and $A=$ area of cross-section.

$$\therefore R(\text{ Thermal resistance })=\frac{1}{KA}$$



$$\therefore \frac{KA(\theta_1-\theta_3)}{l_1+l_2}=\frac{K_1A(\theta_1-\theta_2)}{l_1}$$
$$=\frac{K_2A(\theta_2-\theta_3)}{l_2}$$

and since, in series arrangement equivalent series thermal resistance,

$$R_{eq}=R_1+R_2$$
$$\therefore \quad R_{eq}=\frac{l_1+l_2}{KA}=\frac{l_1}{K_1A}+\frac{l_2}{K_2A}$$
$$\Rightarrow \frac{l_1+l_2}{K}=\frac{l_1}{K_1}+\frac{l_2}{K_2}$$
$$\Rightarrow \quad K=\frac{(l_1+l_2)K_1K_2}{K_2l_1+K_1l_2}$$

20. (c) The total work done by the two-stage system

$=W_1+W_2=(Q_1-Q_2)+(Q_2-Q_3)$

$\therefore$ Efficiency of the system,

$$\eta=\frac{W_1+W_2}{Q_1}$$
$$=\frac{(Q_1-Q_2)+(Q_2-Q_3)}{Q_1}$$
$$=\frac{Q_1-Q_3}{Q_1}$$
$$\Rightarrow \eta=1-\frac{Q_3}{Q_1}\ \ldots.(i)$$

Now, $\frac{Q_1}{T_1}=\frac{Q_2}{T_2}=\frac{Q_3}{T_3}$

$$\Rightarrow \frac{Q_1}{T_1}=\frac{Q_3}{T_3}$$
$$\Rightarrow \frac{Q_3}{Q_1}=\frac{T_3}{T_1}\ \ldots..(ii)$$

From Eqs. (i) and (ii), we have

$$\eta=1-\frac{T_3}{T_1}$$

But given, $T_1 = T, T_2 = \alpha T_1$ and $T_3 = \beta T$

$\therefore\ \eta = 1 - \frac{\beta T}{T} = 1 - \beta$

21. (c) Given,

Let, initial and final volume and pressures are V_1, V_2 and p_1, p_2, respectively.

where, $V_1 = 2\ m^3$

$p_1 = 2 \times 10^5\ N/m^2$

$V_2 = 0.5\ m^3$

and adiabatic constant γ for diatomic

gases $= \frac{C_p}{C_V} = \frac{7}{5} = 1.4$

As we know that,

In adiabatic condition, $p_1 V_1^\gamma = p_2 V_2^\gamma$

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^\gamma = 2 \times 10^5 \left(\frac{2}{0.5}\right)^{1.4}$$

$$= 2 \times 10^5 \times 4^{1.4}$$

$$= 2 \times 10^5 \times 6.96$$

$$= 1392 \times 10^5\ N/m^2$$

Since, work done in adiabatic condition,

$$W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

$$= \frac{2 \times 10^5 \times 2 - 13.92 \times 10^5 \times 0.5}{1.4 - 1}$$

$$= -7.4 \times 10^5\ J$$

22. (c) As per kinetic theory of gases,

Statement I Since, average kinetic energy of gases, $KE_{av} = \frac{3}{2} K_B T$

$\therefore\ KE_{av} \propto T$

Hence, temperature of gases is a measure of average kinetic energy of a molecule is correct.

So, option (a) is the correct.

Statement II As temperature of gas is independent of nature of gas.

Hence, option (b) is the incorrect.

Statement III Since, $v_{rms} = \sqrt{\frac{3RT}{M}}$

where, v_{mms} = root mean square speed, R = universal gas constant

T = temperature

and M = molar mass.

$\because\ v_{rms} \propto \frac{1}{\sqrt{M}}$

As M increases v_{rms} decreases.

23. (d) Given, wave equation, $y = 0.02$

$\sin(5\pi x - 20t)$m

As we know that,

Minimum distance between two

particles $= \frac{\text{wavelength } (\lambda)}{2}$

and according to wave equation,

Wave number, $k = 5\pi\ m^{-1}$

Angular frequency, $\omega = 20 rads^{-1}$

Since, $k = \frac{2\pi}{\lambda}$

$\therefore\ \lambda = \frac{2\pi}{k} = \frac{2\pi}{5\pi}$

$= \frac{2}{5} = 0.4\ m$

$\therefore$ Minimum separation between two particles $= \frac{0.4}{2} = 0.2$ m

24. **(a)** Given, separation between object and screen $D = 90$ cm and lens displacement, $x = 20$ cm Since, $f = \frac{D^2 - x^2}{4D}$

where, f is focal length.

$$f = \frac{90^2 - 20^2}{4 \times 90}$$
$$= \frac{8100 - 400}{360}$$
$$= \frac{770}{36} = 21.38 \text{ cm}$$

25. **(a)** Given, in Young's double slit experiment,

Wavelength, $\lambda = 400 \times 10^{-9}$ m

Separation between slits

$d = 0.001 \text{ m} = 1 \times 10^{-3}$ m

As we know that, $\lambda = d\sin\theta$ and for very small value of $\theta, \sin\theta \approx \theta$

$$\therefore \theta = \frac{\lambda}{d}$$
$$= \frac{400 \times 10^{-9}}{10^{-3}}$$
$$= 4 \times 10^{-4} \text{rad}$$

26. **(d)** Given,

Radius of loop,

$r = 10 \text{ cm} = 10 \times 10^{-2}$ m

Total charge on loop,

$q = 10^{-5}$C

Small are length,

$x = 3.14 \times 10^{-6}$ m

Let, E be the electric field. As we know that,

$$E = \frac{kq'}{r^2} \ldots. \text{(i)}$$

where, k = Coulomb's constant and q' = charge on arc length x.

$$\therefore q' = \frac{q}{2\pi r} x$$
$$= \frac{10^{-5}}{2\pi \times 10 \times 10^{-2}} \times 3.14 \times 10^{-6}$$
$$= \frac{1}{2} \times 10^{-10} \text{C}$$

Substituting in Eq. (i), we get

$$E = \frac{9 \times 10^9 \times \frac{1}{2} \times 10^{-10}}{(10 \times 10^{-2})^2}$$
$$= 4.5 \times 10^{9-10+2} = 45 \text{ N/C}$$

27. **(a)** As we know that,

$$\int E \cdot dS = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

where, E = electric field

dS = surface area

ε_0 = free space permittivity

$$\Rightarrow E \cdot 2\pi rl = \frac{\lambda ql}{\varepsilon_0}$$
$$\Rightarrow E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}$$

where,λ = charge per unit length, and r = distance from axis of cylinder.

$\therefore\ E \propto \frac{1}{r}$

28. (b) Given, outer radius, $b = 2R$ Inner radius, $a = R$

As we know that, capacitance of spherical capacitor,

$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

$$\therefore\ C = 4\pi\varepsilon_0\left(\frac{R\cdot 2R}{2R-R}\right)$$

$$= 4\pi\varepsilon_0\frac{2R^2}{R}$$

$$C = 8\pi\varepsilon_0 R$$

29. (c) Given,

Length of wire be l along X-axis.

Force, $F = IB_0 l(\hat{k}-\hat{j})$

As we know that,

$F = I(B \times l)$

where, I = current

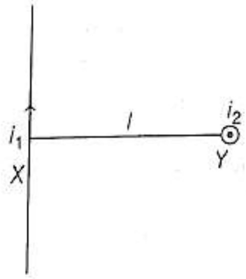
and B = magnetic field.

$\therefore IB_0 l(\hat{k}-\hat{j}) = I(l \times B)$

$\Rightarrow B_0 l(\hat{k}-\hat{j}) = (l \times B)$

$\Rightarrow B = B_0(\hat{\imath}+\hat{\jmath}+\hat{k})$

30. (a) Given, i_1 and i_2 are the current flowing through two perpendicular wires as shown below.



Magnetic field produced by first wire X on second wire Y.

$$B_1 = \frac{\mu_0}{2\pi}\cdot\frac{i_1}{l}$$

$\therefore$ Magnetic force on small length dl of the second wire Y,

$$F = i_2 B_1 dl$$

$$= i_2\cdot\frac{\mu_0}{2\pi}\frac{i_1}{l}\cdot dl$$

$$\Rightarrow\ F = \frac{\mu_0 dl}{2\pi l}\cdot i_1 i_2$$

$$\Rightarrow\ F \propto i_1 i_2$$

31. (a) Given, relative permeability,

$\mu_r = 501$

Current through solenoid,

$I = 2.5\ A$

and number of turns per unit length,

$n = \frac{N}{l} = 900$

As we know that,

$$B = \mu_0 H$$

$$\Rightarrow\ H = \frac{B}{\mu_0}$$

where,

B = magnetic flux density / magnetic induction,

μ_0 = free space permeability and H = magnetising field intensity.
Since, magnetic field due to solenoid,
$B = \mu_0 nI$
Here, μ is permeability of medium and $\mu = \mu_r \mu_0$
where, μ_r is relative permeability.

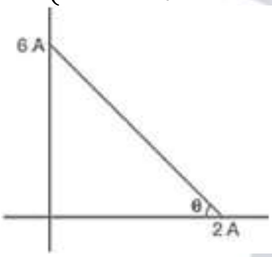
$$\therefore H = \frac{B}{\mu_0} = \frac{n\mu I}{\mu_0}$$
$$= \frac{n\mu_r\mu_0 I}{\mu_0}$$
$$\Rightarrow H = n\mu_r I$$
$$= 900 \times 501 \times 2.5$$
$$= 1.12 \times 10^6 \text{ A/m}$$

32. (a) Given, radius of solenoid be a, Number of turns per unit length be n, Current per unit time I, and distance from axis of solenoid r. Let, electric field be E.

$$\oint E \cdot dl = -\frac{d\phi}{dt}$$

Since,
$$E \cdot 2\pi r = \frac{B \cdot A}{t}$$
$$= \frac{n\mu_0 I \cdot t \cdot \pi a^2}{t}$$
$$\Rightarrow E = \frac{n\mu_0 I a^2}{2r}$$

33. (a) Given, equation of an alternating current,
$i = (2\sin\omega t + 6\cos\omega t)A$

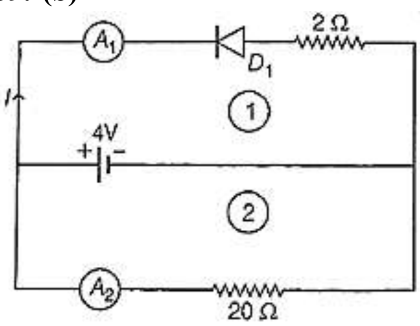


$$i = \sqrt{2^2+6^2}\left[\frac{2}{\sqrt{2^2+6^2}}\sin\omega t + \frac{6}{\sqrt{2^2+6^2}}\cos\omega t\right]$$
$$= \sqrt{40}[\cos\theta\sin\omega t + \sin\theta\cos\omega t]$$
$$i = \sqrt{40}\sin(\omega t + \theta)$$
$$\therefore i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{\sqrt{40}}{\sqrt{2}} = \sqrt{20} = 2\sqrt{5}\text{ A}$$

34. (c) Given, electric field, $E = 300$ V/m
Magnetic field, $H = 7.9$ A/m
Maximum rate of energy flow be
$$\Rightarrow S = HE$$
$$= 7.9 \times 300$$
$$= \frac{79}{10} \times 300$$
$$= 2370 \text{ W/m}^2$$

35. (c) By using de-Broglie wavelength,
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$
$$\therefore \quad \lambda \propto \frac{1}{m}$$

Since, α-particle is the heavier most.
∴ λ of α-particle will be least.
36. (c) Given,
Energy given to hydrogen atom in ground state
$= \Delta E$
Let initial angular momentum be L. Then, final angular momentum be
$$L' = L + \frac{h}{2\pi} \ldots\ldots (i)$$
Since, $L = \frac{nh}{2\pi}$
Angular momentum of ground state ($n = 1$)
$$L = \frac{1 \cdot h}{2\pi} = \frac{h}{2\pi}$$
Hence, from Eq. (i),
$$L' = \frac{h}{2\pi} + \frac{h}{2\pi} = 2 \cdot \frac{h}{2\pi}$$
Final orbit level,
$n_f = 2$
Now, electrons will transit from orbit $n = 1$ to $n = 2$
Since,
$$\Delta E = 13.6 Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \ldots\ldots (ii)$$
where, $Z =$ atomic number, and $E =$ transition energy of electron, while moving n_i to n_f and Z (hydrogen atom) $= 1$
Substituting in Eq. (ii), we get
$$\Delta E = 13.6 \times 1^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$
$$\Rightarrow \Delta E = 13.6 \left(\frac{1}{1} - \frac{1}{4}\right)$$
$$= 13.6 \times \frac{3}{4}$$
$$= 3.4 \times 3$$
$$= 10.2 \text{eV}$$
37. (b) As we know that, Radio -active decay is a very fast process, so moderators are used to slow down the neutrons produced by fission reaction, to sustain chain reaction
38. (c) As we know that, semiconductors do not conduct at room temperature but with increase of temperature, its covalent bands start breaking and its resistivity starts decreasing, therefore conductivity increases and hence, number of electrons in conduction band increases.
39. (b)



According to above circuit diagram. Diode D_1 is in reverse bias, hence no current will flow through ammeter A_1.
40. (a) We know that, Lower side band,
$f_1 = f_c - f_m \ldots\ldots (i)$
and upper side band,
$f_2 = f_c + f_m \ldots\ldots (ii)$
Adding Eqs. (i) and (ii), we get
$$f_c = \frac{f_1 + f_2}{2}$$
Subtracting Eq. (ii) from Eq. (i), we get

$$f_1 - f_2 = -2f_m$$
$$\Rightarrow \quad f_m = \frac{f_1 - f_2}{-2}$$
$$\Rightarrow \quad f_m = \frac{f_2 - f_1}{2}$$
$$\therefore \quad \frac{f_c}{f_m} = \frac{f_1 + f_2}{f_2 - f_1}$$

Chemistry

41. (d) Heisenberg's uncertainty principle states that more precisely we measure the position of a particle, less precisely you can know its velocity and vice-versa. It also states that the product of uncertainty in measurement of velocity and uncertainty in measurement of position, is significant to micro particles having a very high speed. The mathematical expression of the law is given below:

$$\Delta x \times \Delta y \geq \frac{h}{4\pi}$$

Δx = change in position of the particle
Δy = change in momentum of particle
h = Planck's constant

42. (d) We know that,

$$\lambda_e = \frac{h}{mv_e} \ldots (i)$$

Here, m = mass of electron
v_e = velocity of electron
λ_e = wavelength of electron
If wavelength (λ_e) is equal to distance travelled by the electron in one second.
i.e. $\lambda_e = v_e$
Put in Eq. (i), we get

$$\lambda_e = \frac{h}{m\lambda_e}$$
$$\Rightarrow \quad \lambda_e^2 = \frac{h}{m}$$
$$\Rightarrow \quad \lambda_e = \sqrt{\frac{h}{m}}$$

43. (d) Metal oxides which react with both acid as well as bases to produce salts and water are known as amphoteric oxides. $Ga_2O_3, As_4O_{10}, Sb_4O_{10}$, and Al_2O_3, all are amphoteric oxides because they react as acid with base and vice-versa.

44. (b) The first ionisation energy of nitrogen is 1402 kJ/mol. Due to the half- filled stable configuration of nitrogen it has maximum ionisation potential. On left to right in periodic table, effective nuclear charge is increasing and ionisation energy also increasing.

$N \rightarrow 2s^2 2p^3$
(stable half-filled orbital)
$O \rightarrow 2s^2 2p^4$
(partially filled orbital)
easily lose outer
So, B, C, N and O have ionisation energy 1086, 1314, 1402, 1681 respectively.
Hence, nitrogen have 1402 kJ/mol energy.

45. (d) "Greater is the polarisation, greater is the covalent character." For anions to get polarised, the size of an anion must be big in size, to get distorted by the cation. Therefore, bigger the size of the anion, greater the polarisability, greater the covalent character. Among the given ionic compound, the iodide anion has the largest size, it undergoes polarisation to larger extent. Hence, CaI_2 has the largest covalent character.

46. (b)

47. (d) Let, temperature be T.

Most probable velocity $= \sqrt{\frac{2RT}{M}}$

Root mean square velocity $= \sqrt{\frac{3RT}{M}}$

Most probable velocity at 7200 K = RMS velocity of He gas at 300 K .

$$\sqrt{\frac{2RT_1}{M_1}} = \sqrt{\frac{3RT_2}{M_2}}$$

$$\sqrt{\frac{2RT_1}{M_1}} = \sqrt{\frac{3RT_2}{4}}$$

($\therefore$ Atomic mass of He = 4amu)

$$\sqrt{\frac{2 \times R \times 7200}{M_1}} = \sqrt{\frac{3 \times R \times 300}{4}}$$

$$\frac{2 \times 7200}{M_1} = \frac{900}{4}$$

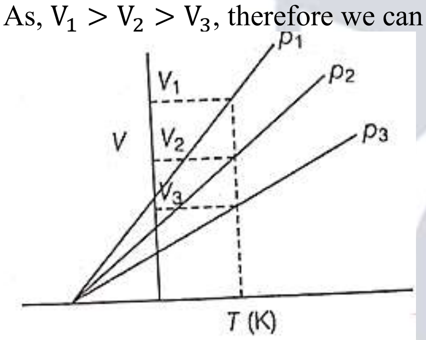
$$M_1 = \frac{2 \times 7200 \times 4}{900}$$

$$= 64(\text{ Mass of } SO_2)$$

48. (a) At a particular temperature, pV = constant

$p_1V_1 = p_2V_2 = p_3V_3$

As, $V_1 > V_2 > V_3$, therefore we can say, $p_1 < p_2 < p_3$.



It is clear that slope of V_1 is more than V_2. So, the growth of the plot between p and T at V_1 volume is higher. Since, for ideal gas p is inversely proportional to volume V. Thus, p_1 will be less than p_2 is lesser than p_3. So, $p_1 < p_2 < p_3$.

49. (b)

50. (b) Number of oxygen in one molecule of $Na_2SO_4 \cdot 10H_2O$

$= 4 + 10 = 14$

Thus, moles of oxygen in ' n ' mole of compound is 14n.

Molar mass of $Na_2SO_4 \cdot 10H_2O$

$= 46 + 32 + 64 + 180$

$= 322$ g/mol

Thus, $\frac{\text{weight of compound}}{\text{molar mass of compound}}$

$= \frac{32.2}{322} = 0.1$ mol

Now, moles of oxygen $= 0.1 \times 14 = 1.4$

So, weight = moles $\times$ molar mass

$= 16 \times 1.4 = 22.4$ g

51. (a) The hydrogenation of benzene forms cyclohexane.

$\underset{\text{Benzene}}{C_6H_6} + 3H_2 \longrightarrow \underset{\text{Cyclohexane}}{C_6H_{12}}$

Enthalpy of hydrogenation of cyclohexene $= -119.5$ kJ/mol

$\therefore$ Calculated enthalpy of benzene

$= 3 \times (-119.5)$

$= -358.5$ kJ/mol

Resonance energy of benzene

$= -150.4$ kJ/mol

As we know,

Actual value of enthalpy = Calculated enthalpy

- Resonance energy

$\therefore$ Actual value of enthalpy

$= (-358.5) - (-150.4)$

$= -208.1$ kJ/mol

Hence, the enthalpy of hydrogenation of benzene is -2081 kJ/mol.

52. (d) Greater the value of $[H^+]$ion, greater the value of ionisation constant, K_a and vice-versa.

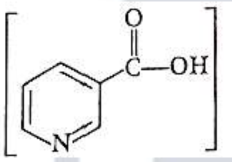
$K_W = K_a \times K_b$

$K_w \propto K_a$

Reaction, $HA \longrightarrow H^+ + A^-$

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

$\Rightarrow K_a \propto [H^+]$

$H_2C_2O_4 >$ Niacin 

$> H_2S > HCN$

Value of K_a 5.6×10^{-2} 1.5×10^{-5} 8.9×10^{-7} 4.9×10^{-10}

Hence, $A \rightarrow 3, B \rightarrow 4, C \rightarrow 2$ and $\rightarrow$ D $\rightarrow 5$.

53. (a) There is no such rule that you need equal concentrations of base and salt of conjugate acid to make a buffer. A buffer solution is one which resists changes in pH when small quantities of an acid or an alkali are added to it.

e.g. $\underset{\text{Base}}{NaOH} \longrightarrow Na^+ + \underset{\text{Conjugate acid}}{OH^{\ominus}}$

$pOH = -\log[OH^-]$ .... (i)

$pH + pOH = 14$

We know that,

$pK_b = -\log[K_b]$ ..... (ii)

Comparing Eq. (i) and (ii), we get

$[pOH = pK_b]$

So, pOH of the buffer solution is same as pK_b of acid.

54. (c)

55. (a) On moving down, the group, the atomic and ionic size of alkali metals increases. As the size increases, the extent of hydration decreases in aqueous solution. The electrical conductivity or ionic mobility is inversely proportional to extent of hydration.

So, extent of hydration

$Li^+ > Na^+ > K^+ > Cs^+$

Hence, the increasing order of electrical conductivity of alkali metals ions in their aqueous solution is

$Cs^+ > K^+ > Na^+ > Li^+$

56. (a) Among B, Al, Ga and In , Boron atom (B) is smallest in size and a large amount of energy is needed to remove 3 electrons. When one electron is removed from the p-orbital,He-like, fulfilled s-orbital is left. This is highly stable. So, the second ionisation enthalpy is quite high.

Again wher one electron is removed a half-filled orbital is left. So, third ionisation enthalpy is also quite high.

$B = 1s^2 2s^2 2p^1$

$[B(H_2O)_6]^{3+}$

$\Rightarrow B^{3+} \rightarrow$ Highly unstable (due to very high ionisation energy)
Since, the total energy needed to make B^{3+} is the total of all ionisation enthalpies an enormous amount of energy is required to form it.

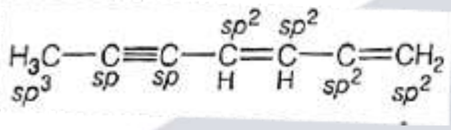
57. (d) (a) Isotope of carbon, i.e. ^{13}C is a natural, stable isotope of carbon with a nucleus containing six protons and seven neutrons. It make up about 1.1% of all natural carbon on Earth.
(b) On moving from top to bottom in IV A group, the melting point decreases. The decreasing order of melting point

$\underset{3550°C}{C} > \underset{1414°C}{Si} > \underset{938°C}{Ge} > \underset{327°C}{Pb} > \underset{232°C}{Sn}$

(c) On Earth, oxygen is most common element, making up about 47% of Earth's mass. Silicon is second making up 28% followed by aluminium (8%), iron (5%) etc.
(d) The resistivity of semiconductors depends strongly on the presence of impurities in the material.
Carbon resistivity (Ωm)
$= 3 \times 10^5 - 60 \times 10^{-5}$
Hence, all the statements are correct.

58. (b) $Cl_2C = CCl_2, CO_2$ (liquid), H_2O_2, benzene, benzol, CCl_4, trichloroethylene are some of the frequently used dry cleaning fluids. However, tetrachloroethylene (perchloro ethylene) is a common solvent used in dry cleaning because it is a highly vapourising organic fluid. But acetaldehyde is not used for dry purpose due to its decomposition property.

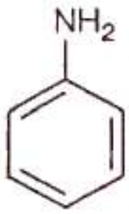
59. (a) Hepta-1, 3-dien-5-yne (IUPAC name)

Structure: 

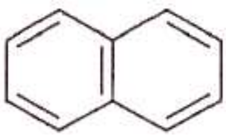
There are 2sp carbons and 4sp^2 carbons present in hepta-1, 3-dien-5-yne structure.

60. (b)

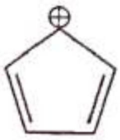
61. (b) Aromatic compounds are compounds that consist of conjugated planar ring system accompanied by delocalised π-electron cloud in place of individual alternating double or single bond compound which follow Huckel's rule or $(4n+2)\pi$ rule are called aromatic compound.



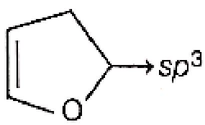
$\rightarrow (4n+2)\pi$ rule follow
$\rightarrow$ Aromatic
(due to phenyl group attached to an amino group)



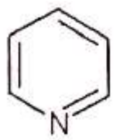
$\rightarrow (4n+2)\pi$ rule follow $\Rightarrow 10e^-$
(Aromatic)



$\rightarrow 4n\pi$ rule $\rightarrow 4e^-$(Non-aromatic
or anti aromatic)



$\rightarrow$ Non-aromatic (due to sp^3 carbon).



$\rightarrow (4n+2)\pi$ rule follow (Aromatic) due to $6e^-$.
Hence, total number of aromatic compound is 3.

62. (c) Atom of B forms HCP lattice, so the atom at corners are shared by 6 unit cell.
So, its contribution is $1/6$.
fcc atom contribute $\frac{1}{2}$ and middle layer atom contribute 1 each.
Effective number of atoms in unit cell

$$= \frac{1}{6} \times 12 + \frac{1}{2} \times 2 + 1 \times 3$$
$$= 2 + 1 + 3 = 6$$

Number of tetrahedral voids $= 2 \times$ number of atoms forming the lattice
$= 2 \times 6 = 12$
Number of atoms of A $= \frac{2}{3} \times 12 = 8$.
Number of atoms of B $= 6$.
Ratio of A: B is 8: 6.
or, in simplest form $= 4:3$
So, formula of the compound is A_4B_3.

63. (c) As we know, the relation between molarity (M) and molality (m).

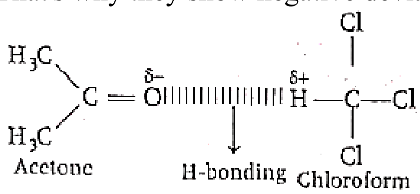
$$m = \frac{1000 \times M}{(1000 \times d) - (M \times \text{ molar mass })} \ldots \text{(i)}$$

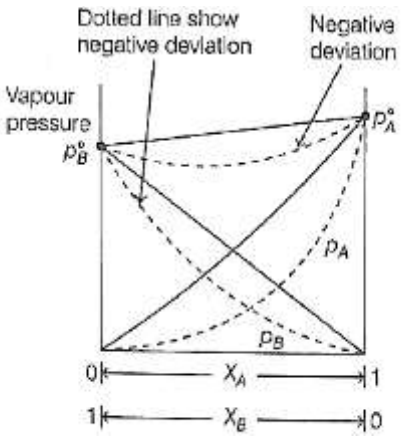
here, m = molality
M = molarity $\Rightarrow M = 2$ (given)
d = density of solution in g/mL
($d = 1.11$ g/mL)
Molar mass of ethylene glycol
$\Rightarrow 62.07$ g/mol
Put all values in equation number (i), we get

$$m = \frac{1000 \times 2}{(1000 \times 1.11) - (2 \times 62.07)} \text{g/mol}$$
$$= \frac{2000}{1110 - 124.14} = \frac{2000}{985.86}$$
$$= 2.028 \cong -2.05$$

Hence, molality of the solution is 2.05 m.

64. (b) Raoult's law is used to estimate the contribution of individual components of a liquid or solid mixture to the total pressure exerted by the system.
(A) Phenol and aniline A solution of phenol and aniline will show negative deviation from Raoult's law. In case of negative deviations from Raoult's law, A − B forces are stronger than A − A and B − B forces. In phenol and aniline amount of H -bonding increase due to increase in number of hydrogen atoms of aniline.
(B) Chloroform and acetone A solution of chloroform and acetone shows negative deviation from Raoult's law because they have an attractive interaction happening between them which results in the formation of hydrogen bonding.
That's why they show negative deviation from Raoult's law.





Both the solutions i.e. A and B, shows negative deviation.

65. (c) According to Faraday's law,

$W = ZQ$

Here, W = weight of substance discharged at an electrode.

Z = Electrochemical equivalent

$Z = \frac{E}{96500}$

Q = Electricity/Charge (Coulomb)

$W = \frac{E \times Q}{96500}$

For A, $2.4 = \frac{\left(\frac{8}{x}\right)\times Q}{96500}$

For B, $1.8 = \frac{\left(\frac{18}{y}\right)\times Q}{96500}$

For C, $7.5 = \frac{\left(\frac{50}{z}\right)\times Q}{96500}$

For ratio of A, B and C, we get,

$x:y:z = \left(\frac{8}{2.4}\right):\left(\frac{18}{1.8}\right):\left(\frac{50}{7.5}\right)$

$= 3.33:10:6.66 = 1:3:2$

By solving equation, $A = 1, B = 3$ and $C = 2$

So, valence of A, B and C are 1,3 and 2.

66. (a) The rate expression is $\frac{dx}{dt}$ or rate $= k[A]^{\frac{1}{2}}[B]^{\frac{3}{2}}$ · order of the reaction with respect to A and B is 1/2 and 3/2 respectively.

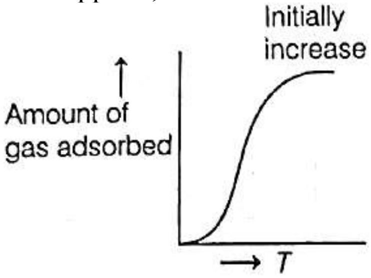
Hence, we count stiochiometry of the A and B as an order of the reaction.

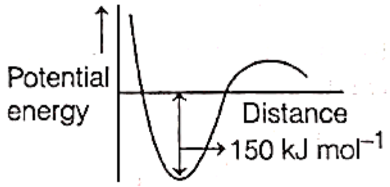
The overall order of the reaction is

$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$

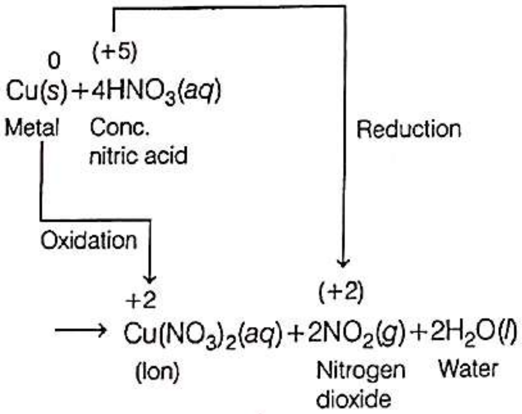
Given rate expression is for second order reaction.

67. (c) Chemisorption requires high activation energy. So, it is referred to as activated adsorption. In chemisorption, adsorption first increases and then decreases with an increase in temperature. The initial increase is due to heat supplied, which act as activation energy required in Chemisorption

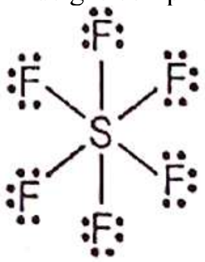




68. (c) Copper is oxidised by concentrated nitric acid to produce $Cu(NO_3)_2$ ions. The nitric acid is reduced to form nitrogen dioxide (NO_2) which is a poisonous brown gas with an irritating odour.



69. (b) SF_6 is extremely stable and chemically inert gas for stearic reasons. In SF_6, the six F atoms protect the sulphur atoms from attack by reagent to such an extent that even thermodynamically most favourable reactions like hydrolysis do not occur. S is completely blocked by fluorine atoms from all directions. SF_6 is a man-made gas comprising of one sulphur and 6 fluoride atoms.



70. (c) The complete hydrolysis of XeF_4 and XeF_6 produces XeO_3 which is xenon trioxide. This xenon trioxide is highly explosive and acts as a powerful oxidising agent in solution.

Complete hydrolysis of XeF_4

$$\underset{\text{Xenon tetrafluoride}}{6XeF_4} + 12H_2O \longrightarrow \underset{\text{Xenon}}{4Xe} + \underset{\substack{\text{Xenon}\\ \text{trioxide}\\ \text{(P)}}}{2XeO_3} + \underset{\substack{\text{Hydrogen}\\ \text{fluoride}}}{24HF} + 3O_2$$

XeF_4 on partial hydrolysis produces $XeOF_2$ (xenon oxyfluoride).

Complete hydrolysis of XeF_6 XeF_6 is one of 3 binary fluoride formed by the xenon. XeO_3 is highly explosive and acts as a powerful oxidising agent in solution.

$$\underset{\substack{\text{Xenon}\\ \text{hexafluoride}}}{XeF_6} + 3H_2O \longrightarrow \underset{\substack{\text{Xenon}\\ \text{trioxide}\\ \text{(Q)}}}{Xe\ O_3} + \underset{\substack{\text{Hydrogen}\\ \text{fluoride}}}{6}$$

71. (b)

72. (c) Oxidation number also called oxidation state is the total number of electrons that an atom either gains or loses in order to form a chemical bond with another form.

$[Pt(NH_3)_2Cl(NO_2)]$

Let x be the oxidation state of platinum. The oxidation state of ammonia (NH_3) and NO_2 is zero.

The oxidation state of chlorine is -1.

$x + 2(0) + 2(-1) + 0 = 0$

$x - 2 = 0 \Rightarrow x = +2$

$[CoCl_2(en)_2]^+$

Let, y be the oxidation state of cobalt. The oxidation state of en is 0 and oxidation state of chlorine is -1.

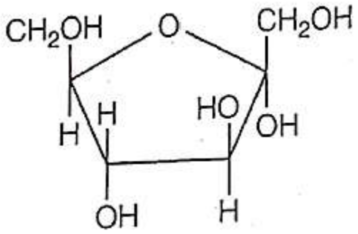
$y + 2(-1) + 0 - 1 = 0$

$y - 2 - 1 = 0 \Rightarrow y - 3 = 0$

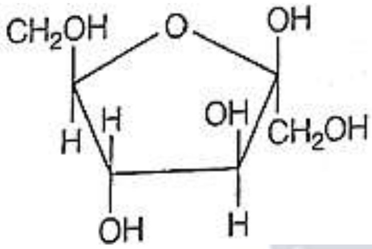
$y = +3$

So, oxidation number of central metal in $[Pt(NH_3)_2Cl(NO_2)]$ is +2 and in $[CoCl_2(en)_2]^+$is +3.

73. **(c)** α-D-fructofuranose It's a structure is analogous to the cyclic structure called as furan, which is a 5 membered ring. It is an enantiomer of an α - L-fructofuranose. It is non-reducing sugar.

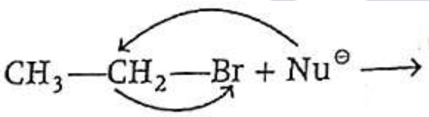


α-D-fructofuranose



ß-D-fructofuranose

74. **(a)**



$CH_3 - CH_2 - Nu + Br^{\ominus}$

Above reaction is known as nucleophilic substitution reaction. Bimolecular nucleophilic substitution reaction follows second-order kinetics. The rate of reaction depends on the concentration of two first-order reactants.

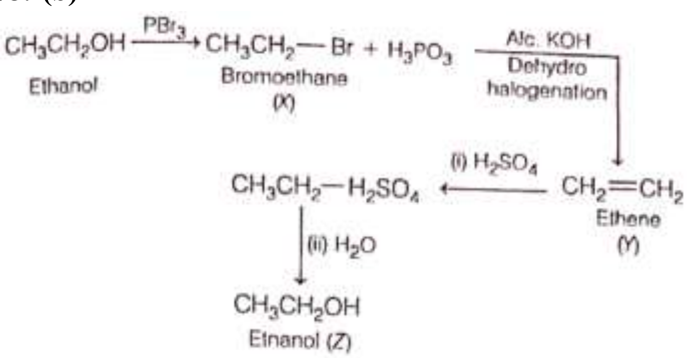
Hydroxide ion $(OH^-)$is stronger nucleophile than acetate ion as in acetate ion, the negative charge is involved in resonance with carboxylic group. Methoxide ion $(CH_3O^-)$is more nucleophilic than hydroxide ion (OH^-) due to $+I$ effect of methyl group.

PhO° is very weak nucleophile because the negative charge is in resonance with ring and charge further decreases.

The order is

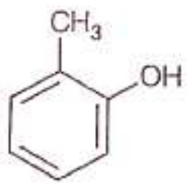
$\underset{(IV)}{CH_3O^{\circ}} > \underset{(III)}{OH^{\ominus}} > \underset{(I)}{PhO^{\ominus}} > \underset{(II)}{CH_3COO^{\ominus}}$

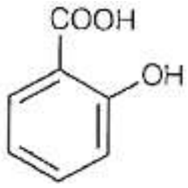
75. **(b)**



In first step, nuclcophilic substitution reaction occur to form bromoethane (X). In next step, dehydrohalogenation reaction gives alkene i.e. ethene (Y). In final step, hydrolysis will be occur to form ethanol (Z).

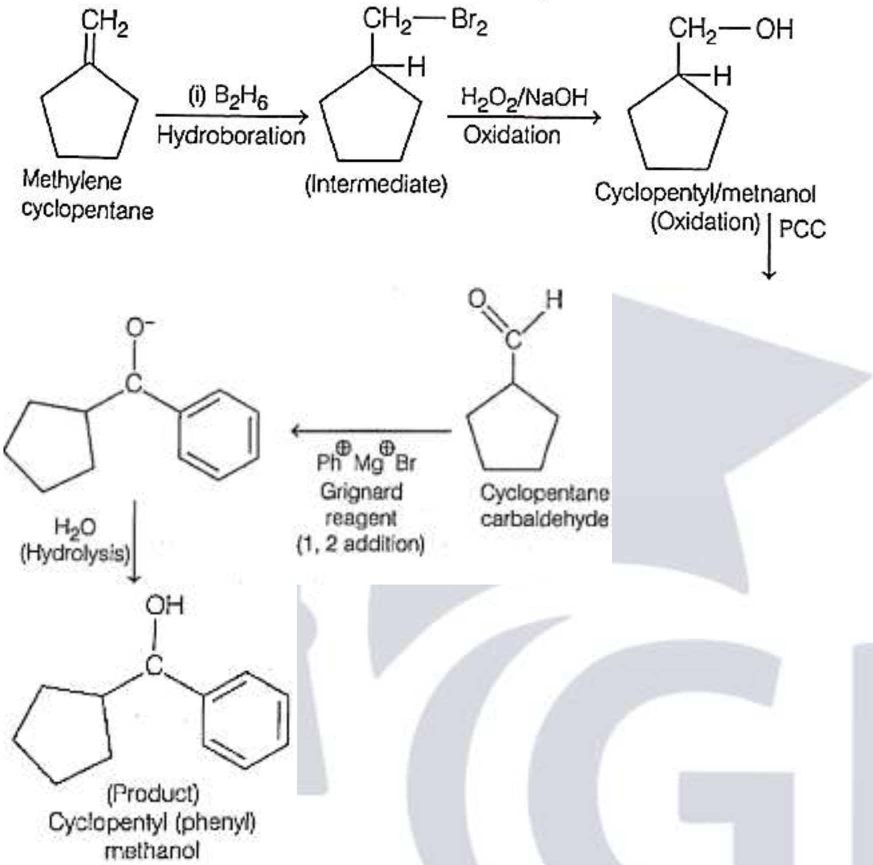
76. **(c)** Phenol when treated with zinc dust, undergo reduction to form benzene. Only phenol gives deoxygenation reaction by using zinc with supply of heat.



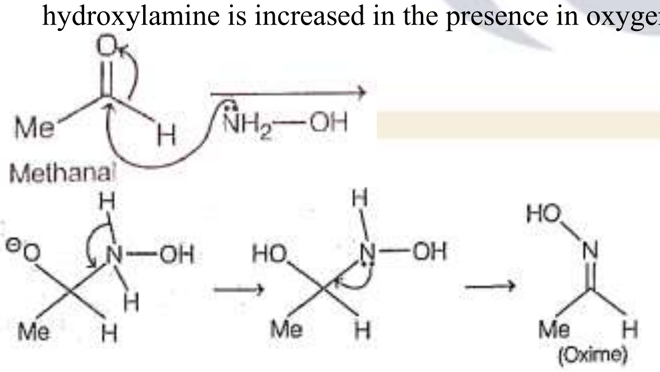


(B) (D)
o-cresol 2-hydroxy benzoic acid
∴ (B) and (D) give deoxygenaration reaction as they contain aromatic alcohol group (-OH).

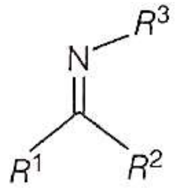
77. **(c)** In given reaction, first two step is hydroboration oxidation reaction which form alcohol. PCC is used for oxidation of alcohol into aldehyde. PbMgBr is used as a Grignard reagent to attack in a carbonyl carbon.



78. **(a)** (A) Reaction of aldehyde and ketones with hydroxylamine gives oximes. The nucleophilicity of nitrogen on hydroxylamine is increased in the presence in oxygen. Methanal reacts with hydroxylamine to give oxime.



(B) Water is added to aldehyde and ketones to form geminal-diol. In a similar reaction alcohols add reversely to aldehyde and ketones to form hemiacetals.
(C) Acetone azine can be prepared from acetone and hydrazine. The hydrazone then condense with molecule of acetone to produce azine.
(D) A Schiff base is a compound with the general structure containing amine $R_1R_2C = NR'$. It is used as catalysts, pigments etc.



So, correct match is A → 3, B → 4, C → 1, D → 2.

79. (a)

80. (b) Given reaction is Hofmann bromide degradation reaction. An amide reacts with bromine and aqueous solution of sodium hydroxide which produces primary amine. This is a degradation reaction as primary amine in the product has one carbon lesser than primary amide (in reactant).

$$\underset{\text{Pentanamide}}{CH_3CH_2CH_2CH_2CONH_2} + Br_2 + \underset{\substack{\text{Sodium}\\\text{hydroxide}}}{4NaOH}$$

$$\longrightarrow \underset{\text{Butan-l-amine}}{CH_3CH_2CH_2CH_2NH_2} + \underset{\substack{\text{Sodium}\\\text{carbonate}}}{Na_2CO_3} + \underset{\substack{\text{Sodium}\\\text{bromide}}}{2NaBr} + 2H_2O$$

Mathematics

81. (c) Given, $f(x+y) = f(x) + f(y)$ and

$f(l) = 10 = 1 \times 10$

Then,

$$\begin{aligned} f(2) &= f(1+1) = f(1) + f(1) \\ &= 10 + 10 = 2 \times 10 \\ f(3) &= f(2+1) = f(2) + f(1) \\ &= 20 + 10 = 3 \times 10 \\ f(4) &= f(3+1) = f(3) + f(1) \\ &= 30 + 10 = 4 \times 10 \end{aligned}$$

$\ldots f(n) = n \cdot 10$

Now,

$$\sum_{r=1}^{n} [f(r)]^2 = [f(1)]^2 + [f(2)]^2 + [f(3)]^2 + \ldots + [f(n)]^2$$

$$= (1 \cdot 10)^2 + (2 \cdot 10)^2 + (3 \cdot 10)^2 + \cdots + (n \cdot 10)^2$$

$$= 10^2(1^2 + 2^3 + 3^2 + \cdots + n^2)$$

$$= 100 \times \frac{n(n+1)(2n+1)}{6}$$

$$\left[\because \sum n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{50}{3} n(n+1)(2n+1)$$

82. (a) If $f(x) = \frac{K^x + K^{-x}}{2}$, $(K \in R^+)$, then

$f(x+y) + f(x-y) = 2f(x)f(y)$

Here, $f(x) = \frac{3^x + 3^{-x}}{2}$, then $a = 2$.

83. (a)

84. (d) Given, system of equations

$$\begin{aligned} 3x - 2y + z &= 0 \\ \lambda x - 14y + 15z &= 0 \\ x + 2y - 3z &= 0 \end{aligned}$$

can be written as

$$\begin{bmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

i.e. $AX = 0$

$\Rightarrow |A| = 0$ (for non-trivial solution of homogeneous equation)

$$\Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$3(42-30)+2(-3\lambda-15)+1(2\lambda+14)=0$

$\Rightarrow 36-6\lambda-30+2\lambda+14=0$

$\Rightarrow -4\lambda+20=0$

$\Rightarrow \lambda=5$

85. (b)

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & (b+a)(b-a) \\ 0 & c-a & (c+a)(c-a) \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$= (b-a)(c-a)[(c+a)-(b+a)]$

expanding along C_1

$= (b-a)(c-a)(c-b)$

$= (a-b)(b-c)(c-a)$

$\therefore\ K=1$

86. (b) Matrix representation of given system of equations,

$$\begin{bmatrix} 5 & -7 & 3 \\ 7 & 10 & -8 \\ 2 & 3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix}$$

By Cramer's rule,

$$y = \beta = \frac{\Delta_2}{\Delta}$$

Where, $\Delta_2 = \begin{vmatrix} 5 & 0 & 3 \\ 7 & 3 & -8 \\ 2 & -4 & -4 \end{vmatrix}$

$= 5(-12-32)-0+3(-28-6)$

$= 5 \times (-44) + 3 \times (-34)$

$= -322$

and $\Delta = \begin{vmatrix} 5 & -7 & 3 \\ 7 & 10 & -8 \\ 2 & 3 & -4 \end{vmatrix}$

$= 5(-40+24)+7(-28+16)$

$+3(21-20)$

$= -161$

$$\therefore\ \beta = \frac{-322}{-161} = 2$$

87. (b)

$a+ib=(2+3i)^4$

$= 2^4 + {}^4C_1 2^3(3i) + {}^4C_2 2^2(3i)^2$

$+ {}^4C_3 2(3i)^3 + (3i)^4$

$= 16+4\cdot 8\cdot 3i+6\cdot 4\cdot 9\cdot(-1)$

$+4\cdot 2\cdot 27(-i)+81$

$= 16+96i-216-216i+81$

$= -119-120i$

$\therefore a=-119, b=-120$

Now, $3b-2a=3(-120)-2(-119)$

$= -360+238=-122$

88. **(c)** (A)$\omega^{1010}+\omega^{2020}$

$=\omega^{336\times3+2}+\omega^{673\times3+1}$

$=(\omega^3)^{336}\cdot\omega^2+(\omega^3)^{673}\cdot\omega$

$=\omega^2+\omega=-1\ [\because \omega^3=1]$

(B) $(1+\omega-\omega^2)(1-\omega+\omega^2)$

$[\because 1+\omega+\omega^2=0]$

$=(-\omega^2-\omega^2)(-\omega-\omega)$

$=(-2\omega^2)(-2\omega)$

$=4\omega^3=4$

(C) $(2+\omega^2+\omega^4)^5=(2+\omega^2+\omega)^5$

$=(1+1+\omega+\omega^2)^5=(1+0)^5=1$

(D) $(3+5\omega+3\omega^2)^3$

$=\left(3+2\omega+3(\omega+\omega^2)\right)^3$

$=(3+2\omega-3)^3=(2\omega)^3=8$

89. **(c)**

$z-4=(8i)^{1/3}$

$z=4+2(i)^{1/3}$

$i=e^{i\pi/2}=e^{i\left(\frac{\pi}{2}+2n\pi\right)}$

$i^{1/3}=e^{i\left(\frac{\pi}{6}+\frac{2n\pi}{3}\right)}, n=0,1,2$

$$\left\{\begin{aligned} & e^{i\frac{\pi}{6}}=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}=\frac{\sqrt{3}}{2}+i\frac{1}{2} \\ & e^{i\left(\frac{\pi}{6}+\frac{2\pi}{3}\right)}=e^{i\frac{5\pi}{6}}=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6} \\ & =\frac{-\sqrt{3}}{2}+i\frac{1}{2} \qquad e^{i\left(\frac{\pi}{6}+\frac{4\pi}{3}\right)}=e^{i\frac{3\pi}{2}} \\ & =\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}=0-i \end{aligned}\right\}$$

For, $n=0, e^{i\pi/6}=\frac{\sqrt{3}}{2}+i\frac{1}{2}$

$n=1, e^{i5\pi/6}=-\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}$

$n=2, e^{i\frac{3\pi}{2}}=0-i$

$z=4+2\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$

$=(4+\sqrt{3})+i=b+i$

$z=4+2\left(\frac{-\sqrt{3}}{2}+\frac{i}{2}\right)$

$=(4-\sqrt{3})+i=c+i$

$z=4+2(0-i)=4-2i=a-2i$

$\Rightarrow a=4, b=4+\sqrt{3}$ and $c=4-\sqrt{3}$

$\therefore \sqrt{abc}=\sqrt{4(4+\sqrt{3})(4-\sqrt{3})}$

$=\sqrt{4(16-3)}=2\sqrt{13}$

90. **(b)** Let $f(x)=x^2+7x+3=0$

Roots are $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$.

Equation having roots α and β is

$f\left(\frac{x}{x+1}\right)=0$

$\Rightarrow \left(\frac{x}{x+1}\right)^2+7\left(\frac{x}{x+1}\right)+3=0$

$\Rightarrow x^2+7x\,(x+1)+3(x+1)^2=0$

$\Rightarrow x^2+7^2+7x+3(x^2+2x+1)=0$

$\Rightarrow 11x^2+13x+3=0$

91. (a)

$y=\frac{x^2+14x+9}{x^2+2x+3}$

$\Rightarrow yx^2+2yx+3y=x^2+14x+9$

$\Rightarrow (y-1)x^2+(2y-14)x+(3y-9)=0$

$\because\ x \in R$

$\Rightarrow$ Discriminant ≥ 0

$\Rightarrow\ (2y-14)^2-4(y-1)(3y-9)\geq 0$

$\Rightarrow\ 4[(y-7)^2-3(y-1)(y-3)]\geq 0$

$\Rightarrow\ [y^2-14y+49-3(y^2-4y+3]\geq 0$

$\Rightarrow\ -2y^2-2y+40\geq 0$

or $y^2+y-20\leq 0$

or $(y+5)(y-4)\leq 0$

$\Rightarrow\ y\in[-5,4]$

92. (a) Let $f(x)=x^4-2x^3+6x-21=0$

Required equation where roots are squares of the roots of $f(x)$ is

$f(\sqrt{x})=0$

$(\sqrt{x})^4-2(\sqrt{x})^3+6\sqrt{x}-21=0$

$\Rightarrow\ x^2-2x\sqrt{x}+6\sqrt{x}-21=0$

$\Rightarrow\ x^2-21=2x\sqrt{x}-6\sqrt{x}$

$\Rightarrow\ x^2-21=2\sqrt{x}(x-3)$

$\Rightarrow\ (x^2-21)^2=4x(x-3)^2$

$\Rightarrow x^4-42x^2+441=4x(x^2-6x+9)$

$\Rightarrow x^4-42x^2+441=4x^3-24x^2+36x$

$\Rightarrow x^4-4x^3-18x^2-36x+441=0$

93. (a) α,β and γ are the roots of

$x^3-x+1=0 \ldots.(i)$

We have to find a polynomial whose roots are

$\frac{1+\alpha}{1-\alpha},\frac{1+\beta}{1-\beta},\frac{1+\gamma}{1-\gamma}$

Let $y=\frac{1+x}{1-x}\Rightarrow x=\frac{y-1}{y+1}$

Substituting $x=\frac{y-1}{y+1}$ in Eq. (i), we

have $\left(\frac{y-1}{y+1}\right)^3-\left(\frac{y-1}{y+1}\right)+1=0$

$\Rightarrow (y-1)^3-(y-1)(y+1)^2+(y+1)^3=0$

$\Rightarrow y^3-1+3y-3y^2-(y^2-1)$

$(y+1)+(y^3+1+3y+3y^2)=0$

$\Rightarrow y^3-1+3y-3y^2-y^3-y^2+y+1$

$+y^3+1+3y+3y^2=0$

$\Rightarrow y^3-y^2+7y+1=0$

$\therefore$ Sum of roots

$$=\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}$$
$$=\frac{-(-1)}{1}=1$$

94. (d)

95.

(c)$^{22}P_{r+1}:{}^{20}P_{r+2}=11:52$

$52\times{}^{22}P_{r+1}=11\times{}^{20}P_{r+2}$

$\Rightarrow 52\times\frac{22!}{(21-r)!}=11\times\frac{20!}{(18-r)!}$

$\Rightarrow \frac{52\times22\times21}{(21-r)(20-r)(19-r)}=11\times1$

$\Rightarrow (21-r)(20-r)(19-r)=52\times2\times21$

$\Rightarrow (21-r)(20-r)(19-r)=14\times13\times12$

$\Rightarrow (21-r)(20-r)(19-r)$

$=(21-7)(20-7)(19-7)$

$\Rightarrow r=7$

96. (a) Greatest term of

$$(1+x)^n=\frac{(n+1)|x|}{1+|x|}$$

$$(2x+3y)^{11}=\left(2\cdot\frac{1}{2}+3\cdot\frac{1}{3}\right)^{11}=(1+1)^{11}$$

$$\text{Greatest term }=\frac{(11+1)|1|}{1+|1|}=\frac{12}{2}=6$$

$$\therefore T_6=T_{5+1}\quad {}^{11}C_5=\frac{11\times10\times9\times8\times7}{1\times2\times3\times4\times5}$$
$$=462$$

97. (d)

1) $(1-x-x^2+x^3)^6$

$=[(1-x)(1-x^2)]^6=(1-x)^{12}(1+x)^6$

In the expansion of $(1-x)^{12}$, coefficient are of the form $(-1)^r\,{}^{12}C_r$ and in $(1+x)^{12}$, coefficient are of the form ${}^6C_r$

Coefficient of x^4 in expansion of

$(1-x-x^2+x^3)^6$

$={}^{12}C_0\times{}^6C_4-{}^{12}C_1\times{}^6C_3+{}^{12}C_2\times{}^6C_2$

$-{}^{12}C_3\times{}^6C_1+{}^{12}C_4\times{}^6C_0$

$=15-240+990-1320+495=-60$

98. (c) Sum of coefficient in the expansion of

$$\left(1+\frac{x}{2}\right)^{12}=\left(1+\frac{1}{2}\right)^{12}=\left(\frac{3}{2}\right)^{12}$$

[put variable = 1]

[$\because$ sum of coefficients in the expansion of $(1+x)^n=2^n$ which can be obtained by putting $x=1$]

99. (c) $3x^2+4x+7$

$=A+B(x-2)+C(x-2)^2$

Given,

$$f(x)=f(a)+f'(a)(x-a)+\frac{f'(a)}{2!}(x-a)^2$$

and $g(x)=3x^2+4x+7$

$$\therefore\ a=2A=g(2),B=g'(2),C=\frac{g'(2)}{2!}$$

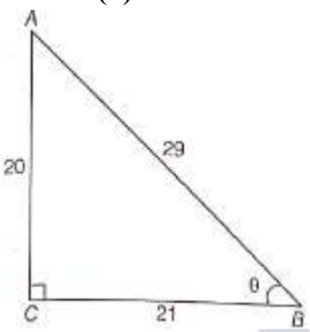
$$\Rightarrow A+B+C=g(2)+g'(2)+\frac{g'(2)}{2!}$$

100. (b)

$$\cot\frac{\pi}{16}\cdot\cot\frac{2\pi}{16}\cdot\cot\frac{3\pi}{16}$$
$$\cdot\cot\frac{4\pi}{16}\cdot\cot\frac{5\pi}{16}\cdot\cot\frac{6\pi}{16}\cdot\cot\frac{7\pi}{16}$$
$$=\left(\cot\frac{\pi}{16}\cdot\cot\frac{7\pi}{16}\right)\cdot\left(\cot\frac{2\pi}{16}\cdot\cot\frac{6\pi}{16}\right)$$
$$\cdot\left(\cot\frac{3\pi}{16}\cdot\cot\frac{5\pi}{16}\right)\cdot\cot\frac{4\pi}{16}$$
$$=\left[\cot\frac{\pi}{16}\cot\left(\frac{\pi}{2}-\frac{\pi}{16}\right)\right]$$
$$\cdot\left[\cot\frac{2\pi}{16}\cdot\cot\left(\frac{\pi}{2}-\frac{2\pi}{16}\right)\right]$$
$$\cdot\left[\cot\frac{3\pi}{16}\cdot\cot\left(\frac{\pi}{2}-\frac{3\pi}{16}\right)\right]\cdot\cot\frac{\pi}{4}$$
$$=\left(\cot\frac{\pi}{16}\cdot\tan\frac{\pi}{16}\right)\cdot\left(\cot\frac{2\pi}{16}\cdot\tan\frac{2\pi}{16}\right)$$

$$\left(\cot\frac{3\pi}{16}\cdot\cot\frac{3\pi}{16}\right).1$$
$$=1\cdot1\cdot1\cdot1=1$$

101. (b)



AB $=29$ units, BC $=21$ units, then

$$\text{AC}=\sqrt{29^2-21^2}=20\text{ units}$$

$$\cos\theta=\frac{\text{BC}}{\text{AB}}=\frac{21}{29}\text{ and }\sin\theta=\frac{\text{AC}}{\text{AB}}=\frac{20}{29}$$

$$\therefore\cos^2\theta-\sin^2\theta=\left(\frac{21}{29}\right)^2-\left(\frac{20}{29}\right)^2=\frac{41}{841}$$

102. (c)

$$\frac{1-\cos2\theta+\sin2\theta}{1+\cos2\theta+\sin2\theta}$$
$$=\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$$
$$=\frac{\sin\theta}{\cos\theta}\left(\frac{\sin\theta+\cos\theta}{\sin\theta+\cos\theta}\right)=\tan\theta$$

103. (a)

$y = 4\sin^2\theta - \cos 2\theta$

$= 4\sin^2\theta - (1 - 2\sin^2\theta)$

$y = 6\sin^2\theta - 1$

$\because\ 0 \le \sin^2\theta \le 1$

$\Rightarrow\ 0 \le 6\sin^2\theta \le 6$

$\Rightarrow\ -1 \le 6\sin^2\theta - 1 \le 5$

or $\ -1 \le y \le 5$

Maximum value of $y = 5 \Rightarrow m = 5$

Minimum. value of $y = -1 \Rightarrow l = -1$

$lm = -5, \frac{m}{l} = -5$

$\therefore\ lm = \frac{m}{l}$

104. **(b)** $\cos\theta = \frac{-1}{\sqrt{2}}$ and $\tan\theta = 1$

$\Rightarrow\ \cos\theta = -\cos\frac{\pi}{4}$ and $\tan\theta = \tan\frac{\pi}{4}$

$\Rightarrow\ \cos\theta = \cos\left(\pi + \frac{\pi}{4}\right)$ and

$\tan\theta = \tan\left(\pi + \frac{\pi}{4}\right)$

[$\because$ in IIIrd quadrant, cosine is negative and tangent is positive]

$\Rightarrow\ \theta = (2n+1)\pi + \frac{\pi}{4}, n = 0,1,2,3, \ldots$

105. **(a)**

$\sin^{-1}x < \cos^{-1}x$

$\Rightarrow\ \sin^{-1}x < \frac{\pi}{2} - \sin^{-1}x$

$\Rightarrow$

$2\sin^{-1}x < \frac{\pi}{2}$

$\Rightarrow\ \sin^{-1}x < \frac{\pi}{4}$

$\Rightarrow$

$\Rightarrow\ x < \sin\frac{\pi}{4}$

$\Rightarrow\ x < \frac{1}{\sqrt{2}} \ldots.(i)$

For $\sin^{-1}x$ domain is $-1 \le x \le 1 \ldots.(ii)$

From Eq. (i) and (ii), we get

$-1 \le x < \frac{1}{\sqrt{2}}$

106. **(a)**

$$\coth^{-1}(x) = \frac{1}{2}\log\left(\frac{x+1}{x-1}\right)$$

$x < -1$ or $x > 1$

$$\text{and } \operatorname{cosech}^{-1}(x) = \log\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right)$$

$x \neq 0$

$\therefore \coth^{-1}(2) + \operatorname{cosech}^{-1}(-2\sqrt{2})$

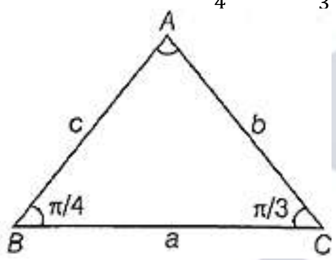
$$= \frac{1}{2}\log\left(\frac{2+1}{2-1}\right)$$

$$+\log\left(\frac{-1}{2\sqrt{2}} + \sqrt{\frac{1}{(-2\sqrt{2})^2} + 1}\right)$$

$$= \frac{1}{2}\log 3 + \log\left(\frac{-1}{2\sqrt{2}} + \frac{3}{2\sqrt{2}}\right)$$

$$= \log\sqrt{3} + \log\frac{1}{\sqrt{2}} = \log\sqrt{\frac{3}{2}}$$

107. **(d)** $\angle B = \frac{\pi}{4}, \angle C = \frac{\pi}{3}$



$$\angle A = \pi - \frac{\pi}{4} - \frac{\pi}{3} = \frac{5\pi}{12}$$

By sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow b = a\frac{\sin B}{\sin A}$$

and $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}a^2\frac{\sin B}{\sin A}\sin C$

$$\therefore \quad 54 + 18\sqrt{3} = \frac{1}{2}a^2\frac{\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$\Rightarrow \quad a^2 = 36(\sqrt{3}+1)^2$$

$$\therefore \quad a = 6(\sqrt{3}+1)$$

108. **(b)**

$r_1 = 12, r_2 = 6, r_3 = 4$

$$\frac{1}{r} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1}{2}$$

$\therefore r = 2$

$a^2 = (r_1 - r)(r_2 + r_3)$

$= (12 - 2)(6 + 4) = 100$

$a = 10$

$b^2 = (r_2 - r)(r_1 + r_3)$

$= (6 - 2)(12 + 4) = 64 \Rightarrow b = 8$

$c^2 = (r_3 - r)(r_1 + r_2)$

$= (4 - 2)(12 + 6) = 36 \Rightarrow c = 6$

$\therefore a + 2b + 3c = 10 + 2 \times 8 + 3 \times 6 = 44$

109. (a)

$r_1 + r_2 + r_3 - r = 4R$

$\therefore\ r_1 + r_2 + r_3 = 4R + r \ldots\ldots (i)$

Let $\frac{a}{4} = \frac{b}{5} = \frac{c}{6} = k$

Then,

$a = 4k, b = 5k, c = 6k$

$$s = \frac{4k + 5k + 6k}{2} = \frac{15k}{2}$$

Area $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\frac{15k}{2}\left(\frac{15k}{2} - 4k\right)\left(\frac{15k}{2} - 5k\right)\left(\frac{15k}{2} - 6k\right)} = 15\sqrt{7}\frac{k^2}{4}$$

$$\therefore\ r = \frac{\Delta}{s} = \frac{15\sqrt{7}\cdot\frac{k^2}{4}}{\frac{15k}{2}} = \sqrt{7}\frac{k}{2} \ldots\ldots (ii)$$

and $R = \frac{abc}{4\Delta} = \frac{(4k)\cdot(5k)(6k)}{4\times 15\sqrt{7}\frac{k^2}{4}} = \frac{8k}{\sqrt{7}} \ldots (iii)$

Now, $\frac{1}{4R}[r_1 + r_2 + r_3] = \frac{4R+r}{4R}$ [from Eq. (i)]

$$= 1 + \frac{r}{4R} = 1 + \frac{\sqrt{7}\cdot k/2}{4\times\frac{8k}{\sqrt{7}}}$$

$$= 1 + \frac{7}{64} = \frac{71}{64}$$

110. (b)

$a = 12, b = 16, c = 20$

$$s = \frac{a+b+c}{2} = \frac{12 + 16 + 20}{2} = 24$$

$$\Delta = \sqrt{24(24 - 12(24 - 16)(24 - 20)} =$$

$$= \sqrt{24 \times 12 \times 8 \times 4} = 96$$

$$r_1 = \frac{\Delta}{s - a} = \frac{96}{24 - 12} = \frac{96}{12} = 8.$$

$$r_2 = \frac{\Delta}{s - b} = \frac{96}{24 - 12} = \frac{96}{8} = 12$$

$$r_3 = \frac{96}{24 - 12} = \frac{96}{12} = 24$$

Ratio of external radii of the triangle opposite to the angles in the order $\angle C, \angle B, \angle A = 24 : 12 : 8 = 6 : 3 : 2$

111. (d) Vectors $(-3\hat{i} + 4\hat{j} + \lambda\hat{k})$ and

$(\mu\hat{i} + 8\hat{j} + 6\hat{k})$

are collinear, then $\frac{-3}{\mu} = \frac{4}{8} = \frac{\lambda}{6} \Rightarrow \frac{-3}{\mu} = \frac{1}{2}$ and $\frac{\lambda}{6} = \frac{1}{2}$

$\Rightarrow\ \mu = -6, \lambda = 3$

$\therefore\ \lambda - \mu = 9$

112. (b)

$$\cos\theta = \frac{0\cdot 1 + (-3)(0) + 2(-2)}{\sqrt{0^2 + (-3)^2 + 2^2}\sqrt{1^2 + 0^2 + (-2)^2}}$$

$$= \frac{-4}{\sqrt{65}}$$

$= \cos^{-1}\left(\frac{-4}{\sqrt{65}}\right)$

113. (b)

$a + b + c = 0 \{\because (a)^2 = a \cdot a\}$

$\Rightarrow (a + b + c)^2 = 0$

$\Rightarrow |a|^2 + |b|^2 + |c|^2$

$+2(a \cdot b + b \cdot c + c \cdot a) = 0$

$\Rightarrow 3^2 + 4^2 + 2^2$

$+2(a \cdot b + b \cdot c + c \cdot a) = 0$

$\Rightarrow 29 + 2(a \cdot b + b \cdot c + c \cdot a) = 0$

$\Rightarrow a \cdot b + b \cdot c + c \cdot a = \frac{-29}{2}$

$\therefore a \cdot b + b \cdot c + c \cdot a + 2(|a| + |b| + |c|)$

$= \frac{-29}{2} + 2 \cdot (3 + 4 + 2)$

$= \frac{-29 + 36}{2} = \frac{7}{2}$

114. (c)

$(a + 2b - c) \cdot [(a - b) \times (a - b - c)]$

$= (a + 2b - c) \cdot (a \times a - a \times b - a \times c$

$-b \times a + b \times b + b \times c)$

$= (a + 2b - c) \cdot (0 - a \times b - a \times c$

$+a \times b + 0 + b \times c)$

$= (a + 2b - c) \cdot (b \times c - a \times c)$

$= a \cdot (b \times c) - a \cdot (a \times c) + 2b \cdot (b \times c)$

$-2b \cdot (a \times c) - c \cdot (b \times c) + c \cdot (a \times c)$

$= a \cdot (b \times c) + 2b \cdot (c \times a)$

$= [abc] + 2[bca] = [abc] + 2\{abc]$

$= 3\{abc]$

115. (c) $a = \hat{i} + \hat{j} + \hat{k}, c = \hat{j} - \hat{k}$

Given, $a \times b = c$ and $a \cdot b = 3$

$\because \quad a \times b = c$

$\Rightarrow \quad a \times (a \times b) = a \times c$

$\Rightarrow \quad (a \cdot b)a - (a \cdot a)b = a \times c$

$\Rightarrow \quad 3a - |a|^2 b = a \times c \ldots.(i)$

$a \times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix}$

$= \hat{i}(-1 - 1) - \hat{j}(-1 - 0) + \hat{k}(1 - 0)$

$= -2\hat{i} + \hat{j} + \hat{k}$

$|a|^2 = 1^2 + 1^2 + 1^2 = 3$

From Eq. (i),

$3(\hat{i} + \hat{j} + \hat{k}) - 3b = -2\hat{i} + \hat{j} + \hat{k}$

$3\hat{i} + 3\hat{j} + 3\hat{k} + 2\hat{i} - \hat{j} - \hat{k} = 3b$

or $b = \frac{5\hat{i}}{3} + \frac{2\hat{j}}{3} + \frac{2\hat{k}}{3}$

116. (d)

$(a \times b) \times c = (a \cdot c)b - (b \cdot c)a$
$= [(2\hat{i} - 3\hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j})(\hat{i} + 2\hat{j} - 3\hat{k})]$
$-[(\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (\hat{i} - \hat{j}](2\hat{i} - 3\hat{j} + \hat{k})$
$= 5(\hat{i} + 2j - 3\hat{k}) + (2\hat{i} - 3\hat{j} + \hat{k})$
$= 7\hat{i} + 7\hat{j} - 14\hat{k}$
$(a \times b) \times c \perp d$
$(a \times b) \times c\}. d = 0$
$\Rightarrow (7\hat{i} + 7\hat{j} - 14\hat{k}) \cdot (a + \hat{i} + j + x\hat{k}) = 0$
$\Rightarrow 7 + 7 - 14x = 0$ or $x = 1$

117. **(b)** $\bar{x} = 10$

$$\frac{2 + 3 + 5 + 18 + 17 + 15 + 13 + x + 9 + 7}{10}$$

$\Rightarrow x = 11$
Deviation from mean $|x_i - \bar{x}|$ we get
8,7,5,8,7,5,3,1,1,3
Mean deviation about mean

$$= \frac{\sum |x_i - \bar{x}|}{n}$$

$$= \frac{8 + 7 + 5 + 8 + 7 + 5 + 3 + 1 + 1 + 3}{10}$$

$= 4.8$

118. **(d)**

119. **(d)** Total number of ways of throwing an unbiased die three times $= 6 \times 6 \times 6 \because \beta_1, \beta_2, \beta_3$ satisfies
$\omega^{\beta_1} + \omega^{\beta_2} = -\omega^{\beta_3}$
or $\omega^{\beta_1} + \omega^{\beta_2} + \omega^{\beta_3} = 0$
Then, $\beta_1, \beta_2, \beta_3$ is a permutation of the numbers of the form $3k, 3n + 1, 3m + 2$ from the set $\{1,2,3,4,5,6\}$
Thus, $\beta_1, \beta_2, \beta_3$ can be chosen in $2 \times 2 \times 2 \times 3\,!$ ways.
Hence, required probability

$$= \frac{2 \times 2 \times 2 \times 3!}{6 \times 6 \times 6} = \frac{2}{9}$$

120. **(b)**

$$P(A/B) = \frac{3}{10} \text{ and } P(B/A) = \frac{4}{5}$$

$$\Rightarrow P(A \cap B) = \frac{3}{10} P(B)$$

$$\text{and } P(A \cap B) = \frac{4}{5} P(A)$$

$$\Rightarrow \frac{3}{10} P(B) = \frac{4}{5} P(A) \text{ or } \frac{P(A)}{P(B)} = \frac{3}{8}$$

Now, $P(A \cup B) = kP(B)$

$$\Rightarrow k = \frac{P(A \cup B)}{P(B)} = \frac{P(A) + P(B) - P(A \cap B)}{P(B)}$$

$$= \frac{P(A)}{P(B)} + 1 - P(A/B)$$

$$= \frac{3}{8} + 1 - \frac{3}{10}$$

$$k = \frac{43}{40} \text{ or } \frac{1}{k} = \frac{40}{43}$$

121. **(a)**

$p = 0.001, n = 2000$

$\because \lambda = np \Rightarrow \lambda = 2$

$\because p(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$p(x = 3) = \frac{e^{-2}2^3}{3!} = \frac{8}{6e^2} = \frac{4}{3e^2}$

122. **(b)** Probability of getting tail $P = \frac{1}{2}$,

Then, probability of getting head $q = \frac{1}{2}$,

$n = 15$

$P(x \geq 3) = 1 - [P(x = 0)$

$+P(x = 1) + P(x = 2)]$

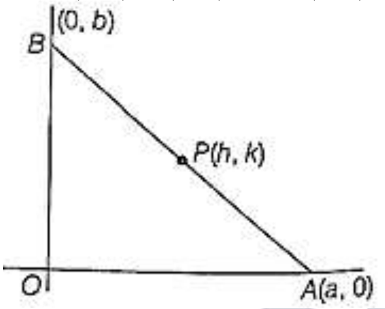
$= 1 - ({}^{15}C_0 + {}^{15}C_1 + {}^{15}C_2)\frac{1}{2^{15}}$

[by binomial distribution]

$= 1 - \left(1 + 15 + 105\frac{1}{2^{15}} = 1 - \frac{121}{2^{15}}\right.$

123. **(a)** $AB = 6$.

Let $A(a, 0), B(0, b)$ and $P(h, k)$ be the mid-point of AB.



Then, $h = \frac{a}{2}, k = \frac{b}{2}$

$\Rightarrow a = 2h, b = 2k$

By Pythagoras theorem,

$OA^2 + OB^2 = AB^2$

$\Rightarrow a^2 + b^2 = 6^2 = 36$

$\Rightarrow (2h)^2 + (2k)^2 = 36$

or $h^2 + k^2 = 9$

Locus of mid-point P is

$x^2 + y^2 = 9$

124. **(a)** Replacing x with $(x - 1)$ and y with $(y + 2)$ in given equation, we have

$2(x - 1)^2 + (y + 2)^2 - 3(x - 1)$

$+5(y + 2) - 8 = 0$

or $2(x^2 - 2x + 1) + (y^2 + 4y + 4)$

$-3x + 3 + 5y + 10 - 8 = 0$

or $2x^2 + y^2 - 7x + 9y + 11 = 0$

125. **(b)** Let the point be $P(x, y)$ which lies on- line

$4x - y - 2 = 0 \ldots.. (i)$

and point P is equidistant from $A(-5,6)$ and $B(3,2$. Then, $PA = PB$

or $PA^2 = PB^2$

$\Rightarrow (x + 5)^2 + (y - 6)^2 = (x - 3)^2 + (y - 2)^2$

$\Rightarrow x^2 + 10x + 25 + y^2 - 12y - 36$

$= x^2 - 6x + 9 + y^2 - 4y + 4$

$\Rightarrow 16x - 8y + 48 = 0$

$\Rightarrow 2x - y + 6 = 0 \ldots. (ii)$

Solving Eqs. (i) and (ii), we get

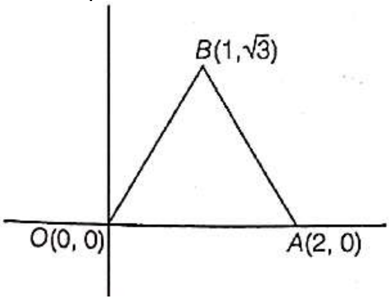
$x = 4, y = 14$

126. (d)

$OA = 2$

$OB = \sqrt{1+3} = 2$

$AB = \sqrt{1+3} = 2$



$OA = OB = AB$

$\therefore$ Triangle is equilateral.

Incentre

$$\left(\frac{0+2+1}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$$

127. (c)

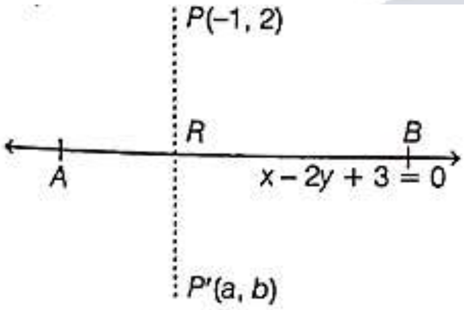


Image of $(-1,2)$ w.r.t. $x - 2y + 3 = 0$

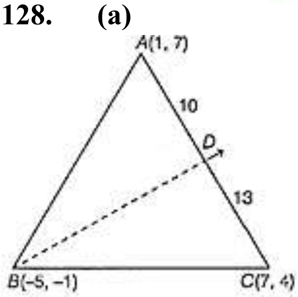
$$\frac{a+1}{1} = \frac{b-2}{-2} = \frac{-2(-1-4+3)}{5}$$

$$a = \frac{-9}{5}, b = \frac{18}{5}$$

Length of perpendicular drawn from P' to $2x + y - 7 = 0$

$$= \left|\frac{-\frac{18}{5} + \frac{18}{5} - 7}{\sqrt{5}}\right| = \frac{7}{\sqrt{5}}$$

128. (a)



$$\frac{AD}{DC}=\frac{BA}{BC}$$

$$AB=\sqrt{(1+5)^2+(7+1)^2}$$

$\because$ $$=\sqrt{36+64}$$

$$=\sqrt{100}=10$$

$$BC=\sqrt{(7+5)^2+(4+1)^2}$$

$$=\sqrt{12^2+5^2}=13$$

So, $\frac{AD}{DC}=\frac{10}{13}$

Coordinate of

$$D=\left(\frac{10\times 7+13\times 1}{10+13},\frac{10\times 4+13\times 7}{10+13}\right)$$

$$=\left(\frac{83}{23},\frac{131}{23}\right)$$

Equation of BD in straight line joining points $B(-5,-1)$ and $D\left(\frac{83}{23},\frac{131}{23}\right)$ is given by

$$\frac{y+1}{\frac{131}{23}+1}=\frac{x+5}{\frac{83}{23}+5}$$

$$\Rightarrow \frac{y+1}{154}=\frac{x+5}{198}$$

$$\Rightarrow \frac{y+1}{22\times 7}=\frac{x+5}{22\times 9}$$

$$\Rightarrow 9y+9=7x+35$$

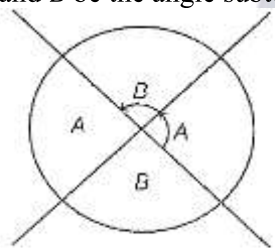
$$7y-9y+26=0$$

129. **(b)** Let $A=5B$

Let A be the angle subtended by bigger sector on centre and $A+B=\pi$

$\Rightarrow$ $B=\frac{\pi}{6}$ [linear pair]

and B be the angle subtended by smaller sector



$$\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

Then, $$\frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}=\cos B$$

or $$\frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$$

130. **(d)** $9x^2-24xy+16y^2+\alpha x+\beta y+6$
$=0$ ...(i)

$$a=9, b=16, h=-12, g=\frac{\alpha}{2}, f=\frac{\beta}{2}, c=6$$

Eq. (i) represents a pair of parallel lines, if

$\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$

$\Rightarrow \frac{9}{-12} = \frac{-12}{16} = \frac{\alpha/2}{\beta/2}$

$\Rightarrow \frac{\alpha}{\beta} = \frac{-3}{4}$

131. **(b)** Given, sides of triangles

$x + y = 6 \ldots..(i)$

$2x + y = 4 \ldots\ldots(ii)$

$x + 2y = 5 \ldots.(iii)$

From Eqs. (i) and (ii), $x = -2, y = 8$

From Eqs. (i) and (iii), $x = 7, y = -1$

From Eqs. (ii) and (iii), $x = 1, y = 2$

Let vertices are $A(-2,8), B(7,-1), C(1,2)$.

Let coordinate of circumcentre be $P(h, k)$.

Then, $PA = PB = PC$

$PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (h+2)^2 + (k-8)^2 = (h-7)^2 + (k+1)^2$

$\Rightarrow h - k + 1 = 0 \ldots\ldots(iv)$

$PB = PC$

$\Rightarrow PB^2 = PC^2$

$\Rightarrow (h-7)^2 + (k+1)^2 = (h-1)^2 + (k-2)^2$

$\Rightarrow -12h + 6k + 45 = 0 \ldots.(v)$

From Eqs. (iv) and (v), we get

$h = \frac{17}{2}, k = \frac{19}{2}$

$\therefore$ Circumcentre $\left(\frac{17}{2}, \frac{19}{2}\right)$

132. **(a)** Given circle, $x^2 - 2x + y^2 = 0$

Centre $C(1, 0)$

The locus of the mid-points of the chords of given circle from point $(0,0)$ is a circle whose extremities of diameter is centre and given point.

Locus of mid-points of the chord is

$(x-1)(x-0) + (y-0)(y-0) = 0$

$\Rightarrow x^2 - x + y^2 = 0$

or $x^2 + +y^2 - x = 0$

133. **(c)** Let $S_1 \equiv x^2 + y^2 + 4x - 6y - 3 = 0$

$\Rightarrow g = 2, f = -3, C_1 = -3$

$\because r = \sqrt{g^2 + f^2 - c}$ and centre

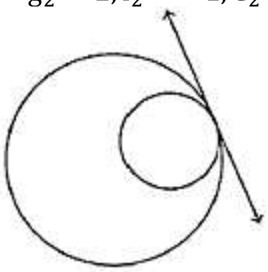
$= (-g, -f)]$

Centre, $C_1(-2,3)$

radius, $r_1 = \sqrt{(-2)^2 + 3^2 + 3} = 4$

and $S_2 \equiv x^2 + y^2 + 4x - 2y + 1 = 0$

$\Rightarrow g_2 = 2, f_2 = -1, C_2 = 1$



Centre, $C_2(-2,1)$
Radius, $r_2 = \sqrt{(-2)^2 + 1^2 - 1} = 2$
Distance between centres
$C_1C_2 = \sqrt{0^2 + 2^2} = 2$
Difference in radius $|r_1 - r_2| = 2$
$\because\ C_1C_2 = |r - r_2|$
$\therefore$ Circle touches each other internally at one point
$\Rightarrow$ Number of tangents $= 1$

134. **(a)**

135. **(b)** Let $S \equiv (x-2)^2 + (y-3)^2 = 25$
Centre $C_1(2,3)$ and radius $r_1 = 5$ and
$S' \equiv x^2 + y^2 - \frac{40}{25}x - \frac{70}{25}y - \frac{160}{25} = 0$
Centre $C_2\left(\frac{4}{5}, \frac{7}{5}\right)$ and radius $r_2 = 3$
If circle S and S′ with centre $C_1(x_1, y_1)$ and $C_2(x_2, y_2)$ and having radius r_1 and r_2 respectively touches each other internally. Then, coordinate of point of contact is $\left(\frac{r_1x_2 - r_2x_1}{r_1 - r_2}, \frac{r_1y_2 - r_2y_1}{r_1 - r_2}\right)$ Putting the values, we have

$$(\alpha, \beta) = \left(\frac{5 \times \frac{4}{5} - 3 \times 2}{5 - 3}, \frac{5 \times \frac{7}{5} - 3 \times 3}{5 - 3}\right)$$
$$= (-1, -1)$$
$$\therefore \alpha + \beta = (-1) + (-1) = -2$$

136. **(d)** Equation of parabola
$(y - k)^2 = 4a(x - h) \ldots. (i)$
Where focus $(h + a, k)$ and vertex (h, k)
Given, vertex $(1, -2) = (h, k)$
$$\Rightarrow \quad h = 1, k = -2$$
$$\text{Focus}\left(\frac{5}{4}, -2\right) = (h + a, k)$$
$$\Rightarrow \quad h + a = \frac{5}{4}$$
$$a = \frac{5}{4} - 1 = \frac{1}{4}$$
Putting values in Eq. (i), we get
$$(y + 2)^2 = 4 \cdot \frac{1}{4}(x - 1)$$
$$(y + 2)^2 = (x - 1)$$
Clearly, point $(10,1)$ satisfies this equation.

137. **(d)** Given equation of parabola,
$y^2 - 4x - 8y - 12 = 0$
or $y^2 - 8y + 16 = 4x + 28$
$(y - 4)^2 = 4(x + 7)$
Comparing with $Y^2 = 4aX$
$Y = y - 4, a = 1, X = x + 7$
parametric form of $Y^2 = 4aX$
$$X = at^2, Y = 2at$$
$$\Rightarrow \quad x + 7 = t^2, y - 4 = 2t$$
$$\text{or} \quad x = -7 + t^2, y = 4 + 2t$$

138. **(d)** Equation of ellipse
$$\frac{(x - \alpha)^2}{a^2} + \frac{(y - \beta)^2}{b^2} = 1$$
Where foci $(\alpha \pm ae, \beta)$
Equation of directrix $x = \alpha \pm \frac{a}{e}$, where e is eccentricity.
Given, $(\alpha + ae, \beta) = (0,0)$ [focus]

$\Rightarrow\ \alpha + ae = 0, \beta = 0$

$\Rightarrow\ \alpha + a \cdot \frac{1}{2} = 0\ \ldots$ (i) $\left[e = \frac{1}{2}\right]$

Also, $x = \alpha + \frac{a}{e} = 4$ [directrix]

$\Rightarrow\ \alpha + 2a = 4$…..(ii)

Solving Eqs. (i) and (ii), we get

$a = 8/3$

From Eq. (i), $\alpha = -\frac{4}{3}$

Also, $\quad e^2 = 1 - \frac{b^2}{a^2}$

$$\Rightarrow \quad \frac{1}{4} = 1 - \frac{b^2}{64/9}$$

$$\Rightarrow\ b^2 = \frac{16}{3}$$

$\therefore$ Equation of ellipse

$$\frac{\left(x + \frac{4}{3}\right)^2}{\frac{64}{9}} + \frac{y^2}{\frac{16}{3}} = 1$$

$\Rightarrow (3x + 4)^2 + 12y^2 = 64$

139. **(d)** For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Foci $(\pm ae, 0)$, when $a > b$ and $(0, \pm be)$, when $b > a$

Let $S \equiv \frac{x^2}{9} + \frac{y^2}{5} = 1$

Foci $(\pm ae, 0)$

$a^2 = 9, b^2 = 5$

$a = \pm 3$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{9-5}{9}} = \frac{2}{3}$$

$\therefore$ Foci, $A(2,0), B(-2,0)$

Again, $S' \equiv \frac{x^2}{5} + \frac{y^2}{9} = 1$

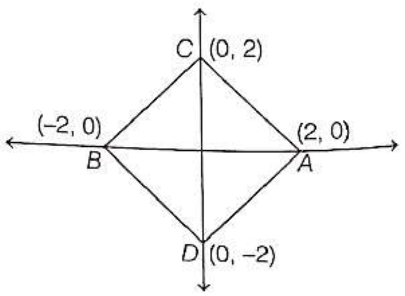
Foci $(0, \pm be)$

$a^2 = 5, b^2 = 9$

$\Rightarrow\ b = \pm 3$

$$e = \sqrt{\frac{b^2 - a^2}{b^2}} = \sqrt{\frac{9-5}{9}} = \frac{2}{3}$$

$C(0,2), D(0,-2)$



Area of quadrilateral

$ADBC = 4 \times$ area of $\triangle OAC$

$$= 4 \times \frac{1}{2} \times 2 \times 2 = 8 \text{ sq units}$$

140. **(c)** Let $P(h, k)$ be the pole of chord of the hyperbola

$$\frac{x^2}{9}-\frac{y^2}{36}=1 \ldots..(i)$$
Then, equation of chord
$$\frac{hx}{9}-\frac{ky}{36}=1 \ldots..(ii)$$
Combining Eqs. (i) and (ii) and maintaining homogeneity
$$\frac{x^2}{9}-\frac{y^2}{36}=\left(\frac{hx}{9}-\frac{ky}{36}\right)^2$$
$\because$ Chord substends right angle at the centre
$\Rightarrow$ Coefficient of x^2 + Coefficient of $y^2=0$
$$\therefore \frac{1}{9}-\frac{h^2}{9^2}-\frac{1}{36}-\frac{k^2}{36^2}=0$$
$$\Rightarrow \frac{h^2}{9^2}+\frac{k^2}{36^2}=\frac{1}{9}-\frac{1}{36}$$
$$\Rightarrow 16h^2+k^2=108$$
Locus of pole (h,k)
$$16x^2+y^2=108$$

141. (d)

142. (a) Let $\alpha=\frac{\pi}{4}, \beta=\frac{\pi}{3}$ and $\gamma=\theta$

Then, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$
or $\sin^2\gamma=\cos^2\alpha+\cos^2\beta$
$$\sin^2\theta=\cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{3}$$
$$=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$
$$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}=\sin\frac{\pi}{3}$$
$$\Rightarrow \theta=\frac{\pi}{3}$$
Direction cosines are $\cos\frac{\pi}{4}, \cos\frac{\pi}{3}, \cos\frac{\pi}{3} \Rightarrow \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$

143. (a) Given, plane $2x-2y+4z+5=0$

Direction ratios of the normal $=2,-2,4$
Equation of line passing through
$$\left(1,\frac{3}{2},2\right)$$
and dr's $2,-2,4$ is given by
$$\frac{x-1}{2}=\frac{y-\frac{3}{2}}{-2}=\frac{z-2}{4}=r$$
$$x=2r+1, y=-2r+\frac{3}{2}, z=4r+2$$
This point also lies on given plane.
$$\therefore 2(2r+1)-2\left(-2r+\frac{3}{2}\right)+4(4r+2+5=0$$
$$24r+12=0 \text{ or } r=\frac{-1}{2}$$
$\therefore$ Required point
$$\left(2\cdot\left(\frac{-1}{2}\right)+1,-2\left(\frac{-1}{2}\right)+\frac{3}{2},4\left(\frac{-1}{2}\right)+2\right)$$
$$=\left(0,\frac{5}{2},0\right)$$

144. (b)

$\lim_{x\to1}\frac{\log x}{1-x}$ [0/0 form]

$=\lim_{x\to1}\frac{1/x}{-1}$ [Using L' Hospital rule]

$=\lim_{x\to1}\left(\frac{-1}{x}\right)=-1$

145. (a)

$f(0)=\lim_{x\to0}f(x)$

$=\lim_{x\to0}\frac{2-\sqrt{x+4}}{\sin 2x}$ [0/0 form]

$=\lim_{x\to0}\frac{-\frac{1}{2\sqrt{x+4}}}{2\cos 2x}$

[Using L'Hospital rule]

$=\lim_{x\to0}\frac{-1}{4\sqrt{x+4}\cos 2x}=\frac{-1}{4\cdot\sqrt{4}\cdot1}=\frac{-1}{8}$

146. (b) $\because$ f(x) is continuous everywhere

$\Rightarrow$ f(x) is continuous at $x=0$ and $x=2$

$\therefore$ LHL at $(x=0)$ = RHL at $(x=0)=f(0)$

$\lim_{x\to0^-}f(x)=\lim_{x\to0^+}f(x)=2$

$\lim_{x\to0^-}(1+\cos x)=\lim_{x\to0^+}(a-x)=2$

$\Rightarrow\ 1+1=a-0=2$

$\Rightarrow a=2$

and LHL at $(x=2)$ = RHL at $(x=2)=f(2)$

$\lim_{x\to2^-}f(x)=\lim_{x\to2^+}f(x)=a-2$

$\Rightarrow\ \lim_{x\to2^-}(a-x)=\lim_{x\to2^+}(x^2-b^2)=2-2$

or $2-2=2^2-b^2=0$

$\Rightarrow\ b^2=4$

$\therefore\ a^2+b^2=4+4=8$

147. (b)

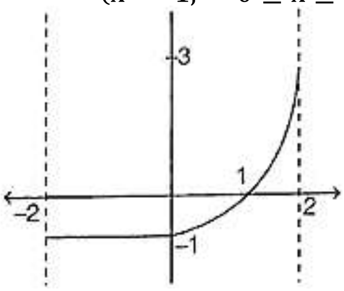
$x=5(1-\sin t),y=5(t+\cos t)$

$\frac{dx}{dt}=5(-\cos t),\frac{dy}{dt}=5(1-\sin t)$

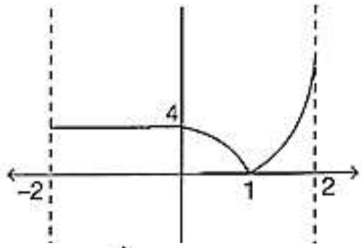
$\frac{dx}{dy}=\frac{dx/dt}{dy/dt}=\frac{-5\cos t}{5(1-\sin t)}=\frac{\cos t}{\sin t-1}$

148. (b)

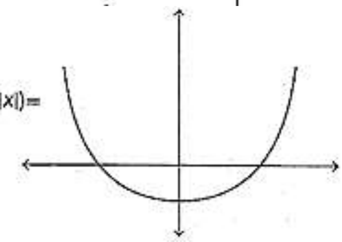
$g(x)=\begin{cases}-1, & -2\le x<0\\ x^2-1, & 0\le x\le 2\end{cases}$



$|g(x)| =$

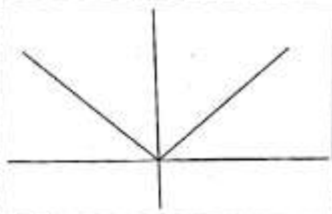


$g(|x|) =$



$f(x) = |g(x)| + g(|x|) + 2$
By graph $|g(x)|$ is making sharp edge at
$x = 1$
$\therefore$ $f(x)$ is not differentiable at $x = 1$.

149. **(b)** We know that, If $f(x)$ is differentiable at $x = a$, then it is continuous at $x = a$
$\Rightarrow$ If $f(x)$ is not continuous at $x = a$, then it is not differentiable at $x = a$
(a) is incorrect and (b) is correct. Now, (c) graph of $f(x) = |x|$



$|x|$ is continuous on R but it makes a corner at $x = 0$
$\therefore$ $|x|$ is not differentiable at $x = 0$ option (c) is correct.
(d) $f(x) = x - [x] = \{x\}$ [fractional put function]
Then, $f'(1) \neq 1$

150. **(a)** $x^2 + xy + y^2 = k \ldots (i)$
Differentiating. w.r.t. x,
$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0 \ldots (ii)$$
or $\frac{dy}{dx} = \frac{-2x-y}{x+2y} \ldots (iii)$
Again, differentiating. Eq. (ii) w.r.t. x,
$$2 + x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} + 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$$
$$2 + (x + 2y)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2\left(\frac{dy}{dx}\right)^2 = 0$$
$$(x + 2y)\frac{d^2y}{dx^2} + 2 - 2\frac{(2x + y)}{x + 2y} + 2\frac{(2x + y)^2}{(x + 2y)^2} = 0$$
$$(x + 2y)^3\frac{d^2y}{dx^2} + 2(x + 2y)^2 - 2(2x + y)(x + 2y) + 2(2x + y)^2 = 0$$
$$(x + 2y)^3\frac{d^2y}{dx^2} + 6(x^2 + y^2 + xy) = 0$$
$$\frac{d^2y}{dx^2} = \frac{-6k}{(x + 2y)^3}$$

151. **(d)**

$(8.01)^{4/3} + (8.01)^2$

$= (8 + 0.01)^{4/3} + (8 + 0.01)^2$

$= 8^{4/3}\left(1 + \frac{0.01}{8}\right)^{4/3} + 8^2\left(1 + \frac{0.01}{8}\right)^2$

$= 16(1 + 0.00125)^{4/3} + 64(1 + 0.00125)^2$

$= 16\left(1 + \frac{4}{3} \times 0.00125\right)$

$+64(1 + 2 \times 0.00125)$

[Using binomial expansion and neglecting higher order terms]

$16 \times \frac{3.005}{3} + 64 \times 1.0025 = 80.186$

152. **(a)** Given curve,

$y = 4x^4 + x \ldots . (i)$

On differentiating Eq. (i) w.r.t. x

$\frac{dy}{dx} = 16x^3 + 1$

Let $P(x, y)$ be a point Eq. on (i).

Slope of tangent at $P(x_1, y_1)$

$m_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 16x_1^3 + 1$

Slope of tangent at $(0,0)$

$m_2 = \left(\frac{dy}{dx}\right)_{(0,0)} = 1$

According to question,

$m_1 m_2 = -1$

$\Rightarrow (16x_1^3 + 1) \cdot 1 = -1$

or $x_1^3 = \frac{-1}{8}$

or $x_1 = \frac{-1}{2}$

$\therefore \quad y_1 = 4\left(\frac{-1}{2}\right)^4 + \left(\frac{-1}{2}\right) = \frac{-1}{4}$

$\therefore \quad P\left(\frac{-1}{2}, \frac{-1}{4}\right)$

153. **(b)** $2x^2 + y^2 = 20 \ldots . (i)$

On differentiating w.r.t. x, we have

$\frac{dy}{dx} = \frac{-2x}{y} \ldots . (ii)$

and $4y^2 - x^2 = 8 \ldots . (iii)$

Differentiating w.r.t. x, we have

$\frac{dy}{dx} = \frac{x}{4y} \ldots . (iv)$

Point of intersection of curves (i) and (ii) from Eq. (iii),

$x^2 = 4y^2 - 8$

Putting in Eq. (i), $2(4y^2 - 8) + y^2 = 20$

$8y^2 - 16 + y^2 = 20$

$9y^2 = 36$

$y^2 = 4$

$\Rightarrow \quad y = \pm 2$

$\therefore \quad x^2 = 4 \cdot (4) - 8 = 8$

or $\quad x = \pm 2\sqrt{2}$

$\therefore$ Point of intersection in IVth quadrant $(2\sqrt{2},-2)$
Slope of tangent at $(2\sqrt{2},-2)$ of curve (i)

$$m_1=\left(\frac{dy}{dx}\right)_{(2\sqrt{2},-2)}=\frac{-2\times2\sqrt{2}}{-2}=2\sqrt{2}$$

Slope of tangent at $(2\sqrt{2},-2)$ of curve (ii)

$$\left.\begin{aligned}m_2&=\left(\frac{dy}{dx}\right)_{(2\sqrt{2},-2)}=\frac{2\sqrt{2}}{4\cdot(-2)}=\frac{-\sqrt{2}}{4}\\&=\frac{-1}{2\sqrt{2}}\end{aligned}\right\}$$

$$\because m_1m_2=2\sqrt{2}\times\frac{-1}{2\sqrt{2}}=-1$$

angle between curves $=\pi/2$

154. (c)

$f(x)=x(x+3)e^{-x/2}=(x^2+3x)e^{-x/2}$

$$f'(x)=(2x+3)e^{-x/2}$$
$$+(x^2+3x)e^{-x/2}\cdot\left(\frac{-1}{2}\right)=0$$
$$e^{-x/2}\left[(2x+3)-\frac{(x^2+3x)}{2}\right]=0$$

or $(x^2-x-6)e^{-x/2}=0$

$\because\quad e^{-x/2}\neq0$ for $x\in[-3,0]$

$\Rightarrow\quad x^2-x-6=0$

$\Rightarrow\quad (x+2)(x-3)=0$

$\Rightarrow\quad x=-2\in[-3,0]$

155. (a)

156. (b)

$$I=\int(x+2)\sqrt{x+3}\,dx$$

Put $x+3=t^2\Rightarrow x=t^2-3$

$dx=2tdt$

$$\therefore\ I=\int(t^2-1)\cdot2t^2dt=2\left(\frac{t^5}{5}-\frac{t^3}{3}\right)+C$$
$$=\frac{2}{15}(3t^5-5t^3)+C$$
$$=\frac{2}{15}[3\cdot(x+3)^2\sqrt{x+3}$$
$$-5(x+3)\sqrt{x+3}]+C$$
$$=\frac{2}{15}\sqrt{x+3}[3(x^2+6x+9)$$
$$=\frac{2}{15}\sqrt{x+3}(3x^2+13x+12)+C$$

157. (a)

$$\int \frac{(1-\cos x)^{2/7}}{(1+\cos x)^{9/7}} dx$$

$$= \int \frac{(1-\cos x)^{2/7}}{(1+\cos x)(1+\cos x)^{2/7}} dx$$

$$= \int \left(\frac{1-\cos x}{1+\cos x}\right)^{2/7} \frac{dx}{1+\cos x}$$

$$= \int \left(\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)^{2/7} \cdot \frac{dx}{2\cos^2\frac{x}{2}}$$

$$= \int \left(\tan^2\frac{x}{2}\right)^{2/7} \cdot \frac{1}{2}\sec^2\frac{x}{2} dx$$

$$= \frac{1}{2}\int \tan^{4/7}\frac{x}{2} \cdot \sec^2\frac{x}{2} dx$$

Let $\tan\frac{x}{2} = t$

$\Rightarrow \sec^2\frac{x}{2} dx = 2dt = \int t^{4/7} dt$

$$= \frac{t^{\frac{4}{7}+1}}{\frac{4}{7}+1} + C = \frac{7}{11} t^{11/7} + C$$

$$= \frac{7}{11} \tan^{11/7}\frac{x}{2} + C$$

158. **(a)** Let

$$I = \int_5^9 \frac{\log 3x^2 dx}{\log 3x^2 + \log(588 - 84x + 3x^2)} \dots\dots (i)$$

$$= \int_5^9 \frac{\log 3x^2 dx}{\log 3x^2 + \log 3(14-x)^2}$$

$$= \int_5^9 \frac{\log 3(14-x)^2 dx}{\log 3(14-x)^2 + \log 3(14-(14-x))^2}$$

$$\left[\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$$I = \int_5^9 \frac{\log 3(14-x)^2 dx}{\log 3(14-x)^2 + \log 3x^2} \dots\dots (ii)$$

Adding Eqs. (i) and (ii), we get

$$2I = \int_5^9 \frac{\log 3x^2 + \log 3(14-x)^2}{\log 3(14-x)^2 + \log 3x^2} dx$$

$$= \int_5^9 dx = 9 - 5 = 4$$

$I = 2$

159. **(c)**

$$\int_{-1}^{1}\frac{\log(1+x)}{1+x^2}dx=\int_{0}^{1}\frac{\log(1+x)}{1+x^2}dx$$
$$+\int_{0}^{1}f(x)dx$$
$$\Rightarrow\int_{-1}^{0}\frac{\log(1+x)}{1+x^2}dx+\int_{0}^{1}\frac{\log(1+x)}{1+x^2}dx$$
$$=\int_{0}^{1}\frac{\log(1+x)}{1+x^2}dx+\int_{0}^{1}f(x)dx$$
$$\Rightarrow\int_{-1}^{0}\frac{\log(1+x)}{1+x^2}dx+\int_{0}^{1}\frac{\log(1+x)}{1+x^2}dx$$
$$=\int_{0}^{1}\frac{\log(1+x)}{1+x^2}dx=\int_{0}^{1}f(x)dx$$
$$\Rightarrow\int_{-1}^{0}\frac{\log(1+x)}{1+x^2}dx=\int_{0}^{1}f(x)dx$$
$$\text{Let } I=\int_{0}^{1}f(x)dx$$
$$=\int_{-1}^{0}\frac{\log(1+x)}{1+x^2}dx$$

Put $x=-y\Rightarrow dx=-dy$

For $x=-1$

$y=1$

$x=0$

$y=0$

$$\therefore I=\int_{1}^{0}\frac{\log(1-y)}{1+y^2}(-dy)$$
$$=\int_{0}^{1}\frac{\log(1-y)}{1+y^2}dy$$

By change of variable property,

$$I=\int_{0}^{1}\frac{\log(1-x)}{1+x^2}dx$$

or

$$\int_{0}^{1}f(x)dx=\int_{0}^{1}\frac{\log(1-x)}{1+x^2}dx$$
$$f(x)=\frac{\log(1-x)}{1+x^2}$$

160. (b)

$$\frac{dy}{dx}=x(1+\cos y)+\sin x(1+\cos y)$$
$$\frac{dy}{dx}=(x+\sin x)(1+\cos y)$$
$$\text{or }\frac{dy}{1+\cos y}=(x+\sin x)dx$$
$$\frac{dy}{2\cos^2\frac{y}{2}}=(x+\sin x)dx$$
$$\frac{1}{2}\sec^2\frac{y}{2}dy=(x+\sin x)dx$$

Integrating both sides, we get

$$\tan\frac{y}{2}=\frac{x^2}{2}-\cos x+C$$