## Physics

**1.** **(b)** Option (a) is incorrect because all basic laws of physics are universal. Option (b) is correct because it follows Noether's first theorem which states that any differentiable symmetry of the action of a physical system has a corresponding conservation law. Option (c) is incorrect as there are only 4 fundamental forces in nature. Option (d) is incorrect as there are examples where physics came out from technology, for example measurement of time (the technology of clocks) enabled Galileo to understand the physics of velocity and acceleration quite differently from Aristotle, giving rise to Galilean / Newtonian physics. Measurement of charge, in the technology of electricity, gave rise to millikans oil drop experiment and the discovery of the electron.

**2.** **(d)** We know, for a formula to be correct only when dimensions of LHS and dimensions of RHS must be equal.

$E = \frac{2E_0L}{L_0} \Rightarrow \frac{E}{E_0} = \frac{2L}{L_0}$ ...... (i)

$\Rightarrow$ Both LHS and RHS of Eq. (i), are dimensionless. Hence, the formula is dimensionally correct.

• In $E = E_0 e^{-\frac{2L}{L_0}}$, clearly the exponent is dimensionless because, L and $L_0$ will have same dimension and dimensions of E and $E_0$ will be equal, since both E and $E_0$ will have dimensions of energy. Thus, this formula is dimensionally correct.

• In $E = Le^{-L/E_0}$, clearly the power of exponential $\frac{-L}{E_0}$ will have dimensions and hence the power of exponent is not dimensionless, hence this equation is dimensionally incorrect.

• In $E = 2\left(\frac{E_0}{L_0}\right) \times e^{-L/L_0}$, here $\frac{E_0}{L_0}$ will have different dimensions from E, hence this formula is also dimensionally incorrect.

**3.** **(a)** Initial velocity,

$u = 0\ m/s$ [ rest]

Final velocity, $v = 10\ m/s$

Time taken, $t = 2\ s$

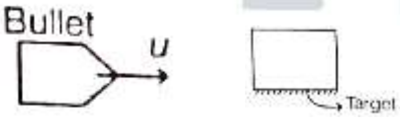
We know,

$$a = \frac{v-u}{t} = \frac{10-0}{2} = 5\ m/s^2$$

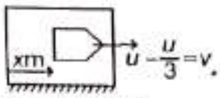
Now, $s = ut + \frac{1}{2}at^2$

$$= 0 + \frac{1}{2} \times 5 \times 4 = 10\ m$$

**4.** **(c)** Let u be initial velocity of bullet,




Before Collision




After Collision

Final velocity of bullet after travelling a distance x is equal to $\left(u - \frac{u}{3}\right) = \frac{2u}{3}$

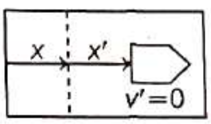
We know, $v^2 = u^2 + 2as$

$$\Rightarrow \left(\frac{2u}{3}\right)^2 = u^2 + 2ax$$

$$\Rightarrow \frac{4u^2}{9} - u^2 = 2ax$$

$$\Rightarrow x = \frac{-5}{18a}u^2 \quad \text{...... (i)}$$

Again after embedding further distance of $x'$, it comes to rest



$v'^2 = v^2 + 2ax'$
(Now, v = initial velocity for $x'$ distance)

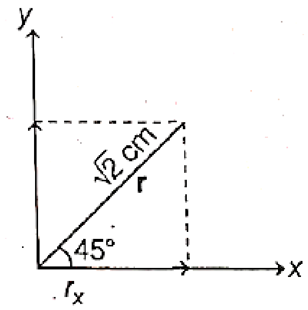
$$\Rightarrow \quad 0 = \frac{4u^2}{9} + 2 \times a \times x'$$

$$\Rightarrow \quad x' = \frac{-4u^2}{18a} \quad \ldots (ii)$$

$$\Rightarrow \quad \frac{x'}{x} = \frac{\frac{-4u^2}{18a}}{\frac{-5u^2}{18a}} = \frac{4}{5}$$

[from Eqs. (i) and (ii)]

**5.** **(a)** Displacement, $s = (10\hat{\mathbf{i}} + 20\hat{\mathbf{j}})$cm Position, $\mathbf{r} = \left(r_x\hat{\mathbf{i}} + r_y\hat{\mathbf{j}}\right) = (1\hat{\mathbf{i}} + 1\hat{\mathbf{j}})$cm

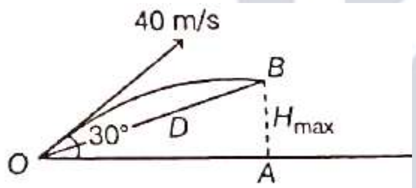



$$r_x = \sqrt{2}\cos 45^\circ = 1 \text{ cm}$$
$$r_y = \sqrt{2}\sin 45^\circ = 1 \text{ cm}$$

Now, dot product of s and r

$$\begin{aligned} \mathbf{s} \cdot \mathbf{r} &= (10\hat{\mathbf{i}} + 20\hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \\ &= (10 + 20)(\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1, \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 1, \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0) \\ &= 30 \text{ cm} \end{aligned}$$

**6.** **(d)** The given situation is shown below




Since, time of flight, $T = \frac{2u\sin\theta}{g}$

$$= \frac{2 \times 40 \times \sin 30^\circ}{10} = \frac{2 \times 40 \times 1}{10 \times 2} = 4 \text{ s}$$

In 2 s , it will be at the topmost point, where it will hit the hillside.
After 2 s , the maximum height ( $H_{max}$ )

$$AB = \quad H_{max} = \frac{u^2\sin^2\theta}{2g}$$
$$= \frac{40^2\sin^2 30^\circ}{2 \times 10} = \frac{1600 \times 1}{2 \times 10 \times 4} = 20 \text{ m}$$

Also, at this time the horizontal distance travelled will be half of the range = OA

$$\text{ie} \frac{R}{2} OA \quad = \frac{R}{2} = \frac{u^2\sin 2\theta}{2g} = \frac{40^2\sin(2 \times 30^\circ)}{2 \times 10}$$
$$= \frac{1600\sin 60^\circ}{20}$$
$$\frac{R}{2} \quad = 80\frac{\sqrt{3}}{2} = 40\sqrt{3}$$

The displacement from the projection point,

$$OB = D = \sqrt{OA^2 + AB^2}$$

(By Pythagorous theorem)

$$\Rightarrow D = \sqrt{(40\sqrt{3})^2 + (20)^2} = \sqrt{4800+400}$$
$$= \sqrt{5200}$$
$$\Rightarrow D = \sqrt{4 \times 13 \times 100} = 2 \times 10\sqrt{13}$$
$$= 20\sqrt{13}\text{ m}$$

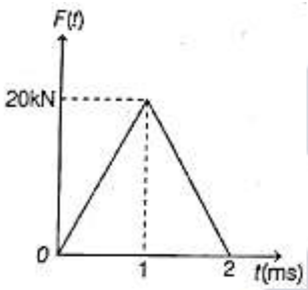
**7. (b)**

**8. (b)** Given, $U(x) = (5x^2 - 4x^3)J$

We know, $F = \frac{-dU(x)}{dx}$

$$\Rightarrow \quad F = \frac{-d}{dx}(5x^2 - 4x^3)$$
$$= -10x + 12x^2$$
$$F = 0 \quad \text{(given)}$$
$$12x^2 - 10x = 0$$
$$\Rightarrow \quad 2x(6x-5) = 0$$
$$2x = 0 \text{ or } 6x - 5 = 0$$
$$\Rightarrow \quad x = 0 \text{ or } x = \frac{5}{6}\text{ m}$$

**9. (a)**




Given,
$m \quad = 100\text{ g} = 01\text{ kg}$
$\Delta t \quad = 2\text{ ms} = 2 \times 10^{-3}\text{ s}$
Initial speed, $u = 10\text{ m/s}$
Let final speed be v.
We know that, area under force-time graph gives impulse or change in momentum.

Area of the triangle $= |\Delta \mathbf{p}|$

(where, $\mathbf{p} =$ momentum)

$$\Rightarrow \frac{1}{2} \times \text{ Base } \times \text{ Height } = |\mathbf{p}_f - \mathbf{p}_i|$$

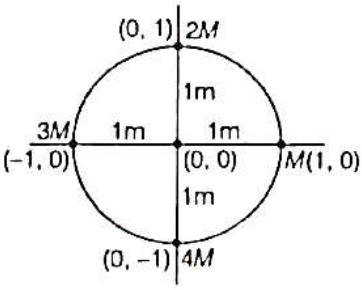
(where, $\mathbf{p}_f =$ final momentum and
$\mathbf{p}_i =$ initial momentum)

$$\Rightarrow \frac{1}{2} \times (2 \times 10^{-3}) \times (20 \times 1000) = 0.1v - 0.1u$$

$(20\text{kN} = 20000\text{ N})$

$$2 \times 10 = 0.1(v-10) \quad (u = 10\text{ m/s})$$
$$\Rightarrow \quad \Rightarrow v - 10 = 200$$
$$\Rightarrow \quad v = 210\text{ m/s}$$

**10. (a)** The given situation is shown as




We know,

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{(3M \times -1) + (2M \times 0) + (M \times 1) + (4M \times 0)}{3M + 2M + M + 4M}$$

$$= \frac{-3M + 0 + M + 0}{10M} = \frac{-2M}{10M} = \frac{-1}{5}$$

Also, $Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$

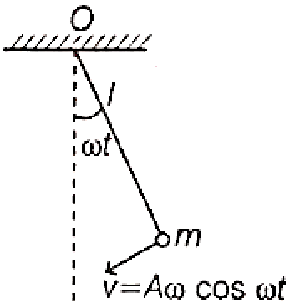
$$= \frac{(3M \times 0) + (2M \times 1) + (M \times 0) + (4M \times -1)}{10M}$$

$$= \frac{0 + 2M + 0 - 4M}{10M} = \frac{-2M}{10M} = \frac{-1}{5}$$

Position of centre of mass from the centre of the circle

$$r_{CM} = X_{CM}\hat{i} + Y_{CM}\hat{j} = \left(\frac{-1}{5}\hat{\mathbf{i}} - \frac{1}{5}\hat{\mathbf{j}}\right)$$

**11. (b)** The given situation is shown as




Let the angle be ωt when the KE is 75% of total energy.
Also, the moment of inertia of the bob of pendulum about the point of suspension
$= I = ml^2$ …..(i)
Also, velocity at that instant
$= l\omega\cos\omega t$ ……(ii)
Now, according to the question,

$$\frac{1}{2}mv^2 = \frac{75}{100} \times \frac{1}{2}I\omega^2$$

where, $\left(\frac{1}{2}I\omega^2\right)$ is the rotational KE of the particle,

$$\frac{1}{2}mv^2 = \frac{75}{100} \times \frac{1}{2} \times ml^2 \times \omega^2 [\text{ from Eq. (i) }]$$

$$\Rightarrow \frac{1}{2}l^2m\omega^2\cos^2\omega t = \frac{3}{4} \times \frac{1}{2} \times ml^2\omega^2$$

[from Eq. (ii)]

$$\Rightarrow \cos^2\omega t = 3/4$$

$$\Rightarrow \cos\omega t = \pm\frac{\sqrt{3}}{2} \Rightarrow \omega t = \frac{\pi}{6}$$

(keeping only positive solution)

$$t = \frac{\pi}{6\omega} = \frac{\pi T}{6 \times 2\pi}$$

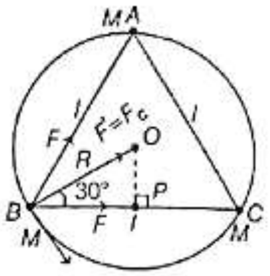
$\left(\text{ Time period, } T = \frac{2\pi}{\omega}\right)$

$$\Rightarrow\ t = \frac{\pi \times 4}{12\pi}$$

(given, $T = 4$ s )

$$\Rightarrow\ t = \frac{1}{3}\ \text{s}$$

**12. (c)** The situation given in the question is depicted below




Let the speed of particle is v for circular motion.
Clearly the gravitational force acting all the 3 masses will be same due to symmetry and let it be F.
Now, considering the mass M placed at point B. Since, it is performing circular motion in radius say R, hence the net gravitational pull on it due to the other two masses are providing the required centripetal force.
Now, net force on M at B,

$$\Rightarrow\quad F' = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ}$$

$$\Rightarrow\quad F' = \sqrt{3}F \quad \ldots\ldots \text{(i)}$$

$$\text{and}\quad F = \frac{GM^2}{l^2}$$

(gravitational force of Newton)
Also, by symmetry of forces the $F'$ will be acting along the bisector of angle at point B i.e. at 30° from each force F.
Now, in $\triangle$ OBP,

$$\cos 30^\circ = \frac{\frac{l}{2}}{R} \quad \left(BP = \frac{BC}{2}\right)$$

$$\frac{\sqrt{3}}{2} = \frac{l}{2R}$$

$$\Rightarrow\ R = \frac{l}{\sqrt{3}} \quad \ldots\ldots \text{(ii)}$$

Now, since $F' = F_{\text{centripetal}}$

i.e. $\quad F' = F_c$

$$\Rightarrow\quad \sqrt{3}F = \frac{Mv^2}{R} \quad [\text{ from Eq. } \cdot (1)]$$

$$\left[F_c = \frac{M1^2}{R}\right]$$

$$\Rightarrow\quad \sqrt{3}\frac{GM^2}{l^2} = \frac{Mv^2}{R}$$

Now, from Eq. (ii),

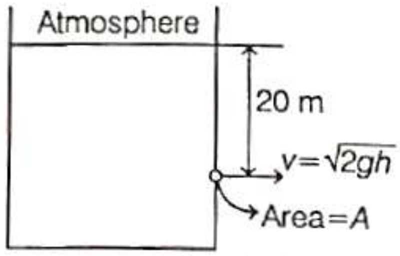
$$\sqrt{3}\frac{GM^2}{l^2} = \frac{Mv^2}{l}\sqrt{3}$$

$$\Rightarrow\quad v^2 = \frac{GM}{l}$$

$$\Rightarrow\quad v = \sqrt{\frac{GM}{l}}$$

**13. (b)** By definitions of shear modulus, it matches with resistance to change against deformation force. Shearing stress is the tangential stress applied on any surface. Elastic fatigue itself means temporary loss of elastic property. Modulus of elasticity is the proportionality constant in Hooke's law.

**14. (c)** The situation given in the question is depicted below




$v = \sqrt{2 \times 10 \times 20} = \sqrt{400} = 20 \text{ m/s}$

Given, rate of flow = area × velocity

$Av = 3.08 \times 10^{-5} \text{ m}^3/\text{s}$

$\Rightarrow \quad A = \dfrac{308 \times 10^{-5}}{20} \text{ m}^2$

Now, area of circular hole

$= \pi R^2 = \pi \left(\dfrac{D}{2}\right)^2$

where, D = diameter of the circle.

$\Rightarrow \pi \left(\dfrac{D}{2}\right)^2 = \dfrac{3.08 \times 10^{-5}}{20} \text{ m}^2$

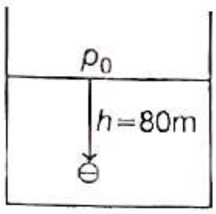
$\Rightarrow \quad D^2 = \dfrac{3.08 \times 10^{-5} \times 4}{20\pi}$

$\Rightarrow D = \sqrt{\dfrac{308 \times 10^{-5} \times 4}{20\pi}}$

$= 1.4 \times 10^{-3} \text{ m}$

$= 1.4 \text{ mm}$

**15. (b)** The given situation is shown below




where, $p_0$ = atmospheric pressure,

R = Air bubble of radius = 1 mm,

$\rho$ = density of liquid = 2000 kg/m$^3$

and T = surface tension of liquid = 0.1 N/m.

Now, we know that, excess pressure inside an air bubble inside any liquid

$= \dfrac{2T}{R}$

Also, pressure inside the air bubble = pressure outside + excess pressure

$$\Rightarrow p_{\text{inside}} = \underset{\substack{\text{atmospheric}\\ \text{pressure}}}{p_0} + \underset{\substack{\text{pressure}\\ \text{due to liquid}\\ \text{column}}}{\rho g h} + \underset{\substack{\text{excess pressure}\\ \text{inside air bubble}}}{\frac{2T}{R}}$$

Difference of pressure inside the air bubble and atomospheric pressure,

$$\Rightarrow p_{\text{inside}} - p_0 = \rho g h + \frac{2T}{R}$$
$$= 2000 \times 10 \times \frac{8}{100} + \frac{2 \times 0.1}{1 \times 10^{-3}}$$
$$= 1600 + (2 \times 100) = 1800 \text{ N/m}^2$$

**16. (a)** $\Delta L_{\text{steel}} = L_S \alpha_s \Delta t$

$\Delta L_{\text{copper}} = L_C \alpha_C \Delta t$

Given, $L_S - L_C = 4$ cm for any temperature

$$L_s \alpha_s \Delta t = L_C \alpha_C \Delta t$$
$$\frac{L_S}{L_C} = \frac{\alpha_C}{\alpha_s} = \frac{1.7 \times 10^{-5}}{11 \times 10^{-5}} = \frac{1.7}{11} = \frac{17}{11}$$

**17. (d)** By Newton's law of cooling,

$\frac{-dT}{dt} = b[T - T_0]$ .......(i)

where, $\frac{-dT}{dt}$ = Cooling rate,

$T$ = Body's temperature,

$T_0$ = Surrounding temperature and $b$ = Constant.

From Eq. (i),

$$\int_{T_i}^{T} \frac{dT}{T - T_0} = -b \int_0^t dt$$

$\Rightarrow \quad T = T_0 + (T_i - T_0)e^{-bt}$ ... ... (ii)

From the first condition of question,

$T = 40°C$

$T_0 = 10°C$

$T_i = 100°C$

$t = 10$ min

Putting these value in Eq. (ii),

$40 = 10 + (100 - 10)e^{-b \times 10}$

$\Rightarrow \frac{30}{90} = e^{-10b}$

$\Rightarrow \frac{1}{3} = e^{-10b}$

$\Rightarrow \ln\left(\frac{1}{3}\right) = -10b \ln m$

$\Rightarrow \ln 1 - \ln 3 = -10b$

$\Rightarrow -\ln 3 = -10b$

$\Rightarrow b = \frac{\ln 3}{10} = \frac{11}{10}$

Now, by next condition of question,

$T = 20°C$

$T_i = 70°C$

$T_a = 10°C$

From Eq. (ii), $20 = 10 + (70 - 10)e^{-bt}$

$$\frac{10}{60} = e^{-\frac{1.1}{10}t}$$

$$\Rightarrow \ln\left(\frac{1}{6}\right) = -\frac{1.1}{6} t \ln e.$$

$$\Rightarrow \ln 1 - \ln 6 = -\frac{1.1}{10} t \Rightarrow -\ln 6 = \frac{-11}{100} t$$

$$\Rightarrow t = \frac{100 \times \ln 6}{11} = \frac{100 \times 1.8}{11} = 16.3 \text{ min}$$

**18. (a)** We know, from first law of thermodynamics,

$\Delta Q = \Delta U + W \quad \ldots\ldots\ldots (i)$

Given, volume of liquid, $V_l = 10^{-3}\ m^3$

And volume of gas or volume of steam,

$V_g = 2001\ m^3$

Now, $V_g \gg V_l$

$$(V_g - V_l) \approx V_g$$

$$W = pV_g = (10^5 \times 2.001)J = 2.001 \times 10^2\ kJ$$

$$= 200\ kJ$$

Now, heat required for phase change to convert the liquid into steam,

$\Delta Q = mL = 1 \times 2000\ kJ$

Put in Eq (i), $\Delta U = \Delta Q - W$

$\Rightarrow \Delta U = (2000 - 200) kJ$

[from Eq. (i)]

$\Rightarrow \Delta U = 1800\ kJ$

**19. (c)** Given, $W = 100\ J$

From ideal gas equation,

$p\Delta V = nR\Delta T \quad \ldots\ldots (i)$

$W = p\Delta V$

Eq. (i) becomes,

$$W = nR\Delta T \Rightarrow \Delta T = \frac{W}{nR} = \frac{100}{nR} \quad \ldots\ldots (ii)$$

Now, we know the relation for change in internal energy,

$\Delta U = nC_V\Delta T$

$$\Delta U = n\frac{R}{\gamma - 1}\Delta T \quad \ldots \text{(iii)}$$

$$\left(C_V = \frac{R}{\gamma - 1}\right)$$

and since it is a monoatomic gas,

$$\gamma = \frac{5}{3}$$

From Eqs. (ii) and (iii)

$$\Rightarrow \Delta U = 150\ J$$

$$\Rightarrow \Delta U = \frac{nR\Delta T}{\frac{5}{3} - 1} = \frac{3nR}{2} \times \frac{100}{nR} = 150\ J$$

Now, the heat given to the gas in the process is

$$\Delta Q = \Delta U + W$$

$$\Rightarrow \Delta Q = 150 + 100 = 250\ J$$

$$\Rightarrow \Delta Q = 250\ J$$

**20. (a)** We know, $v_{rms} = \sqrt{\frac{3RT}{M}}$ (where M = molar mass of gas)

For $N_2$:

$$\Rightarrow 100=\sqrt{\frac{3RT\times 1000}{28}}$$

(for $N_2$ molar mass, $M=\frac{28}{1000}$ kg )

$$\Rightarrow 100=\sqrt{\frac{3000RT}{28}} \quad \ldots\ldots(i)$$

Now, for Hc, $v'_{rms}=\sqrt{\frac{3RT\times1000}{4}}$ (for He molar mass, $M=\frac{4}{100}$ kg )

i.e $v'_{rms}=\sqrt{\frac{3000RT}{4}} \quad \ldots..(ii)$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{v'_{rms}}{100}=\sqrt{\frac{\frac{3000RT}{4}}{\frac{3000RT}{28}}}$$

$$\Rightarrow \frac{v'_{rms}}{100}=\sqrt{\frac{28}{4}}$$

$$\Rightarrow v'_{rms}=100\sqrt{7}\ m/s$$

**21. (b)** The ratio between the maximum and minimum intensities of the resultant wave,

$I_{max}:I_{min}=9:4$

$$\Rightarrow \frac{I_{max}}{I_{min}}=\frac{9}{4}\Rightarrow\frac{(A_1+A_2)^2}{(A_1-A_2)^2}=\left(\frac{3}{2}\right)^2$$

$$\Rightarrow \frac{A_1^2\left(1+\frac{A_2}{A_1}\right)^2}{A_1^2\left(1-\frac{A_2}{A_1}\right)^2}=\left(\frac{3}{2}\right)^2$$

$$\Rightarrow \frac{\left(1+\frac{A_2}{A_1}\right)^2}{\left(1-\frac{A_2}{A_1}\right)^2}=\left(\frac{3}{2}\right)^2\Rightarrow\frac{1+\frac{A_2}{A_1}}{1-\frac{A_2}{A_1}}=\frac{3}{2}$$

$$\Rightarrow \frac{A_1+A_2}{A_1-A_2}=\frac{3}{2}$$

$$\Rightarrow 2A_1+2A_2=3A_1-3A_2$$

$$\Rightarrow 5A_2=A_1\Rightarrow\frac{A_2}{A_1}=\frac{1}{5}=0.20$$

**22. (a)** Refractive index, $\mu=1.5$

Since, one side of given lens is flat and other side is convex with a radius of curvature R.

$R_1=R$ and $R_2=\infty$

Position of object, $u=-60$ m

Position of image, $v=120$ cm

Using lens Maker's formula, we get

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)$$

$$\left(\frac{1}{f}=\frac{1}{v}-\frac{1}{u}\right)$$

$$\Rightarrow \frac{1}{120}-\frac{1}{-60}=(1.5-1)\cdot\frac{1}{R}$$

$$\Rightarrow \frac{3}{120}=\frac{0.5}{R}\Rightarrow\frac{1}{40}=\frac{0.5}{R}$$

$$\Rightarrow R=40\times0.5=20\text{ cm}$$

**23. (c)** In Young's double slit experiment,

$d = 0.2\text{ mm} = 2\times10^{-4}\text{ m}$

$D = 60\text{ cm} = 0.6\text{ m}$

$\lambda = 550\text{ nm} = 5.5\times10^{-7}\text{ m}$

$\mu = \frac{11}{9}$

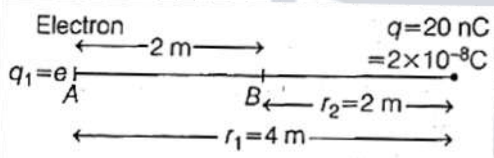
Fringe width when whole apparatus is immersed in liquid,

$$\beta=\frac{D\lambda}{d\mu}=\frac{0.6\times5.5\times10^{-7}}{2\times10^{-4}\times\frac{11}{9}}$$

$$=\frac{9\times0.6\times5.5\times10^{-3}}{2\times11}$$

$$=1.35\times10^{-3}\text{ m}=1.35\text{ mm}$$

**24. (b)** The given situation is shown below.




When electron reaches at point B, then change in its potential energy is equal to gain in kinetic energy.

i.e. $\Delta U = K$

$$\Rightarrow \text{PE at point B} - \text{PE at point A} = \frac{1}{2}m_e v^2$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_1\cdot q}{r_2^2}-\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q}{r_1^2}=\frac{1}{2}m_e v^2$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left[\frac{eq}{r_2^2}-\frac{eq}{r_1^2}\right]=\frac{1}{2}m_e v^2$$

$$\Rightarrow 9\times10^9\left[\frac{eq}{2^2}-\frac{eq}{4^2}\right]=\frac{1}{2}m_e v^2$$

$$\Rightarrow 9\times10^9\left(\frac{3eq}{16}\right)=\frac{1}{2}m_e v^2$$

$$\Rightarrow v=\sqrt{\frac{3eq\times9\times10^9}{8m_e}}$$

$$\Rightarrow v=\sqrt{\frac{3\times1.6\times10^{-19}\times2\times10^{-8}\times9\times10^9}{8\times9\times10^{-31}}}$$

$$=3.46\times10^6$$

Which is nearest to $4\times10^6$ m/s.

**25. (c)** Given, $C_1=C_2=C_3=1\mu\text{ F}=10^{-6}\text{ F}$

$V = 9\text{ V}$

When switch is thrown to the right side, then left side of switch is open. Hence, capacitors $C_2$ and $C_3$ are uneffective. Therefore, only capacitor $C_2$ is in valid connection.
Charge on capacitor $C_1$,
$Q_1 = C_1V = 1 \times 10^{-6} \times 9 = 9 \times 10^{-6}C = 9\mu C$
Now, when switch is thrown to left, then capacitors $C_1, C_2$ and $C_3$ are connected in series and charge of charged capacitor $C_1$ is flowing towards uncharged capacitors $C_2$ and $C_3$.
Since, $C_1 = C_2 = C_3$, hence $V_{C_1} = V_{C_2} = V_{C_3}$
Therefore, $Q_2 = Q_3 = \frac{Q_1}{3} = \frac{9}{3} = 3\mu C$

**26. (a)** We know that, resistance of wire,

$$R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2}$$

$$\Rightarrow \quad R \propto \frac{L}{r^2} \Rightarrow \frac{R_2}{R_1} = \frac{L_2}{L_1} \times \left(\frac{r_1}{r_2}\right)^2$$

Given, $L_1 = L, L_2 = 2L$
$r_1 = r, r_2 = 3r$

$$\frac{R_2}{R_1} = \frac{2L}{L} \times \left(\frac{r}{3r}\right)^2 = 2 \times \frac{1}{9} = \frac{2}{9}$$

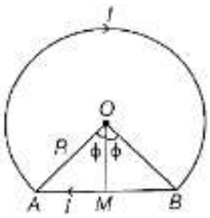
$$\Rightarrow R_2 = \frac{2}{9}R_1 = \frac{2}{9}R$$

**27. (b)** Given, $l_1 = 60$ cm, $l_2 = 40$ cm, $R = 4\Omega$
Internal resistance of the cell,

$$r = R\left(\frac{l_1}{l_2} - 1\right) = 4\left(\frac{60}{40} - 1\right)$$

$$= 4 \times \frac{1}{2} = 2\Omega$$

**28. (a)** The given situation is shown as




Magnitude of magnetic field due to straight current carrying wire AB is given as

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I}{OM}(\sin\phi + \sin\phi)$$

$$= \frac{\mu_0}{4\pi} \times \frac{I}{R\sin\phi}(2\sin\phi) \ (OM = R\sin\phi)$$

$$= \frac{\mu_0}{4\pi} \times \frac{2I}{R} = 10^{-7} \times \frac{2 \times 5}{0.1}$$

$(R = 100 \text{ mm} = 0.1 \text{ m and } I = 5 \text{ A})$

$= 10^{-5}$ T

Magnitude of magnetic field due to current carrying circular arc,

$$B_2 = \frac{\mu_0 I}{2R} \times \frac{(2\pi - 2\phi)}{2\pi}$$

$$= \frac{4\pi \times 10^{-7} \times 5}{2 \times 01} \times \frac{\left(2\pi - \frac{\pi}{2}\right)}{2\pi}$$

$$\left(2\phi = 90^\circ = \frac{\pi}{2}\right)$$

$$= \frac{4 \times 314 \times 10^{-7} \times 5}{0.2} \times \frac{3}{4}$$

$B_2 = 2.36 \times 10^{-5}$ T

Total magnetic field at O,

$B = B_1 + B_2 = 10^{-5} + 2.36 \times 10^{-5}$

$= 10^{-5}(I + 2.36) = 3.36 \times 10^{-5}$ T

$= 33.6 \times 10^{-6}$ T $= 33.6\mu$ T

**29. (a)** Given, inner radius of toroid,

$r_1 = 24$ cm $= 0.24$ m

Outer radius, $r_2 = 26$ cm $= 0.26$ m

Current, $I = 12$ A, $N = 2000$ turns

Magnetic field inside the core of toroid,

$B = \left(\frac{\mu_0 NI}{l}\right)$

where, $l = 2\pi\left(\frac{r_1+r_2}{2}\right) = 2\pi\left(\frac{0.24+0.26}{2}\right)$

$= 2\pi \times 0.25 = 0.5\pi$

$$B = \frac{\mu_0 NI}{0.5\pi}$$

$$= \frac{4\pi \times 10^{-7} \times 2000 \times 12}{0.5\pi}$$

$= 1.92 \times 10^{-2}$ T

**30. (a)** Magnetic dipole moment of planet,

$M = 27 \times 10^{22}\ A - m^2$

Radius of planet,

$R = 300$ km $= 3 \times 10^5$ m

$\frac{\mu_0}{4\pi} = 10^{-7}$

Magnetic field at the equator,

$$B = \frac{\mu_0}{4\pi} \times \frac{M}{R^3} = 10^{-7} \times \frac{27 \times 10^{22}}{(3 \times 10^5)^3}$$

$$= 10^{-7} \times \frac{27 \times 10^{22}}{27 \times 10^{15}} = 1\ T$$

**31. (b)** Given, number of turns per cm in long solenoid, $n = 20$ turns /cm $= 2000$ turns/m

Area of loop, $A = \frac{4}{\pi}$ cm$^2$ $= \frac{4}{\pi} \times 10^4$ m$^2$

$\Delta I = 3 - 1 = 2$ A $\Delta t = 0.2$ s

Magnetic field inside the solenoid,

$B = \mu_0 nI$ .......(i)

Magnetic flux linked with loop kept inside the solenoid normal to it,

$\phi = BA = \mu_0 nLA$ [from Eq. (i)]

According to Faraday's law of electromagnetic induction, magnitude of induced emf is given as

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(\mu_0 nIA) = \mu_0 nA\frac{dI}{dt}$$

$$\Rightarrow |e| = \mu_0 nA.\frac{\Delta I}{\Delta t} = 4\pi \times 10^{-7} \times 2000$$

$$\times \frac{4}{\pi} \times 10^{-4} \times \frac{2}{0.2}$$

$= 3.2 \times 10^{-6}$ V $= 3.2\mu$ V$(\because 10^{-6}$ V $= 1\mu$ V$)$

**32. (b)** Expression of AC current is given by $I(t) = 50\sin(200\pi t)$A

Maximum value of alternating current,

$I_0 = 50$ A

RMS value of alternating current,

$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 25\sqrt{2}$ A

From the given expression of alternating current, we get

$\omega = 200\pi$

$\Rightarrow 2\pi f = 200\pi \Rightarrow f = 100$ Hz

Hence, $I_{rms} = 25\sqrt{2}$ and f = 100 Hz

**33. (b)** Expression for the magnetic field of EM wave is given as

$B = (2 \times 10^{-8})\cos\left[\pi \times 10^{15}\left(t + \frac{y}{c}\right)\right]\hat{k}T$

Magnitude of magnetic field vector,

$B_0 = 2 \times 10^{-8}$ T

Magnitude of electric field vector of EM wave,

$E_0 = B_0 c$

where, c is speed of light

$= 2 \times 10^{-8} \times 3 \times 10^8$

$= 6\ V/m\ (\because c = 3 \times 10^8\ m/s)$

If $\hat{n}$ be the unit vector along the direction of electric field, then we know that unit vector along electric field vector x Unit vector along magnetic field vector = Unit vector along the direction of propagation of EM wave.

i.e.

clearly,

i.e. $\hat{n} \times \hat{k} = -\hat{j}$

$\hat{n} = \hat{i}$

Hence, expression of electric field vector of EM wave,

$E = 6\cos\left[\pi \times 10^{15}\left(t + \frac{y}{c}\right)\right]\hat{i}\ V/m$

**34. (c)** In photoelectric effect, kinetic encrgy of emitted photoelectrons is directly proportional to the frequency of emitted photoelectrons. Hence, on increasing frequency of incident radiation, kinetic energy of emitted photoelectron increases. Hence, statement I is incorrect, and statement IV is correct. In photoelectric effect, photoelectric emission takes place only when frequency of incident radiation is greater than or equal to threshold frequency, no matter how much intensity of incident light is. Hence, statement II is not correct.

In photoelectric emission, maximum kinetic energy of emitted photoelectrons depends only frequency of incident radiation and does not depend on intensity of incident radiation. Hence, statement III is correct.

**35. (c)** Photon energy is dependent on the frequency or wavelength of radiation. It does not depend on the intensity of radiation.

Since, $E = hv = \frac{hc}{\lambda}$

i.e. $E \propto$ vor $E \propto \frac{1}{\lambda}$

where, v = frequency and

$\lambda =$ wavelength.

**36. (c)** Given, transition levels,

$n_1 = 2 n_2 = 3$

Energy emitted during transition

$n_2 = 3$ to $n_1 = 2$ in H-atom

(i.e. for $H_\alpha$ - light)

$E = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$= 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$= 13.6\left[\frac{5}{36}\right]$ eV = 1.89eV

For $H_\alpha$-light to be able to emit photoelectrons from a metal, the work function must be less than or equal to 1.89 eV . Hence, maximum value of work function = 1.89eV

**37. (c)** Let A be the mass number and R be the radius of a nucleus. If m be the average mass of a nucleon, then mass of nucleus, M = mA

Volume of nucleus, $V = \frac{4}{3}\pi R^3$

Nuclear density,

$$\rho = \frac{\text{Mass of nucleus (M)}}{\text{Volume (V)}}$$
$$= \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi R^3}$$
$$= \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} \quad (R = R_0 A^{1/3})$$
$$= \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, nuclear density does not depend on mass number.

Therefore, with increase of mass number A, nuclear density does not change.

**38.** **(a)** When trivalent impurities (Al, Ga, B etc.) are doped in a pure semiconductor material ( Ge or Si), then p-type semiconductor is formed. In p-type semiconductor holes are majority charge carricrs and electrons are minority charge carriers.

**39.** **(d)** The truth table of NAND gate is given as

| A | B | AB | $Y = \overline{AB}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Hence, it is clear from truth table, when $A = 1, B = 1; Y = 0$

**40.** **(b)** Given, height of transmission antenna,

$h = 40$ m

Radius of earth,

$R = 6400 \text{ km} = 6.4 \times 10^6$ m

The maximum distance up to which signal can be broadcasted,

$d_{max} = \sqrt{2Rh}$

Service area, $A = \pi d_{max}^2$

$= \pi \times 2Rh = \pi \times 2 \times 6.4 \times 10^6 \times 40$

$= 512\pi \times 10^6 \text{ m}^2$

## Chemistry

**41.** **(c)** Energy required to transfer an electron in hydrogen atom is given by,

$$\Delta E = \frac{-13.6}{n_2^2} - \frac{-13.6}{n_1^2}$$

Here, $n_1 = 2$ and $n_2 = 3$

$$E_H = \frac{-13.6}{3^2} - \frac{-13.6}{2^2} = 1.89\text{eV} \simeq 1.9\text{eV}$$

**42.** **(c)** We know that, $\lambda = \frac{h}{mv}$

(where, v is velocity.)

So, $v = h/m\lambda$ (where, v is frequency.) and $\lambda = C/v$

In relation, $mv = h/\lambda$, we can substitute the value of $\lambda$.

$$mv = \frac{h}{C/v} \Rightarrow v = \frac{hv}{mC}$$

**43.** **(c)** Helium (He) is the second-most abundant element in the universe, after hydrogen, and accounts for about 25 per cent of the elements in the universe. Most of the helium in the universe was created in the Big Bang, but it is also obtained as the product of hydrogen fusion in stars. The Darmstadtium is a chemical element with the symbol Ds and atomic number 110 . The very small atomic radius of osmium results in a small metal-metal separation. This small atomic separation along with osmium's relatively high atomic number gives rise to osmium's highest density. In group 1A, francium is more electropositive than ceasium but, not stable as it is highly radioactive (maximum half-life of only 22 minutes).

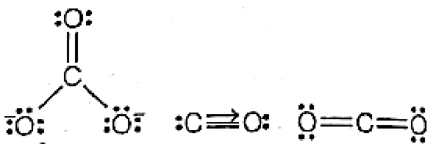
**44. (a)** The electronic configuration of elements are as follows

$\mathrm{Li} = 1s^2 2s^1$; $\mathrm{Be} = 1s^2 2s^2$

$\mathrm{B} = 1s^2 2s^2 2p^1$; $\mathrm{N} = 1s^2 2s^2 2p^3$

All these element belong to second period. The ionisation energy generally increases as you move from left to right across a period. The first ionisation energy of beryllium is greater than that of boron since the order of attraction of electrons towards nucleus is $2s > 2p$. The
beryllium has a stable complete electronic configuration $1s^2 2s^2$ whereas boron has the electronic configuration $1s^2 2s^2 2p^1$ which needs lesser energy than that of beryllium to remove the valence electron. Therefore, the order of first ionisation energy is
$\mathrm{Li} < \mathrm{B} < \mathrm{Be} < \mathrm{N}$.

**45. (d)**



$CO_3^{2-}$ CO $CO_2$

Bond order of $CO_3^{2-} = 1.33$
Bond order of $CO = 3$
Bond order of $CO_2 = 2$
Bond order $\propto \frac{1}{\text{bond length}}$
Therefore, the correct order of bond length is
$CO < CO_2 < CO_3^{2-}$

**46. (c)**

| Molecule | Total number of electrons | Electronic configuration | Bond order (BO)= $\frac{a-b}{2}$ |
|---|---|---|---|
| $CN^+$ | 12 | $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^1 = \pi 2p_y^1 \pi^* 2p_x^0 = \pi^* 2p_y^0$ | $\frac{8-4}{2} = 2$ |
| $NO^-$ | 16 | $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi^* 2p_x^1 = \pi^* 2p_y^1$ | $\frac{10-6}{2} = 2$ |
| $N_2^{2+}$ | 12 | $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^1 = \pi 2p_y^1 \pi^* 2p_x^0 = \pi^* 2p_y^0$ | $\frac{8-4}{2} = 2$ |
| $O_2$ | 16 | $\sigma 1s^2 \sigma^* 1s^2 \sigma^2 s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi^* 2p_x^1 = \pi^* 2p_y^1$ | $\frac{10-6}{2} = 2$ |
| $C_2$ | 12 | $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^1 = \pi 2p_y^1 \pi^* 2p_x^0 = \pi^* 2p_y^0$ | $\frac{8-4}{2} = 2$ |

| | | | |
|---|---|---|---|
| $C_2^+$ | 11 | $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^1 = \pi 2p_y^0 \pi^* 2p_x^0 = \pi^* 2p_y^0$ | $\frac{7-4}{2} = 1.5$ |
| $B_2^{2-}$ | 12 | $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^1 = \pi 2p_y^1 \pi^* 2p_x^0 = \pi^* 2p_y^0$ | $\frac{8-4}{2} = 2$ |

**47. (a)** At high pressure, molecules tend to be more crowded together, If they are closer together, the intermolecular forces are stronger, and cause more deviations from ideal gas behaviour. The ideal gases are assumed to occupy no space and have no attractions for one another.

**48. (c)** The kinetic-molecular theory of g$^2$ ases assumes that

A. The particles are much smaller than the distance between particles. Most of the volume of a gas is therefore empty space.

B. Collisions between gas particles or collisions with the walls of the container are perfectly elastic. None of the energy of a gas particle is lost when it collides with another particle or with the walls of the container.

C. Different particles have different speed.

D. The pressure of a gas results from collisions between the gas particles and the walls of the container.

So, option (A) and (D) are only correct.

**49. (b)** 50%w/w solution of HCl solution means that 50 g of HCl is present in 100 g solution.

$CaCO_3 + 2HCl \longrightarrow CaCl_2 + H_2CO_3$

1 mole of $CaCO_3$ reacts with 2 moles of HCl

100 g of $CaCO_3$ = 1 mole$^2$ $CaCO_3$

200 g of $CaCO_3$ = 2 moles of $CaCO_3$

2 moles of $CaCO_3$ will react with 4 moles of HCl

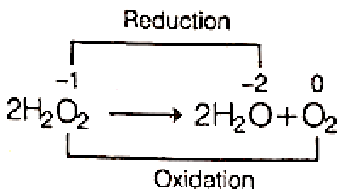
1 mole of HCl = 36.5 g of HCl

4 moles of HCl = $4 \times 36.5$ g of HCl

= 146 g of HCl

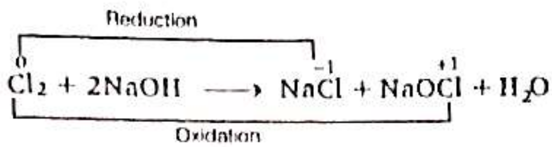
50 g of HCl is present in 100 g solution. 146 g of HCl will be present in 292 g solution.

**50. (a)** Disproportionation reactions are those in which one element undergoes both oxidation and reduction both simultaneously. In the reaction,




Oxygen in reactant $H_2O_2$ is in -1 oxidation state. Oxygen in product $H_2O$ is in -2 oxidation state and in other product $O_2$, it is in 0 oxidation state. So, in this reaction oxygen undergoes oxidation and reduction both simultaneously.

In the reaction




Chlorine in reactant $Cl_2$ is in 0 oxidation state. Chlorine in product NaCl is in -1 oxidation state and in other product NaOCl, Na is in +1 oxldation state, 0 is in -2 oxidation state and Cl is in +1 oxidation state. So, in this reaction chlorine undergoes oxidation and reduction both simultancously.

**51. (b)** $2Na(s) + 2H_2O(t) \longrightarrow$

$2NaOH(aq) + H_2(g)$

1 mole of Na = 23 g of Na

4 moles of Na = 92 g of Na

2 moles of Na reacts with 2 moles of $H_2O$ to form
1 mole of $H_2$.
4 moles of Na will react with 4 moles of $H_2O$ to form 2 moles of $H_2$.
Work done by expansion of gas is given by $-p\Delta V$
$p\Delta V=\Delta n_g RT$
At constant external pressure,
$\Delta V=V_{H_2}-0 \quad =V_{H_2}$
$p\Delta V \quad =\Delta n_g RT$
$p\Delta V=\Delta n_g RT \quad =2\times 8.314\times 300=4.98\text{ kJ}$
Therefore, the work done is 4988.4 J .
Since it is work of expansion so will be with negative sign ( -4988.4 J ).

**52. (c)** For the reaction at equilibrium,

$A(s)\rightleftharpoons B(s)+C(g)$

$K=\frac{[B][C]}{[A]}$

The concentrations of pure solids, pure liquids, and solvents are omitted from equilibrium constant expressions because they do not change significantly during reactions when enough amount is present to reach equilibrium.
Therefore, the expression is written as, $K_C=[C]$
$K_p=pC(g)$, where $pC(g)$ is the partial pressure of C.
$\Delta H$ is dependent on temperature, if the reaction is exothermic, and the temperature is increased, the reaction enthalpy becomes more negative (more heat energy is released). If the reaction is endothermic, and the temperature is increased, the reaction enthalpy becomes more positive (more heat energy is absorbed). $\Delta H$ is independent of catalyst addition. The catalyst only decreases the activation energy of a reaction, thereby increases the reaction rate but does not affect the enthalpy of reaction. So, statements (A), (B) and (D) are only correct.

**53. (d)** The value of $K_w$ increases with rise in temperature because dissociation of water is endothermic process. At 37°C, $K_w$ is $10^{-14}$ and pH of water is 7 . So, at 80°C, $K_w$ will be more than $10^{-14}$ and pH of water will be less than 7.

**54. (a)** Consider LiH as the molten ionic hydrides of s-block,
$LiH\longrightarrow Li^++H^-$
Upon passing electric current, the oxidation reaction that take place at anode.
At anode, $2H^-\longrightarrow H_2(g)+2e^-$
So, $H_2$ gas is liberated at anode.

**55. (b)** Lithium nitrate, on heating, decomposes to give lithium oxide, whereas other alkali metals nitrate decomposes to give their corresponding nitrite.

$2LiNO_3\longrightarrow 2Li_2O+4NO_2+O_2$

$2KNO_3\longrightarrow 2KNO_2+O_2$

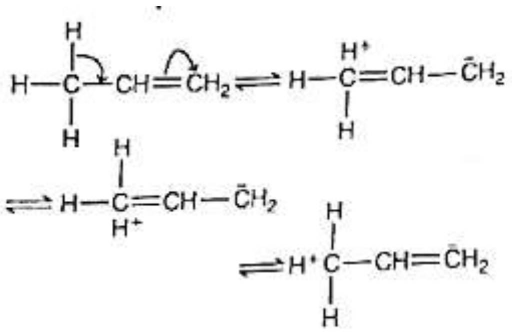
**56. (a)** The reactions in which a more reactive element displaces another element from its compounds are called displacement reactions.

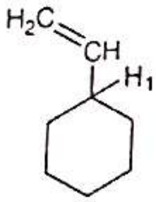
$2Al(s)+2NaOH(l) \quad +6H_2O(l)\rightarrow$
$2Na[Al(OH)_4](s) \quad +3H_2(g)$

The reaction of aluminium with sodium hydroxide gives a salt and hydrogen gas. In this reaction, aluminium displaces sodium from its compound, therefore, it is a displacement reaction.

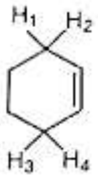
**57. (d)** The electronegativity decreases down the group. C, Ge and Pb all belong to 14 th group. So, the order should be $C>Ge>Pb$. But the actual order is $C>Pb>Ge$. This can be explained on the basis of shielding effect. The valence shell electronic configuration of Ge is [Ar] $3d^{10}4s^2 4p^2$ and that of Pb is [Xe] $6s^2 4f^{14}5d^{10}6p^2$. Due to the poor shielding effect of intervening ' f ' electrons in Pb , the $Z_{eff}$ gets highly increased in Pb . Therefore, the electronegativity of Pb is higher than Ge .

**58. (a)** Hyperconjugation is the stabilising interaction that results from the interaction of the electrons in a σ-bond (usually $C-H$ ) with an adjacent empty or partially filled p-orbital or a π-orbital to give an extended molecular orbital that increases the stability of the system. Consider the molecule propene. where π bond is in conjugation with $30C-II$ bond of adjacent methyl group. So, higher the number of adjacent $\sigma C-H$ bond to π-bond higher will be the hyperconjugation and higher will be the stability.

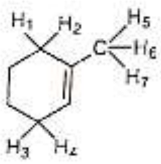




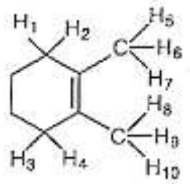
(i)



(ii)



(iii)



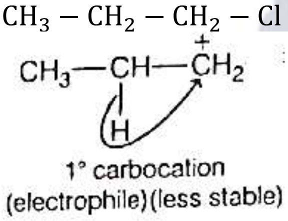
(iv)

The adjacent $\sigma C - H$ bond to $\pi$ bond increases in the order of IV > III > II > I.
Therefore, stability follows the same order.

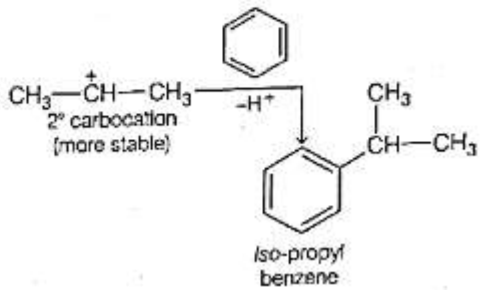
**59. (a)**

**60. (a)** In reaction A,

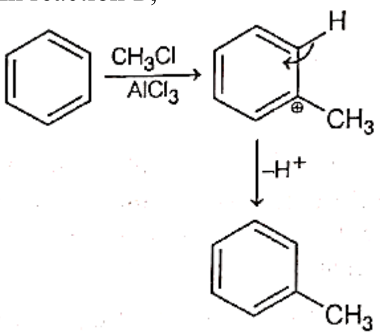
$CH_3 - CH_2 - CH_2 - Cl + AlCl_3 \xrightarrow{-AlCl_4^-}$



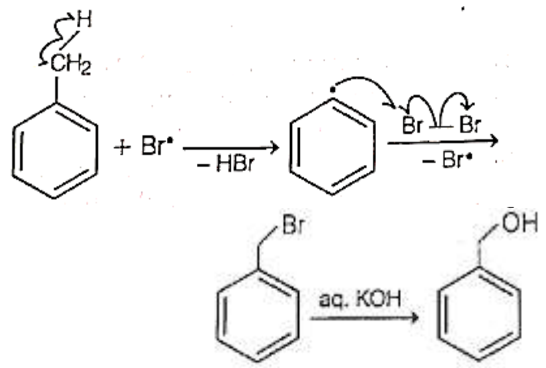
1, 2-hydride shift



In reaction B,



$Br - Br \xrightarrow{hv} 2Br^{\circ}$



**61. (a)** The density is given by

$$\rho = \frac{Z \times M}{N_A \times a^3}$$

Since their is 0.1% Schottky defect, so the Z value will change.

$$Z = 4 - \frac{4 \times 0.1}{100} = 3.996$$

Putting this value of Z in above equation

$$\rho = \frac{3.996 \times 55.8 \text{ g}}{6.023 \times 10^{23} \times (400 \times 10^{-10} \text{ cm})^3}$$

$$= 5.8 \text{ g cm}^{-3}$$

**62. (c)** In ideal solutions, the volume of the solution is the sum of the volumes of the components before mixing.

$V_{solvent} + V_{solute} = V_{solution}$

i.e. there is no change in volume on mixing or $\Delta V_{mix}$ is zero.

The intermolecular interactions between the components ( $A-B$ interactions) are of same magnitude as the intermolecular interactions in pure components ( $A-A$ and $B-B$ interactions). Since, there is no change in magnitude of the intermolecular interactions in the two components present, the heat change on mixing, i.e. $\Delta H_{mix}$ in such solutions must be zero.

$H_2O + CO_2 \longrightarrow H_2CO_3$

This is a reaction in which carbonic acid is formed from water and $CO_2$. So, this is not an ideal solution.

**63. (a)** We have the equation $\pi = iCRT$.As urea is non-electrolyte, $i = 1, \pi$ is the osmotic pressure at temperature T.

$$\pi_1 = C_1RT_1$$

$$400 = C_1R \times 273 \text{ K} \quad \ldots..(i)$$

$$\pi_2 = C_2RT_2$$

$$100 = C_2R \times 293 \text{ K} \quad \ldots\ldots(ii)$$

Dividing Eq. (i) by Eq. (ii),

$$\frac{\pi_1}{\pi_2} = \frac{400}{100} = \frac{C_1 \times 273}{C_2 \times 293}$$

$$\frac{C_1}{C_2} = \frac{400 \times 293}{100 \times 273} = 4.29$$

$C_1$ is 4.29 times $C_2$. So, the dilution factor is 4.3.

**64. (d)** Consider the reduction reaction

$M^{n+} + ne^- \longrightarrow M$

According to Faradays law,

$$W = \frac{M}{n} \times \frac{i \times t}{F} = \frac{M}{n} \times \frac{Q}{F}$$

If W is the weight of substance deposited.

Since W is given as 1 g equivalent, which is equal to $M/n$. Therefore, $Q = F$.

1 F = 96500 coulomb of charge = charge on 1 mole of electrons. Therefore, to deposit 1 g equivalent of substance 1 mole of electrons is needed.

**65. (b)** The temperature coefficient is given by the ratio $\frac{k_2}{k_1}$, where $k_1$ and $k_2$ are the rate constants at temperature $T_1$ and $T_2$ when $T_2 = T_1 + 10$. This is approximately equal to 2 .

$$\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log 2 = \frac{E_a}{2.303 \times 8.314}\left[\frac{1}{300} - \frac{1}{310}\right]$$
$$E_a = 53.5 \text{ kJ mol}^{-1}$$

**66. (c)** According to Hardy Schulze rule, greater the valency of the coagulating ion, greater is its coagulating power. $As_2 S_3$ is a negatively charged sol. For coagulation, oppositely charged sol is needed. Thus, out of the given, $FeCl_3(Fe^{3+})$ is most effective for causing coagulation of $As_2 S_3$ sol.

**67. (c)**

**68. (d)** Gradual increase in boiling points down the group happens as the relative molecular mass of the molecules increases. But the water has anomalously highest boiling point due to strong intermolecular hydrogen bonding in water molecules. Therefore, the order is $H_2O > H_2Te > H_2Se > H_2 S$.

**69. (d)** Carnallite is not the mineral of fluorine, whereas all others are mineral of fluorine. Carnallite (also carnalite) is an evaporite mineral, a hydrated potassium magnesium chloride with formula $KMgCl_3 \cdot 6(H_2O)$.

**70. (a)** The element that can even diffuse through silica glass is He due to very small size.

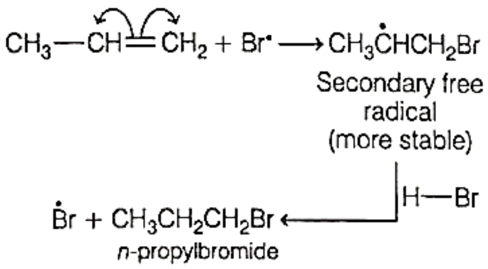
**71. (b)** According to Fajan's rule, smaller is the size of cation and more positive charge it has, stronger is the polarising power of that cation. Hence, that ion has a larger tendency to distort the electron cloud of anion. Hence, it has more tendency to form covalent compound. The metal ions of $FeF_3, VF_5, VF_2$ and $TiF_2$ all belong to fourth period of periodic table. The metal ions exist as $Fe^{3+}, V^{5+}, V^{2+}$ and $Ti^{+2}$ respectively. The anions are same in all the compounds. The $V^{5+}$ has the smallest size and largest charge. So the molecule $VF_5$ has largest covalent character.

**72. (c)** A spectrochemical series is a list of ligands ordered by ligand strength, weak field $I^- < Br^- < S^{2-} < SCN^- <$
$Cl^- < NO_3^- < F^- < C_2O_4^{2-} < H_2O < NCS^- <$
$CH_3CN < NH_3 < en < bipy < phen <$
$NO_2 < PPh_3 < CN^- < CO$ strong field
$NCS^- S^{2-}$ en $SCN^-$
I II III IV
So, according to series III > I > IV > II.

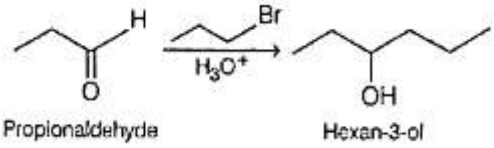
**73. (d)** The structure of proteins is destroyed by the application of heat or chemical agents and the process is called denaturation. It involves the disruption of both the secondary and tertiary structures of a protein, but the primary structure, which involves a sequence of amino acids does not get disturbed and remains intact. This results in the loss of the biological activity of the proteins. The protein in milk is normally suspended in a colloidal solution, which means that the small protein molecules float around freely and independently. On adding lime juice to milk the pH decreases and the small protein molecules gets coagulated and forms large chain. This is denaturing of protein and causes curdling.
Therefore the (A) is false but (R) is true.

**74. (a)** (i) Reaction of HBr with propene in the presence of peroxide gives n-propyl bromide. This addition reaction is an example of an anti-Markownikoff's addition reaction.

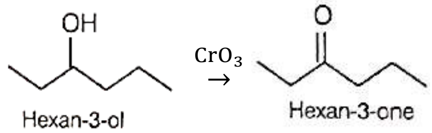
$$R - O - O - R \xrightarrow{\Delta} 2R - O^\circ$$
$$2R - O^* + H - Br \longrightarrow R - OH + Br^*$$



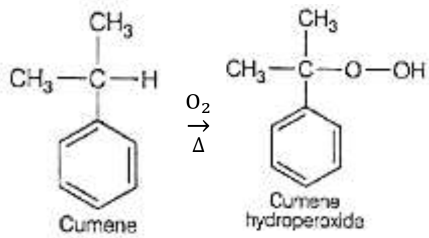

(ii) Reaction of n-propyl bromide with Mg gives propyl magnesium bromide (Grignard reagent).
$$\underset{\text{Propyl bromide}}{CH_3CH_2CH_2Br} + Mg \rightarrow CH_3CH_2CH_2MgBr$$
(iii) Reaction of propyl magnesium bromide with propanal gives nucleophilic additon reaction.

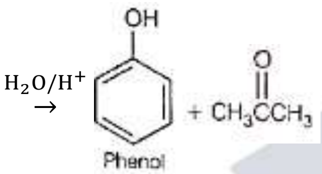



(iv) Reaction of hexan-3-ol with oxidising agent $CrO_3$.

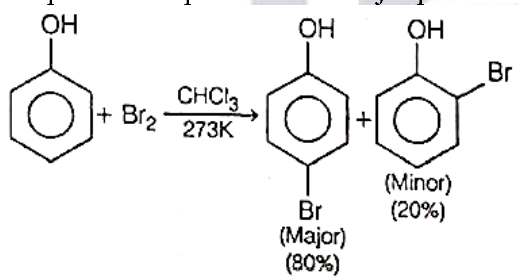



**75.** **(d)** Step (i) and (ii) The reaction of cumene with oxygen results in the formation of peroxide which on acidic treatment gives phenol and acetone.

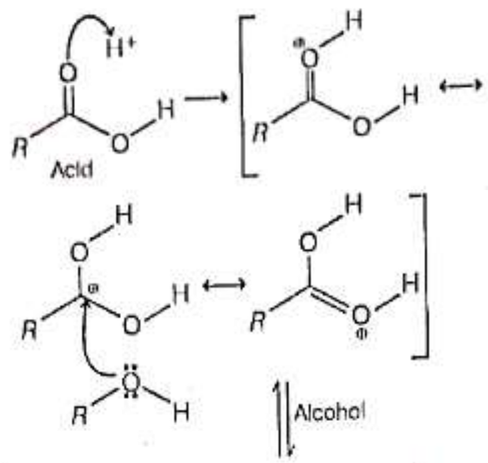


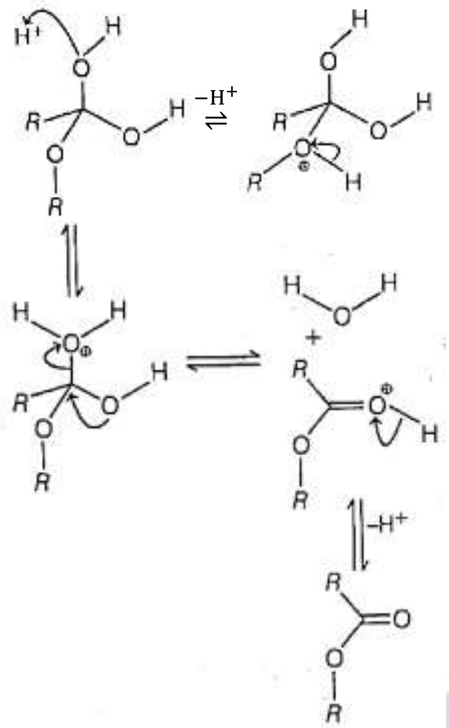




**Step (iii)** Phenol is o,-p-directing group. At low temperature and in the presence of non-polar solvent like $CHCl_3$, phenoxide ion is not generated. The ring doesn't get highly activated for trisubstitution. So the phenol takes part in the electrophilic substitution reaction and gives a mixture of ortho and para bromophenol.
The para-bromophenol is the major product.

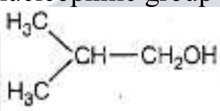



**76.** **(c)** The alcohol acts as nucleophile and attacks the carbonyl carbon of acid in esterification.



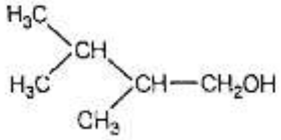


As the bulkiness of the nucleophilic group increases the reactivity towards esterification decreases.



$CH_3 - CH_2OH$
(i)

(ii)



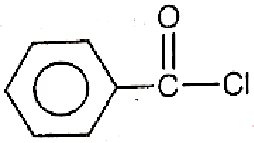
(iii)

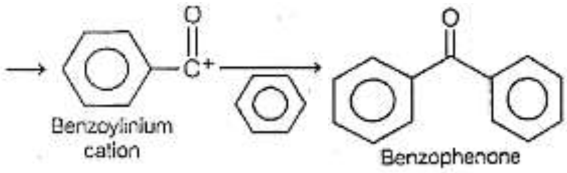
$CH_3 - CH_2 - CH_2OH$
(iv)

The nucleophilicity order is
I > IV > II > III
Therefore, reactivity towards esterification decreases in same order.

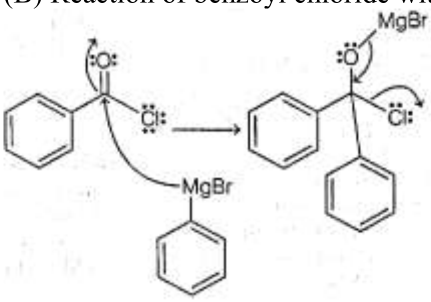
**77. (d)**

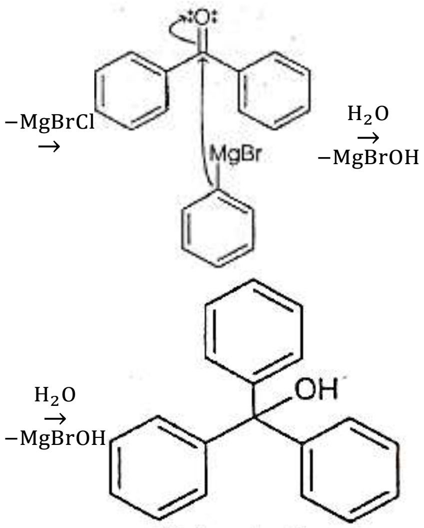
**78. (c)** (A) Reaction of benzoyl chloride with benzene and aluminium chloride gives benzophenone.


$+AlCl_3$ (anhydrous)



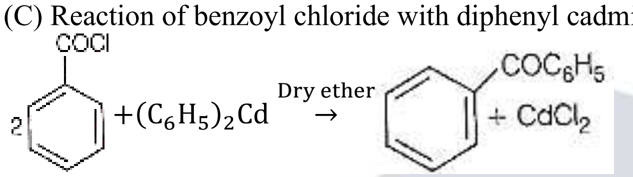

(B) Reaction of benzoyl chloride with phenyl magnesium bromide (excess).



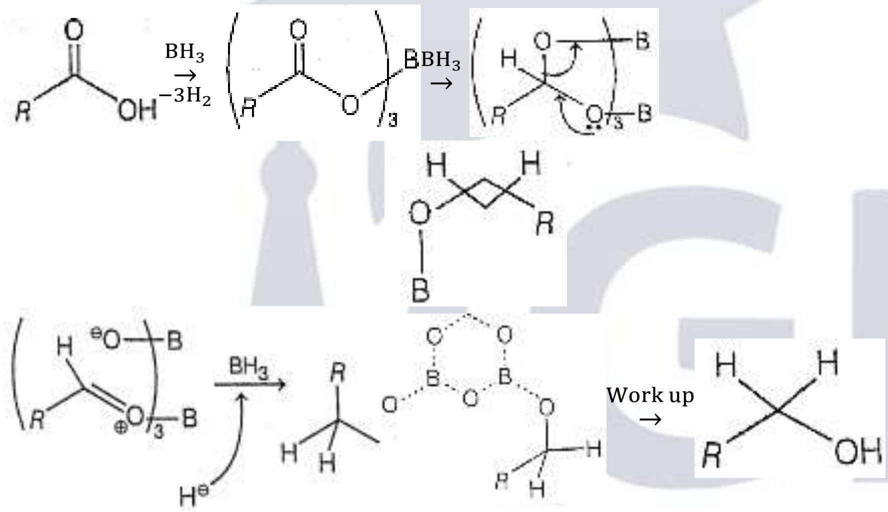


Triphenylmethanol

(C) Reaction of benzoyl chloride with diphenyl cadmium gives benzophenone.



**79. (b)** The reagent that can reduce the carboxylic acid to corresponding alcohol is diborane (disassociated to $BH_3$ in ether or THF solution)



$NaBH_4$ being weak reducing agent does not reduce carboxylic acids but reduces aldehyde and ketones.
The reaction of aldehydes and ketones with zinc amalgam ( $Zn/Hg$ alloy) in concentrated hydrochloric acid, reduces the aldehyde or ketone to a hydrocarbon, is called Clemmensen reduction. It does not reduce carboxylic acid. $H_2$ over Pd/C will reduce alkenes and alkynes all the way to alkanes.

**80. (d)** In Hofmann bromamide degradation reaction, an amide reacts with bromine and an aqueous solution of sodium hydroxide and result in the production of primary amine. The number of ' C ' in product (amine) is always one less than present in parent amide (reactant).

$$\underset{\text{(General amide)}}{R-CO-NH_2} + \underset{\text{(Bromine)}}{Br_2} + \underset{\text{(Sodium hydroxide)}}{4NaOH} \longrightarrow$$

$$R-NH_2 + Na_2CO_3 + 2NaBr + 2H_2O$$

$$\underset{\text{Hexanamide}}{CH_3CH_2CH_2CH_2CH_2CONH_2} + Br_2 + NaOH \longrightarrow$$

$$\underset{\text{Pentanamine}}{CH_3CH_2CH_2CH_2CH_2NH_2} + Na_2CO_3 + 2NaBr + 2H_2O$$

## Mathematics

**81. (a)**

$$f(x)=\frac{\sqrt{6x^2+5x-6}}{\sqrt{4-x}-\sqrt{x+4}}$$

$$6x^2+5x-6\geq 0$$

$$\Rightarrow\ 6x^2+9x-4x-6\geq 0$$

$$\Rightarrow\ 3x(2x+3)-2(2x+3)\geq 0$$

$$\Rightarrow\ (3x-2)(2x+3)\geq 0$$

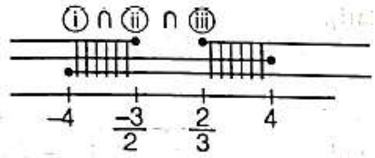
$$x\in\left(-\infty,\frac{-3}{2}\right]\cup\left[\frac{2}{3},\infty\right)\ \ldots\ldots(\text{i})$$

$$\Rightarrow\ 4-x\geq 0\Rightarrow x\leq 4$$

$$\Rightarrow\ x\in(-\infty,4]\qquad\ldots\ldots(\text{ii})$$

$$\Rightarrow\ x+4\geq 0\ x\geq -4$$

$$x\in[-4,\infty)\qquad\ldots\ldots(\text{iii})$$




$$x\in\left[-4,\frac{-3}{2}\right]\cup\left[\frac{2}{3},4\right]$$

**82. (b)** Given, $f(x)=\frac{1}{\sqrt{[x]^2+[x]-2}}$

$$=\frac{1}{\sqrt{\left([x]+\frac{1}{2}\right)^2-\frac{9}{4}}}$$

Maximum value of $\left([x]+\frac{1}{2}\right)^2$ is $\infty$.

So, $f(x)=\frac{1}{\sqrt{\infty-\frac{9}{4}}}\to 0$

$\left([x]+\frac{1}{2}\right)^2_{\min}-\frac{9}{4}$ to be positive, $[x]$ must be 2 .

$$\Rightarrow\ \left(2+\frac{1}{2}\right)^2-\frac{9}{4}=\frac{25}{4}-\frac{9}{4}=\frac{16}{4}=4$$

$$\Rightarrow\ f(x)=\frac{1}{\sqrt{4}}=\frac{1}{2}$$

Range : $\left(0,\frac{1}{2}\right]$

**83. (c)** Given, trace of $A=-4$

$$a+(-1)+(-4)=-4$$

$$\Rightarrow\ a=1$$

$$\text{Now, } AB=\begin{bmatrix}-1&0&17\\-3&10&25\\28&-8&3\end{bmatrix}$$

$$\begin{bmatrix}1&3&5\\5&-1&3\\2&3&-4\end{bmatrix}\begin{bmatrix}b&1&4\\4&c&1\\-3&1&d\end{bmatrix}=\begin{bmatrix}-1&0&17\\-3&10&25\\28&-8&3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}b-3&3c+6&7+5d\\5b-13&8-c&19+3d\\2b+24&3c-2&11-4d\end{bmatrix}$$

$$=\begin{bmatrix}-1&0&17\\-3&10&25\\28&-8&3\end{bmatrix}$$

On comparing,

$$\begin{aligned} b-3=-1 \Rightarrow b &= 2 \\ 3c+6=0 \Rightarrow c &= -2 \\ 7+5d=17 \Rightarrow d &= 2 \\ a+b+c+d=1+2-2+2 &= 3 \end{aligned}$$

**84. (b)**

Given, $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3-b^3 & & c^3 \end{vmatrix}$

$= (b^2c^3 - b^3c^2) - (a^2c^3 - a^3c^2)$

$+(a^2b^3 - a^3b^2)$

$= (a^2b^3 - a^2c^3) + (b^2c^3 - a^3b^2)$

$+(a^3c^2 - b^3c^2)$

$= a^2(b^3-c^3) + b^2(c^3-a^3) + c^2(a^3-b^3)$

**85. (d)** Let $\alpha, \beta, \gamma$ be real numbers, then

$$A = \begin{bmatrix} 7 & 3 & \alpha \\ \beta & 1 & -11 \\ -5 & \gamma & 19 \end{bmatrix}$$

$$\Rightarrow A\begin{bmatrix} 5 \\ -13 \\ 11 \end{bmatrix} = \begin{bmatrix} -290 \\ -119 \\ 210 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 3 & \alpha \\ \beta & 1 & -11 \\ -5 & \gamma & 19 \end{bmatrix}\begin{bmatrix} 5 \\ -13 \\ 11 \end{bmatrix} = \begin{bmatrix} -290 \\ -119 \\ 210 \end{bmatrix}$$

$11\alpha - 4 = -290 \Rightarrow 11\alpha = -286$

$\Rightarrow \alpha = -265\beta - 134 = -119$

$\Rightarrow 5\beta = 15 \Rightarrow \beta = 3 - 13\gamma + 184 = 210$

$\Rightarrow -13\gamma = 26 \Rightarrow \gamma = -2$

$$A = \begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & -11 \\ -5 & -2 & 19 \end{bmatrix}$$

$|A| = 7(-3) - 3(2) - 26(-1)$

$= -27 + 26 = -1$

$$A^{-1} = \frac{\operatorname{adj}(A)}{|A|}$$

$\Rightarrow \operatorname{adj}(A) = A^{-1}|A|$

$$\begin{aligned} (\operatorname{adj} A)^{-1} + \operatorname{adj} A^{-1} &= (\operatorname{adj} A)^{-1} + (\operatorname{adj} A)^{-1} \\ &= 2(\operatorname{adj} A)^{-1} \\ = 2(A^{-1}|A|)^{-1} &= 2A \cdot |A|^{-1} \\ = \frac{2A}{|A|} = \frac{2A}{-1} &= -2A \end{aligned}$$

**86. (a)** Given, $[\alpha\beta\gamma]\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -5 \\ 1 & 2 & 5 \end{bmatrix} = [3 \quad 5 \quad 2]$

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -5 \\ 1 & 2 & 5 \end{bmatrix}$

$[\alpha \quad \beta \quad \gamma] \cdot A = [3 \quad 5 \quad 2]$

$[\alpha \quad \beta \quad \gamma] = [3 \quad 5 \quad 2]A^{-1}$

$|A| = 1(15+10) - 2(10+5) + 3(4-3)$

$|A| = 25 - 30 + 3 = -2$

$$\operatorname{adj}(A) = \begin{bmatrix} 25 & -4 & -19 \\ -15 & 2 & 11 \\ 1 & 0 & -1 \end{bmatrix}$$

Now, $[\alpha \quad \beta \quad \gamma] = [3 \quad 5 \quad 2]\frac{\operatorname{adj} A}{|A|}$

$$[\alpha \quad \beta \quad \gamma] = \frac{1}{-2}[3 \quad 5 \quad 2]\begin{bmatrix} 25 & -4 & -19 \\ -15 & 2 & 11 \\ 1 & 0 & -1 \end{bmatrix}$$

$$[\alpha \quad \beta - \gamma] = -\frac{1}{2}[2 - 2 - 4] = [-1 \quad 1 \quad 2]$$

On comparing $\alpha = -1, \beta = 1$ and $\gamma = 2$

$\alpha^3 + \beta^3 + \gamma^3 = -1 + 1 + 8 = 8$

**87. (c)** Given, $\sqrt{(-3 + 4i)(8 + 6i)}$

$= \sqrt{-48 + 14i} = \sqrt{2}\sqrt{-24 + 7i}$

We know that,

$$\sqrt{z} = \pm\left(\sqrt{\frac{|z| + \operatorname{Re}(z)}{2}} + i\sqrt{\frac{|z| - \operatorname{Re}(z)}{2}}\right) \text{ for}$$

$\operatorname{Im}(z) > 0$

$\sqrt{-24 + 7i}$

$$= \pm\left(\sqrt{\frac{25 - 24}{2}} + i\sqrt{\frac{25 + 24}{2}}\right)$$

$$= \pm\frac{1}{\sqrt{2}}(1 + 7i)$$

$$\sqrt{(-3 + 4i)(8 + 6i)} = \pm(1 + 7i)$$

**88. (c)** We know that,

$$\omega = \frac{-1}{2} + i\frac{\sqrt{3}}{2} \text{ and } \omega^2 = \frac{-1}{2} - i\frac{\sqrt{3}}{2}$$

$\Rightarrow \ -2i\omega = \sqrt{3} + i$ and $2i\omega^2 = \sqrt{3} - i$.

$$\left(\frac{\sqrt{3} + i}{\sqrt{3} - i}\right)^m = 1$$

$$\Rightarrow \left(\frac{-2i\omega}{2i\omega^2}\right)^m = 1 \Rightarrow (-\omega)^m = 1$$

$$\Rightarrow (-1)^m\omega^m = 1$$

$\Rightarrow$ m is even number and multiple of 3 . Clearly, 2028 is even and multiple of 3.

**89. (c)** Here, 1, $\omega$ and $\omega^2$ are the cube root of unity, $n \in N, n > 2$

$1 + \omega + \omega^2 = 0, \omega^3 = 1$

$x^n - x = 0 \rightarrow 1 + \omega$ is a root.

So, $(a + \omega)^n - (a + \omega) = 0$

$\Rightarrow \quad (1 + \omega)^n = (1 + \omega)$

$(1 + \omega)^{n-1} = 1 \Rightarrow (-\omega^2)^{n-1} = 1$

Least value of n must be 7 .

**90. (b)** $A = \left\{x \in R/\sqrt{x^2 - 8x + 15} \in R\right\}$

$\Rightarrow \quad x^2 - 8x + 15 \geq 0$

$\Rightarrow \quad (x - 5)(x - 3) \geq 0$

$\Rightarrow \quad x \in (-\infty, 3] \cup [5, \infty]$

$A = (-\infty, 3] \cup [5, \infty]$

$$B = \left\{x \in R/\frac{x - 3}{2x - 5} < \frac{x - 6}{2x - 11}\right\}$$

$$\Rightarrow \frac{x - 3}{2x - 5} - \frac{x - 6}{2x - 11} < 0$$

$$\Rightarrow \frac{-10x+3}{(2x-5)(2x-11)} < 0$$

$$\Rightarrow \frac{10x-3}{(2x-5)(2x-11)} > 0$$

$$\frac{-ve}{-ve\times-ve}\ \frac{+ve}{-ve\times-ve}\ \frac{+ve}{+ve\times-ve}\ \frac{+ve}{+ve\times+ve} - ve$$

$$-ve\ \frac{3}{10} + ve\ \frac{5}{2} \qquad \frac{11}{2} \ + ve$$

$$\Rightarrow \quad x \in \left(\frac{3}{10}, \frac{5}{2}\right) \cup \left(\frac{11}{2}, \infty\right)$$

$$B = \left(\frac{3}{10}, \frac{5}{2}\right) \cup \left(\frac{11}{2}, \infty\right)$$

$$A \cap B = \left(\frac{5}{2}, 3\right] \cup \left[5, \frac{11}{2}\right)$$

**91.** **(c)** Let $f(x) = 3x - 2x^2 + 1$

Given, extreme value of f(x) is k.

$$\Rightarrow f'(k) = 0\ 3 - 2(2k) = 0\ k = 3/4$$

$$kx^2 + 2x + 1 > 0$$

$$\Rightarrow \frac{3}{4}x^2 + 2x + 1 > 0 \Rightarrow 3x^2 + 8x + 4 > 0$$

$$\Rightarrow \left(x + \frac{2}{3}\right)(x+2) > 0$$

$$\Rightarrow x \in \left(-\infty, \frac{-2}{3}\right) \cup (-2, \infty)$$

**92.** **(d)** Let $\alpha, \beta, \gamma$ be the roots of the equation

$x^3 - 5x^2 - 2x + 24 = 0$

$\alpha + \beta + \gamma = 5 \ldots.(i)$

$\alpha\beta + \beta\gamma + \gamma\alpha = -2 \quad \ldots.(ii)$

$\alpha\beta\gamma = -24 \ldots..(iii)$

$$\frac{\beta\gamma}{\alpha} + \frac{\gamma\alpha}{\beta} + \frac{\alpha\beta}{\gamma}$$

$$= \frac{\beta\gamma\gamma}{-24} \times \beta\gamma + \frac{\gamma\alpha}{-24} \times \gamma\alpha + \frac{\alpha\beta}{-24} \times \alpha\beta \text{ [using Eq. (iii)]}$$

$$= \frac{1}{-24}[(\beta\gamma)^2 + (\gamma\alpha)^2 + (\alpha\beta)^2]$$

$$= \frac{-1}{24}[(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha\beta^2\gamma$$

$=$

$+\alpha\beta\gamma^2 + \alpha^2\beta\gamma)]$

$$= \frac{-1}{24}[(-2)^2 - 2(-24(\alpha + \beta\beta\gamma^2 + \alpha^2\beta\gamma)]$$

$$= \frac{-1}{24}[4 + 2 \times 24 \times 5] = -\frac{61}{6}$$

**93.** **(c)** Let $\alpha, \beta, \gamma$ be the roots of the equation

$3x^3 - 26x^2 + 52x - 24 = 0$

$$\alpha + \beta + \gamma = \frac{26}{3} \quad \ldots.(i)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{52}{3} \ldots..(ii)$$

$\alpha\beta\gamma = 8 \ \ldots\ldots(iii)$

$\alpha, \beta, \gamma$ are in GP $\Rightarrow \beta^2 = \alpha\gamma$

From Eq. (iii), $\beta^3 = 8 \Rightarrow \beta = 2$ .......(iv)

From Eqs. (i) and (iv),

$\alpha + \gamma = \frac{20}{3}\ \alpha\gamma = 4 \Rightarrow \gamma = \frac{4}{\alpha}$

$\alpha + \frac{4}{\alpha} = \frac{20}{3} \Rightarrow 3\alpha^2 - 20\alpha + 12 = 0$

$\Rightarrow (\alpha - 6)(3\alpha - 2) = 0 \quad \alpha = 6, \alpha = \frac{2}{3}$

For $\alpha = 6, \gamma = \frac{2}{3}$ and for $\alpha = \frac{2}{3}, \gamma = 6$

$\alpha = 6, \beta = 2$ and $\gamma = \frac{2}{3}$

or $\alpha = \frac{2}{3}, \beta = 2$ and $\gamma = 6$

Now, $3\alpha + 2\beta + \gamma = 3 \times \frac{2}{3} + 2 \times 2 + 6$

$= 2 + 4 + 6 = 12$

**94. (d)** $p(x)$ is a quadratic polynomial having only purely imaginary roots.

Let $p(x) = x^2 + a^2 = 0, a \in R$

whose roots are $x = \pm ia$

Then, $p(p(x)) = (x^2 + a^2)^2 + a^2$

$= x^4 + 2a^2x^2 + a^4 + a^2$

Discriminant $= 4a^4 - 4(a^4 + a^2)$

$= -4a^2 < 0$

Roots of $p(p(x)$ are non-real.

**95. (b)** Given equation, $4x^3 + 12x^2 - 7x + 165 = 0$ has roots $\alpha, \beta$ and $\gamma$, respectively.

$\alpha + \beta + \gamma = -\frac{12}{4}\ \alpha\beta\gamma = -\frac{165}{4}$

$\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{7}{4}$

Now, $ax^3 + bx^2 + cx + d = 0$ has roots

$\alpha + 5, \beta + 5$ and $\gamma + 5$, respectively.

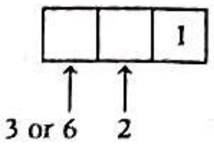
Product of roots $= (\alpha + 5)(\beta + 5)(\gamma + 5)$

$= \alpha\beta\gamma + 5(\alpha\beta + \beta\gamma + \gamma\alpha) + 25$

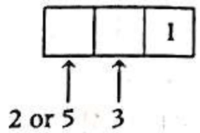
$(\alpha + \beta + \gamma) + 125$

$= \frac{-165}{4} - \frac{35}{4} - \frac{300}{4} + 125 = 0$

**96. (c)** Taking 1 at onc's place,

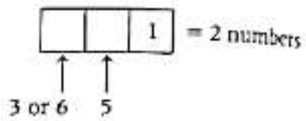



So, two numbers can be formed using the above combination.

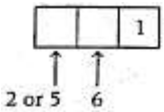



So, two numbers can be formed using the above combination.

Similarly

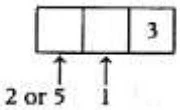



And

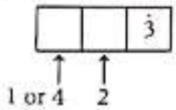



= 2 numbers

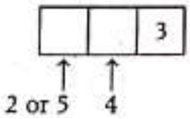
Total numbers that can be formed when 1 is at the unit's place
$= 2 + 2 + 2 + 2 = 8$
Now, taking 3 at one's place,



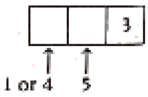

= 2 numbers




= 2 numbers

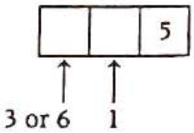



= 2 numbers

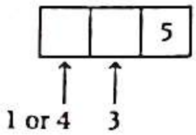



= 2 numbers

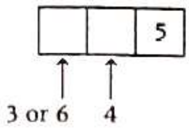
Total numbers that can be formed when 3 is at the unit's place $= 2 + 2 + 2 + 2 = 8$ Now, taking 5 at one's place,



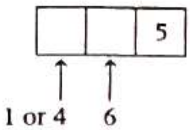

= 2 numbers




= 2 numbers




= 2 numbers




= 2 numbers

Total numbers that can be formed when 5 is at the unit's place $= 2 + 2 + 2 + 2 = 8$
Total number of 3 -digit odd numbers divisible by 3 that can be formed using the digits 1,2,3,4,5,6 when repetition is not allowed $= 8 + 8 + 8 = 24$

**97. (c)** (A) The number of ways of not selecting $(n-r)$ things from **n** different things $= {}^{n}C_{n-r}$

(B) $(n-r+1) \cdot {}^{n}C_{r-1}$

$$= (n-r+1) \times \frac{n!}{(r-1)!\,(n-r+1)!}$$

$$= \frac{(n-r+1) \times n!}{(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!} = r\frac{n!}{r!\,(n-r)!} = r \cdot {}^{n}C_{r}$$

(C) The number of ways of selecting atleast ( $n-r$ ) things from n different things

$$= {}^{n}C_{n-r} + {}^{n}C_{n-r+1} + {}^{n}C_{n-r} \quad +2 + \cdots + {}^{n}C_{n}$$

$$= {}^{n}C_{n} + {}^{n}C_{n-1} + {}^{n}C_{n-2} + \cdots$$

$$+ {}^{n}C_{n-r+1} + {}^{n}C_{n-r}$$

$$= 1 + {}^{n}C_{1} + {}^{n}C_{2} + \cdots + {}^{n}C_{r}$$

(D) $(n-r)({}^{n-1}C_{r-1} + {}^{n-1}C_r)$

$= (n-r)({}^{n}C_r)$

$[{}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}]$

$$= (n-r) \times \frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-r-1)!\,r!}$$

$$= (r+1)\frac{n!}{(r+1)!\,(n-r-1)!} = (r+1)\,{}^{n}C_{r+1}$$

**98. (b)** General term of $\left(ax + \frac{b}{x^2}\right)^{11}$

$T_{r+1} = {}^{11}C_r a^r b^{11-r} x^{3r-22}$

For $x^{-7}$ $3r - 22 = -7$

$\Rightarrow r \quad = 5$

$L \quad = $ Coefficient of $x^{-7} = {}^{11}C_5 a^5 b^6$

General term of $\left(bx^2 + \frac{a}{x}\right)^{11}$

$T_{r+1} = {}^{11}C_r a^{11-r} b^r x^{3r-11}$

For $x^7$

$3r - 11 = 7 \Rightarrow r = 6$

$M =$ Coefficient of $x^7 = {}^{11}C_6 a^5 b^6$

$L + M \quad = ({}^{11}C_5 + {}^{11}C_6) a^5 b^6$

$\quad = {}^{12}C_6 a^5 b^6$

Now, coefficient of $x^6$ in

$$\left(ax^2 + \frac{b}{x}\right)^{12} = {}^{12}C_6 a^6 b^6$$

$$L + M = \frac{1}{a}$$

$$\left[\text{coefficient of } x^{-6} \text{ in } \left(ax^2 + \frac{b}{x}\right)^{12}\right]$$

**99. (c)**

$$\text{Given, } \frac{x^2 - 3x + 2}{(x-4)(x-3)^2}$$

$$= \frac{A}{x-4} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$x^2 - 3x + 2 = A(x-3)^2$

$+B(x-4)(x-3) + C(x-4) \quad \ldots\ldots(i)$

This is an identity so, true for every value of x.

On putting $x = 3$ in Eq. (i), we have

$C = -2$

On putting $x = 4$ in Eq. (i), we have

$A = 6$

On putting $x = 0$ in Eq. (i) and $A = 6$ and $C = -2$, we have $B = -5$

Now, $A + B + C = 6 - 5 - 2 = -1$

**100. (d)** Given,

$$\frac{x^2 + 3}{(x^2+1)(x^2+2)} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 + 2}$$

$\Rightarrow \quad x^2 + 3 = (x^2 + 2)(Ax + B)$

$+(x^2 + 1)(Cx + D)$

$\Rightarrow \quad x^2 + 3 = Ax^3 + Bx^2 + 2Ax$

$+ 2B + Cx^3 + x^2 D + Cx + D$

On comparing,

$A+C=0 \quad \ldots\ldots(i)$
$B+D=1 \quad \ldots\ldots(ii)$
$2A+C=0 \quad \ldots\ldots(iii)$
$2B+D=3 \quad \ldots\ldots(iv)$
From Eqs. (i) and (iii), $A=C=0$
From Eqs. (ii) and (iv), $B=2$ and
$D=-1$
$A+B+C+D=1$

**101.** **(a)** $A>B$ and $A, B$ are acute angles.
Given, $\sin(A-B)=\frac{16}{65}$ and $\sin B=\frac{5}{13}$
Then, $\cos(A-B)=\frac{63}{65}$ and $\cos B=\frac{12}{13}$

$$\sin A=\sin((A-B)+B)$$
$$=\sin(A-B)\cos B+\cos(A-B)\sin B$$
$$=\frac{16}{65}\times\frac{12}{13}+\frac{63}{65}\times\frac{5}{13}=\frac{507}{845}=\frac{3}{5}$$
$$\cos A=\cos((A-B)+B)$$
$$=\cos(A-B)\cos B-\sin(A-B)\sin B$$
$$=\frac{63}{65}\times\frac{12}{13}-\frac{16}{65}\times\frac{5}{13}=\frac{676}{845}=\frac{4}{5}$$
$$\tan A+\cot A=\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}$$
$$=\frac{1}{\sin A\cos A}=\frac{1}{\frac{3}{5}\times\frac{4}{5}}=\frac{25}{12}$$

**102.** **(b)** Given, $\tan A=2/3$
$$\sin 4A=2\sin 2A\cdot\cos 2A$$
$$=2\cdot\frac{2\tan A}{1+\tan^2 A}\times\frac{1-\tan^2 A}{1+\tan^2 A}$$
$$=4\times\frac{\frac{2}{3}}{1+\frac{4}{9}}\times\frac{1-\frac{4}{9}}{1+\frac{4}{9}}=\frac{120}{169}$$

**103.** **(b)**
Given, $\dfrac{\sqrt{2}\cos 45^\circ+\cos 56^\circ+\cos 58^\circ-\cos 66^\circ}{\sqrt{2}\cos 28^\circ\cos 29^\circ\sin 33^\circ}$
$$=\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)+\cos 56^\circ+\cos 58^\circ-\cos 66^\circ}{\sqrt{2}\cos 28^\circ\cos 29^\circ\sin 33^\circ}$$
$$=\frac{(1-\cos 66^\circ)+2\cos 57^\circ\cos 1^\circ}{\sqrt{2}\cos 28^\circ\cos 29^\circ\sin 33^\circ}$$
$$=\frac{2\sin^2 33^\circ+2\sin 33^\circ\cos 1^\circ}{\sqrt{2}\cos 28^\circ\cos 29^\circ\sin 33^\circ}$$
$$=2\sin 33^\circ\left(\frac{\sin 33^\circ+\cos 1^\circ}{\sqrt{2}\cos 28^\circ\cos 29^\circ\sin 33^\circ}\right)$$
$$=\sqrt{2}\left(\frac{\cos 57^\circ+\cos 1^\circ}{\cos 28^\circ\cos 29^\circ}\right)$$
$$=\sqrt{2}\left(\frac{2\cos 29^\circ\cos 28^\circ}{\cos 28^\circ\cos 29^\circ}\right)=2\sqrt{2}$$

**104.** **(a)** For $\theta=\pi/12$

$$x=\log\left(\cot\left(\frac{\pi}{4}+\frac{\pi}{12}\right)\right)$$

$$x=\log\left(\frac{1}{\sqrt{3}}\right)$$

Now, $\cosh x=\frac{e^{x}+e^{-x}}{2}$

$$=\frac{e^{\log\frac{1}{\sqrt{3}}}+e^{-\log\frac{1}{\sqrt{3}}}}{2}$$

$$=\frac{\frac{1}{\sqrt{3}}+\sqrt{3}}{2}=\frac{2}{\sqrt{3}}$$

**105.** **(a)** We know that,

$$\sinh x=\frac{e^{x}-e^{-x}}{2}\text{ and }\cosh x=\frac{e^{x}+e^{-x}}{2}$$

$$2\cosh(x+y)\cdot\sinh(x-y)+\sinh 2y$$

$$=2\cdot\frac{e^{x+y}+e^{-(x+y)}}{2}\cdot\frac{e^{(x-y)}-e^{-(x-y)}}{2}+\frac{e^{2y}-e^{-2y}}{2}$$

$$=\frac{e^{2x}-e^{2y}+e^{-2y}-e^{-2x}+e^{2y}-e^{-2y}}{2}$$

$$=\frac{e^{2x}-e^{-2x}}{2}=\sinh 2x$$

**106.** **(b)** In $\triangle ABC$,

$$(b+c)^2\sin^2\frac{A}{2}+(b-c)^2\cos^2\frac{A}{2}=K(b-\cos 2A)$$

$$\Rightarrow (b+c)^2\sin^2\frac{A}{2}+(b-c)^2\left(1-\sin^2\frac{A}{2}\right)=K(2\sin^2 A)$$

$$\Rightarrow (b-c)^2+\{(b+c)^2-(b-c)^2\}\sin^2\frac{A}{2}=2K\sin^2 A$$

On putting

$$\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}},\ \sin A=\frac{a}{2R}$$

So, $(b-c)^2+4bc\left(\frac{(s-b)(s-c)}{bc}\right)$

$$=2K\left(\frac{a^2}{4R^2}\right)$$

$$\Rightarrow (b-c)^2+4\left(\frac{a+b+c}{2}-b\right)\left(\frac{a+b+c}{2}-c\right)=\frac{Ka^2}{2R^2}$$

$$\Rightarrow (b-c)^2 + (a+c-b)(a+b-c) = \frac{Ka^2}{2R^2}$$

$$\Rightarrow (b-c)^2 + [a-(b-c)][a+(b-c)] = \frac{Ka^2}{2R^2}$$

$$\Rightarrow (b-c)^2 + [a^2-(b-c)^2] = \frac{Ka^2}{2R^2}$$

$$\Rightarrow a^2 = \frac{Ka^2}{2R^2} \Rightarrow K = 2R^2$$

**107.** **(d)** In $\triangle ABC, b = 7, c = 4\sqrt{3}$ and $A = \frac{\pi}{6}$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\Rightarrow \cos\frac{\pi}{6} = \frac{49+48-a^2}{2\times 7\times 4\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{97-a^2}{56\sqrt{3}}$$

$$\Rightarrow 84 = 97-a^2$$

$a^2 = 13$ and $a = \sqrt{13}$

Using sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

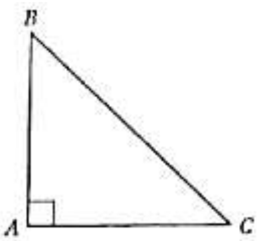
$$\frac{\sin\frac{\pi}{6}}{\sqrt{13}} = \frac{\sin B}{7} = \frac{\sin C}{4\sqrt{3}}$$

$$\sin B = \frac{7}{2\sqrt{13}}, \sin C = \frac{2\sqrt{3}}{\sqrt{13}}$$

$$a\sin B\sin C = \sqrt{13}\times\frac{7}{2\sqrt{13}}\times\frac{2\sqrt{3}}{\sqrt{13}}$$

$$= \frac{7\sqrt{3}}{\sqrt{13}}$$

**108.** **(b)**




In $\triangle ABC$,

$$r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b} \text{ and } r_3 = \frac{\Delta}{s-c}$$

$$r_2 + r_3 = \Delta\left(\frac{1}{s-b} + \frac{1}{s-c}\right)$$

$$= \Delta\left(\frac{(s-c)+(s-b)}{(s-b)(s-c)}\right)$$

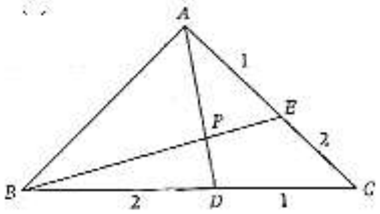
$$= \Delta\left(\frac{2s-b-c}{(s-b)(s-c)} \times \frac{s(s-a)}{s(s-a)}\right)$$

$$= \Delta\left(\frac{as(s-a)}{\Delta^2}\right)$$

$$r_2 + r_3 = \frac{as(s-a)}{\Delta} = \frac{a}{\tan A/2} = a$$

$$\left[A = \frac{\pi}{2}\right]$$

**109.** **(b)**




Let A, B, C, D, E, P have position vectors **a**, **b**, **c**, **d**, **e** and **p** respectively.
Given, D and E divide the line segment BC and AE in the ratio 2: 1.
By section formula,

$$\mathbf{d} = \frac{2c+b}{2+1} \Rightarrow 3d = 2c + b \quad \text{... ... (i)}$$

$$e = \frac{c+2a}{2+1} \Rightarrow 3e = c + 2a \quad \text{... ... . (ii)}$$

From Eq. (i), $3\mathbf{d} - \mathbf{b} = 2c$
From Eq. (ii), $3\mathbf{e} - 2a = \mathbf{c}$
$6e - 4a = 2c$
Equating both values of 2 c ,

$$3d - b = 6\mathbf{e} - 4a$$

$$\Rightarrow 3\,d + 4\mathbf{a} = 6e + b$$

$$\Rightarrow \frac{3\,d + 4a}{3+4} = \frac{6e + b}{6+1}$$

LHS is the position vector of the point which divides segment AD internally in the ratio 3: 4. RHS is the position vector of the point which divides segment BE internally in the ratio 6: 1, But P is the point of intersection of AD and BE. P divides AD internally in the ratio 3 : 4 and P divides BE internally in the ratio 6: 1.

**110.** **(b)** Let position vector of

A: $\hat{i} - 2\hat{j} + 3\hat{k}$

B: $2\hat{i} + 3\hat{j} - 4\hat{k}$

C: $-3\hat{i} + \hat{j} - 5\hat{k}$

D: : $a\hat{i} - 2\hat{j} + 4\hat{k}$

AB: $\hat{i} + 5\hat{j} - 7\hat{k}$

AC: $-4\hat{i} + 3\hat{j} - 8\hat{k}$

AD : $(a-1)\hat{i} + 0\hat{j} + \hat{k}$

Given A, B, C and D are coplanar.
[AB AC AD] = 0

$$\begin{vmatrix} 1 & 5 & -7 \\ -4 & 3 & -8 \\ a-1 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow (a-1)(-40+21)+0+1(3+20)=0$
$\Rightarrow -19(a-1)+23=0$
$\Rightarrow -19a+19+23=0 \Rightarrow a=\dfrac{42}{19}$

**111.** **(c)** Equation of line L: $r = a + \lambda b$
$r = (\hat{i}+2\hat{j}-2\hat{k})+\lambda(2\hat{i}-3\hat{j}-6\hat{k})$
P be any point on line L.
$P(1+2\lambda, 2-3\lambda, -2-6\lambda)$ and
$A(1,2,-2)$
Given, $AP = 21$
$\sqrt{(2\lambda)^2+(-3\lambda)^2+(-6\lambda)^2}=21$
$\Rightarrow |\lambda|(\sqrt{4+9+36})=21$
$\Rightarrow |\lambda|=3 \Rightarrow \lambda=\pm 3$
At $\lambda=3, P(7,-7,-20)$
Position vector of P: $7\hat{i}-7\hat{j}-20\hat{k}$
At $\lambda=-3$ and $P(-5,11,16)$
Position vector of P: $-5\hat{i}+11\hat{j}+16\hat{k}$

**112.** **(d)** Let $a = a\hat{i}+b\hat{j}+c\hat{k}$
a be a vector in the plane containing vectors
$b=\hat{i}+2\hat{j}+\hat{k}$ and $c=2\hat{i}-\hat{j}+\hat{k}$
$a\cdot(b\times c)=0$
$$\begin{vmatrix} a & b & c \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{vmatrix}=0$$
$\Rightarrow 3a+b-5c) \quad \dots\dots \text{(i)}$
$\Rightarrow a\perp(i+j+3k)\Rightarrow a\cdot(i+j+3k)=0$
$a+b+3c=0 \quad \dots\dots \text{(ii)}$
Projection of a on b is $3\sqrt{6}$.
$\dfrac{a\cdot b}{|b|}=3\sqrt{6}\Rightarrow \dfrac{a+2b+c}{\sqrt{1+4+1}}=3\sqrt{6}$
$a+2b+c=18 \quad \dots\dots \text{(iii)}$
On subtracting Eq. (ii) from Eq. (i),
$2a-8c=0\Rightarrow a=4c \quad \dots\dots\text{(iv)}$
On multiplying 2 in Eq. (ii) and subtracting Eq. (iii), we get
$a+5c=-18 \quad \dots\dots\text{(v)}$
From Eqs. (iv) and (v), $c=-2$ and $a=-8$
On putting in Eq. (iii), $b=14$
$a=-8\hat{i}+14\hat{j}-2\hat{k}$
$|a|^2=8^2+14^2+2^2=64+196+4=264$

**113.** **(b)** Given, point on plane $(1,-2,3)$
$a=\hat{i}-2\hat{j}+3\hat{k}$
Perpendicular vector perpendicular to plane
$-\hat{i}+2\hat{j}-3\hat{k}$
Equation of required plane, $r\cdot n=a\cdot n$
$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(-\hat{i}+2\hat{j}-3\hat{k})$
$=(\hat{i}-2\hat{j}+3\hat{k})\cdot(-\hat{i}+2\hat{j}-3\hat{k})$
$\Rightarrow - \quad x+2y-3z=-1-4-9=-14$
$\Rightarrow \quad x-2y+3z=14$

**114.** **(b)** Let $a=\hat{i}+\hat{j}+\hat{k}$
$b=\hat{i}-2\hat{j}+\hat{k}$

$c = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$d = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$l = b \cdot c = 1 - 6 - 2 = -7$

$m = c \cdot a = 1 + 3 - 2 = 2$

$[mb + labd] = [mb \quad b \quad d] + [la \quad b \quad d]$

$\Rightarrow [labd] = l[abb]$

$$= -7\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -7$$

$[1 + 3 + 5] \quad = -63$

**115.** **(d)** Given, mean $= \bar{x}$

Mean of absolute deviation,

$$= \frac{\sum |x - \bar{x}|}{2}$$

$=$ Mean deviation of the data

**116.** **(b)** So, volume of cube having edge of length 5 cm and all faces are painted

$= 5^3 = 125 \text{ cm}^3$

Let cube be divided into ' n ' small cubes of unit volume.

Volume of big cube $= n \times$ volume of unit volume cube

$125 = n \times 1 \Rightarrow n = 125$

Now,

Number of cubes whose 3 faces are painted $= 8$

Number of cubes whose 2 faces are painted $= 36$

Number of cubes whose 1 face is painted $= 54$

Number of cubes whose 0 face is painted

$= 125 - (8 + 36 + 54) = 27$

Required probability

$$= \frac{\text{3 faces painted cubes}}{\text{Cubes whose atleast one face is painted}}$$

$$= \frac{8}{8 + 36 + 54} = \frac{8}{98} = \frac{4}{49}$$

**117.** **(c)** We know that, Prime numbers $= 2,3,5$

Composite numbers $= 4,6$

Pairs of prime numbers on both the die in first throw $= \{(2,2), (2,3), (2,5), (3,2), (3,3), (3,5), (5,2), (5,3), (5,5)\}$

$\Rightarrow$ Total pairs $= 9$

Pairs of composite numbers on both the die in second throw

$= \{(4,6), (4,4), (6,4), (6,6)\}$

Required probability $= \frac{9}{36} \times \frac{4}{36} = \frac{1}{36}$

**118.** **(b)** We have, total balls $= 4 + 5 + 6 = 15$

Number of ways of selecting 3 balls $= {}^{15}C_3$

Number of ways of selecting 3 different coloured balls

$= {}^4C_1 \times {}^5C_1 \times {}^6C_1$

Required probability

$$= \frac{{}^4C_1 \times {}^5C_1 \times {}^6C_1}{{}^{15}C_3} = \frac{24}{91}$$

**119.** **(c)** Given in bag containing 5 black balls and 3 white balls.

Random variable X denotes the number of white balls drawn

$X(0) =$ No white ball is drawn

$$P(0) = \frac{{}^3C_0 \times {}^5C_2}{{}^8C_2} = \frac{10}{28}$$

$X(1) = 1$ white ball is drawn

$$P(X) = \frac{^5C_1 \times {}^3C_1}{^8C_2} = \frac{15}{28}$$

$X(2) =$ 2 white balls are drawn

$$P(X) = \frac{^3C_2 \times {}^5C_0}{^8C_2} = \frac{3}{28}$$

Mean of

$$X = \Sigma X_i P(X_i) \quad = 0 \times \frac{10}{28} + 1 \times \frac{15}{28} + 2 \times \frac{3}{28}$$

$$= \frac{21}{28} = \frac{3}{4}$$

**120.** **(a)** We know that in binomial distribution

Mean $= np$

Variance $= npq$

$$np \quad = 4 \text{ and } npq = \frac{4}{3}$$

$$\Rightarrow q \quad = \frac{1}{3}, p = \frac{2}{3} \text{ and } n = 6$$

$$P(X \quad = 2) = {}^6C_2 p^2 q^4$$

$$= \frac{6 \times 5}{2} \times \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4 = \frac{20}{243}$$

**121.** **(d)** Let $A(5,-3)B(3,-2)C(-1,5)$ be three points.

Let $P(h,k)$ satisfying

$$(PA)^2 + 2(PB)^2 = 3(PC)^2$$

$$(h-5)^2 + (k+3)^2 + 2(h-3)^2 + 2(k+2)^2$$
$$= 3(h+1)^2 + 3(k-5)^2$$

$$\Rightarrow -10h + 25 + 6k + 9 - 12h$$
$$+ 18 + 8k + 8$$
$$= 6h + 3 - 30k + 75$$
$$\Rightarrow -28h + 44k - 18 = 0$$
$$\Rightarrow 14h - 22k + 9 = 0$$

Locus $14x - 22y + 9 = 0$

Only option (d). $\left(2, \frac{37}{22}\right)$ satisfies the locus.

**122.** **(c)** Axes are rotated through $\frac{\pi}{4}$ about origin in positive direction.

$$x \to x\cos\frac{\pi}{4} - y\sin\frac{\pi}{4}, x \to \frac{1}{\sqrt{2}}(x-y)$$

$$y \to x\sin\frac{\pi}{4} + y\cos\frac{\pi}{4}, x \to \frac{1}{\sqrt{2}}(x+y)$$

So, equation $49x^2 + 25y^2 = 1225$ will transform into

$$\frac{49}{2}(x-y)^2 + \frac{25}{2}(x+y)^2 \quad = 1225$$

$$\Rightarrow 37x^2 - 24xy + 37y^2 \quad = 1225$$

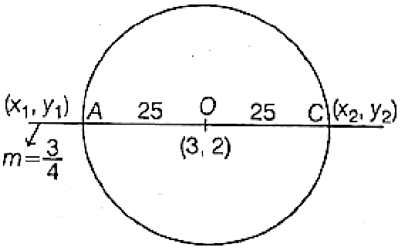
On comparing with $px^2 + qxy + ry^2 = t$

$p = 37, q = -24, r = 37, t = 1225$

$$(p+q+r-15)^2 = (37-24+37-15)^2$$
$$= (35)^2 = 1225$$

So, $(p+q+r-15)^2 = t$

**123.** **(d)**




Slope of diameter, AC $= 3/4$

Slope of AO $= 3/4$

$$\frac{x_1+x_2}{2} = 3 \Rightarrow x_1+x_2 = 6$$

$$\frac{y_1+y_2}{2} = 2 \Rightarrow y_1+y_2 = 4$$

$$\frac{y_2-y_1}{x_2-x_1} = \frac{3}{4} \Rightarrow (y_2-y_1) = \frac{3}{4}(x_2-x_1)$$

$$AC = 50$$

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = 50$$

$$(x_2-x_1)^2+(y_2-y_1)^2 = 2500$$

$$\Rightarrow \frac{9}{16}(x_2-x_1)^2+(x_2-x_1)^2 = 2500$$

$$\Rightarrow \frac{25}{16}(x_2-x_1)^2 = 2500$$

$$\Rightarrow (x_2-x_1)^2 = 1600$$

$$\Rightarrow (y_2-y_1)^2 = \frac{9}{16}(x_2-x_1)^2$$

$$= \frac{9}{16}\times 1600 = 900$$

$$\Rightarrow (x_2-x_1)^2 = 1600$$

$$\Rightarrow (x_1+x_2)^2-(4x_1x_2) = 1600$$

$$\Rightarrow 6^2-4x_1x_2 = 1600 \Rightarrow x_1x_2 = \frac{-1564}{4}$$

$$\Rightarrow (y_2-y_1)^2 = 900$$

$$\Rightarrow (y_1+y_2)^2-4y_1y_2 = 900$$

$$\Rightarrow 4^2-4y_1y_2 = 900 \Rightarrow y_1y_2 = -\frac{884}{4}$$

$$\frac{x_1x_2}{y_1y_2} = \frac{1564}{884} = \frac{23}{13}$$

**124.** **(d)** Given, lines

$$\frac{x}{a}+\frac{y}{b} = 1 \quad \ldots\ldots(i)$$

$$\frac{x}{b}+\frac{y}{a} = 1 \quad \ldots\ldots(ii)$$

Slope of line (i), $m_1 = \frac{-1/a}{1/b} = \frac{-b}{a}$

Slope of line (ii), $m_2 = \frac{-1/b}{1/a} = \frac{-a}{b}$

Then, $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$$= \left|\frac{-\frac{b}{a}+\frac{a}{b}}{1+1}\right| = \left|\frac{a^2-b^2}{2ab}\right|$$

Then, $\sin\theta = \left|\frac{a^2-b^2}{a^2+b^2}\right|$.

**125.** **(a)** Let A be the point of intersection of $x-y+1=0$ and $2x+2y+3=0$

$A\left(-\frac{5}{4}, -\frac{1}{4}\right)$

B is the point of intersection of

$x - y + 1 \quad = 0$ and $3x + 3y + 2 = 0$

$$B\left(\frac{-5}{6}, \frac{1}{6}\right)$$

$$AB \quad = \sqrt{\left(\frac{-5}{6} + \frac{5}{4}\right)^2 + \left(\frac{1}{6} + \frac{1}{4}\right)^2}$$

$$= \sqrt{\left(\frac{10}{24}\right)^2 + \left(\frac{10}{24}\right)^2}$$

$$= \sqrt{2}\left(\frac{10}{24}\right) = \frac{5}{6\sqrt{2}}$$

**126.** **(b)** Sides of triangle are $x = 2$

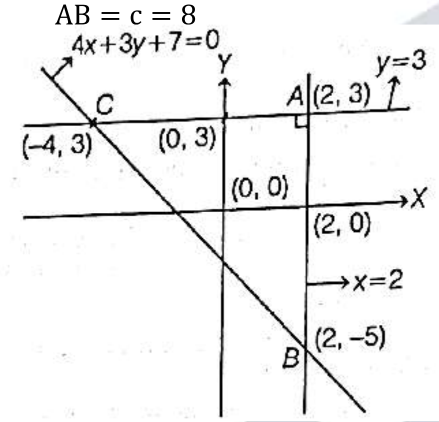
$4x + 3y + 7 = 0$

$y = 3$

$BC = a = \sqrt{6^2 + 8^2} = 10$

$AC = b = 6$

$AB = c = 8$




In centres

In centres

$$\left(\frac{ax_1 + bx_2 + cx_2}{a + b + c + cy_1 + cy_3}\right) a + b + c$$

$$I\left(\frac{(10 \times 2) + (6 \times 2) + (8 \times -4)}{10 + 6 + 8}\right.$$

$$\left.\frac{(10 \times 3) + (6 \times -5) + (8 \times 3)}{10 + 6 + 8} I(0,1)\right)$$

As $\triangle$ ABC is right angled triangle.

So, mid-point of BC is circumcentre

$S(-1, -1)$

$IS = \sqrt{1^2 + 2^2} = \sqrt{5}$

**127.** **(a)** Equation $ax^2 - 4xy - 2y^2 = 0$ represents pair of lines.

Also, $\cos\theta = \frac{1}{5} \Rightarrow \tan\theta = 2\sqrt{6}$

$$\tan\theta \quad = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$2\sqrt{6} \quad = \frac{2\sqrt{4 + 2a}}{a - 2}$$

Squaring both sides,

$(a - 2)^2 \cdot 6 = 4 + 2a$

$\Rightarrow (a - 2)^2 \cdot 3 = 2 + a$

$\Rightarrow 3a^2 - 12a + 12 = 2 + a$
$\Rightarrow 3a^2 - 13a + 10 = 0$
$\Rightarrow (3a - 10)(a - 1) = 0$
$a = 1, \frac{10}{3}$

As $a_2 > a_1 \Rightarrow a_1 = 1, a_2 = \frac{10}{3}$

$a_1 + 3a_2 = 11$

**128.** **(d)** Lines represented by

$4x^2 - 5xy + 3y^2 = 0$ are $L_1$ and $L_2$.

$$3\left(\frac{y}{x}\right)^2 - 5\left(\frac{y}{x}\right) + 4 = 0$$

$$3m^2 - 5m + 4 = 0 < \frac{m_1}{m_2}$$

$$m_1 + m_2 = \frac{5}{3},$$

$$m_1 \cdot m_2 = \frac{4}{3}$$

$$\frac{1}{m_1} + \frac{1}{m_2} = \frac{m_1 + m_2}{m_1 m_2} = \frac{5}{4}$$

Let the slope of line $L_1$ is $m_1$ and the slope of line $L_2$ is $m_2$.
$L_3$ is perpendicular to $L_1 \Rightarrow$ So, slope of
$L_3 = -\frac{1}{m_1}$
$L_4$ is perpendicular to $L_2 \Rightarrow$ So, slope of
$L_4 = -\frac{1}{m_2}$

Let the equation of $L_3: y = \frac{-1}{m_1}x + c_1$ It passes through $(4,3): 3 = \frac{-4}{m_1} + c_1$

$$\Rightarrow c_1 = 3 + \frac{4}{m_1} \quad L_3: y = \frac{-1}{m_1}x + 3 + \frac{4}{m_1}$$

$$\Rightarrow y + \frac{x}{m_1} - \left(3 + \frac{4}{m_1}\right) = 0$$

and Equation of $L_4: y = -\frac{x}{m_2} + c_2$

Passes through $(4,3): 3 = -\frac{4}{m_2} + c_2$

$$\Rightarrow \quad c_2 = 3 + \frac{4}{m_2}$$

$$L_4: y = -\frac{x}{m_2} + 3 + \frac{4}{m_2}$$

$$\Rightarrow \quad y + \frac{x}{m_2} - \left(3 + \frac{4}{m_2}\right) = 0$$

Combined equation of $L_3$ and $L_4$

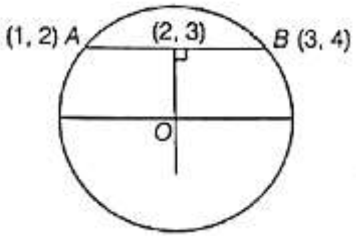
$$\left[y + \frac{x}{m_1} - \left(3 + \frac{4}{m_1}\right)\right]$$
$$\left[y + \frac{x}{m_2} - \left(3 + \frac{4}{m_2}\right)\right] = 0$$

$$\Rightarrow \quad y^2+\frac{xy}{m_2}-y\left(3+\frac{4}{m_2}\right)+\frac{xy}{m_1}+\frac{x^2}{m_1m_2}$$
$$-\quad \frac{x}{m_1}\left(3+\frac{4}{m_2}\right)-\frac{x}{m_2}\left(3+\frac{4}{m_1}\right)$$
$$-y\left(3+\frac{4}{m_1}\right)+\left(3+\frac{4}{m_1}\right)\left(3+\frac{4}{m_2}\right)=0$$
$$\Rightarrow \quad y^2+\frac{x^2}{m_1m_2}+xy\left(\frac{1}{m_1}+\frac{1}{m_2}\right)$$
$$-x\left(\frac{8}{m_1m_2}+3\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\right)$$
$$-y\left(6+4\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\right)$$
$$+\left(9+12\left(\frac{1}{m_1}+\frac{1}{m_2}\right)+\frac{16}{m_1m_2}\right)=0$$
$$\Rightarrow y^2+\frac{5}{4}xy+\frac{3}{4}x^2-x\left(6+3\times\frac{5}{4}\right)$$

$$-y\left(6+4\times\frac{5}{4}\right)$$
$$+\left(9+12\times\frac{5}{4}+16\times\frac{3}{4}\right)=0$$
$$\Rightarrow y^2+\frac{5}{4}xy+\frac{3}{4}x^2-\frac{39}{4}x-11y+36=0$$
$$\Rightarrow 4y^2+5xy+3x^2-39x-44y+144$$
On comparing with
$$ax^2+2hxy+by^2+2gx+2fy+c=0$$
$$a=4, h=\frac{5}{2}, b=3, g=\frac{-39}{2}, f=-22 \text{ and}$$
$$c=144$$
$$af+bg+ch=-88-\frac{117}{2}+360=2135$$

**129.** **(d)** Given, $x^2-y^2+ax+b=0$
On comparing with
$$ax^2+2hxy+by^2+2gx+2fy+c=0$$
$$a=1, h=0, b=-1, g=\frac{a}{2}, f=0 \text{ and}$$
$$c=b$$
If it represents a pair of straight line
Then, $D=0$
$$\Rightarrow abc+2fgh-af^2-bg^2-ch^2=0$$
$$-b+0-0+\frac{a^2}{4}-0=0\Rightarrow a^2=4b$$
From the options, $a=6$ and $b=9$ satisfy above relation.
So, pair of $(a,b)\equiv(6,9)$

**130.** **(d)** Given circle passes through the points $(1,2),(3,4)$.
If its centre lines on the line
$x-y+3=0$
Then, slope of AB $=\frac{4-2}{3-1}=1$




Mid-point of AB $= (2,3)$
Equation of line perpendicular to AB and passing through mid-point of AB
Line

$y-3=-1(x-2)$

$L_1: x+y=5 \quad \ldots \ldots$ (i)

Also, centre lies on line $x-y=-3$ ... (ii)
Point of intersection of Eqs. (i) and (ii) is the centre of circle $x=1, y=4$ Centre, $O(1,4)$

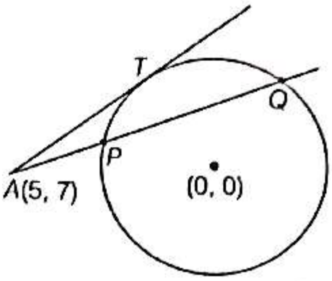
Radius $= AO = \sqrt{0+2^2} = 2$

**131. (c)**

Circle : $x^2+y^2-36=0$

$x^2+y^2=36$

$AP \cdot AQ = (AT)^2$

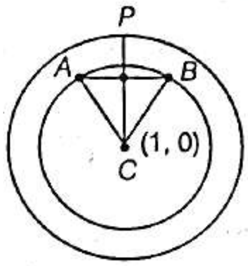



$AT = \text{ Length of tangent } = \sqrt{x_1^2+y_1^2-36}$

$= \sqrt{25+49-36} = \sqrt{38}$

$AP \cdot AQ = (\sqrt{38})^2 = 38$

**132. (a)**




Given, equation of circle
$x^2+y^2-2x-1=0$
Coordinate of C is $(1,0)$.
Radius $= \sqrt{2}$
Let the coordinate of P be $(1+\sqrt{2}\cos\theta, \sqrt{2}\sin\theta)$.
Given equation of another circle
$x^2+y^2-2x=0$
Coordinate center is $(1,0)$ and radius $= 1$
The two given circle are concentric.
Let the circumcenter of CAB be $(h, k)$.
Circumcenter of CAB lies on mid-point of PC

$$h = \frac{1+\sqrt{2}\cos\theta+1}{2}$$

and $k = \frac{\sqrt{2}\sin\theta + 0}{2}$

$$\Rightarrow \cos\theta = \frac{2h-2}{\sqrt{2}} \text{ and } \sin\theta = \frac{2k}{\sqrt{2}}$$

$$\Rightarrow \cos^2\theta + \sin^2\theta = 1$$

$$\Rightarrow \left(\frac{2(h-1)}{\sqrt{2}}\right)^2 + \left(\frac{2k}{\sqrt{2}}\right)^2 = 1$$

$$\Rightarrow 2(h-1)^2 + 2k^2 = 1$$

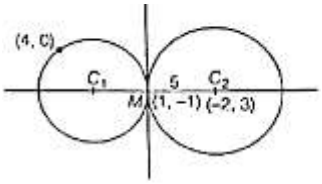
$$\Rightarrow 2h^2 + 2 - 4h + 2k^2 = 1$$

$$\Rightarrow 2h^2 + 2k^2 - 4h + 1 = 0$$

The locus of circumcenter is

$2x^2 + 2y^2 - 4x + 1 = 0$

**133. (d)**




Let centre of circle C is $C_1$

Circle C: $x^2 + y^2 + 4x - 6y - 12 = 0$

With centre $C_2: (-2,3), r = 5$

Tangent at $M(1,-1)$ to the circle C is

$$xx_1 + yy_1 + 4\left(\frac{x+x_1}{2}\right)$$

$$-6\left(\frac{y+y_1}{2}\right) - 12 = 0$$

$$x - y + 2(x+1) - 3(y-1) - 12 = 0$$

$$\Rightarrow 3x - 4y - 7 = 0$$

So, equation of circle C touching the line $3x - 4y - 7 = 0$ at $(1,-1)$

$(x-1)^2 + (y+1)^2 + \lambda(3x - 4y - 7) = 0$

It passes through $(4,0)$

$$(4-1)^2 + (0+1)^2 + \lambda(12 - 0 - 7) = 0$$

$$\Rightarrow 9 + 1 + 5\lambda = 0 \Rightarrow \lambda = -2$$

So equation of

C: $(x-1)^2 + (y+1)^2 - 2(3x - 4y - 7) = 0$

$$x^2 + y^2 - 8x + 10y + 16 = 0$$

Centre $C_1 = (4,-5)$

Radius of C $= \sqrt{16 + 25 - 16} = 5$

**134. (b)** Given,

$$C_1: x^2 + y^2 + 2x + 4y - 20 = 0$$

Centre $O_1(-1,-2), r_1 = \sqrt{1 + 4 + 20} = 5$

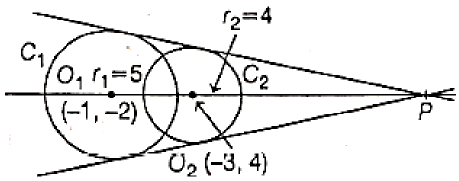
$$C_2: x^2 + y^2 + 6x - 8y + 9 = 0$$

$$\Rightarrow (x+3)^2 + (y-4)^2 = 4^2$$

Centre $O_2: (-3,4), r_2 = \sqrt{9 + 16 - 9} = 4$

$r_1 + r_2 = 9$ and $|r_1 - r_2| = 1$

Two tangents are there, so $n = 2$




P divides $O_1$ and $O_2$ in 5:4 ratio externally

$$P\left(\frac{(-3\times5)-(-1\times4)}{5-4},\frac{(4\times5)-(4\times-2)}{5-4}\right)$$
$$= P(-11,28)$$

L is the length of tangent from P to $C_2$

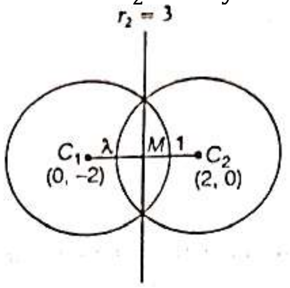
$$l = \sqrt{(-11+3)^2+(28-4)^2-4^2}$$
$$= \sqrt{64+576-16} = \sqrt{624} = 4\sqrt{39}$$
$$\frac{l}{n^2} = \frac{4\sqrt{39}}{4} = \sqrt{39}$$

**135.** **(b)**

Let $S_1: x^2+y^2+4y=0$

$C_1(0,-2), r_1 = 2$

and $S_2: x^2+y^2-4x-5=0, C_2(20)$




Common chord of $S_1$ and $S_2$ is

$S_1 - S_2 = 0$

$4x+4y+5=0$ ... (i)

M is the point of intersection of common chord and line joining $C_1$ and $C_2$.

Let M divides $C_1C_2$ in $\lambda$: 1 ratio internally.

$$M\left(\frac{2\lambda}{\lambda+1},\frac{-2}{\lambda+1}\right)$$

M lies on common chord

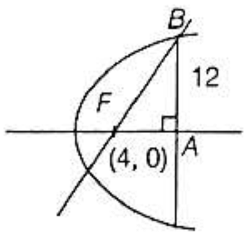
$$\frac{8\lambda}{\lambda+1}-\frac{8}{\lambda+1}+5 = 0$$
$$\Rightarrow 8\lambda-8+5\lambda+5 = 0$$
$$\lambda = 3/13$$

Also M is the centre of the circle whose diameter is common chord of $S_1$ and $S_2$

So, abscissa of

$$M = \frac{2\lambda}{\lambda+1} = \frac{\frac{6}{13}}{\frac{3}{13}+1} = \frac{6}{16} = \frac{3}{8}$$

**136.** **(b)**




Given, $y^2 = 16x \Rightarrow a = 4$

Let B be $(x_1, 12)$ lies on parabola.

$$(12)^2 = 16x_1 \Rightarrow x_1 = \frac{12\times12}{16} = 9$$

Point B(9,12)

Equation of focal chord passing through (4,0) and (2,2)

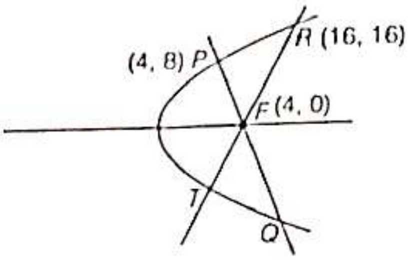
$$y - 0 = \frac{2-0}{2-4}(x-4)$$
$$\Rightarrow y = -x + 4$$
$$x + y = 4 \quad \ldots.(i)$$

Equation of double ordinate :

$x = 9 \quad \ldots..(ii)$

Point of intersection of Eqs. (i) and (ii), $(9, -5)$.

**137. (a)**




Given,

$$y^2 = 16x \; a = 4$$
$$P(4,8) = (at_1^2, 2at_1)$$
$$2at_1 = 8$$
$$8t_1 = 8, \Rightarrow t_1 = 1$$

If PQ is a focal chord, then

$$t_1t_2 = -1 \Rightarrow +1 \times t_2 = -1$$
$$t_2 = -1$$
$$Q(at_2^2, 2at_2) \equiv Q(4, -8)$$
$$R(16,16) \equiv (at_3^2, 2at_3)$$
$$2at_3 = 16$$
$$8t_3 = 16 \Rightarrow t_3 = 2$$

If RT is a focal chord, then $t_3t_4 = -1$

$$2 \times t_4 = -1 \Rightarrow t_4 = \frac{-1}{2}$$
$$T(at_4^2, 2at_4) \equiv T(1, -4)$$
$$QT = \sqrt{3^2 + 4^2} = 5$$

**138. (a)** Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\text{Eccentricity} = \frac{\sqrt{3}}{2} \Rightarrow \sqrt{1 - \frac{b^2}{a^2}} = \frac{\sqrt{3}}{2}$$
$$1 - \frac{b^2}{a^2} = \frac{3}{4} \Rightarrow \frac{b^2}{a^2} = \frac{1}{4}$$
$$b^2 = a^2/4 \quad \ldots (i)$$

Length of latus rectum $= 1$

$\frac{2b^2}{a} = 1 \; b^2 = \frac{a}{2} \quad \ldots\ldots(ii)$

From Eqs (i) and (ii),

$$\frac{a^2}{4} = \frac{a}{2} \Rightarrow a^2 - 2a = 0$$
$$a = 2, a \neq 0 \; b = 1$$

Sum of length of major-axis and minor-axis

$= 2a + 2b = 4 + 2 = 6$

**139. (b)** Given, Focii of ellipse are $f_1(-3,0)$ and $f_2(9,0)$.

Centre of ellipse will be at mid-point of $f_1$ and $f_2$

$$o\left(\frac{-3+9}{2}, \frac{0+0}{2}\right) \equiv O(3,0)$$

$$2ae = 12 \Rightarrow ae = 6\,a \times \frac{1}{3} = 6$$

$$\Rightarrow a = 18 \text{ and } \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{3} \Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{9}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{8}{9} \Rightarrow b^2 = \frac{8}{9} \times (18)^2 = 288$$

$$b = 12\sqrt{2}$$

Equation of ellipse :

$$\frac{(x-3)^2}{(18)^2} + \frac{(y-0)^2}{288} = 1$$

$$x - 3 = a\cos\theta, y - 0 = b\sin\theta$$

$$x - 3 = 18\cos\theta, y = 12\sqrt{2}\sin\theta$$

$$x = 3 + 18\cos\theta, y = 12\sqrt{2}\sin\theta$$

**140.** **(a)** Given, $\frac{x^2}{k-\frac{5}{2}} + \frac{y^2}{\frac{7}{3}-k} = 1$

( k is a real number)

$$a = \sqrt{k - \frac{5}{2}}, b = \sqrt{\frac{7}{3} - k}$$

$$k - \frac{5}{2} > 0, \frac{7}{3} - k > 0$$

$$k > \frac{5}{2}, k < \frac{7}{3} \Rightarrow k \in \left(-\infty, \frac{7}{3}\right) \cup \left(\frac{5}{2}, \infty\right)$$

**141.** **(b)** Given, hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and focus $S(ae, 0)$. $A(\theta_1)$ and $B(\theta_2)$ be two points on hyperbola, equation of chord joining $A(\theta_1)$ and $B(\theta_2)$ is

$$\Rightarrow \frac{x}{a}\cos\left(\frac{\theta_1 - \theta_2}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta_1 + \theta_2}{2}\right) = \cos\left(\frac{\theta_1 + \theta_2}{2}\right)$$

A, S and B are collinear so, S lies on line AB

$$\frac{ae}{a}\cos\left(\frac{\theta_1 - \theta_2}{2}\right) = \cos\left(\frac{\theta_1 + \theta_2}{2}\right)$$

$$\Rightarrow \quad a\cos\left(\frac{\theta_1 + \theta_2}{2}\right) = ae\cos\left(\frac{\theta_1 - \theta_2}{2}\right)$$

On comparing with

$$a\cos\left(\frac{\theta_1 + \theta_2}{2}\right) = k\cos\left(\frac{\theta_1 - \theta_2}{2}\right)$$

$$k = ae \Rightarrow k = a\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{a^2 + b^2}$$

**142.** **(b)** Given,

$A(1,2,3), B(-1,4,6), C(0,-6,4), D(1,1,1)$

$G\left(\frac{1-1+0+1}{4}, \frac{2+4-6+1}{4}, \frac{3+6+4+1}{4}\right) \equiv G\left(\frac{1}{4}, \frac{1}{4}, \frac{14}{4}\right)$

$G_1\left(\frac{-1+0+1}{3}, \frac{4-6+1}{3}, \frac{6+4+1}{3}\right)$

$\equiv G_1\left(0, \frac{-1}{3}, \frac{11}{3}\right)^2$

$AG_1 = \sqrt{1^2 + \left(\frac{7}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$

$= \frac{1}{3}\sqrt{9+49+4} = \frac{\sqrt{62}}{3}$

$AG = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{7}{4}\right)^2 + \left(\frac{2}{4}\right)^2} = \frac{\sqrt{62}}{4}$

$\frac{AG}{AG} = \frac{\sqrt{62}/3}{\sqrt{62}/4} = \frac{4}{3}$

**143.** **(a)** Here,

$P_1: x - y + z + 2 = 0$

$n_1: 1, -1, 1$

$P_2: 2x + y - 2z + 5 = 0$

$n_2: 2, 1, -2$

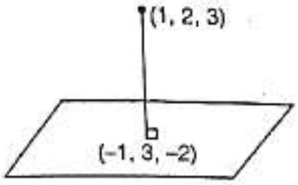
vector along line of intersection of $P_1$ and $P_2 = \mathbf{n}_1 \times \mathbf{n}_2$

$n = \begin{vmatrix} 1 & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 2 & 1 & -2 \end{vmatrix} = 1 + 4\,J + 3k$

$|n| = \sqrt{1+16+9} = \sqrt{26}$

Dc's are $\frac{1}{\sqrt{26}} \cdot \frac{4}{\sqrt{26}}, \frac{3}{\sqrt{26}}$

**144.** **(b)** Dr's of normal to plane




$: 1-(-1), 2-3, 3-(-2)$

$: (2,-1,5) \equiv (a,b,c)$

Plane passes through $(-1,3,-2) \equiv (x_1, y_1, z_1)$.

Equation of plane

$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

$2(x+1) - (y-3) + 5(z+2) = 0$

$2x - y + 5z + 15 = 0$

Perpendicular distance from origin to the plane

$\left|\frac{0-0+0+15}{\sqrt{4+1+25}}\right| = \frac{15}{\sqrt{30}} = \sqrt{\frac{15}{2}}$

**145.** **(c)**

$$\text{Given, } \lim_{x\to 3^-}\frac{x^3-3x^2-4x+12}{2x^3-7x^2+2x+3}$$

$$\lim_{x\to 3^-}\frac{(x-2)(x^2-x-6)}{(x-1)(2x^2-5x-3)}$$

$$=\lim_{x\to 3^-}\frac{(x-2)(x-3)(x+2)}{(x-1)(2x+1)(x-3)}=\frac{1\times 5}{2\times 7}=\frac{5}{14}$$

**146.** **(b)** Given,

$$\lim_{x\to 0}\frac{2^{2x}-2^{x+1}+2-\cos 2x}{x^2}$$

$$=\lim_{x\to 0}\frac{(2^x)^2-2\cdot 2^x+1+1-\cos 2x}{x^2}$$

$$=\lim_{x\to 0}\frac{(2^x-1)^2+2\sin^2 x}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)^2+2\left(\frac{\sin x}{x}\right)^2=(\ln 2)^2+2$$

**147.** **(c)** Given,

$$f(x)=\begin{cases}\dfrac{x^2-16}{x-4}; & x>4\\ 2x; & x\le 4\end{cases}=\begin{cases}x+4; & x>4\\ 2x; & x\le 4\end{cases}$$

$$f'(4^-)=\lim_{h\to 0}\frac{f(4-h)-f(4)}{-h}$$

$$=\lim_{h\to 0}\frac{2(4-h)-8}{-h}=+2$$

$$f'(4^+)=\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}$$

$$=\lim_{h\to 0}\frac{(4+h+4)-8}{h}=1$$

$$f'(4^-)+f'(4^+)=3$$

**148.** **(d)** Given,

$$f(x)=\log_e\left[e^{2x}\left(\frac{3x+5}{5-3x}\right)^{\frac{2}{3}}\right], x\ne\frac{-5}{3},\frac{5}{3}$$

$$f(x)=\log_e e^{2x}+\log_e\left(\frac{3x+5}{5-3x}\right)^{\frac{2}{3}}$$

$$f(x)=2x+\frac{2}{3}[\log_e(3x+5)-\log(5-3x)]$$

$$\frac{df}{dx}=2+\frac{2}{3}\left[\frac{3}{3x+5}-\frac{(-3)}{5-3x}\right]$$

$$\frac{df}{dx}=2+\frac{2}{3}\left[\frac{3}{8}+\frac{3}{2}\right]=2+\frac{2}{3}\times\frac{15}{8}$$

$$=2+\frac{5}{4}=\frac{13}{4}$$

**149.** **(b)** Here, $x=\operatorname{cosec}\theta-\sin\theta$

$x^2 + 4 = (\operatorname{cosec}\theta + \sin\theta)^2$

$y = \operatorname{cosec}^{2022}\theta - \sin^{2022}\theta$

$y^2 + 4 = (\operatorname{cosec}^{2022}\theta + \sin^{2022}\theta)^2$

$\frac{dy}{d\theta} = 2022\operatorname{cosec}^{2021}\theta(-\operatorname{cosec}\theta\cot\theta)$

$-2022\sin^{2021}\theta\cos\theta$

$= -2022\cos\theta\left(\frac{\operatorname{cosec}^{2022}\theta}{\sin\theta} + \sin^{2021}\theta\right)$

$\frac{dy}{d\theta} = -2022\cot\theta(\operatorname{cosec}^{2022}\theta + \sin^{2022}\theta)$

$\frac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta - \cos\theta$

$= -\cot\theta(\operatorname{cosec}\theta + \sin\theta)$

$\frac{dy}{dx} = \frac{2022(\operatorname{cosec}^{2022}\theta + \sin^{2022}\theta)}{\operatorname{cosec}\theta + \sin\theta}$

$\left(\frac{dy}{dx}\right)^2 = \frac{(2022)^2(y^2+4)}{x^2+4}$

$\Rightarrow \frac{k(y^2+4)}{g(x)} = \frac{(2022)^2(y^2+4)}{x^2+4}$

$k = (2022)^2, g(x) = x^2 + 4$

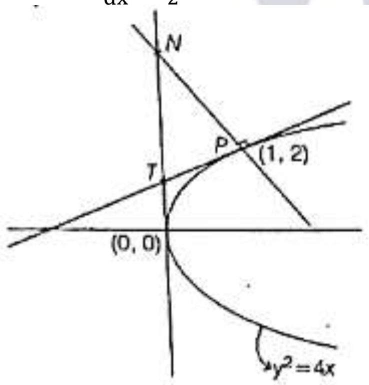
$10 + k - g(2022)$

$= 10 + (2022)^2 - ((2022)^2 + 4) = 6$

**150.** **(d)** Given curves, $y^2 = 4x$

$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

At $(1,2)\frac{dy}{dx} = \frac{2}{2} = 1$




Equation of tangent

$y - 2 = 1(x - 1) \Rightarrow y = x + 1 \Rightarrow T(0,1)$

Slope of normal $= -1$

Equation of normal

$y - 2 = -1(x - 1)\ x + y = 3 \Rightarrow N(0,3)$

Area of $\triangle$ TPN $= \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 0 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix}$

$= \frac{1}{2}|1 - 3| = 1$

**151.** **(d)** Given,

$C_1: y^2 = 4ax, C_2: x^2 + \frac{y^2}{2} = c^2$

Let point of intersection of two curves is

$(x_1, y_1)$. $C_1 : y^2 = 4ax$

$$\Rightarrow 2y\left(\frac{dy}{dx}\right)_1 = 4a \Rightarrow \left(\frac{dy}{dx}\right)_1 = \frac{2a}{y_1}$$

Let angle between two curves be θ.

$$C_2 : x^2 + \frac{y^2}{2} = c^2$$

$$\Rightarrow 2x + \frac{2y}{2}\left(\frac{dy}{dx}\right)_2 = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_2 = -\frac{2x_1}{y_1}$$

$$\tan\theta = \left|\frac{\left(\frac{dy}{dx}\right)_1 - \left(\frac{dy}{dx}\right)_2}{1 + \left(\frac{dy}{dx}\right)_1\left(\frac{dy}{dx}\right)_2}\right|$$

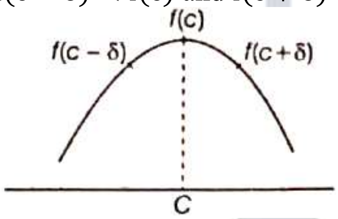
$$= \left|\frac{\frac{2a}{y_1} + \frac{2x_1}{y_1}}{1 - \frac{4ax_1}{y_1^2}}\right| = \left|\frac{\frac{2a + 2x_1}{y_1}}{1 - \frac{y_1^2}{y_1^2}}\right|$$

$$\tan\theta = \left|\frac{\frac{2a + 2x_1}{y_1}}{0}\right| = \infty$$

$$\theta = \frac{\pi}{2}$$

**152.** **(c)** If $c \in (a, b)$

$f(c - \delta) < f(c)$ and $f(c + \delta) < f(c)$




So, f(c) is point of local maxima

Also $[f(\alpha - \delta) - f(\alpha)]f(\alpha + \delta) < 0$

$\Rightarrow f(\alpha - \delta) - f(\alpha) < 0$ and $f(\alpha + \delta) > 0$

or $f(\alpha - \delta) - f(\alpha) > 0$ and $f(\alpha + \delta) < 0$

i.e. No maxima and minima at

$\alpha, \alpha \in (a, b)$

**153.** **(c)** Given,

$$f(x) = \int \frac{2x^3 - 3x^2 + 4x - 5}{x^2} dx$$

$$f(x) = \int 2x - 3 + \frac{4}{x} - \frac{5}{x^2} dx$$

$$f(x) = 2 \cdot \frac{x^2}{2} - 3x + 4\ln x + \frac{5}{x} + c$$

Given, $f(1) = 1$

$$f(1) = 1 - 3 + 4\ln 1 + \frac{5}{1} + c$$
$$1 = 3 + c \Rightarrow c = -2$$
$$f(x) = x^2 - 3x + 4\ln x + \frac{5}{x} - 2$$
$$f(9) = 25 - 15 + 4$$
$$\ln 5 + \frac{5}{5} - 2 = 9 + 4\ln 5$$

**154.** **(d)**

If $x > 0, x \neq (2n+1)\frac{\pi}{2}$

$$\int x\sqrt{x} - e^{\log(\sec x \tan x)} + \frac{3x^2 - 2x + 1}{x^2} dx$$
$$= \int \left(x^{\frac{3}{2}} - \sec x \tan x + 3 - \frac{2}{x} + \frac{1}{x^2}\right) dx$$
$$= \frac{x^{5/2}}{5/2} - \sec x + 3x - 2\ln x - \frac{1}{x} + c$$
$$= \frac{2}{5}x^2\sqrt{x} - \sec x + 3x - 2\ln x - \frac{1}{x} + c$$

**155.** **(d)** Given, $\int (2x-3)\sqrt{3x+2}dx$

Put $3x + 2 = t \Rightarrow 3dx = dt$

$$\frac{1}{3}\int \left[2\left(\frac{t-2}{3}\right) - 3\right]\sqrt{t}dt = \frac{1}{9}\int (2t-13)\sqrt{t}dt$$
$$\Rightarrow \frac{1}{9}\int 2t^{3/2} - 13\sqrt{t}dt$$
$$= \frac{1}{9}\left[2 \cdot \frac{t^{5/2}}{5/2} - 13\frac{t^{3/2}}{3/2}\right] + c$$
$$= \frac{1}{9}\left[\frac{4}{5}t^2 - \frac{26}{3}t\right]\sqrt{t} + c$$
$$= \frac{2}{135}[6(3x+2)^2 - 65(3x+2)]\sqrt{3x+2} + c$$
$$= \frac{2}{135}[54x^2 - 195x - 106]\sqrt{3x+2} + c$$

**156.** **(a)**

Given, $\int_1^4 \left(x + \sqrt{x} + \frac{1}{x}\right) dx - \int_1^{2\log 2} 1dx$

$$= \left[\frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}} + \ln x\right]_1^4 - [x]_1^{2\log 2}$$
$$= \left(8 + \frac{16}{3} + \ln 4\right) - \left(\frac{1}{2} + \frac{2}{3} + 0\right)$$
$$-(2\log 2 - 1) = \frac{17}{2} + \frac{14}{3} = \frac{79}{6}$$

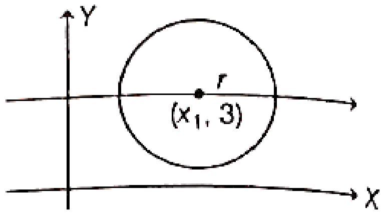
**157.** **(a)**

**158.** **(a)**

$\because (x - x_1)^2 + (y-3)^2 = r^2$ .... (i)

On differentiating,

$$\Rightarrow \begin{array}{l} 2(x - x_1) + 2(y-3) \cdot y' = 0 \\ \Rightarrow x - x_1 = -(y-3)y' \end{array}$$




On putting in Eq. (i),

$$(y-3)^2(y)^2 + (y-3)^2 = r^2$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{r^2}{(y-3)^2}$$

**159.** **(b)** Given equation

$$x\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} + 2x^2\left(\frac{d^2y}{dx^2}\right)^{\frac{5}{3}} + 7\frac{dy}{dx} + y = 0$$

$$\Rightarrow \left[x\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} + 2x^2\left(\frac{d^2y}{dx^2}\right)^{\frac{5}{3}}\right]^3 = \left(7\frac{dy}{dx} + y\right)^3$$

Degree of differential equation = 5

**160.** **(d)**