Physics

1. (b) Given,

F_1 = Gravitational Force

F_2 = Weak Nuclear Force

F_3 = Electromagnetic Force

The ratio of strength of gravitational force, weak nuclear force and electromagnetic force are $1:10^{26}:10^{36}$ respectively.

Thus, the correct order of strength is

$F_1 < F_2 < F_3$

2. (c) Dimensional formula for,

(i) Spring constant,

$F = kx$ (spring/restoring force)

$$kg\frac{m}{s^2} = k(m)$$

$$= \frac{kg}{s^2} = [MT^{-2}] \quad \ldots (I)$$

(ii) For surface energy

Surface energy is the ratio of force per unit length.

$$S.E = \frac{F}{l} = \frac{kgm}{(s)^2 \cdot m}$$

$$= \frac{kg}{(s)^2} = [MT^{-2}] \quad \ldots (II)$$

Thus, from Eqs. (i) and (ii) we can see that spring constant and surface energy have same dimensional formula.

3. (c) Given,

The relation between time and distance x of particle is given as

$t = ax^2 + bx \quad \ldots\ldots(i)$

Differentiate the Eq. (i) w.r.t x,

$dt/dx = 2ax + b$

or $\frac{dx}{dt} = \frac{1}{2ax+b} = v$ (velocity) $\quad \ldots\ldots(ii)$

Again differentiate Eq. (ii) w.r.t t

$\frac{d^2x}{dt^2} = -\frac{1\times 2a}{(2ax+b)^2}\cdot\frac{dx}{dt} \quad \ldots\ldots(iii)$

Put the value of $\frac{dx}{dt}$ in Eq. (iii),

$$a = \frac{d^2x}{dt^2} = -2av^3$$

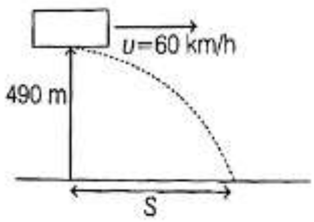
$\Rightarrow a = -2av^3$

4. (b) Given,

Initial speed of plane $= 60\ km/h$

$$= \frac{50}{3}\ m/s$$

Vertical height = 490 m



Horizontal distance

$s = u \times t \quad \ldots(i)$

$(a = 0)\ \left(s = ut + \frac{1}{2}at^2\right)$

For vertical component of motion,
$u = 0$
using equation of motion,

$$s = ut + \frac{1}{2}at^2 (a = g)$$

$$t = \sqrt{\frac{2s}{g}} \Rightarrow t = \sqrt{\frac{2 \times 490}{9.8}}$$

$$t = 10 \text{ s}$$

Put this in Eq. (i)

$$s = \frac{50}{3} \times 10$$

$$s = \frac{500}{3} \text{ m}$$

5. **(a)** Let the angular speed of particle $= \omega$
Equation for centripetal acceleration,
$a = -\omega^2 r$
Now new speed, $\omega' = 2\omega$
New centripetal acceleration,

$$a' = -(\omega)^2 r = -4\omega^2 r$$

$$a' = 4a$$

6. **(c)** Given, let the masses of two bodies are
$m_1 = 1 \text{ g}, m_2 = 4 \text{ g}$
Kinetic energy of both bodies are equal. From the formula of linear momentum
$p = \sqrt{2m(KE)}$
For Ist body $p_1 = \sqrt{2(1)(KE)}$ ......(i)
For 2 nd body $P_2 = \sqrt{2(4)(KE)}$ ......(ii)
Divide Eqs. (i) by (ii)

$$\frac{p_1}{p_2} = \frac{\sqrt{2(KE)}}{\sqrt{8(KE)}}$$

As KE of both are same,

$$\frac{p_1}{p_2} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 1:2$$

Hence, the ratio is 1: 2.

7. **(d)** Given, $s = \frac{t^3}{3}$, mass $= 3$ kg

$$t = 2 \text{ s} \Rightarrow s = \frac{t^3}{3}$$

$$\frac{ds}{dt} = v = t^2 \Rightarrow ds = t^2 dt$$

$$\frac{dv}{dt} = a = 2t$$

$$W = \int F \cdot ds$$

$$W = \int_{t_1}^{t_2} F \cdot t^2 dt \quad \ldots\ldots (i)$$

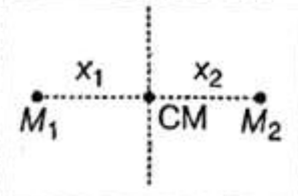
Put the value of F(F = ma, a = 2t) in Eq (i),

$$W = 3\int_0^2 2t^3 dt = 6\int_0^2 t^3 dt = \frac{6}{4}[t^4]_0^2$$

$$= 24 \text{ J} = \frac{6}{4} \times 16$$

8. **(c)** Given,

Two masses m_1 and m_2 moves toward CM by distance.



We know position of CM

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

Shift in position of centre of mass,

$$\Delta x_{CM} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

But $\Delta x_{CM} = 0$ (Given)

$$m_1\Delta x_1 + m_2\Delta x_2 = 0$$

$$\left(\frac{m_1}{m_2}\right)\Delta x_1 = -\Delta x_2$$

$$\Delta x_2 = -\frac{m_1}{m_2}d$$

9. **(c)** Let the radius of earth = R

Mass = M

If radius became n times.

By conservation of angular momentum

$I_1\omega_1 = I_2\omega_2$ ....(i)

where, I is moment of inertia for sphere,

$$I = \frac{2}{5}mR^2$$

ω is angular frequency, $\omega = 2\pi/T$

Put the value in Eq. (i) accordingly

$$\left(\frac{2}{5}MR^2\right)\frac{2\pi}{24\ h} = \frac{2}{5}M(xR)^2\frac{2\pi}{T}$$

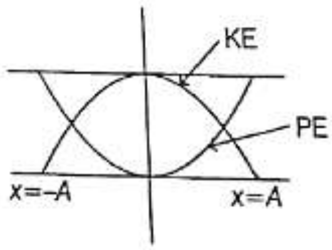
$\Rightarrow T = 24x^2$

10. **(a)** Given,

Particle is executing SHM at $x_1 = 4$ cm from mean position.

$$KE = \frac{1}{3}KE_{max}$$

max potential energy of SHM



$$KE_{max} = TE = \frac{1}{2}KA^2$$

$$KE = \frac{1}{3}(KE_{max}) = \frac{1}{3} \times \frac{1}{2}KA^2$$

$$= \frac{1}{3} \times \frac{1}{2}KA^2$$

$$\frac{1}{2}K(A^2 - x^2) = \frac{1}{6}KA^2$$

$$A^2 - x^2 = \frac{1}{3}A^2$$

At $x = 4$ cm

$$A^2 - 16 = \frac{1}{3}A^2$$

$$A = 2\sqrt{6}\ \text{cm}$$

11. **(a)** Given,

Wire length = 40 cm

Stretched = 01 cm (Elongation)

Strain is given by

$$\text{Strain} = \frac{\text{Elongation}}{\text{Length}} = \frac{0.1}{40} = 25 \times 10^{-4}$$

12. **(a)** Given,

Radius of straw = R

Surface tension of liquid = T

contact angle = 53°

$\cos 53° = 0.6$

We know that,

Force due to surface tension is given by $F = T \times 2\,(\text{length} + \text{thickness}) \times \cos\theta$

As thickness is negligible,

$\Rightarrow \quad F = T \times 21 \times \cos\theta$

$\Rightarrow \quad = 4\pi RT \times 2\pi R \times \cos 53° [l = 2\pi R]$

$\Rightarrow \quad F$

$\Rightarrow \quad = \dfrac{12\pi RT}{5}$

13. **(b)** Terminal velocity It is a velocity (highest) that is obtain by an object falling through liquid.

F_B is buoyant force, F_V is viscous force and F_W is gravitational force.

Among these only F_W acts in downward direction.

At terminal velocity, $\Rightarrow F_W = F_B + F_V$

14. **(c)** Given,

Two bodies are given with

mass = m and 2m

Temperature = 2T and T

Using the formula,

$Q = mc\Delta T$ (where c is specific heat, ΔT is change in temperature and Q is heat given)

For 2 nd body with mass 2m, the change in ΔT is 2T.

$$Q = (2m)c \cdot (2T)$$

$$c = \frac{Q}{4mT} \quad \ldots\ldots (i)$$

Now, for 1 st body with mass m, the change in temperature is

$Q = mc\Delta T$

Put the value of c from Eq. (i)

$$Q = m \cdot \frac{Q}{4mT} \Delta T$$

$\Delta T = 4T$

15. (b) From standard scale, $0^\circ C$ represents the freezing point of water.

$100^\circ C$ represents the boiling point of water.

According to question, the new scale has $20^\circ X$ as melting point
$110^\circ X$ as boiling point.
From above discussions

$$100^\circ C \quad = 90^\circ X \Rightarrow 1^\circ C = \frac{90^\circ}{100} X$$

$$40^\circ C \quad = \frac{90}{100} \times 40 = 36^\circ X$$

Final temperature $= 20^\circ + 36^\circ = 56^\circ X$

16. (c) For isobaric process, pressure of system remain same.

For constant pressure, heat supplied is

$\Delta Q = nC_p \Delta T$ ..... (i)

$n \rightarrow$ number of mole of ideal gas

$C_p \rightarrow$ specific heat at constant pressure

$\Delta T \rightarrow$ change in temperature

Work done in isobaric process

$\Delta W = p\Delta V$

$\Delta W = p(V_2 - V_1)$ $[pV = nRT]$

$\Delta W = nR(T_2 - T_1)$ ... (ii)

$\Delta W = nR\Delta T$

Ratio is given by

$\frac{\Delta W}{\Delta Q} = \frac{nR\Delta T}{nC_p \Delta T} = \frac{R}{C_p}$ ....(iii)

For diatomic gas, $C_p = \frac{7}{2} R$

Put in above equation.

$$\frac{W}{Q} = \frac{\mathbf{2}}{7}$$

Percentage of conversation

$$= \frac{2}{7} \times 100 = 28.6\%$$

17. (d) For Polyatomic gas,

Translational degree of freedom $= 3$

Rotational degree of freedom $= 2$

Vibration degree of freedom $= 1$

The ratio of translational to rotational

$$= \frac{3}{2} = 3:2$$

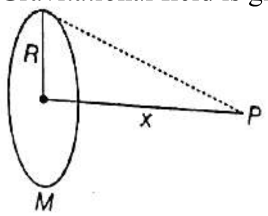
18. (d) Given,

Mass of ring $= m$

Radius $= R$

We have to calculate the distance from its geometrical axis

Gravitational field is given by at point P.



$E_P = \frac{GMx}{(R^2+x^2)^{3/2}}$ ..... (i)

To get the maximum value of E_P

$$\frac{dE_P}{dx} = 0$$

$$\Rightarrow \frac{GM}{(R^2+x^2)^{3/2}} - GMx$$

$$\times \frac{3}{2(R^2+x^2)^{1/2}} \times 2x = 0$$

Solving above expression,

$$3x^2 = R^2 + x^2$$

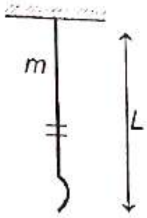
$$2x^2 = R^2$$

$$\Rightarrow x = \frac{R}{\sqrt{2}}$$

19. (a) Speed of wave on the string is given by

$$V = \sqrt{\frac{T}{\mu}} \quad \ldots(i)$$

where T is tension, μ is linear mass density.



Mass of the rope = m

$$\mu = \frac{m}{L} \quad \ldots\ldots(iii)$$

Put the value of T and μ in Eq. (i),

$$v = \sqrt{\frac{mg}{\frac{m}{L}}} = \sqrt{gL}$$

Now, using equation of motion,

$$v^2 = u^2 + 2as$$

(v = 0, rope is connected to ceiling)

$$a = \frac{U^2}{2S} \Rightarrow \frac{gL}{2L}$$

$$a = \frac{g}{2} \quad \text{(because } S = L\text{)}$$

Hence, the value of acceleration is constant.

20. (c) Given,

Wave equation

$$y = (0.02)\sin(\pi x - 8\pi t) \quad \ldots.. (i)$$

Compare the above equation with standard wave equation.

$$y = A\sin(kx - \omega t) \quad \ldots.. (ii)$$

From Eqs. (i) and (ii),

$$A = 0.02$$

$$k = \pi \Rightarrow \omega = 8\pi$$

Velocity is given by $v = \frac{\omega}{k} = \frac{8\pi}{\pi}$

$$\Rightarrow v = 8 \text{ m/s}$$

21. (b) Given,

Image distance, $v = -32$ cm

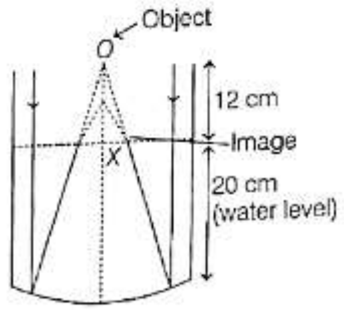
Object (sun) distance, $u = \infty$

Tank is filled upto = 20 cm

Refractive index of water $= \frac{4}{3}$

Using mirror formula,

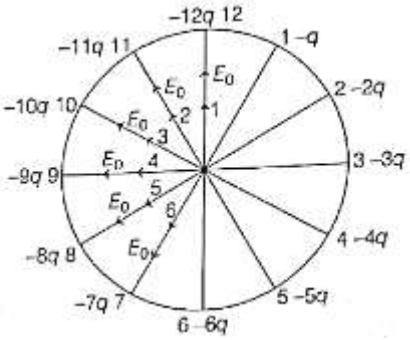
$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \Rightarrow \frac{1}{f}=\frac{-1}{32}+\frac{1}{\infty}$$
$f=-32$ cm
When water is filled upto a height ($h=20$ cm), the image formed by mirror will acts as virtual object for water surface.
The image formed by virtual object.



$$\frac{\mu_{water}}{\mu_{air}}=\frac{\text{Actual height}}{\text{Apparent height}}$$
$$\frac{4}{3}=\frac{XO}{XI} \Rightarrow XO=12 \text{ cm}$$
$$\frac{4}{3}=\frac{12}{XI}=9 \text{ cm}$$
Sunlight will focus at 9 cm above water level.

22. (c) Given,
In air YDSE's central maxima
Y-coordinate $=2$ cm
10th maxima $=5$ cm
Liquid's refractive index $=1.5=3/2$
$$\mu=\frac{c}{v}=\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$
$$\mu=\frac{\lambda_{vacuum}}{\lambda'_{medium}} \Rightarrow \lambda'_{medium}=\frac{2}{3}\lambda_{vacuum}$$
Path difference along X-axis,
$\Delta x=0, \lambda, 2\lambda, \ldots 10\lambda$
For central maxima for 10th maxima For Y-axis,
$$\Delta Y=\frac{\Delta x D}{d}=\frac{n\lambda D}{d} \Rightarrow \Delta Y \propto \lambda$$
Central maxima will remain unchanged ($Y'=2$ cm) and ($n=0$)
$$\frac{\Delta Y}{\Delta Y'}=\frac{\lambda}{\lambda'_{medium}}$$
$$\frac{5-2}{Y'-2}=\frac{3}{2} \Rightarrow Y'=4 \text{ cm}$$
New coordinates will be 2 cm, 4 cm.

23. (b) Given, The charges corresponding to number's of a clock $-q,-2q \ldots 12q$.



Electric field is given by
$E=kq/r^2 \Rightarrow E \propto q$

Let's take a pair of charges ($-9q$ and $-3q$)

The value of E due to $-9q > -3q$

and net magnitude $= -6q = E_0$

Same goes for all pairs with magnitude of $(-6q)$.

The angle between each field,

$\theta = \dfrac{360^\circ}{12} = 30^\circ$

From the property of vector addition, if two equal vector is separated by 120°, then resultant vector is same as initial vector.

For making 120°, take vector 1 to vector 5

Resultant of vector 1 and 5 $= E_0 =$ Vector 3 direction.

Resultant of vector 2 and 6 $= E_0 =$ Vector 4 direction

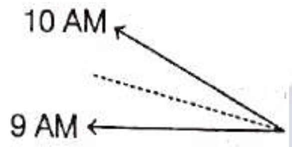
Now, vector (1 and 5) and (2 and 6) are eliminated

Now, vector 3 $= E_0 + E_0 = 2E_0$

Vector 4 $= E_0 + E_0 = 2E_0$

Vector 3 denotes 10 AM

Vector 4 denotes 9 AM



So, the resultant vector of 9 AM and 10 AM is in midway direction.

i.e., 9: 30.

24. (c) Given,

Electric potential, $V = \frac{1}{2}(y^2 - 4x)$ ...(i)

$$\mathbf{B} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}}$$

$$= -2\hat{\mathbf{j}} + y\hat{\mathbf{i}} \quad \ldots\ldots \text{(ii)}$$

The point at which electric field is to be find $(x, y) = (1,1)\text{m}$

$\Rightarrow E = 2\hat{\mathbf{i}} - \hat{\mathbf{j}}$

25. (b) Given,

Potentiometer is connected across A and B,

Balance obtained $= 64$ cm .....(i)

When potentiometer lead at B move toward

C, balance obtained $= 8$ cm ....(ii)

Potential difference $V = kl[k =$

Potential gradient]

For Eq. (i), $V_1 = kl_1 = 64k$

For Eq. (ii), $V_1 - V_2 = 8k$

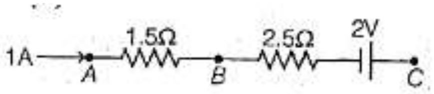
$64k - V_2 = 8k$

$V_2 = 56k\ [V_2 = kl_2]$

$\Rightarrow kl_2$

$\Rightarrow l_2 = 56$ cm

26. (d)

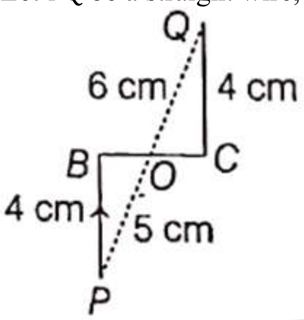


The direction of current is from A to B which means point A is at higher potential.

For section AB.

$V = IR = 1 \times 1.5$
$V = +1.5$ V at A
Now for section BC
$V = IR$ (at C let potential be X)
$$(2V + X) = IR \Rightarrow I = \frac{2V + X}{2.5\Omega}$$
We know from diagram $I = 1A$
For I to be 1 the V/R ratio should be 1 .
$2.5 = 2 + X \Rightarrow X = 2 - 2.5$
$\Rightarrow \quad X = -0.5$ V

27. (b) Given,
Wire PQ carries current $= 10$ A
Magnetic field (uniform) $= 5$ T Which is normal to plane of paper.
Let PQ be a straight wire, then



$[\triangle \text{PBO}, \text{PO}^2 = \text{PB}^2 + \text{BO}^2]$
Length of $PQ = PO + OQ$
$= 5 + 5 = 10$ cm
Force experienced by wire in uniform magnetic field is given by
$F = BIL\sin\theta$
Here, $\theta = 90^\circ$
[Plane and outward (normal) to plane]

$\Rightarrow \; F = 5 \times 10 \times \sin 90^\circ \times (0.\text{m})$
$\Rightarrow \; F = 5$ N

28. (c) Given,
Current in long straight wire $= 18$ A
Distance from wire, where magnetic field is to find $= 12$ cm $= 0.12$ m
Using Ampere's law for long straight wire
$\oint B \cdot dl = \mu_0 I$
$$B = \frac{\mu_0 I}{2\pi r}$$
Put the value of all terms in above equation
$$B = \frac{(4\pi \times 10^{-7}) \times 18}{2\pi \times (0.12)}$$
$\Rightarrow B = 3 \times 10^{-5}$ T

29. (d) Given,
Coercivity of bar magnet $= 4 \times 10^3$ A/m
Turns $= 60$
Length $= 12$ cm
Number of turn per unit length in m.
$$n = \frac{N}{L} = \frac{60}{12 \times 10^{-2}} = 500 \text{ turns /m}$$
A magnetic intensity H of equal magnitude as coercivity is required to demagnetise the bar magnet.
$H = ni$

$4 \times 10^3 = 500 \times i$

$\Rightarrow = 8.0$ A

30. **(b)** Current flowing through solenoid is increasing at constant rate.

Magnetic field associated with solenoid

$$B = \mu_0 nI$$

$$\frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$$

as dI/dt is constant, dB/dt is also constant.

Flux is given by

$$\phi = B \cdot A$$

$$\frac{d\phi}{dt} = A\frac{dB}{dt} \quad (\text{ A is area})$$

As dB/dt is constant, therefore $d\phi/dt$ is also constant.

So, the current induced is constant. Whenever there is a change in flux, the body tries to oppose this change by inducing current in opposite direction.

31. **(b)** Given,

Current change $= \Delta I = 10 - 2 = 8$ A

Time $= 0.2$ s

EMF induced $= 120$ V

Using Faraday's law of mutual induction,

$$|E| = \frac{d\phi}{dt}$$

$$\phi = M\Delta I$$

[M is mutual inductance of pair]

$$E = \frac{M\Delta I}{\Delta T} \quad \ldots\ldots(i)$$

Put the value in Eq. (i)

$$120 = \frac{M \times 8}{0.2}$$

$$M = \frac{120}{40} \Rightarrow M = 3H$$

32. **(d)** Given,

Peak current $= 2$ A

Peak voltage $= 1$ V

Power factor $= 1/2$

$$\text{Power factor} = \frac{\text{Peak voltage}}{\text{Peak current}} = \frac{1}{2}$$

Power factor $= \cos\phi$

where, ϕ is the angle between V and I,

$$\cos\phi = \frac{1}{2}$$

$$\phi = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \phi = 60^\circ$$

33. **(a)** Given,

Frequency of oscillation $= 3$GHz

$= 3 \times 10^9$ Hz

Speed of light, $c = 3 \times 10^8$ m/s

The electromagnetic wave travel with speed of light and is given by

$c = v\lambda$

where λ is the wavelength and v is frequency of wave

$$\lambda = \frac{c}{v} = \frac{3 \times 10^8}{3 \times 10^9}$$

$$\Rightarrow \lambda = 01 \text{ m}$$

34. **(d)** de-Broglie wavelength associated with electron accelerated with a potential difference v is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

$E = eV$ (e is charge, V is potential difference)

$\lambda = \frac{h}{\sqrt{2meV}}$ ...... (i)

Put the value of h, m, e in the equation 1

$\lambda = \frac{1.23}{\sqrt{v}}$ nm

Now, put value of potential difference which is 900 V given

$l = \frac{1.23}{\sqrt{900}}$ nm $= \frac{1.23}{30}$ nm

$\Rightarrow \quad \lambda = 0.04$ nm

35. (b) Blectron is moving in 4 th orbit of Hydrogen.

According to Bohr's postulate, The angular momentum is integral multiple of $\frac{h}{2\pi}$

$L = \frac{nh}{2\pi}$

As electron is moving in 4th orbit, $n = 4$

$L = \frac{4h}{2\pi} \Rightarrow L = \frac{2h}{\pi}$

36. (c) Energy equivalence of a mass is given by

$E = mc^2$

[m is mass of object c is speed of light] m is given as 1 kg

$c = 3 \times 10^8$ m/s

$\Rightarrow \quad E = 1 \times (3 \times 10^8)^2$

$\Rightarrow \quad E = 9 \times 10^{16}$ J

37. (d) Mass number of nucleus = A

Surface area of nucleus = S

Using the relation,

$R = R_0 A^{1/3}$

Radius $\propto A^{1/3}$

Nucleus is spherical in shape, so

Area $= 4\pi R^2$

or $S = 4\pi\left(R_0 A^{1/3}\right)^2$

$\Rightarrow S \propto A^{2/3}$

38. (a) Base current, $I_b = 10\mu$ A

Emitter current, $I_e = 1$ mA

$= 1000\mu$ A

In transistor,

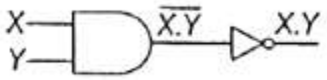
$I_e = I_b + I_c$

where I_c is current in collector arm.

$I_c = I_c - I_b = 1000 - 10$

$\Rightarrow \quad I_c = 990\mu$ A

39. (a) Output of NAND gate = Input of NOT gate



So, the output will be an AND gate.

40. (b) Carrier wave frequency = 6MHz = 6000KHz

Message signal frequency = 10kHz Let C be carrier wave frequency and M is message signal frequency.

So, upper side band is given by

$C + M = 6000 + 10 = 6010$kHz

Lower side band frequency is given by

$C - M = 6000 - 10 = 5990$kHz

Final sideband frequency. 6010kHz, 5990kHz.

Chemistry

41. (d) Given,
Radius of 3rd orbit of hydrogen atom
= Rpm.
Radius of 2 nd orbit of Hc^+ion (x) = ?
Using the formula,

$$r = \frac{n^2}{Z} \times 0.529Å$$

$$R = \frac{9}{1} \times 52.9pm \quad ....(i)$$

(Radius of 3rd orbit of hydrogen)

$$x(Hc^+) = \frac{4}{2} \times 52.9pm \quad ......(ii)$$

From Eq. (i) and (ii),

$$\frac{R}{x} = \frac{9 \times 52.9}{2 \times 52.9}$$

Or $x = \frac{2}{9}R$

42. (b) Given,
Threshold frequency of metal
$(v_0) = 10^{15}s^{-1}$
First frequency $(v_1) = 1.5 \times 10^{15}\ s^{-1}$
Second frequency $(v_2) = 2 \times 10^{15}\ s^{-1}$

$$\frac{(KE_{max})_1}{(KE_{max})_2} = ?$$

Using the equation of photoelectric effect,

$$h\nu = h\nu_0 + KE$$

$$\Rightarrow \text{ or } KE = h(v - v_0)$$

$$\frac{(KE_{max})_1}{(KE_{max})_2} = \frac{h(1.5 \times 10^{15} - 1 \times 10^{15})}{h(2 \times 10^{15} - 1 \times 10^{15})}$$

$$\frac{KE_1}{KE_2} = \frac{0.5 \times 10^{15}}{1 \times 10^{15}}$$

$$\frac{KE_1}{KE_2} = \frac{1}{2} \text{ or } KE_1 : KE_2 = 1:2$$

43. (c) Among the given statements, I and III only are correct while that in II and IV are incorrect. It is because the electronegativity of chlorine is 3 not 3.5. Also, the IUPAC name of element with atomic number 106 is unnilhexium.

44. (d) The correct match is A-III, B-IV, C-II, D-I.

Molecule/Ion	Bond Pair	Lone Pair	Shape
(A) $SnCl_2$	2	1	III. Angular
(B) NH_3	3	1	IV. Trigonal Pyramidal
(c) I_3^-	2	3	II. Linear
(D) SO_3	3	0	I. Trigonal planar

45. (d) The set of molecules given in option (d) has different hybridisation of central atoms.
i. e. SF_4, XcF_4, CH_4.

For
$SF_4 = 4 + 1/2(6 - 4) = 4 + 1$
So, the hybridisation is sp^3d.
$XcF_4 = 4 + 1/2(8 - 4) = 4 + 2$
(Square planar)
So, the hybridisation is sp^3d^2.
$CH_4 = 4 + 1/2(4 - 4) = 4$
So, the hybridisation is sp^3.

46. (b) As we know that, pM = dRT

where, M = molecular mass

d = density

$\frac{760}{760} \times M = 0.543 \times 0.0821 \times 300$

$\Rightarrow M = 1337 \text{ g/mol}$

Let mole fraction of oxygen in mixture be y.

$\chi_{O_2} = y; \chi_{He} = (1 - y)$

Average molecular weight

$= y \times M_{O_2} + (1 - y)M_{He}$

$1337 = y \times 32 + (1 - y)4$

$13.37 = 32y - 4y + 4$

$\Rightarrow y = 0.335$

% by mass of oxygen

$= \frac{0.335 \times 32 \times 100}{0.665 \times 4 + 0.335 \times 32}$

% mass of oxygen $= 8012\% \approx 80\%$

47. (a) (i) $Na_2C_2O_4 \cdot 2H_2O$

$= \frac{2(23) + 2(12) + 4(16) + 2(18)}{2}$

$= \frac{170}{2} = 85 \text{ geq}^{-1}$

(ii) $KMnO_4/H^+ = \frac{39+55+4(16)}{5}$

$= 31.6 \text{ geq}^{-1}$

(iii) $KMnO_4/H_2O = \frac{39+55+64}{3}$

$= 52.6 \text{ geq}^{-1}$

(iv)

$K_2Cr_2O_7/H^+$

$= \frac{2(39) + 2(52) + 7(16)}{6} = \frac{294}{6}$

$= 49 \text{ geq}^{-1}$

It should be noted that molar mass of K is 39 not 19 (which is given in question).

48. (c) Given,

n = 2.5 moles

$V_1 = 2\text{dm}^3;\ V_2 = 20\text{dm}^3$

$W = -16.5 \text{ kJ} = -16.5 \times 10^3 \text{ J}$

$R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$

T =?

Using the formula,

$$W = -nRT\ln\frac{V_2}{V_1}$$

$$-16.5 \times 10^3 = -2.5 \times 8.314 \times T\ln\left(\frac{20}{2}\right)$$

$$\Rightarrow T = \frac{16.5 \times 10^3}{2.5 \times 2.303 \times 8.314}$$

$$= 344.7 \text{ K} \approx 345 \text{ K}$$

49. (a) The solution obtained by mixing the given data is an acidic buffer.

The expression used for finding the pH of acidic buffer is given by

$$\text{pH} = \text{pK}_a + \log\frac{[\text{Salt}]}{[\text{Acid}]}$$

$$\text{pK}_a = 4.76 \text{ (given)}$$

$$[\text{Salt}] = \frac{20 \times 0.5}{100} = 01\text{M}$$

$$[\text{Acid}] = \frac{10 \times 1}{100} = 0.1\text{M}$$

Substituting the values in above equation, we get

$$\text{pH} = 4.76 + \log\frac{0.1}{0.1} = 4.76$$

50. (c) Statements given in II and IV are correct while those in I and III are incorrect. The correct form of statements I and III are :

I. Sodium hydride with water liberates hydrogen gas.,

$$\underset{\text{Sodium Hydride}}{NaH(s)} + \underset{\text{Water}}{H_2O(aq)} \longrightarrow \underset{\text{Sodium hydroxide}}{NaOH(aq)} + \underset{\text{Hydrogen}}{H_2(g)}$$

III. Ammonia and water molecules are electron rich compound.

51. (d) The nitrate of potassium metal on heating liberates KNO_2 and O_2. The reaction involved is as follows:

$$2KNO_3(s) \xrightarrow{\Delta} 2KNO_2(s) + O_2(g)$$

So, nitrate of potassium does not librate NO_2 gas on heating.

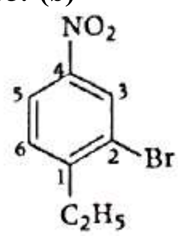
52. (a) The correct match is A-III, B-IV, C-I, D-II.

	List I (Substance)		List II (Use)
(A)	Na_2O_2	(III)	Oxidising agent
(B)	D_2O	(IV)	Moderator
(C)	Cs	(I)	Photoelectric cells
(D)	$Mg(OH)_2$	(II)	Antacid

53. (a) Statements given in I, III and V are correct while those in II and IV are incorrect. It is because, boron has high melting and boiling point. Also, atomic radii increases down the group but atomic radii of gallium is less than that of aluminium due to presence of d-electrons. So, the correct order will be Ga < Al < In.

54. (c) Both (A) and (R) are true and (R) is the correct explanation of (A).

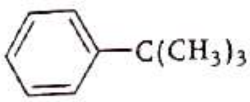
55. (b)



The IUPAC name of the above compound is 2-bromo-1-ethyl-4-nitrobenzenc.

56. (a) Hyperconjugation is not possible in compounds in which α-hydrogen is not present.

Thus,



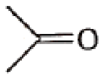
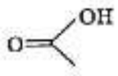
compound does not show hyperconjugation.

57. (b) (I) $(CH_3)_2C = CH_2 \xrightarrow{KMnO_4/H^+}$

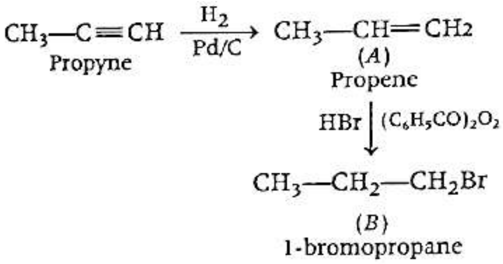
$$\underset{(X)}{CH_3COCH_3} + CO_2 + H_2O$$

(II) $CH_3 - CH = CH - CH_3 \xrightarrow{KMnO_4/H^+} \underset{(Y)}{2CH_3COOH}$

Thus, the functional group in X and Y are

 and 

58. (b)



The major product (B) will be formed as per anti-Markownikoff's addition.

59. (c) Given,

Edge length = xÅ

Atomic mass of Ca = 63.5u

For fcc unit cell i.e., Z = 4

Density = ?

Using the formula,

$$d = \frac{Z \times M}{a^3 \times N_A}$$

$$d = \frac{4 \times 63.5}{6.0 \times 10^{23} \times x^3 \times 10^{-24}} \text{ g cm}^{-3}$$

$$d = \frac{63.5 \times 4 \times 10}{6 \times x^3}$$

or $d = \frac{423}{x^3}$ g/cm^3

60. (a) Given,

$p_A^\circ = 70$ tort

$\chi_B = 0.2$

$p_{total} = 84$ torr

$p_B^\circ = ?$

We know that, $\chi_A + \chi_B = 1$

$\Rightarrow x_A = 1 - 0.2 = 0.8$

Also, from Dalton's law of partial pressure

$p_{total} = p_A + p_B$ ……(i)

Now, using Raoult's law for component A and B, we get

$$p_A = p_A^\circ \cdot \chi_A$$

$$p_A = 70 \times 0.8 = 56 \quad \ldots\ldots \text{(ii)}$$

$$p_B = p_B^\circ \cdot \chi_B = p_B^\circ \times 0.2 \quad \ldots\ldots\text{(iii)}$$

Substitute the value of Eq. (ii) and Eq. (iii) in Eq. (i)

$$84 = 56 + 0.2 \times p_B^\circ$$

$$\Rightarrow p_B^\circ = \frac{84 - 56}{0.2} = 140 \text{ torr}$$

61. (a) In general reduction potential of non-metal electrode increases with decrease in concentration of electrolyte. Hence, the concentration of chloride ion in the solution will be 2.5×10^{-3}.

62. (c) Given,

$n = 1$ (Ist order)
$k = 6.93 \times 10^{-2} \text{ min}^{-1}$
$a = 100$
$a - x = 10$
$t = ?$

Using the formula of rate constant for Ist order reaction,

$$t = \frac{2.303}{k} \log \frac{[A_0]}{[A]}$$

$$t = \frac{2.303}{6.93 \times 10^{-2}} \log \left[\frac{100}{10}\right]$$

$$= \frac{2.303}{6.93 \times 10^{-2}}$$

$$= 33.23 \text{ min} \approx 33 \text{ min}$$

63. (b) Conditions given in I, II and V i.e., high surface area, low temperatures and high pressures favours physical absorption.

64. (a) Sphalerite (ZnS), siderite ($FeCO_3$) and malachite $[Cu_2CO_3(OH)_2]$ are the ores of zinc, iron and Cu respectively. Thus, their atomic numbers are 30, 26 and 29 respectively.

65. (d) By completing the given reaction, we get

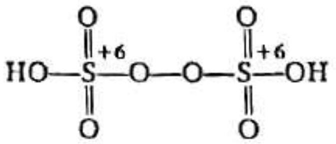
$P_4 + 3NaOH + 3H_2O \longrightarrow PH_3 + 3NaH_2PO_2$

So, $Q = PH_3$

Now, among the given options (reactions) product Q is not obtained in option (d) i.c.,

$PCl_3 + 3H_2O \longrightarrow H_3PO_3 + 3HCl$

66. (c) The formula of peroxydisulphuric acid is $H_2\ S_2O_8$. It's structure is given by



From the structure, we can clearly see that, the oxidation state for both the sulphur atom is +6 and it has two $S - OH$ bonds.

67. (b) Statements given in II and III are correct. While those in I and IV are incorrect. It is because Au and Pt both are soluble in aqua regia. Also, the correct stability order of oxides of halogens are $I > Cl > Br$.

68. (a)

69. (d) The electronic configuration of $Gd(64)$ is

$Gd_{64} = [Xe]4f^7 5d^1 6\ s^2$

$Gd^{3+} = [Xe]4f^7 \Rightarrow x = 7$

Now, electronic configuration of $Yb(70)$ is

$Yb_{70} = [Xe]4f^{14} 6\ s^2$

So, $Yb^{2+} = [Xe]4f^{14} \Rightarrow y = 14$

So, $x + y = 7 + 14 = 21$

70. (a)

Complex	Electronic configuration of metal/ion
(A) $[Co(NH_3)_6]^{3+}$	(II) $t_{2g}^6 e_g^0$
(B) $[CoF_6]^{3-}$	(III) $t_{2g}^4 e_g^2$
(C) $[Ni(CO)_4]$	(IV) $t_2^4 e^6$
(D) $[Fe(CN)_6]^{3-}$	(I) $t_{2g}^5 e_g^0$

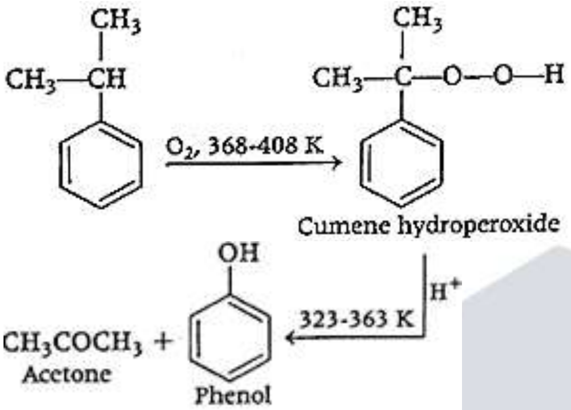
71. **(a)** Elastomeric polymers are those polymeric chains which are held together by weakest intermolecular forces. Among the given polymers four are clastomers i.e., natural rubber, vulcanised rubber, Buna-N and neoprene.

72. **(c)** The incorrect statement about maltose is given in option (c) because it is composed of 2 molecules of α-D-glucose.

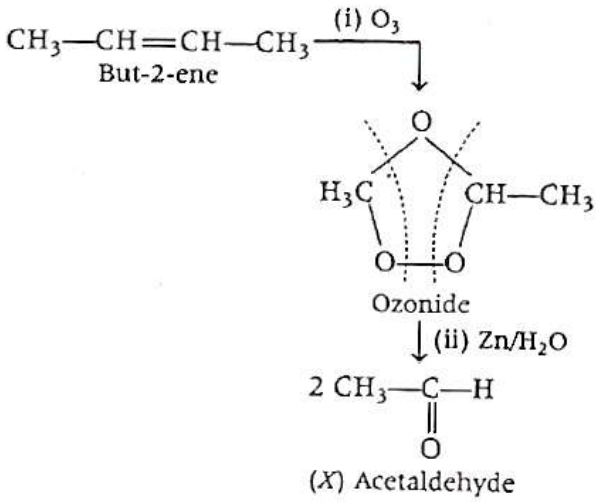
73. **(b)** Chloramphenicol is an antibiotic used for the treatment of bacterial infections. Thus, option (b) is an incorrect match.

74. **(d)** The order of reactivity towards S_N2 reaction for alkyl halide is methyl > primary halide (1°) > secondary halide (2°) > tertiary halide (3°). Thus, option (d) is least reactive towards S_N2 reaction as it is a 3° halide.

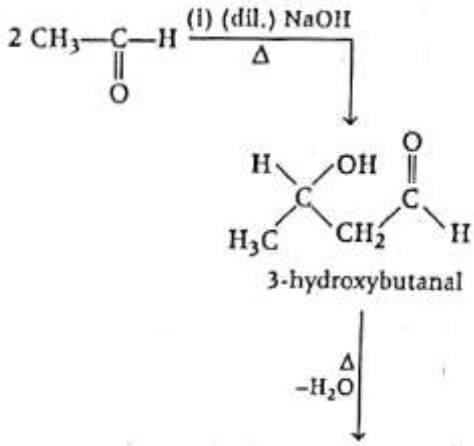
75. **(d)** The oxidation of cumene (isopropyl benzene) in the presence of air gives cumene hydro-peroxide. The reaction between cumene hydroxide and dilute acid gives phenol. The reaction involved is as follows:

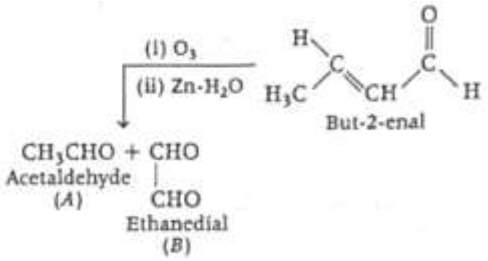


76. **(b)** **(I)**

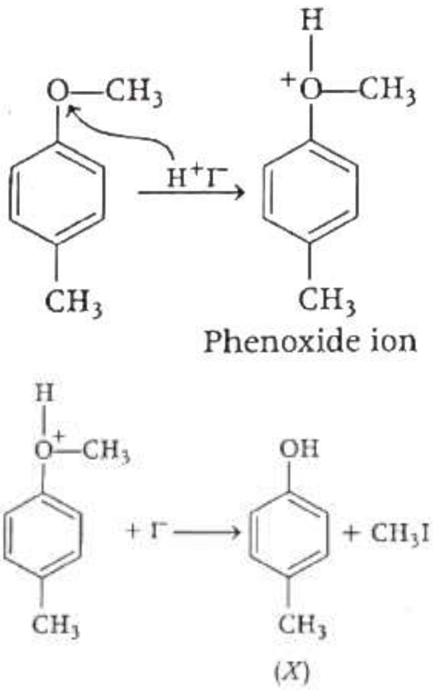


(II)

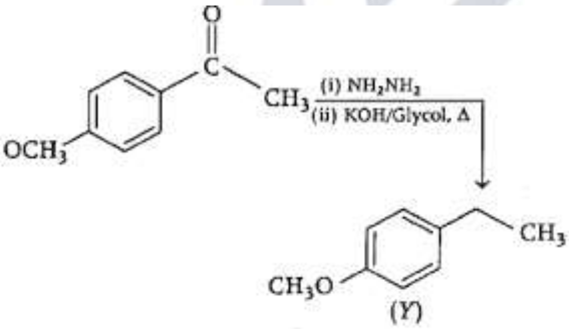



(i) O_3
(ii) Zn-H_2O
But-2-enal
CH_3CHO + CHO | CHO
Acetaldehyde (A)
Ethanedial (B)

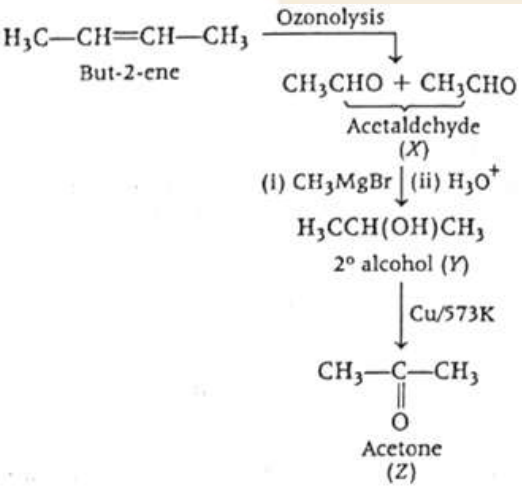
77. (a) I.


Phenoxide ion
(X)

II.

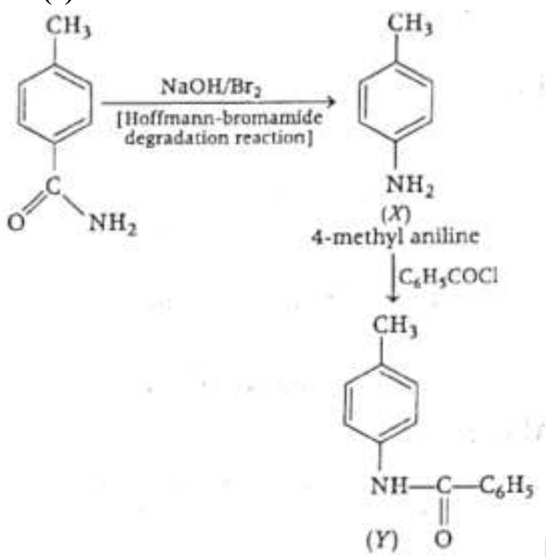

(i) NH_2NH_2
(ii) KOH/Glycol, Δ
(Y)

78. (d)

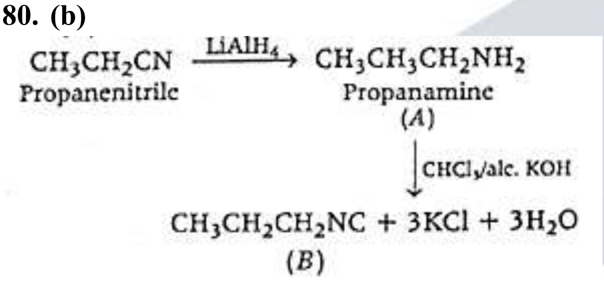

$H_3C—CH═CH—CH_3$
But-2-ene
Ozonolysis
$CH_3CHO + CH_3CHO$
Acetaldehyde (X)
(i) CH_3MgBr (ii) H_3O^+
$H_3CCH(OH)CH_3$
2° alcohol (Y)
Cu/573K
Acetone (Z)

The compound (Z) formed undergoes Wolff-Kishner reduction and it also gives with yellow precipitate with I_2 and NaOH solution. Thus, statements given in I and III are correct.

79. (c)



80. (b)



In the given reaction propanenitrile undergo reduction on reaction with $LiAlH_4$ to give primary amine i.e., propanamine (A) which reacts with $CHCl_3$ in alc. KOH to give an isocyanide (B). This conversion of A to B is carbylamine reaction.

Mathematics

81. (b) $f(x) = \log_{0.5}(x^4 - 2x^2 + 3)$

Let $x^2 = t$

$t^2 - 2t + 3 = y$

$D = b^2 - 4ac = (-2)^2 - 4 \times 1 \times 3 = -8$

No real roots.

y will be always positive

$t_{min} = -\frac{b}{2a} = \frac{-(-2)}{2 \times 1} = 1$

$y_{min} = -\frac{D}{4a} = \frac{-(-8)}{4 \times 1} = 2$

Base is less than 1 .

Graph will decrease for $x > 1$.

$f(x) \quad = \log_{0.5}(2) = \log_{\left(\frac{1}{2}\right)}(2)$

$= \log_{2^{-1}}(2) = -1\log_2 2 = -1$

Range $\in (-\infty, -1]$

82. (a) $I + P + P^2 + P^3 + \cdots + P^n = 0$

$I\left(\frac{P^{n+1} - I}{P - I}\right) = 0$

$P^{n+1} - I = 0 \Rightarrow P^{n+1} = I$

$P^{n+1} \times P^{-1} = I \times P^{-1} \Rightarrow P^n = P^{-1}$

83. (d) Given, $|z_1 + z_2| = |z_1| + |z_2|$

On squaring both sides, we get

$|z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$
$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$
$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0$
$\Rightarrow \arg(z_1) - \arg(z_2) = 0$

84. (c) Given, $1 + i^2 + i^4 + i^6 + \cdots + i^{2024}$
$= 1 + (i^2 + i^4 + i^6 + \cdots + 1012 \text{ terms })$
$= 1 + (-1 + 1 - 1 + 1 - \cdots + 1 - 1 + 1)$
i.e. 506 terms will be +1 and 506 terms will be $-1 = 1 + 0 = 1$

85. (b) Given, $4x^2 - 2x + (k - 4) = 0$
On comparing with
$ax^2 + bx + c = 0$, we get
$a = 4, b = -2$ and $c = k - 4$
Also, given one of the root is reciprocal of the other.
$\alpha = \frac{1}{\beta} \Rightarrow \alpha\beta = 1$
We know that, product of roots
$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$
$\alpha \cdot \beta = \frac{c}{a}$
$1 = \frac{c}{a}$
$\Rightarrow c = a$
$\Rightarrow k - 4 = 4$
$k = 4 + 4 = 8$

86. (d) If $f(x) = x^3 + 3x^2 - 9x + \lambda = (x - \alpha)^2$
$(x - \beta)$, then $(x - \alpha)$ is a factor of order 2.
$(x - \alpha)$ is a factor of order one of
$f'(x)$ i.c. $3x^2 + 6x - 9$
Now, $f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$
$= 3(x + 3)(x - 1)$
$\Rightarrow f'(x) = 0$
$\Rightarrow x = 3,1$
It means either $\alpha = 1$ or $\alpha = -3$
If $\alpha = 1$, then α is a root of
$x^3 + 3x^2 - 9x + \lambda = 0$
$\therefore 1 + 3 - 9 + \lambda = 0 \Rightarrow \lambda = 5$
If $\alpha = -3$, then α is a root of
$x^3 + 3x^2 - 9x + \lambda = 0$
$\therefore -27 + 27 + 27 + \lambda = 0$
$\Rightarrow \lambda = -27$
Hence, $\lambda = 5, -27$

87. (d) Vowels $\rightarrow$ A, U, I
3 vowels occupy 3 places which can be arranged among themselves in 3 ! ways.
Remaining 4 letters can be arranged in 41 ways.
The total number of ways is
$3! \times 4! = 6 \times 24 = 144$

88. (c)

$$\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r\left(x^{\frac{1}{2}}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r$$

$$= {}^{10}C_r\frac{x^{\frac{10-r}{2}}}{x^{2r}}(-k)^r$$

$$= {}^{10}C_r x^{\frac{10-r}{2}-2r}(-k)^r$$

$$= {}^{10}C_r x^{\frac{10-5r}{2}}(-k)^r$$

To be independent of $x, r = 2$

$$405 = {}^{10}C_2(-k)^2$$

$$\Rightarrow \quad 405 = \frac{10\times 9}{2}(-k)^2$$

$$\Rightarrow \quad 405 = 45k^2$$

$$\Rightarrow \quad 9 = k^2$$

$$k = \pm 3$$

89. (d)

(d) $(x-1)(x-2)(x-3)$

$= x^3-6x^2+11x-6$

$x^3-6x^2+11x-6)x^4$

$(x+6$

$x^4-6x^3+11x^2-6x$

$(-)(+)(-)(+)$

$6x^3-11x^2+6x$

$6x^3-36x^2+66x-36$

$(-)\ (+)\ (-)\ (+)$

$25x^2-60x+36$

$x^4 = (x+6)(x-1)(x-2)(x-3)$

$+25x^2-60x+36$

$$\Rightarrow \frac{x^4}{(x-1)(x-2)(x-3)} = (x+6)$$

$$+\frac{25x^2-60x+36}{(x-1)(x-2)(x-3)}$$

$p(x) = (x+6)$

and $\dfrac{25x^2-60x+36}{(x-1)(x-2)(x-3)}$

$$= \frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$$

$$\Rightarrow C = \frac{25(3^2)-60\times 3+36}{(3-1)(3-2)} = \frac{81}{2}$$

$$p\left(\frac{3}{2}\right)+C = \left(\frac{3}{2}+6\right)+\frac{81}{2} = 48$$

90. (a)

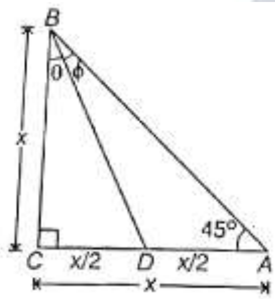
$$81^{\sin^2 x} + 81^{\cos^2 x} = 30$$
$$\Rightarrow 81^{\sin^2 x} + 81^{1-\sin^2 x} = 30$$
$$\Rightarrow 81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30$$
$$\text{Let } 81^{\sin^2 x} = t, t + \frac{81}{t} = 30$$
$$\Rightarrow t^2 - 30t + 81 = 0 \Rightarrow t = 27,3$$
$$\Rightarrow 81^{\sin^2 x} = 27,3 \Rightarrow 3^4 \sin^2 x = 3^3, 3^1$$
$$\Rightarrow 4\sin^2 x = 3,1 \Rightarrow \sin x = \frac{\sqrt{3}}{2}, \frac{1}{2}$$

$[0 \leq x \leq \pi, \sin x \to + \text{positive}]$
$\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{2\pi}{3}$

91. (a) $\sinh x = \frac{e^x - e^{-x}}{2}$

$$\sinh(\log x) = \frac{e^{\log x} - e^{-\log x}}{2}$$
$$\Rightarrow \quad -2 = \frac{\left(x - \frac{1}{x}\right)}{2} \cdot [e^{\ln x} = x]$$
$$\Rightarrow \quad \frac{x^2 - 1}{x} = -4 \Rightarrow x^2 + 4x - 1 = 0$$
$$x = \frac{-4 \pm \sqrt{16 - 4 \times 1 \times (-1)}}{2 \times 1}$$
$$= \frac{-4 \pm \sqrt{20}}{2} = -2 \pm \sqrt{5}$$
$$= (\sqrt{5} - 2), -(2 + \sqrt{5})$$
$$x = \sqrt{5} - 2 \ (x > 0 \text{ for } \log x)$$

92. (b) $\theta + \phi = 45^\circ$ ....(i)



In $\triangle$ BCD,

$$\tan\theta = \frac{CD}{BC} = \frac{\frac{x}{2}}{x} = \frac{1}{2}$$
$$\Rightarrow \cot\theta = 2$$
$$\phi = 45^\circ - \theta \text{ [from Eq. (i)]}$$
$$\Rightarrow \tan\phi = \tan(45^\circ - \theta)$$
$$= \frac{\tan 45^\circ - \tan\theta}{1 + \tan 45^\circ \cdot \tan\theta}$$
$$= \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{3}$$
$$\Rightarrow \cot\phi = 3$$

Possible values of the cotangents of the two angles are 2 and 3.

93. (b) Given, position vectors of **P** and **Q** are

$P = \hat{i} + 2\hat{j} - 7\hat{k}$

$Q = 5\hat{i} - 3\hat{j} + 4\hat{k}$

$PQ = Q - P = 4\hat{i} - 5\hat{j} + 11\hat{k}$

DR' 's of PQ

$a = 4, b = -5$ and $c = 11$

$$\cos\gamma = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{11}{\sqrt{16 + 25 + 121}} = \frac{11}{\sqrt{162}}$$

94. (b) Here, $|a + b + c| = 1$

$|a + b + c|^2 = 1$

$|a|^2 + |b|^2 + |c|^2 + 2a \cdot b + 2b \cdot c$

$+2c \cdot a = 1$

[**a**, **b**, **c** are three -unit vectors]

$\Rightarrow 1^2 + 1^2 + 1^2 + 0 + 2|b||c|\cos\alpha$

$+2|\mathbf{c}||\mathbf{a}|\cos\beta = 1$

$\Rightarrow 1 + 1 + 1 + 2\cos\alpha + 2\cos\beta = 1$

$\Rightarrow 2\cos\alpha + 2\cos\beta = -2$

$\Rightarrow \cos\alpha + \cos\beta = -1$

95. (a) $n(S) = {}^9C_1 \times {}^8C_1 = 72$

$x^2 + bx + c > 0 \Rightarrow b^2 - 4c < 0$

b	c	Number of cases
1	2,3,4, ... ,9	8
2	3,4,5, ... ,9	7
3	4,5,6, ... ,9	6
4	5,6,7,8,9	5
5	7,8,9,	3
6	ϕ	0
		Total = 29

$n(E) = 29$

Required probability $= \frac{29}{72}$

96. (c) $n = 6$

P = Probability of a question marked with correct answer $= 1/2$

$$q = \frac{1}{2}$$

$$P(X \geq 4) = {}^6C_4\left(\frac{1}{2}\right)^6 + {}^6C_5\left(\frac{1}{2}\right)^6 + {}^6C_6\left(\frac{1}{2}\right)^6$$

$$= \frac{1}{64}(15 + 6 + 1) = \frac{22}{64} = \frac{11}{32}$$

97. (c) Here, $P(X = x) = c\left(\frac{2}{3}\right)^x$

$\Sigma P(x) = 1$

$$\Sigma\left(\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2+\cdots\right) = 1$$

$$c\frac{\left(\frac{2}{3}\right)}{1-\frac{2}{3}} = 1$$

$$c(2) = 1, c=\frac{1}{2}$$

98. (a)

$$\text{Let } x = \frac{a\sec t - a\tan t + 0}{3}$$

$$3x = a(\sec t - \tan t)$$

$$\Rightarrow y = \frac{b\tan t + b\sec t + 0}{3}$$

$\Rightarrow 3y = b(\sec t + \tan t)$

$(3x)(3y) = ab(\sec^2 t - \tan^2 t)$

$9xy = ab$

99. (d) Let axes be rotated through an angle θ, then old coordinates are

$x = x\cos\theta - y\sin\theta$

$y = x\sin\theta + y\cos\theta$

$\sqrt{3}(x\cos\theta - y\sin\theta)^2$

$+(\sqrt{3}-1)(x\cos\theta - y\sin\theta)$

$(x\sin\theta + y\cos\theta) - (x\sin\theta + y\cos\theta)^2$

$= 0$

Coefficient of $xy = 0$

$\Rightarrow -2\sqrt{3}\cos\theta\cdot\sin\theta + (\sqrt{3}-1)$

$(\cos^2\theta - \sin^2\theta) - 2\sin\theta\cdot\cos\theta = 0$

$\Rightarrow -\sqrt{3}\sin 2\theta + (\sqrt{3}-1)\cos 2\theta - \sin 2\theta = 0$

$\Rightarrow (\sqrt{3}-1)\cos 2\theta - (\sqrt{3}+1)\sin 2\theta = 0$

$$\Rightarrow \tan 2\theta = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$$

$\Rightarrow 2\theta = 15^\circ \Rightarrow \theta = 7.5^\circ$

100. **(b)** Given, $\tan\theta = 3/4$

$$\cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}$$

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = \pm r$$

$$\frac{x-3}{\frac{4}{5}} = \pm 5 \text{ and } \frac{y-2}{\frac{3}{5}} = \pm 5$$

$x - 3 = \pm 4$ and $y - 2 = \pm 3$

$x = 3 \pm 4$ and $y = 2 \pm 3$

$A = (7,5)$ and $B = (-1,-1)$

101. **(c)** $a^2x^2 + 2xy + 4y^2 = 0$

$A = a^2, H = 1$ and $B = 4$

to be real and distinct

$H^2 - AB > 0$

$\Rightarrow$ (l) $-4a^2 > 0$

$\Rightarrow$ $4a^2 - 1 < 0$

$(2a-1)(2a+1) < 0$

$$\begin{array}{ccc} + & - & + \\ \hline -\frac{1}{2} & \frac{1}{2} & \end{array}$$

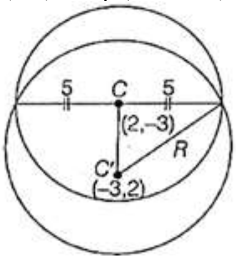
$$-\frac{1}{2} < a < \frac{1}{2}$$

102. **(a)** Given, equation of circle

$x^2 + y^2 - 4x + 6y - 12 = 0$

$C \equiv (2, -3)$

$l(CC) = \sqrt{(-3-2)^2 + (2-(-3)^2}$

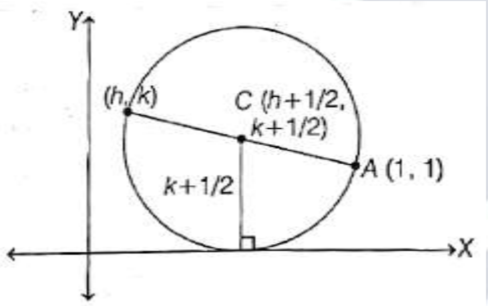


$= \sqrt{25+25} = 5\sqrt{2}$

$R^2 = (5\sqrt{2})^2 + 5^2 = 50 + 25 = 75 = 25 \times 3$

$R = 5\sqrt{3}$

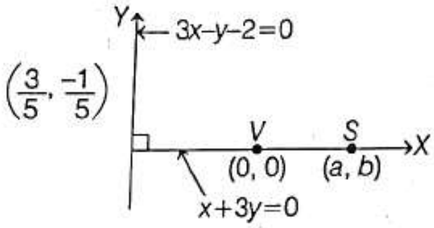
103. **(c)**



$$\begin{aligned} \frac{k+1}{2} &= \frac{1}{2}\sqrt{(h-1)^2 + (k-1)^2} \\ (k+1)^2 &= (h-1)^2 + (k-1)^2 \\ k^2 + 2k + 1 &= (h-1)^2 + k^2 - 2k + 1 \\ (h-1)^2 &= 4k \\ (x-1)^2 &= 4y \end{aligned}$$

104. **(b)** Line of intersection

$$\begin{array}{l} x^2 + y^2 - 6x - 6y + 13 = 0 \\ x^2 + y^2 - 8y + 9 = 0 \\ \underline{(-)\ (-)\ (+) - (-)} \\ -6x + 2y + 4 = 0 \\ 3x - y - 2 = 0 \end{array}$$



$$9x - 3y = 6$$
$$x + 3y = 0$$
$$10x = 6 \text{ [By adding above equation]}$$
$$x = \frac{3}{5}; y = -\frac{1}{5}$$
$$0 = \frac{a + \frac{3}{5}}{2} \Rightarrow a = -\frac{3}{5}$$
$$0 = \frac{b - \frac{1}{5}}{2} \Rightarrow b = \frac{1}{5}$$
$$S(a, b) = \left(\frac{-3}{5}, \frac{1}{5}\right)$$

105. **(b)** Equation of tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ at } P(x_1, y_1)$$
$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \Rightarrow \frac{x(-8)}{100} + \frac{y(3)}{25} = 1$$
$$-2x + 3y = 25$$

Intersection with Y-axis

$$0 + 3y = 25 \Rightarrow y = \frac{25}{3}$$
$$P(x, y) = \left(0, \frac{25}{3}\right)$$

106. **(a)** Hyperbola

$$9x^2 - 16y^2 + 72x - 32y - 16 = 0$$
$$\Rightarrow 9(x^2 + 8x) - 16(y^2 + 2y) = 16$$
$$\Rightarrow 9(x^2 + 8x + 16) - 16(y^2 + 2y + 1)$$
$$\Rightarrow = 16 + 144 - 16$$
$$\Rightarrow 9(x + 4)^2 - 16(y + 1)^2 = 144$$
$$\Rightarrow \frac{(x + 4)^2}{16} - \frac{(y + 1)^2}{9} = 1$$

Conjugate hyperbola

$$\frac{(x + 4)^2}{16} - \frac{(y + 1)^2}{9} = -1$$
$$9x^2 + 72x + 144 - 16y^2 - 32y - 16 = -144$$
$$9x^2 + 72x - 16y^2 - 32y + 288 - 16 = 0$$
$$9x^2 - 16y^2 + 72x - 32y + 272 = 0$$

107. **(a)** Given,

$l + m + n = 0$ .... (i)

$lm = 0$ ....(ii)

$\Rightarrow$ Either $l = 0$ or $m = 0$

When

$$l = 0$$
$$m + n = 0 \quad \text{[from Eq. (i)]}$$
$$\Rightarrow m = -n$$
$$0^2 + (-n)^2 + n^2 = 1$$
$$[l^2 + m^2 + n^2 = 1]$$

$\Rightarrow n = \frac{1}{\sqrt{2}}$ (taking (+) positive sign)

$$\langle l, m, n \rangle = \left\langle 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

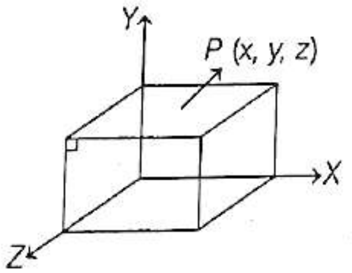
When $m = 0$

$$l + n = 0 \Rightarrow l = -n$$
$$(-n)^2 + 0^2 + n^2 = 1 \Rightarrow n = \frac{1}{\sqrt{2}}$$
$$< l, m, n > = < -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$$

Let θ be the angle between these lines.

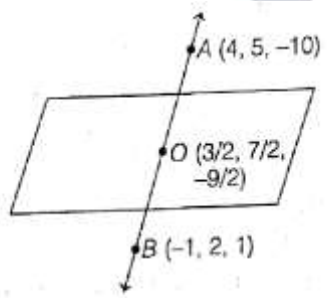
$$\cos\theta = 0 \times \left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \theta = 0 + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$
$$\Rightarrow \theta = \frac{\pi}{3}$$

108. (a)



Perpendicular distance on X-axis
$= y^2 + z^2$
Perpendicular distance on Y-axis
$= x^2 + z^2$
Perpendicular distance on Z-axis
$= x^2 + y^2$
$2(x^2 + y^2 + z^2) = k(x^2 + y^2 + z^2)$
Hence, $2 = k$

109. (b)



$AB = n = b - a = -5\hat{i} - 3\hat{j} + 11\hat{k}$
Plane is, $\bar{\mathbf{r}} \cdot \bar{\mathbf{n}} = \bar{\mathbf{a}} \cdot \bar{\mathbf{n}}$

$$r \cdot (-5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 11\hat{\mathbf{k}}) = -\frac{15}{2} - \frac{21}{2} - \frac{99}{2}$$
$$\Rightarrow \quad -5x - 3y + 11z = -\frac{135}{2}$$
$$\Rightarrow \quad -10x - 6y + 22z + 135 = 0$$
$$\Rightarrow \quad 10x + 6y - 22z - 135 = 0$$

110. (b)

$$\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$$
$$=\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)(\sqrt{x}+1)}{(2x^2+x-3)(\sqrt{x}+1)}$$
$$=\lim_{x\to1}\frac{(2x-3)(x-1)}{(2x+3)(x-1)(\sqrt{x}+1)}$$
$$=\frac{(2-3)}{(2+3)}\times\frac{1}{(\sqrt{1}+1)}=-\frac{1}{5}\times\frac{1}{2}=\frac{-1}{10}$$

111. **(a)** $\mathbf{a}\times\hat{\imath}=\hat{\jmath}+\hat{k}$

$(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})\times\hat{\mathbf{i}}=\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\Rightarrow x\hat{\mathbf{i}}\times\hat{\mathbf{i}}+y\cdot\hat{\mathbf{j}}\times\hat{\mathbf{i}}+z\hat{\mathbf{k}}\times\hat{\mathbf{i}}=\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\Rightarrow\ 0-\hat{y}\times\hat{\mathbf{i}}+z(\hat{\mathbf{k}}\times\hat{\mathbf{i}})=\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\Rightarrow\ -y\hat{\mathbf{k}}+z\hat{\mathbf{j}}=\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\Rightarrow\ y\hat{\mathbf{j}}-y\hat{\mathbf{k}}=\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\Rightarrow\ z=1, y=-1$

$\mathbf{a}\cdot\hat{\mathbf{i}}=1$

$(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})\cdot\hat{\mathbf{i}}=1$

$x=1$

and $\mathbf{a}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Equation of line

$\mathbf{r}=$ (Passing through point) $+\lambda$

(Parallel vector)

$=(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})+t(\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})$

$=(l+t)\hat{\mathbf{i}}+(l-t)\hat{\mathbf{j}}+(l+t)\hat{\mathbf{k}}$

112. **(b)** $x=2\sin t, y=2\cos t$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-2\sin t}{2\cos t}=\text{ Slope of tangent}$$

$$\text{Slope of normal }=\frac{\cos t}{\sin t}$$

$$m_s=\frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}=0$$

Equation of normal

$$(y-2\cos t)\quad=0(x-2\sin t)$$
$$\left(y-2\times\cos\frac{\pi}{2}\right)\quad=0$$
$$\Rightarrow y\quad=0$$

113. **(d)** $I=\int\left(\frac{\sin^8x-\cos^8x}{1-2\sin^2x\cos^2x}\right)dx$

$$I=\int\frac{(\sin^4x-\cos^4x)(\sin^4x+\cos^4x)}{(1-2\sin^2x\cos^2x)}dx$$

$(\sin^2x-\cos^2x)(\sin^2x+\cos^2x)$

$$I=\int\frac{\{(\sin^2x+\cos^2x)^2-2\sin^2x\cos^2x\}}{1-2\sin^2x\cos^2x}dx$$

$$I=\int\frac{(-\cos 2x)(1)(1-2\sin^2x\cos^2x)}{1-2\sin^2x\cos^2x}dx$$

$$I=-\frac{\sin(2x)}{2}+C$$

114. **(b)**

$$\int_{1/2}^{2} |\log_{10} x| dx = \int_{1/2}^{1} (-\log_{10} x) dx$$
$$+ \int_{1}^{2} \log_{10} x dx$$
$$= -\int_{1/2}^{1} (\log_e x \cdot \log_{10} e) dx$$
$$+ \int_{1}^{2} (\log_e x \cdot \log_{10} e) dx$$
$$= -\log_{10} e \int_{1/2}^{1} (\log x) dx + \log_{10} e \int_{1}^{2} (\log x) dx$$
$$= -\log_{10} e [x(\log x - 1)]_{\frac{1}{2}}^{1}$$
$$+\log_{10} e [x(\log x - 1)]_{1}^{2} \text{ (using by parts)}$$
$$= -\log_{10} e \left[1(0-1) - \frac{1}{2}\left(\log\frac{1}{2} - 1\right)\right]$$
$$+\log_{10} e [2(\log 2 - 1) - 1(0-1)]$$
$$= \log_{10} e \left[\left(1 + \frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\right) + (2\log 2 - 1)\right]$$
$$= (\log_{10} e)\left(-\frac{1}{2}\log 2 + 2\log 2 - \frac{1}{2}\right)$$
$$= \log_{10} e \left(\frac{3}{2}\log 2 - \frac{1}{2}\right)$$
$$= (\log_{10} e)\left(\frac{3}{2}\log_e 2 - \frac{1}{2}\log_e e\right)$$
$$= \frac{1}{2}\log_{10} e (\log_e 8 - \log_e e)$$
$$= \frac{1}{2}(\log_{10} e)\left(\log_e\left(\frac{8}{e}\right)\right) = \frac{1}{2}\log_{10}\left(\frac{8}{e}\right)$$

115. **(b)** Equation of tangent

$$y = mx + \frac{a}{m}$$
$$-2 = -m + \frac{1}{m}$$
$$-2m = -m^2 + 1$$
$$m^2 - 2m + 1 = 2 \Rightarrow (m-1)^2 = 2$$
$$(m-1) = \pm\sqrt{2}$$
$$m_1 = 1 + \sqrt{2} \text{ and } m_2 = 1 - \sqrt{2}$$
$$m_1 - m_2 = 2\sqrt{2}$$
$$1 + m_1 m_2 = 1 + 1 - 2 = 0$$
$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
$\tan\theta =$ not defined

$\theta = \pi/2$

116. **(b)**

$$f(x) = x^2 - 4x + 5$$
$$\Delta = (-4)^2 - 4 \times 5$$
$= $ Negative

f(x) will be always positive.

$$x_{min} = -\frac{b}{2a} = \frac{4}{2} = 2$$

f(x) is increasing in $[2, \infty)$

$f(2) = 2^2 - 4 \times 2 + 5 = 1 = \min$

$f(\infty) = \infty = \max$

Range $\in [1, \infty)$

117. **(d)**

$$A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

$|A^2| = 25$

and $|A|^2 = 25$

$$|A| = \pm 5$$
$$5 \times \alpha \times 5 = \pm 5$$
$$\alpha = \pm\frac{1}{5}$$
$$|\alpha| = \frac{1}{5}$$

118. **(c)** Let $z = \frac{1+i\cos\theta}{1-2i\cos\theta}$

$$z = \frac{(1 + i\cos\theta)(1 + 2i\cos\theta)}{(1)^2 - 4i^2\cos^2\theta}$$

z is purely real.

$\Rightarrow \text{Im}(z) = 0$

$$\frac{\cos\theta + 2\cos\theta}{1 + 4\cos^2\theta} = 0$$

$$= \frac{\pi}{2}$$

$\cos^3\theta + \sin^2\theta + \cos\theta + 1$

$= 0 + (1)^2 + 0 + 1 = 2$

119. **(b)** $T_{10} = (\cos\theta + i\sin\theta)^9$

Using De-Moivre theorem,

$\cos(9\theta) + i\sin(9\theta)$

$$= \cos\left(\frac{9\pi}{6}\right) + i\sin\left(\frac{9\pi}{6}\right)$$
$$= \cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)$$
$$= 0 + i(-1)$$
$$= -i$$

120. **(b)** Given, $(x - 2)$ is a common factor of equation $(x^2 + ax + b)$ and $(x^2 + cx + d)$.

$x^2 + ax + b = (x - 2)(x - \alpha)$

$x^2 + ax + b = x^2 - (2 + \alpha)x + 2\alpha$

$2\alpha = b;\ \alpha = \frac{b}{2}$

and $-2 - \alpha = u$

$-2 - \frac{b}{2} = a$

$$x^2+cx+d = (x-2)(x-\beta)$$
$$x^2+cx+d = x^2-(2+\beta)x+2\beta$$
$$d = 2\beta;$$
$$\frac{d}{2} = \beta$$
$$-2-\beta = c$$
$$\frac{b-d}{c-a} = \frac{2\alpha-2\beta}{-2-\beta+2+\alpha}$$
$$= \frac{2(\alpha-\beta)}{(\alpha-\beta)} = 2$$

121. **(d)** $x^3-14x^2+56x-64=0$

Using Hit and Trial, we can say $x=2$ is one of the root

$(x-2)(x^2-12x+32)=0$

$x=2, x^2-12x+32=0$

$\Rightarrow$ $(x-4)(x-8)=0$

$\Rightarrow$ $x=4$ or $x=8$

$\Rightarrow$ 2,4,8 are roots.

$\Rightarrow$ Roots are in GP.

122. **(d)** $\overline{Th}\,\overline{H}\,\overline{T}\,\overline{U}$

Thousand place can be filled in 5 ways (1, 2, 3, 4, 5)

Hundred place can be filled in 5 ways (since repetition is not allowed)

Tens place can be filled in 4 ways.

Unit place can be filled in 3 ways

$5\times5\times4\times3=300$

123. **(c)** $(\sqrt[4]{5}+\sqrt[5]{4})^{100} = \Sigma^{100}C_r 5^{\frac{100-r}{4}} 4^{\frac{r}{5}}$

For terms to be rational, $\frac{r}{5}$ and $\frac{100-r}{4}$ must be integer.

Possible values of r, so that $\frac{r}{5}$ is integer are 0,5,10,15,20, ... ,100.

Out of these values, 0,20,40,60,80,100 makes $\frac{100-r}{4}$ integer, so there are 6 rational terms.

124. **(a)**

125. **(d)** Given, $r_1=2r_2=3r_3$

Let $r_1=2r_2=3r_3=k$

$r_1=k$

$$\Rightarrow \frac{\Delta}{s-a}=k \Rightarrow s-a=\frac{\Delta}{k}$$

$$a=s-\frac{\Delta}{k} \quad \ldots..(i)$$

$2r_2=k$

$$\Rightarrow \frac{2\Delta}{s-b}=k \Rightarrow s-b=\frac{2\Delta}{k}$$

$$\Rightarrow b = s-\frac{2\Delta}{k} \quad \ldots\ldots(ii)$$

$3r_3 = k$

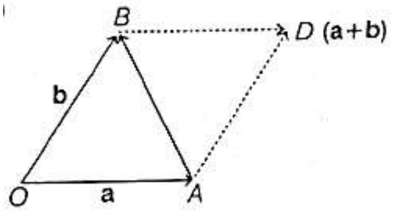
$$\frac{3\Delta}{s-c} = k \Rightarrow s-c=\frac{3\Delta}{k}$$

$$\Rightarrow c=s-\frac{3\Delta}{k} \quad \ldots\ldots(iii)$$

Adding Eqs. (i), (ii) and (iii), we get

$$a+b+c = 3s-\frac{6\Delta}{k}$$
$$\Rightarrow 2s = 3s-\frac{6\Delta}{k} \Rightarrow s=\frac{6\Delta}{k}$$
$$a = \frac{5\Delta}{k}, b=\frac{4\Delta}{k} \text{ and } c=\frac{3\Delta}{k}$$
$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a} = \frac{5}{4}+\frac{4}{3}+\frac{3}{5}$$
$$= \frac{75+80+36}{60}=\frac{191}{60}$$

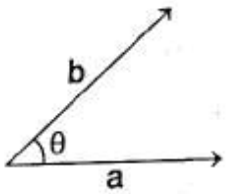
126. (c)



Mid-point of AB is $\frac{\mathbf{a}}{2}+\frac{\mathbf{b}}{2}$ and position vector of C is $\frac{\mathbf{a}}{2}+\frac{\mathbf{b}}{3} \Rightarrow$ C lies inside $\triangle$ OAB.

127. (d)

$$|\mathbf{a}-\mathbf{b}|^2+|\mathbf{a}+2\mathbf{b}|^2=20 \text{ [given]}$$
$$\Rightarrow |\mathbf{a}|^2+|\mathbf{b}|^2-2\mathbf{a}\cdot\mathbf{b}+|\mathbf{a}|^2+4|\mathbf{b}|^2 +4\mathbf{a}\cdot\mathbf{b}=20$$
$$\Rightarrow 2|\mathbf{a}|^2+5|\mathbf{b}|^2+2\mathbf{a}\cdot\mathbf{b}=20$$



$$|a|=1, |b| = 2$$
$$a\cdot b = |a||b|\cos\theta$$
$$= 2\cos\theta$$
$$2+20+4\cos\theta = 20$$
$$\Rightarrow \cos\theta = -\frac{1}{2}$$
$$\Rightarrow \theta=\frac{2\pi}{3}$$

128. (c) In a non-leap year,

Sample space = { Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

$n(S) = 7$

A = { Sunday, Tuesday, Thursday }

$n(A) = 3 \Rightarrow P(A) = 3/7$

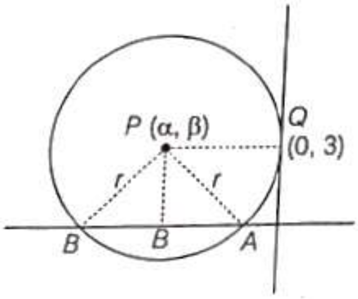
129. (c) Equation of *AB*

$$y+1=\frac{-3+1}{5-2}(x-2)$$

$\Rightarrow\ y+1=-\frac{2}{3}(x-2)$
$\Rightarrow\ 3y+3=-2x+4$
$\Rightarrow\ 2x+3y=1$
$(a,4)$ and $(-2,b)$ lies on AB
$\Rightarrow 2a+12=1$ and $-4+3b=1$
$a=-\frac{11}{2}, b=\frac{5}{3}$

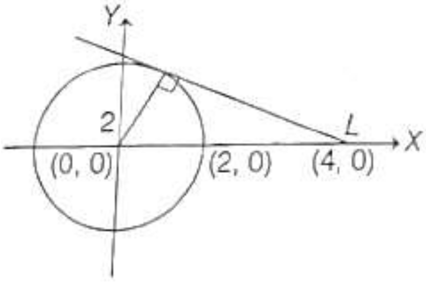
(a,b) i.e. $\left(-\frac{11}{2},\frac{5}{3}\right)$ lies on $2x+6y+1=0$

130. (c)



$\alpha<0 \Rightarrow$ centre of circle lies on 2nd quadrant
circle is touching Y-axis
$\Rightarrow\ r=|\alpha|=-\alpha$ .....(i)
and $\beta=3$
Also, $AB=2$ (given)
In $\triangle$ PQA,
$PQ = 3, PA = r, AQ = 1$
$PQ^2+AQ^2 = PA^2$
$9+1 = r^2 \Rightarrow r=\sqrt{10}$
From Eq. (i), we get $\alpha=-\sqrt{10}$
Centre $\equiv(-\sqrt{10},3)$

131. (a)



Given equation of circle
$x^2+y^2=4$
Let the slope of tangent be m
$y = m(x-4)$
$\Rightarrow y-mx+4m = 0$
Distance of L from origin will be 2 .
$\frac{|4m|}{\sqrt{1+m^2}}=2$

$\Rightarrow \quad 16m^2 = 4m^2 + 4$

$\Rightarrow \quad 12m^2 = 4$

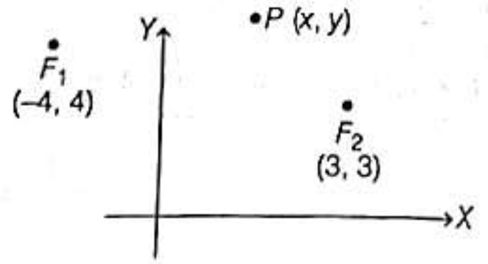
$\Rightarrow \quad m^2 = \frac{1}{3}$

$\Rightarrow \quad m = \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}$

$\sqrt{3}y = \pm(x - 4)$

132. **(a)**

133. **(a)**



$PF_1 + PF_2 = 2a$, where a is length of semi-major axis.

Ellipse passes through (0,0)

$$\Rightarrow \sqrt{(-4)^2 + 4^2} + \sqrt{3^2 + 3^2} = 2a$$

$$\Rightarrow 4\sqrt{2} + 3\sqrt{2} = 2a \Rightarrow a = \frac{7\sqrt{2}}{2} = \frac{7}{\sqrt{2}}$$

$$\Rightarrow F_1F_2 = 2ae$$

$$\Rightarrow \sqrt{(-4-3)^2 + (4-3)^2} = 2 \times \frac{7}{\sqrt{2}} \times e$$

$$\Rightarrow \sqrt{50} = \sqrt{2} \times 7e \Rightarrow e = \frac{5}{7}$$

134. **(b)** $\lim_{x\to\infty} x\left(a^{\frac{1}{x}} + b^{\frac{1}{x}} + c^{\frac{1}{x}} - 3k^{\frac{1}{x}}\right) = 0$

Let $x = \frac{1}{y}$

$$\Rightarrow y \to 0$$

$$\lim_{y\to 0} \frac{a^y + b^y + c^y - 3k^y}{y} = 0$$

As, we know, $\lim_{y\to 0} \frac{a^y - 1}{y} = \log_e a$

$$\Rightarrow \lim_{y\to 0}\left[\frac{a^y - 1}{y} + \frac{b^y - 1}{y} + \frac{c^y - 1}{y} - 3\frac{(k^y - 1)}{y}\right] = 0$$

$$\Rightarrow \log_e a + \log_e b + \log_e c - 3\log_e k = 0$$

$$\Rightarrow \log_e \frac{abc}{k^3} = 0$$

$$\Rightarrow \frac{abc}{k^3} = 1$$

$$\Rightarrow k = (abc)^{\frac{1}{3}}$$

135. **(c)** Given, $\tan y = \cot\left(\frac{\pi}{4} - x\right)$

Differentiating on both sides w.r.t. x, we get

$$\frac{d(\tan y)}{dx} = \frac{d\left(\cot\left(\frac{\pi}{4} - x\right)\right)}{dx}$$

$$\Rightarrow \sec^2 y \frac{dy}{dx} = -\mathrm{cosec}^2\left(\frac{\pi}{4} - x\right) \cdot (-1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\operatorname{cosec}^2\left(\frac{\pi}{4}-x\right)}{\sec^2 y}$$

$$= \frac{\operatorname{cosec}^2\left(\frac{\pi}{4}-x\right)}{1+\cot^2\left(\frac{\pi}{4}-x\right)}$$

$$\left[\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta\right]$$

$$\Rightarrow \cot\left(\frac{\pi}{4}-x\right) = \tan\left(\frac{\pi}{4}+x\right)$$

$$\frac{dy}{dx} = \frac{\operatorname{cosec}^2\left(\frac{\pi}{4}-x\right)}{1+\tan^2\left(\frac{\pi}{4}+x\right)}$$

136. **(d)** Given, $x = 3\sqrt{2}\cos^3\theta$

and $y = 4\tan^2\theta$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d(4\tan^2\theta)}{d\theta}}{\frac{d(3\sqrt{2}\cos^3\theta)}{d\theta}}$$

$$= \frac{4\cdot 2\tan\theta\sec^2\theta}{3\sqrt{2}\cdot 3\cos^2\theta(-\sin\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{-8\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}{9\sqrt{2}\cos^2\frac{\pi}{4}\cdot\sin\frac{\pi}{4}}$$

$$= \frac{-8\times 1\times 2}{9\sqrt{2}\times\frac{1}{2}\times\frac{1}{\sqrt{2}}} = -\frac{32}{9}.$$

137. **(c)**

$$\int \frac{x^2(\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$$

$$= \int \frac{x^2\sec^2 x + x^2\tan x}{(x\tan x+1)^2}dx$$

$$= \int \left[\frac{-x}{x\tan x+1} + \frac{x(x\sec^2 x-1)}{(x\tan x+1)^2} + \frac{2x^2\tan x+2x}{(x\tan x+1)^2}\right]dx$$

$$I = \underbrace{\int \left[\frac{-x}{x\tan x+1} + x\frac{x\sec^2 x-1}{(x\tan x+1)^2}\right]dx}_{I_1} + \underbrace{\int \frac{2x^2\tan x+2x}{(x\tan x+1)^2}dx}_{I_2}$$

$$I = I_1 + I_2$$

$$I_1 = \int \left[\frac{-x}{x\tan x+1} + x\frac{x\sec^2 x-1}{(x\tan x+1)^2}\right]dx$$

$$\int (f(x)+xf'(x))dx = xf(x)+C$$

$$I_1 = \frac{-x^2}{x\tan x+1}$$

$$I_2 = \int \frac{2x^2\tan x + 2x}{(x\tan x + 1)^2}dx = \int \frac{2x}{x\tan x + 1}dx$$
$$= \int \frac{2x\cos x}{x\sin x + \cos x}dx$$

Let $x\sin x + \cos x = t$

$x\cos x dx = dt$

$$I_2 = \int \frac{2dt}{t} = 2\ln t$$
$$I_2 = 2\ln|x\sin x + \cos x|$$
$$I = \frac{-x^2}{x\tan x + 1} + 2\ln|x\sin x + \cos x| + C$$

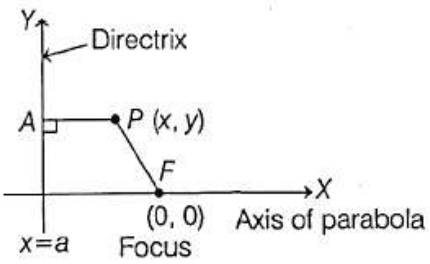
138. (b)

$$I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}dx \quad \ldots\ldots\text{(i)}$$
$$I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}dx$$
$$= \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}dx \quad \ldots\ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}dx$$
$$2I = \int_0^{\pi/2} \frac{1}{\sqrt{2}\left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right)}dx$$
$$2I = \int_0^{\pi/2} \frac{1}{\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)}dx$$
$$2I = \int_0^{\pi/2} \frac{\sec\left(x - \frac{\pi}{4}\right)}{\sqrt{2}}dx$$
$$2I = \frac{\ln\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|_0^{\frac{\pi}{2}}}{\sqrt{2}}$$
$$I = \frac{1}{2\sqrt{2}}[\ln(\sqrt{2} + 1) - \ln(\sqrt{2} - 1)]$$
$$= \frac{1}{2\sqrt{2}}\ln\left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}\right)$$
$$I = \frac{1}{\sqrt{2}}\ln(\sqrt{2} + 1)$$

139. (c)



Let a be any arbitrary constant

$$PF = PA$$
$$\Rightarrow \sqrt{x^2+y^2} = (x-a)$$
$$\Rightarrow x^2+y^2 = x^2+a^2-2ax$$
$$y^2 = a^2-2ax \quad \ldots\ldots(i)$$
$$2y\frac{dy}{dx} = -2a$$
$$a = -y\frac{dy}{dx} \quad \ldots.(ii)$$

On putting the value of a (ii) in Eq. (i), we get

$$y^2 = y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$
$$m = 1, n = 2$$
$$mn - m + n = 3$$

140. (c)

$$f(x) = -|x|$$
$$f(-x) = -|-x|$$
$$f(f(x)) = -|f(x)| = -|-|x|| = f(x)$$
$$= -|x| f(f(f(x))) = f(x)$$
$$f(f(f(x))) = -|ff(x)| = -|-|x||$$
$$= f(x)$$

So, $f\circ f\circ f(x) + f\circ f\circ f(-x) = 2f(x)$

141. (b) $P = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \mathrm{Tr}(P) = a+e+i$

$$P^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \mathrm{Tr}(P^T) = a+e+i$$
$$\Rightarrow \mathrm{Tr}(P) = \mathrm{Tr}(P^T)$$
$$\mathrm{Tr}(P-P^T) + \mathrm{Tr}(P+P^T) + \frac{\mathrm{Tr}(P)}{\mathrm{Tr}(P^T)} + \mathrm{Tr}(P)$$
$$\times \mathrm{Tr}(P^T) = 0$$
$$\Rightarrow \mathrm{Tr}(P) - \mathrm{Tr}(P^T) + \mathrm{Tr}(P) + \mathrm{Tr}(P^T) + \frac{\mathrm{Tr}(P)}{\mathrm{Tr}(P)}$$
$$+\mathrm{Tr}(P)\cdot\mathrm{Tr}(P) = 0$$
$$\Rightarrow (\mathrm{Tr}(P))^2 + 2\mathrm{Tr}(P) + 1 = 0$$
$$(\mathrm{Tr}(P)+1)^2 = 0$$
$$\mathrm{Tr}(P) = -1$$

142. (b)

$$z = (\alpha+i\beta)(2+7i)$$
$$= 2\alpha + 7\alpha i + 2\beta i - 7\beta$$
$$= (2\alpha - 7\beta) + (7\alpha + 2\beta)i$$

z is purely imaginary.

$$\Rightarrow 2\alpha - 7\beta = 0 \Rightarrow \alpha = \frac{7}{2}\beta$$
$$|z|^2 = (7\alpha+2\beta)^2$$
$$= \left(\frac{49}{2}\beta + 2\beta\right)^2 = \left(\frac{53\beta}{2}\right)^2$$

Minimum value of $|z|$ will be at $\beta = 2$

as α and β both are integer.

$|z|^2 = 53^2 = 2809$

143. **(b)** Given,

$e^{4t} - 10e^{3t} + 29e^{2t} - 20e^{t} + 4 = 0$

Let $e^{t} = x$ ….. (i)

$\Rightarrow x^4 - 10x^3 + 29x^2 - 20x + 4 = 0$

$$\Rightarrow x^2 - 10x + 29 - \frac{20}{x} + \frac{4}{x^2} = 0$$

$$\Rightarrow x^2 + \left(\frac{2}{x}\right)^2 - 10\left(x + \frac{2}{x}\right) + 29 = 0$$

$$\Rightarrow \left(x + \frac{2}{x}\right)^2 - 10\left(x + \frac{2}{x}\right) + 25 = 0$$

Let $x + \frac{2}{x} = P$ ….. (iii)

$P^2 - 10P + 25 = 0$

$(p - 5)^2 = 0$

$\Rightarrow P = 5$

Putting the value of P in Eq. (ii), we get

$$x + \frac{2}{x} = 5$$

$$\Rightarrow x^2 - 5x + 2 = 0$$

$$\Rightarrow x = \frac{5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{5 + \sqrt{17}}{2}, \frac{5 - \sqrt{17}}{2}$$

$$e^{t_1} = \frac{5 + \sqrt{17}}{2}$$

$$e^{t_2} = \frac{5 - \sqrt{17}}{2}$$

$$e^{t_1} \cdot e^{t_2} = \frac{25 - 17}{4} = \frac{8}{4} = 2$$

$$e^{t_1 + t_2} = 2$$

$$t_1 + t_2 = \log_e 2$$

There are 4 roots and 2 of them are repeated.

$\Rightarrow t_1 + t_2 + t_3 + t_4 = 2\log_e 2$

144. **(b)** For a number to be divisible by 4 , last 2 digits must be divisible by 4.

Possible cases 12, 16, 24, 32, 36, 52, 56, 64, 72, 76

$4 \times 5 \times 10 = 200$

145. **(a)** Here, $(1 + x)^{101}(1 - x + x^2)^{100}$

$= (1 + x)(1 + x)^{100}(1 - x + x^2)^{100}$

$= (1 + x)(1 + x^3)^{100}$

$= (1 + x^3)^{100} + x(1 + x^3)^{100}$

$= \Sigma^{100}C_r x^{3r} + \Sigma^{100}C_r x^{3r+1}$

$3r = 50; 3r + 1 = 50$

$r = \frac{50}{3}; r = \frac{49}{3}$

$\Rightarrow$ Coefficient of $x^{50} = 0$

146. **(b)** Given, $\sin\alpha + \cos\alpha = m$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = m^2$

$\sin\alpha\cos\alpha = \dfrac{m^2-1}{2}$

$\sin^6\alpha + \cos^6\alpha = (\sin^2\alpha)^3 + (\cos^2\alpha)^3$

$= (\sin^2\alpha + \cos^2\alpha)(\sin^4\alpha + \cos^4\alpha$
$-\sin^2\alpha\cos^2\alpha)$

$= (\sin^2\alpha)^2 + (\cos^2\alpha)^2 + 2\sin^2\alpha\cos^2\alpha$
$- 2\sin^2\alpha\cos^2\alpha - \sin^2\alpha\cos^2\alpha$

$= (\sin^2\alpha + \cos^2\alpha)^2 - 3\sin^2\alpha\cos^2\alpha$

$= 1 - \dfrac{3(m^2-1)^2}{4}$

$= \dfrac{4-3(m^2-1)^2}{4}$

147. **(a)**

$I = \int \sin(101x)(\sin x)^{99}dx$

$= \int \sin(100x + x)(\sin x)^{99}dx$

$= \int \sin(100x)\cos x(\sin x)^{99}dx +$
$\int \cos(100x)\sin^{100}xdx$

$= \int \cos x(\sin x)^{99}\sin 100xdx + I_1$

$I_1 = \int \cos(100x)\sin^{100}xdx$

$= \sin^{100}x\int \cos 100xdx -$
$\int \left[\left(\int \cos 100xdx\right)\left(\dfrac{d\sin^{100}x}{dx}\right)\right]dx$

$= \dfrac{\sin^{100}x\sin 100x}{100} -$
$\int \left(\dfrac{\sin 100x}{100}\cdot 100(\sin x)^{99}\cos x\right)dx$

$I = \int \cos x(\sin x)^{99}\sin 100xdx$
$+ \dfrac{\sin^{100}x\sin 100x}{100} - \int \cos x(\sin x)^{99}$

$\sin 100x \times dx + C$

$I = \int \dfrac{\sin^{100}x\sin(100x)}{100} + C$

$\lambda = 100, \mu = 100$

$\dfrac{\lambda}{\mu} = 1$

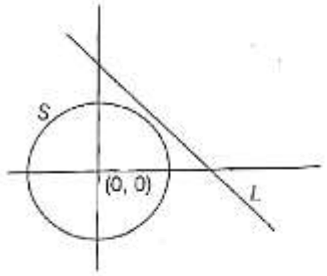
148. **(b)** Given, $P(A) + P(B) = 2P(A\cap B)$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow P(A\cup B) = P(A\cap B)$

This is possible only, if $P(A) = P(B)$

149. **(d)**



To find image of circle in the line find the image of its centre and radius will be equal to radius of given circle.

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-\frac{2(ax_1+by_1)+c}{a^2+b^2}$$

$$\frac{x}{1}=\frac{y}{1}=-2\frac{(-1)}{1+1}$$

$$x=1, y=1$$

Centre of image circle is $(1,1)$ and

radius $=1$

$$(x-1)^2+(y-1)^2=1^2$$

$$\Rightarrow x^2+y^2-2x-2y+1=0$$

150. **(c)** Given, points are $P(-3,5), Q(1,3)$ and Q is inverse of P, then the line perpendicular to PQ will.

Slope of PQ, $m_1=\frac{5-3}{-3-1}=-\frac{1}{2}$

$$m_1m_2=-1\Rightarrow m_2=2$$

$$y-3=2(x-1)$$

$$\Rightarrow 2x-y+1=0$$

151. **(b)** $f(x)=\frac{1-x^2}{1+x^2}, g(x)=\frac{2x}{1+x^2}$

$$\left.\frac{df(x)}{dg(x)}\right|_{x=2}=\frac{\frac{df(x)}{dx}}{\frac{dg(x)}{dx}}$$

$$=\frac{\frac{d\left(\frac{(1-x^2)}{(1+x^2)}\right)}{dx}}{\frac{d\left(\frac{2x}{1+x^2}\right)}{dx}}$$

$$=\frac{\frac{-2x(1+x^2)-(1-x^2)2x}{(1+x^2)^2}}{\frac{2(1+x^2)-2x(2x)}{(1+x^2)^2}}$$

$$=\frac{-2x-2x^3-2x+2x^3}{2+2x^2-4x^2}$$

$$=\frac{-4x}{2-2x^2}$$

$$\left.\frac{df(x)}{dg(x)}\right|_{x=2}=\frac{-8}{-6}=\frac{4}{3}$$

152. **(d)** Here, $f(x)=\frac{x}{5}+\frac{5}{x}$

$\Rightarrow \quad f'(x) = \frac{1}{5} - \frac{5}{x^2} = 0$

$\Rightarrow \quad \frac{5}{x^2} = \frac{1}{5}$

$\Rightarrow \quad x = \pm 5$

$f'''(x) = \frac{10}{x^3}$

$\Rightarrow \quad f''(1) = \frac{10}{125} > 0$

$\Rightarrow \quad x = 5$ is relative minimum

$\Rightarrow f''(-5 \quad = \frac{-10}{125} < 0$

$\Rightarrow x \quad = -5$ relative maximum

$\alpha \quad = -5$

$\sqrt{\alpha^2 + 2\alpha - 6} \quad = \sqrt{25 - 10 - 6} = \sqrt{9} = 3$

153. **(b)**

$\int e^x(\sin^2 2x - 8\cos 4x)dx$

$= \int e^x\left(\frac{1-\cos 4x}{2} + 2\sin 4x\right.$

$\left.-2\sin 4x - 8\cos 4x\right)dx$

$= \int e^x\left(\frac{1-\cos 4x}{2} + 2\sin 4x\right)dx$

$-2\int e^x(\sin 4x + 4\cos 4x)dx$

$= e^x\left(\frac{1-\cos 4x}{2}\right) - 2e^x\sin 4x + C$

$= e^x\left(\frac{1-\cos 4x}{2} - 2\sin 4x\right) + C$

$\Rightarrow f(x) = \frac{1-\cos 4x}{2} - 2\sin 4x$

$f\left(\frac{\pi}{4}\right) = \frac{1+1}{2} - 0 = 1$

154. **(d)** $I = \int_0^{2\pi}[|\sin x| + |\cos x|]dx$

Period of $|\sin x| + |\cos x|$ is $\frac{\pi}{2}$

$\Rightarrow \quad I = 4\int_0^{\frac{\pi}{2}}[|\sin x| + |\cos x|]dx$

For $x \in \left(0, \frac{\pi}{2}\right)$, $\sin x > 0$ and $\cos x > 0$

$\Rightarrow \quad I = 4\int_0^{\frac{\pi}{2}}[\sin x + \cos x]dx$

$\Rightarrow \quad 1 \le \sin x + \cos x \le \sqrt{2}$

$\Rightarrow \quad [\sin x + \cos x] = 1$

$I = 4\int_0^{\frac{\pi}{2}} dx = 4x|_0^{\frac{\pi}{2}} = 2\pi$

155. **(c)**

$\frac{dy}{dx} + yf'(x) - f(x)f'(x) = 0$

$\Rightarrow \frac{dy}{dx} + y\frac{df(x)}{dx} - f(x)\frac{df(x)}{dx} = 0$

$\Rightarrow \dfrac{dy}{df(x)} + y = f(x)$

Integrating factor $= e^{\int df(x)} = e^{f(x)}$

$e^{f(x)}y = \int f(x)e^{f(x)}df(x)$

$e^{f(x)}y = f(x)\int e^{f(x)}df(x)$

$-\int \left(\int e^{f(x)}dx \cdot \dfrac{df(x)}{df(x)}\right) df(x)$

$e^{f(x)}y = f(x)e^{f(x)} - \int e^{f(x)}df(x) + c$

$e^{f(x)}y = f(x)e^{f(x)} - e^{f(x)} + c$

$y = f(x) - 1 + ce^{-f(x)}$

156. **(a)**

$x + ky + 3z = -2$

$4x + 3y + kz = 14$

$2x + y + 2z = 3$

$\begin{vmatrix} 1 & k & 3 \\ 4 & 3 & k \\ 2 & 1 & 2 \end{vmatrix} \neq 0$

$\Rightarrow 1(6 - k) - 4(2k - 3) + 2(k^2 - 9) \neq 0$

$\Rightarrow 2k^2 - 9k \neq 0$

$k(2k - 9) \neq 0$

$k \neq 0$ and $k \neq 9/2$

157. **(a)** $y = e^{a+bx^2}$

At $x = 1, y = 1$

$1 = e^{a+b} \Rightarrow a + b = 0$

$\left.\dfrac{dy}{dx}\right|_{(1,1)} = e^{a+bx^2}\, 2bx$

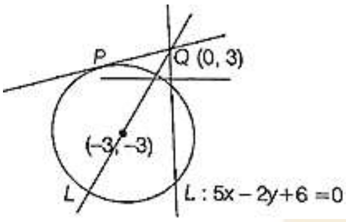
$\Rightarrow e^{a+b} \cdot 2b = -2$

$\Rightarrow b = -1$

$\Rightarrow a = 1$

$2a - 3b = 5$

158. **(a)**



L: $5x - 2y + 6 = 0$

Length of tangent from an external point is $\sqrt{S_1}$

$\Rightarrow$ PQ $= \sqrt{0 + 9 + 0 + 18 - 2}$

$= \sqrt{25} = 5$

159. **(d)**

Given, $I_n = \int \frac{\sin nx}{\sin x} dx$

$$I_{n+1} - I_{n-1} = \int \left(\frac{\sin(n+1)x}{\sin x} - \frac{\sin(n-1)x}{\sin x}\right) dx$$

$$= \int \left(\frac{\sin(n+1)x - \sin(n-1)x}{\sin x}\right) dx$$

$$= \int \frac{2\sin x \cos nx}{\sin x} dx$$

$$= \int 2\cos nx dx = \frac{2}{n} \sin nx$$

160. **(d)** Given, $f(x) \cdot f(-x) = 9$

$$f(-x) = \frac{9}{f(x)} \quad \ldots \ldots (i)$$

$$I = \int_{-23}^{23} \frac{dx}{3 + f(x)} \quad \ldots \ldots (ii)$$

$$I = \int_{-23}^{23} \frac{dx}{3 + f(-x)}$$

$$\left\{\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right\}$$

$$I = \int_{-23}^{23} \frac{dx}{3 + \frac{9}{f(x)}} \quad \text{(from Eq. (i)}\Big]$$

$$I = \frac{1}{3}\int_{-23}^{23} \frac{f(x)}{3 + f(x)} dx \quad \ldots \ldots (iii)$$

Adding Eqs. (ii) and (iii), we get

$$2I = \int_{-23}^{23} \left(\frac{1}{3+f(x)} + \frac{1}{3}\frac{f(x)}{3+f(x)}\right) dx$$

$$= \int_{-23}^{23} \frac{3 + f(x)}{3(3 + f(x))} dx = \frac{1}{3}\int_{-23}^{23} dx$$

$$2I = \frac{46}{3}$$

$$\Rightarrow I = \frac{46}{6}$$