Physics

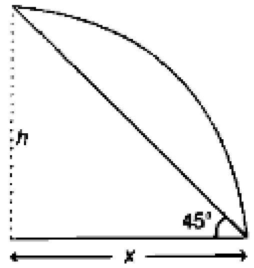
1. **(d)** The electromagnetic force between two protons is much stronger than the gravitational force between them.
Let F_e be the electromagnetic force, and F_g be the gravitational force.
According to the problem: $F_e = 10^n F_g$
We write the formulas for both forces:
Electromagnetic force: $F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$
Gravitational force: $F_g = \frac{Gm^2}{r^2}$
Replace F_e and F_g in the equation: $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = 10^n \frac{Gm^2}{r^2}$
Since r^2 is on both sides, it cancels out:
$$\frac{1}{4\pi\varepsilon_0}e^2 = 10^n \mathrm{Gm}^2$$
Solve for 10^n :
$$10^n = \frac{1}{4\pi\epsilon_0}\frac{e^2}{Gm^2}$$
Now, use the constants:
$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$$
Charge of proton, $e = 1.6 \times 10^{-19}$
Gravitational constant, $G = 6.67 \times 10^{-11}$
Mass of proton, $m = 1.67 \times 10^{-27}$
Plug in the values:
$$10^n = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times (1.67 \times 10^{-27})^2}$$
Calculate the numerator: $9 \times 10^9 \times (1.6 \times 10^{-19})^2 = 9 \times 10^9 \times 2.56 \times 10^{-38} = 2.304 \times 10^{-28}$
Calculate the denominator:
$6.67 \times 10^{-11} \times (1.67 \times 10^{-27})^2 = 6.67 \times 10^{-11} \times 2.79 \times 10^{-54} = 1.86 \times 10^{-64}$
Divide the values: $\frac{2.304\times10^{-35}}{1.86\times10^{-64}} = 1.23 \times 10^{36}$
This means $10^n = 1.23 \times 10^{36}$, so $n = 36$.

2. **(c)** According to principle of homogenity, physical quality can only be added or subtracted if they have the same dimensions.
$\frac{A-C}{B}$ is correct only when A and C must have some dimensions.

3. **(c)** $u = 72\ \mathrm{km/h} = 72 \times \frac{3}{18} = 20\ \mathrm{m/s}$
$a = -5\ \mathrm{m/s^2}$
$\therefore$ using, $v^2 = u^2 + 2a\,\mathrm{s}$
$\Rightarrow 0 = 20^2 + 2(-5)\mathrm{s}$
$$\Rightarrow s = \frac{400}{10} = 40\ \mathrm{m}$$
Distance travelled during reaction time
$=$ Total distance $- s$
$= 50 - 40 = 10$ m
If t be the reaction time, then
$s = ut$
$$t = \frac{s}{u} = \frac{10}{20} = 0.5\ \mathrm{s}$$

4. **(b)** $u = 288\ \mathrm{km/h}$
$$= 288 \times \frac{5}{18} = 80\ \mathrm{m/s}$$
$$h = \frac{1}{2}gt^2$$

$x = ut$



$\therefore \tan 45° = \frac{h}{x}$

$\Rightarrow h = x \Rightarrow \frac{1}{2}gt^2 = ut$

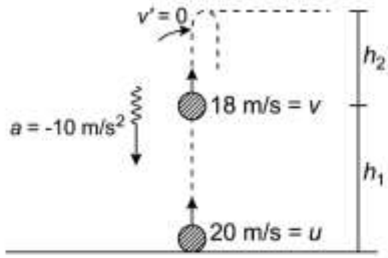
$\Rightarrow t = \frac{2u}{g} = \frac{2 \times 80}{10} \Rightarrow t = 16 \text{ s}$

$\therefore$ Height of bomb, $h = \frac{1}{2}gt^2$

$= \frac{1}{2} \times 10 \times 16^2 = 1280 \text{ m}$

5. **(d)** When a man standing in a lift moving up with a retardation of $a \text{ m/s}^2$, then apparent weight $= m(g+a)$
$= 60(9.8 - 2.8) \therefore a = -2.8 \text{ m/s}^2$
$= 60 \times 7 = 420 \text{ N}$

6. **(a)**



Using $v^2 - u^2 = 2gh$
We have for h_1 (see figure)
$18^2 - 20^2 = 2(-10) \times h_1$
$\Rightarrow h_1 = 3.8 \text{ m}$
Now for h_2,
$0 - 18^2 = 2(-10) \times h_2$
$\Rightarrow h_2 = 16.2 \text{ m}$
Maximum height attained
$= h_1 + h_2 = 3.8 + 16.2$
$= 20 \text{ m}$

7. **(b)** We know that force ($\mathbf{F}$) is directly proportional to acceleration (a) by Newton's second law ($\mathbf{F} = m\mathbf{a}$). Therefore, if the force is always perpendicular to the velocity, then the acceleration is also always perpendicular to the velocity. The work done by a force is given by $W = \int \mathbf{F} \cdot d\mathbf{s}$, where $d\mathbf{s}$ is the displacement.
Since, $\mathbf{F}$ is perpendicular to $\mathbf{v}$ and $d\mathbf{s}$ is in the direction of $\mathbf{v}$, the dot product $\mathbf{F} \cdot d\mathbf{s} = 0$. This means the work done by the force is zero.
According to the work-energy theorem, the net work done on a particle equals the change in its kinetic energy (ΔKE). Since the work done is zero, the change in kinetic energy is zero, meaning the kinetic energy remains constant.

8. **(d)** As we know,
Moment of inertia about axis of cylinder,

$I_{\text{long}} = \frac{1}{2}MR^2$

Moment of inertia about an axis through centre and perpendicular to length.

$$I_{\text{transverse}} = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$$

By using, given ratio,

$$\frac{1}{2}MR^2 = \frac{1}{n}\left(\frac{1}{12}ML^2 + \frac{1}{4}MR^2\right)$$

$$\left(\frac{2n-1}{4}\right)R^2 = \frac{1}{12}L^2$$

$$3(2n-1) = \frac{L^2}{R^2} \Rightarrow \frac{L}{R} = \sqrt{3(2n-1)}$$

9. **(b)** For block m (moving down)

$mg - T = ma$ ....(i)

For block n (moving up)

$T - ng = na$ ....(ii)

From Eqs. (i) and (ii), we get

$mg - ng = ma + na$

$(m-n)g = (m+n)a$

$a = \frac{(m-n)}{(m+n)}g$

$\therefore\ a_m = +a$ and $a_n = -a$

then, $a_{\text{CM}} = \frac{ma-na}{m+n} = \frac{(m-n)}{(m+n)} \times a$

$$a_{\text{CM}} = \left(\frac{m-n}{m+n}\right)^2 g$$

10. **(b)** Given:

$$\frac{U}{K} = \frac{4}{5}$$

The amplitude $A = 6$ cm

Step 1: Write the energy formulas

The total energy is $E = \frac{1}{2}kA^2$

The potential energy is $U = \frac{1}{2}kx^2$

The kinetic energy is $K = \frac{1}{2}k(A^2 - x^2)$

Step 2: Set up the given ratio

We want the distance x from the mean position so that $\frac{U}{K} = \frac{4}{5}$:

$$\frac{\frac{1}{2}kx^2}{\frac{1}{2}k(A^2 - x^2)} = \frac{4}{5}$$

Step 3: Simplify the equation

The $\frac{1}{2}k$ cancels out: $\frac{x^2}{A^2 - x^2} = \frac{4}{5}$

Step 4: Solve for x^2

Multiply both sides by $(A^2 - x^2)$: $5x^2 = 4(A^2 - x^2)$

Expand: $5x^2 = 4A^2 - 4x^2$

Bring $4x^2$ to the left: $5x^2 + 4x^2 = 4A^2$

Combine like terms: $9x^2 = 4A^2$

Step 5: Find x

Divide both sides by 9: $x^2 = \frac{4}{9}A^2$

Take the square root: $x = \frac{2}{3}$ A

Since $A = 6$ cm :

$$x = \frac{2}{3} \times 6 = 4 \text{ cm}$$

The distance from the mean position is 4 cm .

11. **(c)** Applying conservation of energy

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

But $GM = gR^2$

$\therefore \frac{1}{2}mv^2 - \frac{gR^2m}{R} = \frac{-gR^2m}{R+h}$

$\Rightarrow \frac{v^2}{2} - gR = -\frac{gR^2}{R+h}$

$\Rightarrow h = R\left(\frac{v^2}{2gR - v^2}\right)$

$= 6.4 \times 10^6 \left[\frac{(8000)^2}{2 \times 10 \times 6.4 \times 10^6 - (8000)^2}\right]$

$= 6.4 \times 10^6 \text{ m} = 6400 \times 10^3 \text{ m}$

$= 6400 \text{ km}$

12. **(b)** $B = \frac{F}{A} \times \frac{V}{\Delta V}$

$B = \frac{PV}{\Delta V}$

Here,

$V = \frac{4}{3}\pi R^3$

$\Delta V = 4\pi R^2 \Delta R$

$\therefore B = P \times \frac{4}{3}\pi R^3 \times \frac{1}{4\pi R^2 \Delta R}$

$B = \frac{PR}{3\Delta R}$

$\Delta R = \frac{PR}{3B} = \frac{10^7 \times 36 \times 10^{-2}}{3 \times 60 \times 10^9}$

$\Delta R = 2 \times 10^{-5} \text{ m}$

$= 2 \times 10^{-3} \text{ cm}$

13. **(d)** $d = 3 \text{ cm}$

$r = \frac{3}{2} \times 10^{-2} \text{ m}$

$T = 0.035 \text{Nm}^{-1}$

For soap bubble, both inner and outer surface contributes, so increase in surface area is

$\Delta A = 2 \times 4\pi r^2 = 8\pi r^2$

Work done = Surface tension × Increase in area

$W = T \times \Delta A$

$= 0.035 \times 8 \times \pi \times \left(\frac{3}{2} \times 10^{-2}\right)^2$

$\approx 1.98 \times 10^{-4} \text{ J} \approx 198\mu \text{ J}$

14. **(c)** We know that terminal velocity

$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$

$\Rightarrow v \propto r^2$ ....(i)

But mass of sphere

$m = \frac{4}{3}\pi r^3 \rho$

$\Rightarrow r^3 \propto m$

$\Rightarrow r \propto m^{1/3}$ ....(ii)

From Eqs. (i) and (ii), we have

$v \propto (m^{1/3})^2$

$\Rightarrow v \propto m^{2/3}$

$\Rightarrow \frac{v_2}{v_1} = \left(\frac{m_2}{m_1}\right)^{2/3}$

$= \left(\frac{64}{8}\right)^{2/3} = (8)^{2/3}$

$= (2^3)^{2/3} = 2^2 = 4$
$\Rightarrow v_2 = 4v_1 = 4 \times 3 = 12 \text{ cm/s}$

15. (c) The heat transferred through the rod is

$$Q = \frac{kA\Delta T}{l} t$$

$$Q = \frac{90 \text{ W/mk} \times 4 \times 10^{-4} \text{ m}^2 \times (100^\circ\text{C} - 0^\circ\text{C})}{0.2 \text{ m}} \times 420 \text{ s}$$

$$Q = \frac{90 \times 4 \times 10^{-4} \times 100}{0.2} \times 420 \text{ J}$$

$$Q = \frac{35000 \times 10^{-4}}{0.2} \times 420 \text{ J}$$

$$Q = \frac{3.6}{0.2} \times 420 \text{ J}$$

$$Q = 18 \times 420 \text{ J} = 7560 \text{ J}$$

The heat required to melt a mass m of ice is given by $Q = mL_f$

$$m = \frac{Q}{L_f}$$

$$m = \frac{7560 \text{ J}}{336 \times 10^3 \text{ J/kg}}$$

$$m = \frac{7560}{336000} \text{ kg}$$

$$m = 0.0225 \text{ kg}$$

$$m = 22.5 \text{ g}$$

The mass of ice melted is 22.5 g

16. (b) Heat of raise temperature of ice from -20°C to 0°C.

$Q_1 = m \cdot C_{\text{ice}} \cdot \Delta T$
$= 0.008 \times 2100 \times 20 = 336 \text{ J}$

Heat to melt ice at 0°C to water

$Q_2 = m \times L_f = 0.008 \times 336000 = 2{,}688 \text{ J}$

Heat to raise temperature of water from 0°C to 100°C

$Q_3 = m \times C_{\text{water}} \times \Delta T$
$= 0.008 \times 4200 \times 100 = 3360 \text{ J}$

Heat to convert water at 100°C to steam

$Q_4 = M \cdot L_v = 0.008 \times 2.268 \times 10^6$
$= 18144$

Total heat required

$Q_{\text{Total}} = Q_1 + Q_2 + Q_3 + Q_4$
$= 336 + 2688 + 3360 + 18144$
$= 24{,}528 \text{ J(or } 24.5 \text{ kJ)}$

$1 \text{ calorie} = 4.18 \text{ J}$

Total heat in joules

$$Q = \frac{24528}{4.2} \approx 5840\text{cal}$$

17. (a) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$T_2 = T_1 \left[\frac{V_1}{V_2}\right]^{\gamma-1} = 600 \left[\frac{V}{8V}\right]^{\frac{4}{3}-1}$$

$$T_2 = 600 \times \frac{1}{2} = 300 \text{ K}$$

$$W = \frac{nRT_1}{\gamma - 1}\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right]$$

$$= \frac{2 \times 8.3 \times 600}{\frac{4}{3} - 1}\left[1 - \left(\frac{1}{8}\right)^{\frac{4}{3}-1}\right]$$

$$= \frac{3 \times 2 \times 8.3 \times 600}{1}\left[1 - \frac{1}{2}\right]$$

$$= \frac{3 \times 2 \times 8.3 \times 600}{2} = 14940 \text{ J}$$

$= 14.94$ kJ

18. **(b)** Step 1: Recall relations

For an ideal gas,

$C_p - C_v = R$

For a monoatomic gas,

$$C_v = \frac{3}{2}R$$

So,

$$C_p = C_v + R = \frac{3}{2}R + R = \frac{5}{2}R$$

Step 2: Substitute the gas constant

Given:

$R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$

Hence,

$$C_p = \frac{5}{2} \times 8.3 = 2.5 \times 8.3 = 20.75 \text{ J mol}^{-1} \text{ K}^{-1}$$

19. **(d)** The initial fundamental frequency

$$f_1 = \frac{1}{2L_1}\sqrt{\frac{T_1}{\mu}}$$

$$300 = \frac{1}{2L_1}\sqrt{\frac{T_1}{\mu}} \quad \ldots.(i)$$

The new fundamental frequency

$$f_2 = \frac{1}{2L_2}\sqrt{\frac{T_2}{\mu}}$$

$$100 = \frac{1}{2(2L_1)}\sqrt{\frac{T_2}{\mu}}$$

$$100 = \frac{1}{4L_1}\sqrt{\frac{T_2}{\mu}} \quad \ldots.(ii)$$

Divide the Eq. (i) by the Eq. (ii)

$$\frac{300}{100} = \frac{\frac{1}{2L_1}\sqrt{\frac{T_1}{\mu}}}{\frac{1}{4L_1}\sqrt{\frac{T_2}{\mu}}}$$

$$3 = \frac{\frac{1}{2}\sqrt{T_1}}{\frac{1}{4}\sqrt{T_2}} \Rightarrow 3 = \frac{4}{2}\sqrt{\frac{T_1}{T_2}}$$

$$3 = 2\sqrt{\frac{T_1}{T_2}} = \frac{3}{2} = \sqrt{\frac{T_1}{T_2}} = \frac{9}{4} = \frac{T_1}{T_2}$$

$$\frac{T_2}{T_1} = \frac{4}{9}$$

The ratio of the new tension to the initial tension is $\frac{4}{9}$.

20. **(a)** $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Here, $\phi = \frac{\pi}{2}$ and $I_1 = I_2 = I$

$\therefore\ I_R = I + I + 2\sqrt{I \cdot I}\cos\frac{\pi}{2} = 2I$

21. **(b)** $\mu = \sqrt{2}, A = 90^\circ$

For total internal reflection

$\sin i_C = \frac{1}{\mu} = \frac{1}{\sqrt{2}} = \sin 45^\circ$

$\therefore\ i_C = 45^\circ$

For total internal reflection to occur at the second face, the angle of incidence at the second face (r_2) must be greater than or equal to the critical angle. To ensure total internal reflection, we consider the limiting case where $r_2 = \theta_c = 45^\circ$

The angle of the prism is related to the angles of refraction at the two faces by

$A = r_1 + r_2$

Since $A = 90^\circ$ and $r_2 = 45^\circ$

$90^\circ = r_1 + 45^\circ$

$r_1 = 90^\circ - 45^\circ = 45^\circ$

Applying Snell's law at the first face,

$\mu_{\text{air}} \sin i = \mu_{\text{prism}}\ \sin r_1$

\1 $\cdot \sin i = \sqrt{2}\sin 45^\circ$

$\sin i = \sqrt{2} \cdot \frac{1}{\sqrt{2}}$

$= 1 = \sin 90^\circ$

$\therefore\ i = 90^\circ$

22. **(a)** When the final image is at the least distance of distinct vision,

$M = M_0 \cdot M_e$

as we know, $M_e = 1 + \frac{D}{F_e}$

$M_e = 1 + \frac{25}{5} = 6$

So, $24 = M_0 \times 6$

$M_0 = 4$

23. **(d)** Given, $\Delta x = \frac{\lambda}{3}$

$\theta = \pi \cdot \frac{\Delta x}{\lambda}$

$\theta = \frac{\pi \cdot \lambda}{3 \cdot \lambda} = \frac{\pi}{3}$

So, $I' = I_{\max}\ \cos^2\theta$

$I' = I_{\max} \cdot \cos^2\left(\frac{\pi}{3}\right)$

$I' = \frac{I_{\max}}{4}\ \therefore I_{\max} = I$

$I' = \frac{I}{4}$

24. **(a)** Total charge on spherical shell.

$Q = \sigma A = \sigma(4\pi R^2) = 4\pi R^2\sigma$

According to Gauss's law total electric flux through a closed surface.

$\phi = \frac{Q}{c0} = \frac{4\pi R^2\sigma}{c0}$

$\therefore$ Flux through one face

$\phi_1 = \frac{\phi}{6} = \frac{4\pi R^2\sigma}{6\varepsilon_0} = \frac{2\pi R^2\sigma}{3\epsilon_0}$

25. **(c)** When dielectric is filled, we model this as two capacitors in series.

For capacitor with dielectric,

$$C_1=\frac{c_0A}{\frac{d}{2K}}=\frac{2KA_{\varepsilon_0}}{d}$$

For capacitor without dielectric,

$$C_2=\frac{\varepsilon_0A}{\frac{\mathrm{d}}{2}}=\frac{2\varepsilon_0A}{d}$$

For series combination,

$$C=\frac{C_1\times C_2}{C_1+C_2}$$

$$C=\frac{\frac{2KA\varepsilon_0}{d}\times\frac{2\varepsilon_0A}{d}}{\frac{2KA\varepsilon_0}{d}+\frac{2\varepsilon_0A}{d}}$$

$$C=\frac{2\varepsilon_0A}{d\left(1+\frac{1}{K}\right)}$$

Charge stored initially,

$$Q=CV=\frac{2\varepsilon_0A}{d\left(1+\frac{1}{k}\right)}\cdot V$$

Now, the entire capacitor just vacuum,

$$C'=\frac{\varepsilon_0A}{d}$$

So, final voltage, $V'=\frac{Q}{C}$

$$=\frac{2\varepsilon_0AV}{d\left(1+\frac{1}{K}\right)}\times\frac{d}{\varepsilon_0A}$$

$$V'=\frac{2V}{\left(1+\frac{1}{K}\right)}$$

26. **(b)** Given

Drift speed, $v_d=0.3\ \mathrm{ms}^{-1}$

Electric field, $E=2\mathrm{Vm}^{-1}$

We are to find electron mobility μ, where

$$\mu=\frac{v_d}{E}$$

Calculation

$$\mu=\frac{0.3}{2}=0.15\ \mathrm{m}^2\ \mathrm{V}^{-1}\ \mathrm{s}^{-1}$$

$$\mu=0.15=15\times10^{-2}\ \mathrm{m}^2\ \mathrm{V}^{-1}\ \mathrm{s}^{-1}$$

27. **(d)** Resistance of motor

$$R=\frac{V_1^2}{P_1}=\frac{220^2}{242}=200\Omega$$

Current drawn by the motor

$$I=\frac{V_2}{R}=\frac{200}{200}=1\ \mathrm{A}$$

28. **(b)** When a charged particle moves in a magnetic field, the time it takes to make one full round (the time period) is given by:

$$T=\frac{2\pi m}{Bq}$$

Here, m is the mass of the particle, q is its charge, and B is the magnetic field. This means the time period depends on the ratio of mass to charge ($\frac{m}{q}$).

To compare the time periods of a proton (T_p) and an alpha particle (T_α), use this ratio:

$$\frac{T_p}{T_\alpha}=\frac{m_p}{m_\alpha}\times\frac{q_\alpha}{q_p}$$

An alpha particle has a mass four times that of a proton ($m_\alpha=4m_p$) and a charge twice that of a proton ($q_\alpha=2q_p$).

Plug these values in:

$$\frac{T_p}{T_\alpha}=\frac{m_p}{4m_p}\times\frac{2q_p}{q_p}=\frac{1}{4}\times 2=\frac{1}{2}$$

This shows that the time taken by a proton is half that of the alpha particle. So, the ratio of their times is $T_p{:}T_\alpha=1{:}2$

29. (c) Number of turns per unit length

$$n=\frac{N}{L}=\frac{200}{0.5}=400\text{turns/m}$$

$\therefore\ B=\mu_0 nI$

$=4\pi\times 10^{-7}\times 400\times 2.5$

$=4\pi\times 10^{-4}$ T

30. (c) Magnetic susceptibility $\chi=0.6$

We need the ratio $\frac{\mu}{\mu_0}$, where

$\mu=$ permeability of the substance

$\mu_0=$ permeability of free space

Relation between μ and χ :

$\mu=\mu_0(1+\chi)$

Substitute the value:

$$\frac{\mu}{\mu_0}=1+\chi=1+0.6=1.6$$

$$\frac{\mu}{\mu_0}=\frac{1.6}{1}=\frac{8}{5}$$

31. (c) $\phi=2t^2+3t-2$

$$\therefore e=\frac{d\phi}{dt}=\frac{d}{dt}(2t^2+3t-2)=4t+3$$

at $t=3$ s, $e=4\times 3+3$

$e=15$volt

Induced power, $P=\frac{c^2}{R}=\frac{15^2}{7.5}=30$ W

32. (b) $V=V_0\sin\omega t$

$$\Rightarrow\frac{V_0}{2}=V_0\sin\omega t$$

$$\Rightarrow\frac{1}{2}=\sin\omega t\Rightarrow\omega t=\frac{\pi}{6}$$

$$\Rightarrow 2\pi ft=\frac{\pi}{6}\Rightarrow 2\pi\times 50\times t=\frac{\pi}{6}$$

$$\Rightarrow t=\frac{1}{600}\text{ s}$$

33. (d) Velocity of electromagnetic wave in medium

$$v=\frac{c}{\sqrt{\mu_r\varepsilon_r}}=\frac{3\times 10^8}{\sqrt{200\times 8}}$$

$v=7.5\times 10^6$ m/s

$\Rightarrow\ v\lambda=7.5\times 10^6$

$$\Rightarrow\ \lambda=\frac{7.5\times 10^6}{v}=\frac{7.5\times 10^6}{100\times 10^6}$$

$=0.075$ m $=7.5$ cm

34. (a) The maximum kinetic energy

$\mathrm{KE}_{\max}=E-\phi$

$KE_{max} = 4.5eV - 3eV$
$KE_{max} = 1.5eV$
The de-Broglie wavelength is,
$\lambda = \frac{12.27}{\sqrt{KE_{max}}}$Å
$\lambda = \frac{12.27}{\sqrt{1.5}}$Å $\Rightarrow \lambda \approx \frac{12.27}{1.2247}$Å
$\lambda \approx 10.018$Å
The de-Broglie wavelength associated with the photoelectrons is approximately 10.02Å.

35. (c) The frequency for hydrogen spectral lines is found using:
$f = RC\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
Lyman Series (moving to $n_1 = 1$):
First line: Electron goes from $n_2 = 2$ to $n_1 = 1$
$f_1^{(L)} = RC\left(1 - \frac{1}{4}\right) = RC\frac{3}{4}$
Second line: Electron goes from $n_2 = 3$ to $n_1 = 1$
$f_2^{(L)} = RC\left(1 - \frac{1}{9}\right) = RC\frac{8}{9}$
Difference between the two Lyman frequencies:
$f = f_2^{(L)} - f_1^{(L)}$
$= RC\left(\frac{8}{9} - \frac{3}{4}\right)$
$= RC\left(\frac{32-27}{36}\right) = RC\frac{5}{36}$
Balmer Series (moving to $n_1 = 2$):
First line: Electron goes from $n_2 = 3$ to $n_1 = 2$
$f_1^{(B)} = RC\left(\frac{1}{4} - \frac{1}{9}\right) = RC\frac{5}{36}$
Second line: Electron goes from $n_2 = 4$ to $n_1 = 2$
$f_2^{(B)} = RC\left(\frac{1}{4} - \frac{1}{16}\right) = RC\frac{3}{16}$
Difference between the two Balmer frequencies:
$f' = f_2^{(B)} - f_1^{(B)}$
$= RC\left(\frac{3}{16} - \frac{5}{36}\right)$
$= RC\left(\frac{27-20}{144}\right)$
$= RC\left(\frac{7}{144}\right)$
Relationship between differences:
$\frac{f'}{f} = \frac{\left(\frac{7}{144}\right)}{\left(\frac{5}{36}\right)} = \frac{7}{144} \times \frac{36}{5} = \frac{252}{720} = \frac{7}{20}$
$f' = \frac{7}{20}f$

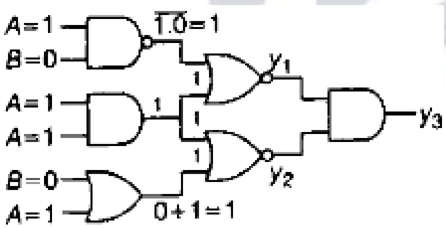
36. (b) Average energy of a neutron emitted in $^{235}_{92}U$ fission is 2 MeV
$\varepsilon = 2MeV$
$= 2 \times 10^6 \times 1.6 \times 10^{-19}$ J
$= 3.2 \times 10^{-13}$ J $= 320 \times 10^{-15}$ J

37. (d) We use this formula to find how much of the substance is left after a certain time:
$N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

It is said that 96.875% of the substance has decayed after 10 days.
This means that only 3.125% of the substance is left, because $100\% - 96.875\% = 3.125\%$.
We can also write 3.125% as a fraction: $3.125\% = \frac{1}{32}$.
So, $\frac{N}{N_0} = \frac{1}{32}$. We set up the formula:
$$\left(\frac{1}{2}\right)^{t/T_{1/2}} = \frac{1}{32}$$
Now, $\frac{1}{32} = \left(\frac{1}{2}\right)^5$, so $t/T_{1/2} = 5$.
So, $t = 5 \times T_{1/2}$.
We know $t = 10$ days. So,
$T_{1/2} = \frac{10}{5} = 2$ days
The half-life of the substance is 2 days.

38. **(a)** Power gain = Current × Voltage gain
$\Rightarrow 1800 = \text{Current gain } (\beta) \times 60$
$$\Rightarrow \beta = \frac{1800}{60} \Rightarrow \beta = 30$$
$$\Rightarrow \frac{\Delta I_C}{\Delta I_B} = 30 \Rightarrow \frac{\Delta I_B - \Delta I_B}{\Delta I_B} = 30$$
$$\Rightarrow \frac{\Delta I_B}{\Delta I_B} - 1 = 30$$
$$\Rightarrow \frac{0.62}{\Delta I_B} = 31$$
$$\Rightarrow \Delta I_B = \frac{0.62}{31} = 0.02 \text{ mA}$$
$\therefore \Delta I_C = \Delta I_E - \Delta I_B = 0.62 - 0.02$
$= 0.60$ mA

39. **(d)**



$y_1 = \overline{1+1} = \overline{1} = 0$
$y_2 = \overline{1+1} = \overline{1} = 0$
$y_3 = y_1 \cdot y_2 = 0.0 = 0$

40. **(b)** Explanation:
Human speech contains frequency components roughly between 100 Hz and 8 kHz , but for commercial telephony, only the most important frequencies for intelligibility are transmitted.
The telephone system limits the transmitted audio bandwidth to approximately 300 Hz − 3400 Hz, though sometimes it's cited as 300 Hz − 3100 Hz, depending on standards and equipment.
This limited range saves bandwidth while still allowing speech to sound natural and be easily understood.
So, the frequency range adequate for commercial telephonic communication is approximately 300 Hz to 3100 Hz(or 3400 Hz).

Chemistry

41. **(b)** Radius of nth orbit of hydrogen and hydrogen like atom, $r_n = \frac{n^2 a_0}{Z}$
For $n = 2, r_n = x\text{pm}, Z = 1$ (for H-atom)
$$x = \frac{2^2(a_0)}{1} \Rightarrow a_0 = \frac{x}{4}$$
For $n = 3, Z = 2, r_n = ?$
(for He^+atom)

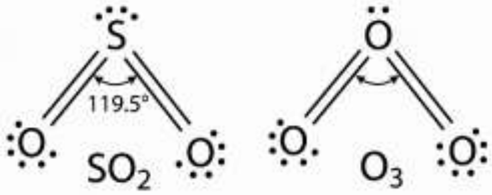
$$r_n = \frac{(3)^2 a_0}{2} = \frac{9a_0}{2}$$
$$\Rightarrow r_n = \frac{9 \times x}{2 \times 4} = \frac{9x}{8}$$

42. **(b)** Given, $\lambda = 310$ nm
$= 310 \times 10^{-9}$ m
$\phi = 3.55\text{eV} \Rightarrow v = x \times 10^5$ m/s
$x = ?$
Energy of incident photon,
$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{310 \times 10^{-9}}$$
$E = 6.412 \times 10^{-19}$ J
Kinetic energy of photoelectrons,
$\text{KE} = E - \phi$
$\text{KE} = 6.412 \times 10^{-19} - 3.55 \times 1.6 \times 10^{-19}$
$\text{KE} = 0.732 \times 10^{-19}$ J
Velocity of photoelectrons,
$$v = \sqrt{\frac{2\text{KE}}{m_e}}$$
$$= \sqrt{\frac{2 \times 0.732 \times 10^{-19}}{9 \times 10^{-31}}}$$
$v = 4 \times 10^5$ m/s
or $x = 4$

43. **(c)** The correct order of increasing atomic radius is F < N < P < Si < Na.
It is because atomic radius decreases across a period. While it increases down the group.

44. **(c)** (i) Elements with atomic number 13,11,9,7 and 16 are aluminium, sodium, fluorine, nitrogen and sulphur respectively. Thus, largest ions is S^{2-}, sulphide ion while smallest ion will be Al^{3+} ion. So, elements X and Y are E and A respectively.

45. **(b)** SO_2 and O_3 has sp^2 hybridisation with bent geometry.



46. **(c)** Statement given in (III) and (IV) are correct. While the statements given in (I) and (II) are incorrect. Their correct forms are
(I) When O_2 is converted to O_2^{2+}, the bond order increases from 2 to 3. (II) In conversion of O_2 to O_2^{2+}, the magnetic property changes from paramagnetic to diamagnetic.

47. **(d)** v_{rms} is given by, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$
Also, it is given that
$$v_{\text{rms}(H_2)} = \sqrt{7} v_{\text{mss}(N_2)}$$
$$\sqrt{\frac{3RT_{H_2}}{M_{H_2}}} = \sqrt{7} \times \sqrt{\frac{3RT_{N_2}}{M_{N_2}}}$$
$$\sqrt{\frac{3T_{H_2}}{2}} = \sqrt{7} \times \sqrt{\frac{3T_{N_2}}{28}}$$
$$\frac{T_{H_2}}{2} = \frac{T_{N_2}}{4} \Rightarrow T_{H_2} = \frac{T_{N_2}}{2}$$

48. (c) Mass of solute = Molarity × Volume (L) × Molar mass

(i) $0.25 \times 1 \times 106 = 26.5$ g

(ii) $0.2 \times 0.25 \times 142 = 7.1$ g

(iii) $1 \times 0.5 \times 158 = 79$ g

(iv) $0.5 \times 0.75 \times 60 = 22.5$ g

Hence, 0.5 L of $1.0 \mathrm{M} \mathrm{KMnO}_4$ has highest amount of solute.

49. (b) Given, $r\mathrm{H}_2(g) + \frac{1}{2}\mathrm{O}_2(g) \longrightarrow \mathrm{H}_2\mathrm{O}(l); \Delta H^\circ = -286$ kJ ....(i)

$\mathrm{C}(s) + \mathrm{O}_2(g) \longrightarrow \mathrm{CO}_2(g); \Delta H^\circ = -394$ kJ ....(ii)

$\mathrm{CH}_4(g) + 2\mathrm{O}_2(g) \longrightarrow \mathrm{CO}_2(g) + 2\mathrm{H}_2\mathrm{O}(l); \Delta H^\circ = -890$ kJ ....(iii)

Reverse reaction given in Eq. (ii),

$\mathrm{CO}_2(\mathrm{~g}) \longrightarrow \mathrm{C}(\mathrm{~s}) + \mathrm{O}_2(\mathrm{~g}); \Delta H^\circ = +394$ kJ ....(iv)

Now, add Eq. (iii) and (iv),

$\mathrm{CH}_4(g) + 2\mathrm{O}_2(g) + \mathrm{CO}_2(g) \longrightarrow \mathrm{CO}_2(s) + 2\mathrm{H}_2\mathrm{O}(l) + \mathrm{C}(s) + \mathrm{O}_2(g)$

$\mathrm{CH}_4(g) + \mathrm{O}_2(g) \longrightarrow \mathrm{C}(s) + 2\mathrm{H}_2\mathrm{O}(l)$

So, $\Delta H^\circ = -890 + 394 = -496$ kJ

50. (c) Given, reaction is

$\mathrm{N}_2\mathrm{O}_4(g) \rightleftharpoons 2\mathrm{NO}_2(g)$

Initial moles 1 0

Degree of dissociation $= \alpha$

At equilibrium $1 - \alpha$ 2α

Partial pressure is given by

$$p_{\mathrm{N}_2\mathrm{O}_4} = \left(\frac{1-\alpha}{1+\alpha}\right)p;\ p_{\mathrm{NO}_2} = \frac{2\alpha}{1+\alpha} \cdot p$$

$$K_p = \frac{\left(p_{\mathrm{NO}_2}\right)^2}{p_{\mathrm{N}_2\mathrm{O}_4}}$$

Substituting the values,

$$\alpha = \left(\frac{\frac{K_p}{p}}{4 + \frac{K_p}{p}}\right)^{1/2}.$$

51. (a) LiAlH_4 = Total charge = 0

Li oxidation number = +1

Al oxidation number = +3

$1 + 3 + x \times 4 = 0$

$x = -1$

$\mathrm{H}_2\mathrm{O} = 2 \times x - 2 = 0, x = +1$

Hence, $X = \mathrm{LiAlH}_4, Y = \mathrm{H}_2\mathrm{O}$

52. (b) (A) KOH (Potassium hydroxide)

KOH is a strong base that absorbs carbon dioxide (CO_2) forming potassium carbonate.

Use: Absorbent for CO_2.

→ (A) → (IV)

(B) Na(l) (Liquid sodium)

Liquid sodium metal is used as a coolant in nuclear reactors because of its high thermal conductivity.

Use: Coolant.

→ (B) → (I)

(C) Li (Lithium)

Lithium is used in electrochemical cells (batteries).

Use: Electrochemical cells.

→ (C) → (III)

(D) $\mathrm{Mg(OH)}_2$ (Magnesium hydroxide)

$\mathrm{Mg(OH)}_2$ is a weak base, used as an antacid ("milk of magnesia").

Use: Antacid.

(D) → (II)

Therefore, correct matching is:
(A)-(IV), (B)-(I), (C)-(III), (D)-(II)

53. (b) The reaction mention is as follows,

(i) $2Na(s) + \underset{\text{(excess)}}{O_2(g)} \longrightarrow \underset{(x)}{Na_2O_2(s)}$

Sodium peroxide O_2^{2-} has all its electrons paired making it diamagnetic.

(ii) $K(s) + O_2(g) \longrightarrow \underset{(y)}{KO_2(s)}$

The superoxide has one unpaired electrons, making it paramagnetic.

54. (d) The correct order of atomic radii in group 13 elements is, Tl > In > Al > Ga.
Usually radii increases down the group, due to addition of shell, but Ga has smaller radii than Al due to poor shielding effect of d-electrons.

55. (c) Amphoteric oxide:
$SnO, PbO, SnO_2, PbO_2, Ga_2O_3$
Acidic oxide : $CO_2,\ B_2O_3$
Neutral : CO

56. (c)

57. (a) The complete reaction mechanism is as follows

$2CH_3CH_2CH_2Br \xrightarrow[\text{Wurtz reaction}]{\text{Na/ Dry ether}} \underset{\text{(Product X)}}{CH_3CH_2CH_2CH_3}$

Catalytic dehydrogenation $\downarrow Cr_2O_3, 773\ K$

$\underset{\substack{\text{1,3 Butadiene} \\ \text{(Y)}}}{CH_2 = CH - CH = CH_2}$

$\xrightarrow[CH_3COCl_3]{Anhy.AlCl_3} CH_2 = CH - \underset{(Z)}{CH}(COCH_3) - CH_3$

So, in product Z, number of sp^2 carbon $(b) = 4$, number of sp^3 carbon $(a) = 4$
So, $a + b = 8$

58. (a) Hyperconjugation involves delocansation or electrons nom CHσ-oond aajacent to a π system or carbocation.
Hence, option (a) shows hyperconjugation.
$CH_3 - CH = CH_2$ molecules shows hyperconjugation as electron from CH bond of methyl group interact with π bond of alkene.

59. (a) Kjeldahl's method can estimate nitrogen only in organic compounds where nitrogen is present in amino or amide forms and not in aromatic rings, azo groups, or nitro groups.
Compound I (aniline) and compound(V) (benzylamine) is suitable.

60. (c) Given that molecular formula is C_6H_{10}. First of all let us calculate the degree of unsaturation for C_6H_{10}.

So, $DBE = C + 1\frac{-H}{2}$

$DBE = 6 + 1\frac{-10}{2} = 2$

Here, DBE of 2 indicates the presence of one triple bond or two double bonds or one ring and one double bond.
So, for alkynes let us see the structure with one triple bond.
Now, the possible number of alkyne monomers of C_6H_{10} are
Hex-1-yne (Terminal alkyne)
Hex-2-yne (Internal alkyne)
Hex-3-yne (Internal alkyne)
3-methyl pent-1-yne (Terminal alkyne)
4-methylpent-1-yne (Terminal alkyne)
4-methylpent-2-yne (Internal alkyne)
3, 3-Dimethyl but-1-yne (Terminal alkyne)
So, terminal alkynes contain acidic hydrogen while internal alkynes contains no acidic hydrogen. Thus x and y are 4,3 respectively.

61. (b) Using the formula to calculate molar mass,

$M = \frac{d \times N_{A \times a^3}}{Z}$

For fcc crystal, $x = 4$,
$= \frac{2 \times 10^{23} \times 2.16 \times 10^{-22}}{4}$
$= 64.8$ g/mol

62. (a) Both Statement (I) and (II) are correct.

63. (c) The cell reaction is,
At anode (Zn), $Zn \longrightarrow Zn^{2+} + 2e^-$
Cathode (H^+), $2H^+ + 2e^- \longrightarrow H_2$
Using Nernst equation,
$$E_{cell} = E^\circ_{cathode} - E^\circ_{anode} - \frac{2.303RT}{F}\log Q$$
$$0.28 = +0.76 - \frac{0.06}{2}\log\left(\frac{0.01}{[H^+]^2}\right)$$
Solving this,
$[H^+] = 10^{-9}$
$pH = -\log[H^+] = 9$

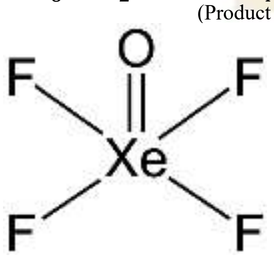
64. (b) $R \longrightarrow P$
Half-life is independent of initial concentration, means it's a first order reaction.
Integrated rate law,
$\ln[R] = -kt + \ln[R_0]$
Thus, $\ln R vs$ time is a straight line with slope $= -k$.
Option (b) graph is incorrect because it $[R]$ vs t.
So, it should be exponential.

65. (b)

66. (d) Reaction (a) i.e
$2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$
copper extraction via roasting.
Reaction (b) i.e
$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2$ is use to remove iron oxide from copper ore.
Reaction (c) i.e
$FeO + SiO_2 \longrightarrow FeSiO_3$ is removal of slag in copper extraction.
$SiO_2 + CaO \longrightarrow CaSiO_3$ is not related to copper extraction.

67. (c) The complete reaction is as follows,
$P + SO_2Cl_2 \longrightarrow \underset{\text{(Product X)}}{PCl_5}$
$PCl_5 + H_2O \longrightarrow \underset{\text{(Product Y)}}{H_3PO_4}$
Complete hydrolysis, Thus, $X = PCl_5, Y = H_3PO_4$.

68. The reaction involved is,
$XeF_6 + H_2O \rightarrow \underset{\text{(Product x)}}{XeOF_4} + 2HF$



Thus the shape of $(X)XeF_4$ is square pyramidal.

69. (a) H_3PO_3 and H_2SO_4 both have basicity of 2. H_3PO_3 has two $P-OH$ bond meaning it can donate two $[OH^-]$ions when in solution. H_2SO_4 also have two ionisation H -atom attached to oxygen, giving it a basicity of 2 .

70. (d) In acidic medium,
$5Fe^{2+} + MnO_4^- + 8H^+ \longrightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$
1 mole of MnO_4^-requires 5 moles of $Fe^{2+} x = 5$
$Cr_2O_7^{2-} + 14H^+ + 6Fe^{2+} \longrightarrow 2Cr^{3+} + 7H_2O + 6Fe^{3+}$

1 mole of $Cr_2O_7^{2-}$ are needed to give 6 moles of Fe^{3+}, $y = 6$.
$x + y = 5 + 6 = 11$

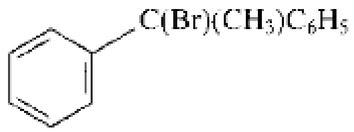
71. **(c)** An ambidentate ligand is a ligand that can coordinate to a central metal atom through two different donor atoms, but not at the same time. SO_4^{2-} is a bidentate ligand.

72. **(c)** Given:
Polymer ' X ' is mainly used for making unbreakable cups and laminated sheets.
We're asked: What are the monomers of polymer X ?
Step 1: Identify the polymer
Unbreakable cups and laminated sheets are typically made from melamine-formaldehyde or phenol-formaldehyde type resins.
However, phenol-formaldehyde resin (commonly known as Bakelite) is specifically known for:
Being hard and brittle
Used in electrical switches, unbreakable crockery, laminated sheets, etc.
Step 2: Match with monomers
Bakelite (phenol-formaldehyde resin) is formed by:
Phenol + Formaldehyde ⟶ Phenol-formaldehyde resin (Bakelite)
Hence, the monomers are phenol and formaldehyde.

73. **(b)** Polypeptide hormones are made up of chains of amino acids.
Insulin is a classic example - it's composed of two peptide chains (A and B chains) linked by disulfide bonds.
Epinephrine is a catecholamine (derived from the amino acid tyrosine).
Estrogen and androgen are steroid hormones derived from cholesterol.
Hence, Insulin is the polypeptide hormone among the options.

74. **(d)** Aspartame is a dipeptide methyl ester composed of aspartic acid and phenylalanine. Its chemical name is L-aspartyl-L-phenylalanine methyl ester.
Because it contains phenylalanine, individuals with phenylketonuria (PKU) must avoid it.
Other options for comparison:
Saccharin - a sulfonamide compound, not derived from amino acids.
Sucralose - a chlorinated derivative of sucrose (sugar), not protein-based.
Alitame - a dipeptide sweetener related to aspartame but has a different amine component (aspartic acid + a different amine, not phenylalanine).

75. **(d)** S_N1 mechanism depends on carbocation stability. The more stable the carbocation formed after the leaving group departs, the faster is the S_N1 reaction.
In



tertiary benzylic carbocation is stabilised by $+I$ effect of CH_3 and 2 phenyl group, which is most stable among the given options.

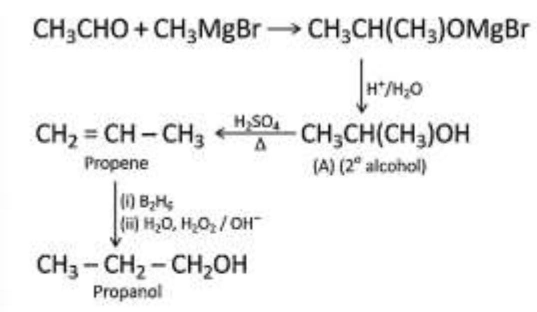
76. **(c)** The complete reaction is as follows,

$$(CH_3)_3CH \xrightarrow{KMnO_4} \underset{\text{Product } x}{(CH_3)_3COH}$$

$$\xrightarrow[573\,K]{Cu} \underset{\substack{\text{(Product } Y)\\ Isobutene}}{(CH_3)_2C = CH_2}$$

Number of sp^3 carbon = 2 (methyl group)
Number of sp^2 carbon = 2 (double bond carbon)

77. **(d)**



Hence, A and C are position isomers.

78. **(c)** The correct increasing order of acidic strength of given compounds is (I)< (II) < (III) < (IV).
As acidity increases with electron withdrawing group.
$-\mathrm{OCH_3} < -\mathrm{CH_3} < -\mathrm{CN} < -\mathrm{NO_2}$

79. **(b)** The correct increasing order of boiling points is
$\underset{\text{(III)}}{\mathrm{CH_3CH_3CH_3}} < \mathrm{CH_3} - \underset{\text{(I)}}{\mathrm{O}} - \mathrm{CH_3} < \underset{\text{(II)}}{\mathrm{CH_3CHO}} < \underset{\text{(iv)}}{\mathrm{CH_3CH_2OH}}$
It is because of strength of intermolecular forces present in each molecule, with hydrogen bonding in alcohols being the strongest and van der Waals' forces in alkanes being the weakest.

80. **(b)** The complete reaction is a follows.
$\underset{\text{(P)}}{\mathrm{C_6\,H_5CH_2NH_2}} \xleftarrow[\mathrm{H_2O}]{\mathrm{LiAlH_4}} \mathrm{C_6H_5CONH_2}$
$\xrightarrow{\mathrm{Br_2/NaOH}} \underset{\text{(Q)}}{\mathrm{C_6\,H_5NH_2}}$

Mathematics

81. **(b)** Given, $f(x) = \tan\left(\frac{\pi}{\sqrt{x+1+4}}\right)$
For $\sqrt{x+1}$ to be defined,
we have $x + 1 \geq 0 \Rightarrow x \geq -1$
$\therefore \sqrt{x+1} + 4 \geq 0 + 4 = 4$
$\Rightarrow 0 < \dfrac{1}{\sqrt{x+1}+4} \leq \dfrac{1}{4}$
$\Rightarrow 0 < \dfrac{\pi}{\sqrt{x+1}+4} \leq \dfrac{\pi}{4}$
$\therefore f(x) = \tan\left(\dfrac{\pi}{\sqrt{x+1}} + 4\right)$, where
$0 < \dfrac{\pi}{\sqrt{x+1}+4} \leq \dfrac{\pi}{4}$
So, the function $f(x)$ is strictly increasing in the interval $\left(0, \frac{\pi}{4}\right]$
And the range of the given function is $\left(\tan(0), \tan\left(\frac{\pi}{4}\right)\right]$, which is $(0,1]$
$\therefore$ The range of $f(x)$ is $(0,1]$

82. **(d)** Given, $f(x) = \sin^{-1}\left(\sqrt{x^2+x+1}\right)$
The domain of $\sin^{-1}\left(\sqrt{x^2+x+1}\right)$ is $[-1,1]$
$\therefore\ -1 \leq \sqrt{x^2+x+1} \leq 1$
Since $\sqrt{x^2+x+1}$ is always,
non-negative, so $0 \leq \sqrt{x^2+x+1} \leq 1$
Let $g(x) = x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$
The minimum value of $g(x)$ is $3/4$, which occurs at $x = -\frac{1}{2}v$
$\therefore x^2 + x + 1 \geq \dfrac{3}{4}$

$\Rightarrow \sqrt{x^2+x+1} \geq \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{\sqrt{3}}{2} \leq \sqrt{x^2+x+1} \leq 1$

$\Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \leq \sin^{-1}\left(\sqrt{x^2+x+1}\right)$

$\Rightarrow \frac{\pi}{3} \leq f(x) \leq \frac{\pi}{2} \leq \sin^{-1}$

Thus, the range of $f(x) = \sin^{-1}(\sqrt{x^2+x+1})$ is $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

83. **(a)** Given series

$1 + (1+3) + (1+3+5) + (1+3+5+7) + \cdots$

to 10 terms.

The nth term of the series is the sum of first n odd numbers.

So $T_n = 1 + 3 + 5 + \cdots + (2n-1) = n^2$

$\therefore\ T_1 = 1^2 = 1$

$T_2 = 2^2 = 4$

$T_3 = 3^2 = 9$

$T_{10} = 10^2 = 100$

Thus, $S_{10} = \sum_{n=1}^{10} n^2$

We know that the sum of the first k squares is

$$\sum_{n=1}^{k} n^2 = \frac{k(k+1)(2k+1)}{6}$$

For $k = 10$

$$S_{10} = \frac{10(10+1)(2 \times 10 + 1)}{6}$$

$$= \frac{10 \times 11 \times 21}{6} = \frac{2310}{6} = 385$$

$\therefore S_{10} = 385$

84. **(b)** Given equations are $x + y - z = 1$,

$2x + 4y - z = 0$ and $3x + 4y + 5z = 18$

The augmented matrix for the system is

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 4 & -1 & 0 \\ 3 & 4 & 5 & 18 \end{bmatrix}$$

We need to transform this matrix into

$$B = \begin{bmatrix} 1 & a & 0 & -1 \\ 0 & 2 & 1 & b \\ 0 & 0 & c & 32 \end{bmatrix}$$

Now, perform the row operation for matrix A, we get

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 1 & 8 & 15 \end{bmatrix}$$

Now, $R_3 \to R_3 - \frac{1}{2}R_2$

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 7.5 & 16 \end{bmatrix}$$

Apply $R_3 \to 2R_3$, we get

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 15 & 32 \end{bmatrix}$$

Again apply $R_1 \to R_1 + R_2$, we get

$A=\begin{bmatrix}1 & 3 & 0 & -1\\ 0 & 2 & 1 & -2\\ 0 & 0 & 15 & 32\end{bmatrix}$

Thus, $a=3, b=-2$ and $c=15$

$\Rightarrow\ c=15$

$\therefore \sqrt{a+b+c}=\sqrt{3+(-2)+15}$

$=\sqrt{1+15}=\sqrt{16}=4$

85. **(d)** Given,$K=\begin{vmatrix}9 & 25 & 16\\ 16 & 36 & 25\\ 25 & 49 & 36\end{vmatrix}$

$=9\begin{vmatrix}36 & 25\\ 49 & 36\end{vmatrix}-25\begin{vmatrix}16 & 25\\ 25 & 36\end{vmatrix}+16\begin{vmatrix}16 & 36\\ 25 & 49\end{vmatrix}$

$=9(1296-1225)-25(576-625)+16(784-900)$

$=9(71)-25(-49)+16(-116)$

$=639+1225-1856=8$

$\therefore K=8$

So, the roots are K and $K+1$, i.e., 8 and 9

Sum of roots $=8+9=17$

$=\dfrac{-b}{a}=\alpha+\beta$

And product of roots $=8\times 9=72$

$=\dfrac{c}{a}=\alpha\beta$

So, the general form of a quadratic equation is $x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow\ x^2-17x+72=0$

86. **(a)** Given, $A=\begin{bmatrix}1 & -3 & -5\\ -2 & 4 & -6\\ 7 & -11 & 13\end{bmatrix}$

$|A|=\begin{vmatrix}1 & -3 & -5\\ -2 & 4 & -6\\ 7 & -11 & 13\end{vmatrix}$

$=1\big(4\times 13-(-6)\times(-11)\big)-(-3)(-2\times 13-(-6)\times 7)-5(-2\times(-11)-7\times 4)$

$=1(52-66)+3(-26+42)-5(22-28)$

$=-14+48+30=64$

Since, $|\text{adj}A|=|A|^{n-1}$, for $n\times n$ matrix A

So, for 3×3 matrix,

$|\text{adj}A|\ =|A|^{3-1}=|A|^2$

$\therefore |\text{adj}A|=|A|^2=(64)^2=4096$

$\sqrt{|\text{adj}A|}\ =\sqrt{4096}=64$

87. **(a)** $\Delta r=\begin{vmatrix}\frac{1}{3r-2} & \frac{2}{3r-5}\\ 0 & \frac{3}{3r+1}\end{vmatrix}$

$=\dfrac{1}{(3r-2)}\times\dfrac{3}{(3r+1)}-\dfrac{2}{3r-5}\times 0$

$=\dfrac{3}{(3r-2)(3r+1)}$

Using partial fractions

$\Delta r=\dfrac{A}{3r-2}+\dfrac{B}{3r+1}$

$\Rightarrow\ 3=A(3r+1)+B(3r-2)$

If $r=\frac{2}{3}$, then

$\Rightarrow\ 3=A\left(3\times\dfrac{2}{3}+1\right)+B\left(3\times\dfrac{2}{3}-2\right)$

$\Rightarrow 3A=3$

$\Rightarrow\ A=1$

If $r = -\frac{1}{3}$, then

$3 = B\left(3 \times \left(-\frac{1}{3}\right) - 2\right) = B(-1-2)$

$\Rightarrow B = -1$

So, $\Delta r = \dfrac{1}{3r-2} - \dfrac{1}{3r+1}$

$\therefore \sum_{r=1}^{33} \Delta r = \sum_{r=1}^{33} \left(\dfrac{1}{3r-2} - \dfrac{1}{3r+1}\right)$

$\Rightarrow \left(\dfrac{1}{1} - \dfrac{1}{4}\right) + \left(\dfrac{1}{4} - \dfrac{1}{7}\right) + \left(\dfrac{1}{7} - \dfrac{1}{10}\right) +$

$\ldots + \left(\dfrac{1}{97} - \dfrac{1}{100}\right)$

$\Rightarrow 1 - \dfrac{1}{100} = \dfrac{99}{100} = 0.99$

88. **(b)** Given $\frac{2+3i}{i-2} - \frac{4i-3}{2+4i} - x + iy$

Now, $\frac{2+3i}{i-2} = \frac{2+3i}{-2+i} \times \frac{(-2-i)}{(-2-i)}$

$\Rightarrow \dfrac{-4-2i-6i-3i^2}{(-2)^2 - i^2} = \dfrac{-1-8i}{5}$

And, $\frac{4i-3}{3+4i} \times \frac{3-4i}{3-4i}$

$\Rightarrow \dfrac{-9+12i+12i-16i^2}{3^2-(4i)^2} = \dfrac{7+24i}{25}$

So, $\frac{2+3i}{i-2} - \frac{4i-3}{3+4i}$

$= \dfrac{-1}{5} - \dfrac{8i}{5} - \dfrac{7}{25} - \dfrac{24}{25}i$

$= \dfrac{-12}{25} - \dfrac{64}{25}i$

On comparing, we get

$x = \dfrac{-12}{25}, y = \dfrac{-64}{25}$

So, $3x + y = 3\left(\frac{-12}{25}\right) - \frac{64}{25}$

$= -\dfrac{36}{25} - \dfrac{64}{25} = \dfrac{-100}{25} = -4$

89. **(c)** Given, $\arg\left(\frac{z-3i}{z+2i}\right) = \frac{\pi}{4}$

Let $z = x + iy$

So, $\frac{z-3i}{z+2i} = \frac{x+iy-3i}{x+iy+2i}$

$\Rightarrow \dfrac{x+i(y-3)}{x+i(y+2)} \times \dfrac{x-i(y+2)}{x-i(y+2)}$

$\Rightarrow \dfrac{x^2 - ix(y+2) + ix(y-3) + (y-3)(y+2)}{x^2+(y+2)^2}$

$\Rightarrow \dfrac{x^2+y^2-y-6-i5x}{x^2+(y+2)^2}$

Let the complex numbers be $A + iB$, where

$A = \dfrac{x^2+y^2-y-6}{x^2+(y+2)^2}$ and $B = \dfrac{-5x}{x^2+(y+2)^2}$

So, $\arg\left(\frac{z-3i}{z+2i}\right) = \left(\frac{\pi}{4}\right)$

$\Rightarrow \tan\left(\dfrac{B}{A}\right) = \dfrac{\pi}{4}$

$\Rightarrow \dfrac{-5x}{x^2+y^2-y-6} = \tan^{-1}\left(\dfrac{\pi}{4}\right) = 1$

$\Rightarrow\ x^2+y^2-y-6=-5x$
$\Rightarrow\ x^2+5x+y^2-y-6=0$, where
$x\neq 0$ and $y\neq -2$

90. **(a)** Given, $x=\omega^2-\omega+2$ where w is a complex cube root of unity.
We know that $1+\omega+\omega^2=0$
$\Rightarrow \omega^2=-1-\omega$
So, $x=(-1-\omega)-\omega+2=1-2\omega$
$$\Rightarrow \omega=\frac{1-x}{2}$$
So, $1+\omega+\omega^2=0$
$$\Rightarrow\ 1+\left(\frac{1-x}{2}\right)+\left(\frac{1-x}{2}\right)^2=0$$
$$\Rightarrow\ 1+\frac{(1-x)}{2}+\frac{(1-x)^2}{4}=0$$
$\Rightarrow\ 4+2(1-x)+1-2x+x^2=0$
$\Rightarrow\ 4+2-2x+1-2x+x^2=0$
$\Rightarrow\ x^2-4x+7=0$

91. **(d)** Let $z=\sqrt{3}-i$
$$|z|=\sqrt{(\sqrt{3})^2+(-1)^2}=\sqrt{3+1}=2$$
And $\theta=\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)=-\frac{\pi}{6}$ or $\frac{11\pi}{6}$
So, $z=2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$
Now, $z^{\frac{3}{7}}=(\sqrt{3}-i)^{\frac{3}{7}}$
First, $z^3=(\sqrt{3}-i)^3$
$$=2^3\left(\cos\left(\frac{-3\pi}{6}\right)+i\sin\left(\frac{-3\pi}{6}\right)\right)$$
$$=8\left(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}\right)\qquad \ldots.(\text{i})$$
Now, $z^{\frac{3}{7}}=(\sqrt{3}-i)^{\frac{3}{7}}$
$$=(8)^{\frac{1}{7}}\left(\cos\left(\frac{-\frac{\pi}{2}+2k\pi}{7}\right)+i\sin\left(\frac{-\frac{\pi}{2}+2k\pi}{7}\right)\right)\qquad \ldots.(ii)$$
for $k=0,1,\ldots,6$
So, $z^3=8(0-i)=-8i$[Using Eq. (i)]
Now, we know that the product of nt h root of a complex number A is given by $(-1)^{n-1}A$, if A is a real number.
Since, $n=7$ is odd, the product of the 7 roots of A is A itself.
$\therefore$ The product of the 7th roots of A is A itself.
$\therefore (\sqrt{3}-i)^{\frac{3}{7}}=z^3$
So, product $=-8i$

92. **(b)** Given, $\sin^2x+b\sin x+c=0$ and
$$\alpha+\beta=\frac{\pi}{2}$$
Let $y=\sin x$, thus we get
$y^2+by+c=0$
The roots of this equation are $\sin\alpha$ and $\sin\beta$.
So, sum of the roots $=\sin\alpha+\sin\beta=-\frac{b}{1}$
Product of the roots $=\sin\alpha\cdot\sin\beta=c$
Since, $\alpha+\beta=\frac{\pi}{2}$
$$\Rightarrow\ \beta=\frac{\pi}{2}-\alpha$$
So, $\sin\alpha+\sin\left(\frac{\pi}{2}-\alpha\right)=-b$

$\Rightarrow \sin\alpha + \cos\alpha = -b$

$\left[\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right]$

$\Rightarrow (\sin\alpha + \cos\alpha)^2 = (-b)^2$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = (-b)^2$

$\Rightarrow 1 + 2c = b^2 [\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow b^2 - 1 = 2c$

93. **(c)** Given, quadratic equation is

$ax^2 + ax + 5 = 0$

For a quadratic equation

$Ax^2 + Bx + C = 0$ to have no real roots,

$\Delta = B^2 - 4AC < 0$

So, $\Delta = a^2 - 4(a)(5) < 0$

$\Rightarrow a^2 - 20a < 0$

$\Rightarrow a(a - 20) < 0$

So, S \quad 0

Thus, a must be an integer.

So, the integral values of a in this range are $1, 2, 3, \ldots, 19$.

The number of such value is

$19 - 1 + 1 = 19$

94. **(b)** Given equation is

$32x^3 - 48x^2 + 22x - 3 = 0$

Since, the roots of this equation are in AP.

So, sum of roots $= a - d + a + a + d = 3a$

$= \frac{-(-48)}{32} = \frac{3}{2} \Rightarrow a = \frac{1}{2}$

Now, sum of the products of roots $= \frac{22}{32}$

$\Rightarrow (a - d)a + a(a + d) + (a - d)(a + d) = \frac{11}{16}$

$\Rightarrow a^2 - ad + a^2 + ad + a^2 - d^2 = \frac{11}{16}$

$\Rightarrow 3a^2 - d^2 = \frac{11}{16} \Rightarrow 3\left(\frac{1}{2}\right)^2 - d^2 = \frac{11}{16}$

$\Rightarrow d^2 = \frac{3}{4} - \frac{11}{16} = \frac{12 - 11}{16} = \frac{1}{16}$

$\Rightarrow d^2 = \frac{1}{16}$

So, the square of the common difference of the root is $\frac{1}{16}$.

95. **(c)** Given, equation is

$x^4 - 2x^3 + x^2 + 4x - 6 = 0$

Let the roots of this equation be α, β, γ and δ.

Sum of the roots

$= \alpha + \beta + \gamma + \delta = \frac{-(-2)}{1} = 2$ …(i)

Sum of the products of roots

$= \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

$= \frac{1}{1} = 1$

Sum of products of roots taken three at a time

$= \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

$= \frac{-4}{1} = -4$

Product of roots $= \alpha\beta\gamma\delta = \frac{-6}{1} = -6$

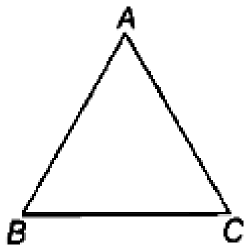
Since, the sum of two roots be zero, i.e.,

$\Rightarrow \alpha + \beta = 0$

$\Rightarrow \beta = -\alpha$
So, $\gamma + \delta = 2$ [from Eq. (i)]
And,
$\alpha(-\alpha) + \alpha\gamma + \alpha\delta + (-\alpha\gamma) + (-\alpha)\delta + \gamma\delta = 1$
$\Rightarrow -\alpha^2 + \alpha\gamma + \alpha\delta - \alpha\gamma - \alpha\delta + \gamma\delta = 1$
$\Rightarrow -\alpha^2 + \gamma\delta = 1$ ... (ii)
And $\alpha(-\alpha)\gamma\delta = -6$
$\Rightarrow -\alpha^2\gamma\delta = -6$ ....(iii)
From Eqs. (ii) and (iii), we get
$\gamma\delta = 1 + \alpha^2$
$\Rightarrow \alpha^2(1 + \alpha^2) = 6$
$\Rightarrow \alpha^4 + \alpha^2 = 6$
Let $y = \alpha^2$, then we get
$y^2 + y - 6 = 0$
$\Rightarrow (y + 3)(y - 2) = 0$
$\Rightarrow y = -3$ and $y = 2$
Since, α^2 must be non-negative,
$y = \alpha^2 = 2$
Then, $\gamma\delta = 1 + \alpha^2 = 1 + 2 = 3$
Now, $(\gamma + \delta)^2 = \gamma^2 + \delta^2 + 2\gamma\delta$
$\Rightarrow \gamma^2 + \delta^2 = (\gamma + \delta)^2 - 2\gamma\delta$
$= 2^2 - 2(3) = 4 - 6 = -2$

96. **(d)** The total number of ways to choose any subset of the 5 options $= 2^5 = 32$
These 32 subsets include choosing 2,3,4 or 5 options.
But choosing options 0 and 1 are not allowed.
Now, subsets with 0 options
$= {}^5C_0 = \frac{51}{0.5!} = 1$
Subsets with 1 options
$= {}^5C_1 = \frac{5!}{114!} = 5$
So, total subsets with options 0 and 1
$= 1 + 5 = 6$
Thus, the number of ways in which the student can answer that questions
$= 32 - 6 = 26$

97. **(c)** Let the triangle be ABC with point (x, y),



where $0 \le x \le 4$ and $0 \le y \le 4$
So, the points (x, y) are
(0,0), (0,1), (0,2), (0,3), (0,4), (1,0), (1,1), (1,2),
(1,3), (1,4), (2,0), (2,1), (2,2), (2,3), (2,4), (3,0), (3,1), (3,2), (3,3), (3,4), (4,0), (4,1), (4,2), (4,3), (4,4).
Total coordinates are 25 , but we need to select only 3 .
Now, the number of triangles with sets of 3 non-collinear points. Total number of ways to choose 3 points $= {}^{25}C_3$
But, we must subtract the number of collinear triplets.
Case I Horizontal lines
Number of ways to choose 3 collinear points
$\Rightarrow {}^5C_3 = 10$
So, in 5 rows $= 5 \times 10 = 50$
Case II Vertical lines

Same as above $\rightarrow$ 5 columns
$= 5 \times 10 = 50$
Now, diagonal from bottom-left to top-right
= Length 3: 2 such diagonals + Length
4 : 2 diagonals + Length 5:1 diagonal
$= 2(\,^3C_3) + 2(\,^4C_3) + \,^5C_3$
$2(1) + 2(4) + 10$
$= 2 + 8 + 10 = 20$
And, diagonals from top-left to bottom-right $= 20$
Total collinear triplets
$= 50 + 50 + 20 + 20 = 140$
So, total number of triangles $= \,^{25}C_3 - 140$
$\Rightarrow\ 2300 - 140 = 2160$

98. **(b)** Total Alphabets are A, D, E, H, L, N Letters before H are A, D, E.
So, the number of letters before H $= 3 \times 5\,!$
$= 3 \times 120 = 360$
Letters before E are A, D.
So, the number of letters before $E = 2 \times 4\,!$
$= 2 \times 24 = 48$
Letters before L are A, D.
So, the number of letters before L
$= 2 \times 3! = 12$
Letters before N is D
So, the number of letters with N only
$\Rightarrow\ 1 \times 1! = 1$
So, total words $= 360 + 48 + 12 + 1 = 421$
Now, only 1 letter remaining, that is D.
So, Rank of HELAND $= 421 + 1 = 422$

99. **(c)** Given, coefficient of 3rd term from beginning in the expansion $\left(ax^2 - \frac{8}{bx}\right)^9$ = Coefficient of 3rd term from end in the expansion $\left(ax - \frac{2}{bx^2}\right)^9$

For $\left(ax^2 - \frac{8}{6x}\right)^9$

$T_3 = \,^9C_2(ax^2)^7\left(\frac{-8}{bx}\right)^2$

$= \frac{9!}{2!\,7!}a^7x^{14} \cdot \frac{64}{b^2x^2}$

$= \frac{36 \cdot a^7}{b^2} \cdot x^{12} \cdot 64$

So, its coefficient is $\frac{3664a^7}{b^2}$

For $\left(ax - \frac{2}{bx^2}\right)^9$

$T_8 = \,^9C_7(ax)^2\left(\frac{-2}{bx^2}\right)^7$

$= \frac{9!}{2!\,7!}a^2x^2 \cdot \frac{-128}{b^7x^{14}}$

So, if coefficient is

$= -36 \times 128\frac{a^2}{b^7}$

Now, $36 \cdot \frac{64a^7}{b^2} = -36 \cdot 128\frac{a^2}{b^7}$

$\Rightarrow 64 \cdot a^5b^5 = -128$

$\Rightarrow a^5b^5 = -2$

100. **(d)** Given, $5^{2n}-48n+k$ is divisible by 24

Let $E(n)=5^{2n}-48n+k$

and $E(n)\equiv 0(\text{mod}24), \forall n \in N$

$\Rightarrow\ 5^{2n}-48n+k\equiv 0(\text{mod}24)$

$\Rightarrow\ k=48n-5^{2n}(\text{mod}24)$

For $n=1$,

$k=48(1)-5^{2\times 1}(\text{mod}24)$

$=48-25(\text{mod}24)$

$=23(\text{mod}24$

For $n=2$

$k=48(2)-5^{1}(\text{mod}24\ )$

$=96-625(\text{mod}24)$

$=529(\text{mod}24)=23(\text{mod}24)$

Thus, divisibility for 24 to hold for all n, we must have $k\equiv 23(\text{mod}24)$

Thus, the least positive integral value of k is 23.

101. **(d)** Given,

$\frac{x^3+3}{(x-3)^3}=a+\frac{b}{x-3}+\frac{c}{(x-3)^2}+\frac{d}{(x-3)^3}$ ....(i)

Let $y=x-3$

$\Rightarrow x=y+3$

Then, $\frac{x^3+3}{(x-3)^3}=\frac{(y+3)^3+3}{y^3}$

$=\dfrac{y^3+9y^2+27y+27+3}{y^3}$

$=\dfrac{y^3+9y^2+27y+30}{y^3}$

$=1+\dfrac{9}{y}+\dfrac{27}{y^2}+\dfrac{30}{y^3}$

$=1+\dfrac{9}{x-3}+\dfrac{27}{(x-3)^2}+\dfrac{30}{(x-3)^3}$

Comparing this with Eq. (i), we get

$a=1, b=9, c=27, d=30$

So, $(a+d)-(b+c)=(1+30)-(9+27)$

$=31-36=-5$

102. **(c)** Given, $\sin A=-\frac{60}{61}, \cot B=-\frac{40}{9}$

Now, $\cos^2 A=1-\sin^2 A$

$\Rightarrow\ 1-\left(-\dfrac{60}{61}\right)^2 \Rightarrow 1-\dfrac{3600}{3721}=\dfrac{121}{3721}$

$\therefore\ \cos A=\pm\dfrac{11}{61}$

But $\sin A<0$ and A and B are not in 4th quadrant.

So, A in third quadrant and B in 2nd quadrant

$\therefore\ \cos A=-\dfrac{11}{61}$

And $\cot A=\frac{\cos A}{\sin A}=\frac{-11}{-60}=\frac{11}{60}$

$6\cot A=\dfrac{66}{60}=\dfrac{11}{10}$

Now, $\tan B=\frac{1}{\cot B}=\frac{-9}{40}$

And, $\sec^2 B=1+\tan^2 B$

$\Rightarrow\ 1+\left(\dfrac{-9}{40}\right)^2=1+\dfrac{81}{1600}=\dfrac{41}{40}$

$\Rightarrow\ \sec B=-\dfrac{41}{40}(\because B \text{ in 2nd quadrant })$

$\Rightarrow\ 4\sec B=\frac{-164}{40}=-\frac{41}{40}$

$\therefore\ 6\cot A+4\sec B=\frac{11}{10}-\frac{41}{10}$

$\Rightarrow\ \frac{-30}{10}=-3$

103. **(a)** Given,

$$f(x)=\frac{2\sin\left(\frac{\pi x}{3}\right)\cos\left(\frac{2\pi x}{5}\right)}{3\tan\left(\frac{7\pi x}{2}\right)-5\sec\left(\frac{4\pi x}{3}\right)}$$

We know that

$\sin(kx)\rightarrow$ period $=\frac{2\pi}{k}$

$\cos(kx)\rightarrow$ period $=\frac{2\pi}{k}$

$\tan(kx)\rightarrow$ period $=\frac{\pi}{k}$

$\sec(kx)\rightarrow$ period $=\frac{2\pi}{k}$

So, for $\sin\left(\frac{\pi x}{3}\right)$, period $=\frac{2\pi}{\frac{\pi}{3}}=6$

$\cos\left(\frac{2\pi x}{5}\right)$ period $=\frac{2\pi}{\frac{2\pi}{5}}=5$

$\tan\left(\frac{7\pi x}{2}\right)$ period $=\frac{\pi}{\frac{7\pi}{2}}=\frac{2}{7}$

$\sec\left(\frac{5\pi x}{3}\right)$ period $=\frac{2\pi}{\frac{5\pi}{3}}=\frac{6}{5}$

Now, the LCM of $6,5,\frac{2}{7}$ and $\frac{6}{5}$ is as follows

LCM of $\left\{\frac{6}{1},\frac{5}{1},\frac{2}{7},\frac{6}{5}\right\}=\frac{\text{LCM of }(6,5,2,6)}{\text{HCF of }(1,1,7,5)}$

$\Rightarrow\frac{30}{1}=30$

So, the period of $f(x)$ is 30.

104. **(d)** Given, $A+B+C=4S$

$\therefore A+B+C=4S=\pi\Rightarrow S=\frac{\pi}{4}$

$\therefore \sin(2S-A)+\sin(2S-B)+\sin(2S-C)-\sin 2S$

$=\sin\left(2\times\frac{\pi}{4}-A\right)+\sin\left(2\times\frac{\pi}{4}-B\right)+\sin\left(2\times\frac{\pi}{4}-C\right)-\sin\left(2\times\frac{\pi}{4}\right)$

$=\sin\left(\frac{\pi}{2}-A\right)+\sin\left(\frac{\pi}{2}-B\right)+\sin\left(\frac{\pi}{2}-C\right)-\sin\left(\frac{\pi}{2}\right)$

$=\cos A+\cos B+\cos C-1$

$=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

105. **(c)** Given equation

$\sqrt{6-5\cos x+7\sin^2 x}-\cos x=0$

$\Rightarrow\sqrt{6-5\cos x+7-7\cos^2 x}-\cos x=0$

$\Rightarrow\sqrt{13-5\cos x-7\cos^2 x}-\cos x=0$

Squaring both sides, we get

$\Rightarrow 13-5\cos x-7\cos^2 x=\cos^2 x$

$\Rightarrow 13-5\cos x-8\cos^2 x=0$

$\Rightarrow 8\cos^2 x+5\cos x+13=0$

Let $y=\cos x$

$\Rightarrow 8y^2+5y-13=0$

$y=\frac{-5\pm\sqrt{25-4(8)(-13)}}{2\times 8}$

$$= \frac{-5 \pm \sqrt{25+416}}{16} = \frac{-5 \pm 21}{16}$$

$$\Rightarrow y = \frac{-5+21}{16} \text{ or } y = \frac{-5-21}{16}$$

$$\Rightarrow y = 1 \text{ or } y = -\frac{26}{16} = -\frac{13}{8} \text{ (not possible)}$$

$\therefore \cos x = 1 \Rightarrow \cos x = 1$

And $\sin^2 x = 1 - \cos^2 x = 0 \Rightarrow \sin x = 0$

So, $\tan x = \frac{\sin x}{\cos x} = \frac{0}{\mathrm{T}} = 0$

$$\sec x = \frac{1}{\cos x} = 1$$

Now, $\tan x + \cot x = 0 + \frac{1}{0} = \infty$

$$\cot x + \operatorname{cosec} x = \frac{1}{0} + \frac{1}{0} = \infty$$

$\tan x + \sec x = 0 + 1 = 1$

$$\sec x + \operatorname{cosec} x = 1 + \frac{1}{0} = \infty$$

106. **(a)** $\tan\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{6}{41}\right)$

$$= \tan^{-1}\left(\frac{\frac{3}{5}+\frac{6}{41}}{1-\frac{3}{5}\cdot\frac{6}{41}}\right) = \tan^{-1}\left(\frac{\frac{153}{205}}{\frac{187}{205}}\right)$$

$$= \tan^{-1}\left(\frac{153}{187}\right)$$

Now, $\tan^{-1}\left(\frac{153}{187}\right) + \tan^{-1}\left(\frac{9}{191}\right)$

$$= \tan^{-1}\left(\frac{\frac{153}{187}+\frac{9}{191}}{1-\frac{153}{187}\cdot\frac{9}{191}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{29223+1683}{35717}}{1-\frac{1377}{35717}}\right)$$

$$= \tan^{-1}\left(\frac{30906}{34340}\right) = \tan^{-1}\left(\frac{9}{10}\right)$$

107. **(c)** Given, $2\tanh^{-1} x = \sinh^{-1}\left(\frac{4}{3}\right)$

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow 2\tanh^{-1}(x) = \ln\left(\frac{1+x}{1-x}\right)$$

So, $\ln\left(\frac{1+x}{1-x}\right) = \sinh^{-1}\left(\frac{4}{3}\right)$

$$= \ln\left(\frac{4}{3} + \sqrt{\left(\frac{4}{3}\right)^2 + 1}\right)$$

$$\left[\because \sinh^{-1}(y) = \ln\left(y + \sqrt{y^2+1}\right)\right]$$

$$= \ln\left(\frac{4}{3} + \sqrt{\frac{16}{9}+1}\right)$$

$$= \ln\left(\frac{4}{3}+\frac{5}{3}\right) \Rightarrow \ln\frac{9}{3} = \ln(3)$$

$$= \frac{1+x}{1-x} = 3 \Rightarrow 1 + x = 3 - 3x$$

$= 4x = 2 \Rightarrow x = \frac{1}{2}$

Now, $\cosh^{-1}\left(\frac{1}{x}\right) = \cosh^{-1}(4$

$= \ln(2 + \sqrt{4-1})$

$\left[\because \cosh^{-1}(z) = \ln\left(z + \sqrt{z^2 - 1}\right)\right]$

$= \ln(2 + \sqrt{3})$

108. **(b)** Given, $a = 4, b = 5$ and $c = 6$

Since, p_1, p_2 and p_3 are altitudes of the sides a, b and c respectively.

$\therefore p = \frac{2\Delta}{\text{base}}$

$\Rightarrow p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$

$\Rightarrow \frac{1}{p_1^2} = \frac{a^2}{4\Delta^2}, \frac{1}{p_2^2} = \frac{b^2}{4\Delta^2}, \frac{1}{p_3^2} = \frac{c^2}{4\Delta^2}$

$\therefore \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2}$

$= \frac{a^2}{4\Delta^2} + \frac{b^2}{4\Delta^2} + \frac{c^2}{4\Delta^2}$

$= \frac{a^2 + b^2 + c^2}{4\Delta^2}$

We know that $s = \frac{a+b+c}{2}$

$= \frac{4+5+6}{2} = \frac{15}{2}$

So, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

(Using heron's formula)

$= \sqrt{\frac{15}{2} \cdot \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2}}$

$= \sqrt{\frac{1575}{16}} = \frac{\sqrt{1575}}{4}$

$= \Delta^2 = \frac{1575}{16}$

And $a^2 + b^2 + c^2 = 4^2 + 5^2 + 6^2$

$= 16 + 25 + 36 = 77$

So, $\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{77}{4\Delta^2}$

$= \frac{77}{4 \times \frac{1575}{16}} = \frac{77}{\frac{6300}{16}}$

$= \frac{77 \times 16}{6300} = \frac{44}{225}$

109. **(c)** Given angles A, B, C are in AP and $A + B + C = 180°$ ($\because$ angles of $\triangle ABC$)

So, $\frac{A+C}{2} = B$

$\therefore 2B + B = 180°$

$\Rightarrow 3B = 180°$

$\Rightarrow B = 60°$

Given that $r_3^2 = r_1 r_2 + r_2 r_3 + r_3 r_1$

Since, $B = 60°$, using the cosine rule for side b.

$b^2 = a^2 + c^2 - 2ac\cos 60°$

$= a^2 + c^2 - 2ac \cdot \frac{1}{2} = a^2 + c^2 - ac$

We know that, $s^2 = r_1r_2 + r_2r_3 + r_3r_1$
And given, $r_3^2 = r_1r_2 + r_2r_3 + r_3r_1$
So, $r_3^2 = s^2$
$\Rightarrow r_3 = s$
Since, $r_3 = \frac{\Delta}{s-c}$
$\Rightarrow\ s = \dfrac{\Delta}{s-c} \Rightarrow \Delta = s(s-c\,)$
$\Rightarrow\ \sqrt{s(s-a)(s-b)(s-c)} = s(s-c)$
Squaring both sides, we get
$\Rightarrow s(s-a)(s-b)(s-c) = s^2(s-c)^2$
$\Rightarrow (s-a)(s-b) = s(s-c)$
$\Rightarrow s^2 - sa - sb + ab = s^2 - sc$
$\Rightarrow ab = s(a+b-c)$
$\Rightarrow ab = \dfrac{(a+b+c)}{2}\cdot(a+b-c)$
$\left[\because s = \dfrac{a+b+c}{2}\right]$
$\Rightarrow 2ab = (a+b)^2 - c^2$
$\Rightarrow 2ab = a^2 + b^2 + 2ab - c^2$
$\Rightarrow a^2 + b^2 - c^2 = 0$
$\Rightarrow c^2 = a^2 + b^2$
So, the triangle is a right-angled triangle with right angle at c.
But, we have $B = 60^\circ$ and $C = 90^\circ$
So, $A = 180^\circ - 60^\circ - 90^\circ = 30^\circ$
Using the sine rule,
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
$\Rightarrow \dfrac{a}{\sin 30^\circ} = \dfrac{b}{\sin 60^\circ}$
$\Rightarrow \dfrac{a}{\frac{1}{2}} = \dfrac{b}{\frac{\sqrt{3}}{2}} \Rightarrow 2a = \dfrac{2b}{\sqrt{3}}$
$\Rightarrow \sqrt{3}a = \ b$
$\therefore b = \sqrt{3}a$

110. **(a)** Given, $\mathrm{A} = \hat{\imath} + 2\hat{\jmath} + 3\hat{\mathrm{k}}, \mathrm{B} = 7\hat{\imath} - \hat{\mathrm{k}},$
$\mathrm{P} = -2\hat{\imath} + 3\hat{\jmath} + 5\hat{\mathrm{k}}$
If P divides AB in the ratio $m{:}\,n$, then its harmonic conjugate Q divides AB in the ratio $-m{:}\,n$.
Now, $\mathrm{AB} = \mathrm{B} - \mathrm{A}$
$= (7-1)\hat{\imath} + (0-2)\hat{\jmath} + (-1-3)\hat{\mathrm{k}}$
$= 6\hat{\imath} - 2\hat{\jmath} - 4\hat{\mathrm{k}}$
Let P divides AB in the ratio $m{:}\,n$, so
$\mathrm{P} = \dfrac{n\mathrm{A} + m\mathrm{B}}{m+n}$
$\Rightarrow -2\hat{\imath} + 3\hat{\jmath} + 5\hat{\mathrm{k}}$
$= \dfrac{n(\hat{\imath} + 2\hat{\jmath} + 3\hat{\mathrm{k}}) + m(7\hat{\imath} - \hat{\mathrm{k}})}{m+n}$
$\Rightarrow (m+n)(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{\mathrm{k}})$
$= n(\hat{\imath} + 2\hat{\jmath} + 3\hat{\mathrm{k}}) + m(7\hat{\imath} - \hat{\mathrm{k}})$
Comparing the coefficients of $\hat{\imath}, \hat{\jmath}$ and $\hat{\mathrm{k}}$ on both sides
$\Rightarrow -2(m+n) = n(1) + m(7)$
$\Rightarrow -2m - 2n = n + 7m$
$\Rightarrow -9m = 3n$
$\Rightarrow -n = -3m$

$\Rightarrow \frac{m}{n} = \frac{1}{-3}$

$\therefore \frac{-m}{n} = \frac{-1}{-3} = \frac{1}{3}$

Now using section formula,

$Q = \frac{3A + 1B}{3 + 1}$

$= \frac{3(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + (7\hat{\imath} - \hat{k})}{4}$

$= \frac{10\hat{\imath} + 6\hat{\jmath} + 8\hat{k}}{4} = \frac{5}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} + 2\hat{k}$

So, sum of scalar components of

$Q = \frac{5}{2} + \frac{3}{2} + 2$

$= \frac{8}{2} + 2 = 4 + 2 = 6$

111. **(d)** Let the points be $A = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $B = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ and plane through points are $P_1 = \hat{\imath}, P_2 = 2\hat{\jmath}, P_3 = 3\hat{k}$

Let the point on the line be

$r(t) = A + t(B - A)$

$\Rightarrow r(t) = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + t[(2\hat{\imath} - \hat{\jmath} - \hat{k})$

$(\hat{\imath} + 2\hat{\jmath} + \hat{k})]$

$= (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + t(\hat{\imath} - 3\hat{\jmath} - 2\hat{k})$

$= (1 + t)\hat{\imath} + (2 - 3)\hat{\jmath} + (1 - 2t)\hat{k}$

Now, vectors in the plane are

$v_1 = P_2 - P_1 = 2\hat{\jmath} - \hat{\imath}$

$v_2 = P_3 - P_1 = 3\hat{k} - \hat{\imath}$

Normal vectors, $n = v_1 \times v_2$

$\Rightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = (6 - 0)\hat{\imath} - (-3 - 0)\hat{\jmath} + (0 + 2\hat{k}$

$\Rightarrow 6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

Using point $P_1 = \hat{\imath}$ to get plane equation is

$n \cdot (r - \hat{\imath}) = 0$

$\Rightarrow \langle 6,3,2 \rangle \cdot \langle x - 1, y, z \rangle = 0$

$\Rightarrow 6x - 6 + 3y + 2z = 0$

$\Rightarrow 6x + 3y + 2z = 6 \quad \ldots.(i)$

Put the value of $r(t) = (1 + t, 2 - 3t, 1 - 2t)$ into the plane Eq. (i), we get

$6x + 3y + 2z = 6$

$\Rightarrow 6(1 + t) + 3(2 - 3t) + 2(1 - 2t) = 6$

$\Rightarrow 6 + 6t + 6 - 9t + 2 - 4t = 6$

$\Rightarrow -7t + 14 = 6 \Rightarrow 7t = 8$

$\Rightarrow t = \frac{8}{7}$

So, $r\left(\frac{8}{7}\right) = \left(1 + \frac{8}{7}\right)\hat{\imath} + \left(2 - 3 \times \frac{8}{7}\right)\hat{\jmath} + \left(1 - 2 \times \frac{8}{7}\right)\hat{k}$

$\Rightarrow \frac{15}{7}\hat{\imath} - \frac{10}{7}\hat{\jmath} - \frac{9}{7}\hat{k}$

112. **(a)** Given, $|a| = 5, |b| = 12 |a - b| = 13$

Now, $|a - b|^2 = 13^2 = 169$

$\Rightarrow |a|^2 + |b|^2 - 2a \cdot b = 169$

$\Rightarrow 5^2 + 12^2 - 2a \cdot b = 169$

$\Rightarrow -2a \cdot b = 169 - 25 - 144 = 0$

$\Rightarrow a \cdot b = 0$

So, $a \perp b$

Now, $|2a+b|^2 = (2a+b)\cdot(2a+b)$

$\Rightarrow 4a\cdot a + 4a\cdot b + b\cdot b$

$= 4|a|^2 + 4a\cdot b + |b|^2$

$= 4(5)^2 + 4\times 0 + (1)^2$

$= 100 + 0 + 144 = 244$

$\therefore\ |2a+b| = \sqrt{244} = 2\sqrt{61}$

113. **(d)** Given, $a = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ and

$b = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$a + 2b = (\hat{\imath} - 2\hat{\jmath} - 2\hat{k}) + 2(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

$\Rightarrow\ (1+4)\hat{\imath} + (-2+2)\hat{\jmath} + (-2+4)\hat{k}$

$\Rightarrow\ 5\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$

$3a - b = 3(\hat{\imath} - 2\hat{\jmath} - 2\hat{k}) - (2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

$\Rightarrow\ (3-2)\hat{\imath} + (-6-1)\hat{\jmath} + (-6-2)\hat{k}$

$\Rightarrow\ \hat{\imath} - 7\hat{\jmath} - 8\hat{k}$

Now, $(a+2b)\times(3a-b) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & 0 & 2 \\ 1 & -7 & -8 \end{vmatrix}$

$\Rightarrow (0-(-14))\hat{\imath} - (-40-2)\hat{\jmath} + (-35-0)\hat{k}$

$\Rightarrow 14\hat{\imath} + 42\hat{\jmath} - 35\hat{k}$

114. **(b)** Given,

$r_1 = (3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}) + t(4\hat{\imath} + 3\hat{\jmath} - \hat{k})$

So, $a_1 = (3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$,

And $r_2 = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + s(6\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$

So, $a_2 = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}), b_2 = (6\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$

Since, shortest distance,

$$d = \frac{|(a_2 - a_1)\cdot(b_1\times b_2)|}{|b_1\times b_2|}$$

Now, $a_2 - a_1 = \langle 1,2,-4\rangle - \langle 3,-5,2\rangle$

$= \langle 1-3, 2-(-5), -4-2\rangle$

$= \langle -2,7,-6\rangle$

$b_1\times b_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 3 & -1 \\ 6 & 3 & -2 \end{vmatrix}$

$= (-6-(-3))\hat{\imath} - (-8-(-6))\hat{\jmath} + (12-18)\hat{k}$

$= -3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$

$|b_1\times b_2| = \sqrt{(-3)^2 + 2^2 + (-6)^2}$

$= \sqrt{9+4+36} = \sqrt{49} = 7$

So, $\quad d = \frac{|\langle -2,7,-6\rangle\cdot\langle -3,}{7}$

$\frac{|6+14+36|}{7} = \frac{56}{7} = 8$

115. **(a)** Given data,

x_i	2	9	8	3	5	7
f_i	5	3	1	6	6	1

x_i	f_i	Cumulative frequency
2	5	5
9	3	8

8	1	9
3	6	15
5	6	21
7	1	22

Total frequency $N=22$ (even)

$\Rightarrow \frac{N}{2}=\frac{22}{2}=11$

Since, the cumulative frequency just greater than $\frac{N}{2}$ i.e., 11 is 15 and the value of x corresponding to 15 is 3 .

$\therefore$ Median $=3$

Now, mean deviation

$=\frac{1}{N} \Sigma f_i \mid x_i-$ Median $\mid$

x_i	f_i	$\|x_i-3\|$	$f_{\|}\|x_i-3\|$
2	5	1	5
9	3	6	18
8	1	5	5
3	6	0	0
5	6	2	12
7	1	4	4

So, $\Sigma f_i\left|x_i-3\right|=44$

So, mean deviation $=\frac{44}{22}=2$

116. **(b)** Total number of ways to choose any 3 distinct squares $={ }^{64} \mathrm{C}_3$

$\Rightarrow \frac{641}{31511}=\frac{64 \times 63 \times 62}{6}=41664$

Since, each row has 8 squares.

In each row, the number of ways to choose 3 adjacent squares $=8-3+1=6$

So, total for 8 rows $=8 \times 6=48$

Similarly, total for 8 columns $=8 \times 6=48$

$\therefore$ Total favourable outcomes

$\Rightarrow 48+48=96$

Hence, required probability

$\Rightarrow \frac{95}{41664}=\frac{1}{434}$

117. **(b)** Number of spade cards $=13$, number of kings $=4$

Prime numbered cards are 2,3,5 and 7

So, total primes cards $=4$ suits $\times 4$

numbers

$\Rightarrow 16$ cards

But, a single card can be both a king and a spade or a prime and a spade.

So, the number of favourable outcomes.

Case I Spade that is not a king or prime $=13-1-4=8$

Case II King that is not a spade or prime $=3$

Case III Prime card that is not a spade or king $=16-4=12$

So, total favourable cases

$\Rightarrow 8 \times 3 \times 12=288$

and total number of ways to choose 3 cards from 52 cards $={ }^{52} \mathrm{C}_3$

$$=\frac{521}{3149}=\frac{52\times51\times50}{6}=22100$$

Thus, required probability

$$=\frac{288}{22100}=\frac{72}{5525}$$

118. **(b)** Since, there are three Urn.

So, $P(\text{Urn}A)=\frac{1}{3}=P(\text{Urn}B)=P(\text{Urn}C)$

Now, probability of drawing a white ball from each Urn

Urn A 6 white balls, 2 black balls.

Total $=8$ balls

$$P(\text{ White }/\text{Urn A})=\frac{6}{8}=\frac{3}{4}$$

Urn B 5 white balls, 3 black balls

Total $=8$ balls

$$P(\text{ White/Urn B })=\frac{5}{8}$$

Urn C 4 white balls, 4 black balls

Total $=8$ balls

$$P(\text{ White }/\text{UrnC})=\frac{4}{8}=\frac{1}{2}$$

So, the total probability of drawing a white ball is

$P(\text{ White })=P(\text{ White }/\text{Um A})\cdot P(\text{Um A})$

$+P(\text{ White }/\text{Um B})\cdot P(\text{Um B})$

$+P(\text{ White }/\text{UmC})\cdot P(\text{UmC})$

$$=\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{5}{8}+\frac{1}{3}\times\frac{1}{2}$$

$$=\frac{1}{4}+\frac{5}{24}+\frac{1}{6}$$

$$=\frac{6+5+4}{24}=\frac{15}{24}=\frac{5}{8}$$

119. **(d)** The numbers on a single die are 1,2,3,4,5 and 6.

So, the

$$\text{the mean}=\frac{1+2+3+4+5+6}{6}$$

$$=\frac{21}{6}=3.5$$

When multiple dice are thrown, the mean of the sum of the numbers appearing on them $=$ the sum of means of individual dice.

Since, each die has mean 3.5.

So, the mean of the sum of the numbers appearing on three dice

$=3.5+3.5+3.5=10.5$

120. **(c)** Given $X\sim B(7,P)$ is a binomial variate and $P(X=3)=P(X=5)$

Using the binomial probability formula

$$P(X=K)=\binom{n}{K}P^K(1-P)^{n-K}$$

Here $n=7$,

So, $P(X=3)=\binom{7}{3}P^3(1-P)^4$

And, $P(X=5)=\binom{7}{5}P^5(1-P)^2$

$$\therefore\binom{7}{3}P^3(1-P)^4=\binom{7}{5}P^5(1-P)^2$$

$\Rightarrow 35P^3(1-P)^4=21P^5(1-P)^2$

$\Rightarrow 35(1-P)^2=21P^2$, where $P\neq0, P\neq1$

$\Rightarrow 5(1-P)^2=3P^2$

$\Rightarrow 5(1-2P+P^2)=3P^2$

$\Rightarrow 5-10P+5P^2-3P^2=0$
$\Rightarrow 2P^2-10P+5=0$
$\Rightarrow P=\dfrac{-(-10)\pm\sqrt{100-4\times2\times5}}{2\times2}$
$=\dfrac{10\pm\sqrt{100-40}}{4}$
$=\dfrac{10\pm2\sqrt{15}}{4}=\dfrac{5\pm\sqrt{15}}{2}$
But $P=\frac{5+\sqrt{15}}{2}>1$, So it can't be a valid probability.
$\therefore P=\dfrac{5-\sqrt{15}}{2}$

121. **(b)** Let the centroid be $G(x,y)$. The coordinates of the centroid of triangles with vertices $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) are
$x=\dfrac{x_1+x_2+x_3}{3}$ and $y=\dfrac{y_1+y_2+y_3}{3}$
Substituting the coordinates of A,B and P.
$x=\dfrac{2+3+t}{3}=\dfrac{5+t}{3}$
$y=\dfrac{3+2+t^2}{3}=\dfrac{5+t^2}{3}$
$\Rightarrow 3x-5=t, y=\dfrac{5+(3x-5)^2}{3}$
$(\because t=2x-5)$
$\Rightarrow 3y=5+9x^2+25-30x$
$\Rightarrow 9x^2-30x-3y+30=0$
$3x^2-10x-y+10=0$
The equation of the locus of the centroid of $\triangle ABC$ is $3x^2-10x-y+10=0$

122. **(a)** Let $S(x,y)=2x^2-xy-y^2-3x+3y$
$\dfrac{\partial S}{\partial x}=4x-y-3,\dfrac{\partial S}{\partial y}=-x-2y+3$
Solve, $\frac{\partial s}{\partial x}=0$ and $\frac{\partial s}{\partial y}=0$, we get
$4x-y-3=0$ and $-x-2y+3=0$
Solving these two equations for $x=h$ and $y=k$
$4h-k-3=0$ and $-h-2k+3=0$
$\therefore h=1,k=1$
So, the new origin is $(h,k)=(1,1)$
We know that a general second-degree equation
$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$, the angle of rotation θ satisfies
$\tan 2\theta=\dfrac{B}{A-C}$
Here, $S\equiv 2x^2-xy-y^2-3x+3y=0$, we have
$A=2B=-1$ and $C=-1$
$\therefore \tan 2\theta=\dfrac{-1}{2-(-1)}=-\dfrac{1}{3}$
Now, $h+k=1+1=2h-k=1-1=0$,
$hk=1\times1=1,\dfrac{h}{3k}=\dfrac{1}{3}$
So, $\tan 2\theta=\frac{-1}{3}=\frac{-h}{3k}$

123. **(c)** Give line is $5x-12y+6=0$
Slope of this line is $m_1=\frac{-5}{-12}=\frac{5}{12}$
Since, line L is perpendicular to this line, the product of their slope is -1 . Let the slope of line L be m_L.

Then, $m_L \times \frac{5}{12} = -1$

$\Rightarrow\ m_L = -\dfrac{12}{5}$

The equation of a line in normal form is $x\cos\theta + y\sin\theta = P$, where P is the distance from the origin to the line and θ is the angle made by the perpendicular from the origin.

We have $P = 2$ unit, So, the equation of line is $x\cos\theta + y\sin\theta = 2$

Slope, $m_L = \frac{-\cos\theta}{\sin\theta} = -\cot\theta$

$\therefore -\cot\theta = \dfrac{-12}{5} \Rightarrow \cot\theta = \dfrac{12}{5}$

For y-intercept, $x = 0$ then $y\sin\theta = 2$

$\Rightarrow\ y = \dfrac{2}{\sin\theta}$

And since y-intercept is positive, so

$\dfrac{2}{\sin\theta} > 0$

$\Rightarrow \sin\theta > 0$

$\Rightarrow \cot\theta = \frac{12}{5} > 0, \theta$ must be in first quadrant.

Now, $\tan\theta = \frac{1}{\cot\theta} = \frac{5}{12}$

So, $\tan\theta + \cot\theta = \frac{5}{12} + \frac{12}{5}$

$= \dfrac{25+144}{60} = \dfrac{169}{60}$

124. **(a)** Let the angle made by Line L with positive X-axis be θ.

So, $B = (Ax + AB\cos\theta, Ay + AB\sin\theta)$

Given, $A(2,3)$ and $AB = 4$, we have

$B = (2 + 4\cos\theta, 3 + 4\sin\theta)$

Since, point B lies on the line $4x - 3y - 19 = 0$, substitute the coordinate of B into this equation.

$4(2 + 4\cos\theta) - 3(3 + 4\sin\theta) - 19 = 0$

$\Rightarrow\ 8 + 16\cos\theta - 9 - 12\sin\theta - 19 = 0$

$\Rightarrow\ 16\cos\theta - 12\sin\theta - 20 = 0$

$\Rightarrow\ 4\cos\theta - 3\sin\theta - 5 = 0$

$\Rightarrow\ \dfrac{4}{5}\cos\theta - \dfrac{3}{5}\sin\theta = 1$

Let $\cos\alpha = \frac{4}{5}$ and $\sin\alpha = \frac{3}{5}$ for some angle α in the first quadrant.

$\therefore \cos\alpha \cdot \cos\theta - \sin\alpha \cdot \sin\theta = 1$

$\Rightarrow \alpha\cos(\alpha + \theta) = 1$

$\Rightarrow \alpha + \theta = 2n\pi$, for some integer n.

$\Rightarrow \alpha + \theta = 0$

$\Rightarrow \alpha = -\theta$ or $\alpha + \theta = 2\pi$

$\Rightarrow \theta = 2\pi - \alpha$

The angle made by the line L with positive X-axis in the anti-clockwise direction is taken as a positive angle, where

$\cos\theta = \dfrac{4}{5}$ and $\sin\theta = -\dfrac{3}{5}$

This angle lies in the fourth quadrant.

So, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-3}{4}$

$\Rightarrow\ \theta = \tan^{-1}\left(\dfrac{-3}{4}\right) = -\tan^{-1}\left(\dfrac{3}{4}\right)$

125. **(a)** Let the equation of the line L be $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are the x and y intercepts, respectively. So, $OA = a$ and $OB = b, a > 0, b > 0$.

Here, the line passes through the point (4,9), So,

$\dfrac{4}{a} + \dfrac{9}{b} = 1$

We want to minimize $a+b$. So consider

$(a+b)\left(\frac{4}{a}+\frac{9}{b}\right)=4+\frac{9a}{b}+\frac{4b}{a}+9$

$=13+\frac{9a}{b}+\frac{4b}{a}$

By AM-GM inequality, for positive numbers,

$\frac{9a}{b}+\frac{4b}{a}\geq 2\sqrt{\frac{9a}{b}\cdot\frac{4b}{a}}$

$\Rightarrow 2\sqrt{36}=2\cdot 6=12$

So, $13+\frac{9a}{b}+\frac{4b}{a}=13+12=25$

But, since $(a+b)\left(\frac{4}{a}+\frac{9}{b}\right)=(a+b)\cdot 1$

$=a+b$ (Using Eq. (i))

$\therefore$ The minimum value of $a+b$ is 25 and this occurs when $\frac{9a}{b}=\frac{4b}{a}$

$\Rightarrow 9a^2=4b^2$ or $3a=2b$ ($\because a,b>0$)

$\Rightarrow b=\frac{3}{2}a$

So, $\frac{4}{a}+\frac{9}{b}=1$

$\Rightarrow \frac{4}{a}+\frac{9}{\frac{3}{2}a}=1 \Rightarrow \frac{4}{a}+\frac{18}{3a}=1$

$\Rightarrow \frac{4}{a}+\frac{6}{a}=1 \Rightarrow \frac{10}{a}=1$

$\Rightarrow a=10$

Then, $b=\frac{3}{2}a=\frac{3}{2}\times 10=15$

So, the minimum value of $OA+OB$ is $a+b=10+15=25$

126. **(d)** Given equation,

$4x^2+12xy+9y^2+2gx+2fy-1=0$

Since, general second-degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines, if

$\Delta=abc+2fgh-bg^2-ch^2-af^2=0$

Here, $a=4, b=9, h=6$

So, $\Delta=4\times 9\times c+2fg\times 6-9g^2$

$-c(6)^2-4f^2=0$

$=36c+12fg-9g^2-36c-4f^2=0$

$=12fg-9g^2-4f^2=0$

$\Rightarrow -(3g-2f)^2=0 \Rightarrow 3g-2f=0$

$\Rightarrow \frac{g}{f}=\frac{2}{3}$ and $\frac{f}{g}=\frac{3}{2}$

So, $\frac{j}{9}+\frac{9}{f}=\frac{3}{2}+\frac{2}{3}=\frac{9+4}{6}=\frac{13}{6}$

127. **(c)** Given equation is

$ax^2+ay^2+2gx+2fy-2=0$

Put $(-1,0)$ into this equation, we get

$a(-1)^2+a(0)^2+2g(-1)+2f(0)-2=0$

$\Rightarrow a-2g-2=0$ ....(i)

Put ($-1,1$), we get

$a(-1)^2+a(1)^2+2g(-1)+2f(1)-2=0$

$\Rightarrow a+a-2g+2f-2=0$

$\Rightarrow 2a-2g+2f-2=0$

$\Rightarrow a-g+f-1=0$ ....(ii)

Put (1, 1), we get

$a(1)^2+a(1)^2+2g(1)+2f(1)-2=0$

$\Rightarrow\ a+g+f-1=0$ ....(iii)
Subtract Eq. (ii) from Eq. (iii),
$(a+g+f-1)-(a-g+f-1)=0$
$\Rightarrow\ 2g=0$
$\Rightarrow\ g=0$
Put $g=0$ into Eq. (i), we get
$a-2(0)-2=0$
$\Rightarrow a=2$

128. **(a)** Given circle, $x-2=5\cos\theta, y+1=5\sin\theta$, where θ is the perimeter.
$\Rightarrow (x-2)^2+(y+1)^2=(5\cos\theta)^2+(5\sin\theta)^2$
$=25(\cos^2\theta+\sin^2\theta)=25$
$\therefore$ The circle has a centre at $(h,k)=(2,-1)$ and a radius, $r_c=5$
Now, the line equation is $x=1+\frac{r}{2}$ and $y=-2+\frac{\sqrt{3}}{2}r$. This is parametric form.
Let $x_1=1, y_1=-2$
The direction vector of the line is $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
So, the slope of the line, $m=\frac{\Delta y}{\Delta x}$
$$\Rightarrow \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$$
Now, the equation of line is
$y-y_1=m(x-x_1)$
$\Rightarrow y-(-2)=\sqrt{3}(x-1)$
$\Rightarrow y+2=\sqrt{3}x-\sqrt{3}$
$\Rightarrow \sqrt{3}x-y-(2+\sqrt{3})=0$
Now, calculate the distance from the center of the circle to line,
$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$$
Here, $(x_0,y_0)=(2,-1)$ and the line is $\sqrt{3}x-y-(2+\sqrt{3})=0$
So, $A=\sqrt{3}, B=-1, C=-(2+\sqrt{3})$
$$d=\frac{|\sqrt{3}(2)+(-1)(-1)-(2+\sqrt{3})|}{\sqrt{(\sqrt{3})^2+(-1)^2}}$$
$$=\frac{|2\sqrt{3}+1-2-\sqrt{3}|}{\sqrt{3+1}}$$
$$=\frac{\sqrt{3}-1}{2}\approx 0.366<5$$
So, the line intersects the circle at two distinct points, meaning it is a secant to the circle.
So, the line is a chord of circle other than diameter.

129. **(b)** The equation of the circle is
$x^2+y^2-6x+2y+c=0$
$\Rightarrow (x-3)^2+(y+1)^2=10-c$
So, the center of the circle is $C(3,-1)$ and the radius is $r^2=10-C$
Now, since the equation of tangent is
$x-2y=0$
Slop, $m_t=1/2$
Slope of the radius, connecting the centre $(3,-1)$ to the point of tangency
$P(x_p,y_p)$ is $m_r=\frac{y_p-(-1)}{x_p-3}=\frac{y_p+1}{x_p-3}$
Now, since radius is perpendicular to the tangent, so
$m_t\cdot m_r=-1$

$\Rightarrow \frac{1}{2} \cdot \frac{y_p + 1}{x_p - 3} = -1 \Rightarrow y_p + 1 = -2x_p + 6$
$\Rightarrow y_p = -2x_p + 5$
But, P lies on the tangent line, its coordinates satisfy $x_p - 2y_p = 0$
$\Rightarrow x_p = 2y_p$
So,$y_p = -2x_p + 5$
$\Rightarrow -2(2y_p) + 5 = -4y_p + 5$
$\Rightarrow 5y_p = 5 \Rightarrow y_p = 1$
So, $x_p = 2y_p = 2 \times 1 = 2$
So, the point P is $(2,1)$.
Now, the distance between $(6,3)$ and $(2,1)$ is
$d = \sqrt{(2-6)^2 + (1-3)^2}$
$\Rightarrow \sqrt{(-4)^2 + (-2)^2} \Rightarrow \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$
$\therefore$ The distance of the point $(6,3)$ from P is $2\sqrt{5}$.

130. **(c)** The equation of the circle is
$x^2 + y^2 - 4x + 2y - 4 = 0$
Comparing this with the general form
$x^2 + y^2 + 2gx + 2fy + c = 0$
We have
$2g = -4$
$\Rightarrow g = -2$ and $2f = 2$
$\Rightarrow f = 1$
So, the center of the circle C is
$(-g, -f) = (2, -1)$
And, the radius is $r = \sqrt{g^2 + f^2 - c}$
$= \sqrt{(-2)^2 + (1)^2 - (-4)}$
$= \sqrt{4+1+4} = \sqrt{9} = 3$
The point from which tangents are drawn is $P = (-3,1)$
Now, the circumcircle of $\triangle PAB$ is the circle with PC as its diameter. The center of this circumcircle is the mid-point of PC.
Mid-point $M = \left(\frac{-3+2}{2}, \frac{1+(-1)}{2}\right)$
$\Rightarrow \left(\frac{-1}{2}, \frac{0}{2}\right) = \left(\frac{-1}{2}, 0\right)$
Distance $PC = \sqrt{(2-(-3))^2 + (-1-1)^2}$
$\Rightarrow \sqrt{5^2 + (-2)^2}$
$\Rightarrow \sqrt{25+4} = \sqrt{29}$
$\therefore$ The radius of the circumcircle
$= \frac{1}{2}$, distance $PC = \frac{\sqrt{29}}{2}$
Now, the equation of a circle with center (h,k) and radius R is
$(x-h)^2 + (y-k)^2 = R^2$
Here, $(h,k) = \left(\frac{-1}{2}, 0\right)$ and $R = \frac{\sqrt{29}}{2}$
So, $\left(x - \left(-\frac{1}{2}\right)\right)^2 + (y-0)^2 = \left(\frac{\sqrt{29}}{2}\right)^2$
$\Rightarrow \left(x + \frac{1}{2}\right)^2 + y^2 = \frac{29}{4}$
$\Rightarrow x^2 + x + \frac{1}{4} + y^2 = \frac{29}{4}$
$\Rightarrow x^2 + y^2 + x - \frac{28}{4} = 0$
$\Rightarrow x^2 + y^2 + x - 7 = 0$

131. **(a)** For $S_1: x^2+y^2-2x+ky+1=0$,

Center $C_1=\left(1,-\frac{k}{2}\right)$

Radius, $r_1=\sqrt{1^2+\left(-\frac{k}{2}\right)^2-1}=\sqrt{\frac{k^2}{4}}=\frac{k}{2}$

For $S_2: x^2+y^2-kx-2y+1=0$

Center $C_2=\left(\frac{k}{2},1\right)$

Radius, $r_2=\sqrt{\left(\frac{k}{2}\right)^2+1^2-1}$

$$\sqrt{\frac{k^2}{4}}=\frac{k}{2}$$

Since, angle θ between two circles is given by $\cos\theta=\frac{d^2-r_1^2-r_2^2}{2r_1r_2}$, where d is the distance between the centers.

Given, $\theta=\cos^{-1}\left(\frac{1}{4}\right)$

$$\Rightarrow \cos\theta=\frac{1}{4}$$

$$d^2=\left(1-\frac{k}{2}\right)^2+\left(\frac{-k}{2}-1\right)^2$$

$$=1-k+\frac{k^2}{4}+1+k+\frac{k^2}{4}$$

$$=2+\frac{k^2}{2}$$

Since $k<0, r_1=\frac{-k}{2}$ and $r_2=\frac{-k}{2}$

Substituting into the formula

$$\frac{1}{4}=\frac{\left(2+\frac{k^2}{2}\right)-\left(\frac{-k}{2}\right)^2-\left(\frac{-k}{2}\right)^2}{2\left(\frac{-k}{2}\right)\left(\frac{-k}{2}\right)}$$

$$\Rightarrow \frac{1}{4}=\frac{2+\frac{k^2}{2}-\frac{k^2}{4}-\frac{k^2}{4}}{2\left(\frac{k^2}{4}\right)}$$

$$\Rightarrow \frac{1}{4}=\frac{4}{k^2}$$

$\Rightarrow k^2=16$

$\Rightarrow k=\pm 4$

But since $k<0, k=-4$

Now, the equation of the radical axis is

$S_1-S_2=0$

$(x^2+y^2-2x+ky+1)-(x^2+y^2-kx$
$-2y+1)=0$

$\Rightarrow -2x+ky+kx+2y=0$

$\Rightarrow -6x-2y=0\ (\because k=-4)$

$\Rightarrow 3x+y=0$

The points satisfying this equation are $(-1,3)$ and $(1,-3)$ lies on the radical axis.

132. **(b)** Let the equation of circle C be

$x^2+y^2+2gx+2fy+c=0$

Since, it passes through $(1,1)$ then

$1^2+1^2+2g+2f+c=0$

$\Rightarrow 2+2g+2f+c=0 \quad \ldots(i)$

Now, $S_1: x^2+y^2+2gx+2fy+c=0$

and $S_2=x^2+y^2-2x=0$

So, $S_1-S_2=0$ is the common chord.

$\Rightarrow (2g + 2x + 2fy + c = 0$
Now, center of $s_2 = x^2 + y^2 - 2x = 0$ is $(1,0)$
Since, the common chord is a diameter of S_2, then centre of S_2 must lies on the common chord.
$\therefore\ (2g + 2(l) + 2f(0) + c = 0$
$\Rightarrow\ 2g + 2 + c = 0 \quad \ldots.\text{(ii)}$
Also, circle C is orthogonal to
$x^2 + y^2 + 2y - 3 = 0$
Here, $g_1 = g, f_1 = f, c_1 = c$ and $g_2 = 0$,
$f_2 = 1, c_2 = -3 \quad \ldots.\text{(iii)}$
Using the orthogonality condition
$2g_1g_2 + 2f_1f_2 = c_1 + c_2$
$\Rightarrow 2g(0) + 2f(1) = c + (-3)$
$\Rightarrow 2f = c - 3$
From Eq. (ii), $c = -2g - 2$
Put this in Eq. (iii), we get
$\Rightarrow 2f = c - 3 = -2g - 2 - 3$
$\Rightarrow 2f = -2g - 5 \quad \ldots.\text{(iv)}$
Substitute $c = -2g - 2$ into Eq. (i), we get
$\Rightarrow 2 + 2g + 2f - 2g - 2 = 0$
$\Rightarrow 2f = 0$
$\Rightarrow f = 0$
Put $f = 0$ in Eq. (iv), we get
$2(0) = -2g - 5$
$\Rightarrow 2g = -5$
$\Rightarrow g = \dfrac{-5}{2}$
The center of circle C is
$(-g, -f) = \left(-\left(-\dfrac{5}{2}\right), 0\right)$
$\Rightarrow \left(\dfrac{5}{2}, 0\right)$

133. **(a)** Given parabola is $y^2 = 32x$,
So, $4a = 32$
$\Rightarrow\ a = 8$
The point P is $(8,16)$. This in parametric form as $(at^2, 2at)$
So, $8t^2 = 8$
$\Rightarrow\ t^2 = 1 \Rightarrow t = \pm 1$
And, $2at = 2(8)t = 16t = 16$
$\Rightarrow\ t = 1$
So, the perimeter for P is $t_1 = 1$.
The equation of the normal to the parabola at a point $P(at_1^2, 2at_1)$ is
$y + t_1x = 2at_1 + at_1^3$
$\Rightarrow\ y + (1)x = 2(8)(1) + 8(1)^3$
$\Rightarrow\ y + x = 16 + 8 = 24$
$\Rightarrow\ y = 24 - x$
Substitute this into parabola equation
$y^2 = 32x$
$\Rightarrow\ (24 - x)^2 = 32x$
$\Rightarrow\ 576 - 48x + x^2 = 32x$
$\Rightarrow\ x^2 - 80x + 576 = 0$
Since, $x = 8$ is one root corresponding to point P. Let the other root be xQ.
Using the sum of roots, $8 + x_Q = 80$
$\Rightarrow x_Q = 72$
So, $y = 24 - x_Q$
$= 24 - 72 = -48$

So, the coordinates of Q are $(72,-48)$
Now, the equation of the tangent to parabola $y^2=4ax$ at a point (x_1, y_1) is
$yy_1=2a(x+x_1)$
$\Rightarrow y(-48)=2(8)(x+72)=16(x+72$
$\Rightarrow -3y=x+72 \Rightarrow x+3y+72=0$
So, the equation of the tangent drawn at Q to the parabola is $x+3y+72=0$

134. **(a)** Given equation is
$x^2-2x-4y+5=0$
$\Rightarrow (x-1)^2-4y+4=0$
$\Rightarrow (x-1)^2=4(y-1)$
So, $(h,k)=(1,1)$ are the vertex
And $4a=4$
$\Rightarrow a=1$
Since, the parabola open upwards (because the y term is positive and x term is squared), the focus is $(h,k+a)=(1,1+1)=(1,2)$ and the directrix is the line $y=k-a=1-1=0$, which is the X-axis.
Since, focal distance of a point on a parabola is its distance from the directrix.
The point $(5,5)$ and directrix is $y=0$.
The distance from a point (x_0,y_0) to a horizontal line $y=c$ is $|y_0-c|$.
$\therefore$ Focal distance $=|5-0|=5$
So, the focal distance of the point $(5,5)$ on the parabola $x^2-2x-4y+5=0$ is 5 .

135. **(a)** The equation of ellipse is
$\frac{x^2}{169}+\frac{y^2}{144}=1$
So, $a^2=169, b^2=144$
$\Rightarrow a=13, b=12$
$\therefore c^25=a^2-b^2 \Rightarrow 169-144=2$
$\Rightarrow c=5$
So, the foci are at $(\pm c,0)$. So, $S=(5,0)$ and $S'=(-5,0)$
Since, the point B is one end of the minor axis lying on the positive Y-axis.
So, $B=(0,\pm b)=(0,12)$
$\therefore$ The vertices of the triangle are $S(5,10), S'(-5,0)$ and $B(0,12)$
$SS'=\sqrt{(-5-5)^2+(0-0)^2}$
$=\sqrt{(-10)^2}=10$
$SB=\sqrt{(0-5)^2+(12-0)^2}$
$=\sqrt{25+144}=\sqrt{169}=13$
$S'B=\sqrt{(0-(-5))^2+(12-0)^2}$
$=\sqrt{25+144}=\sqrt{169}=13$
Since, $SB=S'B$, So SBS' is an isosceles triangle.
Now, the incenter (I_x, I_y) of a triangle with vertices $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ and opposite side lengths a,b and c is
$I_x=\frac{ax_1+bx_2+cx_3}{a+b+c}, I_y=\frac{ay_1+by_2+cy_3}{a+b+c}$
Here, $(x_1,y_1)=S(5,0),(x_2,y_2)=S'(-5,0),(x_3,y_3)=B(0,12)$ and $a=S'B=13$,
$b=SB=13, C=SS'=10$
$\therefore I_x=\frac{13(5)+13(-5)+10(0)}{13+13+10}$
$\Rightarrow \frac{65-65+0}{36}=0$
$I_y=\frac{13(0)+13(0)+10(12)}{13+13+10}$
$\Rightarrow \frac{0+0+120}{36}=\frac{10}{3}$
So, the incenter of the $\triangle SBS'$ is $\left(0,\frac{10}{3}\right)$.

136. **(c)** Let the given focus be $F_1(2,-3)$ and the corresponding directrix be $L_1: 2x+y-5=0$ and its eccentricity,

$e=\frac{\sqrt{5}}{3}$

Let the other focus be $F_2(x_2, y_2)$

Now, the distance from $F_1(2,-3)$ to $2x+y-5=0$ is

$d_1=\frac{|2(2)+(-3)-5|}{\sqrt{2^2+1^2}}$

$=\frac{|4-3-5|}{\sqrt{4+1}}=\frac{|-4|}{\sqrt{5}}=\frac{4}{5}$

Let $P(x, y)$ be any point on the ellipse.

So, $PF_1=\sqrt{(x-2)^2+(y+3)^2}$

And the distance from P to the directrix L_1 is

$PM_1=\frac{|2x+y-5|}{\sqrt{2^2+1^2}}$

$\Rightarrow \frac{|2x+y-5|}{\sqrt{5}}$

Since, $PF_1=e \cdot PM_1$

$\Rightarrow \sqrt{(x-2)^2+(y+3)^2}$

$=\frac{\sqrt{5}}{3} \cdot \frac{|2x+y-5|}{\sqrt{5}}$

$\Rightarrow \frac{|2x+y-5|}{3}$

Squaring on both sides, we get

$(x-2)^2+(y+3)^2=\frac{(2x+y-5)^2}{9}$

$\Rightarrow 9((x-2)^2+(y+3)^2)$

$=4x^2+y^2+25+4xy-20x-10y$

$=9x^2-36x+36+9y^2+54y+81$

$=4x^2+y^2+25+4xy-20x-10y$

$\Rightarrow 5x^2+8y^2-4xy-16x+64y+92=0$

This is equation of ellipse.

Now, slope of directrix is $m_D=-2$ and the line joining the two foci is perpendicular to the directrix.

So, $m_F=\frac{-1}{m_D}=\frac{-1}{-2}=\frac{1}{2}$

Let, the other focus be $F_2(x_2, y_2)$

So, $\frac{y_2-(-3)}{x_2-2}=\frac{y_2+3}{x_2-2}=\frac{1}{2}$

$\Rightarrow 2(y_2+3)=x_2-2$

$\Rightarrow x_2=2y_2+8$

We know that $c=ae$ and the distance between the focus and directrix is

$\frac{a}{e}-c=\frac{a}{e}-ae=\frac{a(1-e^2)}{e}$

$\Rightarrow \frac{a\left(1-\left(\frac{\sqrt{5}}{3}\right)^2\right)}{\frac{\sqrt{5}}{3}}=\frac{4}{\sqrt{5}}$

$\Rightarrow \frac{a\left(1-\frac{5}{9}\right)}{\frac{\sqrt{5}}{3}}=\frac{4}{\sqrt{5}} \Rightarrow a\left(\frac{4}{9}\right) \times \frac{3}{\sqrt{5}}=\frac{4}{\sqrt{5}}$

$\Rightarrow \frac{4a}{3\sqrt{5}}=\frac{4}{\sqrt{5}} \Rightarrow a=3$

$\therefore c = ae = 3 \times \frac{\sqrt{5}}{3} = \sqrt{5}$

The distance between two foci $= 2c = 2\sqrt{5}$

Let $F_2(x_2, y_2)$. So, the distance

$F_1F_2 = \sqrt{(x_2-2)^2 + (y_2+3)^2} = 2\sqrt{5}$

$\Rightarrow \sqrt{(2y_2+8-2)^2 + (y_2+3)^2} = 2\sqrt{5}$

$(\because x_2 = 2y_2 + 8)$

$\Rightarrow \sqrt{4(y_2+3)^2 + (y_2+3)^2} = 2\sqrt{5}$

$\Rightarrow \sqrt{5(y_2+3)^2} = 2\sqrt{5}$

$\Rightarrow \sqrt{5}|y_2+3| = 2\sqrt{5} \Rightarrow |y_2+3| = 2$

So, $y_2 + 3 = 2$ or $y_2 + 3 = -2$

$\Rightarrow y_2 = -1$ or $y_2 = -5$

$\therefore x_2 = 2(-1) + 8 = 6$ and

$x_2 = 2(-5) + 8 = -2$

So, $F_2(6, -1)$ and $F_2(-2, -5)$

Line $2x + y - 5 = 0$

For $F_1(2, -3), 2(2) + (-3) - 5 = 4 - 8 = -4$

For $F_2(6, -1), 2(6) + (-1) - 5 = 12 - 6 = 6$

Since, -4 and 6 have opposite signs, so foci of an ellipse is $F_2(-2, -5)$.

137. **(d)** Since, the product of perpendicular distance from any point on the hyperbola to its asymptotes $= \frac{36}{13}$

So, $\frac{a^2b^2}{a^2+b^2} = \frac{36}{13}$ ....(i)

Also, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow \frac{\sqrt{13}}{3} = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow \frac{13}{9} = 1 + \frac{b^2}{a^2}$

(Squaring on both sides)

$\Rightarrow \frac{b^2}{a^2} = \frac{13}{9} - 1 = \frac{4}{9}$

$\Rightarrow b^2 = \frac{4}{9}a^2$

$\Rightarrow b = \frac{2}{3}a \; (\because a, b > 0)$

Substitute this into Eq. (i), we get

$$\frac{a^2 \cdot \left(\frac{2}{3}a\right)^2}{a^2 + \frac{4}{9}a^2} = \frac{36}{13}$$

$$\Rightarrow \frac{\frac{4}{9}a^4}{\frac{13}{9}a^2} = \frac{36}{13} \Rightarrow \frac{4}{13}a^2 = \frac{36}{13}$$

$\Rightarrow 4a^2 = 36$

$\Rightarrow a^2 = 9$

$\Rightarrow a = 3 \; (\because a > 0)$

And $b = \frac{2}{3}a = \frac{2}{3} \times 3 = 2$

Now, $a - b = 3 - 2 = 1$

138. **(b)** $AB = \sqrt{(1-0)^2 + (5-3)^2 + (6-4)^2}$

$= \sqrt{1+4+4} = \sqrt{9} = 3$

$AC = \sqrt{(-2-0)^2 + (0-3)^2 + (-2-4)^2}$

$= \sqrt{4+9+36} = \sqrt{49} = 7$

Using the angle bisector theorem,

$\frac{BD}{DC} = \frac{AB}{AC} = \frac{3}{7}$

Since, D divides BC in the ratio $3:7$, using the section formula

$D = \left(\frac{7\cdot 1+3\cdot(-2)}{7+3}, \frac{7\cdot 5+3\cdot 0}{7+3}, \frac{7\cdot 6+3\cdot(-4)}{7+3}\right)$

$\Rightarrow \left(\frac{7-6}{10}, \frac{35}{10}, \frac{42-6}{10}\right)$

$\Rightarrow \left(\frac{1}{10}, \frac{35}{10}, \frac{36}{10}\right) = (0.1, 35, 3.6)$

Now,

$AD = \sqrt{(0.1-0)^2+(3.5-3)^2+(3.6-4)^2}$

$\Rightarrow \sqrt{(0.1)^2+(0.5)^2+(-0.4)^2}$

$\Rightarrow \sqrt{0.01+0.25+0.16} = \sqrt{0.42}$

$\Rightarrow \sqrt{\frac{42}{100}} = \frac{\sqrt{42}}{10}$

139. **(a)** Given two lines $2l+m-n=0$

$\Rightarrow\ n = 2l+m$ and $l^2-2m^2+n^2=0$

$\Rightarrow l^2-2m^2+(2l+m)^2=0$

$\Rightarrow l^2-2m^2+4l^2+4lm+m^2=0$

$\Rightarrow\ 5l^2+4lm-m^2=0$

$\Rightarrow 5\left(\frac{l}{m}\right)^2+4\left(\frac{l}{m}\right)-1=$

Let $x=\frac{l}{m}, 5x^2+4x-1=0$

$\Rightarrow x = \frac{-4\pm\sqrt{4^2-4(9)(-1)}}{2(9)}$

$\Rightarrow \frac{-4\pm\sqrt{36}}{10} = \frac{-4\pm 6}{10}$

$\therefore x_1 = \frac{2}{10}$ and $x_2 = \frac{-10}{10} = -1$

Case I $\frac{l}{m} = \frac{1}{5}$

$\Rightarrow\ m = 5l$

So, $n = 2l+m = 2l+5l = 7l$

Direction cosine (l_1, m_1, n_1) are proportional to $(l, 5l, 7l)$, i.e., $(1,5,7)$

Case II $\frac{l}{m} = -1$

$\Rightarrow\ m = -l$

So, $n = 2l+m = 2l-l = l$

Direction cosine (l_2, m_2, n_2) are proportional to $(l, -l, l)$ i.e., $(1,-1,1)$

Now,

$\cos\theta = \frac{l_1l_2+m_1m_2+n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\cdot\sqrt{l_2^2+m_2^2+n_2^2}}$

$\Rightarrow \frac{1\times 1+5\times(-1)+7\times 1}{\sqrt{1^2+5^2+7^2}\cdot\sqrt{1^2+(-1)^2+1^2}}$

$\Rightarrow \frac{1-5+7}{\sqrt{1+25+49}\cdot\sqrt{1+1+!}}$

$\Rightarrow \frac{3}{\sqrt{75}\cdot\sqrt{3}} = \frac{3}{5\cdot 3} \Rightarrow \frac{3}{15} = \frac{1}{5}$

$\therefore \cos\theta = \frac{1}{5}$

140. **(c)** Let $P_1 = (2,1,2)$ and $P_2 = (1,2,1)$

So, $\mathrm{P_1P_2} = P_2 - P_1$

$\Rightarrow (1,2,1) - (2,1,2)$

$\Rightarrow (1-2,2-1,1-2$

$\Rightarrow (-1,1,-1)$

The plane $2x - y + 2z = 1$ has a normal vector $\mathrm{n}_1 = (2,-1,2)$.

Since, the required plane is perpendicular to $2x - y + 2z = 1$, its normal vector $\mathrm{n} = (a,b,c)$ must be perpendicular to n_1.

So, n must be perpendicular to $\mathrm{P_1P_2}$.

$\therefore$ n is parallel to the cross product of $\mathrm{P_1P_2}$ and n_1.

$$\mathrm{n} = \mathrm{P_1P_2} \times \mathrm{n}_1 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -1 \\ 2 & -1 & 2 \end{vmatrix}$$

$= (2-1)\hat{\imath} - (-2+2\hat{\jmath} + (1-2\hat{k}$

$= \hat{\imath} - \hat{k} = \langle 1,0,-1 \rangle$

Now, equation of the point $(2,1,2)$ and the normal vector $(1,0,-1)$ is

$1(x-2+0(y-1)-1(z-2)=0$

$\Rightarrow x-2-z+2=0$

$\Rightarrow x-z=0$

Comparing this with $ax+by+cz+d=0$, we have

$a=1, b=0, c=-1, d=0$

$$\therefore \frac{a+b}{c+d} = \frac{1+0}{-1+0} = -1$$

141. **(d)** Since, $x \to 3^-$, we have $x < 3$.

$\therefore |x| = x$ and $|3-x| = 3-x$

As, $x \to 3^-, 3x \to 9^-$, so $9 - 3x \to 0^+$

Thus, $[9-3x] = 0$ for x slightly less than 3 and $[3x-9] = -1$ for x slightly less than 3

$$\therefore \lim_{x\to 3^-} \frac{(3-x+\sin(3-x))\cos(0)}{(3-x)(-1)}$$

$$= \lim_{x\to 3^-} \frac{(3-x+\sin(3-x))}{(3-x)(-1)}$$

Let $t = 3-x$.

As, $x \to 3^-, t \to 0^+$

So, $\lim\limits_{x\to 3^-} \frac{(3-x+\sin(3-x))}{(3-x)(-1)}$

$$\Rightarrow \lim_{t\to 0^+} \frac{t+\sin t}{-t}$$

$$\Rightarrow \lim_{t\to 0^+} \left(-1 - \frac{\sin t}{t}\right)$$

$$\Rightarrow -1-1 \left[\because \lim_{t\to 0} \frac{\sin t}{t} = 1\right]$$

$\Rightarrow -2$

$$\therefore \lim_{x\to 3^-} \frac{(3-|x|+\sin|3-x|)\cos[9-3x]}{|3-x|[3x-9]}$$

$= -2$

142. **(a)** Given,

$$f(x) = \begin{cases} \dfrac{6^x - 3^x - 2^x + 1}{1-\cos\left(\frac{x}{a}\right)}, & x \neq 0 \\ \log 3 \log 4, & x = 0 \end{cases}$$

Now, $f(0) = \log 3 \log 4$

Now, $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \frac{6^x-3^x-2^x+1}{1-\cos\left(\frac{x}{a}\right)}$

$$\Rightarrow \lim_{x\to 0} \frac{(3^x-1)(2^x-1)}{1-\cos\left(\frac{x}{a}\right)}$$

$$\Rightarrow \lim_{x\to 0}\left[\frac{\frac{3^x-1}{x}\cdot\frac{2^x-1}{x}}{\frac{1-\cos\left(\frac{x}{a}\right)}{x^2}}\right]$$

$$\Rightarrow \frac{(\ln 3)(\ln 2)}{\frac{1}{(2a^2)}}\left[\because \lim_{x\to 0}\frac{a^x-1}{x}=\ln a\right.$$

$$\left.\text{and } \lim_{x\to 0}\frac{1-\cos kx}{x^2}=\frac{k^2}{2}\right]$$

$= 2a^2(\ln 3)\ln(2)$

Since, given function is continuous

So, $\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow\ 2a^2(\ln 3)(\ln 2) = (\ln 3)(\ln 4)$

$\Rightarrow\ (\ln 3)(\ln 2)^2 = 2(\ln 3)(\ln 2)$

$\Rightarrow\ 2a^2 = 2 \Rightarrow a^2 = 1$

$\Rightarrow\ a = 1$ ($\because$ a is a positive real number)

143. **(a)** Given, $f(x) = \sqrt{\cos^{-1}\sqrt{1-x^2}}$

Let $x = \sin\theta$.

Then, $\sqrt{1-x^2} = \sqrt{1-\sin^2\theta}$

$\Rightarrow\ \sqrt{\cos^2\theta} = |\cos\theta|$

Since, $\theta \in \left[0, \frac{\pi}{2}\right]$, so $\cos\theta \geq 0$

$\therefore \sqrt{1-x^2} = \cos\theta$

So, $f(x) = \sqrt{\cos^{-1}(\cos\theta)}$

$\Rightarrow\ \sqrt{\theta} = \sqrt{\sin^{-1}(x)}$

$$f'(x) = \frac{d}{dx}\sqrt{\sin^{-1}(x)}$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}\cdot\frac{1}{2\sqrt{\sin^{-1}(x)}}$$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}\cdot\frac{1}{2\cdot\sqrt{\sin^{-1}\frac{1}{2}}}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}}\cdot\frac{1}{2\cdot\sqrt{\frac{\pi}{6}}}$$

$$\Rightarrow \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{6}}{2\sqrt{\pi}} \Rightarrow \frac{\sqrt{2}}{\sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$$

$$\therefore\ f'(x) = \sqrt{\frac{2}{\pi}}$$

144. **(a)** Given, $y = f(\cosh x)$ and

$f'(x) = \log\left(x+\sqrt{x^2-1}\right)$

Let $u = \cosh x$, so $y = f(u)$

$$\therefore \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= f'(u)\cdot\frac{d}{dx}(\cosh x)$$

$= f'(\cosh x)\cdot\sinh x$

Since, $f'(x) = \log\left(x+\sqrt{x^2-1}\right)$

$\therefore f'(\cosh x) = \log\left(\cosh x + \sqrt{\cosh h^2 x - 1}\right)$
$= \log(\cosh x + |\sinh x|)$
Since, $x > 0, \sinh > 0$
So, $f'(\cosh x) = \log(\cosh x + \sinh x)$
$\Rightarrow \log e^x = x\ [\because \cosh x + \sinh x = e^x]$
So, $\frac{dy}{dx} = x \cdot \sinh x$
$\frac{d^2y}{dx^2} = x \cdot \frac{d}{dx}(\sinh x) + \sinh x \cdot \frac{d}{dx}(x)$
$\Rightarrow x \cdot \cosh x + \sinh x \cdot 1$
$\Rightarrow x\cosh x + \sinh x$

145. **(b)** Given equation
$(x^2 - 3x + 2)e^{\frac{y}{x-1}} = x + 2$
Differentiate both sides w.r.t., we get
$\frac{d}{dx}\left((x^2 - 3x + 2) \cdot e^{\frac{y}{x-1}}\right) = \frac{d}{dx}(x + 2)$
$\Rightarrow (2x - 3)e^{\frac{y}{x-1}} + (x^2 - 3x + 2) \cdot e^{\frac{y}{x-1}}$
$\cdot \frac{d}{dx}\left(\frac{y}{x-1}\right) = 1$
$\Rightarrow (2x - 3)e^{\frac{y}{x-1}} + (x^2 - 3x + 2) \cdot e^{\frac{y}{x-1}}$
$\left[\frac{\frac{dy}{xt}(x-1) - y}{(x-1)^2}\right] = 1$
Now, substitute $x = 0$ into the given equation
$(0^2 - 3(0) + 2)e^{\frac{y}{(0-1)}} = 0 + 2$
$\Rightarrow 2e^{-y} = 2 \Rightarrow e^{-y} = 1$
$\Rightarrow -y = \ln(1) \Rightarrow -y = 0$
$\Rightarrow y = 0$
Substitute $x = 0$ and $y = 0$ in Eq. (i), we get
$(2(0) - 3)e^{\frac{0}{0-1}} + (0^2 - 3(0) + 2)e^{\frac{0}{0-1}}$
$\left[\frac{\frac{dy}{dx}(0-1) - 0}{(0-1)^2}\right] = 1$
$\Rightarrow -3e^0 + 2e^0\left(\frac{-dy}{dx}\right) = 1$
$\Rightarrow -3 + 2\left(\frac{-dy}{dx}\right) = 1$
$\Rightarrow -2\frac{dy}{dx} = 1 + 3 = 4 \Rightarrow \frac{dy}{dx} = \frac{4}{-2} = -2$
$\therefore \left(\frac{dy}{dx}\right)_{x=0} = -2$

146. **(b)** Given $x = \frac{t^2}{1+t^3}, y = \frac{2t^3}{1+t^5}$ and
$t \neq -1$
So, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{4t}{dt}}$
Now, $\frac{dx}{dt} = \frac{(2t)(1+t^5) - t^2 5t^4}{(1+t^5)^2}$
$= \frac{2t - 3t^6}{(1+t^5)^2}$
And, $\frac{dy}{dt} = \frac{(6t^2)(1+t^5) - (2t^3)(5t^4)}{(1+t^5)^2}$

$= \dfrac{6t^2 - 4t^7}{(1+t^5)^2}$

So, $\dfrac{dy}{dx} = \dfrac{\left[\frac{6t^2-4t^7}{(1+t^5)^2}\right]}{\left[\frac{(2t-3t^9)}{(1+t^5)^2}\right]} = \frac{6t^2-4t^7}{2t-3t^6}$

$= \dfrac{2t^2(3-2t^5)}{t(2-3t^5)} = \dfrac{2t(3-2t^5)}{(2-3t^5)}$

147. **(a)** Set the equations equal to each other

$3x^2 - 2x - 1 = x^3 - 1$

$\Rightarrow x^3 - 3x^2 + 2x = 0$

$\Rightarrow x(x-1)(x-2) = 0$

$\Rightarrow x = 0, x = 1$ and $x = 2$

For $x = 0, y = 0^3 - 1 = -1$, Point : $(0,-1)$

For $x = 1, y = 1^3 - 1 = 0$, Point : $(1,0)$

For $x = 2, y = 2^3 - 1 = 7$, Point : $(2,7)$

So, the point of intersection in the first quadrant is $(2,7)$.

For, $y_1 = 3x^2 - 2x - 1$

So, $m_1 = \frac{dy_1}{dx} = 6x - 2$

At $(2,7), m_1 = 6(2) - 2 = 12 - 2 = 10$

For $y_2 = x^3 - 1, m_2 = \frac{dy_2}{dx} = 3x^2$

At $(2,7), m_2 = 3(2^2) = 12$

$\therefore \tan\theta = \left|\dfrac{m_1 - m_2}{1 + mm2}\right|$

$\Rightarrow \left|\dfrac{10-12}{1+(10)(12)}\right|$

$\Rightarrow \left|\dfrac{-2}{1+120}\right| \Rightarrow \left|\dfrac{-2}{121}\right|$

$\Rightarrow \tan\theta = \dfrac{2}{121} \Rightarrow \theta = \tan^{-1}\left(\dfrac{2}{121}\right)$

Thus, the acute angle between the curves at their point of intersection in the first quadrant is $\tan^{-1}\left(\frac{2}{121}\right)$.

148. Given, curve $y = x^3 - 2x^2 + 3x - 2$

So, slope of tangent, $m = \frac{dy}{dx}$

$= 3x^2 - 4x + 3$

Now, the rate of change of the slope of tangent = second derivative $\frac{d^2y}{dx^2}$

$\Rightarrow \dfrac{d}{dx}(3x^2 - 4x + 3) = 6x - 4$

At point (2, 4),

$\frac{d^2y}{dx^2} = 6 \times 2 - 4 = 8$ ...(i)

Now, the abscissa is x. So, the rate of change of the abscissa is $\frac{dx}{dt}$

Now, given $\frac{d}{dt}\left(\frac{dy}{dx}\right) = k\frac{dx}{dt}$

$\Rightarrow \dfrac{d^2y}{dx^2} \cdot \dfrac{dx}{dt} = k\dfrac{dx}{dt} \Rightarrow \dfrac{d^2y}{dx^2} = k$

$\Rightarrow k = 8$ [using Eq. (i)]

149. **(b)** Let $f(x) = \sec(x)$

Since 58° is close to 60° and $\sec(60^\circ) = 2$

Also, $60^\circ = 60 \times 0.0175$ radians $= 1.05$

Now, $f(x) = \sec x$

$\Rightarrow f'(x) = \sec(x)\tan(x)$

$f'(60^\circ) = \sec(60^\circ)\tan(60^\circ)$

$= 2 \times \sqrt{3} = 2\sqrt{3}$
And, change in angle,
$\Delta x = 58^\circ - 68^\circ = -2^\circ$
$= -2 \times 0.0175$ radians
$= -0.035$ radians
Now, $\Delta y \approx f'(x) \cdot \Delta x$
$\approx 2\sqrt{3} \times (-0.035)$
$\approx 2 \times 1.732 \times (-0.035)$
≈ -0.12124
So, $\sec(58^\circ) \approx f(60^\circ) + \Delta y$
$\Rightarrow\ 2 - 0.12124 \approx 1.87876$
So, the approximate value of $\sec(58^\circ)$ is 1.8788.

150. **(b)** Given, $f(x) = x + \log\left(\frac{x-1}{x+1}\right)$
For, $\log\left(\frac{x-1}{x+1}\right)$ to be well-defined, $\frac{x-1}{x+1} > 0$.
This inequality holds when
$\Rightarrow$ Both $x - 1 > 0$ and $x + 1 > 0 \Rightarrow x > 1$
$\Rightarrow$ Both $x - 1 < 0$ and $x + 1 < 0 \Rightarrow x < -1$
So, the domain of $f(x)$ is $(-\infty, -1) \cup (1, \infty)$
Now, $f'(x) = \frac{d}{dx}\left[x + \log\left(\frac{x-1}{x+1}\right)\right]$
$$\Rightarrow 1 + \frac{(x+1)}{(x-1)} \cdot \frac{(x+1)\cdot 1 - 1\cdot(x-1)}{(x+1)^2}$$
$$\Rightarrow 1 + \frac{(x+1)-(x-1)}{(x-1)(x+1)}$$
$$\Rightarrow 1 + \frac{2}{(x-1)(x+1)}$$
Since, $x \in (-\infty, -1) \cup (1, \infty)$, we have $x^2 > 1$
$\Rightarrow x^2 - 1 > 0$
Thus, $\frac{2}{x^2-1} > 0$
So, $f'(x) = 1 + \frac{2}{x^2-1} > 1 + 0 = 1$
Since, $f'(x) > 0$ for all x in the domain, the function $f(x)$ is monotonically increasing.

151. **(d)** Given, $f(x) = |x^2 - 3x + 2| + 2x - 3$ is defined on $[-2,1]$
$x^2 - 3x + 2 = (x-1)(x-2)$
So, on the interval $[-2,1]$
If $x \le 1$, then $x - 1 \le 0$
If $x \le 2$ then $x - 2 \le 0$
$\therefore$ On $[-2,1]$, $(x-1)(x-2) \ge 0$
So, $|x^2 - 3x + 2| = x^2 - 3x + 2$ for $x \in [-2,1]$
$\therefore f(x) = (x^2 - 3x + 2) + 2x - 3$
$= x^2 - x - 1$
Now, $f'(x) = 2x - 1$
For, critical points, set $f'(x) = 0$ we get
$2x - 1 = 0$
$$\Rightarrow x = \frac{1}{2} \in [-2,1]$$
Now, at $x = -2 f(-2) = (-2)^2 - (-2) - 1$
$\Rightarrow 4 + 2 - 1 = 5$
At $x = \frac{1}{2}, f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 1$
$$\Rightarrow \frac{1}{4} - \frac{1}{2} - 1 \Rightarrow \frac{1-2-4}{4} = -\frac{5}{4}$$

At $x = 1, f(1) = 1^2 - 1 - 1 = -1$

So, the absolute minimum value, $m = \frac{-5}{4}$ and the absolute maximum value, $M = 5$

$\therefore M - 4m = 5 - 4 \times \left(-\frac{5}{4}\right)$

$\Rightarrow\ 5 + 5 = 10$

152. **(c)** $I = \int \frac{2\sin x - 3\cos x}{4\cos x - 3\sin x} dx$

Let $t = 4\cos x - 3\sin x$

$\Rightarrow \frac{dt}{dx} = -4\sin x - 3\cos x$

Now, $2\sin x - 3\cos x$

$\Rightarrow A(-4\sin x - 3\cos x) + B(4\cos x - 3\sin x)$

$\Rightarrow (-4A - 3B)\sin x + (-3A + 4B)\cos x$

Comparing the coefficients, we get

$-4A - 3B = 2$ and $-3A + 4B = -3$

Solving these two equations, we get

$A = \frac{1}{25}$ and $B = \frac{-18}{25}$

So, $I = \int \frac{2\sin x - 3\cos x}{4\cos x - 3\sin x} dx$

$\Rightarrow \frac{1}{25} \int \frac{-4\sin x - 3\cos x}{4\cos x - 3\sin x} dx + \left(\frac{-18}{25}\right) \int \frac{4\cos x - 3\sin x}{4\cos x - 3\sin x} dx$

$\Rightarrow \frac{1}{25} \int \frac{dt}{t} - \frac{18}{25} \int dx \Rightarrow \frac{1}{25} \log|t| - \frac{18}{25} x + C$

$\Rightarrow \frac{1}{25} \log|4\cos x - 3\sin x| - \frac{18}{25} x + C$

$= \frac{1}{25} [\log|4\cos x - 3\sin x| - 18x] + C$

153. **(b)** $\int e^{4x}(\sin 3x - \cos 3x) dx$

$= \int e^{4x} \sin 3x dx - \int e^{4x} \cos 3x dx$

Using the standard integrals

$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$

$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx)$

Here, $a = 4, b = 3$

So, $\int e^{4x}(\sin 3x - \cos 3x) dx$

$= \frac{e^{4x}}{4^2 + 3^2}(4\sin 3x - 3\cos 3x) - \frac{e^{4x}}{4^2 + 3^2}(4\cos 3x + 3\sin 3x) + C$

$\Rightarrow \frac{e^{4x}}{25}[4\sin 3x - 3\cos 3x - 4\cos 3x - 3\sin 3x] + C$

$\Rightarrow \frac{e^{4x}}{25}[\sin 3x - 7\cos 3x] + C$

154. **(c)** $I = \int \left(\frac{1 - \log x}{1 + (\log x)^2}\right)^2 dx$

Let $y = \log x$. Then, $x = e^y \Rightarrow dx = e^y \cdot dy$

So, $I = \int \left(\frac{1-y}{1+y^2}\right)^2 e^y dy$

$= \int \left(\frac{1 + y^2 - 2y}{(1 + y^2)^2}\right) e^y dy$

$\Rightarrow \int \left(\frac{1}{1 + y^2} - \frac{2y}{(1 + y^2)^2}\right) e^y dy$

Since, $\int (f(y)+f'(y))e^y dy = f(y)e^y + C$

Here, $f(y)=\frac{1}{1+y^2}$ and $f'(y)=\frac{-2y}{(1+y^2)^2}$

$\therefore\ I = f(y)e^y + C = \frac{1}{1+y^2}e^y + C$

$= \frac{1}{1+(\log x)^2}\cdot e^{\log x} + C$

$= \frac{x}{1+(\log x)^2} + C$

155. **(b)** Given, $\int (x+2)\sqrt{x^2-x+2}dx$

$\Rightarrow\ \frac{1}{3}f(x)+\frac{5}{8}g(x)+\frac{35}{16}h(x)+C$

Let $x+2=A(2x-1)+B$

Comparing coefficients, we get

$2A=1$

$\Rightarrow\ A=\frac{1}{2}, -A+B=2$

$\Rightarrow\ B=2+A=2+\frac{1}{2}=\frac{5}{2}$

So, $x+2=\frac{1}{2}(2x-1)+\frac{5}{2}$

$\int (x+2)\sqrt{x^2-x+2}dx$

$= \int \left(\frac{1}{2}(2x-1)+\frac{5}{2}\right)\sqrt{x^2-x+2}dx$

$= \frac{1}{2}\int (2x-1)\sqrt{x^2-x+2}dx + \frac{5}{2}\int \sqrt{x^2-x+2}dx$

Let $u=x^2-x+2$, then $du=(2x-1)dx$

$= \frac{1}{2}\int \sqrt{u}du + \frac{5}{2}\int \sqrt{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}\,dx$

$= \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{5}{2}\left[\frac{2x-1}{4}\sqrt{x^2-x+2}+\frac{7}{8}\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|\right]+c$

$= \frac{1}{3}(x^2-x+2)^{\frac{1}{2}} + \frac{5(2x-1)}{8}\sqrt{x^2-x+2}+\frac{35}{16}\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|+c$

Comparing with original equation, we get

$f(x)=(x^2-x+2)^{\frac{3}{2}}$

$g(x)=(2x-1)\sqrt{x^2-x+2}$

$h(x)=\ln\left|x-\frac{1}{2}+\sqrt{x^2-x+2}\right|$

So, $f(-1)=[(-1)^2-(-1)+2]^{\frac{3}{2}}$

$=[1+1+2]^{\frac{3}{2}}=4^{\frac{3}{2}}=8$

$g(-1)=(2(-1)-1)\sqrt{(-1)^2-(-1)+2}$

$=-3\sqrt{1+1+2}=-3\sqrt{4}$

$=-3\times 2=-6$

$h\left(\frac{1}{2}\right)=\ln\left|\frac{1}{2}-\frac{1}{2}+\sqrt{\left(\frac{1}{2}\right)^2-\frac{1}{2}+2}\right|$

$\Rightarrow \ln\left|0+\sqrt{\frac{1}{4}-\frac{1}{2}+2}\right|$

$$\Rightarrow \ln\left|\sqrt{\frac{1-2+8}{4}}\right| \Rightarrow \ln\left|\sqrt{\frac{7}{4}}\right| = \ln\left(\frac{\sqrt{7}}{2}\right)$$

So, $f(-1)+g(-1)+h\left(\frac{1}{2}\right) = 8-6+\ln\left(\frac{\sqrt{7}}{2}\right)$

$= 2+\ln\left(\frac{\sqrt{7}}{2}\right)$

156. **(a)** $I = \int_0^2 \sqrt{(x+3)(2-x)}dx$

$$\Rightarrow \int_0^2 \sqrt{-x^2-x+6dx}$$

$$\Rightarrow \int_0^2 \sqrt{-(x^2+x-6)}dx$$

$$\Rightarrow \int_0^2 \sqrt{\frac{25}{4}-\left(x+\frac{1}{2}\right)^2}\,dx$$

Put $u = x+\frac{1}{2} \Rightarrow du = dx$

When $x=0, u=\frac{1}{2}$ and $x=2$

$u = 2+\frac{1}{2} = \frac{5}{2}$

$$I = \int_{\frac{1}{2}}^{\frac{5}{2}} \sqrt{\left(\frac{5}{2}\right)^2 - u^2du}$$

$$\Rightarrow \left[\frac{u}{2}\sqrt{\frac{25}{4}-u^2}+\frac{\frac{25}{4}}{2}\sin^{-1}\left(\frac{u}{\frac{5}{2}}\right)\right]_{\frac{1}{2}}^{\frac{5}{2}}$$

$$\Rightarrow \left[\frac{u}{2}\sqrt{\frac{25}{4}-u^2}+\frac{25}{8}\sin^{-1}\left(\frac{2u}{5}\right)\right]_{\frac{1}{2}}^{\frac{5}{2}}$$

$$\Rightarrow \left\{\frac{\frac{5}{2}}{2}\sqrt{\frac{25}{4}-\frac{25}{4}}+\frac{25}{8}\sin^{-1}(1)\right\} - \left\{\frac{1}{4}\sqrt{\frac{25}{4}-\frac{1}{4}}+\frac{25}{8}\sin^{-1}\left(\frac{1}{5}\right)\right\}$$

$$\Rightarrow 0+\frac{25}{8}\cdot\frac{\pi}{2}-\frac{1}{4}\sqrt{6}-\frac{25}{8}\sin^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \frac{25\pi}{16}-\frac{\sqrt{6}}{4}-\frac{25}{8}\sin^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \frac{25}{8}\left(\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{5}\right)\right)-\frac{\sqrt{6}}{4}$$

$$\Rightarrow \frac{25}{8}\cos^{-1}\left(\frac{1}{5}\right)-\frac{\sqrt{6}}{4}$$

157. **(c)** $\int_0^{\frac{\pi}{4}} x^2\sin 2x dx, I = \int x^2 \sin 2x dx$

Put $t = 2x \Rightarrow dt = 2\cdot dx$

So, $I = \int \frac{t^2\sin t}{8}dt = \frac{1}{8}\int t^2 \sin t dt$

$$= \frac{1}{8}\left[t^2\cdot(-\cos t) - 1\cdot(-2)\cdot\int t\cos t dt\right]$$

$$= \frac{1}{8}\left[-t^2\cos t + 2\int t\cdot\cos t dt\right]$$

$$= \frac{1}{8}[-t^2\cos t + 2(t\cdot\sin t - (-\cos t))]$$

$= \frac{1}{8}[-t^2\cos t + 2t\sin t + 2\cos t]$

$= \frac{1}{8}[(2x)^2\cos(2x) + 2(2x \cdot \sin(2x)) + 2\cos(2x)]$

$= \frac{-x^2\cos(2x) + x\sin(2x)}{2} + \frac{\cos(2x)}{4}$

So, $\int_0^{\frac{\pi}{\pi}} x^2\sin 2x dx$

$= \left[\frac{-x^2\cos(2x) + x\sin(2x)}{2} + \frac{\cos(2x)}{4}\right]_0^{\frac{\pi}{4}}$

$= \left\{\frac{-\left(\frac{\pi}{4}\right)^2 \cos\left(2 \times \frac{\pi}{4}\right) + \frac{\pi}{4}\sin\left(2 \times \frac{\pi}{4}\right)}{2} + \frac{\cos\left(\frac{2\pi}{4}\right)}{4}\right\} - \left\{\frac{0}{2} + \frac{\cos(0)}{4}\right\}$

$\Rightarrow \left(0 + \frac{\pi}{8} \cdot 1 + \frac{1}{4} \cdot 0\right) - \left(0 + \frac{1}{4}\right)$

$= \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}$

158. **(d)** $\int_{-2\pi}^{2\pi} \sin^4 x\cos^6 x dx$

$f(x) = \sin^4 x\cos^6 x$

Since, $\sin x$ is an odd function and $\sin^2 x$ is an even function and $\cos^6 x$ is an even function.

So, $f(x)$ is an even function.

$\therefore \int_{-2\pi}^{2\pi} \sin^4 x\cos^6 x dx = 2\int_0^{2\pi} \sin^4 x\cos^6 x dx$

$\left[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx\right]$

$= 2 \times 2\int_0^{\pi} \sin^4 x\cos^6 x dx$

$\left[\because \int_0^{n\pi} f(x)dx = n\int_0^T f(x)dx\right]$

$= 4\int_0^{\pi} \sin^4 x\cos^6 x dx$

$= 4 \times 2\int_0^{\frac{\pi}{2}} \sin^4 x\cos^6 x dx$

$\left[\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx\right]$

$= 8\int_0^{\frac{\pi}{2}} \sin^4 x\cos^6 x dx$

[$\because$ If m and n are both even, then

$\int_0^{\frac{\pi}{2}} \sin^m x\cos^n x dx =$

$\left.\frac{[(m-1)(m-3).2 \text{ or } 1][(n-1)(n-3) \ldots .2 \text{ or } 1]}{[(m+n)(m+n-2) \ldots .2 \text{ or } 1]} \times \frac{\pi}{2}\right]$

$= 8 \times \frac{3 \times 1 \times 5 \times 3 \times 1}{10 \times 8 \times 6 \times 4 \times 2} \times \frac{\pi}{2}$

$= \frac{3\pi}{64}$

159. **(a)** Given, $\cos x\frac{dy}{dx} = y\sin x - 1$,

$x \neq (2n+1)\frac{\pi}{2}, n \in Z$

$\Rightarrow \frac{dy}{dx} = \frac{y\sin x}{\cos x} - \frac{1}{\cos x}$

$= y\tan x - \sec x$

$\Rightarrow \dfrac{dy}{dx} - y\tan x = -\sec x$

This is in the form $\frac{dy}{dx} + P(x)y = Q(x)$, where, $P(x) = -\tan x$ and $Q(x) = -\sec x$

IF $= e^{\int P(x)dx} = e^{\int -\tan x dx}$

$= e^{\ln|\cos x|} = |\cos x$

So, the general solution is

$y \cdot \mathbb{F} = \int A(x) \cdot \mathrm{IF} dx + C$

$y \cdot \cos x = \int -\sec x \cdot \cos x dx + C$

$= -\int dx + C = -x + Cv$

$= y = \dfrac{-x + C}{\cos x}$

Given, $f(0) = 1$

$\Rightarrow y = 1$ and $x = 0$

So, $1 = \frac{-0+C}{\cos(0)}$

$\Rightarrow 1 = C$

So, $y = \frac{-x+1}{\cos x} = (1 - x)\sec x$

160. **(b)** Given, differential equation

$2dx + dy = (6xy + 4x - 3y)dx$

$\Rightarrow dy = (6xy + 4x - 3y - 2)dx$

$\Rightarrow \dfrac{dy}{dx} = 6xy + 4x - 3y - 2$

$\Rightarrow \dfrac{dy}{dx} = 6xy + 4x - 2 - 3y$

$\Rightarrow 2x(3y + 2) - 1(3y + 2)$

$\Rightarrow (3y + 2)(2x - 1)$

$\Rightarrow \dfrac{dy}{3y + 2} = dx(2x - 1)$

Integrating to both sides w.r.t x, we get

$\int \dfrac{dy}{3y + 2} = \int (2x - 1)dx$

$\Rightarrow \dfrac{1}{3}\log|3y + 2| = \dfrac{2x^2}{2} - x + C_1$

$\Rightarrow \dfrac{1}{3}\log|3y + 2| = x^2 - x + C_1$

$\Rightarrow \log|3y + 2| = 3x^2 - 3x + 3C_1$

$= 3x^2 - 3x + C$

(where $C = 3C_1$)

So, $\log|3y + 2| = 3x^2 - 3x + C$