PART-I PHYSICS

1. **(b)** $V = 300\ \text{V}, C_1 = 2.0\mu\text{F}, C_2 = 8.0\mu\text{F}$,

Net capacitance, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\Rightarrow C_s = \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow C_s = \frac{2\times8}{2+8} = \frac{16}{10} = 1.6\mu\text{F}.$$

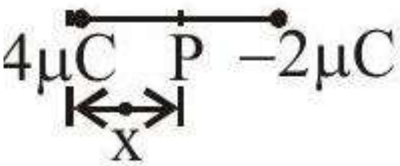
Now total charge,

$Q = V_s \times C_s = 300 \times 1.6 \times 10^{-6} = 4.8 \times 10^{-4}\text{C}.$

$\because$ In series charge is same on capacitors

$\Rightarrow$ Charge on $2\mu\text{F}$ capacitor is $4.8 \times 10^{-4}\text{C}$

2. **(a)**



Let the point P where resultant field is zero be xm from $4\mu\text{C}$ charge and $(1 - x)$ m distance apart from $-2\mu\text{C}$ charge. Since field is zero at this point then,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 0$$

$$\Rightarrow |\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2^2}$$

$$\Rightarrow 0 = \frac{1}{4\pi\epsilon_0}\left[\frac{4\mu C}{x^2} + \frac{(-2\mu C)}{(1-x)^2}\right]$$

$$\Rightarrow \frac{4\mu C}{x^2} = \frac{2\mu C}{(1-x)^2} \Rightarrow \frac{2}{x^2} = \frac{1}{(1-x)^2}$$

$$\Rightarrow 2(1-x)^2 = x^2$$

Taking root $\sqrt{2}(1 - x) = x$

$$\Rightarrow 1.414(1 - x) = x \Rightarrow 1.414 - 1.414x = x$$

$$\Rightarrow 1.414 = (1 + 1.414)x \Rightarrow x = \frac{1.414}{2.414}$$

$$\Rightarrow x = 0.58\ \text{m}$$

3. **(b)** The net electric potential is algebraic sum of potential due to individual point charges.

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times10^{-6}}{0.2} - \frac{2\times10^{-6}}{0.2} + \frac{3\times10^{-6}}{0.3}\right]$$

$$= 9\times10^{9}[5-10+10]\times10^{-6}$$

$$= 9\times10^{3}[5] = 45\times10^{3}\ \mathrm{V} = 45\mathrm{kV}$$

4. **(a)** In a metal, conduction current is due to electrons given by

$$I = nAev$$

$$\Rightarrow \text{drift velocity}, v = \frac{I}{nAe}$$

$$\Rightarrow v = \frac{5}{5\times10^{26}\times4\times10^{-6}\times1.602\times10^{-19}}$$

$$= \frac{1}{4\times1.602\times10^{1}}$$

$$= \frac{10^{-1}}{6.408} = 1.56\times10^{-2}\ \mathrm{m/s}$$

5. **(b)** In series combination of capacitors, charges on both capacitors will be same.

$$V_s = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\Rightarrow 1000 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\Rightarrow 1000 = Q\left(\frac{C_1 + C_2}{C_1C_2}\right)$$

$$\Rightarrow Q = \frac{1000\times C_1C_2}{C_1 + C_2}$$

$$Q = 1000\times\frac{3\times10^{-12}\times6\times10^{-12}}{(3+6)\times10^{-12}} = \frac{18}{9}\times10^{-9}$$

$$= 2\times10^{-9}\mathrm{C}$$

6. **(c)** Parallel combination of 3Ω and 6Ω gives effective resistance,

$R_p = \frac{3\times6}{3+6} = \frac{18}{9} = 2\Omega$. This in series with

2Ω gives net ressistance as 4Ω.

7. **(b)** The value of temperature coefficient of resistance is given by

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{65-50}{50(70-20)}$$

(t_1 and t_2 are in °C)

$$\Rightarrow \alpha = \frac{15}{50\times50} = 0.006/\,^{\circ}\mathrm{C}$$

8. **(b)** In Leclanche cell a strong solution of ammonium chloride acts as an electrolyte.

9. **(d)** In the galvanometer, I_g = max. current through galvanometer, S = shunt resistance, G = galvanometer resistance then

$I_g(\text{ galvanometer }) = I\left(\frac{S}{G+S}\right)$

$$\Rightarrow I = I_g\left(\frac{G+S}{S}\right) = \left(\frac{50+1}{1}\right)I_g = 51I_g$$

10. **(a)** Current in tangent galvanometer

$$I = \frac{H}{G}\tan\theta$$

Where G = galvanometer constant H = earth's horizontal field = constant

$$\therefore \frac{I_1}{I_2} = \frac{\tan\theta_1}{\tan\theta_2} \Rightarrow \frac{1}{I_2} = \frac{\tan 30^\circ}{\tan 60^\circ}$$

$$\Rightarrow I_2 = \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{1.7321}{0.5774} = 2.999\text{Å}$$

$$\Rightarrow I_2 \simeq 3\text{ A}$$

11. **(c)** Lorentz force on a charged particle in presence of magnetic and electic field is

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\Rightarrow \vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

12. **(a)** In a circular coil of n turns, magnetic field is

$B = \frac{\mu_0 nI}{2r} = \frac{4\pi\times10^{-7}\times250\times20\times10^{-3}}{2\times40\times10^{-3}}$ ($\because$ n = no. of turns, I = current through coil, r = radius of coil)

$$\Rightarrow B = \frac{4\pi \times 250 \times 20 \times 10^{-7-3+3}}{2 \times 40}$$

$$= 250 \times 3.14 \times 10^{-7}$$

$$= 785 \times 10^{-7} = 0.785 \times 10^{-4}\text{ tesla}$$

$$= 0.785\text{ gauss}$$

13. **(d)** RMS value of A.C is

$$I_v = \frac{I_0}{\sqrt{2}} = 0.707I_0$$

I_0 = peak value

$\therefore$ it is 70.7% of peak value.

14. **(a)** Q-factor is given by $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\therefore$ If resistance R is decreased, Q increases and inductance L is increased, Q increases.

15. **(c)** Induced emf $e = \frac{-d\phi}{dt}$. Assuming, small change in flux $d\phi = 8 \times 10^{-4}$ Wb change in time dt = 0.5 s

$|e| = \frac{8\times10^{-4}}{0.5} = \frac{80\times10^{-4}}{5} = 16\times10^{-4} = 1.6\times10^{-3}\ \text{V} = 1.6\text{mV}$

16. (c) Current through an LCR circuit is maximum when impedance is minimum. Now impedance

$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ is minimum at resonance frequency when $\omega L = \frac{1}{\omega C}$ and $Z = R =$ minium i.e., inductive reactance (ωL) is equal to capacitive reactance $(1/\omega C)$

17. (a) Our eyes respond to visible range from 400 nm to 700 nm

18. (a) Velocity of electromagnatic wave in space is $c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \Rightarrow c = \frac{1}{\sqrt{2\times8}} = \frac{1}{\sqrt{16}} = \frac{1}{4} = 0.25$

19. (b) According to Brewster's law, reflected light is plane polarised if unpolarised light falls at the interface of air and medium at an angle i_p called polarising angle then

$\mu = \tan i_p \Rightarrow \mu(\text{ glass }) = \tan 60^\circ = \sqrt{3}$

20. (a) Newton's ring arrangement is used for determining the wavelength of monochromatic light. For this the diameter of n^{th} dark ring (D_n) and $(n+p)^{\text{th}}$ dark ring $\left(D_{n+p}\right)$ are measured then

$D_{(n+p)}^2 = 4(n+p)\lambda R$ and $D_n^2 = 4n\lambda R$

$$\Rightarrow \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Here, $n = 10, n+p = 20$;

$\therefore p = 10; R = 1\ \text{m}, D_{10} = 3.96\times10^{-3}\ \text{m}, D_{20} = 5.82\times10^{-3}\ \text{m}$

$$\therefore \lambda = \frac{D_{20}^2 - D_{10}^2}{4pR}$$

$$= \frac{(5.82\times10^{-3})^2 - (3.36\times10^{-3})^2}{4\times10\times1}$$

$= 5646\text{Å}$

21. (b) Angular momentum in any stationary orbit

is $mvr = \frac{nh}{2\pi}$ for 4^{th} orbit, $n = 4$

$$\Rightarrow mvr = \frac{4\ h}{2\pi}$$

22. (d) According to Bohr's, Brackett series is obtained when an electron jumps to 4^{th} orbit from any other outer orbit

23. (a) Total energy of electron

$E = -\frac{KZe^2}{2r} = \text{K.E.} + \text{P.E.}$

Potential energy in the orbit $\text{P.E.} = \frac{-KZe^2}{r}$

$\therefore \text{P.E.} = 2\times E \Rightarrow \text{P.E.} = 2\times(-13.6) = -27.2\text{eV}$

24. (a) Bragg's condition is $2\ d\sin\theta = n\lambda$, for second order $n = 2, \sin\theta = 1$. For longest $\lambda, 2d = 2\lambda \Rightarrow \lambda = d$

25. (b) Photon moves with speed of light ie, $v = c$ and rest mass of a particle is $m_0 = m\sqrt{1 - v^2/c^2}$

hence m_0 (photon) $= 0$

$\therefore$ photon has zero rest mass.

Momentum of photon $= \frac{h}{\lambda}$

26. (c) de Broglie wavelength is given by $\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}}{1000\times10^{-3}\times1} \simeq 6.626\times10^{-34}$ m

27. (b) Let the velocity of a particle be v where mass m is double the rest mass i.e., $m = 2\ m_0$ then

$$m_0 = m\sqrt{1-\frac{v^2}{c^2}} \Rightarrow m_0 = 2\ m_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{2} = \sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{1}{4} = 1-\frac{v^2}{c^2}$$

$$\Rightarrow \frac{v^2}{c^2} = 1-\frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow v = \sqrt{\frac{3}{4}}c$$

28. (d) By Einstein's equation $E = mc^2$ where $m = 2$ kg

$\Rightarrow E = 2\times(3\times10^8)^2$

$= 2\times3\times3\times10^{16} = 18\times10^{16}$ J

29. (b) By definition, the difference between the sum of the masses of neutrons and protons forming a nucleus and mass of nucleus is called mass defect

30. (b) Mean life time $\tau = 1.44$ T where T is half life period of an atom

$\therefore \tau = 1.44\ T = \frac{T}{0.6931} = \frac{3}{0.6931}$ minute

31. (d) (by conservation of charge)

32. (d) Baryons are proton, neutron, lamda,

sigma$(\Sigma^+, \Sigma^0, \Sigma^-)$, $X_i(\equiv^0, \equiv^-)$

33. (c) As donor and acceptor impurities are added to semiconductor to make an extrinsic semiconductor, intrinsic semiconductor is formed by internal generation of $e^-$by breaking up of covalent bonds.

34. (d) In p-type semiconductor, valency = 3, thus there is one ur. Y med bond or hole created. This hole is in valence band and is able to cause hole current. The energy levels of acceptor are in forbidden gap just above valence band

35. (d) In an oscillator, L-C circuit is coupled with transistor amplifier in such a way that there is a positive feed -back to the LC circuit i.e., proper energy supply to LC at proper timings. So that total energy of LC circuit remains same.

36. (c) The gates AND, OR, NAND do not give binary addition, however in EXOR gate truth table is

A	B	Y
0	0	0

0	1	1
1	0	1
1	1	0

This shows it gives perfect binary addition

37. **(d)** In FM, carrier frequency is the constant frequency which is modulated by signal amplitude. It is also called carrier swing. (Centre frequency is f_c in AM wave, frequency deviation $\delta = f_{max} - f_c$, mod ulation factor $= \frac{\delta_{max}}{f_c}$)

38. **(c)** The common antenna is a straight conductor of length $1 = \frac{\lambda}{4}$ held vertically with its lower end touching the ground.

39. **(b)** The vidicon is a storage-type camera tube in which a charge-density pattern is formed by the imaged scene radiation on a photoconductive surface which is then scanned by a beam of low-velocity electrons. The fluctuating voltage coupled out to a video amplifier can be used to reproduce the scene being imaged. The electrical charge produced by an image will remain in the face plate until it is scanned or until the charge dissipates.

40. **(b)** Maximum range of radar $d_{max} \propto \sqrt{1}$ and power transmitted by antenna of length 1 is $p \propto (1/\lambda)^2 \Rightarrow 1 \propto \sqrt{p}$ and $d \propto (p)^{1/4}$

PART-II CHEMISTRY

41. **(b)** The oxidation of ferrous ion by $KMnO_4$ takes place in acidic medium as per following reaction

$2KMnO_4 + 8H_2SO_4 + 10FeSO_4$

$\rightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5Fe_2(SO_4)_3$

$\therefore$ Eq. mass of $KMnO_4$

Molecular mass

change in oxidation number

$$= \frac{\text{Molecular mass}}{5} = \frac{158}{5} = 31.6$$

42. **(b)** In case of lanthanoids, 4f orbitals lie too deep and hence the magnetic effect of the motion of the electron in its orbital is not quenched out. Here spin contribution S and orbital contribution L couple together to give a new quantum number J.

Thus, magnetic moment of lanthanoids is given by, $\mu = g\sqrt{J(J+1)}$

where $J = L - S$ when the shell is less than half fill

$J = L + S$ when the shell is more than half fill

and $g = 1\frac{1}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$

43. **(d)** $Mg^{2+} = 1s^2, 2s^2, 2p^6$ (No unpaired electrons)

$Ti^{3+} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^0, 3d^1$

(One unpaired electrons)

$V^{3+} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^0, 3d^2$

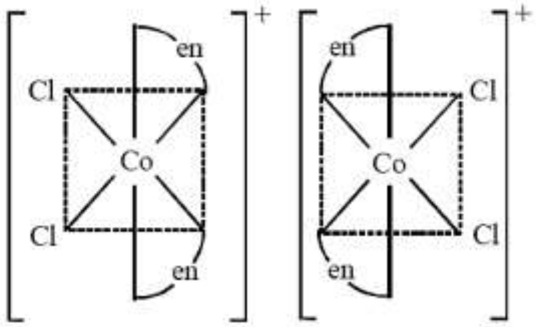
(Two unpaired electrons)

$Fe^{2+} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^0, 3d^6$

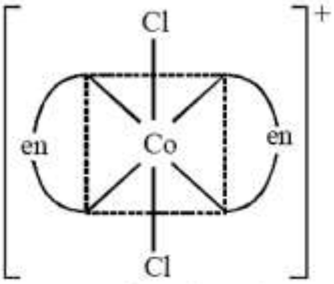
(Four unpaired electrons)

$\therefore Fe^{2+}$ has highest number of unpaired electrons.

44. (d)
45. (c)



d - cis form $\quad$ *l* - cis form



trans form (meso)

46. (b) Neutral ligands are given the same names as the neutral molecule. However, two very important exceptions to this rule are: H_2O Aquo (Aqua) NH_3 Ammine.
47. (a) $Ni(PF_3)_4$ - tetrakis phosphours (III) fluoride nickel(0).
48. (a) The colour of $KMnO_4$ is due to charge transfer. The configuration of manganese in permagnate ion is d^0 but it is coloured because its electrons are photo-exited.
49. (c) In face centred cubic lattice, the atoms are present at eight corners of faces and one each at 6 faces.

$\therefore$ Lattice points belonging to face centred cubic unit cell $= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

50. (a) Schottky defect is caused when equal number of cations and anions are missing from their lattice sites.
51. (a) Polystyrene is thermoplastic substance.
52. (a) Po-Simple cubic lattice

Na – bcc

Cu – fcc

53. (c) $W_{irr} = -P_{ext} \frac{R[P_1 T_2 - P_2 T_1]}{P_1 P_2}$

During expansion in vaccum $P_{ext} = 0$

$\therefore$ work done = 0.

54. (b) As the system is closed, hence the reaction will be reversible, hence according to Le-chatelier principle pressure decreases since the volume is increasing.

55. (d) $W = -2.303nRT\log\frac{V_2}{V_1}$

Given $n = 6$, $T = 27°C = 273 + 27 = 300$ K $V_1 = 1$ L, $V_2 = 10$ L

$$\therefore W = -2.303 \times 6 \times 8.314 \times 300\log\frac{10}{1}$$

$= 34.465$ kJ

56. (a) It is spontaneous process because zinc is more reactive than copper, hence can easily repace Cu from $CuSO_4$.

57. (c)

$$K_p = K_c(RT)^{\Delta n}$$
$$\Delta n = n_{p(g)} - n_{r(g)} = 2 - 2 = 0$$
$$\therefore K_p = K_c$$

58. (d) Ice $\rightleftharpoons$ Water

The volume of ice is more than water Therefore when pressure is increased the equilibrium shifts in forward direction. It favoures melting of ice.

59. (c) It is a first order reaction because rate of reaction $\propto [N_2O_5]$

60. (d) The reactions with low activation energies are always fast whereas the reactions with high activation energy are always slow.

61. (d) For spontaneous reaction free energy change is negative.

$\Delta G = -nFE$

62. (a) Because at infinite dilution the equivalent conductance of strong electrolytes furnishing same number of ions is same.

63. (c) Reducing character i.e tendency to loose electron decreases down the series, hence the correct order is $Zn > Cu > Ag$.

64. (b) At anode the following reaction takes place $CH_4 + 2H_2O \rightarrow CO_2 + 8H^+ + 8e^-$

65. (a) Oxygen atom in alcohol molecule is sp^3 hybridised.

66. (d) In this reaction $LiAlH_4$ acts as reducing

agent. $R-\overset{\overset{O}{\|}}{C}-OH \xrightarrow{LiAlH_4} RCH_2OH$

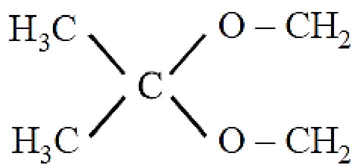
67. (d) 3° alcohols are resistant to oxidation and are oxidised only by strong oxidising agents like conc. HNO_3. They are resistant to oxidation in neutral or alkaline $KMnO_4$.

68. (b)

$$C_2H_5-O-C_2H_5 \xrightarrow[\text{boil, dil. acid}]{+H_2O} \underset{\text{ethyl alcohol}}{2C_2H_5OH}$$

69. (b)

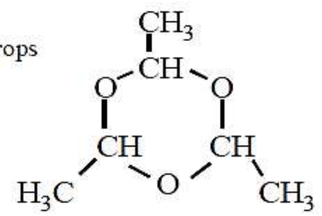
$$CH_3-\overset{O}{\overset{\|}{C}}-CH_3 + \begin{matrix} CH_2OH \\ | \\ CH_2OH \end{matrix} \xrightarrow{HCl}$$



cyclic ketal

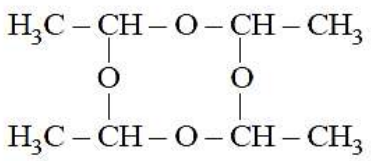
70. (b)

$$3\,CH_3CHO \underset{\text{Dil. } H_2SO_4}{\overset{\text{Conc. } H_2SO_4 \text{ drops, Room temp.}}{\rightleftharpoons}}$$



Para aldehyde
(pleasant smelling liquid,
used as hypnotic and soporofic)
(sleep producing)

$$4\,CH_3CHO \underset{\text{dil } H_2SO_4}{\overset{\text{Conc. } H_2SO_4 \text{ or HCl gas}}{\rightleftharpoons}}$$



Metaldehyde
(used as solid fuel in spirit lamps)

71. (c)

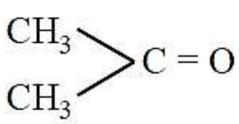
$$2(CH_3COO)_2Ca \xrightarrow{\text{distillation}} \underset{\text{Acetone}}{CH_3COCH_3} + \underset{\text{Calcium carbonate}}{2CaCO_3}$$

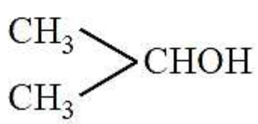
72. (d) Ketones do not react with alcohol.

(i) Aldehydes $\xrightarrow{\text{reduction}}$ 1° alcohol

e.g., $CH_3CHO \longrightarrow CH_3CH_2OH$

(ii) Ketones $\xrightarrow{\text{reduction}}$ 2° alcohol





(iii) Ketones do not reduce Fehling solution but aldehydes do so.

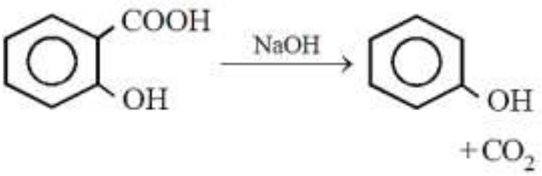
73. (d) The $O-H$ strecting of carboxylic acid absorb in region of $1700-2000\ \text{cm}^{-1}$

74. (b) Formic acid

$$\left(H-\overset{\overset{\displaystyle O}{\|}}{C}-OH\right)\ \text{has}\ -CHO$$

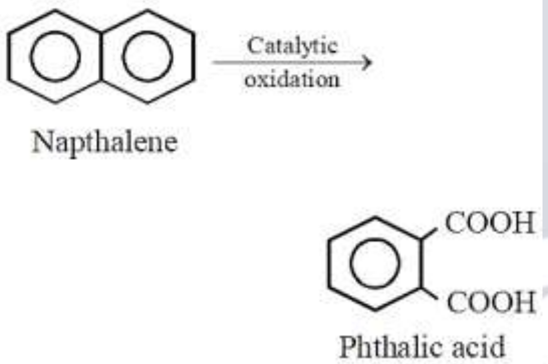
group and therefore it reduces Fehling solution.

75. (c)



This is decarboxylation reaction.

76. (d)



77. (b) In both the cases carbon is sp^3 hybridised and bond angle is $109°28'$.

78. (b) This reaction is known as curtius rearrangement.

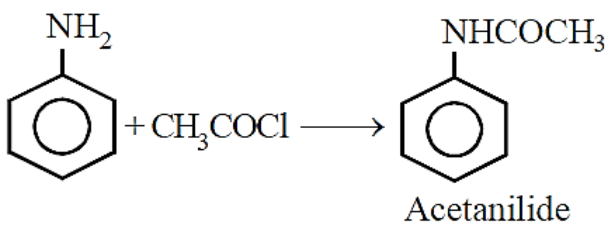
$$RCON_3 \xrightarrow{-N_2} RNCO \xrightarrow{2NaOH} Na_2CO_3 + RNH_2$$

1° amine is formed.

79. (a) It is expected that the basic nature of amines should be in order tertiary > secondary > primary but the observed order in the case of lower members is found to be as secondary > Primary > tetriary. This anomalous behaviour of tetriary amines is due to steric factors i.e crowding of alkyl groups cover nitrogen atom from all sides thus makes the approach and bonding by a proton relatively difficult which results the maximum steric strain in tetiary amines. The electrons are there but the path is blocked, resulting the reduction in basicity. Thus ,the correct order is

$R_2NH > RNH_2 > R_3\ N.$

80. (d) On acetylation aniline is converted into acetamide which is resonance stablised and therefore less reactive.



PART-III MATHEMATICS

81. **(a)** We know that, A is singular if $|A| = 0$

$$|A| = \begin{vmatrix} \frac{2}{x} & -1 & 2 \\ 1 & x & 2x^2 \\ 1 & \frac{1}{x} & 2 \end{vmatrix} = 0$$

$$\Rightarrow |A| = \frac{2}{x}[2x - 2x] + 1[2 - 2x^2] + 2\left[\frac{1}{x} - x\right] = 0$$

$$\Rightarrow \frac{2}{x}[0] + [2 - 2x^2] + \frac{2}{x} - 2x = 0$$

$$\Rightarrow 2x - 2x^3 + 2 - 2x^2 = 0$$

$$\Rightarrow x^3 + x^2 - x - 1 = 0$$

$$\Rightarrow x^2(x+1) - 1(x+1) = 0$$

$$\Rightarrow (x+1)(x^2 - 1) = 0$$

$$\Rightarrow x = \pm 1$$

82. **(b)** Given $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$$\Rightarrow x[x^2 - 12] - 3[2x - 14] + 7[12 - 7x] = 0$$

$$\Rightarrow x^3 - 67x + 126 = 0$$

But given $(x = 9)$ is a root of given determinant.

$\therefore$ $(x + 9)$ is a factor

$$\Rightarrow x^3 + 9x^2 - 9x^2 - 81x + 14x + 126 = 0$$

$$\Rightarrow x^2(x+9) - 9x(x+9) + 14(x+9) = 0 \Rightarrow (x+9)(x^2 - 9x + 14) = 0$$

$$\Rightarrow (x+9)(x^2 - 7x - 2x + 14) = 0$$

$$\Rightarrow (x+9)(x-7)(x-2) = 0$$

$$\Rightarrow x = -9, 7, 2$$

83. **(c)** We have

$$A : B = \begin{vmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 4 & : & \alpha \\ 1 & 4 & 10 & : & \alpha^2 \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & \alpha - 1 \\ 0 & 3 & 9 & : & \alpha^2 - 1 \end{vmatrix}$$

$\begin{bmatrix} \text{applying } R_2 \to R_2 - R_1 \\ \& R_3 \to R_3 - R_1 \end{bmatrix}$

$$\sim \begin{vmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & \alpha - 1 \\ 0 & 0 & 0 & : & \alpha^2 - 3\alpha + 2 \end{vmatrix}$$

[applying $R_3 \to R_3 - 3R_2$]

But the system is consistent

$\therefore \alpha^2 - 3\alpha + 2 = 0$

$\Rightarrow (\alpha - 2)(\alpha - 1) = 0 \Rightarrow \alpha = 2$ or $\alpha = 1$

84. (c) We know that inverse of A does not exist only when $|A| = 0$

$$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & 5 & t \\ 4 & 7-t & -6 \end{vmatrix} = 0$$

$\Rightarrow (-30 - 7t + t^2) - 3(-12 - 4t)$

$+2(14 - 2t - 20) = 0$

$\Rightarrow -30 - 7t + t^2 + 36 + 12t - 12 - 4t = 0$

$\Rightarrow t^2 + t - 6 = 0 \Rightarrow t^2 + 3t - 2t - 6 = 0$

$\Rightarrow t(t + 3) - 2(t + 3) = 0$

$\Rightarrow (t + 3)(t - 2) = 0 \Rightarrow t = 2, -3$

85. (a) Given $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$

By using Hit & trial method, we have $(x - 1)$ is a factor of given equation

$\therefore (x - 1)(x^3 - 2x^2 - 4x - 1) = 0$

$\Rightarrow (x - 1)[x^3 + x^2 - 3x^2 - 3x - x - 1] = 0$

$\Rightarrow (x - 1)[x^2(x + 1) - 3x(x + 1) - 1(x + 1)] = 0$

$\Rightarrow (x - 1)(x + 1)(x^2 - 3x - 1) = 0$

$\therefore x = 1, -1$ or $x^2 - 3x - 1 = 0$

Now $x^2 - 3x - 1 = 0$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\left[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

$\therefore$ non-integer roots of given equation are

$$\frac{1}{2}(3 + \sqrt{13}), \frac{1}{2}(3 - \sqrt{13})$$

86. (b) Given $e^x = y + \sqrt{1+y^2}$

$\Rightarrow e^x - y = \sqrt{1+y^2}$

Squaring both side, we have

$e^{2x} + y^2 - 2e^x y = 1 + y^2$

$\Rightarrow 2e^x y = e^{2x} - 1$

$\Rightarrow y = \dfrac{e^{2x}-1}{2e^x} \Rightarrow y = \dfrac{1}{2}[e^x - e^{-x}]$

87. (d) First term $= a$ & common ratio $= r$

Given $S_\infty = 4$ & $a_2 = \frac{3}{4}$

$\Rightarrow \dfrac{a}{1-r} = 4$

$\& ar = \dfrac{3}{4}\ \left[\because S_\infty = \dfrac{a}{1-r} \& a_n = ar^{n-1}\right]$

$\Rightarrow a = \frac{3}{4r}$ ∴ Equation (1) becomes $\frac{3}{4r(1-r)} = 4$

$\Rightarrow 16r^2 - 16r + 3 = 0$

$\Rightarrow (4r-3)(4r-1) = 0$

$\Rightarrow r = \frac{3}{4}$ or $r = \frac{1}{4}$

when $r = \frac{1}{4}$ then $a = \frac{3}{4\times\frac{1}{4}} = 3$

$\therefore a = 3\ \&\ r = \dfrac{1}{4}$

88. (c) Given: $\alpha \& \beta$ are roots of equation

$ax^2 + bx + c = 0$

$\therefore \alpha + \beta = -\dfrac{b}{a} \& \alpha\beta = \dfrac{c}{a}$

Now, $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$\Rightarrow \alpha^3 + \beta^3 = \left(-\dfrac{b}{a}\right)^3 - 3\dfrac{c}{a}\cdot\left(-\dfrac{b}{a}\right)$

$\Rightarrow \alpha^3 + \beta^3 = -\dfrac{b^3}{a^3} + \dfrac{3bc}{a^2}$

$\Rightarrow \alpha^3 + \beta^3 = \dfrac{-b^3 + 3abc}{a^3}$

89. (b) Given: The vertices of tetrahedron are $P(-1,2,0), Q(2,1,-3), R(1,0,1) \& S(3,-2,3)$

$\therefore$ Volume of tetrahedron $= \frac{1}{6}[\overrightarrow{PQ}\overrightarrow{PR}\overrightarrow{PS}]$

Now,

$\overrightarrow{PQ} = (2+1)\hat{i} + (1-2)\hat{j} + (-3)\hat{k} = 3\hat{i} - \hat{j} - 3\hat{k}$

Similarly, $\overrightarrow{PR} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\&\overrightarrow{PS} = 4\hat{i} - 4\hat{j} + 3\hat{k}$

$\therefore$ Volume of tetrahedron

$$= \frac{1}{6}\begin{vmatrix} 3 & -1 & -3 \\ 2 & -2 & 1 \\ 4 & -4 & 3 \end{vmatrix} = \frac{2}{3}$$

90. (a) We have, $\vec{a} + t\vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k}) + t(-\hat{i} + 2\hat{j} + \hat{k})$

$= (1-t)\hat{i} + (2+2t)\hat{j} + (3+t)\hat{k}$

It is $\perp$ to $c = 3\hat{i} + \hat{j}$

If $3(1-t) + (2+2t) + (3+t)(0) = 0$

$\Rightarrow 3 - 3t + 2 + 2t = 0 \Rightarrow t = 5$

91. (d) The equation of the plane through the line of intersection of the given planes is $(x+y+z-6) + \lambda(2x + 3y + 4z + 5) = 0$ ... (1) If equation (1) passes through (1,1,1), we have

$-3 + 14\lambda = 0 \Rightarrow \lambda = \dfrac{3}{14}$

Putting $\lambda = \frac{3}{14}$ in (1), we obtain the equation of the required plane as

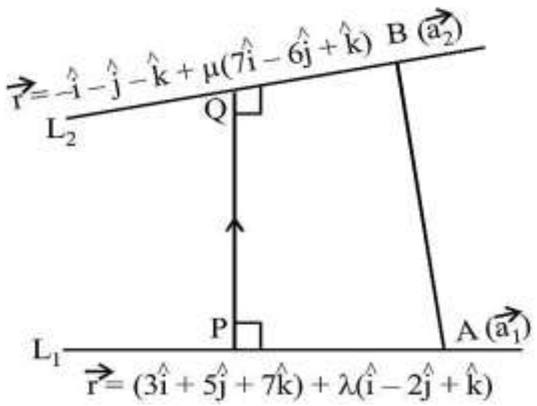
$(x+y+z-6) + \dfrac{3}{14}(2x + 3y + 4z + 5) = 0$

$\Rightarrow 20x + 23y + 26z - 69 = 0$

92. (d) Shortest distance PQ $= \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|(\vec{b}_1 \times \vec{b}_2)|}\right|$

Now, $\vec{a}_2 - \vec{a}_1 = -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$

$\Rightarrow \vec{a}_2 - \vec{a}_1 = -4\hat{i} - 6\hat{j} - 8\hat{k}$



And $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix} \Rightarrow \vec{b}_1 \times \vec{b}_2 = \hat{i}(-2+6) - \hat{j}(1-7) + \hat{k}(-6+14)$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = 4\hat{i} + 6\hat{j} + 8\hat{k}$

$\therefore$ Shortest distance

$$PQ = \left|\frac{(4\hat{i} + 6\hat{j} + 8\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 8\hat{k})}{\sqrt{16+36+64}}\right|$$

$$\Rightarrow PQ = \left|\frac{-16-36-64}{\sqrt{116}}\right|$$

$$= \left|\frac{-116}{\sqrt{116}}\right| = \sqrt{116} = 2\sqrt{29}$$

$\therefore PQ = 2\sqrt{29}$ units

93. (c) Given, $|z - i| + |z + i| \le 4$

$\Rightarrow |z - (0 + i)| + |z - (0 - i)| \le 0$

This equation represent the interior and boundary of ellipse with foci at $(0,1)$& $(0,-1)$, whose major axis is along the y-axis.

94. (b) $\sum_{n=1}^{13}(i^n + i^{n+1}) = \sum_{n=1}^{13} i^n + \sum_{n=1}^{13} i^{n+1}$

$$= i\left(\frac{1-i^{13}}{1-i}\right) + i^2\left(\frac{1-i^{13}}{1-i}\right)$$

$$= i\frac{(1-i)}{(1-i)} - \frac{(1-i)}{(1-i)} = i - 1$$

95. (b) Given : $\sin\theta, \cos\theta, \tan\theta$ are in G.P.

$$\Rightarrow \cos^2\theta = \sin\theta\tan\theta \Rightarrow \cos^3\theta = \sin^2\theta$$
$$\Rightarrow \cos^3\theta = 1 - \cos^2\theta$$
$$\Rightarrow (\cos^3\theta + \cos^2\theta) = 1$$

Cubic both sides, we have

$\cos^9\theta + \cos^6\theta + 3\cos^5\theta \cdot (\cos^3\theta + \cos^2\theta) = 1$

$\Rightarrow \cos^9\theta + \cos^6\theta + 3\cos^5\theta = 1$

[Using equation (1)]

$\Rightarrow \cos^9\theta + \cos^6\theta + 3\cos^5\theta - 1 = 0$

96. (c)

Given : $5\cos C + 6\cos B = 4$

$6\cos A + 4\cos C = 5$

Adding eq. (1) & (2), we have

$9\cos C + 6(\cos A + \cos B) = 9$

$\Rightarrow 9\cos C + 6\left[2\cos\frac{A+B}{2} \cdot \cos\frac{A-B}{2}\right] = 9$

$\Rightarrow 9\cos C - 9 + 12\cos\left(\frac{\pi}{2} - \frac{C}{2}\right) \cdot \cos\frac{A-B}{2} = 0$

$\Rightarrow 9(\cos C - 1) + 12\sin\frac{C}{2} \cdot \cos\frac{A-B}{2} = 0$

$\Rightarrow 9\left[1 - 2\sin^2\frac{C}{2} - 1\right] + 12\sin\frac{C}{2} \cdot \cos\frac{A-B}{2} = 0$

$\Rightarrow -18\sin^2\frac{C}{2} + 12\sin\frac{C}{2} \cdot \cos\frac{A-B}{2} = 0$

$\Rightarrow 3\sin\frac{C}{2} = 2\cos\frac{A-B}{2}$

$\Rightarrow 3\cos\frac{A+B}{2} = 2\cos\frac{A-B}{2}$

$\Rightarrow 3\left(\cos\frac{A}{2} \cdot \cos\frac{B}{2} - \sin\frac{A}{2} \cdot \sin\frac{B}{2}\right)$

$= 2\left[\cos\frac{A}{2} \cdot \cos\frac{B}{2} + \sin\frac{A}{2} \cdot \sin\frac{B}{2}\right]$

$\Rightarrow 5\sin\frac{A}{2} \cdot \sin\frac{B}{2} = \cos\frac{A}{2} \cdot \cos\frac{B}{2}$

$\Rightarrow 5\tan\frac{A}{2} \cdot \tan\frac{B}{2} = 1$

$\Rightarrow \tan\frac{A}{2} \cdot \tan\frac{B}{2} = \frac{1}{5}$

97. (b) Equation of the semielliptical bridge

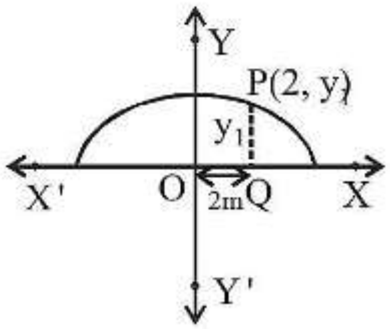
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here, $2a = 9$

$\therefore a = \frac{9}{2}, \; b = 3$

$\therefore \frac{x^2}{\frac{81}{4}} + \frac{y^2}{9} = 1$

$\Rightarrow \frac{4x^2}{81} + \frac{y^2}{9} = 1$



Here, $OQ = 2$ m, let $PQ = y_1$

$\therefore P(2, y_1)$

Since point P lies on the ellipse (2)

$$\therefore \frac{4\times 4}{81} + \frac{y_1^2}{9} = 1$$

$$\Rightarrow \frac{y_1^2}{9} = 1 - \frac{16}{81} = \frac{81-16}{81} = \frac{65}{81}$$

$$\Rightarrow y_1^2 = \frac{65}{9} \Rightarrow y_1 = \frac{\sqrt{65}}{3} \simeq \frac{8}{3}\text{m}$$

Hence, best approximation of the height of

the arch $= \frac{8}{3}$ m.

98. (c) Given: Equations of ellipses

$3x^2 + 5y^2 = 32$

$\&25x^2 + 9y^2 = 450$

Tangents to the ellipse (1) & (2) are passing through the point (3,5)

$\therefore 3(3)^2 + 5(5)^2 - 32 = 27 + 75 - 32 > 0$

So the given point lies outsides the ellipse. Hence, two real tangents can be drawn from the point to the ellipse,

$\&25(3)^2 + 9(5)^2 - 450 = 225 + 225 - 450 = 0$

$\therefore$ The point lie on the ellipse. Hence one real tangent can be drawn.

$\therefore$ No. of real tangents $= 3$

99. (b) The equation of tangent at $\left(ct, \frac{c}{t}\right)$ is $ty = t^3x - ct^4 + c$

If it passes through $\left(ct', \frac{c}{t'}\right)$ then

$$\Rightarrow \frac{tc}{t'} = t^3ct' - ct^4 + c$$

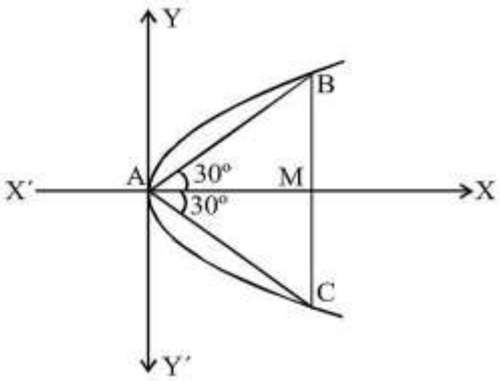
$$\Rightarrow t = t^3t'^2 - t^4t' + t'$$

$\Rightarrow t.t' = t^3t'(t'.t) \Rightarrow t^3t' = -1$

Note: If we take the co-ordinate axes along the asymptotes of a rectangular hyperbola, then the general equation $x^2 - y^2 = a^2$ becomes $xy = c^2$, where c is a constant.

100. **(d)** Let $AB = \ell$, then $AM = \ell\cos 30^\circ = \frac{\ell\sqrt{3}}{2}$

$\&BM = \ell\sin 30^\circ = \frac{\ell}{2}$



So, the coordinates of B are $\left(\frac{\ell\sqrt{3}}{2}, \frac{\ell}{2}\right)$

Since, B lies on $y^2 = 4x$

$\therefore \frac{\ell^2}{4} = 4\left(\frac{\ell\sqrt{3}}{2}\right)$

$\Rightarrow \ell^2 = \frac{16}{2} \cdot \sqrt{3}\ell \Rightarrow \ell = 8\sqrt{3}$

101. **(d)** Let $f(x) = ax + b$

Given $f(2) = 4\&f'(2) = 1$

$\therefore f(2) = a.2 + b = 4 \Rightarrow 2a + b = 4\ \&f'(x) = a \Rightarrow f'(2) = a = 1 \Rightarrow a = 1$

$\therefore 2 \times 1 + b = 4 \Rightarrow b = 2$ [using equation (1)]

$\therefore f(x) = x + 2$

Now, $\lim_{x\to 2} \frac{xf(2) - 2f(x)}{x-2}$

$= \lim_{x\to 2} \frac{4x - 2(x+2)}{x-2} = \lim_{x\to 2} \frac{2x-4}{x-2}$

$= \lim_{x\to 2} \frac{2(x-2)}{(x-2)} = 2$

102. **(d)** Let $f(x) = x^2 + kx + 1$

$f'(x) = 2x + k$

$f(x)$ is strictly increasing on $(1,2)$

if $f'(x) > 0$ for $x \in (1,2)$

$\Rightarrow 2x + k > 0$ for $x \in (1,2)$

$\Rightarrow k > -2x$ for $x \in (1,2)$

Now, $1 < x < 2 \Rightarrow 2 < 2x < 4$

$\Rightarrow -2 > -2x > -4$

$\Rightarrow -4 < -2x < -2$

$\Rightarrow k \geq -2$

Hence least value of $k = -2$.

103. **(c)** Let $y = \left(\frac{1}{x}\right)^x \Rightarrow y = x^{-x}$

Then $\log y = -x\log x$

$\therefore \frac{1}{y}\frac{dy}{dx} = -(1 + \log x)$

or $\frac{dy}{dx} = -y(1 + \log x)$

$\& \frac{d^2y}{dx^2} = -y \cdot \frac{1}{x} - (1 + \log x) \cdot \frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{1}{x}\right)^{x+1} - (1 + \log x) \cdot \frac{dy}{dx}$

Now, $\frac{dy}{dx} = 0 \Rightarrow 1 + \log x = 0$

$\Rightarrow \log x = -1 = -\log e = \log\left(\frac{1}{e}\right)$

$\Rightarrow x = \left(\frac{1}{e}\right)$

Also, $\frac{d^2y}{dx^2}$ at $x = \left(\frac{1}{e}\right)$ is $-e^{1+\frac{1}{e}} < 0$

$\left[\because \frac{dy}{dx} = 0\right]$

So, $x = \left(\frac{1}{e}\right)$ is a point of local maxima.

$\therefore$ Maximum value

$=$ value of y when $x = \left.\frac{1}{e}\right) = e^{\frac{1}{e}}$

104. **(a)** Euler's theorem $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$

Given : $U = \tan^{-1}\frac{x^3+y^3}{x+y}$

$\Rightarrow \tan U = \frac{x^3+y^3}{x+y} = z$ (let)

$n = 3 - 1 = 2$

$$\therefore x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$$

$$\Rightarrow x\frac{\partial}{\partial x}\tan U + y\frac{\partial}{\partial y}\cdot\tan U = 2\tan U$$

$$\Rightarrow x\cdot\sec^2 U\cdot\frac{\partial U}{\partial x} + y\cdot\sec^2 U\cdot\frac{\partial U}{\partial y} = 2\tan U$$

$$\Rightarrow \sec^2 U\cdot\left[x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y}\right] = 2\tan U$$

$$\Rightarrow x\cdot\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = 2\cdot\frac{\sin U}{\cos U}\cdot\cos^2 U$$

$$\Rightarrow x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = \sin 2U$$

105. **(b)** Given: $f'(x) = \frac{x}{\sqrt{1+x}}, f(0) = 0$

$$\Rightarrow \int f'(x)dx = \int \frac{x}{\sqrt{1+x}}dx$$

$$\Rightarrow f(x) = \int \frac{x}{\sqrt{1+x}}dx$$

Let $1 + x = t^2 \Rightarrow x = t^2 - 1$

$\Rightarrow dx = 2t.dt$

$$\therefore f(x) = \int \frac{t^2-1}{t}\cdot 2tdt = 2\int (t^2-1)dt$$

$$\Rightarrow f(x) = 2\left[\frac{t^3}{3} - t\right] + c$$

$$\Rightarrow f(x) = 2\left[\frac{(1+x)^{3/2}}{3} - (1+x)^{1/2}\right] + c$$

But $f(0) = 0 \Rightarrow 2\left[\frac{1}{3} - 1\right] + c = 0$

$$\Rightarrow -\frac{4}{3} + c = 0 \Rightarrow c = \frac{4}{3}$$

$\therefore$ Equation (1) becomes

$$f(x) = 2\left[\frac{(1+x)^{3/2}}{3} - (1+x)^{1/2}\right] + \frac{4}{3}$$

$$\Rightarrow f(x) = \frac{2}{3}\left[(1+x)^{3/2} - 3(1+x)^{1/2} + 2\right]$$

106. **(a)** Let $I = \int_0^{\frac{\pi}{2}} \log(\tan x)dx$

Then, $I = \int_0^{\frac{\pi}{2}} \log\left[\tan\left(\frac{\pi}{2} - x\right)\right] dx$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log(\cot x)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{\tan x}\right) dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log(\tan x)^{-1}dx = -\int_0^{\frac{\pi}{2}} \log(\tan x)dx$

$\Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$

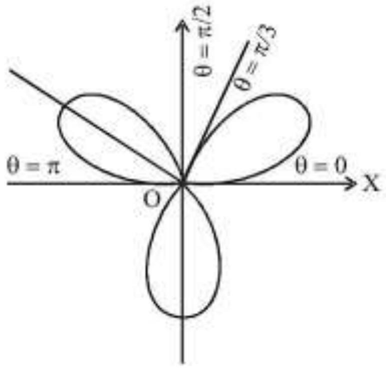
[Using eq. (1)]

107. **(d)** Ifcurve $r = a\sin 3\theta$

To trace the curve, we consider the following table:

$3\theta =$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
$\theta =$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r =$	0	a	0	$-a$	0	a	0

Thus there is a loop between $\theta = 0 \& \theta = \frac{\pi}{3}$ as r varies from $r = 0$ to $r = 0$.



Hence, the area of the loop lying in the positive quadrant $= \frac{1}{2}\int_0^{\frac{\pi}{3}} r^2 d\theta = \frac{1}{2}\int_0^{\frac{\pi}{3}} \sin^2\phi \cdot \frac{1}{3}d\phi$

[On putting, $3\theta = \phi \Rightarrow d\theta = \frac{1}{3}d\phi$] $= \frac{a^2}{6}\int_0^{\frac{\pi}{2}} \sin^2\phi d\phi$

$= \frac{a^2}{6} \cdot \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2\phi}{2} d\phi$

[$\because \cos 2\theta = 1 - 2\sin^2\theta$]

$$= \frac{a^2}{12} \cdot \left[\phi + \frac{\sin 2\phi}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2}{12} \cdot \left[\frac{\pi}{2} + \sin \pi\right] = \frac{a^2 \pi}{24}$$

108. **(b)** Let $I = \int_1^9 e^{\sqrt{t}} dt$

Put $t = x^2 \Rightarrow dt = 2x.dx$

For limit: $x = 1 \& x = 3$

$I = \int_1^3 e^x \cdot 2xdx = 2\int_1^3 e^x \cdot xdx$

$\Rightarrow I = 2\left[\{x \cdot e^x\}_1^3 - \int_1^3 e^x dx\right]$

$= 2[\{xe^x\}_1^3 - \{e^x\}_1^3]$

$\Rightarrow I = 2[3e^3 - e^1 - e^3 + e^1] = [2e^3] \cdot 2 = 4e^3$

109. **(d)** Equation of the family of such parabolas is $(y - k)^2 = 4a(x - h)$

where h&k are arbitrary constants

Differentiating w.r.t. x, we get

$$(y - k)\frac{dy}{dx} = 2a$$

Differentiating again

$$(y - k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

Putting value of $(y - k)$ from (2) in (3), we get

$2a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$, which is required equation.

110. **(b)** Given: $\left(x\operatorname{cosec}\left(\frac{y}{x}\right) - y\right) dx + xdy = 0$

$$\Rightarrow \left(\frac{x}{\sin\left(\frac{y}{x}\right)} - y\right) dx + xdy = 0$$

$$\Rightarrow \left[x - y \cdot \sin\left(\frac{y}{x}\right)\right] dx + x\sin\left(\frac{y}{x}\right) dy = 0$$

Put $\frac{y}{x} = z \Rightarrow y = zx$

$$\therefore \frac{dy}{dx} = z \cdot 1 + x \cdot \frac{dz}{dx} = z + x\frac{dz}{dx}$$

$\therefore$ Equation (1) becomes

$$z + x\frac{dz}{dx} = \frac{zx \cdot \sin z - x}{x\sin z} = z - \operatorname{cosec} z$$

$$\Rightarrow x\frac{dz}{dx} = -\operatorname{cosec} z \Rightarrow -\sin z dz = \frac{dx}{x}$$

$$\Rightarrow \log|x| = \cos z + c$$

$$\Rightarrow \log|x| - \cos\left(\frac{y}{x}\right) = c$$

111. **(c)** If $\frac{d^2y}{dx^2} + 2y = x^2$

$$\Rightarrow (D^2 + 2)y = x^2 \ \left[D \equiv \frac{d}{dx}\right]$$

Particular integral (P.I.) $= \frac{1}{D^2+2} \cdot x^2$

$$= \frac{1}{2\left(1 + \frac{D^2}{2}\right)} \cdot x^2 = \frac{1}{2} \cdot \left(1 + \frac{D^2}{2}\right)^{-1} \cdot (x^2)$$

$\because (1 + D)^{-1} = 1 - D + D^2 - D^3 + \cdots ....$

$\therefore$ P.I. $= \frac{1}{2} \cdot \left[1 - \left(\frac{D^2}{2}\right) + \left(\frac{D^2}{2}\right)^2 - \cdots ..\right](x^2)$

$\Rightarrow$ P.I. $= \frac{1}{2} \cdot \left[x^2 - \frac{D^2}{2}(x^2)\right]$

$\Rightarrow$ P.I. $= \frac{1}{2} \cdot [x^2 - 1]$

112. **(a)** If$(D^2 + 16)y = \cos 4x$

Here the auxiliary equation is $m^2 + 16 = 0 \Rightarrow m = \pm 4$

$\therefore$ Complementary function

$= (A\cos 4x + B\sin 4x)$

& Particular Integral (P.I.)

$$= \frac{1}{D^2 + 16} \cdot \cos 4x$$

But $\frac{1}{D^2+a^2} \cos ax = \frac{x}{2a} \sin ax$

$\therefore$ P.I. $= \frac{x}{2 \times 4} \cdot \sin 4x = \frac{x}{8} \sin 4x$

$\therefore$ Solution y = Complementary function + Particular Integral

$$\Rightarrow y = A\cos 4x + B\sin 4x + \frac{x}{8}\sin 4x$$

113. **(c)**

114. **(d)** Let us make one packet for each of the books on the same size. Now, 3 packets can be arranged in $P(3,3) = 3\,!$ ways

5 large books can be arranged in 5 ! ways 4 medium size books can be arranged in !! ways

3 small books can be arranged in 3! ways

$\therefore$ Required number of ways

$= 3! \times 5! \times 4! \times 3!$ ways

115. **(a)** An identity relation is one in which every element of a set is related to itself only. $\mathbf{a * b = a + b - ab}$

As in identity relation 'a' is related to ' a ', so the correct option will be the one which gives the value of the relation = 'a'. So, equating $a + b - ab = a$, we get $b(1 - a) = 0$. Now putting the values of a, we find b and the option in which $a = b$, will be the answer. For $a = 0$, $b = 0$, so the correct option.

For $a = 1$, $b(1 - 1) = 0 \Rightarrow$ b can have multiple values.

For $a = 2$, $b(1 - 2) = 0 \Rightarrow b = 0$ but $a = 2$.

116. **(a)**

117. **(b)** Let S be the sample space

$\because n(S) = 36$

Events
[sum greater than 3 but not exceeding 6]

$$= \{(2,2),(3,1),(1,3),(4,1),(1,4),(5,1)(1,5),$$
$$(3,2),(2,3),(4,2),(2,4),(3,3)\}$$
$$\Rightarrow n(E) = 12$$
$$\therefore \text{ Required probability } = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

118. **(a)** Let ' p ' denote the probability of winning of team A whenever it plays

$\therefore p = \frac{2}{3} \& q = 1 - \frac{2}{3} = \frac{1}{3}$

Let X denotes the number of winning games out of 4 games i.e. $n = 4$
$\therefore$ The probability of r success

$P(X = r) = {}^nC_r p^r q^{n-r}, r = 0,1,2,3,4$

$\therefore$ Probability of winning more than half

$$\text{games } = P(X > 2)$$
$$= P(X = 3) + P(X = 4)$$

$$= {}^4C_3\left(\frac{2}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^{4-3} + {}^4C_4\left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^{4-4}$$
$$= \frac{4!}{3!\,1!} \cdot \frac{8}{27} \cdot \frac{1}{3} + \frac{4!}{4!\,0!} \cdot \frac{16}{81}$$
$$= \frac{32}{81} + \frac{16}{81} = \frac{48}{81} = \frac{16}{27}$$

119. **(b)** $n =$ total number of ways $= 2^5 = 32$

Since each answer can be true or false & $m =$ favourable number of ways $= {}^5C_4 + {}^5C_5$

$$= \frac{5!}{4!\,1!} + \frac{5!}{5!\,0!} = 5 + 1 = 6 \Rightarrow m = 6$$

Since to pass the quiz, student must give 4 or 5 true answers.

Hence, $p = \frac{m}{n} \Rightarrow p = \frac{6}{32} \Rightarrow p = \frac{3}{16}$

120. **(a)** Since f(x) is the probability density function of random variable X.

$$\therefore \int_{-\infty}^{\infty} f(x) = 1$$

Now we have

$$\int_{-\infty}^{\infty} Ke^{-|x|}dx = 1 \Rightarrow 2\int_{0}^{\infty} K \cdot e^{-|x|}dx = 1$$
$$\Rightarrow 2\int_{0}^{\infty} K \cdot e^{-x}dx = 1$$
$$\Rightarrow -2\,K \cdot [e^{-x}]_{0}^{\infty} = 1 \Rightarrow 2\,K = 1$$
$$\Rightarrow K = \frac{1}{2}$$