PART-I PHYSICS

1. **(d)** The magnetic moment of the ground state of an atom whose open sub-shell is half filled with n electrons is given by

$\mu = \sqrt{n(n+2)} \cdot \mu_B$

where μ_B is the gyromagnetic moment of the atom.

Here, $n = 5$.

$\therefore \mu = \sqrt{5(5+2)} \cdot \mu_B = \sqrt{35}\mu_B$

2. **(a)** Bragg's law gives $2\, d \sin\theta = n\lambda, n =$ order of reflection, $d =$ distance between planes. For same λ and $d, n \propto \theta$; for a given λ, smallest d for least n, θ can be found. If crystal is symmetric reflections from different planes may cancel out.
3. **(b)** According to Moseley's law, square root of frequency of X-ray is plotted against atomic number it gives straight line, the relation is

$\sqrt{f} = c(Z-1)$ where $c =$ constant

(for $\sqrt{f} = 0, Z = +c$, for $Z = 0, \sqrt{f} = -c$)

$\therefore$ Option (b) is correct. as Z can not be negative.

4. **(b)** Balmer series is given for $n_1 = 2$ and $n_2 = 3,4, \ldots$

$$v = Rc\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$$

For Ist line in spectrum $n_2 = 3$

$$v_1 = Rc\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right] = Rc\left[\frac{1}{4} - \frac{1}{9}\right] = f$$

$$\Rightarrow f = \frac{5}{36}Rc \Rightarrow Rc = \frac{36}{5}f$$

For second line $n_2 = 4$

$$v_2 = Rc\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = Rc\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$= Rc\left[\frac{4-1}{16}\right] v_2 = \frac{3}{16}Rc = \frac{3}{16} \times \frac{36}{5}f = 1.35f.$$

5. **(d)** Here, Kinetic energy = Rest energy

we know that

Kinetic energy (Relativistic)

$= (m - m_0)c^2$

and Rest energy $= m_0c^2$,

where $m_0 =$ rest mass, $c =$ velocity of light Also, mass (m) of a particle moving with velocity v is given by

$m = \frac{m_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$.

$\therefore (m - m_0)c^2 = m_0c^2$ provides

$$\left(\frac{m_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} - m_0\right)c^2 = m_0c^2$$

or, $\frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} - 1 = 1$

or, $\frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} = 2$ or, $1 - \frac{v^2}{c^2} = \frac{1}{4}$

or, $\frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore v = \frac{\sqrt{3}}{2}c.$

$$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$\Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow v^2 = \frac{3}{4}c^2$$

$$\Rightarrow v = \sqrt{\frac{3}{2}}c$$

6. **(d)** de-Broglie wavelength (λ) of a particle of mass m and moving with a velocity v is given by, $\lambda = \frac{h}{mv}$, where h is Planck's constant.

When a particle having charge q is accelerated through a potential V then

$$qV = \frac{1}{2}mv^2$$

or, $qV = \frac{m^2v^2}{2\ m} \Rightarrow mv = \sqrt{2mqV}$

$$\therefore \lambda = \frac{h}{\sqrt{2mqV}}$$

Hence, de-Broglie wavelength of electron,

$$\lambda_e = \frac{h}{\sqrt{2\ m_e eV}}$$

and de-Broglie wavelength of proton,

$\lambda_p = \frac{h}{\sqrt{2\ m_p eV}}$, where e represents the charge of electron (or proton)

$$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\left(\frac{m_p}{m_e}\right)}$$

7. **(c)** Relative speed is given by

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

Here, $u' = 0.8c$ and $v = 0.4c$

$$\therefore u = \frac{0.8c + 0.4c}{1 + \frac{(0.8c)(0.4c)}{c^2}} = \frac{1.2c}{1.32} = 0.9c$$

8. **(d)** By Einstein's equation, photoemission occurs when $h\nu > \phi_0$ or $h\nu > h\nu_0$ i.e., frequency of incident photon is greater than threshold frequency.

9. **(c)** Radius of nucleus is given by $R \propto A^{1/3}$ $R = R_0 A^{1/3}$, where A = mass number

10. **(c)** Radio carbon dating is done by measuring ratio of ^{14}C present in the sample, since proportion of ^{14}C and ^{12}C in a body is same; but after death ^{14}C decays. Hence knowing the present ratio of ^{14}C to ^{12}C, sample can be dated.

11. **(b)** α particles are positively charged He nucleus, it can accept $2e^-$, β rays are negatively charged which are similar to e^-, can donate $1e^-$, γ are radiations. Hence ionisation power of α is maximum. γ are most energetic and $\propto$ is least energetic $\therefore$ Penetration power of γ is maximum

12. **(d)** Given half life T = 3.8 days; t = 19 days

$$\frac{N}{N_0} = ? \text{ now } \frac{t}{T} = \frac{19}{3.8} = 5$$

$$\Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T}} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$\Rightarrow N = \frac{N_0}{32} = 0.03\ N_0$$

13. **(a)** If the p-n junctions are identical, then their resistances would be same. In circuit 1 , the first p-n function is forward biased and second is reverse biased. Hence the voltages across them would be different. In circuit 2 both are reverse biased. So potential drop would be same. In circuit 3 both are forward biased; again voltages would be same.

14. **(a)** Zener diode is a semiconductor device and for semiconductors, temperature coefficient is negative

15. **(a)** The truth table corresponds to the logic $\overline{A + B}$ hence it is NOR gate.

16. **(d)** $\bar{\bar{A}}.\bar{B}$ in Boolean algebra, can be written as $\overline{\overline{A}.\overline{B}} = \bar{\bar{A}} + \bar{\bar{B}} = A + B$

17. **(a)** By Dopplers effect, if source moves towards the observer, frequency received will increase.

As $v \propto \frac{1}{\lambda}$

$\therefore$ wavelength will decrease.

18. **(d)** A coaxial cable consists of a conducting wire surrounded by a dielectric space, over which there is a sleeve of coppermesh covered with a shield of PVC insulation. The power transmission is regulated by dielectric. At high frequencies energy loss due to mesh is significant (called skin effect).

19. **(b)** Due to their inherent distortion, platemodulated class C amplifiers are not used as audio amplifiers. Also, class A amplifiers are not used owing to low efficiency. A gridmodulated amplifier has very high frequency. Therefore, in output stage of a TV transmitter, grid-modulated class C amplifier is used.

20. **(b)** The rms value of carrier current is $I_c = 8$ A. The rms value of modulated current

$= I_c + I_m = 8.93 \text{ A} = I_t$

Percentage modulation $= m_a \times 100$

The current relation in AM wave is

$$\frac{I_t^2}{I_c^2} = 1 + \frac{m_a^2}{2} \Rightarrow m_a = \sqrt{\left(\frac{I_t^2}{I_c^2} - 1\right)2}$$

$$\Rightarrow \text{ Modulation index, } m_a = \sqrt{\left(\frac{(8.93)^2}{8^2} - 1\right)2}$$

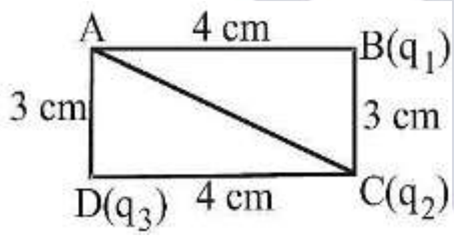
$$= \sqrt{\left(\frac{79.7}{64} - 1\right)2} = \sqrt{(1.24 - 1)2}$$

$$= 0701$$

Percentage modulation $= m_a \times 100 = 70.1\%$

21. **(d)** For a point charge, $E \propto \frac{1}{x^2}$. For positive charges, electric field will decrease in positive direction as distance increases, (case 1), for negative charge, as distance increases field will increase (case 4). As we move from positive charge to negative charge, field will keep on decreasing (case 2), As we move from negative to positive charge, field will keep on increasing (case 3)

22. **(a)**



$AC = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$ cm.

$\therefore$ Potential at A $= V_B + V_C + V_D$.

$$\therefore V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{AB} + \frac{q_2}{AC} + \frac{q_3}{AD}\right)$$

$$V_A = 9 \times 10^9\left(\frac{10 \times 10^{-12}}{4 \times 10^{-2}} + \frac{-20 \times 10^{-12}}{5 \times 10^{-2}} + \frac{10 \times 10^{-12}}{3 \times 10^{-2}}\right)$$

$$= 9 \times 10^9 \times 10^{-10}\left(\frac{10}{4} + \frac{-20}{5} + \frac{10}{3}\right)$$

$$= 0.9\left(\frac{150 - 240 + 200}{60}\right)$$

$= \dfrac{0.9 \times 110}{60} = 1.65 \text{ V}$

23. (d) The capacitance (C) of a parallel plate capacitor with dielectric is given by

$$C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

$C = 100\text{pF} = 100 \times 10^{-12}\text{ F}, \epsilon_0 = 8.85 \times 10^{-12}$

$A = \pi r^2 = 3.14 \times (10^{-2})^2 (r = 1\text{ cm})$

$t = 10^{-3}\text{ m}, d = t, K = 4$

$$C_1 = \frac{8.85 \times 10^{-12} \times 3.14 \times 10^{-4}}{10^{-3} - 10^{-3}\left(1 - \frac{1}{4}\right)}$$

$$= \frac{27.79 \times 10^{-16}}{10^{-3} - 0.75 \times 10^{-3}}$$

$$= \frac{27.79 \times 10^{-13}}{0.25}$$

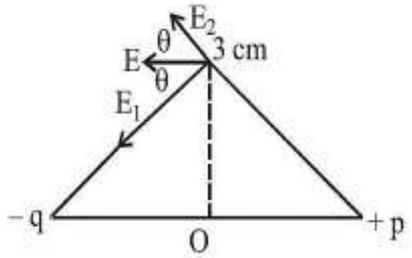
$$= \frac{27.79 \times 10^{-13}}{0.25} = 111.16 \times 10^{-13}\text{ F}$$

(for 1 set)

Required $C = 100 \times 10^{-12}$ F

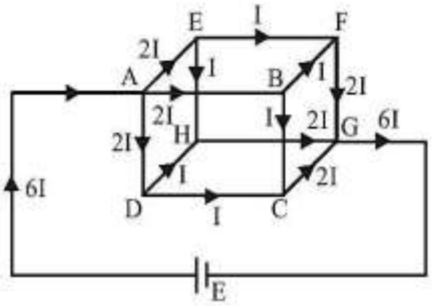
$$\Rightarrow n = \frac{C}{C_1} = \frac{100 \times 10^{-12}}{111.16 \times 10^{-13}} \simeq 10$$

24. (a) The resultant intensity at a point on equatorial line is $\vec{E}$. $\vec{E}$ is parallel and opposite to direction of $\vec{p}$.



25. (d) Let a total current of 6 I enter at A. It divides into three equal parts, each of 2I, along AE, AB and AD. At E, B and D each the current 2I divides into two equal parts, each of I, along EF, EH, BF, BC, DH and DC.

At F, H and C, the two currents each of I, combine together to give a current of 2I at each corner. Thus, at G we get the same current 6I as shown in the figure.



Let r be the value of resistance of each arm of the cube and R be the joint resistance across the conners A and G.

Applying Kirchhoff's law along the loop AEFGA, we get

$2Ir + Ir + 2Ir = E$

or, $5Ir = E$

Also, by Ohm's law,

$6I \times R = E$

$\Rightarrow 6IR = 5Ir$, using (1)

or, $R = \frac{5}{6}r$.

Here, $r = 6\Omega$

$\therefore R = \frac{5}{6} \times 6 = 5\Omega$

26. (c) Since conductor and semiconductor are connected in parallel hence voltage across them is same. If ammeters show same reading hence their resistances $R = \frac{V}{I}$ are same. If voltage is increased by small value then following the same relation $V \propto I$ for constant R, both conductor and semiconductor show same current.

27. (d) Time taken by free electrons to cross the conductor

$t = \frac{\ell}{v_d}$ where drift velocity $v_d = \frac{I}{neA}$

$$\Rightarrow v_d = \frac{1}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-7}}$$
$$= \frac{10^{-2}}{64} \text{ m/s}$$

$$\Rightarrow t = \frac{1}{v_d} = \frac{64}{10^{-2}} = 64 \times 10^2 \text{ s} = 6.4 \times 10^3 \text{sec}$$

28. (b) Temperature coefficient, $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$

$$\Rightarrow (t_2 - t_1) = \frac{R_2 - R_1}{\alpha R_1}$$

$$\Rightarrow t_2 - t_1 = \frac{2 - 1}{1 \times 0.00125}$$

$$t_2 = \frac{1}{0.00125} + t_1 = 800 + 300 = 1100 \text{ K}$$

29. (d) Torque on the coil is $\tau = n$ IB A $\cos\theta$. If coil is set with its plane parallel to direction of magnetic field B, then $\theta = 0°, \cos\theta = 1$

$\Rightarrow \tau = nIBA.\ 1 = nIBA =$ maximum.

Hence, $I =$ maximum (as n, B, A are constant)

30. (d) $E = at + \frac{1}{2}bt^2$ is the parabolic equation for thermo emf. The thermoelectric power is

$$S = \frac{dE}{dt}$$

$\Rightarrow S = a + bt$. The graph between $S\left(=\frac{dE}{dt}\right)$ and straight line.

When $t = 0$, $S = a$ (intercept).

At neutral temperature $\frac{dE}{dt} = 0$ and

$t = t_n \Rightarrow 0 = a + bt_n \Rightarrow t_n = \frac{-a}{b}$ and at cold

junction $t_i = 2t_n = \frac{-2a}{b}$

31. (d) Energy of proton = K.E. = 1MeV = 10^6eV.

Frequency $\nu = \frac{Bq}{2\pi m}$

$$= \frac{6.28 \times 10^{-4} \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 1.7 \times 10^{-27}}$$
$$\simeq 0.9 \times 10^4 \text{ Hz} \simeq 10^4 \text{ Hz}$$

32. (d) Force on a conductor of length ℓ due to a magnetic field of strength $\vec{B}$ is given by $\vec{F} = I(\vec{\ell} \times \vec{B})$

$\therefore |\vec{F}| = I\ell B\sin\theta$

Here, $\theta = 90°$

$\therefore F = I\ell B = I \times \pi R \times B = \pi RIB$

Therefore, a force of magnitude (π RIB) will act on each wire. The direction of the forces on each wire will be same. Thus, total force on the wire AB $= \pi RIB + \pi RIB$

$= 2\pi RIB$

33. (d) In first case, the two inductances are in series hence total inductance $= L_0 + L_0$

$= 2\ L_0$

In second case, the current is same, but no. of turns has doubled since the sense of turning is same.

Thus $L \propto n^2 \Rightarrow L \propto (2n)^2$

hence inductance $L = 4\ L_0$

In third case, the sense of turns is opposite. So net inductance cancel each other $\therefore L = 0$

34. (b) At resonance, impedance of circuit is minimum. When impedance of capacitor and inductor is same, $X_L = X_C$

Resonance frequency $v = \frac{1}{2\pi\sqrt{LC}}$ $v = \frac{1}{2\pi\sqrt{5\times80\times10^{-6}}} = \frac{1}{2\pi\sqrt{400\times10^{-6}}}$

$$= \frac{1}{2\pi \times 20 \times 10^{-3}}$$

$$= \frac{10^3}{40\pi} = \frac{1000}{40\pi} = \frac{25}{\pi}$$

35. (b) $M = 5H, I = 10\ A, t = 5 \times 10^{-4}$ s.

Now, emf induced in secondary is $e = -M\frac{dI}{dt}$, ($-$ve sign shows direction)

$\Rightarrow e = 5 \times \frac{10}{5 \times 10^{-4}} = 1 \times 10^{+5}$ V

36. (d) Average power of an LCR circuit is $P = E_v I_v \cos\phi$. Maximum power is dissipated at resonance when $X_L = X_C$. The resonance frequency is given by

$$v = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14 \times \sqrt{25 \times 10^{-3} \times 400 \times 10^{-6}}}$$

$$= \frac{1}{2 \times 3.14 \times 5 \times 20 \times 10^{-5}\sqrt{10}}$$

$$= \frac{10^5}{3.14 \times 200\sqrt{10}}$$

$$= \frac{10^3}{6.28 \times \sqrt{10}} = \frac{159.2}{3.16} = 50.31 \text{ Hz}$$

37. (d) Coherent sources should have same frequency and wavelength and constant or zero phase difference. Frequency is same only for wave X_1 and X_4 and phase difference $= \delta$

38. (c) Since position of central maximum is same, hence $a\sin\theta = \lambda$ is same for both wavelengths. i.e, $a =$ constant, $\theta =$ same, $\Rightarrow \lambda$ is same for both waves

$\Rightarrow \lambda_1 = \lambda_2$ and $\frac{\lambda_1}{\lambda_2} = 1$

39. (b) For bright fringes, $I_{max} = (a+b)^2$

For dark fringes, $I_{min} = (a-b)^2$

Now $\frac{I_{max}}{I_{min}} = 9 = \frac{(a+b)^2}{(a-b)^2} \Rightarrow \frac{a+b}{a-b} = 3$

$\Rightarrow a + b = 3(a-b) \Rightarrow a + b = 3a - 3b \Rightarrow 2a = 4b$

$\frac{a}{b} = 2 \Rightarrow \frac{a^2}{b^2} = 4$

$\therefore$ Ratio of intensities of the two slits,

$\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{4}{1}$

40. (b) Rising and setting sun appears red because light from the sun travels slightly more distance from the horizon, than when it is overhead. Hence blue light is scattered by dust in the atmosphere because scattering

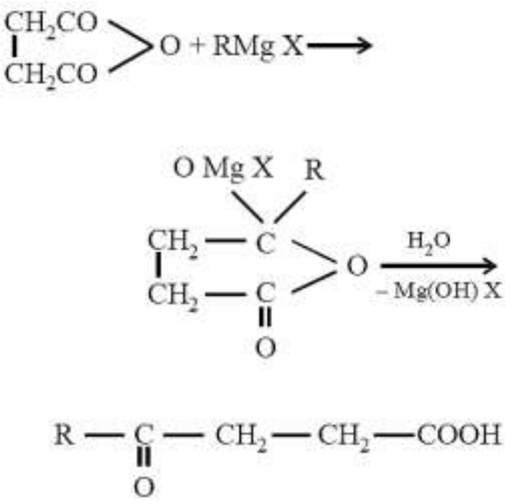
$\propto \frac{1}{\lambda_4}$ and $\lambda_b < \lambda_r$. Red colour is less scattered and reaches us.

PART-II CHEMISTRY

41. (b) Roseumund's reaction -

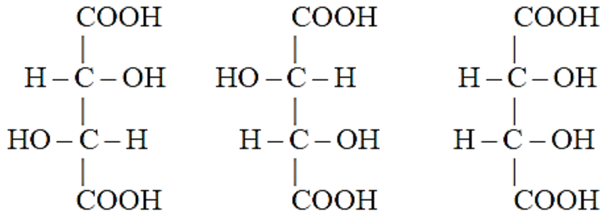
$$RCOCl + H_2 \xrightarrow{Pd/BaSO_4} RCHO + HCl$$

42. (b)

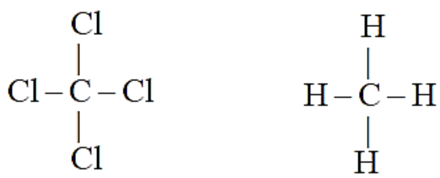


43. **(a)**

(+) – Tartaric acid does not have element of symmetry.



(+) – Tartaric acid (A)   Meso tartaric acid (D)
(Plane of symmetry)



Carbon tetracholride (B)   (C) Methane

44. **(b)** Proton doners are acids. Electron withdrawing groups (like halogens) increase the acidity of carboxylic acids. Therefore, HCOOH is weakest acid among the given choices.

45. **(a)** Toluene is used in the reaction to make both substances (acid and alcohol) miscible.

46. **(c)** Trans esterification is the process of conversion of one ester to another ester.

$$RCOOR' + R''OH \longrightarrow RCOOR'' + R'OH$$

47. **(a)** It is expected that the basic nature of amines should be in order tertiary > secondary > primary but the observed order in the case of lower members is found to be as secondary > Primary > tetriary. This anomalous behaviour of tetriary amines is due to steric factors i.e crowding of alkyl groups cover nitrogen atom from all sides thus makes the approach and bonding by a proton relatively difficult which results the maximum steric strain in tetiary amines. The electrons are there but the path is blocked, resulting the reduction in basicity.

Thus, the correct order is

$$R_2NH > RNH_2 > R_3N.$$

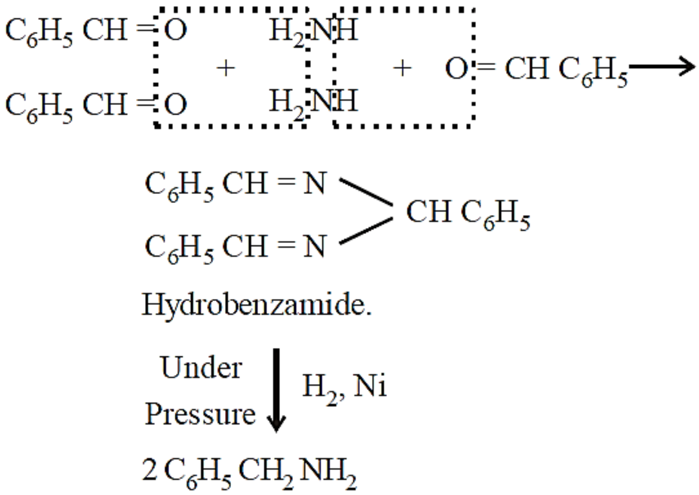
48. (d) $C_6H_5\,N_2^+Cl^- + H_2O \xrightarrow{\text{Boiling}} C_6H_5OH + N_2 + HCl$ Phenol

49. (a) Carbylamine test is given by aliphatic primaryamine.

Aliphatic (or aromatic) 1° amine $+CHCl_3 +$

$KOH \longrightarrow$ Bad smelling isocyanide.

50. (c) Benzaldehyde reacts with ammonia to form hydrobenzamide.



51. (b) Hybridisation $= \frac{1}{2}$ [Number of valence electrons of central atom + no. of monovalent atoms attached to it + negative charge if any - positive charge if any]

$= \frac{1}{2}[6 + 4 + 0 - 0] = 5 = sp^3\,d$

$[\because Te(52) = [Kr]4\,d^{10}5\,s^2 5p^4]$

52. (a) The permanganate ion has an intense purple colour. Mn (+ VII) has a d^0 configuration. So the colour arises from charge transfer and not from d-d spectra. In MnO_4^- an electron is momentarily changing O^{--} to O^- and reducing the oxidation state of the metal from Mn(VII) to Mn(VI). Charge transfer requires that the energy levels on the two different atoms are fairly close.

$O = (8) = \underset{K}{2}, \underset{L}{6}$

$Mn(25) = \underset{K}{2}, \underset{L}{8}, \underset{M}{15}$

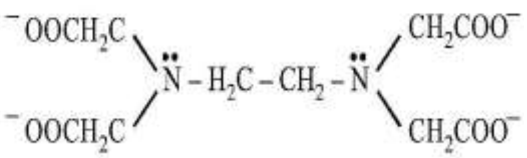
hence the charge transfer occurs from $L \rightarrow M$.

53. (d) ${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{144}_{56}Ba + {}^{92}_{36}Kr + 2\,{}^{1}_{0}n$

Sum of atomic number of reactants = sum of atomic masses of products.

54. (b) AgCl dissolves in a solution of NH_3 but not in water because Ag^+ forms soluble complex ion with NH_3.

55. (b) EDTA is hexadentate ligand. 4 oxygen and 2 nitrogen atoms act as donars.



56. (c) A co-ordinate bond is a dative covalent bond in which two atoms form a bond and one of them provides both electrons

57. (d) $[Ni(NH_3)_6]Cl_2$ $sp^3\ d^2$ hybridisation

2 unpaired electrons

$Na_3[FeF_6]$ $sp^3\ d^2$ hybridisation

3 unpaired electrons

$[Cr(H_2O)_6]SO_4$ d^2sp^3 hybridisation

3 unpaired electrons

$K_4[Fe(CN)_6]$ d^2sp^3 hybridisation

No unpaired electrons

Zero magnetic momoment means all the electrons paired.

58. (c) The IUPAC name is dichloro triphenyl phosphine nickel (II).

59. (c) (d) In $K_4[Fe(CN)_6]$Cu is in +1 oxidation state hence has no unpaired electron hence colourless and diamagnetic.

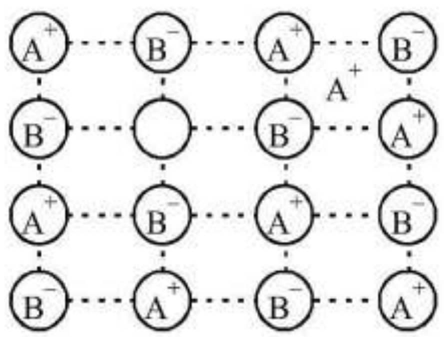
(b) In $(NH_4)_2[TiCl_6]$Ti is in +4 oxidation state. hence has no unpaired electron hence colourless and diamagnetic.

(c) In $VOSO_4$, V is in +4 oxidation state hence has one unpaired electron, thus it is coloured and paramagnetic.

(a) In $K_2Cr_2O_7$, Cr is in +6 oxidation. hence has no unpaired electron and thus it is diamagnetic. Though $K_2Cr_2O_7$ has no unpaired electron but it is coloured. This is due to charge transfer spectrum.

60. (b) On an X-ray diffraction photograph the intensity of the spots depends on electron density of atoms/ions.

61. (a) In Frankels defect an ion leaves its regular site and occupy a position in the space between the lattice sites.



62. (b) 8:8 type of packing is present in CsCl. 6: 6 type of packing is present in NaCl and KCl.

63. (d) When solid melts S increases. because when solid changes into liquid randomness increases.

64. (d) Gibbs Helmholtz Equation-

$\Delta G = \Delta H - T\Delta S$

differentiate this equation w.r.t. temperature at constant pressure

$$\left(\frac{\partial \Delta G}{\partial T}\right)_P = \left(\frac{\partial G_y}{\partial T}\right)_P - \left(\frac{\partial G_x}{\partial T}\right)_P$$

$$= -S_y - (-S_x)$$

$$= -(S_y - S_x) = -\Delta S$$

where ΔS change in entropy

on combining equation (1) & (3) we get

$$\Delta G = \Delta H + T\left(\frac{\partial(\Delta G)}{\partial T}\right)_P$$

Equation (4) is on alternative form of Gibbs Helmholtz equation

Dividing equ. (4) by T^2, we get

$$\frac{\Delta G}{T^2} = \frac{\Delta H}{T^2} + \frac{1}{T}\left[\frac{\partial(\Delta G)}{\partial T}\right]_P$$

on rearrangement, we get

$$\left[\frac{\partial(\Delta G)}{\partial T}\right]_P = -\frac{\Delta H}{T^2}$$

$$\therefore \Delta H = -T^2\left[\frac{\partial(\Delta G)}{\partial T}\right]_P$$

65. **(a)** Since $\Delta G = \Delta A + P.\Delta V$

For a spontaneous process ΔG should be negative which is possible only if

$\Delta A + P.\Delta V$ or $\Delta A + W < 0$.

66. **(c)**

67. **(b)** Endothermic reactions are favoured at high temperature. Therefore, increasing the temperature will shift equilibrium to the right.

68. **(a)** $\ln\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

69. **(a)**

$$H_2 + I_2 \rightleftharpoons 2HI$$

Initial moles 1 1 0

At equilibrium $(1-x)(1-x)$ $2x$

$$K_c = \frac{[HI]^2}{[H_2][I_2]} = \frac{(2x)^2}{(1-x)^2}$$

Given $x = 80\% = 0.80$

$$\therefore K_c = \frac{(2 \times 0.80)^2}{(1-0.80)^2} = 64$$

70. **(d)** The equilibrium constant K_p will change with temperature for the reaction

$H_{2(g)} + I_{2(g)} \rightleftharpoons 2HI_{(g)}$.

Catalyst does not alter the state of equilibrium.

Equilibrium constant depends only upon temperature. The relation of k with temp can be shown as

$$\log\frac{k_2}{k_1} = \frac{\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

71. **(d)** Eq. mass of $Ca^{++} = \frac{\text{Mol.mass}}{2} = \frac{40}{2} = 20$

$$Z = \frac{\text{Eq.mass of water}}{96500} = \frac{20}{96500}$$

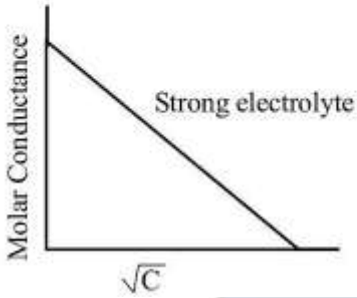
Given $w = 60$ g, $i = 5$amp.

$w = zit$

or $60 = \frac{20}{96500} \times 5 \times t$

$\therefore t = \frac{96500 \times 60}{20 \times 5} = 57900\text{sec} = 16$ h.

72. **(b)** According to Debye - Huckel - Onsagar equation $\Lambda_m = \Lambda_m^o - (A + B\Lambda_m^o)\sqrt{C}$ where A and B are the Debye - Huckel constants. If we plot a graph between molar conductance (Λ_m) against the square roots of the concentration $(\sqrt{C})$ a straight line is obtained



73. **(c)** Molar conductance of $CaCl_2$

$=$ Molar conductance of $Ca^{2++} + 2 \times$ (molar conductance of Cl^-)

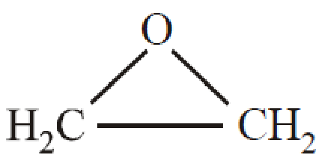
$= 118.88 \times 10^{-4} + 2(77.33 \times 10^{-4})$

$= 273.54 \times 10^{-4}\ m^2 mhomol^{-1}$

74. **(a)** Lower the value of reduction potential, greater will be the reducing power of element.

Since Zn has lowest reduction potention hence Zn is the strongest reducing agent.

75. **(a)** The epoxide ring consists of three membered ring with two carbon atoms and one oxygen.



76. **(c)** Grignard reagent is a sigma bonded organometallic compound in which Mg is bonded with one alkyl and one halogen group. This can be prepared as

$R - X + Mg \xrightarrow{\text{ether}} R - Mg - X$

77. **(a,c)** Presence of electron attracting group like$NO_2, -Cl$ increases the acidity of phenol as it enables the ring to draw more electrons from the phenoxy oxygen and thus releasing easily the proton. Presence of electron releasing group e.g. $-OCH_3$ on benzene sing decreases the acidity of phenol as it strengthens the negative charge on phenoxy oxygen and thus proton release becomes difficult.

Further phenols are much more acidic than alcohols. The acidic nature of phenol is due to the formation of stable phenoxide ion in solution.

$C_6H_5OH + H_2O \rightleftharpoons C_6H_5O^- + H_3O^+$

No resonance is possible in alkoxide ions (RO^-)derived from alcohols. The negative charge is localized on oxygen atom. Thus alcohols are not acidic.

78. (d) Aldol condensation is given by aldehydes which have $\alpha - H$ atoms. So acetaldehyde gives this reaction.

$(CH_3)_3C-\overset{\overset{O}{||}}{C}-H$ Trimethyl acetaldehyde $CH_3-\overset{\overset{O}{||}}{C}-H$ acetaldehyde

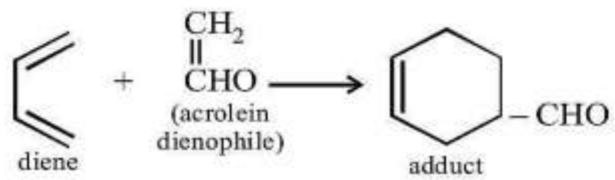
$C_6H_5-\overset{\overset{O}{||}}{C}-H$ benzaldehyde $H-\overset{\overset{O}{||}}{C}-H$ formaldehyde

79. (d)

$$\underset{6}{H_3C}-\underset{5}{CH_2}-\underset{4}{\overset{\overset{O}{||}}{C}}-\underset{3}{H_2C}-\underset{2}{CH_2}-\underset{1}{\overset{\overset{O}{||}}{C}}-OCH_3$$

Methyl 4 – oxohexanoate

80. (b) It is cycloaddition reaction between a conjugated diene and subsituted alkene or α, β unsaturated carboxyl compound.



PART-III MATHEMATICS

81. (c) Since $\vec{a}$ and $\vec{b}$ are coplanar, therefore $\vec{a} \times \vec{b}$ is a vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. Similarly $\vec{c} \times \vec{d}$ is a vector perpendicular to the plane containing $\vec{c}$ and $\vec{d}$. Thus, the two planes will be parallel if their normals i.e. $\vec{a} \times \vec{b}$ and $\vec{c} \times \vec{d}$ are parallel.

$\Rightarrow (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$

82. (d) Given: diagonals $\vec{d}_1 = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{d}_2 = \hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\because$ Area of a parallelogram $= \frac{1}{2}|\, d_1 \times d_2|$

Now, $\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

$= \hat{\imath}(4-6) - \hat{\jmath}(12+2) + \hat{k}(-9-1)$

$\Rightarrow \vec{d}_1 \times \vec{d}_2 = -2\hat{\imath} - 14\hat{\jmath} - 10\hat{k}$

$\therefore |\vec{d}_1 \times \vec{d}_2| = \sqrt{(-2)^2 + (-14)^2 + (-10)^2}$

$= \sqrt{4+196+100} = \sqrt{300} = 2\sqrt{75}$

$\therefore$ Area of parallelogram $= \frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right|$

$= \frac{1}{2} \times 2\sqrt{75} = \sqrt{75}$ square units

83. **(d)** Given: $\cos x + \cos^2 x = 1$

$\Rightarrow \cos x = 1 - \cos^2 x \Rightarrow \cos x = \sin^2 x \Rightarrow \cos^2 x = \sin^4 x \Rightarrow 1 - \sin^2 x = \sin^4 x \Rightarrow \sin^4 x + \sin^2 x = 1$

cubic both sides, we have $\sin^{12} x + \sin^6 x + 3\sin^6 x(\sin^4 x + \sin^2 x) = 1 \Rightarrow \sin^{12} x + \sin^6 x + 3\sin^{10} x + 3\sin^8 x = 1 \Rightarrow \sin^{12} x + 3\sin^{10} x + 3\sin^8 x + \sin^6 x - 1 = 0$

84. **(c)** $(\cos\alpha + i\sin\alpha)^{3/5} = (\cos 3\alpha + i\sin 3\alpha)^{1/5} = [\cos(2k\pi + 3\alpha) + i\sin(2k\pi + 3\alpha)]^{1/5}$

$= \left[\cos\left(\frac{2k\pi+3\alpha}{5}\right) + i\sin\left(\frac{2k\pi+3\alpha}{5}\right)\right],$

where $k = 0,1,2,3,4$

Product of all values.

$$= \left(\cos\frac{3\alpha}{5} + i\sin\frac{3\alpha}{5}\right) \cdot \left(\cos\left(\frac{2\pi+3\alpha}{5}\right)\right.$$

$$\left.+i\sin\left(\frac{2\pi+3\alpha}{5}\right)\right) \cdot \left(\cos\frac{4\pi+3\alpha}{5} + i\sin\right.$$

$$\left.\frac{4\pi+3\alpha}{5}\right) \cdot \left(\cos\frac{6\pi+3\alpha}{5} + i\sin\frac{6\pi+3\alpha}{5}\right)$$

$$\left(\cos\left(\frac{8\pi+3\alpha}{5}\right) + i\sin\left(\frac{8\pi+3\alpha}{5}\right)\right)$$

$$= \cos\left[\frac{3\alpha}{5} + \frac{2\pi+3\alpha}{5} + \frac{4\pi+3\alpha}{5} + \frac{6\pi+3\alpha}{5} +\right.$$

$$\left.\frac{8\pi+3\alpha}{5}\right] + i\sin\left[\frac{3\alpha}{5} + \frac{2\pi+3\alpha}{5} + \frac{4\pi+3\alpha}{5}\right.$$

$$\left.+\frac{6\pi+3\alpha}{5} + \frac{8\pi+3\alpha}{5}\right]$$

$$= \cos\left[\frac{5}{2}\left(2\cdot\frac{3\alpha}{5} + (5-1)\cdot\left(\frac{2\pi}{5}\right)\right)\right]$$

$$+i\sin\left[\frac{5}{2}\left(2\cdot\frac{3\alpha}{5} + (5-1)\cdot\left(\frac{2\pi}{5}\right)\right)\right]$$

$$= \cos\left[\frac{5}{2}\cdot\left(\frac{6\alpha}{5} + \left(\frac{8\pi}{5}\right)\right)\right] +$$

$i\sin\left[\frac{5}{2}\left(\frac{6\alpha}{5} + \frac{8\pi}{5}\right)\right]$

$= \cos(3\alpha + 4\pi) + i\sin(3\alpha + 4\pi)$
$= \cos(4\pi + 3\alpha) + i\sin(4\pi + 3\alpha)$
$= \cos 3\alpha + i\sin 3\alpha$

85. (d) $\frac{(1+i)^2}{i(2i-1)} = \frac{1+i^2+2i}{2i^2-i}$

$$\Rightarrow \frac{1-1+2i}{-2-i} = -\frac{2i}{2+i} \times \frac{(2-i)}{(2-i)} = \frac{-4i+2i^2}{4-i^2}$$

$[\because i^2 = -1]$

$$= \frac{-4i-2}{4+1} = \frac{-2-4i}{5} \Rightarrow \frac{-2}{5} - \frac{4i}{5}$$

$\therefore$ The imaginary part $= -\frac{4}{5}$

86. (a) Given $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$

we know that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x = \pi/2 - \cos^{-1} x$

$\therefore$ Equation (1) becomes.

$$\frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$$

87. (c) Equation of ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where $a > b$.

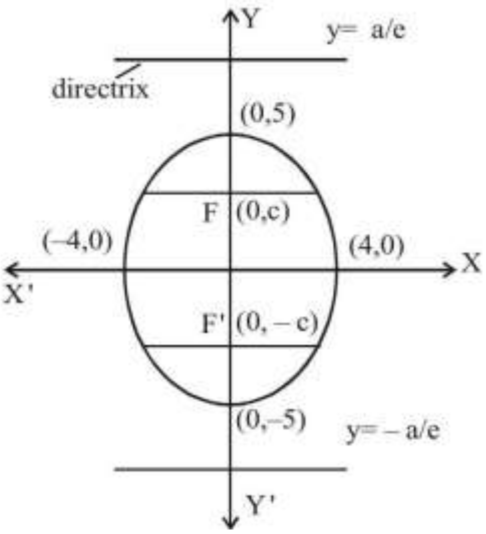
Given, $\frac{x^2}{16} + \frac{y^2}{75} = 1 \Rightarrow b = 4, a = 5$

But $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}}$

$$\Rightarrow e = \frac{3}{5}$$

$\therefore$ equation of directrix $y = \pm\frac{a}{e}$

$$\therefore y = \pm\frac{5}{3/5} \Rightarrow 3y = \pm 25$$



88. **(a)** Since the normal at $(ap^2, 2ap)$ on $y^2 = 4ax$ meets the curve again at $(aq^2, 2aq)$, therefore $px + y = 2ap + ap^3$ passes through $(aq^2, 2aq) \Rightarrow paq^2 + 2aq = 2ap + ap^3$

$\Rightarrow p(q^2 - p^2) = 2(p - q)$

$\Rightarrow p(q + p) = -2$

$\Rightarrow p^2 + pq + 2 = 0$

89. **(d)** Given equation of line, $x - 3y = 1$

and hyperbola $x^2 - 4y^2 = 1$

putting $x = 1 + 3y$ in equation (2), we get

$(1 + 3y)^2 - 4y^2 = 1 \Rightarrow 1 + 9y^2 + 6y - 4y^2 = 1 \Rightarrow 5y^2 + 6y = 0 \Rightarrow y(5y + 6) = 0$

$\Rightarrow y = 0$ or $y = -\frac{6}{5}$

$\therefore x = 1$ for $y = 0 \& x = 1 - \frac{18}{5} = \frac{-13}{5}$

for $y = -6/5$

$\therefore$ the line (1) cuts the hyperbola (2) in at most two point.

$\therefore$ co-ordinates of points are $P(1,0)$ &

$Q\left(-\frac{13}{5}, -\frac{6}{5}\right)$

$$\therefore PQ = \sqrt{\left(1 + \frac{13}{5}\right)^2 + \left(0 + \frac{6}{5}\right)^2}$$

$$= \sqrt{\left(\frac{18}{5}\right)^2 + \frac{36}{25}} = \sqrt{\frac{324 + 36}{25}} = \sqrt{\frac{360}{25}}$$

$\therefore$ length of straight line $PQ = \frac{6\sqrt{10}}{5}$ units.

90. **(c)** Given $x = t^2 + 2t - 1 \& y = 3t + 5$

$\Rightarrow x = t^2 + 2t + 1 - 2 \& y = 3t + 3 + 2$

$\Rightarrow x = (t+1)^2 - 2 \& y = 3(t+1) + 2$

$\Rightarrow (t+1) = \sqrt{x+2}$ (1)

$\therefore$ Equation (2) becomes [using equation (1)]

$y = 3\sqrt{x+2} + 2$

$\Rightarrow y - 2 = 3\sqrt{x+2}$

squaring both sides, we get

$(y-2)^2 = 9(x+2)$

$\Rightarrow Y^2 = 9X$ where $Y = y - 2 \& X = x + 2$

This equation represents a parabola

91. (c) Slope of the normal at (3,4) is the value of

$-\frac{1}{f'(x)}$ at $x = 3$ or $-\frac{1}{f'(3)} = \tan\frac{3\pi}{4} = 1$

$\Rightarrow f'(3) = 1$

92. (c) Given: $f(x) = x^2 e^{-2x}, x > 0$

$f'(x) = x^2 \cdot e^{-2x}(-2) + e^{-2x} \cdot 2x$

put $f'(x) = 0 \Rightarrow 2e^{-2x} \cdot x(-x+1) = 0$

$\Rightarrow x = 1$ or $x = 0$

$f''(x) = (-4x^2 - 6x + 1)e^{-2x}$

$f''(1) = -9e^{-2x} < 0$

$f''(0) = e^{-2x} > 0$

$\therefore$ value of $f(x)$ is maximum at $x = 1$

$\because f(x) = x^2 \cdot e^{-2x} \Rightarrow f(1) = e^{-2} = \frac{1}{e^2}$

93. (d) Given: $(x+y)\sin U = x^2y^2$

$\Rightarrow \sin U = \frac{x^2y^2}{x+y} = v$ (let)

Here $n = 2 - 1 = 1$

Euler's theorem $x \cdot \frac{\partial v}{\partial x} + y \cdot \frac{\partial v}{\partial y} = nv$

$\therefore x\frac{\partial \sin U}{\partial x} + y\frac{\partial \sin U}{\partial y} = \sin U$

$\Rightarrow x\cdot\cos U\frac{\partial U}{\partial x}+y\cdot\cos U\cdot\frac{\partial U}{\partial y}=\sin U\Rightarrow x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}=\frac{\sin U}{\cos U}=\tan U$

$\Rightarrow x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}=\tan U$

94. (c) Equation of the given curve in parametric form,

$x=t^2+1$ and $y=t^2-t-6$

Y-coordinate of the point, where the given curve meets X-axis is 0 .

When $y=0$, then $t^2-t-6=0$

$\Rightarrow t^2-3t+2t-6=0$

$\Rightarrow t(t-3)+2(t-3)=0$

$\Rightarrow (t-3)(t+2)=0$

$\Rightarrow t=3$ or -2

when $t=3$, then $x=10$

when $t=-2$, then $x=5$

Hence, the points where the curve meets the X-axis are $(10,0)$ and $(5,0)$.

Now, $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2t-1}{2t}$

Slope of the tangent at point $(10,0)$

$$m_1=\left.\frac{dy}{dx}\right]_{x=10}=\left.\frac{dy}{dx}\right]_{t=3}=\frac{5}{6}$$

Slope of the tangent at point $(5,0)$,

$$m_2=\left.\frac{dy}{dx}\right]_{x=5}=\left.\frac{dy}{dx}\right]_{t=-2}=\frac{-5}{-4}=\frac{5}{4}$$

If θ be the angle beween two tangents, then

$$\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|=\left|\frac{\frac{5}{4}-\frac{5}{6}}{1+\frac{5}{6}\cdot\frac{5}{4}}\right|$$

$$=\left|\frac{\frac{15-10}{24+25}}{\frac{24}{24}}\right|=\left|\frac{\frac{5}{12}}{\frac{49}{24}}\right|=\left|\frac{10}{49}\right|$$

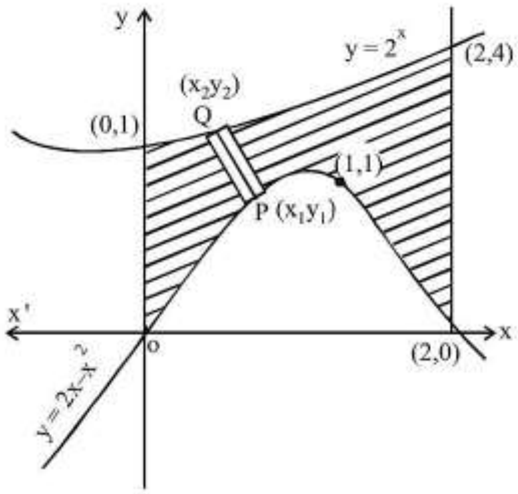
$$\Rightarrow\tan\theta=\pm\frac{10}{49}$$

$$\therefore\ \theta=\tan^{-1}\left(\pm\frac{10}{49}\right)=\pm\tan^{-1}\left(\frac{10}{49}\right)$$

95. (b) $\int_1^4|x-3|dx=\int_1^3-(x-3)dx+\int_3^4(x-3)dx$

$$= -\left[\frac{x^2}{2} - 3x\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$$
$$= -\frac{1}{2}[3^2 - 1^2] + 3[3-1] + \frac{1}{2}\cdot[4^2 - 3^2] - 3[4-3]$$
$$= -\frac{1}{2}(8) + 3(2) + \frac{1}{2}\cdot(7) - 3(1)$$
$$= -4 + 6 + \frac{7}{2} - 3 = \frac{5}{2}$$

96. **(b)** Required area $= \int_0^2 (y_2 - y_1)dx$



$$= \int_0^2 (2^x - 2x + x^2)dx$$
$$= \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2$$
$$= \frac{4}{\log 2} - 4 + \frac{8}{3} - \frac{1}{\log 2} = \frac{3}{\log 2} - \frac{4}{3}$$

97. **(d)** Let $I = \int_0^\infty \frac{dx}{(a^2+x^2)^7}$

Put $x = a\tan\theta \Rightarrow dx = a\sec^2\theta d\theta$

limit at $x = 0 \Rightarrow \theta = 0 \& x = \infty \Rightarrow \theta = \frac{\pi}{2}$

$\therefore I = \int_0^{\frac{\pi}{2}} \frac{a^2\sec^2\theta}{a^{14}(1+\tan^2\theta)^7} d\theta = \frac{1}{a^{13}}\int_0^{\frac{\pi}{2}} \frac{1}{\sec^{12}\theta} d\theta = \frac{1}{a^{13}}\cdot\int_0^{\frac{\pi}{2}} \cos^{12}\theta \cdot d\theta$

But

$$\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \cdot d\theta = \frac{1}{2}B(m,n)$$

$$\& \ B(m,n) = \frac{\sqrt{m}\sqrt{n}}{\sqrt{m+n}} \ \& \sqrt{\frac{1}{2}} = \Gamma\pi$$

$$\therefore I = \frac{1}{a^{13}}\int_0^{\frac{\pi}{2}} \sin^0\theta \cdot \cos^{12}\theta d\theta$$

$\therefore m = \frac{1}{2}, n = \frac{13}{2}$

$\therefore I = \frac{1}{2a^{13}} \frac{\sqrt{\frac{1}{2}} \cdot \sqrt{\frac{13}{2}}}{\sqrt{\frac{1}{2} + \frac{13}{2}}}$

$= \frac{1}{2a^{13}} \cdot \frac{\left[\pi \cdot \frac{11}{2} \cdot \frac{9}{2} \cdot \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\pi}\right.}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= \left(\frac{231}{2048} \cdot \frac{1}{a^{13}}\right) \pi$

98. (c) Let $I = \int e^x \frac{(1-x)^2}{(1+x^2)^2} dx$

$\Rightarrow I = \int e^x \frac{1 + x^2 - 2x}{(1 + x^2)^2} dx$

$= \int e^x \left[\frac{1 + x^2}{(1 + x^2)^2} - \frac{2x}{(1 + x^2)^2}\right] dx$

$\Rightarrow I = \int e^x \frac{1}{(1 + x^2)} dx - 2\int e^x \cdot \frac{x}{(1 + x^2)^2} dx$

$\Rightarrow I = \frac{1}{(1 + x^2)} \cdot e^x - \int e^x \cdot [-(1 + x^2)^{-2}]2xdx$

$-2 \int \frac{e^x x}{(1 + x^2)^2} dx$

$\Rightarrow I = \frac{e^x}{1 + x^2} + 2\int \frac{e^x x}{(1 + x^2)^2} dx$

$-2 \int \frac{e^x x}{(1 + x^2)^2} dx$

$\Rightarrow I = \frac{e^x}{1 + x^2} + C$

99. (c) Given: $x\sin\left(\frac{y}{x}\right) dy = \left[y\sin\left(\frac{y}{x}\right) - x\right] dx \Rightarrow \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)}$

Put $\frac{y}{x} = z \Rightarrow \frac{dy}{dx} = z \cdot 1 + x \cdot \frac{dz}{dx}$

$\therefore x \cdot \frac{dz}{dx} + z = \frac{zx\sin z - x}{x\sin z} = z - \operatorname{cosec} z$

$\Rightarrow x\frac{dz}{dx} = -\operatorname{cosec} z \Rightarrow -\sin z dz = \frac{dx}{x}$

$\Rightarrow \cos z = \log x + c$

$\Rightarrow \cos\left(\frac{y}{x}\right) = \log x + c$

But $y(1)=\frac{\pi}{2} \Rightarrow x=1, y=\frac{\pi}{2} \therefore \cos\frac{\pi}{2}=\log(1)+c \Rightarrow c=0$

$\therefore \cos\left(\frac{y}{x}\right)=\log x$

100. **(c)** The equation of the family of circles of radius r is

$$(x-a)^2+(y-b)^2=r^2$$

Where a& b are arbitrary constants.

Since equation (1) contains two arbitrary constants, we differentiate it two times w.r.t x& the differential equation will be of second order.

Differentiating (1) w.r.t. x, we get

$$2(x-a)+2(y-b)\frac{dy}{dx}=0$$

$$\Rightarrow (x-a)+(y-b)\frac{dy}{dx}=0$$

Differentiating (2) w.r.t. x, we get

$$1+(y-b)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$$

$$\Rightarrow (y-b)=-\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$$

On putting the value of $(y-b)$ in equation (2), we get

$$x-a=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}}$$

Substituting the values of $(x-a)\&(x-b)$ in (1), we get

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2\left(\frac{dy}{dx}\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2}+\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2}{\left(\frac{d^2y}{dx^2}\right)^2}=r^2$$

$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3=r^2\left(\frac{d^2y}{dx^2}\right)^2$$

101. **(c)** Given: $\frac{d^2y}{d^2}+2\frac{dy}{dx}+y=2e^{3x}$

The auxiliary equation is

$D^2+2D+1=0$ or $m^2+2\,m+1=0$

$\Rightarrow (m+1)(m+1)=0 \Rightarrow m=-1,-1$

i.e., repeated roots

$\therefore$ Complementary function $= (c_1 + c_2x)e^{-x}$

Now Particular Integral (P.I.)

$$= \frac{1}{D^2 + 2D + 1} \cdot 2e^{3x}$$

$[D = 3]$

P.I. $= \frac{1}{3^2+2.3+1} \cdot 2 \cdot e^{3x} = \frac{2e^{3x}}{16} = \frac{e^{3x}}{8}$

Solution $y = \text{C. F.} + \text{P. I.}$

$$\Rightarrow y = (c_1 + c_2x)e^{-x} + \frac{e^{3x}}{8}$$

102. **(c)** Given $y dx + (x - y^3)dy = 0$

$$\Rightarrow y\frac{dx}{dy} + x - y^3 = 0 \Rightarrow \frac{dx}{dy} + \frac{1}{y}x = y^2$$

compare this equation to general equation

i.e. $\frac{dx}{dy} + Px = Q \Rightarrow P = \frac{1}{y}, Q = y^2$

$$\therefore \text{I.f.} = e^{\int Pdy} = e^{\int \frac{1}{y}dy} = e^{\log y} = y$$

$$x \times \text{I.f.} = \int Q \times \text{I.f. } dy + C_1$$

$$\Rightarrow x.y = \int y^2.y dy + C_1 \Rightarrow xy = \frac{y^4}{4} + C_1$$

$$\Rightarrow 4xy = y^4 + 4C_1$$

$$\Rightarrow y^4 = 4xy - 4C_1 \Rightarrow y^4 = 4xy + C,$$

where $C = -4C_1$

103. **(b)** Given, $x_1x_2x_3x_4x_5 = 1050$

$\Rightarrow x_1x_2x_3x_4x_5 = 2 \times 3 \times 5^2 \times 7$

Each of 2,3 or 7 can take 5 places and 5^2 can be disposed in 15 ways.

Hence, number of positive integral solution $= 5^3 \times 15 = 1875$

104. **(d)** Number of onto functions: If A & B are two sets having m&n elements respectively such that $1 \le n \le m$ then number of onto functions from A to B is

$$\sum_{r=1}^{n} (-1)^{n-r} \cdot {}^nC_r r^n$$

Given $A = \{1,2,3, - - - - n\}$ & $B = \{a, b, c\}$

$\therefore$ Number of onto functions

$= \sum_{r=1}^{3} (-1)^{3-r} \cdot {}^{3}C_r (r)^n$

$= (-1)^{3-1}\, {}^{3}C_1 (1)^n + (-1)^{3-2}\, {}^{3}C_2 (2)^n$

$+ {}^{3}C_3 (3)^n (-1)^{3-3}$

$= {}^{3}C_1 - {}^{3}C_2 2^n + {}^{3}C_3 3^n$

$= \frac{3!}{2!\,1!} - \frac{3!}{2!\,1!} 2^n + \frac{3!}{3!\,0!} 3^n$

$= 3 - 3 \cdot 2^n + 3^n$

$= 3^n - 3(2^n - 1)$

105. **(b)** Let there are n persons in the room. The total number of hand- shakes is same as the number of ways of selecting 2 out of n.

$\therefore {}^{n}C_2 = 66 \Rightarrow \frac{n(n-1)}{2!} = 66$

$\Rightarrow n^2 - n - 132 = 0$

$\Rightarrow (n-12)(n+11) = 0 \Rightarrow n = 12$

106. **(a)** We know that, let $(G, 0)$ be a group & e be the identity then

$(a * a)^{-1} = a^{-1} o^{-1}$

$= (a^{-1})^{-1} = a$

107. **(d)** No. of tickets $= 9$

No. of odd numbered tickets $= 5$

No. of even numbered tickets $= 4$

Required probability $=$ P {odd, even, odd }

+P (even, odd, even)

$= \frac{{}^{5}C_1}{{}^{9}C_1} \times \frac{{}^{4}C_1}{{}^{8}C_1} \times \frac{{}^{4}C_1}{{}^{7}C_1} + \frac{{}^{4}C_1}{{}^{9}C_1} \times \frac{{}^{5}C_1}{{}^{8}C_1} \times \frac{{}^{3}C_1}{{}^{7}C_1}$

$= \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} + \frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} = \frac{5}{18}$

108. **(a)** Given

$P(A) = \frac{1}{12}, P(B) = \frac{5}{12}, P(B/A) = \frac{1}{15}$

We know that $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow \frac{1}{15} = \frac{P(A \cap B)}{1/12}$

$$\Rightarrow P(A \cap B) = \frac{1}{15 \times 12} = \frac{1}{180}$$

But,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{1}{12} + \frac{5}{12} - \frac{1}{180} = \frac{89}{180}$$

109. **(a)** $\int_{1.5}^{2} f(x)dx$, where $f(x) = \frac{x}{2}$

$$\int_{1.5}^{2} \frac{x}{2}dx = \frac{1}{2} \cdot \int_{1.5}^{2} xdx = \frac{1}{2}\left[\frac{x^2}{2}\right]_{1.5}^{2}$$

$$= \frac{1}{4}[4 - 2.25] = \frac{1}{4} \times [1.75] = \frac{175}{400} = \frac{7}{16}$$

110. **(b)** If $2P(X = 0) + P(X = 2) = 2P(X = 1)$

Let probability distribution of X be given by

$P(X = r) = \frac{m^r \cdot e^{-m}}{r!}$ where $r = 0,1,2, \dots \dots$

$$\therefore 2\left[\frac{m^0 e^{-m}}{0!}\right] + \left[\frac{m^2 \cdot e^{-m}}{2!}\right] = 2\left[\frac{m \cdot e^{-m}}{1!}\right]$$

$$\Rightarrow 2 + \frac{m^2}{2} = 2m \Rightarrow m^2 - 4m + 4 = 0$$

$$\Rightarrow (m-2)^2 = 0 \Rightarrow m = 2$$

111. **(c)** $A(\theta) = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ &$AB = I$

$B = IA^{-1},\ A_{11} = 1,\ A_{12} = +\tan\theta,$

$A_{21} = -\tan\theta, A_{22} = 1$

$$\therefore |A| = \begin{vmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{vmatrix} = 1 + \tan^2\theta = \sec^2\theta \Rightarrow A^{-1} = \frac{1}{\sec^2\theta} \cdot \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$B = \frac{1}{\sec^2\theta} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$B = \frac{1}{\sec^2\theta} \cdot \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$B = \frac{1}{\sec^2\theta} \cdot A(-\theta)$$

$$\Rightarrow (\sec^2\theta) \cdot B = A(-\theta)$$

$$\Rightarrow (\cos^2\theta)^{-1} \cdot B = A(-\theta)$$

112. **(b)** Given: $\begin{vmatrix} 2x+1 & 4 & 8 \\ 2 & 2x & 2 \\ 7 & 6 & 2x \end{vmatrix} = 0$

$\therefore (2x+1)(4x^2-12)-4(4x-14)$

$+8(12-14x)=0$

$\Rightarrow 8x^3-24x+4x^2-12-16x$

$+56+96-112x=0$

$\Rightarrow 8x^3+4x^2-152x+140=0$

$(x+5)$ is a factor of above equation

$\therefore 8x^3+40x^2-36x^2-180x$

$\Rightarrow 8x^2(x+5)-36x(x+5)+28(x+5)=0$

$\Rightarrow (x+5)(8x^2-36x+28)=0$

$\Rightarrow (x+5)4(2x^2-9x+7)=0$

$\Rightarrow 4(x+5)(2x^2-7x-2x+7)=0$

$\Rightarrow 4(x+5)[x(2x-7)-1(2x-7)]=0$

$\Rightarrow 4(x+5)(2x-7)(x-1)=0$

$\Rightarrow x=-5,3.5,1$

113. **(b)** For only one solution $|A|\neq 0$

$$\begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0$$

$\Rightarrow k(k-1)(k+2)\neq 0$

$\Rightarrow k\neq 0, k\neq 1, k\neq -2.\ \therefore k=-1$

114. **(c)** Let

$$A=\begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{vmatrix} \sim \begin{vmatrix} -1 & 2 & 5 \\ 0 & 0 & a+6 \\ 0 & 0 & a+6 \end{vmatrix}$$

$[R_2\to R_2+2R_1, R_3\to R_3+R_1]$

clearly rank of A is 1 if $a=-6$

115. **(c)** Given: $ax^4+bx^2+c=0$

Equation will be real if $D\geq 0$

$\Rightarrow b^2-4ac\geq 0 \Rightarrow b^2\geq 4ac$

116. **(a)** Given: $\log_3 x+\log_3(\sqrt{x})+\log_3(\sqrt[4]{x})$

$+\log_3 \sqrt[8]{x} + \log_3(\sqrt[16]{x}) + --= 4$

$\Rightarrow \log_3 x^{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+---\infty} = 4$

$\Rightarrow \log_3 x^{\frac{1}{1-\frac{1}{2}}} = 4 \left[\because S_\infty = \frac{a}{1-r}\right]$

$\Rightarrow \log_3 x^2 = 4 \Rightarrow x^2 = 3^4 \Rightarrow x = 9$

117. **(a)** $\left(x+\frac{1}{x}\right)^3 + \left(x+\frac{1}{x}\right) = 0$

$$\Rightarrow \left(x+\frac{1}{x}\right) + \left[\left(x+\frac{1}{x}\right)^2 + 1\right] = 0$$

$$\Rightarrow x + \frac{1}{x} = 0 \Rightarrow x^2 + 1 = 0 \Rightarrow x = \pm i$$

Thus, the given equation has no real roots.

118. **(a)** Given: H is the harmonic mean between P&Q

$$\therefore H = \frac{2PQ}{P+Q} \Rightarrow \frac{1}{H} = \frac{P+Q}{2PQ}$$

$$\Rightarrow \frac{2}{H} = \frac{1}{Q} + \frac{1}{P} \Rightarrow \frac{H}{P} + \frac{H}{Q} = 2$$

119. **(c)** We have $(\vec{a}+\vec{b}) \times (\vec{a}\times\vec{b})$

$= \vec{a} \times (\vec{a}\times\vec{b}) + \vec{b} \times (\vec{a}\times\vec{b})$

$= (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} + (\vec{b}\cdot\vec{b})\vec{a} - (\vec{b}\cdot\vec{a})\vec{b}$

$= (\vec{a}\cdot\vec{b})(\vec{a}-\vec{b})$

$+\vec{a} - \vec{b}\left(\vec{b}\cdot\vec{b} = \vec{b}^2 = 1, \vec{a}\cdot\vec{a} = \vec{a}^2 = 1\right)$

$= (\vec{a}\cdot\vec{b}+1)(\vec{a}-\vec{b})$

$= x(\vec{a}-\vec{b})$ where $x = \vec{a}\cdot\vec{b}+1$ is a scalar

$\therefore$ The given vector is parallel to $\vec{a}-\vec{b}$.

120. **(a)** Given : A, B & C are three points with coordinates $(1,2,-1), (2,0,3) \& (3,-1,2)$ respectively.

Now, direction ratio's of

$AB = 2-1, 0-2, 3+1 = 1, -2, 4 \&$

direction ratio's of

$AC = 3-1, -1-2, 2+1 = 2, -3, 3$

we know that

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2} \cdot \sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$$

$$\Rightarrow \cos\theta = \frac{(1)(2) + (-2)(-3) + (4)(3)}{\sqrt{1+4+16} \cdot \sqrt{4+9+9}}$$

$$= \frac{2+6+12}{\sqrt{21} \cdot \sqrt{22}} = \frac{20}{\sqrt{462}}$$

$$\Rightarrow \sqrt{462}\cos\theta = 20$$