## PART-I PHYSICS

1. **(d)** Two beams of light give rise to an interference pattern if they are coherent, they have same wavelength/frequency in same phase or having a constant phase difference and same state of polarisation. Interference pattern can -not be obtained if the two beams are not mono chromatic.
2. **(b) B**y the theory of diffraction at a single slit, the width of the central maximum is given by

$W = \frac{2D\lambda}{a} \Rightarrow W \propto D$,

$W \propto \lambda$ and $W \propto \frac{1}{a}$

Therefore, to increase the width of the central maximum a should be decreased.

3. **(b)** $R_1$ and $R_2$ are the two rays considered for interference. $R_1$ is the result of reflection at denser medium; hence it suffers an additional path difference of $\frac{\lambda}{2}$. Ray $R_2$ originates after reflection at narer medium.

Net path difference $= 2\mu t + \frac{\lambda}{2}$,

where t is the thickness of the soap solution. For constructive interference,

$2\mu t + \frac{\lambda}{2} = m\lambda$, where $m = 0,1,2$ ....

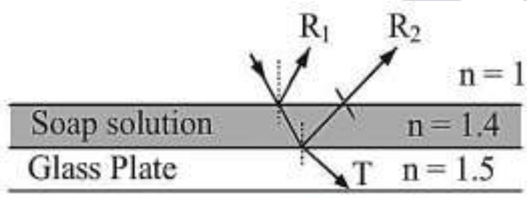
Let the two adjacent reflection maxima be observed at m and $m - 1$. Then

$$2\mu t + \frac{\lambda_1}{2} = m\lambda_1$$

or $2\mu t = \left(m - \frac{1}{2}\right)\lambda_1$

Similarly, $2\mu t = \left(m - \frac{3}{2}\right)\lambda_2$

Solving (1) & (2), we get




$$2\mu t = \left(\frac{\lambda_2}{\frac{\lambda_2}{\lambda_1} - 1}\right)$$

Putting, $\lambda_1 = 420\text{ nm}, \lambda_2 = 630\text{ nm}, \mu = 1.4$

We get, $t = 450\text{ nm}$

4. **(c)** We have,

speed = frequency × wavelength

Also, when a wave passes from one medium to the other, its frequency remains constant.

∴ speed ∝ wavelength

Therefore, when the speed of a wave doubles, its wavelength also doubles.

**5. (c)** Here,

Frequency, $v = 6 \times 10^{14}$ Hz

Work-function, $\phi = 2\text{eV} = 2 \times 1.6 \times 10^{-19}$ J

$= 3.2 \times 10^{-19}$

Maximum energy, $T_{max} = ?$

By Einstein's photo electric equation, we have

$$hv = \phi + T_{max}$$
$$\Rightarrow T_{max} = hv - \phi$$
$$= (6.63 \times 10^{-34} \times 6 \times 10^{14}) - (3.2 \times 10^{-19})$$
$$= (3.97 \times 10^{-19}) - (3.2 \times 10^{-19})$$
$$= 0.77 \times 10^{-19} \text{ J}$$
$$= \frac{0.77 \times 10^{-19}}{1.6 \times 10^{-19}} \text{eV} = 0.49\text{eV}$$

**6. (b)** We have, $d\sin\varphi = n\lambda$

For $\phi = 90°$ and $n = 1$, we get $d = \lambda$

But $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{h^2}{2meV}}$

$$= \sqrt{\left(\frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}\right)}$$

$$= \sqrt{\frac{1.5}{V}} \times 10^{-9} \text{ m} \quad \therefore d = \sqrt{\frac{1.5}{V}} \times 10^{-9}$$

or $5 \times 10^{-10} = \sqrt{\frac{1.5}{V}} \times 10^{-9}$ or $0.5 = \sqrt{\frac{1.5}{V}}$

or $0.5 \times 0.5 = \frac{1.5}{V} \Rightarrow V = \frac{1.5}{0.5 \times 0.5} = 6$ V

No option is matching with the exact answer. But 5 V is approximately equal to the exact potential. Therefore, option (b) should be the correct option.

**7. (b)** Davison and Germer performed an experiment to prove that matter has a wave nature. The experiment was based on electron diffraction.

**8. (c)** We have, $N_t = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$,

When

$N_t$ = number of atoms present after time t

$N_0^t$ = initial number of atoms

$T_{1/2}$ = half life of the nuclide

$\therefore \frac{N_t}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$ or $\frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{2}{T_{1/2}}}$

or $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{2}{T_{1/2}}} \Rightarrow 4 = \frac{2}{T_{1/2}}$

$\Rightarrow T_{1/2} = \frac{2}{4}\text{Hr} = \frac{1}{2}\text{Hr} \;\therefore T_{1/2} = 30$ min.

**9.** **(c)** Due to time dilation the interval between two events at the same point in a moving frame appears to be longer by a factor

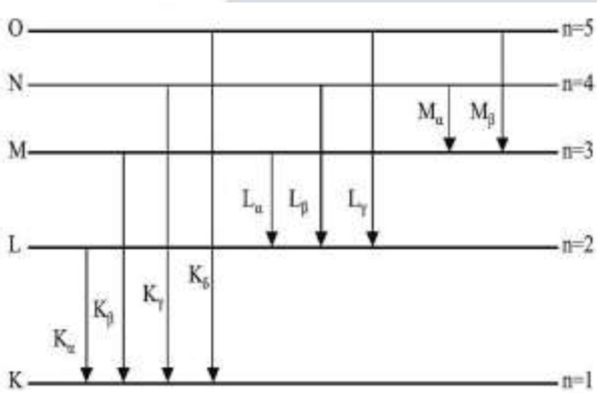
$\gamma\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$ to an observer in a stationary frame.

Time dilation is independent of the direction of velocity and depends only on its magnitude.

**10.** **(c)** The α-decay in the case of $^{234}U$ takes place as follows:

$$^{234}_{92}U \longrightarrow {}^{230}_{90}Th + {}^{4}_{2}He$$

**11.** **(a)** When electron jumps from the level n = 2 to the level n = 1, $K_\alpha$x - rays are emitted. Similarly, $K_\beta$x-rays are emitted when there is atransition of electron from the level n = 3 to the level n = 1. X-ray spectra has been shown below.




**12.** **(b)** Let the number of α and β particles emitted be m and n respectively. Then

$A - 4\,m = A - 8 \Rightarrow m = 2$

[∵ the mass number of a radioactive nuclide decreases by 4 due to emission of one α particle]

Again,

$(Z - 2\,m) + n = Z - 3$

[∵ the atomic number decreases by 2 due to emission of 1α-particle but increases by 1 due to emission of 1β - particle]

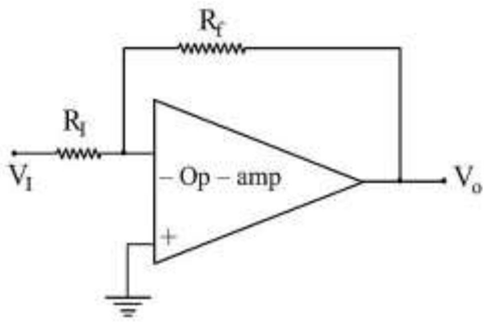
or $-2m + n = -3$

or $2\,m - n = 3$

or $(2 \times 2) - n = 3$ (∵ m = 2)

$\Rightarrow n = 1$

**13. (b)**




For the Op-amp shown above, we have

$$\frac{V_o}{V_I} = -\frac{R_f}{R_I}$$

Comparing this circuit with the given one,

We get $V_I = 1\text{ V}, R_f = 10\text{k}\Omega = 10 \times 10^3 \Omega$

$R_I = 1\text{k}\Omega = 1 \times 10^3 \Omega$

$$\therefore \frac{V_o}{1} = -\frac{10 \times 10^3}{1 \times 10^3} \Rightarrow V_o = -10\text{ V}$$

**14. (d)** The solid is an n-type semiconductor. In an n-type semiconductor, the impurity is pentavalent which is also called the Donar impurity because one impurity atom generate one electron. The Donor energy level lies just below the conduction band as shown in the figure.

Conduction band

Donor levels

Valence band

**15. (a)** We have, $I_D = I_S\left(e^{V_D/V_T} - 1\right)$

where, $V_T = \frac{kT}{e}$

$$\therefore \left(\frac{I_D}{I_S} + 1\right) = e^{\left(\frac{V_o}{V_T}\right)} \text{ or } \ln\left(1 + \frac{I_D}{I_S}\right) = \frac{V_D}{nV_T}$$

$$\therefore V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = V_T \ln\left(1 + \frac{I_D}{I_S}\right)$$

$$= V_T \ln\left(\frac{I_{majority}}{I_S}\right)$$

Here, $n_e = (10^{17} - 10^{16})\text{cm}^{-3}$

$= 10^{16}(10 - 1)\text{cm}^{-3} = 9 \times 10^{16}\text{ cm}^{-3}$

We know that, $n_e \times n_h = n_i^2$

$\therefore n_h = \frac{n_i^2}{n_e} = \frac{(1.4\times10^{10})^2}{9\times10^{16}}\ \text{cm}^3$

Also, $\frac{I_{\text{majority}}}{I_S} \propto \frac{n_e}{n_h}$

$\therefore V_D = \frac{kT}{e}\ln\left[\frac{9\times10^{16}}{\frac{(1.4\times10^{10})^2}{9\times10^{16}}}\right]$

$= \frac{kT}{e}\ln(4\times10^{12})$

**16. (c)** Here, electricfield, $E = 10^6$ V/m

width of depletion region, $d = 2.5\mu m = 2.5\times10^{-6}$ m

$\therefore$ Potential required for breakdown

$(V) = Ed = (10^6\times2.5\times10^{-6})V = 2.5$ V

Contact Potential $= 1.0$ V

$\therefore$ Reverse biased potential for zener breakdown $= (2.5-1.0)V = 1.5$ V

**17. (b)** In a colpitt oscillator, the feed-back network consists of two capacitors and one inductor.

**18. (d)** The reverse saturation of p-n diode depends on the doping concentrations, diffusion length and device temperature.

**19. (a)** Antenna length $= \frac{\lambda}{4} \propto \lambda$

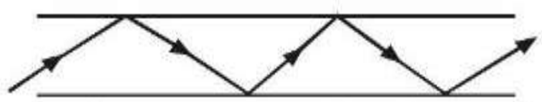
$\because \lambda \propto \frac{1}{v}$ for cosntant velocity,

$\therefore$ Antenna length $\propto \frac{1}{v}$

Here, $v_{AM} < v_{FM}$

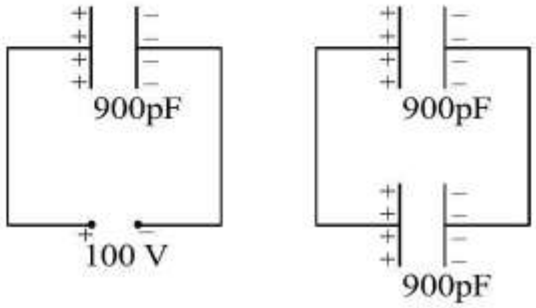
$\therefore$ Antenna length for AM should be longer than that of FM.

**20. (a)** The principle of communication using optical fibers is based on the principle of total internal reflection.



**21. (d)** According to quantization of charge, the charge of any system is an integral multiple of the charge of electron which is the least amount of charge on any system.

**22. (b)** The charge on the capacitor when connected to the battery is given by




$Q = CV = (900 \times 10^{-12}\ \mathrm{F}) \times 100\ \mathrm{V} = 9 \times 10^{-8}\mathrm{C}$ When the battery is replaced by another capacitor of 900pF capacitance, the charge of $9 \times 10^{-8}\mathrm{C}$ is distributed on both. Let $Q_1$ and $Q_2$ be the charge on each of them.

$\therefore Q = Q_1 + Q_2$

$= C_1\ V + C_2\ V$, where V is the common potential.

or $V = \frac{Q}{C_1 + C_2}$

As the two capacitors are in parallel, the equivalent capacitance is given by $C = C_1 + C_2$

$\therefore$ Total energy of the capacitors $= \frac{1}{2} CV^2$

$$= \frac{1}{2} \times (C_1 + C_2) \times \frac{Q^2}{(C_1 + C_2)^2} = \frac{Q^2}{2(C_1 + C_2)}$$

$$= \frac{(9 \times 10^{-8})^2}{2(900 \times 10^{-12} + 900 \times 10^{-12})}$$

$$= \frac{(9)^2 \times 10^{-16}}{2 \times 2 \times 9 \times 10^{-10}} = 2.25 \times 10^{-6}\ \mathrm{J}$$

**23. (c)** We know that $R = \rho \frac{l}{A}$

$\therefore$ For $l = L$ and $A = A$; $R_1 = \rho \frac{L}{A}$

For $l = \frac{L}{2}$ and $A = 2\ A$;

$$R_2 = \rho \frac{\left(\frac{L}{2}\right)}{2\ A} = \frac{1}{4}\left(\rho \frac{L}{A}\right) = \frac{1}{4} R_1$$

For $l = 2\ L$ and $A = \frac{A}{2}$;

$$R_3 = \rho \frac{2\ L}{\left(\frac{A}{2}\right)} = 4\left(\rho \frac{L}{A}\right) = 4R_1$$

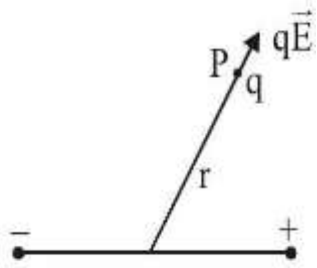
Thus $R_3 > R_1 > R_2$

Therefore, the wire having length 2 L and area

$\frac{A}{2}$ has the maximum resistance.

**24. (d)** We know that the electric field at any point due to an electric dipole varies inversely with the cube of the distance of the point from the centre of the dipole, that is,

$$E \propto \frac{1}{r^3}$$




Also, force on a charge (q) is given by

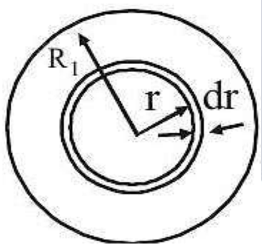
$$F = qE \Rightarrow F \propto \frac{1}{r^3}$$

When the distance of the charge becomes 2 m, i.e. double of its initial value, then new force $(F')$ will become

$$F' = \frac{1}{(2)^3} \cdot F = \frac{F}{8}$$

**25. (c)** Let us first calculate the total charge on the soild sphere.

Let us consider a concentric sphere of radius r and thickness dr. Then volume of thesphere, $dV = 4\pi r^2 dr$ Given, the volume charge density of the sphere $= \rho = \frac{\rho_0}{r}$




∴ Charge on this sphere,

$dQ = \rho \cdot dV = \frac{\rho_0}{r} \cdot 4\pi r^2 dr = 4\pi\rho . dr$. ∴ Total charge on the whole solid sphere,

$$Q_s = \int_0^{R_1} dQ = 4\pi\rho_0 \int_0^{R_1} r dr$$
$$= 4\pi\rho_0 \frac{R_1^2}{2} = 2\pi\rho_0 R_1^2$$

Now, the total charge on the hollow sphere,

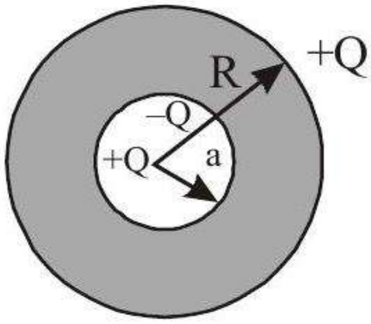
$$Q_h = -(4\pi R_2{}^2)\sigma$$

By question, $Q_s + Q_h = 0$

$$\Rightarrow 2\pi\rho_0 R_1{}^2 = 4\pi R_2{}^2\sigma$$

$$\Rightarrow \left(\frac{R_2}{R_1}\right)^2 = \frac{\rho_0}{2\sigma} \Rightarrow \frac{R_2}{R_1} = \sqrt{\frac{\rho_0}{2\sigma}}$$

**26. (b)** A charge Q will be induced on the inner surface of the solid spherical conductor. An equal but opposite charge will be induced on the outer surface of the conductor. There will be no charge at a position between the inner and outer surface.




**27. (b)** The energy stored in a capacitor is given by $E=\frac{Q^2}{2C}$

Here, $C=\frac{2\pi\epsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)}$, where

$L=$ length of the cylindrical conductor

$R_1=$ inner radius

$R_2=$ outer radius

$\therefore C\propto L\Rightarrow E\propto\frac{Q^2}{L}$

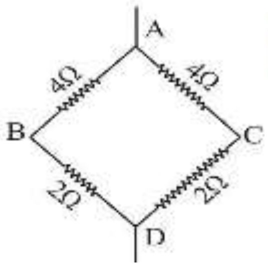
When both Q and L are doubled, by keeping other parameters fixed, the energy stored ( $E'$ ) becomes

$E'\propto\frac{(2Q)^2}{2L}$

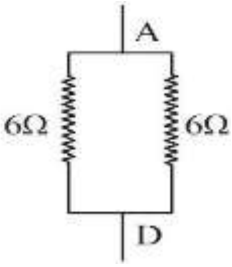
$\propto\frac{2Q^2}{L}$

$\therefore E'=2E$

**28. (c)** Redrawing the figure, we get




(a)




(b)

The 4Ω and 2Ω resistors are in series and the same is the case with another 4Ω and 2Ω resistors. So, the four resistors are equivalent to following two resistors.

Now, in fig (b) these two 6Ω resistors are in parallel.

$\therefore$ Equivalent resistance, $R = \frac{6\times6}{6+6} = \frac{36}{12}$

$= 3\Omega$.

**29.** **(b)** In a conductor, the charge carriers are electrons. As the temperature is increased, the collisions of these conduction electrons with the fixed ions of the lattice of the metal increases and hence the resistance of the conductor also increases.

**30.** **(b)** If there is no potential difference through a metallic wire, the current is zero because the electrons drift in a random direction with a speed of the order of $10^{-2}$ cm/s so that the net charge crossing a particular cross-section in a given time is zero.

**31.** **(d)** If the wire is replaced by another wire of same material but with double the length and half the thickness, the resistance of the wire as a whole will change. Let us calculate this change.

Initial resistance, $R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2}$

Fubak resistance, $R' = \rho\frac{2L}{A'} = \rho\frac{2L}{\pi\left(\frac{r}{2}\right)^2}$

$$= \rho\frac{8L}{\pi r^2} = 8\left(\rho\frac{L}{\pi r^2}\right) = 8R$$

Therefore, the resistance will increase by eight times.

In a meter-bridge, we have $\frac{R}{S} = \frac{l}{100-l}$

where R = unknown resistor

S = known resistor

l = balancing length

$$\therefore l = \frac{100R}{S+R}$$

When the resistance of the new wire is 8R then the new balancing length will be

$$l = \frac{100\times8R}{S+8R} = \frac{100\times8R}{(S+R)+7R}$$

$$= \frac{100\times8R}{\left(\frac{100R}{l}\right)+7R} \quad \therefore l' = \frac{800l}{100+7l}$$

No option is correct.

**32.** **(b)** The transport current flows through the surface of the superconducting wire and not through the entire area of cross-section of the wire.

**33.** **(c)** Here, $E = 3 \times 10^4 NC^{-1}$

$p = 6 \times 10^{-30} Cm$
$\Gamma_{max} = ?$

We have, torque acting on a dipole $\Gamma = pE\sin\theta$ The maximum value of $\Gamma$, i.e. $\Gamma_{max} = pE \therefore \Gamma_{max} = (3 \times 10^4 \times 6 \times 10^{-30}) Nm$

$= 18 \times 10^{-26} Nm$

**34.** **(c)** When a metallic plate swings between the poles of a magnet, eddy currents are set up inside the plate. These currents set up their own magnetic field which opposes the magnetic field of the poles. Thus, the direction of the current opposes the motion of the plate.

**35.** **(b)** When an electrical appliance is switched on, the electrons in the conducting wires move with their drift speed which is very less than the speed of light. But as soon as the switch is on, an electromagnetic wave is set up inside the conductor and the electrical signal is carried by them. The speed of electromagnetic wave is, of course, equal to the speed of light and hence the appliance responds almost immediately after the switch is made on.

**36.** **(b)** The circuit shown is a parallel resonant circuit. The frequency is $f = \frac{1}{2\pi\sqrt{LC}}$ at resonance. Also, at resonance, the capacitative reactance is equal to the inductive reactance. Therefore, equal current will flow through both the bulbs $b_1$ and $b_2$. So, both will glow with same brightness.

**37.** **(b)** In a transformer, the ratio of turns in the windings is given by

$$\frac{N_P}{N_S} = \frac{V_P}{V_S}$$

$$\therefore \frac{N_P}{N_S} = \frac{5kV}{240\ V} = \frac{5000\ V}{240\ V} = 20.8$$

**38.** **(a)** Self-inductance (L) of a solenoid $= \frac{\mu_0 N^2 A}{l}$ where $\mu_o$ = absolute permittivily of space/ air

N = number of turns in the coil A = area of cross-section of the solenoid

l = length of solenoid.

This expression shows that for all three solenoids, the self-inductances will be equal.

**39.** **(b)** We know that the refractive index of a medium is given by

$$\mu = \frac{c_0}{c}$$

Where c = velocity of light in the medium $c_o$ = velocity of light in vacuum

For crown glass,

$$1.5 = \frac{c_o}{2 \times 10^8}$$

For flint glass, $1.8 = \frac{c_o}{c}$

Dividing (2) by (1), we get $\frac{2\times10^8}{c} = \frac{1.8}{1.5}$

or $c = \frac{1.5 \times 2 \times 10^8}{1.8}$ m/s $= 1.67 \times 10^8$ m/s

**40. (c)** Fast moving electrons create electromagnetic waves. So, the diffraction pattern will be observed. Also, the angular width of the central maximum is given by

$W = \frac{2\lambda}{a}$, where a = width of the slit

$\lambda$ = wavelength of the light.

When the speed of electrons will increase, the frequency will increase which will result in decreasing the wavelength as the speed of light is a constant. Therefore, the angular width will decrease.

## PART-II CHEMISTRY

**41. (b)** Aliphatic nitro compounds are prepared by vapour phase nitration of alkanes at 693793 K, under pressure. Alkanes though less reactive do undergo nitration to give a mixture of nitro alkanes resulting through cleavage of carbon-carbon bond alongwith oxidation product like $CO_2, NO_2, H_2O$ etc.

$$\underset{(\text{fuming})}{CH_3CH_3 + HNO_3} \xrightarrow{675K} \underset{(80\%)}{CH_3CH_2NO_2} + \underset{(20\%)}{CH_3 - NO_2}$$

**42. (a)** Amides are hydrolysed rapidly by acids to produce carboxylic acid and ammonium salt;

$$RCONH_2 \xrightarrow{H_3O^+} RCOOH + NH_4^+$$

Hence acetamide will give acetic acid on hydrolysis.

**43. (b)** Diazotisation reactions are shown by primary aromatic amine only as the arene diazonium salt formed is stable at $273 - 278$ K. Compound $C_6H_5CH_2NH_2$ is not an aromatic amine, hence will not give the test/reaction.

**44. (a)** Secondary nitro alkanes upon hydrolysis with boiling HCl gives a ketone & nitrous oxide.

$$2R_2CHNO_2 \xrightarrow[\text{boiling}]{HCl} 2R_2CO + N_2O + H_2O$$

**45. (a)**

**46. (a)** In a colloidal system, the substance present in large amount in the mixture is called the dispersed medium & the solute is called dispersed phase. In case of milk and water solution the dispersed phase is milk protein & fat and water is dispersed medium.

**47. (b)** No. of moles of a compound

$$= \frac{\text{given mass (gm)}}{\text{Molar mass (gm)}}$$

i.e. $\frac{25.6}{342.3} = 0.0748$ moles.

1 mole of sucrose $(C_{12}H_{22}O_{11})$ contains $6.022 \times 10^{23}$ molecules of it.

Hence 0.0748 moles contains

$= 6.022 \times 10^{23} \times 0.0748$
$= 0.4504 \times 10^{23}$ molecules.

1 molecule of sucrose by formula is having 22 atoms of hydrogen.

$\therefore 0.4504 \times 10^{23} \times 10^{23} \times 22 = 9.91 \times 10^{23}$ atoms of hydrogen.

**48.** **(c)** Milk contains lactose as milk sugar. After digestion of milk lactose is broken down by enzymes lactase to form glucose and galactose before it enter the blood stream.

**49.** **(b)** Out of 20 amino acids, the 10 amino acids which human body cannot synthesize are called essential amino acids. The ten essential amino acids are :

1)Valine 2) Leucine 3) Isolucine 4) Histidine 5) Phenylanaline 6) Methionine 7) Tryptophan 8) Lysine 9) Arginine 10) Threonine.

**50.** **(c)** Among the given examples, glucose is an alcohexose, sucrose is a disaccharide, fructose is a ketohexose while ribose is a aldopentose.

**51.** **(d)** To find the oxidation number of a given compound we have to equate the charge on the overall compound with the charge on individual atom of which the compound is made of.

In $KO_3$. K is an alkali metal.

hence its oxidation number is +1 .

$(+1) + 3 \times (x) = 0$ or $x = \frac{-1}{3} \cong -0.33$

hence oxidation number of oxygen i.e. $x = -0.33$.

In $Na_2O_2$, again Na is an alkali metal.

Hence $2 \times (+1) + 2 \times x = 0$

$$x = \frac{-2}{2} = -1$$

**52.** **(c)** In a reaction the alkyl part of grignard reagent acts as a nucleophile as carbon is more electronegative than magnesium. Hence the alkyl part will get attached to the electron deficient species.

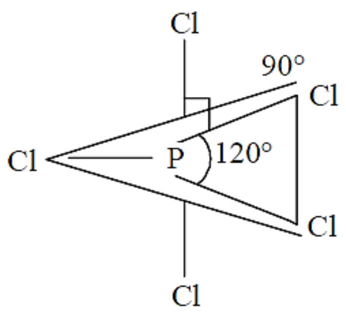
In $PCl_3$, chlorine is more electronegative than phosphorous.
Hence the $Ph^-$will attack the phosphorous in $PCl_3$ to form organic phosphine with formula $Ph_3P$.

$$3PhMgBr + PCl_3 \longrightarrow \underset{\text{Triphenylphosphine}}{Ph_3P} + 3MgBrCl$$

**53.** **(d)** Transition elements or d-block elements have variable oxidation states, they form coloured compounds because of partially filled dorbitals and also because of small size they form interstitial compounds. They are stable elements and does not show radio activity.

**54.** **(a)** In $PCl_5$, phosphorous undergoes $sp^3$ d hybridization and has trigonal bipyramidal geometry. It has two axial chlorine atoms & three equatorial chlorine atoms bonded to the central P.

Hence bond angles for axial are 90°, Cl − P − Cl & for equatorial Cl − P − Cl it is 120°.



**55. (a)** Magnetic moment of a salt depends upon the number of unpaired d-electrons. In $Zn^{2+}$ salt configuration of cation is $4s^0 3d^{10}$. Hence total no. of unpaired electron, n is zero. So magnetic moment i.e.

B.M. $= \sqrt{n(n+2)} = 0$.

**56. (a)** Formula unit = no. of molecules of $CaF_2$.

Moles $= \frac{\text{massin gm}}{\text{molar mass}} = \frac{146.4\text{gm}}{78.08\text{gm}} = 1.875$

Molecules = Mole $\times 6.022 \times 10^{23}$

$= 1.875 \times 6.022 \times 10^{23}$
$= 1.129 \times 10^{24}$ units of $CaF_2$

**57. (c)** For writing IUPAC name of a co-ordination compound we first write the name of (+) ive complex here. $[Co(NH_3)_5Cl]^{2+}$

The names of ligands will come first in alphabatical order, followed by metal ion with its oxidation state written in bracket or parentheis in Roman number i.e. Co (III) here. IUPAC name for cationic complex

→ Pentaamine chloro cobalt (III). This will follow the name of anion with a gap. i.e. Pentaamine chloro cobalt (III) chloride.

**58. (b)** Fe (III) ion from ferric nitrate will react with thiocyanate, $SCN^-$ion to form a blood red complex i.e. $FeSCN^{2+}$. But in presence of water it forms a complex containing five water molecules. i.e. $[Fe(H_2O)_5(SCN^-)]^{2+}$.

**59. (c)** Silver nitrate has been used since the begining of nineteenth century to dye hair. Silver salts darken when exposed to light and silver combines with protein yeilding a dark coloured proteinate.

**60. (b)** Schottky defect is generally shown by compounds which have ionic nature and small difference in the size of cations & anions.

In this defect equal no. of cations & anions is found missing from their lattice sites.

**61. (d)** In $[Co(NH_3)_6]^{3+}$ and $\{CoF_6\}^{3-}$ both the oxidation state of cobalt ion is +3 . In first case $NH_3$ is the neutral ligand which is a strong field ligand.

hence the electrons in Co(+III) i.e. $4\,s^0 3\,d^6$ get paired to form inner orbital complex. Hence no unpaired electron.

On the other hand $F^-$is a weak field ligand hence it forms an outer orbital complex with 4 unpaired electrons.

**62. (a)** $\Delta G^\circ = -115$ kJ at 298 K.

Now, $\Delta G^\circ = -2.303RT\log k_p$

$R = 8.314 JK^{-1}\,mol^{-1}$& $T = 298$ K.

$\Delta G^\circ = -2.303 \times 8.314 \times 298 \times \log k_P$

$$\log k_P = \frac{-115 \times 10^3}{-2.303 \times 8.314 \times 298} = 20.155$$

$= 20.16$

**63. (c)** For a reaction to take place spontaneously the value of $\Delta G$ must be negative i.e. $\Delta G < 0$. Now, $\Delta G = \Delta H - T\Delta S$.

As the reaction is endothermic, so value of $\Delta H$ must be positive, i.e. $\Delta H > 0$.

Hence to have a negative $\Delta G$.

$\Delta H < T\Delta S$. As T&P are constant.

$T\Delta S$ must be positive to give the total value a negative sign.

Hence $\Delta S > 0$.

**64. (b)** Any first order reaction follows the equation

$\log[A] = \frac{-k}{2.303}t + \log[A]_0$

$\because$ it resembles equation of straight line

$y = mx + C$

$y = \log[A]$ i.e. $\log_{10} C$

$m = \frac{-k}{2.303}$ if $x = t \& C = \log[A]_0$

hence the plot is for a $1^{st}$ order reaction.

**65. (b)** The driving forces which are responsible for a process to be spontaneous are:

(i) Tendency for minimizing energy

(ii) Tendency for maximum randomness. i.e. maximum entropy

**66. (b)** For $1^{st}$ order reaction.

$t_{1/2}$ (half life time) $= \frac{0.693}{K}$

Hence $K = \frac{0.693}{t^1/2} = \frac{0.693}{(60+40)\sec} = 0.693 \times$

$10^{-2}\sec^{-1} = 6.93 \times 10^{-3}\sec^{-1}$

**67. (b)** Since $NaNO_3$ is formed by the reaction

$Nacl + KNO_3 \rightarrow NaNO_3 + KCl$

hence, using Kohlrausch's law

$$\overset{o}{\wedge}m_{NaNO_3} = \overset{o}{\wedge}_{NaCl} + \overset{o}{\wedge}_{KNO_3} - \overset{o}{\wedge}_{KCl}$$

$= 128 + 111 - 152 = 87 \text{ S cm}^2 \text{ mol}^{-1}$

**68. (c)** As we know that

$\Delta E_{cell} = E_{cathode} - E_{anode}$

when $E_{cathode} = E_{anode}$

$\Delta E_{cell} = 0$

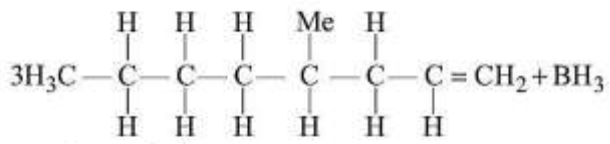
If $\Delta E_{cell} = 0$ no net reaction occurs. The reactants and products are at equilibrium and no current will flow.

Note that it is only possible to obtain electrical work from a system that is not at equilibrium. In order for current to flow, there must be a net reaction occurring. As the oxidation- reduction reaction proceeds toward equilibrium, and the concentrations of the reacting species approach their equilibrium values, the EMF of the cell decreases to zero. When the system is at equilibrium, the cell potential is zero and we have a dead battery.

**69.** **(c)** In $CuSO_4$ solution, oxidation state of Cu is +2 . Hence one mole of copper sulphate will require charge equal to two moles of electrons to form metallic Cu. Mole charge = IF. Hence 2 Faraday is required.

**70.** **(c)** Rusting of iron is generally promoted in an acidic aqueous medium. Alkaline medium prevents availability of $H^+$ions. Sodium phosphate will cause formation of a protective film of iron phosphate on the iron preventing rusting. These solutions are used in car radiators to prevent rusting of iron parts.

**71.** **(a)** Hydroboration-oxidation of alkenes give alcohols containing same number of carbon atoms. The addition follow anti-Markowi Koff's rule. Boron atom act as an electrophile. Main two steps re involved. Reagent used $BH_3$&$NaOH/H_2O_2$.

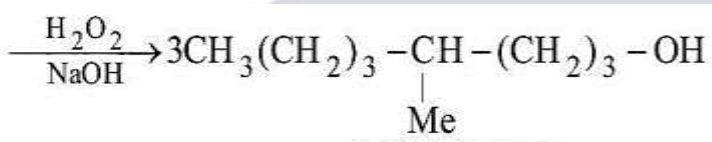


4 - methyl octene

4 - methyl octene

$\longrightarrow R_3 B$

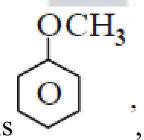
where R = 4-methyl octyl



4–Methyloctanol

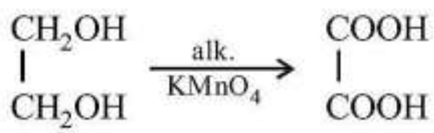
**72.** **(d)** Ethyl alcohol on treating with conc. $H_2SO_4$ undergoes dehydration to form alkene i.e. ethylene

$C_2H_5OH \underset{H_2SO_4}{\overset{Conc}{\rightarrow}} C_2H_4 + H_2O$

**73.** **(b)** Anisole is , Phenyl methyl ether. It can be prepared by treating phenol first with a base like NaOH to form phenoxide ion. The phenoxide ion will then substitute the halide of an R-X molecule, to form methyl phenyl ether.

$C_6H_5ONa + CH_3 - Cl \rightarrow \underset{\text{Anisole}}{CH_3 - O - C_6H_5}$

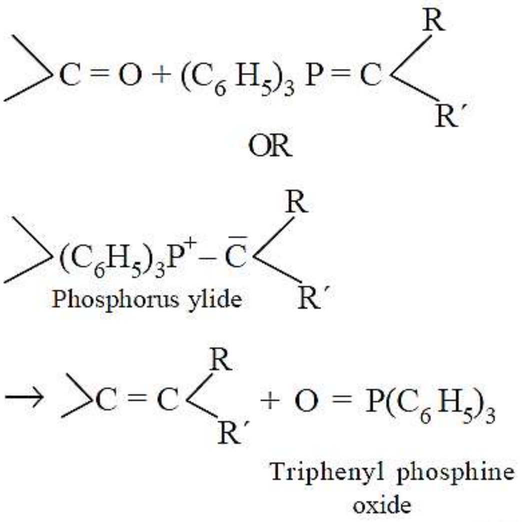
**74.** **(c)** Alkaline Potassium permanganate is a strong oxidising agent. It oxidises ethylene glycol to oxalic acid.



Ethylene glycol Oxalic acid

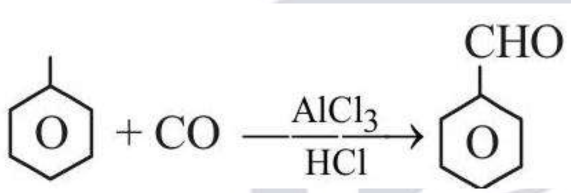
**75.** **(a)** In the structure of diamond each carbon atom in $sp^3$ hybridised & is covalently bonded with four other carbon atom held at the corners of a regular tetrahedron by covalent bonds. This results in a very big three -dimensional polymeric structure in which C - C distance is 154pm and bond angle is 109.5°. Owing to very strong covalent bonds by which atoms are held to gether diamond is the hardest substance known.

**76. (c)** The witting reaction is a chemical reaction of an aldehyde or ketone with triphenyl phosphonium ylide to give an alkene and triphenyl phosphine oxide.




**77. (a)** Cannizaro's reaction is for those aldehydes which does not contain α - hydrogen atom. This is also called self-oxidation - reduction reaction. Among the given carbonyl compounds only HCHO does not have α hydrogen.

**78. (c)**

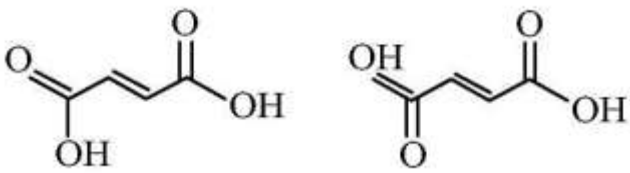


When a mixture of CO and HCl gas is passed through benzene in presence of catalyst consisting anhydrous $AlCl_3$, benzaldehyde is formed.

$$CO + HCl \rightarrow \underset{\text{formyl chloride}}{HOCl}$$

$$C_6H_6 + HOCl \xrightarrow{AlCl_3} C_6H_5CHO + HCl$$

**79. (b)** Maleic acid & fumaric acid are both the isomers of butene dioic acid. Maleic acid is the cis isomer & fumaric is the trans-isomer.



Maleic acid Fumaric acid

**80. (c)** Alkali formate i.e. HCOONa with soda-lime i.e. NaOH + CaO will react to give $Na_2CO_3$ and hydrogen gas is liberated.

$$HCOONa + NaOH \xrightarrow{CaO} Na_2CO_3 + H_2 \uparrow$$

## PART-III MATHEMATICS

**81. (c)** $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$

$(\vec{a}.\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \frac{1}{2}\vec{b}$ [Vector triple product]

$$(|\vec{a}|\cdot|\vec{c}|\cos\theta_2)\vec{b} - (|\vec{a}|\cdot|\vec{b}|\cos\theta_1)\vec{c} = \frac{1}{2}\vec{b}$$

$[\because |\vec{a}||\vec{b}| = |\vec{c}| = 1]$

Equating the coefficients of $\vec{b}$ and $\vec{c}$ on both sides, we get

$\cos\theta_2 = \frac{1}{2}$ and $-\cos\theta_1 = 0$

$\Rightarrow \cos\theta_2 = \cos\frac{\pi}{3}$ and $\cos\theta_1 = \frac{\pi}{2}$

$\Rightarrow \theta_2 = \frac{\pi}{3}$ and $\theta_1 = \frac{\pi}{2}$

**82. (d)** Vector equation of a sphere in central form with centre having position vector $\vec{c}$ and radius R is $|\vec{r} - \vec{c}| = R \Rightarrow |\vec{r} - \vec{c}|^2 = R^2$

$\Rightarrow (\vec{r} - \vec{c})\cdot(\vec{r} - \vec{c}) = R^2$

$\Rightarrow \vec{r}\cdot\vec{r} - \vec{r}\cdot\vec{c} - \vec{c}\cdot\vec{r} + \vec{c}\cdot\vec{c} = R^2$

$\Rightarrow |\vec{r}|^2 - 2(\vec{r}.\vec{c}) + |\vec{c}|^2 = R^2$

$\Rightarrow |\vec{r}|^2 - 2(\vec{r}\cdot\vec{c}) + |\vec{c}|^2 - R^2 = 0$

If $h = |\vec{c}|^2 - R^2$ i.e. $|\vec{c}| > \sqrt{h}$, then

$|\vec{r}|^2 - 2(\vec{r}.\vec{c}) + h = 0$

Hence, the given equation represents a sphere.

**83. (b)** Let $\tan^{-1}x = \theta$

$$\Rightarrow x = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$\Rightarrow x = \sqrt{1-\sin^2\theta} = \sin\theta$

$\Rightarrow x^2(1-\sin^2\theta) = \sin^2\theta$

$\Rightarrow x^2 = \sin^2\theta(1+x^2)$

$\Rightarrow \sin^2\theta = \frac{x^2}{1+x^2} \Rightarrow \sin\theta = \frac{x}{\sqrt{1+x^2}} \Rightarrow \theta = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$$\Rightarrow \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$$

Now, $\sin(\tan^{-1}x) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$

$$= \frac{x}{\sqrt{1+x^2}}$$

**84. (c)** Given that

$$\left|\frac{z-25}{z-1}\right| = 5 \Rightarrow |Z-25| = 5|Z-1|$$

Let $Z = x + iy$, then

$|x + iy - 25| = 5|x + iy - 1|$

$\Rightarrow |(x-25) + iy| = 5 \mid (x - 1 + iy \mid$

Squaring both sides, we get

$(x-25)^2 + y^2 = 25\{(x-1)^2 + y^2\}$

$\Rightarrow x^2 - 50x + 625 + y^2$

$= 25x^2 - 50x + 25 + 25y^2$

$\Rightarrow 24x^2 + 24y^2 - 600 = 0$

$\Rightarrow x^2 + y^2 - 25 = 0$

$\Rightarrow |x + iy|^2 = 25 \Rightarrow |Z|^2 = 5^2$

$\Rightarrow |Z| = 5$

**85. (c)** We have,

$$\frac{-1-3i}{2+i} = \frac{-1-3i}{2+i} \times \frac{2-i}{2-i} = \frac{-2-5i+3i^2}{4-i^2}$$

$= -1 - i$

Now, let us put $-1 = r\cos\theta, -1 = r\sin\theta$

Squaring and adding, $r^2 = 2$ i.e., $r = \sqrt{2}$

So that $\cos\theta = \frac{-1}{\sqrt{2}}, \sin\theta = \frac{-1}{\sqrt{2}}$

Therefore, $\theta = 225°$

Thus, argument is $225°$.

**86. (c)** Since b and C are the roots of $x^2 - 61x + 820 = 0$, so $b + c = 61, bc = 820$

$$A = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \tan A = \frac{4}{3} \Rightarrow \cos A = \frac{3}{5}$$

Now, using the formula,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$\Rightarrow a^2 = b^2 + c^2 - 2bc\cos A$

$= (b + c)^2 - 2bc - 2bc\cos A$

$= (b + c)^2 - 2bc(1 + \cos A)$

$= (61)^2 - 2 \times 820\left(1 + \frac{3}{5}\right)$

$= 3721 - 2624 = 1097$

**87. (d)** Equation of first line,

$\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z-2}{2} = k$ (say)

$\therefore x = k + 6, y = -2k + 2, z = 2k + 2$

Hence, general point on the first line, $P \equiv (k + 6, -2k + 2, 2k + 2)$

Equation of second line,

$\frac{x + 4}{3} = \frac{y}{-2} = \frac{Z + 1}{-2} = l$

$\therefore x = 3l - 4, y = -2l, z = -2l - 1$

Hence, general point on the second line,

$Q \equiv (3l - 4, -2l, -2l - 1)$

Direction ratios of PQ are

$3l - 4 - k - 6, -2l + 2k - 2, -2l - 1 - 2k - 2$

i.e. $3l - k - 10, -2l + 2k - 2, -2l - 2k - 3$

Now $|PQ|$ will be the shortest distance between the two lines if PQ is perpendicular to both the lines. Hence,

$1(3l - k - 10) + (-2)$

$(-2l + 2k - 2) + 2(-2l - 2k - 3) = 0$

and $3(3l - k - 10) + (-2)(-2l$

$+2k - 2) + (-2)(-2l - 2k - 3) = 0$

i.e. $3l - 9k = 12$ or $l - 3k = 4$

and $17l - 3k = 20$

Subtracting equation (i) from (ii), we get $161 = 16 \therefore l = 1$

Putting this value of 1 in equation (i), we get

$-3k = 3, \therefore k = -1$

$\therefore P \equiv (-1 + 6, -2(-1) + 2, 2(-1) + 2) \equiv (5,4,0)$

Similarly, $Q = (-1, -2, -3)$

Hence, shortest distance, PQ,

$= \sqrt{(-1-5)^2 + (-2-4)^2 + (-3-0)^2}$

$= \sqrt{(-6)^2 + (-6)^2 + (-3)^2} = \sqrt{36+36+9}$

$= 9$ units

**88.** **(c)** Since the centre and radius of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ are $(-u, -v, -w)$ and $\sqrt{u^2 + v^2 + w^2 - d}$ respectively. So, for the sphere $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$,

Centre $\equiv \left(-\frac{3}{2}, 0{,}2\right)$, and

Radius $= \sqrt{\left(-\frac{3}{2}\right)^2 + 0^2 + (2)^2 - 1}$

$= \sqrt{\frac{9}{4} + 4 - 1} = \frac{\sqrt{21}}{2}$

**89.** **(b)** Let two fixed points be $A(ae, 0)$ and B (-ae, 0). Let $C(x, y)$ be a moving point such that $AC + CB =$ constant $=$ $2a$ (say)

i.e., $\sqrt{(x-ae)^2 + (y-0)^2}$

$+\sqrt{(x+ae)^2 + (y-0)^2} = 2a$

Or $\sqrt{x^2 + y^2 + a^2e^2 - 2aex}$

$+\sqrt{x^2 + y^2 + a^2e^2 + 2aex} = 2a$

Or $l + m = 2a$
Where, $l^2 = x^2 + y^2 + a^2e^2 - 2aex$
and $m^2 = x^2 + y^2 + a^2e^2 + 2aex$
From, (3) and (4)

$m^2 - l^2 = 4aex$

or

$(m-l)(l+m) = 4aex$
$2a(m-l) = 4aex$ [From(2)]
$m - l = 2ex$

Adding (2) and (5), we get

$m = a + ex$

From (4) and (6),

$a^2 + e^2x^2 + 2aex = x^2 + y^2 + a^2e^2 + 2aex$
$\Rightarrow x^2(1-e^2) + y^2 = a^2(1-e^2)$

Dividing both sides by $a^2(1-e^2)$, we get

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

Or $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2(1-e^2)$
This is the equation of ellipse.

**90. (d)** The equation of the parabola is

$y^2 + 4x + 3 = 0$

or $y^2 = -4\left(x + \frac{3}{4}\right)$

The directrix of the parabola

$Y^2 = -4aX$

On comparing the equation (1) and (2), we

$$\begin{array}{lll} \text{get} & 4a = 4 & \text{and } X = x + \dfrac{3}{4} \\ \text{or} & a = 1 & \text{and } X = x + \dfrac{3}{4} \end{array}$$

Hence the directrix of the parabola (1) is $x + \frac{3}{4} = 1$ or $x - \frac{1}{4} = 0$.

**91. (b)** $g(x).g(y) = g(x) + g(y) + g(xy) - 2...$
Put $x = 1, y = 2$, then

$g(1).g(2) = g(1) + g(2) + g(2) - 2$

$5\,g(1) = g(1) + 5 + 5 - 2$

$4\,g(1) = 8$

$\therefore g(1) = 2$

Put $y = \frac{1}{x}$ in equation (1), we get

$$g(x) \cdot g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right) + g(1) - 2$$
$$g(x) \cdot g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right) + 2 - 2$$

$[\because g(1) = 2]$

This is valid only for the polynomial
$\therefore\ g(x) = 1 \pm x^n$
...(2)
Now $g(2) = 5$

(Given)

$\therefore 1 \pm 2^n = 5$
$\pm 2^n = 4$, [ Using equation (2)]

Since the value of $2^n$ cannot be $-$Ve.

So, $2^n = 4, \Rightarrow n = 2$

Now, put $n = 2$ in equation (2), we get $g(x) = 1 \pm x^2$

$$\therefore \underset{x\to 3}{\mathrm{Lt}}\, g(x) = \underset{x\to 3}{\mathrm{Lt}}\,(1 \pm x^2) = 1 \pm (3)^2$$
$$= 1 \pm 9 = 10, -8$$

**92. (d)** $f(x) = \frac{-e^x + 2^x}{x}$

$$= \frac{1}{x}\left[-\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ..\right) + 1 + \frac{\log 2}{1!}x + \right.$$

$$\left.\frac{(\mathrm{bg}\ 2)^2}{2!}x^2 + \frac{(\log 2)^3}{3!}x^3 + ..\right]$$

$$f(x) = \log 2 - 1 + \frac{x}{2!}\{(\log 2)^2 - 1\}$$

$$+ \frac{x^2}{3!}\{(\log^2)^3 - 1\} + \cdots$$

Putting $x = 0$, we get

$f(0) = \log 2 - 1 + 0 + 0 + \cdots . = -1 + \log 2.$

**93. (b)** Check the continuity of the function

$f(x) = [\tan^2 x]$ at $x = 0$.

L.H.L. ( at $x = 0$)

$= \lim_{x\to 0^-}[\tan^2 x] = \lim_{h\to 0}[\tan^2(0 - h)]$

$= \lim_{h\to 0}[\tan^2\ h] = [\tan^2 0] = [0] = 0$

R.H.L. ( at $x = 0$ )

$= \lim_{x\to 0^+}[\tan^2 x] = \lim_{h\to 0}[\tan^2(0 + h)]$

$= \lim_{h\to 0}[\tan^2\ h] = [\tan^2 0] = [0] = 0$

Now, determine the value of $f(x)$ at $x = 0$.

$f(0) = [\tan^2 0] = [0] = 0$

Hence, $f(x)$ is continuous at $x = 0$.

**94. (c)** Let r and V be the respectively radius and volume of the balloon. Let t represents the time. The rate of increament in radius is $\frac{dr}{dt} = 2$ cm/ minute. The volume of the balloon is given by

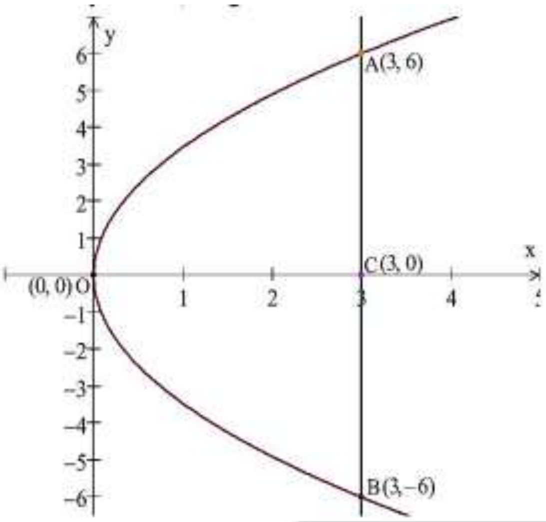
$$V = \frac{4}{3}\pi r^3$$

Differentiating w.r. to t, we get

$$\frac{dV}{dt}=\frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right)$$

Substituting the values of and $\frac{dr}{dt}$, we get $\frac{dV}{dt}=\frac{4}{3}\pi(3\times5^2\times2)=200\pi cm^3/$ minute

**95.** **(a)** On comparing the equation of the parabola $y^2=12x$ with the standard equation, $y^2=4ax$, we get $4a=12$ or $a=3$.




Hence, the focus, point C will be at (3,0) and the extremities of the latus-rectum AB will be at $(a,2a)$ and $(a,-2a)$. So the coordinates of A and B are (3,6) and $(3,-6)$ respectively. Now we need to find the length ( curve AOB) of the parabola. As it is not a straight line so we cannot directly find the length of this curve as we cannot directly apply Pythagorous theorem. Let us consider a small length ds on the parabola. Using pythagorous theorem for this length,

$$ds=\sqrt{(dx)^2+(dy)^2}=\sqrt{\left(\frac{dx}{dy}\right)^2+1dy}$$

From $y^2=12x\Rightarrow x=\frac{y^2}{12}$

$\frac{dx}{dy}=\frac{2y}{12}=\frac{y}{6}$ Putting in (1),

$$s=\int_{-6}^{6}\left[\sqrt{\left(\frac{y}{6}\right)^2+1}\right]dy=2\int_0^6\sqrt{\frac{y^2+36}{36}}dy$$

$$=\frac{2}{6}\int_0^6\sqrt{y^2+6^2}dy$$

Using $\int\sqrt{x^2+a^2}=\frac{x}{2}\sqrt{a^2+x^2}$

$$+\frac{a^2}{2}\log\left(x+\sqrt{a^2+x^2}\right)+C$$

We get $s=\frac{1}{3}\left[\frac{y}{2}\sqrt{6^2+y^2}+\frac{6^2}{2}\right.$

$$\log\left(y+\sqrt{6^2+y^2}\right)+C\Big]_0^6$$

$$=\frac{1}{3}\left[\frac{6}{2}\sqrt{6^2+6^2}+18\log\left(6+\sqrt{6^2+6^2}\right)\right.$$
$$\left.+C-0-18\log\left(0+\sqrt{6^2+0}\right)-C\right]$$

$$=\frac{1}{3}[3.6\sqrt{2}+18\log(6+6\sqrt{2})-18\log 6]$$

$$=6\sqrt{2}+6\log\frac{6(1+\sqrt{2})}{6}$$

$$=6[(\sqrt{2}+\log(1+\sqrt{2})]$$

**96. (d)** $I=\int\frac{x^5}{\sqrt{1+x^3}}dx=\int\frac{x^3\cdot x^2}{\sqrt{1+x^3}}dx$

Let $1+x^3=t^2$, so that $3x^2dx=2tdt$

$$\Rightarrow x^2dx=\frac{2}{3}tdt$$

$$\therefore I=\int\frac{(t^2-1)\frac{2}{3}tdt}{t}=\frac{2}{3}\int t^2-1\Big)dt$$

$$=\frac{2}{3}\left(\frac{t^3}{3}-t\right)+C=\frac{2}{3}\left[\frac{(1+x^3)^{3/2}}{3}-(1+x^3)^{\frac{1}{2}}\right]+C$$

$$=\frac{2}{9}(1+x^3)^{3/2}-\frac{2}{3}(1+x^3)^{1/2}+C$$

**97. (d)** The given curve is

$$\pi\left[4(x-\sqrt{2})^2+y^2\right]=8$$

$$4(x-\sqrt{2})^2+y^2=\frac{8}{\pi}$$

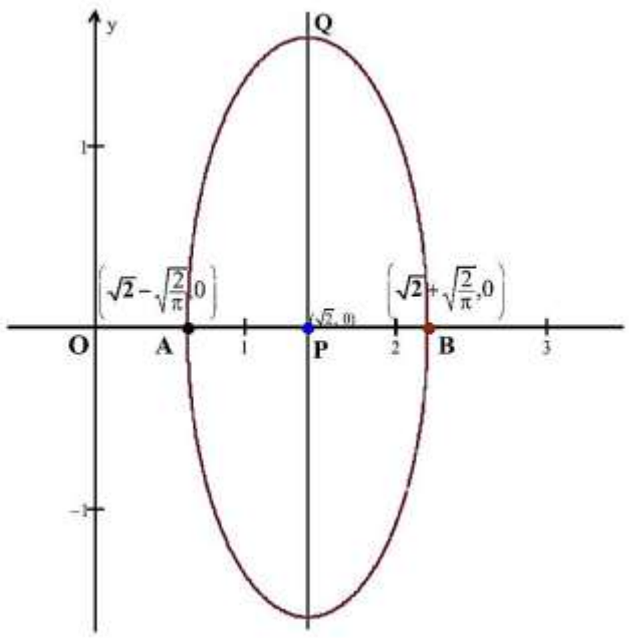
$$(x-\sqrt{2})^2+\left(\frac{y}{2}\right)^2=\frac{2}{\pi}$$

$$\frac{(x-\sqrt{2})^2}{\left(\sqrt{\frac{2}{\pi}}\right)^2}+\frac{y^2}{\left(2\sqrt{\frac{2}{\pi}}\right)^2}=1$$

This is the equation of the ellipse having centre $(\sqrt{2},0)$.

Observe the figure of ellipse (1). The centre

P is $(\sqrt{2},0)$. A and B are $\left(\sqrt{2},-\sqrt{\frac{2}{\pi}},0\right)$ and $\left(\sqrt{2}+\sqrt{\frac{2}{\pi}},0\right)$ respectively.




The required area $= 4 \times$ area of figure PQB

$$= 4 \times \int_{\sqrt{2}}^{\sqrt{2}+\sqrt{\frac{2}{\pi}}} y\,dx$$

$$= 4 \times \int_{\sqrt{2}}^{\sqrt{2}+\sqrt{\frac{2}{\pi}}} \sqrt{\frac{8}{\pi} - 4(x-\sqrt{2})^2 dx}$$

$$= 4 \times 2\int_{\sqrt{2}}^{\sqrt{2}+\sqrt{\frac{2}{\pi}}} \sqrt{\left(\sqrt{\frac{2}{\pi}}\right)^2 - (x-\sqrt{2})^2 dx}$$

$$= 8\left[\frac{x-\sqrt{2}}{2}\sqrt{\frac{2}{\pi} - (x-\sqrt{2})^2} + \frac{(2/\pi)}{2}\sin^{-1}\left(\frac{x-\sqrt{2}}{\sqrt{2/\pi}}\right)\right]_{\sqrt{2}}^{\sqrt{2}+\sqrt{\frac{2}{\pi}}}$$

$$\left[\because \int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C\right]$$

$$= 8\left[\left\{\frac{\sqrt{2}+\sqrt{\frac{2}{\pi}}-\sqrt{2}}{2}\sqrt{\frac{2}{\pi} - \left(\sqrt{2}+\sqrt{\frac{2}{x}}-\sqrt{2}\right)^2} + \frac{1}{\pi}\sin^{-1}\left(\frac{\sqrt{2}+\sqrt{\frac{2}{\pi}}-\sqrt{2}}{\sqrt{\frac{2}{\pi}}}\right)\right\} - \left\{\frac{\sqrt{2}-\sqrt{2}}{2}\sqrt{\frac{2}{\pi} - (\sqrt{2}-\sqrt{2})^2} + \frac{1}{\pi}\sin^{-1}\left(\frac{\sqrt{2}-\sqrt{2}}{\sqrt{2/\pi}}\right)\right\}\right]$$

$$= 8\left[\frac{1}{\sqrt{2\pi}}(0) + \frac{1}{\pi}\sin^{-1}(1) - 0 - 0\right]$$

$$= 8\left(\frac{1}{\pi} \times \frac{\pi}{2}\right) = 4 \text{ square units.}$$

**98. (c)** Let $x = \cos^2\theta$, so that $dx = -2a\sin\theta\cos\theta d\theta$

Now $\int_0^a \sqrt{\frac{a-x}{x}}dx = \int_{\frac{\pi}{2}}^0 \sqrt{\frac{a-a\cos^2\theta}{a\cos^2\theta}}(-a\sin\theta\cos\theta)d\theta$

$\left[\because \theta = \frac{\pi}{2} \text{ at } x = 0; \theta = 0 \text{ at } x = a\right]$

$$= a\int_0^{\pi/2}\sqrt{\frac{1-\cos^2\theta}{\cos^2\theta}}2\sin\theta\cos\theta d\theta$$

$$\left[\because \int_t^0 f(x)dx = -\int_0^t f(x)dx\right]$$

$$= a\int_0^{\pi/2} 2\frac{\sin\theta}{\cos\theta}\cdot\sin\theta\cos\theta d\theta$$

$$= a\int_0^{\pi/2} 2\sin^2\theta d\theta = a\int_0^{\pi/2}(1-\cos 2\theta)d\theta$$

$[\because \cos 2\theta = 1 - 2\sin^2\theta]$

$$= a\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= a\left[\left(\frac{\pi}{2} - \frac{\sin\pi}{2}\right) - (0-0)\right]$$

$$= a\left(\frac{\pi}{2} - 0\right) = \frac{a\pi}{2}$$

**99. (b)** According to the question,

$$\frac{dy}{dt} \propto y \Rightarrow \frac{dy}{dt} = ky$$

Separating the variables, we get $\frac{dy}{dt} = kdt$

Integrating both sides, we get $\int \frac{dy}{y} = \int kdt \log y = kt + M$ (as y cannot be $-$ve ) $\Rightarrow y = e^{kt+M} \Rightarrow y = e^M . e^{kt}$

$y = Ce^{kt}$, where $C = e^M$

Constant k cannot be positive because the population never increases in time. And another constant C cannot be negative because of $e^M > 0$ always.

Hence $y = Ce^{kt}$, for some constants $C \geq 0$ and $k \leq 0$.

**100.** **(b)** The given circle is, $x^2 + y^2 = a^2$

Differentiating with respect to x, we get

$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \; x + y\frac{dy}{dx} = 0$$

$$\Rightarrow \left(x + y\frac{dy}{dx}\right)^2 = 0 \text{ (Squaring both sides)}$$

$$\Rightarrow \; x^2 + 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2 = 0$$

$$\Rightarrow \; -2xy\frac{dy}{dx} = x^2 + y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx}$$

$$x^2 + y^2 + x^2\left(\frac{dy}{dx}\right)^2 + y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$$

$$\left(\text{Adding} y^2 + x^2\left(\frac{dy}{dx}\right)^2 \text{ both sides}\right)$$

$$\Rightarrow x^2 + y^2 = a^2)$$

**101.** **(b)** $\left|\frac{dy}{dx}\right| + |y| + 3 = 0$ Since $\left|\frac{dy}{dx}\right| \geq 0, |y| \geq 0$

$\therefore \left|\frac{dy}{dx}\right| + |y| + 3 \geq 3$

Hence $\left|\frac{dy}{dx}\right| + |y| + 3 = 0$ is not possible.

Therefore, the given differential equation has no solution

**102.** **(b)** The given differential equation is

$$xdy - ydx - \sqrt{x^2 + y^2}dx = 0$$

$$\Rightarrow xdy = \left(y + \sqrt{x^2 + y^2}\right)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(y + \sqrt{x^2 + y^2}\right.}{x}$$

This is the linear differential equation.

Put $y = vx$, so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$. Then

$$v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2x^2}}{x}$$

$$\Rightarrow v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$$

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \log\left(v + \sqrt{1 + v^2}\right) = \log x + \log C$$

$\Rightarrow\ v+\sqrt{1+v^2}=Cx$

$\Rightarrow \frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}=Cx\ [\because y=vx]$

$y+\sqrt{x^2+y^2}=Cx^2$

**103.** **(b)** $\sim Q\rightarrow S\ \therefore \sim S\rightarrow Q$

But $\sim S\rightarrow R$

$\therefore Q\rightarrow R$, True [As $P\rightarrow Q\rightarrow R\rightarrow P$ ]

Hence $\sim Q\rightarrow S$ is true

**104.** **(d)** We know that the number of ways of dividing $(m+n+p)$ things into three groups containing $m, n$ and $p$ things respectively

$=\frac{(m+n+p)!}{m!\,n!\,p!}$

Further if any two groups out of the three have same number of things then number of ways

$=\frac{(m+n+p)!}{m!\,n!\,p!\times 2}$

Hence number of ways to divide 10 students into three teams one containing four students and each remaining two teams contain three

$=\frac{10!}{4!\,3!\,3!\times 2}=\frac{10\times 9\times 8\times 7\times 6\times 5}{3\times 2\times 3\times 2\times 2}=2100$

**105.** **(d)** We observe the following properties:

Reflexivity - Let a be an arbitrary element. Then,

$|a-a|=0\ngtr 0\Rightarrow a\not R\,a$

This, R is not reflexive on R.

Symmetry - Let a and b be two distinct elements, then $(a,b)\in R$

$\Rightarrow |a-b|>0\Rightarrow |b-a|>0$

$(\because |a-b|=|b-a|)$

$\Rightarrow (b,a)\in R$

Thus, $(a,b)\in R\Rightarrow (b,a)\in R$. So, R is symmetric.

Transitivity - Let $(a,b)\in R$ and $(b,c)\in R$.

Then $|a-b|>0$ and $|b-c|>0$

$\Rightarrow\ |a-c|>0\Rightarrow (a,c)\in R$

So, R is transitive.

**106.** **(a)** Let $S = \{a_i\}$ where $i = 1.2 \ldots\ldots n$

From commutative operations,

$a_i * a_j = a_j {}^* a_i$ ... (i) $\forall i, j = 1,2,3 \ldots . n$

where * represents a binary operation ∴ Number of distinct elements in $S \times S$

i.e., $\{a_i\} \times \{a_j\}$ subject to the condition(i) $i = 1,2\ldots n$ $j = 1,2 \ldots n$

$= n\{(a_1, a_1), (a_1, a_2) \ldots \ldots (a_1, a_n),$

$(a_2, a_2), (a_2, a_3), \ldots . (a_2, a_n),$

$\ldots (a_{n-1}, a_{n-1}), (a_{n-1}, a_n), (a_n, a_n)$

$= n + (n-1) + (n-2) + \cdots . +2 + 1 = \dfrac{n(n+1)}{2}$

∴ No. of commutative binary operations

= No. of functions $f: S \times S \to S$ subject to (i)

$= n.n.n\ldots \frac{n(n+1)}{2}$ times $= n^{\frac{n(n+1)}{2}}$

**107.** **(b)** Poisson distribution is a probability distribution which is obtained when the probability (p) of the happening of an event is same in all the trials and there are only two events in each trials generally says successes and failures probability (p) of the happening of the event in trial is very less but number of trials (n) is very large.

Here, $p = 5\% = \frac{5}{100} = \frac{1}{20}$ is very less and $n = 100$, is very large. Hence, one has to employ the Poisson distribution in the given question.

**108.** **(d)** The probability that a component survives is $p = \frac{3}{4}$. Then $q = 1 - p = 1 - \frac{3}{4} = \frac{1}{4}$

$[\because p + q = 1]$

n takes the value 4 and $r = 2$. Hence the required probability is

$$^nC_r p^r q^{n-r} = {}^4C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2$$
$$= 6 \times \frac{3 \times 3 \times 1}{4 \times 4 \times 4 \times 4} = \frac{27}{128}$$

**109.** **(b)** Performance of the class will be best if mean of the marks obtained is maximum but standard deviation of the marks obtained is minimum.

Hence the class which has mean and standard deviation of the marks obtained as 75 and 5 respectively performs best.

**110.** **(b)** Mean, $np = \alpha$; and variance, $npq = \beta$ where n = number of trials and $p + q = 1$.

So, $\frac{npq}{np} = \frac{\beta}{\alpha} \Rightarrow q = \frac{\beta}{\alpha} \Rightarrow (1 - p) = \frac{\beta}{\alpha}$

$\because 0 < p < 1$

$\therefore -1 < -p < 0 \Rightarrow 0 < 1 - p < 1$

$\Rightarrow 0 < \frac{\beta}{\alpha} < 1 \Rightarrow 0 < \beta < \alpha$

**111.** **(b)** $x + y + z = 0$

$2x + 3y + z = 0$

$x + 2y = 0$

From equation (3), we have

$x = -2y$

Putting this value of x in equations (1), we get $-2y + y + z = 0 \Rightarrow y = z$

Hence $x = -2z$

Thus, the solution of the given system of equations is $(-2z, z, z)$, where z is a parameter $(Z \in R)$. Hence the system has infinite number of solutions including zero solution.

**112.** **(d)** Here, $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2$

$= \begin{bmatrix} 0 + ab & 0 + 0 \\ 0 + 0 & ab + 0 \end{bmatrix} = \begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}$

Similarly, $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = \begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}\begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}$

$= \begin{bmatrix} a^2b^2 + 0 & 0 + 0 \\ 0 + 0 & 0 + a^2b^2 \end{bmatrix} = \begin{bmatrix} a^2b^2 & 0 \\ 0 & a^2b^2 \end{bmatrix}$

Now,

$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = I \Rightarrow \begin{bmatrix} a^2\,b^2 & 0 \\ 0 & a^2\,b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a^2b^2 = 1 \Rightarrow ab = 1$

**113.** **(d)** $D = \mathrm{diag}(d_1, d_2, \dots, d_n)$

$$D = \begin{vmatrix} d_1 & 0 & 0 & - & - & 0 \\ 0 & d_2 & 0 & - & - & 0 \\ 0 & 0 & d_3 & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & 0 \end{vmatrix}$$

$$|D| = \begin{vmatrix} d_1 & 0 & 0 & - & - & 0 \\ 0 & d_2 & 0 & - & - & 0 \\ 0 & 0 & d_3 & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & 0 \end{vmatrix}$$

$= d_1\ d_2\ d_3 \ldots \ldots .\, d_n$

$$\text{adj}(D) = \begin{bmatrix} d_2\ d_3\ d_4 \ldots dn & 0 & 0 & - & 0 \\ 0 & d_1\ d_3\ d_4 \ldots dn & 0 & - & 0 \\ 0 & 0 & d_1\ d_2\ d_4 \ldots dn & - & 0 \\ - & - & - & - & - \\ - & - & - & - & - \\ 0 & 0 & 0 & 0 & -d_1\ d_2\ d_3 \ldots d_{n-1} \end{bmatrix}$$

$$d^{-1} = \frac{1}{|D|}\text{adj}(D)$$

$$\text{adj}(D) = \frac{1}{d_1\ d_2\ d_3 \ldots d_n}$$

$$\begin{bmatrix} d_2\ d_3\ d_4 \ldots dn & 0 & 0 & - & 0 \\ 0 & d_1\ d_3\ d_4 \ldots dn & 0 & - & 0 \\ 0 & 0 & d_1\ d_2\ d_4 \ldots dn & - & 0 \\ - & - & - & - & - \\ - & - & - & - & - \\ 0 & 0 & 0 & 0 & -d_1\ d_2\ d_3 \ldots d_{n-1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{d_1} & 0 & 0 & - & - & 0 \\ 0 & \frac{1}{d_2} & 0 & - & - & 0 \\ 0 & 0 & \frac{1}{d_3} & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & \frac{1}{d_n} \end{bmatrix}$$

$$= \begin{bmatrix} d_1^{-1} & 0 & 0 & - & - & 0 \\ 0 & d_2^{-1} & 0 & - & - & 0 \\ 0 & 0 & d_3^{-1} & - & - & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & - & d_n^{-1} \end{bmatrix}$$

$= \text{diag}(d_1^{-1},\ d_2^{-1}, \ldots, d_n^{-1})$

$xyz - acy + cxy + ayz - xyz = 0$

**114.** **(d)**

**115.** **(c)** $x^2 - n = 0 \Rightarrow x = \sqrt{n}$

For each integral value of $x \in [1,40]$, There is a positive root. Hence for 40 integral values of x from 1 to 40 , there are 40 positive roots, out of which only six roots 1,2,3,4,5 and 6 are positive integral roots. Hence, probability of getting positive integral roots $= \frac{6}{40} = \frac{3}{20}$

**116. (a)**

$$x^4 + \sqrt{x^4 + 20} = 22$$
$$x^4 - 22 = -\sqrt{x^4 + 20}$$

Put $x^4 = y$ and square both the sides.

$$(y - 22)^2 = y + 20$$
$$y^2 + 484 - 44y = y + 20$$
$$y^2 - 45y + 464 = 0$$
$$y^2 - 29y - 16y + 464 = 0$$
$$(y - 29)(y - 16) = 0$$
$$y = 16,29$$
$$\therefore x^4 = 16,29 \text{ or } x = \pm 2, \pm 2.31$$

**117. (c)** Since $\alpha, \beta$ be the roots of the equation $x^2 - ax + b = 0$, so each of them must satisfy the equation. Therefore

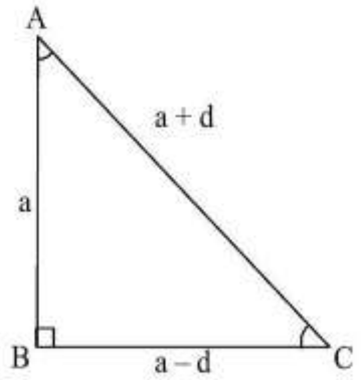
$\alpha^2 - a\alpha + b = 0$ ... (1) and $\beta^2 - a\beta + b = 0$

Now, $A_{n+1} - aA_n + bA_{n-1} = \alpha^{n+1} + \beta^{n+1} - a(\alpha^n + \beta^n) + b(\alpha^{n-1} + \beta^{n-1})$

$= \alpha^{n-1}(\alpha^2 - a\alpha + b) + \beta^{n-1}(\beta^2 - a\beta + b)$

$= \alpha^{n-1}(0) + \beta^{n-1}(0) = 0$ [ From (1) & (2)]

**118. (a)** Let the sides of the triangle be $a - d, a, a + d$, where d is greater than zero. From the figure, it is clear that the angles A and C are acute angles. Now, by the theorem of Pythagorus, $AC^2 = AB^2 + BC^2$ $(a + d)^2 = a^2 + (a - d)^2$ $a^2 + d^2 + 2ad = a^2 + a^2 + d^2 - 2ad$ $4ad = a^2 \Rightarrow d = a/4$




$$\sin A = \frac{a - d}{a + d} = \frac{a - \frac{a}{4}}{a + \frac{a}{4}} = \frac{3}{5}$$

$$\sin C = \frac{a}{a + d} = \frac{a}{a + \frac{a}{4}} = \frac{4}{5}$$

**119. (a)** The plane containing the line of intersection of the planes $\vec{r} \cdot (\hat{\imath} + 3\hat{\jmath} - \hat{k}) = 0$ and $\vec{r} \cdot (\hat{\jmath} + 2\hat{k}) = 0$ is

$\vec{r} \cdot (\hat{\imath} + 3\hat{\jmath} - \hat{k}) + \lambda[\vec{r} \cdot (\hat{\jmath} + 2\hat{k})] = 0$

$\Rightarrow \vec{r} \cdot [\hat{\imath} + (3 + \lambda)\hat{\jmath} + (2\lambda - 1)\hat{k}] = 0$

Since the plane (i) passes through the point $(-1, -1, -1)$ or $(-\hat{\imath} - \hat{\jmath} - \hat{k})$, so this point must satisfy (i). Hence, $(-\hat{\imath} - \hat{\jmath} - \hat{k}) \cdot [\hat{\imath} + (3 \times \lambda)\hat{\jmath} + (2\lambda - 1)\hat{k}] = 0$

$\Rightarrow -1 - (3 + \lambda) - (2\lambda - 1) = 0$

$\Rightarrow -3\lambda - 3 = 0 \Rightarrow \lambda = -1$

Substituting this value for $\lambda$ in (i), we get the

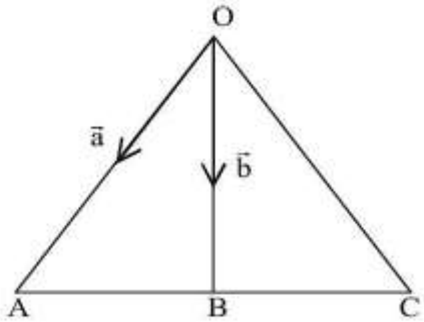
required plane $\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = 0$

**120.** **(d)** In the figure, OAC is a triangle and OB is a median such that

$\overrightarrow{OA} = \vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\overrightarrow{OB} = \vec{b} = 2\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$

$\overrightarrow{OC} = \vec{c}$ (say)




$\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OB}$

$\vec{a} + \vec{c} = 2\vec{b}$

$\Rightarrow \vec{c} = 2\vec{b} - \vec{a} = 2(2\hat{\imath} + 4\hat{\jmath} + 3\hat{k}) - (\hat{\imath} - \hat{\jmath} + \hat{k})$

$= (3\hat{\imath} + 9\hat{\jmath} + 5\hat{k})$

Now, the area of the triangle,

$$\Delta = \frac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OC}| = \frac{1}{2}|\vec{a} \times \vec{c}|$$

Here,

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & k \\ 1 & -1 & 1 \\ 3 & 9 & 5 \end{vmatrix}$$

$= \hat{\imath}(-5 - 9) - \hat{\jmath}(5 - 3) + \hat{k}(9 + 3)$

$= -14\hat{\imath} - 2\hat{\jmath} + 12\hat{k} = 2(-7\hat{\imath} - \hat{\jmath} + 6\hat{k})$

$\therefore |\vec{a} \times \vec{c}| = 2\sqrt{(-7)^2 + (-1)^2 + (6)^2}$

$= 2\sqrt{49+1+36} = 2\sqrt{86}$

$\therefore \Delta = \frac{1}{2} \times 2\sqrt{86} = \sqrt{86}.$