PART-I PHYSICS

1. **(d)** Here, $y = a\sin(bt - cx)$

Comparing this equation with geneal wave equation

$$y = a\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

we get $b = \frac{2\pi}{T}, c = \frac{2\pi}{\lambda}$

(a) Dimensions of $\frac{y}{a} = \frac{[L]}{[L]}$ = Dimensionless

(b) Dimensions of $bt = \frac{2\pi}{T} \cdot t = \frac{[T]}{[T]}$

= Dimensionless

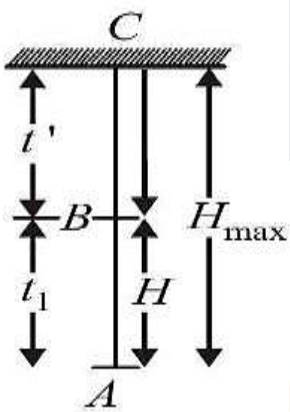
(c) Dimensions of $cx = \frac{2\pi}{\lambda} \cdot x = \frac{[L]}{[L]}$

= Dimensionless

(d) Dimensions of $\frac{b}{c} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T} = [LT^{-1}]$

2. **(b)** Let t' be the time taken by the body to fall from point C to B.

Then $t_1 + 2t' = t_2 \Rightarrow t' = \left(\frac{t_2 - t_1}{2}\right)$



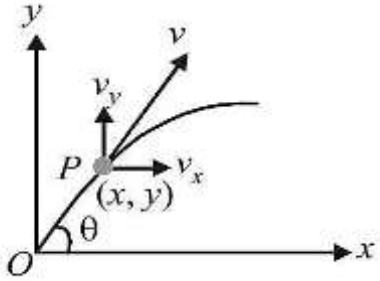
Total time taken to reach point C

$$T = t_1 + t' = t_1 + \frac{t_2 - t_1}{2}$$
$$= \frac{2t_1 + t_2 - t_1}{2} = \left(\frac{t_1 + t_2}{2}\right)$$

Maximum height attained

$$\left.\begin{aligned} H_{max} &= \frac{1}{2}g(T)^2 = \frac{1}{2}g\left(\frac{t_1 + t_2}{2}\right)^2 \\ &= \frac{1}{2}g \cdot \frac{(t_1 + t_2)^2}{4} \end{aligned}\right\}$$

3. **(a)** Momentum, $p = m \cdot v \Rightarrow v = \left(\frac{p}{m}\right)$



Kinetic energy, KE

$$= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p^2}{m^2}\right) = \frac{1}{2m}p^2$$

or, KE $\propto p^2$ $\left(\because \frac{1}{2m} = \text{constant}\right)$

Hence, the graph between KE and p^2 will be linear.

Now, kinetic energy KE $= \frac{1}{2}mv^2$

The velocity component at point P,

$v_y = (u\sin\theta - gt)$ and $v_x = u\cos\theta$

Resultant velocity at point P,

$\vec{v} = v_y\hat{j} + v_x\hat{i}$

$= (u\sin\theta - gt)\hat{j} + u\cos\theta\hat{i}$

$|\vec{v}| = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

$= \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + g^2t^2 - 2ugt\sin\theta}$

$= \sqrt{u^2(\cos^2\theta + \sin^2\theta) + g^2t^2 - 2ugt\sin\theta}$

$\therefore \text{KE} = \frac{1}{2}m(u^2 + g^2t^2 - 2ugt\sin\theta)$

i.e., KE $\propto t^2$

Hence, graph will be parabolic intercept on y-axis.

Hence, the graph between KE and t.

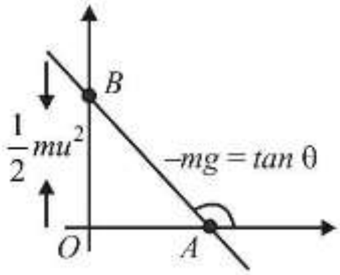
Now, in case of height

KE $= \frac{1}{2}m(v^2)$ and $v^2 = (u^2 - 2gy)$

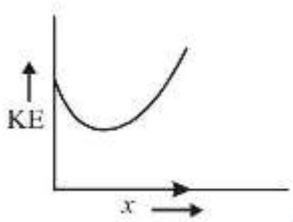
$\therefore \text{KE} = \frac{1}{2}m(u^2 - 2gy)$

$KE = -mgy + \frac{1}{2}mu^2$

Intercept on y-axis $= \frac{1}{2}mu^2$



Now, $KE = \frac{1}{2}mv^2$



$KE = \frac{1}{2}m\left(\frac{x}{t}\right)^2$

i.e., $KE \propto x^2$. Thus graph between KE and x will be parabolic.

4. **(a)** Power of motor initially $= P_0$

Let, rate of flow of motor $= (x)$

Since, power,

$$P_0 = \frac{\text{work}}{\text{time}} = \frac{mgy}{t} = mg\left(\frac{y}{t}\right)$$

If rate of flow of water is increased by n times, i.e., (nx).

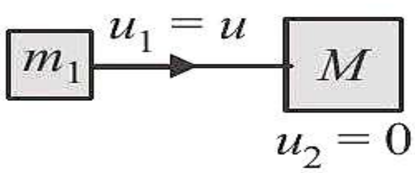
Increased power,

$$P_1 = \frac{mgy'}{t} = mg\left(\frac{y'}{t}\right) = nmgx$$

The ratio of power,

$$\frac{P_1}{P_0} = \frac{nmgx}{mgx}; P_1 : P_0 = n : 1$$

5. **(a)** Mass of the first body $m_1 = 5$ kg and for elastic collision coefficient of restitution, $e = 1$.



Let initially body m_1 moves with velocity v after collision velocity becomes $\left(\frac{u}{10}\right)$.

Let after collision velocity of M block becomes (v_2).

By conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or $5u + M \times 0 = 5 \times \frac{u}{10} + Mv_2$

or $5u = \frac{u}{2} + Mv_2$

Since, $v_1 - v_2 = -e(u_1 - u_2)$

or $\frac{u}{10} - v_2 = -1(u)$ or $\frac{u}{10} + u = v_2$

or $\frac{11u}{10} = v_2$

Substituting value of v_2 in Eq. (i) from Eq. (ii)

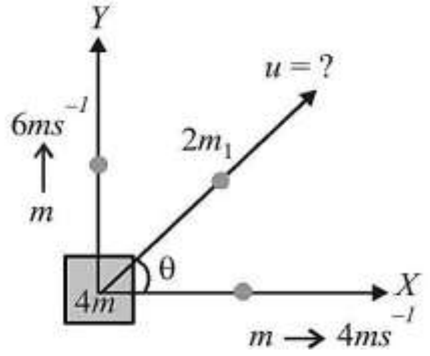
$5u = \frac{u}{2} + M\left(\frac{11u}{10}\right)$

or $5 - \frac{1}{2} = M\left(\frac{11}{10}\right) \Rightarrow M = \frac{9\times10}{2\times11} = \frac{45}{11}$

$= 4.09$ kg

6. **(c)** Let third mass particle (2m) moves making angle θ with X-axis.

The horizontal component of velocity of 2m mass particle $= u\cos\theta$ And vertical component $= u\sin\theta$



From conservation of linear momentum in X-direction

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or $0 = m \times 4 + 2m(u\cos\theta)$

or $-4 = 2u\cos\theta$ or $-2 = u\cos\theta$

Again, applying law of conservation of linear momentum in Y-direction

$0 = m \times 6 + 2m(u\sin\theta)$

$\Rightarrow -\frac{6}{2} = u\sin\theta$ or $-3 = u\sin\theta$

Squaring Eqs. (i) and (ii) and adding,

$$(4) + (9) = u^2\cos^2\theta + u^2\sin^2\theta$$
$$= u^2(\cos^2\theta + \sin^2\theta)$$

or $13 = u^2$

$\therefore\ u = \sqrt{13}\ \text{ms}^{-1}$

7. **(b)** Here, maximum height attained by a projectile

$$h = \frac{v^2R}{2gR - v^2}$$

Velocity of body = half the escape velocity

i.e., $v = \frac{v_e}{2}$

or $v = \frac{\sqrt{2gR}}{2} \Rightarrow v^2 = \frac{2gR}{4} \Rightarrow v^2 = \left(\frac{gR}{2}\right)$

Now, putting value of v^2 in Eq. (i), we get

Height, $h = \frac{\frac{gR}{2}\cdot R}{2gR - \frac{gR}{2}} = \frac{\frac{gR^2}{2}}{\frac{3gR}{2}} = \frac{R}{3}$

8. **(d)** Particle executing SHM.

Displacement $y = 5\sin\left(4t + \frac{\pi}{3}\right)$

Velocity of particle

$$\left(\frac{dy}{dt}\right) = \frac{5d}{dt}\sin\left(4t + \frac{\pi}{3}\right)$$

$$= 5\cos\left(4t + \frac{\pi}{3}\right)\cdot 4 = 20\cos\left(4t + \frac{\pi}{3}\right)$$

Velocity at $t = \left(\frac{T}{4}\right)$

$$\left(\frac{dy}{dt}\right)_{t=\frac{T}{4}} = 20\cos\left(4 \times \frac{T}{4} + \frac{\pi}{3}\right)$$

or $u = 20\cos\left(T + \frac{\pi}{3}\right)$

Comparing the given equation with standard equation of SHM.

$y = a\sin(\omega t + \phi)$

We get, $\omega = 4$

As $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$

or $T = \frac{2\pi}{4} = \left(\frac{\pi}{2}\right)$

Now, putting value of T in Eq. (ii), we get

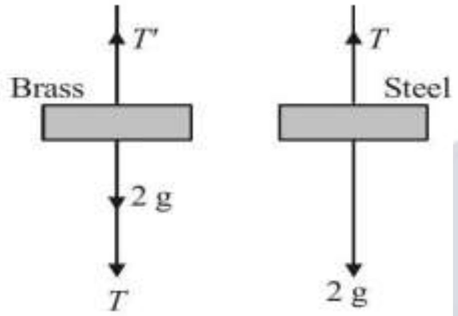
$$u = 20\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = -20\sin\frac{\pi}{3}$$
$$= -10 \times \sqrt{3}$$

The kinetic energy of particle,

$$KE = \frac{1}{2}mu^2 = \frac{1}{2} \times 2 \times 10^{-3} \times (-10\sqrt{3})^2$$
$$= 10^{-3} \times 100 \times 3 = 0.3\ J$$

9. **(d)** Given, $\frac{l_1}{l_2} = a, \frac{r_1}{r_2} = b, \frac{Y_1}{Y_2} = c$

Free body diagram of the two blocks brass and steel are



Let Young's modulus of steel is Y_1 and of brass is Y_2.

$$\therefore Y_1 = \frac{F_1 \cdot l_1}{A_1 \cdot \Delta l_1}$$

and $Y_2 = \frac{F_2 \cdot l_2}{A_2 \cdot \Delta l_2}$

Dividing Eq. (i) by (ii),

$$\frac{Y_1}{Y_2} = \frac{\frac{F_1 \cdot l_1}{A_1 \cdot \Delta l_1}}{\frac{F_2 \cdot l_2}{A_2 \cdot \Delta l_2}}$$

or $\frac{Y_1}{Y_2} = \frac{F_1 \cdot A_2 \cdot l_1 \cdot \Delta l_2}{F_2 \cdot A_1 \cdot l_2 \cdot \Delta l_1}$

Force on steel wire from free body diagram $T = F_1 = (2g)$ newton

Force on brass wire from free body diagram $F_2 = T' = T + 2g = (4g)$ newton

Now, putting the value of F_1, F_2, in Eq. (iii), we get

$$\frac{Y_1}{Y_2} = \left(\frac{2g}{4g}\right) \cdot \left(\frac{\pi r_2^2}{\pi r_1^2}\right) \cdot \left[\frac{l_1}{l_2}\right] \cdot \left(\frac{\Delta l_2}{\Delta l_1}\right)$$

or $c = \frac{1}{2}\left(\frac{1}{b^2}\right) \cdot a\left(\frac{\Delta l_2}{\Delta l_1}\right)$

or $\frac{\Delta l_1}{\Delta l_2} = \left(\frac{a}{2b^2c}\right)$

10. (d) Initially area of soap bubble, $A_1 = 4\pi r^2$ Under isothermal condition radius becomes 2r.

$\therefore$ Area $A_2 = 4\pi(2r)^2 = 16\pi r^2$

Increase in surface area

$\Delta A = 2(A_2 - A_1) = 2(16\pi r^2 - 4\pi r^2) = 24\pi r^2$

Energy spent,

$W = T \times \Delta A = T \cdot 24\pi r^2 = 24\pi T r^2$ J

11. (c) Let radius of big drop = R.

$\therefore \frac{4}{3}\pi R^3 = \frac{4}{3} \times \pi r^3 \cdot 8$

$R = 2r$

Here r = radius of small drops.

Now, terminal velocity of drop in liquid $v_T = \frac{2}{9} \times \frac{r^2}{\eta}(\rho - \sigma)g$

where η is coefficient of viscosity and ρ is density of drop σ is density of liquid.

Terminal speed drop is 6 cm s^{-1}

$\therefore 6 = \frac{2}{9} \times \frac{r^2}{\eta}(\rho - \sigma)g$

Let terminal velocity becomes v' after coalesce, then

$v' = \frac{2}{9}\frac{R^2}{\eta}(\rho - \sigma)g$

Dividing Eq. (i) by (ii), we get

$$\frac{6}{v'} = \frac{\frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g}{\frac{2}{9}\frac{R^2}{\eta}(\rho - \sigma)g} \Rightarrow \frac{6}{v'} = \frac{r^2}{(2r)^2}$$

or $v' = 24$ cm s^{-1}

12. (a) Time period of oscillation,

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{dT}{T} = \frac{1}{2}\frac{dl}{l}$

As, $\frac{dl}{l} = \alpha dt$

$\Rightarrow \frac{dT}{T} = \frac{1}{2}\alpha dt = \frac{1}{2} \times 9 \times 10^{-7} \times (30 - 20) = 4.5 \times 10^{-6}$

$\therefore$ Loss in time $= 4.5 \times 10^{-6} \times 0.5$

$= 2.25 \times 10^{-6}$ s

13. (c) Volume of the metal at 30°C

$$V_{30} = \frac{\text{loss of weight}}{\text{specific gravity } \times g}$$
$$= \frac{(45-25)g}{1.5 \times g} = 13.33 \text{ cm}^3$$

Similarly, volume of metal at 40°C

$$V_{40} = \frac{(45-27)g}{1.25 \times g} = 14.40 \text{ cm}^3$$

Now, $V_{40} = V_{30}[1 + \gamma(t_2 - t_1)]$

or $\gamma = \frac{V_{40} - V_{30}}{V_{30}(t_2 - t_1)} = \frac{14.40 - 13.33}{13.33(40-30)}$

$= 8.03 \times 10^{-3} / ^\circ C$

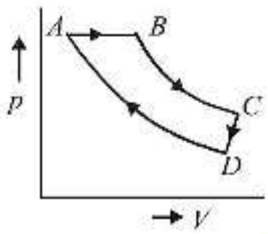
∴ Coefficient of linear expansion of the metal

$$\alpha = \frac{\gamma}{3} = \frac{8.03 \times 10^{-3}}{3} \approx 2.6 \times 10^{-3} / ^\circ C$$

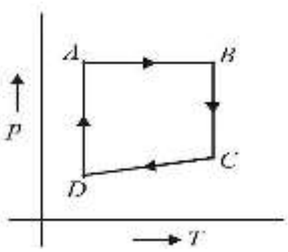
14. (a) Process A → B → C → D → A is clockwise.

During A → B, pressure is constant and B → C, process follows $p \propto \frac{1}{V}$ ie., T is constant. During process C → D, both p and V changes and process D → A

follows $p \propto \frac{1}{V}$ i.e., T is constant.



Hence, equivalent cyclic process is has follows.



15. (b) From first law of thermodynamics

$Q = \Delta U + W$ or $\Delta U = Q - W$

$\therefore \Delta U_1 = Q_1 - W_1 = 6000 - 2500 = 3500$ J

$\Delta U_2 = Q_2 - W_2 = -5500 + 1000 = -4500$ J

$\Delta U_3 = Q_3 - W_3 = -3000 + 1200 = -1800$ J

$\Delta U_4 = Q_4 - W_4 = 3500 - x$

For cyclic process $\Delta U = 0$

$\therefore 3500 - 4500 - 1800 + 3500 - x = 0$

or $x = 700$ J

Efficiency, $\eta = \frac{\text{output}}{\text{input}} \times 100 = \frac{W_1+W_2+W_3+W_4}{Q_1+Q_4} \times 100$

$$= \frac{(2500 - 1000 - 1200 + 700)}{6000 + 3500} \times 100$$

$$= \frac{1000}{9500} \times 100 = 10.5\%$$

16. (c) From first law of thermodynamics

$Q = \Delta U + W$

For cylinder A pressure remains constant.

$\therefore$ Work done by a system

$$W = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$$

For monoatomic gases, $\mu = 1; \gamma = \frac{5}{3}$

$$\therefore W = \frac{1 \times R}{\frac{5}{3} - 1}(442 - 400) = \frac{3}{2} R \times 42$$

or $W = 63R$

But $\Delta U = 0$, for cylinder A

$\therefore Q = 0 + 63R = 63R$

For cylinder B volume is constant,

$\therefore W = 0$ and $Q = \mu C_V \Delta T$

For monoatomic gas

$$C_V = \frac{3}{2} R \Rightarrow Q = 1 \times \frac{3}{2} R\Delta T$$

As heat given on both cylinder is same

$$\therefore 63R = \frac{3}{2} R\Delta T \Rightarrow \Delta T = 42\ K$$

17. (a) From figure, $H = H_1 + H_2$

$$\Rightarrow \frac{3KA(100-T)}{l} = \frac{2KA(T-50)}{l} + \frac{KA(T-0)}{l}$$

$$\Rightarrow 300 - 3T = 2T - 100 + T \Rightarrow 6T = 400$$

$$\Rightarrow T = \frac{200}{3}\ ^\circ C$$

18. (b) Let listener go from A → B with velocity (μ).

And the apparent frequency of sound from source A by listener using Doppler's effect,

$$n' = n\left(\frac{v - v_o}{v + v_s}\right) \Rightarrow n' = 680\left(\frac{340 - u}{340 + 0}\right)$$

The apparent frequency of sound from source B by listener

$$n'' = n\left(\frac{v + v_o}{v - v_s}\right) = 680\left(\frac{340 + u}{340 - 0}\right)$$

Listener hear 10 beats per second.

Hence, $n'' - n' = 10$

$$\Rightarrow 680\left(\frac{340 + u}{340}\right) - 680\left(\frac{340 - u}{340}\right) = 10$$

$$\Rightarrow 2(340 + u - 340 + u) = 10$$

$$\Rightarrow u = 2.5\ ms^{-1}$$

19. (b) When both the wires vibrate simultaneously, beats per second,

$n_1 \pm n_2 = 6$

or $\frac{1}{2l}\sqrt{\frac{T}{m}} \pm \frac{1}{2l}\sqrt{\frac{T'}{m}} = 6$

$$\Rightarrow \frac{1}{2l}\sqrt{\frac{T^1}{m}} - \frac{1}{2l}\sqrt{\frac{T}{m}} = 6$$

$$\Rightarrow \frac{1}{2l}\sqrt{\frac{T'}{m}} - 600 = 6 \Rightarrow \frac{1}{2l}\sqrt{\frac{T'}{m}} = 606$$

Given that fundamental frequency

$$\frac{1}{2l}\sqrt{\frac{T}{m}} = 600$$

Dividing Eq. (i) by (ii),

$$\frac{\frac{1}{2l}\sqrt{\frac{T^1}{m}}}{\frac{1}{2l}\sqrt{\frac{T}{m}}} = \frac{606}{600}$$

$$\Rightarrow \sqrt{\frac{T'}{T}} = (1.01) \Rightarrow \frac{T'}{T} = (1.02)\%$$

$$\Rightarrow T' = T(1.02)$$

Increase in tension

$$\Delta T' = T \times 1.02 - T = (0.02T)$$

$$\therefore \Delta T = 0.02$$

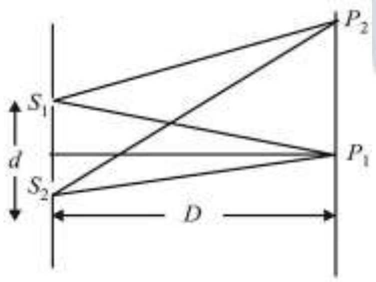
20. (d) Fringe width $\beta = \frac{\lambda D}{d}$

Let be the amplitude of the place where constructive inference takes place.

The position of fringe at p_2.

$$\Rightarrow x = \frac{n\lambda D}{d}$$

Given, $\beta' = \left(\frac{\beta}{4}\right)$



$$\therefore \frac{\lambda D}{4d} = \frac{n\lambda D}{d} \text{ or } n = \frac{1}{4}$$

$$\therefore \frac{I_1}{I_2} = \frac{a^2}{\left(\frac{a}{4}\right)^2} = 16:1$$

21. (a) Position fringe from central maxima

$$y_1 = \frac{n\lambda_1 D}{d}$$

Given, $n = 10$

$$\therefore y_1 = \frac{10\lambda_1 D}{d}$$

For second source

$$y_2 = \frac{5\lambda_2 D}{d}$$

$$\therefore \frac{y_1}{y_2} = \frac{\frac{10\lambda_1 D}{d}}{\frac{5\lambda_2 D}{d}} = \frac{2\lambda_1}{\lambda_2}$$

22. (c) Inteference takes place between two waves having equal frequency and propagate in same direction.

Hence, $y_1 = a\sin(\omega t + \phi_1)$

$y_3 = a'\sin(\omega t + \phi_2)$

will give interference as the two waves have same frequency ω.

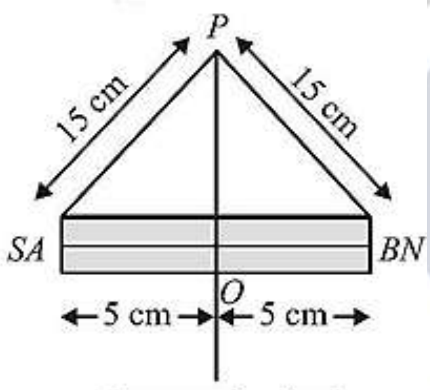
23. (d) The two lenses of an achromatic doublet should have, sum of the product of their powers and dispersive power = zero.

24. (b) Ratio of magnetic moments

$$\frac{M_A}{M_B} = \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2} = \frac{v_s^2 + v_d^2}{v_s^2 - v_d^2}$$
$$= \frac{\left(\frac{1}{20}\right)^2 + \left(\frac{1}{15}\right)^2}{\left(\frac{1}{15}\right)^2 - \left(\frac{1}{20}\right)^2} = \frac{400 + 225}{400 - 225}$$

$M_A : M_B = 25 : 7$

25. (d) Here, length of magnet $= 10\text{ cm} = 10 \times 10^{-2}\text{ m}$, $r = 15 \times 10^{-2}\text{ m}$



$$OP = \sqrt{225 - 25} = \sqrt{200}\text{ cm}$$

Since, at the neutral point, magnetic field due to the magnet is equal to B_H,

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{M}{(OP^2 + AO^2)^{3/2}}$$

$$0.4 \times 10^{-4} = 10^{-7} \times \frac{M}{(200 \times 10^{-4} + 25 \times 10^{-4})^{3/2}}$$

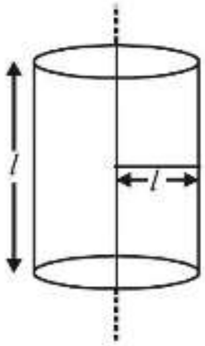
$$\Rightarrow \frac{0.4 \times 10^{-4}}{10^{-7}} \times (225 \times 10^{-4})^{3/2} = M$$

$$\Rightarrow 0.4 \times 10^3 \times 10^{-6}(225)^{3/2} = M$$

$$\Rightarrow M = 1.35\text{ A} - \text{m}$$

26. (a) Charge density or charge per unit length of long wire

$\lambda = \frac{1}{3}\text{Cm}^{-1}$ and $r = 18 \times 10^{-2}$ m



According to Gauss theorem

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E\oint dS = \frac{q}{\varepsilon_0} \text{ or } E \times 2\pi rl = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q}{2\pi\varepsilon_0 rl} = \frac{q/l}{2\pi\varepsilon_0 r}$$

$$= \frac{\lambda \times 2}{2\pi\varepsilon_0 r \times 2} = \frac{\lambda \times 2}{4\pi\varepsilon_0 r}$$

$$= 9 \times 10^9 \times \frac{1}{3} \times 2 \times \frac{1}{18 \times 10^{-2}}$$

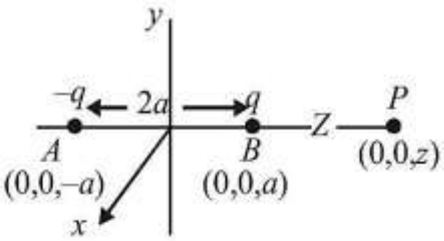
$$= 0.33 \times 10^{11}\text{NC}^{-1}$$

27. **(c)** Potential at P due to $(+q)$ charge of dipole

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(z-a)}$$

Potential at P due to $(-q)$ charge of dipole

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{(z+a)}$$



Total potential at P due to electric dipole $V = V_1 + V_2$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(z-a)} - \frac{1}{4\pi\varepsilon_0} \frac{q}{(z+a)}$$

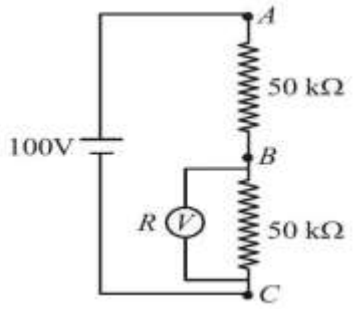
$$= \frac{q}{4\pi\varepsilon_0} \frac{(z+a-z+a)}{(z-a)(z+a)}$$

or $V = \frac{2qa}{4\pi\varepsilon_0(z^2-a^2)}$

28. **(c)** Internal resistance of voltmeter = R. Effective resistance across B and C

$$\frac{1}{R^1}=\frac{1}{R}+\frac{1}{50}=\frac{50+R}{50R}$$

or $R' = \left(\frac{50R}{50+R}\right)$



According to Ohm's law, $V' = IR'$

or $\frac{100}{3} = I\cdot\left(\frac{50R}{50+R}\right)$

or $\frac{100}{3}\left(\frac{50+R}{50R}\right) = I$

Now, total resistance of circuit

$$R'' = 50+\frac{50R}{50+R}$$

or $R'' = \frac{(2500+100R)}{(50+R)}$

Now, $V'' = IR''$

$$\Rightarrow 100 = \frac{100}{3}\left(\frac{50+R}{50R}\right)\frac{2500+100R}{(50+R)}$$

$\Rightarrow 150R = 2500+100R$ or $R = 50k\Omega$

29. **(c)** Here length of potentiometer wire, $l = 10$ m Resistance of potentiometer wire

$R = \rho\times\frac{l}{A}$ or $R = \left(\rho\times\frac{10}{A}\right)$

The value of 2.5 m length wire

$$R' = \frac{\rho\times 10}{A\times 10}\times 2.5 \Rightarrow R' = \left(\frac{2.5\rho}{A\times 10}\right)$$

Potential, $V' = I\times R' = I\left(\frac{2.5\rho}{A\times 10}\right)$

Again the length of potentiometer wire is increased by 1 m.

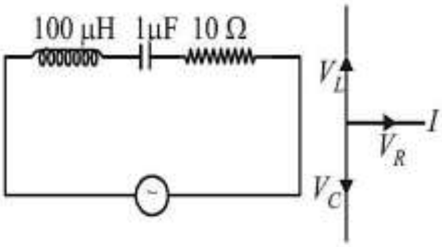
Resistance of null position

$$R'' = \left(\frac{\rho\times l}{11\times A}\right)$$

$\therefore V'' = IR''$ and $V = V' \Rightarrow \frac{I\times2.5\rho}{A\times10} = \frac{\rho\times l}{11\times A}\times I$

or $\frac{2.5\times11}{10} = l = 2.75$ m

30. (c)



Impedance, $Z = \sqrt{(X_L \sim X_C)^2 + R^2}$

or $Z = \sqrt{\left(\omega L \sim \frac{1}{\omega C}\right)^2 + R^2}$

Inductive reactance

$X_L = \omega L = 70\times10^3\times100\times10^{-6} = 7\Omega$

Capacitance reactance

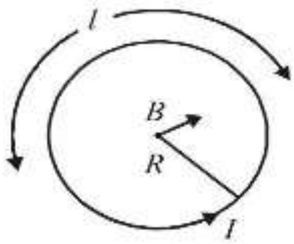
$$X_C = \frac{1}{\omega C} = \frac{1}{70\times10^3\times1\times10^{-6}}$$
$$= \frac{1}{7\times10^{-2}} = \frac{10^2}{7} = \frac{100}{7}$$

As $X_C > X_L$

So, circuit behaves like R-C circuit.

31. (c) At the centre of the loop, magnetic field

$B = \frac{\mu_0}{4\pi}\cdot\frac{I\cdot2\pi R}{R^2}$



For the wire which is looped double let radius becomes r

Then, $\frac{l}{2} = 2\pi r$ or $\frac{l}{4\pi} = (r)$

$\therefore B' = \frac{\mu_0}{4\pi}\cdot\frac{I\cdot2\pi r\times2}{r^2}$

or $B' = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot \frac{l}{2} \cdot 2}{\left(\frac{l}{4\pi}\right)^2}$

or $B' = \frac{\mu_0}{4\pi} \cdot \frac{Il \times 16\pi^2}{l^2}$

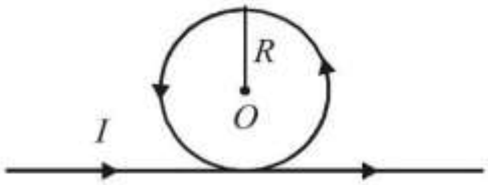
Now, $B = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot l}{\left(\frac{l}{2\pi}\right)^2} \left[R = \frac{l}{2\pi}\right]$

Dividing Eq. (ii) by (iii),

$$\frac{B'}{B} = \frac{\frac{\mu_0}{4\pi} \cdot \frac{I \cdot l \cdot 16\pi^2}{l^2}}{\frac{\mu_0}{4\pi} \cdot \frac{Il \cdot 4\pi^2}{l^2}} = 4$$

or $B' = 4B$

32. (c) Magnetic field due to long wire at O point



$B_1 = \frac{\mu_0}{2\pi}\left(\frac{I}{R}\right)$

(upward)

Magnetic field due to loop at O point

$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot 2\pi R}{R^2}$

$\Rightarrow B_2 = \frac{\mu_0}{2} \cdot \frac{I}{R}$ (in upward direction)

Resultant magnetic field at centre O

$B = B_1 + B_2$

$\Rightarrow B = \frac{\mu_0 I}{2\pi \cdot R}(\pi + 1)T$

33. (b) Work function $W_0 = 3.31 \times 10^{-19}$ J

Wavelength of incident radiation

$\lambda = 5000 \times 10^{-10}$ m

According to Einstein's photoelectric equation $E = W_0 + KE$

$\Rightarrow \frac{hc}{\lambda} = 3.31 \times 10^{-19} + KE$

$KE = -3.31 \times 10^{-19} + \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}}$

$$= -3.31 \times 10^{-19} + \frac{6.62 \times 3}{5} \times 10^{-19}$$

$$= (-3.31 \times 1.324 \times 3) \times 10^{-19}$$

$$= (3.972 - 3.31) \times 10^{-19}$$
$$= 0.662 \times 10^{-19}\ \mathrm{J}$$
$$\text{or } \mathrm{E} = \frac{0.662 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.41\mathrm{eV}$$

34. (d) From Einstein's photoelectric equation

$$\mathrm{E} = \mathrm{W}_0 + \frac{1}{2}\mathrm{mv}^2 \text{ or } \sqrt{\frac{2(\mathrm{E}-\mathrm{W}_0)}{\mathrm{m}}} = \mathrm{v}$$

A charged particle placed in uniform magnetic field experience a force

$$\mathrm{F} = \mathrm{evB} = \frac{\mathrm{mv}^2}{\mathrm{r}}$$

$$\text{or } \mathrm{r} = \frac{\mathrm{mv}}{\mathrm{eB}}$$

$$\text{or } \mathrm{r} = \frac{\mathrm{m}\sqrt{\frac{2(\mathrm{E}-\mathrm{W}_0)}{\mathrm{m}}}}{\mathrm{eB}} \Rightarrow \mathrm{r} = \frac{\sqrt{2\mathrm{m}(\mathrm{E}-\mathrm{W}_0)}}{\mathrm{eB}}$$

35. (d) Here, $N_1 = N_0 e^{-10\lambda t}$ and $N_2 = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{N_1}{N_2} = \frac{1}{e} = e^{-1} = e^{(-10\lambda+\lambda)t} = e^{-9\lambda t}$$

$$\Rightarrow t = \frac{1}{9\lambda}$$

36. (c) Here current flows in circuit A as both (p − n) junction diode act as forward biasing. Total resistnace R.

$$\frac{1}{\mathrm{R}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \quad \text{or } \mathrm{R} = 2\Omega$$

According to Ohm's law

$\mathrm{V} = \mathrm{I_A R}$

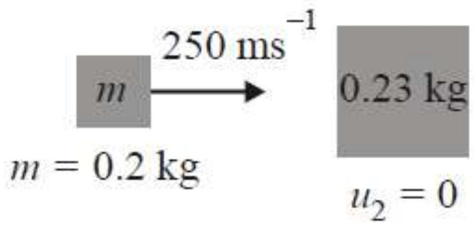
or $8 = \mathrm{I_A} \times 2$ or $\mathrm{I_A} = 4$ A

In circuit B, lower p - n-junction diode is reverse biased. Hence, no current will flow but upper diode is forward biased so current cna flow through it

$\mathrm{V} = \mathrm{I_B R}$

or $8 = \mathrm{I_B} \times 4$ or $\mathrm{I_B} = 2$ A

37. (c) After collision the bullet and block move together and comes to rest after covering a distance of 40 m.



By conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow 0.02 \times 250 + 0.23 \times 0 = 0.02\text{v} + 0.23\text{v}$

$5 + 0 = \text{v}(0.25)$ or $\text{v} = 20\ \text{ms}^{-1}$

Now, by conservation of energy

or $\frac{1}{2}\text{Mv}^2 = \mu\text{R} \cdot \text{d}$

or $\frac{1}{2} \times 0.25 \times 400 = \mu \times 0.25 \times 9.8 \times 40$

$\Rightarrow \mu = \dfrac{200}{9.8 \times 40} = 0.51$

38. (a) Let after the time (t) the position of A is $(0, v_At)$ and position of B $= (v_Bt, 10)$. Distance between them

$y = \sqrt{(0 - v_Bt)^2 + (v_At - 10)^2}$

or $y^2 = (2t)^2 + (2t - 10)^2$

or $y^2 = l = 4t^2 + 4t^2 + 100 - 40t$

$\Rightarrow l = 8t^2 + 100 - 40t$

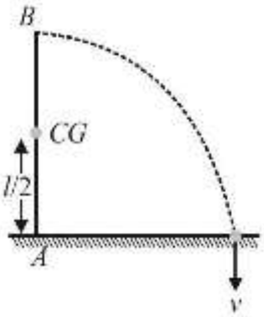
Now, $\frac{dl}{dt} = (16t - 40) = 0$

or

$t = \dfrac{40}{16} = 2.5\ \text{s} \ \because \dfrac{d^2l}{dt^2} = 16 = (+\text{ve})$

So, l will be minimum.

39. (b) Here, potential energy of the metre stick will be converted into rotational kinetic energy.



Because centre of gravity of stick lies at the middle of the rod, PE of metre stick $= \frac{mgl}{2}$

Rotational kinetic energy $E = \frac{1}{2} I\omega^2$ I about point A $= \frac{ml^2}{3}$.

By law of conservation of energy

$$mg\left(\frac{l}{2}\right) = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{ml^2}{3}\left(\frac{v_B}{l}\right)^2$$

By solving, we get $v_B = \sqrt{3gl}$

40. (a) Given, $r = 0.4$ m,

$\alpha = 8 rads^{-1}$,

$m = 4$ kg, $I = ?$

Torque, $\tau = I\alpha$

$mgr = I \cdot \alpha$

$4 \times 10 \times 0.4 = I \times 8$ or $I = \frac{16}{8} = 2$ kg. m^2

PART-II CHEMISTRY

41. (c) Given: $\Delta H_f(H) = 218$ kJ/mol

i.e., $\frac{1}{2} H_2 \longrightarrow H;\ \Delta H = 218$ kJ/mol

or $H_2 \longrightarrow 2H;\ \Delta H = 436$ kJ/mol

$$= \frac{436}{4.18} = 104.3 \text{kcal/mol}$$

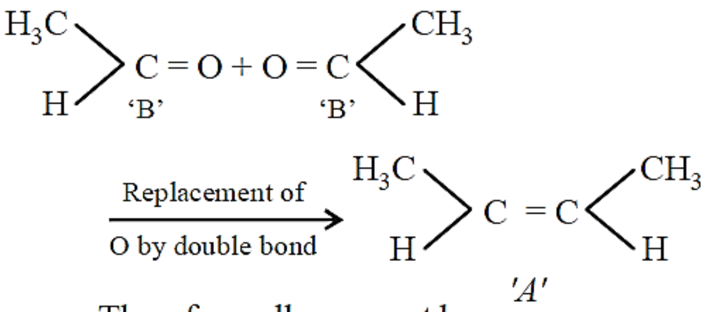
Hence, H − H bond energy is 104.3kcal/mol.

42. (a) In Wacker process,alkene is oxidised into aldehyde.

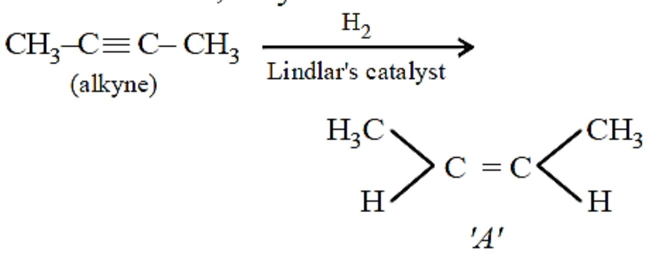
$$CH_2 = CH_2 + \frac{1}{2} O_2 \xrightarrow[H_2O]{PdCl_2 \cdot CuCl_2} \underset{(B)}{CH_3CHO}$$

Since only alkenes produce aldehydes, on ozonolysis hence ' A ' must be an alkene.

Now to find the structure of alkene we should add two molecules of aldehyde and replace O by double bond

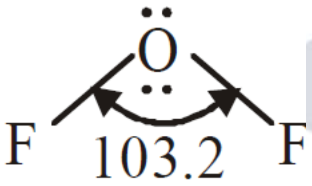


Therefore, alkyne must be



43. **(b)** $2\,F_2 + 2\underset{\text{dilute}}{NaOH} \longrightarrow 2NaF + \underset{(A)}{OF_2} \uparrow + H_2O$

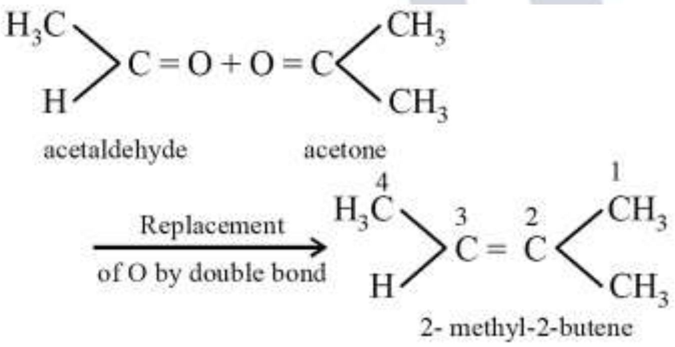
The structure of ' A ' (OF_2) is as



σ bonds made by $O = 2$

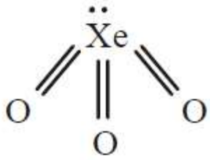
Due to repulsion between two lone pairs of electrons, its shape gets distorted. Therefore, the bond angle in the molecule is 103°.

44. **(a)** To decide the structure of alkene that undergoes ozonolysis, add the products and replace O by double (=) bond. Thus,



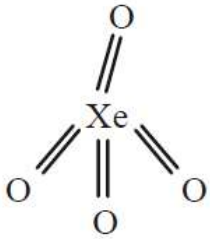
45. **(a)**

Structure of XeO_3



$\Rightarrow 3p\pi\text{-}d\pi$ pi bonds.

Structure of XeO_4



$\Rightarrow 4p\pi\text{-}d\pi$ bonds.

46. (a) According to de-Broglie's equation.

$$\lambda = \frac{h}{mv} \Rightarrow \lambda^2 = \frac{h^2}{m^2v^2}$$

$$\text{or } mv^2 = \frac{h^2}{m\lambda^2}$$

$$\because KE(K) = \frac{1}{2}mv^2$$

$$\therefore KE(K) = \frac{1}{2}\frac{h^2}{m\lambda^2}$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^2 = \left(\frac{5}{3}\right)^2$$

$$\therefore K_1 : K_2 = 25 : 9$$

47. (c) Paramagnetic nature depends upon the number of unpaired electrons. Higher the number of unpaired electrons, higher the paramagnetic property will be.

$Cu^{2+} = [Ar]3d^9$, no. of unpaired electrons $= 1$ $V^{2+} = [Ar]3d^3$, no. of unpaired electrons $= 3$ $Cr^{2+} = [Ar]3d^4$, no. of unpaired electrons $= 4$ $Mn^{2+} = [Ar]3d^5$, no. of unpaired electrons $= 5$ Hence, correct order is

$Cu^{2+} < V^{2+} < Cr^{2+} < Mn^{2+}$

48. (a) $\because$ 1 mol $= 6.023 \times 10^{23}$ atoms

KE of 1 mol $= 6.023 \times 10^4$ J

or KE of 6.023×10^{23} atoms $= 6.023 \times 10^4$ J

$$\therefore \text{KE of 1 atom } = \frac{6.023 \times 10^4}{6.023 \times 10^{23}}$$

$$= 1.0 \times 10^{-19}\ J$$

$$h\nu_{\text{energy}} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}}$$

$$= 3.313 \times 10^{-19}\ J$$

Now since Threshold energy

$= h\nu - KE$

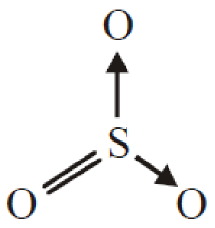
$= 3.313 \times 10^{-19} - 1.0 \times 10^{-19}$

$= 2.313 \times 10^{-19}$ J

Hence minimum amount of energy required to remove an electron from the metal ion will be 2.313×10^{-19} J.

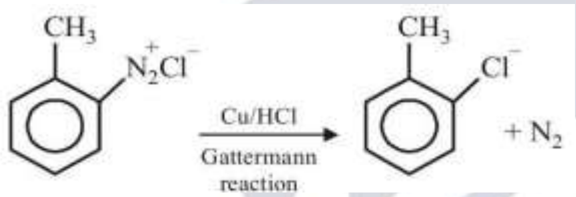
49. (a) The earth's thermosphere also includes the region of the atmosphere, called the ionosphere. The ionosphere is the region of the atmosphere that is filled with charged particles such as O_2^+, O^+, NO^+. The high temperature in the thermosphere can cause molecules to ionize.

50. (b) The formula of sulphuric anhydride is SO_3 and its structure is as follows:

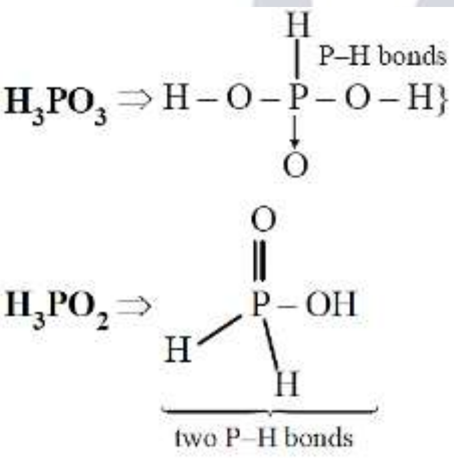


$\Rightarrow 3\sigma, 1p\pi - p\pi, 2p\pi - d\pi$ bonds are present

51. (a)



52. (c)



53. (c) Dipole moment,

$(\mu) = \delta \times d$

where, δ = magnitude of electric charge d = distance between particles (or bond length)

$\therefore \delta = \frac{\mu}{d}$

or $\frac{\delta_{HCl}}{\delta_{HI}} = \frac{\mu_{HCl}}{d_{HCl}} \times \frac{d_{HI}}{\mu_{HI}} = \frac{1.03 \times 1.6}{1.3 \times 0.38} = 3.3:1$

54. (d) $SiCl_4 + 4H_2O \longrightarrow H_4SiO_4 + 4HCl$

$H_4SiO_4 \xrightarrow[1000^\circ C]{\Delta} SiO_2 + 2H_2O$

55. (c) Masspercent of CdinCdCl$_2$

$= \frac{0.9}{1.5} \times 100 = 60\%$

$\therefore$ Mass percent of Cl_2 in $CdCl_2$

$= 100 - 60 = 40\%$

$\because$ 40% part (Cl_2) has atomic weight $= 2 \times 35.5 = 71.0$

$\therefore$ 60% part (Cd) has atomic weight

$= \frac{71.0 \times 60}{40} = 106.5$

56. (c) $2Al + 2NaOH + 2H_2O \longrightarrow 2NaAlO_2 + 3H_2$ sodium meta

Sodium meta aluminate, thus formed, is soluble in water and changes into the complex $[Al(H_2O)_2(OH)_4]^-$, in which coordination number of Al is 6

57. (b) Average kinetic energy per molecule $= \frac{3}{2}kT$

or $\frac{3}{2}\frac{R}{N_0}T = \frac{3}{2} \times \frac{8.314}{6.023\times10^{23}} \times 300 = 6.21 \times 10^{-21} JK^{-1}$ molecule $^{-1}$

58. (c) The species having an $O - O$ bond and O in an oxidation state of $-\frac{1}{2}$ are super oxides and is represented as O_2^-. Usually these are formed by active metals such as KO_2, RbO_2 and CsO_2. For the salts of larger anions (like O_2^-), lattice energy increases in a group. Since, lattice energy is the driving force for the formation of an ionic compound and its stability, the stability of the superoxides from ' K ' to ' Cs' also increases.

59. (a) 30% solution of H_2O_2 is known as perhydrol.

H_2O_2 decomposes as

$2H_2O_2 \longrightarrow 2H_2O + O_2$

Volume strength of 30%H_2O_2 solution is 100 i.e., 1 mL of this solution on decomposition gives 100 mL oxygen.

$SO_2 +$	$\frac{1}{2}O_2 \longrightarrow$	SO_3
1L	$\frac{1}{2}$L	1L
2L	1L	2L

Since, 100 mL of oxygen is obtained by $= 1$ mL of H_2O_2

$\therefore$ 1000 mL of oxygen will be obtained by

$= \frac{1}{100} \times 1000$ mL of $H_2O_2 = 10$ mL of H_2O_2

60. (d) Buffer capacity, $\beta = \frac{dC_{HA}}{d_{pH}}$, where, dC_{HA} = no. of moles of acid added per litre

d_{pH} = change in pH

$$dC_{HA} = \frac{\text{moles of acetic acid}}{\text{volume}}$$

$$= \frac{0.12/60}{250/1000} = \frac{1}{125}$$

$$\therefore \beta = \frac{1/125}{0.02} = \frac{1}{2.5} = 0.4$$

61. (b) (A) Felspar (orthoclase) $(KAlSi_3O_8)$

(B) Asbestos $\{CaMg_3(SiO_3)_4\}$

(C) Pyrargyrite (Ruby silver) (Ag_3SbS_3)

(D) Diaspore $(Al_2O_3 \cdot H_2O)$

62. (c) Along a period first ionisation energy increases. Thus, the first IE of the elements of the second period should follow the order $Be < B < N < O$

But in practice. The first IE of these elements follows the order $B < Be < O < N$ The lower IE of B than that of Be is because in $B(1s^2, 2s^2 2p^1)$, electron is to be removed from 2p which is easy while in $Be(1s^2, 2s^2)$, electron is to be removed from 2s which is difficult. The low IE of O than that of N is because of the half-filled 2p orbitals in N $(1s^2, 2s^2 2p^3)$.

63. (c)

$$CH_3CH_2OH + Cl_2 \xrightarrow[-2HCl]{} \underset{\substack{X \\ \text{Acetaldehyde}}}{CH_3CHO}$$

$$\xrightarrow[-3HCl]{3Cl_2} \underset{\substack{Y \\ \text{chloral}}}{CCl_3CHO}$$

64. (c)

I. $(CH_3COO)_2Ca \xrightarrow[-CaCO_3]{\Delta} CH_3COCH_3$

II. $CH_3COOH \longrightarrow 6HI$

$$\xrightarrow{\text{Red P}} CH_3CH_3 + 3I_2 + 2H_2O$$

III. $\begin{matrix} CH_3COOH \\ CH_3COOH \end{matrix} \xrightarrow{\Delta,\ P_4O_{10}} \begin{matrix} CH_3CO \\ CH_3CO \end{matrix} \Big> O + H_2O$

65. (b) $C = 85.71\% = \frac{85.71}{12} = 7.14; \frac{7.14}{7.14} = 1$

$$H = 14.29\% = \frac{14.29}{1} = 14.29; \frac{14.29}{7.14} = 2$$

$\therefore$ Empirical formula $= CH_2$

and, empirical formula weight $= 12 + 2 = 14$

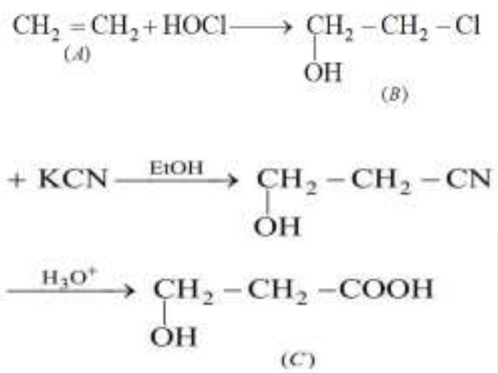
Now since molecular formula weight

$= 2 \times$ vapour density

$= 2 \times 14 = 28$

$\therefore n = \frac{28}{14} = 2$

$\therefore$ Molecular formula $= (CH_2)_2 = C_2H_4$



66. (c) Tripeptides are amino acids polymers in which three individual amino acid units, called residues, are linked together by amide bonds.

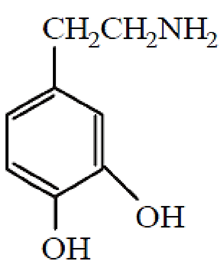
glycine ($NH_2 - CH_2 - COOH$), alanine ($CH_3 - CH(NH_2) - COOH$) and phenyl alanine

$C_6H_5 - CH_2 - CH(NH_2)COOH$ can be linked

in six different ways.

67. (d) A codon is a specific sequence of three adjacent bases on a strand of DNA or RNA that provides genetic code information for a particular amino acid.

68. (c) The IUPAC name of dopamine is 2-(3,4-dihydroxyphenyl) ethylamine and its structure is as follows:



69. (a) Freezing point of a substance is the temperature at which the solid and the liquid forms of the substance are in equilibrium.

70. (b)

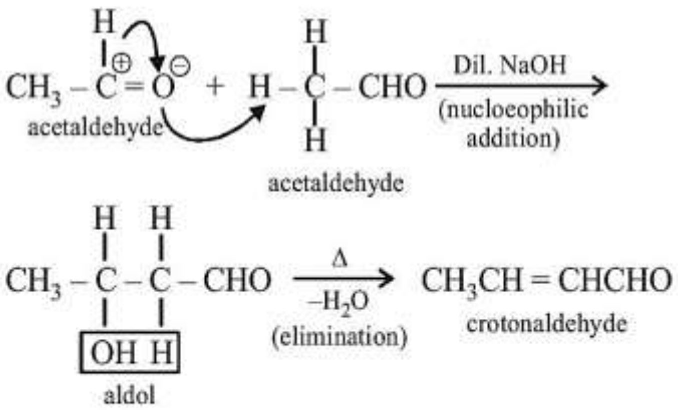
$$C_2H_5-Cl+AgCN \xrightarrow[H_2O]{EtOH/} C_2H_5-NC+AgCl$$

$\Downarrow$

(N-linked to ethyl carbon)

71. (a)

72. (d) Crotonaldehyde is produced by the aldol condensation of acetaldehyde.



73. (b) $BaCl_2 + 2NaOH \longrightarrow Ba(OH)_2 + 2NaCl$

$$\begin{aligned}\lambda^{\infty}_{mBa(OH)_2} &= \lambda^{\infty}_{mBaCl_2} + 2\lambda^{\infty}_{mNaOH} - 2\lambda^{\infty}_{mNaCl}\\ &= 280\times10^{-4} + 2\times248\times10^{-4}\\ &\quad -2\times126\times10^{-4}\\ &= (280+496-252)\times10^{-4}\\ &= 524\times10^{-4}\,Sm^2\ mol^{-1}\end{aligned}$$

74. (b) Density, $d = \frac{MZ}{N_0a^3}$

where, Z = number of atoms in unit cell

$$\therefore Z = \frac{dN_0a^3}{M}$$

$$= \frac{8.92\times6.023\times10^{23}\times(362\times10^{-10})^3}{63.55}$$

$= 4.0$

Thus, metal has face centred unit cell.

75. (c) $N_2 + 2O_2 \rightleftharpoons 2NO_2$

$$K_1 = \frac{[NO_2]^2}{[N_2][O_2]^2}$$

Again, $[NO_2] \rightleftharpoons \frac{1}{2}N_2 + O_2$

$$K_2 = \frac{[N_2]^{1/2}[O_2]}{[NO_2]}$$

or $K_2^2=\frac{[N_2][O_2]^2}{[NO_2]^2}$

Eqs. (i) × (ii), we get $100\times K_2^2=1$

$\Rightarrow K_2^2=\frac{1}{100}$ or $K_2=\frac{1}{10}=0.1$

76. (c) For a first order reaction,

$$t=\frac{2.303}{\lambda}\log_{10}\frac{a}{a-x}$$

Let initial amount of reactant is 100 .

$\frac{t_1}{t_2}=\frac{\log\frac{100}{100-75}}{\log\frac{100}{100-25}}$ [$\because$ λ remains constan t]

$$=\frac{\log\frac{100}{25}}{\log\frac{100}{75}}=\frac{\log 4}{\log 4/3}$$

$$=\frac{\log 4}{\log 4-\log 3}$$

$$=\frac{2\times 0.3010}{2\times 0.3010-0.4771}$$

$$=\frac{0.6020}{0.1249}=4.81$$

77. (c)

$$[\alpha]=\frac{[\alpha]_{\text{observed}}}{l\times C}=\frac{-1.2}{5\times\frac{6.15}{1000}}=-39^\circ$$

78. (a) Let the concentration of potassium acetate is x. According to Henderson's equation,

$$pH=pK_a+\log\frac{[\text{salt}]}{[\text{ acid }]}$$

$$4.8=-\log(1.8\times10^{-5})+\log\frac{x\times 50}{20\times 0.1M}$$

$$4.8=4.74+\log 25x$$

or $\log 25x=0.06$
$25x=1.148$

$\therefore x=0.045M$

79. (b) By ' $2A+\frac{C}{2}-B'$, we get

$Na_2O+SO_3\longrightarrow Na_2SO_4$

$$\Delta H=-2\times 146+\frac{259}{2}-418$$

$\Rightarrow \Delta H = -580.5 \approx 581$ kJ

80. (a) According to Hardy Schulze rule, greater the valency of the coagulating ion, greater is its coagulating power. Thus, out of the given, $AlCl_3 (Al^{3+})$ is most effective for causing coagulation of $As_2 S_3$ sol.

PART-III MATHEMATICS

81. (b) Given, $f(x) = x^3 + 3x - 2$

$\Rightarrow f'(x) = 3x^2 + 3$

Put $f'(x) = 0 \Rightarrow 3x^2 + 3 = 0$

$\Rightarrow x^2 = -1$

$\therefore$ f(x) is either increasing or decreasing.

At $x = 2, f(2) = 2^3 + 3(2) - 2 = 12$

At $x = 3, f(3) = 3^3 + 3(3) - 2 = 34$

$\therefore f(x) \in [12,34]$.

82. (c) The total number of subsets of given set is $2^9 = 512$

Even numbers are $\{2,4,6,8\}$.

Case I : When selecting only one even number $= {}^4C_1 = 4$

Case II : When selecting only two even numbers $= {}^4C_2 = 6$

Case III : When selecting only three even numbers $= {}^4C_3 = 4$

Case IV : When selecting only four even numbers $= {}^4C_4 = 1$

$\therefore$ Required number of ways

$= 512 - (4 + 6 + 4 + 1) - 1 = 496$

[Here, we subtract 1 for due to the null set]

83. (a) The required number of ways = The even number of 0 's i.e., $\{0,2,4,6, \ldots\}$

$$= \frac{n!}{n!} + \frac{n!}{2!\,(n-2)!} + \frac{n!}{4!\,(n-4)!}$$

$$= {}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots = 2^{n-1}$$

84. (a) $\left(1 + \frac{2x}{3}\right)^{3/2} (32 + 5x)^{-1/5}$

$$= \left[1 + \frac{3}{2}\left(\frac{2x}{3}\right)\right] (32)^{-1/5} \left(1 + \frac{5}{32}x\right)^{-1/5}$$

(neglecting higher powers of x)

$$= [1 + x]2^{-1} \left[1 - \frac{1}{5}\left(\frac{5}{32}\right)x\right]$$

(neglecting higher powers of x)

$$= \frac{1}{2}(1+x)\left(1-\frac{x}{32}\right)$$

$$= \frac{(1+x)(32-x)}{64} = \frac{32+31x}{64}$$

(neglecting x^2 term)

85. (c) $(x-a)(x-a-1)+(x-a-1)(x-a-2)$
$+(x-a)(x-a-2)=0$

Let $x-a=t$, then

$t(t-1)+(t-1)(t-2)+t(t-2)=0$

$\Rightarrow t^2-t+t^2-3t+2+t^3-2t=0$

$\Rightarrow 3t^2-6t+2=0$

$$\Rightarrow t = \frac{6\pm\sqrt{36-24}}{2(3)} = \frac{6\pm2\sqrt{3}}{2(3)}$$

$$\Rightarrow x-a = \frac{3\pm\sqrt{3}}{3}$$

$$\Rightarrow x = a+\frac{3\pm\sqrt{3}}{3}$$

Hence, x is real and distinct.

86. (b) $f(x)=x^2+ax+b$ has imaginary roots.
$\therefore$ Discriminant, $D<0 \Rightarrow a^2-4b<0$

$f'(x)=2x+a$

$\Rightarrow f''(x)=2$

Also, $f(x)+f'(x)+f''(x)=0$

$\Rightarrow x^2+ax+b+2x+a+2=0$

$\Rightarrow x^2+(a+2)x+b+a+2=0$

$$\therefore x = \frac{-(a+2)\pm\sqrt{(a+2)^2-4(a+b+2)}}{2}$$

$$= \frac{-(a+2)\pm\sqrt{a^2-4b-4}}{2}$$

Since, $a^2-4b<0$

$\therefore a^2-4b-4<0$

Hence, Eq. (i) has imaginary roots.

87. **(c)** $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $(x-2)(x-1)$.

$\therefore f(2) = 2(2)^4 - 13(2)^2 + a(2) + b = 0$

$\Rightarrow 2a + b = 20$

and $f(1) = 2(1)^4 - 13(1)^2 + a + b = 0$

$\Rightarrow a + b = 11$

On solving Eqs. (i) and (ii), we get $a = 9, b = 2$

88. **(d)** Let a and R be the first term and common ratio of a GP respectively.

So, $T_p = aR^{p-1} = x$

$T_q = aR^{q-1} = y$ and $T_r = aR^{r-1} = z$

$\Rightarrow \log x = \log a + (p-1)\log R$

$\log y = \log a + (q-1)\log R$

and $\log z = \log a + (r-1)\log R$

$$\therefore \begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = \begin{vmatrix} \log a + (p-1)\log R & p & 1 \\ \log a + (q-1)\log R & q & 1 \\ \log a + (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log a & p & 1 \\ \log a & q & 1 \\ \log a & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \log a \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix}$$

$(C_2 \to C_2 - C_3)$

$= 0 + 0 = 0$ ($\because$ two columns are identical)

89. **(c)** Let $z = x + iy$

Given: $\left|\frac{z+2i}{2z+i}\right| < 1$

$$\Rightarrow \frac{\sqrt{(x)^2 + (y+2)^2}}{\sqrt{(2x)^2 + (2y+1)^2}} < 1$$

$\Rightarrow x^2 + y^2 + 4 + 4y < 4x^2 + 4y^2 + 1 + 4y$

$\Rightarrow 3x^2 + 3y^2 > 3 \Rightarrow x^2 + y^2 > 1$

90. **(c)** $(1 + \sqrt{3}i)^n + (1 - \sqrt{3}i)^n$

$$= \left[2\left(\frac{1+\sqrt{3}i}{2}\right)\right]^n + \left[2\left(\frac{1-\sqrt{3}i}{2}\right)\right]^n$$
$= (-2\omega^2)^n + (-2\omega)^n$
$= (-2)^n[(\omega^2)^{3r+1} + (\omega)^{3r+1}]$
($\because$ $n = 3r + 1$, where r is an integer)
$= (-2)^n(\omega^2 + \omega) = -(-2)^n$

91. (d) Let $f(x) = \sin^4 x + \cos^4 x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$= 1 - \frac{1}{4} \cdot 2(\sin 2x)^2$

$= 1 - \frac{1}{4}(1 - \cos 4x) = \frac{3}{4} + \frac{\cos 4x}{4}$

$\therefore$ Period of $f(x) = \frac{2\pi}{4} = \frac{\pi}{2}$

92. (c) $\sin^2 x - \cos 2x = 2 - \sin 2x$

$\Rightarrow 1 - \cos^2 x - (2\cos^2 - 1) = 2 - 2\sin x\cos x$

$\Rightarrow -3\cos^2 x + 2\sin x\cos x = 0$

$\Rightarrow \cos x(2\sin x - 3\cos x) = 0$

$\Rightarrow \cos x = 0, \ (\because 2\sin x - 3\cos x \neq 0)$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{2}$

$\Rightarrow x = (4n \pm 1)\frac{\pi}{2}, n \in Z$

93. (d) $\cos^{-1}\left(-\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right) + 3\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$-4\tan^{-1}(-1)$

$= \pi - \cos^{-1}\left(\frac{1}{2}\right) - 2\left(\frac{\pi}{6}\right) + 3\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$

$+4\tan^{-1}(1)$

$= \pi - \frac{\pi}{3} - \frac{\pi}{3} + 3\left(\pi - \frac{\pi}{4}\right) + 4 \cdot \frac{\pi}{4}$

$= \frac{\pi}{3} + 3 \cdot \frac{3\pi}{4} + \pi = \frac{43\pi}{12}$

94. (c) We know that, $2s = a + b + c$

$$\therefore \frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$$
$$= \frac{2s(2s-2a)(2s-2b)(2s-2c)}{4b^2c^2}$$
$$= 4\frac{s(s-a)}{bc} \times \frac{(s-b)(s-c)}{bc}$$
$$= 4\cos^2\frac{A}{2} \times \sin^2\frac{A}{2} = \sin^2 A$$

95. (c) $l + m + n = 0, \Rightarrow l = -m - n$

and $l^2 + m^2 - n^2 = 0$

$\therefore (-m-n)^2 + m^2 - n^2 = 0$

$\Rightarrow 2m^2 + 2mn = 0$

$\Rightarrow 2m(m+n) = 0$

$\Rightarrow m = 0$ or $m + n = 0$

If $m = 0$, then $l = -n$

$\therefore \frac{l_1}{-1} = \frac{m_1}{0} = \frac{n}{1}$

and if $m + n = 0 \Rightarrow m = -n$, then $l = 0$

$\therefore \frac{l_2}{0} = \frac{m_2}{-1} = \frac{n_2}{1}$

ie., $(l_1, m_1, n_1) = (-1,0,1)$

and $(l_2, m_2, n_2) = (0,-1,1)$

$\therefore \cos\theta = \frac{0+0+1}{\sqrt{1+0+1}\sqrt{0+1+1}} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

96. (a) $m_1 = |\vec{a}_1| = \sqrt{2^2 + (-1)^2 + (1)^2} = \sqrt{6}$

$m_2 = |\vec{a}_2| = \sqrt{3^2 + (-4)^2 + (-4)^2} = \sqrt{41}$

$m_3 = |\vec{a}_3| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$

and $m_4 = |\vec{a}_4| = \sqrt{(-1)^2 + (3)^2 + (1)^2} = \sqrt{11}$

$\therefore m_3 < m_1 < m_4 < m_2$

97. (a) Here, $n = 6$

According to the question

${}^6C_2 p^2 q^4 = 4 \cdot {}^6C_4 p^4 q^2$

$\Rightarrow q^2 = 4p^2 \Rightarrow (1-p)^2 = 4p^2 \Rightarrow 3p^2 + 2p - 1 = 0$

$\Rightarrow (p+1)(3p-1) = 0$

$\Rightarrow p = \frac{1}{3}$ ($\because$ p cannot be negative)

98. **(d)** Given lines are parallel.

$$d = \frac{15-5}{\sqrt{4^2+3^2}} = \frac{10}{5}$$

$\Rightarrow d = 2 =$ diameter of the circle

$\therefore$ Radius of circle $= 1$

$\therefore$ Area of circle $= \pi r^2 = \pi$ sq unit

99. **(c)** $x^2 - 2xy - xy + 2y^2 = 0$

$\Rightarrow (x-2y)(x-y) = 0$

$\Rightarrow x = 2y, x = y$

Also, $x + y + 1 = 0$

On solving Eqs. (i) and (ii), we get

$$A\left(-\frac{2}{3}, -\frac{1}{3}\right), B\left(-\frac{1}{2}, -\frac{1}{2}\right), C(0,0)$$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2}\begin{vmatrix} -\frac{2}{3} & -\frac{1}{3} & 1 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{6}\right] = \frac{1}{2}\left[\frac{1}{6}\right] = \frac{1}{12}$$

100. **(c)** Given pair of lines are

$x^2 - 3xy + 2y^2 = 0$

and $x^2 - 3xy + 2y^2 + x - 2 = 0$

$\therefore (x-2y)(x-y) = 0$

and $(x-2y+2)(x-y-1) = 0$

$\Rightarrow x - 2y = 0, x - y = 0$

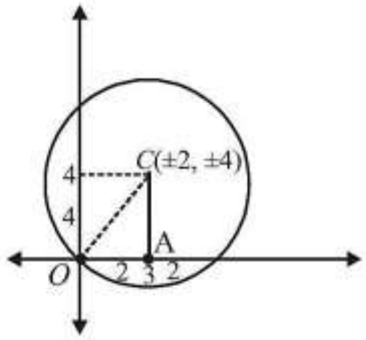
and $x - 2y + 2 = 0, x - y - 1 = 0$

The lines $x - 2y = 0, x - 2y + 2 = 0$ and $x - y = 0, x - y - 1 = 0$ are parallel.

Also, angle between $x - 2y = 0$ and $x - y = 0$ is not $90°$.

$\therefore$ It is a parallelogram.

101. **(a)** In $\triangle OAC$, $OC^2 = 2^2 + 4^2 = 20$



$\therefore$ Required equation of circle is

$(x \pm 2)^2 + (y \pm 4)^2 = 20$

$\Rightarrow x^2 + y^2 \pm 4x \pm 8y = 0$

102. **(d)** Given circles are

$x^2 + y^2 - 2x + 8y + 13 = 0$

and $x^2 + y^2 - 4x + 6y + 11 = 0$

Here $C_1 = (1, -4), C_2 = (2, -3)$

$\Rightarrow r_1 = \sqrt{1 + 16 - 13} = 2$

and $r_2 = \sqrt{4 + 9 - 11} = \sqrt{2}$

So, $d = C_1C_2 = \sqrt{(2-1)^2 + (-3+4)^2} = \sqrt{2}$

$\therefore \cos\theta = \dfrac{d^2 - r_1^2 - r_2^2}{2r_1r_2} = \dfrac{2 - 4 - 2}{2 \times 2 \times \sqrt{2}} = -\dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta = 135°$

103. **(b)** Let the required equation of circle be

$x^2 + y^2 + 2gx + 2fy = 0$

The above circle cuts the given circles orthogonally.

$\therefore 2(-3g) + 2f(0) = 8 \Rightarrow 2g = -\dfrac{8}{3}$

and $-2g - 2f = -7$

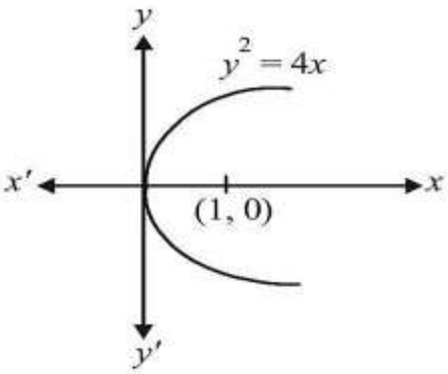
$\Rightarrow 2f = +7 + \dfrac{8}{3} = \dfrac{29}{3}$

$\therefore$ Required equation of circle is

$x^2 + y^2 - \dfrac{8}{3}x + \dfrac{29}{3}y = 0$

or $3x^2 + 3y^2 - 8x + 29y = 0$

104. **(b)** Given curve is $y^2 = 4x$.



Also, point (1,0) is the focus of the parabola.

It is clear from the graph that only one normal is possible.

105. **(a)** $x^2y^2 = c^4$

$\Rightarrow y^2(a^2 - y^2) = c^4$

$\Rightarrow y^4 - a^2y^2 + c^4 = 0$

Let y_1, y_2, y_3 and y_4 are the roots.

$\therefore y_1 + y_2 + y_3 + y_4 = 0$

106. **(b)** Solving $4x - 3y = 5$ and $2x^2 - 3y^2 = 12$

$$\therefore 2\left(\frac{5+3y}{4}\right)^2 - 3y^2 = 12$$

$$\Rightarrow \frac{(25 + 9y^2 + 30y)}{8} - 3y^2 = 12$$

$$\Rightarrow 15y^2 - 30y + 71 = 0$$

$$\Rightarrow y = \frac{30 \pm \sqrt{900 - 4260}}{30} = 1 \pm \frac{\sqrt{-3360}}{30}$$

Also, $2x^2 - 3\left(\frac{4x-5}{3}\right)^2 = 12$

$$\Rightarrow 10x^2 - 40x + 61 = 0$$

$$\Rightarrow x = \frac{40 \pm \sqrt{1600 - 4 \times 10 \times 61}}{2 \times 10}$$

$$= \frac{40 \pm \sqrt{-840}}{20} = 2 \pm \frac{\sqrt{-840}}{20}$$

$\therefore$ Points are $A\left(2 + \frac{\sqrt{-840}}{20}, 1 + \frac{\sqrt{-3360}}{30}\right)$

and $B\left(2 - \frac{\sqrt{-840}}{20}, 1 - \frac{\sqrt{-3360}}{30}\right)$.

$\therefore$ Mid point of AB is (2,1).

107. **(d)** Let $A=(1,0,0)$, $B=(0,1,0)$ and $C=(0,0,1)$

So, $AB=\sqrt{(0-1)^2+(1-0)^2+0^2}=\sqrt{2}$

$BC=\sqrt{0^2+(0-1)^2+(1-0)^2}=\sqrt{2}$

and $CA=\sqrt{(1-0)^2+0^2+(0-1)^2}=\sqrt{2}$

$\therefore$ Perimeter of triangle

$=AB+BC+CA=\sqrt{2}+\sqrt{2}+\sqrt{2}=3\sqrt{2}$

108. **(c)** $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$

$+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

$=(\cos^2\alpha-\sin^2\alpha)+(\cos^2\beta-\sin^2\beta)$

$(\cos^2\gamma-\sin^2\gamma)+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

$=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

109. **(a)** Given equation of sphere is

$x^2+y^2+z^2-12x-4y-3z=0$

Centre of sphere is $\left(6,2,\frac{3}{2}\right)$.

$\therefore$ Radius of sphere $=\sqrt{(6)^2+(2)^2+\left(\frac{3}{2}\right)^2}$

$$=\sqrt{36+4+\frac{9}{4}}=\sqrt{\frac{169}{4}}=\frac{13}{2}$$

110. **(c)** $\lim_{x\to\infty}\left(\frac{x+5}{x+2}\right)^{x+3}=\lim_{x\to\infty}\left(1+\frac{3}{x+2}\right)^{x+3}$

$$=\lim_{x\to\infty}\left[\left(1+\frac{3}{x+2}\right)^{\frac{x+2}{3}}\right]^{\frac{3(x+3)}{x+2}}$$

$$=e^{\lim\limits_{x\to\infty}3\left(\frac{1+\frac{3}{x}}{1+\frac{2}{x}}\right)}=e^3$$

111. **(d)** $f(x)=\begin{cases}\frac{2\sin x-\sin 2x}{2x\cos x}, & \text{if } x\neq 0\\ a, & \text{if } x=0\end{cases}$

$\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{2\sin x-\sin 2x}{2x\cos x}\left(\frac{0}{0}\text{ form }\right)$

$=\lim_{x\to 0}\frac{2\cos x-2\cos 2x}{2(\cos x-x\sin x)}=\lim_{x\to 0}\frac{2-2}{2(1-0)}=0$

Since, $f(x)$ is contineous at $x=0$

$\therefore f(0)=\lim_{x\to 0}f(x)\Rightarrow a=0$

112. **(c)** We have, $x=\cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

and $y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$\Rightarrow x = \tan^{-1} t$ and $y = \tan^{-1} t$

$\therefore y = x \Rightarrow \frac{dy}{dx} = 1$

113. **(b)** $\frac{d}{dx}\left[a\tan^{-1} x + b\log\left(\frac{x-1}{x+1}\right)\right] = \frac{1}{x^4-1}$

On integrating both sides, we get

$$a\tan^{-1} x + b\log\left(\frac{x-1}{x+1}\right)$$

$$= \frac{1}{2}\int \left[\frac{1}{x^2-1} - \frac{1}{x^2+1}\right] dx$$

$$\Rightarrow a\tan^{-1} x + b\log\left(\frac{x-1}{x+1}\right)$$

$$= \frac{1}{4}\log\left(\frac{x-1}{x+1}\right) - \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow a = -\frac{1}{2}, b = \frac{1}{4}$$

$$\therefore a - 2b = -\frac{1}{2} - 2\left(\frac{1}{4}\right) = -1$$

114. **(c)** $y = e^{a\sin^{-1} x}$

On differentiating w.r.t. x, we get

$$y_1 = e^{a\sin^{-1} x} a \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1\sqrt{1-x^2} = ay$$

$$\Rightarrow (1-x^2)y_1^2 = a^2y^2$$

Again differentiating w.r.t. x, we get

$$(1-x^2)2y_1y_2 - 2xy_1^2 = a^2 2yy_1$$

$$\Rightarrow (1-x^2)y_2 - xy_1 - a^2y = 0$$

Using Leibnitz's rule,

$$(1-x^2)y_{n+2} + {}^nC_1 y_{n+1}(-2x) + {}^nC_2 y_n(-2)$$
$$-xy_{n+1} - {}^nC_1 y_n - a^2y_n = 0$$
$$\Rightarrow (1-x^2)y_{n+2} + xy_{n+1}(-2n-1)$$
$$+ y_n[-n(n-1) - n - a^2] = 0$$
$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} = (n^2+a^2)y_n$$

115. **(c)** Given, $f(x) = x^3 + ax^2 + bx + c, a^2 \leq 3b$

$\Rightarrow f'(x) = 3x^2 + 2ax + b$

Put $f'(x) = 0$

$\Rightarrow 3x^2 + 2ax + b = 0$

$$\Rightarrow x = \frac{-2a \pm \sqrt{4a^2 - 12b}}{2 \times 3}$$

$$= \frac{-2a \pm 2\sqrt{a^2 - 3b}}{3}$$

Since, $a^2 \leq 3b$,

$\therefore$ x has an imaginary value.

Hence, no extreme value of x exists.

116. **(a)** Let $I = \int \left(\frac{2-\sin 2x}{1-\cos 2x}\right) e^x dx$

$$= \int \left(\frac{2 - 2\sin x\cos x}{2\sin^2 x}\right) e^x dx$$

$$= \int \operatorname{cosec}^2 x e^x dx - \int \cot x e^x dx$$

$$= -\cot x e^x - \int (-\cot x) e^x dx$$

$$- \int \cot x e^x dx + c$$

$$= -\cot x e^x + c$$

117. **(c)** We know that, if

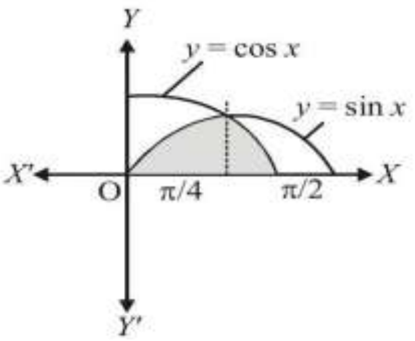
$I_n = \int \sin^n x dx$, then

$$I_n = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n} I_{n-2}$$

where n is a positive integer.

$\Rightarrow nI_n - (n-1)I_{n-2} = -\sin^{n-1} x\cos x$

118. **(d)**



Area, $A_1 = \int_0^{\pi/4} \sin x dx$

$$= -[\cos x]_0^{\pi/4} = 1 - \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}-1}{\sqrt{2}}$$

and area, $A_2 = \int_{\pi/4}^{\pi/2} \cos x dx$

$$= [\sin x]_{\pi/4}^{\pi/2} = \left[1 - \frac{1}{\sqrt{2}}\right] = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\therefore A_1 : A_2 = \frac{\sqrt{2}-1}{\sqrt{2}} : \frac{\sqrt{2}-1}{\sqrt{2}} = 1:1$$

119. **(b)** $\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1$

Put $x + y = z$

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{dz}{dx} - 1 = \sin z \tan z - 1$$

$$\Rightarrow \int \frac{\cos z}{\sin^2 z} dz = \int dx$$

Putting $\sin z = t \Rightarrow \cos z dz = dt$, we have

$$\int \frac{1}{t^2} dt = x - c \Rightarrow -\frac{1}{t} = x - c$$

$$\Rightarrow -\operatorname{cosec} z = x - c \Rightarrow x + \operatorname{cosec}(x+y) = c$$

120. **(c)** $p \Rightarrow (\sim p \vee q)$ is false means p is true and $\sim p \vee q$ is false.

$\Rightarrow$ p is true and both $\sim p$ and q are false.

$\Rightarrow$ p is true and q is false.