PART-I PHYSICS

1. **(b)** Here, length $l = 2\pi r$ or $r = \frac{l}{2\pi}$

Area of circular loop $A = \pi r^2$

Magnetic moment $M = iA = i\pi r^2$

$$M = i\pi \times \frac{l^2}{4\pi^2}$$

$$\therefore l = \sqrt{\frac{4\pi M}{i}}$$

2. **(b)** Current through arms of resistances P and Q in series

$$i_1 = \frac{i \times 330}{330 + 110} = \frac{3}{4}i$$

Here i = total current

Similarly, current through arms of resistances R and S in series

$$i_2 = \frac{i \times 110}{330 + 110} = \frac{1}{4}i$$

Heat developed per second $= i^2R$

$\therefore$ Ratio of heat developed per sec

$$H_P : H_Q : H_R : H_S$$
$$= \left(\frac{3}{4}i\right)^2 \times 100 : \left(\frac{3}{4}i\right)^2 \times 10 :$$
$$\left(\frac{1}{4}i\right)^2 \times 300 : \left(\frac{1}{4}i\right)^2 \times 30$$

$= 30 : 3 : 10 : 1$

3. **(c)** Heat taken by water when its temperature changes from 20°C to 100°C.

$$H_1 = mc(\theta_2 - \theta_1) = 1000 \times 1 \times (100 - 20)\text{cal} = 1000 \times 80 \times 4.2\text{ J}$$

Heat produced in time t due to current in resistor

$$H_2 = Vit = 220 \times 4 \times t\text{ J}$$

According to question,

$$220 \times 4 \times t = 1000 \times 80 \times 4.2$$

$$\Rightarrow t = \frac{1000 \times 80 \times 4.2}{220 \times 4} = 381.8\text{ s} = 6.3\text{ min}$$

4. **(d)** Magnetic field,

$$B = \frac{\mu_0}{4\pi}\frac{2\pi i}{r} = \frac{\mu_0 i}{2r} \text{ or } i = \frac{2Br}{\mu_0}$$

Also, $A = \pi r^2$ or $r = \left(\frac{A}{\pi}\right)^{1/2}$

Magnetic moment,

$$M = iA = \frac{2Br}{\mu_0}A$$

$$= \frac{2BA}{\mu_0} \times \left(\frac{A}{\pi}\right)^{1/2} = \frac{2BA^{3/2}}{\mu_0 \pi^{1/2}}$$

5. **(c)** Here, $\sin\theta = \left(\frac{Y}{D}\right)$ $\therefore \Delta\theta = \frac{\Delta Y}{D}$

Angular fringe width $\theta_0 = \Delta\theta$

(width $\Delta Y = \beta$)

$$\theta_0 = \frac{\beta}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$$

$$\theta_0 = 1^\circ = \frac{\pi}{180}\text{rad and } \lambda = 6 \times 10^{-7}\text{ m}$$

$$d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times 6 \times 10^{-7} = 0.03\text{ mm}$$

6. **(d)** Here, charge $q = \pm 1 \times 10^{-6}C$

$2a = 2.0\text{ cm} = 2.0 \times 10^{-2}\text{ m}$

$E = 1 \times 10^5 NC^{-1}, \tau_{max} = ?$

$W = ?, \theta_1 = 0^\circ, \theta_2 = 180^\circ$

$\tau_{max} = pE = q(2a)E$

$= 1 \times 10^{-6} \times 2.0 \times 10^{-2} \times 1 \times 10^5$

$= 2 \times 10^{-3} Nm$

$W = pE(\cos\theta_1 - \cos\theta_2)$

$= (10^{-6} \times 2 \times 10^{-2})(10^5)(\cos 0^\circ - \cos 180^\circ)$

$= 4 \times 10^{-3}$ J

7. **(b)** Current, $i = \frac{e}{t} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$

Here, $v = \frac{e^2}{\hbar}$ and $r = \frac{\hbar^2}{me^2}$

$$\therefore i = \frac{e(e^2/\hbar)}{2\pi(\hbar^2/me^2)} = \frac{e^3 \times me^2}{2\pi\hbar^3} = \frac{me^5}{2\pi\hbar^3}$$

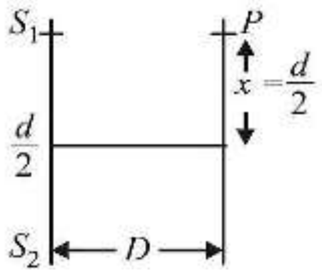
$\because \hbar = \frac{h}{2\pi}$ (given)

$$\therefore i = \frac{me^5}{2\pi\left(\frac{h}{2\pi}\right)^3} = \frac{4\pi^2 me^5}{h^3}$$

8. (d) For dark fringe,

$$\frac{xd}{D} = (2m-1)\frac{\lambda}{2}$$

Here, $m = 5, x = \frac{d}{2}$



$$\therefore \frac{d}{2} \cdot \frac{d}{D} = (2 \times 5 - 1)\frac{\lambda}{2}$$

or $\frac{d^2}{D} = 9\lambda$

$\therefore$ Wavelength, $\lambda = \frac{d^2}{9D}$

9. (d) Generally, temperature of human body is $37^\circ C (= 98.4^\circ F)$ corresponding to which IR and microwave radiations are emitted from the human body.

10. (d) The path of moving proton in a normal magnetic field is circular. If r is the radius of the circular path, then from the figure,

From the symmetry of figure, the angle $\theta = 45^\circ$.

$$AC = 2r\cos 45^\circ = 2r \times \frac{1}{\sqrt{2}} = \sqrt{2}r$$

As $Bqv = \frac{mv^2}{r}$ or $r = \frac{mv}{Bq}$

$$AC = \frac{\sqrt{2}mv}{Bq} = \frac{\sqrt{2} \times 1.67 \times 10^{-27} \times 10^7}{1 \times 1.6 \times 10^{-19}}$$
$$= 0.14 \text{ m}$$

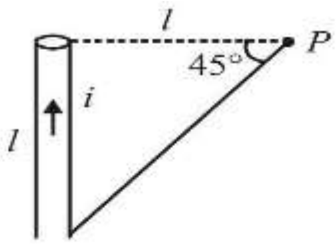
11. (d) In a neutral water molecule, there are 10 electrons and 10 protons.

So, its dipole moment $p = q(2l) = 10e(2l)$ Hence length of the dipole = distance between centres of positive and negative charges

$$2l = \frac{p}{10e} = \frac{6.4 \times 10^{-30}}{10 \times 1.6 \times 10^{-19}} = 4 \times 10^{-12} \text{ m}$$

12. (c) Magnetic field due to finite length of a wire,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$



Here, $\phi_1 = 0°, \phi = 45°$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(\sin 0° + \sin 45°) = \frac{\mu_0}{4\pi} \cdot \frac{i}{\sqrt{2}l}$$

$$\Rightarrow B = \frac{\sqrt{2}\mu_0 i}{8\pi l}$$

13. (c) Zener diode is suitable for voltage regulating purpose. It is used as voltage stabilizer in many applications in electronics.

14. (a) If two independent sources emitting light of the same wavelength are said to be coherent.

15. (d) Induced charge

$$Q = -\frac{nBA}{R}(\cos\theta_2 - \cos\theta_1)$$
$$= -\frac{nBA}{R}(\cos 180° - \cos 0°)$$
$$\Rightarrow B = \frac{QR}{2nA}$$

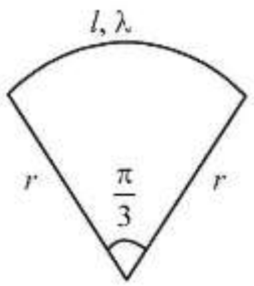
16. (d) From an optical fibre due to absorption or light leaving the fibre area resulting scattering of light sideways by impurities in the glass fibre. And due to this reason a very small part of light energy is lost.

17. (c) Length of the arc $= r\theta = \frac{r\pi}{3}$

Charge on the arc $= \frac{r\pi}{3} \times \lambda$

$\therefore$ Potential at centre v

$$= \frac{kq}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{r\pi\lambda}{3\ r} = \frac{\lambda}{12\varepsilon_0}$$



18. (a) Here$_c = 1.5\text{MHz} = 1500\text{kHz}, f_m = 10\text{kHz}$ $\therefore$ Lower side-band frequency $= f_c - f_m = 1500\text{kHz} - 10\text{kHz} = 1490\text{kHz}$ Upper side-band frequency $= f_c + f_m = 1500\text{kHz} + 10\text{kHz} = 1510\text{kHz}$

19. (c) Equivalent resistance of the circuit $R_{eq} = 100\Omega$

Current through the circuit, $i = \frac{V}{R} = \frac{2.4}{100}$ A

Potential difference across combination of voltmeter and 100Ω resistance

$$= \frac{2.4}{100} \times 50 = 1.2 \text{ V}$$

Since the voltmeter and 100Ω resistance are in parallel, the voltmeter reads the same value i.e., 1.2 V.

20. (c) In space charge limited region, the plate current is given by Child's law $i_p = KV_p^{3/2}$

Thus,

$$\frac{i_{p2}}{i_{p1}} = \left(\frac{V_{p2}}{V_{p1}}\right)^{3/2} = \left(\frac{600}{150}\right)^{3/2} = (4)^{3/2} = 8$$

or, $i_{p2} = i_{p1} \times 8 = 10 \times 8 \text{ mA} = 80 \text{ mA}$

21. (c) Here, $E = \frac{hc}{2} - W_0$ and $2E = \frac{hc}{\lambda'} - W_0$

$$\Rightarrow \frac{\lambda'}{\lambda} = \frac{E + W}{2E + W_0} \Rightarrow \lambda' = \lambda\left(\frac{1 + W_0/E}{2 + W_0/E}\right)$$

Since $\frac{(1+W_0/E)}{(2+W_0/E)} > \frac{1}{2}$ So $\lambda' > \frac{\lambda}{2}$

22. (b) According to Einstein's photoelectric equation,

$$E = W_0 + \frac{1}{2}mv_{max}^2 \Rightarrow v_{max} = \sqrt{\frac{2(hf - W_0)}{m}}$$

If frequency becomes 4f then

$$v' = \sqrt{\frac{2(h \times 4f - W_0)}{m}} = 2\sqrt{\frac{2\left(hf - \frac{W_0}{4}\right)}{m}}$$

$\Rightarrow v' > 2v$

23. (b) In electric field photoelectron will experience force and accelerate opposite to the field so its KE increases (i.e., stopping potential will increase), no change in photoelectric current, and threshold wavelength.

24. (b) Magnetic field inside the cyclindrical conductor $B_{in} = \frac{\mu_0}{4\pi}\frac{2ir}{R^2}$

(R = radius of cylinder and r = distance of observation point from axis of cylinder)

Magnetic field outside the cylinder at a

distance r′ from its axis, $B_{out} = \frac{\mu_0}{4\pi}\frac{2i}{r'}$

$$\Rightarrow \frac{B_{in}}{B_{out}} = \frac{rr'}{R^2} \Rightarrow \frac{10}{B_{out}} = \frac{\left(R - \frac{R}{4}\right)(R + 4R)}{R^2}$$

$$\Rightarrow B_{out} = \frac{8}{3}T$$

25. (a) By using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow 5 = (1.5 - 1)\left(\frac{2}{R}\right)$$

If a lens of refractive index μ_g is immersed in a liquid of refractive index μ_1, then its focal length in liquid

$$\frac{1}{f_l} = \left({}_{\ell}\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow -1 = \left(\frac{1.5}{n} - 1\right)\left(\frac{2}{R}\right)$$

Dividing, (i) by (ii) $-5 = \frac{0.5n}{1.5-n}$

$$\Rightarrow -7.5 + 5n = 0.5n \Rightarrow -7.5 = -4.5n$$

$$\Rightarrow n = \frac{75}{45} = \frac{5}{3}$$

26. (a) Magnetic field

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt} = \frac{1}{9 \times 10^{16} \times 2} \times 10^{10}$$
$$= 5.56 \times 10^{-8}\ \text{T}$$

27. (b) $E = E_0 \sin \omega t$

Voltmeter read rms value

$$\therefore E_0 = \sqrt{2} \times 234\ \text{V} = 331\ \text{V}$$

and $\omega t = 2\pi n t = 2\pi \times 50 \times t = 100\pi t$

Thus, the equation of the line voltage $E = 331\sin(100\pi t)$

28. (a) Power, $P = \frac{V^2}{R}, \Rightarrow R = \frac{V^2}{P}$

For the first bulb,

$$R_1 = \left(\frac{V^2}{P_1}\right) = \left(\frac{(220)^2}{25}\right) = 1936\Omega$$

For the second bulb,

$$R_2 = \left(\frac{V^2}{P_2}\right) = \left(\frac{(220)^2}{100}\right) = 484\Omega$$

Current in series combination is the same in the two bulbs,

$$i = \frac{V}{R_1 + R_2} = \frac{220}{1936 + 484} = \frac{220}{2420} = \frac{1}{11}\ \text{A}$$

If the actual powers in the two bulbs be P_1 and P_2 then

$P_1' = i^2R_1 = \left(\frac{1}{11}\right)^2 \times 1936 = 16\text{ W}$

and $P_2' = i^2R_2 = \left(\frac{1}{11}\right)^2 \times 484 = 4\text{ W}$

Since $P_1' > P'_2$, so, 25 W bulb will glow more brightly.

29. (a) Given : $\lambda = 100\text{ nm} = 1000\text{Å}$

Energy corresponding to 1000Å

$= \frac{12375}{1000} = 12.375\text{eV}$

Now, $7.7 = 12.375 - \phi_0$

or $\phi_0 = 12.375 - 7.7 = 4.675\text{eV}$

In the second case,

Energy corresponding to 2000Å

$= \frac{12375}{2000}\text{eV} = 6.1875\text{eV}$

Now, $4.7 = 6.1875 - \phi_0'$

or $\phi_0' = 6.1875 - 4.7 = 1.4875 \simeq 1.5\text{ V}$

30. (c) As we know, $\frac{1}{2}mv^2 = qV$ or $v = \sqrt{\frac{2qV}{m}}$

Centripetal force $\frac{mv^2}{R} = q \times B \times v$

$\therefore v = \frac{qBR}{m}$

Hence, $\sqrt{\frac{2qV}{m}} = \frac{qBR}{m}$ or $R = \left(\frac{2mV}{q}\right)^{1/2} \times \frac{1}{B}$

Here, V, q and B are constants.

$\therefore R \propto m$

And, $\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

31. (a) According to Bohr's theory of hydrogen atom,

(i) The speed of the electron in the nth orbit $V_n = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0(h/2\pi)}$ or $v_n \propto \frac{1}{n}$

(ii) The energy of the electron in the nth orbit $E_n = \frac{me^4}{8n^2\varepsilon_0^2h^2} = \frac{-13.6}{n^2}\text{eV}$ or $E_n \propto \frac{1}{n^2}$

(iii) The radius of the electron in the nth orbit

$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2} = n^2a_0$ or $r_n \propto n^2$

where $a_0 = \frac{h^2\varepsilon_0}{\pi me} = 5.29 \times 10^{-11}$ m

32. **(b)** Current, $i = qv$

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 qv}{2r}$$
$$= \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 6.6 \times 10^{15}}{2 \times 0.53 \times 10^{-10}}$$
$$= \frac{2\pi \times 1.6 \times 6.6}{5.3} = 12.513 \text{ T}$$

33. **(a)** For interference phase difference must be constant.

34. **(a)** To remove the error, resistance box and the unknown resistance must be interchanged and then the mean reading must be taken.

35. **(c)** Here, $\tan i_c = \frac{r}{h}$ or $r = h\tan i_c$

or $r = h\frac{\sin i_c}{\cos i_c}$ or $r = h\frac{\sin i_c}{\sqrt{1-\sin^2 i_c}}$

But $\sin i_c = \frac{1}{\mu}$

$$\therefore r = h\frac{\frac{1}{\mu}}{\sqrt{1-\frac{1}{\mu^2}}} = \frac{h}{\sqrt{\mu^2-1}} = \frac{12}{\sqrt{\frac{16}{9}-1}}$$

$$= \frac{12 \times 3}{\sqrt{7}} \text{ cm}$$

36. **(b)** Diffraction takes places when the wavelength of waves is comparable with the size of the obstacle in path.

$P_{radio} > P_{light}$

Hence, radio waves are diffracted around building.

37. **(c)** Centripetal force = force of attraction of nucleus on electron.

$$\frac{mv^2}{a_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{a_0^2} \Rightarrow v = \frac{e}{\sqrt{4\pi\varepsilon_0 m a_0}}$$

38. **(d)** As we know, conductivity

$$\sigma = ne(\mu_e + \mu_h)$$
$$= 2 \times 10^{19} \times 1.6 \times 10^{-19}(0.36 + 0.14)$$
$$= 1.6(\Omega\text{m})^{-1}$$

$$R = \rho\frac{l}{A} = \frac{l}{\sigma A} = \frac{0.5 \times 10^{-3}}{1.6 \times 10^{-4}} = \frac{25}{8}\Omega$$

$$i = \frac{V}{R} = \frac{2}{25/8} = 0.64 \text{ A}$$

39. **(b)** Total power $P_t = P_c\left[1 + \frac{ma^2}{2}\right] \because ma^2 = 1$

$$\therefore 1800 = P_c\left[1 + \frac{1}{2}\right] \Rightarrow P_c = 1200 \text{ W}$$

40. (b) Optical path for ray 1 $= n_1 l_1$ Optical path for ray 2 $= n_2 l_2$ Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(n_1 l_1 - n_2 l_2)$$

PART-II CHEMISTRY

41. (d) The correct formula of the given complex is tetraammine aqua chlorocobalt (III) chloride is $[CoCl(H_2O)(NH_3)_4]Cl_2$, because in it the oxidation number of Co is +3 . While in rest other options O. No. of Co is +2

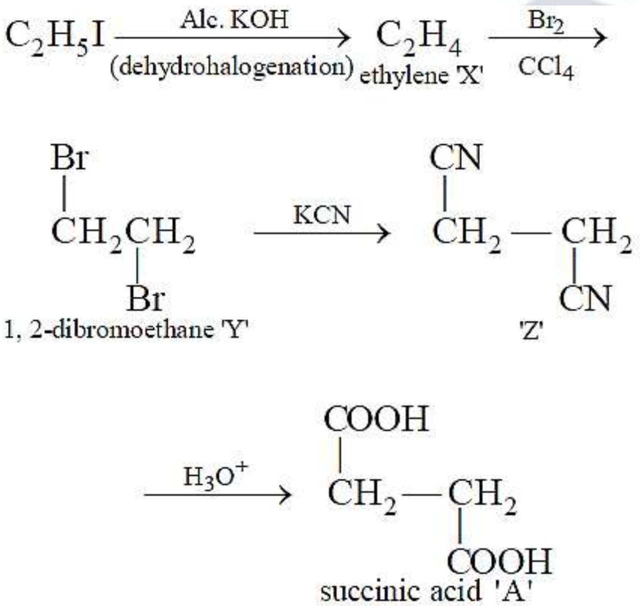
$[CoCl(H_2O)(NH_3)_4]Cl$

$\Rightarrow x + (-1) + 0 + (0 \times 4) + (-1)2 = 0$

$\Rightarrow x - 3 = 0 \Rightarrow x = +3$

42. (c) According to Kohlrausch's law, equivalent conductance at infinite dilution of HF, $\Lambda^\circ_{HF} = \Lambda^\circ_{NaF} + \Lambda^\circ{}_{HCl} - \Lambda^\circ{}_{NaCl}$

43. (a)



44. (c) Let the initial rate be R

and order with respect to A be x and B be y. Thus, rate law can be written as, rate, $R = [A]^x[B]^y$ ...(i)

After doubling the concentration of A, rate becomes 4R,
$4R = [2A]^x[B]^y$

After doubling the concentration of B, rate remains R,

$R = [A]^x[2B]^y$

From Eq. (i) and (ii), we get

$$\frac{R}{4R} = \left(\frac{1}{2}\right)^x \Rightarrow \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^x$$

So, $x = 2$

From Eq. (i) and (iii), we get

$\frac{R}{R} = \left[\frac{1}{2}\right]^y \Rightarrow \left(\frac{1}{1}\right)^o = \left(\frac{1}{2}\right)^y$

So, $Y = 0$

Hence, the rate law is, rate $R = [A]^2[B]^0$

This clearly shows that the order of this reaction is 2 and for second order reaction units of rate constant are $mol^{-1}Ls^{-1}$.

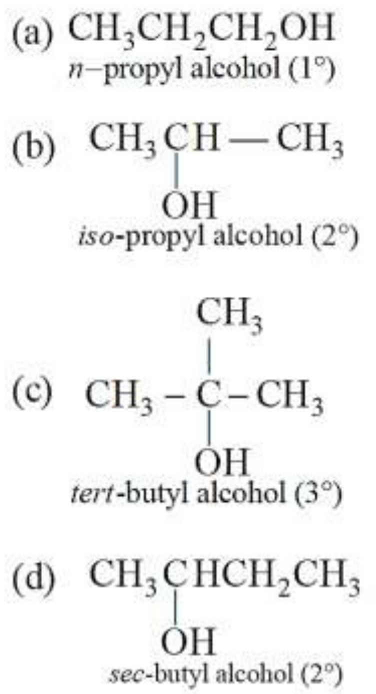
45. (c) As we know that for every 10° rise in temperature, rate constant, k becomes doubled. Hence, on rising the temperature 20°, the rate constant will be four times,

i.e., $k_1 = 4k_2 \Rightarrow k_2 = \frac{1}{4}k_1 = 0.25k_1$

46. (c) The other name of methanol is carbinol. So, the formula of ethyl carbinol is

$\underbrace{CH_3CH_2}_{\text{ethyl}}\underbrace{CH_2OH}_{\text{carbinol}}$

47. (a) In Victor Meyer's test, Red colour is given by primary alcohols (1°) (alcohols havingCH_2OH). The structres of the given Alcohols are



Hence, n-propyl alcohol is a 1° alcohol and gives red colour in Victor-Meyer's test.

48. (b) Heat of formation is equal to enthalpy of a compound.

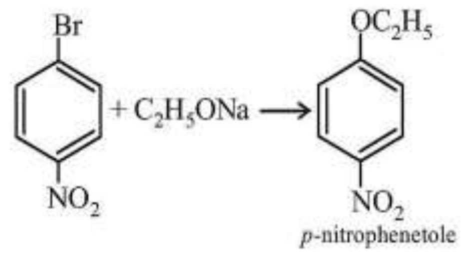
49. (c) ΔS (entropy change) is the measure of randomness and thus in solid, liquid and gas, the order of entropy is

gas > liquid > solid

Thus, ΔS is positive for the reaction given in option (c) because solid $CaCO_3$ is forming gaseous CO_2.

50. (b) We know that basicity is depend on ionic character. So, as the ionic size of lanthanide decreases, the covalent character of their hydroxide increases. Hence, their basicity decreases.

51. (c) Commonly aryl halides do not take part in Williamson's synthesis, due to their high stability but due to the presence of strong electron withdrawing group like $-NO_2$ makes the $C-X$ bond weaker and substitution of $-Br$ takes place by $-OR$.



52. (a) The complete nuclear reaction is

$$_{Z}X^{A} \longrightarrow 76Y^{235} + \underset{\alpha-\text{ particle}}{3\ {}_{2}He^{4}} + \underset{\beta-\text{ particle}}{{}_{-1}e^{0}} + \underset{\gamma-\text{ particle}}{\gamma}$$

On observing the reaction, mass number of 'X' is

$A = 235 + 12 + 0 = 247$

On observing the reaction, atomic number of 'X' is

$Z = 76 + 6 - 1 = 81$

Hence, element 'X' is ${}_{81}X^{247}$.

53. (b) For the given reaction,

$2A(g) + B_2(g) \rightleftharpoons 2AB_2(g)$

the equilibrium constant,

$$K_p = \frac{p_{AB_2}^2}{p_A^2 \cdot p_{B_2}} = 16$$

For the other given reaction,

$$AB_2(g) \rightleftharpoons A(g) + \frac{1}{2}B_2(g)$$

The equilibrium constant,

$$K'_p = \frac{p_A \cdot p_{B_2}^{1/2}}{p_{AB_2}}$$

On squaring Eq. (ii), we obtain,

$$(K'_p)^2 = \frac{p_A^2 \cdot p_{B_2}}{p_{AB_2}^2}$$

Now, from Eq. (i) and (iii), we obtain,

$$K_p \cdot (K'_p)^2 = 1 \Rightarrow 16.(K'_p)^2 = 1$$

$$(\because K_p = 16.0)$$

$$\Rightarrow (K'_p) = \left(\frac{1}{16}\right)^{1/2} \Rightarrow K'_p = \frac{1}{4} = 0.25$$

54. (d) When the size of cation is much smaller than the anion then frenkel defect is observed. Hence, AgBr, AgI and ZnS all exhibit Frenkel defect.

55. (d) Crystals show good cleavage when, the constitutents are arranged in orderly pattern, i.e., in planes,

56. (d) On ionisation they give different ions

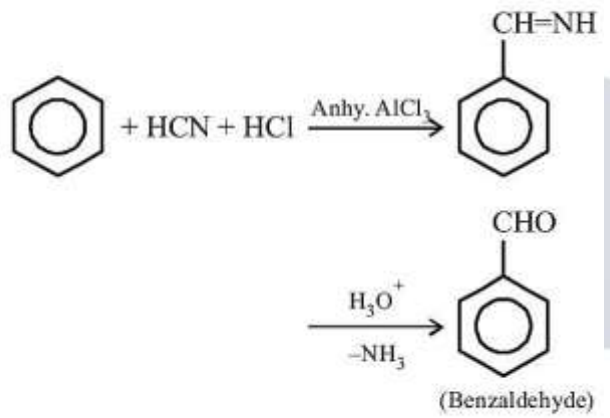
$$[Co(NH_3)_4Cl_2]NO_2 \longrightarrow [Co(NH_3)_4Cl_2]^+ + NO_2^-$$

$$[Co(NH_3)_4ClNO_2]Cl \longrightarrow [Co(NH_3)_4ClNO_2]^+ + Cl^-$$

So, they show ionisation isomerism.

57. (c) We know that complex compound having no unpaired electron is colourless. Among the given complexes, $K_4[Fe(CN)_6]$ has no unpaired electron as $CN^-$is a strong field ligand and causes pairing of electrons. So, it is colourless.

58. (c)

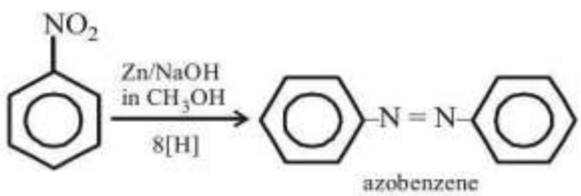


This is Gattermann aldehyde synthesis.

59. (a) Aldol is β-hydroxybutyraldehyde (or 3-hydroxybutanal) i.e.

$$CH_3-\underset{\substack{|\\OH}}{CH}-CH_2CHO$$

60. (b) Nitrobenzene can be converted into azobenzene, on reduction in the presence of Zn/NaOH in CH_3OH.



61. (c) Due to the presence of electron withdrawing group like Ph group decreases the electron density of nitrogen and hence, the lone pair of nitrogen are not available for donation.

So, $(C_6H_5)_3$ N is least basic due to the presence of three electron withdrawing Ph(C_6H_5) groups.

62. (b) Coordination number of Ni in $[Ni(C_2O_4)_3]^{4-}$ is 6 because $C_2O_4^{2-}$ (oxalate) is a bidentate ligand and each have two sites to coordinated with the central atom.

63. (b) Chlorophyll is rich source of Mg, the green pigment of plants.

64. (c) An alloy of 80%Ag and 20% other metals, usually copper is sterling silver.

65. (b) As$CuSO_4$ reacts with KI to give white precipitate of Cu_2I_2 and to evolve $I_2 \cdot CuSO_4$ does not react with KCl.

66. (b) Due to highest reduction potential of fluorine. Transition metals exhibit highest oxidation states in their fluoride.

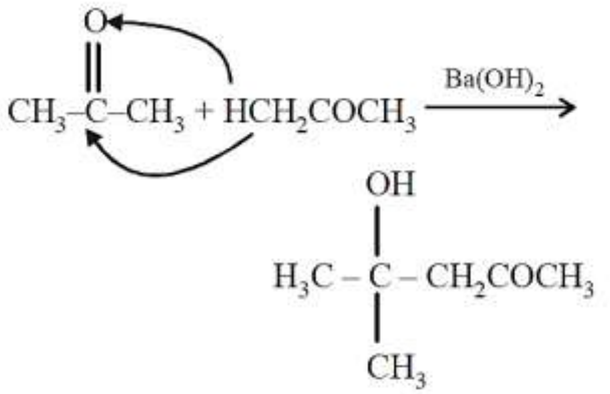
67. (d)

$$\begin{array}{llll} Cu^{2+} & +2e^{-} & \longrightarrow & Cu \\ 1\text{ mol} & 2F & & 1\text{ mol} \\ \frac{1}{2}\text{ mol} & 1F & & \frac{1}{2}\text{ mol} \\ 1\text{ gequi.} & 1F & & 1\text{g equi.} \end{array}$$

Thus, to reduce 4 g equivalent of Cu^{2+} into Cu4 F are required.

68. (c) Fuel cell, which convert chemical energy of fuels like H_2, O_2, CH_4, etc, is converted into electric energy, e.g., $H_2 - O_2$ fuel cell.

69. (d) In Aldol condensation propanone gives diacetone alcohol in presence of $Ba(OH)_2$



4-hydroxy-4-methyl pentanone-2
(diacetone alcohol)

70. (b) $CH_3CONH_2 \xrightarrow{NaOBr} CH_3NH_2 + Na_2CO_3$

71. (b) Radiographer to protect themself from radiation worn lead apron

72. (d) ΔG° and K_p are related as

$$\Delta G^\circ = -RT\ln K_p$$

$$\Rightarrow \ln K_p = \frac{\Delta G^\circ}{RT} \Rightarrow K_p = e^{-\Delta G^\circ/RT}$$

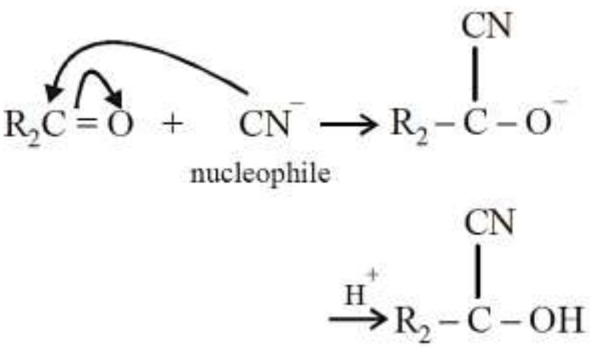
73. (c) $A_2(g) + 2B(g) \rightleftharpoons C(g) + QkJ$

Since, the reaction is exothermic, So, it is favoured by low temperature.

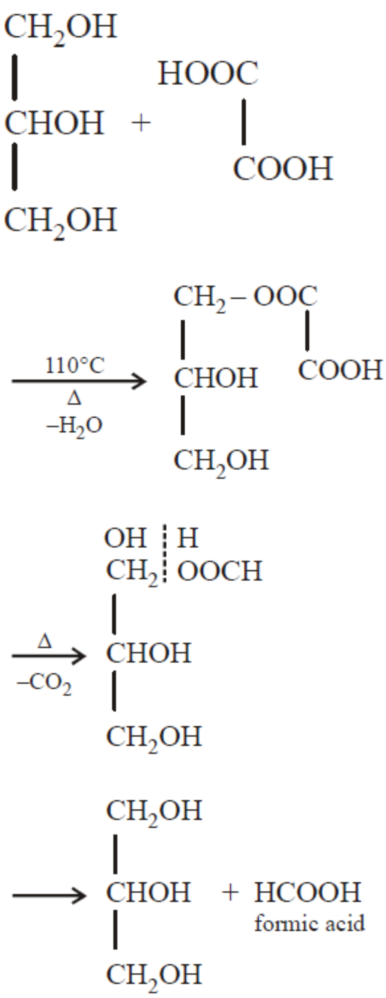
In addition, the number of moles of products is lesser than the number of moles of reactants, thus high pressure favors the forward reaction.

74. (a) Larger the value of K more the reaction moves towards completion.

75. (b) As CN is a nucleophile. So it is an example of nucleophilic addition



76. (a)



77. (d) As we know that,

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{6000} = 1.155 \times 10^{-4}$$

$$t = \frac{2.303}{k} \log \frac{N_0}{N} = \frac{2.303}{1.155 \times 10^{-4}} \log \frac{100}{25}$$
$$= 12000 \text{ yr (age of piece of wood)}$$

78. (b) $\frac{\text{Radius of cation, } r^+}{\text{Radius of anion, } r^-} = \frac{95}{181} = 0.525$

As this value lies in between $0.414 - 0.732$, thus, the coordination number of Na^+ion will be 6 .

79. (a) $ROH + H_2CN_2 \xrightarrow{HBF_4} ROCH_3 + N_2$

80. (d) The compounds having - CHO group reduces Tollen's reagent, Fehling solution etc. Thus, formic acid (HCO/OH) has reducing property.

PART-III MATHEMATICS

81. (d) $F(x+2)=2F(x)-F(x+1)$

Putting $x=0$, we get

$$F(2)=2F(0)-F(1)$$
$$\Rightarrow F(2)=2(2)-3$$

$\{\because F(0)=2, F(1)=3\}$

$\Rightarrow F(2)=4-3 \Rightarrow F(2)=1$

Putting $x=1$, in eq. (i), we get

$$F(3)=2F(1)-F(2)$$
$$=2(3)-1\ \{\because F(1)=3, F(2)=1\}$$
$$\Rightarrow F(3)=5$$

Putting $x=2$, in eq. (i), we get

$$F(4)=2F(2)-F(3)$$
$$=2(1)-5\ \{\because F(2)=1, F(3)=5\}$$
$$\Rightarrow F(4)=-3$$

Putting $x=3$, in eq. (i), we get

$$F(5)=2F(3)-F(4)$$
$$=2(5)+3\ \{\because F(3)=5, F(4)=-3\}$$
$$\Rightarrow F(5)=13$$

82. (c) The number of binary operations on a set S having n elements in n^{n^2}.

83. (a) $(3-5x)^{11}=3^{11}\left(1-\frac{5x}{3}\right)^{11}$

$$=3^{11}\left(1-\frac{5}{3}\cdot\frac{1}{5}\right)^{11}\quad\left\{\because x=\frac{1}{5}\right\}$$
$$=3^{11}\left(1-\frac{1}{3}\right)^{11}$$

Now, $r=\frac{|x|(n+1)}{|x|+1}=\frac{\left|-\frac{1}{3}\right|(11+1)}{\left|-\frac{1}{3}\right|+1}=\frac{4}{\frac{4}{3}}$

$\Rightarrow r=3$

Therefore, $3\text{rd}(T_3)$ and $(3+1)=4$ th (T_4) terms are numerically greatest in the expansion of $(3-5x)^{11}$.

So, greatest term $=T_3$

$$=3^{11}\left|{}^{11}C_2(1)^9\left(-\frac{1}{3}\right)^2\right|=3^{11}\left|\frac{11\times10}{1.2.9}\right|$$
$$=55\times3^9$$

and $T_4 = 3^{11}\left|{}^{11}C_3(1)^8\left(-\frac{1}{3}\right)^3\right|$

$= 3^{11}\left|\frac{11\times10\times9}{1.2.3}\cdot\left(-\frac{1}{27}\right)\right| = 55\times3^9$

$\therefore$ Greatest term (numerically)

$= T_3 = T_4 = 55\times3^9$

84. (a) We have, $\sin(e^x) = 5^x + 5^{-x}$

Let $5^x = t$, then eq. (i), reduces to

$\sin(e^x) = t + \frac{1}{t}$

$\Rightarrow \sin(e^x) = t + \frac{1}{t} - 2 + 2$

$\Rightarrow \sin(e^x) = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 + 2$

$\{\because 5^x > 0, \therefore \sqrt{5^x} = \sqrt{t}$ exists $\}$

$\Rightarrow \sin(e^x) \geq 2$

which is not possible as also $\sin\theta \leq 1$. Thus, given equation has no solution.

85. (c) $a^x = b^y = c^z = d^u$

Let, $a^x = b^y = c^z = d^u = k$

$\Rightarrow a = k^{1/x}, b = k^{1/y}, c = k^{1/z}, d = k^{1/u}$

a, b, c, d are in GP.

$\therefore \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\Rightarrow \frac{k^{1/y}}{k^{1/x}} = \frac{k^{1/z}}{k^{1/y}} = \frac{k^{1/u}}{k^{1/z}}$ {using Eq. (i) }

$\Rightarrow k^{\frac{1}{y}-\frac{1}{x}} = k^{\frac{1}{z}-\frac{1}{y}} = k^{\frac{1}{u}-\frac{1}{z}}$

$\Rightarrow \frac{1}{y} - \frac{1}{x} = \frac{1}{z} - \frac{1}{y} = \frac{1}{u} - \frac{1}{z}$

$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{u}$ are in A.P.

$\Rightarrow x, y, z, u$ are in H.P.

86. (b) Given: $|z| - z = 1 + 2i$

If $z = x + iy$, then this equation reduces to

$|x + iy| - (x + iy) = 1 + 2i$

$\Rightarrow \left(\sqrt{x^2+y^2}-x\right)+(-iy)=1+2i$

On comparing real and imaginary parts of both sides of this equation, we get

$\sqrt{x^2+y^2}-x=1$

$\Rightarrow \sqrt{x^2+y^2}=1+x \Rightarrow x^2+y^2=(1+x)^2$

$\Rightarrow x^2+y^2=1+x^2+2x$

$\Rightarrow y^2=1+2x$

and $-y=2$

$\Rightarrow y=-2$

Putting this value in eq. (i), we get

$(-2)^2=1+2x$

$\Rightarrow 2x=3 \Rightarrow x=\frac{3}{2}$

$\therefore z=x+iy=\frac{3}{2}-2i$

87. **(c)** $z=\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)$

$\Rightarrow z=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$\Rightarrow z=\frac{1-3-2\sqrt{3}i}{1+3}=\frac{-2-2\sqrt{3}i}{4}$

$\Rightarrow z=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$\Rightarrow z=\cos 240^\circ - i\sin 240^\circ$

Thus, $\arg(z)=240^\circ$

88. **(d)** $f(x)=\sqrt{\log_{10} x^2}$ is real, if

$\log_{10} x^2 \geq 0$

$\Rightarrow x^2 \geq 1$

$\Rightarrow x<-1$ and $x>1$

$\Rightarrow x \in (-\infty,-1] \cup [1,\infty)$

89. **(c)** The given expression is

$2x^2+mxy+3y^2-5y-2$

Comparing the given expression with $ax^2+2hxy+by^2+2gx+2fy+c$, we get $a=2, h=\frac{m}{2}, b=3, c=-2, g=0, f=-\frac{5}{2}$

The given expression is resolvable into linear factors, if

$abc+2fgh-af^2-bg^2-ch^2=0$

$(2)(3)(-2)+2(0)-2\left(\frac{25}{4}\right)-0-(-2)\frac{m^2}{4}=0$

$\Rightarrow -12-\frac{25}{2}+\frac{m^2}{2}=0$

$\Rightarrow \frac{m^2}{2}=\frac{49}{2} \Rightarrow m^2=49 \Rightarrow m=\pm 7$

90. (c) $\det(B^{-1}AB)=\det(B^{-1})\det A \det B$

$=\det(B^{-1})\cdot\det B\cdot\det A$

$=\det(B^{-1}B)\det A=\det I\cdot\det A$

$=1.\det A=\det A$

91. (c) Since $f(x), g(x)$ and $h(x)$ are the polynomials of degree 2 ,

therefore $f'''(x)=g'''(x)=h'''(x)=0$

Now, $\Delta(x)=\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

$\Rightarrow \Delta'(x)=\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

$+\begin{vmatrix} f(x) & g(x) & h(x) \\ f''(x) & g''(x) & h''(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

$+\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f'''(x) & g'''(x) & h'''(x) \end{vmatrix}$

$\Rightarrow \Delta'(x)=0+0+0=0$

$\Rightarrow \Delta(x)=$ constant

Thus, $\Delta(x)$ is the polynomial of degree zero.

92. (d) Let E_1, E_2 and E_3 denote the events of selecting boxes A, B, C respectively and A be the event that a screw selected at random is defective. Then,

$P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{1}{3}$

$P\left(\frac{A}{E_1}\right)=\frac{1}{5}, P\left(\frac{A}{E_2}\right)=\frac{1}{6}, P\left(\frac{A}{E_3}\right)=\frac{1}{7}$

By Baye's rule, the required probability

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$\Rightarrow P\left(\frac{E_1}{A}\right)=\frac{\frac{1}{3}\cdot\frac{1}{5}}{\frac{1}{3}\cdot\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{6}+\frac{1}{3}\cdot\frac{1}{7}}$$

$$=\frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{6}+\frac{1}{7}}=\frac{42}{107}$$

93. (c)

$$\frac{\cos\theta}{1+\sin\theta}=\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{1+\cos\left(\frac{\pi}{2}-\theta\right)}$$

$$=\frac{2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}$$

$$=\frac{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}=\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

94. (b) $3\sin\theta+5\cos\theta=5\Rightarrow 3\sin\theta=5(1-\cos\theta)$

$$\Rightarrow 3.2\sin\frac{\theta}{2}\cos\frac{\theta}{2}=5.2\sin^2\frac{\theta}{2}$$

$$\left(\begin{array}{l}\because \sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\\ \text{and } 1-\cos\theta=2\sin^2\frac{\theta}{2}\end{array}\right)$$

$$\Rightarrow \tan\frac{\theta}{2}=\frac{3}{5}$$

Now, $5\sin\theta-3\cos\theta$

$$=5\cdot\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}-3\cdot\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$=5\cdot\frac{2\cdot\frac{3}{5}}{\left(1+\frac{9}{25}\right)}-3\cdot\frac{\left(1-\frac{9}{25}\right)}{\left(1+\frac{9}{25}\right)}$$

$$=\frac{6-3\cdot\frac{16}{25}}{1+\frac{9}{25}}=\frac{150-48}{34}=\frac{102}{34}=3.$$

95. (a) $\sin^{-1}\left\{\sin\frac{5\pi}{6}\right\}=\sin^{-1}\left\{\sin\left(\pi-\frac{\pi}{6}\right)\right\}$

$[\because \sin(\pi-\theta)=\sin\theta]$

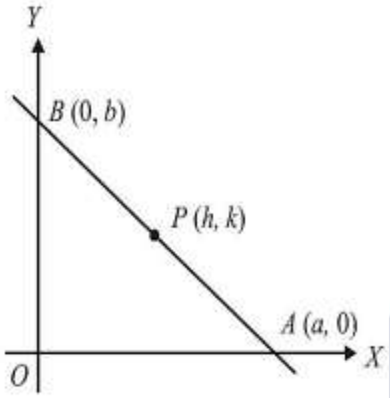
$= \sin^{-1}\left\{\sin\frac{\pi}{6}\right\}$

[$\because$ Principal value $\in [0, \pi/2]$]

$= \frac{\pi}{6}$

which is the required principal value.

96. **(a)** Let both of the ends of the rod are on x-axis and y-axis. Let AB be rod of length l and coordinates of A and B be $(a, 0)$ and $(0, b)$, respectively.



Let $P(h, k)$ be the mid point of the rod AB.

Then, $h = \frac{0+a}{2} = \frac{a}{2}$

$\left. k = \frac{b+0}{2} = \frac{b}{2} \right\}$

In$\triangle$ OAB,

$OA^2 + OB^2 = AB^2$

$\Rightarrow a^2 + b^2 = l^2$

$\Rightarrow (2h)^2 + (2k)^2 = l^2$ [using eq. (i)]

$\Rightarrow h^2 + k^2 = \frac{l^2}{4}$

$\therefore$ The equation of locus is

$x^2 + y^2 = \frac{l^2}{4}$

97. **(a)** Let $L_1 \equiv 2x + y - 1 = 0$

$L_2 \equiv 3x + 2y - 5 = 0$

The equation of straight line passing through the intersection point of the lines L_1 and L_2 is given by

$L_1 + \lambda L_2 = 0$

$\Rightarrow (2x + y - 1) + \lambda(3x + 2y - 5) = 0$

Since, this line passes through the origin also

$(0+0-1)+\lambda(0+0-5)=0$

$\Rightarrow -5\lambda = 1 \Rightarrow \lambda = -\frac{1}{5}$

Required line is

$(2x+y-1)-\frac{1}{5}(3x+2y-5)=0$

$\Rightarrow \left(2-\frac{3}{5}\right)x+\left(1-\frac{2}{5}\right)y-1+1=0$

$\Rightarrow \frac{7}{5}x+\frac{3}{5}y=0 \Rightarrow 7x+3y=0$

98. (c) Let coordinates of P be (h, k), then

$h=\frac{2(10\cos\theta)+3(5)}{2+3}=4\cos\theta+3$

and $k=\frac{2(10\sin\theta)+3(0)}{2+3}=4\sin\theta$

[Using the internal section formula]

$\Rightarrow \frac{h-3}{4}=\cos\theta$ and $\frac{k}{4}=\sin\theta$

Squaring and adding both of these equations,

$\frac{(h-3)^2}{16}+\frac{k^2}{16}=\cos^2\theta+\sin^2\theta$

$\Rightarrow (h-3)^2+k^2=16$

Therefore, locus of point P is

$(x-3)^2+y^2=16$ which is a circle.

99. (c) The equation of any normal to the parabola

$y^2=-8x$ is $y=mx+4m+2m^3$

(using equation of normal of parabola in slope form $y=mx-2am-am^3$ and $a=-2$) The given normal is

$2x+y+k=0 \Rightarrow y=-2x-k$

Comparing eqs. (i) and (ii), we get

$m=-2$ and $-4m-2m^3=k$

$\Rightarrow k=8+16=24$

100. (a) $\lim_{n\to\infty}\left[\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots..+\frac{1}{n(n+1)}\right]$

$=\lim_{n\to\infty}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots....\right.$

$$+\left(\frac{1}{n}-\frac{1}{n+1}\right)\Big]$$

$$=\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)=\lim_{n\to\infty}\frac{n}{n+1}$$

$$=\lim_{n\to\infty}\frac{n}{n\left(1+\frac{1}{n}\right)}=\lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)}=1$$

101. **(d)** Let $P(x_1, y_1)$ be a point on the curve

$y^2 = 4ax$

On differentiating $y^2 = 4ax$ w.r.t. ' x ', we get

$$2y\frac{dy}{dx}=4a$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)}=\frac{2a}{y_1}$$

Thus, the equation of normal at (x_1, y_1) is

$$y-y_1=-\frac{y_1}{2a}(x-x_1)$$

$$\Rightarrow y_1x+2ay=y_1(x_1+2a)$$

But $lx + my = 1$

is also a normal.

Therefore, coefficients of eqs. (ii) and (iii), must be proportional.

i.e., $\frac{y_1}{l}=\frac{2a}{m}=\frac{y_1(x_1+2a)}{1}$

$\Rightarrow y_1=\frac{2al}{m}$ and $x_1=\frac{1}{l}-2a$

Putting these values of x_1 and y_1 in eq. (i), we get

$$\left(\frac{2al}{m}\right)^2=4a\left(\frac{1}{l}-2a\right)$$

$$\Rightarrow \frac{4a^2l^2}{m^2}=\frac{4a-8a^2l}{l}$$

$$\Rightarrow al^3=m^2-2am^2l \Rightarrow al^3+2alm^2=m^2$$

102. **(a)** $\int f(x)dx = f(x)$

$$\Rightarrow \frac{d}{dx}f(x)=f(x)$$

$$\left[\because f(x)=\int \frac{d}{dx}f(x)dx\right]$$

Now, $\int \{f(x)\}^2 dx = \int f(x) \cdot f(x) dx$

$= f(x)\int f(x)dx - \int \left[\frac{d}{dx} f(x) \int f(x)dx\right] dx$

(integrating by parts)

$= f(x)f(x) - \int f(x)f(x)dx$

$\Rightarrow 2\int \{f(x)\}^2 dx = \{f(x)\}^2$

$\Rightarrow \int \{f(x)\}^2 dx = \frac{1}{2}\{f(x)\}^2$

103. **(a)** Let $I = \int \sin^{-1}\left\{\frac{2x+2}{\sqrt{4x^2+8x+13}}\right\} dx$

$\Rightarrow I = \int \sin^{-1}\left\{\frac{2x+2}{\sqrt{4x^2+8x+4+9}}\right\} dx$

$\Rightarrow I = \int \sin^{-1}\left\{\frac{2x+2}{\sqrt{(2x+2)^2+3^2}}\right\} dx$

Substituting $2x + 2 = 3\tan\theta$,

$\Rightarrow 2dx = 3\sec^2\theta d\theta$, we get

$I = \int \sin^{-1}\left\{\frac{3\tan\theta}{3\sec\theta}\right\} \cdot \frac{3}{2}\sec^2\theta d\theta$

$\Rightarrow I = \frac{3}{2}\int \sin^{-1}(\sin\theta) \cdot \sec^2\theta d\theta$

$\Rightarrow I = \frac{3}{2}\int \theta\sec^2\theta d\theta$

$\Rightarrow I = \frac{3}{2}\left[\theta\tan\theta - \int \tan\theta d\theta\right]$

(integrating by parts)

$I = \frac{3}{2}[\theta\tan\theta - \log|\sec\theta|] + c$

$$= \frac{3}{2}\left[\tan^{-1}\left(\frac{2x+2}{3}\right) \cdot \left(\frac{2x+2}{3}\right) - \log\sqrt{1+\left(\frac{2x+2}{3}\right)^2}\right] + c$$

$= (x+1)\tan^{-1}\left(\frac{2x+2}{3}\right)$

$-\frac{3}{4}\log\left(\frac{4x^2+8x+13}{9}\right) + c$

104. **(c)** The equation of ellipse is

$3x^2 + 2y^2 + 6x - 8y + 5 = 0$

$\Rightarrow 3(x^2 + 2x) + 2(y^2 - 4y) + 5 = 0$

$\Rightarrow 3(x^2 + 2x + 1) + 2(y^2 - 4y + 4)$

$+5 - 3 - 8 = 0$

$\Rightarrow 3(x+1)^2 + 2(y-2)^2 = 6$

$\Rightarrow \frac{(x+1)^2}{2} + \frac{(y-2)^2}{3} = 1$

Comparing with

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, we get

$h = -1, k = 2, a^2 = 2, b^2 = 3$

Here, centre $(h, k) = (-1,2)$

And using $a^2 = b^2(1 - e^2)$

$2 = 3(1 - e^2) \Rightarrow e = \frac{1}{\sqrt{3}}$

And foci are $(h, k + be)$ and $(h, k - be)$

$= (-1,2 + 1)$ and $(-1,2 - 1)$

$= (-1,3)$ and $(-1,1)$

105. **(b)** We have the two hyperbolas as

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Any tangent to the hyperbola eq(i)

$y = mx + c$

where $c = \pm\sqrt{a^2m^2 - b^2}$

But this tangent touches the parabola eq. (ii) also

$\therefore \frac{(mx+c)^2}{a^2} - \frac{x^2}{b^2} = 1$

$\Rightarrow b^2(m^2x^2 + c^2 + 2mcx) - a^2x^2 = a^2b^2$

$\Rightarrow (b^2m^2 - a^2)x^2 + 2mcb^2x + b^2(c^2 - a^2) = 0$

For the tangency, it should have equal roots

$(2mcb^2)^2 = 4(b^2m^2 - a^2) \cdot b^2(c^2 - a^2)$

$\Rightarrow 4m^2c^2b^4 = 4b^2(b^2m^2c^2 - b^2m^2a^2 - a^2c^2 + a^4)$

$\Rightarrow \ c^2 = a^2 - b^2m^2$

$\Rightarrow \ a^2m^2 - b^2 = a^2 - b^2m^2$ [using Eq. (iii)]

$\Rightarrow \ (a^2 + b^2)m^2 = a^2 + b^2$

$\Rightarrow \ m^2 = 1 \Rightarrow m = \pm 1$

Hence, the equation of common tangent are

$y = \pm x \pm \sqrt{a^2 - b^2}$

106. **(d)** We have

$f(x) = \log_x \cos x$

$f(x)$ is defined for $\cos x > 0$.

$x > 0, x \neq 1$

$\cos x > 0 \Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

Also, $x > 0, x \neq 1$

$\therefore$ Domain of f is $\left(0, \frac{\pi}{2}\right) - \{1\}$

107. **(b)** $y = \sin^{-1}\left(\frac{x^2}{1+x^2}\right)$

For y to be defined $\left|\frac{x^2}{1+x^2}\right| < 1$

which is true for all $x \in R$.

Now, $y = \sin^{-1}\left(\frac{x^2}{1+x^2}\right)$

$\Rightarrow \frac{x^2}{1 + x^2} = \sin y \Rightarrow x = \sqrt{\frac{\sin y}{1 - \sin y}}$

For the existance of x

$\sin y \geq 0$ and $1 - \sin y > 0$

$\Rightarrow 0 \leq \sin y < 1 \Rightarrow 0 \leq y < \frac{\pi}{2}$

Thus, range of the given function is

$\left[0, \frac{\pi}{2}\right)$.

108. **(c)** $x = \sec\theta - \cos\theta$

$\Rightarrow \frac{dx}{d\theta} = \sec\theta \tan\theta + \sin\theta,$

$y = \sec^n\theta - \cos^n\theta$

$$\Rightarrow \frac{dy}{d\theta} = n\sec^{n-1}\theta\sec\theta\tan\theta + n\cos^{n-1}\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = n\frac{(\sec^n\theta\tan\theta + \cos^{n-1}\theta\sin\theta)}{(\sec\theta\tan\theta + \sin\theta)}$$

$$\Rightarrow \frac{dy}{dx} = n\frac{(\sec^n\theta + \cos^n\theta)\tan\theta}{(\sec\theta + \cos\theta)\tan\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{n(\sec^n\theta + \cos^n\theta)}{(\sec\theta + \cos\theta)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{n^2\{(\sec^n\theta - \cos^n\theta)^2 + 4\}}{(\sec\theta - \cos\theta)^2 + 4}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{n^2(y^2+4)}{x^2+4}$$

$$\Rightarrow (x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

109. **(d)** $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \cdots\ldots\infty}}}}$

$$\Rightarrow y^2 = x + \sqrt{y + \sqrt{x + \sqrt{y + \cdots\ldots.\infty}}}$$

$$\Rightarrow y^2 = x + \sqrt{y+y} \Rightarrow y^2 = x + \sqrt{2y}$$

$$\Rightarrow (y^2 - x)^2 = 2y$$

On differentiating both sides w.r.t. x, we get

$$2(y^2 - x)\left(2y\frac{dy}{dx} - 1\right) = 2\frac{dy}{dx}$$

$$\Rightarrow 2(y^3 - xy)\frac{dy}{dx} - (y^2 - x) = \frac{dy}{dx}$$

$$\Rightarrow (2y^3 - 2xy - 1)\frac{dy}{dx} = y^2 - x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$$

110. **(a)** $\int_1^x \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{6}$

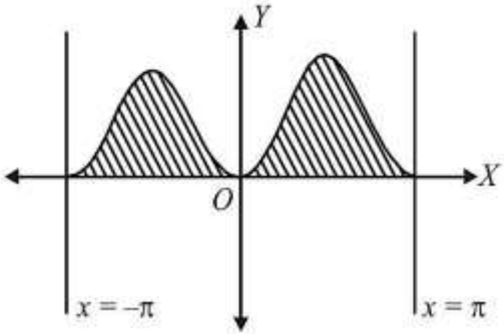
$$\Rightarrow [\sec^{-1} t]_1^x = \frac{\pi}{6}$$

$$\Rightarrow \sec^{-1} x - \sec^{-1} 1 = \frac{\pi}{6}$$

$$\Rightarrow \sec^{-1} x - 0 = \frac{\pi}{6} \Rightarrow x = \sec\frac{\pi}{6}$$

$\Rightarrow x = \frac{2}{\sqrt{3}}$

111. **(c)** Required area = Shaded area



$= \int_{-\pi}^{\pi} |\sin x| dx = 2\int_0^{\pi} |\sin x| dx$

$= -2[\cos x]_0^{\pi} = -2(\cos \pi - \cos 0)$

= 4 sq units

112. **(d)** Length of normal = c

$\Rightarrow y\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = c$

$\Rightarrow y^2\left[1 + \left(\frac{dy}{dx}\right)^2\right] = c^2$

Clearly, this is the differential equation of degree 2

113. **(d)** $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + \hat{j}$

Now,

$\vec{a} + t\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + (-t\hat{i} + 2t\hat{j} + t\hat{k}) = (2 - t)\hat{i} + (2 + 2t)\hat{j} + (3 + t)\hat{k}$

Since, $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$

$(\vec{a} + t\vec{b}) \cdot \vec{c} = 0$

$\Rightarrow \{(2 - t)\hat{i} + (2 + 2t)\hat{j} + (3 + t)\hat{k}\} \cdot (3\hat{i} + \hat{j}) = 0$

$\Rightarrow 3(2 - t) + 2 + 2t = 0$

$\Rightarrow 6 - 3t + 2 + 2t = 0 \Rightarrow t = 8$

114. **(c)** The given line is

$\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$

On comparing it with $\vec{r} = \vec{a} + t\vec{b}$, we get
$\vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 4\hat{k}$

Also, the plane is

$\vec{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 5$

On comparing it with $\vec{r} \cdot \vec{n} = d$, we get

$\vec{n} = \hat{i} + 5\hat{j} + \hat{k}$ and $d = 5$

Since, $\vec{b} \cdot \vec{n} = (\hat{i} - \hat{j} + 4\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k})$

$= 1 - 5 + 4 = 0$

$\therefore$ Given line is parallel to the given plane. Now, distance between the line and the plane is given by required distance

$$= \left|\frac{\vec{a} \cdot \vec{n} - d}{|\vec{n}|}\right|$$

$$= \frac{|(2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k}) - 5|}{\sqrt{1 + 25 + 1}}$$

$$= \frac{|2 - 10 + 3 - 5|}{\sqrt{27}} = \frac{10}{3\sqrt{3}}$$

115. **(a)** The equation of the sphere concentric with the sphere
$x^2 + y^2 + z^2 - 4x - 6y - 8z - 5 = 0$ is

$x^2 + y^2 + z^2 - 4x - 6y - 8z + c = 0$

Since, this sphere eq. (i) passes through origin, therefore

$0 + 0 + 0 - 0 - 0 - 0 + c = 0$

$\Rightarrow c = 0$

Hence, the required equation of sphere is

$x^2 + y^2 + z^2 - 4x - 6y - 8z = 0$

116. **(b)** We have the lines

$$\frac{x - 1}{2} = \frac{y + 1}{3} = \frac{z - 1}{4}$$

and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$

Let a point $(2r + 1, 3r - 1, 4r + 1)$ be on the line Eq. (i). If this is an intersection point of both the lines, then it will lie on Eq. (ii), also

$$\therefore \frac{2r + 1 - 3}{1} = \frac{3r - 1 - k}{2} = \frac{4r + 1}{1}$$

Taking first and third part of eq. (iii), we get $2r - 2 = 4r + 1 \Rightarrow r = -\frac{3}{2}$

Taking second and third part of eq. (iii), we get

$3r - 1 - k = 8r + 2$

$\Rightarrow 3r - 1 - k - 8r - 2 = 0 \Rightarrow k = -5r - 3$

$\Rightarrow k = -5\left(-\frac{3}{2}\right) - 3 \left(\because r = -\frac{3}{2}\right)$

$\Rightarrow k = \frac{15}{2} - 3 \Rightarrow k = \frac{9}{2}$

117. **(a)** Given curves are $y = 3^x$

and $y = 5^x$

intersect at the point $(0,1)$.

Now, differentiating eqs. (i) and (ii) w.r.t. x, we get

$\frac{dy}{dx} = 3^x \log 3$ and $\frac{dy}{dx} = 5^x \log 5$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = \log 3$ and $\left(\frac{dy}{dx}\right)_{(0,1)} = \log 5$

$\Rightarrow m_1 = \log 3$ and $m_2 = \log 5$

Angle between these curves is given by

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$\Rightarrow \tan\theta = \frac{\log 3 - \log 5}{1 + \log 3 \cdot \log 5}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\log 3 - \log 5}{1 + \log 3 \log 5}\right)$

118. **(d)** We have the given equation as

$\lambda x^2 + 4xy + y^2 + \lambda x + 3y + 2 = 0$

On comparing this equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we

get

$a = \lambda, h = 2, b = 1, g = \frac{\lambda}{2}, f = \frac{3}{2}, c = 2$

Since, given equation represents a parabola

$\therefore h^2 = ab \Rightarrow 4 = \lambda \cdot 1 \Rightarrow \lambda = 4$

119. **(c)** The given two circles are

$2x^2 + 2y^2 - 3x + 6y + k = 0$

$\Rightarrow x^2 + y^2 - \frac{3}{2}x + 3y + \frac{k}{2} = 0$

and $x^2 + y^2 - 4x + 10y + 16 = 0$

Since, general equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

Therefore, comparing eqs. (i) and (ii) with eq. (iii), we get

$g_1 = -\frac{3}{4}, f_1 = \frac{3}{2}, c_1 = \frac{k}{2}$

and $g_2 = -2, f_2 = 5, c_2 = 16$

Both the circles cut orthogonally,

$\therefore 2(g_1g_2 + f_1f_2) = c_1 + c_2$

$\Rightarrow 2\left(\frac{3}{2} + \frac{15}{2}\right) = \frac{k}{2} + 16$

$\Rightarrow 18 = \frac{k}{2} + 16 \Rightarrow \frac{k}{2} = 2 \Rightarrow k = 4$

120. **(a)** $A(-2,1), B(2,3)$ and $C(-2,-4)$ are three given points.

Slope of the line BA

$m_1 = \frac{1-3}{-2-2} = \frac{1}{2}$

(Using slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$)

Slope of the line BC

$m_2 = \frac{-4-3}{-2-2} = \frac{7}{4}$

Now, angle between AB and BC is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{\frac{1}{2} - \frac{7}{4}}{1 + \frac{1}{2}\cdot\frac{7}{4}}\right|$$

$$\Rightarrow \tan\theta = \left|-\frac{10}{15}\right| \Rightarrow \tan\theta = \left|-\frac{2}{3}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left|-\frac{2}{3}\right| \Rightarrow \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

[$\because |-x| = x$]