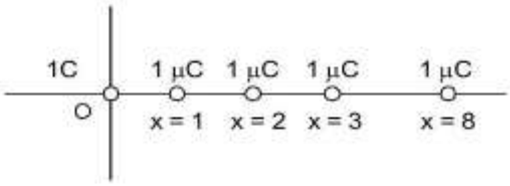
PART- I PHYSICS

1. **(b)** As charge on glass rod is + (ve) so charge on gold leaves will also be + (ve). Due to X-rays more electrons from leaves will be emitted, so leaves becomes more positive and diverge further.
2. **(b)** In figure there is the schematic diagram of distribution of charges on x-axis



Using principle of superposition total force acting on 1C charge

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times1\times10^{-6}}{(1)^2}+\frac{1\times1\times10^{-6}}{(2)^2} + \frac{1\times1\times10^{-6}}{(4)^2}+\frac{1\times1\times10^{-6}}{(8)^2}+\cdots\infty\right]$$

$$=\frac{10^{-6}}{4\pi\varepsilon_0}\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots\infty\right]$$

$$=9\times10^{-6}\times10^{9}\left(\frac{1}{1-\frac{1}{4}}\right)$$

$$=9\times10^{9}\times10^{-6}\times\frac{4}{3}=12000\text{ N}$$

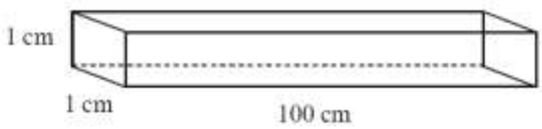
3. **(a)** Here, net flux is zero, as there is no charge residing inside the cube.
4. **(a)** Energy released on discharge of capacitor

$$U=\frac{1}{2}CV^2$$

$$=\frac{1}{2}\times4\times10^{-6}\times(100)^2=0.02\text{ J}$$

5. **(a)** Here length $l = 1\text{ cm} = 10^{-2}\text{ m}$

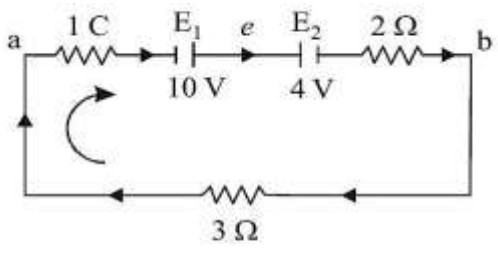
Area of cross-section, $A = 1\text{ cm}\times100\text{ cm}$



Resistance, $R=\rho\frac{\ell}{A}=3\times10^{-7}\times\frac{10^{-2}}{10^{-2}}$

$=3\times10^{-7}\Omega$

6. **(d)** The current in the circuit will be clockwise Since $E_1(10\text{ V}) > E_2(4\text{ V})$



Applying Kirchhoff's junction rule

$-1 \times i - 10 - 4 \times -2 \times i - 3i = 0$

$i = 1$ A

(a to b via e)

Current $I = \frac{V}{R} = \frac{10-4}{6} = 1$ A

7. **(a)** Power $P = \frac{V^2}{R} = \frac{220 \times 220}{100} = 484$ W

8. **(b)** When switched off, resistance of an incandescent lamp increases.

9. **(d)** Using Faraday's law of electrolysis, Mass $m = Zit$

$= 0.216 \times 10^{-3} \times 5 \times 3600$

$= 2.27$ g

10. **(c)** $P = 100$ W and $V = 125$ V

$$P = VI \Rightarrow I = \frac{P}{V} = \frac{100}{125} \text{ A}$$

Mass of chlorine liberated = zit

$$= 0.367 \times 10^{-6} \times \frac{100}{125} \times 60 = 17.6\text{mg}$$

11. **(b)** Current, $i = \frac{q}{t} = 100 \times e$

Magnetic field, $B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r}$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2\pi \times 100e}{r}$$

$$\frac{\mu_0 \times 200 \times 1.6 \times 10^{-19}}{4 \times 0.8} = 10^{-17}\mu_0$$

12. **(a)** Torque, $T = ni(A \times B)$

13. **(a)** Magnetic Flux, $\phi = B \cdot A$

Change in flux, $d\phi = B \cdot dA$

$= 0.05(101 - 100) \times 10^{-4}$

$= 5 \times 10^{-6}$ Wb

Charge $dQ = \frac{d\phi}{R}$

$$= \frac{5 \times 10^{-6}}{2} = 2.5 \times 10^{-6}\text{C}$$

14. **(d)** Induced emf $e = M\frac{di}{dt}$

$$= \frac{\mu_0 N_1 N_2 A}{l} \cdot \frac{di}{dt}$$

$$= \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.30}$$

$$\times \frac{|2-(-2)|}{0.25}$$

$= 48.2 \times 10^{-3}$ V $= 48$mV

15. (b) Here, impedence $z = \sqrt{R^2 + X_L^2}$

$= \sqrt{(10)^2 + (2\pi \times 60 \times 2)^2} = 753.7$

$\therefore$ Current $i = \frac{V}{Z} = \frac{120}{753.7} = 0.159$ A

16. (b) As we know, $Q = ne$

Number of electron $n = \frac{Q}{e}$

$$\frac{6.35 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.9 = 4$$

17. (c)

18. (b) Since, $\lambda_r > \lambda_y > \lambda_g$

the wevelength of red light is greater than threshold wavelength and hence no electrons are emitted.

19. (c) If E is the energy of incident photon and W_0 the work function, then

Available energy $= E - W_0$

$$E - W_0 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2(E - W_0)}{m}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{1-0.5}{2.5-0.5}} = \frac{1}{2}$$

20. (c) According to question $\lambda_e = \lambda_P$

or $\frac{h}{\sqrt{2m_e Q_e V}} = \frac{h}{\sqrt{2m_p Q_p V_p}}$

$\therefore m_e Q_e V = m_p Q_p V_p$

$$\therefore V_p = \left(\frac{m_e}{m_p}\right)\left(\frac{Q_e}{Q_p}\right)V$$

$$= \left(\frac{1}{2000}\right)(1)V$$

$= \frac{V}{2000}$ volt

21. (d) As we know, $\frac{1}{2}mv^2 = eV$

$$\Rightarrow v = \sqrt{\frac{2eV}{m}}$$

$$p = mv = \sqrt{2meV}$$

Now, $p_e = \sqrt{2m_e \times e \times 100}$

and $p_\alpha = \sqrt{4m_\alpha \times e \times 100}$

$$\therefore \frac{p_e}{p_\alpha} = \sqrt{\frac{m_e}{2m_\alpha}}$$

22. (a)

23. (c) Frequency deviation is the frequency of an FM transmitter without signal input.

24. (d) Here, $N_0 = \frac{3}{5} N_0 e^{-\lambda t}$

$$e\lambda^t = \frac{5}{3}$$

$$\therefore \log e^{\lambda t} = \log_e \frac{5}{3} \text{ or } \lambda t = \log_e \frac{5}{3}$$

$$\text{or } t = \frac{1}{\lambda} \log_e \frac{5}{3}$$

$$= \frac{T}{0.693} \times 0.5 \left(\because T = \frac{0.693}{\lambda} \right)$$

$$= \frac{5570 \times 0.5}{0.693} \text{yr} = 4018.7\text{yrs}$$

25. (d) Field inside the metallic shell is zero.

26. (b) Potential at A

$$= +\frac{q}{\sqrt{a^2 + b^2}} - \frac{q}{\sqrt{a^2 + b^2}} = 0$$

27. (a) Potential difference between two points in an electric field

$$V_A - V_B = \frac{W}{q_0}$$

where W is work done by moving charge q_0 from point A to B

Here, $W = 2$ J, $q_0 = 20c$

$$V_A - V_B = \frac{2}{20} = 0.1 \text{ volt}$$

28. (b) Maximum electric field, $E_{max} = \frac{1}{4\pi\varepsilon_0} \frac{Q_{max}}{R^2}$

$$3 \times 10^6 = \frac{9 \times 10^9 \times Q_{max}}{2.5 \times 2.5}$$

$$\text{or } Q_{max} = \frac{3 \times 10^6 \times 2.5 \times 2.5}{9 \times 10^9}$$

$= 2 \times 10^{-3} C$

29. (a) In parallel combination, the potential difference across each individual capacitor is the same as the potenital difference across the combination.

So, potential difference across capacitor of capacity C is $\frac{V}{2}$.
Charge on capacitor, $C = \frac{Q}{V}$

$\therefore$ Charge on capacitor of capacity $C = \frac{CV}{2}$

30. (a) It is a parallel combination of three resistances, each of 10Ω.

$\therefore$ The effective resistance between points A and B.

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$$

or $R = \frac{10}{3} \Omega$

31. (b) Here, $E_1 \propto 64$

$E_1 - E_2 \propto 8$

$E_2 \propto l$

$\therefore\ 64 - l = 8$ or $l = 64 - 8 = 56$ cm

32. (b) $E_{mag} = \frac{B^2}{2\mu_0}$

33. (c) $\frac{k}{\omega}$ is independent of λ

34. (a) Shunt $S = \frac{GI_g}{I - I_g}$

$$= \frac{0.81 \times 1}{10 - 1} = \frac{0.81}{9} \Omega = 0.09\Omega$$

35. (a) Final current, $I = \frac{E}{R} = 10$ A

Energy stored in the magnetic field

$$U = \frac{1}{2} Li$$

$$\frac{1}{2} \times 5 \times (10)^2 = 250 \text{ J}$$

36. (a) ${}^{A}_{Z}X \rightarrow {}^{4}_{2}He + {}^{A-2}_{Z-2}Y$

${}^{A-4}_{Z-2}Y \rightarrow e^{+} + {}^{A-4}_{Z-3}Y'$

During β^{+}emission. ${}_{1}p^{1} \rightarrow {}_{0}n^{1} + \beta^{+}$

The proton changes into neutron. So, charge number decreases by 1 but mass number remains unchanged.

37. (a) Intensity, $I \propto 4a^2 \cos^2 \frac{\phi}{2}$

In the first case, $\phi = 2\pi$

$\therefore\ I' \propto 4a^2$

In the second case, $\phi = -\frac{2\pi}{3}$

$\therefore\ I' \propto 4a^2\cos^2\frac{2\pi}{3}$ or $I' \propto a^2$

$\frac{I'}{I} = \frac{1}{4}$ or $I' = \frac{1}{4}$

38. (c) Here, $\theta p + r = 90°$

$\Rightarrow r = 90° - \theta p$

or $r = 90° - 53°4' = 36°56$

39. (a) Potential gradient $\frac{dv}{dx} = \frac{e}{(R+R_h+r)} \cdot \frac{R}{L}$

$= \frac{2}{(15+5+0)} \times \frac{5}{1} = 0.5\ \mathrm{V/m}$

$= 0.005\ \mathrm{V/cm}$

40. (b) For 100% efficiency

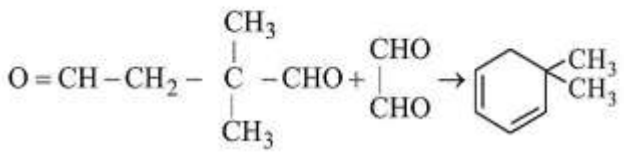
$V_s i_s = V_p i_p$

$1100 \times 2 = 220 \times i$

i.e., Primary current, $i_p = 10\ \mathrm{A}$

PART-II CHEMISTRY

41. (b) The products formed as a result of ozonolysis suggests



2, 2 - dimethyl butane glyoxal (C_8H_{12}) 1,4 - dial

42. (b) The electronic configuration of Ni is

$Ni(28) = [Ar]3\ d^8, 4\ s^2$

$Ni^{2+} = [Ar]3\ d^8$

Both Ni and Ni^{2+} have two unpaired electrons.

CO and $CN^-$are strong field ligands and thus unpaired electrons get paired. Hence, $Ni(CO)_4$ and $[Ni(CN)_4]^{2-}$ are diamagnetic. $Cl^-$is a weak field ligand hence, no pairing of e^-will take place. hence $NiCl_4^{2-}$ is paramagnetic.

43. (b) The equivalent conductance of $BaCl_2$ can be calculated as

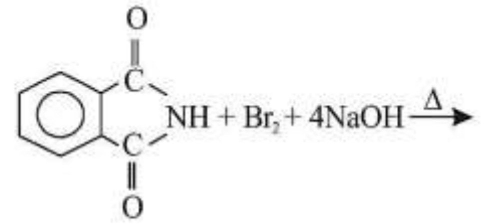
$\Lambda^\infty_{(BaC_2)} = \Lambda^\infty_{(Ba^{2+})} + 2\Lambda^\infty_{(Cl^-)}$

$= 127 + 2 \times 76$

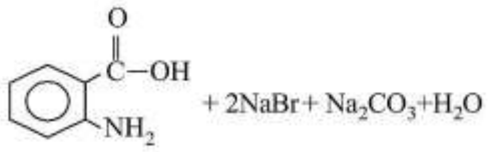
$= 127 + 152$

$= 279 \text{Scm}^2 / \text{equiv.}$

44. (d)

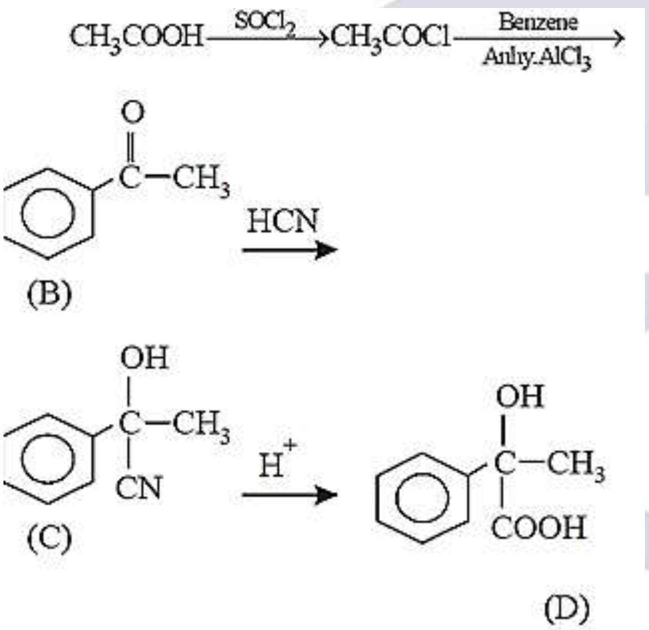


phthalimide



anthranilic acid

45. (b)



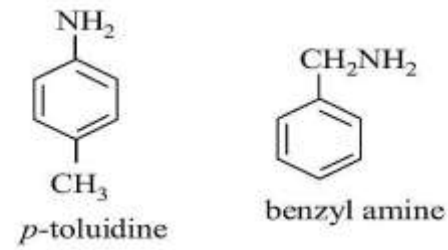
46. (d) Secondary alcohol, when shaken with a mixture of anthydrous $ZnCl_2$ and conc. HCl (Lucas regent) gives an oily layer product after five minutes.

$$H_3C-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2CH_3 \xrightarrow{\text{Lucas reagent}}$$

oily layer product after 5 min.

The amines in which amino group is directly attached to benzene ring undergo diazotisation reaction.

47. (b) p-toluidine contains amino group attached directly to benzene ring, thus undergo diazotisation reaction and gives red dye. In benzyl amine, the amino group is not directly attached to benzene ring. Hence, it will not undergo diazotisation reaction.



Thus, these two can be distinguished by dye test.

48. **(c)** $C_2H_5Br + Na \longrightarrow CH_3\dot{C}H_2 + NaBr$ Intermediate free radical $CH_3CH_2^+$ combines to form $CH_3CH_2 - CH_2CH_3$ (as a main product) and also $CH_2 == CH_2$ and CH_3CH_3 by disproportion

$CH_3\dot{C}H_2 + CH_3\dot{C}H_2 \rightarrow CH_3CH_2 - CH_2CH_3$

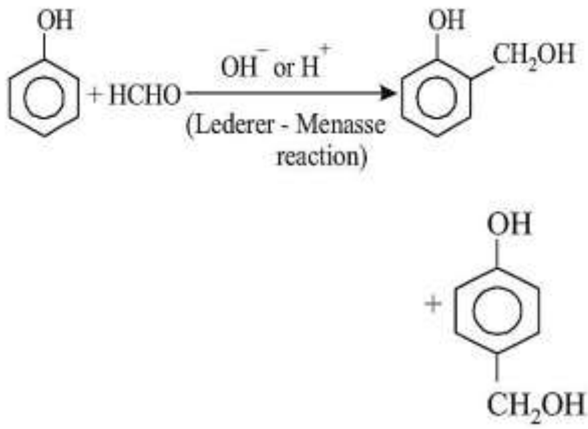
$CH_3\dot{C}H_2 + CH_3\dot{C}H_2 \longrightarrow CH_2 = CH_2 + CH_3 - CH_3$

n-butane (disproportion reaction)

49. **(a)** Ethers, In the presence of air and light form peroxides

50. **(c)** Glycerine is used as a preservative because it keeps the fruit moist

51. **(b)**

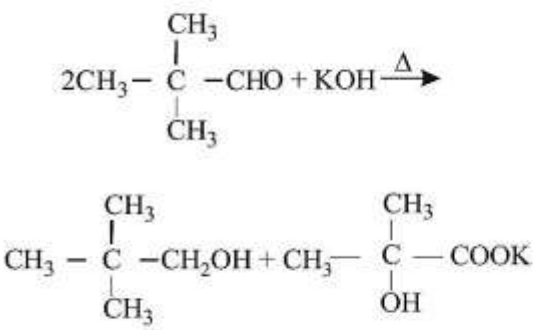


52. **(a)** In presence of SeO_2 compounds containing active methylene (i.e., CH_2 next to the carbonyl group) oxidises to another CO group.

$$CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3 \xrightarrow{SeO_2} CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-\overset{\overset{\displaystyle O}{\|}}{C}-H$$

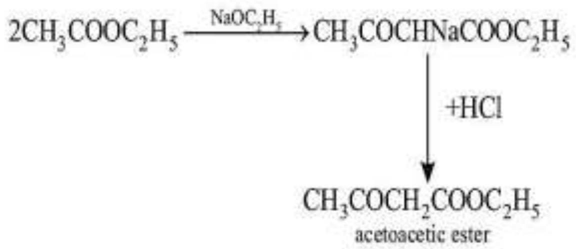
$$+Se+H_2O$$

53. **(c)** Aldehydes which have no $\alpha - H$ atom give Cannizzaro's reaction, $(CH_3)_3C - CHO$ does not contain $\alpha - H$ atom, hence it will give Cannizzaro reaction.



54. (a) Secondary structure involves α-helical and β-pleated sheet like structure. Where as Primary strucure involves sequence of α-amino acids polypeptide chain.

55. (b)



56. (d) In aniline, p-methoxyaniline and p-methyl aniline, the lone pair of electrons on the N atom is delocalised on the benzene ring while in benzylamine it is delocalised, and more available for donation. Hence benzylamine is most basic among the given.

57. (c) Mn_2O_7 dissolves in water to give permanganic acid which is purple in colour.

$$Mn_2O_7 + H_2O \longrightarrow \underset{\text{Purple}}{2HMnO_4}$$

58. (b) " 925 fine silver" means 925 parts of pure Ag in 1000 parts of an alloy. Therefore, in terms of percentage, it will be 92.5%Ag and 7.5%Cu.

59. (a) Spectrochemical series. According to the strength of splitting is as:

$$CO > CN^- > NO_2^- > \text{en} > NH_3 > \text{py} > NCS^-$$

$$> H_2O > O^{2-} > Ox^{2-} > OH^- > F > Cl^-$$

$$> SCN^- > S^{2-} > Br^- > I^-$$

Crystal field stabilisation energy (CFSE) for octahedral complex, $\Delta°$ depends on the strength of negative ligand. [Higher the strength more will be the CFSE]

60. (b) KCN forms complex with $Cu^+$and Cd^{2+} as $K_3[Cu(CN)_4]$ and $K_2[Cd(CN)_4]$ respectively. On passing H_2 S, only Cd^{2+} complex gets decomposed to yellow CDS.

61. (a) EAN =at. no. of central atom - oxidation state +2 (number of ligands)

$$= 27 - 3 + 2(6)$$

$$= 27 - 3 + 12 = 36$$

62. (d) $[Co(NO_2)(NH_3)_5]Cl_2$ Pentamminenitrito-N-cobalt (III) chloride.

63. (d) I-131 radio-isotope is used for thyroid disorders.

64. (b) Tetragonal crystal system has the unit cell dimensions

$$a = b \neq c, \alpha = \beta = \gamma = 90°$$

65. (c) A crystalline solid undergoes deformation of its geometry by application of pressure or heat.

66. (c) $\therefore 100 \times p = V_2 \times \frac{40p}{100}$

$$V_2 = 250 \text{ cc (Total volume)}$$

$$\text{Volume of bulb B} = 250 - 100 = 150\text{cc}$$

67. (a)

68. (b)

69. (b) $\Delta T_f = iK_f m \; m = \frac{10\times1000}{74.55\times100} = 1.34 \text{ m}$

[Molecular Mass of KCl = 74.55]

$$i = \frac{\Delta T_f}{K_f\, m} = \frac{4.5^\circ C}{(1.86^\circ C/m)(1.34\ m)} = 1.8$$

70. (c) $2NO_2 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} N_2O_4$

Rate $= -\frac{1}{2}\frac{d[NO_2]}{dt} = k_1[NO_2]^2 - k_2[\,N_2O_4]$

$\therefore$ Rate of disappearance of NO_2 i.e.,

$$-\frac{d[NO_2]}{dt} = 2k_1[NO_2]^2 - k_2[\,N_2O_4]$$

71. (c) For every 10° rise in temperture, Rate gets double. Hence, in this case rate constant will become four time. (d) i.e.,

$k_1 = 4k_2\ k_2 = 0.25k_1$

as the Rise in temperature given is 20°C

72. (d) At $T_1 = 200$ K, if $k_1 = k$

then at $T_2 = 400$ K, $k_2 = 10k$

$$\log\frac{10k}{k} = \frac{E_a}{2.303R}\left(\frac{400-200}{400\times 200}\right)$$

$E_a = 921.2R$

73. (a) $CO_2(\,g) + C(s) \rightleftharpoons 2CO(g)$

Initial 0.5 atm

pressure

At equi $(0.5 - x)$

$\therefore$ Total no. of moles at equilibrium

$= (0.5 - x) + 2x = 0.8$

$\therefore\ x = 0.3$ atm.

$\therefore\ p_{CO_2} = 0.5 - 0.3 = 0.2$

$P_{CO} = 2x = 2 \times 0.3 = 0.6$ atm

$$K = \frac{p_{CO}^2}{p_{CO_2}} = \frac{(0.6)^2}{0.2} = 1.8 \text{ atm}$$

74. (a) $2\,A + B \rightleftharpoons C, \Delta H = x$ cals.

Since the reaction is endothermic and $n_p < n_r$. Hence, high pressure and high temperature will favour forward reaction.

75. (b) The cell reaction is

$Mg + Sn^{2+} \rightarrow Mg^{2+} + Sn$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{2}\log\frac{[Mg^{2+}]}{[Sn^{2+}]}$$

$$= (2.34 - 0.14) - \frac{0.0591}{2}\log\frac{10^{-2}}{10^{-1}} = 2.23\text{ V}$$

76. (a) Explosive compound has high heat content i.e., it is formed by absorption of heat.

Hence, ΔH°_{f} is positive.

77. (b) $C_3H_8(g) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O(l)$

$\Delta n_g = n_p - n_r$

$= 3 - 6 = -3$

$\Delta H = \Delta E + \Delta n_g RT$

$\Delta H = \Delta E - 3RT$

$\Delta H - \Delta E = -3RT$

78. (a) $\because \Delta G = \Delta H - T\Delta S$

For a spontaneous reaction, ΔG should be negative. Hence, $\Delta H = +ve, \Delta S = +ve$ and $T\Delta S > \Delta H$

79. (b) Gas (Y)SO_2 which can be obtained from Na_2SO_3(A) and gas (Z) is H_2S which can be obtained from Na_2S(B).

This can be easily understood by the following reactions.

$$\underset{(A)}{Na_2SO_3} + 2HCl \rightarrow 2NaCl + \underset{(Y)}{SO_2} + H_2O$$

$$\underset{(B)}{Na_2S} + 2HCl \rightarrow 2NaCl + \underset{(Z)}{H_2S}$$

$$K_2Cr_2O_7 + H_2SO_4 + 3SO_2 \rightarrow K_2SO_4 + Cr_2(SO_4)_3 + H_2O$$

$$Pb(CH_3COO)_2 + H_2S \rightarrow \underset{\text{black}}{PbS\downarrow} + 2CH_3COOH$$

80. (d) Higher the bond order higher will be the stability.

When the bond order of two molecules are same, the molecule with least number of antibonding electrons is more stable.

Molecular orbital configuration of H_2^+ is $H_2^+(1) = \sigma 1s^1$ BO $= \frac{1-0}{2} = 0.5$

Molecular orbital configuration of He_2^+ is

$He_2^+(3) = \sigma 1s^2, \overset{*}{\sigma} 1s^1$

$$BO = \frac{2-1}{2} = 0.5$$

Hence, stability of H_2^+ >stability of He_2^+.

PART-III MATHEMATICS

81. (a) The equation of the bisectors of the angle between the lines given by $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

And the equation of the bisectors of the angle between the lines given by $a^2x^2 + 2h(a + b)xy + b^2y^2 = 0$ is

$$\frac{x^2 - y^2}{a^2 - b^2} = \frac{xy}{h(a + b)}$$

$$\Rightarrow \frac{x^2 - y^2}{a - b} = \frac{x}{h}$$

From eqs. (i) and (ii), it is clear that both the pair of straight lines have the same bisector, hence, the given two pairs of straight lines are equally inclined.

82. (c) $A = \{1,2,3,4\}$ and $B = \{1,3,5\}$

$\therefore R = \{(1,3), (1,5), (2,3), (2,5), (3,5), (4,5)\}$ and $R^{-1} = \{(3,1), (5,1), (3,2), (5,2), (5,3), (5,4)\}$

$\therefore ROR^{-1} = \{(3,3), (3,5), (5,3), (5,5)\}$

83. (c)

$$(1 - i\sqrt{3})^{100} = 2^{100}\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{100}$$

$$= 2^{100}\omega^{100} = 2^{100}\omega$$

$$= 2^{100}\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) = -2^{99} + 2^{99}\sqrt{3}i$$

Now, $x + iy = (1 - i\sqrt{3})^{100}$

$$= -2^{99} + 2^{99}\sqrt{3}i$$

$$\Rightarrow x = -2^{99}, y = 2^{99}\sqrt{3}$$

$$\therefore (x, y) = \left(-2^{99}, -2^{99}\sqrt{3}\right)$$

84. (a) Given, $a_n = 3(2^n)$

$a_{n+1} = 3(2^{n+1})$

$$\therefore r = \frac{a_{n+1}}{a_n} = \frac{3(2^{n+1})}{3(2^n)} = 2$$

85. (c) a, b, c are in HP.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in AP.

$\Rightarrow 1 + \frac{b+c}{a}, 1 + \frac{a+c}{b}, 1 + \frac{a+b}{c}$ are in AP.

$\Rightarrow \frac{b+c}{a}, \frac{a+c}{b}, \frac{a+b}{c}$ are in AP.

$\Rightarrow \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in HP.

86. (d) $\frac{x^2+2x+7}{2x+3} < 6$

$$\Rightarrow \frac{x^2+2x+7}{2x+3} - 6 < 0$$

$$\Rightarrow \frac{x^2-10x-11}{2x+3} < 0$$

$$\Rightarrow \frac{(x-11)(x+1)}{2x+3} < 0$$

$$\Rightarrow \frac{(x-11)(x+1)(2x+3)}{(2x+3)^2} < 0$$

$$\Rightarrow (x-11)(x+1)(2x+3) < 0$$

$$\Rightarrow x \in \left(-\infty, -\frac{3}{2}\right) \cup (-1,11)$$

87. (a) The number of faces is equal to the number of colours.

Therefore the number of ways of painting them is 1 .

88. (d) Equation of line perpendicular to

$3x + y - 3 = 0$ is

$x - 3y + c = 0$

Since, it passes through (2,2).

$\therefore c = 4$

$\therefore$ Equation of line perpendicular to

$3x + y - 3 = 0$ is $x - 3y + 4 = 0$

For y - intercept, put $x = 0$

$\therefore y = \frac{4}{3}$

89. (b) The centres of the given circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ are $C_1(0,0)$ and $C_2(3,4)$ respectively. Their radii are $r_1 = 2$ and $r_2 = 7$ respectively. $C_1C_2 = 5 <$ sum of radii

But $C_1C_2 =$ difference of radii

Thus, the given circles touch each other internally.

Hence, number of common tangent is only one.

90. (d) $1 - e^{\frac{1}{x}-1} > 0$

$\Rightarrow e^{\frac{1}{x}-1} < 1 \Rightarrow \frac{1}{x} - 1 < \log 1$

$\Rightarrow \frac{1}{x} - 1 < 0 \Rightarrow \frac{1}{x} < 1$

$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$

91. (d) $\lim_{x\to 0} \frac{\sqrt{1-\cos x}}{x} = \lim_{x\to 0} \frac{\sqrt{2}\sin\frac{x}{2}}{x}$

$= \sqrt{2}\lim_{x\to 0} \frac{\left|\sin\frac{x}{2}\right|}{x}$

It can be easily seen that LHL at $x = 0$ is

$x = -\frac{1}{\sqrt{2}}$ and RHL at $x = 0$ is $\frac{1}{\sqrt{2}}$.

Hence, $\lim_{x\to 0} \frac{\sqrt{1-\cos x}}{x}$ does not exist.

92. (a) Let $I = \int \frac{x^2+4}{x^4+16} dx$

$= \int \frac{1+\frac{4}{x^2}}{x^2+\frac{16}{x^2}} dx = \int \frac{1+\frac{4}{x^2}}{\left(x-\frac{4}{x}\right)^2+8} dx$

Putting $x - \frac{4}{x} = t$,

So that $\left(1+\frac{4}{x^2}\right) dx = dt$

$\therefore I = \int \frac{dt}{t^2+(2\sqrt{2})^2}$

$= \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{t}{2\sqrt{2}}\right) + C$

$\Rightarrow I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x-\frac{4}{x}}{2\sqrt{2}}\right) + C$

$= \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2-4}{2x\sqrt{2}}\right) + C$

93. (a) $\int_{\pi/4}^{3\pi/4} \frac{1}{1+\cos x} dx = \int_{\pi/4}^{3\pi/4} \frac{1-\cos x}{\sin^2 x} dx$

$= \int_{\pi/4}^{3\pi/4} (\operatorname{cosec}^2 x - \cot x \operatorname{cosec} x) dx$

$= [-\cot x + \operatorname{cosec} x]_{\pi/4}^{3\pi/4}$

$= (1+\sqrt{2}) - (-1+\sqrt{2}) = 2$

94. (b) Let a and b are two numbers.

Then, $A = \frac{a+b}{2}$

Also, a, p, q, b are in GP.

$\therefore \frac{p}{a} = \frac{q}{p} = \frac{b}{q}$

$\Rightarrow \frac{p^2}{q} = a$ and $\frac{q^2}{p} = b$

$\therefore \frac{p^2}{q} + \frac{q^2}{p} = a + b = 2A$ [Using eq. (i)]

95. (b) $f(x) = x^2 + ax + b$, then

$f(x + c) = (x + c)^2 + a(x + c) + b$

$= x^2 + (2c + a)x + c^2 + ac + b$

which shows that the roots of $f(x)$ are transformed to $(x - c)$ i.e., roots of $f(x + c) = 0$ are $c - c$ and $d - c$.

Hence, one of the roots of the equation $f(x + c)$ is $(d - c)$.

96. (a) For the sum of the coefficients in the expansion of $(6a - 5b)^n$, put $a = b = 1$

$\therefore$ Sum of the coefficients $= (6 - 5)^n$

$= 1^n = 1$

97. (b) $(7.995)^{1/3} = (8 - 0.005)^{1/3}$

$= 2\left(1 - \frac{0.005}{8}\right)^{1/3}$

$= 2\left[1 + \left(\frac{1}{3}\right)\left(-\frac{0.005}{8}\right)\right] = 2[1 - 0.000208]$

$= 1.9996$

98. (b) $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1} - ax - b\right) = 0$

$\Rightarrow \lim_{x\to\infty}\left[\frac{x^2(1 - a) - (a + b)x - b + 1}{x + 1}\right] = 0$

$\Rightarrow 1 - a = 0$ and $a + b = 0 \Rightarrow a = 1, b = -1$

99. (b) Let $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$, then projection of **a** on **b** is equal to

$\frac{a.b}{|\mathbf{b}|} = \frac{4 + 8 + 7}{\sqrt{16 + 16 + 49}} = \frac{19}{9}$

100. (a) Here, $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$

$\Rightarrow |\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = 0$

$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2\{\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}\} = 0$

$\Rightarrow \mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}$

$= -\frac{1}{2}\{|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2\} < 0$

$\Rightarrow m < 0$

101. **(a)** Let γ is the required angle, then

$\cos^2 45° + \cos^2 60° + \cos^2\gamma = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\gamma = \frac{1}{2} \Rightarrow \gamma = 60°$

102. **(a)** $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

$= \cos\theta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\theta \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$= I\cos\theta + J\sin\theta$

103. **(c)** According to the definition of determinant, determinant is a number associated to a square matrix.

104. **(c)** α, β, γ are the roots of given equation,

Therefore $\alpha + \beta + \gamma = -a$

$\alpha\beta + \beta\gamma + \gamma\alpha = 0$

and $\alpha\beta\gamma = -b$

Now, $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = -(\alpha + \beta + \gamma)$

$(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$

$= -(\alpha + \beta + \gamma)[(\alpha + \beta + \gamma)^2$

$-3(\alpha\beta + \beta\gamma + \gamma\alpha)]$

$= -(-a)(a^2 - 0) = a^3$

105. **(b)** Substituting $x = X + 1$ and $y = Y - 2$ in given equation, we get

$2(X + 1)^2 + (Y - 2)^2 - 4(X + 1) + 4(Y - 2) = 0 \Rightarrow 2X^2 + Y^2 = 6$

106. **(a)** Given $f(x) = \begin{cases} x^2, & x \le 0 \\ 2\sin x, & x > 0, \end{cases}$

$f'(x) = \begin{cases} 2x, & x < 0 \\ \text{non differentiable}, & x = 0 \\ 2\cos x, & x > 0 \end{cases}$

So, $x = 0$ is a critical point $f(0^-) > 0$ as well as $f(0^+) > 0$ and $f(0) = 0$

Hence, it is a point of minima.

107. **(a)** $x * a = b \Rightarrow (x * a) * a^{-1} = b * a^{-1}$

$\Rightarrow x * (a * a^{-1}) = b * a^{-1}$

$\Rightarrow x * e = b^{*}a^{-1}$

$\Rightarrow x = b^{*}a^{-1}$

108. **(b)** Let A and B be two events such that

A = getting number 2 at least once

B = getting 7 as the sum of the numbers on two dice

Here,

$A = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$

$(1,2), (3,2), (4,2), (5,2), (6,2)\}$

and

$B = \{(2,5), (5,2), (6,1), (1,6), (3,4), (4,3)\}$

$\therefore P(A) = \frac{11}{36}, P(B) = \frac{6}{36}$

$P(A \cap B) = \frac{2}{36}$

$\therefore$ Required probability

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{6/36} = \frac{2}{6} = \frac{1}{3}$

109. **(b)** Any chord PQ which bisected point $R(h, k)$ is $T = S$

i.e., $ky - 2a(x + h) = k^2 - 4ah$

Since, it is a focal chord, so it must pass through focus $(a, 0)$.

$\therefore k(0) - 2a(a + h) = k^2 - 4ah$

$\Rightarrow k^2 = 2ah - 4a^2$

Hence, locus is

$y^2 = 2a(x - a)$

110. **(d)** $\sin 12^\circ \sin 48^\circ \sin 54^\circ$

$= \sin 12^\circ \sin(60^\circ - 12^\circ)\sin(90^\circ - 36^\circ)$

$= \frac{\sin 12^\circ \sin(60^\circ - 12^\circ)\sin 72^\circ \cos 36^\circ}{\sin 72^\circ}$

$= \frac{[\sin 12^\circ \sin(60^\circ - 12^\circ)\sin(60^\circ + 12^\circ)\cos 36^\circ]}{\sin 72^\circ}$

$$= \frac{\sin 12^\circ(\sin^2 60^\circ - \sin^2 12^\circ)\cos 36^\circ}{\sin 72^\circ}$$

$$= \frac{\sin 12^\circ\left(\frac{3}{4} - \sin^2 12^\circ\right)\cos 36^\circ}{\sin 72^\circ}$$

$$= \frac{3\sin 12^\circ - 4\sin^3 12^\circ)}{4} \cdot \frac{\cos 36^\circ}{\sin 72^\circ}$$

$$= \frac{\sin 36^\circ \cos 36^\circ}{4\sin 72^\circ} = \frac{1}{2}\frac{\sin 72^\circ}{4\sin 72^\circ} = \frac{1}{8}$$

111. **(b)** We have, $\Delta = \frac{\sqrt{3}}{4}a^2, s = \frac{3a}{2}$

Inradius $r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}$

Circumradius $R = \frac{abc}{4\Delta} = \frac{a^3}{\sqrt{3}a^2} = \frac{a}{\sqrt{3}}$

and exradii $r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{3}/4a^2}{a/2}$

$$= \frac{\sqrt{3}}{2}a$$

$\therefore$ Required ratio $= r : R : r_1$

$= \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}}{2}a = 1 : 2 : 3.$

112. **(b)** $p \vee q$ is false only when both p and q are false

113. **(a)** Since, each letter can be posted in any one of the five different letter boxes. So, a letter can be posted in 5 ways. Since, there are six letters and each can be posted in 5 ways. So, total number of ways $=$ $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5$

114. **(b)** Since, (1,4), (2,6) and (3,6) are the elements of $A \times B$, therefore 1,2,3 are the elements of A and 4,6 are the elements of B.

Also, $A \times B$ has 6 elements

$\therefore$ $A = \{1,2,3\}$

and $B = \{4,6\}$

$\therefore B \times A = \{(4,1), (4,2), (4,3), (6,1), (6,2), (6,3)\}$

115. **(b)** Let $f^{-1}(-5) = x$. Then, $f(x) = -5$

$\Rightarrow x^2 + 1 = -5 \Rightarrow x^2 = -6 \Rightarrow x = \pm\sqrt{-6}$

which does not belong to R.

$\therefore f^{-1}(-5) = \phi$

116. **(c)** Given: $P(X = 1) = P(X = 2)$

$$\frac{e^{-\lambda}\lambda}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$
$$\Rightarrow \lambda = 2$$
$$\therefore P(X = 4) = \frac{e^{-2}2^4}{4!} = \frac{2}{3e^2}$$

117. **(d)** Required area $= \int_3^5 (3x - 5)dx$

$$= \left[\frac{3x^2}{2} - 5x\right]_3^5 = \left[\frac{75}{2} - 25\right] - \left[\frac{27}{2} - 15\right]$$

$$= \frac{48}{2} - 10 = 14 \text{ sq units}$$

118. **(c)** Given differential equation is

$$\left(1 + 4\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^2y}{dx^2}$$

$$\left(1 + 4\frac{dy}{dx}\right)^2 = 4^3\left(\frac{d^2y}{dx^2}\right)^3$$

Here, order is 2 and degree is 3

119. **(d)** $\frac{dy}{dx} = (4x + y + 1)^2$

Put $4x + y + 1 = v$

$$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 4$$

$$\frac{dv}{dx} - 4 = v^2$$

($\because$ From eq. (i))

$$\Rightarrow \frac{dv}{v^2 + 4} = dx$$

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{v}{2}\right) = x + C$$

$$\Rightarrow \tan^{-1}\left(\frac{4x + y + 1}{2}\right) = 2x + C$$

$$\Rightarrow 4x + y + 1 = 2\tan(2x + C)$$

120. **(d)** Given system of equations are

$2x + y - 5 = 0$

$x - 2y + 1 = 0$

and $2x - 14y - a = 0$

This system is consistent

$$\therefore \begin{vmatrix} 2 & 1 & -5 \\ 1 & -2 & 1 \\ 2 & -14 & -a \end{vmatrix} = 0$$

$\Rightarrow 2(2a + 14) - 1(-a - 2) - 5(-14 + 4) = 0$

$\Rightarrow 4a + 28 + a + 2 + 50 = 0$

$\Rightarrow 5a = -80 \Rightarrow a = -16$