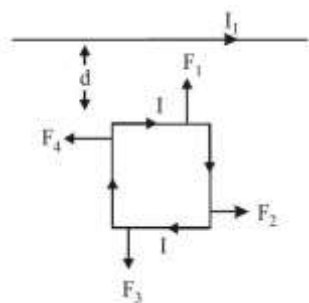


PART-I PHYSICS

1. (d)



$$F_2 = -F_4$$

$$F_1 = \frac{\mu_0 I_1 I l}{2\pi d}$$

$$F_2 = \frac{\mu_0 I_1 I l}{2\pi(d+l)}$$

$$F_1 > F_3$$

$$F_{\text{net}} = F_1 - F_3$$

So, wire attract loop.

$$\begin{aligned} 2. \text{ (a) Here, } V_0 &= \frac{E-V}{e} = \frac{h(\nu-\nu_0)}{e} \\ &= \frac{6.62 \times 10^{-34}(8.2 \times 10^{14} - 3.3 \times 10^{14})}{1.6 \times 10^{-19}} \\ &= \frac{6.62 \times 10^{-34}}{1.6} \times 4.9 \times 10^{33} \\ &= \frac{6.62 \times 4.9 \times 10^{-1}}{1.6} \end{aligned}$$

$$V_0 = 2 \text{ volt}$$

3. (a) In forward biasing, resistance of p – n junction diode is zero, so whole voltage appears across the resistance.

$$4. \text{ (d) BE of } \text{Li}^7 = 39.20\text{MeV}$$

$$\text{and } \text{He}^4 = 28.24\text{MeV}$$

$$\text{Hence binding energy of } {}_2\text{He}^4 = 56.84\text{MeV}$$

$$\text{Energy of reaction} = 56.84 - 39.20$$

$$= 17.28\text{MeV}$$

$$5. \text{ (b) } \sqrt{v} \propto (Z - b)$$

6. (d) When reactance of inductance is more than the reactance of condenser, the current will lag behind the voltage.

$$\text{Thus } \omega L < \frac{1}{\omega C} \text{ or } \omega > \frac{1}{\sqrt{LC}}$$

$$\text{or } n > \frac{1}{2\pi\sqrt{LC}} \text{ or } n > n_r$$

n_r = resonant frequency

7. (c) Capacitance, $C_A = \frac{K_1 \epsilon_0 A}{\frac{d}{2}}$, $C_B = \frac{K_2 \epsilon_0 A}{\frac{d}{2}}$

$$C_{eq} = \frac{C_1}{C_2} = \frac{2 K_1 K_2}{K_1 + K_2}$$

$$= \frac{C_A C_B}{C_A + C_B} = \left(\frac{2 K_1 K_2}{K_1 + K_2} \right) \frac{\epsilon_0 A}{d} \left(\because e = \frac{\epsilon_0 A}{d} \right)$$

8. (d) Intensity of the electric field, $E = \frac{dV}{dx} = 6x$

Potential (v) = $3x^2 + 5$

$\therefore E$ at $x = -2$

$$= 6(-2) = -12 \text{ V/m}$$

9. (c) Volume of 8 small drops = Volume of big drop

$$\therefore \left(\frac{4}{3} \pi r^3 \right) \times 8 = \frac{4}{3} \pi R^3$$

$$\Rightarrow 2r = R$$

...(i)

According to charge conservation

$$8q = Q$$

Potential of one small drop (V') = $\frac{q}{4\pi\epsilon_0 r}$

Similarly, potential of big drop (V) = $\frac{Q}{4\pi\epsilon_0 R}$

$$\text{Now, } \frac{V'}{V} = \frac{q}{Q} \times \frac{R}{r} \Rightarrow \frac{V'}{20} = \frac{9}{8q} \times \frac{2r}{r}$$

$$\therefore V' = 5 \text{ V}$$

10. (b) Threshold energy of $AE_A = h\nu_A$

$$= 6.6 \times 10^{-34} \times 1.8 \times 10^{14}$$

$$= 11.88 \times 10^{-20} \text{ J}$$

$$= \frac{11.88 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 0.74 \text{ eV}$$

Similarly, $E_B = 0.91 \text{ eV}$

As the incident photons have energy greater than E_A but less than E_B

So, photoelectrons will be emitted from metal A only.

11. (a) Balanced wheat stone bridge condition $\frac{P}{Q} = \frac{R}{S}$

No, current flows through galvanometer Now, P and R are in series, so

$$\text{Resistance } R_1 = P + R$$

$$= 10 + 15 = 25\Omega$$

Similarly, Q and S are in series, so

$$\text{Resistance } R_2 = R + S$$

$$= 20 + 30 = 50\Omega$$

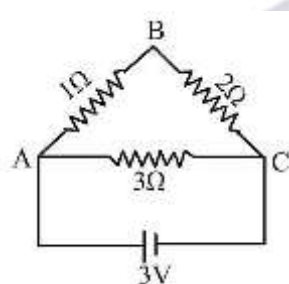
Net resistance of the network as R_1 and R_2 are in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore R = \frac{25 \times 50}{25 + 50} = \frac{50}{3}\Omega$$

$$\text{Hence, current, } I = \frac{V}{R} = \frac{6}{50/3} = 0.36 \text{ A}$$

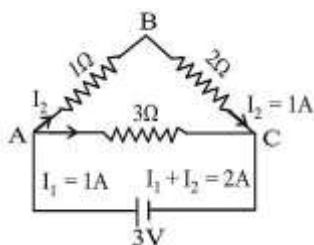
12. (b) The arrangement is shown in figure.



Here, two resistance of 1Ω and 2Ω are in series, which form 3Ω which is in parallel with 3Ω resistance.

Therefore, the effective resistance

$$\frac{(1 + 2) \times 3}{(1 + 2) + 3} = \frac{3}{2}\Omega$$

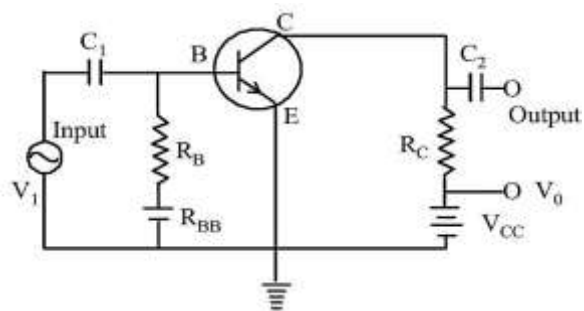


$\therefore$ Current in the circuit,

$$I = \frac{3}{(3/2)} = 2 \text{ A}$$

$$\therefore \text{Current in } 3\Omega \text{ resistor} = \frac{I}{2} = 1 \text{ A}$$

13. (b) In CE amplifier, the input signal is applied across base-emitter junction.



14. (c) de-Broglie wavelength of an electron

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} \text{ or } \lambda \propto \frac{1}{\sqrt{K}}$$

$$\therefore \frac{\lambda'}{\lambda} = \frac{1}{\sqrt{3K}} = \frac{\sqrt{K}}{1} = \frac{1}{\sqrt{3}}$$

$$\text{or } \lambda' = \frac{\lambda}{\sqrt{3}}$$

i.e. de-Broglie wavelength will change by

factor $\frac{1}{\sqrt{3}}$.

15. (a) We know,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \Rightarrow \frac{N}{10000} = \left(\frac{1}{2}\right)^{\frac{10}{20}}$$

$$\Rightarrow N = \frac{10000}{\sqrt{2}} = \frac{10000}{1.414} = 7070$$

16. (c) As X-rays pass through the intestine without casting a clear shadow.

17. (b) Given: $A = 0.3 \text{ m}^2$, $n = 2 \times 10^{25} / \text{m}^3$

$$q = 3t^2 + 5t + 2$$

$$i = \frac{dq}{dt} = 6t + 5 = 17$$

$$\text{Drift velocity, } v_d = \frac{i}{neA}$$

$$= \frac{17}{2 \times 10^{25} \times 1.6 \times 10^{-19} \times 0.3}$$

$$= \frac{17}{0.96 \times 10^6} = 1.77 \times 10^{-5} \text{ m/s}$$

18. (b) Capacitance $1\mu\text{F}$ and $C\mu\text{F}$ are connected in series,

$$\therefore C_{eq} = \frac{C}{1 + C}$$

Given, $V = 120 \text{ V}$ and $q = 80\mu\text{C}$

$$\therefore q = C_{eq}V$$

$$80 = \frac{C}{C+1} \times 20$$

$$\text{or } C = 2\mu\text{F}$$

Energy stored in the capacitor of capacity C

$$U = \frac{1}{2} \frac{q^2}{C}$$

$$= \frac{1}{2} \times \frac{(80 \times 10^{-6})^2}{2 \times 10^{-6}}$$

$$= \frac{1}{2} \times \frac{80 \times 10^{-6} \times 80 \times 10^{-6}}{2 \times 10^{-6}}$$

$$U = 1600\mu\text{J}$$

19. (d) Conducting surface behaves as equipotential surface.

20. (a) $I = n_e q_e + n_p q_E = 1 \text{ mA}$ (towards right)

21. (a) 1 faraday deposited 1 g equivalent

22. (c) The magnetic field

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi(qv)}{r}$$

$$= 10^{-7} \times \frac{2 \times 3.14 \times (1.6 \times 10^{-19} \times 1.6 \times 10^{15})}{0.53 \times 10^{-10}}$$

$$= 12.5 \text{ Wb/m}^3$$

23. (c) The magnetic field, $B = \frac{\mu_0}{4\pi} \cdot \frac{2N}{d^3}$

$$= 10^{-7} \times \frac{2 \times 1.25}{(0.5)^3} = 2 \times 10^{-6} \text{ N/A-m}$$

24. (c) Transformation ratio, $\frac{I_p}{I_s} = \frac{n_s}{n_p}$

$$\text{i.e. } \frac{3}{I_s} = \frac{3}{2} \text{ or, } I_s = 2 \text{ A}$$

25. (b) Voltage

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(20)^2 + (16)^2}$$

$$= 25.6 \text{ V}$$

26. (a) Electron goes to its first excited state ($n = 2$) from ground state ($n = 1$) after absorbing 10.2eV energy

$$\therefore \text{Increase in momentum} = \frac{h}{2\pi}$$

$$= \frac{6.6 \times 10^{-34}}{6.28}$$

$$= 1.05 \times 10^{-34} \text{ J-s}$$

27. (b) Using $R = R_0 A^{1/3}$

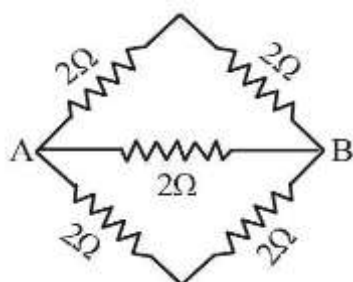
$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\frac{R}{R_{\text{He}}} = \left(\frac{A}{4}\right)^{1/3}$$

$$(14)^{1/3} = \left(\frac{A}{4}\right)^{1/3} \Rightarrow A = 56$$

$$\text{So, } = 56 - 30 = 26$$

28. (a) Given circuit is a balanced Wheatstone bridge.



Equivalent resistance of upper arms
 $= 2 + 2 = 4\Omega$

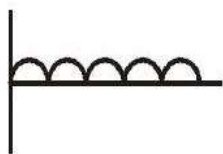
Equivalent resistance of lower arms

$$= 2 + 2 = 4\Omega$$

$$\therefore R_{AB} = \frac{4 \times 4}{4 + 4} = 2\Omega$$

29. (a) Mutual conductance $g_m = \frac{\mu}{R_p} = \frac{20}{10 \times 10^3} = 2 \times 10^{-3} = 2 \text{ milli mho}$

30. (c) Full-wave rectifier output wave form



31. (a) Mass defect

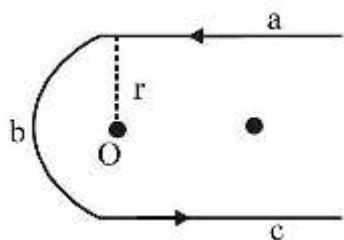
$\Delta m = \text{Total mass of } \alpha\text{-particles}$

– mass of ^{12}C nucleus

$$= 3 \times 4.002603 - 12 = 12.007809 - 12$$

$$= 0.007809 \text{ unit}$$

32. (a) Field due to a straight wire of infinite length is $\frac{\mu_0 i}{4\pi r}$ if the point is on a line perpendicular to its length while at the centre of a semicircular coil is $\frac{\mu_0 \pi i}{4\pi r}$



$$\therefore B = B_a + B_b + B_c$$

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 \pi i}{4\pi r} + \frac{\mu_0 i}{4\pi r}$$

$$= \frac{\mu_0 i}{4\pi r} (\pi + 2) \text{ out of the page}$$

33. (b) Stopping potential = Maximum KE

$$eV = KE_{\max}$$

34. (a) Current $i = \frac{E}{R+r}$

$$2 = \frac{E}{2+r}$$

$$0.5 = \frac{E}{9+r}$$

From Eqs. (i) and (ii), we have

$$\frac{2}{0.5} = \frac{9+r}{2+r} \Rightarrow 4 = \frac{9+r}{2+r}$$

$$3r = 1 \therefore r = \frac{1}{3\Omega}$$

35. (d) We know, induced emf

$$e = -L \frac{di}{dt}$$

During 0 to $\frac{T}{4}$, $\frac{di}{dt} = \text{constant}$

So, $e = -ve$

For $\frac{T}{4}$ to $\frac{T}{2}$, $\frac{di}{dt} = 0$

i.e., $e = 0$

For $\frac{T}{4}$ to $\frac{3T}{4}$, $\frac{di}{dt} = \text{constant}$

i.e., $e = +ve$

36. (d) Current gain, $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{(20-100)\text{mA}}{(300-100)\text{mA}}$

$$= \frac{10 \times 10^{-3}}{200 \times 10^{-6}} = 50$$

37. (a) Field B not applied only force. Field E will apply a force opposite to velocity of the electron hence, speed will decrease.

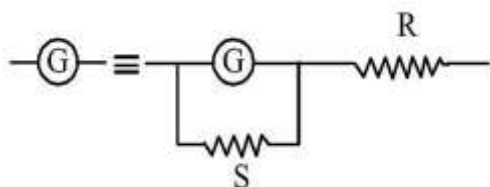
38. (a) We know magnetic field

$$\beta = \frac{\mu_0 i}{2R}$$

$$q = it \Rightarrow i = \frac{q}{t} = qf$$

$$\therefore \beta = \frac{\mu_0 qf}{2R}$$

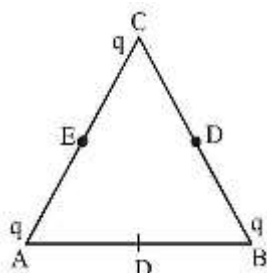
39. (c) If resistance remains same so current will be unchanged.



$$G = \frac{GS}{G+S} + R \Rightarrow R = G - \frac{GS}{G+S}$$

$$\text{or, } R = \frac{G^2}{G+S}$$

40. (c) Here, AC = BC



$$V_D = V_E = V$$

$$W = Q[V_E - V_D]$$

$$W = Q[V - V]$$

$$W = 0$$

PART-II CHEMISTRY

41. (a) $\therefore \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ (By ideal gas equation)

$$\text{or } \frac{1.5 \times V_1}{288} = \frac{1 \times V_2}{298}$$

$$\therefore V_2 = 1.55 V_1$$

i.e, volume of bubble will be almost 1.6 times to initial volume of bubble.

42. (b) (A) Plaster of Paris = $\text{CaSO}_4 \cdot \frac{1}{2} \text{H}_2\text{O}$

(B) Epsomite = $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$

(C) Kieserite = $\text{MgSO}_4 \cdot \text{H}_2\text{O}$

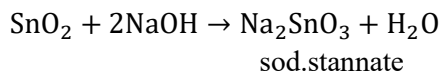
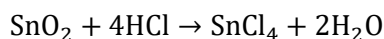
(D) Gypsum = $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$

43. (c)

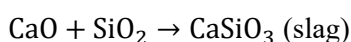
44. (a) $\text{C}_2\text{H}_5\text{OH}$ being a weaker nucleophile, when used as a solvent in case of hindered 1° halide, favours $\text{S}_\text{N}1$ mechanism while $\text{C}_2\text{H}_5\text{O}^-$ being a strong nucleophile in this reaction favours $\text{S}_\text{N}2$ mechanism.

45. (b)

46. (a) SnO_2 reacts with acids as well as bases to form corresponding salts. So it is an amphoteric oxide.

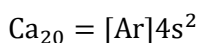
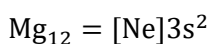


47. (c) A slag is an easily fusible material which is formed when gangue still present in the roasted or the calcined ore combines with the flux. For example, in the metallurgy of iron, CaO (flux) combines with silica gangue to form easily fusible calcium silicate (CaSiO_3) slag.



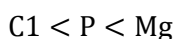
48. (b) Atomic radii increases, as the number of shells increases. Thus, on moving down a group atomic radii increases.

The electronic configuration of the given element is

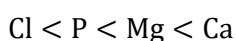


On the other hand, on increasing the number of electron in the same shell, the atomic radii decreases because effective nuclear charge is increases.

In Mg , P and Cl , the number of electrons are increasing in the same shell, thus the order of their atomic radii is



In case of Ca , the electron is entering in higher shell. So, its atomic radii is highest. Thus, the order of radii is

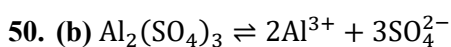


49. (b) The reaction-



Initial	1	1	0	0
At equilibrium	$1 - 0.50$	$1 - 0.25$	0.75	0.25

$$K = \frac{(0.75)^3(0.25)}{(0.50)^2(0.75)}$$



We can calculate the equivalent conductance only for ions, so the equivalent conductance at infinite dilution,

$$\Lambda_{\text{eq}}^\infty = \Lambda_{\text{Al}^{3+}} + \Lambda_{\text{SO}_4^{2-}}^\circ$$

51. (c)

$$\left[\begin{array}{l} w(\text{given mass of methane}) = 6 \text{ g} \\ \text{temperature, } T = 129 + 273 = 402 \text{ K} \\ \text{mol mass of methane, } M = 12.01 + 4 \times 1.01 \\ = 16.05 \end{array} \right]$$

From, ideal gas equation,

$$pV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$p = \frac{6}{16.05} \times \frac{8.314 \times 402}{0.03} = 41648 \text{ Pa}$$

52. (c) (A) If $K_p > Q$ and goes in forward direction than reaction is spontaneous

(B) Given, $\Delta G^\circ < RT \ln Q$, thus, $\Delta G^\circ = +ve$ and hence, the reaction is nonspontaneous.

(C) At equilibrium, $K_p = Q$

$$(D) T > \frac{\Delta H}{\Delta S}$$

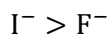
$$\text{or } T\Delta S = \Delta H$$

This is valid condition for spontaneous endothermic reactions (as $\Delta G \geq \Delta H - T\Delta S$)

53. (b) The size of cation is in order of-



and the size of anions in the order of-



Thus, when the cation is largest and anion is smallest, the ratio of their sizes is maximum.

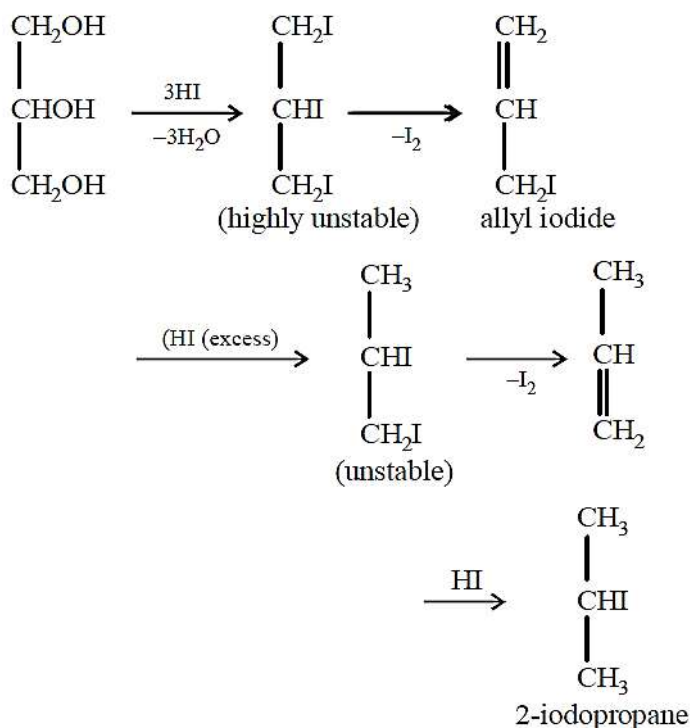
Hence, cation to anion size ratio is maximum for CsF .

54. (c) Electron deficient species are known as electrophiles.

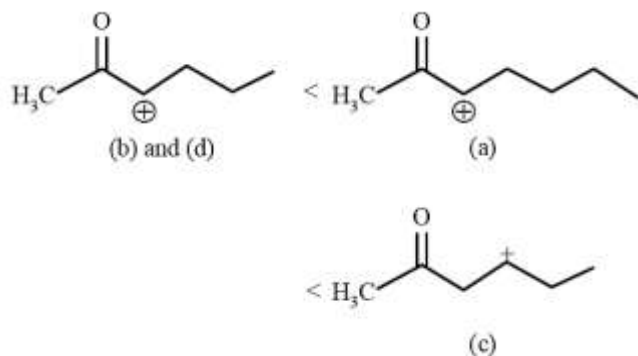
Among the given, H_3O^+ has lone pair of electrons for donation, so it is not electron deficient and hence, not an electrophile.

55. (d) Contact process is used for sulphuric acid, steel is manufactured by Bessemer's process, Leblanc process is used for the production of NaOH while Haber's process is used for NH_3 production.

56. (a)



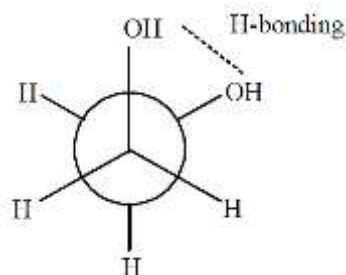
57. (a) In nuclear reactors heavy water is used as a moderator. It has higher boiling point as compared to the ordinary water. Thus, it is more associated as compared to ordinary water. The dielectric constant is however higher for H_2O , thus, H_2O is a more effective solvent as compare to heavy water (D_2O).
58. (c) Dehydration of alcohols involve formation of carbocation intermediate. Higher the stability of carbocation, higher is the ease of dehydration. The order of stability of carbocation, is



Hence, compound given in option (c) readily undergoes dehydration.

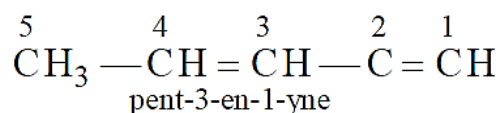
59. (c) Compounds having tetrahedral geometry does not exhibit isomerism due to presence of symmetry elements. Here, $[\text{Ni}(\text{NH}_3)_2\text{Cl}_2]$ has tetrahedral geometry.

60. (d)



This conformation is most stable due to intramolecular H-bonding.

61. (b)



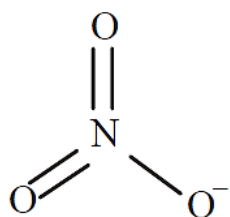
62. (d) The most common oxidation state exhibited by lanthanoids is +3 .

63. (a) In NO_3^- ,

$$H = \frac{1}{2} [5 + 0 - 0 + 1] = 3. \text{ So, } sp^3$$

hybridization.

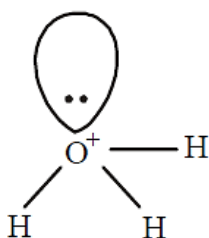
Thus, it has trigonal planar geometry.



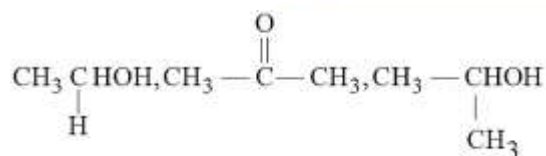
In H_3O^+ ,

$$H = \frac{1}{2} [6 + 3 - 1 + 0] = 4; \text{ So, } sp^3$$

hybridization and it has pyramidal geometry due to the presence of one lone pair of electrons.



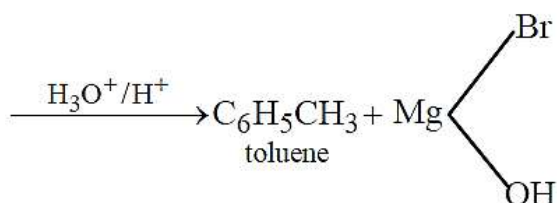
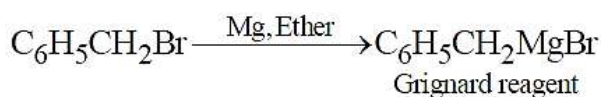
64. (c) Compounds having either $\text{CH}_3\text{C}(=\text{O})-$ group or $\text{CH}_3\text{CHOH}-$ group, give iodoform when warmed with I_2 and NaOH . Thus, compounds



give iodoform when heated with I_2 and NaOH . (Note: NaOI oxidises $\text{CH}_3\text{CH}_2\text{OH}$ to CH_3CHO , and gives positive iodoform test.)

65. (d) In aqueous medium, fructose is enolised and converted into aldehyde in basic medium. Generally, all aldehydes reduce Tollen's reagent, thus fructose can also reduce Tollen's reagent.

66. (c)



67. (a) Fat soluble vitamins are A, D and E. Whereas vitamin-B complex is soluble in water.

68. (c) In the process denaturation secondary and tertiary structures of protein destroyed but primary structure remains undisturbed. Heat, acid and alkali denature DNA molecule and double strand of DNA converts into single strand.

69. (c)

44 gCO₂ = 1 molCO₂ = N_A molecules of CO₂

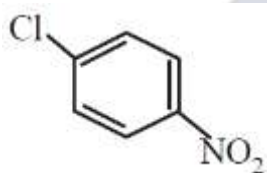
48 gO₃ = 1 molO₃ = N_A molecules of O₃

8 gH₂ = 4 molH₂ = 4 × N_A molecules of H₂

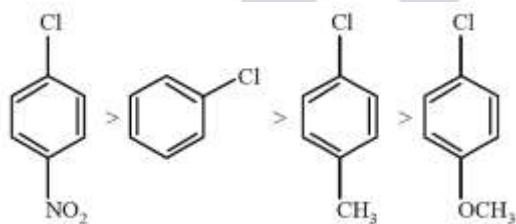
64 gSO₂ = 1 molSO₂ = N_A molecules of SO₂

N_A = 6.023 × 10²³

70. (a)



It has electron withdrawing group –NO₂ which reduces the double bond character between carbon of benzene ring and chlorine. Hence, the correct order of nucleophilic substitution reactions are,



71. (d) Freezing point depression (ΔT_f) = $iK_f m$ $\text{HA} \rightarrow \text{H}^+ + \text{A}^-$ $1 - \alpha \quad \alpha \quad \alpha$ $1 - 0.3 \quad 0.3 \quad 0.3$ $i = 1 - 0.3 + 0.3 + 0.3$ $i = 1.3$

$\therefore \Delta T_f = 1.3 \times 1.86 \times 0.1 = 0.2418^\circ\text{C}$

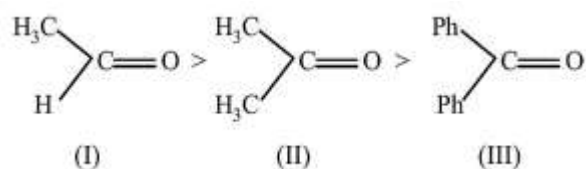
$T_f = 0 - 0.2418^\circ\text{C}$

$= -0.2418^\circ\text{C}$

72. (a) As positive charge on the central metal atom increases, the less readily the metal can donate electron density into the antibonding π -orbitals of C – O ligand to weaken the C – O bond. Thus, the C – O bond would be strongest in Mn(CO)₆⁺

73. (d) Since alkyl group has +I-effect and aryl group has +R-effect, Hence greater the number of alkyl and aryl groups attached to the carbonyl group, its reactivity towards nucleophilic addition reaction. Secondly, as the steric crowding on carbonyl group increases, the reactivity decreases accordingly.

$\therefore$ Correct reactivity order for reaction with PhMgBr is



74. (c) Radius ratio of NaCl like crystal = $\frac{r^+}{r^-}$

$$= 0.414 \text{ or } r^- \frac{100}{0.414} = 241.5 \text{ pm}$$

75. (b) $\frac{1}{2} \text{A} \rightarrow \text{B}; \Delta H = 150 \text{ kJ/mol}$

$3\text{B} \rightarrow 2\text{C} + \text{D}; \Delta H = -125 \text{ kJ/mol}$

$\text{E} + \text{A} \rightarrow 2\text{D}; \Delta H = +350 \text{ kJ/mol}$

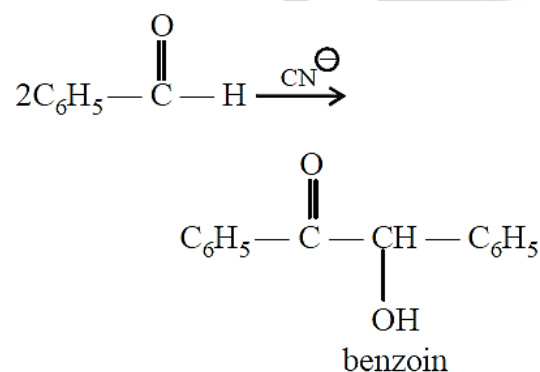
By $[2 \times (\text{i}) + (\text{ii})] - (\text{iii})$, we have

$\text{B} + \text{D} \rightarrow \text{E} + 2\text{C}$

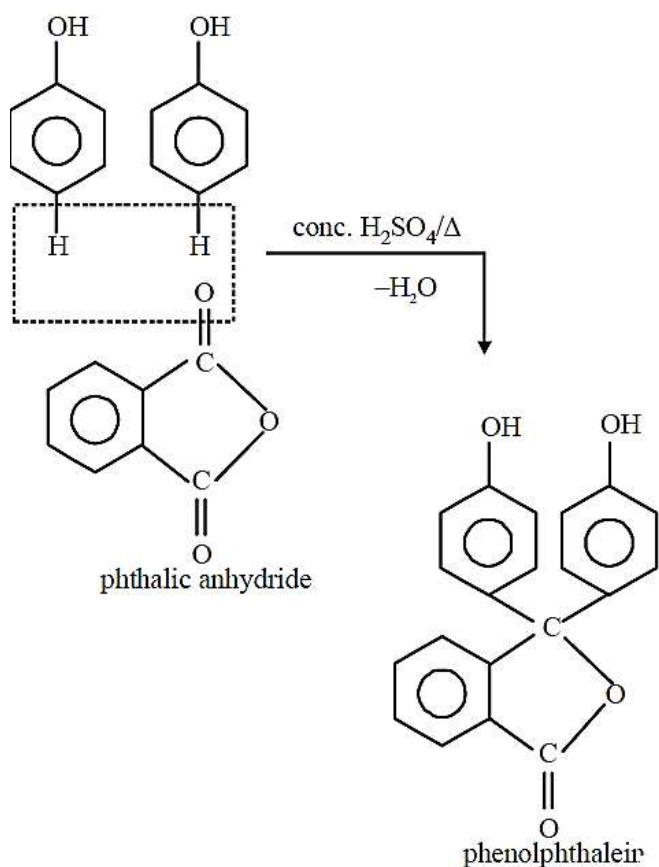
$$\therefore \Delta H = 150 \times 2 + (-125) - 350$$

$$= -175 \text{ kJ/mol}$$

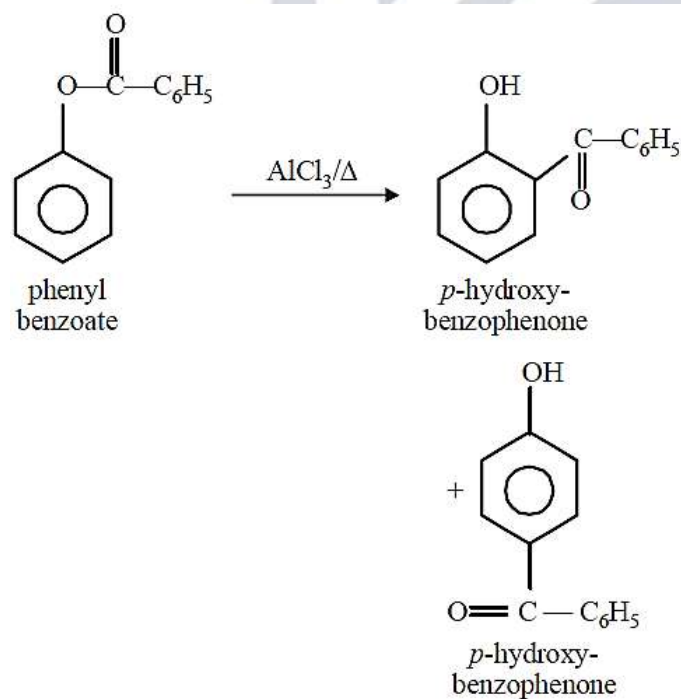
76. (d) (a) Benzoin condensation: Heating ethanolic solution with strong alkali like KCN or NaCN, benzoin is obtained.



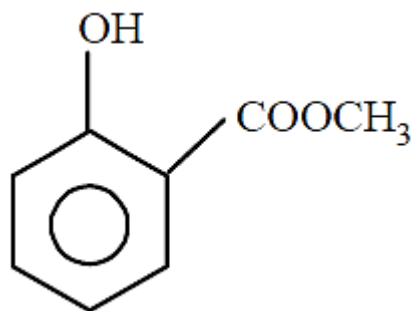
(b) Formation of phenolphthalein phenol is treated with phthalic anhydride in the presence of conc. H_2SO_4 , it gives phenolphthalein, an indicator.



(c) Fries rearrangement Phenyl benzoate heated with anhydrous AlCl_3 in the presence of inert solvent gives ortho- and Para-hydroxybenzophenone. In this rearrangement, there is only a benzoyl group migration from the phenolic oxygen to an ortho- and para-position.

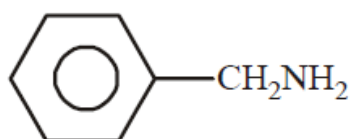


(d) Methylsalicylate



(A chief constituent of oil of wintergreen)

77. (b)



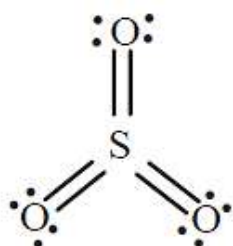
Compound is most basic due to localised lone pair of electrons on nitrogen atom. While in other compounds, because of resonance, the lone pair of electrons on nitrogen atom gets delocalised over benzene ring and thus is less easily available for donation.

78. (d) Formal charges help in selection of the lowest energy structure from a number of possible Lewis structures for a given species. Generally the lowest energy structure is the one with the smallest formal charges on the atoms.

Formal charge on an atom

$$= \text{total no. of valence electrons} - \text{non-bonding electrons} - \frac{1}{2} \times \text{bonding electrons}.$$

For Lewis structure of SO_3



Formal charge on S atom

$$= 6 - 0 - \frac{1}{2} \times 12 = 0$$

Formal charge on three O atoms

$$= 6 - 4 - \frac{1}{2} \times 4 = 0$$

79. (a) IE_1 of Na = - Electron gain enthalpy of Na^+ ion = -5.1eV

80. (a) For zero order reaction,

$$\text{Rate} = k[\text{Reactants}]^0$$

$$\therefore \text{Rate} = k$$

and unit of $k = \text{molL}^{-1} \text{s}^{-1}$

PART-III MATHEMATICS

81. (a) $\frac{dy}{dx} + \frac{2yx}{1+x^2} = \frac{1}{(1+x^2)^2}$

which is a linear differential equation.

Here, $P = \frac{2x}{1+x^2}$, $Q = \frac{1}{(1+x^2)^2}$

Now, IF = $e^{\int P dx}$

$$= \int \frac{2x}{1+x^2} dx = e^{\log(1+x^2)} = (1+x^2)$$

$\therefore$ Solution of differential equation is

$$y \cdot (1+x^2) = \int \frac{1}{(1+x^2)^2} \cdot (1+x^2) dx + C$$

$$\Rightarrow y(1+x^2) = \int \frac{1}{1+x^2} dx + C$$

$$\Rightarrow y(1+x^2) = \tan^{-1} x + C$$

82. (b)

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz) \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz)[x(y^2-z^2) - y(x^2-z^2)$$

$$+z(x^2-y^2)] = 0$$

$$\Rightarrow (1+xyz)(x-y)(y-z)(z-x) = 0$$

$$\Rightarrow 1+xyz = 0 \Rightarrow xyz = -1$$

83. (c) $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$ we know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 0.6 = P(A) + P(B) - 0.2$$

$$\Rightarrow P(A) + P(B) = 0.8$$

$$\Rightarrow 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.8$$

$$\Rightarrow -[P(\bar{A}) + P(\bar{B})] = 0.8 - 2$$

$$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.2$$

$$84. (b) Q = \frac{R \sin \alpha}{\sin(\alpha + \beta)}$$

$$\text{Also, } (5P)^2 = (4P)^2 + (3P)^2$$

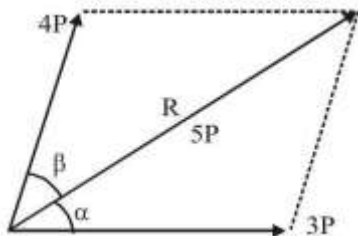
$$+ 2(4P)(3P)\cos(\alpha + \beta)$$

$$\Rightarrow 25P^2 = 16P^2 + 9P^2 + 24P^2\cos(\alpha + \beta)$$

$$\Rightarrow 24P^2\cos(\alpha + \beta) = 0$$

$$\Rightarrow \cos(\alpha + \beta) = 0 = \cos 90^\circ$$

$$\Rightarrow \alpha + \beta = 90^\circ$$



$$\text{Now, } 4P = \frac{5P \sin \alpha}{\sin 90^\circ}$$

$$\Rightarrow \sin \alpha = \frac{4}{5}$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{4}{5}\right)$$

$$85. (b) 2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\Rightarrow \frac{2\cos x}{1 - \cos^2 x} = 2\operatorname{cosec} x$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x$$

$$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$$

$$86. (d) \text{ Given conditions are } a + x = 1 \text{ and } ax = 0.$$

These two conditions will be true, if $x = a'$.

$$87. (a) (x + y) \cdot (x + 1) = x + x \cdot y + y$$

Replace '.' by '+', '+' by '!', '1' by '0', we get $(x \cdot y) + (x \cdot 0) = x \cdot (x + y) \cdot y$

88. (b) $f(x) = (x - 1)(x - 2)(x - 3)$

$$\Rightarrow f(1) = f(2) = f(3) = 0$$

$\therefore f(x)$ is not one-one.

For each $y \in \mathbb{R}$, there exists $x \in \mathbb{R}$ such that $f(x) = y$.

$\therefore f$ is onto.

Note that if a continuous function has more than one roots, then the function is always many-one.

89. (c) Let z_1, z_2 and z_3 be affixes of points A, B and C, respectively. Since, z_1, z_2 and z_3 are in AP, therefore

$$2z_2 = z_1 + z_3$$

$$\Rightarrow z_2 = \frac{z_1 + z_3}{2}$$

So, B is the mid-point of the line AC.

$\Rightarrow$ A, B and C are collinear.

$\therefore z_1, z_2$ and z_3 lie on a line.

90. (c)

$$x = 1 + a + a^2 + \dots \infty = \frac{1}{1-a}$$

$$y = 1 + b + b^2 + \dots \infty = \frac{1}{1-b}$$

$$\text{and } z = 1 + c + c^2 + \dots \infty = \frac{1}{1-c}$$

Since, a, b and c are in AP.

$\Rightarrow 1 - a, 1 - b$ and $1 - c$ are also in AP.

$\Rightarrow \frac{1}{1-a}, \frac{1}{1-b}$ and $\frac{1}{1-c}$ are in HP.

$\therefore x, y$ and z are in HP.

Note that if the common ratio of a GP is not less than 1, then we do not determine the sum of an infinite GP that series.

91. (a) Let $f(x) = -3 + x - x^2$

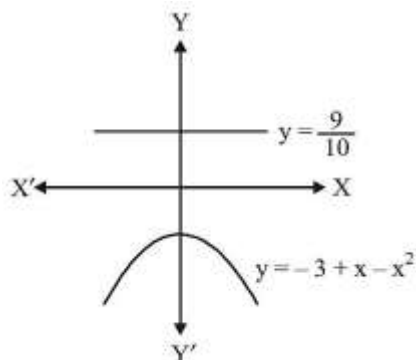
Then, $f(x) < 0$ for all x because coefficient of $x^2 < 0$ and $\text{disc} < 0$. Thus, LHS of the given equation is always positive whereas the RHS is always less than zero.

Hence, the given equation has no solution.

Alternate Solution:

Given, equation is

$$\frac{9}{10} = -3 + x - x^2$$



Let $y = \frac{9}{10}$, therefore

$$y = -3 + x - x^2$$

$$y = -\left[x^2 - x + \frac{1}{4}\right] - 3 + \frac{1}{4}$$

$$\Rightarrow y + \frac{11}{4} = -\left(x - \frac{1}{2}\right)^2$$

It is clear from the graph that two curves do not intersect. Hence, no solution exists.

92. (c) The centre of the required circle lies at the intersection of $2x - 3y - 5 = 0$ and $3x - 4y - 7 = 0$. Thus, the coordinates of the centre are $(1, -1)$.

Let r be the radius of the circle.

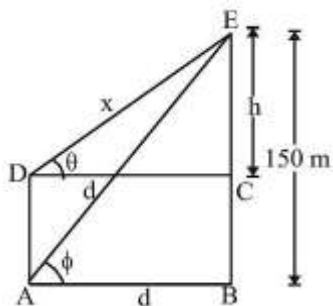
$$\pi r^2 = 154$$

$$\Rightarrow \frac{22}{7} r^2 = 154 \Rightarrow r = 7$$

Hence, the equation of required circle is $(x - 1)^2 + (y + 1)^2 = 7^2$

$$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$$

93. (d) Given : $\tan \theta = \frac{4}{3}$ and $\tan \phi = \frac{5}{2}$



In $\triangle ABE$

$$\tan \phi = \frac{150}{d}$$

$$\Rightarrow d = 150 \cot \phi$$

$$= 150 \times \frac{2}{5} = 60 \text{ m}$$

In $\triangle DCE$,

$$\tan \theta = \frac{h}{d}$$

$$\Rightarrow \frac{4}{3} = \frac{h}{d} \Rightarrow h = \frac{4}{3} \times 60 \Rightarrow h = 80 \text{ m}$$

Now in $\triangle DCE$,

$$DE^2 = DC^2 + CE^2$$

$$\Rightarrow x^2 = 60^2 + 80^2 = 10000$$

$$\Rightarrow x = 100 \text{ m}$$

94. (a) Given equation $x^2 + px + q = 0$ has roots α and α^2

$$\Rightarrow \text{Sum} = \alpha + \alpha^2 = -p \text{ and Product} = \alpha^3 = q$$

$$\Rightarrow \alpha(\alpha + 1) = -p$$

$$\Rightarrow \alpha^3[\alpha^3 + 1 + 3\alpha(\alpha + 1)] = -p^3$$

$$\Rightarrow q(q + 1 - 3p) = -p^3$$

$$\Rightarrow p^3 - (3p - 1)q + q^2 = 0$$

95. (c) $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$

Above expansion can be rewritten as

$$[(x-3) + 2]^{100} = (x-1)^{100} = (1-x)^{100}$$

$\therefore x^{53}$ will occur in T_{54} .

$$\Rightarrow T_{54} = {}^{100}C_{53} (-x)^{53}$$

$\therefore$ Required coefficient is $-{}^{100}C_{53}$.

96. (b) Equation of family of concentric circles to

the circle $x^2 + y^2 + 6x + 8y - 5 = 0$ is

$$x^2 + y^2 + 6x + 8y + \lambda = 0$$

which is similar to

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Thus, the point $(-3, 2)$ lies on the circle

$$x^2 + y^2 + 6x + 8y + c = 0$$

$$\therefore (-3)^2 + (2)^2 + 6(-3) + 8(2) + c = 0$$

$$\Rightarrow 9 + 4 - 18 + 16 + c = 0 \Rightarrow c = -11$$

97. (a) $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$

and $\mathbf{c} = 7\mathbf{i} + 9\mathbf{j} + 11\mathbf{k}$

Let $\mathbf{A} = \mathbf{a} + \mathbf{b}$

$$= (\mathbf{i} + \mathbf{j} + \mathbf{k}) + (\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$$

$$= 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$$

and $\mathbf{B} = \mathbf{b} + \mathbf{c}$

$$= (\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) + (7\mathbf{i} + 9\mathbf{j} + 11\mathbf{k})$$

$$= 8\mathbf{i} + 12\mathbf{j} + 16\mathbf{k}$$

$\therefore$ Area of parallelogram

$$= \frac{1}{2} |\mathbf{A} \times \mathbf{B}|$$

($\because$ A and B are diagonals)

$$= \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{vmatrix}$$

$$= \frac{1}{2} |\mathbf{i}(64 - 72) - \mathbf{j}(32 - 48) + \mathbf{k}(24 - 32)|$$

$$= \frac{1}{2} |-8\mathbf{i} + 16\mathbf{j} - 8\mathbf{k}|$$

$$= \sqrt{(-4)^2 + (8)^2 + (-4)^2}$$

$$= \sqrt{96} = 4\sqrt{6} \text{ sq units}$$

98. (a) We know that, $\text{tr}(\mathbf{A}) = \sum_{i=1}^n a_{ii}$

$\therefore$ If $\mathbf{A} = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$, then

$$\text{tr}(\mathbf{A}) = 1 + 7 + 9 = 17$$

99. (a) Given, $\begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ \cos(\alpha + \beta) & -\sin(\alpha + \beta) & 1 \end{vmatrix}$

[Applying $R_3 \rightarrow R_3 - R_1(\cos \beta) + R_2(\sin \beta)$]

$$= \begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ 0 & 0 & 1 + \sin \beta - \cos \beta \end{vmatrix}$$

$$= (1 + \sin \beta - \cos \beta)(\cos^2 \alpha + \sin^2 \alpha)$$

$$= 1 + \sin \beta - \cos \beta, \text{ which is independent of } \alpha.$$

100. (d) $4\sin^2 x - 12\sin x + 7$

$$= 4(\sin^2 x - 3\sin x) + 7$$

$$= 4\left[\left(\sin x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$$

$$= 4\left(\sin x - \frac{3}{2}\right)^2 - 9 + 7$$

$$= 4\left(\sin x - \frac{3}{2}\right)^2 - 2$$

We know that, $-1 \leq \sin x \leq 1$

$$\Rightarrow -\frac{5}{2} \leq \sin x - \frac{3}{2} \leq -\frac{1}{2}$$

$$\Rightarrow \frac{1}{4} \leq \left(\sin x - \frac{3}{2}\right)^2 \leq \frac{25}{4}$$

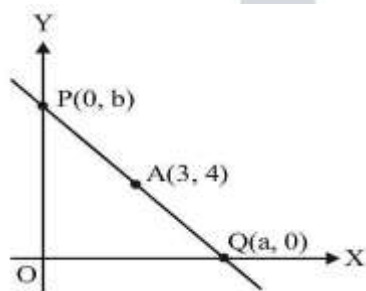
$$\Rightarrow 1 \leq 4\left(\sin x - \frac{3}{2}\right)^2 \leq 25$$

$$\Rightarrow -1 \leq 4\left(\sin x - \frac{3}{2}\right)^2 - 2 \leq 23$$

101. (b) A is mid-point of line PQ.

$$\therefore 3 = \frac{a + 0}{2} \Rightarrow a = 6$$

$$\text{and } 4 = \frac{0 + b}{2} \Rightarrow b = 8$$



Thus, equation of line is

$$\frac{x}{6} + \frac{y}{8} = 1$$

$$\Rightarrow 4x + 3y = 24$$

102. (d) The tangent at (1,7) to the curve $x^2 = y - 6$

is

$$x = \frac{1}{2}(y + 7) - 6$$

$$\Rightarrow 2x = y + 7 - 12$$

$$\Rightarrow y = 2x + 5$$

which is also tangent to the circle

$$x^2 + y^2 + 16x + 12y + c = 0$$

i.e., $x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0 \Rightarrow 5x^2 + 60x + 85 + c = 0$, which must have equal roots.

Let α and β are the roots of the equation.

$$\text{Then } \alpha + \beta = -12 \Rightarrow \alpha = -6$$

$$(\because \alpha = \beta)$$

$$\therefore x = -6, y = 2x + 5 = -7$$

$\therefore$ Point of contact is $(-6, -7)$.

103. (c) The intersection point of lines $x - 2y = 1$

and $x + 3y = 2$ is $\left(\frac{7}{5}, \frac{1}{5}\right)$

Since, required line is parallel to $3x + 4y = 0$.

Therefore, the slope of required line is $-\frac{3}{4}$.

$\therefore$ Equation of required line which passes

through $\left(\frac{7}{5}, \frac{1}{5}\right)$ is given by

$$y - \frac{1}{5} = -\frac{3}{4}\left(x - \frac{7}{5}\right)$$

$$\Rightarrow \frac{3x}{4} + y = \frac{21}{20} + \frac{1}{5}$$

$$\Rightarrow 3x + 4y - 5 = 0$$

104. (c) Let $I = \int \frac{dx}{\sin x - \cos x + \sqrt{2}}$

$$= \int \frac{dx}{\sqrt{2}\left(\sin x \sin \frac{\pi}{4} - \cos x \cos \frac{\pi}{4} + 1\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos\left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2\sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{-1}{\sqrt{2}} \int \operatorname{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx$$

$$= \frac{-1}{2\sqrt{2}} \cdot \frac{-\cot\left(\frac{x}{2} + \frac{\pi}{8}\right)}{\frac{1}{2}} + C$$

$$= \frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$$

105. (b) Let $I = \int_0^1 \sqrt{\frac{1-x}{1+x}} dx$

$$= \int_0^1 \frac{1-x}{\sqrt{1-x^2}} dx$$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx - \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$

$$= [\sin^{-1} x]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$

Put $t^2 = 1 - x^2 \Rightarrow 2t dt = -2x dx$

$$\Rightarrow t dt = -x dx$$

$$\therefore I = (\sin^{-1} 1 - \sin^{-1} 0) + \int_1^0 \frac{t}{t} dt$$

$$= \frac{\pi}{2} + [t]_1^0 = \frac{\pi}{2} - 1$$

106. (c) Let $I = \int_0^1 x \left| x - \frac{1}{2} \right| dx$

$$= -\int_0^{1/2} x \left(x - \frac{1}{2} \right) dx + \int_{1/2}^1 x \left(x - \frac{1}{2} \right) dx$$

$$= \int_0^{1/2} \left(\frac{x}{2} - x^2 \right) dx + \int_{1/2}^1 x \left(x^2 - \frac{x}{2} \right) dx$$

$$= \left[\frac{x^2}{4} - \frac{x^3}{3} \right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{x^2}{4} \right]_{1/2}^1$$

$$= \left(\frac{1}{16} - \frac{1}{24} \right) + \left(\frac{1}{3} - \frac{1}{4} - \frac{1}{24} + \frac{1}{16} \right)$$

$$= \left(\frac{6-4}{96} \right) + \left(\frac{32-24-4+6}{96} \right)$$

$$= \frac{12}{96} = \frac{1}{8}$$

107. (b) Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2}$

$$= 1.$$

It is given that it passes through (7,0) and (0,-5).

Therefore, $a^2 = 49$ and $b^2 = 25$

The eccentricity of the ellipse is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{25}{49}} = \sqrt{\frac{24}{49}} = \frac{2\sqrt{6}}{7}$$

108. (c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$... (i)

On differentiating w.r.t. x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^2}{a^2 y} \text{ and}$$

$$x^2 - y^2 = c^2$$

On differentiating w.r.t. x, we get

$$2x - 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

The two curves will cut at right angles, if

$$\left(\frac{dy}{dx}\right)_{c_1} \times \left(\frac{dy}{dx}\right)_{c_2} = -1$$

$$\Rightarrow -\frac{b^2 x}{a^2 y} \cdot \frac{x}{y} = -1$$

$$\Rightarrow \frac{x^2}{a^2} = \frac{y^2}{b^2}$$

$$\Rightarrow \frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{1}{2}$$

[using eq. (i)]

On substituting these values in $x^2 - y^2 = c^2$, we get

$$\frac{a^2}{2} - \frac{b^2}{2} = c^2$$

$$\Rightarrow a^2 - b^2 = 2c^2$$

109. (c) Let P(x, y) be any point on the conic. Then,

$$\sqrt{(x-1)^2 + (y+1)^2} = \sqrt{2} \left(\frac{x-y+1}{\sqrt{2}} \right)$$

$$\Rightarrow (x-1)^2 + (y+1)^2 = (x-y+1)^2$$

$$\Rightarrow 2xy - 4x + 4y + 1 = 0$$

110. (b) Required numbers

$$= 5! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right] = 44$$

Note that if r ($0 \leq r \leq n$) objects occupy the original places and none of the remaining $(n - r)$ objects occupies its original places then the number of such arrangements = ${}^nC_r \cdot (n - r)!$

$$\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^{n-2} \frac{1}{(n - r)!} \right]$$

111. (b) $T_n = \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n!}$

$$= \frac{\sum n^2}{n!} = \frac{n(n + 1)(2n + 1)}{6n!}$$

$$= \frac{1}{6} \left(\frac{2n^3 + 3n^2 + n}{n!} \right)$$

$$= \frac{1}{6} \left(2 \cdot \frac{n^3}{n!} + \frac{3n^2}{n!} + \frac{n}{n!} \right)$$

$\therefore$ Sum of the series

$$= \frac{1}{6} \left(2 \sum_{n=1}^{\infty} \frac{n^3}{n!} + 3 \sum_{n=1}^{\infty} \frac{n^2}{n!} + \sum_{n=1}^{\infty} \frac{n}{n!} \right)$$

$$= \frac{1}{6} (2 \times 5e + 3 \times 2e + e)$$

$$= \frac{1}{6} (10e + 6e + e) = \frac{17}{6} e$$

112. (b) $\log_a(1 + x) = \log_e(1 + x) \log_a e$

$$= \log_a e \left[\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \right]$$

So, the coefficient of x^n in $\log_a(1 + x)$ is $\frac{(-1)^{n-1}}{n} \log_a e$.

113. (b) Let the equation of the required plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

This meets the coordinate axes at A, B and C, the coordinates of the centroid of $\triangle ABC$

are $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3} \right)$

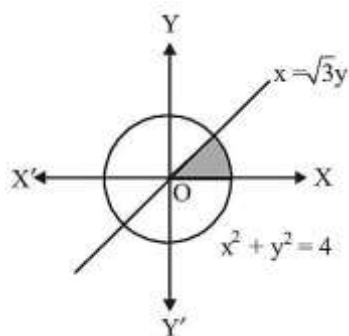
$$\therefore \frac{a}{3} = 1, \frac{b}{3} = 2, \frac{c}{3} = 3$$

$$\Rightarrow a = 3, b = 6, c = 9$$

Hence, the equation of the plane is

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$$

114. (c) Required area



$$= \int_0^1 (x_2 - x_1) dy$$

$$= \int_0^1 (\sqrt{4 - y^2} - \sqrt{3}y) dy$$

$$= \left[\frac{1}{2} y \sqrt{4 - y^2} + \frac{1}{2} (4) \sin^{-1} \frac{y}{2} - \frac{\sqrt{3} y^2}{2} \right]_0^1 = \frac{\sqrt{3}}{2} + 2 \sin^{-1} \left(\frac{1}{2} \right) - \frac{\sqrt{3}}{2} - 2 \sin^{-1} 0$$

$$= \frac{\sqrt{3}}{2} + 2 \left(\frac{\pi}{6} \right) - \frac{\sqrt{3}}{2} = \frac{\pi}{3} \text{ sq units}$$

115. (b) Let $y = \lim_{x \rightarrow \infty} \left(\frac{\pi}{2} - \tan^{-1} x \right)$

Taking log on both sides, we get

$$\log y = \lim_{x \rightarrow \infty} \frac{1}{x} \log \left(\frac{\pi}{2} - \tan^{-1} x \right) \text{ form } \left(\frac{0}{\infty} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{\left(-\frac{1}{1 + x^2} \right)}{\frac{\pi}{2} - \tan^{-1} x}$$

(using L'Hospital's rule)

$$= \lim_{x \rightarrow \infty} \frac{\frac{2x}{(1 + x^2)^2}}{-\left(\frac{1}{1 + x^2} \right)}$$

(using L'Hospital's rule)

$$= \lim_{x \rightarrow \infty} \frac{-2x}{1 + x^2} = 0 \Rightarrow y = e^0 = 1$$

116. (c) $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$$\text{So, } \lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{f \rightarrow \frac{\pi}{2}} f(x)$$

$$x \rightarrow \frac{\pi^-}{2} \quad x \rightarrow \frac{\pi^+}{2}$$

$$\Rightarrow m \frac{\pi}{2} + 1 = \sin \frac{\pi}{2} + n$$

$$\Rightarrow m \frac{\pi}{2} + 1 = 1 + n \Rightarrow \frac{m\pi}{2} = n$$

$$117. \quad (c) f(x) = \frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$$

$\sqrt{4-x^2}$ is defined for $4-x^2 \geq 0$.

$$\Rightarrow x^2 \leq 4$$

$$\Rightarrow -2 \leq x \leq 2$$

and $\sin^{-1}(2-x)$ is defined for $-1 \leq 2-x \leq 1$

$$\Rightarrow -3 \leq -x \leq -1$$

$$\Rightarrow 1 \leq x \leq 3$$

Also, $\sin^{-1}(2-x) = 0$ for $x = 2$

$$\therefore \text{Domain of } f(x) = [-2, 2] \cap [1, 3] - \{2\} = [1, 2)$$

$$118. \quad (c) (1+y^2)dx + (1+x^2)dy = 0$$

$$\Rightarrow \frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$$

On integrating, we get

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} C$$

$$\Rightarrow \frac{x+y}{1-xy} = C$$

$$\Rightarrow x+y = C(1-xy)$$

$$119. \quad (a) \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$\Rightarrow \rho \left(\frac{d^2y}{dx^2}\right) = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

On squaring both sides, we get

$$\rho^2 \left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3$$

Clearly, it is a second order differential equation of degree 2.

Note that the higher order derivative is in the transcendental, then we do not determine the degree of that equation.

$$120. \quad (b) \text{ Let } R = \{(a, b) : a, b \in \mathbb{N}, a - b = 3\}$$

$$= \{(n+3, n) : n \in \mathbb{N}\}$$

$$= \{(4, 1), (5, 2), (6, 3), \dots\}$$

