PART-I PHYSICS

1. **(d)** As we know, velocity of electromagnetic wave, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s

It is independent of amplitude of electromagnetic wave, frequency and wavelength of electromagnetic wave.

2. **(b)** According to Moseley's law $\sqrt{v} = a(z-b)$ or $v = a^2(z-b)^2$

or $\frac{c}{\lambda} = a^2(z-b)^2$

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{(z_2-1)^2}{(z_1-1)^2}$$

Here $\lambda_1 = \lambda, \lambda_2 = 4\lambda, z_1 = 11$ and $z_2 = ?$

$$\therefore \frac{1}{4\lambda} = \frac{(z_2-1)^2}{(11-1)^2}$$

or $(z_2-1)^2 = 25$ or $z_2 = 6$

3. **(b)** As we know, conductivity,

$$\sigma = \frac{1}{\rho} = \rho(\mu_e n_e + \mu_n n_n)$$

$$= 1.6 \times 10^{-19}[0.36 + 0.17](2.5 \times 10^{19})]$$

$$= 2.12 \mathrm{Sm}^{-1}$$

4. **(d)** If a radio receiver amplifies all the signal frequencies equally well, it is said to have high fidelity.

5. **(b)** Given, $y = 3\sin\pi\left(\frac{t}{2} - \frac{x}{4}\right)$

$$= 3\sin\left(\frac{\pi t}{2} - \frac{\pi x}{4}\right)$$

Comparing it with standard equation

$$y = r\sin\frac{2\pi}{\lambda}(vt - x)$$

$$= r\sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

We have, $\frac{2\pi v}{\lambda} = \frac{\pi}{2}$ or $v = \frac{\lambda}{4}$ and $\frac{2\pi}{\lambda} = \frac{\pi}{4}$ or $\lambda = 8$ m $\therefore v = \frac{8}{4} = 2$ m/s

So, the distance travelled by wave in t second $= vt = 2 \times 5 = 10$ m

6. **(a)** Gravitational intensity,

$$I = -\frac{dv}{dx} = -\frac{d}{dx}\left(\frac{k}{x^2}\right) = \frac{2k}{x^3}$$

7. **(d)** For Cu wire, $1_1 = 2.2$ m, $r_1 = 1.5$ mm $= 1.5 \times 10^{-3}$n

$Y_1 = 1.1 \times 10^{11}$ N/m^2

For steel wire, $1_2 = 1.6$ m, $r_2 = 1.5$ mm

$= 1.5 \times 10^{-3}$ m

$Y_2 = 2.0 \times 10^{11}\ N/m^2$

Let F be the stretching force in both the wires the

For Cu wire, $Y_1 = \frac{F}{\pi r_1^2} \times \frac{l_1}{\Delta l_1}$

$$\Rightarrow F = \frac{Y_1 \pi r_1^2 \times \Delta l_1}{l_1}$$

$$= \frac{1.1 \times 10^{-11}}{2.2} \times \frac{22}{7} \times (1.5 \times 10^{-3})^2 \times 0.5 \times 10^3$$

$= 1.8 \times 10^2$ N

8. **(a)** Mean speed, $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 0.92 v_{rms}$

rms speed, $v_{rms} = \sqrt{\frac{3kT}{m}}$

Most probable speed v_p

$$= \sqrt{\frac{2kT}{m}} = 0.816 v_{rms}$$

i.e., $v_p < \bar{v} < v_{rms}$

9. **(b)** Let two balls collide at a height s from the ground after t second when second ball is thrown upwards.

$\therefore$ Time taken by first ball to reach the point of collision $= (t + 2)$s

$$s = 39.2(t+2) + \frac{1}{2}(-9.8)(t+2)^2$$
$$= 39.2(t+2) - 4.9(t+2)^2 \ \ldots \text{(i)}$$

For second ball

$$s = 39.2t + \frac{1}{2}(-9.8)t^2$$

$$= 39.2t - 4.9t^2$$

From eqs. (i) and (ii)

$$39.2(t+2) - 4.9(t+2)^2 = (39.2)t - 4.9t^2$$

On solving we get, $t = 3$ s

From Eq. (ii),

$$s = 39.2 \times 3 - 4.9 \times (3)^2 = 117.6 - 44.1 = 73.5\text{m}$$

10. **(b)** Magnetic flux, $\phi = \mathbf{B} \cdot \mathbf{A} = \frac{F}{I\lambda} \cdot \mathbf{A}$

$= \dfrac{[M^1\ L^{+1}\ T^{-2}]L^2}{[\ A.L]} = [M^1\ L^2\ T^{-2}\ A^{-1}]$

11. **(c)** Given, $P = P_0 \exp(-\alpha t^2)$

As P and P_0 have the same units, therefore αt^2 must be dimensionless for which

$\alpha = \dfrac{1}{T^2} = T^{-2}$

12. **(d)** Given, $U = \frac{M}{r^6} - \frac{N}{r^{12}}$

$\therefore F = \dfrac{-du}{dr} = \dfrac{-d}{dr}\left(\dfrac{M}{r^6} - \dfrac{N}{r^{12}}\right)$

$= -\left(\dfrac{-6M}{r^7} + \dfrac{12\ N}{r^{13}}\right) = \left(\dfrac{6M}{r^7} - \dfrac{12\ N}{r^{13}}\right)$

For equilibrium position, $F = 0$

$\therefore \frac{6M}{r^7} = \frac{12\ N}{r^{13}}$ or $r^6 = \frac{2\ N}{M}$

Hence, $U = \frac{M}{(2N/M)} - \frac{N}{(2N/M)^2} = \frac{M^2}{4N}$

13. **(b)** Total number of revolutions = area under n-t graph

$= \frac{1}{2} \times 8 \times \frac{1800}{60} + 8 \times \frac{600}{60} + \frac{1}{2} \times 16 \times \frac{600}{60} = 120 + 80 + 80 = 280$

14. **(c)** From the graph we can say, upto $t = 2.0$ s, the body moves with a constant velocity

Slope of position-time graph $= \frac{4}{2} = 2\ m/s$

After $t = 2.0$ s, position-time graph is parallel to time axis i.e., body comes to rest.

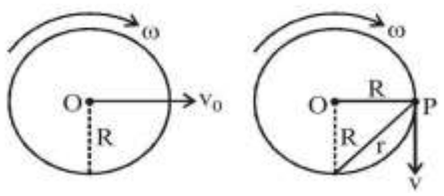
$\therefore$ Change in velocity $= dx = 2\ m/s$ Impulse = Change in momentum $= mdv = 0.1 \times 2 = 0.2$ kg m/s

15. **(d)** Pressure $= \frac{\text{force}}{\text{area}} = \frac{F}{L^2}$

$\therefore \dfrac{\Delta p}{p} = \dfrac{\Delta F}{F} + \dfrac{2\Delta L}{L} = 4\% + 2(2\%)$

or percentage error, $= 8\%$

16. **(d)** The situation can be shown as



Here $v_0 = R\omega$

At. P, $v = r\omega$

$= \sqrt{(R^2 + R^2)}\omega = \sqrt{2}R\omega = \sqrt{2}v_0$

17. (b) Using principle of conservation of linear momentum,

$3\ \mathrm{m} \times \mathrm{v} = \sqrt{(\mathrm{m} \times 60)^2 + (\mathrm{m} \times 60)^2}$

$= \mathrm{m} \times 60\sqrt{2}$

$\Rightarrow \mathrm{v} = 20\sqrt{2}\ \mathrm{m/s}$

18. (a) Common velocity, $\mathrm{v} = \frac{\mathrm{m_1 v_1 + m_2 v_2}}{\mathrm{m_1 + m_2}}$

$= \frac{2 \times 6 + 2 \times 0}{2 + 2} = 3\ \mathrm{m/s}$

Initial kinetic energy ($\mathrm{E_1}$)

$= \frac{1}{2}\mathrm{m_1 v_1^2} + \frac{1}{2}\mathrm{m_2 v_2^2} = \frac{1}{2} \times 2 \times (6)^2 = 36\ \mathrm{J}$

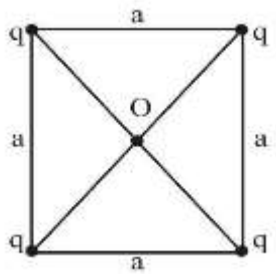
$= \frac{1}{2}(\mathrm{m_1 + m_2})\mathrm{V}^2 = \frac{1}{2}(2 + 2)(3)^2 = 18\ \mathrm{J}$

$\therefore$ Heat evolved $= (36 - 18)\mathrm{J} = 18\ \mathrm{J}$

19. (a) From given condition

$\frac{\mathrm{Gmm}}{(2\mathrm{R})^2} = \frac{\mathrm{mv}^2}{\mathrm{R}}, \mathrm{v} = \frac{\mathrm{Gm}}{4\mathrm{R}} = \frac{1}{2}\sqrt{\frac{\mathrm{Gm}}{\mathrm{R}}}$

20. (d) At the centre O of the square due to four equal charges q at the corners, potential

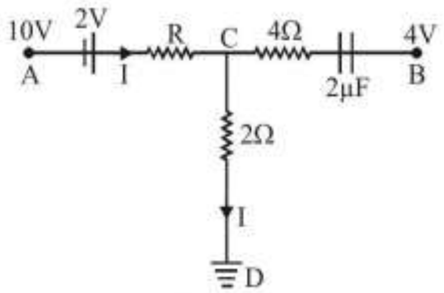


$\mathrm{V} = \frac{4\mathrm{q}}{4\pi\varepsilon_0(\mathrm{a}\sqrt{2})/2} = \frac{\sqrt{2}\mathrm{q}}{\pi\varepsilon_0 \mathrm{a}}$

$\therefore \mathrm{W_{0\infty}} = -\mathrm{W_{\infty 0}}$

$= -(-\mathrm{q})\mathrm{V} = \frac{\sqrt{2}\mathrm{q}^2}{\pi\varepsilon_0 \mathrm{a}}$

21. (a) In the steady state, current through capacitor



Using Kirchhof's voltage law to the circuit ACD

We have, $10 - 2 + 1 \times R + 1 \times 2 = 0$ or $R = 10\Omega$

Potential difference across C and D

$V_C - V_D = 2 \times 1 = 2$ V

As $V_D = 0$ V

So, $V_C = 2$ V

Potential difference across capacitor $= 4 - 2 = 2$ V

$\therefore$ Charge on capacitor $Q = CV = 2\mu F \times 2 = 4\mu C$

22. **(a)** KE of an electron in nth orbit: $K_n \propto \frac{1}{n^2}$ and PE of an electron in nth orbit:

$U_n \propto \dfrac{1}{n^2}$

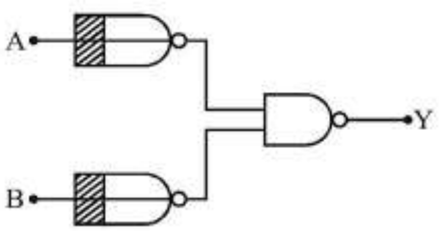
$\therefore$ When an electron passes from state $n = 2$ to $n = 1$

$\frac{K_2}{K_1} = \frac{1^2}{2^2} = \frac{1}{4}$ or $K_1 = 4\,K_2$

$\dfrac{U_2}{U_1} = \dfrac{1^2}{2^2} = \dfrac{1}{4}$

or $U_1 = 4U_2$

23. **(b)** To obtain OR gate from NAND gates we need two NOT gates obtained from NAND gates and one NAND gate as figure.



Boolean expression $\gamma = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$ OR gate

24. **(b)** Magnetic field $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$

Here, $2\pi = \theta$

$\therefore B = \frac{\mu_0}{4\pi}\frac{\theta I}{r}$

25. **(a)** Since axes are perpendicular so midpoint lies on axial line of one magnet and on equitorial line of other magnet

$$\therefore B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = \frac{10^{-7} \times 2 \times 1}{1^3} = 2 \times 10^{-7}\ T$$

and $B_2 = \frac{\mu_0}{4\pi}\frac{M}{d^3} = \frac{10^{-7}\times 1}{1^3} = 10^{-7}\ T$

As $B_1 \perp B_2$

$\therefore$ Resultant magnetic field

$$= \sqrt{B_1^2 + B_2^2} = \sqrt{5} \times 10^{-7}\ T$$

26. **(a)** In the given circuit, two condensors and the inductor are in series.

$\therefore L_s = L + L = 2\ L$

and $\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_s = \frac{C}{2}$

$\therefore$ Natural frequency of the circuit

$$v = \frac{1}{2\pi\sqrt{L_s C_s}} = \frac{1}{2\pi\sqrt{2\ L} \times \sqrt{C/2}}$$

$$= \frac{1}{2\pi\sqrt{LC}}$$

27. **(a)** In the beginning due to gravity pull, the lead shot will be accelerated and hence will move, with increasing velocity for some time When the viscous force balance the gravity pull, then the shot will move with constant velocity. As in the beginning, the velocity of shot is not fully linear with the effective distance covered by the shot.

28. **(b)** Refractive index of a medium $\mu = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

29. **(c)** Lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or $\frac{u}{v} - 1 = \frac{u}{f}$

$$\Rightarrow \frac{1}{m} - 1 = \frac{u}{f}$$

or $\frac{1}{m} = \left(\frac{1}{f}\right)u + 1$

this is the equation of a straight line whose slope

$$\frac{1}{f} = \frac{b}{c} \therefore f = \frac{c}{b}$$

30. **(d)** Amplitudes A_1 and A_2 are added as vectors. Angle between the two vectors is the phase difference $(\beta_1 - \beta_2)$ between them.

∴ Resultant wave,

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\beta_1 - \beta_2)}$$

31. **(a)** Here, case (i) $e(3V_0) = \frac{hc}{\lambda} - \phi_0$

Case (ii) $eV_0 = \frac{hc}{2\lambda} - \phi_0$

From eqs. (i) and (ii),

$$\frac{3hc}{2\lambda} - 3\phi_0 = \frac{hc}{\lambda} - \phi_0$$

or $\frac{3hc}{2\lambda} - \frac{hc}{\lambda} = 3\phi_0 - \phi_0 = \frac{hc}{2\lambda} = 2\phi_0$

or, $\phi_0 = \frac{hc}{4\lambda}$

∴ Threshold wavelength

$$\lambda_0 = \frac{hc}{\phi_0} = \frac{hc}{hc} \times 4\lambda = 4\lambda$$

32. **(b)** In an isobaric process, $p =$ constant Hence, $V \propto T$

i.e., $V = \left(\frac{nR}{P}\right)T$

∴ $V - T$ graph is a straight line with slope $\propto \frac{1}{p}$

$(\text{slope})_2 > (\text{slope}\ _1$

∴ $p_2 < p_1$

33. **(a)** Total energy

$$U = 2\left(\frac{n_1}{2}RT\right) + 4\left(\frac{n_2}{2}RT\right)$$

For $O_2, n = 5$ and for $Ar, n_2 = 3$

$$\therefore U = 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right) = 11RT$$

34. **(b)** Given, speed of each pulse $= 2$ cm/s

Therefore distance travelled by both pulses in 2 s = 4 cm toward each other. On their superposition, the resultant displacement at every point will be zero.

Hence, total energy will be purely kinetic.

35. **(d)** Time interval between two successive maxima = time interval between two

successive beats $= \frac{1}{n_1 - n_2}$

36. **(b)** Here, $h = \frac{2\ s\cos 0^\circ}{r\rho g} = \frac{2\ s}{r\rho g}$

According to question,

$$\frac{h}{2}=\frac{2\,s\cos\theta'}{rpg}$$

Dividing eq. (ii) by (i) we get,

$$\frac{1}{2}=\cos\theta'$$

or $\theta' = 60°$

37. (a) **G**iven, $V_1 = V_2 = 300\ V; V_3 = ?, i = ?$

As, $V=\sqrt{V_3^2+(V_1-V_2)^2}$

$$\therefore 220=\sqrt{V_3^2}=V_3 \Rightarrow V_3=220\ V$$

$$\therefore I=\frac{V_3}{R}=\frac{220}{100}=2.2\ A$$

38. (b) We have, $W=-MB(\cos\theta_2-\cos\theta_1)$

So, $W_1=-MB(\cos 90°-\cos 0°)=MB$

and $W_2=-MB(\cos 60°-\cos 0°)=\frac{1}{2}MB$

As $W_1 = nW_2$

$$\therefore n=\frac{W_1}{W_2}=\frac{MB}{\frac{1}{2}MB}=2$$

39. (b) Capacitance, $C=\frac{\varepsilon_0\,A}{d}$

As one-fourth of capacitor is filled with dielectric of constant K, then,

$$C_1=\frac{K\varepsilon_0\,A/4}{d}$$

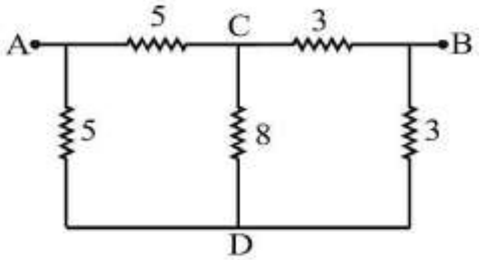
and $C_2=\frac{\varepsilon 3\,A/4}{d}$

Both C_1 and C_2 are in parallel.

$$\therefore C_P=C_1+C_2=\frac{K\varepsilon_0\,A}{4\,d}+\frac{3\varepsilon_0\,A}{4\,d}$$

$$=(K+3)\frac{\varepsilon_0\,A}{4\,d}=(K+3)\frac{C}{4}$$

40. (c) The equivalent circuit of the given circuit is



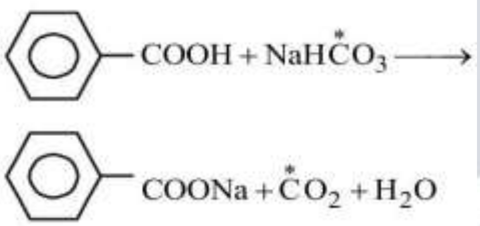
This is a balanced Wheatstone bridge. Therefore, the arm CD becomes in effective. Hence 5Ω and 3Ω are in series and they together are in parallel with $(5 + 3)\Omega$

$\therefore$ Net resistance $= \frac{(5+3)+(5+3)}{(5+3)\times(5+3)} = 4\Omega$

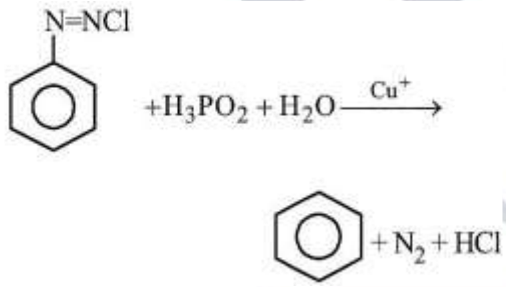
PART-II CHEMISTRY

41. **(a)** Benzoin condensation is performed by aromatic aldehydes (i.e., compounds in which $-\text{CHO}$ group is directly attached with benzene ring).

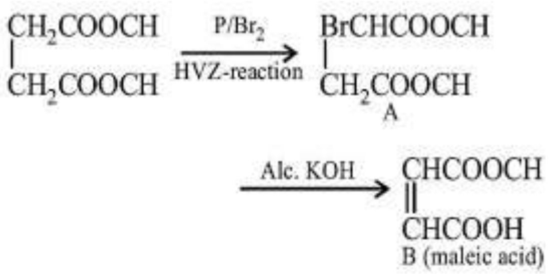
42. **(a)**

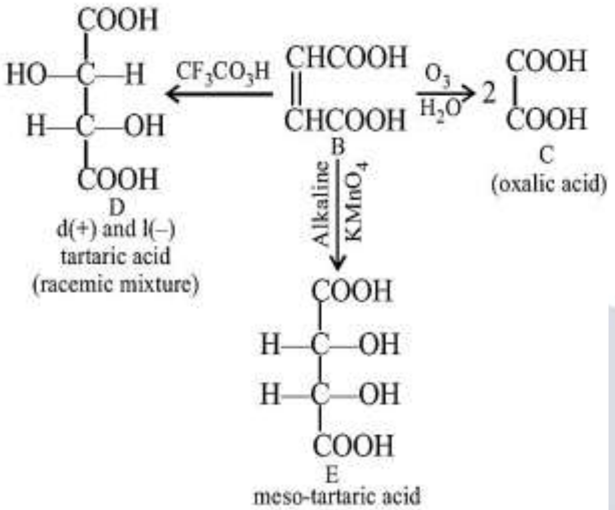


43. **(d)**

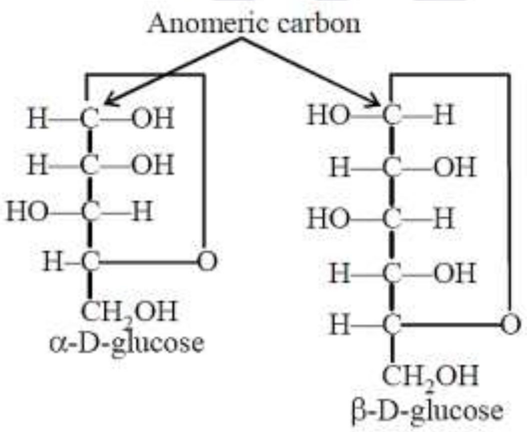


44. **(c)**





45. (d) When two cyclic forms of a carbohydrate differ in configuration only at hemiacetal carbons, they are said to be anomers. Thus, anomers are cyclic forms of carbohydrates that are epimeric at hemiacetal carbon and this carbon ($C-1$ of aldose) is called anomeric carbon, e.g.,



46. (c)

47. (d) $[CH_3COOH] = \text{millimoles of } CH_3COOH = 0.1 \times 10 = 1.0$

$[CH_3COONa] = \text{millimoles of } CH_3COONa = 0.1 \times 20 = 2.0$

From, Henderson Hasselbalch equation,

$$pH = pK_a + \log \frac{[\text{conjugate base }]}{[\text{acid }]}$$

$$= 4.74 + \log \frac{2}{1} = 4.74 + 0.30 = 5.04$$

48. (b) $\lambda^{\infty}(Ag^{+})$ =transport number of

$Ag^{+} \times A^{\infty}_{(AgNO_3)}$

$= 0.464 \times 133.3 = 61.9\Omega^{-1}\ cm^2$ equiv $^{-1}$

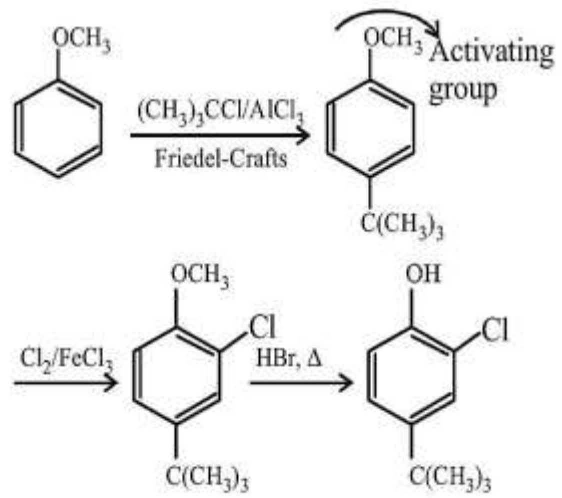
By Kohlrausch's law

$\Lambda^{\infty}(AgNO_3) = \lambda^{\infty}_{(Ag^{+})} + \lambda^{\infty}_{(NO_3^{-})}$

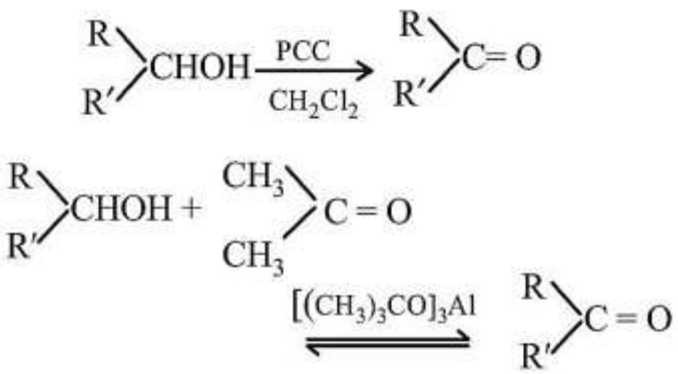
$\therefore \Lambda^{\infty}_{(NO_3^{-})} = \Lambda^{\infty}_{(AgNO_3)} - \lambda^{\infty}_{(Ag^{+})}$

$= 133.3 - 61.9 = 71.4\Omega^{-1}\ cm^2$ equiv $^{-1}$

49. (d)

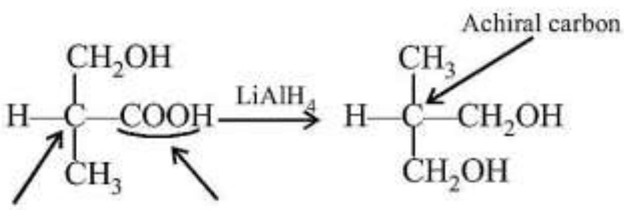


50. (c) Oxidation of Ketones, yield secondary alcohol

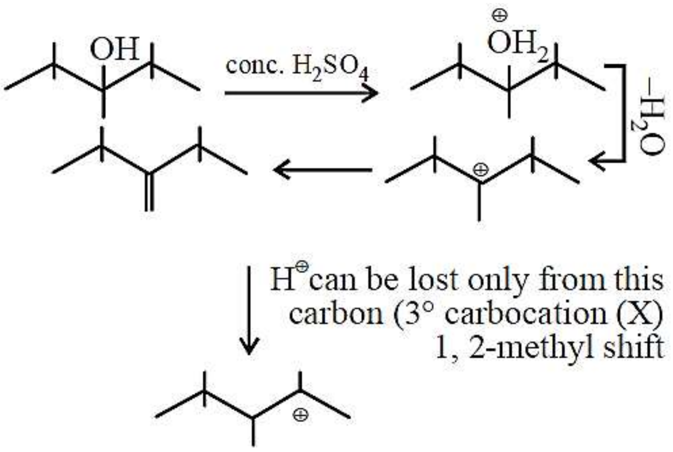


51. (c) Since, X $\xrightarrow{NaHCO_3}$ CO_2

Hence, it must contain $-COOH$ group.

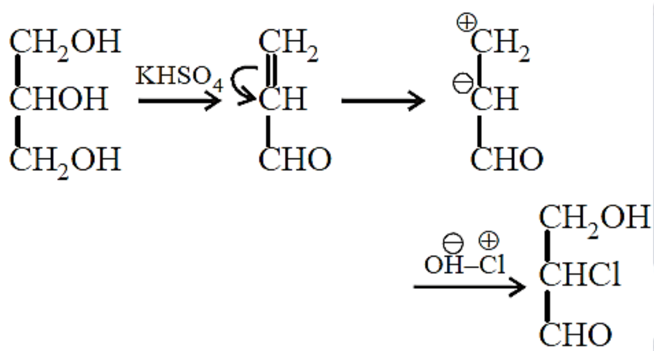


52. (a)

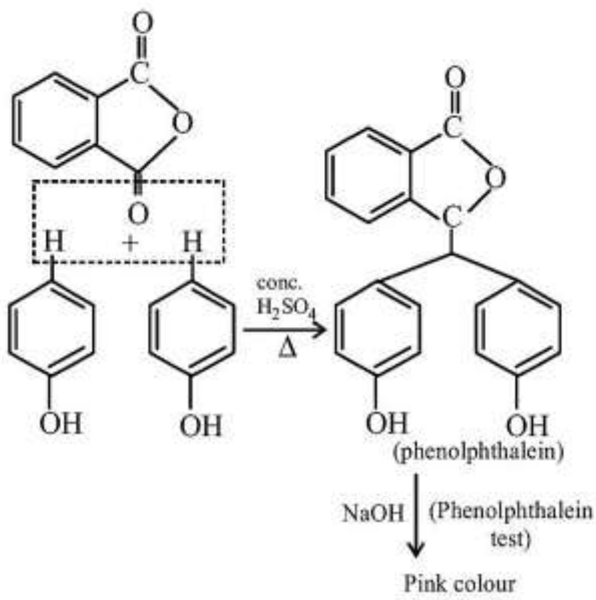


Y is less soluble than (X) due to lack of symmetry Chiral carbon. This is reduced to $-CH_2OH$

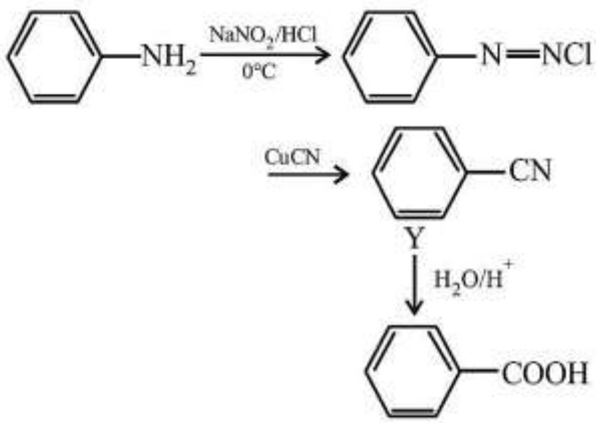
53. (b)



54. (a)



55. (d)



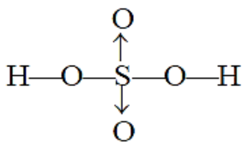
56. (a)

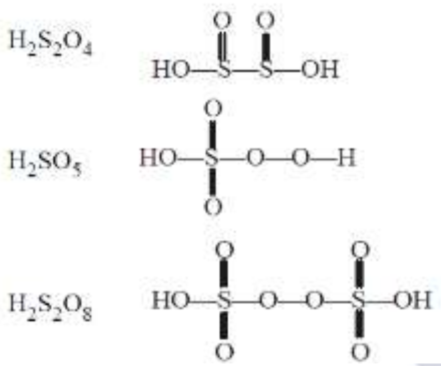
57. (c) $NH_4Cl\ (s) \xrightarrow{\Delta} NH_3 \uparrow + HCl \uparrow$

Graham's law of diffusion says, lighter gas will diffuse most rapidly. Therefore, NH_3 will be (mol. wt. = 17) diffused rapidly than HCl (mol. wt. = 36.5).

58. (b) Peroxy acids contain - $-O-$ linkage.

$H_2S_2O_3$





59. (d) Volume of one molecule

$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.54 \times 10^{-8})^3\ cm^3$

$= 1.53 \times 10^{-23}\ cm^3$

Volume of molecules in 1.65 gAr

$= \frac{1.65}{40} \times N_0 \times 1.53 \times 10^{-23} = 0.380\ cm^3$

Volume of solid containing 1.65 gAr $= 1\ cm^3$

$\therefore$ Empty space $= 1 - 0.380 = 0.620$

$\therefore$ Per cent of empty space $= 62\%$

60. (d) In adiabatic expansion

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

for CO_2 (triatomic gas) is, $\gamma = 1.33$

$$\therefore \left(\frac{150}{300}\right) = \left(\frac{10}{V_2}\right)^{0.33}$$

$$\left(\frac{1}{2}\right) = \left(\frac{10}{V_2}\right)^{0.33}$$

$$\left(\frac{1}{2}\right)^3 = \frac{10}{V_2}$$

$$\frac{1}{8} = \frac{10}{V_2}$$

$$V_2 = 80\ \text{L}$$

61. (c)

62. (b) Energy values are additives.

$$E = E_1 + E_2$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

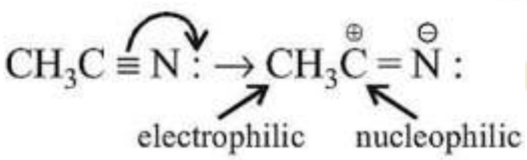
$$\frac{1}{300} = \frac{1}{760} + \frac{1}{\lambda_2}$$

$$\frac{1}{\lambda_2} = \frac{1}{300} - \frac{1}{760}$$

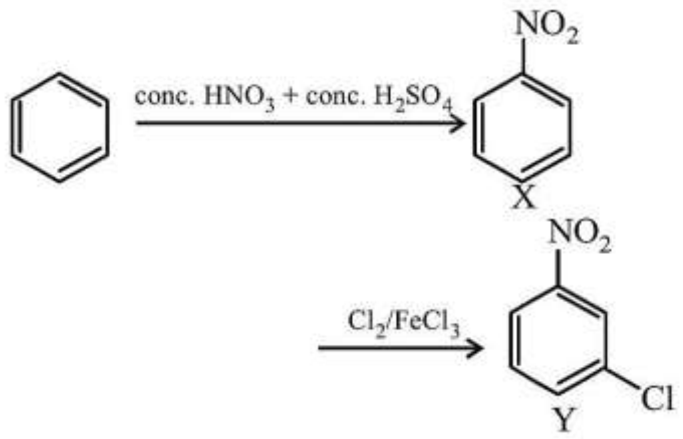
$$\lambda = 495.6\ \text{nm} = 496\ \text{nm}$$

63. (a)

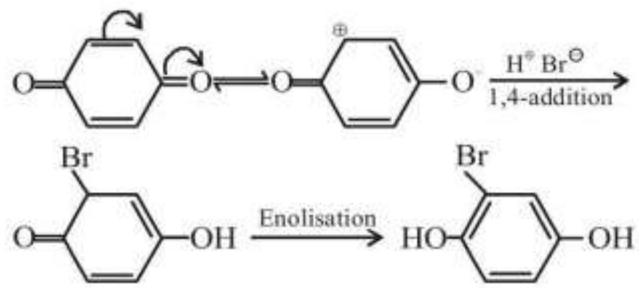
64. (b)



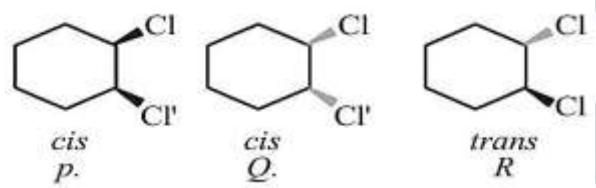
65. (c)



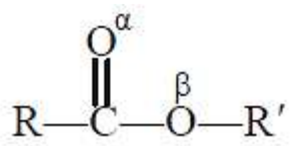
66. (d)



67. (a)

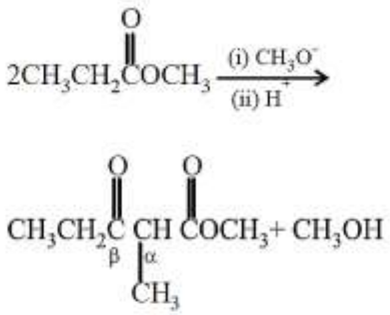


68. (a)



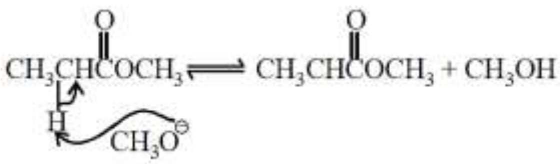
$\alpha - O$ oxygen atom can donate lone pair of electron more easily, therefore, it is more basic than β-oxygen.

69. (d) When two molecules of an ethylacetate undergo condensation reaction, in presence of sodium ethoxide involving the reaction is called as Claisen condensation and product is a β-keto ester.

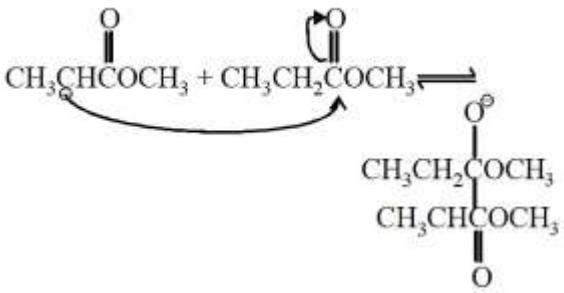


Mechanism

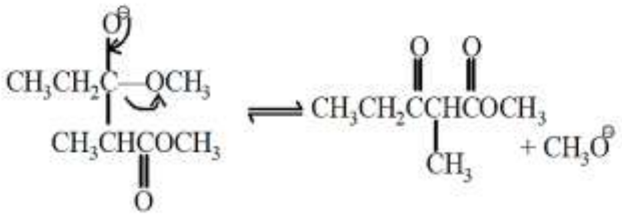
Step I



Step II

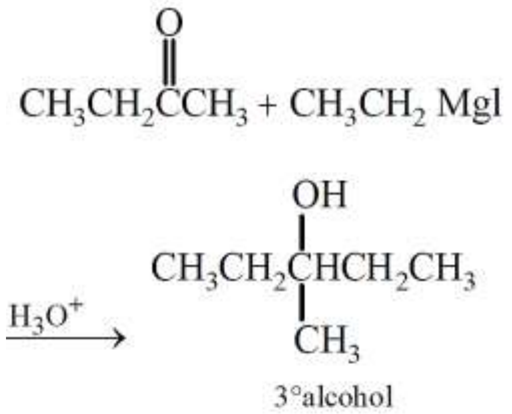


Step III



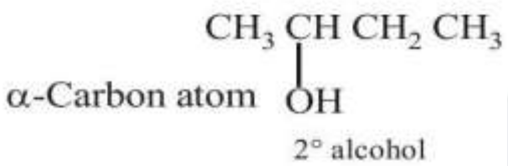
70. (b) B is a tertiary alcohol based on given properties.

(a) $CH_2 - CH_2 + CH_3CH_2MgI \xrightarrow{H_3O^+} \underset{1^\circ \text{ alcohol}}{CH_3CH_2CH_2CH_2OH}$



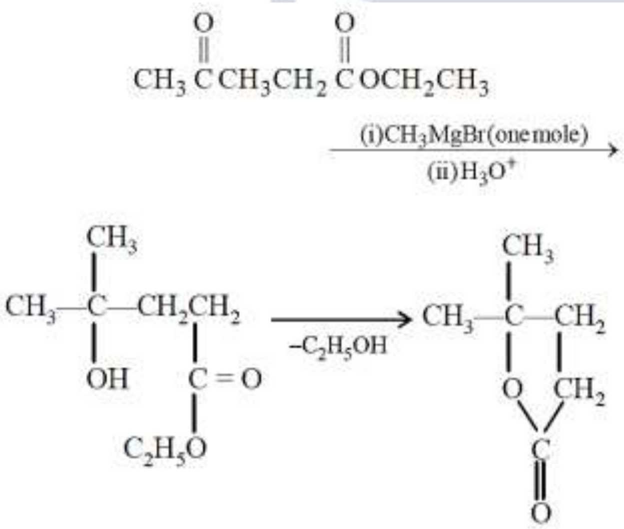
(b)

(c) $CH_3CHO + CH_3CH_2MgI \xrightarrow{H_3O^+}$



(d) $HCHO + CH_3CH_2MgI \xrightarrow{H_3O^+} \underset{1^\circ \text{ alcohol}}{CH_3CH_2CH_2OH}$

71. **(c)** Keto group is more reactive for addition of Grignard reagent.



72. **(b)** $E^\circ_{cell} = E^\circ_{ox} + E^\circ_{red}$

$= -E^\circ_{Co/Co^{2+}} + E^0_{Ce^{4+}/Ce^{3+}}$

$1.89 = -(-0.28) + E^0_{Ce^{4+}/Ce^{3+}}$

$\therefore E^\circ_{Ce^{4+}/Ce^{3+}} = -1.879 - 0.28 = 1.61 \text{ V}$

73. **(c)** Due to electrolysis

$2H_2O(l) + 2e^- \rightarrow H_2(g) + 2OH^-(aq)$

$2Cl^-(aq) \rightarrow Cl_2(g) + 2e^-$

$2H_2O(l) + 2Cl^-(aq) \rightarrow H_2(g) + Cl_2(g)$

$+2OH^-(aq)$

OH^- formed $=$ NaOH formed $=$ Z it

$$= \frac{E}{96500} \times i \times t$$

$$= \frac{40}{96500} \times 30 \times 1 \times 60 \times 60 = 44.77\ g$$

$$= \frac{44.77}{40} = 1.12\ mol$$

Cl_2 formed $= \frac{1}{2}$ mol of NaOH

$$= \frac{1.12}{2} = 0.56\ mol$$

$= 0.56 \times 22.4$ L at STP $= 12.54$ L

74. (b) $K_B = Ae^{-E_a/RT}$

$= 10^{12} \times 4.35 \times 10^{-8}$

$= 4.35 \times 10^4\ s^{-1}$

Also equilibrium constant, $k = \frac{k_A}{k_B} = 10^4$

$\therefore k_A = k_B \times 10^4 = 4.35 \times 10^8\ s^{-1}$

75. (a) $\Delta G = \Delta H - nFT\left(\frac{dE}{dT}\right)_P$ and $\Delta G = \Delta T - T\Delta S$

$$\therefore \frac{\Delta S}{nF} = \left(\frac{dE}{dT}\right)_P$$

or $\frac{96.5}{2 \times 96500} = \left(\frac{dE}{dT}\right)_P$

$$\therefore \left(\frac{dE_{cell}}{dT}\right)_P = \frac{1 \times 10^{-3}}{2} = 5 \times 10^{-4} VK^{-1}$$

76. (c) We have to compare wavelength of transition in the H-spectrum with the Balmer transition n = 4 to n = 2 of He^+ spectrum.

$\because \lambda_H = \lambda_{He^+}$

$$\therefore R_H Z_H^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = R_H Z_{He^+}^2 \left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$1 \times \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 4 \times \left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 4 \times \frac{4-1}{16}$$

$$\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]=\frac{3}{4}$$

If $n_1 = 1$, then $n_2 = 2,3,\ldots$

For first line $n_2 = 2, n_1 = 1$

$$\left[\frac{1}{1^2}-\frac{1}{2^2}\right]=\frac{1}{1}-\frac{1}{4}=\frac{3}{4}$$

Hence, trnasition $n_2 = 2$ to $n_1 = 1$ will give spectrum of the same wavelength as that of Balmer transition, $n_2 = 4$ to $n_1 = 2$ in He^+.

77. (b) Energy of the electron in the nth orbit in terms of R_H is $E_n = -\frac{R_H Z^2}{n^2}$

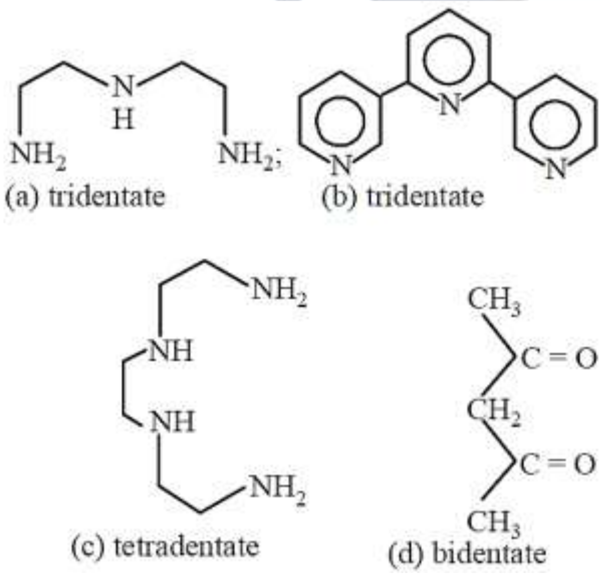
where, Z = atomic number, n^2 = degeneracy

For H-atom, $E_n = -\frac{R_H (1)^2}{n^2}$

$$-\frac{R_H}{9}=-\frac{R_H}{n^2}$$

$\therefore n^2 = 9$

78. (d)
79. (c)



80. (b) Effective atomic number EAN = Atomic number - oxidation number +2 × coordination number

For $[Al(C_2O_4)_3]^{3-}$

$Z = 13$

$ON = 3$

$CN = 6$

$\therefore EAN = 13 - 3 + 2 \times 6 = 22$

PART-III MATHEMATICS

81. (a) $A = \{x: |x| < 3, x \in I\}$

$A = \{x: -3 < x < 3, x \in I\} = \{-2, -1, 0, 1\}$

Also, $R = \{(x, y): y = |x|\}$

$\therefore R = \{(-2,2), (-1,1), (1,1), (0,0), (2,2)\}$

82. (b) $\frac{dy}{dx} = \frac{y'(x) - y^2}{f(x)}$

$\Rightarrow yf'(x)dx - f(x)dy = y^2dx$

$$\Rightarrow \frac{yf'(x)dx - f(x)dy}{y^2} = dx$$

$$\Rightarrow d\left\{\frac{f(x)}{y}\right\} = dx$$

On integration, we get

$$\frac{f(x)}{y} = x + C$$

$\Rightarrow f(x) = y(x + C)$

83. (a) Let $\Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 2(2b-a-c) \\ x+4 & x+5 & x+2c \end{vmatrix}$$

(using $R_2 \to 2R_2 - R_1 - R_3$)

But a, b and c are in AP using $2b = a + c$, we get

$$\Delta = \frac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 0 \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$$

Since, all elements of R_2 are zero.

84. (b) $P(A) = \frac{1}{3}, P(A \cup B) = \frac{3}{4}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\leq P(A) + P(B)$

$$\Rightarrow \frac{3}{4} \leq \frac{1}{3} + P(B)$$

$$\Rightarrow P(B) \geq \frac{5}{12}$$

Also, $B \leq A \cup B \Rightarrow P(B) \leq P(A \cup B) = \frac{3}{4}$

$\therefore \frac{5}{12} \leq P(B) \leq \frac{3}{4}$

85. (a) $2\tan^{-1}(\operatorname{cosec} \tan^{-1} x - \tan \cot^{-1} x)$

$$= 2\tan^{-1}\left[\operatorname{cosec}\left\{\operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}\right\} - \tan\left\{\tan^{-1}\frac{1}{x}\right\}\right]$$

$$= 2\tan^{-1}\left[\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right]$$

$$= 2\tan^{-1}\left\{\frac{\sqrt{1+x^2-1}}{x}\right\}$$

$$= 2\tan^{-1}\left\{\frac{\sec\theta - 1}{\tan\theta}\right\} \text{ (put } x = \tan\theta \text{)}$$

$$= 2\tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$$

$$= 2\tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}\right\}$$

$$= 2\tan^{-1}\tan\frac{\theta}{2}$$

$$= 2\cdot\frac{\theta}{2} = \theta = \tan^{-1} x$$

86. (b) $\sim(p \Leftrightarrow q) \equiv \sim[(p \Rightarrow q) \wedge (q \Rightarrow p)] \equiv \sim(p \Rightarrow q) \vee (\sim(q \Rightarrow p))$

($\because$ De-Morgan's law)

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

87. (b)

p	**q**	**$\sim$ p**	**$\sim$ p $\vee$ q**	**$\sim$ ($\sim$ p $\vee$ q)**
F	T	T	T	F

$\therefore$ Truth value of $\sim(\sim p \vee q)$ is F.

88. (c) Surface area of sphere,

$S = 4\pi r^2$

and $\frac{dr}{dt} = 2$

$$\therefore \frac{ds}{dt} = 4\pi \times 2r\frac{dr}{dt} = 8\pi r \times 2 = 16\pi r$$

$$\Rightarrow \frac{ds}{dt} \propto r$$

89. (d) For $(a, b), (c, d) \in N \times N$

(a, b) R (c, d)

$\Rightarrow ad(b + c) = bc(a + d)$

Reflexive : $ab(b + a) = ba(a + b), \forall ab \in N$

$\therefore (a, b)R(a, b)$

So, R is reflexive,

Symmetric: $ad(b + c) = bc(a + d)$

$\Rightarrow bc(a + d) = ad(b + c)$

$\Rightarrow cd(d + a) = da(c + b)$

$\Rightarrow (c, d)R(a, b)$

So, R is symmetric.

Transitive: For (a, b), (c, d), (e, f) $\in$ N $\times$ N Let (a, b) R (c, d), (c, d) R (e, f)

$\therefore ad(b + c) = bc(a + d), cf(d + e) = de(c + f)$

$\Rightarrow adb + adc = bca + bcd$... (i)

and $cfd + cfe = dec + def$ ...(ii)

On multiplying eq. (i) by ef and eq. (ii) by ab and then adding, we have adbef + adcef + cfdab + cfeab

= bcaef + bcdef + decab + defab

$\Rightarrow adcf(b + e) = bcde(a + f)$

$\Rightarrow af(b + e) = be(a + f)$

$\Rightarrow (a, b)R(e, f)$

So, R is transitive.

Hence R is an equivalence relation.

90. (c) $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$

$$\Rightarrow \arg\left(\frac{x - 2 + iy}{x + 2 + iy}\right) = \frac{\pi}{3}$$

$$\Rightarrow \arg(x - 2 + iy) - \arg(x + 2 + iy) = \frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x - 2}\right) - \tan^{-1}\left(\frac{y}{x + 2}\right) = \frac{\pi}{3}$$

$$\Rightarrow \frac{4y}{x^2 + y^2 - 4} = \sqrt{3}$$

$\Rightarrow \sqrt{3}(x^2 + y^2) - 4y - 4\sqrt{3} = 0$

which is an equation of a circle.

91. (d) We know that, GM $\geq$ HM

$$\Rightarrow (a_1 \cdot a_2 \cdot a_3)^{1/3} \geq \frac{3}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)}$$

$$\Rightarrow (a_1 \cdot a_2 \cdot a_3) \geq \frac{27}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^3}$$

$$\Rightarrow (a_1 \cdot a_2 \cdot a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^3 \geq 27$$

92. (b) $|x^2 - x - 6| = x + 2$, then

Case I : $x^2 - x - 6 < 0$

$\Rightarrow (x - 3)(x + 2) < 0$

$\Rightarrow -2 < x < 3$

In this case, the equation becomes

$x^2 - x - 6 = -x - 2$

or $x^2 - 4 = 0$

$\therefore x = \pm 2$

Clearly, $x = 2$ satisfies the domain of the equation in this case. So, $x = 2$ is a solution.

Case II: $x^2 - x - 6 \geq 0$

So, $x \leq -2$ or $x \geq 3$

In this case, the equation becomes

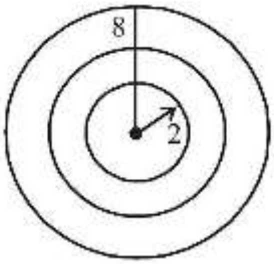
$x^2 - x - 6 = 0 = x + 2$

i.e., $x^2 - 2x - 8 = 0$ or $x = -2,4$

Both these values lie in the domain of the equation in this case, so $x = -2,4$ are the roots.

Hence, roots are $x = -2,2,4$.

93. (a) Let (h, k) be any point in the set, then equation of circle is $(x - h)^2 + (y - k)^2 = 9$



(h, k) lies on $x^2 + y^2 = 25$, then $h^2 + k^2 = 25 \Rightarrow 2 \leq$ Distance between the two circles ≤ 8

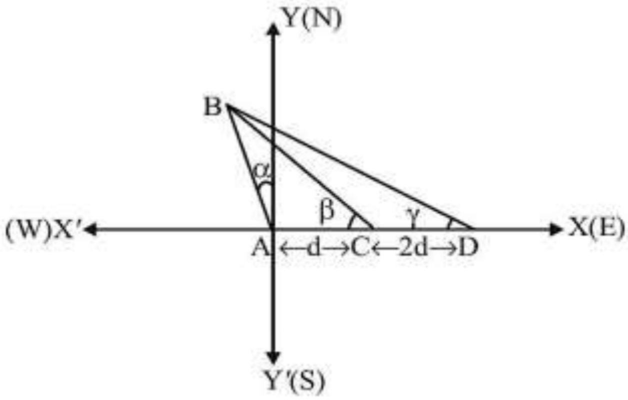
$\Rightarrow 2 \leq \sqrt{h^2 + k^2} \leq 8$

$\Rightarrow 4 \leq h^2 + k^2 \leq 64$

$\therefore$ Locus of (h, k) is $4 \leq (x^2 + y^2) \leq 64$

94. (c) By m −n theorem at C

$(d + 2\ d)\cot\beta = d\cot\gamma - 2\ d\cot(90^\circ + \alpha)$



$3\ d\cot\beta = d\cot\gamma + 2\ d\tan\alpha$

$\Rightarrow 3\cot\beta = \cot\gamma + 2\tan\alpha$

$\therefore 2\tan\alpha = 3\cot\beta - \cot\gamma$

95. (c) Multiplying $x^2 - ax + b = 0$ by x^{n-1}, we get

$x^{n+1} - ax^n + bx^{n-1} = 0$

α, β are roots of $x^2 - ax + b = 0$, therefore they will satisfy (i).

Also, $\alpha^{n+1} - a\alpha^n + b\alpha^{n-1} = 0$

and $\beta^{n+1} - a\beta^n + b\beta^{n-1} = 0$

On adding eqs. (ii) and (iii), we get

$(\alpha^{n+1} + \beta^{n+1}) - a(\alpha^n + \beta^n) + b\left(\alpha^{n-1} + \beta^{(n-1)}\right) = 0$

$\Rightarrow V_{n+1} - aV_n + bV_{n-1} = 0 (\because \alpha^n + \beta^n = V_n)$

$\Rightarrow V_{n+1} = aV_n - bVV_{n-1}$

96. (a) $\sum_{r=0}^{n} (-1)^r.\ {}^nC_r$

$$\left(\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\cdots \text{ upto m terms}\right)$$

$$=\sum_{r=0}^{n}(-1)^{r\cdot n}C_r\frac{1}{2^r}+\sum_{r=0}^{n}(-1)^{r\cdot}\cdot C_r\cdot\frac{3^r}{2^{2r}}$$

$$+\sum_{r=0}^{n}(-1)^{r\cdot n}C_r\cdot\frac{7^r}{2^{3r}}+\cdots$$

$$=\left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{4}\right)^n+\left(1-\frac{7}{8}\right)^n+$$

... upto m terms $=\frac{1}{2^n}+\frac{1}{4^n}+\frac{1}{8^n}+\cdots$ upto m terms

$$=\frac{\frac{1}{2^n}\left(1-\frac{1}{2^{mn}}\right)}{\left(1-\frac{1}{2^n}\right)}=\frac{2^{mn}-1}{2^{mn}(2^n-1)}$$

97. (c) Angle of intersection between two circles is given by

$$\cos\theta=\frac{r_1^2+r_2^2-d^2}{2r_1r_2}=\frac{\frac{17}{2}+13-\frac{10}{4}}{2\sqrt{\frac{17}{2}}\cdot\sqrt{13}}$$

$\int$ here, $r_1=\sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2+8}$

$$=\sqrt{\frac{17}{2}}$$

$r_2=\sqrt{13}$

and $d=c_1c_2=\sqrt{\frac{10}{2}}$

$\Rightarrow \cos\theta=\left(\frac{19}{\sqrt{442}}\right)$

or $\tan\theta=\left(\frac{9}{19}\right)$

$\Rightarrow \theta=\tan^{-1}\left(\frac{9}{19}\right)$

98. (a) $\mathbf{b}_1 \parallel \mathbf{a} \Rightarrow \mathbf{b}_1=\mathbf{a}(\mathbf{i}+\mathbf{j})$

$\mathbf{b}_2=\mathbf{b}-\mathbf{b}_1=(3-a)\mathbf{i}-a\mathbf{j}+4\mathbf{k}$

Also, $\mathbf{b}_2\cdot\mathbf{a}=0$

$\Rightarrow (3-a)-a\Rightarrow a=\frac{3}{2}$

$\therefore \mathbf{b}_1=\frac{3}{2}(\mathbf{i}+\mathbf{j})$

99. **(b)** The given matrix is $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$, using $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$ $\Delta = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x_3 - x_1 & y_3 - y_1 & 0 \end{bmatrix} = 0$

($\because$ points are collinear i.e., area of triangle $= 0$)

$\Rightarrow \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} = 0$

So, the rank of matrix is always less than 2 .

100. **(d)** On solving the determinant, we have

$1(1 - \cos^2 \beta) - \cos(\alpha - \beta)[\cos(\alpha - \beta)$

$-\cos \alpha \cdot \cos \beta] + \cos \alpha[\cos \beta \cdot \cos(\alpha - \beta)$

$-\cos \alpha]$

$= 1 - \cos^2 \beta - \cos^2 \alpha - \cos^2(\alpha - \beta) + 2\cos \alpha \cdot \cos \beta \cdot \cos(\alpha - \beta)$

$= 1 - \cos^2 \beta - \cos^2 \alpha + \cos(\alpha - \beta)$

$[2\cos \alpha \cdot \cos \beta - \cos(\alpha - \beta)]$

$= 1 - \cos^2 \beta - \cos^2 \alpha + \cos(\alpha - \beta)\cos(\alpha + \beta)$

$[\cos(\alpha + \beta) + \cos(\alpha - \beta) - \cos(\alpha - \beta)]$

$= 1 - \cos^2 \beta - \cos^2 \alpha + \cos^2 \alpha \cdot \cos^2 \beta$

$-\sin^2 \alpha \cdot \sin^2 \beta$

$= 1 - \cos^2 \beta - \cos^2 \alpha(1 - \cos^2 \beta)$

$-\sin^2 \alpha \cdot \sin^2 \beta$

$= 1 - \cos^2 \beta - \cos^2 \alpha \cdot \sin^2 \beta - \sin^2 \alpha \cdot \sin^2 \beta$

$= (1 - \cos^2 \beta) - \sin^2 \beta(\sin^2 \alpha + \cos^2 \alpha)$

$= \sin^2 \beta - \sin^2 \beta 1) = 0$

101. **(b)** $-\sqrt{7^2 + 5^2} \leq (7\cos x + 5\sin x) \leq \sqrt{7^2 + 5^2}$

$\Rightarrow -\sqrt{74} \leq (2\text{ K} + 1) \leq \sqrt{74}$

$\Rightarrow -8.6 \leq (2\text{ K} + 1) \leq 8.6$

$\Rightarrow -9.6 \leq 2\text{ K} \leq 7.6$

$\Rightarrow -4.8 \leq \text{K} \leq 3.8$

So, integral values of K are

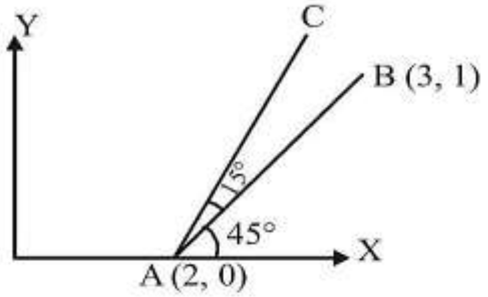
$-4, -3, -2, -1, 0, 1, 2, 3$ (eight values)

102. **(a)** Slope of AB $= \frac{1}{1}$

$\Rightarrow \tan\theta = m_1 = 1$ or $\theta = 45°$
Thus, slope of new line is $\tan(45° + 15°)$

$= \tan 60° = \sqrt{3}$

($\because$ it is rotated anti-clockwise, so the angle will be $45° + 15° = 60°$)



Hence, the equation is $y = \sqrt{3}x + c$ But it passes through $(2,0)$,

So, $c = -2\sqrt{3}$

Thus, required equation is $y = \sqrt{3}x - 2\sqrt{3}$

103. **(a)** Solving the equation of line and curve, we get

$$x^2 - 2\left\{\frac{2-2x}{\sqrt{6}}\right\}^2 = 4$$

$$\Rightarrow x^2 - \frac{1}{3} \times 4(1 + x^2 - 2x) = 4$$

$$\Rightarrow 3x^2 - 4 - 4x^2 + 8x = 12$$

$$\Rightarrow x^2 - 8x + 16 = 0$$

$$\Rightarrow (x-4)^2 = 0 \Rightarrow x = 4$$

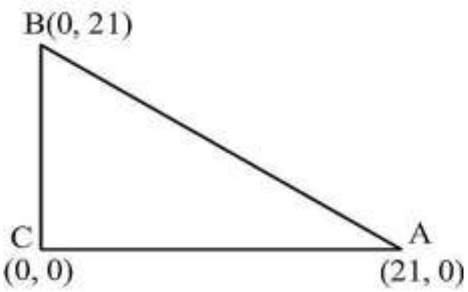
and $\sqrt{6} \cdot y = 2 - 2(4) = -6$

$$\Rightarrow y = -\sqrt{6}$$

$\therefore$ Point of contact is $(4, -\sqrt{6})$.

104. **(b)** $x + y = 21$

The number of integral solutions to the equations are $x + y < 21$, i.e., $x < 21 - y$



$\therefore$ Number of integral coordinates

$= 19 + 18 + \cdots + 1$

$= \dfrac{19(19+1)}{2} = \dfrac{19 \times 20}{2} = 190$

105. **(c)** $\int (1 + x - x^{-1}) e^{x+x^{-1}} dx$

$= \int \left[x . e^{x+x^{-1}} \left(1 - \frac{1}{x^2}\right) + e^{x+x^{-1}}\right] dx$

$\left[\because \int xf'(x) + f(x) dx = xf(x) + C\right]$

$\therefore \int (1 + x - x^{-1}) e^{x+x^{-1}} dx = xe^{x+x^{-1}} + C$

106. **(a)** $f(x) = x - [x], -1 \le x < 0$

$\Rightarrow f(x) = x + 1$

When $0 \le x < 1$

$\Rightarrow f(x) = x$

$\int_{-1}^{1} f(x) dx = \int_{-1}^{0} f(x) dx + \int_{0}^{1} f(x) dx$

$= \int_{-1}^{0} (x+1) dx + \int_{0}^{1} x dx$

$= \left[\frac{x^2}{2} + x\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$

$= 0 - \left[\frac{(-1)^2}{2} - 1\right] + \frac{1}{2} = 1$

107. **(c)** $\int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right]^{1/2} dx$

$= \int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1} - \frac{x-1}{x+1}\right)^2\right]^{1/2} dx$

$= \int_{-1/2}^{1/2} \left|\frac{4x}{x^2 - 1}\right| dx$

$= \int_{-1/2}^{0} \left|\frac{4x}{1 - x^2}\right| dx + \int_{0}^{1/2} \left|\frac{4x}{1 - x^2}\right| dx$

$= -4\int_{-1/2}^{0} \frac{x}{1 - x^2} dx + 4\int_{0}^{1/2} \frac{x}{1 - x^2} dx$

$= 2\{\log(1 - x^2)\}_{-1/2}^{0} - 2\{\log(1 - x^2)\}_{0}^{1/2}$

$= -2\log\left(1 - \frac{1}{4}\right) - 2\log\left(1 - \frac{1}{4}\right)$

$= -4\log\frac{3}{4} = 4\log\frac{4}{3}$

108. **(d)** Let $P(x_1, y_1)$ be a point on the ellipse.

$$\frac{x^2}{18}+\frac{y^2}{32}=1$$

$\Rightarrow \frac{x_1^2}{18}+\frac{y_1^2}{32}=1$ ... (i)

The equation of the tangent at (x_1, y_1) is $\frac{x_1}{18}+\frac{yy_1}{32}=1$. This meets the axes at A $\left(\frac{18}{x_1}, 0\right)$ and B $\left(0, \frac{32}{y_1}\right)$. It is given that slope of the tangent at (x_1, y_1) is $-\frac{4}{3}$

So, $-\frac{x_1}{18}\cdot\frac{32}{y_1}=-\frac{4}{3}$

$$\Rightarrow \frac{x_1}{y_1}=\frac{3}{4}$$

$$\Rightarrow \frac{x_1}{3}=\frac{y_1}{4}=K$$

$\therefore x_1=3\text{ K}$ and $y_1=4\text{ K}$

Putting x_1, y_1 in (i), we get $K^2=1$

$\therefore$ Area of $\triangle$ OAB $=\frac{1}{2}$ OA. OB

$$=\frac{1}{2}\cdot\frac{18}{x_1}\cdot\frac{32}{y_1}=\frac{1}{2}\frac{(18)(32)}{(3\text{ K})(4\text{ K})}=\frac{24}{K^2}$$

$=24$ sq units $(\because K^2=1)$

109. **(d)** Let mid-point of part PQ which is in between the axis is $R(x_1, y_1)$, then coordinates of P and Q will be $(2x_1, 0)$ and $(0, 2y_1)$, respectively.

$\therefore$ Equation of line PQ is $\frac{x}{2x_1}+\frac{y}{2y_1}=1$

$$\Rightarrow y=-\left(\frac{y_1}{x_1}\right)x+2y_1$$

If this line touches the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

then it will satisfy the condition,

$$c^2=a^2m^2+b^2$$

So, $(2y_1)^2=a^2\left(\frac{-y_1}{x_1}\right)^2+b^2$

$$\Rightarrow 4y_1^2=\left\{\frac{a^2y_1^2}{x_1^2}\right\}+b^2$$

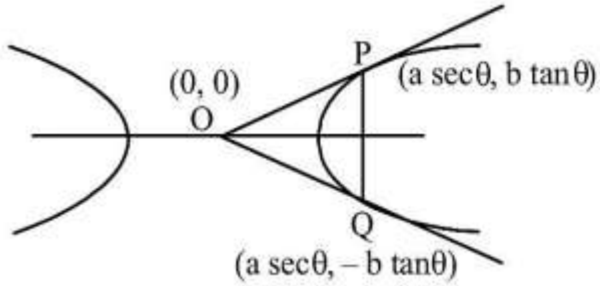
$$\Rightarrow 4=\left(\frac{a^2}{x_1^2}\right)+\left(\frac{b^2}{y_1^2}\right)\Rightarrow\left(\frac{a^2}{x_1^2}\right)+\left(\frac{b^2}{y_1^2}\right)=4$$

$\therefore$ Required locus of (x_1, y_1) is

$$\left(\frac{a^2}{x^2}+\frac{b^2}{y^2}\right)=4$$

110. **(d)** Let $P(a\sec\theta, b\tan\theta), Q(a\sec\theta, -b\tan\theta)$ be end points of double ordinates and $(0,0)$ is the centre of the hyperbola.

So, $PQ = 2\,b\tan\theta$



$OQ = OP = \sqrt{a^2\sec^2\theta + b^2\tan^2\theta}$

Since, $OQ = OP = PQ \therefore 4\,b^2\tan^2\theta = a^2\sec^2\theta + b^2\tan^2\theta$

$\Rightarrow 3\,b^2\tan^2\theta = a^2\sec^2\theta$

$\Rightarrow 3\,b^2\sin^2\theta = a^2$

$\Rightarrow 3a^2(e^2-1)\sin^2\theta = a^2$

$\Rightarrow 3(e^2-1)\sin^2\theta = 1$

$$\Rightarrow \frac{1}{3(e^2-1)} = \sin^2\theta < 1, (\because \sin^2\theta < 1)$$

$$\Rightarrow \frac{1}{e^2-1} < 3 \Rightarrow e^2 - 1 > \frac{1}{3} \Rightarrow e^2 > \frac{4}{3}$$

$$\therefore e > \frac{2}{\sqrt{3}}$$

111. **(a)** There are $3+4+5=12$ points in a plane.

The number of required triangles

= (The number of triangles formed by these 12 points) -(The number of triangles formed by the collinear points)

$= {}^{12}C_3 - ({}^{3}C_3 + {}^{4}C_3 + {}^{5}C_3)$

$= 220 - (1+4+10) = 205$

112. **(c)** $(a+bx)e^{-x}$

$= (a+bx)$

$$\left\{1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + (-1)^n\frac{x^n}{n!} + \cdots\right\}$$

$\therefore$ The coefficient of $x^r = a$.

$$\frac{(-1)^r}{r!}+b\frac{(-1)^{r-1}}{(r-1)!}=\frac{(-1)^r}{r!}(a-br)$$

113. **(a)** $\sum_{x=1}^{1999}\log_n x$

$=\log_{(1999)!}1+\log_{(1999)!}2+\cdots+\log_{(1999)!}1999$

$=\log_{(1999)!}(1.2.3\ldots\ldots1999)$

$=\log_{(1999)!}(1999)!=1$

114. **(d)** Since, the line is equally inclined to the axes and passes through the origin, its direction ratios are 1,1,1. So, its equation is $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$.

A point P on it is given by (a,a,a). So, equation of the plane through $P(a,a,a)$ and perpendicular to OP is

$1(x-a)+1(y-a)+1(z-a)=0$

($\because$ OP is normal to the plane)
i.e., $x+y+z=3a$

$$\Rightarrow\frac{x}{3a}+\frac{y}{3a}+\frac{z}{3a}=1$$

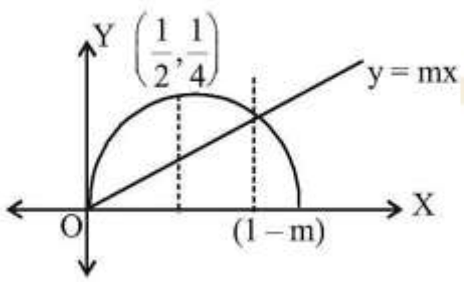
Intercepts on axes are $3a,3a$ and $3a$, therefore sum of reciprocals of these intercepts.

$$=\frac{1}{3a}+\frac{1}{3a}+\frac{1}{3a}=\frac{1}{a}$$

115. **(b)** The equation of curve is $y=x-x^2$
$\Rightarrow x^2-x=y$

$$\Rightarrow\left(x-\frac{1}{2}\right)^2=-\left(y-\frac{1}{4}\right)$$

which is a parabola whose vertex is $\left(\frac{1}{2},\frac{1}{4}\right)$



Hence, finding the point of intersection of the curve and the line,

$x-x^2=mx\Rightarrow x(1-x-m)=0$

i.e., $x=0$ or $x=1-m$

$$\therefore\frac{9}{2}=\int_0^{1-m}(x-x^2-mx)dx$$

$$= \left\{\frac{x^2}{2}-\frac{x^3}{3}-m\frac{x^2}{2}\right\}_0^{1-m}$$

$$= (1-m)\frac{(1-m)^2}{2}-\frac{(1-m)^3}{3}=\frac{(1-m)^3}{6}$$

$$\therefore (1-m)^3=\frac{6\times 9}{2}=27$$

$$\Rightarrow 1-m=(27)^{1/3}=3$$

$$\Rightarrow m=-2$$

Also, $(1-m)^3-(3)^3=0$

$\therefore (1-m)^3=3^3 \Rightarrow 1-m=3$

or $m=-2$

116. **(c)** $\lim_{x\to 0}\frac{1}{x}[\log f(1+x)-\log f(1)]$

$$= e^{\lim\frac{f'(1+x)/f(1+x)}{1}}$$

$$= e^{f'(1)/f(1)} = e^{613} = e^2$$

117. **(b)**

$$f(x)=\begin{cases}(1+|\sin x|)^{a/|\sin x|}, & -\frac{\pi}{6}<x<0\\ b & ,x=0\\ e^{\tan 2x/\tan 3x} & ,0<x<\frac{\pi}{6}\end{cases}$$

For f(x) to be continuous at $x=0$

$$\lim_{x\to 0^-} f(x)=f(0)=\lim_{x\to 0^+} f(x)$$

$$\lim_{x\to 0^-}(1+|\sin x|)^{a/|\sin x|}$$

$$= e^{\lim_{x\to 0}\left\{|\sin x|\frac{a}{|\sin x|}\right\}} = e^a$$

Now, $\lim_{x\to 0^+} e^{\tan 2x/\tan 3x}$

$$= \lim_{x\to 0^+} e^{\left(\frac{\tan 2x}{2x}\times 2x\right)}/\left(\frac{\tan 3x}{3x}\times 3x\right)$$

$$= \lim_{x\to 0^+} e^{2/3} = e^{2/3}$$

Since, f(x) is continuous at $x=0$.

$$\therefore e^a = e^{2/3} \Rightarrow a=\frac{2}{3}$$

and $b=e^{2/3}$

118. **(b)** $x+2\ge 0$, i.e., $x\ge -2$ or $-2\le x$

$\log_{10}(1-x) \neq 0$

$\Rightarrow 1-x \neq 1 \Rightarrow x \neq 0$

Again, $1-x>0$

$\Rightarrow 1>x \Rightarrow x<1$

Combining all the results for values of x, we get

$-2 \leq x<0$ and $0<x<1$

119. **(b)** $(1+y^2)+\left(x-e^{\tan^{-1} y}\right)\frac{dy}{dx}=0$

$$(1+y^2)\frac{dx}{dy}+x=e^{\tan^{-1} y}$$

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1} y}}{(1+y^2)}$$

$$IF=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1} y}$$

$$\Rightarrow x.\, e^{\tan^{-1} y}=\int \frac{e^{\tan^{-1} y}}{1+y^2}\cdot e^{\tan^{-1} y}\cdot dy$$

$$\Rightarrow x\left(e^{\tan^{-1} y}\right)=\frac{e^{2\tan^{-1} y}}{2}+c$$

$$\therefore 2xe^{\tan^{-1} y}=e^{2\tan^{-1} y}+K$$

120. **(c)** $\frac{dy}{dx}=\frac{y}{x}-\sin^2\left(\frac{y}{x}\right)$

Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}\, v+x\frac{dv}{dx}=v-\sin^2 v$

$$\Rightarrow -\operatorname{cosec}^2 v dv=\frac{dx}{x}$$

Integrating both sides, we get

$$-\int \operatorname{cosec}^2 v dv=\int \frac{dx}{x}$$

$$\Rightarrow \cot v=\log x+C$$

$$\cot\frac{y}{x}=\log x+C$$

Curve passes through the point $\left(1,\frac{\pi}{4}\right)$

$\therefore C=1$

$$\Rightarrow \cot\frac{y}{x}=\log x+\log_e e$$

$\Rightarrow \cot \frac{y}{x} = \log xe$

$\Rightarrow y = x\cot^{-1}(\log xe)$