PART-I PHYSICS

1. **(b)** Amplification factor of a triode, $\mu = \frac{\Delta V_p}{\Delta V_g}$

$\Rightarrow \Delta V_p = -\mu \times \Delta V_S = -50(-20) = 10\ V$

2. **(a)** As we know, energy $E = \Delta mc^2$

$= 0.5 \times 10^{-3} \times (3 \times 10^8)^2$

$= 4.5 \times 10^{13}\ J$

$= \frac{4.5 \times 10^{13}}{3.6 \times 10^6} = 1.25 \times 10^7 kWh$

3. **(b)** Here, diode in lower branch is forward and in upper branch is reversed biased

$\therefore\ i = \frac{5}{20 + 30} = \frac{5}{50}\ A$

4. **(b)** Power consumed by motor $= 5\ kW$

$= 5 \times 10^3\ W = 5000\ W$

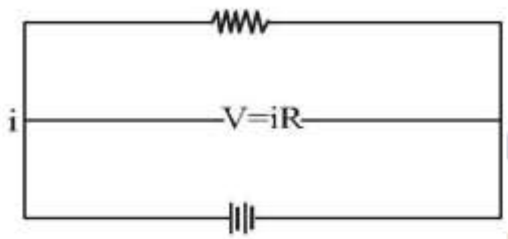
Power used in lifting water $= \frac{mgh}{t}$

$= 7.5 \times 9.8 \times 4.7 = 3454.5\ kW$

Efficiency $= \frac{\text{Power used}}{\text{Power consumed}} \times 100\%$

$= \frac{3454.5}{5000} \times 100 = 69\%$

5. **(c)** For a closed-circuit cell supplies a constant current in the circuit.



For cell $E = V + Ir$

For $V = 50\ V$

$E = 50 + 11r$

Similarly, for $V = 60\ V$

$E = 60 + r$

From eqs. (i) and (ii), we get $E = 61\ V$

6. **(b)** The ionosphere can reflect electromagnetic waves of frequency less than 40MHz but not of frequency more than 40MHz.

7. **(a)** $\lambda_{red} > \lambda_{violet}$

As wavelength λ less than that of yellow colour and hence can initiate photoelectric effect irrespective of intensity.

8. **(c)** Radius of circular path

$$r = \frac{mV}{B} = \frac{V}{\left(\frac{e}{m}\right)B}$$

$$r = \frac{6 \times 10^7}{1.7 \times 10^{11} \times 15 \times 10^{-2}}$$

$$= 2.35 \times 10^{-2}\ \text{m} = 2.35\ \text{m}$$

9. **(a)** As we know, Self-inductance of the solenoid

$$L = \frac{\mu_r \cdot \mu_0\, N^2\, A}{I}$$

$$= \frac{600 \times 4\pi \times 10^{-7} \times (2000)^2 \times (1.5) \times 10^{-4}}{0.3}$$

$$= 1.5\text{H}$$

10. (d) Current $I = \frac{V}{R} = \frac{100}{10} = 10\ A$

Energy, $E = \frac{1}{2}LI^2$

$$= \frac{1}{2} \times 5 \times (10)^2 = 250\ J$$

11. **(c)** Given : $V = 750 \times 10^{-3}$ V;

$I_g = 15 \times 10^{-3}$ A and $I = 25$ A

Using, $a = \frac{y}{I_g}$

$$= \frac{750 \times 10^{-3}}{15 \times 10^{-3}} = 50\Omega$$

$$I_g = \frac{S}{S + a} \times I$$

$$15 \times 10^{-3} = \left(\frac{S}{S + 50}\right) \times 25$$

$\therefore S = 0.03\Omega$

12. **(d)** Here, $r = R\left(\frac{I_1}{I_2} - 1\right) = 2\left(\frac{125}{100} - 1\right)$

$$= 2\left(\frac{5}{4} - 1\right) = 2 \times \frac{1}{4} = 0.05\Omega$$

13. **(d)** Resistances $10\Omega, 60\Omega$ and 100Ω are in series and they together are in parallel to 200Ω resistance. When a potential difference of 15 V is applied across 200Ω then current through it

$$I = \frac{15}{200} = 7.5 \times 10^{-2}\ A$$

14. (b) Energy stored in capacitor = energy stored in inductance

i.e., $\frac{1}{2}CV^2 = \frac{1}{2}LI^2$

$$\Rightarrow C = \frac{LI^2}{V^2} = \frac{1 \times (2)^2}{(400)^2} = 25\mu F$$

15. (d) Boolean expression for Logic gate-1
B. C = Y,

Boolean expression for Logic gate-II

$$A + (\vec{B} \cdot \vec{C}) = Y''$$

Boolean expression for Logic gate-III

$$A + \vec{B} \cdot \vec{C} = Y$$

16. (b) In series combination equivalent resistance,

$$R = R_1 + R_2$$

$$= 6 + 4 = 10k\Omega$$

Error in combination,

$$\Delta R = \Delta R_1 + \Delta R_2$$

$$= \frac{10}{100} \times 6 + \frac{10}{100} \times 4$$

$$= 0.6 + 0.4 = 1$$

$$\frac{\Delta R}{R} = \frac{1}{10} = 10\%$$

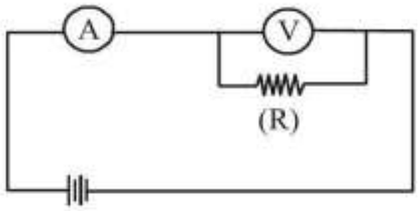
17. (b) If the number of electrons increase, their number of collision, increasing the thermal and electrical resistance. So, electrical and thermal conductivities both decrease.

18. (b) Radius of path $r_{time} = \frac{1}{\beta}\sqrt{\frac{2mv}{q}}$

$$\therefore \frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha}{m_p}}\sqrt{\frac{q_p}{q_\alpha}}$$

or, $\frac{r_\alpha}{10} = \sqrt{\frac{4}{2}} \Rightarrow r_\alpha = 10\sqrt{2}$ cm.

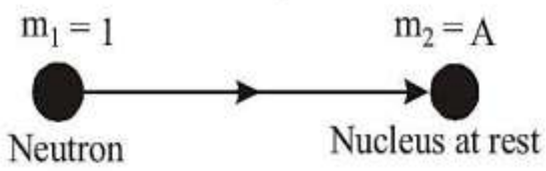
19. (d) The effective resistance will decrease when resistance R is connected in parallel with the voltmeter.



According to Ohm's law, $V = IR$ or, $R = \frac{V}{I}$

Here, as R decreases, so V decrease and I should increase

20. **(c)** Fraction retained by nucleus



$$\left(\frac{\Delta k}{k}\right)_{\text{retained}} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)^2 = \left(\frac{A-1}{A+1}\right)^2$$

After collision kinetic energy retained by neutron $\left(\frac{A-1}{A+1}\right)^2 E$

21. **(d)** Here, $\tan\delta' = \frac{\tan\delta}{\cos\theta} = \frac{\tan 45°}{\cos 30°}$

$$\tan\delta = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$$

or $\delta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

22. **(b)** Luminous efficiency for the same power supply, 40 W fluorescent tube gives more light. Hence, 40 W fluorescent tube has greater

23. **(b)** Resistance, $R = \frac{\Delta V}{\Delta I} = \frac{2.3-0.3}{10\times10^{-3}}$

$$R = \frac{2}{10} \times 10^3 = 0.2 \times 10^3 \Omega = 0.2 k\Omega$$

24. **(c)** Here, number of electrons $n_e = 3.13 \times 10^{15}$ and number of protons $n_p = 3.12 \times 10^{15}$ Current $I = n_e q_e + n_p q_p = 3.13 \times 10^{15} \times 1.6 \times 10^{-17} + 3.12 \times 10^{15} \times 1.6 \times 10^{-19}$

$= 1 \times 10^{-3} = 1$ mA

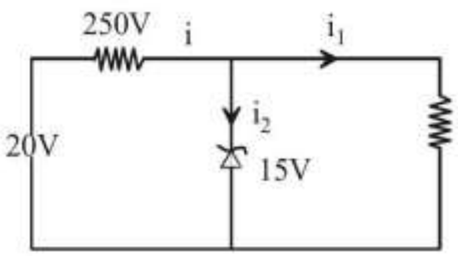
Now, due to excess charge on electrons, the direction of the current will be towards right.

25. **(b)** In conductor separation between conduction and valence bands is zero and in insulator, it is greater than 1eV.

Hence, in semiconductor the separation between conduction and valence band is 1eV.

26. **(c)** Potential of big drop $= q \times n^{2/3} = 100$ V

27. **(d)**



For $R = 1k\Omega$

$$i_1 = \frac{15}{1}\ \text{mA} = 15\ \text{mA}$$

$$R = 250\Omega$$

$$i_{250} = \frac{20-15}{250} = \frac{5}{250} = 20\ \text{mA}$$

$$i_{Zener} = 20 - 15 = 5\ \text{mA}$$

28. (d) After n half-lives

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/7}$$

$$N = \frac{N_0}{e} = \frac{cN_0}{cN} = \left(\frac{1}{2}\right)^{5/7}$$

$$\Rightarrow \frac{1}{e} = \left(\frac{1}{2}\right)^{5/7}$$

Taking log on both sides, we get

$$\log 1 - \log e = \tfrac{5}{7}\log\tfrac{1}{2} - 1 = \tfrac{5}{7}(-\log 2)$$

$$T = 5\log_e 2$$

Now, let t ' be the time after which activity reduces to half

$$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{1/5\log_e 2}$$

$$t' = 5\ \text{olog}\ {}_e 2$$

29. (b) As we know $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right)$

$$\frac{1}{\lambda} = R\left(\frac{9-4}{36}\right) = \frac{5R}{36}$$

$$\therefore \lambda = \frac{36}{5R}$$

30. (d) From Einstein's Photoelectric equation,

$$ev_0 + \phi = \frac{hc}{\lambda}$$

and $ev_0 + \phi_0 = \frac{hc}{\lambda'}$

$$\frac{ev_0' + \phi}{eV_0 + \phi} = \frac{\lambda}{\lambda'} = \frac{100}{200} = \frac{1}{2}$$

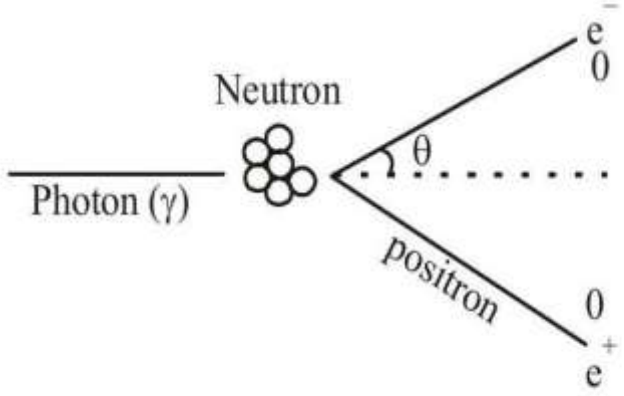
$$2ev_{0\ 0} + 2\phi = ev_0 + \phi$$

$$ev_0' = \frac{ev_0 - \phi}{2} = \frac{7.7 - 4.7}{2} = 1.5\ \text{V}$$

31. **(d)** As we know, Photoelectric current depends on the intensity of incident radiation i.e., $i \propto I$

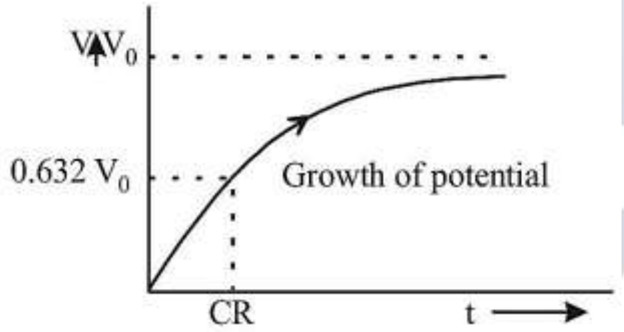
But, intensity of radiation $I \propto \frac{1}{d^2}$ so, $i \propto \frac{1}{d^2}$

32. **(a)** The creation of an elementary particle and its antiparticle usually from a photon (or another neutral boson) is called Pair production. This is allowed, provided there is enough energy available to create the pair.



33. **(a)** For charging the capacitor, $q = q_0\left(1 - e^{\frac{-t}{CR}}\right)$

And, Potential difference $V = V_0\left(1 - e^{-t/CR}\right)$



34. **(c)** Here, $i = \frac{E}{R+r}$

$0.5 = \dfrac{E}{11 + r} \Rightarrow E = 5.5 + 0.5r$

$0.9 = \frac{E}{5+r}$ or, $E = 4.5 + 0.9r$

On solving we get $r = 2.5\Omega$

35. **(c)** Power of battery, when charged is given by $P_1 = V_1 I_1$

Electrical energy dissipated id given by $E_1 = P_1 t_1$

$E_1 = V_1 I_1 t_1 = 15 \times 10 \times 8 = 1200\text{Wh}$

Similarly, the electrical energy dissipated during the discharge of battery is given by $E_2 = V_2 I_2 t_2$

$= 14 \times 5 \times 15 = 1050\text{Wh}$

Hence, watt-hour efficiency of the battery

$\Rightarrow \eta = \dfrac{E_2}{E_1} \times 1000.875 \times 100 = 87.5\%$

36. (b) Here the ratio, $\frac{B_{Centre}}{B_{axis}} = \left(1 + \frac{x^2}{R^2}\right)^{3/2}$

Also, $B_{axis} = \frac{1}{8} B_{centre}$

$$\frac{8}{1} = \left(1 + \frac{x^2}{R^2}\right)^{3/2} \Rightarrow 4 = 1 + \frac{x^2}{R^2}$$

$$3 = \frac{x^2}{R^2} \Rightarrow X^2 = 3R^2$$

or, $x = \sqrt{3}R$

37. (c) Inside a magnet, magnetic lines of force move from south pole to north pole.

38. (b) E.M.F. $e = M\frac{di}{dt} = 0.005 \times \frac{d}{dt}(i_0 \sin \omega t) = 0.005 \times i_0 \cos \omega t\ e_{max} = 0.005 \times 10 \times 100\pi = 5\pi$

39. (a) Impedance of L-C-R circuit will be minimum for a resonant frequency so,

$$v_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{1 \times 10^{-3} \times 0.1 \times 10^{-6}}} = \frac{10^5}{2\pi} \text{Hz}$$

40. (a) Energy $= \frac{12375}{5000} = 2.475\text{eV} = 4 \times 10^{-19}$ J

Minimum intensity to which the eye can respond.

1_{eye} = (photon flux) × energy of a photon

$1_{eye}^{eye} = (5 \times 10^4) \times 4 \times 10^{-19}$

$= 2 \times 10^{-14}\ \text{W/m}^2$

Now, lesser the intensity required by a detector for detection more sensitive it will

be $= \frac{1_{ear}}{1_{eye}} = \frac{10^{-13}}{2\times10^{-14}} = 5$

PART-II CHEMISTRY

41. (c) The organic compounds containing sulphur when react with sodium metal give Na_2S. The Na_2S when react with lead acetate forms black ppt. of PbS.

$Na_2S + (CH_3COO)_2Pb \longrightarrow$

$PbS + 2CH_3COONa$

Black

ppt

42. (b) Volume strength = 5.6 × normality = 5.6 × 1.5 = 8.4 L

43. (a) On subtracting eqn. (ii) from (i) we get

$MnO_4^- + 4H^+ + 3e^- \longrightarrow MnO_2 + 2H_2O$

$\therefore \ -E_3 = \dfrac{-1.51 \times 5 + 2 \times 1.23}{3}$

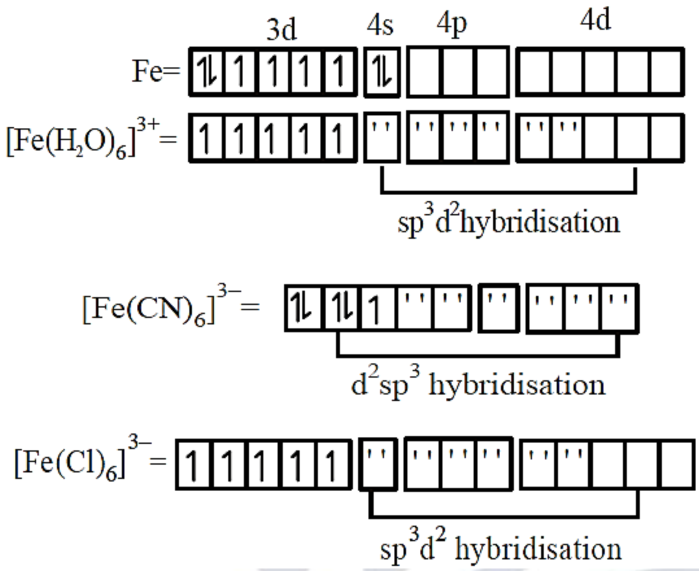
$\therefore \ E_3 = 1.70 \text{ V}$

44. (b) Volume of unit cell $(V) = a^3$

$\therefore \ = (3.04 \times 10^{-8} \text{ cm})^3$

$= 2.81 \times 10^{-23} \text{ cm}^3$

45. (c)



46. (a) A compound having maximum electronegative element will form strong hydrogen bond. F is the most negative element among halogens hence form strongest hydrogen bond.

47. (b) Given $k_f = 1.1 \times 10^{-2}, k_b = 1.5 \times 10^{-3}$

$k_c = \dfrac{k_f}{k_b} = \dfrac{1.1 \times 10^{-2}}{1.5 \times 10^{-3}} = 7.33$

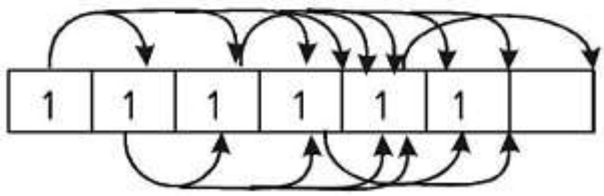
48. (c) Molarity of base $= \frac{\text{normality}}{\text{Acidity}} = \frac{0.1}{1} = 0.1$

$M_1 V_1 = M_2 V_2$

$0.1 \times 19.85 = M_2 \times 20$

$M_2 = 0.09925 \approx 0.099$

49. (d)



$5 + 4 + 3 + 2 + 1 = 15$

$\therefore$ Equation of normal of $x = 0$ and $y = 1$ is $y - 1 = -1(x - 0)$

$\Rightarrow y-1=-x \Rightarrow x+y=1$

50. (a) Given,

(a) $2C+3H_2 \longrightarrow C_2H_6; \Delta H=-21.1\text{kcal}$

(b) $C+O_2 \longrightarrow CO_2; \Delta H=-94.1\text{kcal}$

(c) $H_2+\frac{1}{2}O_2 \longrightarrow H_2O; \Delta H=-68.3\text{kcal}$

Now, eqs. $2 \times$ (b)) $+ 3 \times$ (c) - (a)

$$C_2H_6+\frac{3}{2}O_2 \longrightarrow 2CO_2+3H_2O$$

Heat of combustion of Ethane $\Delta x=2(-94.1)$

$+3(-68.3)-(-21.1)$

$=(-188.2)+(-204.9)-21.1$

$=-372\text{kcal}$

51. (a) $Ag_2CrO_4 \longrightarrow 2Ag^{+}+CrO_4^{2-}$

S 2 S S

$K_{SP}=(2s)^2 s=4s^2 \cdot s=4s^3$

$$S=\left(\frac{K_{sp}}{4}\right)^{1/3} \Rightarrow \left(\frac{32 \times 10^{-12}}{4}\right)^{1/3}$$

$=2 \times 10^{-4}M$

52. (a) We know that,

$\kappa=\Lambda_{eq}.C$

Given, Λ eq $=91.0\Omega^{-1}\text{ cm}^2\text{eq}^{-1}$

$\kappa=(91\Omega^{-1}\text{ cm}^2\text{eq}^{-1})$

$$\left(\frac{2.54}{159/2 \times 1000}\text{ eq.cm}^{-3}\right)$$

$=2.9 \times 10^{-3}\Omega^{-1}\text{ cm}^{-1}$

53. (b) It is known,

$$\frac{(t_{1/2})_1}{(t_{1/2})_2}=\left[\frac{a_2}{a_1}\right]^{(n-1)}$$

Here, $n=$ order of the reaction

Given, $(t_{1/2})_1=0.1\text{ s}, a_1=400$

$(t_{1/2})_2=0.8\text{ s}, a_2=50$

On putting the values,

$$\frac{0.1}{0.8} = \left[\frac{50}{400}\right]^{(n-1)}$$

Taking log on both sides

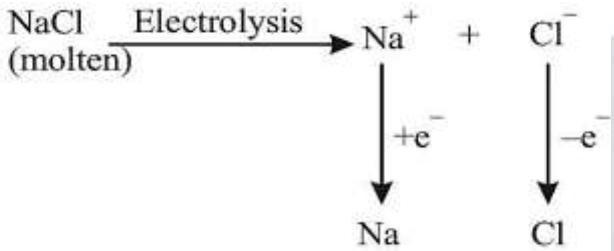
$$\log\frac{0.1}{0.8} = (n-1)\log\frac{50}{400}$$

$$\log\frac{1}{8} = (n-1)\log\frac{1}{8}$$

$$n - 1 = 1 \Rightarrow n = 2$$

54. (d) $2Na + 2H_2O \longrightarrow 2NaOH + H_2 \uparrow$

$2NaOH + CO_2 \longrightarrow Na_2CO_3 + H_2O$ $Na_2CO_3 + 2HCl \longrightarrow 2NaCl + H_2O + CO_2$



55. (b) The purest form of iron is wrought or malleable.
56. (d) P^+ having maximum nuclear charge per electron. Therefore, its size is smallest.
57. (d) $8Al \longrightarrow 8Al^{3+} + 24e^-$

$9Fe^{8/3+} + 24e^- \longrightarrow 9Fe$

Total 24 electrons are transferred

58. (b) On increases the number of lone pairs of electrons, bond angle decreases. Therefore, order of bond angle is

$NH_4^+ > NH_3 > NH_2^-$

(no/p) (one/p) (two/p)

59. (c) $\frac{n'_{He}}{n'CH_4} = \frac{1}{2}\sqrt{\frac{16}{4}} = \frac{1}{1}$

$$\frac{n'_{He}}{n'_{SO_2}} = \frac{1}{3}\sqrt{\frac{64}{4}} = \frac{4}{3}$$

So, molar ratio will be,

$n'He : n'CH_4 : n'SO_2 = 4 : 4 : 3.$

60. (a) Angular momentum, mvr

$$= \frac{nh}{2\pi} = \frac{3 \times h}{2\pi} = \frac{1.5\ h}{\pi}$$

$$= 3\ h\left[\because h = \frac{h}{2\pi}\right]$$

61. (d) Silver carbonate on being strongly heated decomposes as

$Ag_2CO_3 \xrightarrow{\Delta} 2Ag + CO_2 + \frac{1}{2}O_2$

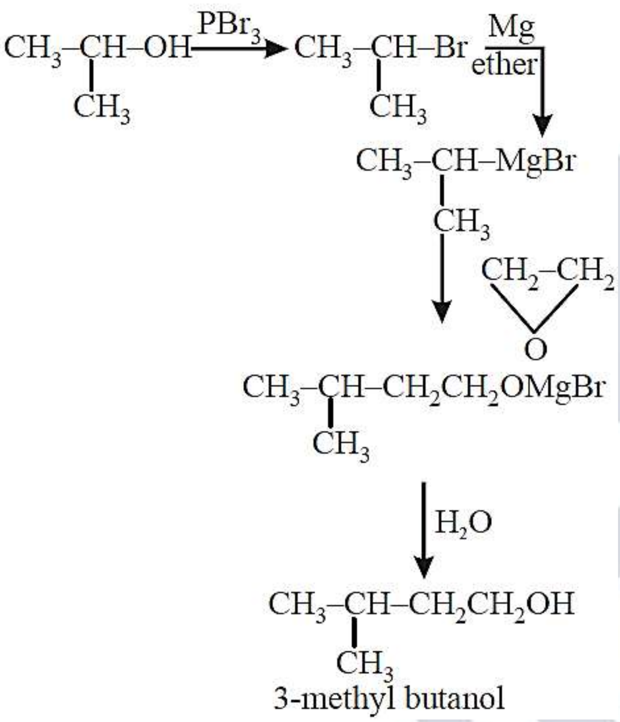
276 g 216 g

As 276 g of Ag_2CO_3 gives = 216 g of Ag

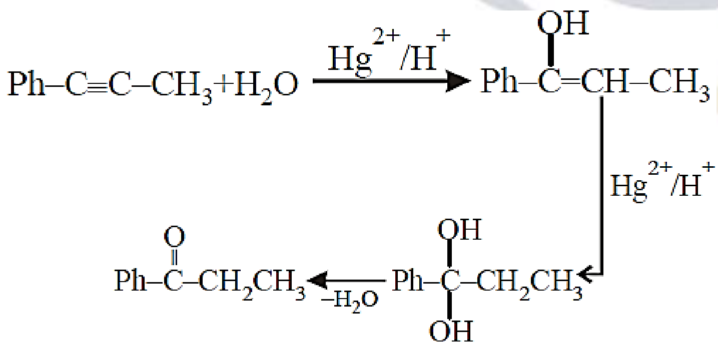
Hence, 2.76 g of Ag_2CO_3 will give

$= \frac{2.76 \times 216}{276} = 2.16$ g

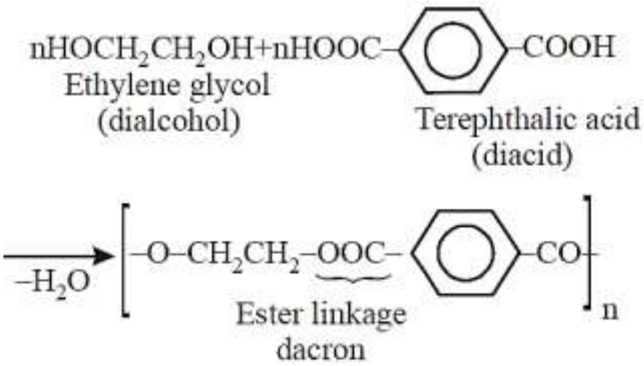
62. (c)



63. (a)



64. (b) Condensation of diacid with dialcohol leads to ester linkage,



65. **(b)** Aldehydes and α-hydroxy ketones give positive Tollen's test. Glucose being a polyhydroxy aldehyde and fructose an α-hydroxy ketone give positive Tollen's test.

66. **(b)** Peptisation is the process in which freshly prepared precipitate disintegrates into colloidal solution.

67. **(a)** $Fe^{2+}(24) = [Ar]3d^6 4s^0$ It has 4 unpaired electrons

$Cu^{+}(28) = [Ar]3d^{10} 4s^0$ It has 0 unpaired electron

$Zn(30) = [Ar]3d^{10} 4s^2$ It has 0 unpaired electron

$Ni^{3+}(25) = [Ar]3d^7 4s^0$ It has 3 unpaired electrons

68. **(b)**

$$CuSO_4 + 2KI \longrightarrow CuI_2 + K_2SO_4$$

$$2CuI_2 \longrightarrow \underset{\text{Cuprous iodide}}{2CuI} + I_2$$

69. **(a)** The correct order of ionic radii is $Cr^{3+} > Mn^{3+} > Fe^{3+} > Sc^{3+}$.

70. **(c)** We know that, $\Delta T_b = \frac{1000 \times K_b \times w}{W \times M}$

$$M = \frac{1000 \times K_b \times w}{W \times \Delta T_b}$$

$$\Delta T_b = \frac{1000 \times K_b \times 10}{100 \times 100}$$

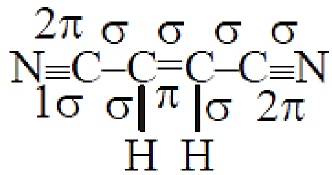
$$\Delta T_b = K_b$$

71. **(b)** Iftwo different alkyl halides ($R_1 - X$ and $R_2 - X$) are used, a mixture of three alkanes is obtained which is difficult to separate.

72. **(b)** On moving down the group, the nature of the oxides of group 13 elements changes from weakly acidic to amphoteric and amphoteric to basic. Tl_2O in aqueous solution gives TlOH which soluble and a strong base.

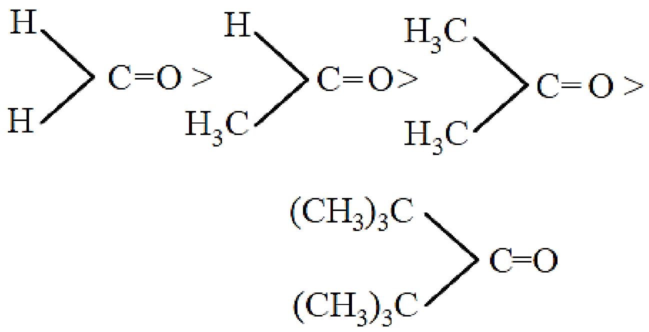
73. **(c)** According to Langmuir's adsorption isotherm, the mass of gas striking a given area of surface is proportional to the pressure of the gas as $\frac{x}{m} = \frac{kp}{1+kp}$

74. **(c)**

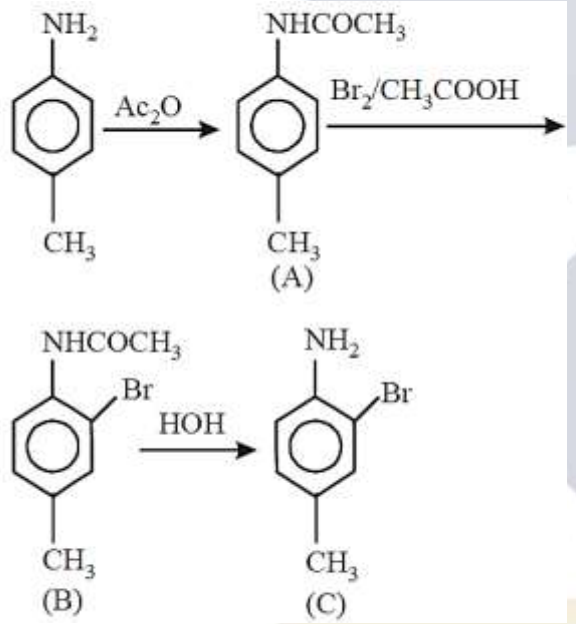


7 sigma and 5 pi

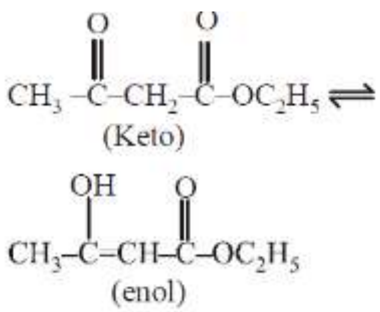
75. (a) As the number and the size of the alkyl groups increases, reactivity decreases. Hence, the correct order of reactivity is



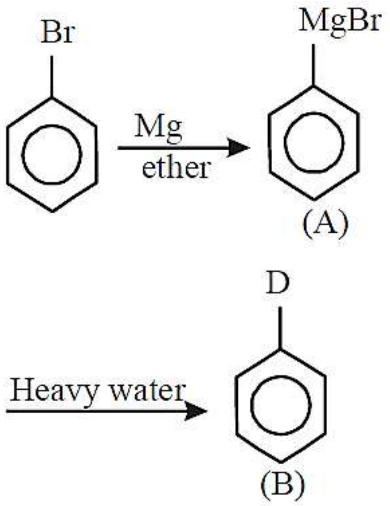
76. (d)



77. (b)



78. (b)



79. **(a)** Nucleophiles are the species which have tendency to donate a pair of electrons. They can be neutral or negatively charged. The nucleophilic power depends on the tendency of species to donate the electrons. Due to the presence of +I effect, it increases. Hence, higher the +I effect, higher the nucleophilic power.

80. **(a)** Heroin is a narcotic analgesic and not used as tranquilizer

PART-III MATHEMATICS

81. **(a)** $\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy}$

$\Rightarrow 2xydy = (x^2 + 1)dx + y^2dx$

$$\Rightarrow \frac{xd(y^2) - y^2dx}{x^2} = \left(\frac{x^2+1}{x^2}\right) \cdot dx$$

$$\Rightarrow \int d\left(\frac{y^2}{x}\right) = \int \left(1 + \frac{1}{x^2}\right) dx$$

$$\Rightarrow \frac{y^2}{x} = x - \frac{1}{x} + C$$

$\Rightarrow y^2 = (x^2 - 1 + Cx)$

When $x = 1, y = 0$

$\Rightarrow 0 = 1 - 1 + C$

$\Rightarrow C = 0$

$\therefore$ The solution is $x^2 - y^2 = 1$ i.e., hyperbola.

82. **(a)** $x \cdot \frac{dy}{dx} + y = x \cdot \frac{f(xy)}{f'(xy)}$

i.e., $\frac{d}{dx}(xy) = x\frac{f(x,y)}{f'(x,y)}$

$\Rightarrow \dfrac{f'(xy)}{f(xy)} d(xy) = xdx$

$\Rightarrow \int \dfrac{f'(xy)}{f(xy)} d(xy) = \int xdx$

$\Rightarrow \log[f(xy)] = \dfrac{x^2}{2} + C$

$\Rightarrow f(xy) = e^{(x^2/2+C)}$

$= e^{\frac{x^2}{2}} \cdot e^C = k. e^{\frac{x^2}{2}}$

83. **(b)** The differential equation of the rectangular hyperbola $xy = c^2$ is

$y + x\dfrac{dy}{dx} = 0$

$\Rightarrow x\dfrac{dy}{dx} = -y$

84. **(c)** Given: $|\mathbf{a}| = 2\sqrt{2}, |\mathbf{b}| = 3$

One diagonal is $5\mathbf{a} + 2\mathbf{b} + \mathbf{a} - 3\mathbf{b} = 6\mathbf{a} - \mathbf{b}$ Length of one diagonal

$= |6\mathbf{a} - \mathbf{b}|$

$= \sqrt{36\mathbf{a}^2 + \mathbf{b}^2 - 2 \times 6|\mathbf{a}| \cdot |\mathbf{b}| \cdot \cos 45^\circ}$

$= \sqrt{36 \times 8 + 9 - 12 \times 2\sqrt{2} \times 3 \times \dfrac{1}{\sqrt{2}}}$

$= \sqrt{288 + 9 - 12 \times 6} = \sqrt{225} = 15$

other diagonal is $(5\mathbf{a} + 2\mathbf{b}) - (\mathbf{a} - 3\mathbf{b})$

$= 4\mathbf{a} + 5\mathbf{b}$

Its length is

$= \sqrt{(4\mathbf{a})^2 + (5\mathbf{b}^2) + 2 \times |4\mathbf{a}||5\mathbf{b}|\cos 45^\circ}$

$= \sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

85. **(b)** $\mathbf{r}.\mathbf{a} = \alpha(\mathbf{a}.\mathbf{b} \times \mathbf{c}) + \beta(\mathbf{a}.\mathbf{c} \times \mathbf{a}) + \gamma(\mathbf{a}.\mathbf{a} \times \mathbf{b})$

$= \alpha[\mathbf{abc}] + 0 + 0$

Similarly, $\mathbf{r}.\mathbf{b} = \beta[\mathbf{abc}]$ and $\mathbf{r}.\mathbf{c} = \gamma[\mathbf{abc}]$

$\therefore \dfrac{1}{2}\mathbf{r}.(\mathbf{a} + \mathbf{b} + \mathbf{c}) = \dfrac{1}{2}(\mathbf{r}.\mathbf{a} + \mathbf{r}.\mathbf{b} + \mathbf{r}.\mathbf{c})$

$= \dfrac{1}{2}(\alpha + \beta + \gamma)[\mathbf{abc}]$

$= \frac{1}{2}(\alpha + \beta + \gamma) \times 2 = \alpha + \beta + \gamma$

86. (c) p, q and r are reciprocal vectors of **a**, **b** and c respectively.

So, $\mathbf{p}.\mathbf{r} = 1, \mathbf{p}.\mathbf{b} = 0 = \mathbf{p}.\mathbf{c}$

$\mathbf{q} \cdot \mathbf{a} = 0, \mathbf{q} \cdot \mathbf{b} = 1, \mathbf{q} \cdot \mathbf{c} = 0$

$\mathrm{r.a} = 0, \mathrm{r.b} = 0, \mathrm{r.c} = 1$

$\therefore$ (la + mb + nc). (lp + mq + nr)

$= l^2 + m^2 + n^2$

87. (a) Let $I = 7^n + 7^m$, then we observe that $7^1, 7^2, 7^3$ and 7^4 ends in 7,9,3 and 1 , respectively. Thus, 7^1 ends in 7,9,3 or 1 according as i is of the form $4k + 1, 4k + 2, 4k - 1$, respectively.

If S is the sample space, then $n(S) = (100)^2$

$7^m + 7^n$ is divisible by 5 , if

(i) m is of the form $4k + 1$ and n is of the form $4k - 1$ or

(ii) m is of the form $4k + 2$ and n is of the form 4k or
(iii) m is of the form $4k - 1$ and n is of the form $4k + 1$ or

(iv) m is of the form 4k and n is of the form $4k + 1$ or

So, number of favourable ordered pairs $(m, n) = 4 \times 25 \times 25$

$\therefore$ Required probability $= \frac{4\times25\times25}{(100)^2} = \frac{1}{4}$

88. (b) Let **x** denote the sum of the numbers obtained when two fair dice are rolled.

So, X may have values 2,3,4,5,6,7,8,9, 10,11 and 12 .

$P(X = 2) = P(1,1) = \frac{1}{36}$

$P(X = 3) = P\{(1,2), (2,1)\} = \frac{2}{36}$

$P(X = 4) = \frac{3}{36}, P(X = 5) = \frac{4}{36}$

$P(X = 6) = \frac{5}{36}; P(X = 7) = \frac{6}{36}$

$P(X = 8) = \frac{5}{36}$

$P(X = 9) = \frac{4}{36}; P(X = 10) = \frac{3}{36}; P(X = 11)$
$= \frac{2}{36}$

$P(X = 12) = \frac{1}{36}$

$\therefore$ Probability distribution table is given below

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Mean $\overline{X} = \sum XP(X)$

$$= \frac{\begin{bmatrix} 2\times 1 + 3\times 2 + 4\times 3 + 5\times 4 + 6\times 5 + 7\times 6 + \\ 8\times 5 + 9\times 4 + 10\times 3 + 11\times 2 + 12\times 1 \end{bmatrix}}{36}$$

$$= \frac{252}{36} = 7$$

Variance $= \sum X^2 P(X) - \overline{X}^2 = \frac{\begin{bmatrix} 2^2\times 1+3^2\times 2+4^2\times 3+5^2\times 4 \\ +6^2\times 5+7^2\times 6+8^2\times 5+9^2\times \\ 4+10^2\times 3+11^2\times 2+12^2\times 1 \end{bmatrix}}{36} - 7^2$

$$= \frac{1974}{36} - 49$$

$$= \frac{1974 - 1764}{36}$$

$$= \frac{210}{36} = \frac{35}{6}$$

$\therefore$ Variance $= \frac{35}{6}$

And, SD $= \sqrt{\frac{35}{6}}$

89. (d) Given digits are 1, 2, 3, 4 .

Possibilities for units place digit

(either 1 or 3) = 2

Possibilities for ten's digit = 3

Possibilities for hundred's place digit = 2

Possibilities for thousand place's digit = 1

$\therefore$ Number of favourable outcomes

$= 2 \times 3 \times 2 \times 1 = 12$

Number of numbers formed by 1,2,3,4

(without repetitions) = 4 !

$\therefore$ Required probability $= \frac{12}{4\times 3\times 2} = \frac{1}{2}$

90. (b) Vertices of $\triangle$ ABC are $A(0,4,1), B(2,3,-1)$ and $C(4,5,0)$.

$$AB = \sqrt{(2-0)^2 + (3-4)^2 + (-1-1)^2}$$
$$= \sqrt{4+1+4} = 3$$
$$BC = \sqrt{(4-2)^2 + (5-3)^2 + (0+1)^2}$$
$$= \sqrt{4+4+1} = 3$$

and $CA = \sqrt{(4-0)^2 + (5-4)^2 + (0-1)^2}$

$= \sqrt{16+1+1} = 3\sqrt{2}$

$AB^2 + BC^2 = AC^2$

$\therefore$ $\triangle$ ABC is a right angled triangle.

We know that, the ortho centre of a right -angled triangle is the vertex containing the right angle.

$\therefore$ Orthocentre is point $B(2,3,-1)$.

91. (a) Given curve is

$y = (1+x)^y + \sin^{-1}(\sin^2 x)$

On differentiating w.r.t x, we get

$$\frac{dy}{dx} = (1+x)^y\left[\frac{y}{1+x} + \log(1+x)\frac{dy}{dx}\right] + \frac{2\sin x\cos x}{\sqrt{1-\sin^4 x}}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{at(0,1)} = 1[\because \text{ at } x=0, y=1]$$

Slope of normal at $(x=0) = -1$

$\therefore$ Equation of normal at $x=0$ and $y=1$ is

$y-1 = -1(x-0)$

$\Rightarrow y-1 = -x \Rightarrow x+y = 1$

92. (c) It is clear that $f(x)$ has a definite and unique value for each $x \in [1,5]$.

Thus, for every point in the interval $[1,5]$, the value of $f(x)$ exists.

So, $f(x)$ is continuous in the interval $[1,5]$.

Also, $f'(x) = \frac{-x}{\sqrt{25-x^2}}$, which clearly exists

for all x in an open interval $(1,5)$.

So, $f'(x)$ is differentiable in $(1,5)$.

So, there must be a value $c \in [1,5]$ such that

$$f'(c) = \frac{f(5)-f(1)}{5-1} = \frac{0-\sqrt{24}}{4}$$

$= \frac{0 - 2\sqrt{6}}{4} = \frac{-\sqrt{6}}{2}$

But $f'(c) = \frac{-c}{\sqrt{25-c^2}}$

$\Rightarrow \frac{-c}{\sqrt{25 - c^2}} = -\frac{\sqrt{6}}{2}$

$\Rightarrow 4c^2 = 6(25 - c^2)$

$\Rightarrow 4c^2 = 150 - 6c^2 \Rightarrow 10c^2 = 150$

$\Rightarrow c^2 = 15 \Rightarrow c = \pm\sqrt{15}$

$\therefore c = \sqrt{15} \in [1,5]$

93. **(c)** $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$

$|A| = 21 - 20 = 1 \therefore\ A(\text{adj } A) = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A|.I$

94. **(b)** Volume V of a cube of side x is given by $V = x^3$

$\Rightarrow \frac{dv}{dx} = 3x^2$

Let the change in x be $\Delta x =$ K% of

$x = \frac{kx}{100}$

Now, the change in volume,

$\Delta V = \left(\frac{dV}{dx}\right)\Delta x = 3x^3(\Delta x)$

$= 3x^2\left(\frac{kx}{100}\right) = \frac{3x^3 \cdot k}{100}$

$\therefore$ Approximate change in volume

$= \frac{3kx^3}{100} = \frac{3k}{100} \cdot x^3$
$= 3$ K% of original volume

95. **(c)** The system has non- zero solution, if

$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$

$\Rightarrow 1(-k - 3) - k(3 + 1) - 1(-9 + k) = 0$

$\Rightarrow -6k + 6 = 0$

$\Rightarrow k=1$

96. (d) The points (1,2,3) and $(2,-1,0)$ lie on the opposite sides of the plane $2x+3y-2z-k=0$ So, $(2+6-6-k)(4-3-k)<0$

$\Rightarrow (k-1)(k-2)<0$

$\therefore 1<k<2$

97. (d) $\Delta(x)=\begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$

Applying $C_3 \rightarrow C_3+C_2-C_1$

$\Delta(x)=\begin{vmatrix} 1 & \cos x & 0 \\ 1+\sin x & \cos x & 0 \\ \sin x & \sin x & 1 \end{vmatrix}$

$=\cos x-\cos x(1+\sin x)$

$[\because$ expanding along $C_3]=-\cos x \cdot \sin x=-\frac{1}{2}\sin 2x$

$\therefore \int_0^{\pi/4}\Delta(x)dx=-\frac{1}{2}\int_0^{\pi/4}\sin 2x dx$

$=-\frac{1}{2}\left[-\frac{\cos 2x}{2}\right]_0^{\pi/4}$

$=\ +\frac{1}{2\times 2}\left[\cos\frac{\pi}{2}-\cos 0^{\circ}\right]$

$=\frac{1}{4}(0-1)=-\frac{1}{4}$

98. (b) $f'(x)$ is differentiable $\forall x \in [1,6]$ By Lagrange's mean value theorem,

$f'(x)=\frac{f(6)-f(1)}{6-1}$

$f'(x)\geq 2\forall x\in[1,6]$

(given)

$\Rightarrow \frac{f(6)+2}{5}\geq 2$

$[\because f(1)=-2]$

$\Rightarrow f(6)\geq 10-2 \Rightarrow f(6)\geq 8$

99. (b) $\Delta_r=\begin{vmatrix} 2r-1 & m_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix} \therefore \sum_{r=0}^m \Delta_r=\begin{vmatrix} \sum_{r=0}^m(2r-1) & \sum_{r=0}^m {}^mC_r & \sum_{r=0}^m 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$

$=\begin{vmatrix} m^2-1 & 2^m & m+1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$

$= 0$ ($\because$ two rows are identical)

100. **(c)** $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = r$ (say)

$\Rightarrow x = 2r + 1, y = 3r - 1, z = 4r + 1$

Since, the two lines intersect.

So, putting above values in second line, we get

$$\frac{2r+1-3}{1} = \frac{3r-1-k}{2} = \frac{4r+1}{1}$$
$$2r - 2 = 4r + 1$$
$\Rightarrow r = -3/2$

Also $3r - 1 - k = 8r + 2$

$$\Rightarrow k = -5r - 3 = \frac{15}{2} - 3 = \frac{9}{2}$$

101. **(a)** Let $f(x) = \frac{x}{\log x}$

$$\Rightarrow f'(x) = \frac{\log x - 1}{(\log x)^2}$$

For maxima and minima, put $f'(x) = 0$

$\log x - 1 = 0$

$\Rightarrow x = e$

Now, $f''(x)$

$$= \frac{(\log x)^2 \cdot \frac{1}{x} - (\log x - 1) \cdot \frac{2\log x}{x}}{(\log x)^4}$$

$$\Rightarrow f''(e) = \frac{\frac{1}{e} - 0}{1} = \frac{1}{e} > 0$$

$\therefore$ $f(x)$ is minimum at $x = e$.

Hence, minimum value of $f(x)$ at $x = e$ is

$$f(e) = \frac{e}{\log e} = e$$

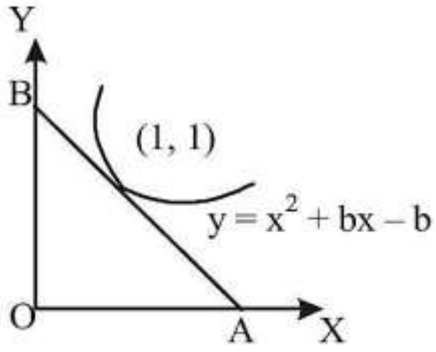
102. **(c)** Given curve is $y = f(x) = x^2 + bx - b$

$\Rightarrow f'(x) = 2x + b$

The equation of tangent at point $(1,1)$ is

$$y - 1 = \left(\frac{dy}{dx}\right)_{(1,1)} (x - 1)$$

$\Rightarrow y-1=(b+2)(x-1)$
$\Rightarrow (2+b)x-y=1+b$
$\Rightarrow \frac{x}{\left(\frac{1+b}{2+b}\right)}-\frac{y}{(1+b)}=1$



So, $OA=\frac{1+b}{2+b}$

and $OB=-(1+b)$

Now, area of $\triangle$ AOB $=\frac{1}{2}\times$

$\frac{(1+b)[-(1+b)]}{(2+b)}=2$

$\Rightarrow 4(2+b)+(1+b)^2=0$

$\Rightarrow b^2+6b+9=0$

$\Rightarrow (b+3)^2 \Rightarrow b=-3$

103. (c)

p	**q**	**p**	**p ⇒ q**	**~ p^q**	**(p ⇒ q) ⇔ (~ p^q)**
T	T	F	T	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	F	F

Hence, given statement is neither tautology nor contradiction.

104. (b) $x+iy$

$$=\frac{3}{2+\cos\theta+i\sin\theta}=\frac{3(2+\cos\theta-i\sin\theta)}{(2+\cos\theta)^2+\sin^2\theta}$$

$$=\frac{6+3\cos\theta-3i\sin\theta}{4+\cos^2\theta+4\cos\theta+\sin^2\theta}$$

$$= \frac{6 + 3\cos\theta - 3i\sin\theta}{5 + 4\cos\theta}$$

$$= \left(\frac{6 + 3\cos\theta}{5 + 4\cos\theta}\right) + \left(\frac{-3\sin\theta}{5 + 4\cos\theta}\right)$$

On equating real and imaginary parts, we get

$$x = \frac{3(2 + \cos\theta)}{5 + 4\cos\theta}$$

And $y = \frac{-3\sin\theta}{5+4\cos\theta}$

$$\therefore x^2 + y^2 = \frac{9[4 + \cos^2\theta + 4\cos\theta + \sin^2\theta]}{(5 + 4\cos\theta)^2}$$

$$= \frac{9}{5 + 4\cos\theta} = 4\left(\frac{6 + 3\cos\theta}{5 + 4\cos^2\theta}\right) - 3$$

$= 4x - 3$

105. **(b)** Let S: $(\sim p \wedge q) \vee (p \wedge \sim q)$

$\Rightarrow \sim S: \sim [(\sim p \wedge q) \vee (p \wedge \sim q)]$

$\Rightarrow \sim S: \sim (\sim p \wedge q) \wedge \sim (p \wedge \sim q)$

$\Rightarrow \sim S: (p \vee \sim q) \wedge (\sim p \vee q)$

106. **(b)** The sum of ordinates of feet of normals drawn from a point to the parabola, $y^2 = 4ax$ is always zero. Now, as normals at three points P, Q and R of parabola $y^2 = 4ax$ meet at (h, k).

$\Rightarrow$ The normals from (h, k) to $y^2 = 4ax$ meet the parabola at P, Q and R.

$\Rightarrow$ y-coordinate y_1, y_2, y_3 of these points and R will be zero.

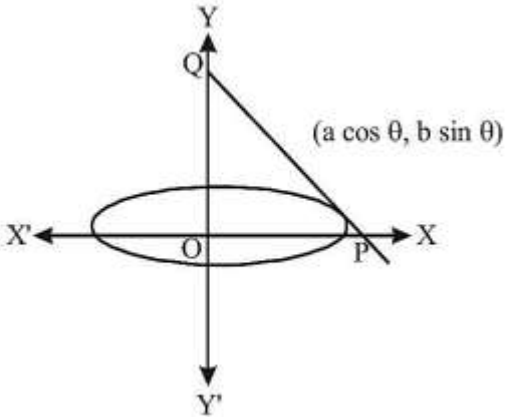
$\Rightarrow$ y-coordinate of the centroid of $\triangle$ PQR

i. e., $\frac{y_1+y_2+y_3}{3} = \frac{0}{3} = 0$

$\therefore$ centroid lies on $y = 0$

107. **(c)** Equation of tangent at $(a\cos\theta, b\sin\theta)$ to the ellipse is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$



Coordinates of P and Q are

$\left(\frac{a}{\cos\theta}, 0\right)$ and $\left(0, \frac{b}{\sin\theta}\right)$, respectively

Area of $\triangle$ OPQ $= \frac{1}{2}\left|\frac{a}{\cos\theta} \times \frac{b}{\sin\theta}\right| = \frac{ab}{|\sin 2\theta|}$

$\therefore$ Minimum area ab

108. **(b)** The equation of any normal

to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $ax\cos\phi + by\cot\phi$

$= a^2 + b^2$

$\Rightarrow ax\cos\phi + by\cot\phi - (a^2 + b^2) = 0$

The straight line $lx + my - n = 0$ will be normal to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then eq. (i) and $lx + my - n =$

0 represent the same line,

$$\therefore \frac{a\cos\phi}{l} = \frac{b\cot\phi}{m} = \frac{a^2 + b^2}{n}$$

$$\Rightarrow \sec\phi = \frac{na}{l(a^2 + b^2)}$$

and $\tan\phi = \frac{nb}{m(a^2+b^2)}$

$$\frac{n^2a^2}{l^2(a^2 + b^2)^2} - \frac{n^2b^2}{m^2(a^2 + b^2)^2} = 1$$

$[\because \sec^2\phi - \tan^2\phi = 1] \Rightarrow \frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

But given equation of normal is

$$\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2 + b^2)^2}{k}$$

$\therefore k = n^2$

109. **(d)** Given: $a = \cos\alpha + i\sin\alpha$

$b = \cos\beta + i\sin\beta$

and $c = \cos\gamma + i\sin\gamma$

Now, $\frac{b}{c} = \frac{\cos\beta + i\sin\beta}{\cos\gamma + i\sin\gamma} \times \frac{\cos\gamma - i\sin\gamma}{\cos\gamma - i\sin\gamma}$

$\cos\beta \cdot \cos\gamma + \sin\beta \cdot \sin\gamma + i$

$[\sin\beta \cdot \cos\gamma - \sin\gamma \cdot \cos\beta]$

$\Rightarrow \frac{b}{c} = \cos(\beta - \gamma) + i\sin(\beta - \gamma)$

Similarly, $\frac{c}{a} = \cos(\gamma - \alpha) + i\sin(\gamma - \alpha)$

and $\frac{a}{b} = \cos(\alpha - \beta) + i\sin(\alpha - \beta)$

On adding Eqs. (i), (ii) and (iii), we get $\cos(\beta - \alpha) + \cos(\gamma - \alpha) + \cos(\alpha + \beta) + i\,[\sin(\beta - \gamma) + \sin(\gamma - \alpha) + \sin(\alpha - \beta)] = 1$

$\left[\because \frac{b}{c} + \frac{c}{a} + \frac{a}{b} = 1\right]$

On equating real parts, we get

$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = 1$

110. **(b)** $|z + 4| \le 3 \Rightarrow -3 \le z + 4 \le 3$

$\Rightarrow -6 \le z + 1 \le 0$

$\Rightarrow 0 \le -(z + 1) \le 6$

$\Rightarrow 0 \le |z + 1| \le 6$

Hence, the greatest and least values are 6 and 0

111. **(c)** On homogenising $y^2 - x^2 = 4$ with the help of the line $\sqrt{3}x + y = 2$, we get

$$y^2 - x^2 = 4\frac{(\sqrt{3}x + y)^2}{4}$$

$\Rightarrow y^2 - x^2 = 3x^2 + y^2 + 2\sqrt{3}xy$

$\Rightarrow 4x^2 + 2\sqrt{3}xy = 0$

On comparing with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 4, h = \sqrt{3}$ and $b = 0$

We know that,

$$\tan\theta = 2\frac{\sqrt{h^2 - ab}}{a + b} = \frac{2 \cdot \sqrt{3 - 0}}{4 + 0}$$

$\therefore$ The angle between the lines is

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

112. **(c)** Let $z = x + iy$, then

$z + iz = x + iy + i(x + iy) = (x - y) + i(x + y)$ and $iz = i(x + iy) = -y + ix$,

Then, the area of the triangle formed by these lines is

$$\Delta = \frac{1}{2}\left\|\begin{matrix} x & y & 1 \\ (x-y) & (x+y) & 1 \\ -y & x & 1 \end{matrix}\right\|$$

Applying $R_2 \rightarrow R_2 - (R_1 + R_3)$,

$$\Delta = \frac{1}{2}\left\|\begin{matrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 1 \end{matrix}\right\| = \frac{1}{2}(x^2 + y^2)$$

$$\Rightarrow \frac{1}{2}|z|^2 = 200$$
$$|z|^2 = 400 \Rightarrow |z| = 20$$
$$\therefore 3|z| = 3 \times 20 = 60$$

113. **(b)** Given hyperbola is $25x^2 - 16y^2 = 400$ If $(6,2)$ is the midpoint of the chord, then equation of chord is $T = S_1$

$\Rightarrow 25(6x) - 16(2y) = 25(36) - 16(4)$

$\Rightarrow 75x - 16y = 450 - 32$

$\Rightarrow 75x - 16y = 418$

114. **(b)** Let the equation of the plane is

$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

Then, $A(\alpha, 0,0), B(0, \beta, 0)$ and $C(0,0, \gamma)$ are the points on the coordinate axes, The centroid of the triangle is $(1,2,4)$.

$\therefore \frac{\alpha}{3} = 1 \Rightarrow \alpha = 3$

$\frac{\beta}{3} = 2 \Rightarrow \beta = 6$

and $\frac{\gamma}{3} = 4 \Rightarrow \gamma = 12$

$\therefore$ The equation of the plane is

$$\frac{x}{3} + \frac{y}{6} + \frac{z}{12} = 1$$
$$\Rightarrow 4x + 2y + z = 12$$

115. **(b)** The given equation of the plane is $3x + 4y - 5z - 60 = 0$. It can be written in the

form $\frac{x}{20} + \frac{y}{15} + \frac{z}{-12} = 1$

which meets the coordinate axes at the points $A(20,0,0), B(0,15,0)$ and $C(0,0,-12)$. The coordinates of the origin are $O(0,0,0)$. Therefore, volume of the tetrahedron OABC is =

$$= \frac{1}{6}\begin{vmatrix} 20 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & -12 \end{vmatrix} = \frac{1}{6}|20 \times 15 \times (-12)| = 600$$

116. **(b)** $\int_0^{2\pi}(\sin x + |\sin x|)dx$

$= \int_0^{\pi}(\sin x + \sin x)dx +$

$\int_{\pi}^{2\pi}(\sin x - \sin x)dx$

$= 2\int_0^{\pi}\sin x dx + 0 = -2\int_0^{\pi}\cos x dx$

$= -2(\cos\pi - \cos 0) = -2(-1-1) = 4$

117. **(c)** $\int_0^{\sqrt{2}}[x^2]dx = \int_0^1[x^2]dx + \int_1^{\sqrt{2}}[x^2]dx$

$= \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx$

$= [x]_1^{\sqrt{2}} = \sqrt{2} - 1$

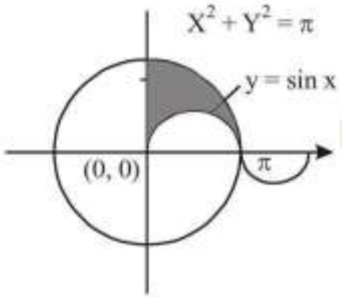
118. **(a)** $I(m,n) = I = \int_0^1 t^m(1+t)^n dt$

$$\Rightarrow I(m,n) = \left[(1+t)^n \cdot \frac{t^{m+1}}{m+1}\right]_0^1$$

$$-\frac{n}{m+1}\int_0^1 (1+t)^{n-1} \cdot t^{m+1}. dt$$

$$= \frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$$

119. **(a)** $x^2 + y^2 = \pi^2$ is a circle of radius π and centre at origin.



Required area

$= \text{Area of circle (1st quadrant)} - \int_0^{\pi}\sin x dx$

$$= \frac{\pi \cdot \pi^2}{4} - [-\cos x]_0^{\pi} = \frac{\pi^3}{4} + (\cos\pi - \cos 0)$$

$$= \frac{\pi^3}{4} + (-1-1) = \frac{\pi^3}{4} - 2 = \frac{\pi^3 - 8}{4}$$

120. (d)

$|x| = 1$

$\therefore\ x = \pm 1$

$\therefore\ y = xe^{|x|} = \begin{Bmatrix} x \cdot e^{-x}, -1 < x < 0 \\ x.e^{x}, 0 < x < 1 \end{Bmatrix}$

$\therefore$ Required area $= \left|\int_{-1}^{0} xe^{-x}dx + \int_{0}^{1} xe^{x}dx\right|$

$= \left|[-x.e^{-x} - e^{-x}]_{-1}^{0} + \{x.e^{x} - e^{x}\}_{0}^{1}\right|$

$= |\{(0-1) - (1.e - e)\}| + |\{(e - e) - (0 - 1)\}|$

$= 1 + 1 = 2$ squnits