PART-I PHYSICS

1. **(a)** As r increase, the potential energy increases. Thus, it decreases kinetic energy of hydrogen atom. So, when an atom jumps from one energy level to the higher level, its potential energy increases and kinetic energy decreases.
2. **(b)** Given: $T_{1/2} = 4.47 \times 10^8 \text{yr}$

$$\frac{N}{N_0} = \frac{60}{100} = \left(\frac{1}{2}\right)^n \Rightarrow 2^n = \frac{10}{6}$$

Apply logarithm on both sides $n\log 2 = \log 10 - \log 6$

$\Rightarrow n \times 0.3 = 1 - 0.778 = 0.22$

$$\Rightarrow n = \frac{0.222}{0.3} = 0.74$$

So, $t = nT_{1/2} = 0.74 \times 4.47 \times 10^8$

or, $t = 3.3 \times 10^8 \text{yr}$

3. **(b)** $Y(n, \alpha)$ the nucleus splits into α-particle and neutrons

i.e. $ZY^A + {}_0n^1 \longrightarrow {}_3L_i{}^7 + {}_2He^4$

So $A + 1 = 7 + 4 \Rightarrow A = 10$

and $Z + 0 = 3 + 2$ or $Z = 5$

Hence, the nucleus of element Y is boron ${}_5Y^{10} = {}_5B^{10}$.

4. **(c)** Energy of electron in nth orbit of hydrogen atom

$$E_n = -\frac{13.6}{n^2}\text{eV}$$

$$\Rightarrow 3.4 = -\frac{13.6}{n^2} \Rightarrow n^2 = 4$$

or, $n = 2$

Angular momentum of electron

$$L = \frac{nh}{2\pi} = \frac{2h}{2\pi} = \frac{h}{\pi}$$

5. **(b)** As, we know de-Broglic wavelength, $\lambda = \frac{h}{p}$

$$\therefore \lambda \propto \frac{1}{p} \Rightarrow \frac{\Delta p}{p} = -\frac{\Delta\lambda}{\lambda} \therefore \left|\frac{\Delta p}{p}\right| = \left|\frac{\Delta\lambda}{\lambda}\right|$$

$$\Rightarrow \frac{p'}{p} = \frac{0.20}{100} = \frac{1}{500}$$

or, $p = 500p'$

6. **(a)** The emitted electrons may lie near the surface and can have a maximum amount of energy E_0. If they are from deep inside, then energy is less than E_0.

7. **(a)** The n-type semiconductor has excess of free electrons for conduction. The total number of electrons in an atom is equal to the total number of protons in the nucleus. So, n-type semiconductor is neutral.

8. **(c)** The output is obtained for half cycle only in half wave rectifier. Therefore, frequency of the ripple is same as that of the input i.e. 50 Hz.

9. **(c)** Given: $\Delta I_C = 1\ \text{mA} = 10^{-3}\ \text{A}$

$\Delta I_b = 15\mu\text{A} = 15 \times 10^{-6}\ \text{A}$

$R_L = 5\text{k}\Omega = 5 \times 10^3 \Omega.$

$Ri = 330\Omega$

The voltage gain of an amplifier

$$A_r = \frac{\Delta I_C \times R_L}{\Delta I_b \times R_i}$$

$$= \frac{10^{-3} \times 5 \times 10^3}{15 \times 10^{-6} \times 330} \approx 1010$$

10. **(a)** As we know, output of OR gate

$Y = A + B$

Output of AND gate

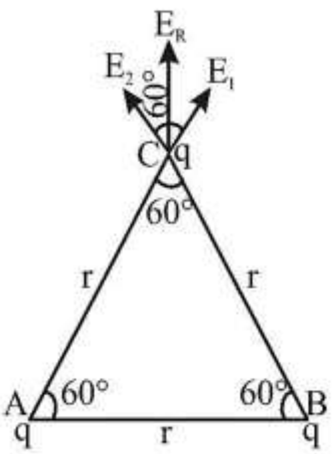
$Y' = Y.C$

$\Rightarrow Y' = (A + B).C$

If $C = 0$ irrespective of A and B, then output Y must be zero.

11. **(c)** Due to charge at A and B magnitude of intensity of electric field at point C

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



Net intensity at point C is

$$E_R = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 60^\circ}$$

$$= \sqrt{E_1^2 + E_1^2 + 2E_1^2 \times \frac{1}{2}} = \sqrt{3}E_1 = \frac{\sqrt{3}q}{4\pi\varepsilon_0 r^2}$$

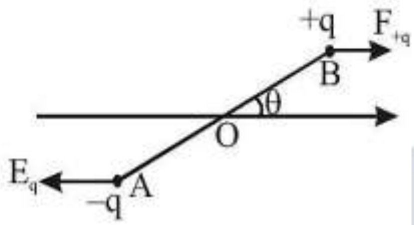
12. (b) Torque when the wire is brought in a uniform field E.

$\tau = qEL\sin\theta$

$= qEL\theta$ [$\because$ θ is very small]

Moment of inertia of rod AB about O

$$I = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{L^2}{2}$$



$\tau = I\alpha.$

$$\therefore\ \alpha = \frac{\tau}{I} = \frac{qEL\theta}{\frac{mL^2}{2}}$$

$$\Rightarrow\ \omega^2\theta = \frac{2qEL\theta}{mL^2}\ [\because\ \theta = \omega^2\theta]$$

$$\Rightarrow\ \omega^2 = \frac{2qE}{mL}$$

Time period of the wire

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mL}{2qE}}$$

The rod will become parallel to the field in time $\frac{T}{4}$.

$$\therefore\ t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$$

13. (d) The energy stored in the capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}C \times (200)^2 = 2C \times 10^4\ J$$

This energy is used to heat up the block. Let $\Delta\theta$ be the rise in temperature, then heat energy

$$Q = ms\Delta\theta = 0.1 \times 250 \times 0.4 = 10\ J$$

Now,

$2C \times 10^4 = 10$

$\Rightarrow C = \dfrac{10}{2 \times 10^4} = 5 \times 10^{-4} = 500\mu F$

14. (b) Capacitance of air capacitor

$C_0 = \dfrac{\varepsilon_0 A}{d} = 3\mu F$

When a dielectric of permittivity ε_r and dielectric constant K is introduced between the plates, then

Capacitance, $C = \frac{K\varepsilon_0 A}{d} = 15\mu F$

Dividing eq. (ii) by (i), we get

$\dfrac{C}{C_0} = \dfrac{d}{\dfrac{\varepsilon_0 A}{d}} = \dfrac{15}{3}$

$\Rightarrow K = 5$

$\therefore$ permittivity of the medium

$\varepsilon_r = \varepsilon_0 K$

$\varepsilon_r = 8.85 \times 10^{-12} \times 5 = 0.44 \times 10^{-10}$

15. (d) using, $R = \rho \frac{1}{A}$

$R_1 : R_2 : R_3 = \dfrac{1_1}{A_1} : \dfrac{1_2}{A_2} : \dfrac{1_3}{A_3}$

$= \dfrac{1_1^2}{V_1} : \dfrac{l_2^2}{V_2} : \dfrac{l_3^2}{V_3}$

$= \dfrac{1_1^2}{(m_1\ d)} : \dfrac{1_2^2}{(m_2\ d)} : \dfrac{1_3^2}{(m_3\ d)}$

$= \dfrac{1_1^2}{m_1} : \dfrac{1_2^2}{m_2} : \dfrac{1_3^2}{m_3}$

$= \dfrac{5^2}{2} : \dfrac{3^2}{3} : \dfrac{2^2}{5} = 125 : 30 : 8$

16. (b) Resistance of lamp

$R_0 = \dfrac{V^2}{P} = \dfrac{(30)^2}{90} = 10\Omega$

Current in the lamp

$I = \dfrac{V}{R_0} = \dfrac{30}{10} = 3\ A$

As the lamp is operated on 120 V DC, then resistance becomes

$$R' = \frac{V'}{i} = \frac{120}{3} = 40\Omega$$

For proper glow, a resistance R is joined in series with the bulb $R' = R + R_0$

$$\Rightarrow R^{\alpha} = R' - R_0 = 40 - 10 = 30\Omega$$

17. (d) Let us Consider a cell of emf E and balancing length 1_1

$E = kl_1$

potential difference is balanced by length 1_2. $V = kl_2$

Internal resistance of the cell

$$r = \left(\frac{E - V}{V}\right)R = \left(\frac{E}{V} - 1\right)R = \left(\frac{l_1}{l_2} - 1\right)R$$

$$= \left(\frac{560}{560 - 60} - 1\right)10 = \left(\frac{56}{50} - 1\right)10$$

$$= \frac{6}{5} = 1.2\Omega$$

18. (d) Let the emf of each source be E. When they are connected in series, the current in the circuit

$$I = \frac{E_{tot}}{R_{tot}} = \frac{E + E}{r_1 + r_2 + R} = \frac{2E}{r_1 + r_2 + R}$$

$\therefore$ potential drop across the cell of internal

resistance r_2, $\left(\frac{2E}{r_1+r_2+R}\right)r_2$

Hence, $E - \frac{2E}{(r_1+r_2+R)}r_2 = 0 \Rightarrow \quad r_1 + r_2 + R = 2r_2$

$$R = r_2 - r_1$$

19. (a) Here, the magnetic force (Bqv) will provide the necessary centripetal force $\left(\frac{mv^2}{r}\right)$

$$\therefore Bqv = \frac{mv^2}{r}$$

$$\Rightarrow Bqr = mv$$

For electron and proton, the magnetic field B, charge q and radius r, all same.

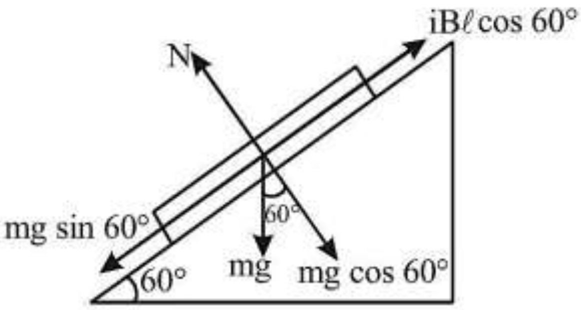
$\therefore$ mv = constant

i.e. $m_e v_e = m_p v_p$

$$v_P = \left(\frac{m_e}{m_p}\right)v_e = \left(\frac{9 \times 10^{-31}}{1.8 \times 10^{-27}}\right)3 \times 10^6$$

$$= 1.5 \times 10^3 \text{ m/s}$$

20. (a) Here two forces acting on the rod simultaneously.



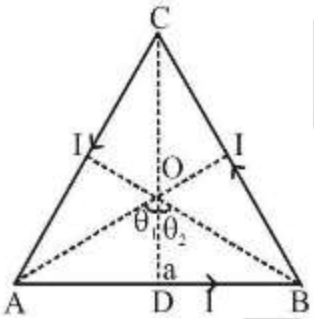
From FBD, mg sin 60 = Bil cos 60°

$B = \frac{mg}{il}\tan 60^\circ$

$= \frac{0.01 \times 10}{173 \times 0.1} \times \sqrt{3} = 1\ T$

21. (c) Due to current through side AB Magnetic field at the centre O

$B_1 = \frac{\mu_0 I}{4\pi a}[\sin\theta_1 + \sin\theta_2]$



As the magnetic field due to each of the three sides is the same in magnitude and direction.

∴ Total magnetic field at O is sum of all the fields.

i.e. $B = 3B_1 = \frac{3\mu_0 I}{4\pi a}[\sin\theta_1 + \sin\theta_2]$

Here, $\tan\theta_1 = \frac{AD}{OD} \Rightarrow \tan 60^\circ = \frac{\frac{\ell}{2}}{a}$

$\Rightarrow a = \frac{\ell}{2\sqrt{3}} = \frac{9 \times 10^{-2}}{2\sqrt{3}}$

Now B

$= 3 \times \frac{4\pi \times 10^{-7} \times 2}{4\pi \times \frac{9 \times 10^{-2}}{2\sqrt{3}}}[\sin 60^\circ + \sin 60^\circ]$

$= \frac{4\sqrt{3}}{9} \times 10^{-5}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right]$

$= 1.33 \times 10^{-5}\ T$

22. (b) The direction of dB is the direction of vector $dl \times r$. From right hand screw rule, if we place a right -handed screw at the point where the magnetic field is needed to be determined and turn its handle from dl to r, then the direction in which the screw advances gives the direction of field dB.

23. (b) Given: current sensitivity $= 10 \text{div/mA}$ and there are 100 division on the scale.

$\therefore$ Current required for full scale deflection.

$$I_g = \frac{1}{10} \times 100 \text{ mA} = 10 \text{ mA} = 0.01 \text{ A}$$

Also voltage sensitivity $= 2 \text{div/mV}$

$\therefore$ voltage required for full scale deflection

$$V_g = \frac{1}{2} \times 100 \text{mV} = 0.05 \text{ V}$$

Galvanometer resistance is given by

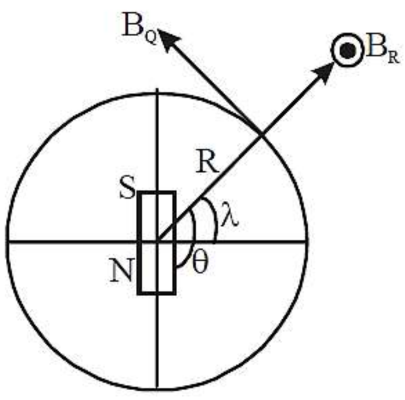
$$G = \frac{V_g}{I_g} = \frac{0.05}{0.01} = 5\Omega$$

24. (d) For a dipole at position (R, Q)

$$B_R = \frac{\mu_0}{4\pi} \cdot \frac{2M\cos\theta}{R^3}$$

and $B_Q = \frac{\mu_0}{4\pi} \cdot \frac{M\sin\theta}{R^3}$

Also $\tan\phi = \frac{B_V}{B_H} = -\frac{B_R}{B_Q}$



Dividing eq. (i) by (ii)

$$\frac{B_R}{B_Q} = \frac{2\cos\theta}{\sin\theta} = 2\cot\theta$$

From eq. (iii) and (iv)
$\tan\phi = -2\cot\theta$
From figure, $\theta = 90° + \lambda$

$\therefore \tan\phi = -2\cot(90 + \lambda)$
$\tan\phi = 2\tan\lambda$

25. **(b)** The hysteresis loop i.e. area of B-H curve is a measure of energy dissipated per cycle per unit volume of the specimen. It depends on the nature of magnetic material.

26. **(d)** Work done by magnet to turn from angle θ_1 to θ_2

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

$$= MB(\cos 0^\circ - \cos 45^\circ)$$

$$= MB\left(1 - \frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)MB$$

Also torque acting on the magnet

$$\tau = MB\sin 45^\circ = \frac{MB}{\sqrt{2}}$$

$$W = (\sqrt{2}-1)\cdot\tau \Rightarrow \tau = \frac{W}{(\sqrt{2}-1)}$$

27. **(c)** From Lentz's law, the direction of induced emf in a circuit is such that it opposes the magnetic flux that produces it. So, if the magnetic flux linked with a closed circuit increases the induced current flows in a direction so as to develop a magnetic flux in the opposite direction of original flux.

If the magnetic flux linked with a closed circuit decreases then the induced current flows in the same direction of original flux. So, the induced emf has not direction of its own.

28. **(d)** Given: $N = 20$

$B = 10^3$ gauss

$= 10^3 \times 10^{-4}\ T = 0.1\ T$

$A = 5\ cm^2 = 5 \times 10^{-4}\ m^2$

$\theta = 80^\circ$

$\therefore$ Flux through the coil

$$\phi = NBA\cos\theta$$

$$= 20 \times 0.1 \times 5 \times 10^{-4} \times \cos 30^\circ$$

$$= 10 \times 10^{-4} \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \times 10^{-4} = 865 \times 10^{-4} wb$$

29. **(c)** In LC circuit, if $X_L = X_C$ then $\omega = \frac{1}{\sqrt{LC}} I_0 \infty$, so $Z = \frac{E_0}{I_0} = 0$.

As $\frac{1}{\sqrt{LC}}$ is the natural

frequency of LC circuit, therefore for an LC circuit if the frequency of applied AC becomes equal to the natural frequency of an AC circuit then the amplitude of current becomes infinite due to zero impedance.

30. **(d)** Maximum current flows in the circuit in resonance condition Current in the LCR circuit

$$i = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

For current to be maximum denominator should be minimum

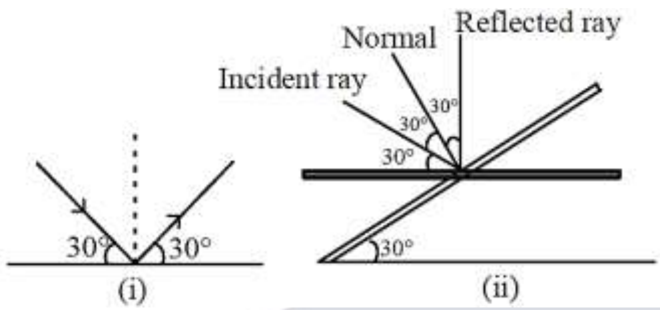
$(X_L - X_C)^2 = 0$

$$\Rightarrow X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\therefore L = \frac{1}{\omega^2 C} = \frac{1}{(100)^2 \times 10 \times 10^{-6}}$$

$$L = \frac{1}{10} H = 0.1H = 100mH$$

31. **(c)** When a light ray falls on a mirror at an angle 30°, then the reflected ray will make the same angle with the plane as shown in Fig. (i)



In order to make the reflected ray vertical, the mirror should be rotated at an angle of 60°.
So, the mirror should be tilted by

$$\frac{60°}{2} = 30° \text{ Fig. (ii)}$$

32. **(d)** For a compound microscope, magnifying power

$MP = m_e \times m_0$

When the final image is at least distance of distance vision then

$$M_e = 1 + \frac{D}{f_e}$$

$$\therefore MP = m_0 \left[1 + \frac{D}{f_e}\right]$$

$$\Rightarrow -35 = m_0 \left[1 + \frac{25}{10}\right]$$

$$\Rightarrow -35 = m_0 \times 35$$

$$\Rightarrow m_0 = -10$$

The negative sign shows that the image formed by objective is inverted.

33. **(d)** Using prism formula,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where, A = angle of prism

δ_m = angle of minimum deviation

Given, $\mu = \cot\left(\frac{A}{2}\right) = \frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

So, from Eq. (i)

$$\frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A}{2} + \frac{\delta_m}{2}\right)$$

$$\Rightarrow \delta_m = \pi - 2\,A = 180^\circ - 2\,A$$

34. **(d)** Given:

$P_1 = 2D; P_2 = 3D$

$$d = \frac{1}{3}\ m$$

We know that $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2}$

Equivatent power,

$$\therefore\ P = P_1 + P_2 - dP_1.P_2$$

$$= 2 + 3 - \frac{1}{3} \times 2 \times 3 = 3D$$

35. **(d)** First ray optical path = $\mu_1\ L_1$ second ray optical path = $\mu_2\ L_2$ So, phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{\lambda} \times \Delta x$

$$\therefore\ \Delta\phi = \frac{2\pi}{\lambda}(\mu_1\ L_1 - \mu_2\ L_2)$$

36. **(d)** Let the intensity of unpolarised light be I_0, so the intensity of first polaroid is $\frac{I_0}{2}$.

On rotating through 60°, the intensity of light from second polaroid

$$I = \left(\frac{I_0}{2}\right)(\cos 60)^2 = \frac{I_0}{2}\frac{1}{4} = \frac{I_0}{8} = 0.125 I_0$$

∴ percentage of incident light transmitted through the system = 12.5%.

37. **(b)** As the electromagnetic wave is the crossed field of electric and magnetic waves, so the direction of propagation of EM wave is the direction of vector E × B. Here E is upward and (E × B) is towards north. So, from right hand thumb rule B will be along east.

38. **(c)** An electromagnetic wave is the wave radiated by an accelerated charge and propagates through space as coupled electric and magnetic field. These fields are oscillating perpendicular to each other.

39. **(d)** From Rayleigh's law of scattering, intensity

$I \propto \frac{1}{\lambda^4}$

$\therefore \ \frac{I_1}{I_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$

$\Rightarrow \frac{I_1}{I_2} = \left(\frac{330}{880}\right)^4 = \left(\frac{3}{8}\right)^4 = \frac{81}{4096}$

$I_2 = \frac{4096}{81} Å = (50.557) Å$

40. (c) As we knows, radius of an atom,

$\gamma_A \approx 10^{-10}$ m

radius of nucleus,. $\gamma_B \approx 10^{-15}$ m

So, ratio of their volumes

$$\frac{V_A}{V_N} = \frac{\frac{4}{3}\pi r_A^3}{\frac{4}{3}\pi r_N^3} = \left(\frac{r_A}{I_N}\right)^3 = \left(\frac{10^{-10}}{10^{-15}}\right)^3$$

$\therefore \ V_A : V_N = 10^{15} : 1$

PART-II CHEMISTRY

41. (b) Balanced equations are

$3Cu + 8HNO_3 \longrightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O$

$Cu + 4HNO_3 \longrightarrow Cu(NO_3)_2 + 2NO_2 + 2H_2O$

Here NO and NO_2 are evolved in equal volumes, hence, on adding Eqs. (i) and (ii)

$$4Cu + 12\ HNO_3 \longrightarrow 4Cu(NO_3)_2 + 2NO + 2NO_2 + 6H_2O$$

or $2Cu + 6HNO_3 \longrightarrow 2Cu(NO_3)_2 + NO + NO_2 + 3H_2O$

Hence, coefficients x and y of Cu and HNO_3 are 2 and 6 respectively.

42. (c) $[S^{2-}] = 10^{-23}$ mol L^{-1}.

$[M^{2+}] = 10^{-2}M$

Ionic product, $K_{1P} = [M^{2+}][S^{2-}] = 10^{-25}$,

$\therefore$ ionic product is greater than K_{sp} of CuS and CdS.

Therefore, all except MnS and FeS are precipitated.

43. (c) $K = \frac{[CO(g)][H_2(g)]}{[H_2O(g)]}$

Concentration will increase, on halving the volume. There are two terms in numerator. So to keep K constant, concentration of $[H_2O]$ should increase much more.

44. (d) During smelting process of copper from copper pyrites reactions are

$Cu_2O + FeS \longrightarrow Cu_2S + FeO$

$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2$

45. (d) $SnCl_2 + HgCl_2 \longrightarrow SnCl_4 + Hg$

$(xCl_2)\ (yCl_2)\ (xCl_2)\ (y)$

$$HgO \xrightarrow[>400^\circ C]{\Delta} \underset{(y)}{Hg} + \frac{1}{2}O_2$$

So, ore of γ is HgS i.e., Cinnabar.

46. (a) (ΔG°) reaction $= \Delta G_f^0$ (Products)

$-\Delta G_f^0$ (reactants) $\therefore\ 264.4 = \left[2\Delta G_f^0(H^+) + 2\Delta G_f^0(C^l)\right]$

or $264.4 = -\left[\Delta G_f^0(H_2) + \Delta G_f^0(Cl_2)\right]$

$= \left[0 + 2\Delta G_f^0(Cl^-)\right] + [0 + 0]$

or, $-262.4 = 2\Delta G_f^0(Cl^-)$

or, $\Delta G_f^0(Cl^-) = -131.2 kJmol^{-1}$.

47. (a) 2 L of $3MAgNO_3$ will contains 6 moles of $AgNO_3$.

3 L of $1MBaCl_2$ will contain 3 moles of $BaCl_2$.

$2AgNO_3 + BaCl_2 \longrightarrow 2AgCl$

$+Ba(NO_3)_2$.

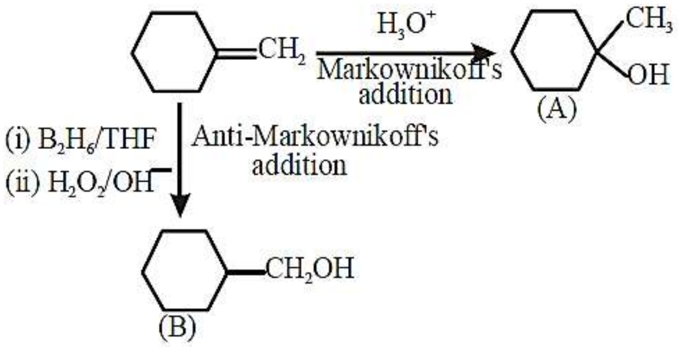
So, 6 moles of $AgNO_3$ will react with 3 moles on $BaCl_2$ it means, two solution will react completely to form 3 moles of

$Ba(NO_3)_2 \equiv 6$ moles of NO_3^- ions in $2 + 3 = 5$ L solution

Hence, molarity of $NO_3^- = \frac{6}{5} = 1.2M$

48. (b) AsO_3^{3-}, ClO_3^{3-}, and SO_3^{2-} have sp^2 hybridisation and hence are non-planar species, while NO_3^-, CO_3^{2-} and BO_3^{3-} have sp^2 hybridisation and hence are planar species

49. (d)



50. (b) $(KE)_1 = hv_1 - hv_0$

$(KE)_2 = hv_2 - hv_0$

As, $(KE)_1 = 2 \times (KE)_2$

$\therefore hv_1 - hv_0 = 2(hv_2 - hv_0)$

or, $hv_0 = 2hv_2 - hv_1$

or, $v_0 = 2v_2 - v_1$

$= 2 \times (2 \times 10^{16}) - (3.2 \times 10^{16})$

$= 0.8 \times 10^{16}$ Hz $= 8 \times 10^{15}$ Hz

51. (d) Let initially, pressure of $C_6H_6(g)$ is p_1 mm and for $H_2(g)$ is p_2 mm

$\therefore p_1 + p_2 = 60$ mm

After the reaction pressure of

$C_6H_6(g) = 0$ (as all C_6H_6 has reacted)

$H_2(g) = p_2 - 3p_1$

So, total pressure $= p_2 - 3p_1 + p_1 = 30$

mm

$p_2 - 2p_1 = 30$ mm

On solving equation (i) and (ii)

$p_1 = 10$ mm, $p_2 = 50$ mm

Fraction of C_6H_6 by volume = moles

fraction fraction of pressure $= \frac{10}{60} = \frac{1}{6}$

52. (c) In the unit cell number of Cu atoms (fcc/ccp)

$= 8 \times \frac{1}{6} + 6 \times \frac{1}{2} = 4$

As Ag atoms occupying edge centred

$= 12 \times \frac{1}{4} = 3$

and Au atoms are presents at the body centred = 1

$\therefore$ formula, Cu_4Ag_3Au.

53. (b) As we know that $-nFE^\circ{}_{cell} = -RT \ln k$ or $E^\circ_{cell} = \frac{RT}{nF} \ln k$.

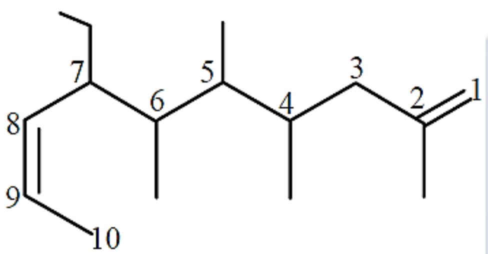
Plot of ln k or E°_{cell} will have slope

$= \frac{1}{2}\frac{RT}{F}$

54. (d) Since 2° propyl carbocation is little more stable than allyl carbocation and ethyl carbanion is more stable than isopropyl carbanion.

55. (a)

56. (b)



The correct IUPAC name is 7-ethyl- 2,4,5, 6-tetramethyldeca-1, 8-diene

57. (a) M. wt. of caffeine = 194u

% of N present in one molecular of caffeine is 28.9% of

$194u = \frac{28.9}{100} \times 194 = 56u$

Mass of one N atom = 14 m

$= 1$ N atom ($\because$ 14 m = 14u)

$\therefore$ $56u = \frac{56}{14}$ N atom = 4 N atom

58. (b)

$$\underset{X}{CaCO_3} \xrightarrow{\Delta} CaO + \underset{\text{Colourless gas}}{CO_2} \uparrow$$

$$\underset{\text{Residue}}{CaO} + H_2O \longrightarrow \underset{Y}{Ca(OH)_2}$$

$$\underset{Y}{Ca(OH)_2} + \underset{\text{Excess}}{2CO_2} \longrightarrow \underset{Z}{Ca(HCO_3)_2}$$

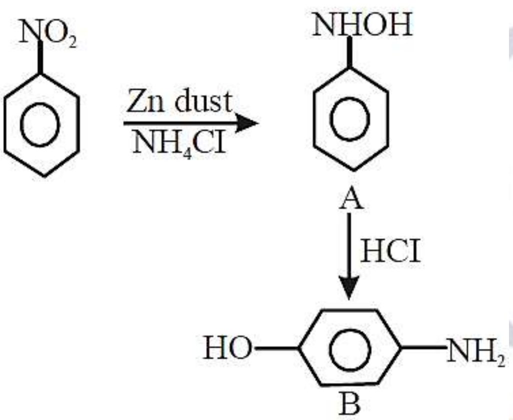
$$\underset{Z}{Ca(HCO_3)_2} \xrightarrow{\Delta} \underset{x}{CaCO_3} + CO_2 \uparrow + H_2O$$

59. (b)

$$\underset{\text{Acid}}{Zn(OH)_2} + \underset{\text{Base}}{2O^-H} \longrightarrow \underset{\text{Salt}}{ZnO_2^{2-}} + \underset{\text{Water}}{2H_2O}$$
$$\underset{\text{Base}}{Zn(OH)_2} + \underset{\text{Acid}}{2H^+} \longrightarrow \underset{\text{Salt}}{Zn^{2+}} + \underset{\text{Water}}{2H_2O}$$

The amphoteric character of $Zn(OH)_2$ is represented by I and III

60. (a)



61. (b) Increasing order of stability

(a) $[Cu(NH_3)_4]^{2+} < [Cu(en)_2]^{2+} < [Cu(\text{ trein })^{2+}$.

Their formation of entropy increases in the same order. Ligand denticity is increased.

(b) $[Fe(H_2O)_6]^{3+} < [Fe(NO_2)_6]^{3-} < [Fe(NH_3)_6]^{3+}$.

$\because$ NH_3 is weaker ligand than $NO_2{}^-$.

$\therefore$ The correct stability order is

$[Fe(H_2O)^{3+} < [Fe(NH_3)_6]^{3+} < [Fe(NO_2)_6]^{3-}$

(c) $[Co(H_2O)_6]^{3+} < [Rh(H_2O)_6]^{3+} < [Ir(H_2O)_6]^{3+}$

Z_{eff} value increases from Co^{3+} to Ir^{3+}.

(d) $[Cr(NH_3)_6]^{1+} < [Cr(NH_3)_6]^{2+} < [Cr(NH_3)_6]^{3+}$

O. S of Cr atom increases from +1 to +3 .

62. (d) The amount of energy required to take out an electron from the monopositive cation is called second ionisation energy

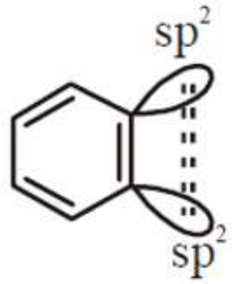
$M(g) \longrightarrow M^{2+}(g) + 2e^-$

$M(g) \longrightarrow M^{+}(g) + e^-$

On subtracting eq(iii) form eq. (v) we get,

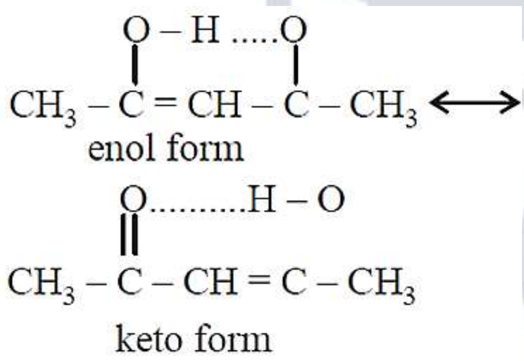
$M \longrightarrow M^{2+} + e^-$.

63. (d)



In benzene, the triple bond consists of one $sp^2 - sp^2 \sigma$-bond, one $sp^2 - sp^2, \pi$-bond and one $p - p\pi$-bond.

64. (b) Resonance stabilisation of enol form is



65. (d) The minimum m. wt. must contain at least one S atom.

$\therefore\ \% S = \dfrac{\text{weight of one S} - \text{ atom}}{\text{minimum m.wt.}} \times 100$

$4 = \dfrac{32}{\text{minimum m.wt.}} \times 100$

minimum m. wt. $= \frac{32}{4} \times 100 = 800$

66. (b) When the concentration of the adsorbate is less on the surface as compare to its concentration in the bulk is called negative adsorption. Add from left in this adsorption, concentration of dilute KCl solution is less on the surface of blood charcoal as compare to its concentration in solution.

67. (a)

Interval	Conc.change	Rate

$0-17$ min	$0.069-0.052=0.017M$	$\frac{0.017}{17}=0.001$
$17-34$ min	$0.052-0.035=0.017M$	$\frac{0.017}{17}=0.001$
$34-51$ min	$0.035-0.018=0.017M$	$\frac{0.017}{17}=0.001$

$\because$ Rate remains constant. So, it is independent of concentration, the reaction is of zero order.

According to rate law

Rate $=K(\text{ conc. })^0=0.001M/\text{min}$

68. (d) $hv=hv_0+ev_0$.

$v_0=\frac{h}{e}v-\frac{h}{e}v_0$

On comparing this equation with the straight- line equation, i.e $y=mx+c$

The slope of $v_0 vsv$ is

(v_0 is stopping potential)

$(\text{ slope })_1=\frac{h}{e}$

Likewise,

$hv=hv_0+K_{max}$.

Or

$K_{max}\ =hv-hvm_0$

Thus, slope of $K_{max} vsv$ is

$(\text{ slope })_2=h\therefore\frac{(\text{ slope })_2}{(\text{ slope })_1}=\frac{h}{h/e}=e$

69. (c) Both P_2O_5 and PCl_5 give H_3PO_4 With excess of water.

$P_2O_5+3H_2O\longrightarrow 2H_3PO_4$

$PCl_5+4H_2O\longrightarrow H_3PO_4+5HCl$

70. (c) $Al(OH)_3$ dissolves in NaOH solution to
give $Al(OH)_4^-$ion which is supposed to have the octahedral complex species $[Al(OH)_4(H_2O)_2]^-$in aqueous solution.

$Al(OH)_3+NaOH(aq)\longrightarrow[Al(OH)_4$

$(H_2O)_2-(aq)+Na+(aq)$

71. (b) In the absence of d-orbitals F_2 does not combine with $F^-$to form F_3^-. ion.

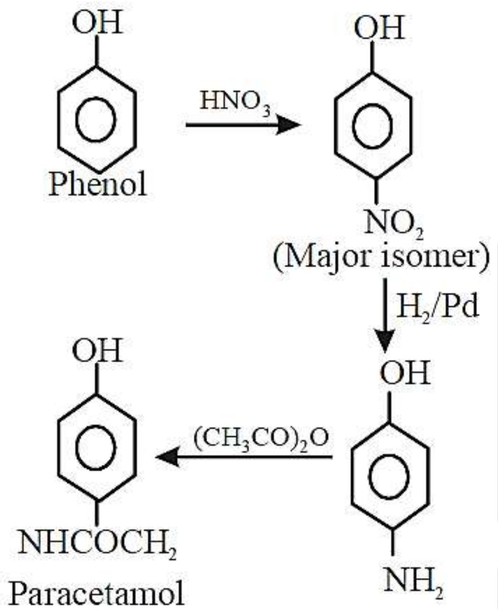
72. (a) According to Mulliken, electronegativity of an atom is average of ionization energy and electron affinity (in eV).

$$n_m = \frac{IE + EA}{2}$$

If ionization energy and electron affinity are in kcalmol^{-1}.

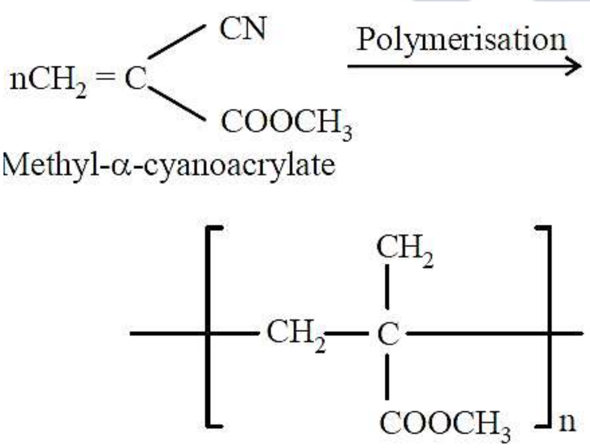
$$n = \frac{IE + EA}{125} = \frac{275 + 86}{125} = 288$$

73. (a) For the preparation of paracetamol



74. (b) A compound which gives a negative test with ninhydrin, it cannot be a protein or an amino acid. As, it gives a positive test with Benedict's solution. So, it must be a monosaccharide but not a lipid.

75. (c)

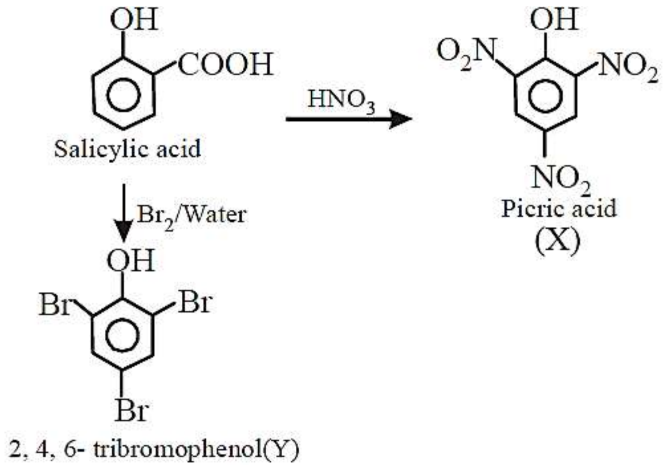


Poly (methyl α-cyanoacrylate)
Super glue of crazy glue

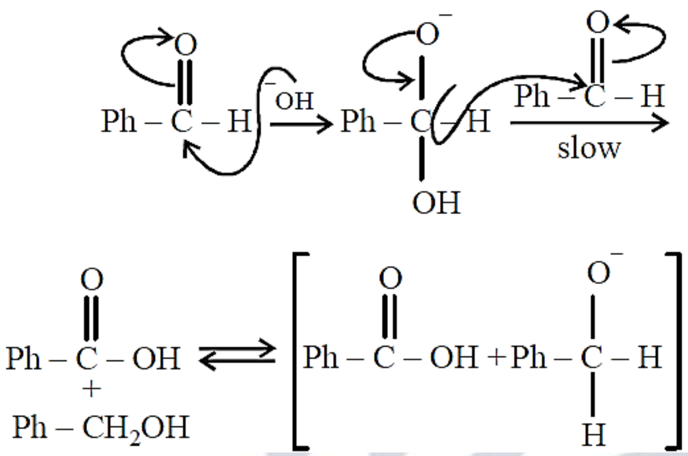
76. (a) Nitration and bromination of salicylic acid, give picric acid (X) and 2, 4, 6tribromophenol (Y) respectively.

(1) Decarboxylation

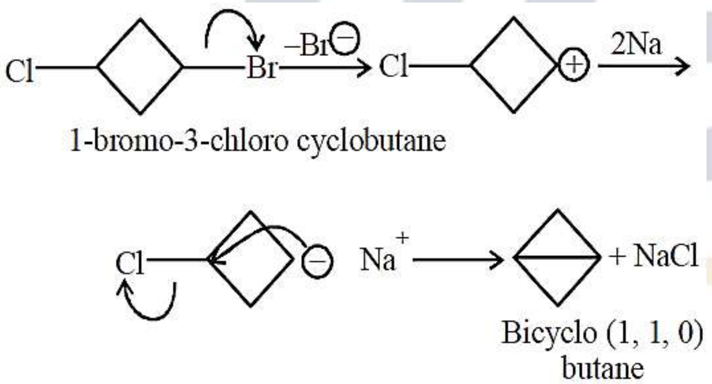
Bromination



77. (b) Hydride ion transfer to the carbonyl group is the slowest or the rate determining step.



78. (d) This is Wurtz reaction. Bromides have high reactivity than chlorides in Wurtz reaction therefore, reaction occurs from Br atoms,



79. (d)

$$CH_3CH_2CH_2OH \xrightarrow[160^\circ - 180^\circ C]{Conc\ H_2SO_4} CH_3CH = CH_2$$

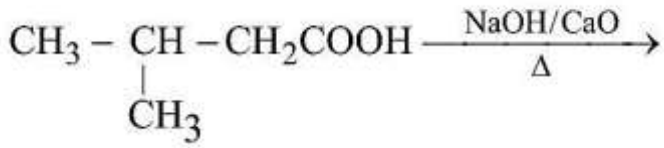
$$\xrightarrow{Br_2} \xrightarrow{alc.\ KOH} \underset{(Y)}{CH_3CHBr - CH_2Br}$$

$$[\underset{(A)}{CH_3C(Br) = CH_2} + \underset{(B)}{CH_3CH = CHBr}]$$

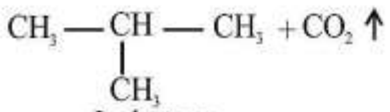
$$\xrightarrow[-HBr]{NaNH_2} \underset{(Z)}{CH_3C \equiv CH}$$

Alcoholic KOH brings about dehydrobromination of Y and give a mixture of vinyl bromide (A and B) while $NaNH_2$ being a strong base than alc. KOH readily brings about dehydrobromination of less reactive vinyl bromide to give propyne $CH_3C \equiv CH$ i.e. (Z).

80. (d) 3-methylbutanoic acid gives isobutane on decarboxylation i.e.,



3-methylbutanoic acid



Isobutane

While Wurtz reaction of C_2H_5Br gives. n-butane and hydrolysis of n-butyl magnesium bromide gives n-butane but reduction of propanol with HI/P gives propane

PART-III MATHEMATICS

81. (d) The given differential equation is $(3x + 4y + 1)dx + (4x + 5y + 1)dy = 0$

Comparing eq. (i) with $Mdx + Ndy = 0$, we get

$M = 3x + 4y + 1$

and $N = 4x + 5y + 1$

Here, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 4$

Hence, eq. (i) is exact and solution is given by

$\int (3x + 4y + 1)dx + \int (5y + 1)dy = C$

$\Rightarrow \frac{3x^2}{2} + 4xy + x + \frac{5y^2}{2} + y - C = 0$

$\Rightarrow 3x^2 + 8xy + 2x + 5y^2 + 2y - 2C = 0$

$\Rightarrow 3x^2 + 2.4xy + 2x + 5y^2 + 2y + C' = 0$

where, $C' = -2C$

On comparing eq. (ii) with standard form of conic section

$ax^2 + 2\,hxy + by^2 + 2gx + 2fy + C = 0$

We get,

$a = 3, h = 4, b = 5$

Here, $h^2 - ab = 16 - 15 = 1 > 0$

Hence, the solution of differential equation represents family of hyperbolas.

82. (b) $\Delta(r) = \begin{vmatrix} r & r^3 \\ 1 & n(n+1) \end{vmatrix}$

$\Rightarrow \sum_{r=1}^{n} \Delta(r) = \begin{vmatrix} \sum_{r=1}^{n} r & \sum_{r=1}^{n} r^3 \\ 1 & n(n+1) \end{vmatrix}$

$= \begin{vmatrix} \frac{n(n+1)}{2} & \frac{[n(n+1)]^2}{2} \\ 1 & n(n+1) \end{vmatrix}$

$= \frac{[n(n+1)]^2}{2} - \frac{[n(n+1)]^2}{4}$

$= \frac{[n(n+1)]^2}{2} = \sum_{r=1}^{n} r^3$

83. (b) $P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A} \cap B\right)$

$= P(A \cap B)P\left(\frac{C}{A} \cap B\right)$

$= \frac{P(A \cap B)P[C \cap (A \cap B)]}{P(A \cap B)}$

$= P(A \cap B \cap C)$

84. (d) $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & -5 & -3 \\ 0 & -9 & -2 \end{bmatrix}$

[Applying $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$]

$\approx \begin{bmatrix} 1 & 3 & 1 \\ 0 & -5 & -3 \\ 0 & 0 & \frac{17}{5} \end{bmatrix} \left[R_3 \to R_3 - \frac{9}{5}R_2\right]$

$\therefore$ rank$(A) = 3$

85. (b) The probability of getting a double six in one throw of two dice

$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$\therefore\ p = \frac{1}{36},$

$q = 1 - p$

$= 1 - \frac{1}{36} = \frac{35}{36}$

Now, $(p+q)^m$

$= q^n + {}^nC_1 q^{n-1}p + {}^nC_2 q^{n-2}p^2 + \cdots.$

$+ {}^nC_r q^{n-r}p^r + \cdots.. + p^n$

The probability of getting atleast one double six in n throws with two dice.

$= (q+p)^n - q^n$

$= 1 - q^n = 1 - \left(\frac{35}{36}\right)^n$

$\therefore\ 1 - \left(\frac{35}{36}\right)^n > 0.99$

$\Rightarrow \left(\frac{35}{36}\right)^n < 0.01 \Rightarrow n(\log 35 - \log 36) < \log 0.01$

$\Rightarrow n[15441 - 15563] < -2$

$\Rightarrow\ -0.0122n < -2$

$\Rightarrow 0.0122n > 2 \Rightarrow n > 163.9$

So, the least value of n is 164

86. (d) The given system of equations will be consistent with unique solution, when

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & k \end{vmatrix} \neq 0$$

$\Rightarrow\ 1(2k-12) + 1(3-k) + 1(4-2) \neq 0$

$\Rightarrow k - 12 + 3 + 2 \neq 0 \Rightarrow k - 7 \neq 0 \Rightarrow k \neq 7$

87. (d) Given: $|z| = \frac{1}{4}$

Let $z' = -1 + 8z$

$\Rightarrow\ z = \frac{z'+1}{8} \Rightarrow |z| = \frac{|z'+1|}{8}$

$\Rightarrow \frac{1}{4} = \frac{|z'+1|}{8} \Rightarrow |z'+1| = 2$

$\therefore\ z'$ lies on a circle with centre $(-1,0)$ and radius 2

88. (c) If the line $y = mx + c$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

$c^2 = \frac{m^2(a^2-b^2)^2}{a^2+b^2m^2}$

[Here, $m = 2, a^2 = 9$ and $b^2 = 16$]

$$= \frac{(2)^2(9-16)^2}{9+16\times(2)^2}$$

$$= \frac{4\times 49}{9+64} = \frac{4\times 49}{73} = \frac{196}{73}$$

$$\therefore\ c = \frac{14}{\sqrt{73}}$$

89. (b) Given equation is $x^2 + x + 1 = 0$

$\Rightarrow x = \omega$ and $x = \omega^2$

Case I : When $x = \omega$

Then

$$\sum_{n=1}^{6}\left[x^n + \frac{1}{x^n}\right]^2 = \sum_{n=1}^{6}\ [\omega^n + \omega^{2n}]^2 \left[\because \frac{1}{\omega} = \omega^2\right]$$
$$= (\omega+\omega^2)^2 + (\omega^2+\omega^4)^2 + (\omega^3+\omega^6)^2$$
$$+(\omega^4+\omega^8)^2 + (\omega^5+\omega^{10})^2 + (\omega^6+\omega^{12})^2$$

$= (-1)^2 + (-1)^2 + (2)^2 + (-1)^2$

Case II: When $x = \omega^2$

$+(-1)^2 + (2)^2 = 12$

Then

$$\sum_{n=1}^{6}\left[x^n + \frac{1}{x^n}\right]^2 = \sum_{n=1}^{6}\ [\omega^{2n} + \omega^n]^2 \left[\because \frac{1}{\omega^2} = \omega\right]$$

$= 12$

90. (b) Correct result is as follows:

$(\sim p \vee \sim q) \Rightarrow (r \wedge s)$

or $\sim (p \wedge q) \Rightarrow r \wedge s$

91. (a) $A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 2 & -3 \\ 0 & 1 & 2 \end{bmatrix}$

$\Rightarrow |A| = 1.(4+3) - 3(-2+0) + 1(-1-0)$

$= 7 + 6 - 1 = 12$

So, $\text{adj}(\text{adj}\,A) = |A|^{n-2} = A$

$= (12)^{3-2}\ A = 12\ A$

$= 12\begin{bmatrix} 1 & 3 & 1 \\ -1 & 2 & -3 \\ 0 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 12 & 36 & 12 \\ -12 & 24 & -36 \\ 0 & 12 & 24 \end{bmatrix}$

92. (c) $\phi_3(m) = 3 + 2m - 7m^2 + 2m^3$

$\phi_2(m) = -14m + 7m^2$

$\phi_3'(m) = 2 - 14m + 6m^2$

Now, putting $\phi_3(m) = 0$, we have

$3 + 2m - 7m^2 + 2m^3 = 0$

$\Rightarrow (1 - m)(1 + 2m)(3 - m) = 0$

$\Rightarrow m = -\frac{1}{2}, 1, 3$

We know that

$c\phi_n'(m) + \phi_{n-1}(m) = 0$, which in the given case becomes

$c(2 - 14m + 6m^2) + (-14m + 7m^2) = 0$

$\Rightarrow c = \frac{14m - 7m^2}{2 - 14m + 6m^2}$

So, when $m = -\frac{1}{2}, c = -\frac{5}{6}$

When $m = 1, c = -\frac{7}{6}$

When $m = 3, c = -\frac{3}{2}$

$\therefore$ Asymptotes are $y = -\frac{1}{2}x - \frac{5}{6}$,

$y = x - \frac{7}{6}$ and $y = 3x - \frac{3}{2}$.

93. (b) The structure (N,.) satisfies the closure property, associativity and commutativity but the identity element 0 does not belong to N.

So, N is a semi-group.

94. (a) $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$

$= \frac{1}{2}\int \frac{dx}{\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x}$

$= \frac{1}{2}\int \frac{dx}{\cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x}$

$$= \frac{1}{2} \int \frac{dx}{\cos\left(x - \frac{\pi}{3}\right)}$$

$$= \frac{1}{2} \int \sec\left(x - \frac{\pi}{3}\right) dx$$

$$= \frac{1}{2} \log \tan\left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4}\right) + C$$

$$= \frac{1}{2} \log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$$

95. (d) Given sphere is

$x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$

Centre $\equiv (3,6,1)$

Here, one end of diameter is $(2,7,3)$.

Let the other end of the diameter be (x, y, z)

Centre of the sphere will be the mid-point of the ends of diameter.

So, $(3,6,1) = \left(\frac{2+x}{2}, \frac{7+y}{2}, \frac{3+z}{2}\right) \Rightarrow 2 + x = 6 \Rightarrow x = 4$

$\Rightarrow 7 + y = 12 \Rightarrow y = 5$

and $3 + z = 2 \Rightarrow z = -1$

Therefore, $(x, y, z) \equiv (4,5,-1)$

96. (d) Given lines are

$x = my + n, z = py + q$

and $x = m'y + n', z = p'y + q'$

Above equations can be rewritten as

$$\frac{x - n}{m} = \frac{y - 0}{1} = \frac{z - q}{p}$$

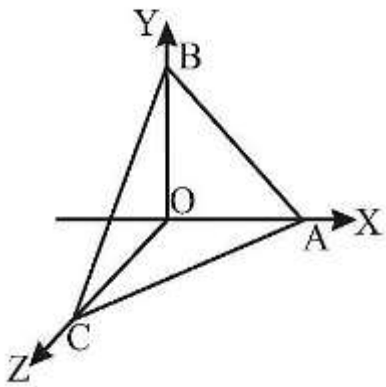
and $\frac{x-n'}{m'} = \frac{y-0}{1} = \frac{z-q'}{p'}$

Lines will be perpendicular, if

$mm' + 1 + pp' = 0$

$\Rightarrow mm' + pp' = -1$

97. (c)



Vector perpendicular to face OAB $= \vec{n}_1$

$= \overrightarrow{OA} \times \overrightarrow{OB}$

$= (\hat{i} - 2\hat{j} + \hat{k}) \times (-2\hat{i} + \hat{j} + \hat{k})$

$= (-2 - 1)\hat{i} + (-2 - 1)\hat{j} + (1 - 4)\hat{k}$

$= -3\hat{i} - 3\hat{j} - 3\hat{k}$

Vector perpendicular to face ABC $= \vec{n}_2$.

$= \overrightarrow{AB} \times \overrightarrow{AC}$

$= (-3\hat{i} + 3\hat{j}) \times (\hat{j} + \hat{k})$

$= -3\hat{i} + 3\hat{j} - 3\hat{k}$

Since, angle between faces is equal to angle between their normals.

$$\therefore \cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|n_1||n_2|}$$
$$= \frac{(-3)(3) + (-3)(3) + (-3)(-3)}{\sqrt{9+9+9}\sqrt{9+9+9}}$$
$$= \frac{-9-9+9}{\sqrt{27}\sqrt{27}} = -\frac{1}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{-1}{3}\right)$$

98. (b) Let α, β and γ be the angles made by the line segment OP with X-axis, Y-axis and Z-axis, respectively.

Given: $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{3}$

We know that, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\therefore \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{3} + \cos^2\gamma = 1$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$$

$\Rightarrow \frac{1}{2}+\frac{1}{4}+\cos^2\gamma = 1$

$\Rightarrow \ \cos^2\gamma = \frac{1}{4}$

$\Rightarrow \ \cos\gamma = \frac{1}{\sqrt{2}}$

$\therefore \ \gamma = \frac{\pi}{4}$

Hence, direction cosines are $\cos\alpha, \cos\beta, \cos\gamma$

i.e. $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{2}}$.

99. (a) $\sim p \vee q$ means $F \vee F = F, \sim r$ means F

$\therefore \ [(\sim p \vee q) \wedge \sim r] \Rightarrow p$ means T

100. **(b)** Let $f(x) = x^{25}(1-x)^{75}, x \in [0,1]$

$\Rightarrow f'(x) = 25x^{24}(1-x)^{75} - 75x^{25}(1-x)^{74}$
$= 25x^{24}(1-x)^{74}\{(1-x)-3x\}$
$= 25x^{24}(1-x)^{74}(1-4x)$
$+ + \frac{1}{1}$

We can see that $f'(x)$ is positive for $x < \frac{1}{4}$ and $f'(x)$ is negative for $x > \frac{1}{4}$.

Hence, $f(x)$ attains maximum at $x = \frac{1}{4}$.

101. **(b)** $\left|z+\frac{1}{4}\right|$

$= \left|z-\left(\frac{-1}{4}\right)\right| \geq |z| - \left|\frac{-1}{4}\right|$
$= \left|(-z)-\frac{1}{4}\right| \geq \left|3-\frac{1}{4}\right| = \frac{11}{4}$
$\therefore \left|z+\frac{1}{4}\right| \geq \frac{11}{4}$

102. **(b)** Equation of the normal at point $(at_1^2, 2at_1)$ on parabola is $y = -t_1x + 2at_1 + at_1^3$

It also passes through $(at_2^2, 2at_2)$

So, $2at_2 = -t_1(at_2^2) + 2at_1 + at_1^3$

$\Rightarrow \ 2t_2 - 2t_1 = -t_1(t_2^2 - t_1^2)$

$\Rightarrow t_1 + t_2 = \frac{-2}{t_1}$

$\Rightarrow \ t_2 = -t_1 - \frac{2}{t_1}$

103. **(c)** $\cos\theta = \frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$

$= \frac{1\times 2+(-1)\times(-1)+2\times(1)}{\sqrt{1+1+4}\sqrt{4+1+1}}$

$= \frac{2+2+2}{6} = \frac{6}{6} = 1$

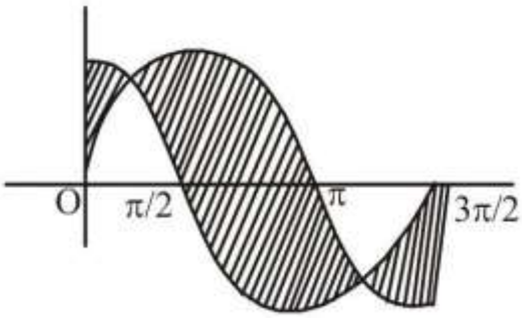
So, $\theta = 0°$ or $\theta = 2\pi$

$\because\ \sec 2\pi = 1$

$\therefore\ 2\pi = \sec^{-1}(1)$

$\Rightarrow \theta = \sec^{-1}(1)$

104. **(a)**



Required area

$= \int_0^{\pi/4}(\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx$

$\int_{5\pi/4}^{3\pi/2}(\cos x - \sin x)dx$

$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4} + [\sin x + \cos x]_{5x/4}^{3\pi/4}$

$= (4\sqrt{2} - 2)$ sq units

105. **(b)** $[a+b-c]\cdot[(a-b)\times(b-c)$

$= (a+b-c).[a\times b - a\times c - b\times b + b\times c] = a.(a\times b) - a.(a\times c) + a.(b\times c) + b.(a\times b) - b(a\times c) + b.(b\times c) - c.(a\times b)$

$+c.(a\times c) - c.(b\times c)$

$= a.(b\times c) - b.(a\times c) - c.(a\times b)$

$= [abc] - [bac] - [c\ a\ b\ ]$

$= [abc] + [abc] - [abc]$

$= [abc] = a.(b\times c)$

106. **(a)** Surface area A of a cube of side x is given by $A = 6x^2$.

On differentiating w.r.t. x, we get

$$\frac{dA}{dx} = 12x$$

Let the change in x be $\Delta x = m\%$ of x

$$= \frac{mx}{100}$$

Change in surface area,

$$\Delta A = \left(\frac{dA}{dx}\right)\Delta x = (12x)\Delta x$$
$$= 12x\left(\frac{mx}{100}\right) = \frac{12x^2\ m}{100}$$

∴ The approximate change in surface area

$$= \frac{2\ m}{100} \times 6x^2$$

$= 2\ m\%$ of original surface area

107. **(d)** Given equation of rectangular hyperbola is $x^2 - y^2 = 8^2$

Length of latusrectum $= 2 \times (8) = 16$ and eccentricity $= \sqrt{2}$

The asymptotes are perpendicular lines.

So, $x \pm y = 0$

Now, directrices are

$$x = \pm\frac{8}{\sqrt{2}} = \pm 4\sqrt{2}$$

108. **(a)** Equation of hyperbola is

$3x^2 - 2y^2 = 6$

$$\Rightarrow \frac{x^2}{2} - \frac{y^2}{3} = 1$$

So, $a^2 = 2$ and $b^2 = 3$

Given, equation of line is $x - 3y = 3$.

∴ Slope of given line $= \frac{1}{3}$

∴ Slope of line perpendicular to given line, $m = -3$

The equation of tangents are

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$
$$= -3x \pm \sqrt{2 \times 9 - 3}$$
$$= -3x \pm \sqrt{18 - 3}$$
$$= -3x \pm \sqrt{15}$$

109. **(d)** $\lim_{x\to\pi/4} \frac{\tan x - 1}{\cos 2x} = \lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4}+h\right)-1}{\cos 2\left(\frac{\pi}{4}+h\right)}$

$$\left[\because x = \frac{\pi}{4} + h\right]$$

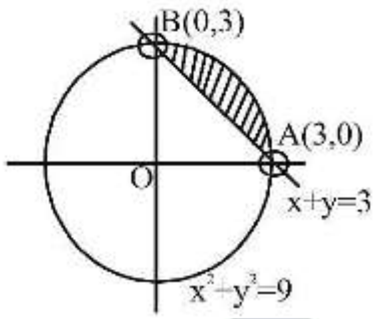
$$= \lim_{h\to 0} \frac{\left(\frac{1+\tan h}{1-\tan h}\right) - 1}{\cos\left(\frac{\pi}{2} + 2h\right)}$$

$$= \lim_{h\to 0} \frac{1 + \tan h - 1 + \tan h}{-\sin 2h(1 - \tanh)}$$

$$= \lim_{h\to 0} \frac{-2\tan h}{2\sin h\cos h(1 - \tan h)}$$

$$= \lim_{h\to 0} \frac{-1}{\cos^2 h(1 - \tan h)} = -1$$

110. **(c)**



Area of required region

$= \frac{1}{4} \times$ Area of circle - Area of ΔOAB

$$= \frac{1}{4} \times \pi \times (3)^2 - \frac{1}{2} \times 3 \times 3$$

$$= 9\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

111. **(c)** $[a + b, b + c, c + a]$

$= (a + b) \cdot [(b + c) \times (c + a)]$

$= (a + b) \cdot [b \times c + b \times a + c \times c + c \times a]$

$= (a + b). (b \times c + b \times a + c \times a)$

$= a. (b \times c) + a. (b \times a) + a. (c \times a) + b.$

$(b \times c) + b. (b \times a) + b. (c \times a)$

$= a. (b \times c) + b. (c \times a)$

$= [abc] + [abc]$

$= [abc] + [abc] = 2[abc]$

112. **(a)** Let $I = \int_0^{\pi/2} (\log \tan x) \cdot \sin 2x dx$ ... (i)

$I = \int_0^{\pi/2} \log \tan\left(\frac{\pi}{2} - x\right) \sin 2\left(\frac{\pi}{2} - x\right) dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$\Rightarrow\ I = \int_0^{\pi/2} \log \cot x \cdot \sin 2x dx$

$[\because \sin(\pi - 2x) = \sin 2x]$

On adding eqs (i) and (ii), we get

$2I\int_0^{\pi/2} \log \tan x \cdot \sin 2x dx + \int_0^{\pi/2} \log \cot x \sin 2x dx = \int_0^{\pi/2} \sin 2x \log(\tan x \cdot \cot x) dx\ [\because \log m + \log n = \log(m.n)]$

$= \int_0^{\pi/2} \sin 2x \log 1 dx$

$\Rightarrow I = 0 [\because \log 1 = 0]$

$\therefore \int_0^{\pi/2} \sin 2x \log(\tan x) dx = 0$

113. **(c)** Here, mean $= 4$ and variance $= 2$

$\Rightarrow np = 4$ and $npq = 2$

So, $\frac{npq}{np} = \frac{2}{4} \Rightarrow q = \frac{1}{2}$

Then, $p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

Mean $= np = 4$

$\Rightarrow\ n \times \frac{1}{2} = 4 \Rightarrow n = 8$

$\therefore\ P(X = r) = {}^nC_r p^r q^{n-r}$

${}^8C_r \left(\frac{1}{2}\right)^8 \left[\because p = q = \frac{1}{2}\right]$

The required probability of atleast 7 successes is

$P(X \geq 7) = P(X = 7) + P(X = 8)$

$= ({}^8C_7 + {}^8C_8)\left(\frac{1}{2}\right)^8$

$= \left(\frac{8!}{7!\,1!} + \frac{8!}{8!\,0!}\right)\left(\frac{1}{2}\right)^8$

$= (8+1)\left(\frac{1}{2}\right)^8 = \frac{9}{256}$

114. **(b)** Given, lines are $\frac{x-7}{3} = \frac{y+4}{-16} = \frac{z-6}{7}$

and $\frac{x-10}{3} = \frac{y-30}{8} = \frac{4-z}{5}$

The vector form of given lines are

$r = 7\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

and $r = 10\hat{i} + 30\hat{j} + 4\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

On comparing these equations with $r = a_1 + \lambda b_1$ and $r = a_2 + \mu b_2$, we get

$\vec{a}_1 = 7\hat{i} - 4\hat{j} + 6\hat{k}$

$\vec{a}_2 = 10\hat{i} + 30\hat{j} + 4\hat{k}\; b_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$

and $\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$

Shortest distance $= \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|b_1 \times b_2|}\right|$

$= \left|\frac{(3\hat{i} + 34\hat{j} - 2\hat{k}) \cdot (24\hat{i} + 36\hat{j} + 72\hat{k})}{84}\right|$

$= \left|\frac{72+1224-144}{84}\right| = \left|\frac{1152}{84}\right| = \frac{288}{21}$ units

115. **(a)** Equation of plane passing through (2,2,1) is

$a(x-2) + b(y-2) + c(z-1) = 0$

Since, above plane is perpendicular to

$3x + 2y + 4z + 1 = 0$

and $2x + y + 3z + 2 = 0$

$\therefore\; 3a + 2b + 4c = 0$

and $2a + b + 3c = 0$

[$\because$ for perpendicular, $a_1 a_2$

$+ b_1 b_2 + c_1 c_2 = 0$]

On multiplying eq. (iii) by 2 , we get

$4a + 2b + 6c = 0$

On subtracting eq. (iv) from eq. (ii), we get

$\Rightarrow\; c = \frac{-a}{2}$

On putting $c = \frac{-a}{2}$ in eq. (iii), we get $b = \frac{-a}{2}$

On putting $b = \frac{-a}{2}$ and $c = \frac{-a}{2}$ in eq. (i),

we get $a(x-2) - \frac{a}{2}(y-2) - \frac{a}{2}(z-1) = 0$

$\Rightarrow \frac{a}{2}[2(x-2) - (y-2) - (z-1)] = 0$

$\Rightarrow 2x - 4 - y + 2 - z + 1 = 0$

$\Rightarrow 2x - y - z - 1 = 0$

116. **(c)** Suppose, A: a male is selected B: a smoker is selected Given:

$P(A \cup B) = \frac{7}{10}, P(A \cap B) = \frac{2}{5}$ and $P\left(\frac{A}{B}\right) = \frac{2}{3}$

The probability of selecting a smoker..

$$P(B) = \frac{P(A \cap B)}{P\left(\frac{A}{B}\right)}$$
$$= \frac{2 \times 3}{5 \times 2} = \frac{3}{5}$$

The probability of selecting a non-smoker So, $P(B) = 1 - P(B)$

$= 1 - \frac{3}{5} = \frac{2}{5}$

The probability of selecting a male

$$P(A) = P(A \cup B) + P(A \cap B) - P(B)$$
$$= \frac{7}{10} + \frac{2}{5} - \frac{3}{5}$$
$$= \frac{7 + 4 - 6}{10} = \frac{1}{2}$$

Probability of selecting a smoker, if a male is first selected, is given by

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$= \frac{2}{5} \times \frac{2}{1} = \frac{4}{5}$

117. **(d)** Given: $f(t) = \frac{\sin t}{t}$

At $t = 0$, we will check continuity of the function

$LHL = f(0 - h)$

$= \lim_{h \to 0} \frac{\sin(0 - h)}{(0 - h)} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$

$RHL = f(0 + h)$

$$\lim_{h\to 0}\frac{\sin(0+h)}{(0+h)}$$

$$=\lim_{h\to 0}\frac{\sin h}{h}=1$$

and $f(0)=1$

$LHL = RHL = f(0)$

So, the function is continuous at $t=0$
Now, we check the function is maximum or minimum.

$$f'(t)=\frac{1}{t}\cos t-\frac{1}{t^2}\sin t$$

and $f''(t)=\frac{-1}{t}\sin t-\frac{1}{t^2}\cos t-\frac{1}{t^2}\cos t+\frac{2}{t^3}\sin t$

$$=\frac{-\sin t}{t}-\frac{2\cos t}{t^2}+\frac{2\sin t}{t^3}$$

For maximum or minimum value of f(x), put

$f'(x)=0$

$$\Rightarrow \frac{\cos t}{t}-\frac{\sin t}{t^2}=0\Rightarrow\frac{\tan t}{t}=1$$

Now $\lim_{t\to 0}f''(t)$

$$=-\lim_{t\to 0}\left(\frac{\sin t}{t}\right)-2\lim_{t\to 0}\left(\frac{t\cos t-\sin t}{t^3}\right)$$

$\left[\frac{0}{0}\text{ form}\right]$

$$=-1-2\lim_{t\to 0}\left(\frac{\cos t-t\sin t-\cos t}{3t^2}\right)$$

[using L' Hospital rule]

$$=-1+\frac{2}{3}\lim_{t\to 0}\frac{\sin t}{t}$$

$$=-1+\frac{2}{3}\times 1=\frac{-1}{3}<0$$

So, function f(t) is maximum at $t=0$

118. **(d)** Consider the function f defined by

$$f(x)=a_0\frac{x^{n+1}}{n+1}+a_n\frac{x^n}{n}+\cdots+a_{n-1}\frac{x^2}{2}+a_nx$$

Since, f(x) is a polynomial, so it is continuous and differentiable for all x. f(x) is continuous in the closed interval [0,1] and differentiable in the open interval (0,1).

Also, $f(0)=0$

and

$f(1)=\frac{a_0}{n+1}+\frac{a_1}{n}+\cdots.+\frac{a_{n-1}}{2}+a_n=0$[say]

i.e. $f(0)=f(1)$

Thus, all the three conditions of Rolle's theorem are satisfied. Hence, there is atleast one value of x in the open interval $(0,1)$ where $f'(x)=0$

i.e. $a_0x^n+a_1x^{n-1}+\cdots.+a_n=0$

119. **(d)** Let $f(x)=\log(1+x)-\frac{x}{1+x}$

$$\therefore\ f'(x)=\frac{1}{1+x}-\frac{(1+x)\cdot 1-x\cdot 1}{(1+x)^2}$$
$$=\frac{1}{1+x}-\frac{1}{(1+x)^2}=\frac{x}{(1+x)^2}$$

which is positive. $[\because x>0]$

$\therefore$ f(x) is monotonic increasing, when $x>0$.

$\Rightarrow f(x)>f(0)$

Now, $f(0)=\log 1-0=0$

$\therefore f(x)>0$

$$\Rightarrow\ \log(1+x)-\frac{x}{1+x}>0$$

$$\Rightarrow\frac{x}{1+x}<\log(1+x)$$

Also, for $x>0$,

$x^2>0\Rightarrow x^2+x>x$

$\Rightarrow x(x+1)>x$

$$\Rightarrow\ x>\frac{x}{x+1}$$

From eqs. (i) and (ii), we get

$$\frac{x}{x+1}<\log(1+x)<x$$

$[\because \log(1+x)<x \text{ for } x>0]$

120. **(c)** We can write given differential equation as,

$(D^2-1)x=k$

where, $D\equiv\frac{d}{dy}$

Its auxiliary equation is $m^2 - 1 = 0$, so that $m = 1, -1$

Hence, $CF = C_1e^y + C_2e^{-y}$.

where C_1, C_2 are arbitrary constants

Now, also $PI = \frac{1}{D^2-1}k$

$$= k \cdot \frac{1}{D^2 - 1} e^{0.y}$$

$$= K \cdot \frac{1}{0^2 - 1} e^{0.y} = -K$$

So, solution of eq. (i) is

$x = C_1e^y + C_2e^{-y} - k$

Given that $x = 0$, when $y = 0$

So, $0 = C_1 + C_2 - k$ (From (ii))

$\Rightarrow C_1 + C_2 = k$ ....(iii)

Multiplying both sides of eq. (ii) by e^{-y}, we get
$x. e^{-y} = C_1 + C_2e^{-2y} - ke^{-y}$

Given that $x \to m$ when $y \to \infty$, m being a finite quantity.

So, eq (iv) becomes

$x \times 0 = C_1 + C_2 \times 0 - (k \times 0)$

$\Rightarrow C_1 = 0$ ....(v)

From eqs. (iv) and (v), we get

$C_1 = 0$ and $C_2 = k$

Hence, eq. (ii) becomes

$x = ke^{-y} - k = k(e^{-y} - 1)$