PART-I PHYSICS

1. **(d)** When work is done upon a system by a conservative force then its potential energy increases.
2. **(a)** de-Broglie wavelength is given by $\lambda = \frac{h}{p}$, where h = Planck's constant and p = momentum

Also, energy (E) and momentum are related as

$$E = \frac{p^2}{2m}$$

$$\therefore p = \sqrt{2mE}$$

$\therefore \lambda = \frac{h}{\sqrt{2mE}} \times \frac{1}{\sqrt{E}}$ as h and m are constants

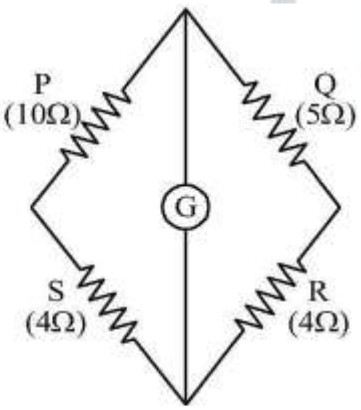
Hence, $\frac{\lambda_o}{\lambda'} = \sqrt{\frac{E'}{E}} = \sqrt{\frac{2E}{E}} = \sqrt{2}$

$\therefore \lambda' = \frac{\lambda_o}{\sqrt{2}}$.

3. **(a)** For no current through the galvanometer, the wheatstone bridge should be balanced. For this, we must have

$$\frac{P}{Q} = \frac{S}{R}$$

This condition is satisfied with only option (a).



When a 5Ω resistor is connected in series with Q, the equivalent resistance in the P-arm becomes 10Ω.

$$\therefore \frac{P}{Q} = \frac{10}{10} = 1$$

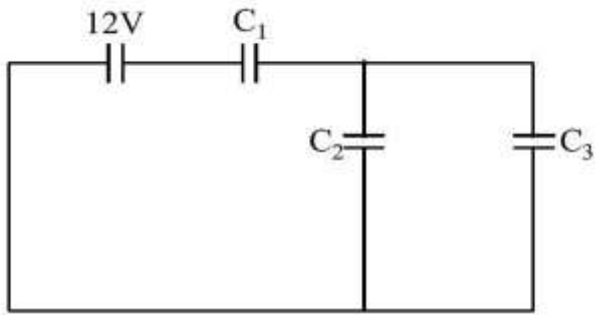
and $\frac{S}{R} = \frac{4}{4} = 1$

$\Rightarrow \frac{P}{Q} = \frac{S}{R}$.

4. **(a)** When a dielectric slab of dielectric constant $K = \frac{3}{2}$ is inserted between the plates of C_2, its new capacitance (C_2') becomes $C_2' = \frac{3}{2}C$

Equivalent capacitance of C_2' and C_3 is

$$C_{eq} = C_2' + C_3 = \frac{3}{2}C + C = \frac{5C}{2}$$



Now, C_{eq} and C_1 are in series. Therefore, their equivalent capacitance is

$$C_{eq} = \frac{C_{eq} \times C_1}{C_{eq} + C_1} = \frac{\frac{5C}{2} \times C}{\frac{5C}{2} + C}$$

$$= \frac{5C^2}{7C} = \frac{5C}{7}.$$

5. **(c)** Terminal velocity,

$$v_T = \frac{2r^2(d_1 - d_2)g}{9\eta}$$

$$\frac{v_{T_2}}{0.2} = \frac{(10.5 - 1.5)}{(19.5 - 1.5)} \Rightarrow v_{T_2} = 0.2 \times \frac{9}{18}$$

$\therefore v_{T_2} = 0.1 \text{ m/s}$

6. **(b)** Mganetic field at the centre of the circular loop increases as it moves towards the current carrying wire as we know it varies inversely with the distance. Therefore, induced current in the loop should be such that it can reduce the increased magnetic field. This is possible only when the induced current is clockwise.

7. **(b)** $|e| = L\frac{dI}{dt}$

Here, $e = 20 \text{ mV} = 20 \times 10^{-3} \text{ V}$

$$\frac{dI}{dt} = \frac{6-2}{2} = 2 \text{ A/s}$$

$L = ?$

$\Rightarrow 20 \times 10^{-3} = L \times 2$

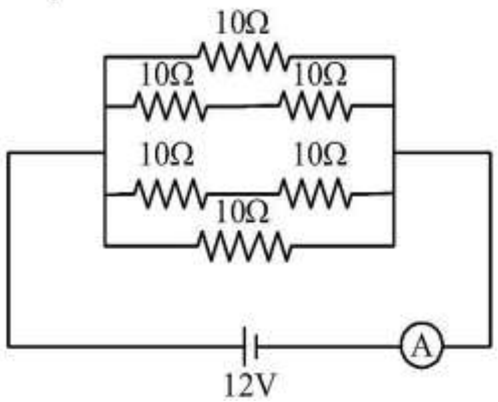
$\therefore L = 10 \times 10^{-3}\text{H} = 10\text{mH}.$

8. **(a)** For the time $t < t_1$, area of loop changes. Hence, magnetic flux linked with it changes during this time and emf is induced thereby.

9. **(c)** Hologram is based on the phenomenon of interference.

10. **(a)** An equivalent of the given network is as shown in the figure.

If R_p be the net resistance, then



$$\frac{1}{R_p}=\frac{1}{10}+\left(\frac{1}{10+10}\right)+\left(\frac{1}{10+10}\right)+\frac{1}{10}$$

$$=\frac{1}{10}+\frac{1}{20}+\frac{1}{20}+\frac{1}{10}$$

$$=\frac{6}{20}=\frac{3}{10}$$

$$\therefore\ R_p=\frac{10}{3}\Omega$$

Hence, current flowing through ammeter is

$$I=\frac{V}{R_p}=\frac{12}{\left(\frac{10}{3}\right)}=3.6\ A$$

11. **(c)** Energy released in transition from state n_1 to n_2 is equal to the energy absorbed in transistion from state n_2 to n_1.

12. **(b)** Magnetic field $\propto$ no. of turns per unit length

$\therefore$ Required ratio $=1:n$.

13. **(d)** R_1, R_2 and R_3 are in series. Therefore, same current will flow through all the resistors.

14. **(a)** As the charged particle is at rest, no force will act on it due to the magnetic field produced by the conductor at the site of the charge. Hence, it will remain at rest.

15. **(b)** $\frac{\Delta\ell}{\ell}=\alpha\Delta T=10^{-5}\times100=10^{-3}$

$$\frac{\Delta\ell}{\ell}\times100\%=10^{-3}\times100$$

$$=10^{-1}=0.1\%$$

16. **(d)** When wavelength decreases, frequency increases. Also, we know that cut-off voltage (or stopping potential) increases when frequency increases.

Hence, option (d) is correct. Note that work function and threshold frequency are constant for a given metal.

17. **(b)** Clearly,

$Y=(A+B).C=(A\cdot C)+(B\cdot C)$

For

$A=0, B=0\&C=1,\ Y=(0.1)+(0.1)=0$

$A = 1, B = 0 \& C = 1,\ Y = (1.1) + (0.1) = 1$

$A = 1, B = 0 \& C = 0,\ Y = (0.1) + (0.0) = 0$

$A = 0, B = 1 \& C = 0,\ Y = (0.0) + (1.0) = 0$

So, option (b) is correct.

18. (b) We have

$h\nu = W_o + E$, where E is the energy of emitted photoelectron

$$\Rightarrow \frac{hc}{\lambda} = W_o + E$$

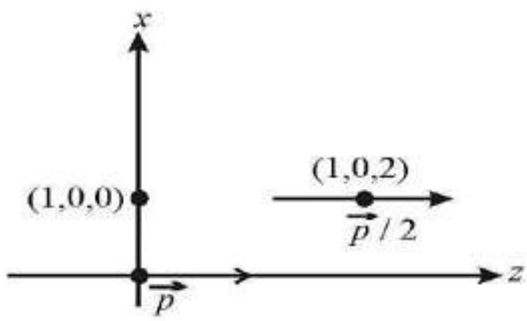
As hc and W_o are constant,

$$E \propto \frac{1}{\lambda}$$

Therefore, as λ is doubled, E will become half.

19. (a) $E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3} = \frac{q}{\frac{4}{3}\pi R^3} \times \frac{r}{3\varepsilon_0} = \frac{\rho \times r}{3\varepsilon_0}$

20. (b) The given point is on axis of $\frac{\vec{p}}{2}$ dipole and at equatorial line of $\vec{p}$ dipole so that field at given point is $(\vec{E}_1 + \vec{E}_2)$



$$\vec{E}_1 = \frac{2K(p/2)}{2^3} = \frac{Kp}{8}(+\hat{k})$$

$$\vec{E}_2 = \frac{Kp}{1}(-\hat{k})$$

$$\vec{E}_1 + \vec{E}_2 = -\frac{7}{8}Kp(-\hat{k}) = -\frac{7p}{32\pi\epsilon_0}\hat{k}$$

21. (c) $Y = \frac{F/A}{\Delta\ell/\ell} = \frac{\frac{250 \times 9.8}{50 \times 10^{-6}}}{\frac{0.5 \times 10^{-3}}{2}}$

$$= \frac{250 \times 9.8}{50 \times 10^{-6}} \times \frac{2}{0.5 \times 10^{-3}} \Rightarrow 19.6 \times 10^{10}\ N/m^2$$

22. (d) $R_1 = R_0(1 + \alpha_1 t) + R_0(1 + \alpha_2 t)$

$$= 2R_0\left(1 + \frac{\alpha_1 + \alpha_2}{2}t\right)$$

Comparing with $R = R_0(1 + \alpha t)$

$$\alpha = \frac{\alpha_1 + \alpha_2}{2}$$

23. (a) Given; $J = ar^2$.

$$i = \int_1^2 J \times 2\pi r dr = \int_{R/3}^{R/2} ar^2 \times 2\pi r dr$$

$$= 2\pi a \int_{R/3}^{R/2} r^3 dr = 2\pi a \left|\frac{r^4}{4}\right|_{R/3}^{R/2}$$

$$= \frac{\pi a}{2}\left[\left(\frac{R}{2}\right)^4 - \left(\frac{R}{3}\right)^4\right]$$

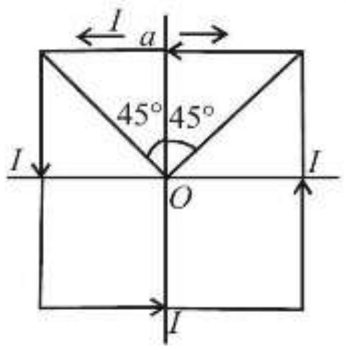
$$= \frac{\pi a R^4}{2} \times \frac{65}{81 \times 16} = \frac{65\pi a R^4}{2592}$$

24. (b) Decreasing R increases current in XY and there by increases the potential drop across XP and the balance point may be obtained. The current may be increased also by increasing V.

25. (b) Field due to one side of loop at O

$$= \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}(2\sin 45^\circ)$$

Field at O due to all four sides is along unit vector $\hat{e}_z$



∴ Total field

$$= 4.\frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}(2\sin 45^\circ) = \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

26. (c) The angular momentum L of the particle is given by

$L = mr^2\omega$ where $\omega = 2\pi n$.

∴ Frequency $n = \frac{\omega}{2\pi}$;

Further $i = q \times n = \frac{\omega q}{2\pi}$

Magnetic moment, $M = iA = \frac{\omega q}{2\pi} \times \pi r^2$;

$$\therefore M = \frac{\omega q r^2}{2}$$

So, $\frac{M}{L}=\frac{\omega q r^2}{2mr^2\omega}=\frac{q}{2m}$

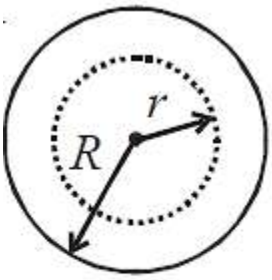
27. (c) Using Ampere's law, we have

$\oint \vec{B}\cdot d\vec{\ell}=\mu_0 i_{in}$

or

$$B\times 2\pi r=\mu_0\frac{i}{\pi R^2}\pi r^2$$

$$\therefore\ B=\frac{\mu_0}{2\pi}\cdot\frac{ir}{R^2}$$



28. (a) From Kirchhoff's current law,

$i_3=i_1+i_2=3\sin\omega t+4\sin(\omega t+90^\circ)$

$=\sqrt{3^2+4^2+2(3)(4)\cos 90^\circ}\sin(\omega t+\phi)$

where $\tan\phi=\frac{4\sin 90^\circ}{3+4\cos 90^\circ}=\frac{4}{3}$

$\therefore i_3=5\sin(\omega t+53^\circ)$

29. (b) We know that, $Z=\frac{E_0}{I_0}$

Given, $E_0=220$ V and $I_0=10$ A

so $Z=\frac{220}{10}=22\text{ohm}$

$$\phi=\left[\frac{\pi}{6}-\left(-\frac{\pi}{6}\right)\right]=\frac{\pi}{3}$$

$$p_a=\frac{E_0}{\sqrt{2}}\times\frac{I_0}{\sqrt{2}}\times\cos\phi$$

$$=\frac{220}{\sqrt{2}}\times\frac{10}{\sqrt{2}}\times\cos\frac{\pi}{3}=550\text{ W}$$

30. (b) $I=I_0\left(1-e^{-t/\tau}\right)$

where $\tau\rightarrow$ time constant

$$\therefore\ \frac{3}{4}I_0=I_0\left(1-e^{-t/\tau}\right)$$

$\Rightarrow \dfrac{3}{4} = 1 - e^{-t/\tau} \Rightarrow e^{-t/\tau} = \dfrac{1}{4}$

$\Rightarrow \dfrac{-t}{\tau} \ln e = \ln \dfrac{1}{4} \Rightarrow \dfrac{-4}{\tau} = -2\ln 2$

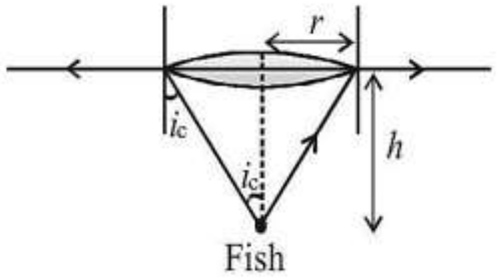
$\Rightarrow \tau = \dfrac{2}{\ln 2}$

31. (a) $e = -\frac{d\phi}{dt} = [-2 + 8t + 0]$

$\Rightarrow 8t = 2 \Rightarrow t = \dfrac{1}{4}\text{sec} = 0.25\text{sec}$

32. (d) $\sin i_c = \frac{1}{\mu} = \frac{r}{\sqrt{r^2+h^2}}$

Using $h = 12$ cm, $\mu = 4/3$



We get $r = \frac{36}{\sqrt{7}}$ cm.

33. (c) Apply conservation of momentum, $m_1 v_1 = (m_1 + m_2)v$

$v = \dfrac{m_1 v_1}{(m_1 + m_2)}$

Here $v_1 = 36\ \text{km/hr} = 10\ \text{m/s}$,

$m_1 = 2$ kg, $m_2 = 3$ kg

$v = \dfrac{10 \times 2}{5} = 4\ \text{m/s}$

K.E. (initial) $= \frac{1}{2} \times 2 \times (10)^2 = 100$ J

K.E. (Final) $= \frac{1}{2} \times (3+2) \times (4)^2 = 40$ J

Loss in K.E. $= 100 - 40 = 60$ J

Alternatively use the formula

$-\Delta E_k = \dfrac{1}{2} \dfrac{m_1 m_2}{(m_1 + m_2)} (u_1 - u_2)^2$

34. (d) $P = 2\left[\frac{100}{10} + \frac{100}{-20} + 0\right]$

$P = 10$ dioptre.

35. (a) $X_0 = \frac{\beta}{\lambda}(\mu - 1)t \Rightarrow 5\beta = \frac{\beta(0.45)t}{5890\times10^{-10}}$

$\therefore t = \dfrac{5 \times 5890 \times 10^{-10}}{0.45} = 6.544 \times 10^{-4}$ cm

36. (c) According to Malus'law$_2$

$I = I_0 \cos\theta = I_0(\cos 60°)$

$= I_0 \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$

37. (b) $t_{1/2} = 3.8$ day

$\therefore\ \lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{3.8} = 0.182$

If the initial number of atom is $a = A_0$ then after time t the number of atoms is $a/20 = A$. We have to find t.

$t = \dfrac{2.303}{\lambda}\log\dfrac{A_0}{A} = \dfrac{2.303}{0.182}\log\dfrac{a}{a/20}$

$= \frac{2.303}{0.182}\log 20 = 16.46$ days

38. (a) Nuclear radius, $r \propto A^{1/3}$ where A is mass number $\frac{r_{Cu}}{r_{Al}} = \left(\frac{A_{Cu}}{A_{Al}}\right)^{1/3} = \left(\frac{64}{27}\right)^{1/3}$

$\Rightarrow r_{Cu} = 3.6\left(\frac{4}{3}\right) = 4.8\text{Fermi}$

39. (d)

As $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$\therefore\ \dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right)$

Multiply both sides by λ

$1 = \lambda R\left(1 - \frac{1}{n^2}\right)$ or $\frac{1}{\lambda R} = 1 - \frac{1}{n^2}$

or $\frac{1}{n^2} = 1 - \frac{1}{\lambda R} = \frac{\lambda R - 1}{\lambda R}$

or $n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$

40. (a)

$a = \mu g = \dfrac{6}{10}$ [using $v = u + at$]

$\Rightarrow \mu = \dfrac{6}{10 \times g} = \dfrac{6}{10 \times 10} = 0.06$

PART-II CHEMISTRY

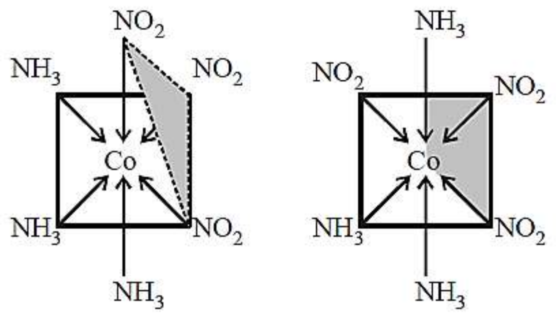
41. (d) The common name of Pentanoic acid is valeric acid.

42. (d)

43. (c) The number of atoms of the ligands that are directly bound to the central metal atom or ion by coordinate bond is known as the coordination number of the metal atom or ion. Hence the coordination no. of the given compound will be 6 .

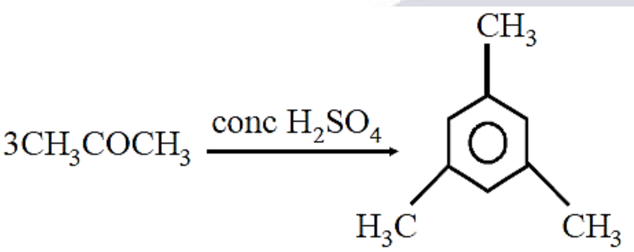
44. (a) Complexes of the type $M_{A_3\ B_3}$ exist in two geometrical forms which are nämed as facial (fac-) and meridonal (mer-isomers).

$[Co(NH_3)_3(NO_2)_3]$ may be represented in fac- and mer-isomeric forms as follows.



fac-isomer **mer-isomer**

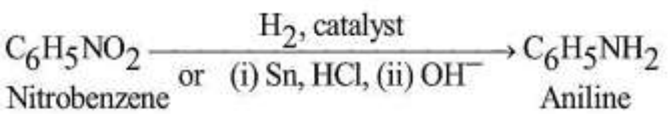
45. (c, d) Acetone on polymerisation give mesitylene



3 molecules of acetaldehyde produce paraldehyde $(CH_3CHO)_3$ and 4 molecules of it produce metaldehyde $(CH_3CHO)_4$.

46. (a)

47. (a) The most widely used method for preparing aromatic amines is the reduction of the nitro group to the amino group. This reduction can be achieved by catalytic hydrogenation, or most frequently with an acid and a metal (Fe, Zn, Sn) or a metal salt like $SnCl_2$.



48. (a) The electronic configuration for Rb (37) is $Rb(37) = 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^1$

For $5\ s^1, n = 5, \ell = 0, m = 0, s = \frac{1}{2}$

49. (b) XeF_4 disproportionates in water giving solid XeO_3 on evaporation.

$6XeF_4 + 12H_2O \longrightarrow 2XeO_3 + 24HF + 4Xe + 3O_2$

50. (b) Number of A ions in the unit cell

$= \frac{1}{8} \times 8 = 1$

Number of B ions in the unit cell

$= \frac{1}{2} \times 6 = 3$

Hence empirical formula of the compound $= AB_3$

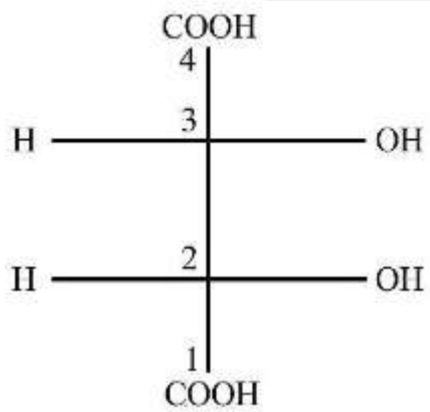
51. **(d)** The electrical resistance of metals depends upon temperature. Electrical resistance decreases with decrease in temperature and becomes zero near the absolute temperature. Material in this state is said to possess super conductivity.

52. **(a)** An electron releasing group stablilises the carbocation by dispersing its positive charge and thus activates the ring while an electron-withdrawing group destabilises the carbocation by intensifying its positive charge and thus deactivates the ring.

$-NH_2$ being electron releasing group releases electron and thus tend to neutralise positive charge of the ring and itself becomes somewhat positive. The dispersal of positive charge of the ring stabilises carbocation and hence facilitates its formation which ultimately results an increased rate of reaction. Now since such factor is not present in benzene, its carbocation is less stable than (A). Hence aniline undergoes electrophilic substitution at a faster rate than benzene.

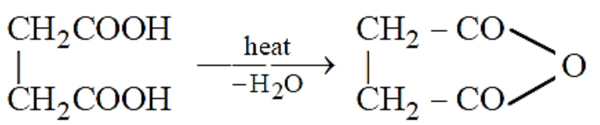
On the other hand in (C) the $-NO_2$ group (an electron withdrawing group) intensifies positive charge on the ring and thus destabilises the carbocation and hence its formation becomes difficult which ultimately results a slower reaction. Now since substituent is not present in benzene, its carbocation (B) is relatively more stable than (C). Hence nitrobenzene undergoes electrophilic substitution at a slower rate than benzene.

53. **(c)**

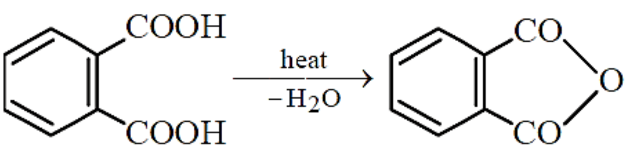


2S, 3R

54. **(b)**



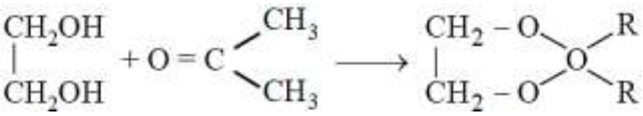
Succinic anhydride



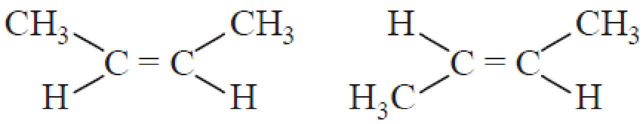
Phthalic anhydride

Oxalic on heating produces formic acid

55. **(a)**



56. (a)



cis-2-butene trans-2-butene

57. (c) Aldehydes and Ketones containing α-hydrogen undergo aldol condensation, since benzophenone does not have α-hydrogen hence do not undergo aldol condensation whereas acetophenone show this reaction due to presence of $\alpha - H$ atom.

58. (b) Isonitriles on reduction with $LiAlH_4$ give 2° amines

$$R - N \overset{\rightarrow}{=} C \xrightarrow{LiAlH_4} RNHCH_3$$

59. (c) Keratin

60. (c) For an isothermal process

$$\Delta S = 2.303 nR \log \frac{V_2}{V_1}$$

$$= 2.303 \times 1 \times 2 \log \frac{10}{1}$$

$$= 4.6 \text{cal} \deg^{-1} \text{mol}^{-1}$$

61. (b) Energy of activation for forward reaction $(E_a) = 85$ kJ/mol

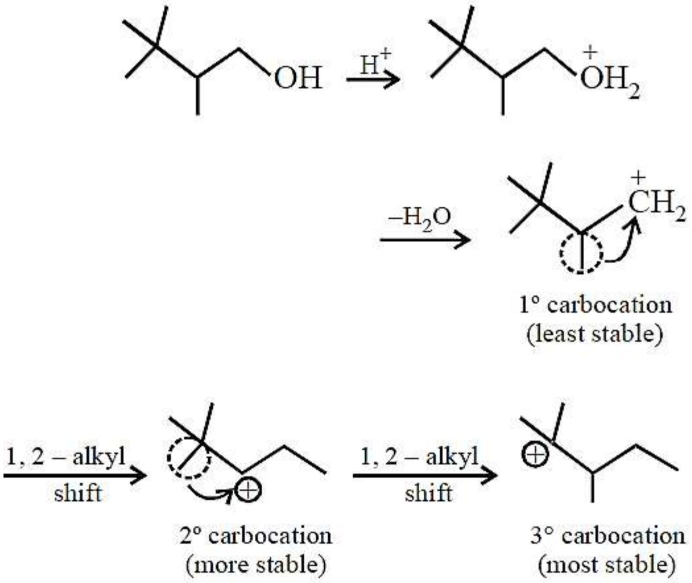
∴ Energy of activation for backward reaction $= E_a - \Delta H = 85 - 20 = 65$ kJ mol^{-1}

62. (a) Since the formation of amnonia is an exothermic reaction hence on increasing temperature, reaction will proceed in backward direction i.e. formation of NH_3 decreases.

63. (a) The atoms having half-filled and fully filled orbitals are comparatively more stable, hence more energy is required to remove
the electron from such atoms. Therefore group 15 have more I.E. than gp. 16 elements.

64. (c) Because of +I effect, t-butyl group destabilises the carbanion.

65. (d)



66. (c) We know $\Delta G^\circ = -nFE^\circ$

$$\therefore\ E^\circ = -\frac{\Delta G^\circ}{nF}$$
$$= +\frac{237.2 \times 1000\ \text{J}}{2 \times 96500} = 1.23\ \text{V}\ [\because n = 2]$$

67. (b) Solubility $S = \frac{1000k}{\lambda^0_{AgCl}} = \frac{1000 \times 2.6 \times 10^{-6}}{\lambda^0_{Ag^+} + \lambda^0_{Cl^-}}$

$$= \frac{2.6 \times 10^{-3}}{63 + 67} = 2 \times 10^{-5}\ \text{mol L}^{-1};\ K_{sp} = S^2$$

68. (c) For a reaction to be at equilibrium $\Delta G = 0$.

Since $\Delta G = \Delta H - T\Delta S$ so at equilibrium

$\Delta H - T\Delta S = 0$

or $\Delta H = T\Delta S$

For the reaction

$\frac{1}{2}X_2 + \frac{3}{2}Y_2 \longrightarrow XY_3;\ \Delta H = -30\ \text{kJ}$

(given)

Calculating ΔS for the above reaction, we get

$$\Delta S = 50 - \left[\frac{1}{2} \times 60 + \frac{3}{2} \times 40\right] \text{JK}^{-1}$$
$$= 50 - (30 + 60)\text{JK}^{-1} = -40\text{JK}^{-1}$$

At equilibrium, $T\Delta S = \Delta H\ [\because \Delta G = 0]$

$\therefore T \times (-40) = -30 \times 1000 [\because 1\ \text{kJ} = 1000\ \text{J}]$

or $T = \frac{-30 \times 1000}{-40}$ or 750 K

69. (b)

70. (d) $\underset{1-\alpha}{PCl_{5(g)}} \rightleftharpoons \underset{\alpha}{PCl_{3(g)}} + \underset{\alpha}{Cl_{2(g)}}$

$\alpha = \frac{x}{a}$ (degree of dissociation)

Total moles $= 1 - \alpha + \alpha + \alpha = 1 + \alpha$

$$K_p = \frac{P_{PCl_3} \times P_{Cl_2}}{P_{PCl_5}} = \frac{\left[\frac{\alpha}{1+\alpha} \cdot p\right]\left[\frac{\alpha}{1+\alpha} \cdot p\right]}{\frac{1-\alpha}{1+\alpha} \cdot p}$$

$$= \frac{\alpha^2 p}{1 - \alpha^2} \Rightarrow \alpha = \left(\frac{K_p}{K_p + P}\right)^{1/2}$$

71. (b) $RT\ln K = -\Delta G^\circ = T\Delta S^\circ - \Delta H^\circ$;

$$\ln K = \frac{\Delta S^\circ}{R} - \frac{\Delta H^\circ}{RT}$$

Thus, a plot of ln K versus $1/T$ (abscissa) will be straight line with slope equal to

$\frac{-\Delta H^\circ}{R}$ and intercept $\frac{\Delta S^\circ}{R}$.

72. (c)

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{a_2}{a_1}\right)^{n-1}; \frac{120}{240} = \left(\frac{4 \times 10^{-2}}{8 \times 10^{-2}}\right)^{n-1}; n = 2$$

73. (c) From slow step:

rate $= k_2[A][B]$

From fast step:

$K_e = \frac{[A]^2}{[A_2]}$ or $[A] = K_e^{1/2}[A]^{1/2}$

From (i) and (ii)

rate $= k_2 k_e^{1/2}[A_2]^{1/2}[B] = k[A_2]^{1/2}[B]$

74. (c) Using Arrhenius equation,

$K = A.e^{-\frac{E_a}{Rt}}$, we get

$$\log k = \log A - \frac{E_a}{2.303RT}$$

$$\log k_1 = \log A - \frac{E_{a(1)}}{2.303RT}$$

and $\log k_2 = \log A - \frac{E_{a(2)}}{2.303RT}$

or $\log\frac{k_2}{k_1} = \frac{1}{2.303RT}\left[E_{a(1)} - E_{a(2)}\right]$

(from (i) and (ii))

$$= \frac{1}{2.303 \times 8.314 \times 300}(76000 - 57000)$$

or $\log \frac{k_2}{k_1} = \frac{19000}{2.303 \times 8.314 \times 300}$

$$= \frac{190}{6.9 \times 8.314}$$

or $\frac{k_2}{k_1} = 2000$ [taking antilog]

75. (a) $\bar{v} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$$= R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$$

76. (d) The products of the concerned reaction react each other forming back the reactants.

$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$

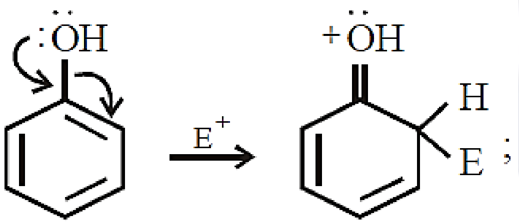
77. (c) Although in both cases products are CH_3I and C_6H_5OH; the two reactions follow different mechanism.

$$C_6H_5-O-CH_3 \xrightarrow[S_N2]{HI(g)} CH_3I + C_6H_5OH$$

$$C_6H_5-O-CH_3 \xrightarrow[S_N1]{conc.\,HI} CH_3I + C_6H_5OH$$

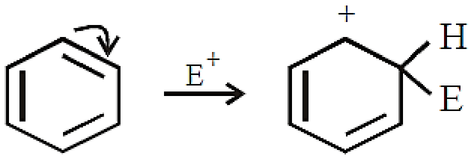
Remember that during S_{N_1} reaction, CH_3^+ is formed because it is more stable than $C_6H_5^+$.

78. (c)



+ M effect in phenol activates benzene ring

Oxonium ion (more stable)

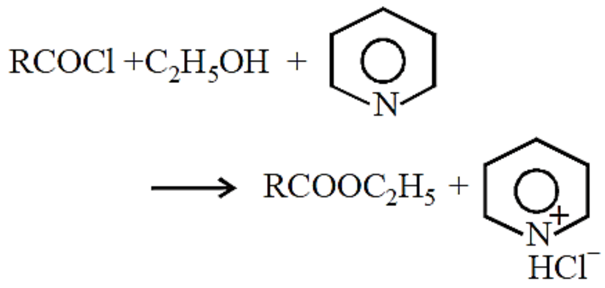


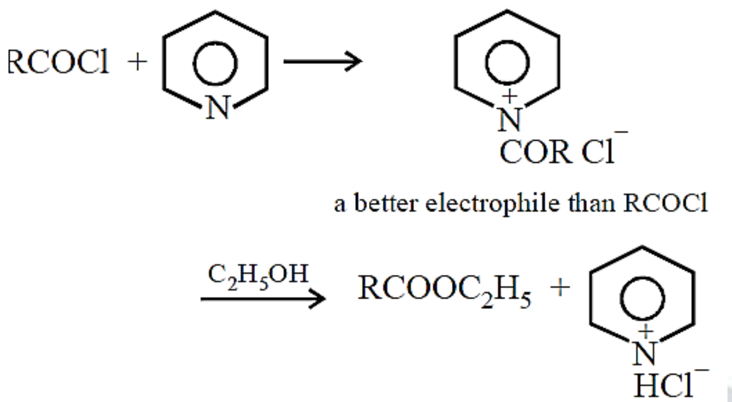
Carbonium ion (less stable)

High stability of oxonium ion (oxocation) is because here every atom (except H) has a complete octet of electrons, while in carbocations, carbon bearing positive charge is having six electrons.

79. (c) Baeyer - Villiger oxidation involves the conversion of a cyclic ketone to a lactone, or an acyclic ketone into ester.

80. (c)





PART-III MATHEMATICS

81. (b) $1+y^2+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

$\Rightarrow (1+y^2)dx=\left(e^{\tan^{-1}y}-x\right)dy$

$$\Rightarrow \frac{dx}{dy}=\frac{e^{\tan^{-1}y}-x}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy}+\frac{1}{1+y^2}\cdot x=\frac{e^{\tan^{-1}y}}{1+y^2}$$

Which is the linear different equation of the form $\frac{dx}{dy}+Rx=S$, where R and S are functions of y or constant (s)

$\therefore$ I.F $=\int\frac{1}{e^{1+y^2}}dy=e^{\tan^{-1}y}$

Hence required solution is

$x.(\text{I.F.})=\int\frac{e^{\tan^{-1}y}}{1+y^2}(\text{ I.F})\,dy$

$$\Rightarrow x\cdot e^{\tan^{-1}y}=\int\frac{e^{\tan^{-1}y}}{1+y^2}\left(e^{\tan^{-1}y}\right)dy$$

$$\Rightarrow x\cdot e^{\tan^{-1}y}=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy$$

Put $t=\tan^{-1}y$

$$\therefore \frac{dt}{dy}=\frac{1}{1+y^2}\Rightarrow dt=\frac{1}{1+y^2}\cdot dy$$

$$\therefore \int \frac{e^{2\tan^{-1}y}}{1+y^2}dy = \int e^{2t}\cdot dt = \frac{e^{2t}}{2} + K$$

Hence equation (1) becomes,

$$xe^{\tan^{-1}y} = \frac{1}{2}e^{2t} + K$$

$$\Rightarrow xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + K$$

$$\Rightarrow 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + K$$

82. **(a)** $\overrightarrow{AO} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\overrightarrow{AC} = -2\hat{\imath} - \hat{\jmath} + \hat{k}$

Angle between faces OAB and ABC

$=$ Angle between $\overrightarrow{AO}$ and $\overrightarrow{AC}$

If Q be the angle between $\overrightarrow{AO}$ and $\overrightarrow{AC}$,

then

$$\cos\theta = \frac{\overrightarrow{AO}\cdot\overrightarrow{AC}}{|\overrightarrow{AO}||\overrightarrow{AC}|}$$

$$= \frac{1\times(-2) + 2\times(-1) + 1\times 1}{\sqrt{1+4+1}\cdot\sqrt{4+1+1}} = \frac{-3}{6}$$

$$= -\frac{1}{2} = \cos 120^\circ$$

$$\therefore \theta = 120^\circ$$

83. **(c)** Given ellipse : $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

Now $b^2 = a^2(1-e^2)$

$$\Rightarrow b^2 = 16(1-e^2), \Rightarrow \frac{b^2}{16} = 1 - e^2$$

$$\Rightarrow e^2 = 1 - \frac{b^2}{16} = \frac{16-b^2}{16} \Rightarrow e = \frac{\sqrt{16-b^2}}{4}$$

$$\text{Foci} = (\pm ae, 0) = \left(\pm\sqrt{16-b^2}, 0\right)$$

Given hyperbola : $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$\Rightarrow \frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

Now, $b^2 = a^2(e^2 - 1)$

$\Rightarrow \left(\frac{9}{5}\right)^2 = \left(\frac{12}{5}\right)^2 (e^2 - 1)$

$\Rightarrow \left(\frac{9}{12}\right)^2 = e^2 - 1,$

$\Rightarrow e^2 = 1 + \frac{81}{144} = \frac{144 + 81}{144}$

$\Rightarrow e = \frac{15}{12} = \frac{5}{4}$

Foci $= (\pm ae, o) = (\pm 3,0)$

Since foci of the given ellipse and hyperbola coincide, therefore

$\sqrt{16 - b^2} = 3 \Rightarrow 16 - b^2 = 9$

$\therefore b^2 = 7$

84. **(a)** $f'(3) = \tan\frac{3\pi}{4} = -\tan\frac{\pi}{4} = -1$

85. **(b)** Required probability

$= P(\text{ First win }) \times P(\text{ First win }) \times P(\text{ Second win})$

$+P(\text{ First Defeat }) \times P(\text{ First win }) \times P(\text{ Second win})$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

86. **(b)** $\vec{a} + b + \vec{c} = 0$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow 9 + 4 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 7$

87. **(d)** Let $r(x) = f(x) \cdot g(x)$

$= x^2.2x = 2x^3$

$r'(x) = 6x^2$

Put $6x^2 = 0, \therefore x = 0$

$\text{Max}\, r(x) = 2(2)^3 = 16$

or $\text{Max}(f(x), g(x)) = 16$

$I(x) = \int_0^2 16dx$

$I(x) = [16x]_0^2 = 32 - 0 = 32$

88. **(b)** $B = ABA^{-1}$ (Given)

But $B = BAA^{-A}$

$\therefore ABA^{-1} = BAA^{-1} \Rightarrow AB = BA$

Now $(A + B)(A - B) = A^2 - AB + BA - B^2$

$= A^2 - AB + AB - B^2$ [$\because AB = BA$]

$= A^2 - B^2$

89. (b) $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7 = (-2\omega^2)^7$
$= -128(\omega^4)^3\omega^2 = -128\omega^2$

90. (d) Here, $r = (2\hat{i} + 3\hat{j} + \hat{k}) - (i + 2\hat{j} + 3\hat{k})$
$\Rightarrow r = i + \hat{j} - 2\hat{k}$ and $F = i + \hat{j} + \hat{k}$

Then, the required moment is given by

$r \times F = (i + \hat{j} - 2\hat{k}) \times (i + \hat{j} + \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j}$$

$\therefore$ Moment about given point $= 3\hat{i} - 3\hat{j}$

91. (b) $g(x). g(y) = g(x) + g(y) + g(xy) - 2$
Put $x = 1, y = 2$, then

$g(1). g(2) = g(1) + g(2) + g(2) - 2$

$5\ g(1) = g(1) + 5 + 5 - 2$

$4\ g(1) = 8\ \therefore g(1) = 2$
Put $y = \frac{1}{x}$ in equation (1), we get

$$g(x) \cdot g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right) + g(1) - 2$$

$$g(x) \cdot g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right) + 2 - 2$$

[$\because g(1) = 2$]

This is valid only for the polynomial

$\therefore g(x) = 1 \pm x^n$

Now $g(2) = 5$

(Given)

$\therefore 1 \pm 2^n = 5$ [Using equation (2)] $\pm 2^n = 4, \Rightarrow 2^n = 4, -4$

Since the value of 2^n cannot be $-$Ve.

So, $2^n = 4, \Rightarrow n = 2$

Now, put $n = 2$ in equation (2), we get $g(x) = 1 \pm x^2$

$\therefore \underset{x\to 3}{\mathrm{Lt}}\, g(x) = \underset{x\to 3}{\mathrm{Lt}}(1 \pm x^2) = 1 \pm (3)^2 = 1 \pm 9 = 10, -8$

92. (c) Any tangent to parabola $y^2 = 8x$ is $y = mx + \frac{2}{m}$

It touches the circle $x^2 + y^2 - 12x + 4 = 0$, if the length of perpendicular from the centre $(6,0)$ is equal to radius $\sqrt{32}$.

$$\therefore \frac{6\,m + \frac{2}{m}}{\sqrt{m^2 + 1}} = \pm\sqrt{32}$$
$$\Rightarrow \left(3\,m + \frac{1}{m}\right)^2 = 8(\,m^2 + 1)$$
$$\Rightarrow (3\,m^2 + 1)^2 = 8(\,m^4 + m^2)$$
$$\Rightarrow m^4 - 2\,m^2 + 1 = 0 \Rightarrow m = \pm 1$$

Hence, the required tangents are $y = x + 2$ and $y = -x - 2$.

93. (b) Given $e^x = y + \sqrt{1 + y^2}$

$\Rightarrow e^x - y = \sqrt{1 + y^2}$

Squaring both side, we have

$e^{2x} + y^2 - 2e^x y = 1 + y^2$

$\Rightarrow 2e^x y = e^{2x} - 1$

$\Rightarrow y = \frac{e^{2x} - 1}{2e^x} \Rightarrow y = \frac{1}{2}[e^x - e^{-x}]$

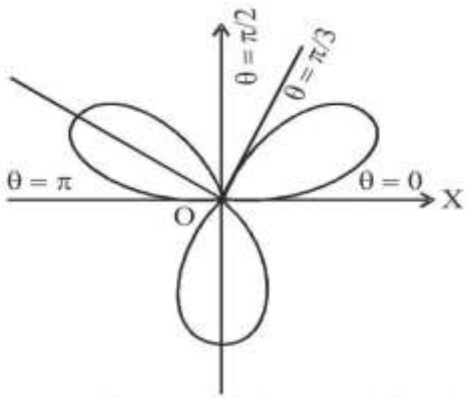
94. (d) If curve $r = a\sin 3\theta$

To trace the curve, we consider the following table:

$3\theta =$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
$\theta =$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r =$	0	a	0	$-a$	0	a	0

Thus there is a loop between $\theta = 0 \& \theta = \frac{\pi}{3}$

as r varies from $r = 0$ to $r = 0$.



Hence, the area of the loop lying in the

positive quadrant $=\frac{1}{2}\int_0^{\frac{\pi}{3}} r^2\, d\theta$

$=\frac{1}{2}\int_0^{\frac{\pi}{3}} \sin^2\phi\cdot\frac{1}{3}\, d\phi$

[On putting, $3\theta=\phi \Rightarrow d\theta=\frac{1}{3}\, d\phi$]

$=\frac{a^2}{6}\int_0^{\frac{\pi}{2}} \sin^2\phi d\phi$

$=\frac{a^2}{6}\cdot\int_0^{\frac{\pi}{2}} \frac{1-\cos 2\phi}{2}\, d\phi [\because \cos 2\theta=1-2\sin^2\theta]$

$=\frac{a^2}{12}\cdot\left[\phi+\frac{\sin 2\phi}{2}\right]_0^{\frac{\pi}{2}}$

$=\frac{a^2}{12}\cdot\left[\frac{\pi}{2}+\sin\pi\right]=\frac{a^2\pi}{24}$

95. (a) Replacing each hexadecimal digit by the corresponding 4-digit binary numeral, we have $(ABCD)_{16}=(1010101111001101)_2$

96. (c) Let the normal at ' t_1 ' cuts the parabola again at the point ' t_2 '. the equation of the normal at $(at_1{}^2, 2at_1)$ is $y+t_1x=2at_1+at_1{}^3$

Since it passes through the point ' t_2 ' i.e $(at_2^2, 2at_2)$

$\therefore\ 2at_2+at_1t_2^2=2at_1+at_1^3$

$\Rightarrow\ 2a(t_1-t_2)+at_1(t_1^2-t_2^2)=0$

$\Rightarrow 2+t_1(t_1+t_2)=0\ (\because t_1-t_2\neq 0)$

$\Rightarrow 2+t_1^2+t_1t_2=0$

$\Rightarrow t_1t_2=-(t_1^2+2) \Rightarrow t_2=-\left(t_1+\frac{2}{t_1}\right)$

97. (b) $s=t^3-12t^2+6t+8$

$$\frac{ds}{dt} = 3t^2 - 24t + 6$$

$$\frac{d^2 s}{dt^2} = 6t - 24$$

Acceleration $= 0$

$\Rightarrow 6t - 24 = 0$

$\therefore\ t = 4$

Required velocity $= \left.\frac{ds}{dt}\right]_{t=4}$

$= 3 \times (4)^2 - 24 \times 4 + 6$

$= 48 - 96 + 6 = 42$ units

98. **(a)** $X = a.b$

99. **(b)** Let $\cos^{-1}\frac{1}{8} = \theta$, where $0 < \theta < \frac{\pi}{2}$.

$$\Rightarrow \frac{1}{2}\cos^{-1}\frac{1}{8} = \frac{1}{2}\theta$$

$$\Rightarrow \cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right) = \cos\frac{1}{2}\theta$$

Now, $\cos^{-1}\frac{1}{8} = \theta \Rightarrow \cos\theta = \frac{1}{8}$

$$\Rightarrow 2\cos^2\frac{\theta}{2} - 1 = \frac{1}{8}$$

$$\Rightarrow \cos^2\frac{\theta}{2} = \frac{9}{16} \Rightarrow \cos\frac{\theta}{2} = \frac{3}{4}$$

$$\left[\because 0 < \frac{\theta}{2} < \frac{\pi}{4}, \text{ so } \cos\frac{\theta}{2} \neq -\frac{3}{4}\right]$$

100. **(c)** Two constraints are $x \geq 0, y \geq 0$ and the third one will be of the type $ax + by \leq c$.

101. **(b)** Let y denote the number of bacteria at any instant $t \cdot$ then according to the question

$$\frac{dy}{dt} \alpha y \Rightarrow \frac{dy}{y} = kdt$$

k is the constant of proportionality, taken to be + ve on integrating (i), we get

$\log y = kt + c$

c is a parameter. let y_0 be the initial number of bacteria

i.e., at $t = 0$ using this in (ii), $c = \log y_0$

$\Rightarrow \log y = kt + \log y_0$

$$\Rightarrow \log\frac{y}{y_0} = kt$$

$y = \left(y_0 + \frac{10}{100}y_0\right) = \frac{11y_0}{10}$, when $t = 2$

So, from (iii), we get $\log\frac{\frac{11y_0}{10}}{y_0} = k$

$\Rightarrow k = \frac{1}{2}\log\frac{11}{10}$

Using (iv) in (iii) $\log\frac{y}{y_0} = \frac{1}{2}\left(\log\frac{11}{10}\right)t$

let the number of bacteria become 1,00 , 000 to 2,00,000 in t_1 hours. i.e., $y = 2y_0$ when $t = t_1$ hours. from (v)

$\log\frac{2y_0}{y_0} = \frac{1}{2}\left(\log\frac{11}{10}\right)t_1 \Rightarrow t_1 = \frac{2\log 2}{\log\frac{11}{10}}$

Hence, the reqd. no. of hours $= \frac{2\log 2}{\log\frac{11}{10}}$

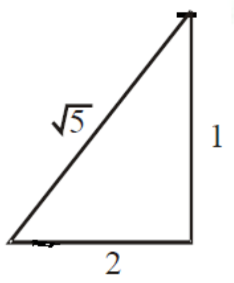
102. **(b)** Consider $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cot^{-1}3$

We have, $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \cot^{-1}2$

∴ From equation (i), we have

$\cos^{-1}2 + \cot^{-1}3 = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)$

$= \tan^{-1}\left(\frac{5/6}{\frac{6-1}{6}}\right) = \tan^{-1}1 = \frac{\pi}{4}$



103. **(b)** We have,

abcd $= \cos(2\alpha + 2\beta + 2\gamma + 2\delta) + i\sin(2\alpha + 2\beta + 2\gamma + 2\delta)$

$\therefore \sqrt{abcd} = [\cos(2\alpha + 2\beta + 2\gamma + 2\delta)$

$+ i\sin(2\alpha + 2\beta + 2\gamma + 2\delta)]^{1/2}$

or $\sqrt{abcd}$

$= \cos(\alpha + \beta + \gamma + \delta) + i\sin(\alpha + \beta + \gamma + \delta) \ldots.(1)$

[De Moivre's Theorem]

$\therefore \frac{1}{\sqrt{abcd}} = \cos(\alpha+\beta+\gamma+\delta) - i\sin(\alpha+\beta+\gamma+\delta)$

Adding (1) and (2), we obtain

$$\sqrt{abcd} + \frac{1}{\sqrt{abcd}} = 2\cos(\alpha+\beta+\gamma+\delta)$$

104. **(a)** Standard deviation $\sigma = \sqrt{npq} \geq 0$

Now mean $= np = 25$ and $q < 1$

So $\sigma = \sqrt{npq} < \sqrt{np} = 5$

$\therefore 0 \leq \sigma < 5$

105. **(d)** Number of ways

$= [({}^{3}C_3 + {}^{4}C_3 + {}^{5}C_3 + {}^{6}C_3 + {}^{7}C_3) \times 2 + {}^{8}C_3] \times 2$

$= 392$

106. **(d)** $\because$ rank(AB) $\leq$ rank(A)

and rank(AB) $\leq$ rank(B)

Therefore rank(AB) $\leq$ min(rank A, rank B)

107. **(a)** Let $A \equiv (a, 0, 0), B \equiv (0, b, 0), C \equiv (0, 0, c)$, then

equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Its distance from the origin, $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$ ... (i)

If (x, y, z) be centroid of $\triangle$ ABC, then

$$x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3}$$

Eliminating a,b,c from (i) and (ii) required locus is

$x^{-2} + y^{-2} + z^{-2} = 9p^{-2}$

108. **(c)** There are 26 red cards and 26 black cards

i.e., total number of cards = 52

P (both cards of different colours)

$= P(B)P(R) + P(R)P(B)$

$$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51} = 2\left(\frac{26}{52} \times \frac{26}{51}\right) = \frac{26}{51}$$

109. **(a)** The equation of the hyperbola is

$x^2 - 2y^2 - 2x + 8y - 1 = 0$

or $(x-1)^2 - 2(y-2)^2 + 6 = 0$

or $\frac{(x-1)^2}{-6}+\frac{(y-2)^2}{3}=1$

or $\frac{(y-2)^2}{3}-\frac{(x-1)^2}{6}=1$

or $\frac{Y^2}{3}-\frac{X^2}{6}=1$

where $X=x-1$ and $Y=y-2$

$\therefore$ The centre $=(0,0)$ in the $X-Y$ co-ordinates.

$\therefore$ The centre $=(1,2)$ in the x-y co-ordinates, using (2).

If the transverse axis be of length 2a, then $a=\sqrt{3}$, since in the equation (1) the transverse axis is parallel to the y-axis.

If the conjugate axis is of length 2b, then $b=\sqrt{6}$.

But $b^2=a^2(e^2-1)$

$\therefore 6=3(e^2-1),\ \therefore e^2=3$ or $e=\sqrt{3}$.

The length of the transverse axis $=2\sqrt{3}$.

The length of the conjugate axis $=2\sqrt{6}$.

Latus rectum $=\frac{2\,b^2}{a}=\frac{2\times 6}{\sqrt{3}}=4\sqrt{3}$

110. **(c)** Probability of getting a blue ball at any draw

$=p=\frac{10}{20}=\frac{1}{2}$

P [getting a blue ball 4th time in 7th draw] = P [getting 3 blue balls in 6 draw] $\times$ P [a blue ball in the 7 th draw]. $= {}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3\cdot\frac{1}{2}$

$=\frac{6\times5\times4}{1\times2\times3}\left(\frac{1}{2}\right)^7=20\times\frac{1}{32\times4}=\frac{5}{32}$

111. **(d)** For the first circle centre $=(3,7)$

Radius $r_1=\sqrt{3^2+7^2-48}=\sqrt{10}$

For the second circle, centre $(3,0)$; radius $r_2=3$

So, $r_1+r_2<d$ (distance between the centres) $\therefore$ Circle don't cut and hence the number of common tangents $=4$.

112. **(d)** We have,

$\cos^2\theta+\sin\theta+1=0\Rightarrow 1-\sin^2\theta+\sin\theta+1=0$

$\Rightarrow \sin\theta=-1(\because \sin\theta\neq 2)\Rightarrow\theta=3\pi/2$

$\therefore \theta\in\left(\frac{5\pi}{4},\frac{7\pi}{4}\right)$

113. **(c)** $\lim_{x\to0}f(x)=\lim_{x\to0}\left[(1+x)^{1/x}\right]^2=e^2=f(0)$

114. **(d)** $\log y = \frac{x}{\log x} \log 2$

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=e} = 0$

115. **(d)** $\int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

Put $1 + \frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5} dx = dt$

So, integral is

$$I = -\frac{1}{4}\int \frac{dt}{t^{3/4}} = -t^{\frac{1}{4}} + c = -\left(1 + \frac{1}{x^4}\right)^{1/4} + c$$

116. **(a)** The number of words starting from A are $5! = 120$

The number of words starting from I are $5\,! = 120$

The number of words starting from KA are $4! = 24$

The number of words starting from KI are $4! = 24$

The number of words starting from KN are $4! = 24$

The number of words starting from KRA are $3! = 6$

The number of words starting from KRIA are $2! = 2$

The number of words starting from KRIN are $2! = 2$

The number of words starting from KRISA are $1! = 1$

The number of words starting from KRISNA are $1! = 1$

Hence, rank of word 'KRISNA

$= 2(120) + 3(24) + 6 + 2(2) + 2(1) = 324$

117. **(b)** The lines are $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$

and $\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$

Here,

$\vec{a}_1 = -2\hat{j} + \hat{k}, b_1 + 6\hat{i} + 6\hat{j} + \hat{k}, \vec{a}_2 = -\hat{i},$

$\vec{b}_2 = 12\hat{i} + 6\hat{j} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

Shortest distance $= \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 - \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

$$= \frac{|(-\hat{\imath} + 2\hat{\jmath} - \hat{k}) \cdot (-12i + 18\hat{\jmath} - 36\hat{k})|}{\sqrt{(-12)^2 + (18)^2 + (-36)^2}}$$

$$= \frac{| + 12 + 36 + 36|}{\sqrt{1764}} = \frac{84}{42} = 2$$

118. **(b)** Given $f(x) = 2 - |x - 5|$

Domain of $f(x)$ is defined for all real values of x.

Since, $|x - 5| \geq 0 \Rightarrow -|x - 5| \leq 0$

$\Rightarrow 2 - |x - 5| \leq 2 \Rightarrow f(x) \leq 2$

Hence, range of $f(x)$ is $(-\infty, 2]$.

119. **(a)** If A and B are two sets having m and n elements such that

$1 \leq n \leq m = \sum_{r=1}^{n} (-1)^{n-r n} C_r r^m$

Number of surjection from A to B

$= \sum_{r=1}^{n} (-1)^{2-r\ 2} C_r (r)^4$

$= (-1)^{2-1 2} C_1 (1)^4 + (-1)^{2-2 2} C_2 (2)^4 = -2 + 16$

$= 14$

120. **(d)** Let $I = \int_a^b xf(x)dx$

Let $a + b - x = z \Rightarrow -dx = dz$

When $x = a, z = b$ and when $x = b, z = a$

$\therefore\ I = -\int_b^a (a + b - z)f(z)dz$

$I = (a + b)\int_b^a f(x)dx - \int_a^b xf(x)dx$

$I = (a + b)\int_{b_b}^{a} f(x)dx - I;$

$2I = (a + b)\int f(x)dx$

Hence, $I = \left(\frac{a+b}{2}\right) \int_a^b f(x)dx$

PART-IV ENGLISH

121. **(b)**
122. **(a)**
123. **(d)**
124. **(a)** "Whether" is correct because the question concerns a choice not a condition. With the expression "the number of" a singular verb is needed and hence "was" is correct. "Liable" is used in expressions such as "liable to prosecution" and not for expressions of possibility.
125. **(b)**