PART-I PHYSICS

1. (b) We know that, $R = \frac{\rho \ell}{A}$

or $R = \frac{\rho \ell^2}{\text{Volume}} \Rightarrow R \propto \ell^2$

According to question $\ell_2 = n\ell_1$

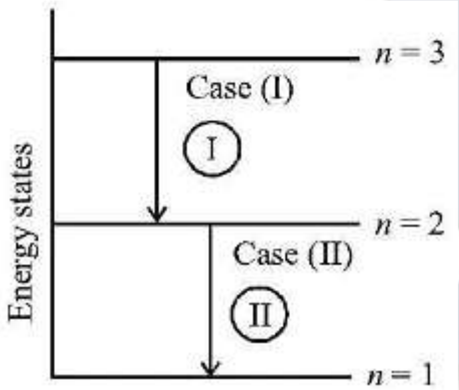
$\frac{R_2}{R_1} = \frac{n^2 l_1^2}{l_1^2}$ or, $\frac{R_2}{R_1} = n^2 \Rightarrow R_2 = n^2 R_1$

2. (b) Time constant is L/R

Given, $L = 40H \& R = 8\Omega$

$\therefore \tau = 40/8 = 5\text{sec.}$

3. (a)
4. (c)
5. (d)
6. (c)



The wave number $(\bar{v})$ of the radiation

$$= \frac{1}{\lambda}$$

$$= R_\infty \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Now for case (I) $n_1 = 3, n_2 = 2$

$\frac{1}{\lambda_1} = R_\infty \left[\frac{1}{9} \frac{-1}{4}\right], R_\infty =$ Rydberg constant

$$\frac{1}{\lambda_1} = R_\infty \left[\frac{4-9}{36}\right] = \frac{-5R_\infty}{36}$$

$$\Rightarrow \lambda_1 = \frac{-36}{5R_\infty}$$

$$\frac{1}{\lambda_2} = R_\infty \left[\frac{1}{4} - \frac{1}{1}\right] = \frac{-3R_\infty}{4} \Rightarrow \lambda_2 = \frac{-4}{3R_\infty}$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{-36}{5R_\infty} \times \frac{3R_\infty}{-4} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{27}{5}$$

7. **(c)** Voltage amplification

$$A_v = \beta \frac{R_o}{R_i} = 60 \times \frac{5000}{500} = 600$$

8. **(a)** Force required $= \frac{\text{change in momentum}}{\text{time taken}}$

$$= \frac{(50 \times 10^{-3} \times 30) \times 400 - (5 \times 0)}{60} = 10 \text{ N}$$

9. **(c)** $\frac{dN}{dt} = KN$

$9750 = KN_0$

$975 = KN$

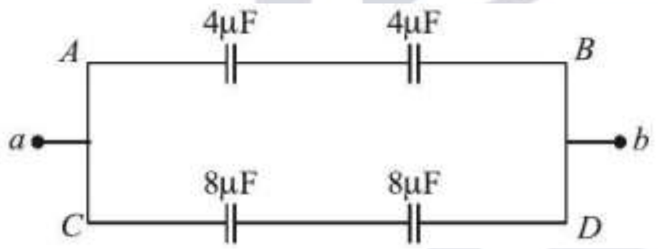
Dividing (i) by (ii)

$$\frac{N}{N_0} = \frac{1}{10}$$

$$K = \frac{2.303}{t} \log \frac{N_0}{N} = \frac{2.303}{5} \log 10$$

$= 0.4606 = 0.461$ per minute

10. **(a)** Rearranging the circuits, we get the following circuit.



$\therefore$ equivalent capacitance between A and B,

$$C_{AB} = \frac{4 \times 4}{4 + 4} = 2\mu F$$

and equivalent capacitance between C and D,

$$C_{CD} = \frac{8 \times 8}{8 + 8} = 4\mu F$$

$\therefore C_{ab} = 2\mu F + 4\mu F = 6\mu F$

11. **(b)** $e = M\frac{di}{dt} = 0.005 \times \frac{d}{dt}(I_0 \sin \omega t)$

$= 0.0005 \times I_0 \omega \cos \omega t$

$\therefore e_{max} = 0.005 \times 10 \times 100\pi = 5\pi$ [$\because \cos \omega t = 1$]

12. **(c)** If a is the amplitude of the wave then

$$\frac{I_{max}}{4} = a^2 = a^2 + a^2 + 2aa\cos\phi$$

$$\text{or} \quad \cos\phi = -\frac{1}{2}$$

$$\text{or} \quad \phi = \frac{2\pi}{3}.$$

Corresponding path difference,

$$\Delta x = \frac{\phi \times \lambda}{2\pi} = \frac{(2\pi/3) \times \lambda}{2\pi} = \frac{\lambda}{3}$$

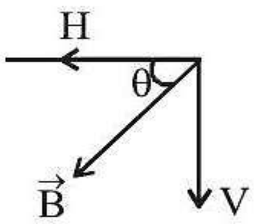
$$\text{So } d\sin\theta = \frac{\lambda}{3}$$

$$\text{or } \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right).$$

13. (a) $I = \frac{8-4}{1+2+9} = \frac{4}{12} = \frac{1}{3}A;$

$V_P - V_Q = 4 - \frac{1}{3} \times 3 = 3$ volt

14. (c) Horizontal component of earth's field, $H = B\cos\theta$, since, $\theta = 60°$



$3.6 \times 10^{-5} = B \times \frac{1}{2} \Rightarrow B = 7.2 \times 10^{-5}$ Tesla

15. (b) $\eta = 10^{-2}$ poise

$$v = 18 \text{ km/h} = \frac{18000}{3600} = 5 \text{ m/s}$$

$l = 5$ m

Strain rate $= \frac{v}{l}$

Coefficient of viscosity,

$$\eta = \frac{\text{shearing stress}}{\text{strain rate}}$$

$\therefore$ Shearing stress $= \eta \times$ strain rate

$$= 10^{-2} \times \frac{5}{5} = 10^{-2}\text{Nm}^{-2}$$

16. (b) From question,

$B_0 = 20\text{nT} = 20 \times 10^{-9}$ T

$\vec{E}_0 = \vec{B}_0 \times \vec{C}$

$|\vec{E}_0| = |\vec{B}_0|.|\vec{C}| = 20 \times 10^{-9} \times 3 \times 10^{8} = 6$ V/m.

($\because$ velocity of light in vacuum $C = 3 \times 10^{8}\ ms^{-1}$)

17. (b) Forward bias resistance

$$= \frac{\Delta V}{\Delta I} = \frac{0.1}{10 \times 10^{-3}} = 10\Omega$$

Reverse bias resistance $= \frac{10}{10^{-6}} = 10^{7}\Omega$

Ratio of resistances

$$= \frac{\text{Forward bias resistance}}{\text{Reverse bias resistance}} = 10^{-6}$$

18. (a) In physics, collision does not means that one particle strike another particle. Infact, two particles may not even touch each other & may still said to be colliding.
The necessary requirements of collision are (i) A large force for a relatively short time (i.e., an impulse) acts on each colliding particle.

(ii) The motion of the particles (at least one of the particle) is changed abruptly.

(iii) The total momentum (as also the total energy) of particles remains conserved.

19. (a) Current sensitivity $= \frac{nBA}{K}$

where K is constant of torsional rigidity.

20. (b) Let ℓ_0 be the unstretched length and ℓ_3 be the length under a tension of 9 N. Then

$$Y = \frac{4\ell_0}{A(\ell_1 - \ell_0)} = \frac{5\ell_0}{A(\ell_2 - \ell_0)}$$
$$= \frac{9\ell_0}{A(\ell_3 - \ell_0)}$$

These give

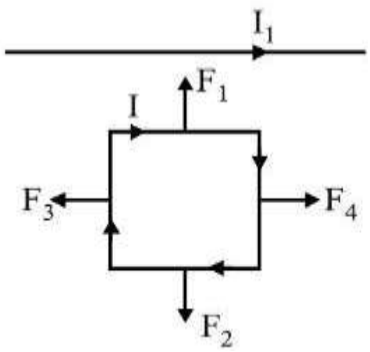
$$\frac{4}{\ell_1 - \ell_0} = \frac{5}{\ell_2 - \ell_0} \Rightarrow \ell_0 = 5\ell_1 - 4\ell_2$$

Further, $\frac{4}{\ell_1 - \ell_0} = \frac{9}{\ell_2 - \ell_0}$

Substituting the value of ℓ_0 and solving,

we get $\ell_3 = 5\ell_2 - 4\ell_1$

21. (d) $F_1 > F_2$ as $F \propto \frac{1}{d}$, and F_3 and F_4 are equal and opposite. Hence, the net attraction force will be towards the conductor.



22. (c) Heat gain = heat lost

$C_A(16-12) = C_B(19-16) \Rightarrow \frac{C_A}{C_B} = \frac{3}{4}$

and $C_B(23-19) = C_c(28-23) \Rightarrow \frac{C_B}{C_C} = \frac{5}{4}$

$\Rightarrow \frac{C_A}{C_C} = \frac{15}{16}$

If θ is the temperature when A and C are mixed then,

$C_A(\theta - 12) = C_C(28 - \theta)$

$\Rightarrow \frac{C_A}{C_C} = \frac{28-\theta}{\theta - 12}$

On solving equations (i) and (ii) $\theta = 20.2°C$

23. (d) Impedence of a capacitor is $X_C = 1/\omega C$

$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 2 \times 10^{-6}} = \frac{5000}{\pi}$.

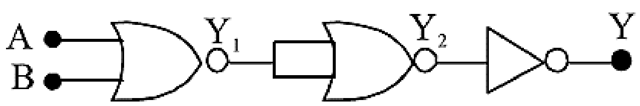
24. (a) $C_p - C_v = R \Rightarrow C_p = C_v + R$

$\because \gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{C_v}{C_v} + \frac{R}{C_v}$

$\Rightarrow \gamma = 1 + \frac{R}{C_v} \Rightarrow \frac{R}{C_v} = \gamma - 1$

$\Rightarrow C_v = \frac{R}{\gamma - 1}$

25. (c)
26. (b)



$Y_1 = \overline{A+B}$
$Y_2 = \overline{Y_1 + Y_1} = \overline{Y_1} = A + B$
$Y = \overline{Y_2} = \overline{A+B}$ i.e. NOR gate

27. (d) When drops combine to form a single drop of radius R.

Then energy released,

$$E = 4\pi TR^3\left[\frac{1}{r}-\frac{1}{R}\right]$$

If this energy is converted into kinetic energy then

$$\frac{1}{2}mv^2 = 4\pi R^3\,T\left[\frac{1}{r}-\frac{1}{R}\right]$$

$$\frac{1}{2}\times\left[\frac{4}{3}\pi R^3\rho\right]v^2 = 4\pi R^3\,T\left[\frac{1}{r}-\frac{1}{R}\right]$$

$$v = \sqrt{\frac{6\,T}{\rho}\left[\frac{1}{r}-\frac{1}{R}\right]}$$

28. (a) $B = 2\left[\frac{\mu_0}{4\pi}\cdot\frac{i}{d\cos 45^\circ}\{\sin(-45^\circ)+\sin 90^\circ\}\right] = \frac{\mu_0 i}{\sqrt{2}\pi d}\left[-\frac{1}{\sqrt{2}}+1\right].\otimes$

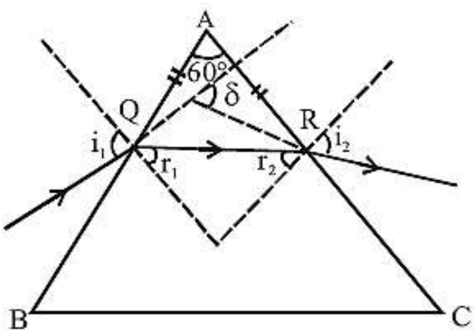
29. (d) BE of $_2He^4 = 4\times 7.06 = 28.24 MeV$

BE of $^7_3Li = 7\times 5.60 = 39.20 MeV$

$^7_3Li + {}^1_1H \rightarrow {}_2He^4 + {}_2He^4 + Q$

Therefore, $Q = 56.48 - 39.20 = 17.28 MeV$.

30. (a)



Given AQ = AR and $\angle A = 60^\circ$
$\therefore\ \angle AQR = \angle ARQ = 60^\circ$
$\therefore\ r_1 = r_2 = 30^\circ$

Applying Snell's law on face AB.
$\sin i_1 = \mu \sin r_1$

$\Rightarrow\ \sin i_1 = \sqrt{3}\sin 30^\circ = \sqrt{3}\times\frac{1}{2} = \frac{\sqrt{3}}{2}$
$\therefore\ i_1 = 60^\circ$

Similarly, $i_2 = 60^\circ$
In a prism, deviation

$\delta = i_1 + i_2 - A = 60^\circ + 60^\circ - 60^\circ = 60^\circ$

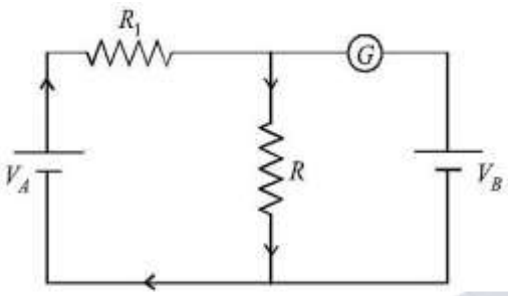
31. (d) $eV_0 = \frac{hc}{\lambda_0} - W_0$ and $eV' = \frac{hc}{2\lambda_0} - W_0$

Subtracting them we have

$$e(V_0 - V') = \frac{hc}{\lambda_0}\left[1 - \frac{1}{2}\right] = \frac{hc}{2\lambda_0}$$

or $V' = V_0 - \frac{hc}{2e\lambda_0}$

32. (b)



Since deflection in galvanometer is zero so current will flow as shown in the above diagram.

current, $I = \frac{V_A}{R_1+R} = \frac{12}{500+100} = \frac{12}{600}$

So $V_B = IR = \frac{12}{600} \times 100 = 2\ V$

33. (b) Let pole strength = m

So, $M = m\ell$

When wire is in form of arc, then the distance between poles $= \frac{2\ell}{\pi}$

So, $M' = \frac{m2\ell}{\pi} = \frac{2M}{\pi}$

34. (c) Let m = mass of boy, M = mass of man v = velocity of boy, V = velocity of man

$$\frac{1}{2}MV^2 = \frac{1}{2}\left[\frac{1}{2}mv^2\right]$$

$$\frac{1}{2}M(V+1)^2 = 1\left[\frac{1}{2}mv^2\right]$$

Putting $m = \frac{M}{2}$ and solving $V = \frac{1}{\sqrt{2}-1}$

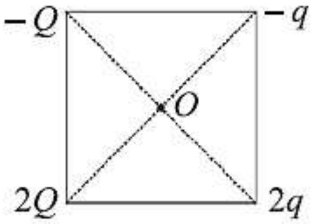
35. (b) $w_a = \lambda/d \Rightarrow w_a \alpha \lambda$

$$\frac{(w_a)_{water}}{w_a} = \frac{\lambda_{water}}{\lambda} = \frac{\lambda}{\mu_{water}\lambda}$$

$$(w_a)_{water} = \frac{2 \times 3}{4} = 0.15^\circ$$

36. (a) Let the side length of square be ' a ' then potential at centre O is

$$V = \frac{k(-Q)}{\left(\frac{a}{\sqrt{2}}\right)} + \frac{k(-q)}{\frac{a}{\sqrt{2}}} + \frac{k(2q)}{\frac{a}{\sqrt{2}}} + \frac{k(2Q)}{\frac{a}{\sqrt{2}}} = 0$$



$$= -Q - q + 2q + 2Q = 0 \Rightarrow Q + q = 0$$
$$\Rightarrow Q = -q$$

37. (b) As $V_L = V_C = 300V$, resonance will take place

$$\therefore V_R = 220V; I = \frac{220}{100} = 2.2 \text{ A}$$

reading of $V_3 = 220V$ and reading of $A = 2.2$ A

38. (b) $\frac{50-49.9}{5} = K\left(\frac{50+49.9}{2} - 30\right)$

$$\frac{40 - 39.9}{t} = K\left[\frac{40 + 39.9}{2} - 30\right]$$

From equations (i) and (ii), we get $t \approx 10$ s.

39. (b) Since diode is in forward bias, so the value of current flowing through AB,

$$i = \frac{\Delta V}{R} = \frac{4 - (-6)}{1 \times 10^3} = \frac{10}{10^3} = 10^{-2} \text{ A}$$

40. (b) Induced emf produced between the centre and a point on the disc is given by

$$e = \frac{1}{2}\omega B R^2$$

Putting the values,

$\omega = 60 \text{rad/s}, B = 0.05 \text{ Wb/m}^2$

and $R = 100 \text{ cm} = 1 \text{ m}$

We get $e = \frac{1}{2} \times 60 \times 0.05 \times (1)^2 = 1.5$ V

PART-II CHEMISTRY

41. (b) I. $E = \frac{Z^2}{n^2} \times 13.6\text{eV}$

or $\frac{I_1}{I_2} = \frac{Z_1^2}{n_1^2} \times \frac{n_2^2}{Z_2^2}$

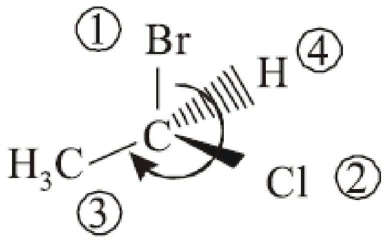
Given $I_1 = -19.6 \times 10^{-18}, Z_1 = 2, n_1 = 1, Z_2 = 3$ and $n_2 = 1$
Substituting these values in equation (ii).

$$-\frac{19.6 \times 10^{-18}}{I_2} = \frac{4}{1} \times \frac{1}{9}$$

$$\text{or } I_2 = -19.6 \times 10^{-18} \times \frac{9}{4}$$

$$= -4.41 \times 10^{-17} \text{ J/ atom}$$

42. (a) Clockwise rotation.



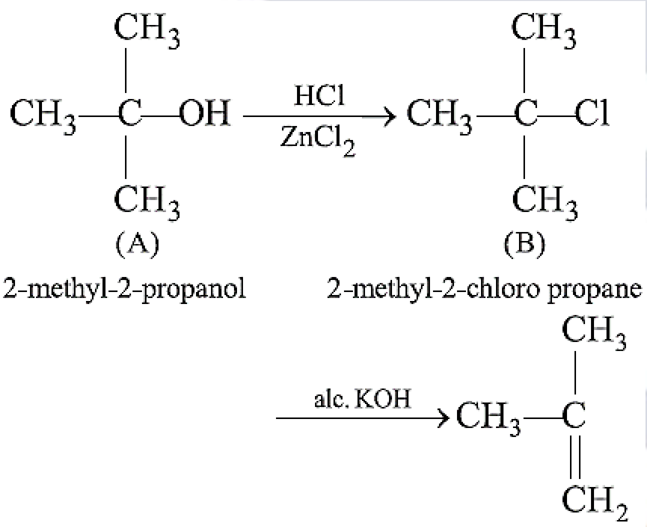
Hence configuration is R.

If the eye travel in a clockwise direction, the configuration is R as the order of priority is $Br > Cl > CH_3 > H$

43. (a) $K_2Cr_2O_7 + 4NaCl + 6H_2SO_4 \xrightarrow{\text{Heat}} 2CrO_2Cl_2 + 6NaHSO_4 + 3H_2O$

44. (a) Since, liquid is passing into gaseous phase so entropy will increase and at 373 K during the phase transformation it remains at equilibrium. So, $\Delta G = 0$.

45. (a) Reaction involved is given as:

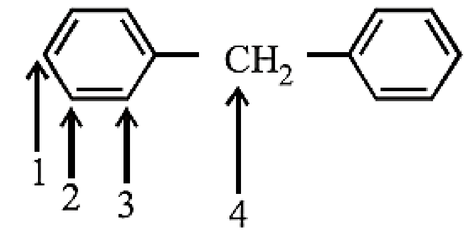


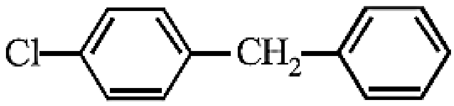
46. (d) I_2 and Na_2CO_3 react with acetophenone $(C_6H_5COCH_3)$ to give yellow ppt. of CHI3 but benzophenone $(C_6H_5COC_6H_5)$ does not and hence can be used to distinguish between them.

47. (d) Decomposition of carbonates and hydrated oxides.

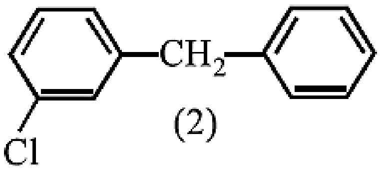
48. (d) When volume is increased the conc. decreases & the equilibrium shifts in the direction where more moles are formed.

49. (b) In diphenylmethane monochlorination at following positions will produce structured isomers

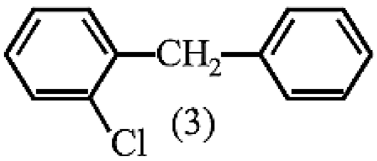




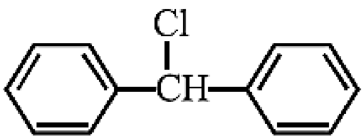
(1)



(2)



(3)



(4)

50. (b) $\Delta H_{S-S} + 2\Delta H_{H-S} = 239$

$2\Delta H_{H-S} = 175$

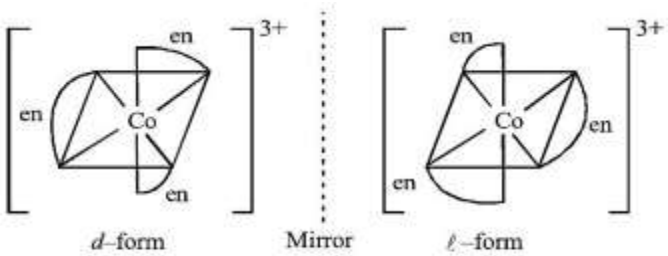
Hence,

$\Delta H_{S-S} = 239 - 175 = 64 \text{kcalmol}^{-1}$

Then, ΔH for $8\ S_{(g)} \rightarrow S_{8(g)}$ is $8 \times (-64) = -512 \text{kcal}$

51. (b) In Wolf-Kishner reduction NH_2NH_2/OH is used. Br^-can be replaced by OH^-.

52. (b) Option (b) shows optical isomerism $[Co(en)_3]^{3+}$



Complexes of Zn^{++}cannot show optical isomerism as they are tetrahedral complexes with plane of symmetry.

$[Co(H_2O)_4(en)]^{3+}$ have two planes of symmetry hence it is also optically inactive. $[Zn(en)_2]^{2+}$ cannot show optical isomerism

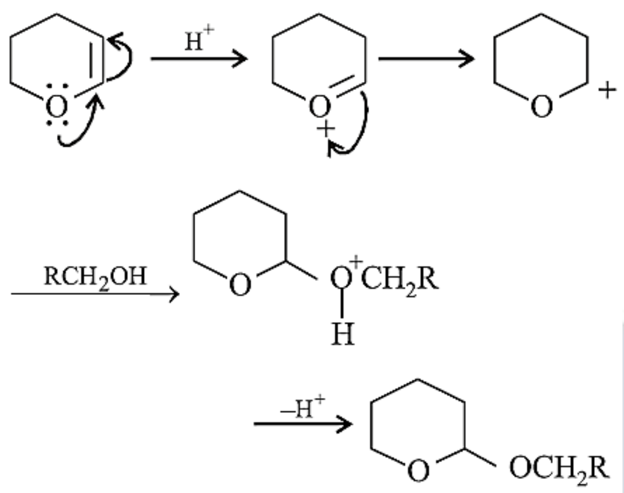
53. (a) There are two atoms in a bcc unit cell.

So, number of atoms in

12.08×10^{23} unit cells

$= 2 \times 12.08 \times 10^{23} = 24.16 \times 10^{23}$ atoms.

54. (b)



an acetal

55. (a) Apply Nernst equation to the reaction

$Pb + Sn^{2+} \rightarrow Pb^{2+} + Sn$

or $E° + \frac{0.059}{2} \log \frac{[Sn^{2+}]}{[Pb^{2+}]} = E_{cell}$

or $\log \frac{[Sn^{2+}]}{[Pb^{2+}]} = \frac{0.01 \times 2}{0.059} = 0.3 (\because E_{cell} = 0)$

or $\frac{[Sn^{2+}]}{[Pb^{2+}]} = \text{antilog}(0.3)$

56. (b) Methyl vinyl ether under anhydrous condition at room temperature undergoes addition reaction.

$$CH_2 = CH - OCH_3 \xrightarrow{HBr} CH_3-\underset{\substack{|\\Br}}{CH}-O-CH_3$$

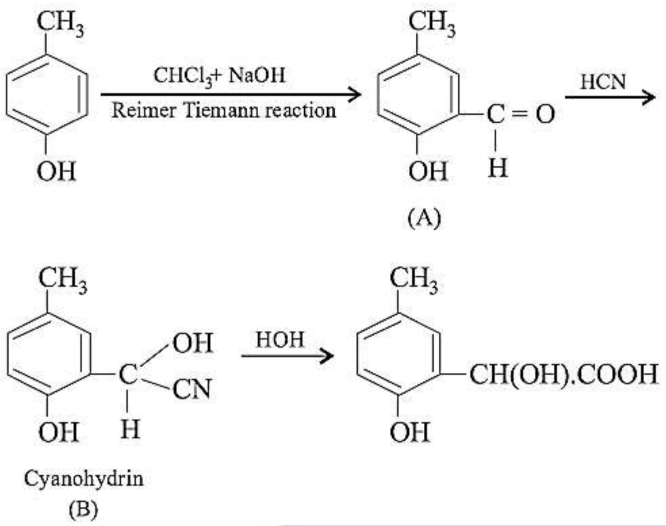
57. (d)

$$\underset{\text{acetic anhydride}}{(CH_3CO)_2O} + \underset{\text{diethyl ether}}{C_2H_5OC_2H_5} \xrightarrow{AlCl_3} 2CH_3COOC_2H_5$$

58. (a) $\Lambda_{eq} = \kappa \times \frac{1000}{N} = \frac{1}{R} \times \frac{l}{a} \times \frac{1000}{N}$

$$= \frac{1}{R} \times \text{ cell constant } \times \frac{1000}{N}$$
$$= \frac{1}{220} \times 0.88 \times \frac{1000}{0.01}$$
$$= 400 \text{mhocm}^2 \text{ geq}^{-1}$$

59. (c)



60. (c) Ionic radii $\propto \frac{1}{z}$

$$\text{Thus, } \frac{z_2}{z_1} \Rightarrow \frac{1.06}{(\text{ Ionic radii of } Lu^{3+})} = \frac{71}{57}$$
$$\Rightarrow \text{ Ionic radii of } Lu^{3+} = 0.85 \text{Å}$$

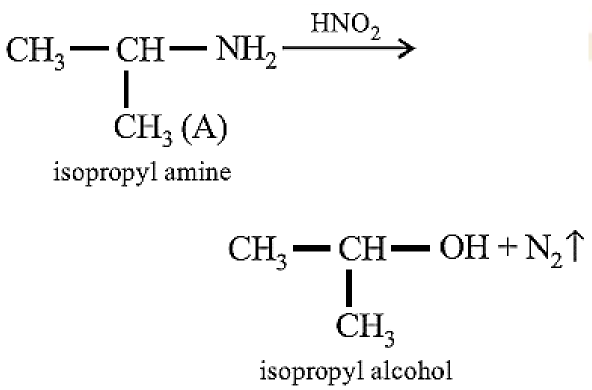
61. (a) From the given data, we have

Number of Y atoms in a unit cell = 4

Number of X atoms in a unit cell $= 8 \times \frac{2}{3} = \frac{16}{3}$

From the above we get the formula of the compound as $X_{16/3}Y_4$ or X_4Y_3

62. (a)

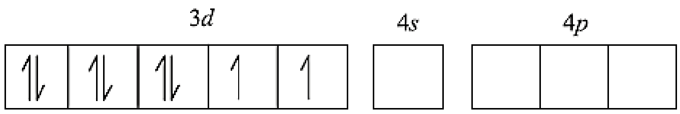


63. (b) $2\ N_2O_5 \rightarrow 4NO_2 + O_2$

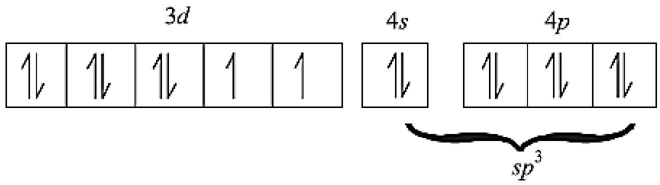
from the unit of rate constant it is clear that the reaction follow first order kinetics. Hence by rate law equation, $r = k[N_2O_5]$ where $r = 1.02 \times 10^{-4}, k = 3.4 \times 10^{-5}$ $1.02 \times 10^{-4} = 3.4 \times 10^{-5}[N_2O_5]$ $[N_2O_5] = 3M$

64. (b)

$[NiCl_4]^{2-}$, O.S. of Ni $= +2$
$Ni(28) = 3d^8 4s^2$



Cl^- being weak ligand it cannot pair up the two electrons present in $3d$ orbital



No. of unpaired electrons = 2
Magnetic moment, $\mu = 2.82$ BM.

65. (d) If rate $= k[A]^x[B]^y[C]^z$

From first two given data

$8.08 \times 10^{-3} = k[0.2]^x[0.1]^y[0.02]^z$ ....

$2.01 \times 10^{-3} = k[0.1]^x[0.2]^y[0.02]^z$.

Divide (1) by (2) we get, $4 = 2^x(1/2)^y$

Similarly, from second and third data

$(9)^y(9)^z = 3$

$2y + 2z = 1$.

From first and fourth data $4^z = 8 = 2^3$

$2z = 3$. So $z = 3/2, y = -1, x = 1$

66. (d) In the silver plating of copper, $K[Ag(CN)_2]$ is used instead of $AgNO_3$. Copper being more electropositive readily precipitate silver from their salt solution

$Cu + 2AgNO_3 \longrightarrow Cu(NO_3)_2 + Ag$

whereas in $K[Ag(CN)_2]$ solution a complex anion $[Ag(CN)_2]^-$ is formed and hence Ag^+ are less available in the solution and therefore copper cannot displace Ag from its complex ion.

67. (d) The given reaction is known as Liebermann Nitroso reaction.
68. (b) Given $E_1 = 25eV E_2 = 50eV$

$$E_1 = \frac{hc}{\lambda_1}\ E_2 = \frac{hc}{\lambda_2} \therefore \frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1}$$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{25}{50} = \frac{1}{2} \quad \therefore \lambda_1 = 2\lambda_2$

69. (d) The reaction involves the conversion of NO_2 to $-NH_2$ group (reduction) which occurs in presence of Fe/HCl.

70. (b) Phosphate is linked to 3rd &5th carbon of corresponding sugar

71. (c) The $H-X$ bond strength decreases from HF to HI. i.e. $HF > HCl > HBr > HI$. Thus HF is most stable while HI is least stable. The decreasing stability of the hydrogen halide is also reflected in the values of dissociation energy of the $H-X$ bond

$H-F$	$H-Cl$	$H-Br$	$H-I$
$135 kcal mol^{-1}$	$103 kcal mol^{-1}$	$87 kcal mol^{-1}$	$71 kcal mol^{-1}$

72. (b) $\Delta G = \Delta H - T\Delta S$

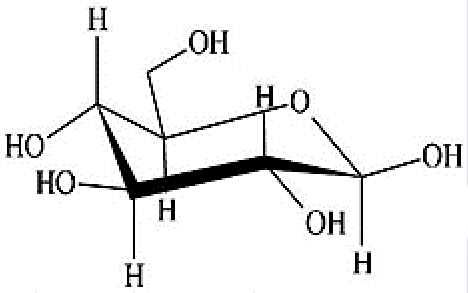
At equilibrium, $\Delta G = 0$

$\Rightarrow 0 = (170 \times 10^3 \text{ J}) - T(170 JK^{-1})$

$\Rightarrow T = 1000 \text{ K}$

For spontaneity, ΔG is $-$ve, which is possible only if $T > 1000$ K

73. (b)

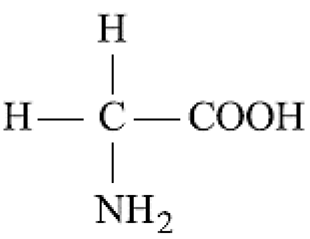


It is a β-pyranose hence it is an aldohexose.

74. (c) $2SO_2 + O_2 \rightleftharpoons 2SO_3 \quad K = 278$ (given) $SO_3 \rightleftharpoons SO_2 + \frac{1}{2}O_2 \quad K' = \left(\sqrt{\frac{1}{K}}\right)$

$= \sqrt{\frac{1}{278}} = \sqrt{35.97 \times 10^{-4}} = 6 \times 10^{-2}$

75. (c) With the exception of glycine all the 19 other common amino acids have a uniquely different functional group on the central tetrahedral alpha carbon



glycine

76. (a) $_7N = 1s^2 2s^2 2p^3$; $_{15}P = 1s^2 2s^2 2p^6 3s^2 3p^3$ In phosphorous the 3d-orbitals are available. Hence phosphorus can form pentahalides also but nitrogen cannot form pentahalide due to absence of d-orbitals.

77. (c) For the reaction

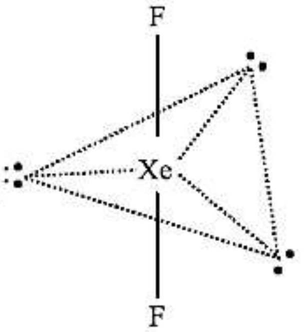
$2BaO_2(s) \rightleftharpoons 2BaO(s) + O_2(g);$
$\Delta H = +ve$

In equilibrium $K_p = P_{O_2}$
Hence, the value of equilibrium constant depends only upon partial pressure of O_2. Further on increasing temperature formation of O_2 increases as this is an endothermic reaction. Hence, pressure of O_2 is dependent on temperature.

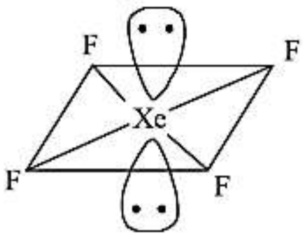
78. (c)

$$C_6H_5NH_2 \xrightarrow[0^\circ C-5^\circ C]{NaNO_2/HCl} \underset{(X)}{C_6H_5N_2^+Cl^-}$$
$$\xrightarrow[H_2O]{HNO_2} \underset{(Y)}{C_6H_5NO_2} + N_2 + HCl$$

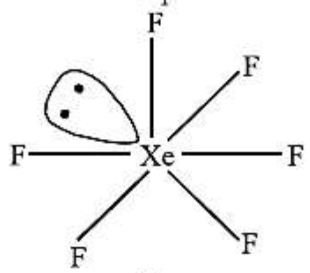
79. (c)

XeF_2 : 

3*l*p

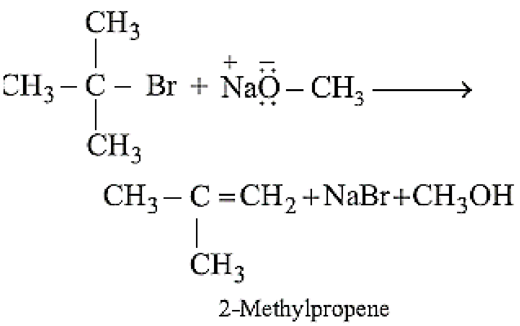
XeF_4 : 

2*l*p

XeF_6 : 

1*lp*

80. (c) If a tertiary alkyl halide is used, an alkene is the only reaction product and no ether is formed. For example, the reaction of CH_3ONa with $(CH_3)_3C-Br$ gives exclusively 2-methylpropene.



It is because alkoxides are not only nucleophiles but strong bases as well. They react with alkyl halides leading to elimination reactions.

PART-III MATHEMATICS

81. (c) $12\cot^2\theta - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow 12(\operatorname{cosec}^2\theta - 1) - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow 12\operatorname{cosec}^2\theta - 31\operatorname{cosec}\theta + 20 = 0$

$\Rightarrow 12\operatorname{cosec}^2\theta - 16\operatorname{cosec}\theta - 15\operatorname{cosec}\theta + 20 = 0$

$\Rightarrow (4\operatorname{cosec}\theta - 5)(3\operatorname{cosec}\theta - 4) = 0$

$\Rightarrow \operatorname{cosec}\theta = \frac{5}{4}, \frac{4}{3} \;\therefore \sin\theta = \frac{4}{5}, \frac{3}{4}.$

82. (c) Letr $(\cos\theta + i\sin\theta)$

$$= \frac{1+i\sqrt{3}}{\sqrt{3}+1} = \frac{1}{\sqrt{3}+1} + i\frac{\sqrt{3}}{\sqrt{3}+1}$$

$$\Rightarrow r\cos\theta = \frac{1}{\sqrt{3}+1}; r\sin\theta = \frac{\sqrt{3}}{\sqrt{3}+1}$$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}.$

83. (c) $\lim_{x\to0} \frac{x^3\cot x}{1-\cos x}$

$$= \lim_{x\to0}\left(\frac{x^3\cot x}{1-\cos x} \times \frac{1+\cos x}{1+\cos x}\right)$$

$$= \lim_{x\to0}\left(\frac{x}{\sin x}\right)^3 \times \lim_{x\to0}\cos x \times \lim_{x\to0}(1+\cos x) = 2$$

84. (b) Connective word is 'or'.

85. (c) Each of the three prizes can be given to any of the four children.

$\therefore$ Total number of ways of distributing prizes

$= 4 \times 4 \times 4 = 64$

Number of ways in which one child gets all prizes $= 4$

$\therefore$ Number of ways in which no child gets

all the three prizes $= 64 - 4 = 60$

86. (a) (I) $P(\text{not } A) = 1 - 0.42 = 0.58$

(II) $P(\text{ not } B) = 1 - P(B) = 1 - 0.48 = 0.52$

(III) $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) -$

$P(A \cap B) = 0.42 + 0.48 - 0.16 = 0.74$

87. (b) The given equation of curve is:

$y^2 + 4x - 6y + 13 = 0$

which can be written as: $y^2 - 6y + 9 + 4x + 4 = 0$

$\Rightarrow (y^2 - 6y + 9) = -4(x + 1)$

$\Rightarrow (y - 3)^2 = -4(x + 1)$

Put $Y = y - 3$ and $X = x + 1$
On comparing $Y^2 = 4aX$

Length of focus from vertex, $a = -1$

At focus $X = a$ and $Y = 0 \Rightarrow x + 1 = -1$

$\Rightarrow x = -2$

$\therefore y - 3 = 0 \Rightarrow y = 3$

$\therefore$ Focus is $(-2,3)$.

88. **(c)** Since the parabola $y^2 = 4ax$ passes through the point $(1, -2)$,
$\therefore (-2)^2 = 4a(1) \Rightarrow a = 1$

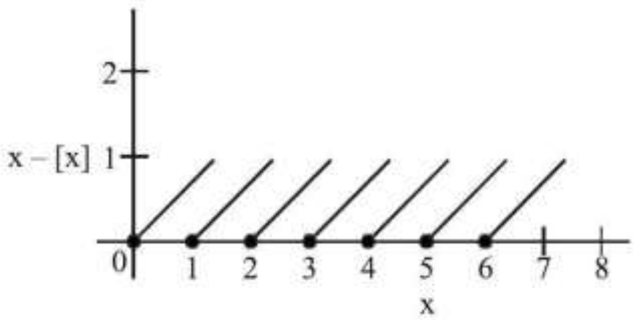
Equation of tangent to the parabola at

$(1, -2)$ is

$yy_1 = 2a(x + x_1)$ or

$y(-2) = 2(1)(x + 1)$ or $x + y + 1 = 0$

89. **(c)** The graph of the function $f(x) = x - [x]$ for the interval $(0,7)$ is shown below:



It is obvious from the above graph that the function $x - [x]$ is discontinuous at the points $x = 1,2,3,4,5,6$. Therefore no. of points of discontinuity of the given function in the given interval are 6 .

90. **(c)** $v = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = \frac{4}{3}\pi\frac{d}{dt}r^3 = \frac{4}{3}\pi 3r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

when $r = 10$ cm;

$\frac{dv}{dt} = 4\pi(10)^2 \cdot (0.02) = 8\pi cm^3/s.$

91. (a) Given $f(x) = \frac{4x^2+1}{x}$ Thus $f'(x) = 4 - \frac{1}{x^2}$

f(x) will be decreasing if $f'(x) < 0$

Thus $4 - \frac{1}{x^2} < 0$

$$\Rightarrow \frac{1}{x^2} > 4 \Rightarrow \frac{-1}{2} < x < \frac{1}{2}$$

Thus interval in which f(x) is decreasing,

is $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

92. (c) Let a be the major axis and b, the minor axis of the ellipse, then 3 minor axis = major axis. $\Rightarrow 3\,b = a$

Eccentricity is given by :

$b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 9\,b^2(1 - e^2)$

$$\Rightarrow \frac{1}{9} = (1 - e^2) \Rightarrow e^2 = \frac{8}{9} \Rightarrow e = \frac{2\sqrt{2}}{3}$$

93. (c) We have $a = 3$ and $\frac{b^2}{a} = 4 \Rightarrow b^2 = 12$

Hence, the equation of the hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{12} = 1$$

$\Rightarrow 4x^2 - 3y^2 = 36 \Rightarrow 4x^2 - 3y^2 - 36 = 0$

94. (c) For function to be continuous:

$f(0 + h) = f(0 - h) = f(0)$

$f(0 + h) = \lim_{h\to 0} h\sin 1/h = 0 \times$ (a finite

quantity) $= 0$

$f(0 - h) = \lim_{h\to 0} - h\sin(1/-h) = 0 \times$ (a finite quantity) $= 0$

Also, $\lim_{x\to 0} x\sin 1/x = 0 \times$ (a finite quantity)

$= 0$

$\Rightarrow$ function is continuous at $x = 0$

For function to be differentiable :

$f'(0 + h) = f'(0 - h)$

$$f'(0 + h) = \frac{f(0 + h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h\sin\frac{1}{h} - 0}{h} = \lim_{h\to 0} \sin\left(\frac{1}{h}\right)$$

which does not exist.

95. (d) We have,

$$\frac{3x+1}{(x-3)(x-5)} = \frac{-5}{x-3} + \frac{B}{x-5}$$

$3x + 1 = -5(x-5) + B(x-3)$

Put $x = 5$

$3(5) + 1 = B(5-3)$

$16 = 2$ B or $B = 8$

96. (d) $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ have vector form

$= (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

Required equation in vector form is

$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \mu(3\hat{i} + 7j + 2\hat{k})$

97. (a) $\because$ The line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane

$x + 3y - \alpha z + \beta = 0$

$\therefore$ Pt$(2,1,-2)$ lies on the plane i.e. $2 + 3 + 2\alpha + \beta = 0$

or $2\alpha + \beta + 5 = 0$ ....(i)

Also normal to plane will be perpendicular to line,

$\therefore 3 \times 1 - 5 \times 3 + 2 \times (-\alpha) = 0 \Rightarrow \alpha = -6$

From equation (i) then, $\beta = 7$

$\therefore (\alpha, \beta) = (-6, 7)$

98. (c) Let $\theta = \sin^{-1}\left[\sin\frac{5\pi}{3}\right]$

$$\Rightarrow \sin\theta = \sin\frac{5\pi}{3} = \sin\left[2\pi - \frac{\pi}{3}\right]$$

$$\Rightarrow \sin\theta = -\sin\frac{\pi}{3} = \sin\left(\frac{-\pi}{3}\right)$$

$(\because \sin(-\theta) = -\sin\theta)$

Therefore, principal value of $\sin^{-1}\left[\sin\frac{5\pi}{3}\right]$ is $\frac{-\pi}{3}$, as principal value of $\sin^{-1}x$ lies between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$.

99. **(b)** We have $[1 \quad x \quad 1]\begin{bmatrix}1 & 3 & 2\\0 & 5 & 1\\0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\1\\-2\end{bmatrix} = 0$

$$\Rightarrow [1 \quad 5x+6 \quad x+4]\begin{bmatrix}x\\1\\-2\end{bmatrix} = 0$$

$\Rightarrow x + 5x + 6 - 2x - 8 = 0 \Rightarrow 4x - 2 = 0 \Rightarrow x = \frac{1}{2}$

100. **(d)** Here $AA^T = \begin{pmatrix}2 & -1\\-7 & 4\end{pmatrix}\begin{pmatrix}2 & -7\\-1 & 4\end{pmatrix} \neq \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$(BB^T)_{11} = (4)^2 + (1)^2 \neq 1$

$(AB)_{11} = 8 - 7 = 1, (BA)_{11} = 8 - 7 = 1$

$\therefore AB \neq BA$ may be not true.

Now, $AB = \begin{pmatrix}2 & -1\\-7 & 4\end{pmatrix}\begin{pmatrix}4 & 1\\7 & 2\end{pmatrix}$

$= \begin{pmatrix}8-7 & 2-2\\-28+28 & -7+8\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$\therefore (AB)^T = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

101. **(d)** Since Rolle's theorem is satisfied

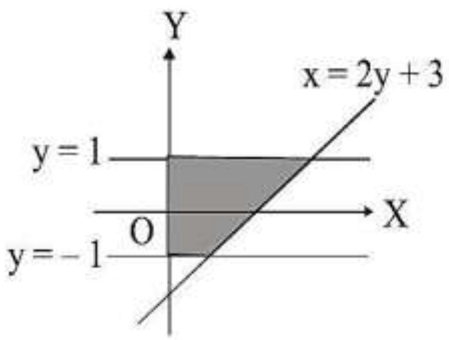
$\therefore f'(c) = 0 \Rightarrow e^c \sin c + \cos^c = 0$

$\Rightarrow e^c\{\sin c + \cos c\} = 0$

$\therefore \sin c + \cos c = 0 \ (\because e^c \neq 0)$

$\Rightarrow \tan c = -1 \Rightarrow c = \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

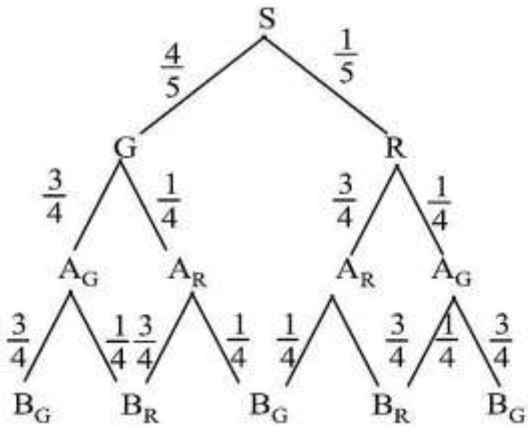
102. **(c)** We have $x = 2y + 3$, a straight line



Required area = Area of shaded region

$= \int_{-1}^{1}(2y+3)dy = [y^2 + 3y]_{-1}^{1} = 6$ sq. units

103. **(c)** From the tree diagram, it follows that



$P(B_G) = \frac{46}{80}, P(G) = \frac{4}{5}$

$P(B_G/G) = \frac{10}{16} = \frac{5}{8}$

$\therefore P(B_G \cap G) = \frac{5}{8} \times \frac{4}{5} = \frac{1}{2}$

$\begin{bmatrix} \because & P(B_G \cap G) \\ = & P\left(\frac{B_G}{G}\right) \times P(G) \end{bmatrix}$

Now, $P(G/B_G)$

$= \frac{P(B_G \cap G)}{P(B_G)} = \frac{1}{2} \times \frac{80}{40} = \frac{20}{23}$

104. **(c)** $\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 0 & 12 & 9 \\ 1 & 2 & 2 \end{vmatrix}$

$= 1(12 \times 2 - 2 \times 9) - 4(0 \times 2 - 1 \times 9) + 3(0 \times 2 - 1 \times 12)$

$= 1(24 - 18) - 4(0 - 9) + 3(0 - 12)$

$= 6 + 36 - 36 = 6$

105. **(b)** $\begin{vmatrix} 1 & a & -1 \\ 2 & -1 & a \\ a & 1 & 2 \end{vmatrix} = 0$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - aR_1$, we get

$\begin{vmatrix} 1 & a & -1 \\ 0 & -1-2a & a+2 \\ 0 & 1-a^2 & 2+a \end{vmatrix} = 0$

Expanding along C_1, we get

$(a+2)(a^2 - 2a - 2) = 0 \Rightarrow a = -2, 1 \pm \sqrt{3}$

106. **(d)** The I.F. of the differential equation $\frac{dy}{dx} + Py = Q$ is $e^{\int Pdx}$. Here $P = 5$ therefore I.F. $= e^{\int 5dx}$. Hence $A = 5$.

107. **(b)** xy-plane is perpendicular to z - axis. Let the vector

$\vec{a} = 3i + 4j + 5k$ make angle θ with z - axis, then it makes $90 - \theta$ with xy-plane. unit vector along z-axis is k.

So, $\cos\theta = \frac{\vec{a}\cdot\hat{k}}{|\vec{a}|\cdot|\hat{k}|} = \frac{(3i+4j+5k)\cdot k}{|3i+4j+5k|}$

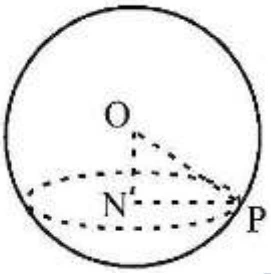
$= \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

Hence angle with xy- plane $\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

projection of $\vec{a}$ on xy plane $= |\vec{a}| \cdot \cos\frac{\pi}{4}$

$= 5\sqrt{2} \times \frac{1}{\sqrt{2}} = 5.$

108. **(b)** The sphere $x^2 + y^2 + z^2 = 49$ has centre at the origin $(0,0,0)$ and radius 7 .



Disance of the plane

$2x + 3y - z - 5\sqrt{14} = 0$

from the origin

$$= \frac{|2(0) + 3(0) - (0) - 5\sqrt{14}|}{\sqrt{2^2 + 3^2 + (-1)^2}}$$

$$= \frac{|-5\sqrt{14}|}{\sqrt{14}} = \frac{5\sqrt{14}}{\sqrt{14}} = 5$$

Thus in Figure; $OP = 7, ON = 5$

$NP^2 = OP^2 - ON^2 = (7)^2 - (5)^2$

$= 49 - 25 = 24$

$\therefore\ NP = 2\sqrt{6}$ Hence the radius of the circle

$= NP = 2\sqrt{6}$

109. **(a)** $\left(\frac{1+i}{\sqrt{2}}\right)^{2/3} = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^{2/3}$

$= (\cos 45° + i\sin 45°)^{2/3}$

$= \left(\cos\frac{2}{3} \times 45° + i\sin\frac{2}{3}45°\right)$

$= \cos 30^\circ + i \sin 30^\circ$

$= \frac{\sqrt{3}}{2} + i \times \frac{1}{2} = \frac{1}{2}(\sqrt{3} + i)$

110. **(c)** $\because$ Equation of ellipse is $9x^2 + 16y^2 = 144$ or $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Comparing this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then we get $a^2 = 16$ and $b^2 = 9$ and comparing the line $y = x + \lambda$ with $y = mx + c$

$\therefore m = 1$ and $c = \lambda$

If the line $y = x + \lambda$ touches the ellipse $9x^2$

$+16y^2 = 144$, then $c^2 = a^2 m^2 + b^2$

$\Rightarrow \lambda^2 = 16 \times 1^2 + 9 \Rightarrow \lambda^2 = 25$

$\therefore \lambda = \pm 5$

111. **(a)** Let the equation of asymptotes be
$2x^2 + 5xy + 2y^2 + 4x + 5y + \lambda = 0$

This equation represents a pair of straight lines,

$\therefore abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\therefore 4\lambda + 25 - \frac{25}{2} - 8 - \lambda \times \frac{25}{4} = 0$

$\Rightarrow -\frac{9\lambda}{4} + \frac{9}{2} = 0 \Rightarrow \lambda = 2$

Putting the value of λ in eq. (1), we get $2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$

this is the equation of the asymptotes.

112. **(c)** $x^3 - 3xy^2 + 2 = 0$
differentiating w.r.t. x :

$3x^2 - 3x(2y)\frac{dy}{dx} - 3y^2 = 0$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 3y^2}{6xy}$ and $3x^2y - y^3 - 2 = 0$

differentiating w.r.t. x

$\Rightarrow 3x^2\frac{dy}{dx} + 6xy - 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{6xy}{3x^2 - 3y^2}\right)$

Now, product of slope

$$=\frac{3x^2-3y^2}{6xy}\times-\left(\frac{6xy}{3x^2-3y^2}\right)=-1$$

$\therefore$ they are perpendicular.

Hence, angle

$=\pi/2$

113. **(d)** Let $f(x)=x^4-62x^2+ax+9$

$\Rightarrow f'(x)=4x^3-124x+a$

It is given that function f attains its maximum value on the interval $[0,2]$ at $x=1$.

$\therefore f'(1)=0\Rightarrow 4\times1^3-124\times1+1=0$

$\Rightarrow 4-124+a=0\Rightarrow a=120$

Hence, the value of a is 120 .

114. **(b)** $I=\int_{-1}^{1}(x-[x])dx=\int_{-1}^{1}xdx-\int_{-1}^{1}[x]dx$

$$=\left[\frac{x^2}{2}\right]_{-1}^{1}-\left[\int_{-1}^{0}[x]dx+\int_{0}^{1}[x]dx\right]$$

$$=\frac{1}{2}[1-1]-\left[\int_{-1}^{0}(-1)dx+\int_{0}^{1}0.dx\right]$$

$$\left[\begin{array}{c}\text{If }-1\le x<0,[x]=-1\\ \text{If }0\le x<1,[x]=0\end{array}\right]$$

$=0-[-\mathbf{x}]_{-1}^{0}-0=0-[-0-(-1)]=1$

115. **(d)** $\int_{0}^{\pi}|\sin^4 x|dx=\int_{0}^{\pi/2}\sin^4 xdx$

Applying gamma function,

$$2\int_{0}^{\pi/2}\sin^4 xdx=2\frac{\Gamma(5/2)\cdot\Gamma(1/2)}{2\cdot\Gamma(6/2)}=\frac{3\pi}{8}$$

Aliter : Using Walli's formula

$$=2\left[\frac{3.1}{4.2}\right]\frac{\pi}{2}=\frac{3\pi}{8}$$

116. **(c)** $y=cx+c^2-3c^{3/2}+2$

Differentiating above with respect to x, we get $\frac{dy}{dx}=c$.

Putting this value of c in (i), we get

$$y=x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2-3\left(\frac{dy}{dx}\right)^{3/2}+2$$

Clearly its order is ONE and after removing the fractional power we get the degree FOUR.

117. **(a)** $\frac{dv}{dt}+\frac{k}{m}v=-g \Rightarrow \frac{dv}{dt}=-\frac{k}{m}\left(v+\frac{mg}{k}\right)$

$$\Rightarrow \frac{dv}{v+mg/k}=-\frac{k}{m}dt \Rightarrow \log\left(v+\frac{mg}{k}\right)$$

$$=-\frac{k}{m}t+\log C$$

$$\Rightarrow v+\frac{mg}{k}=Ce^{-kt/m} \Rightarrow v=Ce^{-kt/m}-\frac{mg}{k}$$

118. **(b)** Let $A(1,0,1), B(0,2,2)$ and $C(3,3,0)$ be the given points, then

$\overrightarrow{AB}=-\hat{\imath}+2\hat{\jmath}+\hat{k}, \overrightarrow{BC}=3\hat{\imath}+\hat{\jmath}-2\hat{k}$

$$\overrightarrow{AB}\times\overrightarrow{BC}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -1 & 2 & 1\\ 3 & 1 & -2\end{vmatrix}=-5\hat{\imath}+\hat{\jmath}-7\hat{k}$$

$\therefore$ unit vector $\perp$ to the plane: ABC

$$=\pm\frac{1}{5\sqrt{3}}(-5\hat{\imath}+\hat{\jmath}-7\hat{k})$$

119. **(c)** $\because (\vec{a}\times\vec{b})\times\vec{a}=(\vec{a}\cdot\vec{a})\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}$

$\therefore\ (\hat{\jmath}-\hat{k})\times(\hat{\imath}+\hat{\jmath}+k)=(\sqrt{3})^2(\vec{b})-(\hat{\imath}+\hat{\jmath}+k)$

$\Rightarrow 3\hat{b}=3\hat{\imath} \Rightarrow \hat{b}=\hat{\imath}$

120. **(a)**

Given, $P(\overline{A\cup B})=\frac{1}{6}$

$\Rightarrow P(A\cup B)=1-\frac{1}{6}=\frac{5}{6}$

$P(\bar{A})=\frac{1}{4} \Rightarrow P(A)=1-\frac{1}{4}=\frac{3}{4}$

We know,

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4} \Rightarrow P(B)=\frac{1}{3}$

$\because P(A)\neq P(B)$ so they are not equally likely.

Also $P(A)\times P(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}=P(A\cap B)$
So A&B are independent.

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121. **(b)**
122. **(c)**
123. **(b)**
124. **(d)** 'Mushrooming' should be used which should serve as an adjective.
125. **(a)**