PART-I PHYSICS

1. (c) $R_1 = \frac{\rho \ell_1}{A_1}$, now $\ell_2 = 2\ell_1$ and $r_2 = 2r_1$

$A_1 = \pi r_1^2$

$A_2 = \pi(r_2)^2 = \pi(2r_1)^2 = 4\pi r_1^2 = 4\ A_1$

$$R_2 = \frac{\rho \ell_2}{A_2}$$

$$\therefore\ R_2 = \frac{\rho(2\ell_1)}{4A_1} = \frac{\rho \ell_1}{2A_1} = \frac{R_1}{2}$$

∴ Resistance is halved, but specific resistance remains the same.

2. (d) $P_2 = P - P_1 = \frac{100}{80} - \frac{100}{20} = -3.75D$

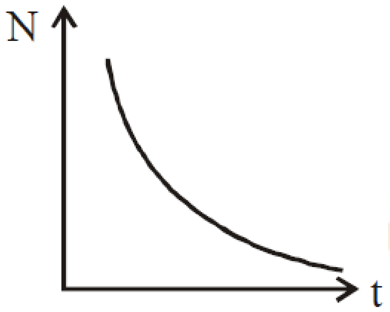
3. (d) de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2.m \cdot (K \cdot E)}}$$

$$\therefore \lambda \propto \frac{1}{\sqrt{K.E}}$$

If K.E is doubled, then de-Broglie wavelength becomes $\frac{\lambda}{\sqrt{2}}$

4. (c) No. of nuclide at time t is given by $N = N_o e^{-\lambda t}$
Where, N_o = initial nuclide
Thus this equation is equivalent to $y = ae^{-kx}$
So correct graph is in option (c)



5. (c) Ist excited state corresponds to $n = 2$ 2nd excited state corresponds to $n = 3$
As $E \propto \frac{1}{n^2}$

$$\therefore \frac{E_1}{E_2} = \frac{n_2^2}{n_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$$

6. (d) (A) is a NAND gate so output is $\overline{1 \times 1} = \overline{1} = 0$

(B) is a NOR gate so output is

$\overline{0 + 1} = \overline{1} = 0$

(C) is a NAND gate so output is $\overline{0 \times 1} = \bar{0} = 1$

(D) is a XOR gate so output is $0 \oplus 0 = 0$

7. **(a)** Work done in displacement of charge, $W = q\Delta V$

where ΔV = potential difference between the displacement of charge

Hence, work done in complete rotation is zero.

8. **(a)** Equivalent capacitance of two parallel capacitors $10\mu F$ and $6\mu F = (10+6)\mu F = 16\mu F$ This $16\mu F$ capacitor is in series combination with $4\mu F$ capacitor,

$\therefore$ Equivalent capacitance of the entire combination $= \frac{16\times4}{16+4} = \frac{64}{20} = 3.2\mu F$

9. **(c)** Half-life, $T_h = \frac{0.693}{\lambda}, T_m = \frac{1}{\lambda}$

Clearly, $T_h < T_m$.

10. **(a)** $I_c = 5.488 \text{ mA}, I_e = 5.6 \text{ mA}$

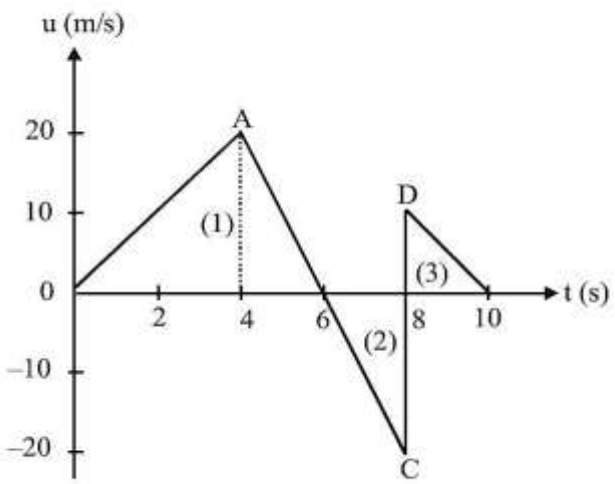
$$\alpha = \frac{I_c}{I_e}$$
$$\alpha = \frac{5.488}{5.6}$$
$$\beta = \frac{\alpha}{(1-\alpha)} = 49$$

11. **(a)** Magnetic field due to long wire,

$B = \frac{\mu_0 i}{2\pi r}$ or $B \propto \frac{1}{r}$

When r is doubled, the magnetic field becomes half, i.e., now the magnetic field will be 0.2 T.

12. **(a)**



Total distance covered in 10 s = Area 1 + Area 2+Area 3

$$\Rightarrow \frac{1}{2}\times 6\times 20 + \frac{1}{2}\times 2\times 20 + \frac{1}{2}\times 2\times 10 = 90 \text{ m}$$

Total displacement in 10s = Area 1 – Area 2 + Area 3

$\Rightarrow \frac{1}{2} \times 6 \times 20 - \frac{1}{2} \times 2 \times 20 + \frac{1}{2} \times 2 \times 10 = 50 \text{ m}$

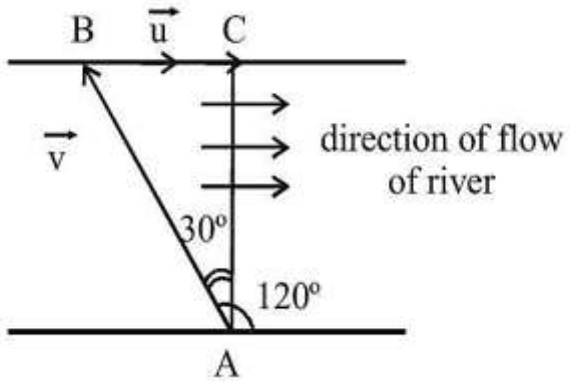
13. (c) In a uniform electric field, net force $= 0$, but torque $\neq 0$.

14. (c) Let the speed of water $= \vec{u}$

Speed of swimmer $= \vec{v} = 0.5 \text{ m/sec}$

Angle between $\vec{v}$ and $\vec{u}$ is $120°$. Then

$\sin\theta = \frac{\vec{u}}{\vec{v}} \Rightarrow \frac{u}{0.5} = \frac{1}{2}$ or $u = 0.25 \text{ ms}^{-1}$



15. (a) Radius of drop, $r = 0.3 \text{ mm} = 0.03 \text{ cm}$

Terminal velocity, $v = 1 \text{ m/s} = 100 \text{ cm/sec}$

Viscosity of air, $\eta = 8 \times 10^{-5}$ poise

Viscous force, $f = 6\pi\eta rv$

$F = 6 \times 3.14 \times (8 \times 10^{-5}) \times 0.03 \times 100 = 4.52 \times 10^{-3}$ dyne

16. (d) Rate of transmission of heat

$= \frac{\text{Temperature difference}}{\text{Thermal Resistance}}$

$\therefore \frac{dQ}{dt} = \frac{d\theta}{R}$

Here, $\frac{dQ}{dt} = \frac{(\theta - \theta_2)}{R_2} = \frac{\theta_1 - \theta}{R_1}$

$\Rightarrow \frac{\theta - \theta_2}{R_2} = \frac{\theta_1 - \theta}{R_1}$

$\Rightarrow R_1\theta - R_1\theta_2 = R_2\theta_1 - R_2\theta$

$\Rightarrow \theta(R_1 + R_2) = R_2\theta_1 + R_1\theta_2$

$\therefore \theta = \frac{(R_2\theta_1 + R_1\theta_2)}{(R_1 + R_2)}$

17. (b) Apply law of conservation of energy,

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow mv^2 = kx^2$ or $0.5 \times (1.5)^2 = 50 \times x^2$

$\therefore x = 0.15$ m

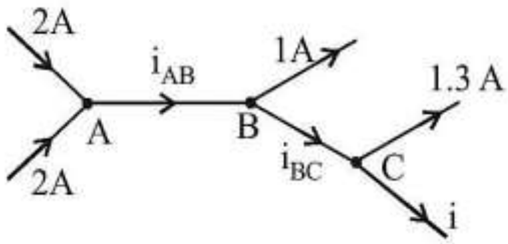
18. **(b)** Taking the same mass of sphere if radius is increased then moment of inertia, angular velocity and rotational kinetic energy will change but according to law of conservation of momentum, angular momentum will not change.

19. **(c)** For a perfectly rigid body strain produced is zero for the given force applied, so Y = stress / strain = Infinite (∞)

20. **(a)** According to Kirchhoff's first law,

At junction A, $i_{AB} = 2 + 2 = 4$ A

At junction B, $i_{AB} = i_{BC} + 1 \Rightarrow i_{BC} = 4 - 1 = 3$ A

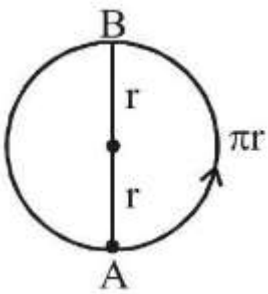


At junction C, $i = i_{BC} - 1.3 = 3 - 1.3 = 1.7$ A

21. **(d)**

22. **(b)** When a particle cover half of circle of radius r, then displacement is AB = 2r

Distance = half of circumference of circle = πr



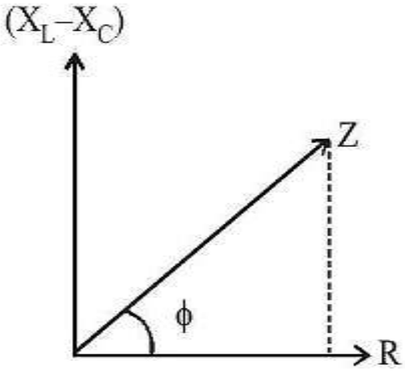
23. **(b)** As the magnet falls, the magnetic flux linked with the ring increases. This induces emf in the ring which opposes the motion of the falling magnet, so a < g.

24. **(b)** Change in momentum = F × t

$= mg \times 10$
$\Rightarrow 0.5 \times 9.8 \times 10$
$\Rightarrow$ 49 N-sec

25. **(b)** Given, $R = X_L = 2X_C$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(2X_C)^2 + (2X_C - X_C)^2}$$
$$= \sqrt{4X_C^2 + X_C^2}$$



$$= \sqrt{5}X_C = \frac{\sqrt{5}R}{2}\ [\because R = 2X_C]$$

$$\tan\phi = \frac{(X_L - X_C)}{R} = \frac{(2X_C - X_C)}{2X_C}$$

$$\tan\phi = \frac{1}{2};\ \phi = \tan^{-1}\left(\frac{1}{2}\right)$$

26. (d) According to the law of conservation of momentum,

$P_{(initial)} = P_{(final)}$

$$\Rightarrow mu_1 + Mu_2 = mv_1 + Mv_2$$

$$\Rightarrow 0 + 0 = 10 \times 10 \times 10^{-3} \times 800 + [100 - 10 \times 10 \times 10^{-3}j$$

$$\Rightarrow v = \frac{-800}{999} \Rightarrow -0.8\ m/sec$$

Negative sign indicates that the velocity is opposite to the direction of bullets.

27. (a) Escape velocity does not depend upon the mass of the body.
28. (b) By energy conservation

$$(K.E)_i + (P.E)_i = (K.E)_f + (P.E)_f$$
$$(K.E)_i = 0, (P.E)_i = mgh, (P.E)_f = 0$$
$$(K.E)_f = 1/2I\omega^2 + 1/2mv_{cm}^2$$

For solid cylinder, moment of inertia, $I = 1/2mR^2$

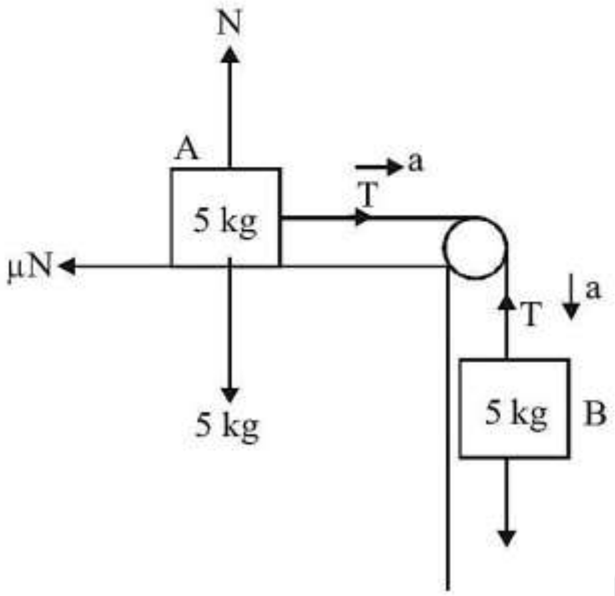
$$(K.E)_i + (P.E)_i = (K.E)_f + (P.E)_f$$

$$\text{so, } mgh = 1/2(1/2mR^2)\left(\frac{v_{cm}^2}{R^2}\right) + 1/2mv_{cm}^2$$

$$\Rightarrow v_{cm} = \sqrt{4gh/3}$$

29. (c) For block A,

$T - \mu N = 5a$ and $N = 5\ g$



For block B,

$5\ g - T = 5a$

$\Rightarrow T = 36.75\ N, a = 2.45\ m/sec^2$

30. **(b)** Stress $= \frac{\text{Force}}{\text{Area}}$ $\therefore$ Stress $\propto \frac{1}{\pi r^2}$

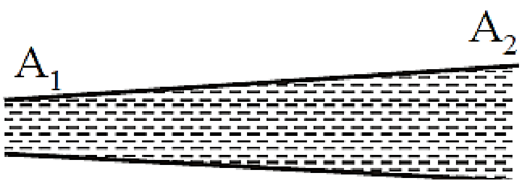
$\frac{S_B}{S_A} = \left(\frac{r_A}{r_B}\right)^2 = (2)^2 \Rightarrow S_B = 4S_A$

31. **(b)** The theorem of continuity is valid.

$\therefore A_1 v_1 \rho = A_2 v_2 \rho$

As the density of the liquid can be taken as uniform.

$\therefore A_1 v_1 = A_2 v_2$



$\Rightarrow$ Smaller the area, greater the velocity.

32. **(a)** The square of the period of planet is directly proportional to the cube of the semi-major axis of the elliptical orbit.

$\Rightarrow T^2 \propto a^3$

33. **(b)** Energy of incident light, $E = \frac{12400}{\lambda(\text{Å})}$

$E = \frac{1240}{400} = 3.1\text{eV}$

$\because E = W_0 + K_{max}$

$\Rightarrow 3.1 = W_0 + 1.68$

$\Rightarrow W_0 = 1.42eV$

34. (b) For path iaf,

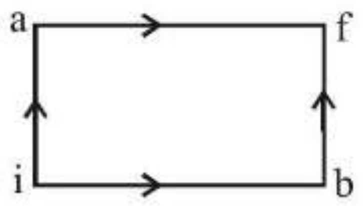
$Q = 50cal$

$W = 20cal$

By first law of thermodynamics,

$\Delta U = Q - W = 50 - 20 = 30cal.$

For path ibf



$Q' = 36cal$

$W' = ?$

or, $W' = Q' - \Delta U'$

Since, the change in internal energy does not depend on the path, therefore

$\Delta U' = 30cal$

$\therefore\ W' = Q' - \Delta U' = 36 - 30 = 6cal.$

35. (a) The differential equation of simple harmonic motion is

$$\frac{d^2y}{dt^2} + 2y = 0 \text{ or } \frac{d^2y}{dt^2} = -2y$$

Standard equation of simple harmonic motion is

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

Comparing eq.(i) and (ii),

$\omega^2 = 2$ or $\omega = \sqrt{2}$

As we know, $\omega = \frac{2\pi}{T}$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{2}} = \pi\sqrt{2}sec$

36. (c) Fundamental freq. for,

Tube A $\Rightarrow f_A = \frac{v}{2L}$

Tube B $\Rightarrow f_B = \frac{v}{4L}$

Now,

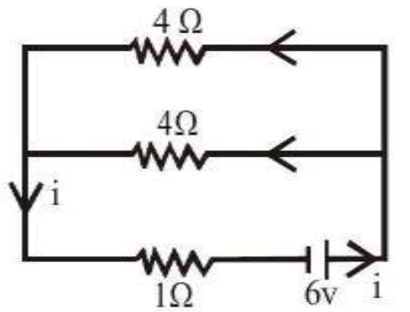
$$\frac{f_A}{f_B} = \frac{v}{2L} \times \frac{4L}{v} = \frac{2}{1}$$

$f_A : f_B = 2 : 1$

37. (d) Two 4Ω resistors are in parallel combination. Their equivalent resistance

$$= \frac{4 \times 4}{4+4} = \frac{16}{8} = 2\Omega$$

∴ Total resistance of the



network $= 2 + 1 = 3\Omega$

∴ Current through 1Ω resistor, $i = \frac{6}{3} = 2$ A

38. (c) Force on moving charge while moving in magnetic field is; $\vec{F} = q(\vec{v} \times \vec{B})$ where $\vec{F}$ is perpendicular to $\vec{v}$.

Work done/sec $= \vec{F} \cdot \vec{v} = Fv\cos 90° = 0$

39. (b) Fringe width, $\beta = \frac{\lambda D}{d}$

When λ alone is doubled then fringe width becomes

$$\beta' = \frac{(2\lambda)D}{d} = 2\beta$$

40. (a) In the normal adjustment of YDSE, Path difference between the waves at central location is always zero. So maxima is obtained at central position. Central point on the screen is bright.

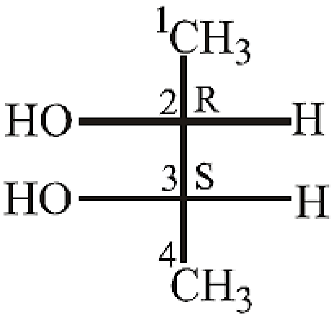
PART-II CHEMISTRY

41. (b)

$$W = 2.303nRT\log\frac{V_2}{V_1}$$
$$= 2.303 \times 1 \times 8.314 \times 10^7 \times 298\log\frac{20}{10}$$
$$= 298 \times 10^7 \times 8.31 \times 2.303\log 2$$

$W = 2.303nRT\log\frac{V_2}{V_1}$

42. (a)



43. (b)

	Electron	Proton	Neutron	α-particle
e	1 unit	1 unit	zero	2 units
m	1/1837 unit	1 unit	1 unit	4 -units
e/m	1837	1	zero	1/2

44. (b) Decreases from +6 to +3 .

$K_2Cr_2O_7 + 4H_2SO_4 \rightarrow K_2SO_4 + Cr_2(SO_4)_3 + 4H_2O + 3O$

$$\frac{[H_2S + [O] \rightarrow S + H_2O] \times 3}{K_2Cr_2O_7 + 4H_2SO_4 + 3H_2S \rightarrow}$$

$$K_2SO_4 + \overset{+3}{Cr_2}(SO_4)_3 + 7H_2O + 3S$$

45. (b)

$$\begin{aligned} N_2 + 3H_2 &\rightarrow 2NH_3 \\ \frac{-\Delta[N_2]}{\Delta t} &= -\frac{1}{3}\frac{\Delta[H_2]}{\Delta t} = \frac{1}{2}\frac{\Delta[NH_3]}{\Delta t} \\ \therefore \frac{-\Delta[H_2]}{\Delta t} &= \frac{3}{2} \times \frac{\Delta[NH_3]}{\Delta t} = \frac{3}{2} \times 2 \times 10^{-4} \\ &= 3 \times 10^{-4}\ \text{mol L}^{-1}\ \text{S}^{-1} \end{aligned}$$

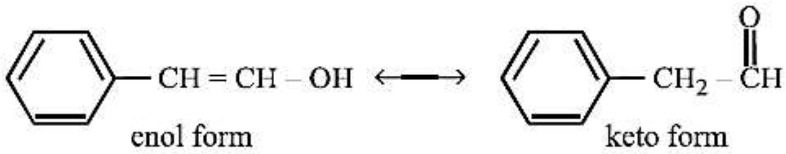
46. (a) $K_p = K_c[RT]^{\Delta n}g\ \Delta n = 2 - 4 = -2\ T = 673\ K,\ K_c = 0.5,$

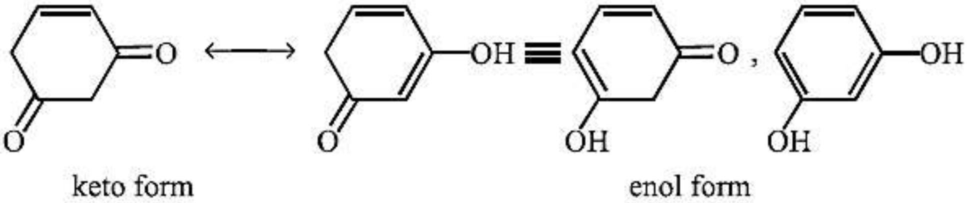
$R = 0.082\ L.atm.mol^{-1}\ K^{-1}$

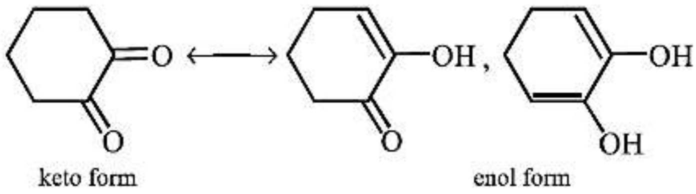
$K_p = 0.5 \times (0.082 \times 673)^{-2}$

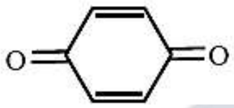
$= 1.64 \times 10^{-4}$ atm.

47. (d)

(a) 

(b) 

(c) 

(d)  cannot tautomerise

48. (b)

$$Cu^{2+} + 1e^- \rightarrow Cu^+ ; \Delta G_1^\circ = -n_1 E_1^o F]$$
$$Cu^+ + 1e^- \rightarrow Cu ; \Delta G_2^o = -n_2 E_2^o F$$
$$Cu^{2+} + 2e^- \rightarrow Cu \ \Delta G^\circ = \Delta G_1^o + \Delta G_2^o$$
$$-nE^\circ F = -1n_1 \ E_1^o F + (-1) n_2 E_2^o F$$
$$-nE^\circ F = -F(n_1 E_1^o + n_2 E_2^o F)$$
$$E^\circ = \frac{n_1 E_1^o + n_2 E_2^o}{n} = \frac{0.15 \times 1 + 0.50 \times 1}{2} = 0.325$$

49. (b) An oxygen- helium mixture is used for artificial respiration in deep sea diving instead of air because nitrogen present in air dissolves in blood under high pressure when sea diver goes into deep sea. When he comes to the surface nitrogen bubbles out of the blood due to decrease in pressure, causing pain. This disease is called "bends".

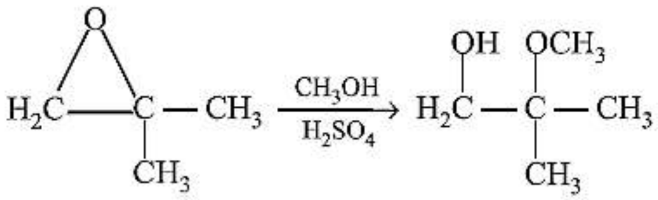
50. (d) $E_1 = h.\frac{c}{\lambda_1}$; $E_2 = h.\frac{c}{\lambda_2}$

$$\frac{E_1}{E_2} = \frac{hc}{\lambda_1} \times \frac{\lambda_2}{hc} = \frac{\lambda_2}{\lambda_1} = \frac{4000}{2000} = 2$$

51. (c)

$$\begin{array}{l} CH_2OH \\ | \\ CH_2OH \end{array} + HIO_4 \rightarrow 2HCHO + HIO_3 + H_2O$$

52. (a)



53. (c) As the system is closed and insulated, no heat enters. or leaves the system, i.e. $Q=0$;

$\therefore \Delta E=Q+W=W$

54. (a) When pressure is low

$$\left[P+\frac{a}{V^2}\right](V-b)=RT$$

$$\text{or } PV=RT+Pb-\frac{a}{V}+\frac{ab}{V^2}$$

$$\text{or } \frac{PV}{RT}=1-\frac{a}{VRT}$$

$$Z=-\frac{a}{VRT}$$

$$\left(\because \frac{PV}{RT}=Z\right)$$

55. (d) $CuSO_4+2KCN \rightarrow Cu(CN)_2+K_2SO_4$

$2Cu(CN)_2 \rightarrow Cu_2(CN)_2$

$$\text{or } 2CuCN+\underset{\text{cyanogen}}{CN-CN}$$

$CuCN+3KCN \rightarrow K_3[Cu(CN)_4]$

colourless

56. (a) $\Delta G=\Delta H-T\Delta S, T=27+273=300$ K

$\Delta G=(-285.8)-300(-0.163)=-236.9$ kJ mol^{-1}

57. (a)

$$\underset{A}{CH_3CHOHCH_3} \xrightarrow[\text{dil}.H_2SO_4]{K_2Cr_2O_7} \underset{B}{CH_3COCH_3}$$

$$\xrightarrow[H_2O]{CH_3MgI} (CH_3)_3COH$$

58. (b) $n\lambda=2\,d\sin\theta$

59. (c) For following reaction, $2NO(g)+O_2(g) \rightarrow 2NO_2(g)$

When the volume of vessel is reduced to $\frac{1}{3}$ then concentration of reactant becomes three times.

The rate of reaction for first order reaction is proportional to the concentration. So rate of reaction will increase three times.

60. (c) (a) Nitration is difficult to be carried out, further the $-NO_2$ group will go to m-position to the $>C=O$ group

(b) Nitrobenzene, being deactivated toward electrophilic substitution will not undergo Friedal Craft reaction.

(c) Benzene easily undergoes Friedal Craft reaction forming the required product.

61. (c) Among the transition metals, Mn forms maximum no. of oxides.

MnO	Mn_3O_4	Mn_2O_3	MnO_2	Mn_2O_7
basic	amphoteric	amphoteric	amphoteric	acidic

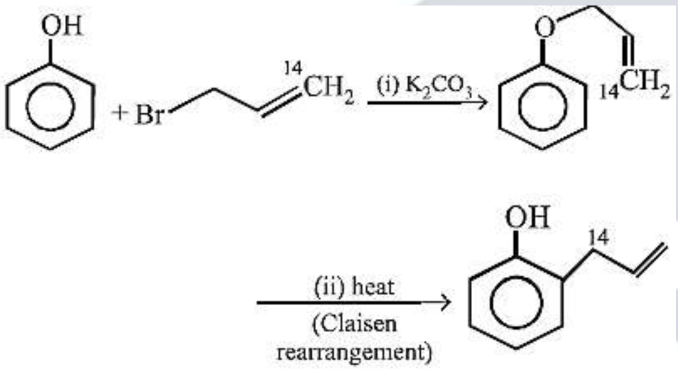
62. (c) The compound A with formula C_3H_6O gives iodoform test, it is propanone. It forms a compound B having carbon atoms three times to the number of carbon atoms in propanone, it is 2, 6-dimethyl-2, 5-heptadien-4-one.

$$\underset{(A)}{CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}}=O}+CH_3-\underset{\displaystyle O}{\underset{\|}{C}}-CH_3+O=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH_3 \xrightarrow{\text{dil HCl}}$$

$$CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH-\underset{\displaystyle O}{\underset{\|}{C}}-CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH_3$$

(B)
2, 6-Dimethyl-2, 5-heptadien-4-one

63. (b)



64. (b) $\Delta S_f=\frac{\Delta H_f}{T}=\frac{6025\ \mathrm{J\ mol^{-1}}}{273\ \mathrm{K}}=22.1\mathrm{JK^{-1}\ mol^{-1}}$

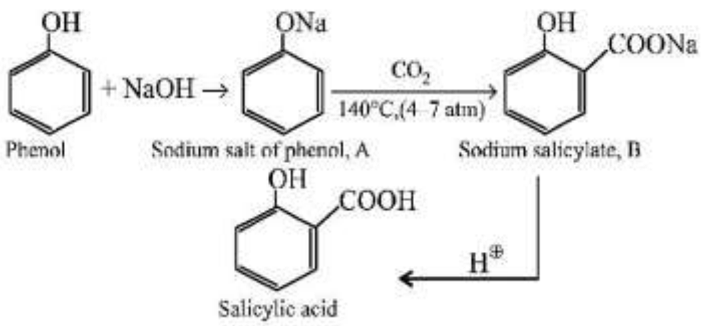
65. (b) $\log\frac{K_2}{K_1}=\frac{E_a}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$; If $\frac{K_2}{K_1}=2$

$$\log 2=\frac{E_a}{2.303\times 8.314}\left[\frac{1}{300}-\frac{1}{310}\right]$$

$$E_a=0.3010\times 2.303\times 8.314\left(\frac{300\times 310}{10}\right)$$

$$=53598.59\mathrm{Jmol^{-1}}=54\ \mathrm{kJ}$$

66. (d) Treatment of sodium salt of phenol with CO_2 under pressure brings about substitution of the- COOH group for the hydrogen of the ring. This is called as Kolbe's reaction



67. (b)

$$CH_3CH_2COOH \xrightarrow{SOCl_2} CH_3CH_2COCl + SO_2 + HCl$$
(X)

$$CH_3CH_2COCl + NH_3 \rightarrow CH_3CH_2CONH_2 + HCl$$
(Y)

$$CH_3CH_2CONH_2 + Br_2/NaOH \rightarrow \underset{\text{Ethylamine, (Z)}}{CH_3CH_2NH_2} + CO_2$$

68. (a) Energy needed to convert 1 mole of sodium (g) to sodium ions = 495 kJ

∴ Energy needed to convert 3 moles of Na(g) to Na^+ions $= 495 \times 3 = 1485$ kJ

69. (a) According to Heisenberg's uncertainty principle,

$$\Delta x . m\Delta v = \frac{h}{4\pi}$$

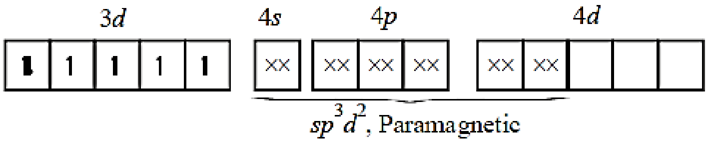
$$\text{or } m = \frac{h}{4\pi\Delta x . \Delta v}$$

$$= \frac{6.625 \times 10^{-34} Js}{4 \times 3.143 \times 10^{-10}\ m \times 5.27 \times 10^{-24}\ ms^{-1}}$$

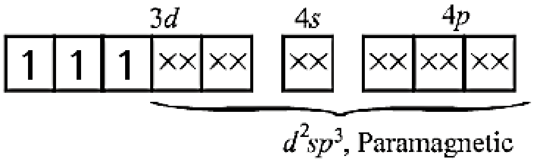
$$\because [J = Kgm^2\ s^2]$$

$$= 0.099 \text{ kg.}$$

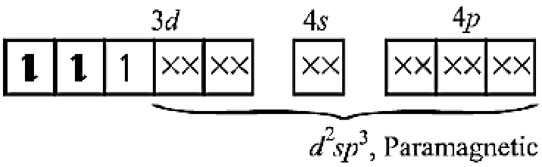
70. (a) (i) $[Fe(H_2O)_6]^{2+}$



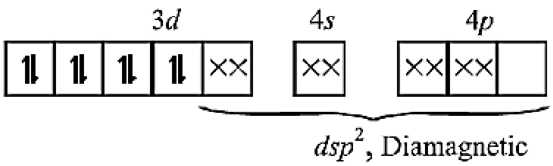
(ii) $[Cr(CN)_6]^{3-}$



(iii) $[Fe(CN)_6]^{3-}$



(iv) $[Ni(CN)_4]^{2-}$



Thus complexes (i), (ii) and (iii) are paramagnetic species.

71. (b) $\underset{\text{White}}{P_4} + 3NaOH + 3H_2O \rightarrow \underset{\text{Phosphine}}{PH_3} + 3NaH_2PO_2$

72. (b) In hydrogen-oxygen fuel cell, following reactions take place to create potential difference between the two electrodes.

$2H_2(g) + 4OH^-(aq) \rightarrow 4H_2O(l) + 4e^-$

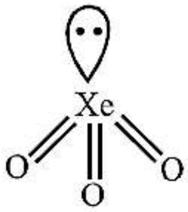
$O_2(g) + 2H_2O(l) + 4e^- \rightarrow 4OH^-(aq)$

Overall reaction $= 2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$

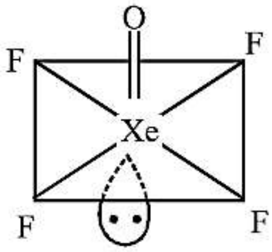
the net reaction is the same as burning (combustion) of hydrogen to form water.

73. (d)

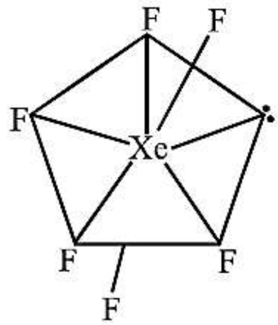
XeO_3:



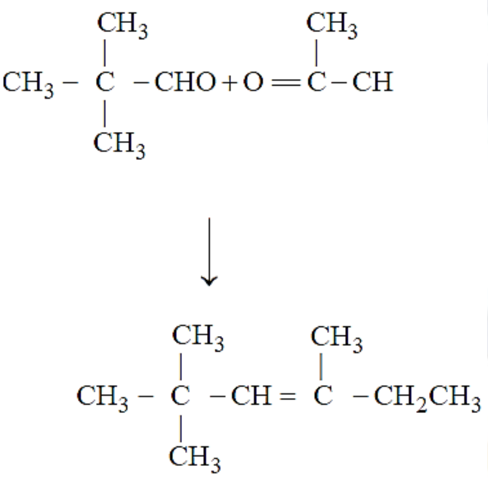
$XeOF_4$:



XeF_6:

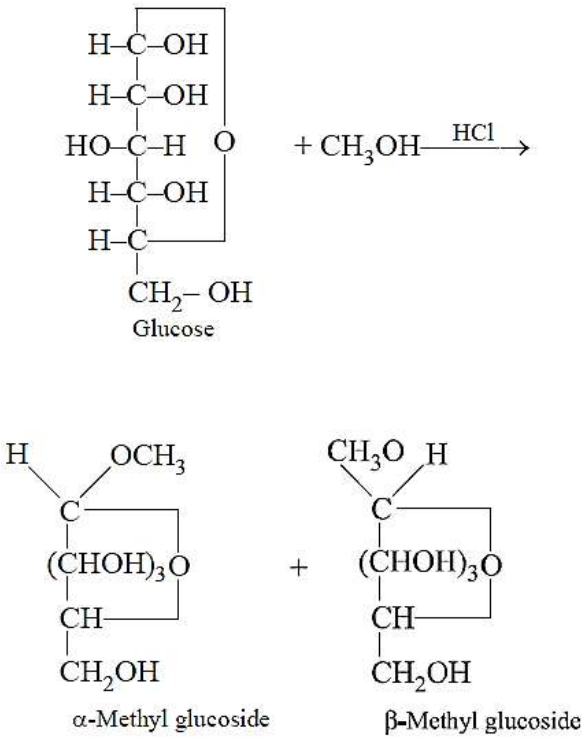


74. (a)



2,2,4 trimethyl-3-hexene

75. (c) A ring structure



76. (d) When acetylene is passed into methanol at $160-200^{\circ}\mathrm{C}$ in the presence of a small amount (1-2%) of potassium methoxide and under pressure just high enough to prevent boiling, methyl vinyl ether is formed.

The mechanism is believed to involve nucleophildic attack in the first step.

$$HC\equiv CH + CH_3O^- \rightarrow C^-H = CHOCH_3$$

$$\xrightarrow{CH_3OH} \underset{\text{Methyl vinyl ether}}{CH_2 = CHOCH_3} + CH_3O^-$$

77. (b)

$$(CH_3)_2.CH.\underset{\displaystyle NH_2}{CH}-COOH \rightleftharpoons (CH_3)_2-CH.\underset{\displaystyle NH_3^+}{CH}-COO^-$$

78. (a) The most probable complex which gives three mole ions in aqueous solution may be $[Co(NH_3)_5NO_2]Cl_2$.

79. (c) Endothermic reactions are those which involve absorption of heat. High activation energy means potential energy of product must be much greater than reactants.

80. (d) $K_c = \frac{[SO_3]^2}{[SO_2]^2[O_2]}$

If volume is reduced to half, K_c will decrease to half. Thus, to maintain the equilibrium, the reaction should shift in the forward direction, i.e. towards right. Also, to attain equilibrium back, the rate of forward direction will become double the rate of backward direction.

PART-III MATHEMATICS

81. (a) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-n}{1+n} \Rightarrow \sec 2\theta = \frac{1+n}{1-n}$

Since, n is non-square natural number $\therefore n \neq 1$

$\therefore \sec 2\theta = \frac{1+n}{1-n} =$ rational number

82. (d) $z^{1/3} = a - ib \Rightarrow z = (a-ib)^3$

$\therefore x + iy = a^3 + ib^3 - 3ia^2 b - 3ab^2$. Then

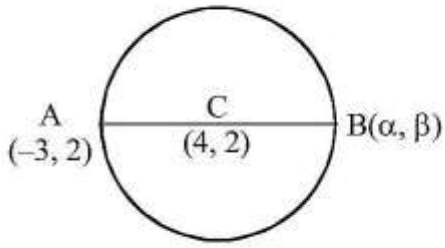
$x = a^3 - 3ab^2 \Rightarrow \frac{x}{a} = a^2 - 3b^2$

$y = b^3 - 3a^2 b \Rightarrow \frac{y}{b} = b^2 - 3a^2$

So, $\frac{x}{a} - \frac{y}{b} = 4(a^2 - b^2)$

83. (d) The centre of the given circle is $C \equiv (4,2)$

Let $A \equiv (-3,2)$



If (α, β) are the coordinates of the other end of the diameter, then, as the middle ploint of the diameter is the centre, $\therefore$ $\frac{\alpha-3}{2} = 4$ and $\frac{\beta+2}{2} = 2 \Rightarrow \alpha = 11, \beta = 2$

Thus, the coordinates of the other end of diameter are $(11,2)$

84. (c) The system is homogenuous system. $\therefore$ it has either unique solution or infinite many solution depend on $|A|$

$$\therefore |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix} = 2 \times 1 + 1(-3) + (4 - 3)$$
$$= 2 - 3 + 1 = 0$$

85. (c) Let $\omega = \frac{-1+\sqrt{3}i}{2}$

Now, $1 + \omega + \omega^2 = 0$, and $\omega^3 = 1$

then $(3 + \omega + 3\omega^2)^4 = (3 + \omega + 3\omega^2 + 2\omega - 2\omega)^4$

$= (3 + 3\omega + 3\omega^2 - 2\omega)^4 = [3(1 + 2 + \omega^2) - 2\omega]^4$

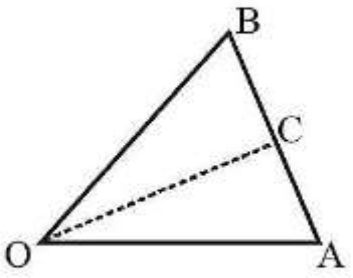
$= (0 - 2\omega)^4 = 16\omega^4 = 16\omega\ (\because \omega^3 = 1)$

86. (a) The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x - 4y + 4 = 0$ and $3x - 4y - \frac{7}{2} = 0$ and so it is equal to $\frac{4}{\sqrt{9+16}} + \frac{7/2}{\sqrt{9+16}} = \frac{3}{2}$. Hence radius is $\frac{3}{4}$.

87. (a) We have,

$OA = |2\hat{i} + 2\hat{j} + \hat{k}| = 3; OB = |2\hat{i} + 4\hat{j} + 4\hat{k}| = 6$

Since the internal bisector divides opposite side in the



ratio of adjacent sides

$\therefore \frac{AC}{BC} = \frac{3}{6} = \frac{1}{2}$

where OC is the bisector of $\angle BOA$.

$\therefore$ Position vector of C is

$$\frac{2(2\hat{i} + 2\hat{j} + \hat{k}) + (2\hat{i} + 4\hat{j} + 4\hat{k})}{2+1} = 2\hat{i} + \frac{8}{3}\hat{j} + 2\hat{k}$$

$$\therefore OC = \left|2\hat{i} + \frac{8}{3}\hat{j} + 2\hat{k}\right| = \sqrt{\frac{136}{9}}$$

88. (c) $y = \tan^{-1}\frac{4x}{1+5x^2} + \tan^{-1}\frac{2+3x}{3-2x}$

$$= \tan^{-1}\frac{5x - x}{1 + 5x \cdot x} + \tan^{-1}\frac{\frac{2}{3} + x}{1 - \frac{2}{3} \cdot x}$$

$$= \tan^{-1} 5x - \tan^{-1} x + \tan^{-1}\frac{2}{3} + \tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = \frac{5}{1 + 25x^2}$$

89. (c) $AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$AB = \begin{bmatrix} abc - abc & b^2c - b^2c & bc^2 - bc^2 \\ -a^2c + a^2c & -abc + abc & -ac + ac \\ a^2b - a^2b & ab^2 - ab^2 & abc - abc \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

90. (d) Let the centre of the circle be (h, k). Since the centre lies on the line $y = x - 1$

$\therefore k = h - 1$

Since the circle passes through the point (7,3), therefore, the distance of the centre from this point is the radius of the circle.

$$\therefore\ 3 = \sqrt{(h-7)^2 + (k-3)^2}$$

$\Rightarrow\ h = 7$ or $h = 4$

For $h = 7$, we get $k = 6$ and for $h = 4$, we get $k = 3$

Hence, the circles which satisfy the given conditions are:

$$(x-7)^2+(y-6)^2=9$$
$$\text{or } x^2+y^2-14x+12y+76=0$$
$$\text{and } (x-4)^2+(y-3)^2=9 \text{ or}$$
$$x^2+y^2-8x-6y+16=0$$

91. (a) The function $f(x)=\sin x$ is differentiable for all $x \in R$. Therefore, the number of points in the interval $(-\infty,\infty)$ where the function is not differentiable are zero.

92. (a) The given line is $\frac{x-2}{2}=\frac{2y-5}{-3}=z+1, \Rightarrow \frac{x-2}{2}=\frac{y-\frac{5}{2}}{-\frac{3}{2}}=\frac{z+1}{0}$

This shows that the given line passes through the point $\left(2,\frac{5}{2},-1\right)$ and has direction ratios $\left(2,\frac{-3}{2},0\right)$. Thus, given line passes through the point having position vector $\vec{a}=2\hat{\imath}+\frac{5}{2}\hat{\jmath}-\hat{k}$ and is parallel to the vector $\vec{b}=\left(2\hat{\imath}-\frac{3}{2}\hat{\jmath}-0\hat{k}\right)$. So, its vector equation is $\vec{r}=\left(2\hat{\imath}+\frac{5}{2}\hat{\jmath}-\hat{k}\right)+\lambda\left(2\hat{\imath}-\frac{3}{2}\hat{\jmath}-0\hat{k}\right)$.

Hence, $p=0$

93. (c) We have

$$L=\lim_{x\to 2}\frac{\sqrt{(x+7)}-3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)}-2\sqrt[3]{(3x-5)}}\left(\frac{0}{0}\text{ form}\right)$$

Let $x-2=t$ such that when $x\to 2, t\to 0$. Then

$$L=\lim_{t\to 0}\frac{(t+9)^{\frac{1}{2}}-3(2t+1)^{\frac{1}{2}}}{(t+8)^{\frac{1}{3}}-2(3t+1)^{\frac{1}{3}}}\left(\frac{0}{0}\text{ form}\right)$$
$$=\frac{3}{2}\lim_{t\to 0}\frac{\left(1+\frac{t}{9}\right)^{\frac{1}{2}}-(2t+1)^{\frac{1}{2}}}{\left(1+\frac{t}{8}\right)^{\frac{1}{3}}-(3t+1)^{\frac{1}{3}}}\left(\frac{0}{0}\text{ form}\right)$$
$$=\frac{3}{2}\lim_{t\to 0}\frac{\frac{1}{2}\frac{t}{9}-(2t)\frac{1}{2}}{\frac{1}{3}-(3t)\frac{1}{3}}=\frac{3}{2}\frac{\frac{1}{18}-1}{\frac{1}{24}-1}=\frac{34}{23}.$$

94. (d) Given $V=\pi r^2 h$.

Differentiating both sides, we get

$$\frac{dV}{dt}=\pi\left(r^2\frac{dh}{dt}+2r\frac{dr}{dt}h\right)=\pi r\left(r\frac{dh}{dt}+2h\frac{dr}{dt}\right)$$
$$\frac{dr}{dt}=\frac{1}{10}\text{ and }\frac{dh}{dt}=-\frac{2}{10}$$
$$\frac{dV}{dt}=\pi r\left(r\left(-\frac{2}{10}\right)+2h\left(\frac{1}{10}\right)\right)=\frac{\pi r}{5}(-r+h)$$

Thus, when $r=2$ and $h=3$,

$$\frac{dV}{dt}=\frac{\pi(2)}{5}(-2+3)=\frac{2\pi}{5}$$

95. (c) Given a, b, c are in A.P.

$\therefore 2b = a + c$

Now, $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+(a+c) \\ x+3 & x+4 & x+c \end{vmatrix}$$

[using equation (i)]

$$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+3 & x+4 & x+c \end{vmatrix} = \frac{1}{2}\cdot 0 = 0$$

[Applying $R_2 \to R_2 - (R_1 + R_3)$]

96. (a) We have, $\sin x = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3}$ Hence, $\sin x = \sin\frac{4\pi}{3}$, which gives $x = n\pi + (-1)^n\frac{4\pi}{3}$, where $n \in Z$.

97. (c) The given equation can be rewritten as $(y-1)^2 = 4\left(x - \frac{1}{2}\right)$ which is a parabola with its vertex $\left(\frac{1}{2}, 1\right)$ axis along the line $y = 1$, hence axis parallel to x-axis.

Its focus is $\left(\frac{1}{2} + 1, 1\right)$, i.e., $\left(\frac{3}{2}, 1\right)$.

98. (b) Given, $\sin y = x\sin(a+y)$

$$\Rightarrow x = \frac{\sin y}{\sin(a+y)}$$

On differentiating w.r. to y, we get

$$\frac{dx}{dy} = \frac{d}{dy}\left[\frac{\sin y}{\sin(a+y)}\right]$$

$$= \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin a}{\sin^2(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

99. (d) Let $I = \int (27e^{9x} + e^{12x})^{1/3}dx$

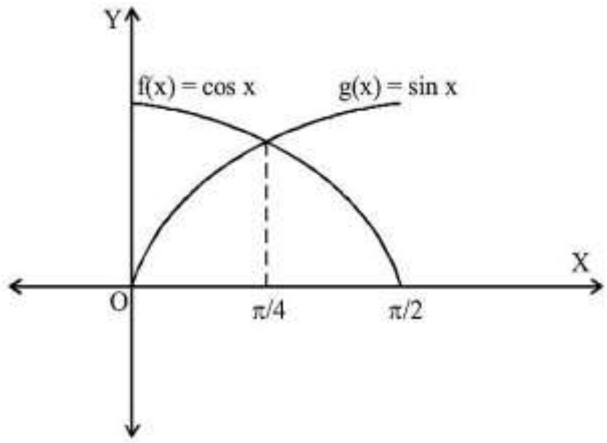
$$= \int e^{3x}(27 + e^{3x})^{1/3}dx$$

Put $27 + e^{3x} = t \Rightarrow 3e^{3x}dx = dt$

$$\therefore I = \frac{1}{3}\int t^{1/3}dt = \frac{1}{3} \times \frac{t^{4/3}}{4/3} + C$$

$$= \frac{1}{4}(27 + e^{3x})^{4/3} + C$$

100. (b) $y = |\cos x - \sin x|$



Required area $= 2\int_0^{\pi/4} (\cos x - \sin x)dx$

$= 2[\sin x + \cos x]_0^{\pi/4} = 2\left[\frac{2}{\sqrt{2}} - 1\right]$

$= (2\sqrt{2} - 2)$ sq. units

101. **(c)** From given equations

$$\frac{x}{2} = \cos t + \sin t$$
$$\frac{y}{5} = \cos t - \sin t$$

Eliminating t from (i) and (ii), we have $\frac{x^2}{4} + \frac{y^2}{25} = 2 \Rightarrow \frac{x^2}{8} + \frac{y^2}{50} = 1$, which is an ellipse.

102. **(a)** Since, $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 676$

$$\Rightarrow (|\vec{a}| \cdot |\vec{b}| \sin\theta \hat{n})^2 + (|\vec{a}| \cdot |\vec{b}| \cos\theta)^2 = 676$$
$$\Rightarrow |\vec{a}|^2 \cdot |\vec{b}|^2 (\sin^2\theta + \cos^2\theta) = 676$$
$$\Rightarrow |\vec{a}|^2 (2)^2 = 676 \Rightarrow |\vec{a}|^2 = 169$$
$$\Rightarrow |\vec{a}| = 13$$

103. **(d)** Equation of plane bisecting the angle containing origin is (making constant term of same sign)

$$\frac{-3x + 6y - 2z - 5}{\sqrt{3^2 + 6^2 + 2^2}} = +\left[\frac{4x - 12y + 3z - 3}{\sqrt{4^2 + 12^2 + 3^2}}\right]$$
$$\text{or } \frac{-3x + 6y - 2z - 5}{7} = \frac{4x - 12y + 3z - 3}{13}$$
$$\text{or } 67x - 162y + 47z + 44 = 0$$

104. **(d)** Let one side of quadrilateral be x and another side be y so, $2(x + y) = 34$

or, $(x + y) = 17$

We know from the basic principle that for a given perimeter square has the maximum area, so, $x = y$ and putting this value in equation (i)

$x = y = \frac{17}{2}$

Area $= x \cdot y = \frac{17}{2} \times \frac{17}{2} = \frac{289}{4} = 72.25$

105. **(b)** $\int \frac{\sin x}{\sin(x-\alpha)}dx = \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx$

$$= \int \frac{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)}dx$$

$$= \int \{\cos\alpha + \sin\alpha\cot(x-\alpha)\}dx$$

$$= (\cos\alpha)x + (\sin\alpha)\log\sin(x-\alpha) + C$$

$\therefore A = \cos\alpha, B = \sin\alpha$

106. **(b)** The equation of the chord, having mid-point as (x_1, y_1), of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by

$T = S_1$

where, $T = \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$ and

$$S_1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$$

According to the question,

$(x_1, y_1) = (5,3)$ and $a^2 = 16, b^2 = 25$

as $25x^2 - 16y^2 = 400$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{25} = 1 \therefore \frac{5x}{16} - \frac{3y}{25} = \frac{25}{16} - \frac{9}{25}$$

[Using (i)]

$\Rightarrow 125x - 48y = 625 - 144 \Rightarrow 125x - 48y = 481$

107. **(b)** $I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}}dx$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$$

$$\text{Then, } I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}}dx$$

Adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}dx$$

$$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \Rightarrow I = \pi/12$$

108. **(c)** $|x-2| = \begin{cases} x-2, x \geq 2 \\ 2-x, x < 2 \end{cases}$

$$\Rightarrow |x-2| - (x-2) = \begin{cases} 0, x \geq 2 \\ 4-2x, x < 2 \end{cases}$$

$\Rightarrow$ given expression is defined for $(-\infty, 2)$

109. **(a)** Required number of ways

$= \frac{6!}{3!\,3!} = \frac{720}{6 \times 6} = 20$

110. **(d)** For $x < 1, f(x) = \frac{x^2-1}{x^2+2x-3} = \frac{x+1}{x+3}$

$\therefore \lim_{x \to 1^-} f(x) = \frac{1}{2}$

For $x > 1, f(x) = \frac{x^2-1}{x^2-2x+1} = \frac{x+1}{x-1}$

$\therefore \lim_{x \to 1^+} f(x) = \infty$

$\therefore$ the function is not continuous at $x = 1$.

111. **(a)** $f(x) = x^3 + bx^2 + cx + d, 0 < b^2 < c$

$\therefore\ f'(x) = 3x^2 + 2bx + c$

Discriminant $= 4b^2 - 12c = 4(b^2 - 3c) < 0$

$\therefore f'(x) > 0 \forall x \in R$

Thus, $f(x)$ is strictly increasing $\forall x \in R$

112. **(c)** Rearranging the equation, we have

$$dx - ydy + \sqrt{(x^2+y^2)}(xdx + ydy) = 0$$
$$\Rightarrow dx - ydy + \frac{1}{2}\sqrt{(x^2+y^2)}d(x^2+y^2) = 0$$

On integrating, we get

$x - \frac{y^2}{2} + \frac{1}{2}\int \sqrt{t}dt = C, \left\{t = \sqrt{(x^2+y^2)}\right\}$

or $x - \frac{y^2}{2} + \frac{1}{3}(x^2+y^2)^{3/2} = C$

113. **(c)** Since $x\frac{dy}{dx} - y = x^4 - 3x$

$\therefore \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$

Hence

$IF = e^{\int Pdx} = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

114. **(b)** $P(A) = 1/4, P(A/B) = \frac{1}{2}, P(B/A) = 2/3$

By conditional probability,

$P(A \cap B) = P(A)P(B/A) = P(B)P(A/B)$

$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

115. **(d)** We have, $f(x) = 2x - 3, g(x) = x^3 + 5$

$(\text{ fog })x = f(x^3 + 5) = 2(x^3 + 5) - 3 = 2x^3 + 7$

Let $y = (\text{ fog })x = 2x^3 + 7$

$$\Rightarrow x = \left(\frac{y-7}{2}\right)^{1/3} \Rightarrow (\text{fog})^{-1}x = \left(\frac{x-7}{2}\right)^{1/3}$$

116. **(c)** The inverse of the proposition $(p \wedge \sim q)$

$\rightarrow r$ is
$\sim (p \wedge \sim q) \rightarrow \sim r \equiv \sim p \vee \sim (\sim q) \rightarrow \sim r$
$\equiv \sim p \vee q \rightarrow \sim r$

117. **(d)** (i) $f(x) = \sqrt{x}$ is continuous in $[4,9]$

(ii) $f'(x) = \frac{1}{2\sqrt{x}}$

Thus $f(x)$ is differentiable in $(4,9)$

(iii) $f(4) \neq f(9)$. All the three conditions of LMV theorem satisfied then there exist at least one $c \in (4,9)$ such that.

$$f'(c) = \frac{f(b) - f(a)}{b - a} \Rightarrow \frac{1}{2\sqrt{c}} = \frac{1}{5} \Rightarrow c = \frac{25}{4}$$

118. **(b)** We have

$$|x| + |x-1| = \begin{cases} -x - (x-1) = -2x + 1, \text{ if } x \leq 0 \\ x - (x-1) = 1, \text{ if } 0 \leq x \leq 1 \\ x + x - 1 = 2x - 1, \text{ if } x \geq 1 \end{cases}$$

$\therefore \int_{-1}^{3} (|x| + |x-1|)dx$

$= \int_{-1}^{0} (-2x + 1)dx + \int_{0}^{1} 1dx + \int_{1}^{3} (2x - 1)dx$

$= [-x^2 + x]_{-1}^{0} + [x]_0^1 + [x^2 - x]_1^3 = 9$

119. **(c)** a, b, c are in A.P. $\Rightarrow 2b = a + c$

Now,

$e^{1/c} \times e^{1/a} = e^{(a+c)/ac} = e^{2b/ac} = \left(e^{b/ac}\right)^2$

$\therefore e^{1/c}, e^{b/ac}, e^{1/a}$ in G.P. with common ratio

$$= \frac{e^{b/ac}}{e^{1/c}} = e^{(b-a)/ac} = e^{d/(b-d)(b+d)}$$

$= e^{d/(b^2 - d^2)}$

$[\because a, b, c$ are in A.P. with common difference $d \therefore b - a = c - b = d]$

120. **(a)** We have $T_2 = 14a^{\frac{5}{2}}$

$$\Rightarrow {}^{n}C_1 \left(a^{\frac{1}{13}}\right)^{n-1} \left(a^{\frac{3}{2}}\right) = 14a^{\frac{5}{2}}$$

$\Rightarrow na\frac{n-1}{13}+\frac{3}{2}=14a^{\frac{5}{2}} \Rightarrow n=14$

$\Rightarrow \frac{^{n}C_3}{^{n}C_2}=\frac{^{14}C_3}{^{14}C_2}=\frac{12}{3}=4$

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121. **(c)** 'Neither' is used for two things. For more than two things, 'none' is used.
122. **(c)** Impeccable means 'in accordance with the highest standard; faultless'.
123. **(d)** Ameliorate means 'make (something bad or unsatisfactory) better.
124. **(d)** The idiom 'a bolt from the blue' means 'an event or a piece of news which is sudden and unexpected; a complete surprise'.
125. **(b)** The second part of the first sentence clearly states that the German students and the professor are quite friendly with each other and no strict relation whatsoever is seen between them which is generally seen between a professor and his students.