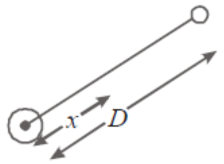
PART-I PHYSICS

1. **(d)** $\frac{Gm_e}{x^2} = \frac{Gm_m}{(D-x)^2}$

or $\frac{G(81m)}{x^2} = \frac{m}{(D-x)^2}$

$\therefore\ x = \frac{9D}{10}$.



2. **(c)** We know that Young's modulus

$$Y = \frac{F}{\pi r^2} \times \frac{L}{\ell}$$

Since Y, F are same for both the wires, we have,

$\frac{1}{r_1^2}\frac{L_1}{\ell_1} = \frac{1}{r_2^2}\frac{L_2}{\ell_2}$ or, $\frac{\ell_1}{\ell_2} = \frac{r_2^2 \times L_1}{r_1^2 \times L_2} =$

$$\frac{(D_2/2)^2 \times L_1}{(D_1/2)^2 \times L_2}$$

or, $\frac{\ell_1}{\ell_2} = \frac{D_2^2 \times L_1}{D_1^2 \times L_2} = \frac{D_2^2}{(2D_2)^2} \times \frac{L_2}{2\,L_2} = \frac{1}{8}$

So, $\ell_1 : \ell_2 = 1:8$

3. **(c)** $[x] = [bt^2]$. Hence $[b] = [x/t^2] = km/s^2$.
4. **(c)**
5. **(c)**
6. **(c)** As mass of B is twice that of A.

And, $\frac{r_1}{r_2} = \frac{m_2}{m_1}$

7. **(d)** $\mu_k < \mu_S$ coefficient of static friction is always greater than kinetic friction.
8. **(c)** Distance travelled in the nth second is

given by $d_n = u + \frac{a}{2}(2n-1)$

put $u = 0, a = \frac{4}{3}\ ms^{-2}, n = 3$

$$\therefore d = 0 + \frac{4}{3 \times 2}(2 \times 3 - 1) = \frac{4}{6} \times 5 = \frac{10}{3}\ m$$

9. **(d)** $h\nu = W + \frac{1}{2}mv^2$ or $\frac{hc}{\lambda} = W + \frac{1}{2}mv^2$ Here $\lambda = 3000Å = 3000 \times 10^{-10}$ m and $W = 1eV = 1.6 \times 10^{-19}$ joule $\therefore \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{3000 \times 10^{-10}}$

$$= (1.6 \times 10^{-19}) + \frac{1}{2} \times (9.1 \times 10^{-31})v^2$$

Solving we get, $v \cong 10^6$ m/s

10. (a) $\gamma_{real} = \gamma_{app.} + \gamma_{vessel}$

So $(\gamma_{app.} + \gamma_{vessel})_{glass} = (\gamma_{app} + \gamma_{vessel})_{steel}$

$\Rightarrow 153 \times 10^{-6} + (\gamma_{vessel})_{glass}$

$= 144 \times 10^{-6} + (\gamma_{vessel})_{steel}$

Further,

$$\begin{aligned} &(\gamma_{vessel})_{steel} = 3\alpha = 3 \times (12 \times 10^{-6}) \\ &= 36 \times 10^{-6} / {}^\circ C \\ &\Rightarrow 153 \times 10^{-6} + (\gamma_{vessel})_{glass} \\ &= 144 \times 10^{-6} + 36 \times 10^{-6} \\ &\Rightarrow (\gamma_{vessel})_{glass} = 3\alpha = 27 \times 10^{-6} / {}^\circ C \\ &\Rightarrow \alpha = 9 \times 10^{-6} / {}^\circ C \end{aligned}$$

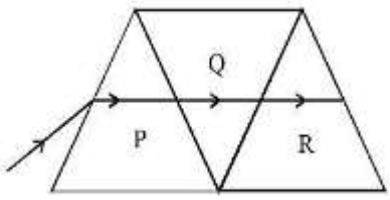
11. (b)

12. (a) Horizontal range is same when angle of projection is θ or $(90^\circ - \theta)$.

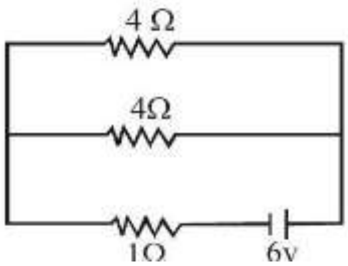
13. (a)

14. (a) $a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}} = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7} g\sin\theta$

15. (c) When the ray suffers minimum deviation, it becomes parallel to the base of prism P. As prisms Q and R are of same material and have identical shape, therefore, the ray continues to be parallel to base of Q and R. Hence final deviation of the ray remains the same as before.



16. (d) Two 4Ω resistors are in parallel combination. Their equivalent resistance $= \frac{4\times4}{4+4} = \frac{16}{8} = 2\Omega$



$\therefore$ Total resistance of the network $= 2 + 1 = 3\Omega$

$\therefore$ Current through 1Ω resistor $= \frac{6}{3} = 2$ A

17. (b) Root-mean square-velocity is given by

$v_{rms} = \sqrt{\frac{3RT}{M}}$ i.e., $v_{rms} \propto \sqrt{\left(\frac{T}{M}\right)}$

$$\frac{(v_{rms})O_2}{(v_{rms})H_2} = \sqrt{\left[\frac{T_{O_2}}{T_{H_2}} \times \frac{M_{H_2}}{M_{O_2}}\right]}$$

$$= \sqrt{\left[\left(\frac{1200}{300}\right) \times \left(\frac{2}{32}\right)\right]} = \frac{1}{2}$$

$$\therefore \ (v_{rms})O_2 = (v_{rms})H_2 \times \frac{1}{2} = \frac{1930}{2} =$$

965 m/s

18. **(b)** Lenz's law helps to identify the direction of induced current.

19. **(d)** Capacity of parallel plate capacitor

$C = \frac{k\varepsilon_0 A}{d}$ (For air $k_r = 1$)

So, $\frac{\varepsilon_0 A}{d} = 8 \times 10^{-12}$

If $d \to \frac{d}{2}$ and $k_r \to 6$ then new capacitance

$$C' = 6 \times \frac{\varepsilon_0 A}{d/2} = 12\frac{\varepsilon_0 A}{d} = 12 \times 8\text{pF} = 96\text{pF}$$

20. **(a)** In $x = A\cos \omega t$, the particle starts oscillating from extreme position. So at $t = 0$, its potential energy is maximum.

21. **(c)** Ratio of number of half- life taken is given as:

After 16 days

$$n_{A1/2} = \frac{16}{4} = 4; n_{B1/2} = \frac{16}{8} = 2$$

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N_A}{N_B}$$

$$= \frac{1}{2^4} : \frac{1}{2^2} = 2^2 : 2^4$$

$$= 4 : 16, = 1 : 4$$

22. **(c)** Given: Length (l) $= 7$ m

Mass (M) $= 0.035$ kg and tension (T) $= 60.5$

N. We know that mass of string per unit length (m)

$$= \frac{0.035}{7} = 0.005 \text{ kg/m}$$

and speed of

$$\text{wave} = \sqrt{\frac{T}{m}} = \sqrt{\frac{60.5}{0.005}} = 110 \text{ m/s}$$

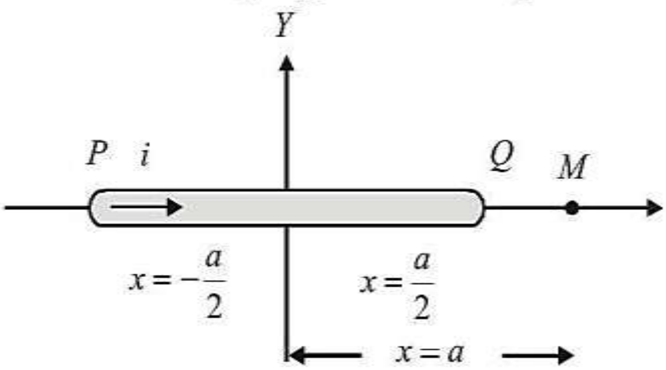
23. **(d)** Output of upper AND gate $= \overline{A}B$

Output of lower AND gate $= A\bar{B}$

$\therefore$ Output of OR gate, $Y = A\bar{B} + B\bar{A}$

This is boolean expression for XOR gate.

24. (d) Magnetic field at a point on the axis of a current carrying wire is always zero.



25. (a) $\lambda = \frac{v}{n} = \frac{340}{170} = 2\text{m}, n' = \frac{340}{340-17} \times 170\ n' = 178.9$ Hz

Now $\lambda' = \frac{v}{n'} = \frac{340}{178.9} = 1.9$

$\Rightarrow \lambda - \lambda' = 2 - 1.9 = 0.1$ m

PART-II CHEMISTRY

26. (a)

$$\underset{}{4NaCl} + K_2Cr_2O_4 + \underset{(Conc.)}{3H_2SO_4} \longrightarrow \underset{(A)}{2CrO_2Cl_2} + K_2SO_4 + 2NaSO_4 + 3H_2O$$

$$\underset{(A)}{CrO_2Cl_2} + NaOH \longrightarrow \underset{(B)}{Na_2CrO_4} + H_2O + NaCl$$

$$Na_2CrO_4 + \underset{(B)}{H_2SO_4} + \underset{(dilute)}{H_2O_2} \longrightarrow \underset{(C)}{CrO_5} + Na_2SO_4 + H_2O$$

The sum of total no. of atoms in one molecules each of A, B&C $= 5 + 7 + 6 = 18$

27. (c)

$$XeF_6 + H_2O \xrightarrow[\text{hydrolysis}]{\text{partial}} \underset{'X'}{\overset{+6}{Xe}OF_4} + 2HF$$

$$\underset{'X'}{XeOF_4} + H_2O \longrightarrow \underset{'Y'}{\overset{+6}{Xe}O_2F_2} + 2HF$$

28. (a)

$$\rho = \frac{ZM}{N_A V}$$

$$Z = \frac{\rho N_A V}{M} = \frac{2 \times 6 \times 10^{23} \times (5 \times 10^{-8})^3}{75}$$

$Z = 2$, which represents bcc structure

$$\therefore r = \frac{\sqrt{3}}{4}a = \frac{\sqrt{3}}{4} \times 5 = 2.165\text{Å} = 216.5\text{pm} \approx$$

217pm

29. (b) According to Arrhenius equation,

$k = Ae^{-E_a/RT}$

$\therefore$ when $E_a = 0, k = A$

Also $\ln k$ vs $1/T$ is a straight line with slope $= -E_2/R$.

$\therefore$ Statements (ii) and (v) are correct.

30. (a) In the electrolysis of cupric sulphate, the reaction that occurs at cathode is

$Cu^{2+} + 2e^- \longrightarrow Cu$

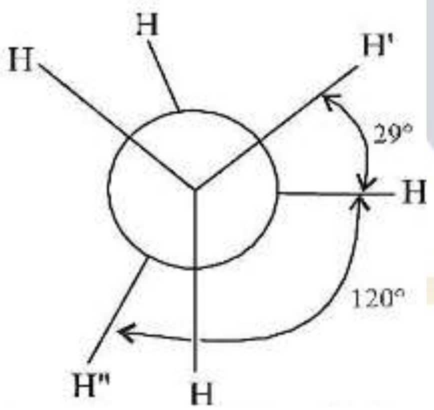
Thus 2 F or $2 \times 96500C$ of electricity is required to deposit

$= 1$ mol of Cu $= 63.5$ g of Cu

It means that to deposit 63.5 g of Cu, the amount of electricity required $= 2 \times 96500C$

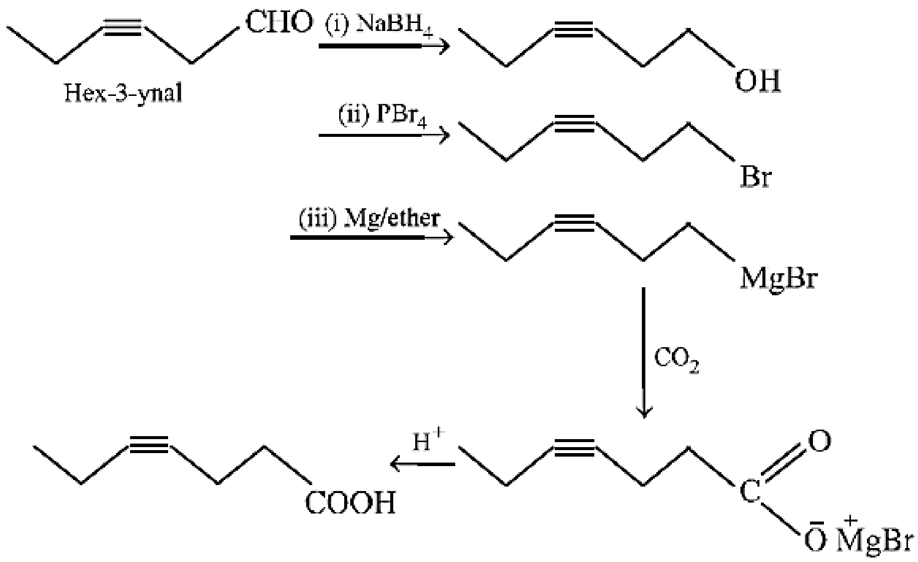
So, to deposit 0.634 g of Cu, the amount of electricity required $= \frac{2 \times 96500}{63.5} \times 0.634 \approx 1930C$

31. (b)



$\therefore$ Angle between H′ and H″ $= 120° + 29° = 149°$

32. (d)



33. (d)

$$CH_3CH_2OH \xrightarrow{P+I_2} \underset{A}{CH_3CH_2I}$$

$$\xrightarrow[\text{Ether}]{\text{Mg}} \underset{(B)}{CH_3CH_2MgI}$$

$$\xrightarrow{HCHO} \underset{(C)}{H-\overset{\overset{\displaystyle CH_2CH_3}{|}}{\underset{\underset{\displaystyle H}{|}}{C}}-OMgI}$$

$$\xrightarrow{H_2O} \underset{\substack{(D)\\ n\text{–Propylalcohol}}}{H-\overset{\overset{\displaystyle CH_2CH_3}{|}}{\underset{\underset{\displaystyle H}{|}}{C}}-OH}$$

34. (d)

35. (d)

36. (d) $\frac{e}{m}$ for (i) neutron $= \frac{0}{1} = 0$

(ii) α-particle $= \frac{2}{4} = 0.5$

(iii) proton $= \frac{1}{1} = 1$

(iv) electron $= \frac{1}{1/1837} = 1837$

37. (d) $Sc^{3+}(Z = 18): 1s^2, 2s^2p^6, 3s^2p^6d^0, 4s^0$; no unpaired electron.

$Cu^{+}(Z = 28): 1s^2, 2s^2p^6, 3s^2p^6d^{10}, 4s^0$; no unpaired electron.

$Ni^{2+}(Z=26): 1s^2, 2s^2p^6, 3s^2p^6d^8, 4s^0$
unpaired electrons are present.
$Ti^{3+}(Z=19): 1s^2, 2s^2p^6, 3s^2p^6d^1, 4s^0$;
unpaired electron is present
$Co^{2+}(Z=25): 1s^2, 2s^2p^6, 3s^2p^6d^7, 4s^0$;
unpaired electrons are present

So from the given options the only correct combination is Ni^{2+} and Ti^{3+}

38. (b) The square planar complex of the type $[Mabcd]^{nt}$, where all four ligands are different, has 3 geometrical isomers. But ifone of the ligands is ambidentate, then $2 \times 3 = 6$ geometrical isomers are possible. But if two ligands are ambidentate, then $4 \times 3 = 12$ geometrical isomers are possible. In the given example, NO_2^- and SCN^- are ambidentate ligands.

39. (a) Equilibrium constant has no relation with catalyst. Catalyst only affects the rate of the reaction.

Catalyst, V_2O_5 in the given reaction, is used to speed up the reaction.

40. (d) $t_{1/2} \propto \frac{1}{a^2}$

We know that $t_{1/2} \propto \frac{1}{a^{n-1}}$

i.e. $n = 3$

Thus, reaction is of 3rd order.

41. (d)
42. (d) Keto-enol tautomerism is shown by carbonyl compounds having α-hydrogen atom.
43. (d)
44. (c)
45. (d)
46. (b) Fructose has 3 chiral centres and hence number of optical isomers are $2^3 = 8$
47. (d) By Heisenberg uncertainty Principle $\Delta x \times \Delta p = \frac{h}{4\pi}$ (which is constant)

As Δx for electron and helium atom is same thus momentum of electron and helium will also be same therefore, the momentum of helium atom is equal to $5 \times 10^{-26}\ kg \cdot m \cdot s^{-1}$.

48. (b) $A \rightleftharpoons B$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$\Delta G^\circ = -2.303RT^\circ \log_{10} K$

$-2.303RT\log_{10} K = \Delta H^\circ - T\Delta S^\circ$

$2.303\ RT \log_{10} K = T\Delta S^\circ - \Delta H^\circ$

$$\log_{10} K = \frac{T\Delta S^\circ - \Delta H^\circ}{2.303RT}$$
$$= \frac{298 \times 10 + 54.07 \times 1000}{2.303 \times 8.314 \times 298}$$
$$= 9.998 \approx 10$$

49. (b) Let, $C(s) + O_2(g) \longrightarrow CO_2(g); \Delta H = R$

and $CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g); \Delta H = S$

Subtracting equ. (i) from equ (ii), we get

$$C(s)+\frac{1}{2}O_2(g)\rightarrow CO(g); \Delta H=R-s$$

Hence, heat of formation of $CO = R - S$

50. (c) Both the doubly bonded carbon atoms are identical.

PART-III MATHEMATICS

51. (d) Since Rolle's theorem is satisfied

$\therefore f'(c)=0 \Rightarrow e^c \sin c+e^c \cos c=0$

$$\Rightarrow e^c\{\sin c+\cos c\}=0$$

$$\therefore \sin c+\cos c=0\ (\because e^c \neq 0)$$

$$\Rightarrow \tan c=-1 \Rightarrow c=\tan^{-1}(-1)=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

52. (a) Consider first two equations:

$2x+3y=-4$ and $3x+4y=-6$

We have $\Delta=\begin{vmatrix}2 & 3\\ 3 & 4\end{vmatrix}=-1 \neq 0$

$$\Delta_x=\begin{vmatrix}-4 & 3\\ -6 & 4\end{vmatrix}=2 \text{ and } \Delta_y=\begin{vmatrix}2 & -4\\ 3 & -6\end{vmatrix}=0$$

$\therefore x=-2$ and $y=0$

Now this solution satisfies all the equations, so the equations are consistent with unique solution.

53. (b) The lines are $\frac{x}{6}=\frac{y+2}{6}=\frac{z-1}{1}$ and $\frac{x+1}{12}=\frac{y}{6}=\frac{z}{-1}$

Here, $\vec{a}_1=-2\hat{j}+\hat{k}, \vec{b}_1=6\hat{i}+6\hat{j}+\hat{k}, \vec{a}_2=-\hat{i}$,

$$\vec{b}_2=12\hat{i}+6\hat{j}-\hat{k}$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & k\\ 6 & 6 & 1\\ 12 & 6 & -1\end{vmatrix}=-12\hat{i}+18\hat{j}-36\hat{k}$$

$$\text{Shortest distance }=\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1-\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$$

$$=\frac{|(-\hat{i}+2\hat{j}-\hat{k})\cdot(-12i+18\hat{j}-36\hat{k})|}{\sqrt{(-12)^2+(18)^2+(-36)^2}}$$

$$=\frac{|+12+36+36|}{\sqrt{1764}}=\frac{84}{42}=2$$

54. (c) $y^2=x(2-x)^2 \Rightarrow y^2=x^3-4x^2+4x$

$$\Rightarrow 2y\frac{dy}{dx}=3x^2-8x+4 \Rightarrow \frac{dy}{dx}=\frac{3x^2-8x+4}{2y}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_P = \frac{3-8+4}{2} = -\frac{1}{2}$$

$\therefore$ Equation of tangent at P is: $y - 1 = -\frac{1}{2}(x-1)$

$\Rightarrow x + 2y - 3 = 0$

Using $y = \frac{3-x}{2}$ in (i),

we get: $\left(\frac{3-x}{2}\right)^2 = x^3 - 4x^2 + 4x$

$\Rightarrow 4x^3 - 17x^2 + 22x - 9 = 0$

which has two roots 1,1

(Because of (ii) being tangent at (1,1)).

Sum of 3 roots $= \frac{17}{4} \therefore$ 3 rd root $= \frac{17}{4} - 2 = \frac{9}{4}$

Then, $y = \frac{3-\frac{9}{4}}{2} = \frac{3}{8} \therefore$ Q is $\left(\frac{9}{4}, \frac{3}{8}\right)$

55. (b) For $x \neq 0$, we have,

$$f(x) = x + \frac{x/1+x}{1-\frac{1}{1+x}} = x + \frac{x/1+x}{x/1+x} = x+1$$

For $x = 0, f(x) = 0$. Thus, $f(x) = \begin{cases} x+1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 1 \neq f(0)$.

So, f(x) is discontinuous and hence not differentiable at $x = 0$.

56. (c) Comparing the equation of the circle

$(x+5)^2 + (y-3)^2 = 36$

with $(x-h)^2 + (y-k)^2 = r^2$

$\therefore \ -h = 5$ or $h = -5, k = 3, r^2 = 36 \Rightarrow r = 6$

$\therefore$ Centre of the circle is $(-5,3)$ and radius $= 6$

57. (b) If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{k}$

$|\vec{c}| = \sqrt{(-1)^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

$|\vec{c}| \cdot \vec{a} = \sqrt{5} \cdot (2\hat{i} - 2\hat{j} + \hat{k})$

$\therefore |\vec{c}| \cdot \vec{a} = 2\sqrt{5}\hat{i} - 2\sqrt{5}\hat{j} + \sqrt{5}\hat{k}$

58. (c) One vertex of square is $(-4,5)$ and equation of one diagonal is $7x - y + 8 = 0$

Diagonal of a square are perpendicular and bisect each other

Let the equation of the other diagonal be $y = mx + c$ where m is the slope of the line and c is the y intercept.

Since this line passes through $(-4,5)$

$\therefore\ 5 = -4m + c$

Since this line is at right angle to the line

$7x - y + 8 = 0$ or $y = 7x + 8$, having slope $= 7$,

$\therefore\ 7 \times m = -1$ or $m = \frac{-1}{7}$

Putting this value of m in equation (i) we get

$$5 = -4 \times \left(\frac{-1}{7}\right) + c$$

or $5 = \frac{4}{7} + c$ or $c = 5 - \frac{4}{7} = \frac{31}{7}$

Hence equation of the other diagonal is

$y = -\frac{1}{7}x + \frac{31}{7}$ or $7y = -x + 31$

or $x + 7y - 31 = 0$ or $x + 7y = 31$.

59. (b) $p \Rightarrow q \equiv \sim p \vee q$

60. (b) Put $x^{3/2} = t \Rightarrow \frac{3}{2}x^{1/2}dx = dt$

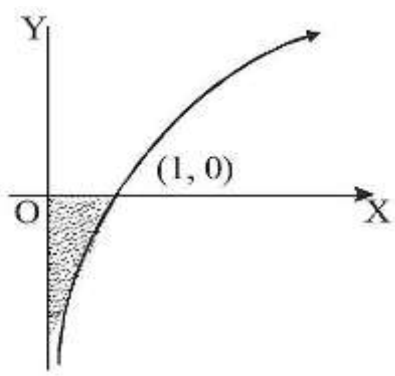
$\therefore$ integral is

$$\int \frac{\frac{2}{3}dt}{\sqrt{1-t^2}} = \frac{2}{3}\sin^{-1}t + C = \frac{2}{3}\sin^{-1}\left(x^{3/2}\right) + C$$

61. (d) In the given sets, the set of all primes is an infinite set.

62. (c) $f(x) = \sqrt{(x-2)(x-3)} + \sqrt{-(x-4)(x+2)}$ The first part is real outside $(2,3)$ and the second is real in $[-2,4]$ so that the domain is $[-2,2] \cup [3,4]$.

63. (b) Observing the graph of $\log x$, we find that the required area lies below x-axis between $x = 0$ and $x = 1$.



So required area $= \left|\int_0^1 \log x dx\right| = |(x\log x - x)|_0^1 = |-1| = 1$

64. (a) Let m_1 and m_2 be slope of curve $y = x^2$ and $6y = 7 - x^3$ respectively.

Now, $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

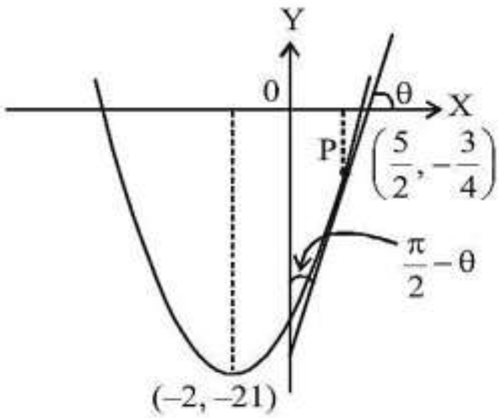
$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2$ i.e. $m_1 = 2$

and $6y = 7 - x^3 \Rightarrow 6\frac{dy}{dx} = -3x^2$

$$\Rightarrow \frac{dy}{dx} = -\frac{3}{6}x^2 = -\frac{1}{2}x^2$$
$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{2}(1)^2 = -\frac{1}{2}$$
$$\therefore m_2 = -\frac{1}{2} \quad \therefore m_1\, m_2 = 2.-\frac{1}{2} = -1$$

$\therefore$ Angle of intersection is $90°$ i.e. $\frac{\pi}{2}$

65. (b)



$y = x^2 + 4x + 4 - 4 - 17$

$y = (x+2)^2 - 21 \Rightarrow$ Vertex is $(-2, -21)$

Also $y = x^2 + 4x - 17 \Rightarrow \frac{dy}{dx} = 2x + 4$

$\Rightarrow$ Slope of tangent at $\left(\frac{5}{2}, -\frac{3}{4}\right)$

$m = \frac{dy}{dx} = 2 \times \frac{5}{2} + 4 = 9; \theta = \tan^{-1} 9$

$\therefore$ angle by y-axis $= \frac{\pi}{2} - \tan^{-1} 9 = \cot^{-1} 9$

66. (d) $(1+i)^4 \times \left(1 + \frac{1}{i}\right)^4 = (1+i)^4 \times (1-i)^4 = 2^4$

67. (d) We have $R = \{(x,y) : |x^2 - y^2| < 16\}$

Let $x = 1, |x^2 - y^2| < 16 \Rightarrow |1 - y^2| < 16$

$\Rightarrow |y^2 - 1| < 16 \Rightarrow y = 1,2,3,4$

Let $x = 2, |x^2 - y^2| < 16 \Rightarrow |4 - y^2| < 16$

$\Rightarrow |y^2 - 4| < 16 \Rightarrow y = 1,2,3,4$

Let $x = 3, |x^2 - y^2| < 16 \Rightarrow |9 - y^2| < 16$

$\Rightarrow |y^2 - 9| < 16 \Rightarrow y = 1,2,3,4$

Let $x = 4, |x^2 - y^2| < 16 \Rightarrow |16 - y^2| < 16$

$\Rightarrow |y^2 - 16| < 16 \Rightarrow y = 1,2,3,4,5$

Let $x = 5, |x^2 - y^2| < 16 \Rightarrow |25 - y^2| < 16$

$\Rightarrow |y^2 - 25| < 16 \Rightarrow y = 4,5$

$\therefore\ R = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,$

$2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1),$

$(4,2), (4,3), (4,4), (4,5), (5,4), (5,5)\}.$

68. **(a)** $\int \frac{2dx}{(e^x+e^{-x})^2} = \int \frac{2e^{2x}}{(e^{2x}+1)^2} dx$

$$= -\frac{1}{(e^{2x}+1)} + c = -\frac{e^{-x}}{e^x + e^{-x}} + c$$

69. **(a)** $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$

$= \frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{2+3}{1-2.3}\right)$ (as $2.3 > 1$)

$$= \frac{5\pi}{4} + \tan^{-1}(-1) = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$$

70. **(b)** Let y denote the number of bacteria at any instant t then according to the question

$$\frac{dy}{dt} \alpha y \Rightarrow \frac{dy}{y} = kdt$$

k is the constant of proportionality, taken to be + ve on integrating (i), we get $\log y = kt + c$

c is a parameter. let y_0 be the initial number of bacteria

i.e., at $t = 0$ using this in (ii), $c = \log y_0$

$$\Rightarrow \log y = kt + \log y_0 \Rightarrow \log \frac{y}{y_0} = kt$$

$y = \left(y_0 + \frac{10}{100} y_0\right) = \frac{11y_0}{10}$, when $t = 2$

So, from (iii), we get $\log \frac{\frac{11y_0}{10}}{y_0} = k$

$$\Rightarrow k = \frac{1}{2} \log \frac{11}{10}$$

Using (iv) in (iii) $\log \frac{y}{y_0} = \frac{1}{2}\left(\log \frac{11}{10}\right) t$

let the number of bacteria become 1,00,000 to 2,00,000 in t_1 hours. i.e., $y = 2y_0$ when $t = t_1$ hours. from (v)

$$\log \frac{2y_0}{y_0} = \frac{1}{2}\left(\log \frac{11}{10}\right) t_1 \Rightarrow t_1 = \frac{2\log 2}{\log \frac{11}{10}}$$

Hence, the reqd. no. of hours $= \frac{2\log 2}{\log \frac{11}{10}}$

71. (a) The given lines are:

$y = (2-\sqrt{3})x + 5$ and $y = (2+\sqrt{3})x - 7$

Therefore, slope of first line $= m_1 = 2-\sqrt{3}$ and slope of second line $= m_2 = 2+\sqrt{3}$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$$

$$= \left|\frac{2\sqrt{3}}{2}\right| = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3} = 60^\circ$$

72. (a) If θ is the angle between line and plane then $\left(\frac{\pi}{2} - \theta\right)$ is the angle between line and normal to plane given by

$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j} + \sqrt{\lambda}\hat{k})}{3\sqrt{4+1+\lambda}}$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{2 - 2 + 2\sqrt{\lambda}}{3 \times \sqrt{5+\lambda}}$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} = \frac{1}{3} \Rightarrow 4\lambda = 5 + \lambda \Rightarrow \lambda = \frac{5}{3}$$

73. (a) Given equation of line is

$r = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})\ (x\hat{i} + y\hat{j} + z\hat{k}) = (2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}$

Any point on the line is

$(2+3\lambda, -1+4\lambda, 2+2\lambda)$

Since it also lie on the plane $r \cdot (\hat{i} - \hat{j} + \hat{k})$

So, $[(2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$

$\Rightarrow 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5 \Rightarrow \lambda = 0$

Therefore, coordinate of the point of intersection of line and plane is $(2, -1, 2)$.

$\therefore$ Distance

$d = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$

74. (a) $I = \int_{\log\sqrt{\pi/2}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right) dx$

Put $e^{2x} = t \Rightarrow 2e^{2x}dx = dt$

When $x = \log\sqrt{\pi/2}, t = e^{2\log\sqrt{\pi/2}}$

$= e^{\log \pi/2} = \frac{\pi}{2}$

When $x = \log\sqrt{\pi}, t = e^{2\log\sqrt{\pi}} = \pi$

$$\therefore I = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\sec^2\left(\frac{1}{3}t\right) dt = \frac{1}{2}\cdot\frac{1}{\frac{1}{3}}\left[\tan\frac{t}{3}\cdot\right]_{\pi/2}^{\pi}$$

$$= \frac{3}{2}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right] = \frac{3}{2}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = \sqrt{3}$$

75. **(c)** We have $\frac{|x+3|+x}{x+2} > 1$

$$\Rightarrow \frac{|x+3|+x}{x+2} - 1 > 0 \Rightarrow \frac{|x+3|-2}{x+2} > 0$$

Now, two cases arise :

Case **I**: When $x + 3 \geq 0$, i.e. $x \geq -3$. Then,

$$\frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{x+3-2}{x+2} > 0$$
$$\Rightarrow \frac{x+1}{x+2} > 0$$
$$\Rightarrow \{(x+1) > 0 \text{ and } x+2 > 0\}$$
$$\text{or } \{x+1 < 0 \text{ and } x+2 < 0\}$$
$$\Rightarrow \{x > -1 \text{ and } x > -2\}$$
$$\text{or } \{x < -1 \text{ and } x < -2\}$$
$$\Rightarrow x > -1 \text{ or } x < -2$$
$$\Rightarrow x \in (-1, \infty) \text{ or } x \in (-\infty, -2)$$

Case II : When $x + 3 < 0$, i.e. $x < -3$

$$\frac{|x+3|-2}{x+2} > 0 \Rightarrow \frac{-x-3-2}{x+2} > 0$$
$$\Rightarrow \frac{-(x+5)}{x+2} > 0 \Rightarrow \frac{x+5}{x+2} < 0$$
$$\Rightarrow (x+5 < 0 \text{ and } x+2 > 0) \text{ or } (x+5 > 0)$$

and $x + 2 < 0$)

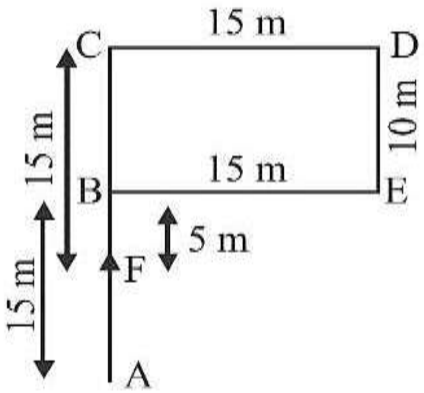
$\Rightarrow (x < -5$ and $x > -2)$ or $(x > -5$ and $x < -2)$ it is not possible.

$\Rightarrow x \in (-5, -2)$

Combining (i) and (ii), the required solution is $x \in (-5, -2) \cup (-1, \infty)$.

PART-IV ENGLISH & LOGICAL REASIONING

76. **(b)** See the last sentence of the fourth paragraph.
77. **(c)** Virtually, the whole passage deals with F.A.S.T. membership requirements. The other choices are too narrow to be main ideas.
78. **(a)** See the first paragraph.
79. **(c)** Let the fixed point from where Jatin starts his journey be A. Also, his walking directions are as follows.



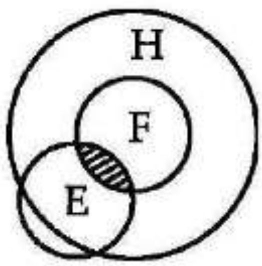
$\therefore$ AF = AB − FB = 15 − 5 = 10 meters

So, Jatin is 10 meters away from the starting point.

80. (b) F-Mohan's family members

E-Employed members

H − Honest members



Here, shaded area denotes the employed members of Mohan's family members, who are honest.