PART – I PHYSICS

1. **(b)** Electrostatic force acting between two point charges in vacuum

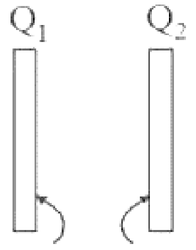
$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

Electrostatic force acting between two point charges in a medium of dielectric constant K is

$$F' = \frac{1}{4\pi(K\epsilon_0)}\frac{q_1q_2}{(r')^2} = \frac{25}{4\pi(5\epsilon_0)}\frac{q_1q_2}{(r)^2}$$

$$\Rightarrow F' = 5\,F$$

2. **(b)**



$V = \frac{Q}{C}$

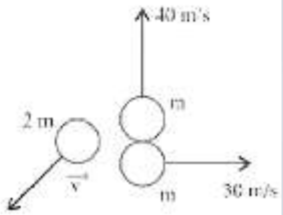
By Gauss law

Charge on inner plates $= \left|\frac{Q_1-Q_2}{2}\right|$

$$\frac{Q_1-Q_2}{2} - \frac{(Q_1-Q_2)}{2}$$

So, $V = \left(\frac{Q_1-Q_2}{2C}\right) = \left(\frac{4-2}{2\times1}\right) = 1\text{ V}$

3. **(b)** By law of conservation of momentum



$$|\vec{P}_i| = |\vec{P}_f|$$

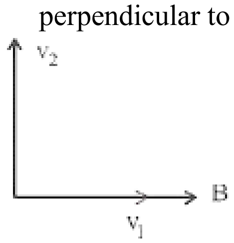
$$0 = m(30\hat{\imath} + 40\hat{\jmath}) + 2m\vec{v}$$

$$\vec{v} = -15\hat{\imath} - 20\hat{\jmath}$$

$$\text{So, } |\vec{v}| = \sqrt{-15^2 + (-20)^2}$$

$$= \sqrt{625} = 25\text{ m/s}$$

4. **(c)** Let v_1 is component of velocity parallel to the magnetic field and v_2 is the component of velocity perpendicular to the magnetic field.



Due to component v_1, magnetic force $F = qv_1 B\sin\theta = 0 (\because \theta = 0)$ So v_1 remains unchanged but due to component v_2 magnetic force act towards centre i.e. moving it circular $F = qvB\sin 90 = qvB$. So path is helical with the axis parallel to magnetic field B.

5. **(a)** In meter bridge experiment, it is assumed that the resistance of the L shaped plate is negligible, but actually it is not so. The error created due to this is called end error. To remove this the resistance box and the unknown resistance must be interchanged and then the mean reading must be taken.

6. **(b)** When metal ball is passing through magnetic field, eddy current will produce and it will oppose the motion, so it will take more time.

7. (c) For A, $u = -15$ cm, $f = -20$ cm

So, $\frac{1}{V_A} + \frac{1}{-15} = \frac{1}{-20} \Rightarrow \frac{1}{V_A} = \frac{1}{15} - \frac{1}{20}$

$$\Rightarrow \frac{1}{V_A} = \frac{20-15}{300} \Rightarrow \frac{1}{V_A} = \frac{5}{300} \Rightarrow V_A = \frac{300}{5} = 60 \text{ cm}$$

For B, $u = -25$ cm, $f = -20$ cm

So, $\frac{1}{V_B} + \frac{1}{-25} = \frac{1}{-20} \Rightarrow \frac{1}{V_B} = -\frac{1}{20} + \frac{1}{25}$

$$\Rightarrow \frac{1}{V_B} = \frac{-5}{500} \Rightarrow V_B = -100 \text{ cm}$$

So, distance between image of A and B are $(100 - (-60))$cm i.e. 160 cm

8. (b) Applying energy conservation between points A & B

$$\frac{1}{2} m V_L^2 = \frac{1}{2} m V_H^2 + mg(2\,L)$$

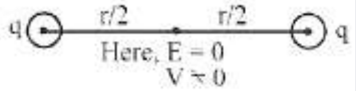
For complete circle,

$$V_L = \sqrt{5gL}$$

$$\therefore V_H = \sqrt{gL}$$

$$\frac{(K \cdot E)_A}{(K \cdot E)_B} = \frac{\frac{1}{2} mV_L^2}{\frac{1}{2} mV} = \frac{\frac{1}{H}(\sqrt{5gL})^2}{\frac{1}{2}\ m(\sqrt{gL})^2} = \frac{5}{1}$$

9. (a) For a group of positive charge, net potential at any point is never zero but electric field can be zero.



10. (a) For the longest wavelength to emit photo electron

$$\frac{hc}{\lambda} = \phi \Rightarrow \lambda = \frac{hc}{\phi} \Rightarrow \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4 \times 1.6 \times 10^{-19}} = 310 \text{ nm}$$

11. (c) Field at the center of a circular coil of radius r is

$$B = \frac{\mu_0 I}{2r}$$

12. (c) Refraction index of the material of the prism

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \cot\frac{A}{2} = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} \left[\because \mu = \cot\frac{A}{2}\right]$$

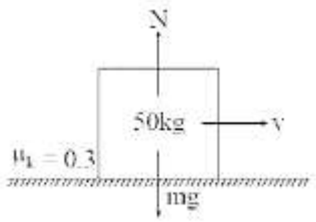
$$\Rightarrow \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right) \Rightarrow \frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{\delta_m}{2}$$

$$\Rightarrow \delta_m = \pi - 2A$$

13. (b)

14. (d) $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \frac{(\sqrt{9}+\sqrt{4})^2}{(\sqrt{9}-\sqrt{4})^2} = \frac{25}{1}$

15. (b)



$F_k = \mu_k N = 0.3 \times 50 \times 9.8 = 147$ N $[\because\ N = mg]$

16. (a) From Brewster's law

$\mu = \tan i_p$

$\Rightarrow \mu = \tan 60^\circ = \sqrt{3}$

Using Snell's law of refraction $\mu = \frac{\sin i}{\sin r}$

$$\Rightarrow \sin r = \frac{\sin i}{\mu} = \frac{\sin 60^\circ}{\sqrt{3}} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow r = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

17. (b) X rays are emitted when target metal is bombarded with high energy electron.

18. (c) Using, $P = \sqrt{2mE}$

Here $E_1 = E_2 \Rightarrow \frac{P_1^2}{2\,m_1} = \frac{P_2^2}{2\,m_2} \Rightarrow \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$

19. (b) Using Gauss's law, $\phi = \frac{q}{\epsilon_0}$

Here, q = charge inside the closed surface

$$\therefore \phi = \frac{q + (-2q) + 5q}{\epsilon_0}$$

$$\Rightarrow \phi = \frac{4q}{\epsilon_0}$$

20. (c) Radius of nucleus, $R = R_0\, A^{\frac{1}{3}} \ldots\ldots (i)$

Density of nucleus $= \frac{\text{Mass of nucleus}}{\text{volume of nucleus}}$

$\rho = \frac{m \times A}{\frac{4}{3}\pi R^3}$ Here m = mass of proton or neutron

from equation (i), we have

$$\rho = \frac{m \times A}{\frac{4}{3}\pi R_0^3\, A} \Rightarrow \rho \propto A^0$$

Hence density of nucleus is independent of mass number.

21. (b) We have

$\Delta l_A = \Delta l_B$

$$\Rightarrow \frac{Fl_A}{Y_A A_1} = \frac{Fl_B}{Y_B A_2} \Rightarrow \frac{5}{\gamma_A \times 2.5 \times 10^{-5}} = \frac{6}{\gamma_B \times 3 \times 10^{-5}}$$

$$\Rightarrow \frac{Y_A}{Y_B} = \frac{15 \times 10^{-5}}{15 \times 10^{-5}} = \frac{1}{1}$$

22. (c) From formula, drift velocity, $V_d = ne\, V_d\, A$

$$\Rightarrow n = \frac{1}{Ae\, V_d} = \frac{10}{5 \times 10^{-6} \times 1.6 \times 10^{-19} \times 2 \times 10^{-3}}$$

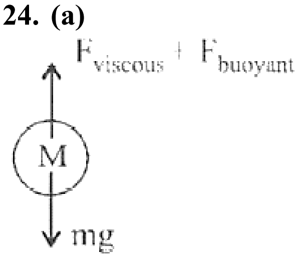
$$= 625 \times 10^{25}$$

23. (c) Using Ampere's circuital law, magnetic field for long straight wire of circular cross section is given by $B = \frac{\mu_0 Ir}{2\pi a^2}$ (For $r \angle a$) $\therefore B \propto r$

For $r > a$

$$B = \frac{\mu_0 I}{2\pi r} \therefore B \propto \frac{1}{r}$$

24. (a)



At terminal velocity, ball move with constant velocity

$\therefore F_{\text{net}} = 0$

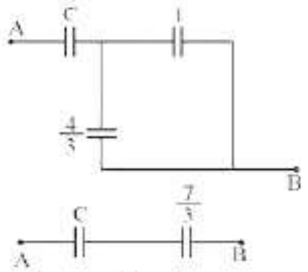
$\Rightarrow F_{\text{viscous}} + F_{\text{buoyant}} = \text{ Weight of ball}$

$\Rightarrow F_{\text{viscous}} + \rho_0 V g = \rho V g$

$\Rightarrow F_{\text{viscous}} = \rho V g - \rho_0 V g = (\rho - \rho_0) V g$

$= \rho V g\left(1 - \frac{\rho_0}{\rho}\right) = \text{Mg}\left(1 - \frac{\rho_0}{\rho}\right)$

25. (a)



For series combination

$$\frac{1}{\text{C}_{\text{eq}}} = \frac{1}{\text{C}_1} + \frac{1}{\text{C}_2} \Rightarrow \frac{\frac{7\text{C}}{3}}{\frac{7}{3} + \text{C}} = \frac{1}{2} \Rightarrow 14\text{C} = 7 + 3\text{C} \Rightarrow \text{C} = \frac{7}{11} \mu \text{ F}$$

26. (c) In Rutherford model, e^-revolve around the nucleus in circular path. Now, according to classical theory of electromagnetism, an accelerating charge radiates energy in form of EM waves and here motion of $e^-$is circular so it is also accelerated, therefore it will radiate energy and finally collapse to nucleus.

27. (b) Given, Height of cylinder, $\text{h} = 20$ cm Acceleration due to gravity, $\text{g} = 10 \text{ ms}^{-2}$

Velocity of efflux $v = \sqrt{2gh}$

Where h is the height of the free surface of liquid from the hole

$\Rightarrow v = \sqrt{2 \times 10 \times 20} = 20 \text{ m/s}$

28. (b) Resistivity (ρ) is given by

$$\rho = \frac{\text{m}}{\text{ne}^2 \tau}$$

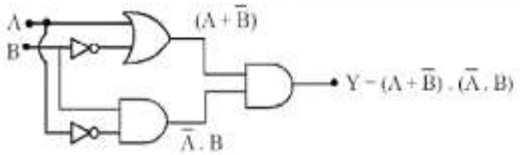
With increase in temperature, number density n increases and relaxation time τ decreases. But n is dominant over τ. Hence, resistivity (ρ) decreases.

29. (c) We know that

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$$

$$\Rightarrow 2 \times 10^{-6} = 2 \times 10^{-7} \times \frac{x^2}{0.2} \Rightarrow 2 = x^2 \Rightarrow x = \sqrt{2} = 1.4$$

30. (c)



Substituting different values of A and B, we get following truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

Hence, output is always zero.

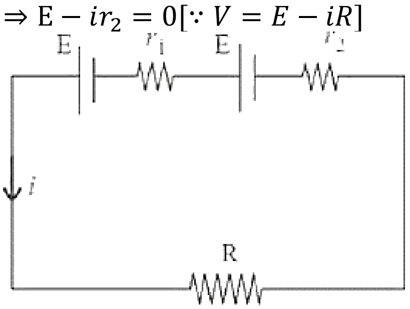
31. (d)

32. (a) We have,

$$i = \frac{E_{net}}{R_{net}} = \frac{2E}{R + x_1 + x_3}$$

As. P.d across second cell = 0

$\Rightarrow E - ir_2 = 0[\because V = E - iR]$



$$\Rightarrow E - \frac{2Er_2}{R + r_1 + r_2} = 0$$

$$\Rightarrow R + r_1 + r_2 - 2r_3 = 0$$

$$\Rightarrow R = r_2 - r_1$$

33. (c)

Number of moles of gas, $n = \dfrac{44.8}{22.4} = 2$

$\Delta Q = nC_V\Delta T[\because V = \text{ constant } \Rightarrow C = C_V]$

$= 2.\frac{3}{2}R \times 20 = 60R = 60 \times 8.3 = 498$ J.

34. (a) Given,

Capacitive reactance, $X_C = 100\Omega$

Inductive reactance, $X_L = 200\Omega$

Resistance, $R = 100\Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{100^2 + (200 - 100)^2} = 100\sqrt{2}\Omega$

RMS value of current,

$$i_{rms} = \frac{V_{rms}}{Z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2 \text{ A}$$

35. (d) We know that

$\Delta l = l_0\alpha\Delta T$

$\Rightarrow (6.241 - 6.230) = 6.230 \times 1.4 \times 10^{-5} \times \Delta T$

$\Rightarrow 0.011 = 6.230 \times 1.4 \times 10^{-5} \times \Delta T$

$\Rightarrow \Delta T = 126.1$

$\Rightarrow T_f - 27 = 126.1$

$\Rightarrow T_f = 153.11^\circ$C.

So nearest option is (d).

PART – II CHEMISTRY

36. (a) $\overline{v} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For second line in lyman series

$n_2 = 3$

$$\therefore \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = R\left[\frac{1}{1} - \frac{1}{9}\right] = \frac{8R}{9}$$

37. (d) In case of anions having same charge as the size of anion increases, polarisibility of anion also increases.

38. (c) In case of transition element, the order of filling of electrons in various orbital is $3p < 4s < 3d$. Thus, $3d$ orbital is filled only when $4s$ orbital gets completely filled.

39. (b)

$$PCl_5 \rightleftharpoons PCl_3 + Cl_2$$

Moles at equilibrium	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Mole fraction at equilibrium	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Partial pressure at equilibrium	$\frac{P}{3}$	$\frac{P}{3}$	$\frac{P}{3}$

$$K_p = \frac{\frac{P}{3} \times \frac{P}{3}}{P/3} = \frac{P}{3} \Rightarrow P = 3\ \mathrm{K_p}$$

40. (d) Cannizzaro reaction is given by aldehydes having no α-hydrogen atom in the presence of conc. alkali. Aldol condensation is given by aldehydes and ketones having at least one α-atom in presence of alkali or in presence of acids.

41. (a) The reaction given is

$Cr_2O_7^{2-} + Fe^{2+} + C_2O_4^{2-} \longrightarrow Cr^{3+} + Fe^{3+} + CO_2$

$\overset{+6}{C}rO_7^{2-} \longrightarrow 2Cr^{3+}$

On balancing

$14H^+ + Cr_2O_7^{2-} + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$ .... (i)

$Fe^{2+} \longrightarrow Fe^{3+} + e^-$ .... (ii)

$C_2O_4^{2-} \longrightarrow 2CO_2 + 2e^-$ ... ... (iii)

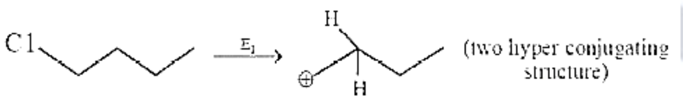
On adding all three equations, we get

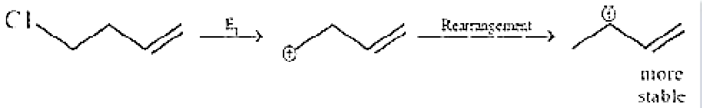
$Cr_2O_7^{2-} + Fe^{2+} + C_2O_4{}^{2-} + 14H^+ + 3e^- \longrightarrow$

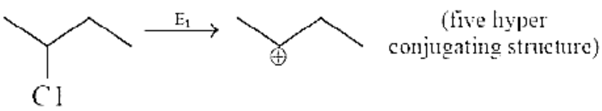
$2Cr^{3+} + Fe^{3+} + 2CO_2 + 7H_2O$

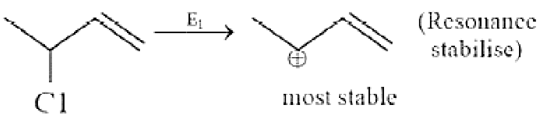
42. (b) For tri or higher substituted benzene derivatives, the compounds are named by identifying substituent positions on the ring by following the lowest locant rule.

43. (a) E_1 reaction proceeds via carbocation formation, therefore greater the stability of carbocation, faster will be the E_1 reaction.

(A) 

(B) 

(C) 

(D) 

Thus correct decreasing order of the given halides towards dehydrohalogenation by E_1 is:

D > B > C > A

44. (c) Molar conductance of solution is related to specific conductance as follows :

$$\Lambda_m = \kappa \times \frac{1000}{C}$$

$$\Lambda_m = (6.3 \times 10^{-2} \mathrm{ohm^{-1}\ cm^{-1}}) \times \frac{1000}{(0.1\ \mathrm{mol/cm^3})}$$

$$= 6.3 \times 10^{-2} \times 10^4 \mathrm{ohm^{-1}\ cm^2\ mol^{-1}} = 630 \mathrm{ohm^{-1}\ cm^2\ mol^{-1}}$$

45. (b) Hydrazine (NH_2NH_2) does not contain carbon and hence on fusion with Na metal, it cannot form NaCN ; consequently hydrazine does not show Lassaigne's test for nitrogen.

46. (a) Salicylic acid, because it stabilizes the corresponding salicylate ion by intramolecular H -bonding.

47. (c) $\Lambda_r H = \Sigma_c H$ (Reactant) $- \Sigma\Lambda_c H$ (Product)
$= 3 \times (-1300) - (-3268) = -632$ kJ mol^{-1}

48. (c) A conformation is defined as the relative arrangement of atoms or groups around a central atom, obtained by the free rotation of one part of the molecule with respect to rest of the molecule. For a complete rotation of 360°, one part may rotate through any degree say $0.1^\circ, 0.5^\circ, 1^\circ$ etc. giving rise to infinite number of relative arrangements of group (atom) around a central atom, keeping other part fixed.

49. (b) The sub-shell are $3d, 4d, 4p$ and $5s, 4d$ has highest energy as $n + \ell$ value is maximum for this.

50. (d) In BrF_5 molecule, there are 5 bond pair and one lone pair of electrons with the central atom.

51. (a) $H = U + PV$ (By definition)
$\Delta H = \Delta U + \Delta(PV)$ at constant pressure
$\Delta H = \Delta U + P\Delta V$

52. (c) Generally, non-metal oxides are acidic in nature and metal oxides are basic in nature, Al_2O_3 is amphoteric.

53. (c) Number of meq. of the acid $= 0.04 \times 100 = 4$
Number of meq. of the base $= 0.02 \times 100 = 2$
$\therefore$ Number of meq. of the acid left on mixing $= 4 - 2 = 2$
Total volume of the solution $= 200$ mL
$\therefore$ No. of meq of the acid present in 1000 mL of the solution
$= 10$
or No. of eq. of the acid in 1000 mL of the solution
$= \frac{10}{1000} = 0.01$
Since the acid is monobasic and completely ionises in solution
$0.01 NHCl = 0.01 MHCl$
Thus $[H^+] = 0.01$
$\therefore pH = -\log(0.01) = -(-2) = 2$

54. (d) [A] is 

55. (b) Number of total ions present in the solution is known as van't Hoff factor (i).
for KCl, $i = 2$
for NaCl, $i = 2$
for $K_2SO_4, i = 3$

56. (b) $k = \frac{0.693}{45} \text{min}^{-1} = \frac{2.303}{t_{99.9\%}} \log \frac{a}{a - 0.999a}$ or
$t_{99.9\%} = \frac{2.303 \times 45}{0.693} \log 10^3 = 448 \text{ min} \approx 7.5 \text{hrs}$

57. (d) Hardy-Schulze rule explains the factors that affect the coagulation or precipitation by an electrolyte.

58. (d)

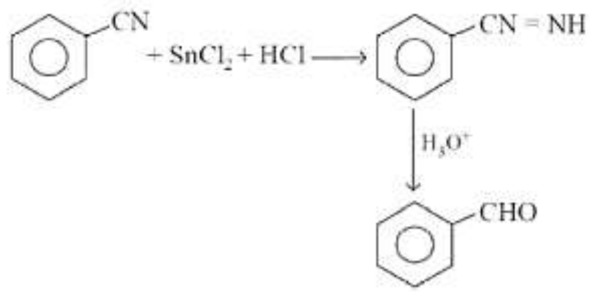
59. (d) Fluorine does not show variable oxidation state as it is the most electronegative element and shows only -1 state.

60. (a) $V = 3d^3 4s^2; V^{2+} = 3d^3 = 3$ unpaired electrons
$Cr = 3d^5 4s^1; Cr^{2+} = 3d^4 = 4$ unpaired electrons
$Mn = 3d^5 4s^2; Mn^{2+} = 3d^5 = 5$ unpaired electrons
$Fe = 3d^6 4s^2; Fe^{2+} = 3d^6 = 4$ unpaired electrons Hence the correct order of paramagnetic behaviour
$V^{2+} < Cr^{2+} = Fe^{2+} < Mn^{2+}$

61. (d) On going from left to right in lanthanoid series ionic, size decreases i.e.,
$Ce^{3+} > Tb^{3+} > Er^{3+} > Lu^{3+}$

62. (d) Oxidation number of hydrogen when it is bonded to metals in binary compounds is -1

63. (b)

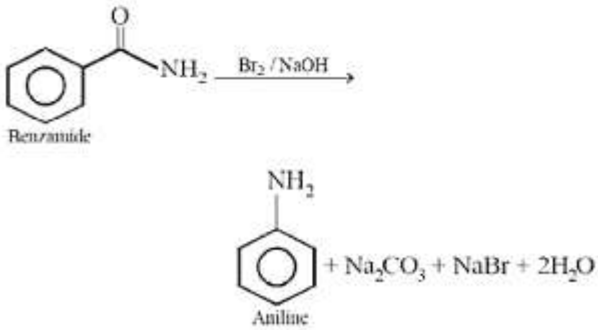


64. (d) All compounds are tertiary alkyl but bond formed between carbon and iodine $(\mathrm{C}-\mathrm{I})$ is weakest bond due to higher difference in size of carbon and iodine.

65. (b) Due to partial double bond character of $\mathrm{C}-\mathrm{OH}$ bond.

66. (d) This reaction proceeds in the presence of anhydrous $\mathrm{AlCl_3}$ or CuCl.

67. (b) Hoffmann bromamide degradation reaction:



68. (b) Ascorbic acid is the chemical name of vitamin C.

69. (b) Nylon $-6,6$ has strong intermolecular H -bonding.

70. (c) Pyridine $(\mathrm{C_5H_5\ N:})$ is a neutral unidentate ligand.

PART – III MATHEMATICS

71. (a) Since, total possible subsets of sets A and B are 2^m and 2^n, respectively.

According to given condition,

$2^m-2^n=56$

$\Rightarrow 2^n(2^{m-n}-1)=2^3\times(2^3-1)$

On comparing both sides, we get

$2^n=2^3$ and $2^{m-n}=2^3$

$\Rightarrow n=3$ and $m-n=3$

$\Rightarrow m=6$ and $n=3$

Now, $m+n=6+3=9$

72. (d) $f(x)=\cos^{-1}\left(\frac{\sqrt{2x^2+1}}{x^2+1}\right)$

$$\therefore = \sin^{-1}\left(\frac{x^2}{x^2+1}\right)=\sin^{-1}\left(1-\frac{1}{x^2+1}\right)$$

$$f(x)=\sin^{-1}\left[\sin^{-1}\left(1-\frac{1}{0+1}\right),\sin^{-1}\left(1-\frac{1}{\infty+1}\right)\right]$$

or $f(x)\in\left[0,\frac{\pi}{2}\right)$

73. (a) We are given that A, P, B are 3×3 matrices and

$B| = 5, |BA^T| = 15, |P^TAP| = -27$

$\because |-B| = 5 \Rightarrow |B| = (-1)^3 \times 5 \Rightarrow |B| = -5$

$|BA^T| = |B| \cdot |A^T| = -5 \times |A^T| \Rightarrow 15 = (-5)|A^T|$

$\Rightarrow |A^T| = -3$

$\because \ |P^TAP| = -27 \Rightarrow |P^T| \cdot |A| \cdot |P| = -27$

$\Rightarrow |P^T| \cdot (-3)|P| = -27 \{\because \ |A| = |A^T|\}$

$\Rightarrow (-3)|P|^2 = -27 \Rightarrow |P|^2 = 9 \Rightarrow |P| = \pm 3$

74. (c) Here, $f(x)$ is continuous on R.

At $x = 0$,

$\Rightarrow f(0) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

$f(0) = \sin 0 = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x^2 + a) = a$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (\sin x) = 0 \ [\because a = 0]$

Now, At $x = 3$,

$f(3) = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$

$f(3) = b(3) + 3 = 3b + 3$

$\lim_{x \to 3^+} f(x) = -3$

$3b + 3 = -3 \Rightarrow b = -2$

$a + b = 0 - 2 = -2$

75. (c) $f(x) = \begin{cases} 3 - x & \text{if} \quad x < -3 \\ 6 & \text{if} \quad -3 \le x \le 3 \\ 3 + x & \text{if} \quad x > 3 \end{cases}$

Firstly check continuity of $f(x)$

At $x = -3$

L.H.L $= \lim_{h \to 0} f(-3 - h) = \lim_{h \to 0} 3 - (-3 - h) = 6$

R.H.L $= \lim_{h \to 0} f(-3 + h) = 6$

$f(-3) = 6$

$\Rightarrow f(x)$ is continuous at $x = -3$

At $x = 3$

L.H.L $= \lim_{h \to 0} f(3 - h) = 6$

R.H.L $= \lim_{h \to 0} f(3 + h) = \lim_{h \to 0} 3 + (3 + h) = 6$

$f(3) = 6$

$\Rightarrow \ f(x)$ is continuous at $x = 3$

$\Rightarrow \ f(x)$ is continuous everywhere

$\Rightarrow \ \alpha = 0$

Now check differentiability of $f(x)$ at $x = -3$

L.H.D $= -1$, R.H.D $= 0$

$\Rightarrow \ f(x)$ is not differentiable at $x = -3$

Now check differentiability of $f(x)$ at $x = 3$

L.H.D $= 0$ R.H.D $= 1$

$\rightarrow \ f(x)$ is not differentiable at $x = 3$

$\Rightarrow \ \beta = 2$

$\Rightarrow \ \alpha + \beta = 2$

$\Rightarrow$ Option (c) is correct.

76. (c) Let $x = \cos^2 \theta$

Now, $\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)\right\}$

$$= \frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}\right)\right\}$$

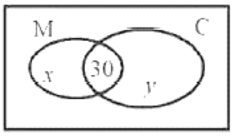
$$= \frac{d}{dx}\{\sin^2(\cot^{-1}(\cot\theta))\}$$

$$= \frac{d}{dx}\sin^2\theta$$

$$= \frac{d}{dx}\left(\frac{1-\cos 2\theta}{2}\right) = \frac{d}{dx}\left(\frac{1-x}{2}\right)$$

$$= \frac{d}{dx}\left(\frac{1}{2}-\frac{x}{2}\right) = -\frac{1}{2}$$

77. (a) Let the number of students who take only Math be x and only Chemistry be y.



So, from the Venn diagram, we have total number of students who take Math $= x + 30$ and take Chemistry $= y + 30$.
According to question, we have

$$30 = \frac{10}{100}(x+30)$$

$$\Rightarrow\ x = 270 \text{ and } 30 = \frac{12}{100}(30+y)$$

$$\Rightarrow\ y = 220$$

$$x + y + 30 = 270 + 220 + 30 = 520$$

78. (a) We have given that,

$$x + 2y + z = 1\ x + 3y + 4z = k$$

$$x + 5y + 10z = k^2$$

$$\therefore\ \Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 1 & 5 & 10 \end{vmatrix}$$

$$= 1(30-20) - 2(10-4) + 1(5-3) = 10 - 12 + 2 = 0$$

$$\Rightarrow \Delta = 0$$

$\therefore$ Given system of equation is consistent.

Therefore, $\Delta_1 = 0$

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 1 \\ k & 3 & 4 \\ k^2 & 5 & 10 \end{vmatrix} = 0$$

$$1(30-20) - 2(10k - 4k^2) + (5k - 3k^2) = 0$$

$$10 - 20k + 8k^2 + 5k - 3k^2 = 0$$

$$5k^2 - 15k + 10 = 0 \Rightarrow k^2 - 3k + 2 = 0$$

$$\Rightarrow (k-2)(k-1) = 0 \Rightarrow k = 2,1$$

Hence, the real values of k i.e.

$A = 2$ and $B = 1$

$A + B = 2 + 1 = 3$

79. (a) Let r, l and h denote respectively the radius, slant height and height of the cone at any time t.

Then, $l^2 = r^2 + h^2$

$$\Rightarrow 2l\frac{dl}{dt} = 2r\frac{dr}{dt} + 2h\frac{dh}{dt}$$
$$\Rightarrow l\frac{dl}{dt} = r\frac{dr}{dt} + h\frac{dh}{dt}$$
$$\Rightarrow l\frac{dl}{dt} = 7 \times 3 + 24 \times (-4)\mathrm{R}$$
$$\left[\because \frac{dh}{dt} = -4 \text{ and } \frac{dr}{dt} = 3\right]$$
$$\Rightarrow l\frac{dl}{dt} = -75$$

Where $r = 7$ and $h = 24$, we have

$$l^2 = 7^2 + 24^2$$
$$\Rightarrow l = 25$$
$$\therefore l\frac{dl}{dt} = -75 \Rightarrow \frac{dl}{dt} = -3$$

Let S denote the lateral surface area Then,

$$\frac{dS}{dt} = \pi\frac{d(rl)}{dt} = \pi\left[\frac{dr}{dt}l + r\frac{dl}{dt}\right]$$
$$= \pi\{3 \times +7 \times (-3)\} = 54\pi \text{ cm}^2/\text{min}$$

80. (d) Let a right angled triangle OAB , with h hypoteneous

Let $\angle \mathrm{OAB} = \theta$

now $\mathrm{OA} = \mathrm{hcos}\, e_\theta$ and $\mathrm{OB} = \mathrm{hsin}_\theta$

$\Rightarrow \text{Area} = \frac{1}{2} \times \mathrm{OA} \times \mathrm{OB} = \frac{1}{4}\mathrm{h}^2(2\sin_\theta \cdot \cos\theta) = \frac{1}{4}\mathrm{h}^2\sin 2_\theta$

For maximum $_\theta = 45^\circ \Rightarrow \frac{1}{4}h^2$

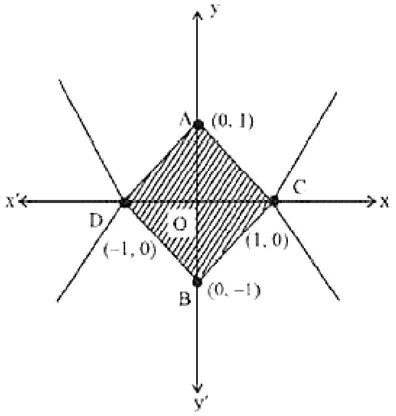
81. (a) $I = \int_0^3 x \cdot f(x^2)dx$

Put $x^2 = t$

$$\Rightarrow 2x \cdot dx = dt$$
$$\Rightarrow I = \frac{1}{2}\int_0^9 f(t)dt = \frac{1}{2} \cdot 4 = 2$$

82. (c) The given curves are

$y - 1 = -|x|$ and $y + 1 = |x|$



Here, $\mathrm{AC} = \sqrt{1+1} = \sqrt{2}$

Area of ACBD = 4 Area of AOC

$$= 4 \times \frac{1}{2} \times \mathrm{OA} \times \mathrm{OC} = 4 \times \frac{1}{2} \times 1 \times 1 = 2$$

83. (b) Given vectors are $\overrightarrow{\mathrm{AB}} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$

and $\overrightarrow{\mathrm{AC}} = 2\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$

Centroid G of triangle has position vector $\overrightarrow{AG}$ can be obtained by putting A as O, B as $2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and C as $2\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$

So, $\overrightarrow{\mathrm{AG}} = \frac{\vec{0}+\mathrm{B}+\mathrm{C}}{3} = \frac{0+2\hat{\imath}+2\hat{\jmath}+\hat{k}+2\hat{\imath}+4\hat{\jmath}+4\hat{k}}{3}$

$\overrightarrow{AG} = \dfrac{4\hat{\imath} + 6\hat{\jmath} + 5\hat{k}}{4}$

Now, $\frac{22}{7} \times |\overrightarrow{AG}|^2 + 5 = 33 + 5 = 38$

84. **(b)** Given lines l_1, l_2 and l_3 are

$$l_1: \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$$

$$l_2: \frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{\alpha}{2}} = \frac{z+5}{2}$$

$$l_3: \frac{x-1}{-3} = \frac{y-\frac{1}{2}}{-2} = \frac{z-0}{4}$$

$$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$$

angle between $l_2 \& l_3$

$$\cos\theta = \frac{\left|1 \times (-3) + (-2)\left(\frac{\alpha}{2}\right) + 2 \times 4\right|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$$

$$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$$

put $\alpha = 3$

$$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

85. **(b)**
$$f(x) = \ln\left(\frac{x^2+e}{x^2+1}\right) = \ln\left(\frac{x^2+1-1+e}{x^2+1}\right)$$
$$= \ln\left(1 + \frac{e-1}{x^2+1}\right)$$

Clearly range is (0,1]

86. **(b)** We have e $< \pi$ and

$$f'(x) = \frac{\frac{1}{\pi+x}\log(e+x) - \frac{1}{e+x}\log(\pi+x)}{\{\log(e+x)\}^2}$$

$$= \frac{(e+x)\log(e+x) - (\pi+x)\log(\pi+x)}{(\pi+x)(e+x)\{\log(e+x)\}^2}$$

In $[0,\infty)$, denominator > 0 and numerator < 0, since, e + x $< \pi$ + x.

Hence, $f(x)$ is decreasing in $[0,\infty)$.

87. **(c)** $\int_{-\pi}^{\pi} x^2(\sin x)dx$

$x^2(\sin x)$ is an odd function

By property of odd & even function

For $f(-x) = -f(x)$ then $\int_{-a}^{a} f(x)dx = 0$

$$\rightarrow \int_{-\pi}^{\pi} x^2(\sin x)dx = 0$$

88. **(a)** We have $x^4\frac{dy}{dx} + x^3y + \operatorname{cosec}(xy) = 0$

$$\Rightarrow x^3\left[x\frac{dy}{dx} + y\right] + \operatorname{cosec}(xy) = 0$$

If $u = xy \Rightarrow \frac{du}{dx} = y + x\frac{dy}{dx}$

$\therefore$ Differential equation becomes

$$x^3\frac{du}{dx} + \operatorname{cosec} u = 0 \ \Rightarrow -\sin u du = x^{-3}dx$$

Now, integrate of getting equation,

$$\Rightarrow \int -\sin u du = \int x^{-3}dx$$

$$\Rightarrow x^{-2} + 2\cos u = c \Rightarrow x^{-2} + 2\cos(xy) = c$$

89. **(b)** The appearance of $\frac{x}{y}$ in the given equation suggests the substitution $\frac{x}{y} = u$

$\Rightarrow dx = udy + ydu$

So, the given equation can be written as $(1 + 2e^u)$

$(udy + ydu) + 2e^u(1 - u)dy = 0$, which is a variable separable form.

$$\Rightarrow \frac{(1 + 2e^u)}{u + 2e^u}du + \frac{dy}{y} = 0$$

$$\Rightarrow \int \left(\frac{1 + 2e^u}{u + 2e^u}\right)du + \int \frac{dy}{y} = 0$$

$$\Rightarrow \log|u + 2e^u| + \log|y| = \log|c|$$

$$\Rightarrow (u + 2e^u)y = c$$

$\Rightarrow \left(x + 2ye^{x/y}\right) = c;$ comparing with given general solution, we get $\lambda = 2$.

90. **(c)** As we know, $\sum_{i=1}^{8} P(x_i) = 1$

$0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$9K + 10K^2 = 1$

$10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K + 1) - 1(K + 1) = 0$

$(K + 1)(10K - 1) = 0$

$\therefore K = -1, \frac{1}{10}$

As the probability cannot be negative. So K must be greater than 0

$\therefore K = \frac{1}{10}$

$P(0 < x < 5) = P(X = 1) + P(X = 2)$

$+P(X = 3) + P(X = 4)$

$= K + 2K + 2K + 3K = 8K$

$= \frac{8}{10}$

91. **(a)** Given lines are $x + y - 5 = 0 \& x - 2y + 3z - 5 = 0$

Direction ratio of line $= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

= Equation of line: $= \frac{x-1}{1} = \frac{x-2}{-4} = \frac{x-4}{-3} = \lambda$

Distance of perpendicular from point $(1, -2, 5)$.

Point $\mathrm{P}(1 + \lambda, 2 - 4\lambda, 4 - 3\lambda)$ and $\mathrm{Q}(1,2,4)$.

$\overrightarrow{PQ}\cdot(\hat{i}-4\hat{j}-3\hat{k})=0, \lambda=\frac{1}{2}$.

Then, point $P\left(\frac{1}{2},2,\frac{-5}{2}\right)$

So, $\overrightarrow{PQ}=\sqrt{\frac{21}{2}}$

92. (a)

$$-\sqrt{2}\leq\sin x+\cos x\leq\sqrt{2}$$
$$\Rightarrow 0\leq|\sin x+\cos x|\leq\sqrt{2}$$
$$\Rightarrow -\sqrt{2}\leq 2|\sin x+\cos x|-\sqrt{2}\leq\sqrt{2}$$

93. (d) Given, $I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x+\frac{\pi}{4}}{2-\cos 2x}dx$

$$I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x}{2-\cos 2x}dx+\frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{2-\cos 2x}dx$$

Let, $I_1=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x}{2-\cos 2x}dx$

$$\because f(-x)=\frac{-x}{2-\cos 2(-x)}$$
$$=\frac{-x}{2-\cos 2x}=-f(x)$$

$f(x)$ is an odd function. Thus, $I_1=0$

Let $I_1=\frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{2-\cos 2x}dx$

$$g(-x)=\frac{\pi}{4(2-\cos 2(-x))}$$
$$=\frac{\pi}{4(2-\cos 2x)}=g(x)$$

$\therefore$ $g(x)$ is an even function

Thus, $I_2=2\times\frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{dx}{2-\cos 2x}$

Now, $I=I_2=\frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{dx}{2-\frac{1-\tan^2 x}{1+\tan^2 x}}$

$$I=\frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{\sec^2 x}{1+3\tan^2 x}dx$$

Put $\tan x=t\Rightarrow\sec^2 x dx=dt$

$$\Rightarrow\ I=\frac{\pi}{4}\int_0^1\frac{dt}{1+3t^2}=\frac{\pi}{2}\times\frac{1}{\sqrt{3}}\left[\tan^{-1}\sqrt{3}t\right]_0^1$$

$$I=\frac{\pi}{2\sqrt{3}}\left[\tan^{-1}(\sqrt{3})-\tan^{-1}(0)\right]=\frac{\pi^2}{6\sqrt{3}}$$

94. (d) Given that A and B are independent events.

$\therefore\ P(A\cap B)=P(A)\cdot P(B)$

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)\cdot P(B)}{P(B)}=P(A)\ \ldots\ldots\text{(i)}$$

And $P(B\mid A)=\frac{P(B\cap A)}{P(A)}=\frac{P(B)\cdot P(A)}{P(A)}=P(B)$

$$P(A \mid B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(A) - P(A \cap B)}{P(B^c)}$$
$$= \frac{P(A) - P(A) \cdot P(B)}{P(B^c)} = \frac{P(A)[1 - P(B)]}{P(B^c)}$$
$$P(A \mid B^c) = \frac{P(A) \cdot P(B^c)}{P(B^c)} = P(A) \ldots\ldots \text{(ii)}$$

From eqns. (i) and (ii)

$P(A \mid B) = P(A \mid B^c)$

95. (b) Equation of tangent of parabola $y = x^2$ be

$$\mathrm{tx} = \mathrm{y} + \mathrm{at}^2 \ldots . (i)$$
$$\mathrm{y} = \mathrm{tx} - \frac{\mathrm{t}^2}{4}$$

Solve with $\mathrm{y} = -(\mathrm{x} - 2)^2$

$$tx - \frac{t^2}{4} = -(x - 2)^2$$
$$x^2 + x(t - 4) - \frac{t^2}{4} + 4 = 0$$

Here, Discriminant $= 0$.

$(t - 4)^2 - 4 \cdot \left(4 - \frac{t^2}{4}\right) = 0 \Rightarrow t^2 - 4t = 0 \Rightarrow t = 0$ or $t = 4$

Put value of t in eq. (i), then $y = 4(x - 1)$.

We know that $\sim (p \rightarrow q) \equiv p \wedge \sim q$

96. (d) $\therefore \sim ((p \wedge r) \rightarrow (r \vee q)) \equiv (p \wedge r) \wedge [\sim (r \vee q)]$

$\equiv (p \wedge r) \wedge (\sim r \wedge \sim q)$

97. (b) The letters other than vowels are: PRMTTN

Number of permutations with no two vowels together is

$\frac{6!}{2!} \times {}^7\mathrm{C}_5 \times 5!$

Further among these permutations the number of cases in which T's are together is $5! \times {}^6\mathrm{C}_5 \times 5!$

So the required number

$= \frac{6!}{2!} {}^7\mathrm{C}_5 \times 5! - 5! \times {}^6\mathrm{C}_5 \times 5! = 57 \times (5!)^2$

98. (c) Given expression is

$$(1 + x)^{101}(1 - x + x)^{100}$$
$$= (1 + x)[(1 + x)(1 - x + x^2)]^{100}$$
$$= (1 + x)(1 + x^3)^{100}$$

$=$ coefficient of x^{50} in $(1 + x)(1 + x^3)^{100}$

$=$ coefficient of x^{50} in $(1 + x^3)^{100}$

coefficient of x^{49} in $(1 + x^3)^{100}$

In the expansion of $(1 + x^3)^{100}$

Since 50 and 49 are not multiple of 3 .

So, coefficient of x^{50} and 49 in the expansion of $(1 + x^3)^{100}$ is 0 .

Coefficient of x^{50} in

$(1 + x)^{100}(1 - x + x^2)]^{101}$ is 0

$1/(q + r), 1/(r + p), 1/(p + q)$ are in A.P.

99. (b) $\Rightarrow \frac{1}{r+p} - \frac{1}{q+r} = \frac{1}{p+q} - \frac{1}{r+p}$

$\Rightarrow q^2 - p^2 = r^2 - q^2 \Rightarrow p^2, q^2, r^2$ are in A.P.

100. **(b)** Let the coordinates of foot of perpendicular from the point P(2,3) (say) on line y $3x + 4$ be $\mathrm{A}(\alpha, \beta)$.

Slope of AP, $m_1 = \frac{\beta - 3}{\alpha - 2}$

Slope of given line, $m_2 = 3$
Since, both are perpendicular.
$\therefore\ m_1 \times m_2 = -1$
$\Rightarrow \dfrac{\beta - 3}{\alpha - 2} \times 3 = -1$
$\Rightarrow 3\beta = -\alpha + 11 \ldots.(\text{i})$
Also, the point $A(\alpha, \beta)$ is on the given line. So, $A(\alpha, \beta)$ satisfy the equation of the line.
$\therefore \beta = 3\alpha + 4 \ldots.(\text{ii})$
On solving (i) and (ii), we get
$\alpha = -\dfrac{1}{10}, \beta = \dfrac{37}{10}$
So, coordinates of foot of perpendicular are $\left(\frac{-1}{10}, \frac{37}{10}\right)$

101. **(a)** The given situation is possible with the 2 circles. Considering centre at C_1, then x-intercept $= PQ = 6$
y-intercept $= 0$
for $x^2 + y^2 + 2gx + 2fy + c = 0 \ldots\ldots(\text{i})$
x-intercept $= 2\sqrt{g^2 - c} = 6, g^2 - c = 9$
y-intercept $= 2\sqrt{f^2 - c} = 0, f^2 = c$
$PB = 1/2PQ = 3$
$OA = C_1 B = 4$
In $\triangle PC_1B$,
$PC_1 = \sqrt{C_1B^2 + PB^2} = \sqrt{3^2 + 4^2}$
$PC_1 = 5$
radius $= \sqrt{g^2 + f^2 - c} = 5$
$g^2 + f^2 - c = 25$
$f^2 = c, g^2 = 25 \Rightarrow g = \pm 5$
$g^2 - c = 9$
$f^2 + 9 = 25, f = \pm 4$
and $c = f^2 = 16$
Equation of circle is
$x^2 + y^2 \pm 10x \pm 8y + 16 = 0$
In given option correct is
$x^2 + y^2 \pm 10x - 8y + 16 = 0$

102. **(b)** Here, the mid-point of AC is
$\left(\frac{4+2}{2}, \frac{-2-5}{2}, \frac{1+10}{2}\right) = \left(3, -\frac{7}{2}, \frac{11}{2}\right)$ and that of BD is
$\left(\dfrac{7-1}{2}, \dfrac{-4-3}{2}, \dfrac{7+4}{2}\right) = \left(3, -\dfrac{7}{2}, \dfrac{11}{2}\right)$
So, the diagonals AC and BD bisect each other.
$\Rightarrow ABCD$ is a parallelogram.
As $|AB| = \sqrt{3^2 + 2^2 + 6^2} = 7$ and
$|AD| = \sqrt{5^2 + 1^2 + 3^2} = \sqrt{35} \neq |AB|$
Therefore, ABCD is not a rhombus and naturally, it cannot be a square.

103. **(b)**
$A + B + C + D = 2\pi$
$\therefore A + B = 2\pi - (C + D)$
$\therefore \tan(A + B) = \tan[2\pi - (C + D)]$
$= -\tan(C + D)$
$\Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\dfrac{\tan C + \tan D}{1 - \tan C \tan D}$
$\Rightarrow (\tan A + \tan B)(1 - \tan C \tan D)$
$= -(1 - \tan A \tan B)(\tan C + \tan D)$

$\Rightarrow \tan A+\tan B-\tan A \tan C \tan D-\tan B \tan C \tan$
$D=-[(\tan C+\tan D-\tan A \tan B \tan C-\tan A \tan B \tan D)]$
$\Rightarrow \tan A+\tan B+\tan C+\tan D$
$=\tan A \tan B \tan C+\tan A \tan C \tan D$
$+\tan A \tan B \tan D+\tan B \tan C \tan D$
Dividing both sides by $\tan A \tan B \tan C \tan D$, we get

$$\frac{\tan A+\tan B+\tan C+\tan D}{\tan A \tan B \tan C \tan D}$$
$$=\frac{1}{\tan D}+\frac{1}{\tan B}+\frac{1}{\tan C}+\frac{1}{\tan A}$$
$$=\cot D+\cot B+\cot C+\cot A$$
$$\Rightarrow \frac{\sin A+\tan B+\tan C+\tan D}{\cot A+\cot B+\cot C+\cot D}$$
$$=\tan A \tan B \tan C \tan D.$$

104. **(d)** Let $\omega=\mathrm{x}+\mathrm{iy}$

If $\frac{\omega-\bar{\omega} z}{1-z}$ is real, then $\frac{x+i y-(x-i y) z}{1-z}$ is real

$$\Rightarrow \frac{y(1+z)}{1-z}=0$$
$$\Rightarrow z=-1 \because y \neq 0 \text{ and } 1-z \neq 0 \therefore |z|=1$$

105. **(a)** Since, roots are equal

$$\therefore\{2(b c+a d)\}^2=4(a^2+b^2)(c^2+d^2)$$
$$\Rightarrow 4 b^2 c^2+4 a^2 d^2+8 a b c d$$
$$=4 a^2 c^2+4 a^2 d^2+4 b^2 c^2+4 b^2 d^2$$
$$\Rightarrow 4 a^2 d^2+4 b^2 c^2-8 a b c d=0$$
$$\Rightarrow 4(a d-b c)^2=0$$
$$\Rightarrow a d=b c \Rightarrow \frac{a}{b}=\frac{c}{d}$$

106. **(d)**

$$L=\lim_{x \rightarrow 0} \frac{35^x-7^x-5^x+1}{(e^x-e^{-x}) \ln (1-3 x)}$$
$$=\lim_{x \rightarrow 0} \frac{(1-7^x)(1-5^x) e^x}{(e^{2 x}-1) \ln (1-3 x)}$$
$$L=\frac{(\ln 5)(\ln 7)}{-6}$$

107. **(c)**

$$z_r=\cos \frac{r \alpha}{n^2}+i \sin \frac{r \alpha}{n^2} \; z_l=\cos \frac{\alpha}{n^2}+i \sin \frac{\alpha}{n^2};$$
$$z_2=\cos \frac{2 \alpha}{n^2}+i \sin \frac{2 \alpha}{n^2}; \ldots \Rightarrow z_n=\cos \frac{n \alpha}{n^2}+i \sin \frac{n \alpha}{n^2}$$

consider $\lim_{n \rightarrow \infty}(z_1 z_2 z_3 \ldots \ldots z_n)$

$$=\lim_{n \rightarrow \infty}\left[\cos \left\{\frac{\alpha}{n^2}(1+2+3+\cdots .+n)\right\}\right.$$
$$\left.+i \sin \left\{\frac{\alpha}{n^2}(1+2+3+\cdots+n)\right\}\right]$$
$$=\lim_{n \rightarrow \infty}\left[\cos \left\{\frac{\alpha n(n+1)}{2 n^2}\right\}+i \sin \left\{\frac{\alpha n(n+1)}{2 n^2}\right\}\right]$$
$$=\lim_{n \rightarrow \infty}\left[\cos \left\{\frac{\alpha\left(1+\frac{1}{n}\right)}{2}\right\}+i \sin \left\{\frac{\alpha\left(1+\frac{1}{n}\right)}{2}\right\}\right]$$
$$=\cos \frac{\alpha}{2}+i \sin \frac{\alpha}{2}=e^{\frac{i \alpha}{2}}$$

108. **(b)** Since, $f(x)$ is a polynomial function satisfying

$$f(x) \cdot f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$$
$$\therefore f(x)=x^n+1 \text{ or } f(x)=-x^n+1$$

If $f(x)=-x^n+1$, then $f(4)=-4^n+1 \neq 65$
So, $f(x)=x^n+1$ Since, $f(4)=65 \therefore 4^n+1=65$

$\Rightarrow n = 3 \therefore f(x) = x^3 + 1 \Rightarrow f'(x) = 3x^2$
$\therefore f'(l_1) = 3l_1^2, f'(l_2) = 3l_2^2, f'(l_3) = 3l_3^2$
Since, l_1, l_2, l_3 are in GP.
$\therefore f'(l_1), f'(l_2), f'(l_3)$ are also in GP.

109. **(b)** Total no. of students in four schools $= 12 + 20 + 13 + 17 = 62$.
Now, one student is selected at random.
$\therefore$ Total outcomes $= 62_{C_b}$
Now, no. of students in school $B_2 = 20$.
No. of ways to select a student from $B_2 = 20\mathrm{C}_1$
$\therefore$ Required probability $= \frac{20}{\mathrm{C}_1}\,{}^{62}\mathrm{C}_1. = \frac{20}{62} = \frac{10}{31}$

110. **(a)** $x \in R \Rightarrow x - x + \sqrt{5} = \sqrt{5}$ is an irrational number.
$\therefore (x, x) \in R$
So, R is reflexive.
$(\sqrt{5}, 1) \in R$ because $\sqrt{5} - 1 + \sqrt{5} = 2\sqrt{5} - 1$
which is an irrational number.
But $(1, \sqrt{5}) \notin R$
$\therefore R$ is not symmetric.
We have, $(\sqrt{5}, 1), (1, 2\sqrt{5}) \in R$ because
$\sqrt{5} - 1 + \sqrt{5} = 2\sqrt{5} - 1$
If $1 - 2\sqrt{5} + \sqrt{5} = 1 - \sqrt{5}$ are irrational numbers.
Also, $(\sqrt{5}, 2\sqrt{5}) \in R$ and $\sqrt{5} - 2\sqrt{5} + \sqrt{5} = 0$
which is not an irrational numbers.
$\therefore (\sqrt{5}, 2\sqrt{5}) \notin R$
So, R is not transitive.

PART – IV APTITUDE TEST

111. **(b)** On two items, savings and house rent, he has to invest more than Rs 1000.

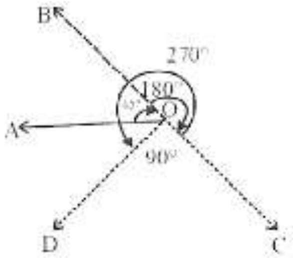
112. **(d)** Savings per month $= 6000 \times 23\% =$ Rs 1380
Annual savings $= 1380 \times 12 =$ Rs16,560

113. **(c)** $10\% = 500 \Rightarrow 100\% =$ Rs. 5000

114. **(d)**

115. **(d)** $3 + 6 = 9$

116. **(d)** Clearly, the man initially faces in the direction OA. On moving 45° clockwise, he faces in the direction OB . On further moving 180° clockwise, he faces in the direction OC. Finally, on moving 270° anticlockwise, he faces in the direction OD, which is South-west. Hence, the answer is (d)



117. **(d)** Without knowing the sex of C, we can't be determined whether B is sister of C or B is brother of C. Similarly without knowing the sex of B we can't be determined whether C is sister of B or C is brother of B . Therefore, both (i) and (ii) are necessary.

118. **(a)** Manav's rank from the top $= 15 - 4 = 11$ th.
Manav's rank from the bottom
$= 20 + 1 - 11 = 10$th

119. **(d)** Some pens are books.
Some books are pens. (conversion) (I-type)

All pens are keys. (A-type) Some books are keys. (I +A = I-type conclusion) Some keys are books. (Conversion) ∴ II follows. Some pens are books. (I-type)

All books are ledgers. (A-type)
Some pens are ledgers.
(I + A = I-type)
Some ledgers are pens. (conversion) (I-type)

All pens are keys. (A-type) Some ledgers are keys.
($I + A = I$-type)
∴ I follows.

120. **(a)** Year 2016 'was a leap year'.
∴ Number of days in 2016 = 366.
= 52 weeks +2 odd days
Now, 1st January 2016 was Friday
So, 30th December, 2016 was also Friday.
Hence, 31th December, 2016 was Saturday.

PART – V
ENGLISH

121. **(b)** The phrase 'on tenterhooks' means a state of suspense or agitation because of uncertainity about a future event.

122. **(d)** The passage clearly shows that money alone can't give happiness.

123. **(a)** All these three points given in the option are discussed in the passage.

124. **(c)** 'Contentment, the key of happiness' suits the best as the title of the passage.

125. **(c)** This fact is clearly mentioned in the passage.