Physics

1. (d)

$$\frac{x-10}{130-10}=\frac{40}{100}$$
$$100x-1000=4800$$
$$100x=5800$$
$$x=58^\circ$$

2. (a)

Given $\phi=4t^2+6t+9$ and $t=2$ s

We know that $\varepsilon=\frac{d\phi}{dt}$

Here, $\frac{d\phi}{dt}=8t+6$

$$\left(\frac{d\phi}{dt}\right)_{t=2}=8\times 2+6$$
$$\varepsilon=22\text{ V}$$

3. (c)

The velocity of water below which the flow remains streamline flow is known as critical velocity.

4. (a)

Given $v=3\times 10^8\text{ ms}^{-1}, d=10\text{ m}$ and $\mu=1.5$

Here,

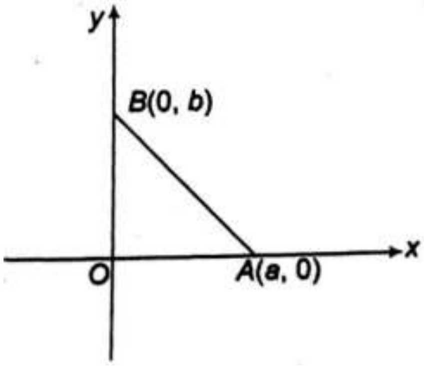
$$t=\frac{d}{c/\mu}$$
$$t=\frac{10\times 10^{-2}}{2\times 10^8}$$
$$\left(\therefore \frac{c}{\mu}=\frac{3\times 10^8}{1.5}=2\times 10^8\text{ ms}^{-1}\right)$$
$$t=0.5\times 10^{-9}\text{ s}$$

5. (a)

We know that

and

$$W=q\Delta V$$
$$V=\frac{Q}{4\pi\varepsilon_0 r}$$

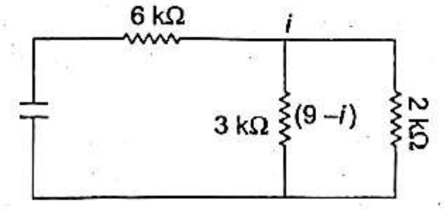


Hence,

$$W=q\left[\frac{Q}{4\pi\varepsilon_0 b}-\frac{Q}{4\pi\varepsilon_0 a}\right]$$
$$=\frac{Qq}{4\pi\varepsilon_0}\left[\frac{a-b}{ab}\right]$$

6. (a)

From KCL we have



$$i=\frac{72}{8\times10^{3}}=9\times10^{-3}\text{ A}$$

$i\times6=(9-i)\times3$

$i=3\text{ mA}$

7. (d)

Given $\mathbf{E}=(2\mathbf{i}+3\mathbf{j}+\mathbf{k})\text{NC}^{-1}$ and $\mathbf{S}=10\mathbf{i}\ \mathbf{m}^2$

We know that

$\phi=\mathbf{E}\cdot\mathbf{S}$

$\phi=(2\mathbf{i}+3\mathbf{j}+\mathbf{k})\cdot(10\mathbf{i})$

$\phi=20\text{Nm}^2\text{C}^{-1}$

8. (c)

Given $v=340\text{ ms}^{-1}, u_s=20\text{ ms}^{-1}$ and $v_0=640\text{ Hz}$

From Doppler's law,

$$v=\left(\frac{340}{340-20}\right)640$$

$v=680\text{ Hz}$

9. (d)

Given $i=10\text{ A}, B=0.15\text{ T}, \theta=45^\circ$ and $l=2\text{ m}$ Here,

$F=ilB\sin\theta$

$=10\times2\times0.15\sin45^\circ$

$$=\frac{3}{\sqrt{2}}\text{ N}$$

10. (b)

Given,

$$x_1=A\sin\left(\omega t+\frac{\pi}{6}\right)$$

$$x_2=A\cos(\omega t)$$

$$x_2=A\cos t\left(\omega t+\frac{\pi}{2}\right)$$

Phase difference

$\Delta\phi=\phi_2-\phi_1$

$$\Delta\phi=\frac{\pi}{2}-\frac{\pi}{6}$$

$$\Delta\phi=\frac{3\pi-\pi}{6}=\frac{\pi}{3}$$

11. (a)

We know $H=\frac{V^2}{R}$ Here, in second condition V' becomes $\frac{V}{3}$ and R' becomes $2R$. So,

$$H'=\frac{\left(\frac{V}{3}\right)^2}{2R}=\frac{V^2}{18R}$$

$$H'=\frac{H}{18}$$

12. (d)

Given $Z_A:Z_B=1:2$ We know

$v\propto Z^2$

Hence,

$$\frac{v_A}{v_B}=\frac{(1)^2}{(2)^2}$$

$v_A:v_B=1:4$

13. (b)

de-Broglie wavelength

$$\lambda = \frac{h}{mv}$$

Here,

$$\lambda_e = \frac{h}{m_e \frac{c}{2}} \text{ and } \lambda_p = \frac{h}{m_p c}$$

Given,

$$\frac{\lambda_e = \lambda_p}{m_e \frac{c}{2}} = \frac{h}{m_p c}$$

$$\frac{m_e}{m_p} = 2$$

Ratio of KE

$$\frac{K_e}{K_p} = \frac{\frac{1}{2} m_e v_e^2}{\frac{1}{2} m_p v_p^2}$$

$$\frac{K_e}{K_p} = \frac{1}{2}$$

14. (b)

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$E = \frac{\sigma}{\varepsilon_0}$ towards negative charged plate.

15. (d)

Given, $m = 2$ kg, $\mu = 0.2$ and $g = 10\ \text{ms}^{-2}$

Here,

$$ma = \mu mg$$

$$a = \mu g$$

$$a = 0.2 \times 10$$

$$a = 2\ \text{m/s}^2$$

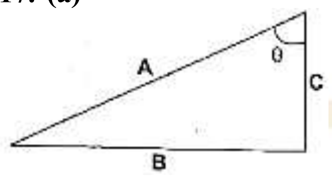
16. (c)

Dimension of angular momentum

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

$$= [\text{L}][\text{MLT}^{-1}] = [\text{ML}^2\ \text{T}^{-1}]$$

17. (a)



$$\theta = \cos^{-1}\left(\frac{|\mathbf{C}|}{|\mathbf{A}|}\right)$$

$$\theta = \cos^{-1}\left(\frac{3}{5}\right)$$

18. (b)

Given,

$$x = 37 + 27t - t^3$$

$$v = \frac{dx}{dt} = 27 - 3t^2$$

According to problem,

$$v = 0 \Rightarrow 27 - 3t^2 = 0$$

Here, $t = 3$ s

$$x = 37 + 27 \times 3 - (3)^2 = 91\ \text{m}$$

19. (c)

At ground, $E=\frac{1}{3}mu^2$

At highest point,

$$E'=\frac{1}{2}m(u\cos 60^\circ)^2$$

$$E'=\frac{E}{4}$$

20. (c)

Given $S=30\text{ cm}=30\times10^{-2}$ m and loss in velocity is 50%

Equation of motion

$$v^2=u^2+2as$$

Here, $$\frac{u^2}{4}=u^2+2a\times30\times10^{-2}$$

$$0=\frac{u^2}{4}+3a\times x$$

On solving Eqs. (i) and (ii), we get

$x=10$ cm

21. (b)

For 1st situation,

$E=\frac{1}{2}k(10\times10^{-2})^2$……..(i)

For 2nd situation,

$E'=\frac{1}{2}k(20\times10^{-2})^2$……..(ii)

On comparing Eqs. (i) and (ii), we get

$E'=4E$

22.(b)

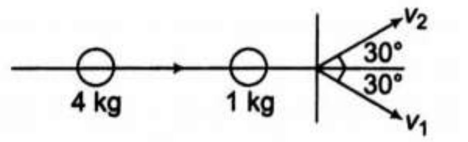
By Keplar's law,

Hence

$$T^2\propto R^3$$

$$T=D\left(\frac{L_2}{L_1}\right)^{3/2}$$

23.(a)



Along y-axis momentum remains zero. Here

$4v_1\sin 30^\circ=v_2\sin 60^\circ$

$$\frac{v_1}{v_2}=\frac{\sqrt{3}}{4}$$

24. (c)

Given, binary number $=1011001$ Its equivalent decimal number

$=2^\circ+0+0+2^3+2^4+2^6$

$=1+8+16+64=89$

25. (d)

Here,

$$v_1=\frac{(2n-1)}{4l_1}v$$

$$v_2=\frac{n}{2l_2}v$$

Hence,

$v_1=v_2$

$$\frac{l_1}{l_2}=\frac{3}{4}$$

26. (b)

R increases with temperature and slope of $V-I$ graph gives resistance so $T_1 < T_2$.

27. (a)

$(m+1)$ vernier division $= m$ main scale division One division on vernier scale $= \left(\frac{m}{m+1}\right)$ division on main scale

Vernier constant $= \left(1-\frac{m}{m+1}\right) d = \frac{d}{m+1}$

28. (c)

Given, $h = 80\ \text{m}, v = 8\ \text{ms}^{-1}$ and $g = 10\ \text{ms}^{-2}$ Here,

$$h = \frac{1}{2} g t^2$$

$$80 = \frac{1}{2} \times 10 \times t^2$$

$$t = 4\ \text{s}$$

and $x = vt = 8 \times 4 = 32\ \text{m}$

29. (d)

In balanced Wheatstone bridge,

$$\frac{1}{10} = \frac{1}{x} + \frac{1}{30}$$

$$x = 15\Omega$$

30. (c)

We know that

Given

$$R = \frac{V^2}{P}$$

and

$V = 200\ \text{V}, P = 40\ \text{W}$

Hence,

$V' = 100\ \text{V}$

$$R = \frac{V^2}{P} = \frac{200 \times 200}{50}$$

$$P' = \frac{V^2}{R} = \frac{100 \times 100 \times 50}{200 \times 200}$$

$$P' = 12.5\ \text{W}$$

31. (d)

Capacitors in series

$2C_1 \times V_1 = C \times V_2$

$2V_1 = V_2$

and $V_1 + V_2 + V_1 = 60$

Solving Eqs. (i) and (ii), we get

$V_2 = 30\ \text{V}$

32. (d)

Capacitors in series

$2C_1 \times V_1 = C \times V_2$ ....(i)

$2V_1 = V_2$

and $V_1 + V_2 + V_1 = 60$.........(ii)

Solving Eqs. (i) and (ii), we get

$V_2 = 30\ \text{V}$

33. (d)

Balancing equation

$W = mg - V\rho g$

Hence,

$44 = m - 1.2\ V$

$50 = m - V$

On solving Eqs. (i) and (ii) we get

$m = 80\ \text{g}$

34. (b)

Ist case

$hv = hv_0 + eV_0 \ldots$(i)

$\frac{hv}{2} = hv_0 + \frac{eV_0}{4} \ldots\ldots$(ii)

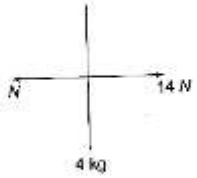
Solving Eqs. (i) and (ii), we get

$V_0 = \frac{v}{3}$

35. (b)

We know that, $F = ma$

$a = \frac{F}{m} = \frac{14}{7} = 2 \text{ ms}^{-2}$



Hence, from the figure

$14 - N = 4a$

14-N=8

N = 6N

36. (c)

We know

$\Delta l = \frac{F \times l}{A \times Y} = \frac{4L}{\pi d^2}$

Here $\frac{4}{\pi}$ is a constant Hence, $\Delta l \propto \frac{L}{d^2}$

37. (d)

Given $v_1 = 3x$ and $v_2 = 2(x+5)$

So here,

Ist case

$\frac{1}{-x} + \frac{1}{-3x} = \frac{1}{f} \ldots\ldots\ldots\ldots$(i)

$\frac{1}{-(x+5)} + \frac{1}{-2(x+5)} = \frac{1}{f} \ldots\ldots$(ii)

On solving Eqs. (i) and (ii) we get $f = 30$ cm

38. (a)

Heat required to convert ice to water at 100°C

$Q = m \times L + ms\Delta T = 18000\text{cal}$

Amount of heat left $= 4320\text{cal}$

$m \times L = 4320$

$m = 8$ g steam

39. (d)

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$$

$$v = \frac{2\pi}{\lambda} VA$$

$$(v_p)_{\max} = 3v$$

$$\lambda = \frac{2\pi}{3} A$$

40. (b)

We know that

$N = N_0 e^{-\lambda t}$

For A,

$N_A = N_0 e^{-5\lambda t}$

For B,

$N_B = N_0 e^{-\lambda t}$

Given,

$$\frac{N_A}{N_B}=\frac{1}{e^2}$$

$$\frac{N}{N_B}=\frac{1}{e^{4\lambda t}}$$

$$t=\frac{1}{2\lambda}$$

CHEMISTRY

41. (a)

In the electrochemical series, metals are arranged in increasing order of their standard reduction potential. The standard reduction potentials of Li^+/Li and Cu^{2+}/Cu are -3.05 V and +.034 V respectively. Therefore, Li occupies higher position in series as compared to Cu .

42. (d)

$${}_{11}\mathrm{Na}^{24} \longrightarrow \underset{\text{(stable)}}{12\mathrm{Mg}^{24}} + {}_{-1}\beta^0$$

43. (a)

$B_2 = 5+5 = 10e^-$

$$=\sigma 1s^2 \overset{*}{\sigma} 1s^2, \sigma 2s^2 \overset{*}{\sigma} 2s^2, \pi 2p_x^1, \pi 2p_y^1$$

Due to the presence of 2 unpaired electrons, in π bonding orbitals, B_2 shows paramagnetic behaviour.

44. (c)

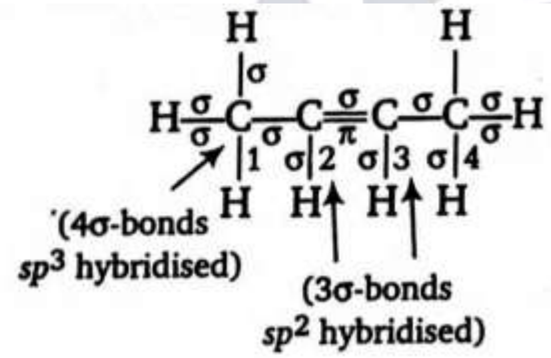
Ammonium acetate is a salt of weak acid and weak base. When solutions of such salts diluted resulting pH of solution is calculated from following relationship

$$\mathrm{pH}=7+\frac{1}{2}\mathrm{p}K_a-\frac{1}{2}\mathrm{p}K_b$$

$\because$ $\mathrm{p}K_a \simeq \mathrm{p}K_b$

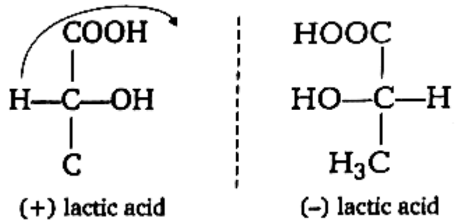
$\therefore$ pH of diluted solution $=7$

45. (c)

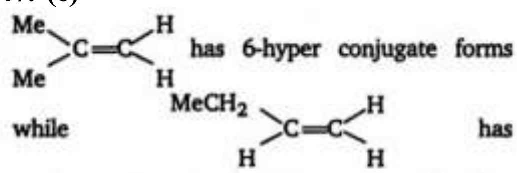


46. (d)

Both are optical isomers because they rotate the plane of polarised light in opposite directions.



47. (c)



2-hyperconltjugate forms. Therefore, $Me_2C=CH_2$ is more stable than $MeCH_2CH=CH_2$.

48. (a)

Electrophiles are electron deficient species (neutral or cationic). They attack on electron rich C-atom.

49. (b)

$$\mathrm{Me_3CONa} \underset{-\mathrm{NaCl}}{\xrightarrow[(1^\circ \text{ alkyl halide })]{\mathrm{EtCl}}} \mathrm{Me_3COEt}$$

This reaction is known as Williamson's synthesis.

50. (a)

Elevation in boiling point, $\Delta T_b = i \times K_b \times m$

Molality of NaCl solution $= \frac{n}{W} \times 1000 = \frac{\frac{58.5}{58.5}}{W_{H_2O}} \times 1000 = \frac{1000}{W_{H_2O}}$

Molality of

$$C_6H_{12}O_6 \text{ solution } = \frac{\frac{180}{180} \times 1000}{W_{H_2}O}$$

$$= \frac{1000}{W_{H_2}O}$$

Both the solutions have same molarity but the values for NaCl and glucose are 2 and 1 respectively.

$\therefore \ \Delta T_{b(\mathrm{NaCl})} = 2 \times \Delta T_{b(C_6H_{12}O_6)}$

51. (c)

Ratio of number of moles of CH_4 and H_2 are

$$CH_4 : H_2 = \frac{x}{16} : \frac{x}{2}$$
$$= 1:8$$

Hence, partial pressure exerted by

$$H_2 = \frac{8}{1+8} \times p_{\text{total}} = \frac{8}{9} \times p_{\text{total}}$$

52. (d)

Acetone-chloroform

53. (c)

Equilibrium constant is independent of the concentration of reactants.

54. (b)

$\Delta G_{(T,P)} = \Delta H - T\Delta S$ determines the existence of a reaction.

55. (a)

Molecularity of a reaction can never be fractional. It is always a whole number.

56. (d)

From Heisenberg uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

or

$$\Delta x \cdot m\Delta v \geq \frac{h}{4\pi}$$

$$\Delta x \cdot \Delta v \geq \frac{h}{4\pi m}$$

or

This principle is also applicable for pairs like energy-time ($\Delta E \cdot \Delta t$) and angular moment-angle ($\Delta w \cdot \Delta \theta$) along with position-moment ($\Delta x \cdot \Delta p$).

Thus,

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

57. (b)

$_{82}\mathrm{Pb} = 6s^2 6p^2$

$_{83}\mathrm{Bi} = 6s^2 6p^3$

Due to inertness of $6s^2$ electrons in Pb and Bi , they show bivalency and trivalency respectively instead of tetra and pentavalency.

58. (c)

In acidic medium,

$K_2\overset{+6}{C}r_2O_7 \longrightarrow Cr^{3+}$

Equivalent weight of

$$K_2Cr_2O_7 = \frac{\text{Molecular weight}}{\text{Change in oxidation state of Cr}}$$

$$= \frac{M}{2(6-3)} = \frac{M}{6}$$

59. (a)

Species	Atomic number	Number of electrons
Ca^{2+}	20	$20-2=18$
Cl^-	17	$17+1=18$
S^{2-}	16	$16+2=18$

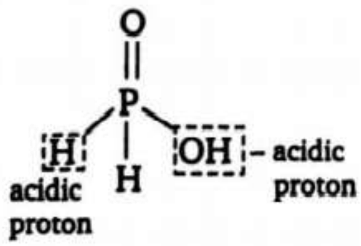
For isoelectronic species, an increase in atomic number, result in decreased size. Thus, order of radius will be $Ca^{2+} < Cl^- < S^{2-}$

60. (a)

$$\overset{\pi}{\longleftarrow}$$
$$C \underset{\sigma}{\overset{\pi}{=}} O$$

In CO molecule, $C-O\sigma$ moment and $O-C$ π-moment cancels each other. So that it is non-polar.

61. (c)

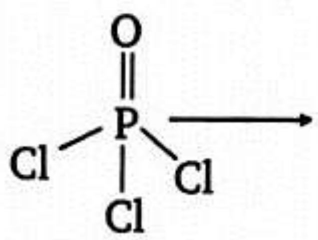


There are two acidic protons in H_3PO_3.

62. (d)

$$K_2Cr_2O_7 + H_2SO_4 + \underset{K_2SO_4}{4H_2O_2} \rightarrow K_2SO_4 + 5H_2O + \underset{\text{dark blue}}{2CrO_5}$$

63. (c)



4σ bonds $\Rightarrow sp^3$ hybridisation without lone pair of electrons.

64. (d)

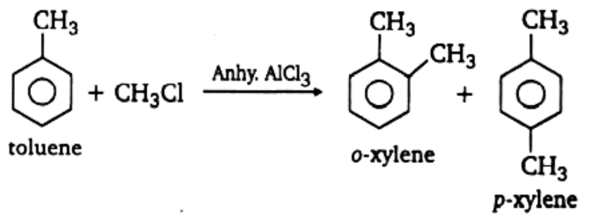
$$CH_3CH_2CH_2CH(OH)CH_3 \xrightarrow{[O]} \underset{\text{keto methyl group}}{CH_3CH_2CH_2COCH_3}$$

$$CH_3CH_2CH_2\overset{O}{\overset{||}{C}}-CH_3 + 3I_2 + 4NaOH \rightarrow CH_3CH_2CH_2\overset{O}{\overset{||}{C}}-ONa + \underset{\text{iodoform}}{CHI_3} + 3H_2O + 3NaI$$

65. (a)

$$\underset{\text{ethyl acetate}}{CH_3COOCH_2CH_3} \xrightarrow[\text{Tischenko reaction}]{Al(OC_2H_5)_3} \underset{\text{ethyl acetate}}{CH_3COOCH_2CH_3}$$

66. (d)



Friedel-crafts reaction is an electrophilic substitution reaction and presence of an electron releasing substituent like Me, makes the benzene nucleus more reactive towards such reactions)

Thus toluene is most reactive among the given

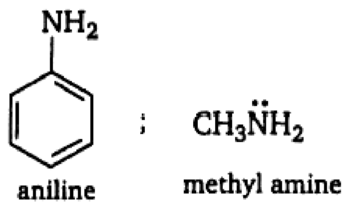
67. (b)

Phenol is a weak acid. It reacts with NaOH to produce salt.

$$C_6H_5OH + NaOH \longrightarrow \underset{\textit{sodium phenoxide}}{C_6H_5ONa} + H_2O$$

But it is not sufficiently acidic to evolve CO_2 from $NaHCO_3$ solution.

68. (b)



The lone pair of electrons, present in N -atom in aniline involve in ring resonance. So it is less basic than methyl amine.

69. (a)

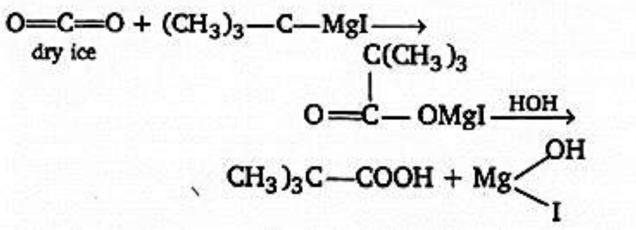
S_{N^1} reaction is most effective in 3° carbon.

$$\underset{\text{tert-butyl chloride}(3^\circ)}{CH_3)_3-C-Cl} \quad \underset{\text{1-chlorobutane}(1^\circ)}{CH_3CH_2CH_2CH_2Cl}$$

$$\underset{\text{2-methyl 1 -chloropropane}(1^\circ)}{Cl-CH_2-CH\,(CH_3)-CH_3}$$

$$\underset{\text{2-chlorobutane}(2^\circ)}{CH_3CH_2CH(Cl)CH_3}$$

70. (d)



71. (b)

$$\begin{array}{lcc} & CH_3COOH & + \quad NaOH \\ \text{Initial} & 20\times0.1=2\text{ mmol} & 10\times0.1=1\text{ mmol} \\ \text{At time } t & (2-1)=1\text{ mmol} & (1-1)=0\text{ mmol} \end{array}$$

$$\longrightarrow \underset{1\text{ mmol}}{CH_3COONa} + H_2O$$

From Henderson's equation,

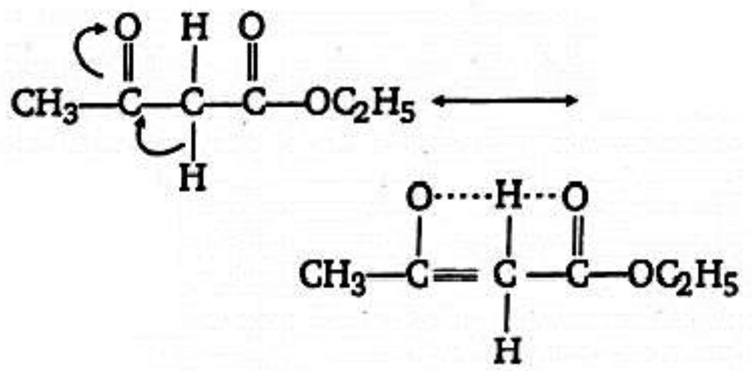
$$pH = pK_a + \log\frac{[CH_3COONa]}{[CH_3COOH]}$$

$$= 4.74 + \log\frac{1}{1} = 4.74$$

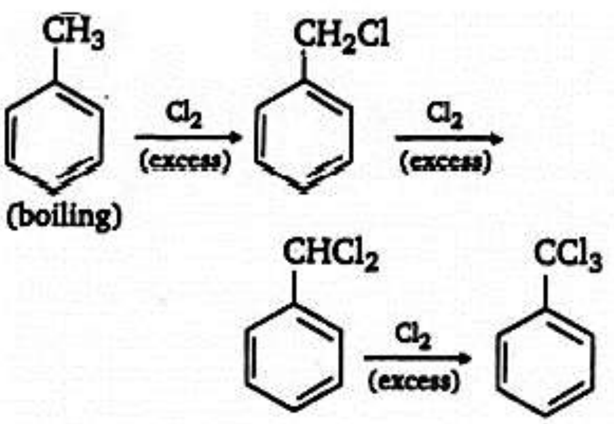
72. (a)

In Fe-complexes, NO behaves as NO^+.

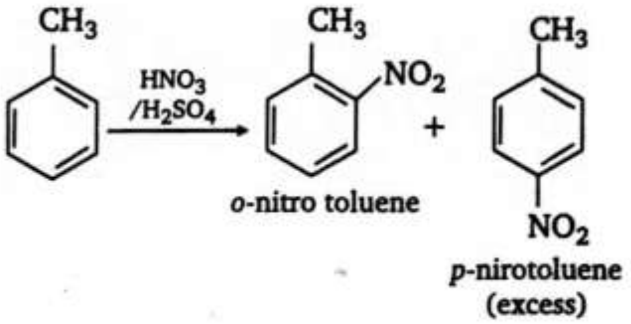
73. (b)

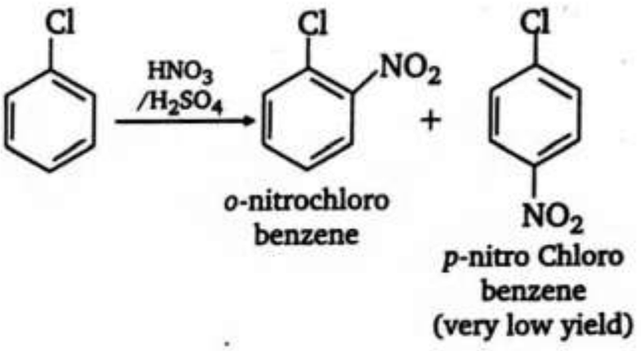


74. (b)

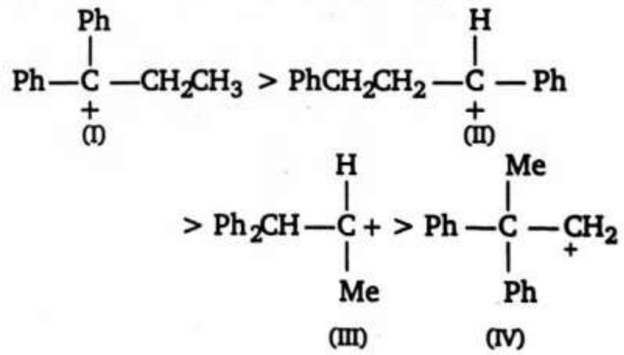


75. (a)





76. (b)



The order of stability of carbocation is 3° benzylic > 2° benzylic > 2° > 1°.
Thus, the correct order of stability will be

77. (a)

Water evaporates into steam. In this physical state its disorder or randomness increases.

78. (d)

$$m=\frac{ECt}{F}$$
$$=\frac{\frac{63.5}{2}\times C\times t}{96500}=\frac{31.75\times C\times t}{96500}$$

79. (c)

$$\text{Second ionisation energy } = 13.6\times\frac{2^2}{1^2}$$
$$=54.4\text{eV}$$

80. (c)

$$w=\frac{NEV}{1000}=\frac{1}{20}\times\frac{63\times 1000}{1000}=\frac{63}{20}\text{ g}$$

Mathematics

81. (a)

Since, $(\alpha+\sqrt{\beta})$ and $(\alpha-\sqrt{\beta})$ are the roots of the equation $x^2+px+q=0$

$\therefore$ Sum of roots $= -p$

$\Rightarrow \quad (\alpha+\sqrt{\beta})+(\alpha-\sqrt{\beta}) = -p$

$\Rightarrow \quad 2\alpha = -p \Rightarrow \alpha = -\dfrac{p}{2}$

and product of roots $= q$

$(\alpha+\sqrt{\beta})(\alpha-\sqrt{\beta}) = q$

$\Rightarrow \quad \alpha^2-\beta = q$

$\Rightarrow \quad \beta = \alpha^2 - q = \left(-\dfrac{p}{2}\right)^2 - q = \dfrac{p^2}{4} - q$

$\Rightarrow \quad p^2 - 4q = 4\beta$

$\therefore$ The given equation

$(p^2-4q)(p^2x^2+4px)-16q = 0$

$4\beta(p^2x^2+4px)-16(\alpha^2-\beta) = 0$

$[\alpha^2-\beta = q]$

$\beta(4\alpha^2x^2-8\alpha x)-4(\alpha^2-\beta) = 0$

$\alpha^2\beta x^2 - 2\alpha\beta x + \beta = \alpha^2$

$\Rightarrow (\alpha x\sqrt{\beta}-\sqrt{\beta})^2 = \alpha^2$

$\Rightarrow \alpha x\sqrt{\beta}-\sqrt{\beta} = \pm\alpha$

$\therefore x = \dfrac{1}{\alpha} \pm \dfrac{1}{\sqrt{\beta}}$

$\Rightarrow x = \left(\dfrac{1}{\alpha}+\dfrac{1}{\sqrt{\beta}}\right)$ and $\left(\dfrac{1}{\alpha}-\dfrac{1}{\sqrt{\beta}}\right)$

82. (c)

Given $\log_2(x^2+2x-1) = 1$

$\Rightarrow \quad \log_2(x^2+2x-1) = \log_2 2$

$\Rightarrow \quad x^2+2x-1 = 2$

$\Rightarrow \quad x^2+2x-3 = 0$

$\Rightarrow \quad x^2+3x-x-3 = 0$

$\Rightarrow \quad x(x+3)-1(x+3) = 0$

$\Rightarrow \quad (x+3)(x-1) = 0$

$\Rightarrow \quad x = 1, -3$

Since, $x = 1$ and $x = -3$ are satisfy the given equation therefore the number of solutions of the equation are two.

83. (a)

$$1+\frac{1}{2}\,{}^nC_1+\frac{1}{3}\,{}^nC_2++\frac{1}{n+1}\,{}^nC_n$$

$$=\frac{1}{n+1}\left[(n+1)+\frac{(n+1)n}{2!}\right.$$
$$\left.+\frac{(n+1)n(n-1)}{3!}++1\right]$$

$$=\frac{1}{n+1}[\,{}^{n+1}C_1+\,{}^{n+1}C_2++\,{}^{n+1}C_{n+1}]$$

$$=\frac{1}{n+1}[\,{}^{n+1}C_0+\,{}^{n+1}C_1+\,{}^{n+1}C_2$$

$$=\frac{1}{n+1}(2^{n+1}-1)$$

84. (c)

$$\textstyle\sum_{r=2}^{\infty}\frac{(r-1)r}{2r!} = \sum_{r=2}^{\infty}\frac{1}{2(r-2)!}$$

$$=\frac{1}{2}\left[\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\infty\right] = \frac{1}{2}e$$

85. (a)

Given, $P=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}$

$\therefore\ Q=PP^{T}=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 2 & 3\\ 1 & 1\end{bmatrix}$

$=\begin{bmatrix}1\times1+2\times2+1\times1 & 1\times1+2\times3+1\times1\\ 1\times1+3\times2+1\times1 & 1\times1+3\times3+1\times1\end{bmatrix}$

$=\begin{bmatrix}1+4+1 & 1+6+1\\ 1+6+1 & 1+9+1\end{bmatrix}=\begin{bmatrix}6 & 8\\ 8 & 11\end{bmatrix}$

$\therefore$ The determintant of $Q=\begin{bmatrix}6 & 8\\ 8 & 11\end{bmatrix}$

$=66-64=2$

86. (b)

Since, $1!=1,2!=2,3!=6,4!=24,5!=120$ Since, all terms from 5 ! onwards are divisible by 15 and $1!+2!+3!+4!=33$ $\therefore$ The required remainder after dividing by 15 will be 3 .

87. (b)

Let $\Delta=\begin{vmatrix}-1 & \cos R & \cos\theta\\ \cos R & -1 & \cos P\\ \cos Q & \cos P & -1\end{vmatrix}$

Multiplying C_1 by p and then doing
$C_1\rightarrow C_1+qC_2+rC_3$, we get

$\Delta=\frac{1}{p}\begin{vmatrix}-p & \cos R & \cos Q\\ p\cos R & -1 & \cos P\\ p\cos Q & \cos P & -1\end{vmatrix}$

$=\frac{1}{p}\begin{vmatrix}-p+q\cos R+r\cos Q & \cos R & \cos Q\\ p\cos R-q+r\cos P & -1 & \cos P\\ p\cos Q+q\cos P-r & \cos P & -1\end{vmatrix}$

$=\frac{1}{p}\begin{vmatrix}0 & \cos R & \cos Q\\ 0 & -1 & \cos P\\ 0 & \cos P & -1\end{vmatrix}=0$

Using the projection formula
$p=q\cos R+r\cos Q$

88. (a)

The system of equations are

$x+3y+5z=\alpha x$

$5x+y+3z=\alpha y$

$3x+5y+z=\alpha z$

$(1-\alpha)x+3y+5z=0\ \ldots\ldots$ (i)

$5x+(1-\alpha)y+3z=0\ \ldots\ldots$ (ii)

$3x+5y+(1-\alpha)z=0\ \ldots\ldots$ (iii)

For infinite number of solutions, we must have

$\begin{vmatrix}1-\alpha & 3 & 5\\ 5 & 1-\alpha & 3\\ 3 & 5 & 1-\alpha\end{vmatrix}=0$

$\Rightarrow\begin{vmatrix}9-\alpha & 3 & 5\\ 9-\alpha & 1-\alpha & 3\\ 9-\alpha & 5 & 1-\alpha\end{vmatrix}=0,$

$C_1\rightarrow C_2+C_3.$

Taking ($9-\alpha$) common from the first column, we get

$$\Rightarrow \quad (9-\alpha)\begin{vmatrix} 1 & 3 & 5 \\ 1 & 1-\alpha & 3 \\ 1 & 5 & 1-\alpha \end{vmatrix} = 0$$

$$\Rightarrow \quad (9-\alpha)\begin{vmatrix} 1 & 3 & 5 \\ 0 & -\alpha-2 & -2 \\ 0 & 2 & -\alpha-4 \end{vmatrix} = 0$$

$$\Rightarrow \quad (9-\alpha)\begin{vmatrix} \alpha+2 & 2 \\ -2 & \alpha+4 \end{vmatrix} = 0$$

$$\Rightarrow \quad (9-\alpha)(\alpha^2+6\alpha+8+4) = 0$$

$$(\alpha-9)(\alpha^2+6\alpha+12) = 0$$

$\Rightarrow \alpha = 9$ is the only real value of α.

89. (d)

Let $A = \{a_1, a_2, a_3, a_4\}$

$B = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$

$\therefore n(A) = 4, n(B) = 7$

$\therefore$ The total number of infections $= {}^7P_4$

$= \frac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840.$

90. (b)

$P = \sum_{r=0}^{5} C_{2r}$

$= {}^{10}C_0 + {}^{10}C_2 + + {}^{10}C_{10} = \frac{2^{10}}{2} = 2^9$

$Q = \sum_{r=0}^{3} d_{2r+1} = d_1 + d_3 + d_5 + d_7$

$= {}^7C_1 + {}^7C_3 + {}^7C_5 + {}^7C_7 = \frac{2^7}{2} = 2^6$

$\therefore \frac{P}{Q} = \frac{2^9}{2^6} = 2^3 = 8$

91. (d)

Since, these are 52 distinct cards in decks and each distinct card is 2 in number.

Therefore, 2 decks will also contain only 52 distinct cards two each.

$\therefore$ Probability that the player gets all distinct cards

$= \frac{{}^{52}C_{26} \times 2^{26}}{{}^{104}C_{26}}$

92. (d)

Total number of selections of three balls in which balls of both colours are drawn = 2 red balls and 1 white ball +1 red ball and 2 white balls

$= {}^8C_2 \times {}^5C_1 + {}^8C_1 \times {}^5C_2$

$= 140 + 80 = 220$

$\therefore$ Required probability

$= \frac{220}{{}^{13}C_3} = \frac{220 \times 6}{13 \times 12 \times 11} = \frac{10}{13}$

93. (b)

Let F denotes fair coin T denotes two headed H denotes head occurs.

$\therefore P(F) = \frac{3}{4}, P(T) = 1 - \frac{3}{4} = \frac{1}{4}$

$$\therefore P\left(\frac{T}{H}\right) = \frac{P\left(\frac{H}{T}\right) \cdot P(T)}{P\left(\frac{H}{T}\right) \cdot P(T) + P\left(\frac{H}{F}\right) \cdot P(F)}$$

(By Baye's theorem).

$$= \frac{1 \cdot \frac{1}{4}}{1 \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{3}{4}} = \frac{2}{5}$$

94. (a)

According to question,

$$f(g(x)) = g(f(x))$$
$$\Rightarrow f(x+1) = g(x^2+2x-3)$$
$$\Rightarrow (x+1)^2 + 2(x+1) - 3 = x^2+2x-3+1$$
$$\Rightarrow x^2+1+2x+2x+2-3 = x^2+2x-2$$
$$\Rightarrow x^2+4x = x^2+2x-2$$
$$\Rightarrow x^2+4x-x^2-2x+2 = 0$$
$$\Rightarrow 2x+2 = 0$$
$$\Rightarrow 2x = -2$$
$$\Rightarrow x = -1$$

95. (a)

Since, a, b and c are in AP .

$\therefore\ 2b = a + c$

Given, quadratic equation,

$ax^2 - 2bx + c = 0$

$\Rightarrow ax^2 - (a+c)x + c = 0\ (2b = a + c)$

$\Rightarrow ax^2 - ax - cx + c = 0$

$\Rightarrow\ ax(x-1) - c(x-1) = 0$

$\Rightarrow\ (x-1)(ax-c) = 0$

$\Rightarrow\ x = 1, \frac{c}{a}$

96. (a)

$$y^2 + 4x + 4y + k = 0$$
$$y^2 + 2 \times 2y + 4 - 4 + 4x + k = 0$$
$$(y+2)^2 = -4x - k + 4$$
$$(y+2)^2 = -4\left(x - \frac{4+k}{4}\right)$$

$\therefore$ Latusrectum = 4 units

97. (a)

Two circles are orthogonally if and only if

$2(g_1g_2 + f_1f_2) = c_1 + c_2$

$\Rightarrow 2[(1 \times 0 + (k)k] = 6 + k$

$\Rightarrow 2k^2 = 6 + k$

$\Rightarrow 2k^2 - k - 6 = 0$

$\Rightarrow 2k(k-2) + 3(k-2) = 0$

$\Rightarrow 2k^2 - 4k + 3k - 6 = 0$

$\Rightarrow (k-2)(2k+3) = 0$

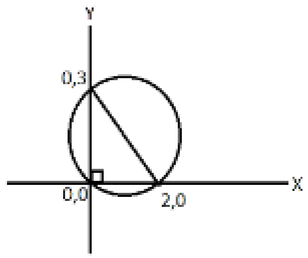
$k = 2 - \frac{3}{2}$

98. (c)

Since, join of (2,0) and (0,3) subtends 90° at (0,0).

$\Rightarrow$ It is a diameter.

$\therefore$ Equation is



$$(x-2)(x-0) + (y-0)(y-3) = 0$$
$$x^2 + y^2 - 2x - 3y = 0$$

$(2k, 3k)$ lies on it

$\Rightarrow \quad 4k^2 + 9k^2 - 4k - 9k = 0$

$\Rightarrow \quad 13k^2 = 13k$

$\Rightarrow \quad k = 1$

Since, $k \neq 0$ otherwise $(2k, 3k)$ will be $(0,0)$.

99. (a)

Since, $AM : BM = b : a$

$\therefore$ M divides AB externally in the ratio $b : a$

$\therefore\ x = \frac{b \cdot a\cos\beta - ab\cos\alpha}{b-a}$.......(i)

$y = \frac{ba\sin\beta - ab\sin\alpha}{b-a}$.........(ii)

Divide Eq. (i) by Eq. (ii), we get

$$\frac{x}{y} = \frac{\cos\beta - \cos\alpha}{\sin\beta - \sin\alpha} = \frac{2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}{-2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}$$

$$\therefore x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = 0$$

100. (d)

$$\frac{x^2}{9} + \frac{y^2}{1} = 1 \Rightarrow e = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

Two foci are $(\pm ae, 0)$ i.e., $(\pm 2\sqrt{2}, 0)$

Let $P(h, k)$ by any point on the ellipse

$$\therefore \frac{k-0}{h-2\sqrt{2}} \times \frac{k-0}{h+2\sqrt{2}} = -1$$

[From given condition]

$\Rightarrow \quad h^2 - 8 = -k^2$

$\Rightarrow \quad x^2 + y^2 = 8$

101. (d)

Given differential equation,

$\frac{dy}{dx} = \frac{x+y+1}{2x+2y+1}$......(i)

Put $x + y = v$

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1$$

$\therefore$ Eq. (i) becomes,

$$\Rightarrow \frac{dy}{dx} = \frac{v+1}{2v+1}$$

$$\Rightarrow \frac{dv}{dx} - 1 = \frac{v+1}{2v+1} \Rightarrow \frac{dv}{dx} = \frac{v+1}{2v+1} + 1$$

$$\Rightarrow \frac{v+1+2v+1}{2v+1} \Rightarrow \frac{dv}{dx} = \frac{3v+2}{2v+1}$$

$$\Rightarrow \left[\frac{2}{3}(3v+2) - \frac{1}{3}\right]$$

$$\Rightarrow \left[\frac{2}{3} - \frac{1}{3} \times \frac{1}{(3v+2)}\right] dv = dx$$

Integrating both sides, we get

$$\Rightarrow \int \left(\frac{2}{3} - \frac{1}{3} \times \frac{1}{(3v+2)}\right) dv = \int dx + C'$$

Where c is a constant of integration.

$$\Rightarrow \quad \frac{2}{3}v-\frac{1}{3}\frac{\log(3v+2)}{3}=x+C'$$

$$\Rightarrow \quad \frac{2}{3}(x+y)-\frac{1}{9}\log(3x+3y+2)=x+C'$$

$$\Rightarrow \quad 6x+6y-\log(3x+3y+2)=9x+C$$

$$\Rightarrow \quad 3x-6y+\log(3x+3y+2)=C$$

102. (d)

Let $I=\int_{\pi/6}^{\pi/2}\left(\frac{1+\sin 2x+\cos 2x}{\sin x+\cos x}\right)dx$

$$=\int_{\pi/6}^{\pi/2}\left(\frac{1+2\sin x\cos x+2\cos^2 x-1}{(\sin x+\cos x)}\right)dx$$

$$=\int_{\pi/6}^{\pi/2}\frac{2\cos x(\sin x+\cos x)}{(\sin x+\cos x)}dx$$

$$=\int_{\pi/6}^{\pi/2}2\cos x dx=2[\sin x]_{\pi/6}^{\pi/2}$$

$$=2\left(\sin\frac{\pi}{2}-\sin\frac{\pi}{6}\right)=2\left(1-\frac{1}{2}\right)=2\times\frac{1}{2}=1$$

103. (d)

$$I=\int_0^{\pi/2}\frac{1}{1+(\tan x)^{101}}dx$$

$$=\int_0^{\pi/2}\frac{dx}{1+\left\{\tan\left(\frac{\pi}{2}-x\right)\right\}^{101}}$$

Let $$=\int_0^{\pi/2}\frac{dx}{1+(\cot x)^{101}}$$

$$=\int_0^{\pi/2}\frac{\tan x^{101}}{\tan x^{101}+1}dx$$

$$=\int_0^{\pi/2}\frac{1+\tan x^{101}-1}{1+\tan x^{101}}=[x]_0^{\pi/2}-I$$

$$\Rightarrow I=\frac{\pi}{2}-I\Rightarrow 2I=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

104. (d)

Given, $3x\log_e x\frac{dy}{dx}+y=2\log_e x$

Dividing both sides by $3x\log_e x$, we get

$$\frac{dy}{dx}+\frac{1}{3x\log_e x}y=\frac{2\log_e x}{3x\log_e x}$$

$$\Rightarrow \frac{dy}{dx}+\frac{1}{3x\log_e x}y=\frac{2}{3x}$$

which is linear form $\frac{dy}{dx}+Py=Q$, where P and Q are function of x and the integrating factor is given by the following formula $e^{\int Pdx}$.

$$\therefore \text{ IF }=e^{\int\frac{1}{3x\log_e x}dx}$$

Put $\log_e x=t\Rightarrow\frac{1}{x}dx=dt$

$$=e^{1/3}\int\frac{dt}{t}=e^{1/3}\log t$$

$$=e^{\log t^{1/3}}=t^{1/3}=(\log_e x)^{1/3}$$

105. (c)

$\tan x+\sec x=2\cos x$

$$\Rightarrow \frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x$$

$$\Rightarrow \sin x+1=2\cos^2 x=2(1-\sin^2 x)$$

$= 2 - 2\sin^2 x$
$\Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0$
$\Rightarrow \quad 2\sin^2 x + 2\sin x - \sin x - 1 = 0$
$\Rightarrow \quad 2\sin x(\sin x + 2) - 1(\sin x + 1) = 0$
$\Rightarrow \quad (\sin x + 2)(2\sin x - 1) = 0$
$\Rightarrow$ $\sin x = -2$, which is not possible.
or $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$
$x \in [0, \pi]$
$\therefore$ The number of solution $= 2$.

106. (d)

$$I = \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$$

Let $= \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + 2\sin x\cos x} dx$

$$= \int_0^{\pi/4} -\frac{\sin x + \cos x}{(\sin x - \cos x)^2 - 4} dx \ldots\ldots\ldots (i)$$

Put $\sin x - \cos x = t$
$\Rightarrow$ $(\cos x + \sin x)dx = dt$
when $x = 0 \Rightarrow t = -1$
and $x = \frac{\pi}{4} \Rightarrow t = 0$
$\therefore$ Eq.(i) becomes,

$$I = -\int_{-1}^{0} \frac{dt}{t^2 - 4} = -\frac{1}{4}\left[\log\left|\frac{t-2}{t+2}\right|\right]_{-1}^{0}$$

$$= -\frac{1}{4}(\log 1 - \log 3)$$

$$= -\frac{1}{4}(0 - \log 3) = \frac{1}{4}\log 3$$

107. (a)

Given,

$$y = \left(\frac{3^x - 1}{3^x + 1}\right)\sin x + \log_e(1 + x), x > -1$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(\frac{3^x - 1}{3^x + 1}\right)\sin x\right] + \frac{d}{dx}\log_e(1 + x)$$

$$= \left(\frac{3^x - 1}{3^x + 1}\right)\frac{d}{dx}\sin x + \sin x\frac{d}{dx}\left(\frac{3^x - 1}{3^x + 1}\right)$$

$$+ \frac{1}{1 + \mathrm{x}}\frac{\mathrm{d}}{\mathrm{dx}}(1 + \mathrm{x})$$

$$= \frac{\left(3^x - 1\right.\frac{d}{1 + x\, dx}(1 + x)}{3^x + 1}\cos x + \sin x$$

$$\frac{(3^x + 1)\frac{d}{dx}(3^x - 1) - (3^x - 1)\frac{d}{dx}(3^x + 1)}{(3^x + 1)^2}$$

$$+ \frac{1}{1 + x}$$

$$= \left(\frac{3^x - 1}{3^x + 1}\right)\cos x + \sin x(3^x + 1)(3^x \log_e 3)$$

$$\frac{-(3^x - 1)(3^x \log_e 3)}{(3^x + 1)^2} + \frac{1}{1 + x}$$

$$\therefore \left(\frac{dy}{dx}\right)_{\text{at } x=0} = 0 + 0 + \frac{1}{1+0} = 1$$

108. (d)

$$f(x) = \frac{x}{8} + \frac{2}{x}$$

$$\therefore f'(x) = \frac{1}{8} - \frac{2}{x^2} = \frac{x^2 - 16}{8x^2}$$

For maximum or minimum $f'(x)$ must be vanish.

$\therefore \quad f'(x) = 0$

$$\Rightarrow \frac{x^2 - 16}{8x^2} = 0 \Rightarrow x = 4, -4$$

$x \in [1,6]$

$\therefore \quad x \neq -4$

Also, in $[1,4], f'(x) < 0 \Rightarrow f(x)$ is decreasing. In $[4,6], f'(x) > 0 \Rightarrow f(x)$ is increasing.

$$f(1) = \frac{1}{8} + \frac{2}{1} = \frac{17}{8}$$

$$f(8) = \frac{6}{8} + \frac{2}{6} = \frac{3}{4} + \frac{1}{3} = \frac{13}{12}$$

Hence, maximum value of $f(x)$ in $[1,6]$ is $\frac{17}{8}$.

109. (b)

$$\frac{d}{dx}\left\{\tan^{-1}\frac{\cos x}{1 + \sin x}\right\}$$

$$= \frac{1}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2} \frac{d}{dx} \frac{\cos x}{1 + \sin x}$$

$$= \frac{(1 + \sin x)^2}{1 + \sin^2 x + 2\sin x + \cos^2 x}$$

$$= \frac{1}{2 + 2\sin x} \times [-(\sin x + \sin^2 x + \cos^2 x)]$$

$$= \frac{1}{2(1 + \sin x)} \times -(1 + \sin x) = -\frac{1}{2}$$

110. (c)

$$\int_{-2}^{2} (1 + 2\sin x)e^{|x|} dx$$

$$= \int_{-2}^{2} e^{|x|} dx + 2\int_{-2}^{2} \sin x e^{|x|} dx$$

(Since, first function is even and second function is odd).

$$= 2\int_{0}^{2} e^x dx + 2{,}0 = 2[e^x]_0^2 = 2(e^2 - 1)$$

111. (c)

We have the identity,

$$|z| = \left|z + \frac{2}{z} - \frac{2}{z}\right| \le \left|z + \frac{2}{z}\right| + \left|\frac{2}{z}\right|$$

$\Rightarrow \quad |z| \le 2 + \dfrac{2}{|z|}$

$\Rightarrow \quad |z|^2 \le 2|z| + 2$

$\Rightarrow \quad |z|^2 - 2|z| + 1 \le 3$

$\Rightarrow \quad (|z| - 1)^2 \le 3$

$\Rightarrow \quad -\sqrt{3} \le |z| - 1 \le \sqrt{3}$

$\Rightarrow \quad -\sqrt{3} + 1 \le |z| \le \sqrt{3} + 1$

Hence, the maximum value $= \sqrt{3} + 1$

112. (d)

Let $z = \frac{3}{2} + i\frac{\sqrt{3}}{2}$.

$$r = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$$

$$\theta = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\frac{3}{2} + \frac{i\sqrt{3}}{2} = \sqrt{3}e^{\frac{i\pi}{6}}$$

$$\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = \left(\sqrt{3}e^{\frac{i\pi}{6}}\right)^{50}$$

$$= (\sqrt{3})^{50}\left(e^{\frac{i\pi}{6}}\right)^{50} = 3^{25}e^{\frac{i50\pi}{6}}$$

$$\Rightarrow \left(\frac{3}{2} + \frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}e^{\frac{i25\pi}{3}}$$

$$= 3^{25}\left(\cos\frac{25\pi}{3} + i\sin\frac{25\pi}{3}\right)$$

$$= 3^{25}(\cos 1500 + i\sin 1500)$$

$$= 3^{25}[\cos(360 \times 4 + 60) + i\sin(360$$

$$= 3^{25}(\cos 60 + i\sin 60)$$

$$\Rightarrow \left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \ldots\ldots (i)$$

According to question,

$$\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$$

$$3^{25}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 3^{25}(x + iy)$$

which is true only when $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$

113. (b)

Let $\omega = \frac{z-1}{z+1}$

Then, using componendo and dividendo, we get

$$z=\frac{1+\omega}{1-\omega}$$

$\Rightarrow\ |z|=\left|\frac{\omega+1}{\omega-1}\right|$

Put . $|z|=1$

$\Rightarrow\ |\omega+1|=|\omega-1|$.........(i)

Let $\omega=u+iv$

Then, $|\omega+1|=|u+iv+1|$

$=|(u+1)+iv|=\sqrt{(u+1)^2+v^2}$

and $|\omega-1|=\sqrt{(u-1)^2+v^2}$

$\therefore$ From Eq. (i)

$$\sqrt{(u+1)^2+v^2}=\sqrt{(u-1)^2+v^2}$$

$\Rightarrow \qquad (u+1)^2+v^2=(v-1)^2+v^2$

$\Rightarrow \qquad u=0$

$\therefore\ \omega=\frac{z-1}{z+1}$ is a pure imaginary number.

114. (c)

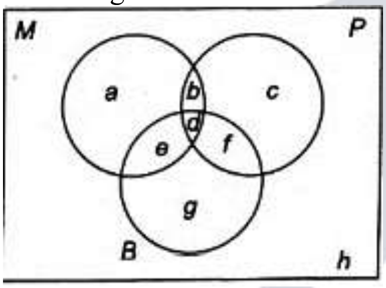
According to question,

$a+b+c+d+e+f+g+h=100$ ... (i)

$h=1$ ... (ii)

$a+b+d+e=50$ ... (iii)

Venn diagram shows failed students



$b+c+d+f=45$ ... (iv)

$d+e+f+g=40$ ... (v)

$b+e+f=32$ ... (vi)

On solving, the above equations, we get $d=2$

115. (d)

The total number of arrangements of 2 letters of English alphabet

$=26\times 26$

The total number of arrangements of 4 digits number in which first digit is not zero

$=9\times 10\times 10\times 10$

$\therefore$ The total number of vehicles with distinct registration number

$=26\times 26\times 9\times 10\times 10\times 10$

$=26^2\times 9\times 10^3$

116. (a)

These are 10 letters in the word IRRATIONAL in which these are 2 $'T$, 2 ' R ' and 2 ' A '.

$\therefore$ The total number of words $=\dfrac{10!}{2!\,2!\,2!}=\dfrac{10!}{(2!)^3}$

117. (d)

Four speakers will address the meeting in 41 ways $=24$ different ways in which half number of cases will be such that P speaks before Q and half number of case will be such that P speaks after Q

$\therefore$ Required number of ways $=\frac{24}{2}=12$

118. (b)

The number of diagonals in a regular polygon of n sides is given by the following formula ${}^nC_2-n$

$\therefore$ The number of diagonals in a regular polygon of 100 sides

$= {}^{100}C_2 - 100 = \dfrac{100 \cdot 99}{2} - 100$

$= \dfrac{9900 - 200}{2} = \dfrac{9700}{2} = 4850$

119. (b)

According to question,

${}^nC_1$, ${}^nC_2$ and ${}^nC_3$ are in AP.

$\Rightarrow \quad \dfrac{2n(n-1)}{2!} = n + \dfrac{n(n-1)(n-2)}{3!}$

$\Rightarrow \quad n^2 - 9n + 14 = 0$

$\Rightarrow \quad (n-7)(n-2) = 0$

$\Rightarrow \quad n = 7$ since $n \neq 2$

$\therefore$ The sum of the coefficients of odd powers of x in the expansion of $(1+x)^n$ is

$\dfrac{2^n}{2} = \dfrac{2^7}{2} = 2^6 = 64$

120. (c)

Given, $f(x) = ax^2 + bx + c, g(x) = px^2 + qx + r$ Since, $f(1) = g(1)$

$\Rightarrow \quad a + b + c = p + q + r$

$\Rightarrow \quad f(2) = g(2)$

$4a + 2b + c = 4p + 2q + r$

Subtracting Eq. (ii) from Eq. (i), we get

$3a + b = 3p + q$

$f(3) - g(3) = 2$

$\Rightarrow \quad (9a + 3b + c) - (9p + 3q + r) = 2$

$\Rightarrow \quad 3(3a + b) + c - 3(3p + q) - r = 2$

$\Rightarrow \quad r - r = 2$

$(3a + b = 3p + q)$

From Eq. (i),

$(a-p) + (b-q) + (c-r) = 0$

$\Rightarrow (a-p) + (b-q) + 2 = 0$

From Eq. (ii),

$4(a-p) + 2(b-q) + c - r = 0$

$\Rightarrow \quad 2(a-p) + (b-q) + 1 = 0$

121. (b)

$1\text{X}1! + 2 \times 2! + \cdots + 50 \times 50!$

$=(2-1)1! + (3-1)2! + (4-1)3! + \cdots + (51-1)50!$

$= (2! - 1!) + (3! - 2!) + (4! - 3!) + \cdots + (51! - 50!)$

$= 51! - 1! = 51! - 1$

122. (d)

Let the numbers be

$a - 5d, a - 3d; a - d, a + d, a + 3d, a + 5d$

$\therefore a - 5d + a - 3d + a - d + a + d + a$

$+3d + a + 5d = 3$

$(\because \text{ Sum } = 3)$

$\Rightarrow$

$= 6a = 3 \Rightarrow a = \dfrac{1}{2}$

Also, given $T_1 = 4T_3$, where T_1, T_3 are respectively, first and third terms of AP.

$\Rightarrow \quad a - 5d \quad = 4(a - d)$

$\Rightarrow \quad d \quad = -3a = -\dfrac{3}{2}$

$\therefore$ The fifth term

$a + 3d = \dfrac{1}{2} + 3\left(-\dfrac{3}{2}\right) = \dfrac{1}{2} - \dfrac{9}{2} = -4$

123. (b)

Given series,

$$1+\frac{1}{3}+\frac{1\cdot 3}{3\cdot 6}+\frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9}+\frac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 9\cdot 12}+\cdots\infty$$

$$=1+\frac{1}{3}+\frac{\frac{1}{2}\cdot\frac{3}{2}}{1\cdot 2}\left(\frac{2}{3}\right)^2+\frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}}{1\cdot 2\cdot 3}\left(\frac{2}{3}\right)^3\ldots\infty$$

$$=1+\frac{1}{2}\cdot\frac{2}{3}+\frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2!}\left(\frac{2}{3}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{3!}\left(\frac{2}{3}\right)\ldots\infty^3$$

$$=\left(1-\frac{2}{3}\right)^{-1/2}=3^{1/2}=\sqrt{3}$$

124. (b)

Let α be the common roots

$\therefore\quad \alpha^2+\alpha+a=0$
and $\quad \alpha^2+a\alpha+1=0$ $\ldots\ldots(i).(ii)$

$$\frac{\alpha^2}{1-a^2}=\frac{\alpha}{a-1}=\frac{1}{a-1}[a\neq 1]$$

Eliminating α, we get

$$(a-1)^2=(1-a^2)(a-1)$$

$\Rightarrow\quad a-1)=1-a^2$

$\Rightarrow\quad a^2+a-2=0$

$\Rightarrow\quad a^2+2a-a-2=0$

$\Rightarrow\quad a(a+2)-1(a+2)=0$

$\Rightarrow\quad (a+2)(a-1)=0$

$\Rightarrow$

$==\mathrm{a}-2\qquad [\because a\neq 1]$

125. (c)

Let a be the first term and r be the common ratio of a GP.

$\therefore$ P th,Q th and R th terms of a GP are respectively ar^{P-1}, ar^{Q-1} and ar^{R-1}.

According to question,

$ar^{p-1}=64\ldots\ldots\ldots$ (i)

$ar^{Q-1}=27\ldots\ldots\ldots$ (ii)

$ar^{R-1}=36\ldots\ldots\ldots$ (iii)

Dividing Eq. (i) by Eq. (ii), we get

$r^{P-Q}=\left(\frac{4}{3}\right)^3\ldots\ldots\ldots$(iv)

Dividing Eq. (ii) by Eq. (iii), we get

$$r^{Q-R}=\frac{3}{4}$$

$\Rightarrow\quad r^{3Q-3R}=\left(\frac{3}{4}\right)^3\ldots\ldots\ldots.(v)$

Multiplying Eq. (iv) and Eq. (v), we get

$r^{P-Q}\times r^{3Q-3R}=1$

$\Rightarrow\quad r^{P-Q+3Q-3R\ldots=1}$

$\Rightarrow\quad r^{P+2Q-3R}=r^0$

$\Rightarrow\quad P+2Q-3R=0$

$\Rightarrow\quad P+2Q=3R$

126. (c)

We know that $|\sin^{-1}x|\leq\frac{\pi}{2}$ Hence, from the given relation we observe that each of $\sin^{-1}x,\sin^{-1}y$ and $\sin^{-1}z$ will be $\frac{\pi}{2}$ so that $x=y=z=\sin\frac{\pi}{2}=1$.

$\therefore x^9+y^9+z^9-\dfrac{1}{x^9y^9z^9}$

$=(1)^9+(1)^9+(1)^9-\dfrac{1}{(1)^9(1)^9(1)^9}$

$=1+1+1-\dfrac{1}{1\times1\times1}$

$=3-1=2$

127. (d)

We know that in $\triangle ABC$

$\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}=2R$

$\therefore r^2\sin P\sin Q=pq$

$\Rightarrow r^2\cdot\frac{p}{2R_1}\cdot\frac{q}{2R_1}=pq$, where R_1 is circumradius of $\triangle PQR$.

$\Rightarrow\quad r^2=4R_1^2$

$\Rightarrow\quad r=2R_1$

$\Rightarrow\quad 2R_1\sin R=-2R_1$

$\Rightarrow\quad R_1=90^\circ$

$\therefore\triangle PQR$ is right angled.

128. (b)

In $\triangle PQR, P+Q+R=180^\circ$

$\therefore 2pr\sin\left(\dfrac{P-Q+R}{2}\right)=2pr\sin\dfrac{180^\circ-Q-Q}{2}$

$=2pr\sin(90^\circ-Q)$

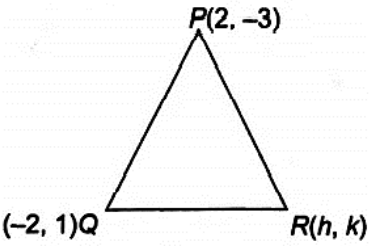
$=2pr\cos Q$

$\left(\text{In }\Delta PQR\cos Q\quad=\dfrac{p^2+r^2-q^2}{2pr}\right)\Bigg)$

$=2pr\left(\dfrac{p^2+r^2-q^2}{2pr}\right)=p^2+r^2-q^2$

129. (a)

Let the vertices of R be $(\,h,k\,)$.



$\therefore$ Centroid of triangle is given by

$\left(\dfrac{x_1+x_2+x_3}{3},\dfrac{y_1+y_2+y_3}{3}\right)$

i.e., $\left(\frac{2+h-2}{3},\frac{-3+1+k}{3}\right)$

i.e., $\left(\frac{h}{3},\frac{-2+k}{3}\right)$

Since, the point $\left(\frac{h}{3},\frac{-2+k}{3}\right)$ lies on the line $2x+3y=1$, therefore the point $\left(\frac{h}{3},\frac{-2+k}{3}\right)$ must be satisfy the equation of the given line.

i.e., $2\left(\frac{h}{3}\right)+3\left(\frac{-2+k}{3}\right)=1$

$\Rightarrow\quad\dfrac{2h-6+3k}{3}=1$

$\Rightarrow\quad 2h-6+3k=3$

$\Rightarrow\quad 2h+3k=9$

Replace $h = x$ and $k = y$, we get the required locus of R.
i.e.,
$2x + 3y = 9$

130. (b)

$\lim_{x\to 0} \frac{\pi^x - 1}{\sqrt{1+x} - 1}$ ....$\left[\frac{0}{0}\text{ form }\right]$

$$= \lim_{x\to 0} \frac{\pi^x \log_e \pi}{\frac{1}{2\sqrt{1+x}}}$$
$$= \lim_{x\to 0} 2\sqrt{1+x}(\pi^x \log_e \pi)$$
$$= 2\sqrt{1}(\pi^0 \log_e \pi) = 2\log_e \pi = \log_e \pi^2$$

131. (d)

Given, $f(x)f'(x) < 0$
$\Rightarrow f(x)$ and $f'(x)$ must be of opposite sign.
(i) Let $f(x) = e^{-x}$
$\therefore\ f'(x) = -e^{-x}$
$\Rightarrow f(x) > 0$ and $f'(x) < 0, \forall x \in R$
(ii) Let $f(x) = -e^{-x}$
$\therefore\ f'(x) = e^{-x}$
$\Rightarrow f(x) < 0$ and $f'(x) > 0, \forall x \in R$
But $|f(x)| = |\pm e^{-x}| = e^{-x}$ in both cases
$\therefore \frac{d}{dx}|f(x)| = -e^{-x} < 0$ in both case, $\forall x \in R$
$\Rightarrow |f(x)|$ must be a decreasing function.

132. (b)

If we take $f(x) = 4x^4$, then
(i) $f(x)$ is continuous in $(-2,2)$
(ii) $f(x)$ is differentiable in $(-2,2)$
(iii) $f(-2) = f(2)$
So, $f(x) = 4x^4$ satisfies all the conditions of Rolle's theorem therefore $\exists$ a point c such that $f'(c) = 0$
$\Rightarrow\ 16c^3 = 0 \Rightarrow c = 0 \in (-2,2)$

133. (c)

Let $y = e^{mx}$ be the solution of given differential

$$\Rightarrow\ \frac{dy}{dx} = me^{mx} \Rightarrow \frac{d^2y}{dx^2} = m^2 e^{mx}$$
$$\therefore\ 25\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + y = 0$$
$$\Rightarrow\ 25m^2e^{mx} - 10me^{mx} + e^{mx} = 0$$
$$\Rightarrow\ e^{mx}(25m^2 - 10m + 1) = 0$$

$\Rightarrow$ Auxiliary equation

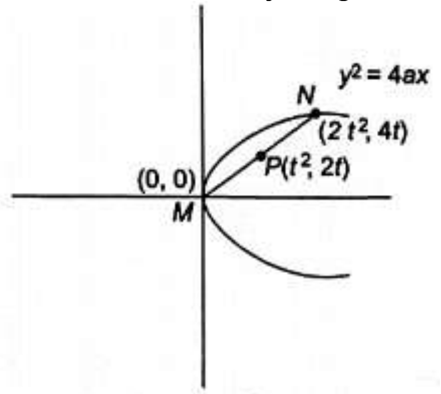
$$\Rightarrow 25m^2 - 10m + 1 = 0$$
$$\Rightarrow\ e^{mx} \neq 0$$
$$\Rightarrow\ (5m)^2 - 2(5m) \times 1 + 1 = 0$$
$$\Rightarrow\ (5m - 1)^2 = 0$$
$$\Rightarrow\ m = \frac{1}{5}, \frac{1}{5}$$

Since, roots are real and equal.
$\therefore$ General solution is $y = (c_1 + c_2 x)e^{x/5}$.........(i)
$y(0) = 1 \Rightarrow c_1 = 1$
$y(1) = 2e^{1/5} \Rightarrow 2e^{1/5} = (c_1 + c_2)e^{1/5}$
$\Rightarrow\ c_1 + c_2 = 2 \Rightarrow c_1 = 1$
Putting the value of c_1 and c_2 in Eq. (i), we get particular solution
$y = (1 + x)e^{x/5}$

134. (b)

Let the chord be MN.

Let other end of chord joining from vertex to it lying on $y^2 = 8x$ be $N(2t^2, 4t)$



$\therefore$ Mid-point of $MN = (t^2, 2t)$

$\therefore \quad x = t^2, y = 2t \Rightarrow x = \left(\frac{y}{2}\right)^2$

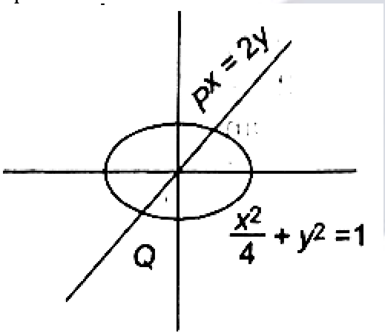
$\Rightarrow \quad y^2 = 4x$

is the required locus of P.

135. (d)

Solving $x = 2y$ and…….(i)

$\frac{x^2}{4} + y^2 = 1$……..(ii)



Put $x = 2y$ in Eq. (ii), we get

$$\frac{(2y)^2}{4} + y^2 = 1 \quad \Rightarrow \frac{4y^2}{4} + y^2 = 1$$

$$\Rightarrow \quad 2y^2 = 1$$

$$\Rightarrow y = \pm\frac{1}{\sqrt{2}}$$

$\therefore$ From Eq. (i), $x = \pm\sqrt{2}$

$\therefore \; P\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$ and $Q\left(-\sqrt{2}, -\frac{1}{\sqrt{2}}\right)$

$\therefore$ Equation of circle with PQ as diameter is

$$(x - \sqrt{2})(x + \sqrt{2}) + \left(y - \frac{1}{\sqrt{2}}\right)\left(y + \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow x^2 - 2 + y^2 - \frac{1}{2} = 0 \Rightarrow x^2 + y^2 = \frac{5}{2}$$

136. (b)

Let $P(\sqrt{10}\cos\theta, \sqrt{8}\sin\theta)$ be the required point on $\frac{x^2}{10} + \frac{y^2}{8} = 1$

Whose distance from centre (0,0) is 3 units.

$$\therefore 10\cos^2\theta + 8\sin^2\theta = 9 = 9(\sin^2\theta + \cos^2\theta)$$

$$\Rightarrow \tan^2\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$$

137. (d)

Let e be the eccentricity of hyperbola and length of conjugate axis be $2b$. Since, vertex $(a, 0)$ bisects the join of centre $(0,0)$ and focus $(ae, 0)$.

$$a = \frac{ae + 0}{2} \Rightarrow e = 2$$

$b^2 = a^2(e^2 - 1) = 3a^2$

$\therefore$ Required hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{3a^2} = 1 \Rightarrow 3x^2 - y^2 = 3a^2$$

138. (d)

We know by the definition of hyperbola, "A hyperbola is the locus of a point which moves in such a way that the difference between two fixed points remains constant,

i.e.,

$|PS - PS'| = 2a$

139. (b)

Given lines are, $3x + 4y = 9$.........(i)

$y = mx + 1$........................(ii)

Solving Eqs. (i) and (ii), we get

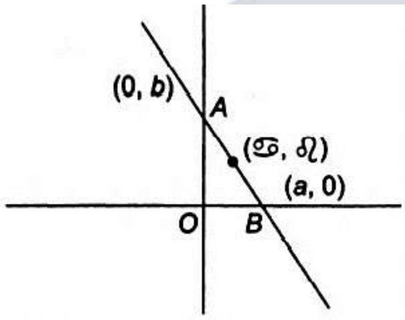
$$x = \frac{5}{3 + 4m}$$

If $m = -1$, then $x = -5$ and $m = -2$, then $x = -1$ are the only possibilities.

140. (c)

Let the line be $AB = \frac{x}{a} + \frac{y}{b} = 1$......(i)

$\therefore$ The coordinates of A and B are respectively $(a, 0)$ and $(0, b)$



Since, (α, β) is the mid-point of AB

$$\therefore \alpha = \frac{a}{2}, \beta = \frac{b}{2}$$

$\therefore$ $a = 2\alpha$ and $b = 2\beta$

$$\therefore \text{ From Eq. (i) }, \frac{x}{2\alpha} + \frac{y}{2\beta} = 1 \Rightarrow \frac{x}{\alpha} + \frac{y}{\beta} = 2$$

141. (c)

The given series is in GP. Hence, its sum

$$S = \frac{1\{(1 + x)^{20+1} - 1\}}{(1 + x) - 1} = \frac{(1 + x)^{21} - 1}{x}$$

Therefore, the required coefficient of x^{10} in the expansion of $\frac{(1+x)^{21}-1}{x}$

= Coefficient of x^{11} in the expansion of

$(1 + x)^{21} - 1$

$= {}^{21}C_{11}$

142. (b)

$$\text{Augmented matrix } [A: B] = \begin{bmatrix} 1 & -1 & -2 & 6 \\ -1 & 1 & 1 & \mu \\ \lambda & 1 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -2 & 6 \\ 0 & 0 & -1 & \mu + 6 \\ \lambda + 1 & 0 & -1 & 9 \end{bmatrix}$$

$$\text{Applying } R_2 \to R_2 + R_1, R_3 \to R_3 + R_1 \sim \begin{bmatrix} 1 & -1 & -2 & 6 \\ 0 & 0 & -1 & \mu + 6 \\ \lambda + 1 & 0 & 0 & 3 - \mu \end{bmatrix}$$

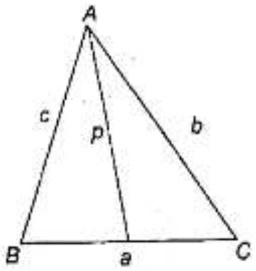
Infinite number of solutions for $\lambda = -1$ and $\mu = 3$.

143. (a)

$$P\left(\frac{B}{A\cup B^c}\right)=\frac{P(B\cap(A\cup B^c))}{P(A\cup B^c)}$$
$$=\frac{P(A\cap B)}{P(A)+P(B^c)-P(A\cap B^c)}$$
$$=\frac{P(A)-P(A\cap B^c)}{P(A)+P(B^c)-P(A\cap B^c)}$$
$$=\frac{0.7-0.5}{0.7+0.6-0.5}=\frac{0.2}{0.8}=\frac{1}{4}$$

144. (b)

According to question, $S=\frac{1}{2}a\cdot p$



$$=\frac{1}{2}b\cdot q=\frac{1}{2}c\cdot r$$
$$\therefore \quad \frac{1}{p}=\frac{a}{2S},\frac{1}{q}=\frac{b}{2S},\frac{1}{r}=\frac{c}{2S}$$
$$\therefore \quad \frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{1}{2S}(a+b+c)=\frac{2t}{2S}=\frac{t}{S}$$

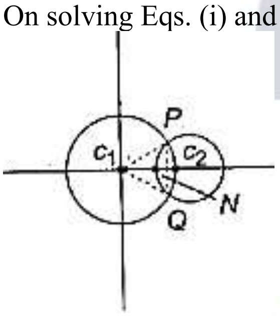
145. (c)

$x^2+y^2=4\ldots\ldots\ldots\ldots(\text{i})$

$(x-2)^2+y^2=1\ldots\ldots\ldots(\text{ii})$

$C_1=(0,0),C_2=(2,0)$

On solving Eqs. (i) and (ii), we get



$$x=\frac{7}{4},y=\pm\frac{\sqrt{15}}{4}$$
$$C_1N=\frac{7}{4},PQ=\frac{2\sqrt{15}}{4}=\frac{\sqrt{15}}{2}$$
$$\frac{\text{Area }(\Delta C_1PQ)}{\text{Area }(\Delta C_2PQ)}=\frac{\frac{1}{2}\cdot PQ\cdot C_1N}{\frac{1}{2}\cdot PQ\cdot C_2N}$$
$$=\frac{C_1N}{C_2N}=\frac{\frac{7}{4}}{2-\frac{7}{4}}=\frac{\frac{7}{1}}{4}=7:1$$

146. (b)

Given lines are,

$x+2y=4\ldots\ldots(\text{i})$

$2x+y=4\ldots\ldots(\text{ii})$

and (i) and (ii), we get
Solving Eqs.

$$x=\frac{4}{3}, y=\frac{4}{3}$$

$$AB: \frac{x}{a}+\frac{y}{b}=1 \ldots \ldots . . \text{(iii)}$$

Let
Meets x-axis at A and y-axis at B.
If the straight line (iii) passes through the point $\left(\frac{4}{3}, \frac{4}{3}\right)$, then $\frac{4}{3a}+\frac{4}{3b}=1$

$\Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{3}{4}$.........(iv)

$$\therefore \; h=\frac{a+0}{2}, k=\frac{0+b}{2}$$

$$\Rightarrow \quad a=2h, b=2k$$

$$\therefore \text{ From Eq. (iv), } \frac{1}{2h}+\frac{1}{2k}=\frac{3}{4}$$

$$\Rightarrow \quad 2h+2k=3hk$$

Replace x, y from h, k respectively, we get required locus
i.e.,
$2(x+y)=3xy$

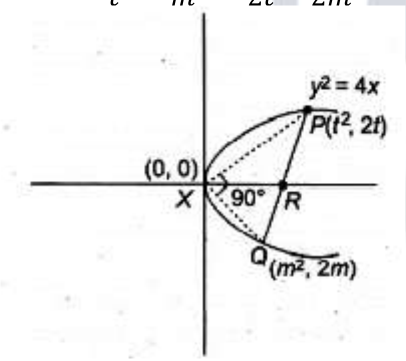
147. (c)

According to question, it is given that slope of PX and QX are perpendicular to each other, i.e., slope of $PX \times$ slope of $XQ=-1$

$$\Rightarrow \quad \frac{2t}{t^2} \times \frac{2m}{m^2}=-1$$

$$\Rightarrow \quad t_m=-4$$

$$PQ: \frac{x-t^2}{t^2-m^2}=\frac{y-2t}{2t-2m}$$



Let PQ meets axis of parabola i.e., x-axis at $R(\alpha, 0)$, then

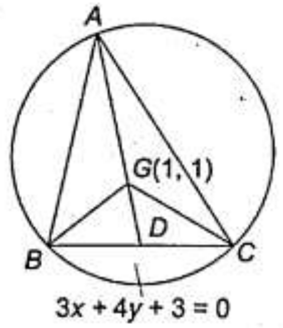
$$\frac{\alpha-t^2}{(t+m)(t-m)}=\frac{0-2t}{2(t-m)}$$

$$\Rightarrow \quad \alpha-t^2=-t^2-tm=-t^2+4$$

$$\Rightarrow \quad \alpha=4 \Rightarrow AR=4$$

148. (b)

Since, triangle is equilateral therefore incentre (1,1) lies on the centroid of the $\triangle ABC$.
$\therefore GD=$ Length of perpendicular from the point $G(1,1)$ to the line $3x+4y+3=0$



$$= \frac{3(1) + 4(1) + 3}{\sqrt{3^2 + 4^2}} = 2$$

$AG = 2GD = 4$

$\therefore$ Equation of circumcircle with centre at $(1,1)$ and radius $= 4$ units

$$(x-1)^2 + (y-1)^2 = 4^2$$

$\Rightarrow\ x^2 + y^2 - 2x - 2y - 14 = 0$

149. (d)

$$\lim_{x\to\infty} \frac{(n!)^{1/n}}{n} = \lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{1/n}$$

We have, $\frac{n!}{n^n} = \frac{1\cdot 2\cdot 3\, n}{n\cdot n\cdot n\, n}$

$$\therefore \left\{\frac{n!}{n^n}\right\}^{1/n} = \left\{\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{r}{n}\cdots\frac{n}{n}\right\}^{1/n}$$

$$\Rightarrow \lim_{n\to\infty} \left\{\frac{n!}{n^n}\right\}^{1/n} = \lim_{n\to\infty} \left\{\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{r}{n}\cdots\frac{n}{n}\right\}^{1/n}$$

Let

$$A = \lim_{n\to\infty} \left\{\frac{n!}{n^n}\right\}^{1/n}$$

Then,

$$A = \lim_{n\to\infty} \left\{\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{r}{n}\cdots\frac{n}{n}\right\}^{1/n}$$

$$\Rightarrow \log A = \lim_{n\to\infty} \frac{1}{n}\Sigma\log\left(\frac{r}{n}\right) = \int_0^1 \log x dx$$

$$= \left[x\log x - \int \frac{1}{x}\cdot x dx\right]_0^1$$

Integrating by parts

$$= [x\log x - x]_0^1 = -1$$

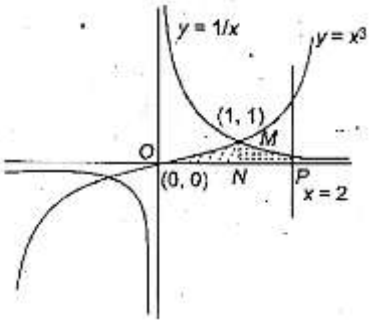
$\Rightarrow A$

$\Rightarrow e^{-1} = \frac{1}{e}$

150. (b)

First of all we draw the graph,

$\because\ y = x^3, y = \frac{1}{x}, x = 2$



$\therefore$ Required area i.e., (OMPNO)

$$= \int_0^1 x^3 dx + \int_1^2 \frac{1}{x} dx$$

$$= \left[\frac{x^4}{4}\right]_0^1 + [\log_e x]_1^2 = \frac{1}{4} + \log_e 2$$

151. (b)

Let $y = x^y \Rightarrow \log y = y\log x$ Differentiating both sides w.r.t. x, we

$$\frac{1}{y}\frac{dy}{dx} = y \times \frac{1}{x} + \log x \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} - \log x \frac{dy}{dx} = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1 - y\log x}{y}\right) = \frac{y}{x}$$

$$\Rightarrow \frac{xdy}{y} = \frac{y^2}{y}$$

152. (d)

$\sin^{-1} x$ is defined, if $-1 \leq x \leq 1$

In first quadrant $0 \leq x \leq 1$ and $x(1-x) \geq 0$

$\therefore\ y = \sin^{-1} x + x(1-x)$ ..(i)

Lies above $y = \sin^{-1} x - x(1-x)$ ..9II0

On solving, we get

$$2x(1-x) = 0$$

$$x = 0,1$$

$$\therefore \text{ Required area } = \int_0^1 (y_1 - y_2) dx$$

$$= \int_0^1 [\{\sin^{-1} x + x(1-x)\}$$

$$-\{\sin^{-1} x - x(1-x)\}] dx$$

$$= 2\int_0^1 (x - x^2) dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$$

153. (c)

$$\int_1^5 [|x-3| + |1-x|] dx$$

$$= \int_1^5 |x-3| dx + \int_1^5 |1-x| dx$$

$$= \int_1^3 |x-3| dx + \int_3^5 |x-3| dx + \int_1^5 |1-x| dx$$

$$= \int_1^3 -(x-3) dx + \int_3^5 (x-3) dx$$

$$+ \int_1^5 -(1-x) dx$$

$$= \int_1^3 (3-x) dx + \int_3^5 (x-3) dx + \int_1^5 (x-1) dx$$

$$= \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^5 + \left[\frac{x^2}{2} - x\right]_1^5$$

$$= \left(3 \times 3 - \frac{9}{2}\right) - \left(3 \times 1 - \frac{1}{2}\right) + \left(\frac{5 \times 5}{2} - 3 \times 5\right)$$

$$-\left(\frac{3\times 3}{2}-3\times 3\right)+\left(\frac{5\times 5}{2}-5\right)-\left(\frac{1}{2}-1\right)$$

$$=\left(9-\frac{9}{2}\right)-\left(3-\frac{1}{2}\right)+\left(\frac{25}{2}-15\right)-\left(\frac{9}{2}-9\right)$$

$$+\left(\frac{25}{2}-5\right)-\left(-\frac{1}{2}\right)$$

$$=\frac{9}{2}-\frac{5}{2}-\frac{5}{2}+\frac{9}{2}+\frac{15}{2}+\frac{1}{2}$$

$$=\frac{9-5-5+9+15+1}{2}=\frac{24}{2}=12$$

154. (b)

According to question,

$f''(x)=g''(x)$

Integrating w.r.t. x, we get

$f'(x)=g'(x)+C_1$

Put $x=1\Rightarrow f'(1)=g'(1)+C_1$

$\Rightarrow\quad 4=6+C_1$

$\therefore\quad C_1=-2$

$\therefore\quad f'(x)=g'(x)-2$

Again, integrating w.r.t. x, we get

$f(x)=g(x)-2x+C_2$

$\Rightarrow\quad 3=9-4+C_2\Rightarrow C_2=-2$

$\therefore\quad f(x)=g(x)-2x-2$

Put $x=1$, we get

$f(1)-g(1)=-2(1)-2=-4$

155. (a)

Let $I=\int_{-1}^{1}(|x|-2[x])dx$

$$=\int_{-1}^{0}(|x|-2[x])dx+\int_{0}^{1}(|x|-2[x])dx$$

$$=\int_{-1}^{0}(-x-2(-1))dx+\int_{0}^{1}(x-2(0))dx$$

$$=\int_{-1}^{0}(-x+2)dx+\int_{0}^{1}xdx$$

$$=\left(-\frac{x^2}{2}+2x\right)_{-1}^{0}+\left[\frac{x^2}{2}\right]_0^1$$

$$=-\left(-\frac{1}{2}+2(-1)\right)+\frac{1}{2}$$

$$=\frac{1}{2}+2+\frac{1}{2}=1+2=3$$

156. (a)

Let $z=x+iy$.

$$\therefore\ \frac{z-2}{z+2}=\frac{x+iy-2}{x+iy+2}$$

$$=\frac{(x-2)+iy}{(x+2)+iy}\times\frac{(x+2)-iy}{(x+2)-iy}$$

$$=\frac{(x-2)(x+2)+iy(x+2)-iy(x-2)-i^2y^2}{(x+2)^2-(iy)^2}$$

$$=\frac{x^2-4+ixy+2iy-ixy+2iy+y^2}{(x+2)^2+y^2}$$

$$= \frac{(x^2+y^2-4)+4iy}{(x+2)^2+y^2} = \frac{x^2+y^2-4}{(x^2+4+4x+y^2)}$$

$$+ \frac{4iy}{x^2+4+4x+y^2}$$

$$\therefore \arg\left(\frac{z-2}{z+2}\right) = \tan^{-1}\frac{4y}{x^2+4+4x+y^2}$$

$$\times \frac{x^2+4+4x+y^2}{x^2+y^2-4} = \frac{\pi}{3} \text{ (given)}$$

$$\therefore \tan\frac{\pi}{3} = \frac{4y}{x^2+y^2-4}$$

$$\sqrt{3} = \frac{4y}{x^2+y^2-4}$$

$x^2+y^2-4-\frac{4}{\sqrt{3}}y = 0$, which represents a circle.

157. (c)

Let $a^p = b^q = c^r = k$

$\therefore\ a = k^{1/p}, b = k^{1/q}, c = k^{1/r}$

Since, a, b, c are in GP.

$$\frac{b}{a} = \frac{c}{b}$$

$$\frac{k^{1/q}}{k^{1/p}} = \frac{k^{1/r}}{k^{1/q}}$$

$$k^{1/q-1/p} = k^{1/r-1/q}$$

$$\frac{1}{q}-\frac{1}{p} = \frac{1}{r}-\frac{1}{q}$$

$\therefore\ \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in GP.

$\Rightarrow\ p, q, r$ in HP.

158. (d)

$$S_1 = 1+\frac{1}{2}+\frac{1}{2^2}+\infty = \frac{1}{1-\frac{1}{2}} = 2$$

$$S_2 = 2+2\cdot\frac{2}{3}+2\left(\frac{2}{3}\right)^2+\cdots\infty = \frac{2}{1-\frac{2}{3}} = 6$$

$$S_3 = 3 + 3\left(\frac{3}{4}\right) + 3\left(\frac{3}{4}\right)^2 + \infty = \frac{3}{1-\frac{3}{4}} = 12$$

$$S_4 = 4 + 4\left(\frac{4}{5}\right) + 4\left(\frac{4}{5}\right)^2 + \infty = \frac{4}{1-\frac{4}{5}} = 20$$

$$\therefore \sum_{k=1}^{\infty} \frac{(-1)^k}{S_k} = -\frac{1}{S_1} + \frac{1}{S_2} - \frac{1}{S_3} + \frac{1}{S_4} - \infty$$

$$= -\frac{1}{2} + \frac{1}{6} - \frac{1}{12} + \frac{1}{20} - \infty$$

$$= -\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} - \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} - \infty$$

$$= -\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) -$$

$$= -1 + 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \infty\right)$$

$$= -1 - 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots \infty\right) + 2\infty$$

$$= -1 - 2\log_e 2 + 2 = 1 - \log_e 4$$

159. (b)

Since, roots are opposite sign .

$\therefore$ Product of the roots < 0

$\Rightarrow \quad a^2 - 4a < 0$

$\Rightarrow \quad a(a-4) < 0$

$\Rightarrow \quad 0 < a < 4$

160. (c)

Given, $\log_e(x^2 - 16) \leq \log_e(4x - 11)$ if and only if

$x^2 - 16 \leq 4x - 11$

$\Rightarrow \quad x^2 - 4x - 5 \leq 0$

$\Rightarrow \quad x^2 - 5x + x - 5 \leq 0$

$\Rightarrow \quad x(x-5) + 1(x-5) \leq 0$

$\Rightarrow \quad (x-5)(x+1) \leq 0$

Sign scheme of $x^2 - 4x - 5 \leq 0$

$\Rightarrow \; -1 \leq x \leq 5$