Physics

1. (d)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Given,

$T = 100°\text{C} = 373\ \text{K}$

$$\therefore\ v = \sqrt{\frac{3R \times 373}{M}}$$

For

$$\sqrt{3}v = \sqrt{\frac{3RT}{M}}$$

$$\therefore\quad \frac{\sqrt{3}v}{v} = \frac{\sqrt{3RT/M}}{\sqrt{3R \times 373/M}}$$

$$\Rightarrow\quad \sqrt{3} = \sqrt{\frac{T}{373}}$$

$$\Rightarrow\quad 3 = \frac{T}{373}$$

$\Rightarrow\quad T = 1119\ \text{K}$

or $\quad T = 846°\text{C}$

2. (a)

$$[P] = \left[\frac{a}{V^3}\right] \Rightarrow [\text{ML}^{-1}\ \text{T}^{-2}] = \frac{a}{[\ \text{L}^3]^3}$$

$$\Rightarrow\ a = [\text{ML}^8\ \text{T}^{-2}]$$

$$[V] = [b^2] \Rightarrow [\text{L}^3] = b^2 \Rightarrow b = [\text{L}^{3/2}]$$

3. (b)

$$\begin{aligned} W_{\text{constant pressure}} &= p \times \Delta V \\ &= 400 \times 10^3 \times 0.3 \times 10 \times 10^{-2} \\ &= 400 \times 10 \times 3 \\ &= 12000 = 12\ \text{kJ} \end{aligned}$$

4. (d)

In first case $A = 0, B = 0$

$\therefore$ Output of NOR gate, $Y = \overline{A + B} = 1$

This output is the input for NAND gate, i.e., $Y = 1$ and $C = 0$

$\therefore \qquad D = \overline{Y \cdot C} = 1$

In second case $\quad A = 1, B = 0$

$\therefore$ Output of NOR gate, $Y = \overline{A + B} = 0$

This output is the input for NAND gate i.e., $Y = 0$ and $C = 1$

$\therefore\ D = \overline{Y \cdot C} = 1$

5. (d)

Pressure difference $= \frac{4T}{r}$

For smaller soap, $p_{\text{atm}} - p_{in} = \frac{4T}{r}$

For bigger soap, $p_{\text{atm}} - p_{i2} = \frac{4T}{2r}$

As the pressure inside smaller bubble is greater than pressure inside bigger bubble, so air flows from smaller to bigger and thus radius of the smaller bubble will decrease and that of the bigger bubble will increase.

6. (b)

We have, $\theta = 2\pi n = \dfrac{\left(\dfrac{v_f^2}{r^2} - \dfrac{v_i^2}{r^2}\right)}{\left(2\dfrac{a}{r}\right)}$

$\Rightarrow \quad n = \dfrac{v_f^2 - v_i^2}{(2ar)(2\pi)} \approx 95$

7. (d)

Given; $E_n = 13.6\text{eV}$

Energy of an electron in n^{th} state

$E_n = \dfrac{-13.6z^2\text{eV}}{n^2}$

∴ Energy of an electron in $n = 2$ state

$E_2 = \dfrac{18 - 6z^2}{(2)^2} = -3.4\text{eV}$

So,

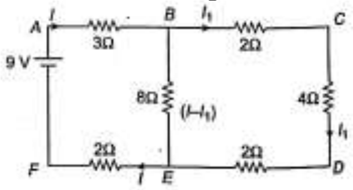
$PE = 2E_{n=2}$

$= 2 \times (-3.4)$

$\approx -6.8\text{eV}$

8. (b)

The current through the various branches of the circuit will be shown as



According to Kirchhoff's second law in closed circuit BCDEB

$2l_1 + 4l_1 + 2l_1 - 8(I - l_1) = 0$

$\Rightarrow \quad 16l_1 - 8I = 0$

$\Rightarrow \quad 1 = \dfrac{8I}{16} \Rightarrow l_1 = \dfrac{1}{2}I \ldots.(i)$

In closed circuit $ABEFA$

$-9 + 3l + 8(l - l_1) + 2l = 0$

$\Rightarrow 13l - 8l_1 = 9 \Rightarrow 13l - 8\left(\dfrac{1}{2}l\right) = 9$

$\Rightarrow l = \dfrac{9}{9} = 1\text{ A} \ldots\ldots\text{(ii)}$

So current through 4Ω resistor $I_1 = \frac{1}{2} \times 1 = 0.5\text{ A}$

9. (c)

The force experienced by the wire placed in magnetic field, $F = Bil$

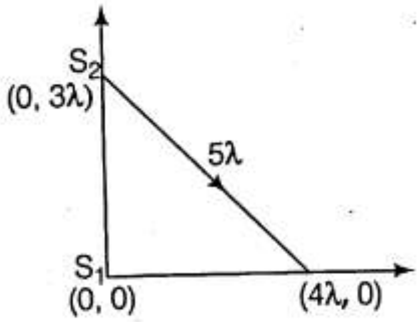
$= (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\left(1\hat{\mathbf{i}} \times \dfrac{1}{2}\right)$

$= \hat{\mathbf{i}} \times \hat{\mathbf{i}} + 4 \times \dfrac{1}{2}(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = 2\hat{\mathbf{k}}\text{ N}$

Direction of this force can be find out by Fleming's left hand rule which is along pasitive z-axis.

10. (b)

The given situation can be show as

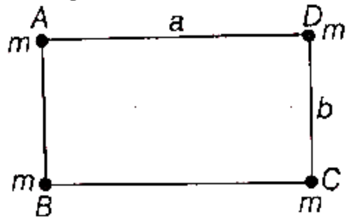


The intensity will be maximum at those given points where the path difference between the two interfering waves is an integral multiple of wavelength i.e., $\Delta x = n\lambda$

For the the given points, the intensity will be maximum for $(4\lambda, 0)$.

11. (a)

Then given situation can be shown as



Moment of inertia of the system about side of length b say CD is

= M.I. of mass at A about CD + M.I. of mass at B about CD + M.I. of mass at C about CD + M.I. of mass of D about CD

$= m(a)^2 + m(a)^2 + m(0)^2 + m(0)^2 = 2ma^2$

12. (b)

The de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mk}}$ Given,

$$h = 6.6 \times 10^{-34}\ \text{J} - \text{s}$$

$$m = 1 \times 10^{-30}\ \text{kg}$$

$K = 200\text{eV} = 200 \times 1.6 \times 10^{-19}\ \text{J}$

Substituting all these values

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1 \times 10^{-30} \times 200 \times 1.6 \times 10^{-19}}}$$

$$= 0.825 \times 10^{-10} = 8.25 \times 10^{-11}\ \text{m}$$

13. (c)

We have, $\frac{N(t)}{N_0} = e^{-\lambda t}$ or $N(t) = N_0 e^{-\lambda t}$

$\therefore$ For the given condition,

$$\frac{N_0}{2} = N_0 \mathrm{e}^{-\lambda t_1} \text{ and } \frac{N_0}{10} = N_0 \mathrm{e}^{-\lambda t_2}$$

$$\Rightarrow \frac{1}{2} = \mathrm{e}^{-\lambda t_1} \text{ and } \frac{1}{10} = \mathrm{e}^{-\lambda t_2}$$

or $\mathrm{e}^{\lambda t_1} = 2$ and $\mathrm{e}^{\lambda t_2} = 10$

Taking log on both sides,

$$\lambda t_1 = \log 2 \Rightarrow t_1 = \frac{\log 2}{\lambda}$$

or $t_1 = \frac{\log 2 \times T}{\log 2}$

and $\lambda t_2 = \log 10$

$$\Rightarrow t_2 = \frac{\log 10}{\lambda}$$

or $t_2 = \frac{\log 10 \times T}{\log 2}$

$$\therefore (t_2 - t_1) = T\left[\frac{\log 10}{\log 2} - 1\right] = T\left[\frac{\log 10 - \log 2}{\log 2}\right]$$

$$= T\left[\frac{\log 5}{\log 2}\right]$$

$$\Rightarrow (t_2 - t_1) = T\log\left[\frac{5}{2}\right]$$

14. (a)

The gravitation force between two masses

$$F = \frac{Gm_1m_2}{r}$$

here, $m_1 = m$

and $m_2 = (M - m)$

$$\therefore \quad F = \frac{Gm(M-m)}{r^2}$$

For maximum gravitational force, $\frac{dF}{dm} = 0$

$$\therefore \frac{d}{dm}[m(M-m)] = 0$$

By solving, we get $m = \frac{M}{2}$

So, $\frac{m}{(M-m)} = \frac{1}{1}$

15. (d)

Mass of bullet $= m_1 = m$

Initial speed of bullet $= u_1 = v$

Mass of block $= m_2 = M$

Initial speed of block $= u_2 = 0$

Let the common velocity of the bodies after collision $= V$

According to conservation of linear momentum

$$m \times v + M \times 0 = (m + M)V$$

$$V = \frac{mv}{(m+M)}$$

$\therefore$ Heat generated $=$ loss in KE

$=$ Initial KE - Final KE

$$= \frac{1}{2}mv^2 - \frac{1}{2}(m+M)v^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2v^2}{(m+M)}$$

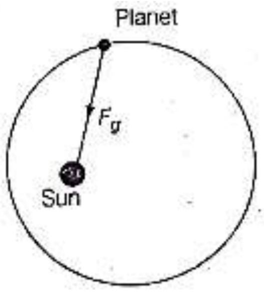
$$= \frac{1}{2}mv^2\left(1 - \frac{m}{m+M}\right)$$

$$= \frac{1}{2}mv^2\left(\frac{m+M-m}{m+M}\right)$$

$$= \frac{1}{2}\frac{mM}{(m+M)}v^2$$

16. (d)

A planet revolves around the sun; is an elliptical orbit under the effect of gravitational pull on the planet.



So, torque, $\mathbf{C}=\mathbf{r}\times\mathbf{F}=rF\sin 180^{\circ}\mathbf{n}=0$

As $\mathbf{C}=\frac{\mathrm{dL}}{\mathrm{dt}}$; so $\mathrm{L}=$ a constant

$\Rightarrow$ Angular momentum is constant.

17. (a)

Given, mass of the particle $=M$,
Charge on the particle $=q$
Electric field $=E$
Initial velocity, $u=0$

$\therefore$ Acceleration, $a=\frac{F}{M}=\frac{qE}{M}$

$\therefore$ Distance travelled in electric field,

$$S=ut+\frac{1}{2}at^2$$

$$S=\frac{1}{2}\left(\frac{qE}{M}\right)t^2$$

Also, kinetic energy $T=\frac{1}{2}M\left(\frac{qEt}{M}\right)^2$

So,

$$\frac{T}{S}=\frac{\frac{1}{2}M\left(\frac{qEt}{M}\right)^2}{\frac{1}{2}\left(\frac{qE}{M}\right)t^2}=qE$$

$\Rightarrow$ Ratio of $\frac{T}{S}$ remains constant with time t.

18. (c)

Amount of heat required, $Q=\int dQ$

$$=\int_{T_1=20}^{T_2=30}\mathrm{mcdT}$$

Given, $C=DT^3$

$$\therefore\ Q=\int_{20}^{30} mDT^3dT=mD\int_{20}^{30}T^3dT$$

$$=mD\frac{1}{4}[(30)^2-(20)^2]=\frac{65}{4}\times 10^4 mD$$

19. (b)

Here, $v=-60$ cm, $u=-12$ cm

$\therefore$ By using the relation, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

We have, $\frac{1}{f}=\frac{1}{-60}-\frac{1}{-12}$

$\Rightarrow\ \frac{1}{f}=\frac{1}{15}$ cm or $\frac{100}{15}$ m

So, the power of lens, $P=\frac{100}{15}=+\frac{20}{3}$D

20. (b)

At the distance of closest approach, the entire KE of a particle is converted into electric potential energy.
i.e., $\mathrm{KE}=\mathrm{PE}$

Here, KE of the particle = work done in moving the particle in uniform electric field (E) through a distance (D)
$=qE\times D$

PE of the particle $=$ potential $\times$ charge

$$= \frac{q}{4\pi\varepsilon_0 r_0} \times Q$$

$$\therefore \quad qED = \frac{qR}{4\pi\varepsilon_0 r_0} \Rightarrow r_0 = \frac{Q}{4\pi\varepsilon_0 ED}$$

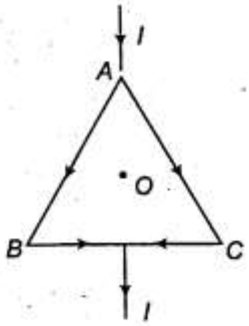
21. (a)

The horizontal component of earth's magnetic field is given by, $H = R\cos\delta$
where, δ is the angle of dip so $H_1 = R\cos 30^\circ$
and $H_2 = R\cos 45^\circ$

$$\therefore \frac{H_1}{H_2} = \frac{R\cos 30^\circ}{R\cos 45^\circ} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$$

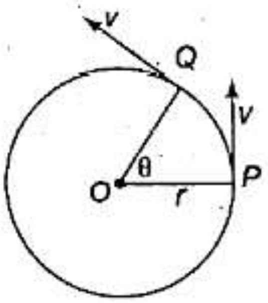
22. (d)

The given situation can be shown as
The magnetic field at centroid O = Magnetic field due to left part + Magnetic field due to right part



$\therefore B = B_1 + B_2 = 0$
($\because$ direction of B due to both parts is different of O)

23. (c)



$\therefore$ Change in magnitude of velocity

$$|\Delta v| = \sqrt{v^2 + v^2 - 2v^2\cos\theta}$$
$$= 2v\sin\frac{\theta}{2}$$

24.(d)

When the switch S is closed, the common potential across the two capacitors becomes V, so charge on isolated plates remains same.

$$\therefore \quad C_0V_0 = (C_0 + CV$$
$$\Rightarrow \quad CV = C_0V_0 - C_0V$$
$$\Rightarrow \quad C = \frac{C_0(V_0 - V)}{V}$$

25. (c)

Work done $= U_f - U_i$

$$= \frac{1}{4\pi\varepsilon_0} \times 2 \times 5 \times \left(\frac{1}{2} - \frac{1}{2}\right) = 0$$

26. (c)

The drift velocity is given as

$$v_d = \frac{i}{neA} = \frac{E}{R \times neA}$$
$$= \frac{E \times A}{\rho l \times neA} = \frac{E}{\ln e}$$

When length of wire changed to 21 , the new drift velocity,

$$\therefore\ v_d' = \frac{E}{\rho \times 2/\times ne}$$
$$\therefore\ v_d' = \frac{E/\rho 2 \ln e}{E/\rho \ln e} = \frac{1}{2}$$
$$\Rightarrow\ v_d' = \frac{v_d}{2}$$

27. (a)

Given, $M = 200A - m^2 B = 0.30 NA^{-1} M^{-1}$

and

$\theta = 30^\circ$

We know that the Torque,

$$\tau = M \times B$$
$$\Rightarrow\ |\tau| = MB\sin\theta = 200 \times 0.3 \times \frac{1}{2}$$
$$= 100 \times 0.3 = 30\ \mathrm{N-m}$$

28. (b)

Fraction $= \frac{\Delta U}{\Delta Q} = \frac{C_V}{C_\rho}$

For monoatomic gas $\frac{C_p}{C_V} = \gamma = \frac{5}{3}$

So, $\frac{C_V}{C_\rho} = \frac{1}{\gamma} = \frac{3}{5}$

29. (d)

Given for prism P_1

$\mu = 1.54, A = 4^\circ$

For prism P_2

$\mu' = ?, A' = 3^\circ$

For no deviation $= \delta + \delta' = 0$

$\Rightarrow\ (\mu - 1)A = (\mu' - 1)A'$

$\Rightarrow\ (1.54 - 1)4 = (\mu' - 1)3$

On solving we get $\mu' = 1.72$

30. (a)

As the water is flowing through the horizontal tube, so in the streamline flow of water, the sun of static pressure and dynamic pressure is constant i.e.,

$$P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho v_1^2$$
$$\Rightarrow\ v_1 = \sqrt{\frac{P}{\rho} + v^2}$$

31. (d)

Elastic energy per unit volume of wire.

$$\mu = \frac{1}{2} \times \text{ Young's modulus } \times (\text{ Strain })^2$$
$$\therefore\ \frac{u_1}{u_2} = \left\{\frac{(\text{ Strain })_1}{(\text{ Strain })_2}\right\}^2 = \frac{I^2}{L^2} \times \frac{4L^2}{4I^2} = 1:1$$

32. (c)

The terminal velocity, $v = \frac{2r^2(\rho-\sigma)g}{9\eta}$ As, all other parameters are constant, therefore $v \propto r^2$

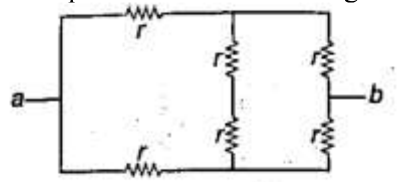
$$\Rightarrow\ \frac{v_R}{v_{3R}} = \frac{(R)^2}{(3R)^2} = \frac{1}{9}$$

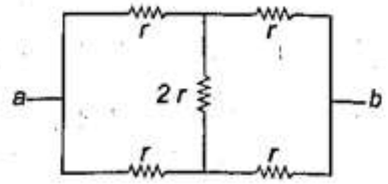
33. (a)

The mass defect, $\Delta m = 2(m_p + m_n) - m_{\text{He}}$
$= 2(1.00783 + 1.00867) - 4.00300$
$= 0.0300\text{amu}$
So, the binding energy, $E = \Delta mc^2$
$= 0.03 \times 931\text{MeV} \approx 27.9\text{MeV}$

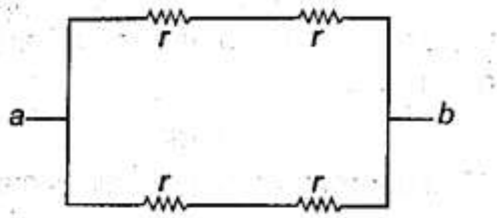
34. (d)

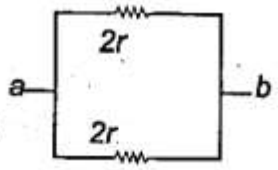
The equivalent circuit for the given electrical network is





This is a balanced Wheatstone bridge, so the arm containing $2r$ not in use. So, we have
So, the equivalent resistance between a and b is





$\frac{1}{r'} = \frac{1}{2r} + \frac{1}{2r} \Rightarrow r' = \frac{2r}{2} = r$

35. (a)

Linear magnification, $m = \frac{f}{f+u}$
As given that magnification is same for both cases this is possible if the two different values of u are of opposite sign.

i.e., $\frac{f}{f-16} = \frac{-f}{f-8}$
$\Rightarrow 16 - f = f - 8$
$\Rightarrow 2f = 24$
or $t = 12$ cm

36. (c)

By using the relation, $v = v\lambda$
We have, $\lambda = \frac{v}{v} = \frac{300}{500} = \frac{3}{5}$ m
The phase difference, $\phi = \frac{2\pi}{\lambda}(\Delta x)$
$\Rightarrow \frac{\pi}{3} = \frac{2\pi \times 5}{3}(\Delta x) \Rightarrow \Delta x = \frac{1}{10} \text{ m} = 10 \text{ cm}$

37. (c)

Let the velocity of third fragment is v'. Then by the conservation of linear momentum
$5(0) = M \times 2v - 2M \times v + 2M \times v'$
$\Rightarrow v' = 0$
i.e., the third fragment will be at rest.

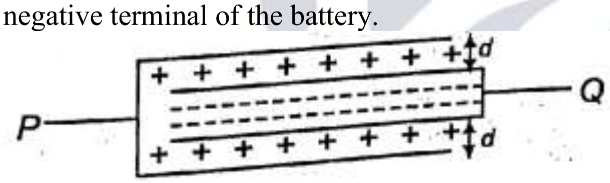
38. (a)

Given, $x(t) = 2t^3 - 3t^2 + 4t$
So, velocity, $v = \frac{dx}{dt} = (6t^2 - 6t + 4)$
and acceleration $a = \frac{dv}{dt} = (12t - 6)$
when acceleration is zero, i.e., $(12t - 6) = 0$
or
$$t = \frac{6}{12} = \frac{1}{2} s$$
∴ Velocity of the particle at zero acceleration is
$$v = 6\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right)4 = 2.5 \text{ m/s}$$

39. (b)
Let the length of closed pipe is L_1 and that of pipe is L_2.
Fundamental frequency of closed pipe, $v_1 = \frac{V}{4L_1}$
and frequency of second harmonic of open pipe,
$$v_2 = 2 \times \frac{V}{2L_2}$$
Given, $\quad v_1 = v_2$
i.e., $\quad \frac{v}{4L_1} = \frac{2v}{2L_2} \Rightarrow \frac{L_1}{L_2} = \frac{1}{4}$

40. (c)
Given, $I = 20\sin(100\pi t + 0.05\pi)$
The root mean square value of alternating current $= I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 10\sqrt{2}$ A
Also, $\omega = 100\pi$
$\Rightarrow\ 2\pi f = 100\pi$ or $f = 50$ Hz

41. (a)
Suppose the pair of plates is connected to positive terminal of the battery and the pair of plates Q is connected to the negative terminal of the battery.



From figure, it is clear that we have two capacitors C_1 and C_2. Positive plates of C_1 are connected to positive plate of C_2 and negative plate to negative. Therefore C_1 and C_2 are in parallel.
So, $C_p = C_1 + C_2 = 2C = \frac{2\varepsilon_0 a}{d}$

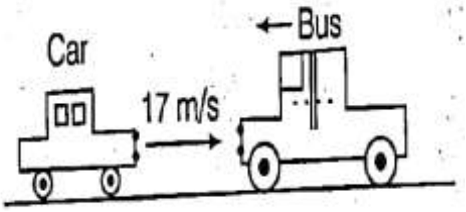
42. (d)
We have
Power = Rate of doing work = $\frac{dW}{dt}$
or, $P = \frac{dW}{dt}$ = rate of change in KE
i.e., $P = \frac{dW}{dt} = \frac{d(KE)}{dt}$ = constant

43. (a)
In an $n - p - n$ or $p - n - p$ transistor, the left hand side thick layer of the transistor is heavily doped known as emitter and right hand side thick layer of the transistor is moderately doped known as collector.

44. (a)
Given, $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
So, $\mathbf{C} = \frac{\hat{\imath}+2\hat{\jmath}+2\hat{k}}{\sqrt{1+4+4}} \times \sqrt{3^2 + 6^2 + 2^2}$
$$= \frac{\hat{\imath} + 2\hat{\jmath} + 2\hat{k}}{3} \times \sqrt{49} = \frac{7}{3}(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$$

45. (b)
Given, velocity of sound, $v = 340$ m/s

Velocity of listner, $v_L = 17$ m/s
Velocity of source $= v_s$
Frequency of horn emitted
$v = 640$ Hz



The apparent frequency

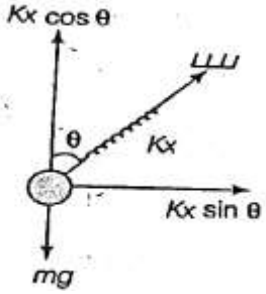
$$v' = v\frac{(v+v_L)}{v-v_s}$$

$$680 = 640\left(\frac{340+17}{340-v_S}\right)$$

On solving we get $v_S = 4$ m/s

46.(a)

The given situation can be shown as



From figure, $Kx\sin\theta = m\omega^2(L+x)\sin\theta$

$\Rightarrow \quad Kx = m\omega^2(L+x)$

Also, $\sqrt{\frac{K}{m}} = \omega_0$

$\Rightarrow \quad K = m\omega_0^2 \ldots\ldots\ldots(i)$

Substituting the value in Eq. (i)

$m\omega_0^2 x \quad =$

$\Rightarrow \quad x\omega^2(L+x)$

$x \quad = \frac{\omega^2 L}{\omega_0^2 - \omega^2}$

47. (d)

Given, $\rho = K\cdot r$

By Gauss's theorem

$$E(4\pi r^2) = \frac{\int \rho \times 4\pi r^2 dr}{\varepsilon_0}$$

$$= \frac{\int Kr \times 4\pi r^2 dr}{\varepsilon_0} \Rightarrow E = \frac{Kr^2}{4\varepsilon_0}$$

Here r

So,

$$E = \frac{KR^2}{4\varepsilon_0}$$

48. (a)

Given, $v = (3\hat{\imath} + 10\hat{\jmath})$m/s

$\Rightarrow v_x = 3$ and $v_y = 10$

$\therefore$ Maximum height attained, $H = \frac{v_y^2}{2g}$

$$= \frac{10 \times 10}{2 \times 10} = 5 \text{ m}$$

Range $= v_x \times T = v_x \times \frac{2v_y}{g} = \frac{3 \times 2 \times 10}{10} = 6$ m

49. (b)

We have, $H = I^2 R$

According to given condition, $l_1^2 R_1 = H$

and

$l_2^2 R_2 = 4\text{H}$

$\Rightarrow \frac{E^2}{(R_1+r)^2} R_1 = H$ and $\frac{E^2}{(R_2+r)^2} R_2 = 4H$

$$\therefore \frac{R_2}{(R_2+r)^2} = \frac{4R_1}{(R_1+r)^2}$$

$\Rightarrow \sqrt{R_2}(R_1 + r) = 2\sqrt{R_1}(R_2 + r)$

On solving,

$$r = \frac{\sqrt{R_1 R_2}[\sqrt{R_1} - 2\sqrt{R_2}]}{2\sqrt{R_1} - \sqrt{R_2}}$$

50. (b)

Given, $B = 2t + 4t^2$

at

$t = 0, B_1 = 0$

and at

$t = 2, B_2 = 2 \times 2 + 4(2)^2$

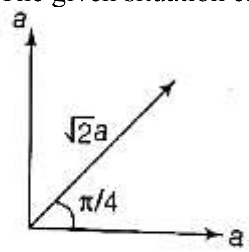
$= 4 + 16 = 20 \text{ Wb/m}^2$

We have, $\Delta Q = \frac{\Delta\phi}{R} = \frac{\pi r^2 (B_2 - B_1)}{R}$

$$= \frac{\pi r^2 [20 - 0]}{R} = \frac{20\pi r^2}{R}$$

51. (a)

The given situation can be shown as



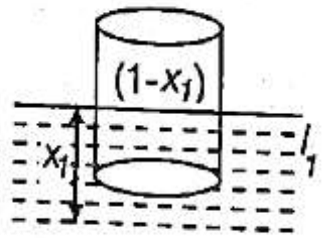
Fon second SHM

Ratio of amplitude $= \frac{a_1}{a_2} = \frac{\sqrt{3}}{\sqrt{2}}$

and phase difference, $\frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$

52. (a)

The given situation is shown as



As the Bouyant force in both the cases are same

$\therefore$ On solving

$\rho_1 x_1 g = \rho_1 x_2 g + \rho_2 (1 - x_2) g$

$$\frac{\rho_1}{\rho_2} = \frac{(1 - x_2)}{(x_1 - x_2)}$$

53. (c)

Restoring force on displacement of x

$$F = K\left[\frac{q}{(d-x)^2} - \frac{Qq}{(d+x)^2}\right]$$

$$= KQq\left[\frac{1}{(d-x)^2} - \frac{1}{(d+x)^2}\right]$$

$$= KQq\left[\frac{4dx}{(d^2-x^2)^2}\right]$$

$$= KQq\left[\frac{4dx}{d^4}\right] \text{ If } (d \gg x)$$

$$\Rightarrow F = KQq\left[\frac{4x}{d^3}\right]$$

Acceleration, $a = \frac{F}{m} = \frac{4KQqx}{Md^3}$

or $\omega^2 = \frac{4KQq}{Md^3}$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{Md^3}{4KQq}}$$

$$= 2\sqrt{\frac{\pi^3 Md^3 \varepsilon_0}{Qq}}$$

54. (d)

Here,

$h\nu_1 = \phi_0 = eV_1 \ldots\ldots\ldots\ldots(i)$

and

$h\nu_2 = \phi_0 + eV_2 \ldots\ldots(ii)$

From Eqs. (i) and (ii), we have

$$h(\nu_2 - \nu_1) = e(V_2 - V_1)$$

$$\Rightarrow \nu_2 = \frac{e}{h}N_2 - V_1) + \nu_1$$

55. (b)

The number of degrees of freedom for the mixture is

$$f_{\text{mix}} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$$

$$= \frac{3\times 3 + 1\times 5}{4} = \frac{7}{2}$$

$$\therefore \gamma = 1 + \frac{2}{f}$$

$$\Rightarrow \gamma = 1 + \frac{4}{7} = \frac{11}{7}$$

56. (b,d)

Magnetic moment, $m = IA$

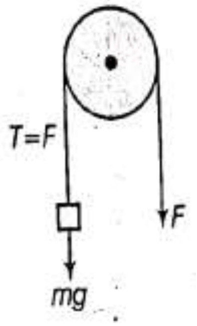
$$= \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

Angular momentum $= 2\,\mathrm{m}\frac{\mathrm{dA}}{\mathrm{dt}}$

57. (a,b,d)

The free body diagram

$F - mg = 2$



$\Rightarrow\ F = 2 + mg = 3$ N

also, $a = \frac{\text{unbalanced force}}{\text{mass}} = \frac{2}{0.1} = 20\ \text{m/s}^2$

$\therefore\ S = \frac{1}{2}at^2 = \frac{1}{2} \times 20 \times 1 = 10$ m

Hence, work done by tension

$= F \times 10 = 3 \times 10 = 30$ J

So, the work done against gravity

$= mg \times S = 1 \times 10 = 10$ J

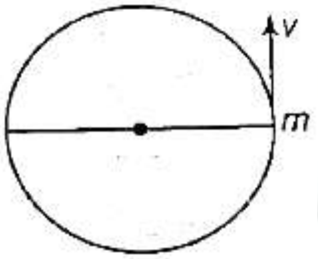
58. (a,c,d)

The charged particle will move with uniform velocity in that space if $E = 0$ and $B \neq 0$, since a moving charge in a magnetic field experience a force. Also if $E \neq 0, B \neq 0$; the charged particle will move with uniform velocity in that space.

If $E = vB$, then particle again moves with uniform velocity.

59. (a,c,d)

We have, $v = \frac{1}{2}\omega l$



$T = \frac{2v}{g} = \frac{\omega}{g}$ or $n\frac{2\pi}{\omega} = \frac{\omega}{g}$

(as completes n rotations with in T)

$\therefore\ n = \frac{1\omega^2}{2\pi g}$

Distance travelled $= 2h = \frac{2v^2}{2g} = \frac{I^2\omega^2}{4g}$

60. (b,c)

The lens will behave like a convex lens or a concave lens depending upon the value of n_1 and n_2.

Chemistry

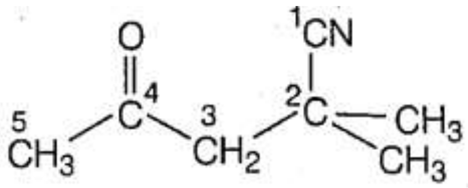
61. (b)

For. MX_2 type salt,

Solubility product, $K_{sp} = 4\ s^3; 3.2 \times 10^{-8} = 4\ s^3$

or $s = \sqrt[3]{\frac{3.2\times10^{-8}}{4}} = 2 \times 10^{-3}$ mol/L

62. (c)



2, 2-dimethyl-4-oxopentanenitrile

63. (d)

In $SOCl_2$, the $Cl-S-Cl$ and $Cl-S-O$ bond angles are respectively 96° and 106°.

64. (d)

A planar carbocation is generated when $SbCl_5$ remove $Cl^-$from the substrate. This carbocation is subsequently attacked by Cl^-(nucleophile) from both the sides (i.e. from top and bottom) to produce a racemic mixture.

65. (b)

$CH_3COOC_2H_5 + H_2O \rightleftharpoons CH_3COOH + C_2H_5OH$

If above reaction is carried out with large excess of ester, the rate of reaction does not depend upon concentration of both the reactants. Due to which, the reaction becomes of zero order.

66. (d)

Medium	Colour of Litmus Paper
Acidic	Red
Neutral	Violet
Basic	Blue

67. (a)

Baeyer's reagent is 1% cold dilute alkaline potassium permanganate. It is used to identify unsaturation. All unsaturated compounds lose its purple colour

68. (b)

Equivalent conductance (Λ_{eq}) is defined as the conducting power of all the ions produced by one gram equivalent of an electrolyte in a given solution.In case of weak electrolytes (as CH_3COOH, NH_4OH, AgCl etc.), ionisation is very small compared to strong electrolytes (as KCl , NaOH etc.), hence Λ_{eq} of weak electrolyte is low. Moreover, ionisation of $H^+$is maximum among given ions. Thus, correct of order of Λ_{eq} is

$H^+ > OH^- > K^+ > CH_3COO^-$

69. (a)

Ostwald process of for manufacture of nitric acid,

$$\underset{\text{ammonia}}{4NH_3} + SO_2 \xrightarrow[800-900^\circ C]{\text{Pt gauge}} \underset{\substack{\text{nitric oxide} \\ \Delta H=-21.5\text{kcal}}}{4NO} + 6H_2O$$

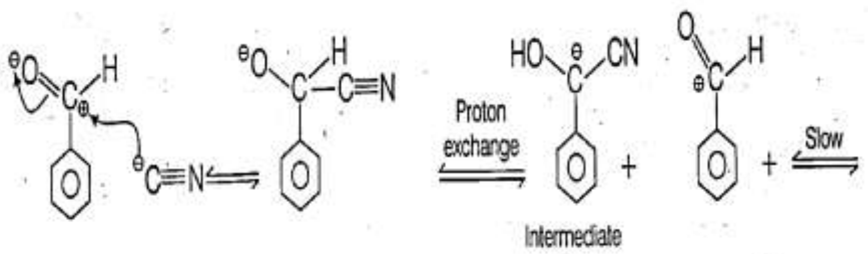
$$2NO + O_2 \xrightarrow{50^\circ C} \underset{\text{nitrogen dioxide}}{2NO_2}$$

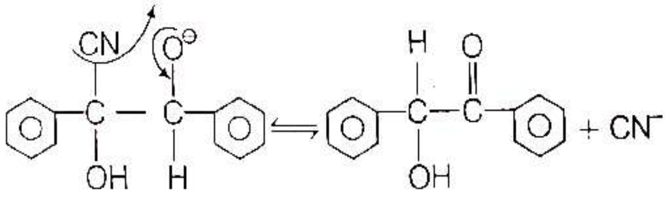
$$4NO_2 + 2H_2O + O_2 \longrightarrow \underset{\text{nitric acid}}{4HNO_3}$$

70. (c)

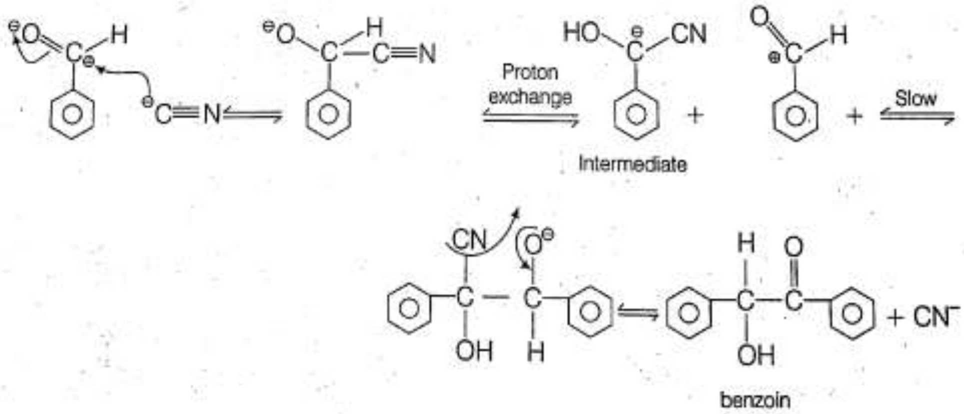
When benzaldehyde is heated with aqueous ethanolic NaCN or KCN , it dimerises to form an α-hydroxy ketone called benzoin, and this reaction is formed as benzoin condensation.

It involves self condensation of an aromatic aldehyde in the presence of CN^-as catalyst.



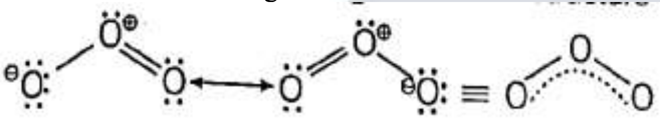


benzoin



71. (a)

Ozone shows following resonance structure



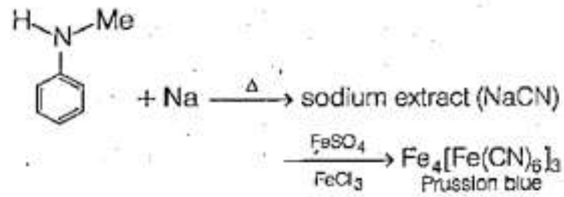
It contains bond angle 116.8° and $O - O$ bond length $1.28 Å$.

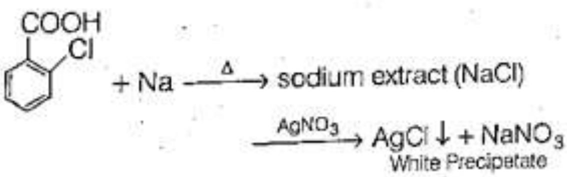
72. (c)

The entropy is the measure of disorder or randomness in a system. When a system changes from one state to another, the change of entropy, dS is given by

$$dS = \frac{\delta q_{\text{rev}}}{T}$$

73. (d)





Hydrozine, $NH_2 \cdot NH_2$ does not respond Lassaigne's test because it does not contain any carbon and hence, NaCN is not formed.

Compound, $NH_2 - \underset{\underset{S}{|}}{C} - NH_2$

contains both$N$ and S, hence, it will give red colour in Lassaigne test.

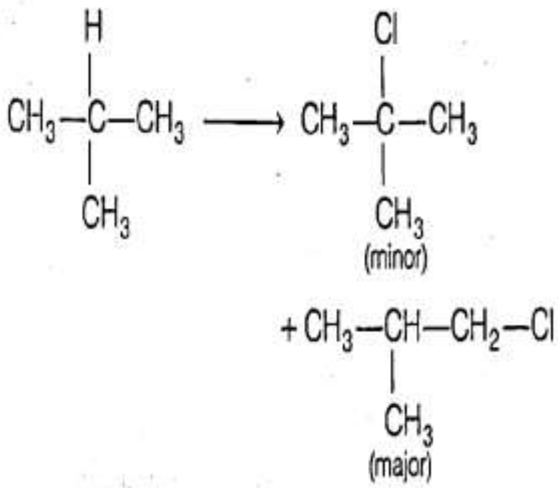
74. (d)

$2CaO + Cl_2 \xrightarrow{\text{Red hot}} CaCl_2 + O_2 \uparrow$

75. (c)

From Arrhenius equation
or
$$\ln\frac{k_2}{k_1}=\frac{E_a}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$
or
$$\ln\frac{k_2}{k_1}=\frac{600R}{R}\left(\frac{1}{300}-\frac{1}{600}\right)$$
$$\ln\frac{k_2}{k_1}=600\left(\frac{2-1}{600}\right)$$
or
$$\ln\frac{k_2}{k_1}=1 \text{ or } \frac{k_2}{k_1}=e$$

76. (c)

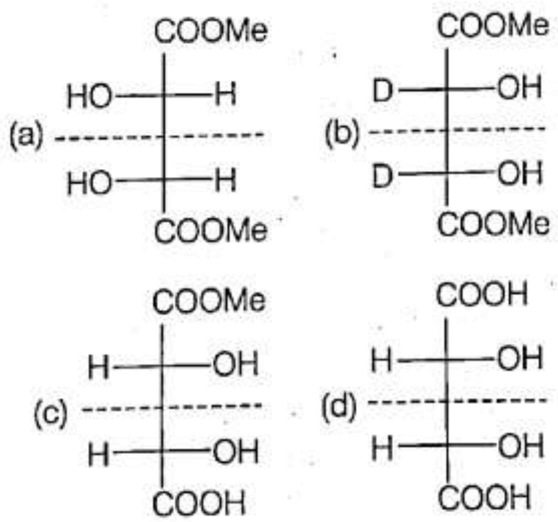


There are nine primary hydrogens and one tertiary hydrogen in 2-methyl propane. Tertiary hydrogen atoms react with Cl about 5.5 times as fast as primary. Even though the primary hydrogens are less reactive, there are so many of them that the primary product is the major product.

77. (d)
$$n=\frac{\text{total time }(t)}{\text{half}-\text{life }(t/2)}=\frac{22920}{5730}=4$$
$$\text{Left amount, } N=N_0\left(\frac{1}{2}\right)^n$$
$$=N_0\left(\frac{1}{2}\right)^4=\frac{N_0}{16}$$
$$\therefore\ \text{Decayed fraction } =N_0-\frac{N_0}{16}$$
$$=\frac{16N_0-N_0}{16}=\frac{15N_0}{16}$$

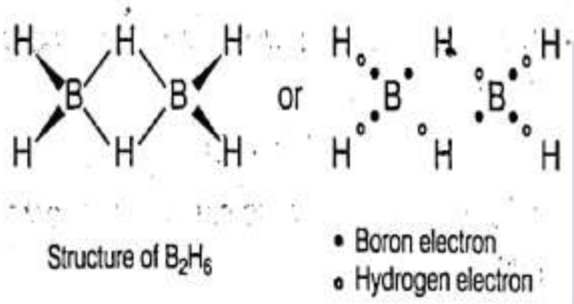
78. (c)
At very low pressure, compressibility factor, Z is approximatedly one.
∴ van der Waals' gas may behave ideally.

79. (c)



Only molecule (c) is optically active. Other moelcules are meso, due to presence of plane of symmetry.

80. (d)



In diborone, odd—electron bonds are found in the bridges. From figure, it is clear that 4 electrons are present for bonding in the bridges.

81. (a)

$$\underset{\text{sodium nitropruside}}{Na_2S + Na_2[Fe(CN)_5NO]} \longrightarrow Na_4[Fe(CN)_5 \underset{\text{purple colour}}{[NOS]}$$

$$\underset{\text{tetra anionic complex of iron (II) co—ordinating}}{Na_4[Fe(CN)_5NOS]} \rightleftharpoons 4Na^+ + \underset{\text{to one NOS}^-\text{ions.}}{[Fe(CN)_5NOS]^{4-}}$$

82. (b)

$NaOH \rightleftharpoons Na^+ + OH^-\ [OH^-] = 10^{-8}M$

$H_2O \rightleftharpoons H^+ + OH^-\ [OH^-] = 10^{-7}M$

$\therefore [OH^-]_{total} = (10^{-8} + 10^{-7})M$

$= 10^{-7}(1.1)$

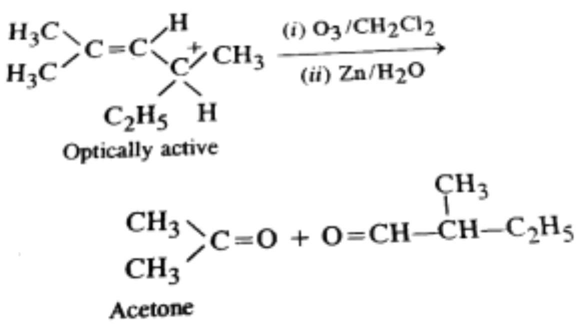
$= 1.1 \times 10^{-7}$

$\therefore \quad pOH = \log 1.1 \times 10^{-7} = 6.98$

$\therefore \quad pH = 14 - 6.98$

$= 7.02$

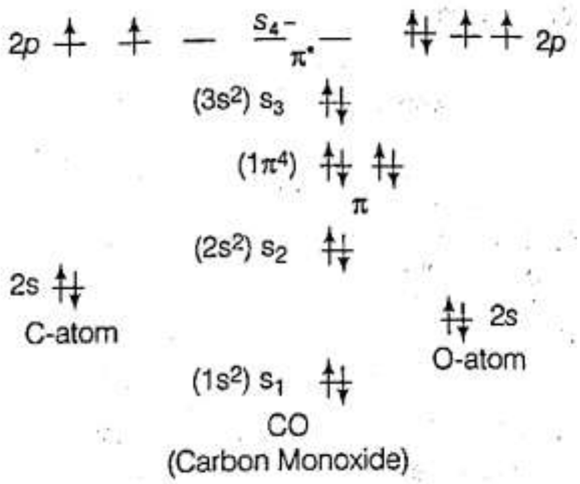
83. (b)



84. (c)

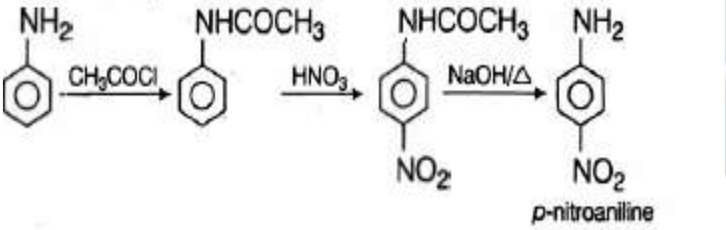
On mixing of two different ideal gases under isothermal reversible conditions, ΔS_{mix} is always positive i. e. , increasing.

85. (a)



86. (a)

Direct nitration of aniline with nitric acid gives a complex mixture of mono, di—and tri—nitro compounds and oxidation products. If $-NH_2$ group is protected by acetylation and then nitrated with nitrating mixture, p—isomer is the main product.



87. (b)

$\because \quad \Delta T_f$

$\therefore \quad i \times K_f \times m$

$$\therefore \quad i = \frac{\Delta T_f}{K_f \times m} = \frac{0.19}{1.86 \times 0.1} = 1.02$$

Again from, $\alpha = \frac{i-1}{n-1} = \frac{1.02-1}{2-1}$

$= 0.02 = 2.0 \times 10^{-2}$

$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

for $\quad Ka = C\alpha^2$

$= 0.1 \times (2 \times 10^{-3})^2$

$= 4 \times 10^{-5}$

88. (a)

Ore chomite is $FeCr_2O_4$.

89. (d)

H_2SO_4 saturated with SO_3 is called oleum or sulphan.

$H_2SO_4 + SO_3 \longrightarrow H_2S_2O_7$

90. (b)

From first law of thermodynamics

$\Delta E = q + W$

where, work do net $(W) = p\Delta V$

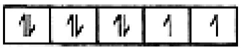
$= p \times V \quad (\because \Delta V = 0)$

$= 0$

91. (a)

Species having unpaired electrons are paramagnetic

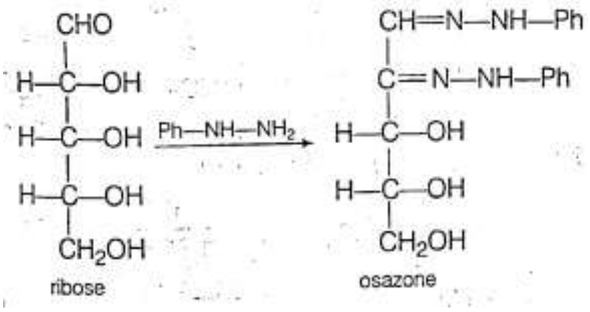
$Ni^{2+} = [Ar]3d^8$



(i) In $[NiCl_4]^{2-}$ and $[Ni(H_2O)_6]^{2+}$ ligands $Cl^-$and H_2O are weak ligands, therefore no pairing will be possible. Thus, there are two unpained electrons:
(ii) In $\mathbb{N}\mathring{\imath}(Ph_3)_2Cl_2]$ although PPh_3 has d-acceptance nature but presence of Cl , makes electrons unpaired.

92. (d)

Ribose forms ozone with $Ph-NH-NH_2$ whereas in deoxyribose, one - OH group is missing, due to which, it cannot form ozone.

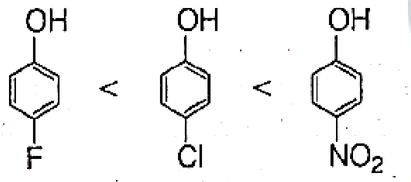


93. (c)

$10 \text{ million} = 10^{-7}$
$\therefore$ 10 million $^{\text{th}}$ part of $1.33 \text{ cm}^3 = 1.33 \times 10^{-7} \text{ cm}^3$
$= 1.33 \times 10^{-7}$ mL
For pure water, $[H^+] = 10^{-7}$ mol/L
$\Rightarrow$ 1 L water contains $[H^+] = 10^{-7}$ mol
or 1 mL water contains $[H^+] = \frac{10^{-7}}{1000} = 10^{-10}$ mol or 10 million $^{\text{th}}$ part of 1.33 cm^3 water contains $[H^+]$
$= 1.33 \times 10^{-7} \times 10^{-10}$ mol
$= 1.33 \times 10^{-17}$ mol$^-$
$\therefore$ Number of H^+ions $= 1.33 \times 10^{-17} \times N_A$
$= 1.33 \times 10^{-17} \times 6.022 \times 10^{23}$
$= 8.009 \times 10^6 = 8.01$ million

94. (c)

The order of electron withdrawing tendency from benzene ring i.e.,
$-F < -Cl < -NO_2$
$\therefore$ Correct order of acidic strength of substituted phenols will be



95. (d)

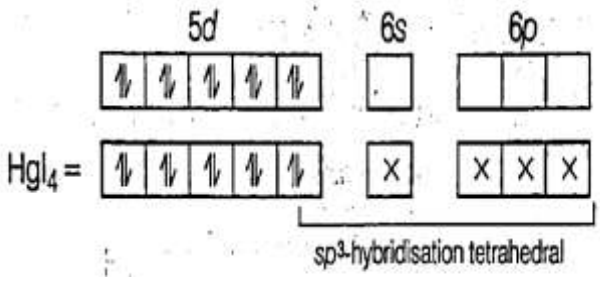
For isothermal expansion of an ideal gas
$\Delta T = 0$
$\therefore$ From $\Delta U = nC_v\Delta T$,
$\Delta U = 0$ and, from
$\Delta H = nC_p\Delta t = 0$
From first law of thermodynamics,
as
$\Delta U = Q + W$
$\Delta U = 0 \Rightarrow Q \neq 0$
and
$W \neq 0$
$\therefore$ Parameters are
$\Delta U = 0, Q \neq 0; W \neq 0$
$\Delta H = 0$

96. (a)

$HgCl_2 + 4KI \longrightarrow K_2[HgI_4] + 2KCl$

$Hg = [Xe]4f^{14}, 5d^{10}6s^2$

$Hg^{2+} = [Xe]4f^{14}, 5d^{10}, 6s^{\circ}$



97. (b)

Molecules having whole number for degree of unsaturation can exist in free state as stable compounds.

Degree of unsaturation $= \frac{\Sigma n(v-2)}{2} + 1$ where, $n =$ number of atoms of a particular type $v =$ valency of the atom.

(a) For $C_7 - H_9O$,

$$DU = \frac{7(4-2) + 9(1-2) + 1(2-2)}{2} + 1 = 3 \cdot 5$$

(b) For $C_8H_{12}O$,

$$DU = \frac{8(4-2) + 9(1-2) + 1(2-2)}{2} + 1 = 3 \cdot 0$$

(c) For $C_6H_{11}O$,

$$DU = \frac{6(4-2) + 11(1-2) + 1(2-2)}{2} + 1 = 1.5$$

(d) For $C_{10}H_1O_2$,

$$DU = \frac{10(4-2) + 17(1-2) + 2(2-2)}{2} + 1 = 2.5$$

$\therefore C_8H_{12}O$ will exist in free state or a stable compound.

98. (c)

Given, $\kappa = 1.25 \times 10^{-3}$ S cm^{-1}

$$\rho = \frac{1}{\kappa} = \frac{1}{1.25 \times 10^{-3}} S^{-1} \text{ cm}$$

From . $R = \rho \frac{l}{A}$

$$800 = \frac{1}{1.25 \times 10^{-3}} \times \frac{1}{A}$$

$$\left(\text{where}, \frac{1}{A} = \text{ cell constant} \right)$$

$$\therefore \frac{1}{A} =$$

$$=$$

$$=1 \text{ cm}^{-1}$$

99. (b)

Orange solid is ammonium dichromate.

$$\underset{\text{orange solid}}{(NH_4)_2Cr_2O_7} \xrightarrow{\Delta} \underset{\text{colourless gas}}{N_2 \uparrow} + \underset{\text{green solid}}{Cr_2O_3} + 4H_2O$$

$Cr_2O_3 + 2Al \longrightarrow 2Cr + Al_2O_3$

100. (b)

2,2-dimethyl butane is prepared from Corey House alkane synthesis.

$$(Me_3C)_2CuLi + MeCH_2Br \underset{Br}{\xrightarrow{S_N2}}$$

$Me_3C - CH_2CH_3 + Me_3CCuLi$

101. (b)

From Gibbs Helmholtz equation,

$\Delta G = \Delta H - T\Delta S$

A process will be spontaneous, if
$\Delta G = -ve$

102. (a)

No option is correct.

As the number of lone pair of electrons increases, bond angle decreases.

NO_2^+ ion is isoelectronic with CO_2 molecule. It is a linear ion and its central atom (N^+) undergoes sp-hybridisation. Hence, its bond angle is 180°. In NO_2^- ion, N -atom undergoes sp^2 hybridisation. The angle between hybrid orbital should be 120° but one lone pair of electrons is lying on N -atom, hence bond angle decreases to 115°.

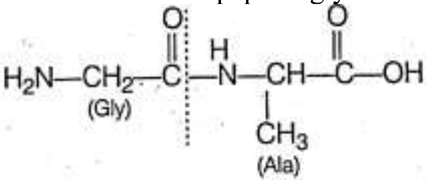
In NO_2 molecule, N -atom has one unpaired electron in sp^2-hybrid orbital. The bond angle should be 120° but actually, it is 132°. It may be due to one unpaired electron in sp^2-hybrid orbital.

Therefore, the increasing order of bond angle is

$NO_2^- < NO_2 < NO_2^+$
115° 132° 180^0

103. (c)

The structure of dipeptide gly-ala is



104. (d)

. $\lambda_m^\infty = \lambda_m^\infty\,(\text{oxalate}) + \lambda_m^\infty(\text{Na}^+) + \lambda_m^\infty(\text{K}^+)$

$= (148 \cdot 2 + 73 \cdot 5 + 50 \cdot 1)$

$= 271.85\ \text{cm}^2\ \text{mol}^{-1}$

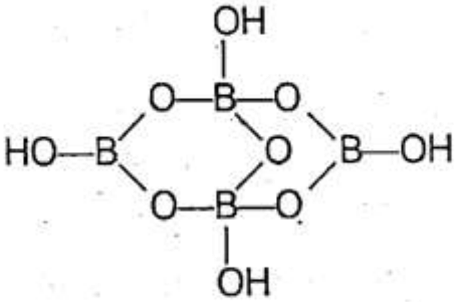
$\lambda_{\text{eq}}^\infty = \dfrac{\lambda_m^\infty}{n \cdot \text{ factor}} = \dfrac{271.8}{2}$

$= 135.9\text{Scm}^2\text{eq}^{-1}$

105. (c)

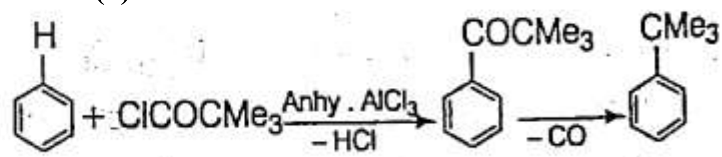
Since, ionic character is inversely proportional to polarising power of cation therefore correct order is
$AlCl_3 > GaCl_3 > BCl_3$

106. (a)



5B-O-Bbonds
4B-OH bonds

107. (b)



108. (b)

$CH_3COOO^- + H^+ \longrightarrow CH_3COOH$

Initially	0·01	0·001	0·01
at equi.	0·01 − 0·001 = 0·009		0·01 + 0·001 = 0·011

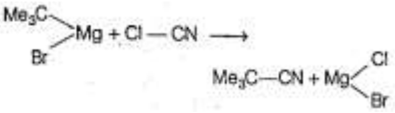
$\text{pH} = \text{p}K_\text{a} + \log\dfrac{(\text{ salt })}{(\text{ acid })} = 4.75 + \log\dfrac{0.009}{0.011}$

$= 4.66$

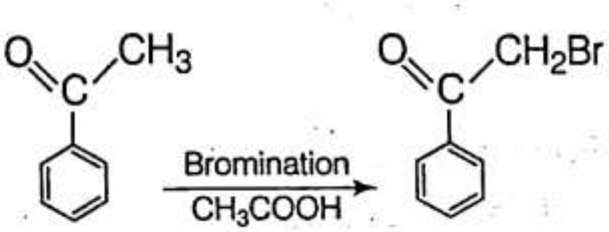
109. (a)

$3HClO_3 \xrightarrow{\Delta} HClO_4 + Cl_2 + 2O_2 + H_2O$

110. (c)



111. (d)



112. (b)

$\Delta G^\circ = -2.303RT\log K$

$= -2.303 \times 8.314 \times 298 \times \log 0.15$

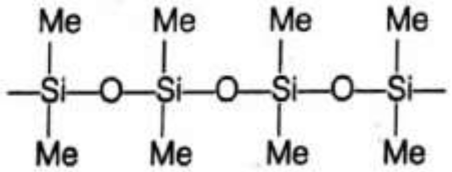
$= -2.303 \times 8.314 \times 298 \times (-0.82)$

$= 4678.7\ \mathrm{J} = 4.67\ \mathrm{kJ}$

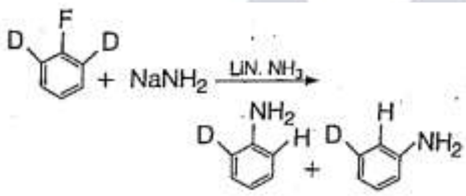
113. (a)

Silicon oil in a polymer of trimethylchloro silane and dimethyldichloro silane. These are useful as broad spectrum antifoaming agents.

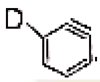
$Si(Mel_3)Cl + Si(Me)_2Cl_2 + H_2O \xrightarrow{\text{Polymerisation}}$



114. (d)



This reaction proceeds via benzyne mechanism

with intermediate as 

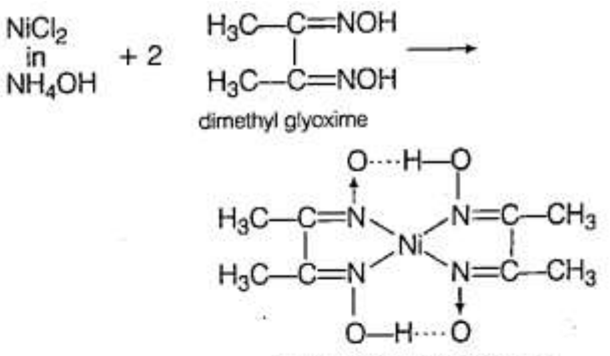
115. (d)

All the quantum numbers (n, l, m, s) are required to describe an electron of an atom completely.
e.g., n describes the position and energy of the electron in an orbit or shell. I is used to describe subshell and the shape of the orbital occupied by the electron.
m describes the preferred orientation of orbitals in space.
s describes the spining of an electron on its axis.

116. (a,b,c,d)

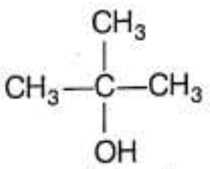
Estimation of Ni^{2+} is carried out as.

Filtrate of group III $+NH_4OH + NH_4Cl \xrightarrow{\Delta}$ passing H_2 S gas → black ppt of NiS .
This black ppt of NiS is soluble in conc. HCl in presence of oxidising agent like $KClO_3$
$NiS + \underset{\text{Conc}}{2HCl} + O \longrightarrow NiCl_2 + H_2 \cdot O + S$
Now this $NiCl_2$, in basic medium, treated with dimethyl glyoxime, cherry red ppt of nickel (II) dimethyl glyoximate is obtained, in which two dimethyl glyoximate units are hydrogen bonded to each other and each unit forms a six-membered chelate ring with Ni^{2+}.



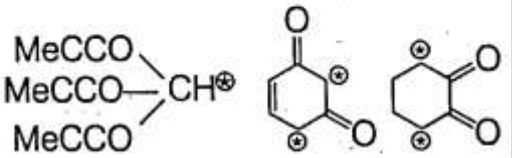
nickel (II) dimethyl glyoximate
cherry red ppt

117. (b,c)

$CH_3CH_2CH_2CH_2OH$ 
t-butanol

nbutanol
More branching results high solubility and low boiling point.

118. (a,b,d)



(a) (b) (d)

Availability of acidic $\boldsymbol{\alpha}-$ **H**-atoms at (*) marked positions, enable the compounds to show ketoenol tautomerism.

119. (a,c,d)

Lintz and Donawitz (L.D.) process is an important process for manufacturing of steel. In India, Rourkela steel plant is based upon this process.
This process is carried out in a converter similar to Bessemer converter having lining (usually refractory made of MgO and (CaO).
In this method scrap iron can be used along with flux in converter : On passing oxygen gas at 4-12 atm pressure into converter, temperature of $2000-2500^{\circ}\mathrm{C}$ range produced which separates C , Si , Mn etc., impurities as slag from iron quickly remaining pure iron is mixed with spiegeleisen (alloy $\mathrm{Fe}, \mathrm{Mn}, \mathrm{C}$) to produce better quality steel.

120. (a,d)

$$2\mathrm{NO}_2(g)+\mathrm{F}_2(g) \longrightarrow 2\mathrm{NO}_2\ \mathrm{F}(g)$$

$$\text{Rate of reaction } =-\frac{1}{2}\left[\frac{dp(\mathrm{NO}_2)}{dt}\right] =-\left[\frac{dp(\mathrm{F}_2)}{dt}\right] =+\frac{1}{2}\left[\frac{dp(\mathrm{NO}_2\ \mathrm{F})}{dt}\right]$$

Mathematics

121. (b)

The given equation
$ax^2+bx+1=0 \ldots\ldots\ldots..(i)$
has real roots.
$\therefore$ Discriminant $(D) \geq 0$
$\Rightarrow\ b^2-4a \geq 0 \ldots\ldots\ldots(ii)$

From Eq. (ii), we observe that
a has to be 1 and b has to be 2 .
So, the required probability $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

122. (c)

Ist turn Total number of face card $=12$
Total number of elements in sample space, $n(s)=52$
$\therefore P_1$ (no face card in first turn) $=\frac{52-12}{52}=\frac{40}{52}$
IInd turn P_2 (no face card in second turn)

$$=\frac{(52-13)}{(52-1)}=\frac{39}{51}$$

IIIrd turn P_3 (face card in third turn)

$$=\frac{(52-40)}{(51-1)}=\frac{12}{50}$$

$\therefore$ Required P (face card on third turn)

$$=P_1\times P_2\times P_3$$

$$=\frac{40}{52}\times\frac{39}{51}\times\frac{12}{50}=\frac{12}{85}$$

123. (d)

Let $E\rightarrow$ Event of head showing up
$E_1\rightarrow$ Event of biased coin chosen
$E_2\rightarrow$ Event of unbiased coin chosen
Now, $P(E_2)=\frac{1}{2}$ and $P(E_1)=\frac{1}{2}$
Also, $P\left(\frac{E}{E_2}\right)=\frac{1}{2}$ and $P\left(\frac{E}{E_1}\right)=\frac{3}{4}$
(by conditional probability)
By Baye's theorem

$$P\left(\frac{E_2}{E}\right)=\frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_1)\cdot P\left(\frac{E}{E_1}\right)}$$

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{3}{4}}=\frac{2}{5}$$

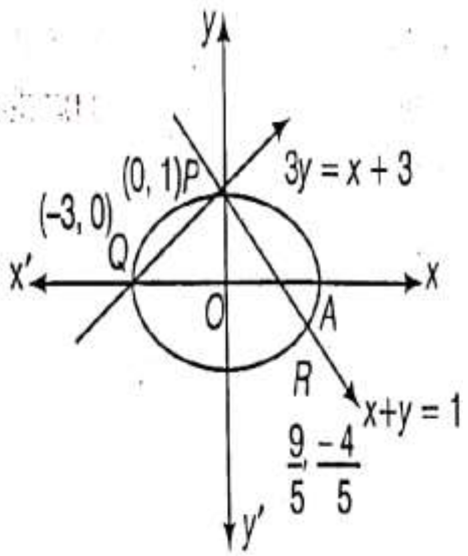
124. (b)

Given equation of ellipse

$$x^2+9y^2=9$$

$$\Rightarrow \frac{x^2}{9}+\frac{y^2}{1}=1 \quad \ldots\ldots\ldots\ldots(i)$$

and equation of lines
$x+y=1$ and $3y=x+3$
or $\frac{x}{(-3)}+\frac{y}{1}=1$



$\therefore$ Area of $\triangle PQR = \left|\frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ -3 & 0 & 1 \\ 9/5 & -4/5 & 1 \end{vmatrix}\right|$

$= \frac{1}{2}\left[\frac{24}{5} + \frac{12}{5}\right]$

$= \frac{1}{2} \times \frac{36}{5} = \frac{18}{5}$

125. (a)

Given equation of lines are

$3tx - 2y + 6t = 0$

and $3x + 2ty - 6 = 0$

On multiplying Eq. (i) by t and

Eq. (ii), we get

$(3t^2 + 3)x + 6t^2 - 6 = 0$

$\Rightarrow$

$\Rightarrow \quad x = \dfrac{2(1 - t^2)}{(1 + t^2)}$

$\Rightarrow \quad x + xt^2 = 2 - 2t^2$

$\Rightarrow \quad (x + 2)t^2 = (2 - x)$

$\Rightarrow \quad t^2 = \dfrac{2 - x}{2 + x} \ldots\ldots\ldots (iii)$

On multiplying Eq. (ii) by t and then subtract from Eq. (i), we get

$(-2 - 2t^2)y + 6t + 6t = 0$

$12t = 2(1 + t^2)y$

On squaring both sides, we get

$144t^2 = 4y^2(1 + t^2)^2$

$\Rightarrow 144\left(\frac{2-x}{2+x}\right) = 4y^2\left(1 + \frac{2-x}{2+x}\right)^2$ [form Eq. (iii)]

$\Rightarrow \; 36\left(\dfrac{2 - x}{2 + x}\right) = y^2\left(\dfrac{4}{2 + x}\right)^2$

$\Rightarrow \; 36\dfrac{(2 - x)}{(2 + x)} = \dfrac{16y^2}{(2 + x)^2}$

$\Rightarrow \; 36(4 - x^2) = 16y^2$

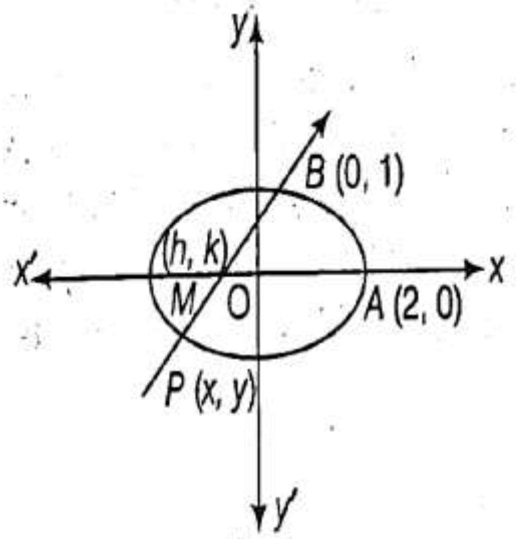
$\Rightarrow \; 9(4 - x^2) = 4y^2$

$\Rightarrow \quad 36 - 9x^2 = 4y^2$

$\Rightarrow \quad 9x^2 + 4y^2 = 36$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$ which represents an ellipse.

126. **(c)**



Given equation of an ellipse is

$x^2 + 4y^2 = 4$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{1} = 1 \ldots\ldots$(i)

$\therefore$ Coordinate of positive end of minor axis is $B(0,1)$. Let mid-point of the chord BP is $M(h,k)$

Then,

$(h,k) = \left(\frac{0+x}{2}, \frac{1+y}{2}\right)$

$\Rightarrow\ h = \frac{x}{2} \Rightarrow x = 2h$

and

$k = \frac{1+y}{2} \Rightarrow y = 2k - 1$

$P(x,y) \equiv \{2h, (2k-1)\}$

Since, the point ' P ' lies cllipes so form Eq. (i), we get

$(2h)_1^2 + 4(2k-1)^2 = 4$

$\Rightarrow \quad 4h^2 + 4(2k-1)^2 = 4$

$\Rightarrow \quad h^2 + 4k^2 + 1 - 4k = 1$

$\Rightarrow \quad h^2 + 4k^2 - 4k = 0$

Thus, required locus is an ellipse whose equation is

$x^2 + 4y^2 - 4y = 0$

$\Rightarrow \frac{(x-0)^2}{1} + \frac{\left(y - \frac{1}{2}\right)^2}{\left(\frac{1}{4}\right)} = 1$

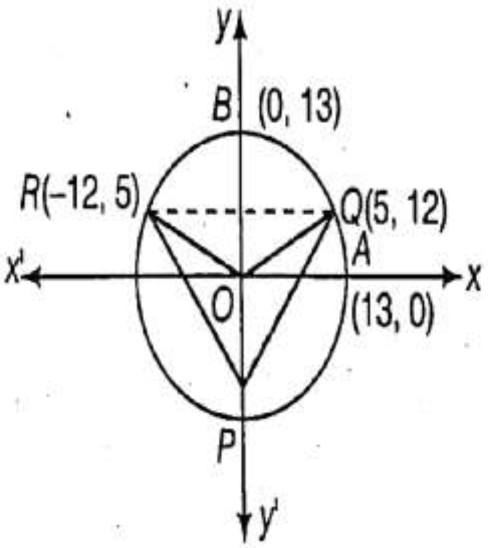
whose centre $\left(0, \frac{1}{2}\right)$ and major and minor axis $\frac{1}{2}$

127. **(b)**

Given equation of circle is

$x^2 + y^2 = 169$

Its centre $= (0,0)$ and radius $= 13$



Now, slope of $OR = \frac{-5}{12} = m_1 (\because \text{slope} = \frac{y_2 - y}{x_2 - x})$

and slope of $OQ = \frac{12}{5} = m_2$

$\because\ m_1 \cdot m_2 = -1 \Rightarrow \angle ROQ = \frac{\pi}{2}$

We know that, angle made by the chord of circle at circumference is equal to the half of the angle made by the same chord at the centre of circle.

$\therefore\ \angle QPR = \frac{1}{2} \cdot \angle ROQ = \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$

128. (b)

Let P be any point, whose coordinate is (h, k). Given,

P moves, so that the sum of squares of its distances from the points $A(1,2)$ and $B(-2,1)$ is 6 .

i.e., $(PA)^2 + (PB)^2 = 6$

$\Rightarrow\ (h-1)^2 + (k-2)^2 + (h+2)^2 + (k-1)^2 = 6$

$\Rightarrow\ h^2 + 1 - 2h + k^2 + 4 - 4k + h^2 + 4 + 4h$

$+k^2 + 1 - 2k = 6$

$\Rightarrow\ 2h^2 + 2k^2 + 2h - 6k + 4 = 0$

$\Rightarrow\ h^2 + k^2 + h - 3k + 2 = 0$

$\therefore$ Required locus is

$x^2 + y^2 + x - 3y + 2 = 0$

Which represent a circle.

Whose centre is $\left(\frac{-1}{2}, \frac{3}{2}\right)$

and radius $= \sqrt{\frac{1}{4} + \frac{9}{4} - 2} = \sqrt{\frac{5}{2} - 2} = \frac{1}{\sqrt{2}}$

129. (c)

Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$……..(i)

When, circle (i) passes through the origin

Then,

$c = 0$…………(ii)

When, circle (i) passes through the point (2,6).

Then, $4 + 36 + 4g + 12f + 0 = 0$

$\Rightarrow\ 4g + 12f + 40 = 0$

$\Rightarrow\ g + 3f = -10$………..(iii)

When, circle (i) passes through the point (6,2)

Then, $36 + 4 + 12g + 4f + 0 = 0$ [from Eq. (i)]

$\Rightarrow\ 12g + 4f + 40 = 0$

$\Rightarrow\ 3g + f + 10 = 0 \dots\dots\dots..$ (iv)

On solving Eqs. (iii) and (iv), we get

$g = \frac{-5}{2}$ and $f = \frac{-5}{2}$

$\therefore$ Equation of circle becomes

$x^2 + y^2 - 5x - 5y = 0 \ldots \ldots \ldots$ (v)

Circle cut the x-axis.

So, put $y = 0$ in Eq. (v), we get

$$x^2 - 5x = 0 \Rightarrow x(x-5) = 0$$

$$\Rightarrow \quad x = 5$$

So, the circle cut the x-axis at point $P(5,0)$.

$\therefore$ The length of $OP = 5$

130. (d)

Given equation of lines are

and

$x - 2y = t$...........(i)

$x + 2y = \frac{1}{t}$..........(ii)

On multiplying Eqs. (i) and (ii), we get

$$(x-2y)(x+2y) = t \times \frac{1}{t}$$

$$\Rightarrow \quad x^2 - 4y^2 = 1$$

$$\Rightarrow \quad \frac{(x-0)^2}{1} - \frac{(y-0)^2}{(1/4)} = 1$$

which represent a hyperbola.

Here, $a^2 = 1$ and $b^2 = \frac{1}{4}$

$\therefore$ Eccentricity $(e) = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{1+\frac{1}{4}}{1}} = \frac{\sqrt{5}}{2}$

and focus

$$= (\pm ae, 0) = \left(\pm 1 \times \frac{\sqrt{5}}{2}, 0\right) = \left(\pm \frac{\sqrt{5}}{2}, 0\right)$$

and centre $= (0,0)$

131. (d)

Let $A = \{1,2, \ldots ,11\}$

$\therefore$ $n(A) = 11$ and $B = \{1,2, \ldots ,10\}$

$\therefore$ $n(B) = 10$

$\therefore$ Hence number of onto function

$= {}^{n(A)}C_{n(B)} \times n(B)! \times n(B)$

$= {}^{11}C_{10} \times 10! \times 10$

$= (11 \times 10!) \times 10 = 111 \times 10$(i)

132. (d)

Let $p(x) = ax^2 + bx + c$.........(i)

Given, constant term 'c' = 1

$\therefore$ $p(x) = ax^2 + bx + 1$............(ii)

Now, by given condition, $p(1) = 2$ (remainder)

$\Rightarrow a + b + 1 = 2$.....(iii)

$\Rightarrow a + b = 1$........(iv)

and $p(-1) = 4$

(remainder)

$\Rightarrow \quad a - b + 1 = 4$

$\Rightarrow \quad a - b = 3$(iv)

On adding Eqs. (iii) and (iv), we get

$2a = 4 \Rightarrow a = 2$ form eys (iii) $b = -1$

On putting the values of a and b in Eq. (ii), we get

$p(x) = 2x^2 - x + 1 = 0$

$\therefore$ Sum of the roots $= -\frac{(\text{ Coefficient of } x)}{(\text{ Coefficient of } x^2)} = \frac{-(-1)}{2}$
$= \frac{1}{2}$

133. (a)

$\lim_{x\to 0}\left\{\frac{1}{x^2}+\frac{(2013)^x}{e^x-1}-\frac{1}{e^x-1}\right\}$
$= \lim_{x\to 0}\left\{\frac{1}{x^2}+\frac{(2013)^x-1}{e^x-1}\right\}$
$= \lim_{x\to 0}\left\{\frac{1}{x^2}+\frac{(2013)^x-1}{x}\cdot\frac{x}{e^x-1}\right\}.$
$= \lim_{x\to 0}\frac{1}{x^2}+\lim_{x\to 0}\frac{(2013)^x-1}{x}\cdot\lim_{x\to 0}\frac{x}{e^x-1}$
$= +\infty+\log(2013)\cdot 1$
$= +\infty$

134. (d)

When eleven apples are distributed among a girl and a boy, then the girl receives atleast 14 apples or the boys receives atleast 8 apples. (by hypothesis)

135. (c)

Given, $z_1 = 2+3i$ and $z_2 = 3+4i$
Now, we have

$|z-z_1|^2+|z-z_2|^2 = |z_1-z_2|^2$ (let $z = x+iy$)
$\Rightarrow |(x+iy)-(2+3i)|^2+|(x+iy)-(3+4i)|^2$
$= |(2+3i)-(3+4i)|^2$
$\Rightarrow |(x-2)+i(y-3)|^2+|(x-3)+i(y-4)|^2$
$= |-1-i|^2$
$\Rightarrow (x-2)^2+(y-3)^2+(x-3)^2+(y-4)^2 = 1+1$
$\Rightarrow\ x^2+4-4x+y^2+9-6y$
$\Rightarrow\ +x^2+9-6x+y^2+16-8y = 2$
$\Rightarrow\ 2x^2+2y^2-10x-14y+36 = 0$
$\Rightarrow\ -x^2+y^2-5x-7y+18 = 0$

which represent a circle with centre $\left(\frac{5}{2}\frac{7}{2}\right)$ and radius $\sqrt{\frac{25}{4}+\frac{49}{4}-18} = \frac{1}{\sqrt{2}}$

136. (a)

Let $a-2d, a-d, a, a+d, a+2d$ are in AP ; then $\frac{1}{a-2d}, \frac{1}{a-d}, \frac{1}{a}, \frac{1}{a_1+d}, \frac{1}{a+2d}$ are in HP.

Given, middle term $= 1$

$\Rightarrow \frac{1}{a} = 1 \Rightarrow a = 1$

and $\frac{\text{Second term}}{\text{Fourth term}} = \frac{2}{1}$

$\Rightarrow \frac{1}{a-d}\times a+d = \frac{2}{1}$

$\Rightarrow\ a = 3d \Rightarrow d = \frac{1}{3}\ (\because a = 1)$

$\therefore$ Sum of first three terms

$= \frac{1}{1-\frac{2}{3}}+\frac{1}{1-\frac{1}{3}}+1 = 3+\frac{3}{2}+1 = \frac{11}{2}$

137. (c)

Given, $P = \begin{pmatrix}\cos\frac{\pi}{4} & -\sin\frac{\pi}{4}\\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix} \Rightarrow\ P = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$

Now,

$$P^2 = P\cdot P = \frac{1}{2}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$$
$$= \frac{1}{2}\begin{pmatrix}1-1 & -1-1\\ 1+1 & -1+1\end{pmatrix}$$
$$= \frac{1}{2}\begin{pmatrix}0 & -2\\ 2 & 0\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$
$$P^3 = P\cdot P^2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\cdot\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}0-1 & -1-0\\ 0+1 & -1+0\end{pmatrix}$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}-1 & -1\\ 1 & -1\end{pmatrix}$$

Also, given $X = \begin{pmatrix}1/\sqrt{2}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$$\therefore P^3X = \frac{1}{\sqrt{2}}\begin{pmatrix}-1 & -1\\ 1 & -1\end{pmatrix}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$
$$= \frac{1}{2}\begin{pmatrix}-1-1\\ 1-1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}-2\\0\end{pmatrix} = \begin{pmatrix}-1\\0\end{pmatrix}$$

138. (b)

Given equation is $x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{1-4}}{2}\left(\because x = \frac{b \pm \sqrt{b^2-4ac}}{2a}\right)$$
$$= \frac{1 \pm i\sqrt{3}}{2} = \frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}$$
$$\Rightarrow -x = \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$$
$$\Rightarrow +x = \omega, -\omega^2$$

Since, (α, β) are the roots of given equation.

Then, $\alpha = -\omega$ and $\beta = -\omega^2$

$$\therefore \alpha^{2013} + \beta^{2013} = -(\omega)^{2013} + (-\omega^2)^{2013}$$
$$= -\omega^{2013} - \omega^{4026}$$
$$= -(\omega^3)^{671} - (\omega^3)^{1342}$$
$$= -(1)^{671} - (1)^{1342}\ (\because \omega^3 = 1)$$
$$= -1 - 1 = -2$$

139. (a)

Given equation, is $x + y + z = 10$

where, x, y and z are positive integers.

$\therefore$ Required number of solutions $= {}^{(10-1)}C_{(3-1)}$

$$= {}^9C_2 = \frac{9\times 8}{2} = 36$$

140. (d)

Let $I = \int_{-1}^{1}\left\{\frac{x^{2013}}{e^{|x|}(x^2+\cos x)} + \frac{1}{e^{|x|}}\right\}dx$

$$\Rightarrow\ = \int_{-1}^{1}\frac{x^{2013}}{e^{|x|}(x^2+\cos x)}dx + \int_{-1}^{1}\frac{1}{e^{|x|}}dx$$

Here, $\frac{x^{2013}}{e^{|x|}(x^2+\cos x)}$ is an odd function and $\frac{1}{e^{|x|}}$ is an even function.

$$\left\{\because \int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx; & f(x) \text{ is even} \\ 0, & f(x) \text{ is odd}\end{cases}\right\}$$

$\therefore\ I = 0 + 2\int_0^1 e^{-x}dx = -2(e^{-x})_0^1 = -2(e^{-1}1)$

$= 2(1 - e^{-1})$

141. (a)

Given, $0 \le P, Q \le \frac{\pi}{2}$

and $\sin P + \cos Q = 2$

This equation hold only

when, $P = \dfrac{\pi}{2}$

and $Q = 0$

$$\text{LHS} = \sin P + \cos Q = \sin\frac{\pi}{2} + \cos 0$$

$$= 1 + 1 = 2 = \text{RHS}$$

$$\therefore \tan\left(\frac{P+Q}{2}\right) = \tan\left(\frac{\frac{\pi}{2}+0}{2}\right) = \tan\frac{\pi}{4} = 1$$

142. (b)

Given, $f(x) = 2^{100}\cdot x + 1$

and $g(x) = 3^{100}\cdot x + 1$

Now, $f\{g(x)\} = x$

$\Rightarrow\ f(3^{100}\cdot x + 1) = x$

$\Rightarrow\ 2^{100}\{3^{100}\cdot x + 1\} + 1 = x$

$\Rightarrow\ 6^{100}\cdot x + 2^{100} + 1 = x$

$\Rightarrow\ x(1 - 6^{100}) = (1 + 2^{100})$

$\Rightarrow\ x = \dfrac{1 + 2^{100}}{1 - 6^{100}}$

Hence, $fog(x) = x$ represent a singleton set:

143. (b)

$$\lim_{x\to 0}\left\{\frac{\sqrt{1+x}}{x} - \sqrt{1+\frac{1}{x^2}}\right\}$$

$$= \lim_{x\to 0}\left\{\frac{\sqrt{1+x} - \sqrt{1+x^2}}{x}\right\}\left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x}} - \frac{x}{\sqrt{1+x^2}}}{1}\ \text{(Use L-Hospital rule)}$$

$$= \frac{1}{2\sqrt{1+0}} - 0 = \frac{1}{2}$$

144. (c)

Given

$\cos^2 75° + \cos^2 45° + \cos^2 15° - \cos^2 30°$

$$= \left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)^2$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{4} + \frac{1}{2} + \frac{1}{2} + \frac{\sqrt{3}}{4} - \frac{3}{4} - \frac{1}{4} = \frac{3}{2} - 1 = \frac{1}{2}$$

145. (a)

Let $f(\theta) = \sin^6\theta + \cos^6\theta$

$\Rightarrow\ f(\theta) = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)$

$(\sin^4\theta + \cos^4\theta - \sin^2\theta \cdot \cos^2\theta)$
$[\because a^3 + b^3 = (a+b)(a^2+b^2-ab)]$
$= 1 \cdot \{(\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta \cdot \cos^2\theta\}$
$= 1 \cdot \left(1 - \frac{3}{4} \cdot 4\sin^2\theta \cdot \cos^2\theta\right)$
$= 1 - \frac{3}{4}(\sin 2\theta)^2 (\because \sin 2A = 2\sin A\cos A)$
$= 1 - \frac{3}{8}(1 - \cos 4\theta)$
$= 1 - \frac{3}{8} + \frac{3}{8}\cos 4\theta$
$f(\theta) = \frac{5}{8} + \frac{3}{8} \cdot \cos 40(1)$
$-1 \leq \cos 4\theta \leq 1$
$\Rightarrow \frac{-3}{8} \leq \frac{3}{8}\cos 4\theta \leq \frac{3}{8}$
$\Rightarrow \frac{5}{8} - \frac{3}{8} \leq \frac{5}{8} + \frac{3}{8}\cos 4\theta \leq \frac{5}{8} + \frac{3}{8}$
$\Rightarrow \frac{1}{4} \leq f(\theta) \leq 1$
[from Eq. (i)]
So, the maximum value is 1 and minimum value is $\frac{1}{4}$.

146. (d)

Given, $z = x + iy$
Now, $\frac{z-1}{z-i} = \frac{(x+iy)-1}{(x+iy)-i}$
$= \frac{(x-1)+iy}{x+i(y-1)} \times \frac{x-i(y-1)}{x-i(y-1)}$
$= \frac{x(x-1)+ixy-i(x-1)(y-1)+y(y-1)}{x^2+(y-1)^2}$
$= \frac{(x^2+y^2-x-y)+i(xy-xy+y+x-1)}{(x^2+y^2+1^{-2y})} \ldots (i)$
$= \left(\frac{x^2+y^2-x-y}{x^2+y^2-2y+1}\right) + i\left(\frac{x+y-1}{x^2+y^2-2y+1}\right).$
Given, $\frac{z-1}{z-i}$ is real.
So, its imaginary part should be zero.
i.e. $\frac{x+y-1}{x^2+y^2-2y+1} = 0$
$\Rightarrow x + y = 1$
$(\because x^2+y^2-2y+1 \neq 0)$
which represent a straight line.

147. (a)

Given that, a, b and c are in AP
$\therefore \quad 2b = a + c$
$\Rightarrow \quad c = 2b - a$(i)
and equation of straight line is
$ax + 2by + c = 0$
$\Rightarrow ax + 2by + (2b - a) = 0$
$\Rightarrow a(x-1) + b(2y+2) = a \cdot 0 + b \cdot 0$
On comparing, we get
and $\quad x - 1 = 0$
$\Rightarrow \quad 2y + 2 = 0$
$\Rightarrow \quad y = -1$
Hence, the required fixed point is $(1, -1)$.

148. (b)

Given equation is

$2x^2 + 5xy - 12y^2 = 0 \ldots\ldots\ldots (i)$
$\Rightarrow 2x^2 + 8xy - 3xy - 12y^2 = 0$
$\Rightarrow 2x(x + 4y) - 3y(x + 4y) = 0$
$\Rightarrow (x + 4y)(2x - 3y) = 0$
$\Rightarrow x + 4y = 0$ and $2x - 3y = 0$
which represent a pair of straight lines.
Compair Eq. (i) with $ax^2 + 2hxy + by^2 = 0$
we get $a = 2$ and $b = -12$
$\because\ a + b \neq 0$
So, lines are not perpendicular to each other:
Hence, it is a pair of non-perpendicular intersecting straight lines

149. (c)

Given equaitson of circle is

$$3x^2 + 3y^2 - 9x + 6y + 5 = 0$$

$$\Rightarrow x^2 + y^2 - 3x + 2y + \frac{5}{3} = 0$$

$$\text{Centre} = \left(\frac{3}{2}, -1\right)$$

$$\text{and radius} = \sqrt{\frac{9}{4} + 1 - \frac{5}{3}}$$

$$= \sqrt{\frac{19}{12}} = \frac{1}{2}\sqrt{\frac{19}{3}}$$

We know that, centre of the circle is the mid-point of the diameter.
Lives one and of point of dianetev in $(1,2)$ Let the other end point of diameter is (h, k).

Then, $\left(\frac{3}{2}, -1\right) = \left(\frac{1+h}{2}, \frac{2+k}{2}\right)$

$\Rightarrow \quad \frac{1+h}{2} = \frac{3}{2}$

$\Rightarrow \quad 1 + h = 3$

$\Rightarrow \quad h = 2$

and $\quad \frac{2+k}{2} = -1$

$\Rightarrow \quad 2 + k = -2$

$\Rightarrow \quad k = -4$

So, the other end point is $(2, -4)$.

150. (d)

Given equation of hyperbola and line are $\frac{x^2}{9} - \frac{y^2}{25} = 1$ and $y = x$ respectively.
For intersection point of both curve put $y = x$, we

$$\frac{x^2}{9} - \frac{x^2}{25} = 1$$

$$\Rightarrow \quad x^2 = \frac{9 \times 25}{16} = \left(\frac{15}{4}\right)^2$$

$$\Rightarrow \quad x = \pm\frac{15}{4} \text{ and } y = \pm\frac{15}{4}$$

$\therefore$ Intersetion points $P\left(\frac{15}{4}, \frac{15}{4}\right)$

and $Q\left(\frac{-15}{4}, \frac{-15}{4}\right)$

Since, PQ is major axis, then its length

$$= 2\sqrt{2} \cdot \frac{15}{4} = \frac{15}{\sqrt{2}}$$

and length of minor axis is $\frac{5}{\sqrt{2}}$ (given)

i.e., Major axis, $2a = \frac{15}{\sqrt{2}} \Rightarrow a = \frac{15}{2\sqrt{2}}$

and minor axis, $2b = \frac{5}{\sqrt{2}} \Rightarrow b = \frac{5}{2\sqrt{2}}$

$\therefore$ Eccentricity of an ellipse

$$= \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$= \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{9}$$

151. (a)

$\lim_{x \to 0} x \sin e^{(1/x)}$

LHL $= f(0-0) = \lim\limits_{h \to 0} (-h) \sin e^{(-1/h)}$

$= -0 \times \sin(e^{-\infty}) = -0 \times \sin(0) = 0$

$= 0 \times$ (a finite number between -1 to $+1$)

$= 0 \; (\because -1 \leq \sin x \leq 1)$

$\because$ LHL = RHL

$\therefore \lim\limits_{x \to 0} x \sin(e^{1/x})$ exist and equal to 0

152. (b)

$$1000\left[\frac{1}{1 \times 2} \times \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{999 \times 1000}\right]$$

$$= 1000\left\{\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{999} - \frac{1}{1000}\right)\right]$$

$$= 1000\left(1 - \frac{1}{1000}\right)$$

$$= 1000 \times \frac{999}{1000}$$

$$= 999$$

153. (c)

Given, $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

The characteristic equation of P is

$$|P - \lambda| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & -1-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\{(1+\lambda)(2+\lambda)\} = 0$$

$$\Rightarrow (1-\lambda^2)(2+\lambda) = 0$$

$$\Rightarrow 2 - 2\lambda^2 + \lambda - \lambda^3 = 0$$

$$\Rightarrow \lambda^3 + 2\lambda^2 - \lambda - 2 = 0$$

We know that, Caylay Hamilton theorem states that 'Every square matrix satisfy its characteristic equation'.

$\therefore P^3 + 2P^2 - P - 2I = 0$

$\Rightarrow P^3 + 2P^2 = P + 2I$

154. (d)

Let

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Apply $C_1 \to C_1 - bC_3$ and $C_2 \to aC_3 + C_2$

$$\Delta = \begin{vmatrix} 1+a^2-b^2+2b^2 & 2ab-2ab & -2b \\ 2ab-2ab & 1-a^2+b^2+2a^2 & 2a \\ 2b-b+a^2b+b^3 & -2a+a-a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} (1+a^2+b^2) & 0 & -2b \\ 0 & (1+a^2+b^2) & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & (1-a^2-b^2) \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & (1-a^2-b^2) \end{vmatrix}$$

$$= (1+a^2+b^2)^2\{(1-a^2-b^2+2a^2)+2b^2\}$$

$$= (1+a^2+b^2)^2(1+a^2+b^2) = (1+a^2+b^2)^3$$

155. (b)

Given equation is, $x^2+ax+b=0, (b \neq 0)$ its roots are α and β.

Then, sum of roots $= \alpha+\beta = -a$........(i)

Product of roots $= \alpha \cdot \beta = b$.............(ii)

Now,

$$\left(\alpha-\frac{1}{\beta}\right)+\left(\beta-\frac{1}{\alpha}\right) = (\alpha+\beta)-\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$= -a - \dfrac{(-a)}{b}$ [from Eqs.(i) and (ii)]

$= -a + \dfrac{a}{b} = \dfrac{a}{b}(1-b)$ ... ...(iii)

and $\left(\alpha-\frac{1}{\beta}\right)\left(\beta-\frac{1}{\alpha}\right) = \alpha\beta - 1 - 1 + \frac{1}{\alpha\beta}$

$= b + \frac{1}{b} - 2$[from Eq. (ii)]......(iv)

$= \frac{1}{b}(b^2-2b+1) = \frac{1}{b}(b-1)^2$......(v)

$\therefore$ Required of quadratic equation whose roots are $\left(\alpha-\frac{1}{\beta}\right)$ and $\left(\beta-\frac{1}{\alpha}\right)$ is

$$x^2 - \left\{\left(\alpha-\frac{1}{\beta}\right)+\left(\beta-\frac{1}{\alpha}\right)\right\}x$$

$$+\left\{\left(\alpha-\frac{1}{\beta}\right)\left(\beta-\frac{1}{\alpha}\right)\right\} = 0$$

On putting the values from Eqs. (i) and (ii), we get

$$x^2 - \frac{a}{b}\cdot(1-b)x + \frac{1}{b}(b-1)^2 = 0$$

$\Rightarrow\ bx^2 + a(b-1)x + (b-1)^2 = 0, b \neq 0$

156. (c)

Given,

In ellipse the distance between the foci

$=$ Length of the latusrectum

$\Rightarrow \quad 2ae = \frac{2b^2}{9a}$

$\Rightarrow \quad a^2e = b^2 \Rightarrow e = \frac{b^2}{a^2} \ldots\ldots\ldots (i)$

e^2

$\Rightarrow \quad e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow$ (i)

e^2

[from Eq. (i)]

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{-1 \pm \sqrt{1+4}}{2}$

(by quadratic formula)

$\Rightarrow \quad e = \frac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow \quad e = \frac{\sqrt{5}-1}{2}, (\because 1 > e > 0)$

157. (a)

Let $S_1 \equiv x^2 + y^2 - 6x - 8 = 0$

and $S_2 \equiv x^2 + y^2 - 6 = 0$

Now, the equation of the circle passing through the point $(1,1)$ and the point of intersection of S_1 and S_2 is

$S_1 + \lambda S_2 = 0$

$\Rightarrow \quad (x^2 + y^2 - 6x - 8) + \lambda(x^2 + y^2 - 6) = 0$

$\Rightarrow \quad (1+\lambda)x^2 + (1+\lambda)y^2 - 6x$

$+(-8 - 6\lambda) = 0$ ...(i)

Since, Eq. (i) passes through the point (1,1).

$\because$ Put $x = 1, y = 1$ in Eq. (ii), we get

$\Rightarrow \quad (1+\lambda) + (1+\lambda) - 6 + (-8 - 6\lambda) = 0$

$\Rightarrow$

$\Rightarrow \quad -4\lambda - 12 = 0$

$\lambda = -3$

On putting the value of ' λ ' in Eq. (i), we get

$-2x^2 - 2y^2 - 6x + 10 = 0$

$\Rightarrow \quad x^2 + y^2 + 3x - 5 = 0$

158. (d)

The distance from the point $(-1,2)$ of the line which passes through $(2,-3)$

$= \sqrt{(2+1)^2 + (-3-2)^2}$

$= \sqrt{9+25} = \sqrt{34} < 8$

Given that, the distance between the point $(-1,2)$ and the line is 8 .

But the maximum distance of the line passing through $(2,-3)$ from $(-1,2)$ is $\sqrt{34}$. So, there is no such line possible.

159. (c)

Let the six terms of GP are

$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$

Now, according to the question

$\frac{a}{r^5} \cdot \frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 \cdot ar^5 = 1000$

$\Rightarrow \quad a^6 = (10)^3 \Rightarrow a^2 = 10$(i)

Also, given fourth term $= ar = 1$

$\Rightarrow \quad a^2r^2 = 1$

$\Rightarrow \quad r^2 = \frac{1}{10}$

[from Eq. (i)]

$\therefore$ Last term $= ar^5 = [a^2(r^2)^5]^{1/2}$

$= \left[10 \times \frac{1}{10^5}\right]^{1/2}$ [from Eqs. (i) and (ii)]]

$= \left(\frac{1}{10^4}\right)^{1/2} = \frac{1}{100}$

160. (c)

Given that, (α, β) are roots of the quadratic equation

$ax^2 + bx + c = 0$

and $3b^2 = 16ac \ldots\ldots (i)$

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$......(ii)

From Eq. (i), we get

$3b \cdot b = 16 \cdot a \cdot c$

$\Rightarrow \quad 3\left(\frac{b}{a}\right)^2 = 16\left(\frac{c}{a}\right)$ (divide by a^2)

$\Rightarrow \quad 3(\alpha + \beta)^2 = 16\alpha\beta$ [from Eq. (ii)]

$\Rightarrow \quad 3\alpha^2 + 3\beta^2 + 6\alpha\beta = 16\alpha\beta$

$\Rightarrow \quad 3\alpha^2 + 3\beta^2 = 10\alpha\beta$

$\Rightarrow \quad 3\left(\frac{\alpha}{\beta}\right) + 3\left(\frac{\beta}{\alpha}\right) = 10$

Let $\frac{\alpha}{\beta} = x$

Then, $3x + \frac{3}{x} = 10$

$\Rightarrow \quad 3x^2 - 10x + 3 = 0$

$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$

$\Rightarrow \quad 3x(x - 3) - 1(x - 3) = 0$

$\Rightarrow \quad (x - 3)(3x - 1) = 0$

$\Rightarrow \quad x = \frac{1}{3}, 3 \Rightarrow \frac{\alpha}{\beta} = \frac{1}{3}, 3$

$\therefore \quad \beta = 3\alpha$ or $\alpha = 3\beta$

161. (d)

Let the relation defined as

$R = \{(A, B) \mid B = P^{-1}AP\}$.........(i)

For reflexive, $A = 1^{-1}Al.$

$\Rightarrow \quad (A, A) \in R$

$\Rightarrow \quad R$ is reflexive

For symmetric

Let $(A, B) \in R$

$\because \; B = P^{-1}AP$

$\Rightarrow \; PB = AP \Rightarrow PBP^{-1} = A$

$\Rightarrow \; A = (P^{-1})^{-1}B(P^{-1})$

$\Rightarrow \; (B, A) \in R \Rightarrow R$ is symmetric.

For transitive

Let $(A, B) \in R, (B, C) \in R$

$\because \; A = P^{-1}BP$ and $B = Q^{-1}CQ$

$\Rightarrow \; A = P^{-1}Q^{-1}CQP = (QP)^{-1}C(QP)$

$\Rightarrow \; (A, C) \in R$

$\Rightarrow$ R is transitive.
Since, R is reflexive, symmetric and transitive.
So, R is an equivalence relation.

162. (d)

Let the given relation defined as
$R = \{(a, b) \mid \sin^2 a + \cos^2 b = 1\}$
For reflexive, $\sin^2 a + \cos^2 a = 1$
$(\because \sin^2\theta + \cos^2\theta = 1, \forall\theta \in R)$
$\Rightarrow$ ${}_aR_a \Rightarrow (a, a) \in R$
$\Rightarrow$ R is reflexive.
For symmetric, $\sin^2 a + \cos^2 b = 1$
$\Rightarrow$ $1 - \cos^2 a + 1 - \sin^2 b = 1$
$\Rightarrow$ $\sin^2 b + \cos^2 a = 1$
$\Rightarrow$ bRa
Hence, R is symmetric.
For transitive
Let aRb, bRc
$\Rightarrow$ $\sin^2 a + \cos^2 b = 1$
and $\sin^2 b + \cos^2 c = 1$
On adding Eqs. (i) and (ii), we get
$\sin^2 a + (\sin^2 b + \cos^2 b) + \cos^2 c = 2$
$\Rightarrow$ $\sin^2 a + \cos^2 c + 1 = 2$
$\Rightarrow$ $\sin^2 a + \cos^2 c = 1$
Hence, R is transitive also.
Therefore, relation R is an equivalence relation.

163. (b)

Given curve is, $x^2 + 4xy + 8y^2 = 64 \ldots\ldots\ldots$ (i)
On differentiating w.r.t x, we get

$$2x + 4\left(y + x\frac{dy}{dx}\right) + 16y\frac{dy}{dx} = 0$$

$$\Rightarrow 2x + 4y + (4x + 16y)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(x + 2y)}{2(x + 4y)}$$

Since, tangent are parallel to x-axis only.

i.e., $\frac{dy}{dx} = 0$

$$\Rightarrow -\frac{(x + 2y)}{2(x + 4y)} = 0 \Rightarrow x + 2y = 0 \text{(ii)}$$

Now, on putting the valus of x from Eqs. (i) in (ii), we get
$4y^2 - 8y^2 + 8y^2 = 64$
$\Rightarrow$ $y^2 = 16$
$\Rightarrow$ $y = \pm 4$
From Eq. (ii)
When
$y = 4, x = -8$
and when
$y = -4, x = 8$
Hence required points are $(-8,4)$ and $(8, -4)$

164. (c)

Given,

$$f(x) = \begin{cases} x^3 - 3x + 2, & x < 2 \\ x^3 - 6x^2 + 9x + 2, & x \geq 2 \end{cases}$$

LHL

$= f(2-0) = \lim_{h\to 0}(2-h)^3 - 3(2-h) + 2$
$= (2)^3 - 6 + 2 = 8 - 6 + 2 = 4$
$\text{RHL} = f(2+0) = \lim_{h\to 0}(2+h)^3 - 6(2+h)^2$
$+9(2+h) + 2$
$= (2)^3 - 6(2)^2 + 9(2) + 2$
$= 8 - 24 + 18 + 2 = 4$
$\text{LHL} = \text{RHL}$
$\therefore \lim_{x\to 2} f(x)$ exist
and
$f(2) = (2)^3 - 6(2)^2 + 9(2) + 2$
$= 8 - 24 + 18 + 2 = 4$
$\therefore LHL = RHL = f(2)$
So, $f(x)$ is continuous at $x = 2$

Now, $f'(x) = \begin{cases} 3x^2 - 3, & x < 2 \\ 3x^2 - 12x + 9, & x \geq 2 \end{cases}$

$\therefore Lf'(2) = 3(2)^2 - 3 = 12 - 3 = 9$
and $Rf'(2) = 3(2)^2 - 12(2) + 9$
$= 12 - 24 + 9 = -3$
$\because Lf'(2) \neq Rf'(2)$
$\therefore f(x)$ is not differentiable at $x = 2$
Hence, f is continuous but not differentiable at $x = 2$.

165. (a)

Given,

$$I = \int_0^{\pi/4} (\tan^{n+1} x)dx + \frac{1}{2}\int_0^{\pi/2} \tan^{n-1}\left(\frac{x}{2}\right)dx$$

In second integral, put $t = \frac{x}{2} \Rightarrow dx = 2dt$
$\Rightarrow$ Also, when $x = 0$ then $t = 0$,
When $x = \pi/2$, then $t = \pi/4$
Then, $I = \int_0^{\pi/4} (\tan^{n+1} x)dx + \int_0^{\pi/4} \tan^{n-1} tdt$

$$l = \int_0^{\pi/4} \tan^{n+1} x \cdot dx + \int_0^{\pi/4} \tan^{n-1} xdx$$

$$\left\{\because \int_a^b f(x)dx = \int_a^b f(y)dy\right\}$$

$$\Rightarrow \quad l = \int_0^{\pi/4} (\tan^{n+1} x + \tan^{n-1} x)dx$$

$$l = \int_0^{\pi/4} \tan^{n-1} x \cdot (\tan^2 x + 1)dx$$

$$l = \int_0^{\pi/4} \tan^{n-1} x(\sec^2 x)dx$$

Put $\quad t = \tan x$
$\Rightarrow \quad dt =$
$= \sec^2 xdx$
Also, when $x = 0$, then $t = 0$
when $x = \pi/4$, then $t = 1$

$$I = \int_0^1 t^{n-1}dt = \left[\frac{t^n}{n}\right]_0^1 = \frac{1}{n}$$

166. (c)

$\lim_{x\to\infty} \sum_{n=1}^{1000} (-1)^n x^n$
$= \lim_{x\to\infty} \{-x + x^2 - x^3 + x^4 + \cdots + x^{1000}\}$

$$= \lim_{x\to\infty} (-x)\cdot\left\{\frac{(-x)^{1000}-1}{(-x-1)}\right\} = \lim_{x\to\infty}\frac{x^{1001}-x}{x+1}$$

$$= \lim_{x\to\infty}\frac{x^{1000}-1}{1+\left(\frac{1}{x}\right)} = +\infty$$

167. (d)

Given

$$f(\theta) = (1+\sin^2\theta)(2-\sin^2\theta)$$
$$= 2+2\sin^2\theta-\sin^2\theta-\sin^4\theta$$
$$= -\sin^4\theta+\sin^2\theta+2$$
$$= -(\sin^4\theta-\sin^2\theta-2)$$
$$= -\left\{\sin^4\theta-\sin^2\theta+\frac{1}{4}-\frac{9}{4}\right\}$$
$$= +\frac{9}{4}-\left(\sin^2\theta-\frac{1}{2}\right)^2 \quad \ldots.(i)$$

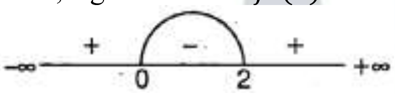
$$\because\ -1\le\sin\theta\le 1 \Rightarrow 0\le\sin^2\theta\le 1$$
$$\Rightarrow -\frac{1}{2}\le\sin^2\theta-\frac{1}{2}\le\frac{1}{2}$$
$$\Rightarrow 0\le\left(\sin^2\theta-\frac{1}{2}\right)^2\le\frac{1}{4}$$
$$\Rightarrow \ge -\left(\sin^2\theta-\frac{1}{2}\right)^2\ge-\frac{1}{4}$$
$$\Rightarrow \frac{9}{4}\ge\frac{9}{4}-\left(\sin^2\theta-\frac{1}{2}\right)^2\ge\frac{9}{4}-\frac{1}{4}$$
$$\Rightarrow 2\le f(\theta)\le\frac{9}{4}\ldots\ldots\ldots\ldots(from\ Eq.(i))$$

168. (a)

Given function is, $f(x) = e^x(x-2)^2$

$$\Rightarrow f'(x) = e^x(x-2)^2+2(x-2)e^x$$
$$= e^x(x-2)(x-2+2) = x(x-2)e^x$$

Now, sign scheme of $f'(x)$ is



So, f is increasing in $(-\infty,0)$ and $(2,\infty)$ and decreasing in $(0,2)$.

169. (b)

Let the number of terms, $n = 2m$

Now, by condition

$$\frac{\text{Largest coefficient in }(1+x)^n}{\text{Second largest coefficient in }(1+x)^n} = \frac{11}{10}$$

$$\Rightarrow \frac{{}^{2m}C_m}{{}^{2m}C_{m-1}} = \frac{11}{10}$$

$$\Rightarrow \frac{(m-1)!\,(m+1)!}{m!\,m!} = \frac{11}{10}$$

$$\Rightarrow \frac{1}{m}\frac{m(m-1)!\,(m+1)m!}{m!\,m!} = \frac{11}{10}$$

$$\Rightarrow \frac{m+1}{m}\frac{m!\,m!}{m!\,m!} = \frac{11}{10}$$

$$\Rightarrow 10m+10 = 11m \Rightarrow m = 10$$

$$\therefore \quad n = 20$$

Hence, total number of term

$= n+1 = 20+1 = 21$

170. (a)

Let the tine number is in AP are
$(a-2d),(a-d),a,(a+d),(a+2d)$
where, $d \neq 0$
Given, 1st, 3rd and 4th terms are in GP.
$\Rightarrow \quad a^2=(a-2d)(a+d)$
$\Rightarrow \quad a^2=a^2-2ad+ad-2d^2$
$\Rightarrow \quad 2d^2+ad=0 \Rightarrow d(2d+a)=0$
$\because \quad d \neq 0$
$\therefore \quad a+2d=0 \Rightarrow a=-2d$
Hence, terms are
$-4d,-3d,-2d,-d,0$
$\therefore$ The fifth term is always 0 .

171. (c)

Given that,
$f(x)=e^{(x)^{\frac{1}{x}},x>0}$
Taking log on both sides, we get
$\log+(x)=(x)^{\frac{1}{x}}=g(x)$ (say) ....(i)
Here, $\quad g(x)=x^{\frac{1}{x}}$
$\Rightarrow \quad \log g(x)=\frac{1}{x}\log x$
On differentiating w.r.t. x, we get

$$\frac{1}{g(x)}\cdot g'(x)=\frac{x\cdot\frac{1}{x}-\log x}{x^2}$$
$$=\left(\frac{1-\log x}{x^2}\right)$$

$\Rightarrow \quad g'(x)=x^{\left(\frac{1}{x}-2\right)}(1-\log x)$
For maximum or minimum of $g(x)$ put $g'(x)=0$
$\Rightarrow \quad x^{\left(\frac{1}{x}-2\right)}(1-\log x)$
$\Rightarrow \quad \log x=1$
$\Rightarrow \quad =\log e$
and $\quad g''(x)|_{x=e}>0$
So, $g(x)$ is minimum at $x=e$
$\therefore g(x)$ increases in $(0,e)$ and decreases in (e,∞), it will be minimum at either 2 or 5 $\therefore 2^{\frac{1}{2}}>5^{\frac{1}{5}} \Rightarrow$ Minimum value of $f(x)=e^{(5)^{\frac{1}{5}}}$

172. (b)

Given,
$f(x)=2|x-1|+|x-2|$

$$\Rightarrow f(x)=\begin{cases}-2(x-1)-(x-2), x<1\\ 2(x-1)-(x-2), 1\leq x<2\\ 2(x-1)+(x-2), x\geq 2\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}-3x+4, & x<1\\ x, & 1\leq x<2\\ 3x-4, & x\geq 2\end{cases}$$

$$f'(x)=\begin{cases}-3, & x<1\\ 1, & 1\leq x<2\\ 3, & x\geq 2\end{cases}$$

So, $f(x)$ will be minimum at $x=1$ and the minimum value is 1 .

173. (b)

Given series is,

$$\frac{1}{1\times 2}\ {}^{25}C_0+\frac{1}{2\times 3}\ {}^{25}C_1+\frac{1}{3\times 4}\ {}^{25}C_2 + \cdots + \frac{1}{26\times 27}\ {}^{25}C_{25}$$

$$\because \int_0^x (1+x)^{25}dx=\int_0^x [{}^{25}C_0+{}^{25}C_1x + {}^{25}C_2x^2+\cdots+{}^{25}C_{25}x^{25}]dx$$

On integrating w.r.t. x, taking limits 0 to x, we get

$$\left[\frac{(1+x)^{26}}{26}\right]_0^x$$

$$=\left[{}^{25}C_0x+{}^{25}C_1\cdot\frac{x^2}{2}+{}^{25}C_2\frac{x^3}{3}+\cdots+{}^{25}C_{25}\cdot\frac{x^{26}}{26}\right]_0^x$$

$$\Rightarrow\left\{\frac{1}{26}(1+x)^{26}-\frac{1}{26}\right\}$$

$$={}^{25}C_0x+{}^{25}C_1\cdot\frac{x^2}{2}+\cdots+{}^{25}C_{25}\cdot\frac{x^{26}}{26}$$

Again, integrating w.r.t. x, taking limits 0 to 1 , we get

$$\frac{1}{26}\int_0^1[1+x)^{26}-1\Big]dx$$

$$=\int_0^1\left[{}^{25}C_0x+{}^{25}C_1\cdot\frac{x^2}{2}+\cdots+{}^{25}C_{25}\frac{x^{26}}{26}\right]dx$$

$$\Rightarrow\frac{1}{26}\left[\frac{(1+x)^{27}}{27}-x\right]_0^1$$

$$=\left[{}^{25}C_0\frac{x^2}{2}+{}^{25}C_1\cdot\frac{x^3}{2\times 3}+\cdots+{}^{25}C_{25}\frac{x^{27}}{26\times 27}\right]_0^1$$

$$\Rightarrow\frac{1}{26}\left\{\frac{2^{27}}{27}-1-\frac{1}{27}\right\}=\frac{1}{2}\ {}^{25}C_0+\frac{1}{2\times 3}\cdot\ {}^{25}C_1$$

$$\therefore\frac{1}{1\times 2}\cdot\ {}^{25}C_0+\frac{1}{2\times 3}\ {}^{25}C_1+\frac{1}{3\times 4}\ {}^{25}C_2 + \cdots+\frac{1}{26\times 27}\cdot\ {}^{25}C_{25}$$

$$=\frac{2^{27}-28}{26\times 27}$$

174. (b)

Given that, P, Q and R are angles of an isosceles triangle and $\angle P=\frac{\pi}{2}$

$\therefore\ Q=R=\frac{\pi}{4}(\because P+Q+R=180^\circ)$

Now, $\left(\cos\frac{P}{3}-i\sin\frac{P}{3}\right)^3+(\cos Q+i\sin Q)$
$(\cos R-i\sin R)$
$+(\cos P-i\sin P)(\cos Q-i\sin Q)$
$(\cos R-i\sin R)$

$$=\left(e^{-\frac{P}{3}}\right)^3+e^{iQ}\cdot e^{-iR}+e^{-ip}e^{-iQ}\cdot e^{-iR}\ (\because \cos\theta+i\sin\theta=e^{i\theta})$$

$$=e^{-iP}+e^{iQ-R)}+e^{-i(P+Q+R)}$$

$$=e^{-i\pi/2}+e^{(0)}+e^{-i\pi}\ Q=R=\frac{\pi}{2}P=\frac{\pi}{2})$$

$= -i + 1 + (-1) = -i$

175. (a)

Let the function, $f(x) = a^{kx}$

Which define in $f: R \to R$ and injective also.

Now, we have

$$f(x)f(y) = f(x+y)$$

$\Rightarrow \quad a^{kx} \cdot a^{ky} = a^{k(x+y)}$

$\Rightarrow \quad a^{k(x+y)} = a^{k(x+y)}$

$\because$ $f(x), f(y)$ and $f(z)$ are in GP

$\therefore \quad f(y)^2 = f(x) \cdot f(z)$

$\Rightarrow \quad a^{2ky} = a^{kx} \cdot a^{kz}$

$\Rightarrow \quad e^{2ky} = e^{k(x+z)}$

On comparing, we get

$2ky = k(x+z) \Rightarrow 2y = x + z.$

$\Rightarrow x, y$ and z are in AP .

176. (c)

Let $I = \int_1^2 e^x \left(\log_e x + \frac{x+1}{x}\right) dx$

$\Rightarrow \; I = \int_1^2 \left(e^x \cdot \log_e x + e^x + \frac{e^x}{x}\right) dx$

$\Rightarrow \; I = \int_1^2 e^x \log_e x dx + \int_1^2 e^x dx + \int_1^2 \frac{e^x}{x} dx$

$\Rightarrow \; I = \int_1^2 e^x \log_e x dx + [e^x]_1^2 + [e^x \log_e x]_1^2 - \int_1^2 e^x \log_e x dx$

$\Rightarrow \; I = (e^2 - e^1) + (e^2 \log_e 2 - 0)$

$\quad = e^2(1 + \log_e 2) - e$

177. (c)

Given equation is

$$\frac{1}{2}\log_{\sqrt{3}}\left(\frac{x+1}{x+5}\right) + \log_9 (x+5)^2 = 1$$

$\Rightarrow \frac{1}{2}\left(\frac{1}{1/2}\right)\log_3\left(\frac{x+1}{x+5}\right) + \frac{1}{2}\log_3 (x+5)^2 = 1$

$\Rightarrow \frac{2}{2}\log_3\left(\frac{x+1}{x+5}\right) + \frac{1}{2}\cdot 2\log_{2^n} b = \frac{1}{n}\log_2 b\Big)$

$\Rightarrow \log_3\left\{\left(\frac{x+1}{x+5}\cdot(x+5)\right)\right\} = \log_3 3$

$\Rightarrow \log_3 3$

$\therefore \because \log m + \log n = \log mn$ and $\log_n n = 1$

$\Rightarrow \; (x+1) = 3 \Rightarrow x = 2$

So, only one solution is possible.

178. (c)

Given,

$$P = 1 + \frac{1}{2 \times 2} + \frac{1}{3 \times 2^2} + \cdots$$

and $Q = \frac{1}{1\times 2} + \frac{1}{3\times 4} + \frac{1}{5\times 6} + \cdots$

Now, $\frac{P}{2} = \frac{(1/2)^1}{1} + \frac{(1/2)^2}{2} + \frac{(1/2)^3}{3} + \cdots$

$\Rightarrow \; -p/2 = -\left(\frac{1/2}{1}\right)^1 - \frac{(1/2)^2}{3} - \frac{(1/2)^3}{3} \ldots$

$\Rightarrow \; -P/2 = \log_e\left(1 - \frac{1}{2}\right)$

$\Rightarrow \; -\frac{P}{2} = \log_e \frac{1}{2}$

$\Rightarrow \; P = 2\log_e 2$………..(i)

Now, $Q = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \cdots$

$\Rightarrow \; Q = \log_e 2 \ldots\ldots$(ii)

Now, from Eqs. (i) and (ii), we get

$P = 2Q$

179. (d)

Given equation of parabola is

$$y$$

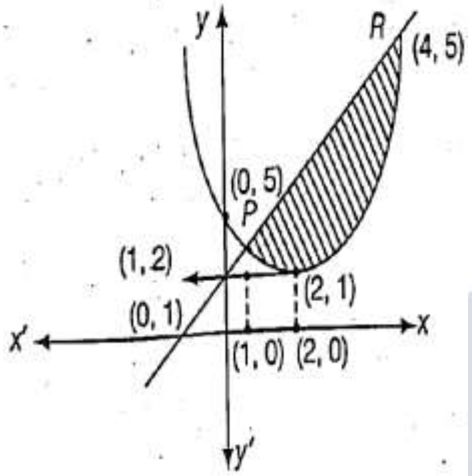
$$\Rightarrow \quad y = x^2 - 4x + 5$$

$$\Rightarrow \quad (x-2)^2 \qquad = (x-1)^2 + 1 \text{(i)}$$

and equation of line is

$$y = x + 1$$

$$\Rightarrow x - y = -1 \text{(ii)}$$



On putting the value of $(y-1)$ from Eqs. (ii) in (i), we get

$$(x-2)^2 = x \Rightarrow x^2 + 4 - 4x = x$$

$$\Rightarrow x^2 - 5x + 4 = 0$$

$$\Rightarrow x^2 - 4x - x + 4 = 0$$

$$\Rightarrow \; x(x-4) - 1(x-4) = 0$$

$\Rightarrow \; (x-1)(x-4) = 0$ or $x = 1,4$

then from is (ii) $\; y = 2,5$

$$\therefore \text{Required area } = \int_1^4 \{(x+1) - (x^2 - 4x + 5)\}dx$$

$$= \int_1^4 (-x^2 + 5x - 4)dx = \left[\frac{-x^3}{3} + \frac{5x^2}{2} - 4x\right]_1^4$$

$$= \left[-\frac{64}{3} + 40 - 16 + \frac{1}{3} - \frac{5}{2} + 4\right].$$

$$= \left(-21 - \frac{5}{2} + 28\right) = \frac{9}{2}$$

180. (c)

Given,

$$f(x) = \sin x + 2\cos^2 x, x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

$\therefore \; f'(x) = \cos x - 4\cos x \cdot \sin x$

and $f''(x) = -\sin x - 4\cos 2x$.

For maximum or minimum of $f(x)$

Put $\quad f'(x) = 0$

$$\Rightarrow \quad \cos x - 4\cos x \cdot \sin x = 0$$

$$\Rightarrow \quad \cos x(1 - 4\sin x) = 0$$

$$\Rightarrow \quad \cos x = 0 = \cos\frac{\pi}{2} \text{ and } \sin x \neq \frac{1}{4}$$

$$\therefore \quad x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \Rightarrow x = \frac{\pi}{2}$$

Now, $f''\left(\frac{\pi}{2}\right)=-\sin\frac{\pi}{2}-4\cos\pi$

$=-1+4=3>0(\text{ min})$

So, $f(x)$ is minimum at $x=\frac{\pi}{2}$

and its minimum value is

$f\left(\frac{\pi}{2}\right)=\sin\frac{\pi}{2}+2\cos^2\frac{\pi}{2}=1-2\times 0=1$

181. (d)

Total sample space, $n(S)=4^3\cdot 2^2$ and total number of favourable cases

$n(E)=({}^3C_1\cdot 3+{}^2C_1\cdot 1)+1$

Required probability $=\dfrac{n(E)}{n(S)}$

$=\dfrac{{}^3C_4\cdot 3+{}^2C_1\cdot 1+1}{4^3\cdot 2^2}=\dfrac{3\cdot 3+2+1}{4^3\cdot 4}$

$=\dfrac{12}{64\cdot 4}=\dfrac{3}{64}$

182. (a)

Given differential equation is

$(y^2+2x)\dfrac{dy}{dx}=y\Rightarrow\dfrac{dx}{dy}=\left(\dfrac{y^2+2x}{y}\right)$

$\Rightarrow\dfrac{dx}{dy}=y+\dfrac{2x}{y}$

$\Rightarrow\dfrac{dx}{dy}-\dfrac{2}{y}\cdot x=y$

$\text{IF}=e^{\int -\frac{2}{y}dy}=e^{-2\log y}=y^{-2}=\frac{1}{y^2}$

$\therefore$ Complete solution is

$x\cdot\frac{1}{y^2}=\int y\cdot\frac{1}{y^2}dy+C$

$\Rightarrow\frac{x}{y^2}=\int\frac{dy}{y}+C=\log_e y+C\ldots\ldots\text{(i)}$

$\Rightarrow x=y^2\log_e y+Cy^2$

At $+x=1, y=1$ then from Eq. (i), we get

$1=0+C\Rightarrow C=1$

$\therefore$ From Eq. (i), we get

$x=y^2(\log_e y+1)$

Which is the required solution.

183. (c)

Let the general equation of tangent.

Which passes through the point (x,y) is

$(Y-y)=\frac{dy}{dx}(X-x)$

$\Rightarrow Y-y=x\frac{dy}{dx}-x\frac{dy}{dx}\ldots\ldots\text{(i)}$

for length of Y-intercept, put $X=0$ in Eq. (i), we get

$y-y=-x\dfrac{dy}{dx}$

$\Rightarrow y=y-x\dfrac{dy}{dx}\text{(ii)}$

Now, according to the guestion

Y-intercept $=3\times$ ordinate of the point of contact

$\Rightarrow \quad y - x\dfrac{dy}{dx} = 3y$

$\Rightarrow \quad -x\dfrac{dy}{dx} = 2y$

$\Rightarrow \quad \displaystyle\int \frac{dy}{y} = -\int \frac{2dx}{x}$ (on integrating)

$\Rightarrow \log y = -2\log x + \log C$

$\Rightarrow \log y + \log x^2 = \log C$

$\Rightarrow yx^2 = C$, where C is a constant.

184. (a)

Given differential equation is

$y\sin\left(\dfrac{x}{y}\right)dx = \left\{x\sin\left(\dfrac{x}{y}\right) - y\right\}dy$

$\Rightarrow \frac{dx}{dy} = \frac{x\sin\left(\frac{x}{y}\right)-y}{y\sin\left(\frac{x}{y}\right)} = \frac{x}{y} - \frac{1}{\sin\left(\frac{x}{y}\right)}$ ...(i)

On putting $v = \dfrac{x}{y} \Rightarrow x = w$

$\Rightarrow \dfrac{dx}{dy} = v \cdot 1 + y\dfrac{dv}{dy}$ in Eq. (i), we get

$\Rightarrow v + y\dfrac{dv}{dy} = v - \dfrac{1}{\sin v}$

$\Rightarrow y\dfrac{dv}{dy} = -\dfrac{1}{\sin v}$

$\Rightarrow -\int \sin v dv = \int \frac{dy}{y}$ (on integrating)

$\Rightarrow \cos v = \log y + C$.......(i)

$\Rightarrow \cos\left(\frac{x}{y}\right) = \log y + C$.

Given at $x = \frac{\pi}{4}, y = 1$ then from Eq. (i)

$\Rightarrow \cos\left(\dfrac{\pi}{4}\right) = \log(1) + C$

$\Rightarrow \left(C = \dfrac{1}{\sqrt{2}}\right)$

On putting the value of C in Eq. (i), we get

$\cos\left(\dfrac{x}{y}\right) = \log_e y + \dfrac{1}{\sqrt{2}}$

185. (b)

Given lines are

$x + y = 4$ and $x - y = 2$

On solving these lines, we get

$x = 3$ and $y = 1$

Now, the equation of line which passes through the intersection point $(3,1)$ having slope $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ is $(y-1) = \frac{3}{4}(x-3)$

$\Rightarrow -4y - 4 = 3x - 9$

$\Rightarrow 3x - 4y = 5$

Now for the intersection point of the line (i) with parabola $y^2 = 4(x-3)$. Put $y = \left(\frac{3x-5}{4}\right)$,

then we get $\frac{(3x-5)^2}{16} = 4(x-3)$

$\Rightarrow 9x^2 + 25 - 30x = 64x - 192$

$\Rightarrow 9x^2 - 94x + 217 = 0$

$\Rightarrow x = \dfrac{94 \pm \sqrt{8836 - 7812}}{18} = \dfrac{94 \pm \sqrt{1024}}{18}$

$\Rightarrow x = \frac{94\pm32}{180} = \frac{126}{18}$ or $\frac{62}{18}$

$\Rightarrow\ x_1 = \dfrac{21}{3} = 7$

and $x_2 = \frac{31}{9}$

$\therefore\ |x_1 - x_2| = \left|7 - \dfrac{31}{9}\right| = \left|\dfrac{32}{9}\right| = \dfrac{32}{9}$

186. (d)

Given, $\sin^2\theta + 3\cos\theta = 2$

$\Rightarrow\ 1 - \cos^2\theta + 3\cos\theta = 2$

$\Rightarrow\ \cos^2\theta - 3\cos\theta + 1 = 0$

$\Rightarrow \cos\theta = \frac{3\pm\sqrt{9-4}}{2}$ (by quadratic formula)

$\Rightarrow\ \cos\theta = \dfrac{3\pm\sqrt{5}}{2}$

Here, $\cos\theta = \frac{3-\sqrt{5}}{2}$

$\left(\because \cos\theta \neq \dfrac{3+\sqrt{5}}{2} - 1 \leq \cos\theta \leq 1\right)$

and $\sec\theta = \frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3-\sqrt{5}}$

$= \dfrac{2(3+\sqrt{5})}{4} = \dfrac{3+\sqrt{5}}{2}$

Now, $\cos^3\theta + \sec^3\theta = (\cos\theta + \sec\theta)$

$(\cos^2\theta + \sec^2\theta - \cos\theta\cdot\sec\theta)$

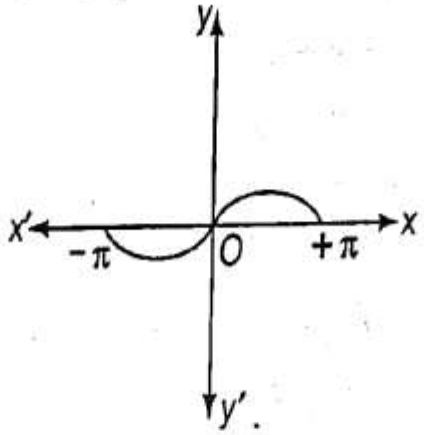
$= \left(\dfrac{3-\sqrt{5}+3+\sqrt{5}}{2}\right)$

$\{(\cos\theta + \sec\theta)^2 - 3\cos\theta\cdot\sec\theta\}$

$= 3\cdot\{(3)^2 - 3\} = 3(9-3) = 3\times 6 = 18$

187. (d)

Let $I = \int_{-\pi/2}^{\pi/2} [\sin x\cdot\cos x]dx = \int_{-\pi/2}^{\pi/2} \left[\frac{1}{2}\sin 2x\right]dx$



Put $\theta = 2x \Rightarrow d\theta = 2dx$

Also, when $x = -\pi/2$, then $\theta = -\pi$

when $x = \frac{\pi}{2}$, then $\theta = \pi$

Then, $I = \frac{1}{2}\int_{-\pi}^{\pi}\left[\frac{1}{2}\sin\theta\right]d\theta$

$= \dfrac{1}{2}\left[\int_{-\pi}^{0}(-1)dx + \int_{0}^{\pi}(0)dx\right]$

$= \dfrac{1}{2}[-x]_{-\pi}^{0} + 0 = -\dfrac{\pi}{2}$

188. (a)

Given,

$$x = 1 + \frac{1}{2\times 1!} + \frac{1}{4\times 2!} + \frac{1}{8\times 3!} + \cdots$$

$\Rightarrow$ $$x = 1 + \frac{(1/2)^1}{1!} + \frac{(1/2)^2}{2!} + \frac{(1/2)^3}{3!} + \cdots$$

$\Rightarrow x = e^{1/2} \Rightarrow x^2 = e$(i)

and $y = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots$

$\Rightarrow\ y = 1 + \frac{(x^2)^1}{1!} + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \cdots$

$\Rightarrow y = e^{x^2} = e^e$..........from Eq.(i)

[from Eq. (i)]

Taking log on both sides, we get

$\log_e y = e$

189. (a)

Given, $P = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$

$$P^2 = P\cdot P = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} = P$$

$\therefore P^4 = P^2 = P$

$\Rightarrow P^5 = P^2 = P$

190. (c)

Given infinite series is

$$\frac{1^2+2^2}{3!} + \frac{1^2+2^2+3^2}{4!} + \frac{1^2+2^2+3^2+4^2}{5!} + \cdots$$

nth term, $t_n = \frac{1^2+2^2+3^2+4^2+\cdots+(r+1)^2}{(r+2)!}$

Now, $S_n = \Sigma t_n = \sum_{r=1}^{n} \frac{1^2+2^2+3^2+\cdots+(r+1)^2}{(r+2)!}$

$$= \sum_{r=1}^{n} \frac{(r+1)(r+2)(2n+3)}{6(r+2)!}$$

$$\left[\because \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$= \sum_{r=1}^{n} \frac{(2r+3)}{6\cdot r!} = \frac{1}{6}\sum_{r=1}^{n}\left\{\frac{2}{(r-1)!} + \frac{3}{r!}\right\}$

$= \frac{1}{6}\left\{\left(\frac{2}{1} + \frac{3}{1!}\right) + \left(\frac{2}{1!} + \frac{3}{2!}\right) + \left(\frac{2}{2!} + \frac{3}{3!}\right) + \cdots\right\}$

$= \frac{1}{6}\left\{\left(\frac{2}{1!} + \frac{2}{1!} + \frac{2}{2!} + \cdots\right) + \left(\frac{3}{1!} + \frac{3}{2!} + \frac{3}{3!} + \cdots\right)\right\}$

$= \frac{1}{6}\left\{2\left(1 + \frac{1}{1!} + \frac{1}{2!} + \cdots\right) + 3\left(\frac{1}{1!} + \frac{1}{2!} + \cdots\right)\right\}$

$= \frac{1}{6}\{2e + 3(e-1)\}$

$= \frac{1}{6}\{2e + 3e - 3\}$

$= \frac{1}{6}(5e-3) = \frac{5e}{6} - \frac{1}{2}$

191. (a)

Let $l = \int_{\pi/6}^{\pi/3} \frac{(\sin x - x\cos x)}{x(x+\sin x)} dx$

$$\Rightarrow 1 = \int_{\pi/6}^{\pi/3} \frac{(x+\sin x) - x(1+\cos x)}{x(x+\sin x)} dx$$

$$\Rightarrow 1 = \int_{\pi/6}^{\pi/3} \left(\frac{1}{x} - \frac{1+\cos x}{x+\sin x}\right) dx$$

$$\Rightarrow 1 = [\log x]_{\pi/6}^{\pi/3} - \int_{\pi/6}^{\pi/3} \frac{1+\cos x}{x+\sin x} \cdot dx$$

put $t = x + \sin x$

$dt = (1+\cos x)dx$ in IInd term

$$\Rightarrow 1 = \left(\log\frac{\pi}{3} - \log\frac{\pi}{6}\right) - \int_{\left(\frac{\pi}{6}+\frac{1}{2}\right)}^{\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)} \frac{dt}{t}$$

$$\Rightarrow 1 = \log 2 - [\log t]_{\left(\frac{\pi}{6}+\frac{1}{2}\right)}^{\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)}$$

$$\Rightarrow 1 = \log 2 - \left[\log\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right) - \log\left(\frac{\pi}{6}+\frac{1}{2}\right)\right]$$

$$\Rightarrow 1 = \log 2 - \log\left(\frac{2\pi+3\sqrt{3}}{\pi+3}\right)$$

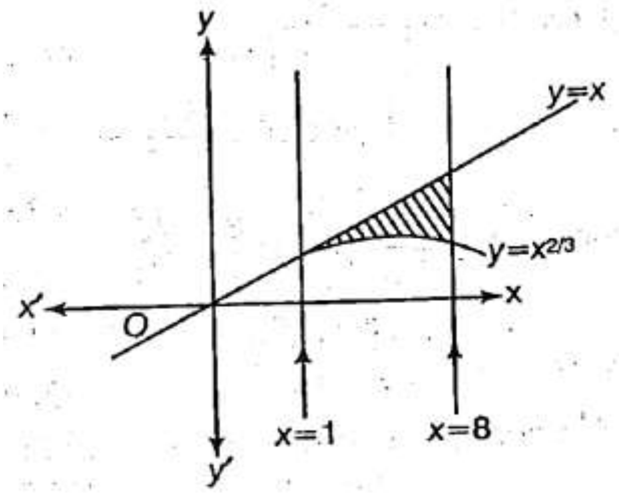
$\left(\because \log m - \log n = \log\frac{m}{n}\right)$

$$1 = \log\left(\frac{2(\pi+3)}{2\pi+3\sqrt{3}}\right)$$

$$= \log\left(\frac{2\pi+6}{2\pi+3\sqrt{3}}\right)$$

192. (d)

Given, $f(x) = x^{2/3}, x \geq 0$ and line $y = x$



a

$\therefore$ Required area $A = \int_{x=1}^{8} (x - x^{2/3})dx$

$$= \left[\frac{x^2}{2} - \frac{3}{5}x^{5/3}\right]_1^8 = \left(32 - \frac{3}{5}\times 32\right) - \left(\frac{1}{2} - \frac{3}{5}\right)$$

$$= 32 \times \frac{2}{5} - \frac{(5-6)}{10} = \frac{64}{5} + \frac{1}{10}$$

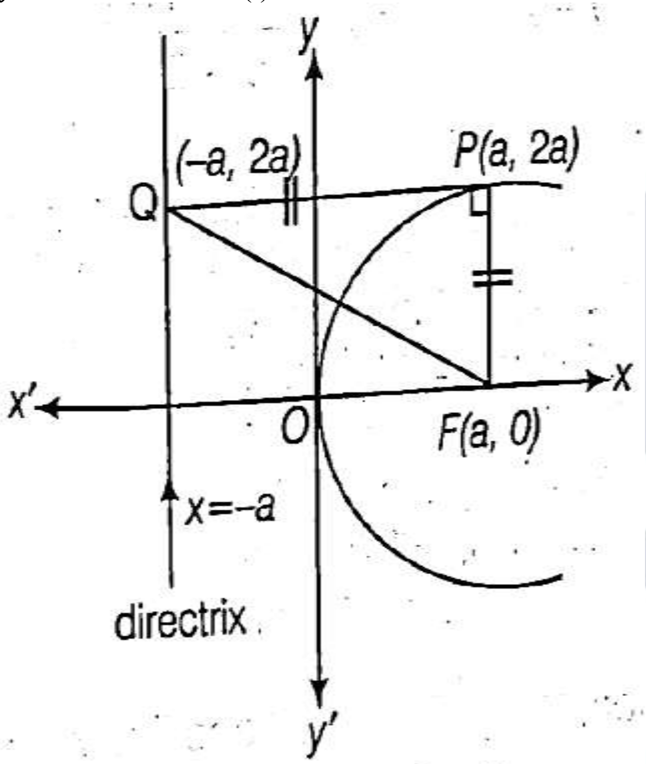
$$= \frac{128+1}{10} = \frac{129}{10}$$

193. (d)
Given function is

$$f(x) = x\left\{\frac{1}{x-1}+\frac{1}{x}+\frac{1}{x+1}\right\}, x>1$$
$$= x\left\{\frac{2x}{x^2-1}+\frac{1}{x}\right\} = \frac{2x^2}{x^2-1}+1$$
$$= \left\{\frac{2}{\left(1-\frac{1}{x^2}\right)}+1\right\} > 3\ (\because x>1)$$

$\therefore\ f(x) > 3$

194. (a)
Equation of parabola is
$y^2 = 4ax$............(i)



Let the parametric coordinate of point P on the parabola is $(a, 2a)$.
Now,

$QF = 2\sqrt{2}a$
$PQ = 2a$ and $PF = 2a$

we observe that, $QF^2 = PQ^2 + PF^2$
$\Rightarrow\ 8a^2 = 4a^2 + 4a^2 = 8a^2$
So, $\triangle\ QPF$ form a right angle isoceles triangle.
In which, $\angle PQF = \angle PFQ$
$\Rightarrow\ \tan\angle PQF = \tan\angle PFQ$
$\Rightarrow\ \dfrac{\tan\angle PQF}{\tan\angle PFQ} = 1$

195. (d)
Given function is

$$F(x) = \int_0^x \frac{\cos t}{(1+t^2)}dt, 0\le x\le 2\pi$$

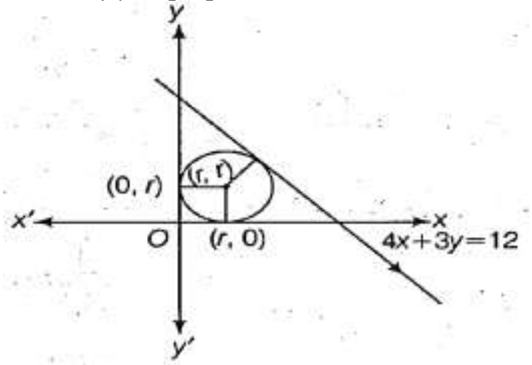
On differentiation w.r.t. x., (apply Leibnitz rule)
$F'(x) = \frac{\cos x}{1+x^2}\times 1 = \frac{\cos x}{1+x^2}$, where $(1+x^2)>0$
Here, $\cos x > 0 \Rightarrow x\in\left(0,\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2}, 2\pi\right)$

and $\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

So, F is increasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$ and decreasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

196. (b,c)

Radius (r) = perpendicular distance on line $4x + 3y = 12$ from center



$\Rightarrow \quad r = \frac{|4r + 3r - 12|}{\sqrt{16 + 9}}$

$\Rightarrow \quad 17r - 12 \,|$

$\Rightarrow \quad = 5r$

$\therefore \quad 7r - 12$

and $\quad = \pm 5r$

$2r$

$= 12 \Rightarrow r = 6$

$12r \quad = 12 \Rightarrow r = 1$

(i) When center is $(1,1)$ and radius is 1 , then equation of circle is

$(x - 1)^2 + (y - 1)^2 = 1$

$\Rightarrow x^2 + y^2 - 2x - 2y + 1 = 0$

(ii) When center is $(6,6)$ and radius is 2 , then equation of circle is

$(x - 6)^2 + (y - 6)^2 = 36$

$\Rightarrow x^2 + y^2 - 12x - 12y + 36 = 0$

197. (a,d)

Equation of parabola is $y^2 = x$ and line $y = mx$

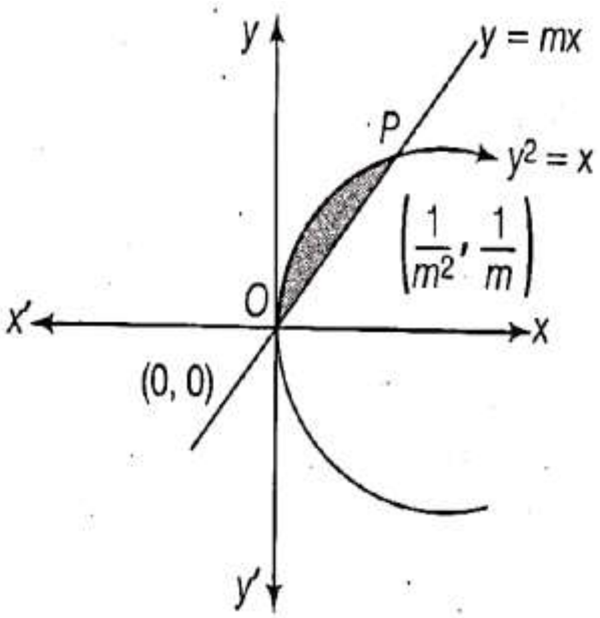
For intersection point of both curves put $x = y^2$, we get

$y = my^2 \Rightarrow y(my - 1) = 0$

$\Rightarrow \quad y = 0 \text{ or } y = \frac{1}{m}$

Then, $\quad x = 0 \text{ or } x = \frac{1}{m^2}$

$\therefore$ Intersection points are $(0,0)$ and $P\left(\frac{1}{m^2}, \frac{1}{m}\right)$



∴ Required area

$$= \int_0^{1/m} \left|\left(\frac{y}{m} - y^2\right)\right| dy = \left|\left[\frac{y^2}{2m} - \frac{y^3}{3}\right]_0^{1/m}\right|$$

$$= \left|\frac{1}{2m^3} - \frac{1}{3m^3}\right| = \left|\frac{1}{6m^3}\right| = \frac{1}{48} \text{ (given)}$$

$\Rightarrow \frac{1}{6m^3} = \pm\frac{1}{48} \Rightarrow m^3 = \pm 8$

Now, if $m^3 = 8$

$\Rightarrow m^3 = (2)^3 \Rightarrow m = 2$

If $m^3 = -8$

$\Rightarrow m^3 = (-2)^3 \Rightarrow m = -2$

198. (a,b)

Given equation is

$x^2 - bx + c = 0$.........(i)

and roots are $\sin\alpha$ and $\cos\alpha$

Also $\sin\alpha + \cos\alpha = b$

and $\sin\alpha \cdot \cos\alpha = c$

$\because \sin^2\alpha + \cos^2\alpha = (\sin\alpha + \cos\alpha)^2$

$\Rightarrow 1 = b^2 - 2c$, [from Eqs. (ii) and (iii)]

$\because -\sqrt{1+1} \le (\sin\alpha + \cos\alpha) \le \sqrt{1+1}$

$\Rightarrow -\sqrt{2} \le b \le \sqrt{2}$

and $2\sin\alpha \cdot \cos\alpha = x2c$

[from Eq. (iii)]

$\Rightarrow \sin 2\alpha = 2c (\because -1 \le \sin 2\alpha \le 1)$

$\Rightarrow 2c \le 1 \Rightarrow c \le \frac{1}{2}$

199. (b,c)

Given system of equations is

$x+y+z=0$
$\alpha x+\beta y+\gamma z=0$
$\alpha^2 x+\beta^2 y+\gamma^2 z=0$

The coefficient matrix, $A=\begin{bmatrix}1 & 1 & 1\\ \alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\end{bmatrix}$

Now, $|A|=\begin{vmatrix}1 & 1 & 1\\ \alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\end{vmatrix}=(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

(i) The system of equations has a unique solution, if α,β and γ are distinct
i.e.,$|A|\neq 0$
(ii) The system of equations has infinite number of solutions, if any two of α,β and γ are equal.
i.e.,$|A|=0$

200. (c,d)

we know that,
if $f(-x)=f(x)$, then function is even and if $f(-x)=-f(x)$, then function is odd.
(a) $f(x)=x^3\sin x$
$f(-x)=(-x)^3\sin(-x)$
$=-x^3(-\sin x)$
$=x^3\sin x=f(x)$
So, $f(x)$ is even.
(b) $f(x)=x^2\cos x$
$f(-x)=(-x)^2\cos(-x)=x^2\cos x=f(x)$
So, $f(x)$ is even.
(c) $f(x)=e^x x^3\sin x$
$f(-x)=e^{-x}(-x)^3\sin(-x)$
$=e^{-x}x^3\sin x\neq f(x)$
$\therefore$ $f(x)$ is not even.
(d) $f(x)=x-[x]$
$f(-x)=(-x)-[-x]\neq f(x)$
$\therefore f(x)$ is not even.