Physics

1. (a)

Vector X of the form $\alpha A + \beta B$

$\mathrm{X} = \alpha \mathrm{A} + \beta \mathrm{B}$

$= \alpha(\hat{\imath} + \hat{\jmath} - 2\hat{k}) + \beta(\hat{\imath} - \hat{\jmath} + \hat{k})$

$\mathrm{X} = \hat{\imath}(\alpha + \beta) + \hat{\jmath}(\alpha - \beta) + \hat{k}(-2\alpha + \beta)$

A vector X is perpendicular to C , i.e., $\mathrm{X} \cdot \mathrm{C} = 0$

$[\hat{\imath}(\alpha + \beta) + \hat{\jmath}(\alpha - \beta)$

$+\hat{k}(-2\alpha + \beta)] \cdot [2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}) = 0$

or $2(\alpha + \beta) - 3(\alpha - \beta) + 4(-2\alpha + \beta) = 0$

or $2\alpha + 2\beta - 3\alpha + 3\beta - 8\alpha + 4\beta = 0$

or $-9\alpha + 9\beta = 0$ or $\alpha = \beta$

or $\frac{\alpha}{\beta} = \frac{1}{1}$ or $\alpha : \beta = 1 : 1$

2. (d)

The combination of three charges in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{6} \Rightarrow C = \frac{6}{4} = 1.5\mu \text{ F}$$

The charge of this circuit

$q = CV = 1.5 \times 120$

$q = 180\mu C$

The potential difference across the 3μ F

$q = CV$

$$V = \frac{q}{C} = \frac{180}{3} = 60 \text{ V}$$

3. (a)

$$1\text{ce } R = \frac{V}{i_g} - G = \frac{120}{0.01} - 10$$

$R = 11990\Omega$

To convert a galvanometer into a voltmeter, high resistance should connect in series.

4. (b)

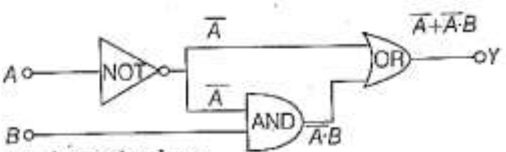
For an open pipe, the frequency

$$f = \frac{V}{2l} \Rightarrow 5100 = \frac{340}{2 \times l}$$

$$l = \frac{340}{5100 \times 2} \Rightarrow l = \frac{2}{30 \times 2} = \frac{1}{30} \text{ m}$$

$$l = \frac{100}{30} = \frac{10}{3} \text{ cm}$$

5. (b)



By absorption laws

$Y = \bar{A} + \bar{A} \cdot B$

or $Y = (\bar{A} \cdot B) + \bar{A} = \bar{A} \cdot (B + 1) = \bar{A} \cdot 1 = \bar{A}$

6. (d)

We know that $\Delta U = nC_v \Delta T$

$\Delta T=(T_2-T_1)$
$T_1=0^{\circ}\mathrm{C}=273\ \mathrm{K}$
$T_2=100^{\circ}\mathrm{C}=373\ \mathrm{K}$
$n=1$ (monoatomic gas)

where $R=8.32\ \mathrm{J\ mol^{-1}\ K^{-1}}=1\times\frac{3}{2}R\times(373-273)$

$=1\times\frac{3}{2}\times 8.32\times 100=3\times 8.32\times 50$

$\Rightarrow 1248\ \mathrm{kJ}$

or $=1.25\times 10^3\ \mathrm{J}$

7. (c)

Energy required to removed electron in the ($n=2$) stage

The energy of the photon

$$=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right)\Rightarrow 13.6\mathrm{eV}\left(\frac{1}{1}-\frac{1}{4}\right)$$

$$=13.6\mathrm{eV}\left(\frac{4-1}{4}\right)=13.6\mathrm{eV}\times\frac{3}{4}=\frac{40.8}{4}\mathrm{eV}$$

$=10.2\mathrm{eV}$

8. (d)

We know that

The parallel plate capacitor $C=\frac{\varepsilon_0\cdot A}{d}$

When d (distance between two parallel plate) increase, then C will be decrease $\because\ Q=CV$ where $Q=$ constant

$\therefore$ The voltage across the capacitor increases.

9. (b)

We know that

Kinetic energy $K=\frac{1}{2}mu^2$ where $u=\frac{dy}{dt}=\omega a\cos\omega t$

So, $K=\frac{1}{2}m\omega^2a^2\cos^2\omega t$

Hence, kinetic energy varies periodically with double the frequency of SHM. So, when a particle executing SHM oscillates with a frequency v, then the kinetic energy of particle changes periodically with a frequency of $2v$.

10. (c)

(a)

$$\text{Planck's constant }=\frac{\text{Energy}}{\text{Frequency}}=\frac{[\mathrm{ML^2\ T^{-2}}]}{[\mathrm{T^{-1}}]}$$
$$=[\mathrm{ML^2\ T^{-1}}]$$

Angular momentum $=$ Moment of inertia $\times$ Angular velocity

$=[\mathrm{ML^2}]\times[\mathrm{T^{-1}}]=[\mathrm{ML^2\ T^{-1}}]$

(b)

Impulse $=$ Force $\times$ Time

$=[\mathrm{MLT^{-2}}][\mathrm{T}]=[\mathrm{MLT^{-1}}]$

and linear momentum $=$ Mass $\times$ Velocity

$=[\mathrm{M}][\mathrm{LT^{-1}}]=[\mathrm{MLT^{-1}}]$

(c)

Moment of inertia $=$ Mass $\times$ (Distance $)^2$

$=[\mathrm{ML^2}]$

and moment of force $=$ Force $\times$ Distance

$=[\mathrm{MLT^{-2}}][\mathrm{L}]=[\mathrm{ML^2\ T^{-2}}]$

(d) Energy = $[ML^2\ T^{-2}]$ and torque = $[ML^2\ T^{-2}]$

So, (c) option has different dimensions.

11. (a)
For vertically moment
(for h_1)
or

$$h_1 = u\sin\theta t_1 - \frac{1}{2}gt_1^2$$

$$t_1 = \frac{h_1 + \frac{1}{2}gt_1^2}{u\sin\theta}$$

$$h_2 = u\sin\theta t_2 - \frac{1}{2}gt_2^2$$

$$t_2 = \frac{h_2 + \frac{1}{2}gt_2^2}{u\sin\theta}$$

or
Divide Eq. (i) by Eq. (ii)

$$\frac{t_1}{t_2} = \frac{h_1 + \frac{1}{2}gt_1^2/u\sin\theta}{h_2 + \frac{1}{2}gt_2^2/u\sin\theta}$$

$$\Rightarrow h_1t_2 - h_2t_1 = \frac{g}{2}(t_1t_2^2 - t_1^2t_2)$$

The time of flight of the ball

$$T = \frac{2u\sin\theta}{g} = \frac{2}{g}(u\sin\theta)\ \text{[from Eq. (i)]}$$

$$= \frac{2}{g}\left[\frac{h_1 + \frac{1}{2}gt_1^2}{t_1}\right] = \frac{2}{t_1}\left[\frac{h_1}{g} + \frac{t_1^2}{2}\right]$$

$$= \frac{h_1}{t_1} \times \frac{2}{g} + t_1 = \frac{h_1}{t_1} \times \left(\frac{t_1t_2^2 - t_1^2t_2}{h_1t_2 - h_2t_1}\right) + t_1$$

$$= \frac{h_1t_1t_2^2 - h_1t_1^2t_2 + h_1t_1^2t_2 - h_2t_1^3}{t_1(h_1t_2 - h_2t_1)}$$

$$= \left(\frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}\right)$$

12. (c)
Stokes established that if a sphere of radius a moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is $F = 6\pi\eta av$

13. (d)
Given, $305X - 25X = 100^\circ C$
($\because$ X is the new unit of temperature)

$$(305 - 25)X = 100^\circ C \Rightarrow 280X = 100^\circ C$$

$\therefore 1^\circ C = 2.8X$

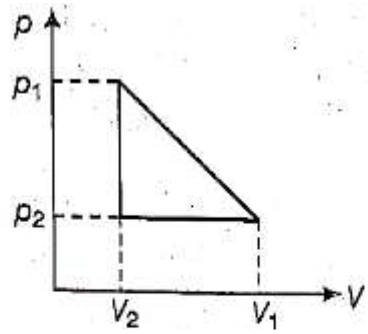
The specific heat capacity of water

$$= 4200\frac{\text{joule}}{\text{kg}^\circ\text{C}} = 4200 \times \frac{\text{joule}}{\text{kg} \times 2.8X}$$

$$= 1500\ \text{J/kg}^{-}\text{X}$$

$$= 1.5 \times 10^3\ \text{J kg}^{-1}\text{X}^{-1}$$

14. (a)
For the cyclic process



$$\text{Heat absorbed} = \text{Work done}$$
$$= \text{Area } = \frac{1}{2}(\Delta p) \times \Delta V$$
$$= \frac{1}{2}(p_1 - p_2) \times (V_1 - V_2)$$
$$= \frac{1}{2}(p_1 - p_2)(V_1 - V_2)$$

15. (c)

float with 40% above the water surface

16. (b)

We know that

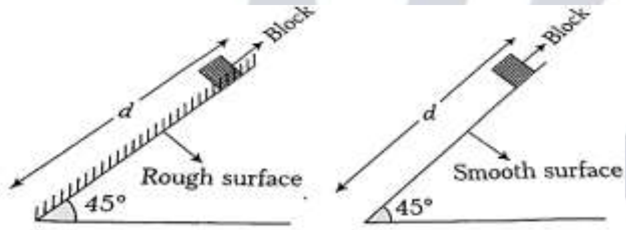
Acceleration, $a_{CM} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \times g (\because m_1 > m_2)$

So, resultant external force,

$$F = (m_1 + m_2) a_{CM}$$
$$= (m_1 + m_2) \times \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \times g$$
$$= \frac{(m_1 - m_2)^2}{(m_1 + m_2)} \times g$$

17. (a)

If the same wedge is made rough then time taken by it to come down becomes n times more.
The coefficient of friction



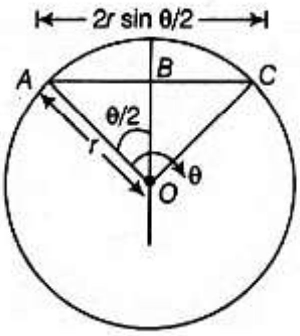
$$\mu = \left[1 - \frac{1}{n^2}\right] \tan\theta \Rightarrow \mu = \left[1 - \frac{1}{2^2}\right] \tan 45^\circ$$
$$\mu = \frac{4-1}{4} \Rightarrow \mu = \frac{3}{4}$$

18. (c)

Strain $= \alpha \cdot \Delta T$
Stress $Y \propto \Delta T$
where $L =$ length of the rod
$\alpha =$ coefficient of linear thermal expansion
$Y =$ Young's modulus of its material
So, the longitudinal stress developed in the rod is directly proportional to $\frac{\Delta T}{Y}$.

19. (d)

In $\triangle AOB \sin\frac{\theta}{2} = \frac{AB}{AO}$



$(\because AO = r)$

$AB = AO\sin\frac{\theta}{2} \Rightarrow AB = r\sin\frac{\theta}{2}$

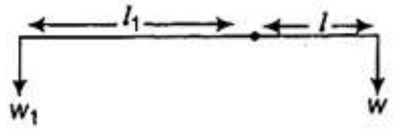
$AC = AB + BC \; (\because AB = BC)$

$= r\sin\frac{\theta}{2} + r\sin\frac{\theta}{2}$

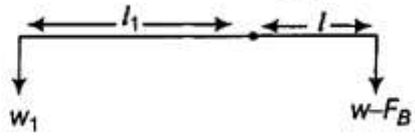
$AC = 2r\sin\frac{\theta}{2}$

So, the magnitude of the corresponding linear displacement will be $2r\sin\frac{\theta}{2}$ ...

20. (c)



$$Wl = W_1 l_1 \quad \text{...(i)}$$



$W - F_B = Wl\left(1 - \frac{1}{\rho}\right)$

where ρ = specific gravity

$W\left(1 - \frac{1}{\rho}\right) = W_1 l_2$

$\left(1 - \frac{1}{\rho}\right) = \frac{W_1 l_2}{Wl}$ [from Eq.(i)]

$\left(1 - \frac{1}{\rho}\right) = \frac{W_1 l_2}{W_1 h}$

$1 - \frac{1}{\rho} = \frac{l_2}{l_1} \Rightarrow \frac{1}{\rho} = 1 - \frac{l_2}{h}$

$\frac{1}{\rho} = \frac{l_1 - l_2}{h}$ or $\rho = \frac{l_1}{h - l_2}$

21. (c)

Let, thickness of layer be x

So, volume $V = \text{Area} \times x$

$V = A \times x$

$x = V/A$

$\therefore \quad 2r = \frac{V}{A} \Rightarrow r = \frac{V}{2A}$

and $\quad \Delta P = \frac{T}{r}$

We know that $F = \Delta P \times A$

$$= \frac{T}{r} \times A$$

$(\because x = 2r)$

$$F = \frac{T}{\left(\frac{V}{2A}\right)} \times A (\text{from } EQ.1)$$

$$T = \frac{F \times V}{2A^2}$$

where $F = 16 \times 10^5$ dyne, $V = 0.04\ \text{cm}^3$,

$$A = 20\ \text{cm}^2 = \frac{16 \times 10^5 \times 0.04}{2 \times 20^2}$$

$$= \frac{8 \times 10^5 \times 4}{20^2 \times 100} = \frac{8 \times 10^5 \times 4}{400 \times 100}$$

$$= 8 \times 10^5 \times 10^{-4} = 80 \text{ dyne }/\text{cm}$$

$$= 80 \text{ dyne cm}^{-1}$$

22. (d)

Given, $r = 0.05\ \text{nm} = 0.05 \times 10^{-9}\ \text{m}$

$n = 10^{16}$ revolutions

$e = 1.6 \times 10^{-19}\text{C}$

We know that

The magnetic moment $M = Ai$

$$M = \pi r^2 \times ne$$

$$M = 3.14 \times (0.05 \times 10^{-9})^2 \times 10^{16} \times 1.6 \times 10^{-19}$$

$$M = 0.1256 \times 10^{-18} \times 10^{16} \times 10^{-19}$$

or $M = 1.26 \times 10^{-23}\text{Am}^2$

23. (c)

We know that $\tau = NBA\omega \sin \omega t$ where $N =$ number of loops $= 1$

$$B = \frac{\mu_0 I}{2b} \text{ newton/amp-m}$$

$$A = \pi a^2 \text{ metre}^2$$

$$\therefore\ \tau = \frac{\mu_0 I}{2b} (\pi a^2) \omega \sin \omega t$$

$$= \frac{\pi a^2 \mu_0 I}{2b} \cdot \omega \sin \omega t$$

24. (d)

Focal length of a convex lens by displacement method Focal length of the convex lens

$$f = \frac{a^2 - b^2}{4a}$$

where $a =$ distance between the image and object and $b =$ distance between two positions of lens

Fact The distance between the two points should be greater than four time the focal length of the convex lens, i.e., $d > 4f$.

So, the maximum possible focal length of this convex lens is $\frac{d}{4}$.

25. (a)

The given

$$\mathbf{A} = a(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$\mathbf{B} = a(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$$

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$$

$$= a(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) - a(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$\mathbf{AB} = a(-4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

Work done $= q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathbf{k}} \cdot \mathbf{AB}$ (along to Z-axis)

$$= q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathbf{k}} \cdot a(-4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = \frac{3q\sigma a}{2\varepsilon_0}$$

$(\because \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1 \text{ and } \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0)$

26. (a)

We know that Intensity of magnetisation

$I=\frac{M}{V}$

[where $M=$ magnetic moment, $V=$ volume]

So

$$M=IV=5.0\times10^4\times\frac{12}{100}\times\frac{1}{(100)^2}$$
$$=60\times10^4\times10^{-6}=0.6\,\mathrm{Am}^2$$

27. (b)

We know that

$$S_{\text{nth}}=u+\frac{1}{2}a(2n-1)$$
$$S_{3\text{rd}}=0+\frac{1}{2}a(2\times3-1)=\frac{5}{2}a(\text{ for } n=3\text{ s})$$
$$S_{4\text{th}}=0+\frac{1}{2}a(2\times4-1)=\frac{7}{2}a\ (\text{ for } n=4\text{ s})$$

So, the percentage increase

$$=\frac{S_{4\text{th}}-S_{3\text{rd}}}{S_{3\text{rd}}}\times100$$
$$=\frac{\frac{7}{2}a-\frac{5}{2}a}{\frac{5}{2}a}\times100$$
$$=\frac{\frac{2a}{2}}{\frac{5}{2}a}\times100=2\times20=40\%$$

28. (b)

Given, Kinetic energy $=E$

Mass $=m$

Magnetic field $=B$

Charge $=q$

We know that

$F=qvB\sin\theta$

(motion of a charged particle in a uniform magnetic field)

If

$\theta=90^\circ$

Then

$F=qvB$

We know that also (centripetal force)

$F=\frac{mv^2}{r}$

From Eqs. (i) and (ii), we get

$$qvB=\frac{mv^2}{r},r=\frac{mv}{qB}\left[\because E=\frac{1}{2}mv^2,v=\sqrt{\frac{2E}{m}}\right]$$

$$\therefore\ r=\frac{m\sqrt{\frac{2E}{m}}}{qB}\Rightarrow r=\frac{\sqrt{2Em}}{qB}$$

29. (a)

We know that The potential energy of the satellite

$U=-\frac{GM_em}{R_e}$......(i)

The kinetic energy of the satellite

$K=\frac{1}{2}\frac{GM_em}{R_e}$.........(ii)

The total energy

$$E = U + K = -\frac{GM_e m}{R_e} + \frac{1}{2}\frac{GM_e m}{R_e}$$

$$E = -\frac{1}{2}\frac{GM_\theta m}{R_e}$$

$$2E = -\frac{GM_e m}{R_\theta} \Rightarrow -2E = U$$

So,

$$\mathrm{PE} = -2(\mathrm{TE})$$

$$\mathrm{PE} = -2E$$

30. (d)

Thermal radiation is the heat waves travel along straight lines with the speed of light.

31. (d)

The band gap of 5 eV corresponds to that of an insulator.

32. (c)

We know that total kinetic energy of a body rolling without slipping

$K_{\text{total}} = K_{\text{rot}} + K_{\text{trans}}$

For solid spherical ball,

$I = \frac{2}{5}mR^2$ (along to diameter)

and $v = R\omega$, where R is radius of spherical ball

So,

$$K_{\text{total}} = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 + \frac{1}{2}mR^2\omega^2$$

$$= \frac{7}{10}mR^2\omega^2$$

$$K = \frac{7}{10}mv^2$$

Potential energy = Kinetic energy

$$mgh = \frac{7}{10}mv^2$$

$$v^2 = \frac{10}{7}gh \quad \text{............(i)}$$

For vertical projection,

$$v^2 = u^2 + 2gh'$$

$$\frac{10}{7}gh = 0 + 2gh' \Rightarrow h' = \frac{5}{7}h$$

33. (d)

We know that The maximum intensities

$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ ....(i)

The minimum intensities

$I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ ....(ii)

So, the ratio of the maximum and minimum of intensities is

$$\frac{I_{\max}}{I_{\min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{4I} + \sqrt{I})^2}{(\sqrt{4I} - \sqrt{I})^2}$$

$$= \left(\frac{3\sqrt{I}}{\sqrt{I}}\right)^2$$

$$= \frac{9}{1} = 9:1$$

34. (b)

$I_{\text{initial}} = 0,$

$I_{\text{final}} = \frac{3}{150} = 0.02 \text{ A}$

So, change in $I = 0.02 \text{ A} = 20 \text{ mA}$

35. (a)

$\left[\frac{nh}{2\pi qB}\right]$

where n and 2π dimensionless quantity

So,

$[h] = [mvr]$

$[qB] = \left[\frac{qvB}{v}\right] = \left[\frac{F}{v}\right]$

$\therefore \left[\frac{mvr}{F/v}\right] = \frac{[mvr][v]}{[F]}$

$= \frac{[mv^2 r]}{\left[\frac{mv^2}{r}\right]} = [r^2] = \text{ Area}$

36. (b)

I_B is in μ amp, I_C is in m amp and V_{CE} in volt.

37. (c)

Given,

$$\begin{aligned}
Y &= 4\cos^2\left(\frac{t}{2}\right)\cdot\sin(1000t) \\
&= 2\times 2\cos^2\left(\frac{t}{2}\right)\cdot\sin(1000t) \\
&= 2(1+\cos t)\sin(1000t) \\
&= 2\sin(1000t) + 2\cos t\cdot\sin(1000t) \\
&= 2\sin(1000t) + 2\sin(1000t)\cdot\cos t \\
&= 2\sin(1000t) + \sin(1000t+t) \\
&+\sin(1000t-t) \\
&= 2\sin A\cdot\cos B = \sin(A+B) + \sin(A-B)] \\
&= 2\sin(1000t) + \sin(1001t) + \sin(999t)
\end{aligned}$$

So, $n = 3$

38. (d)

Assuming temperature of the body and cubica box is same initially, i.e., T and finally it becomes $T/2$.
Because, temperature of body and surrounding remains same.
Hence, no net loss of radiation occur through the body. This total energy remains constant.

39. (c)

Let, the charge on the inner shell be q' The total charge will be zero

So,

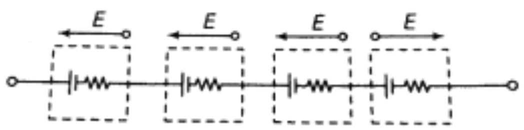
$\frac{kq'}{r_1} + \frac{kq}{r_2} = 0$

$q' = -\frac{Kq/r_2}{K/r_1}$

$q' = -\left(\frac{r_1}{r_2}\right)q$

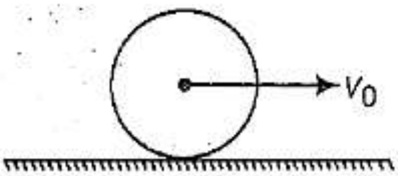
$(\because r_2 > r_1)$

40. (a)



Total emf of the cell $= 3E - E = 2E$

Total internal resistance $= 4r$
$\therefore$ Total resistance of the circuit $= 4r + R$
So, the current in the external circuit $\left(\because i = \frac{V}{R}\right)$
$$\therefore \; i = \frac{2E}{4r+R}$$
41. (a)
We know that
Relation between temperature gradient (TG) and thermal conductivity (K)
So,
$$\frac{d\theta}{dt} = -KA\frac{d\theta}{dx} = -KA \times (\text{TG})$$
$$\text{TG} \propto \frac{1}{K}\left(\frac{d\theta}{dt} = \text{ constant}\right)$$
or
$$K \propto \frac{1}{T}$$
$$K_1 \propto \frac{1}{t_1} \qquad \ldots\ldots\ldots\text{(i)}$$
$$K_2 \propto \frac{1}{t_2} \ldots \ldots . . (ii)$$
From the Eqs. (i) and (ii), we get
$$\frac{K_1}{K_2} = \frac{1/t_1}{1/t_2}$$
$K_1 : K_2 = t_2 : t_1$
42. (b)
Isotope of the original nucleus is produced.
43. (c)
By Doppler's Effect
When observer is moving with velocity v_0, towards a source at rest then approach frequency
$$N_{\text{Approach}} = N\left(\frac{v+v_0}{v}\right)$$
where $N = 850$ Hz, $v = 340\ \text{ms}^{-1}, v_0 = 72\text{kmh}^{-1}$
$$= 20\ \text{ms}^{-1} = 850\left(\frac{340+20}{340}\right)$$
Similarly, when observer is moving away from source, then
$$N_{\text{Separation}} = N\left(\frac{v-v_0}{v}\right) = 850\left(\frac{340-20}{340}\right)$$
The different of the two frequency,
$N_{\text{Approach}} - N_{\text{Separation}}$
$$= 850\left(\frac{360}{340}\right) - 850\left(\frac{320}{340}\right)$$
$$= \frac{850}{340} \times 40 = \frac{850}{8.5} = 100\ \text{Hz}$$
44. (c)
$E_\gamma \geq 100\text{keV}$
$E_X = 100\text{eV}$ to 100keV
$E_V = 2.48\text{eV}$
So, we can say that
$E_\gamma > E_X > E_v$
45. (a)
The intermediate image formed by the objective of a compound microscope is real, inverted and magnified.
46. (c)
Let, the final velocity be v



So, angular momentum will remain conserved along point of contact
By conservation of angular momentum
Angular momentum will remain conserved along point of contact

$$I\omega = \text{ constant}$$
$$mv_0 r = mvr + \frac{2}{5}mr^2 \times \omega \left(\because \omega = \frac{v}{r}\right)$$
$$mv_0 r = mvr + \frac{2}{5}mr^2\left(\frac{v}{r}\right)$$
$$v_0 = v + \frac{2}{5}v$$
$$v_0 = \frac{7}{5}v \Rightarrow v = \frac{5}{7}v_0$$

47. (c)

de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mK}}$
The kinetic energy of the electron
$K_{\text{electron}} = \frac{1}{2m} \cdot \frac{h^2}{\lambda^2}$........(i)
where $h =$ Planck constant
$\lambda =$ wavelength
The photon energy
$E_{\text{photon}} = \frac{hc}{\lambda}$.........(ii)
From Eqs. (ii) and (i), we get

$$\frac{E_{\text{photon}}}{K_{\text{electron}}} = \frac{hc/\lambda}{h^2/2m \cdot \lambda^2} = \frac{hc \cdot \lambda^2 \times 2m}{h^2 \cdot \lambda}$$

where $m = 0.5\text{MeV} = 5 \times 10^5\text{eV}$

$$\frac{h}{\lambda c} = 50 \times 10^3\text{eV} = \frac{2m\lambda c}{h}$$
$$= \frac{2m}{h/\lambda c} = \frac{2 \times 5 \times 10^5}{50 \times 10^3}\left(\because m = \frac{h}{c\lambda}\right)$$
$$= 20:1$$

48. (d)

By Snell's law,

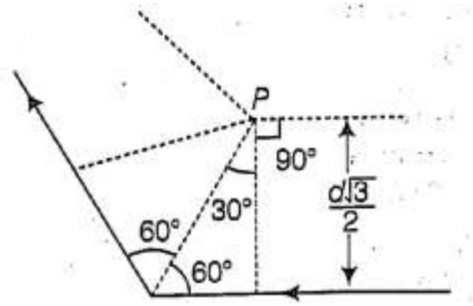
$$\mu \sin i = \text{ constant}$$
$$\mu \sin i = (\mu - m\Delta\mu)\sin r_i$$

where $\mu = 1.5, i = 30^\circ, r = 90^\circ, \Delta\mu = 0.015$
$1.5\sin 30^\circ = (1.5 - m \times 0.15)\sin 90^\circ$

$$\frac{3}{2} \times \frac{1}{2} = (15 - m \times 0.015) \times 1$$
$$\frac{3}{4} = \frac{3}{2} - \frac{15m}{1000}$$
$$15m = \left(\frac{3}{2} - \frac{3}{4}\right) \times 1000$$
$$15m = \frac{6-3}{4} \times 1000 \Rightarrow m = \frac{3}{4} \times \frac{1000}{15}$$
$$m = \frac{3000}{60} \Rightarrow m = 50$$

49. (d)

We know that



$$B_{net} = 2\left[\frac{\mu_0 i}{4\pi r}(\sin\theta_1 + \sin\theta_2)\right]$$

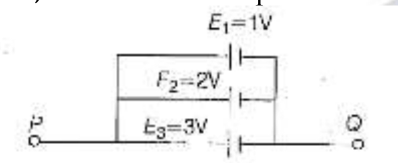
$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{i}{\frac{d\sqrt{3}}{2}} \times (\sin 90^\circ + \sin 30^\circ)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{2i}{d\sqrt{3}} \times \left(1 + \frac{1}{2}\right)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{2i}{d\sqrt{3}} \times \frac{3}{2}\right] = \frac{\sqrt{3}\mu_0 i}{2\pi d}$$

50. (b)

1Ω, 2Ω and 1Ω are in parallel



So, the required internal resistance

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$\frac{1}{r} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1}$$

$$\frac{1}{r} = \frac{2+1+2}{2} \Rightarrow r = \frac{2}{5}\Omega$$

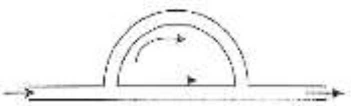
The potential difference between points P and Q

$$E_{diff} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{1/r} = \frac{\frac{1}{1} + \frac{2}{2} + \frac{3}{1}}{5/2}$$

$$= \frac{\frac{2+2+6}{2}}{5/2} = \frac{10/2}{5/2} = \frac{5}{5} \times 2 = 2\text{ V}$$

51. (a)

Path difference $= (\pi r - 2r)$



$$= (\pi - 2)\Gamma = n\lambda(\text{ where } f = \text{ frequency })$$

$$v = f \times \lambda$$

$$\Rightarrow \frac{v}{\lambda} = f \Rightarrow f = \left[\frac{v}{(\pi - 2)r}\right]n$$

So, multiples integral $= \left[\frac{v}{(\pi - 2)r}\right]$

52. (a)

By Archimedes' Principle

$F = v\rho_w g \Rightarrow (w_1 - w_2)g = v\rho_w g$

Let, the total volume be v and first metal weight be x

$$w_1 - w_2 = (v_1 + v_2)\rho_w$$
$$w_1 - w_2 = v_1\rho_w + v_2\rho_w$$
$$w_1 - w_2 = \left(\frac{x}{\rho_1}\rho_w + \frac{w_1 - x}{\rho_2}\rho_w\right) \quad \left(\because v = \frac{m}{\rho}\right)$$
$$w_1 - w_2 = \frac{x\rho_2\rho_w + (w_1 - x)\rho_w\rho_1}{\rho_1\rho_2}$$
$$w_1\rho_1\rho_2 - w_2\rho_1\rho_2 = x\rho_2\rho_w + w_1\rho_w\rho_1 - x\rho_w\rho_1$$
$$x(\rho_2 - \rho_1)\rho_w = \rho_1[w_1(\rho_2 - \rho_w) - w_2\rho_2]$$
$$x = \frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_2 - \rho_w) - w_2\rho_2]$$

53. (b)

$m_1s_1\Delta t + m_2s_2\Delta t = \text{Work done}$

$m_1s_1\Delta t + m_2s_2\Delta t = P_1t_1$

where $m_1 = 0.5$ kg

Specific heat $s_1 = 4200$ J/kg – K

$\Delta t = \Delta t_1 = \Delta t_2 = 3$ K

$P_1 = P_2 = 10$ W

$t_1 = 15 \times 60 = 900$ s

$s_2 =$ Specific heat capacity of container

$$0.5 \times 4200 \times (3 - 0) + m_2s_2 \times (3 - 0) = 10 \times 15 \times 60$$
$$2100 \times 3 + m_2s_2 \times 3 = 9000$$
$$m_2s_2 = \frac{9000 - 6300}{3}$$
$$m_2s_2 = 900$$

Similarly, in case of oil

$m_1s_0\Delta t + m_2s_2\Delta t = P_2t_2$

where $s_0 =$ specific heat capacity of oil

$$P_1 = P_2 = 10 \text{ W}$$
$$2 \times s_0 \times 2 + 900 \times 2 = 10 \times 20 \times 60$$
$$4s_0 + 1800 = 12000$$
$$4s_0 = 12000 - 1800$$
$$s_0 = \frac{10200}{4} = 2550$$
$$= 2.55 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$$

54. (d)

For the best first condition, $f_1 = 10$cmu $= -30$ cm then, $\frac{1}{f_1} = \frac{1}{v} - (1)(u) \Rightarrow = \frac{1}{v} + \frac{1}{30}$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} \Rightarrow v = \frac{30}{2} = 15$$

For the second condition, when concave lens is placed, (where $F =$ focal length of combination)

$$\therefore \frac{1}{F} = \frac{1}{60} + \frac{1}{30} = F = \frac{60}{3} = 20 \text{ cm}$$

The magnitude of focal length of concave lens,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{20} = \frac{1}{10} + \frac{1}{f_2}$$

$f_2 = -20$ (negative sign for concave lens)

55. (b)

We know that

$$K = \frac{1}{2}I\omega^2$$

where

$K =$ kinetic energy

$I =$ moment of inertia

$\omega =$ angular speed

So,

$$\omega \propto \frac{1}{\sqrt{I}}$$

$$\omega_1 : \omega_2 : \omega_3 = \frac{1}{\sqrt{I_1}} : \frac{1}{\sqrt{I_2}} : \frac{1}{\sqrt{I_3}}$$

$$= \frac{1}{\sqrt{1}} : \frac{1}{\sqrt{1}} : \frac{1}{\sqrt{2}} = \sqrt{2} : \sqrt{2} : 1$$

56. (a)

We know that

Internal energy, $U = \frac{nfRT}{2} = \frac{5R}{2}\left(\alpha t + \frac{1}{2}\beta t^2\right)$

Differentiate with respect to t

$$\frac{dU}{dt} = \frac{5R}{2}[\alpha + \beta t]$$

But , $dQ = nC_p dT$

$$\frac{dQ}{dt} = nC_p \times \frac{dT}{dt} \Rightarrow i^2 r = \frac{7}{2}R \times [\alpha + \beta t]$$

$$i = \sqrt{\frac{7R}{2r}(\alpha + \beta t)}$$

$$pV = nRT$$

or $V = \frac{nRT}{p} = \frac{nR}{p}\left(\alpha t + \frac{1}{2}\beta t^2\right)$

$$X = \frac{nR}{pA}\left(\alpha t + \frac{1}{2}\beta t^2\right)$$

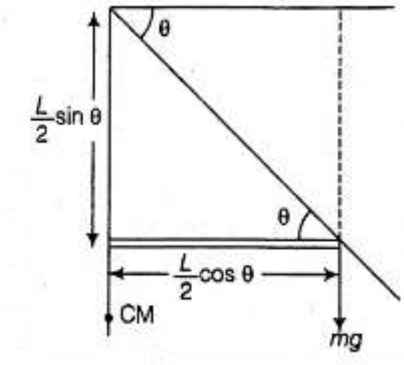
$$V = \frac{n}{pA}(\alpha + \beta t)$$

and acceleration $= \frac{nR}{pA} \times \beta$

So, the rate of increase in internal energy is $\frac{5}{2}R(\alpha + \beta t)$ and the piston moves upwards with constant acceleration.

57. (a,c)

Loss in potential energy = gain in kinetic energy



$$\Rightarrow mg\frac{L}{2}\sin\theta = \frac{1}{2}I\omega^2$$

So,

$\omega \propto \sqrt{\sin\theta}$

and

$v \propto \sqrt{\sin\theta}$

$\left(\because K = \frac{1}{2}I\omega^2\right)$

The speed of end B is proportional to $\sqrt{\sin\theta}$ and

$$U = mgh - mg\frac{L}{2}(1 - \sin\theta)\ (\because \tau - Ir)$$

$$\Rightarrow\ mg\frac{L}{2}\cos\theta = \frac{mI^2}{3} \times \alpha$$

$\alpha \propto \cos\theta$

The anyular auteleraturi is proportional to oos 9

58. (a,d)

There is no significant time delay between the absorption of a suitable radiation and the emission of electrons and the maximum kinetic energy of electrons does not depend on the intensity of radiation.

So, (a) and (d) are correct options.

59. (b,c,d)

We know that

$$C_1 = \frac{K\varepsilon_0 A}{2d}, C_2 = \frac{\varepsilon_0 A}{2d}$$

and $C_{\text{eq}} = \frac{\varepsilon \cdot A}{2d}(K+1)$

$$C_{\text{eq}} = \frac{C_0}{2}(K+1)$$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{K}{1}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{K}{1}$$

$Q_1 = \frac{KQ}{K+1}$ and $Q_2 = \frac{Q}{K+1}$

So,

$$E = \frac{\sigma}{\varepsilon_0 K} \Rightarrow \frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} \times \frac{K_1}{1}$$

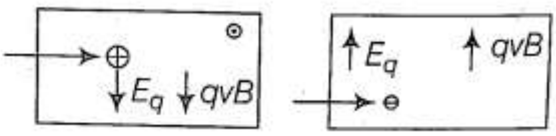
$$= \frac{Q_1}{Q_2} \times \frac{K_2}{K_1} = \frac{K}{1} \times \frac{1}{K}$$

$$= 1:1$$

So, (b), (c) and (d) are correct options.

60. (c,d)

Neither electrons nor protons will go through the slit irrespective of their speed.



And electrons will always be deflected upwards irrespective of their speed.

So, (c) and (d) are correct options.

CHEMISTRY

61. (d)

In 1885, Balmer for the first time showed that the wave numbers of spectral lines present in the visible region in hydrogen spectrum are given by.

$$\bar{v}(\text{ cm}^{-1}) = 109677\left(\frac{1}{(2)^2} - \frac{1}{n^2}\right)$$

Here, $n = 3,4,5, \ldots$

Thus, Balmer spectrum of hydrogen was discovered first and it lies in the visible region.

62. (b)

Atomic number of Cu is 29 .

Thus, its electronic configuration can be written as

$Cu_{29} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^1$

(Since, fully filled orbitals are more stable, so one electron from $4s$ orbital transfers into $3d$ orbital.)

The electronic configuration of neon is

$Ne_{10} = 1s^2, 2s^2, 2p^6$

Thus, using inert gas, the configuration of Cu can also be represented as
$Cu_{29} = [Ne]3s^2, 3p^6, 3d^{10}, 4s^1$

63. (c)

According to de-Broglie, wavelength, $\lambda = \frac{h}{mv}$.

Given, $m = 100$ g, $v = 100$ cm s^{-1}

and

$h = 6.6 \times 10^{-34}$ J − s

$= 6.6 \times 10^{-27}$ erg − s

On substituting values, we get

$$\lambda = \frac{6.6 \times 10^{-27}}{100 \times 100} = 6.6 \times 10^{-31} \text{ cm}$$

64. (c)

Ideal gas equation is

$$pV = nRT \text{ or } V = \frac{nRT}{p} + C$$

or $V = \frac{RT}{p} + 0$

(for 1 mol)

When V vs T is plotted, a straight line is obtained, slope of which is given by R/p. Thus,

$$\text{Slope } = \frac{R}{p} \Rightarrow X = \frac{R}{2}$$

$$R = 2X \text{ L atm mol}^{-1} \text{ K}^{-1}$$

65. (b)

According to Graham's law of diffusion, Rate of diffusion, $r \propto \frac{1}{\sqrt{M}} = \frac{V}{t}$

[Here, $M =$ molecular mass,
$V =$ volume and $t =$ time]

Thus, for H_2 gas,

$$\frac{200}{30} = \frac{1}{\sqrt{2}}$$

For O_2 gas,

$$\frac{50}{t} = \frac{1}{\sqrt{32}}$$

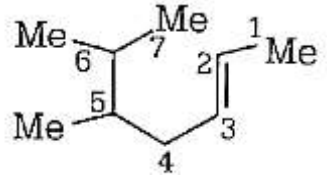
From Eqs. (i) and (ii), we get

$$\frac{200 \times t}{30 \times 50} = \frac{\sqrt{32}}{\sqrt{2}}$$

$$t = \frac{\sqrt{16} \times 30 \times 50}{200}$$

$$= \frac{4 \times 30}{4} = 30 \text{ min}$$

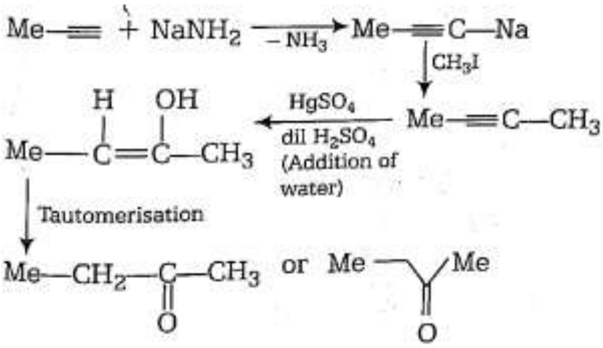
66. (a)



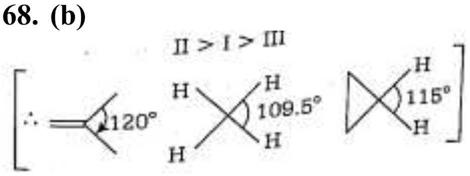
The longest chain contains 7C-atom, word is hept and suffix -ene is addec bond along with its position. Since, groups are also present at 5 and 6 the correct IUPAC name of the co 5,6-dimethylhept-2-ene.

67. (d)

(Here, Me means methyl group) Thus, the correct set of reagents is $NaNH_2/CH_3I; HgSO_4/$ dil H_2SO_4.



68. (b)



$H-C-H$ bond angle is about $120°$, when C is bonded through a double bond but in case of CH_4, the $H-C-H$ bond angle is only $109°28'$. Cyclic compounds have a $H-C-H$ bond angle greater than $109°28'$ due to hindrance. Thus, the correct order of $H-C-H$ bond angle

69. (a)

When a positron is emitted, a proton is converted into neutron and neutrino. Thus, the number of protons and hence the atomic number decreases by one unit.

$$\underset{\text{Proton}}{{}_1^1H} \longrightarrow \underset{\text{Neutron}}{{}_0^1n} + \underset{\text{Positron}}{{}_{+1}e^0} + \underset{\text{Neutrino}}{\nu}$$

70.(c)

During the β-emission, a neutron gets converted into a proton along with the formation of antineutrino and electron.

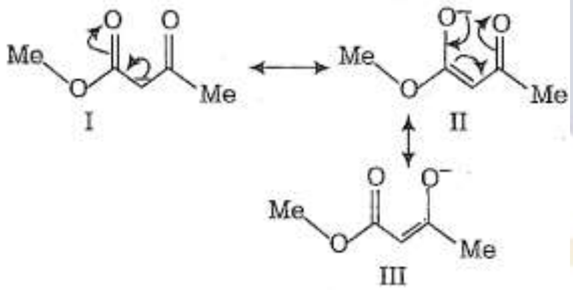
$${}_0n^1 \rightarrow \underset{\text{Proton}}{{}_1H^1} + \underset{\beta\text{ particle}}{{}_{-1}e^0} + \underset{\text{Anti neutrino}}{\bar{\nu}} + \gamma\text{ - ray}$$

71. (a)

Higher the value of a, more will be the tendency to get liquefy. Since, value of a is highest for P_1 thus, P is the most liquefiable gas among the given.

72. (d)

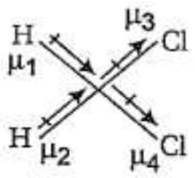
Resonating structures involve movement of electrons, but they do not involve movement of atom or group of atoms.



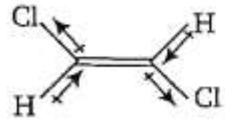
Thus, structure IV is not a resonating structure of others.

73.(a)

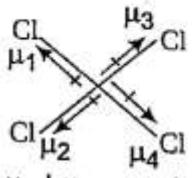
Symmetrical molecules and symmetrical trans-alkene have a net dipole moment zero. CH_2Cl_2 is not a symmetrical molecule, thus it will have a permanent dipole.



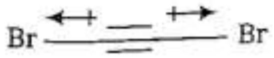
$\mu_1+\mu_2-\mu_3-\mu_4=0$
μ_3 and $\mu_4 > \mu_1$ and μ_2
Thus, $\mu_{net} \neq 0$



Symmetrical
trans-alkene
$\mu_{net} = 0$

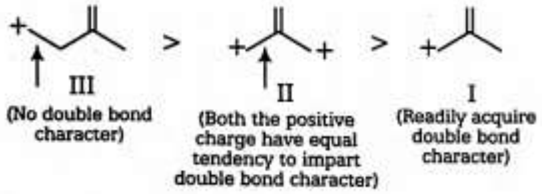


$\mu_1+\mu_2 = \mu_3+\mu_4$
$\therefore \mu_{net}=0$



$\mu_{net}=0$

74. (c)
The length of carbon-carbon single bond is always greater than that of the carbon-carbon double bond. In III, positive charge is not in conjugation of double bond, so the bond does not acquire a double bond character. However, chances of acquiring double bond character (of the indicated bond) is much more in I than that in case of II.
Thus, the correct order of decreasing bond length is



75. (b)
Emission of a positron results in the conversion of a proton into neutron, along with the release of neutrino. Thus, the number of protons decreases and of neutron increases, i.e., n/p ratio increases and the nuclei becomes stable.

$$\underset{\text{Proton}}{{}_1^1\mathrm{H}} \longrightarrow \underset{\text{Neutron}}{{}^1 n} + \underset{\text{Positron}}{{}^{+1} e^0} + v$$

76. (b)
${}_{32}\mathrm{Ge}^{76}$ and ${}_{34}\mathrm{Se}^{76}$ have the same mass number but different atomic number, so they are isobars.
Number of neutrons,
in
${}_{14}\mathrm{Si}^{30} = 30 - 14 = 16$
in
${}_{16}\mathrm{S}^{32} = 32 - 16 = 16$
Because of the presence of same number of neutrons, ${}_{14}\mathrm{Si}^{30}$ and ${}_{16}\mathrm{S}^{32}$ are called isotones.

77. (c)
Let, the unknown radioactive substance be ${}_xA^Y$.
${}_xA^Y + {}_6\mathrm{C}^{12} \longrightarrow {}_{98}\mathrm{Cf}^{246} + {}_0n^1$
On comparing atomic numbers, we get
$x + 6 = 98 + 0$
$x = 98 - 6 = 92$
On comparing mass numbers, we get
$y + 12 = 246 + 1$
$y = 247 - 12$
$= 235$
Since, uranium has the atomic number 92 , thus, the given unknown substance was ${}_{92}\mathrm{U}^{235}$.

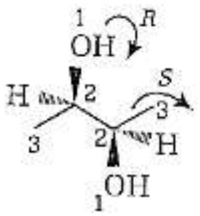
78. (c)
Given, rate $= k[\mathrm{H}^+]^n$
Initial pH $= 3$ so $[\mathrm{H}^+] = 1 \times 10^{-3}$, initial rate $= r_1$
Final pH $= 1$ so $[\mathrm{H}^+] = 1 \times 10^{-1}$,

Final rate $= r_2 = 100r_1$
On substituting values, we get
$r_1 = k[1\times10^{-3}]^n$
$r_2 = 100r_1 = k[10^{-1}]^n$
Dividing eqs (i) by (ii),
$$\frac{r_1}{100r_1} = \left[\frac{1\times10^{-3}}{1\times10^{-1}}\right]^n$$
$$\frac{1}{100} = \left[\frac{10^{-2}}{1}\right]^n \Rightarrow \left[\frac{1}{100}\right]^1 = \left[\frac{1}{100}\right]^n$$
$\therefore n = 1$
Thus, the reaction is of first order.

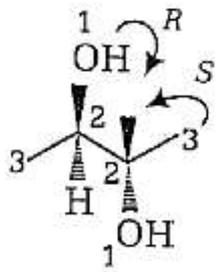
79. (c)

Given,
$\Delta H = -400$ kJ mol^{-1}
$\Delta S = -20$ kJ mol^{-1} K^{-1}
Gibbs Helmholtz equation is
$\Delta G = \Delta H - T\Delta S$
For a reaction to be spontaneous, ΔG must be less than 0 , i.e., negative.
$0 \geq \Delta H - T\Delta S$
$\therefore$ $T\Delta S \leq \Delta H$
$$T \leq \frac{\Delta H}{\Delta S} \leq \frac{-400}{-20} \leq 20 \text{ K}$$

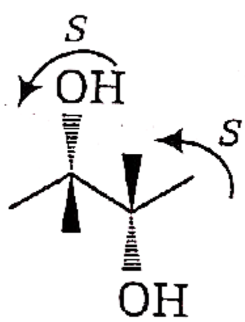
80. (d)



$2-R, 3-R$ configuration, so it is a chiral compound.



2-R, 3-S configuration, i.e., it has plane of symmetry or in other words, rotation of half part is cancelled by the other half, so it is not a chiral compound.



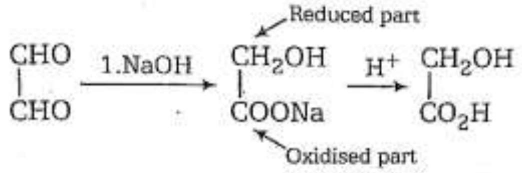
$2-S, 3-S$ configuration, so it is also a chiral compound.

81. (c)

In reaction M, there is only an intermediate step, so it is a fast reaction. Further, in reaction M energy of product is much lesser than that of the reactant, as compared to as shown in reaction N. Thus, reaction M is faster and more exothermic than reaction N.

82. (c)

Aldehydes lacking $\alpha-\mathrm{H}$ atom, undergo Cannizzaro reaction if treated with concentrated alkali (NaOH), i.e., one mole of the aldehyde is reduced to alcohol and other mole is oxidised to salt of acid. Thus, the given aldehyde (glyoxal) undergoes intramolecular Cannizzaro reaction, i.e., its half part is oxidised and half is reduced.



83. (b)

If Cl_2 is passed through hot aqueous NaOH , it results in the formation of sodium chloride and sodium chlorate as

$6\mathrm{NaOH}+3\mathrm{Cl}_2 \longrightarrow 5\mathrm{NaCl}+\mathrm{NaClO}_3+3\mathrm{H}_2\mathrm{O}$

Hot

In NaCl , let oxidation state of Cl be x.

$+1+x=0$ or $x=-1$

In NaClO_3, let oxidation state of Cl be y.

$$+1+y+(-2)3=0$$

$$\therefore \quad y-5=0$$

$$\therefore \quad y=+5$$

Thus, the obtained compounds have Cl in -1 and +5 oxidation states.

84. (a)

' 10 $\mathrm{VH}_2\mathrm{O}_2$ ' means 1 L of this solution wim produce 10 LO_2 at STP.

$$\underset{68\text{ g}}{2\mathrm{H}_2\mathrm{O}_2} \longrightarrow 2\mathrm{H}_2\mathrm{O}+\underset{22.4\text{ L atSTP}}{\mathrm{O}_2}$$

$\because$ 22.4 L of O_2 is obtained from $\mathrm{H}_2\mathrm{O}_2=68$ g

$\therefore$ 10 L of O_2 will be obtained from

$$\mathrm{H}_2\mathrm{O}_2=\frac{68}{22.4}\times 10=30.36\text{ g}$$

$\therefore$ 1000 mL of the given solution contains 30.36 g $\mathrm{H}_2\mathrm{O}_2$ and 100 mL of the given solution contains

$$\frac{30.36\times 100}{1000}=3.03\text{ g}\mathrm{H}_2\mathrm{O}_2$$

Thus, % strength of $\mathrm{H}_2\mathrm{O}_2$ is $3.03(\approx 3)$.

85. (b)

Entropy of vaporisation (ΔS)

$$=\frac{\text{enthalpy of vaporisation }(\Delta H)}{\text{boiling point (in K)}}$$

Given, $\Delta H=24.64$ kJ mol^{-1} and boiling point $=35+273=308$ K

$$\therefore \Delta S=\frac{24.64\times 10^3\text{ J mol}^{-1}}{308\text{ K}}$$

$$=80\mathrm{JK}^{-1}\text{ mol}^{-1}$$

86. (a)

Given,

$\mathrm{C}+\mathrm{O}_2 \longrightarrow \mathrm{CO}_2; \Delta H^{\circ}=-x$ kJ ... . (i)

$2\mathrm{CO}+\mathrm{O}_2 \longrightarrow 2\mathrm{CO}_2; \Delta H^{\circ}=-y$ kJ ... . (ii)

Required equation is

$$\mathrm{C}+\frac{1}{2}\mathrm{O}_2 \longrightarrow \mathrm{CO}, \Delta H_1^{\circ}=?$$

On reversing eq. (ii), we get

$2\mathrm{CO}_2 \longrightarrow 2\mathrm{CO}+\mathrm{O}_2; \Delta\mathrm{H}_2^{\circ}=+y$ kJ.....(iii)

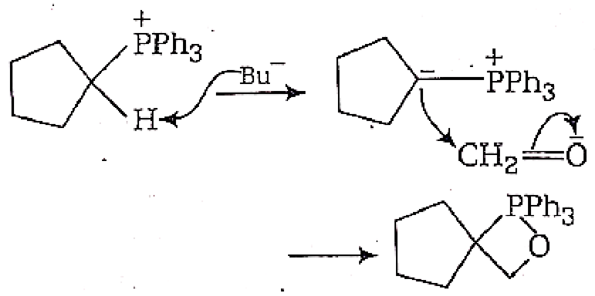
Dividing eq. (iii) by 2 gives

$\mathrm{CO}_2 \longrightarrow \mathrm{CO}+\frac{1}{2}\mathrm{O}_2; \Delta H_3^{\circ}=+\frac{Y}{2}$ kJ.....(iv)

On adding eq. (i) and (iv), we get (required equation)

$$C+\frac{1}{2}O_2 \longrightarrow CO; \Delta H_1^\circ = \Delta H^\circ + \Delta H_3^\circ$$
$$= \left(-x+\frac{y}{2}\right) \text{kJ}$$
$$= \frac{y-2x}{2} \text{ kJ}$$

87. (a)



88. (a)

Only carboxylic acids and their derivatives like anhydride are capable to give effervescence with aqueous $NaHCO_3$ solution, because they are more acidic as compared to H_2CO_3 (carbonic acid).

$$\text{II} CH_3COOH + NaHCO_3 \longrightarrow CH_3COONa + H_2O + CO_2$$
$$\text{I} (CH_3CO)_2O + 2NaHCO_3 \longrightarrow 2CH_3COONa + H_2O + CO_2$$

89. (c)

The molecular formula of ethane is C_2H_6. To get its fourth homologue, we have to add $4 \times CH_2 = C_4H_8$ to it.

$C_2H_6 + C_4H_8 = C_6H_{14}$

C_6H_{14} is the molecular formula of hexane.

Thus, hexane is the fourth homologue of ethane.

90. (b)

The given statement is true as electrons are shifted more towards the more electronegative atom.

91. (b)

Hydrides of N, O and F because of the small size and high electronegativity of elements, have ability to form extensive intermolecular (i.e., between two molecules) hydrogen bonding. Thus, a large amount of energy is required to break these bonds, i.e., the melting and boiling points of hydrides of these elements are abnormally high.

92. (a)

From the Faraday's IInd law, we know that

$$W \propto Zit, W \propto \frac{E}{96500} it$$

or it $\propto \frac{1}{E}$ or electricity $\propto \frac{1}{E}$

Thus, the amount of electricity required for $ZnSO_4, AlCl_3$ and $AgNO_3$ is in the ratio of $2:3:1$. (Here, 2, 3 and 1 are the valencies of metal atom in the given compounds).

Alternatively

In $ZnSO_4$,

$$\underset{1 \text{ mol}}{Zn^{2+}} + \underset{\text{or 2 Faraday}}{2e^-} \longrightarrow Zn$$

In $AlCl_3$,

$$\underset{1 \text{ mol}}{Al^{3+}} + \underset{3 \text{ F}}{3e^-} \longrightarrow Al$$

In $AgNO_3$,

$$\underset{1 \text{ mol}}{Ag^{+}} + \underset{1 \text{ F}}{e^-} \longrightarrow Ag$$

Thus, the ratio of amounts of electricity required is $2:3:1$

93. (d)

Since, AgI is a negatively charged sol, so according to Hardy Schulze law, higher the positive charge on the metal atoms, more is the tendency of metal atom to cause coagulation of negatively charged colloid.

The order of positive charge is

$Na^+ < Ba^{2+} < Al^{3+}$

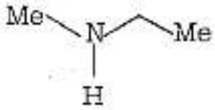
Thus, the order of amount of electrolytes required is in the order
$NaNO_3 > Ba(NO_3)_2 > Al_2(NO_3)_3$
Note In paper, the formula of aluminium nitrate is incorrect. Correct formula of aluminium nitrate is $Al_2(NO_3)_3$.

94. (b)

$$\Delta H = -nC_p\Delta T = -2 \times \frac{5}{2} R(225 - 125)$$
$$= -5R(100) = -500R$$

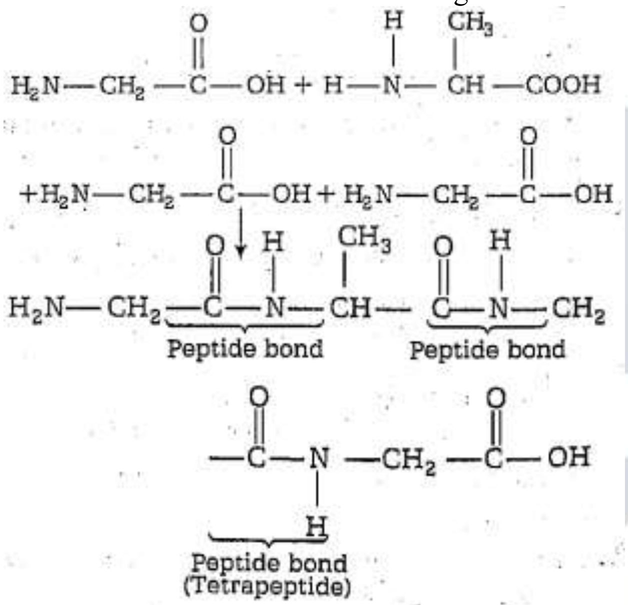
95.(b)

Since, the amine gives white precipitate with benzene sulphonyl chloride which is insoluble in $aq.$ NaOH, so it must be a secondary amine (i.e., amine having $RNHR$ group).

Among the given compound, only  is a secondary amine, so it represents the structure of given amine.

96. (d)

Combination of four amino acid units gives a linear tetrapeptide, in which three peptide bonds are present.



97. (d)

Among the given gases, N_2O, CO_2 and CH_4 have the ability to trap thermal radiation but O_2 does not. That's why it is not a green house gas.
∴ Note O_2 is the gas, presence of which is essential for life and in absence of which life is not possible.

98. (d)

10^{-4}MKOH $= 10^{-4}$M$[OH^-]$ We know that
$$[H^+][OH^-] = 1 \times 10^{-14}$$
$$[H^+] \times 10^{-4} = 1 \times 10^{-14}$$
$$[H^+] = \frac{1 \times 10^{-14}}{1 \times 10^{-4}} = 1 \times 10^{-10}\text{M}$$
$$\therefore \text{ pH} = -\log[H^+] = -\log(1 \times 10^{-10})$$
$$= 10$$

99. (c)

$$\text{Number of atoms } = \frac{\text{mass}}{\text{molar mass}} \times N_A$$
$$\times \text{ Number of atoms in 1 mole}$$
∴ Number of atoms in 4.25 gNH_3
$$= \frac{4.25}{17} \times N_A \times 4$$
$$= N_A$$
Number of atoms in 8 g

$O_2 = \frac{8}{32} \times N_A \times 2 = \frac{N_A}{2}$

Number of atoms in 2 g

$H_2 = \frac{2}{2} \times N_A \times 2 = 2N_A$

Number of atoms in 4 g

$He = \frac{4}{4} \times N_A \times 1 = N_A$

Thus, 2 gH_2 contains the maximum number of atoms among the given.

100. (a)

SO_3^{2-} and SO_4^{2-} when treated with $BaCl_2$, give white ppt of $BaSO_3$ and $BaSO_4$ respectively.

$BaCl_2 + SO_3^{2-} \longrightarrow \underset{\text{Barium Sulphide}}{BaSO_3} + 2Cl^-$

$BaCl_2 + SO_4^{2-} \longrightarrow \underset{\text{Barium sulphate}}{BaSO_4} + 2Cl^-$

Out of these two, SO_3^{2-} is soluble in HCl but SO_4^{2-} does not.

101. (b)

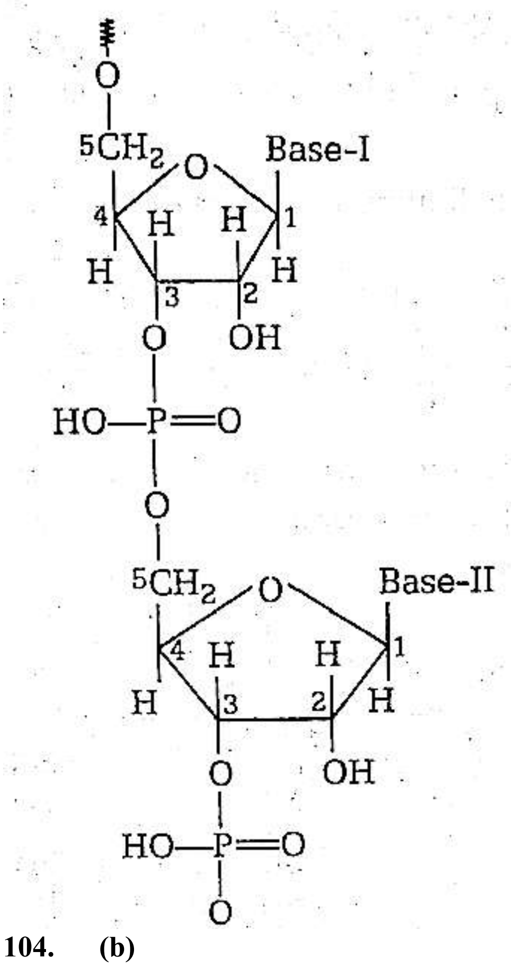
Eating animal poisoned with mercury (Hg^{2+}) causes deformity, known as minamata (minimata) disease which is characterised by diarrhoea, hemolysis, impairment of various senses, meningitis and death.

102. (a)

H_3PO_2 is the reagent that converts $-N_2^+Cl^-$. group into H . During this process N_2 and HCl gases are released.

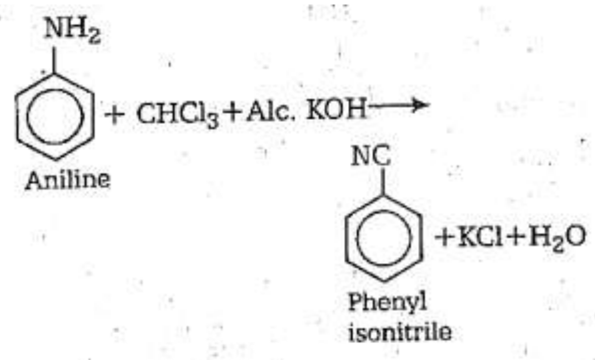
103. (a)

In DNA, the consecutive deoxynucleotides are linked through phosphodiester linkage.



104. (b)

Aniline is a primary amine (contains $-NH_2$ group). When it is treated with chloroform under alkaline conditions, it results in the formation of a bad smelling compound, known by the name isonitrile or carbylamine. This reaction is known as carbylamine reaction.



105. (b)

Species with more negative E° (standard reduction potential) generally acts as reducing agent while with less negative E° acts as oxidising agent. Thus, the overall reaction is

$Ag^+ + Fe^{2+} \longrightarrow Fe^{3+} + Ag$

$$\begin{aligned}\Delta E^\circ &= E^\circ{}_{OA} - E^\circ{}_{RA}\\ &= -0.3995 - (-0.7120)\text{V}\\ &= -0.3995 + 0.7120 \text{ V}\\ &= +0.3125 \text{ V}\end{aligned}$$

106. (c)

van der Waals' equation is

$\left(p + \frac{a}{V^2}\right)(V - b) = RT$

(for 1 mole)

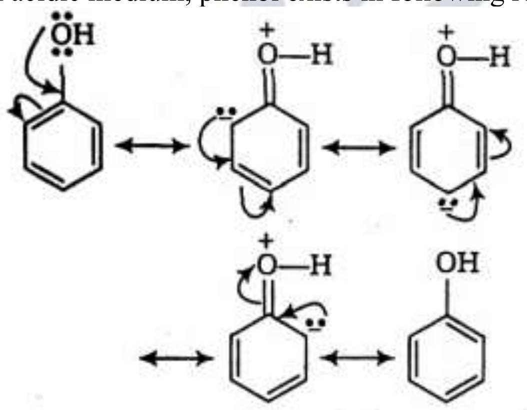
If a is negligible, $p + \frac{a}{V^2} \approx p$

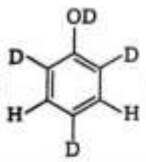
$p(V - b) = RT \Rightarrow pV - pb = RT$

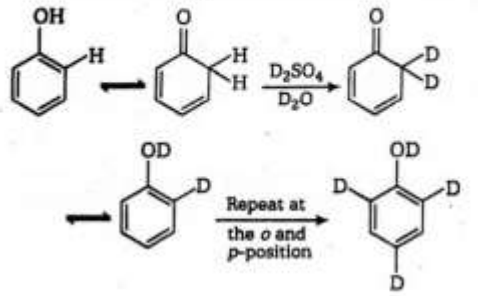
or $\frac{pV}{RT} - \frac{pb}{RT} = 1 \Rightarrow \frac{pV}{RT} = 1 + \frac{pb}{RT}$

107. (a)

In acidic medium, phenol exists in following resonating structures.

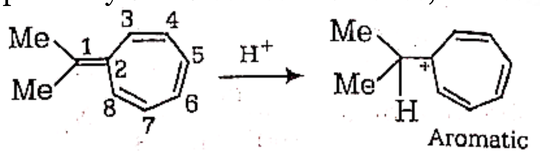


Since, o/p positions are electron rich sites, so electrophile will attack on these site, i.e., hydrogen of these sites get exchanged by D (deuterium). Hence, the final product of the reaction is  which is formed as



108. (a)

Addition of a proton at C^1 results in the formation of tropylium carbocation (an aromatic species with more stability due to delocalisation of π electrons), thus, it is the most reactive site towards protonation.
Addition of proton at any other carbon atom, interupt in the delocalisation of π electrons by disturbing planarity of molecules and hence, makes it less stable. Thus, these all are less reactive towards protonation.



109. (b)

Abstraction of (H_a) hydrogen atom results in the formation of 3° allylic carbocation whereas abstraction of H_c gives 2° allylic carbocation (which is less stable as compared to former one). However, abstraction of H_b gives only a 2° carbocation which is less stable as compared to former two. More the stability of generated carbocation, higher is the tendency of abstraction of hydrogen.
Thus, the order of decreasing ease of abstraction of hydrogen atoms from the given molecule is
$H_a > H_c > H_b$

110. (d)

Degree of dissociation, $\alpha = \frac{\lambda_c^\circ}{\lambda_m^\circ} = \frac{150}{500} = 0.3$

Given,
$C = 0.007\text{M}$
Hydrofluoric acid dissociates in the following manner

$HF \longrightarrow$	H^+	$+ F^-$	
C	0	0	Initially
$C - C\alpha$	$C\alpha$	$C\alpha$	At time t

$= C(1-\alpha)$
Dissociation constant,

$$K_a = \frac{[H^+][F^-]}{[HF]} = \frac{C\alpha \cdot C\alpha}{C(1-\alpha)} = \frac{C\alpha^2}{(1-\alpha)}$$

On substituting values, we get

$$K_a = \frac{0.007 \times (0.3)^2}{(1-0.3)} = \frac{63 \times 10^{-3} \times 10^{-2}}{0.7}$$
$$= 9 \times 10^{-4}\text{M}$$

111. (c)

Elevation of boiling point,

$$\Delta T_b = \frac{W \times K_b \times 1000}{M \times W}$$

(Here, W and W = weights of solute and solvent respectively,
M = molecular weight of solute and
K_b = constant)

On substituting values, we get
or

$$0.05=\frac{w \times 0.5 \times 1000}{100 \times 100}$$

$$w=\frac{0.05 \times 100 \times 100}{0.5 \times 1000}=1 \mathrm{~g}$$

112. (b)

Molarity

$$=\frac{\text{mass } \times 1000}{\text{molecular weight } \times \text{ volume of solution}}$$

Molecular weight of ethyl alcohol,

$$\mathrm{C_2H_5OH}=12 \times 2+5+16+1$$

$$=24+22=46 \mathrm{~g} \mathrm{~mol}^{-1}$$

On substituting the values, we get

$$0.5 \mathrm{M}=\frac{\text{mass } \times 1000}{46 \times 100}$$

$$\text{Mass }=\frac{0.5 \times 46}{10}=2.3 \mathrm{~g}$$

∴ Volume of ethyl alcohol required,

$$V=\frac{\text{mass of ethyl alcohol}}{\text{density of ethyl alcohol}}=\frac{2.3 \mathrm{~g}}{1.15 \mathrm{~g/cc}}=2 \mathrm{cc}$$

113. (c)

In NF_3, N is less electronegative as compared to F but in NH_3, it is more electronegative than H . And in case of same central atom, as the electronegativity of other atoms increases, bond angle decreases. Thus, bond angle in NF_3 is smaller than that in NH_3 because of the difference in the polarity of N in these molecules.

114. (b)

Age of archaeological sample,

$$t=\frac{2.303}{k} \log \left(\frac{N_0}{N}\right)$$

(Here, N_0 = initial activity,
N = remaining activity)
and

$$k=\frac{0.693}{t_{1/2}} \left(\text{ where } t_{1/2}=\text{ half-life }\right)$$

$$=\frac{0.693}{5770} \mathrm{yr}^{-1}$$

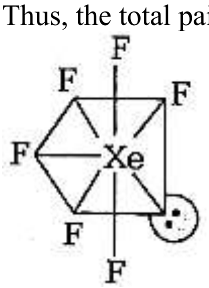
$$\therefore \quad t=\frac{2.303 \times 5770}{0.693} \log \left(\frac{15}{5}\right)$$

$$=19175.0 \log 3 \mathrm{yr}$$

$$=9148 \mathrm{yr} \approx 9200 \mathrm{yr}$$

115. (c)

In XeF_6, Xe, the central atom contains, 8 valence electrons. Out of which 6 are utilised with fluorine in bonding (i.e., it contains six bond pairs of electrons) while one pair
remains as lone pair.
Thus, the total pairs = 6 + 1 = 7



Hence, its shape is pentagonal bipyramid according to VSEPR theory.

116. (b,d)

Given, molecular weight of $X = M_X$
Mol. wt. of $Y = M_Y$ and $M_Y > M_X$
Root mean square velocities $= C_X$ and C_Y respectively
Average KE/ molecule $= E_X$ and E_Y respectively
We know that,
Root mean square velocity,

$$C = \sqrt{\frac{3RT}{M}} \text{ or } C \propto \sqrt{\frac{1}{M}}$$

$$\therefore \quad \frac{C_X}{C_Y} = \frac{M_Y}{M_X}$$

Since $M_Y > M_X$

$$\therefore \quad C_X > C_Y$$

$$\text{KE/molecules } = \frac{3}{2}nRT$$

$$\Rightarrow \quad E_X \neq E_Y \neq \frac{3}{2}RT$$

$$\text{Further, } E_X = E_Y = \frac{3}{2}k_B T$$

117. (b,c)

For a process to be spontaneous,

$$\Delta S_{\text{system}} + \Delta S_{\text{surrounding}} > 0$$

$$(\Delta G_{\text{system}})_{T,p} < 0$$

$$(\Delta U_{\text{system}})_{T,V} < 0$$

118. (a,b,d)

Since, H_2SO_4 and H_3PO_4 both are strong acids so their presence makes the medium more acidic. The Fe^{3+} ions form complex, $[FeHPO_4]^+$with these acid and hence, the concentration of Fe^{3+} ion reduces which results in lowering in the formal reduction potential.

119. (a,b,d)

Electronic configurations of cuprous (Cu^+) and cupric (Cu^{2+}) ions are as follows

$Cu^+ = [Ar]3d^{10}4s^0$

$Cu^{2+} = [Ar]3d^9 4s^0$

Thus, electronic configuration of $Cu^+$is more stable but it is less stable because in Cu^{2+} due to its small size the nuclear charge is sufficient to hold 27 electrons but in Cu^+such a condition is not true.
Further, the IInd IP of Cu is not very high as compared to its Ist IP. Consequently a large amount of lattice energy is released for cupric compounds as compared to Cu^+compounds.

120. (b,c,d)

Both the given molecules contain 6 carbon atoms with an aldehyde group, so these are called aldohexoses. In X, the -OH attached to second last carbon in on right hand side while in Y it is on left hand side, so X is D-sugar and Y is L-sugar. X and Y are the mirror images of each other, i.e., these are enantiomers.

Mathematics

121. (b)

Given equation is

$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

On squaring both sides, we get

$$(x+1) + (x-1) - 2\sqrt{x^2-1} = 4x - 1$$

$$\Rightarrow \quad 2x - 2\sqrt{x^2-1} = 4x - 1$$

$$\Rightarrow \quad -2\sqrt{x^2-1} = 2x - 1$$

Again, squaring both sides, we get

$$4(x^2-1)=4x^2+1-4x$$
$$\Rightarrow \quad -4=+1-4x$$
$$\Rightarrow \quad 4x=5$$
$$\Rightarrow \quad x=\frac{5}{4}$$

But, when we put $x=\frac{5}{4}$ in Eq. (i), we get

$$\sqrt{\frac{5}{4}+1}-\sqrt{\frac{5}{4}-1}=\sqrt{4\times\frac{5}{4}-1}$$

$$\Rightarrow \sqrt{\frac{9}{4}}-\sqrt{\frac{1}{4}}=\sqrt{5-1} \Rightarrow \frac{3}{2}-\frac{1}{2}=2$$

$\Rightarrow 1=2$, which is not true.

Hence, no value of x satisfy the given equation.

122. (c)

Let $z=x+iy$

$$\therefore |z|^2+|z-3|^2+|z-i|^2$$

$$=|x+iy|^2+|(x-3)+iy|^2$$
$$+|x+i(y-1)|^2$$
$$=x^2+y^2+(x-3)^2+y^2+x^2+(y-1)^2$$
$$=x^2+y^2+x^2-6x+9+y^2+x^2+y^2+1-2y$$
$$=3x^2+3y^2-6x-2y+10$$
$$=3(x^2-2x+1)+3\left(y^2-\frac{2}{3}y+\frac{1}{9}\right)$$
$$=3(x-1)^2+3\left(y-\frac{1}{3}\right)^2+\frac{20}{3}$$

It is minimum, when $x-1=0$ and $y-\frac{1}{3}=0$

$$\therefore \quad x=1 \text{ and } y=\frac{1}{3}$$

$$\therefore \quad z=1+\frac{1}{3}i$$

123. (a)

Given, $f(x)=\begin{cases}2x^2+1, & x\le 1\\ 4x^3-1, & x>1\end{cases}$

$$\therefore \int_0^2 f(x)dx=\int_0^1 f(x)dx+\int_1^2 f(x)dx$$
$$=\int_0^1 (2x^2+1)dx+\int_1^2 (4x^3-1)dx$$
$$=\left[\frac{2x^3}{3}+x\right]_0^1+\left[\frac{4x^4}{4}-x\right]_1^2$$
$$=\frac{2}{3}(1)^3+1-(0+0)$$
$$+[(2)^4-2-\{(1)^4-1\}]$$
$$=\frac{2}{3}+1+[16-2-0]$$
$$=\frac{2}{3}+15=\frac{2+45}{3}$$
$$=\frac{47}{3} \text{ sq units}$$

124. (b)

$$\lim_{x\to 0}\frac{2a\sin x-\sin 2x}{\tan^3 x}$$
$$=\lim_{x\to 0}\frac{2a\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)-\left(2x-\frac{(2x)^3}{3!}+\frac{(2x)^5}{5!}-\cdots\right)}{\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+\cdots\right)^3}$$
$$=\lim_{x\to 0}\frac{(2a-2)x+\left(-\frac{2a}{3!}+\frac{2^3}{3!}\right)x^3+\left(\frac{2a}{5!}-\frac{2^5}{5!}\right)x^5+\cdots}{x^3\left(1+\frac{x^2}{3}+\frac{2}{15}x^4+\cdots\right)^3}$$

Since, it is given that given limit is exist, then
$2a-2=0\Rightarrow \alpha=1$

125. (c)

Given, $\log_{101}\log_7(\sqrt{x+7}+\sqrt{x})=0$

$\therefore\quad \log_7(\sqrt{x+7}+\sqrt{x})=(101)^0$

$\Rightarrow\quad \log_7(\sqrt{x+7}+\sqrt{x})=1$

$\Rightarrow\quad (\sqrt{x+7}+\sqrt{x})=7^1$

$\Rightarrow\quad \sqrt{x+7}+\sqrt{x}=7$

On squaring both sides, we get

$$(x+7)+x+2\sqrt{x^2+7x}=49$$

$\Rightarrow\quad 2x-42=-2\sqrt{x^2+7x}$

$\Rightarrow\quad x-21=-\sqrt{x^2+7x}$

Again, squaring both sides, we get

$$x^2+441-42x=x^2+7x$$

$\Rightarrow\quad 49x=441$

$\Rightarrow\quad x=\frac{441}{49}\Rightarrow x=9$

126. (c)

Given equation is

$$(1+x^2)\frac{dy}{dx}+y=\theta^{\tan^{-1}x}$$

or it can be rewritten as

$$\frac{dy}{dx}+\frac{1}{(1+x^2)}y=\frac{e^{\tan^{-1}x}}{1+x^2}$$

It is a linear differential equation of the form of

$$\frac{dy}{dx}+Py=Q$$

where, $P=\frac{1}{1+x^2}$ and $Q=\frac{e^{\tan^{-1}x}}{1+x^2}$

$\therefore$ Integrating factor, IF $=e^{\int Pdx}$

$=e^{\int\frac{1}{1+x^2}dx}=e^{\tan^{-1}x}$

127. (d)

Given, $\sqrt{Y}=\cos^{-1}x\Rightarrow Y=(\cos^{-1}x)^2$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=2(\cos^{-1}x)\times\frac{-1}{\sqrt{1-x^2}}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2}=-2\frac{\sqrt{1-x^2}\times\frac{-1}{\sqrt{1-x^2}}-\cos^{-1}x\times\left(\frac{1}{2}\right)\frac{(-2x)}{(1-x^2)^{1/2}}}{\left(\sqrt{1-x^2}\right)^2}$$

$$=-2\left[\frac{-1+\frac{x\cos^{-1}x}{(1-x^2)^{1/2}}}{(1-x^2)}\right]$$

$$\frac{d^2y}{dx^2}=\left[\frac{2-\frac{2x\cos^{-1}x}{(1-x^2)^{1/2}}}{(1-x^2)}\right]$$

$$\Rightarrow\quad(1-x^2)\frac{d^2y}{dx^2}=2+x\frac{dy}{dx}$$

$$\Rightarrow\quad(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$$

But, it is given

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}\qquad=c$$

$$\therefore c\qquad\qquad\qquad\qquad=2$$

128. (c)

Let $y=20^{301}$

Number of digits = Integral part of $(301\log_{10}20)+1$

$=$ Integral part of $[301(\log_{10}10+\log_{10}2)]+1$

$=$ Integral part of $[301(1+0.3010)]+1$

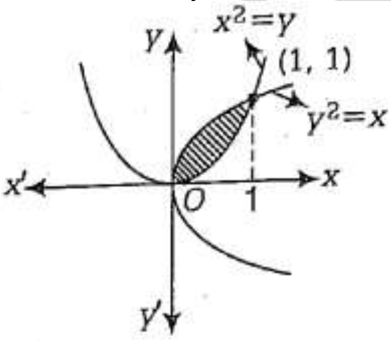
$=$ Integral part of $[301\times1.3010]+1$

$=$ Integral part of $[391.601]+1$

$=391+1=392$

129. (a)

Given curves are $y=x^2$ and $x=y^2$, which is the form of parabola.



The point of intersection, $x=(x^2)^2$

$\Rightarrow\quad x=x^4\Rightarrow x(1-x^3)=0$

$\Rightarrow\quad x=0$ and $1=x^3$

$\Rightarrow\quad x=0$ and $x=1$

When $x=0$, then $y=0$

When $x=1$, then $y=1^2=1$

$\therefore$ The point of intersection is (0,0) and (1,1).

$\therefore$ Area of shaded region

$$=\int_0^1(y_2-y_1)dx$$

$$=\int_0^1\left[\sqrt{x}-x^2\right]dx=\left[\frac{x^{3/2}}{3/2}-\frac{x^3}{3}\right]_0^1$$

$$=\frac{2}{3}(1)^{3/2}-\frac{(1)^3}{3}-0-0$$

$$=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}\text{ sq units}$$

130. (b)

Given, $f(x) = 3x^2 + 1$

Let

$y = 3x^2 + 1$

$$\Rightarrow \quad 3x^2 = y - 1 \Rightarrow x^2 = \frac{y-1}{3}$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{y-1}{3}}$$

$$\therefore \quad f^{-1}(x) = \pm\sqrt{\frac{x-1}{3}}$$

When $x \in [1,6]$

Then,

$$f^{-1}(x) \in \left[-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right]$$

131. (a)

$$\tan\frac{\pi}{5} + 2\tan\frac{2\pi}{5} + 4\cot\frac{4\pi}{5}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + 2\frac{\sin\frac{2\pi}{5}}{\cos\frac{2\pi}{5}} + 4\frac{\cos\frac{4\pi}{5}}{\sin\frac{4\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + 2\frac{\sin\frac{2\pi}{5}}{\cos\frac{2\pi}{5}} + \frac{4\left(\cos^2\frac{2\pi}{5} - \sin^2\frac{2\pi}{5}\right)}{2\sin\frac{2\pi}{5}\cos\frac{2\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + 2\frac{\sin\frac{2\pi}{5}}{\cos\frac{2\pi}{5}} + \frac{2\left(\cos^2\frac{2\pi}{5} - \sin^2\frac{2\pi}{5}\right)}{\sin\frac{2\pi}{5}\cos^2\frac{2\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + 2\left[\frac{\sin^2\frac{2\pi}{5} + \cos^2\frac{2\pi}{5} - \sin^2\frac{2\pi}{5}}{\cos\frac{2\pi}{5}\sin\frac{2\pi}{5}}\right]$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + \frac{2\cos\frac{2\pi}{5}}{\sin\frac{2\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + \frac{2\left(\cos\frac{\pi}{5} - \sin^2\frac{\pi}{5}\right)}{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}\cos^2\frac{\pi}{5} - \frac{\sin^2\pi}{5}}{\sin\frac{\pi}{5}\cos\frac{\pi}{5}}$$

$$= \frac{\cos^2\frac{\pi}{5}}{\sin\frac{\pi}{5}\cos\frac{\pi}{5}}$$

$$= \cot\frac{\pi}{5}$$

132. (c)

Given,

$f'(x) \le 5$

$$\therefore \frac{f(7) - f(2)}{x - 2} \le 5$$

[by Lagrange mean value theorem]

$\Rightarrow \quad \dfrac{f(7)-3}{7-2} \leq 5$

$\Rightarrow \quad f(7) \leq 5(7-2)+3$

$\therefore \quad f(7) \leq 28$

133. (b)

Given, number of elements of A and B are p and q, respectively.

$\therefore$ The number of relations from the set A to the set B is 2^{pq}.

134. (a)

Given equation can be rewritten as

$$x^2 - \frac{r^2}{pq}x + 1 = 0$$

$$\therefore \ \tan A + \tan B = \frac{r^2}{pq}$$

and $\tan A \tan B = 1$

We know, $A + B + C = 180°$

$\Rightarrow \ (A+B) = 180° - C$

$\Rightarrow \tan(A+B) = \tan(180° - C)$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$$\Rightarrow \frac{r^2/pq}{1-1} = -\tan C$$

$\Rightarrow \infty = -\tan C \Rightarrow C = 90°$

Hence, $\triangle ABC$ is a right angled triangle.

135. (b)

Given equation of parabola is

$y^2 = 12x$

On differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 12 \Rightarrow \frac{dy}{dx} = \frac{6}{y}$$

Since, the normal to the curve is parallel to the line $y = 4x + 3$.

$\therefore$ Slope of normal curve = Slope of line

$\Rightarrow \quad -\dfrac{y}{6} = 4$

$\Rightarrow \quad y = -24$

From Eq. (i), we get

$$(-24)^2 = 12x$$

$\Rightarrow \quad 24 \times 24 = 12x$

$\Rightarrow \quad x = 48$

$\therefore$ Normal point on a curve is $(48, -24)$.

$\therefore$ Distance from $(48, -24)$ to the line $4x - y + 3 = 0$ is

$$\frac{4 \times 48 + 24 + 3}{\sqrt{4^2 + 1^2}} = \frac{219}{\sqrt{17}}$$

136. (c)

Given equation of ellipse is

$$\frac{x^2}{144} + \frac{y^2}{25} = 1$$

Here, $a^2 = 144$ and $b^2 = 25$

Now,

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{25}{144}}$$
$$= \sqrt{\frac{119}{144}} = \frac{\sqrt{119}}{12}$$
$\therefore$ Foci of an ellipse $= (\pm ae, 0)$
$$= \left(\pm 12 \times \frac{\sqrt{119}}{12}, 0\right)$$
$$= (\pm\sqrt{119}, 0)$$
Since, the circle with centre $(0, \sqrt{2})$ and passing through foci $(\pm\sqrt{119}, 0)$ of the ellipse.

$\therefore$ Radius of circle $= \sqrt{(\sqrt{119} - 0)^2 + (0 - \sqrt{2})^2}$
$$= \sqrt{119 + 2} = \sqrt{121} = 11$$

137. (a)
Given lines are
$x + y = 0$ ……… (i)
$5x + y = 4$ …….(ii)
$x + 5y = 4$ …… (iii)
and
On solving above equations pairwise, we get
$A(1, -1), B\left(\frac{2}{3}, \frac{2}{3}\right), C(-1,1)$
Now,
$$AB = \sqrt{\left(\frac{2}{3} - 1\right)^2 + \left(\frac{2}{3} + 1\right)^2}$$
$$= \sqrt{\frac{1}{9} + \frac{25}{9}} = \sqrt{\frac{26}{9}} = \frac{\sqrt{26}}{3}$$
$$BC = \sqrt{\left(-1 - \frac{2}{3}\right)^2 + \left(1 - \frac{2}{3}\right)^2}$$
$$= \sqrt{\frac{25}{9} + \frac{1}{9}} = \sqrt{\frac{26}{9}} = \frac{\sqrt{26}}{3}$$
and
$$CA = \sqrt{(1+1)^2 + (-1-1)^2}$$
$$= \sqrt{4 + 4} = 2\sqrt{2}$$
Here, we see that $AB = BC$
Hence, given lines form an isosceles triangle.

138. (a)
Given,
$\sin^{-1}\left(\frac{x}{13}\right) + \operatorname{cosec}^{-1}\left(\frac{13}{12}\right) = \frac{\pi}{2}$……..(i)
Let $\operatorname{cosec}^{-1}\frac{13}{12} = y$
Then, $\operatorname{cosec} y = \frac{13}{12} \Rightarrow \sin y = \frac{12}{13}$

$\therefore\ \cos y = \sqrt{1-\sin^2 y}$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}}$$

$$= \sqrt{\frac{25}{169}} = \frac{5}{13} \Rightarrow y = \cos^{-1}\frac{5}{13}$$

Eq. (i) becomes,

$\sin^{-1}\left(\frac{x}{13}\right) + \cos^{-1}\left(\frac{5}{13}\right) = \frac{\pi}{2}$.........(i)

We know that

$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$

$\therefore$ Both angles of Eq. (i) should be same.

$\therefore\ x = 5$

139. (b)

Given curve is

$(7x+5)^2 + (7y+3)^2 = \lambda^2(4x+3y-24)^2$

$49x^2 + 25 + 70x + 49y^2 + 9 + 42y$

$\lambda^2(16x^2 + 9y^2 + 576 + 24xy - 144y - 192x)$

$(49-16\lambda^2)x^2 + (49-9\lambda^2)y^2 + (70+192\lambda^2)x$

$+(42+144\lambda^2)y - 24\lambda^2 xy + (25-576\lambda^2) = 0$

On comparing with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = 49 - 16\lambda^2, b = (49-9\lambda^2), h = -12\lambda^2$

Condition for parabola is $ab = h^2$.

$\therefore\quad (49-16\lambda^2)(49-9\lambda^2) = (-12\lambda^2)^2$

$\Rightarrow\quad 2401 - 441\lambda^2 - 784\lambda^2 + 144\lambda^4 = 144\lambda^4$

$\Rightarrow\quad 2401 - 1225\lambda^2 = 0$

$\Rightarrow\quad \lambda^2 = \frac{2401}{1225}$

$\Rightarrow\quad \lambda^2 = \frac{(49)^2}{(35)^2}$

$\Rightarrow\quad \lambda = \pm\frac{49}{35}$

$\Rightarrow\quad \lambda = \pm\frac{7}{5}$

140. (a)

Given,

$f(x) = x + \frac{1}{2} = \frac{2x+1}{2}$

$\therefore\quad f(2x) = 2x + \frac{1}{2}$

$\Rightarrow\quad f(2x) = \frac{4x+1}{2}$

and

$f(4x) = 4x + \frac{1}{2} \Rightarrow f(4x) = \frac{8x+1}{2}$

Since, $f(x), f(2x)$ and $f(4x)$ are in HP.

$\therefore \frac{1}{f(x)}, \frac{1}{f(2x)}$ and $\frac{1}{f(4x)}$ are in AP.

$\Rightarrow \frac{1}{f(2x)} = \frac{\frac{1}{f(x)}+\frac{1}{f(4x)}}{2}$

$\Rightarrow \frac{2}{4x+1} = \frac{\frac{2}{2x+1}+\frac{2}{8x+1}}{2}$

$\Rightarrow \frac{2}{4x+1} = \frac{10x+2}{(2x+1)(8x+1)}$

$\Rightarrow (2x+1)(8x+1) = (5x+1)(4x+1)$

$\Rightarrow 16x^2 + 10x + 1 = 20x^2 + 9x + 1$

$\Rightarrow 4x^2 - x = 0 \Rightarrow x(4x-1) = 0$

$\Rightarrow x = \frac{1}{4}$ $[\because x \neq 0]$

Hence, one real value of x for which the three unequal terms are in HP.

141. (c)

Given, $f(x) = 2x^2 + 5x + 1 \ldots\ldots$(i)

Also, $f(x) = a(x+1)(x-2) + b(x-2)(x-1)$
$+c(x-1)(x+1)$

$= a(x^2 - x - 2) + b(x^2 - 3x + 2)$
$+c(x^2-1)$

$\Rightarrow f(x) = (a+b+c)x^2 + (-a-3b)x$
$+(-2a+2b-c) \ldots \ldots$ (ii)

On equating the coefficients of x^2, x and constant term in Eqs. (i) and (ii), we get

$a+b+c = 2, -a-3b = 5$

and $-2a+2b-c = 1$

On solving above equations, we get

$a = -\frac{35}{4}, b = \frac{5}{4}$ and $c = \frac{38}{4}$

Hence, exactly one choice for each of b and c.

142. (a)

Given, α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

and $$\alpha + h + \beta + h = -\frac{q}{p}, (\alpha+h)(\beta+h) = \frac{r}{p}$$

Now, $(\alpha+h) - (\beta+h) = \alpha - \beta$

$\Rightarrow [(\alpha+h) - (\beta+h)]^2 = (\alpha-\beta)^2$

$\Rightarrow [(\alpha+h) + (\beta+h)]^2 - 4(\alpha+h)(\beta+h)$
$= (\alpha+\beta)^2 - 4\alpha\beta$

$\Rightarrow \frac{q^2}{p^2} - \frac{4r}{p} = \frac{b^2}{a^2} - \frac{4c}{a}$

$\Rightarrow \frac{q^2-4pr}{p^2} = \frac{b^2-4ac}{a^2}$

$\therefore \frac{b^2-4ac}{q^2-4pr} = \frac{a^2}{p^2}$

Hence, the ratio of the square of their discriminants is $a^2 : p^2$.

143. (d)

Since, α is a root of

$x^2 + 3p^2x + 5q^2 = 0$

$\therefore \alpha^2 + 3p^2\alpha + 5q^2 = 0.$ (i)

and β is a root of

$x^2 + 9p^2x + 15q^2 = 0$

$\therefore \beta^2 + 9p^2\beta + 15q^2 = 0 \ldots\ldots$ (ii)

Let $f(x) = x^2 + 6p^2x + 10q^2$

Then, $f(\alpha) = \alpha^2 + 6p^2\alpha + 10q^2$

$= (\alpha^2 + 3p^2\alpha + 5q^2) + 3p^2\alpha + 5q^2$
$= 0 + 3p^2\alpha + 5q^2$ [from Eq. (i)]
$\Rightarrow f(\alpha) > 0$
and $f(\beta) = \beta^2 + 6p^2\beta + 10q^2$
$= (\beta^2 + 9p^2\beta + 15q^2) - (3p^2\beta + 5q^2)$
$= 0 - (3p^2\beta + 5q^2)$ [from Eq. (ii)]
$\Rightarrow f(\beta) < 0$
Thus, $f(x)$ is a polynomial such that $f(\alpha) > 0$ and $f(\beta) < 0$.
Therefore, there exists γ satisfying $\alpha < \gamma < \beta$ such that $f(\gamma) = 0$.

144. (a)

Equation of tangent in slope form of parabola $y^2 = 8\sqrt{3}x$ is
$y = mx + c$.......(i)
where,
$c = \frac{a}{m}$
$\therefore\ c = \frac{2\sqrt{3}}{m}$......(ii)
Also, tangent to the hyperbola
$4x^2 - y^2 = 4$
or
$\frac{x^2}{1} - \frac{y^2}{4} = 1$ is
$c^2 = a^2m^2 - b^2$
$c^2 = 1m^2 - 4$
$\Rightarrow \left(\frac{2\sqrt{3}}{m}\right)^2 = m^2 - 4$ [from Eq.(ii)]
$\Rightarrow \frac{12}{m^2} = m^2 - 4$
$\Rightarrow m^4 - 4m^2 - 12 = 0$
$\Rightarrow m^4 - 6m^2 + 2m^2 - 12 = 0$
$\Rightarrow m^2(m^2 - 6) + 2(m^2 - 6) = 0$
$\Rightarrow (m^2 + 2)(m^2 - 6) = 0$
$\Rightarrow m^2 - 6 = 0$ and $m^2 + 2 \neq 0$
$\Rightarrow m^2 = 6$
$\Rightarrow m = \pm\sqrt{6}$
i.e., $m = \sqrt{6}$ as m is positive slope.
$\therefore$ From Eq. (i),
$$y = \sqrt{6} \times + \frac{2\sqrt{3}}{\sqrt{6}}$$
$\Rightarrow y = \sqrt{6} \times + \sqrt{2}$

145. (a)

Given equation of parabola is
$y^2 = 64x$
The point at which the tangent to the curve is parallel to the line is the nearest point on the curve.
On differentiating both sides of Eq. (i), we get
$$2y\frac{dy}{dx} = 64$$
$$\Rightarrow \frac{dy}{dx} = \frac{32}{y}$$
Also, slope of the given line is $-\frac{4}{3}$.
$\therefore\ -\frac{4}{3} = \frac{32}{y} \Rightarrow y = -24$
From Eq. (i), $(-24)^2 = 64x \Rightarrow x = 9$
$\therefore$ Hence, the required point is $(9, -24)$.

146. (a)
We know that $|z-z_1|+|z-z_2|=k$ will represent an ellipse, if $|z_1-z_2|<k$.
Hence, the equation $|z-z_1|+|z-z_2|=2|z_1-z_2|$ represent an ellipse.
147. (a)
Given, $f(x)=\frac{\tan\left\{\pi\left[x-\frac{\pi}{2}\right]\right\}}{2+[x]^2}$
Since, $\left[x-\frac{\pi}{2}\right]$ is an integer for all x, therefore $\pi\left[x-\frac{\pi}{2}\right]$ is an integral multiple of π for all x.
Hence, $\tan\left\{\pi\left[x-\frac{\pi}{2}\right]\right\}=0$ for all x
Also, $2+[x]^2\neq 0$ for all x
Hence, $f(x)=0$ for all x.
Hence, $f(x)$ is continuous and derivable for all x.
148. (c)
Given, $f(x)=a\sin|x|+be^{|x|}$
We know that $\sin|x|$ and $e^{|x|}$ is not differentiable at $x=0$.
Therefore, for $f(x)$ to differentiable at $x=0$, we must have $a=b=0$.
149. (a)
The general term in $\left(ax^2+\frac{1}{bx}\right)^{13}$ is
$$T_{r+1}={}^{13}C_r(ax^2)^{13-r}\left(\frac{1}{bx}\right)^r$$
$$={}^{13}C_r a^{13-r}\times b^{-r}(x)^{26-3r}$$
For coefficient of x^8, put $26-3r=8$
$\Rightarrow\ 3r=18\Rightarrow r=6$
$\therefore\ T_7={}^{13}C_6a^{13-6}b^{-6}x^8$
$={}^{13}C_6a^7b^{-6}x^8$
Now, the general term in $\left(ax-\frac{1}{bx^2}\right)^{13}$ is
$$T'_{r+1}={}^{13}C_r(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$$
$$={}^{13}C_ra^{13-r}\times b^{-r}\times(x)^{13-3r}(-1)^r$$
For coefficient of x^{-8}, put $13-3r=-8$
$3r=21\Rightarrow r=7$
$\Rightarrow T'_8=(-1)^7\,{}^{13}C_7a^{13-7}b^{-7}x^{-8}$
$=(-1)^7\,{}^{13}C_7a^6b^{-7}x^{-8}$
According to the given condition,
Coefficient of x^8 in $\left(ax^2+\frac{1}{bx}\right)^{13}$
$=$ Coefficient of x^{-8} in $\left(ax-\frac{1}{bx^2}\right)^{13}$

$\therefore\ {}^{13}C_6a^7b^{-6}=-\,{}^{13}C_7a^6b^{-7}$
$$\Rightarrow\ {}^{13}C_7\frac{a^7}{b^6}=-\,{}^{13}C_7\frac{a^6}{b^7}$$
$$\Rightarrow\frac{a^7}{a^6}=-\frac{b^6}{b^7}\Rightarrow a=-\frac{1}{b}\Rightarrow ab+1=0$$
150. (a)
Given, $I=\int_0^2 e^{x^4}(x-\alpha)dx=0$
$$\Rightarrow\int_0^2 e^{x^4}x\,dx=\int_0^2 e^{x^4}\alpha\,dx$$
Here, we see that $\int_0^2 e^{x^4}x\,dx$ gives us an area between two curves e^{x^4} and x from $x=0$ to $x=2$. Similarly, $\int_0^2 e^{x^4}\alpha\,dx$ gives us a same area, so α should lies between 0 to 2, i.e., $\alpha\in(0,2)$.
151. (c)
Given differential equation can be rewritten as

$$\frac{dy}{dx}=\frac{y}{x}+\frac{x\phi\left(\frac{y^2}{x^2}\right)}{y\phi'\left(\frac{y^2}{x^2}\right)}$$

Put $y=vx \Rightarrow \frac{dY}{dx}=v+x\frac{dv}{dx}$

∴ Given equation becomes,

$$v+x\frac{dv}{dx}=\frac{vx}{x}+\frac{x\phi\left(\frac{v^2x^2}{x^2}\right)}{vx\phi'\left(\frac{v^2x^2}{x^2}\right)}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{\phi(v^2)}{v\phi'(v^2)}$$

$$\Rightarrow \frac{v\phi'(v^2)}{\phi(v^2)}dv=\frac{dx}{x}$$

On integrating both sides, we get

$$\frac{1}{2}\log\phi(v^2)=\log x+\log c_1$$

$$\Rightarrow \log\phi(v^2)=2\log xc_1$$

$$\Rightarrow \phi(v^2)=(xc_1)^2 \Rightarrow \phi\left(\frac{y^2}{x^2}\right)=x^2c_1^2$$

$$\Rightarrow \phi\left(\frac{y^2}{x^2}\right)=x^2c[\text{ put } c_1^2=c]$$

152. (a)

Since, α and β are the roots of

$f(x)=x^2+bx+c$

$\therefore\ \alpha+\beta=-b$ and $\alpha\beta=c$

$$\therefore\ f\left(\frac{\alpha+\beta}{2}\right)=\left(\frac{\alpha+\beta}{2}\right)^2+b\left(\frac{\alpha+\beta}{2}\right)+c$$

$$=\left(-\frac{b}{2}\right)^2+b\left(-\frac{b}{2}\right)+c$$

$$=\frac{b^2}{4}-\frac{b^2}{2}+c=-\frac{b^2}{4}+c$$

Now, $\frac{dy}{dx}=f'(x)=2x+b$

At point $\left(\frac{\alpha+\beta}{2},f\left(\frac{\alpha+\beta}{2}\right)\right)$,

i.e., $\left(-\frac{b}{2},-\frac{b^2}{4}+c\right)$,

$$\frac{dy}{dx}=2\left(-\frac{b}{2}\right)+b=0$$

Hence, the slope of the tangent to the curve and the positive direction of x-axis is 0°.

153. (d)

Given function is

$f(x)=x^2+bx+c$

It is a quadratic equation in x.

So, we will get a parabola either downward or upward.

Hence, it is a many one mapping and not onto mapping.

Hence, it is neither one-to-one nor onto mapping.

154. (a)

Given,

$$A=\begin{bmatrix}\cos\left(\frac{2\pi}{n}\right) & \sin\left(\frac{2\pi}{n}\right) & 0\\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

Now,

$$A\times A=\begin{bmatrix}\cos\left(\frac{2\pi}{n}\right) & \sin\left(\frac{2\pi}{n}\right) & 0\\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}\times$$

$$\begin{bmatrix}\cos\left(\frac{2\pi}{n}\right) & \sin\left(\frac{2\pi}{n}\right) & 0\\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\cos^2\left(\frac{2\pi}{n}\right)-\sin^2\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)+\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & 0\\ -\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right)-\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & -\sin^2\left(\frac{2\pi}{n}\right)+\cos^2\left(\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos\left(2\times\frac{2\pi}{n}\right) & 2\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & 0\\ -2\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & \cos\left(2\times\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos\left(2\times\frac{2\pi}{n}\right) & \sin\left(2\times\frac{2\pi}{n}\right) & 0\\ -\sin\left(2\times\frac{2\pi}{n}\right) & \cos\left(2\times\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

Similarly,

$$A^n = \begin{bmatrix} \cos\left(2^{n-1} \times \frac{2\pi}{n}\right) & \sin\left(2^{n-1} \times \frac{2\pi}{n}\right) & 0 \\ -\sin\left(2^{n-1} \times \frac{2\pi}{n}\right) & \cos\left(2^{n-1} \times \frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

and

$$A^{n-1} = \begin{bmatrix} \cos\left(2^{n-2} \times \frac{2\pi}{n}\right) & \sin\left(2^{n-2} \times \frac{2\pi}{n}\right) & 0 \\ -\sin\left(2^{n-2} \times \frac{2\pi}{n}\right) & \cos\left(2^{n-2} \times \frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\neq I$$

$\therefore A^m \neq I$ for any positive integer m.

155. (c)

Total sample space $= \{B_1B_2, B_1G_1, G_1B_2\}$

Favourable ways $= \{B_1G_1, G_1B_2\}$

$\therefore$ Required probability $= \frac{2}{3}$

156. (d)

We know that

$$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \cdots + {}^nC_nx^n$$

and

$$(x+1)^n = {}^nC_0x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + \cdots + {}^nC_n$$

On multiplying Eqs. (i) and (ii), we get

$$(1+x)^{2n} = ({}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \cdots + {}^nC_nx^n) \times ({}^nC_0x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + \cdots + {}^nC_n)$$

Coefficient of x^n in RHS

$= ({}^nC_0)^2 + ({}^nC_1)^2 + \cdots + ({}^nC_n)^2$

and coefficient of x^n in LHS $= {}^{2n}C_n$

$$\therefore ({}^nC_0)^2 + ({}^nC_1)^2 + \cdots + ({}^nC_n)^2 = \frac{2n!}{n!\,n!}$$

$$\Rightarrow ({}^nC_1)^2 + \cdots + ({}^nC_n)^2 = \frac{(2n)!}{n!\,n!} - 1 = {}^{2n}C_n - 1$$

157. (a)

Let $S = 1! + 2! + 3! + 4! + \cdots + 11!$

Here, we see that from 4! to 11!, we always get a 12 factor, so it is always divisible by 12 .

Now,

$1! + 2! + 3! = 1 + 2 + 6 = 9$

Hence, when S is divided by 12 , the remainder is 9.

158. (c)

3 consonants can be selected from 7 consonar

$= {}^7C_3$ ways

2 vowels can be selected from 4 vowels :
$= {}^{4}C_{2}$ ways
∴ Required number of words
$= {}^{7}C_{3} \times {}^{4}C_{2} \times 5!$
[selected 5 letters can be arrange in 5!, so get, different words]
$= 35 \times 6 \times 120 = 25200$

159. (b)

We know that
$(1+x)^{n} = {}^{n}C_{0} + x\,{}^{n}C_{1} + x^{2}\,{}^{n}C_{2} + \cdots + x^{n}\,{}^{n}C_{n}$
On integrating both sides from 0 to 2 , we get

$$\left[\frac{(1+x)^{n+1}}{n+1}\right]_{0}^{2} = \left[x\,{}^{n}C_{0} + \frac{x^{2}}{2}\,{}^{n}C_{1} + \frac{x^{3}}{3}\,{}^{n}C_{2} + \cdots + \frac{x^{n+1}}{n+1}\,{}^{n}C_{n}\right]_{0}^{2}$$

$$\Rightarrow \frac{(3)^{n+1}}{n+1} - \frac{1}{n+1} = 2\,{}^{n}C_{0} + \frac{2^{2}}{2}\,{}^{n}C_{1} + \frac{2^{3}}{3}\,{}^{n}C_{2} + \cdots + \frac{2^{n+1}}{n+1}\,{}^{n}C_{n} - 0$$

$$\Rightarrow \frac{2}{1}\,{}^{n}C_{0} + \frac{2^{2}}{2}\,{}^{n}C_{1} + \frac{2^{3}}{3}\,{}^{n}C_{2} + \cdots + \frac{2^{n+1}}{n+1}\,{}^{n}C_{n} = \frac{3^{n+1}-1}{n+1}$$

160. (d)

Given, $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
Continuity at $x = 0$,

$$\text{LHL} = \lim_{x\to 0^{-}} x\sin\frac{1}{x} = 0$$

$$\text{RHL} = \lim_{x\to 0^{+}} x\sin\frac{1}{x} = 0$$

and $f(0) = 0$
∴ $LHL = RHL = f(0)$
Hence, $f(x)$ is continuous for all values of x. Differentiability at $x = 0$

$$\text{LHD} = \lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{h\sin\frac{1}{h}}{h}$$

$$= -\lim_{h\to 0}\sin\frac{1}{h}$$

$$= -\sin\left(\frac{1}{0}\right) = \text{ some definite value}$$

Thus, $f(x)$ is not differentiable at $x = 0$.
Now, $f(0) = 0$ and $f(1) = 1\sin 1$
∴ $f(0) \neq f(1)$, Rolle's theorem is not satisfy.
Now,

$$f'(c) = \frac{f(1)-f(0)}{b-0} = \frac{1\sin(1)-0}{1-0} = \sin 1 = \sin 1$$

which lies between 0 to 1 .
Hence, their exist a real number $c \in (0,1)$. Hence, Lagrange's mean value theorem is satisfy on [0,1].

161. (c)

Since, a, b and c are in GP.
∴ $b^{2} = ac$

Given equation is
$(\log_e a)x^2 - (2\log_e b)x + (\log_e c) = 0$
Put $x = 1$, we get
$\log_e a - 2\log_e b + \log_e c = 0$
$\Rightarrow 2\log_e b = \log_e a + \log_e c$
$\Rightarrow \log_e b^2 = \log_e ac$
$\Rightarrow b^2 = ac$, which is true.
Hence, one of the root of given equation is 1 .
Let another root be α.

$$\therefore \text{ Sum of roots, } 1 + \alpha = \frac{2\log_e b}{\log_e a} = \frac{\log_e b^2}{\log_e a}$$

$$\alpha = \frac{\log_e ac}{\log_e a} - 1$$

$$= \frac{(\log_e a + \log_e c)}{\log_e a} - 1$$

$$= \frac{\log_e c}{\log_e a} = \log_a c$$

Hence, roots are 1 and $\log_a c$.

162. (a)

Let D denotes dancing, P denotes painting and S denotes singing.

$\therefore \quad n(D \cup P \cup S) = 265,$
$n(S) = 200, n(D) = 110, n(P) = 55,$
$n(S \cap D) = 60, n(S \cap P) = 30$
and $\quad n(D \cap P \cap S) = 10$
$\because \quad n(D \cup P \cup S) = n(D) + n(P) + n(S) - n(D \cap P)$
$-n(P \cap S) - n(S \cap D) + n(D \cap P \cap S)$
$\therefore \quad 265 = 110 + 55 + 200 - n(D \cap P)$
$\Rightarrow \quad 265 = 285 - n(D \cap P) - 30 - 60 + 10$
$\Rightarrow \quad n(D \cap P) = 20$
$\therefore$ Only person who like dancing and painting
$= n(D \cap P) - n(D \cap P \cap S)$
$= 20 - 10 = 10$

163. (c)

Given function is $y = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$

Here, domain is $\left[0, \frac{\pi}{4}\right]$

When, $x = 0$

When $x = \frac{\pi}{4}, Y = 3\sin\left(\sqrt{\frac{\pi^2}{16} - \frac{\pi^2}{16}}\right)$

$= 3\sin(0) = 0$

Hence, range of given function is $\left[0, \frac{3}{\sqrt{2}}\right]$.

164. (a)

$$\lim_{x\to 0} \frac{\int_0^{x^2} \cos(t^2)dt}{x\sin x} \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x\to 0} \frac{\cos(x^4) \times 2x}{\sin x + x\cos x} \quad L'Hospital'rulr$$

$$= \lim_{x\to 0} \frac{2[\cos x^4 - x\sin(x^4) \times 4x^3}{\cos x + \cos x - x\sin x} \quad L'Hospital'rulr$$

$$= \frac{2[\cos 0 - 0]}{\cos 0 + \cos 0 - 0} = \frac{2}{1+1} = 1$$

165. (d)

Given, $f'(4)=5$

Now,

$$\lim_{x\to 2}\frac{f(4)-f(x^2)}{x-2}$$

$$=\lim_{x\to 2}\frac{0-f'(x^2)\cdot 2x}{1}$$

$$=\frac{-f'(4)\cdot 2\times 2}{1}$$

$$=-(5)\times 4=-20$$

$\left[\frac{0}{0}\text{ form }\right]$

166. (c)

We know that

$$\sin n\pi=0, \forall n\in N$$

$$\therefore E=\sum_{n=1}^{\infty}\sin\left(\frac{n!\,\pi}{720}\right)=\sin\left(\frac{\pi}{720}\right)+\sin\left(\frac{2!\,\pi}{720}\right)$$

$$+\sin\left(\frac{3!\,\pi}{720}\right)+\sin\left(\frac{4!\,\pi}{720}\right)+\sin\left(\frac{5!\,\pi}{720}\right)$$

$$+\sin\left(\frac{6!\,\pi}{720}\right)+\cdots+\sin\left(\frac{720!\,\pi}{720}\right)$$

$$=\sin\left(\frac{\pi}{720}\right)+\sin\left(\frac{\pi}{320}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{30}\right)$$

$$+\sin\left(\frac{\pi}{6}\right)+\sin\pi+\cdots+\sin\left(\frac{720!\,\pi}{720}\right)$$

Here, we see that after five terms of the above series all angles are multiple of π, i.e., $\sin n\pi$, so all further values are zero.

$$\therefore\ E=\sin\left(\frac{\pi}{720}\right)+\sin\left(\frac{\pi}{320}\right)+\sin\left(\frac{\pi}{120}\right)$$

$$+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{6}\right)$$

167. (b)

Given, $I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

Then, $\det(I)=1$

If we take I as

$$A_1=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$$

Then,

$\det(I_1)=-1$

Similarly, there are four other possibilities,

$$\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix},\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

$$\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix},\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$$

who will give a determinant either -1 or 1 . Hence, there are six distinct choices for P and $\det(P)=\pm 1$.

168. (b)

Given expression can be rewritten as

$$2(1-x)^{-1}\times 2^{-1}\left(1-\frac{x}{2}\right)^{-1}$$

$$=[1+x+x^2+x^3+\cdots]$$

$$\left[1+\left(\frac{x}{2}\right)+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^3+\cdots\right]$$

$\therefore$ Coefficient of x^3 in the above expression is

$$= \left[\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) + 1\right]$$
$$= \left[\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1\right]$$
$$= \left[\frac{1+2+4+8}{8}\right] = \frac{15}{8}$$

169. (b)

Given,

$$f(x) = \frac{x}{1!} + \frac{3}{2!}x^2 + \frac{7}{3!}x^3 + \frac{15}{4!}x^4 + \cdots$$
$$= \frac{(2^1-1)x}{1!} + \frac{(2^2-1)x^2}{2!} + \frac{(2^3-1)x^3}{3!} + \frac{(2^4-1)x^3}{3!}x^4 + \cdots.$$
$$= \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \cdots$$
$$= -\left(\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \right.$$
$$= 1 + \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \cdots$$
$$\Rightarrow -\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + .\right.$$
$$f(x) = e^{2x} - e^x$$

When we put $x = 0$, we get

$f(0) = e^0 - e^0 = 1 - 1 = 0$

Hence, exactly one real solution exists.

170. (c)

Let $S = 1 + \frac{8}{2!} + \frac{21}{3!} + \frac{40}{4!} + \frac{65}{5!} + \cdots$

Again, let $S_1 = 1 + 8 + 21 + 40 + 65 + \cdots + T_n$

and

$$S_1 = +1 + 8 + 21 + 40 + \cdots + T_n$$
$$0 = 1 + 7 + 13 + 19 + 25 + \cdots - T_n$$

$T_n = 1 + 7 + 13 + 19 + 25 + \cdots + n$ terms

$$= \frac{n}{2}[2(1) + (n-1)6]$$
$$= n[1 + 3(n-1)] = n(3n-2)$$
$$S = \sum \frac{n(3n-2)}{n!}$$
$$= \sum \frac{3n-2}{(n-1)!}$$
$$= \sum \frac{3n-3+1}{(n-1)!}$$
$$S = \sum \frac{3}{(n-2)!} + \sum \frac{1}{(n-1)!}$$
$$= 3e + e$$
$$= 4e$$

We know

$2 < e < 3$

$\therefore \quad 8 < 4e < 12$

$\Rightarrow \quad 8 < S < 12$

171. (c)

We have,

$n\sqrt{2}-1<[n\sqrt{2}]\le n\sqrt{2}$ $[\because x-1\le[x]\le x]$

$\Rightarrow\sqrt{2}-\frac{1}{n}<[n\sqrt{2}]\le 1$

$\therefore$ By Sandwich theorem,

$\lim_{n\to\infty}\left(\sqrt{2}-\frac{1}{n}\right)=\sqrt{2}$

172. (c)

Given, $f'(0)=1$ and $f''(0)$ does not exist. Also, given $g(x)=xf'(x)$

$\therefore\ g'(x)=xf''(x)+f'(x)$

Put $x=0$, we get

$g'(0)=0f''(0)+f'(0)$

$=0+1=1$

173. (b)

Given, $z_1\ne\pm1$

Since, z_2 and z_3 can be obtained by rotating vector representing through $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Respectively

$\therefore\ z_2=z_1\omega$

and $z_3=z_1\omega^2$

$\therefore\ z_1z_2z_3=z_1\times z_1\omega\times z_1\omega^2$

$=z_1^3\omega^3$

$=z_1^3$ $[\because\omega^3=1]$

174. (a)

Since, z_1,z_2 and z_3 are the vertices of an equilateral triangle, therefore

$|z_1-z_2|=|z_2-z_3|$

$=|z_3-z_1|=k$

Also,

$\alpha=\frac{1}{2}(\sqrt{3}+i)$

$\Rightarrow\ |\alpha|=\frac{1}{2}\sqrt{3+1}=\frac{1}{2}\times2=1$

Let

$A=\alpha z_1+\beta, B=\alpha z_2+\beta$

and

$C=\alpha z_3+\beta$

Now,

$|AB|=|\alpha z_2+\beta-(\alpha z_1+\beta)|$

$=|\alpha(z_2-z_1)|$

$=|\alpha||z_2-z_1|$

$=|1||z_2-z_1|$

$=1|z_2-z_1|$

$=|z_2-z_1|=k$

Similarly, $BC=CA=k$

Hence, the points $\alpha z_1+\beta,\alpha z_2+\beta$ and $\alpha z_3+\beta$ are the vertices of an equilateral triangle.

175. (a)

Given curve is

$y=(\cos x+y)^{1/2}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{2(\cos x+y)^{1/2}}\times\left(-\sin x+\frac{dy}{dx}\right)$$
$$=\frac{\left(-\sin x+\frac{dy}{dx}\right)}{2y}$$
$$\Rightarrow 2y\frac{dy}{dx}=-\sin x+\frac{dy}{dx}$$
$$\Rightarrow (2y-1)\frac{dy}{dx}=-\sin x$$
Again, differentiating both sides w.r.t. x, we get
$$(2y-1)\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(2\frac{dy}{dx}\right)=-\cos x$$
$$\Rightarrow (2y-1)\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(2\frac{dy}{dx}\right)+\cos x=0$$

176. (b)

Given equation is
$$1+z+z^3+z^4=0$$
$$\Rightarrow (1+z)+z^3(1+z)=0$$
$$\Rightarrow (1+z)(1+z^3)=1$$
$$\Rightarrow z=-1, z^3=-1$$
$$\Rightarrow z=-1, z=-1, -\omega_- -\omega^2$$
Hence, roots are in cubic roots of unity.
Hence, these roots are the vertices of an equilateral triangle.

177. (a)

$$\sum a^3\sin(B-C)$$
$$=\sum k^3\sin^3 A\sin(B-C)$$
$$=\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k\Big]$$
$$=\sum k^3[\sin^2 A\sin(B+C)\sin(B-C)]$$
$$k^3\left[\left\{\sin^2 A\times\frac{1}{2}(\cos 2C-\cos 2B)\right\}\right.$$
$$+\sin^2 B\times\frac{1}{2}(\cos 2A-\cos 2C)$$
$$\left.+\sin^2 C\times\frac{1}{2}(\cos 2B-\cos 2A)\right]$$
$$=\frac{k^3}{2}[\sin^2 A(1-2\sin^2 C-1+2\sin^2 B)+$$
$$\sin^2 B(1-2\sin^2 A-1+2\sin^2 C)$$
$$+\sin^2 C(1-2\sin^2 B-1+2\sin^2 A)]$$
$$=\frac{k^3}{2}[-2\sin^2 A\sin^2 C+2\sin^2 A\sin^2 B$$
$$-2\sin^2 B\sin^2 A+2\sin^2 B\sin^2 C$$
$$-2\sin^2 C\sin^2 B+2\sin^2 C\sin^2 A]$$
$$=\frac{k^3}{2}[0]=0$$

178. (a)

Given, α and β are the roots of $x^2-x-1=0$
$\therefore\quad \alpha+\beta=1$
and $\quad \alpha\beta=-1$

Now, consider option (a),
$S_n + S_{n-1} = S_{n+1}$
Put $n = 2$, we get, $S_2 + S_1 = S_3$
$\therefore\ (\alpha^2+\beta^2)+(\alpha+\beta) = \alpha^3+\beta^3$

$[\because S_n = \alpha^n + \beta^n$, given $]$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta + (\alpha+\beta)$
$= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$
$\Rightarrow\ 1^2 - 2(-1) + 1 = 1^3 - 3(-1)(1)$
$\Rightarrow\ -1+2+1 = 1+3$
$\Rightarrow\ 4 = 4$, which is true.

179. (c)

Required probability

$$= {}^{12}C_2 \times {}^{10}C_2 \times {}^{8}C_2 \times {}^{6}C_2 \times {}^{4}C_2 \times {}^{2}C_2 \times \left(\frac{1}{6}\right)^{12}$$
$$= \frac{12!}{10!\times 2!} \times \frac{10!}{8!\times 2!} \times \frac{8!}{6!\times 2!} \times \frac{6!}{4!\times 2!}$$
$$= \frac{12!}{2^6 \times 6^{12}}$$

180. (d)

Sum of roots, $\alpha+\beta = -p$ and $\alpha\beta = q$

$$\therefore\ (\alpha^3+\beta^3) = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$
$$= (-p)^3 - 3q(-p)$$
$$= -p^3 + 3pq$$

and $\alpha^4+\alpha^2\beta^2+\beta^4 = (\alpha^4+\beta^4)+(\alpha\beta)^2$
$$= (\alpha^2+\beta^2)^2 - (\alpha\beta)^2$$
$$= [(\alpha+\beta)^2 - 2\alpha\beta]^2 - (\alpha\beta)^2$$
$$= [(-p)^2 - 2q]^2 - 3(q)^2$$
$$= [p^2-2q]^2 - 3q^2$$
$$= p^4 - 4p^2q + 4q^2 - q^2$$
$$= p^4 - 4p^2q + 3q^2$$
$$= p^4 - 3p^2q - p^2q + 3q^2$$
$$= p^2(p^2-3q) - q(p^2-3q)$$
$$= (p^2-q)(p^2-3q)$$

181. (a)

Given, differential equation is
$$\frac{dy}{dx} + \frac{y}{x\log_e x} = \frac{1}{x}$$
It is a linear equation of the form
$$\frac{dy}{dx} + Py = Q$$
where
$$P = \frac{1}{x\log_e x} \text{ and } Q = \frac{1}{x}$$
$\therefore$ Integrating factor, IF $= e$
$$= e^{\int \frac{1}{x\log_e x}dx}$$
$$= e^{\log(\log_e x)}$$
$$= \log_e x$$
$\therefore$ Solution of differential equation is
$$y \times \log_e x = \int \frac{1}{x}\log_e x\,dx$$
$$\Rightarrow y \times \log_e x = \frac{(\log_e x)^2}{2} + C \ldots\ldots\ldots (i)$$

When $y=1$ and $x=e$, then

$$1\times\log_e e=\frac{(\log_e e)^2}{2}+C$$

$$\Rightarrow \quad 1=\frac{1}{2}+C$$

$$\Rightarrow \quad C=\frac{1}{2}$$

On putting $C=\frac{1}{2}$ in Eq. (i), we get

$$y\times\log_e x=\frac{(\log_e x)^2}{2}+\frac{1}{2}$$

$$\Rightarrow 2y=\log_e x+\frac{1}{\log_e x}$$

182. (c)

Given, $f(x)=\max\{x+|x|, x-[x]\}$

$$=\begin{cases}2x, & x\geq 0\\ x-[x], & x\leq 0\end{cases}$$

$$\therefore \int_{-3}^{3} f(x)dx=\int_{-3}^{0} x-[x]dx+\int_{0}^{3} 2xdx$$

$$=3\int_{-1}^{0}(1+x)dx+2\int_{0}^{3} xdx$$

[$\because x-[x]$ is a periodic function at $x=1$]

$$=3\left[x+\frac{x^2}{2}\right]_{-1}^{0}+2\left[\frac{x^2}{2}\right]_{0}^{3}$$

$$=3\left[0-0-\left(-1+\frac{1}{2}\right)\right]+3^2-0$$

$$=3\left[\frac{1}{2}\right]+9=\frac{3}{2}+9$$

$$=\frac{21}{2}$$

183. (a)

Given, $$X_n=\left\{z=x+iy: |z|^2\leq\frac{1}{n}\right\}$$
$$=\left\{x^2+y^2\leq\frac{1}{n}\right\}$$

$$\therefore X_1=\{x^2+y^2\leq 1\},$$

$$X_2=\left\{x^2+y^2\leq\frac{1}{2}\right\}$$

$$X_3=\left\{x^2+y^2\leq\frac{1}{3}\right\}$$

Hence, $\bigcap_{n=1}^{\infty} X_n$ is a singleton set.

184. (c)

We know that in a Lagrange mean value theorem there exist $c\in(a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\therefore \quad f'(\theta h) = \frac{f(h)-\cos 0}{h-0}$$

$$\Rightarrow \quad -\sin(\theta h) = \frac{\cos h - \cos 0}{h-0}$$

$$= \frac{\cos h - 1}{h}$$

$[\because f(x) = \cos x]$

$$= \frac{\left(1-\frac{h^2}{2!}\right)-1}{h}$$

[neglecting higher power of h]

$$\Rightarrow \quad -\sin(\theta h) = \frac{-\frac{h^2}{2}}{h}$$

$$\Rightarrow \quad \sin(\theta h) = \frac{\frac{h}{2}}{1}$$

$$\Rightarrow \quad \theta h = \sin^{-1}\left(\frac{h}{2}\right)$$

$$\Rightarrow \quad \theta = \frac{\sin^{-1}\frac{h}{2} \times \frac{1}{2}}{h \times \frac{1}{2}}$$

185. (a)

Let equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Given, foci, $(\pm 8,0) = (\pm ae, 0)$

$\Rightarrow \quad ae = 8$

and length of latusrectum $= \frac{2b^2}{a}$

$$\therefore \quad 24 = \frac{2b^2}{a}$$

$\Rightarrow \quad b^2 = 12a$

$\therefore$ From Eq. (ii),

$$a^2e^2 = 64$$

$$\Rightarrow \quad a^2\left(\frac{a^2+b^2}{a^2}\right) = 64$$

$$\Rightarrow \quad a^2 + b^2 = 64$$

$$\Rightarrow \quad a^2 + 12a = 64$$

$$\Rightarrow \quad a^2 + 12a - 64 = 0$$

$$\Rightarrow \quad a^2 + 16a - 4a - 64 = 0$$

$$\Rightarrow \quad a(a+16) - 4(a+16) = 0$$

$$\Rightarrow \quad (a+16)(a-4) = 0$$

$$\Rightarrow$$

$$a = 4$$

$[\because a$ cannot be negative $]$

On putting $a = 4$in Eq. (iii), we get

$b^2 = 12 \times 4 \Rightarrow b^2 = 48$

$\therefore$ From Eq. (i),

$$\frac{x^2}{16}-\frac{y^2}{48}=1$$
$$\Rightarrow 3x^2-y^2=48$$

186. (c)

Let, E_1 = Student does not know the answer E_2 = Student knows the answer and E = Student answer correctly.

$\therefore \quad P(E_1)=1-p,$

$\quad P(E_2)=p$

$P\left(\frac{E}{E_2}\right)=1$ and $P\left(\frac{E}{E_1}\right)=\frac{1}{5}$

$\therefore$ The probability that student did not tick the answer randomly

= The probability that student tick the answer correctly

$$=\frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)}$$
$$=\frac{p(1)}{(1-p)\frac{1}{5}+p(1)}$$
$$=\frac{p}{\frac{1-p+5p}{5}}=\frac{5p}{1+4p}$$

187. (c)

Let $\frac{2\pi r}{7}=\theta$

$\Rightarrow \quad 2\pi r=3\theta+4\theta$

$\Rightarrow \quad 4\theta=2\pi r-3\theta$

$\Rightarrow \quad \sin 4\theta=\sin(2\pi r-3\theta)$

$\Rightarrow \quad \sin 4\theta=-\sin 3\theta$

$\Rightarrow \quad 2\sin 2\theta\cos 2\theta=-[3\sin\theta-4\sin^3\theta]$

$\Rightarrow \quad 2\times 2\sin\theta\cos\theta(2\cos^2\theta-1)$

$\quad =-3\sin\theta+4\sin^3\theta$

$\Rightarrow \quad \sin\theta[8\cos^3\theta-4\cos\theta+3-4(1-\cos^2\theta)]=0$

$\Rightarrow \quad 8\cos^3\theta+4\cos^2\theta-4\cos\theta-1=0$

Thus, $\cos\frac{2\pi}{7}, \cos\frac{4\pi}{7}$ and $\cos\frac{6\pi}{7}$ are the roots of the above equation.

$$\therefore \quad \cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=-\frac{1}{2}$$

188. (d)

Given, $M=\int_0^{\pi/2}\frac{\cos x}{(x+2)}dx$

and $N=\int_0^{\pi/4}\frac{\sin x\cos x}{(x+1)^2}dx$

$$\Rightarrow N=\int_0^{\pi/4}\frac{1}{2}\frac{\sin 2x}{(x+1)^2}dx$$

Put $2x=t\Rightarrow dx=\frac{dt}{2}$

$$N=\int_0^{\pi/2}\frac{\sin t}{4\left(\frac{t}{2}+1\right)^2}dt$$
$$=\int_0^{\pi/2}\frac{\sin t}{(t+2)^2}dt$$
$$=\int_0^{\pi/2}\frac{\sin x}{(x+2)^2}dx$$

$M - N = \int_0^{\pi/2} \cos_{11} \times \frac{1}{(x+2)} dx$

$= \left[\frac{\sin x}{x+2}\right]_0^{\pi/2} - \int_0^{\pi/2} - \frac{\sin x}{(x+2)^2} dx - \frac{\sin x}{(x+2)^2} dx$

$= \frac{\sin \frac{\pi}{2}}{\frac{\pi}{2} + 2} = \frac{1}{\frac{\pi+4}{2}} = \frac{2}{\pi + 4}$

189. (d)

Given relation is defined as $\theta R \phi$ such that $\sec^2\theta - \tan^2\phi = 1$

For Reflexive

When $\theta R \theta$

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow 1 = 1$, which is true.

Thus, it is reflexive.

For Symmetric

When $\theta R \phi$

$\sec^2\theta - \tan^2\phi = 1$

$\Rightarrow \quad (1 + \tan^2\theta) - (\sec^2\phi - 1) = 1$

$\Rightarrow \quad 2 + \tan^2\theta - \sec^2\phi = 1$

$\Rightarrow \quad \sec^2\phi - \tan^2\theta = 1$

$\Rightarrow \quad \phi R \theta$

Thus, it is symmetric.

For Transitive

When $\theta R \phi$ and $\phi R \psi$, then

and

$\sec^2\theta - \tan^2\phi = 1$

$\sec^2\phi - \tan^2\psi = 1$

Now, $\theta R \psi$

Then,

$\sec^2\theta - \tan^2\psi = 1$

$\Rightarrow \sec^2\theta - \tan^2\psi + 1 = 1 + 1$

$\Rightarrow \sec^2\theta - \tan^2\psi + \sec^2\phi - \tan^2\phi = 1 + 1$

$\Rightarrow \theta R \phi$ and $\phi R \psi$

Thus, it is transitive.

Hence, it is an equivalence relation.

190. (a)

We know that

AM $\geq$ GM

$\therefore \frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x} 2^{\cos x}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2\sqrt{2^{\sin x + \cos x}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2 \times 2^{\frac{\sin x + \cos x}{2}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1 + \frac{\sin x + \cos x}{2}}$

But $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) \geq -\sqrt{2}$

$\therefore 2^{\sin x} + 2^{\cos x} \geq 2^{1 - \frac{\sqrt{2}}{2}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1 - \frac{1}{\sqrt{2}}}, \forall x \in R$

Hence, minimum value is $2^{1-\frac{1}{2}}$.

191. (d)

Let the relation defined as

$R = \{(A, B): B = PAQ^{-1}\}$

For reflexive

$A = IAI^{-1}$

$\Rightarrow (A, A) \in R$

$\Rightarrow R$ is reflexive

For symmetric

Let $(A, B) \in R$

$\therefore \quad B = PAQ^{-1}$

$\Rightarrow \quad BQ = PA(Q^{-1}Q)$

$\Rightarrow \quad BQ = PA$

$\Rightarrow \quad P^{-1}(BQ) = P^{-1}P(A)$

$\Rightarrow \quad P^{-1}BQ = A$

$\Rightarrow \quad (B, A) \in R$

$\Rightarrow R$ is symmetric.

For transitive

Let $(A, B) \in R, (B, C) \in R$

Then, $A = PBQ^{-1}$ and $B = RCS^{-1}$

$\Rightarrow \quad A = PRCS^{-1}Q^{-1}$

$\Rightarrow \quad A = (PR)C(QS)^{-1}$

$\Rightarrow \quad (A, C) \in R$

$\Rightarrow R$ is transitive.

Hence, R is an equivalence relation.

192. (a)

Case I Let $\alpha = \omega$ and $\beta = \omega^2$

$$\therefore S = \sum_{n=0}^{302} (-1)^n \left(\frac{\omega}{\omega^2}\right)^n$$

$$= \sum_{n=0}^{302} (-1)^n (\omega^2)^n$$

$$=1 - \omega^2 + \omega^4 - \omega^6 + \omega^8 - \omega^{10} + \omega^{12} + \cdots + \omega^{600} - \omega^{602} + \omega^{604}$$

$$=1 - \omega^2 + \omega - 1 + \omega^2 - \omega + 1 + \cdots$$

$$=0 + \cdots + 1 - \omega^2 + \omega$$

$$= -\omega^2 - \omega^2 = -2\omega^2 \; [\because 1 + \omega + \omega^2 = 0]$$

Case II Let $\alpha = \omega^2$ and $\beta = \omega$

$$\therefore S = \sum_{n=0}^{302} (-1)^n \left(\frac{\omega^2}{\omega}\right)^n$$

$$= \sum_{n=0}^{302} (-1)^n \left(\frac{\omega^4}{\omega^3}\right)^n$$

$$= \sum_{n=0}^{302} (-1)^n (\omega)$$

$$= 1 - \omega + \omega^2 - \omega^3 + \omega^4 - \omega^5 + \omega^6 - \cdots$$

$$= +\omega^{300} - \omega^{301} + \omega^{302}$$

$$= 1 - \omega + \omega^2 - 1 + \omega - \omega^2 + 1$$

$$- \cdots + 1 - \omega + \omega^2$$

$$= 0 + \cdots + 1 + \omega^2 - \omega$$

$$= -\omega - \omega = -2\omega$$

193. (b)

Let

$$S = 1 + 10 + 21 + 34 + 49 + \cdots + t_n'$$
$$S = 1 + 10 + 21 + 34 + \cdots + t_n'$$
$$0 = 1 + 9 + 11 + 13 + 15 + \cdots - t_n'$$

$$\Rightarrow t_n' = 1 + [9 + 11 + 13 + 15 + \cdots (n-1) \text{ term }]$$
$$= 1 + \left[\frac{n-1}{2}\{2 \times 9 + (n-2)2\}\right]$$
$$= 1 + (n-1)[9 + n - 2]$$
$$= 1 + (n-1)(n+7)$$
$$\therefore t_n = \frac{t_n'}{n!} = \frac{1 + (n-1)(n+7)}{n!}$$

$$= \frac{1 + (n-1)(n+7)}{n!}$$
$$= \frac{1 + n^2 + 6n - 7}{n!} = \frac{n^2 + 6n - 6}{n!}$$
$$= \frac{n}{(n-1)!} + \frac{6}{(n-1)!} - \frac{1}{n!}$$
$$= \frac{n-1+1}{(n-1)!} + \frac{6}{(n-1)!} - \frac{1}{n!}$$
$$= \frac{1}{(n-2)!} + \frac{7}{(n-1)!} - \frac{1}{n!}$$
$$\therefore \lim_{n\to\infty} t_n = \lim_{n\to\infty}\left[\frac{1}{(n-2)!} + \frac{7}{(n-1)!} - \frac{1}{n!}\right] = 0$$

194. (b)

Let time taken from point A to P is t.

When a particle moving from point A to P, using first equation of motion

$v = u + at$

When final velocity is v_1 and time t_2, then

$$v_1 = u + at$$
$$\Rightarrow \quad v_1 - u = at \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

When final velocity is v_2 and time $T/2$, then

$v_2 = u + \frac{aT}{2}$......(ii)

When a particle moving from point P to B using first equation of motion

$v = u + at$

When intial velocity v_1 and time $T - t$, then

$$0 = v_1 - a(T - t)$$
$$\Rightarrow \quad v_1 = aT - at \ldots\ldots\ldots (iii)$$

When initial velocity v_2 and time $\frac{T}{2}$, then

$$0 = v_2 - \frac{aT}{2}$$
$$\Rightarrow v_2 = \frac{aT}{2} \Rightarrow aT = 2v_2$$

$\therefore$ From Eq. (iii),

$v_1 = 2v_2 - (v_1 - u)$

[from Eqs. (i) and (iv)]

$$\Rightarrow\ 2v_1 = 2v_2 + u$$
$$\Rightarrow\ v_1 = v_2 + \frac{u}{2}$$
$$\therefore\ v_1 > v_2$$

195. (a)

$\therefore$ Required probability

$$= \frac{{}^{13}C_1 \times {}^{4}C_2 \times {}^{12}C_1 \times {}^{4}C_3}{{}^{52}C_5}$$
$$= \frac{13 \times 6 \times 12 \times 4}{\frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1}}$$
$$= \frac{13 \times 6 \times 12 \times 4}{52 \times 51 \times 10 \times 49 \times 2}$$
$$= \frac{6}{4165}$$

196. (a,d)

We know that $u(x)$ and $v(x)$ are two independent solutions of the given differential equation, then their linear combination is also the solution of the given equation.
Here, we see that $y = 5u(x) + 8v(x)$ is a linear combination and $y = c_1\{u(x) - v(x)\} + c_2 v(x)$ is also a linear combination of two independent solutions.

197. (a)

Given, $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 10$ We know that $(|\sin x| + |\cos x|) \in [1, \sqrt{2}]$
$\therefore\ y = 1$
The point of intersection of given curve is
$$x^2 + 1^2 = 10$$
$$\Rightarrow \quad x^2 = 9$$
$$\Rightarrow \quad x = \pm 3$$
$\therefore$ Point of intersection is $(\pm 3, 1)$
Now,
$$x^2 + y^2 = 10$$
$$\Rightarrow \quad 2x + 2y\frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$$
At point $(-3,1)$
$$\frac{dy}{dx} = \frac{3}{1} = 3 \Rightarrow m_1 = 3$$
Slope of line $y = 1$ is $m_2 = 0$
$\therefore$ Angle between two curves is
$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = 3$$
$$\theta = \tan^{-1}(3)$$

198. (a,d)

Given,
$$f(x) = \begin{cases} \int_0^x |1-t|dt, & x > 1 \\ x - \frac{1}{2}, & x \le 1 \end{cases} \quad \text{........(i)}$$

Now, for $x > 1$, $\int_0^x |1-t|dt$

$$= \int_0^1 (1-t)dt + \int_1^x (t-1)dt$$

$$= \left[t - \frac{t^2}{2}\right]_0^1 + \left[\frac{t^2}{2} - t\right]_1^x$$

$$= \left[1 - \frac{1}{2} - 0\right] + \left[\frac{x^2}{2} - x - \frac{1}{2} + 1\right]$$

$$= \frac{1}{2} + \frac{x^2}{2} - x - \frac{1}{2} + 1$$

$$\Rightarrow \frac{x^2}{2} - x + 1$$

$\therefore$ Eq. (i) becomes,

$$f(x) = \begin{cases} \frac{x^2}{2} - x + 1, & x > 1 \\ x - \frac{1}{2}, & x \le 1 \end{cases}$$

Continuity at $x = 1$,

LHL $= \lim_{x \to 1^-} f(x)$

$$= \lim_{x \to 1} \left(x - \frac{1}{2}\right) = 1 - \frac{1}{2}$$

RHL $= \lim_{x \to 1^+} f(x)$

$$= \lim_{x \to 1} \left(\frac{x^2}{2} - x + 1\right)$$

$$= \frac{1}{2} - 1 + 1 = \frac{1}{2}$$

and

$$f(1) = 1 - \frac{1}{2} = \frac{1}{2}$$

$\therefore\ f(1) = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

Hence, $f(x)$ is continuous at $x = 1$.

Differentiability at $x = 1$,

$$\text{LHD} = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{(1-h) - \frac{1}{2} - \left(1 - \frac{1}{2}\right)}{-h} = \lim_{h \to 0} \frac{-h}{-h} = 1$$

$$\text{RHD} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(1+h)^2}{2} - (1+h) + 1 - \left(1 - \frac{1}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1 + h^2 + 2h}{2} - 1 - h + 1 - \frac{1}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2} + \frac{h^2}{2} + h - h - \frac{1}{2}}{h}$$

$$= \lim_{h \to 0} \frac{h^2}{2h} = 0$$

$\therefore$

Hence, $f(x)$ is not differentiable at $x = 1$.

199. (d)

Centres and constant terms in the circles $x^2 + y^2 - 5 = 0, x^2 + y^2 - 8x - 6y + 10 = 0$ and $x^2 + y^2 - 4x + 2y - 2 = 0$ are $C_1'(0,0), c_1 = -5$, $C_2'(4,3), c_2 = 10$ and $C_3'(2,-1), c_3 = -2$

Also, centre and constant of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

is

$C_4'(-g,-f)$ and $c_4 = c$

Since, the first circle intersect all the three at the extremities of diameter, therefore they are orthogonal to each other.

$\therefore \quad 2(g_1g_2 + f_1f_2) = c_1 + c$

$\therefore \quad 2[-g(0) + (-f)(0)] = c - 5$

$\Rightarrow \quad c = 5$

$2[-g(4) + (-f)(3)] = c + 10$

$\Rightarrow \quad -2(4g + 3f) = 5 + 10$

$\Rightarrow \quad 4g + 3f = -\frac{15}{2} \ldots\ldots (i)$

and $2[-g(2) + (-f)(-1)] = c - 2$

$\Rightarrow \quad 2[-2g + f] = 5 - 2$

$\Rightarrow \quad -2g + f = \frac{3 \ldots\ldots\ldots\ldots \text{(ii)}}{2}$

On solving Eqs. (i) and (ii), we get

$f = -\frac{9}{10}$ and $g = -\frac{12}{10}$

$\therefore \quad fg = \frac{-9}{10} \times -\frac{12}{10} = \frac{27}{25}$

and $\quad 4f = 4 \times \frac{-9}{10}$

$\Rightarrow \quad 4f = \frac{-36}{10}$

$\Rightarrow \quad 4f = 3g$

200. (c.d)

Given, $P(A) = 0.7$ and $P(B) = 0.6$

We know that

$P(A \cap B) \leq P(B)$

$\therefore \quad P(A \cap B) \leq 0.6 \ldots\ldots \text{(i)}$

From Booley's inequality

$$P(\cap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$

$\therefore \quad P(A \cap B) \geq [(P(A) + P(B) - (2-1)]$

$\Rightarrow \quad P(A \cap B) \geq [0.7 + 0.6 - 1]$

$\Rightarrow \quad P(A \cap B) \geq 0.3 \ldots\ldots \text{(ii)}$

From Eqs. (i) and (ii), we get

$0.3 \leq P(A \cap B) \leq 0.6$

Hence, options (c) and are (d) always incorrect.