Physics

1. (c)

The gravitational force acting between the two masses m_1 and m_2 is given by

$$F_G = \frac{Gm_1m_2}{r^2}$$

Force on mass m_1,

$$F_1 = \frac{Gm_1m_2}{r^2} = m_1a_1$$

where, a_1 = acceleration

$$\Rightarrow a_1 = \frac{Gm_2}{r^2} \Rightarrow a_1 \propto m_2$$

and similarly, $a_2 \propto m_1$

2. (b)

Let the final temperature after the masses in themal contact is θ, then from the principle of calorimetry.

Heat lost = Heat gained

$$\Rightarrow 3ms(60-\theta) = ms(\theta-50) + ms(\theta-40)$$

$$\Rightarrow 3(60-\theta) = \theta - 50 + \theta - 40$$

$$\Rightarrow 180 - 3\theta = 2\theta - 90$$

$$\theta = \frac{270}{5} = 54°\text{C}$$

3. (a)

As we know that for the earth and satellite system,

Kinetic energy, $K = \frac{GMm}{2a}$

Potential energy, $V = -\frac{GMm}{a}$

and total energy, $E = -\frac{GMm}{2a}$

where, a = radius of the orbit of the satellite and m = mass of the satellite

On the basis of above three expressions for the energies, we have

$$K = \frac{GMm}{2a} = -\left(-\frac{GMm}{2}\right) = -\frac{V}{2}$$

4. (b)

From the lens formula (for first lens)

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1} \Rightarrow \frac{1}{2} = \frac{1}{v_1} - \frac{1}{(-4)}$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} = \frac{1}{v_1} = \frac{3}{4} \qquad \text{(i)}$$

$$\Rightarrow v_1 = \frac{4}{3}, u_2 = 3 - \frac{4}{3} = 5/3\text{(i)}$$

This image will be treated as the source for second lens, then again from lens formula, we have

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

$$\Rightarrow \quad \frac{1}{1} = \frac{1}{v_2} + \frac{5}{3} \text{ Eq(i)}$$

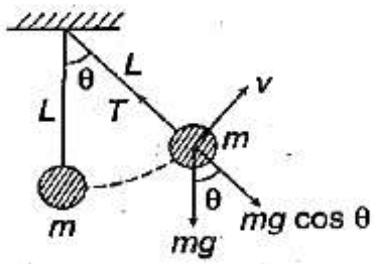
$$\Rightarrow \quad 1 - 5/3 = \frac{1}{v_2} \Rightarrow v_2 = -3/2$$

This is the final image distance from 2nd lens: So, the overall distance of image from the primary source(or object)

Let $d = 4 + 3 - 1.5 = 5.5$

5. (b)

The situation is given below



For motion along a vertical circular track, the required centripetal force is along the radius and towards the centre of the circle is given by

$$T - mg\cos\theta = \frac{mv^2}{L}$$

$$\Rightarrow T = \frac{mv^2}{L} + mg\cos\theta$$

6. (c)

Let the initial length of the metal wire is L.

The strain at tension T_1 is $\Delta L_1 = L_1 - L$

The strain at tension T_2 is $\Delta L_2 = L_2 - L$

Suppose, the Young's modulus of the wire is Y, then

$$\frac{\frac{T_1}{A}}{\frac{\Delta L_1}{L}} = \frac{\frac{T_2}{A}}{\frac{\Delta L_2}{L}}$$

where, A is an cross-section of the wire. assume to be same at all the situations.

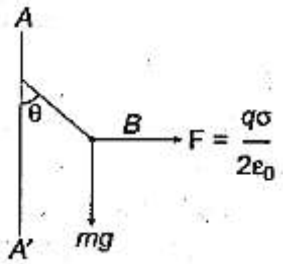
$$\Rightarrow \frac{T_1}{A} \times \frac{L}{\Delta L_1} = \frac{T_2}{A} \times \frac{L}{\Delta L_2}$$

$$\Rightarrow \frac{T_1}{(L_1 - L)} = \frac{T_2}{(L_2 - L)}$$

$$\therefore T_1(L_2 - L) = T_2(L_1 - L); L = \frac{T_2L_1 - T_1L_2}{T_2 - T_1}$$

7. (d)

The diagram is as follows



The electric field due to charged infinite conducting sheet is $E = \frac{\sigma}{\varepsilon_0}$.

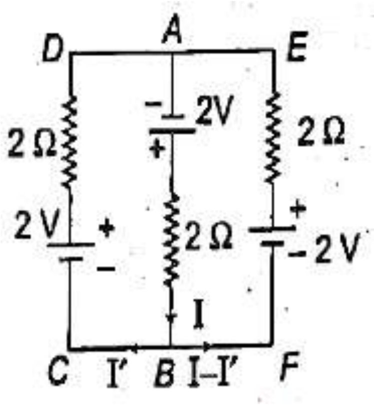
Now, force (electric force) on the charged ball is $F = qE = \frac{q\sigma}{\varepsilon_0}$

The resultant of electric force and mg balance the tension produced in the string.

So, $\tan\theta = \frac{F_e}{mg} = \frac{\frac{q\sigma}{\varepsilon_0}}{mg} = \frac{q\sigma}{\varepsilon_0 mg}$

8. (a)

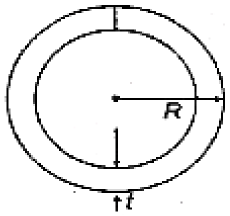
The circuit diagram can be redrawn as



For the loop $ABCDA$,
$+2-2I'+2-2I=0$…….(i)
For the loop $ABFEA$,
$2-2I+2-2(I-I')=0$
$4-2I-2I+2I'=0$
$4-4I+2I' \quad =0$
$2=2I-I'$(ii)
From Eqs. (i) and (ii), we get
$2=2I-I', 2=I+I'$
$4=3I, I=4/3=1.33$ A

9. (b)

The density of material is ρ.



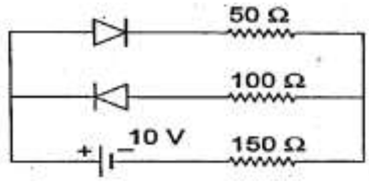
The hollow sphere will float if its weight is less than the weight of the water displaced by the volume of the sphere This implies mass of the sphere is less than that for the same volume of water. Now, mass of spherical cell
$m_1=4\pi R^2\times t\times\rho$
While the mass of water having same volume
$m_2=\frac{4}{3}\pi R^3\times\rho_g=\frac{4}{3}\pi R^3$
where, $\rho_g=$ density of water $=\frac{1\text{ kg}}{\text{m}^3}$
For the floatation of sphere,
$m_1\leq m_2$
$4\pi R^2\times t\times\rho\leq\frac{4}{3}\pi R^3$
$\Rightarrow t\rho\leq\frac{R}{3}\Rightarrow t\leq\frac{R}{3\rho}$

10. (c)

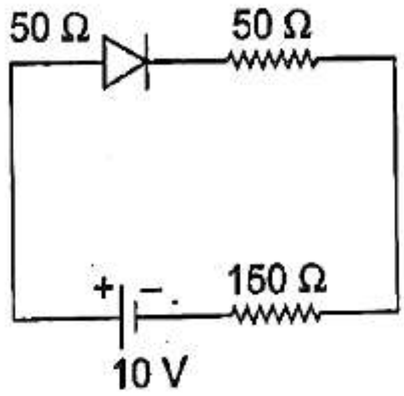
As we know that, $R_1=\rho\frac{1}{a}=\rho\frac{1^2}{V}$
where, $l=$ length of wire
$a=$ area of cross-section of the wire
and $V=$ volume of the wire $R_1\propto l^2$
$\Rightarrow\frac{R_1}{R_2}=\left(\frac{l_1}{l_2}\right)^2=\left(\frac{1}{2}\right)^2\Rightarrow R_2:R_1=4:1$

11. (d)

The circuit is



As the lower diode attached to 100Ω resistance is in reversed biased so, it is non-conducting.
Now, the circuit can be redrawn as;



∴ Current through the circuit, $I = \frac{V}{R}$

$= \frac{10}{250} = 0.04$ A

12. (c)

As the rms speed is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{Mi}}$$

$$\Rightarrow v_{\text{rms}} \propto \sqrt{\frac{T}{M}}$$

When temperature is doubled and molecules dissociates into atoms, then

$$\frac{v_{\text{rms}_1}}{v_{\text{rms}_2}} = \frac{\sqrt{\frac{T}{M}}}{\sqrt{\frac{2T}{M/2}}} = \frac{\sqrt{\frac{T}{M}}}{\sqrt{\frac{4T}{M}}}$$

$$= \frac{\sqrt{T}}{\sqrt{4T}} = \frac{1}{2}$$

If v_{rms_1} is v, then v_{rms_2} will be $2v$.

13. (c)

Radius of circular path followed by charged particle is given by

$$R = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB} \; [\because p = mv = \sqrt{2mK}]$$

where, K is kinetic energy of particle. Charged particle q is accelerated through some potential difference V, such that kinetic energy of particle is

$$K = qV$$

$$\therefore \quad R = \frac{\sqrt{2mqV}}{qB}$$

As the two charged particles of same magnitude and being accelerated through same potential, enters into a uniform magnetic field region, then $R \propto \sqrt{m}$

So,

$$\frac{R_1}{R_2}=\sqrt{\frac{m_A}{m_B}}\Rightarrow\frac{m_A}{m_B}=\left(\frac{R_1}{R_2}\right)^2$$

14. (b)

As large number of particles is situated at a distance R from the origin. If particles are uniformly distributed and make a circular boundary around the origin, then centre of mass will be at the origin. While if the particles are not uniformly distributed, then centre of mass will lie between particle and origin. This implies the distance between centre of mass and origin is always less than equal to R.

15. (b)

Given, length of conductor $l=0.1$ m

Mangetic field, $B=0.1$ T

Velocity of conductor, $v=15$ m/s

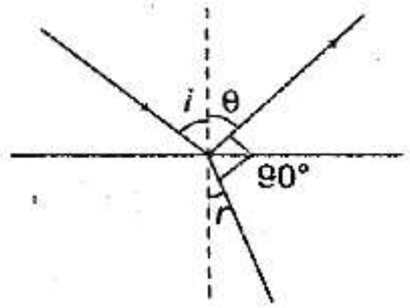
The angle between **v** and **B** is 90°

When **v** and **B** are mutually perpendicular, then emf (induced) is given by.

$$\varepsilon=vBl=15\times0.1\times0.1=\frac{15}{100}=0.15\text{ V}$$

16. (b)

As situation can be diagrammatically as below



From law of reflection

$i=\theta$

Now, $\theta+r+90^\circ=180^\circ$

$\Rightarrow\ i+r+90^\circ=180^\circ$

$r=90^\circ-i$

Also, from Snell's law

$$\frac{\sin i}{\sin r}=\mu\Rightarrow\frac{\sin i}{\sin(90^\circ-i)}=\frac{\sin i}{\cos i}=\mu$$

$\Rightarrow\tan i=\mu$

17. (a)

Total angular momentum about O is given as,

$$\begin{aligned}\mathbf{L}&=\mathbf{L}_1+\mathbf{L}_2=m_1V_1r_1+m_2V_2r_2\\&=-6.5\times2.2\times1.5+3.1\times3.6\times2.8\\&=-21.45+31.248=9.8\text{kgm}^2/\text{s}\end{aligned}$$

18. (d)

The height raised by liquid in capillary tube

$$h=\frac{2l\cos\theta}{\rho gh}$$

As in freely falling platform, a body experience weightlessness.

So, the liquid will rise upto to length of the capillary.

i.e. height raised by the liquid will be 20 cm .

19. (a)

We have,

Apparent frequency,

$$\begin{aligned}v&=\left(\frac{v+u_0}{v-u_s}\right)v_0=\left(\frac{333+33}{333-33}\right)\times1000\\&=\frac{366}{300}\times1000=1220\text{ Hz}\end{aligned}$$

20. (d)

The energy of the photon

$$E=\frac{hc}{\lambda}$$

$$=\frac{4\times10^{-15}\text{eVs}\times3\times10^{8}\text{ m/s}}{300\times10^{-9}\text{ m}}$$

$$=\frac{4\times10^{2}}{300}\text{eV}=\frac{4}{3}\text{eV}=1.33\text{eV}$$

The ionisation energy is 13.6 eV which is greater than energy of photon, so atom can not come into excited state and will remain in ground state.

21. (b)

The component of velocity of B along x-direction

$$\mathbf{v}_{Bx}=20\cos60^\circ=20\times\frac{1}{2}=10\text{ m/s}$$

$$\mathbf{v}_A=10\hat{\mathbf{i}}$$

$$\mathbf{v}_B=10\hat{\mathbf{i}}+10\sqrt{3}\hat{\mathbf{j}}$$

$$\mathbf{v}_{BA}=\mathbf{v}_B-\mathbf{v}_A=10\hat{\mathbf{i}}+10\sqrt{3}\hat{\mathbf{j}}-10\hat{\mathbf{i}}=10\sqrt{3}\hat{\mathbf{j}}$$

22. (c)

When light is refracted its frequency will remain unchanged.

23. (c)

As we know that, the rate of cooling is

$$\frac{d\theta}{dt}=bA(\theta-\theta_0)$$

where, $bA=$ constant

$\Rightarrow \frac{d\theta}{dt}\propto\theta-\theta_0.$

Also $-\frac{dT}{dt}\propto T-T_0$

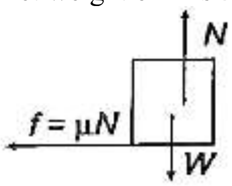
where, $T=$ temperature

and $t=$ time

So, the rate of heat absorbed by cooled body is proportional to t_2-t_1.

24. (b)

Let weight of A is W'. From the free body diagram, For equilibrium of the system,



$T\cos\theta=\mu N=\mu W$….(i)

$T\sin\theta=W'$…..(ii)

where, $T=$ tension in the thread lying between knot and the support.

On divindg Eq. (ii) by Eq. (i), we get

$$\frac{T\sin\theta}{T\cos\theta}=\frac{W'}{\mu W}$$

$$\Rightarrow\quad \tan\theta=\frac{W'}{\mu W}$$

$$\Rightarrow\quad W'=\mu W\tan\theta$$

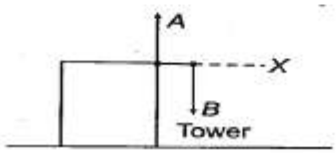
25. (c)

The output of digital circuit can be given as, $Y=\overline{AB}+\bar{C}$

It will be same as, $Y=\bar{A}+\bar{B}+\bar{C}$

26. (c)

The situation is diagrammatically as below



When A is projected with a vertical speed after sometime it comes on the same level with same speed as it was projected. Now, the downward speeds of A and B at the level- X is same. So, on reaching the ground, velocity of A and B are same

27. (b)

Given that, $2\mathrm{eV} \leq \phi \leq 5\mathrm{eV}$

Wavelength corresponding to minimum and maximum values of work function are

$$\lambda_{\max} = \frac{hc}{E} = \frac{4 \times 10^{-15} \times 3 \times 10^{8}}{2}$$

$$= 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

and $\lambda_{\min} = \frac{hc}{E} = \frac{4\times10^{-15}\times3\times10^{8}}{5}$

$= 2.4 \times 10^{-7}$ m $= 240$ nm

Clearly, wavelength range would be

$240 \leq \lambda \leq 600$

Thus, wavelength 650 nm is not suitable for photoelectric effect.

28. (a)

The change in path length $= (\mu - 1)t$. This path length change the position of bright fringe upto seventh bright fringe. The thickness of plastic substance inserted in the path of rays will must be 7 (on assuming fringe width to be units).

29. (c)

Let the length of closed organ pipe l be. Then,

$$v_n = (2n - 1)\frac{V}{4l}$$

Its third harmonics (put $n = 2$) in above question,

$v_3 = \frac{3}{4}\frac{v}{l}$.......(i)

Now, the length of open organ pipe is 21 , then

$$v_n = n\frac{v}{2(2l)} = \frac{nv}{4i}$$

Its fundamental frequency,

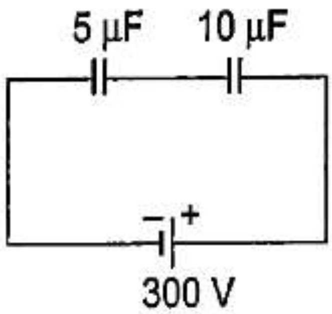
$v_0 = \frac{v}{2l} = 100$.....(ii)

From Eq. (i) and (ii), we get

$$v_3 = \frac{3v}{4l} = 3 \times 100 = 300 \text{ Hz}$$

30. (c)

According to question the figure can be drawn as below



The equivalent capacitance,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5} + \frac{1}{10}$$

$$= \frac{2+1}{10} = \frac{3}{10}$$

$$\Rightarrow C_{eq} = \frac{10}{3}\mu \text{ F}$$

Now, the energy stored in the capacitor is

$$U=\frac{1}{2}CV^2$$
$$=\frac{1}{2}\times\frac{10}{3}\times10^{-6}\times300\times300$$
$$=\frac{3}{10\times2}=\frac{3}{20}=0.15\text{ J}$$

31. (b)

Height of the cylinder from the level PQ is $H=h+\frac{h}{2}=\frac{3h}{2}$

To cover maximum distance along the plane PQ, height of the hole is $=\frac{3h}{2}\times\frac{1}{2}=\frac{3h}{4}$

This height corresponds to hole (2).

32. (c)

Given, $P=\frac{AT-BT^2}{V}$

$\Rightarrow\quad PV=AT-BT^2$

$\Rightarrow\quad P\Delta V=A\Delta T-BT\Delta T$

On integrating, we get

$$\text{Work }=\int PdV=A\int_{T_1}^{T_2}dT-B\int_{T_1}^{T_2}TdT$$
$$=A(T_2-T_1)-\frac{B}{2}[(T_2)^2-(T_1)^2]$$

33. (d)

When switch is in position a, then capacitor will be charged, such that

Charge on capacitor, $q=CE$

Energy stored in capacitor, $U=q^2/2C$

When switch is in position b, then circuit becomes an LC oscillator.

Let amplitude of current in LC circuit be I_o. From conservation of energy,

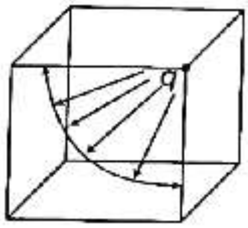
maximum electrical energy

= maximum magnetic energy

$$\frac{q^2}{2C}=\frac{1}{2}LI_0^2[\because q=CE]$$
$$\Rightarrow\quad \frac{C^2E^2}{C}=LI_0^2$$
$$\therefore\quad I_0=E\sqrt{\frac{C}{L}}$$

34. (d)

Consider the diagram

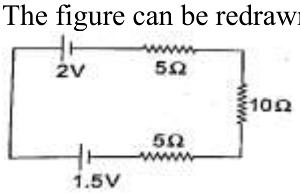


In above the flux coverage of three face is shown in figure. Which is like a quadrant in any plane. So, flux will be

$$\phi=\frac{1}{4}\times\frac{q}{6\varepsilon_0}=\frac{q}{24\varepsilon_0}$$

35. (c)

The figure can be redrawn as,



The current through the circuit

$$i = \frac{\text{net emf}}{\text{effective resistance}}$$
$$= \frac{2-1.5}{5+5+10} = \frac{0.5}{20}$$
$$= \frac{1}{40} = 0.025 \text{ A}$$

The terminal potential difference of the batteries

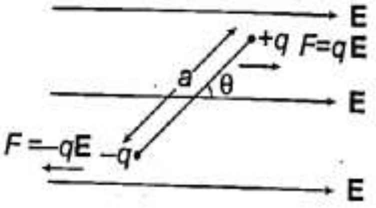
$$V_A = \varepsilon_A - ir_A = 2 - 0.025 \times 5$$
$$= 2 - 0.0125 = 1.875 \text{ V}$$

and

$$V_B = \varepsilon_B + ir_B$$
$$= 1.5 + 0.025 \times 5$$
$$= 1.5 + 0.0125 = 1.625 \text{ V}$$

36. (b,c,d)

The situation can be diagrammatically as,



The dipole moment,

$\mathbf{P} = 2qa(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}})$

The net force on the dipole is always zero while net torque on the dipole is not zero.

37. (b,c)

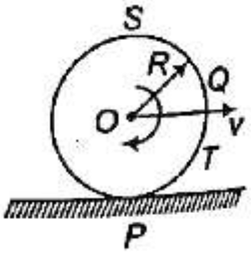
For Fraunhofer diffraction, incident beam should be parallel.
This can be achieved when
(a) Source is placed at focus of converging lens.
(b) Source is at infinite distance.

38. (b,d)

Whenever, there is a change in the magnetic flux crossing through the loop, an emf induced in it and hence there is an amount of induced current through the loop. If loop is rotated about one of its diameter or the loop is deformed the flux through it varies and causing the emf induced in it.

39. (a,c,d)

Consider the rolling motion of a circular disc as shown in the adjacent diagram. Velocity of point of contact (P) is
$v_P = v - \omega R = v - v = 0$



$[\because v = \omega R]$
velocity of point ' S ', $V_S = V + \omega R = 2V$
velocity of centre ' O ', $v_O = v - \omega(0) = v$
Velocity of a point ' T ' at distance R from P is $V_T = \omega R = Qv$.
Thus possible velocities are $v, 2v$ and 0 .

40. (a,b,d)

The de-Broglie equation is

$$\lambda=\frac{h}{mv}=\frac{h}{\sqrt{2mk}}\Rightarrow m=\frac{h}{\lambda v}$$

$$\Rightarrow m\propto\frac{1}{\lambda}, \text{ if } \frac{h}{v}= \text{ constant}$$

Chemistry

41. (a)

Flame colours are produced from the movement of the electrons in the metal ions present in the compounds. Calcium gives brick red colour. Strontium gives crimson colour. Barium gives apple green colour.

42. (a)

Extraction of gold (Au) involves the following sequence of reaction,

$$\text{Gold ore} \xrightarrow[\text{CN}^-,\text{H}_2\text{O},\text{O}_2]{\text{Roasting}} \text{HO}^- + \text{Au(CN)}_2^-$$

$$\xrightarrow{\text{Zn}} \text{Zn(CN)}_4^{2-} + \text{Au}$$

Hence, $X=\text{Au(CN)}_2^-, Y=\text{Zn(CN)}_4^{2-}$

43. (a)

Atomic number of cerium = 58

Electronic configuration of Ce

$=[\text{Xe}]4f^2, 5d^0, 6s^2$

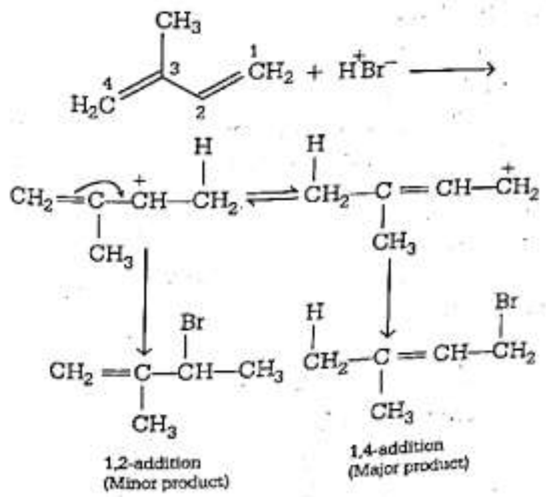
Electronic configuration of Ce^{3+}

$=[\text{Xe}]4f^1$

44. (b)

$\text{HBr}\rightleftharpoons \text{H}^+ + \text{Br}^-$

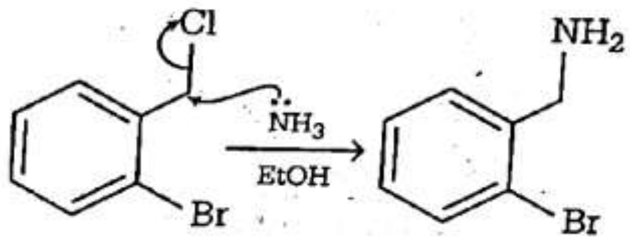
For 1,3-butadiene at room temperature, there is formation of 1,4-addition product.



Again, $\frac{h}{v}$ will be constant only when v is constant. Thus, in present case the de-Broglie wavelength of heavier particle will be smaller. When the velocities of two particles will be same or both move with same kinetic energy (K).

45.(c)

It is a substitution reaction, so the product formed is



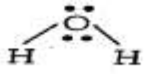
46. (a)

When sulphuryl chloride (SO_2Cl_2) reacts with white phosphorus (P_4), it give phosphorus pentachloride (PCl_5) and sulphur dioxide (SO_2). The reaction is as follows:

$\text{P}_4 + 10\text{SO}_2\text{Cl}_2 \longrightarrow 4\text{PCl}_5 + 10\text{SO}_2$

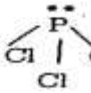
47. (a)

The number of lone pairs of electrons on the central atoms of $H_2O, SnCl_2, PCl_3$ and XeF_2 are 2, 1,1 and 3 respectively.

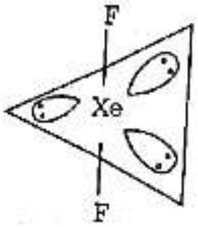


Number of lone pairs = 2

$Cl-\ddot{S}n-Cl$ Number of lone pair $=1$



Number of lone pair = 1

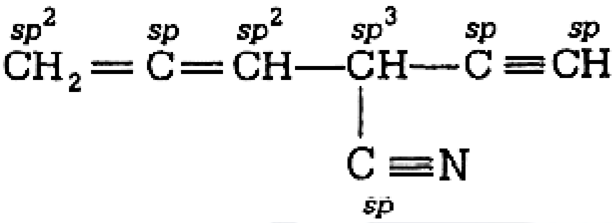


Number of lone pairs $=3$.

48. (a)

Salts such as $NaCl, HgCl_2, CuCl_2$ are soluble in water while salts such as Hg_2Cl_2, CuCl and AgCl are insoluble in water.

49. (c)



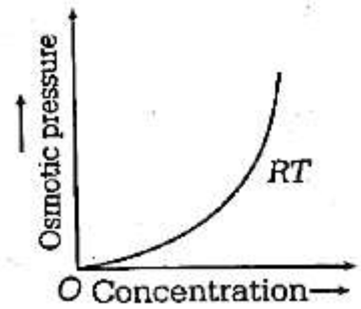
Thus, number of sp-hybridised $\mathbf{C}=4$

50. (c)

Rate of reaction is doubled when concentration of A is doubled. Again, rate of reaction becomes four times when concentration of A is increased by four times. It is clearly shown that there is no effect on rate of reaction on increasing the concentration of B.

Thus, order with respect to A is 1 and order with respect to B is 0 . Total order of reaction $=1$.

51. (b)



Slope, $RT=291R$ or $T=291$ K

$\therefore$ Temperature $=291-273=18^{\circ}C$

52. (b)

As from the formula, $v_{\mathrm{rms}}=\sqrt{\frac{3RT}{M}}$

Given that $(v_{\mathrm{rms}})_{\mathrm{CO}}=1000 \mathrm{~m/s}$

$(\text{Temp.})_{\mathrm{CO}}=27^{\circ}C=300 \mathrm{~K}$

$(\text{Temp.})_{N_2}=600 \mathrm{~K}$

Now putting the values, we get

$$\frac{(v_{\mathrm{ms}})_{\mathrm{CO}}}{(v_{\mathrm{rms}})_{N_2}}=\sqrt{\frac{3R}{3R} \times \frac{T_{\mathrm{CO}}}{M_{\mathrm{CO}}} \times \frac{M_{N_2}}{T_{N_2}}}$$

$$\frac{1000}{(v_{ms})_{N_2}} = \sqrt{\frac{300}{28} \times \frac{28}{600}} = \frac{1}{\sqrt{2}}$$

or, $(v_{rms})_{N_2} = 1000 \times 1.414$
$= 1414 \text{ m/s}$

53. (b)

A gas can be liquefied at temperature T when it is lower than critical temperature, T_c and pressure p is greater than critical pressure.

Hence, $T < T_c$ and $p > p_c$.

54. (c)

The dispersed phase is liquid and dispersion medium is gas in the fog.

55. (a)

On the basis of electropositivity of metal, the order of basic character is
$K_2O > BaO > CaO > MgO$

56. (a)

In aqueous alkaline solution, two electron reduction of HO_2^- gives $\overline{O}H$. The reaction is as follows:

$H_2O_2 \rightleftharpoons H^+ + HO_2^-$
$HO_2^- + H_2O + 2e^- \rightleftharpoons 3O^-H^\circ$

57. (a)

Cold $FeSO_4$ solution on absorption of NO develops brown colour due to the formation of $[Fe(H_2O)_5(NO)]SO_4$.

$$FeSO_4 + NO + 5H_2O \rightarrow \left[\underset{\textit{Brown ring complex}}{Fe(H_2O)_5 \quad (NO)}\right] SO_4$$

This complex has 3.89 BM magnetic moment shows that it has 3 unpaired electrons.

58. (a)

Group II A Group III A.

Be	B
Mg	Al

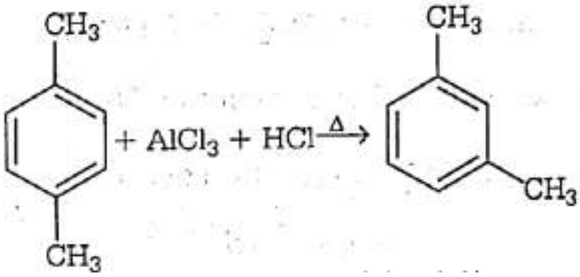
The electronic configuration of boron is $2s^2 2p^1$. And electronic configuration of $B^+$is $2s^2$. Hence, it is difficult to remove second electron from $2s^2$ shell because half-filled and fully filled orbitals are more stable than others.

59. (c)

fully filled orbitals are more stable than others.

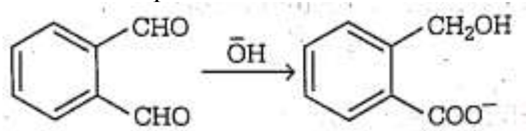
59. 15% will form racemic mixture with another 15%. Hence, the enantiomeric excess is $= (85 - 15) = 70\%$

60. (b)



61. (c)

It is an example of intramolecular Cannizzaro reaction.



62. (a)

Given that,
Mass of single Ag-atom $= m$
Unit cell length $= a$
Total atoms present per unit cell of fcc lattice $= 4$
Therefore, mass of unit cell becomes 4 m .

Volume of unit cell $= a^3$.
Density $(\rho) = \frac{\text{Mass of unit cell}}{\text{Volume of unit cell}} = \frac{4m}{a^3}$

63. (a)

For the reaction,
$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$
Given that, $\Delta G^\circ = -690.9R, T = 300$ K
From the formula, $\Delta G^\circ = -RT\ln K$
$-690.9R = -R \times 300 \times \ln K$
or $\ln k = \frac{690.9}{300} = 2.303$
or $= 2.303\log K = 2.303$
or $K = 10^1 = 10$
Unit of $K = (\text{atm})\Delta^n = (\text{atm})^{2-3} = (\text{atm})^{-1}$
$\therefore\ K = 10\ \text{atm}^{-1}$

64. (a)

As we know that,
Equivalent conductance, (λ)
$= \frac{\text{Specific conductance } (K) \times 1000}{\text{Conduct}}$
Concentration
or, $\frac{\lambda}{K} = \frac{1000}{\text{conc.}}$
or,
$\frac{\lambda}{K} = \frac{1000\Omega^{-1}\ \text{cm}^2\text{eq}^{-1}}{0.01\Omega^{-1}\ \text{cm}^{-1}}$
(Given, conc. $= 0.01$ N)
or,
$\frac{\lambda}{K} = 10^5\ \text{cm}^3\text{eq}^{-1}$

65. (b)

From the formula,
Surface tension, (γ)
$= \frac{\Delta W}{\Delta A} = \frac{\text{J}}{\text{m}^2} = \frac{\text{kg m}^2\ \text{s}^{-2}}{\text{m}^2} = \text{kg s}^{-2}$
Form the formula, $F = \eta \cdot A \cdot \frac{dV}{dx}$
or, $\dot{\eta}$ (coefficient of viscosity)
$= \frac{F}{A \cdot \frac{dV}{dx}} = \frac{\text{N}}{\text{m}^2 \cdot \frac{\text{ms}^{-1}}{\text{m}}} = \text{Nm}^{-2} \cdot \text{ s}$
or $\text{kgms}^{-2} \cdot \text{ m}^{-2} \cdot \text{ s} = \text{kgm}^{-1}\ \text{s}^{-1}$

66. (d)

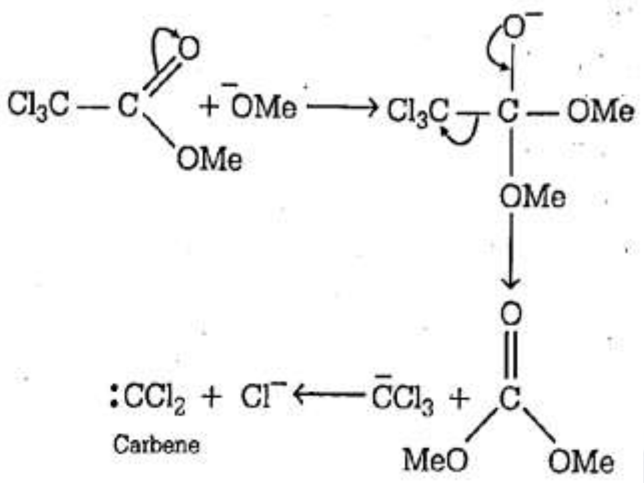
Given, that pH $= 5.74$, p$K_a = 4.74$
Suppose that volume of acid solution $= x$ L
Volume of salt solution $= y$ L
From Henderson equation,
$\text{pH} = \text{p}K_a + \log\frac{[\text{ Salt }]}{[\text{ Acid }]}$
or, $\text{pH} - \text{p}K_a = \log\frac{[CH_3COONa]}{[CH_3COOH]}$
or, $5.74 - 4.74 = 1 = \log\frac{[CH_3COONa]}{[CH_3COOH]}$
or $\frac{CH_3COONa]}{[CH_3COOH]} = 10$

or $\frac{[CH_3COOH]}{[CH_3COONa]} = \frac{1}{10} = \frac{\frac{0.1x}{x+y}}{\frac{0.1y}{x+y}}$

Thus, $\frac{x}{y} = \frac{1}{10}$

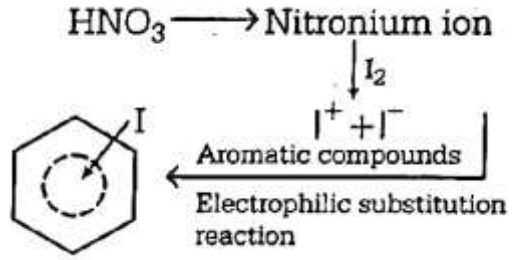
67. (b)

$NaOMe \rightleftharpoons Na^+ + O^-Me$



68. (d)

Best reagent for nuclear iodination of aromatic compounds is I_2/HNO_3.



69. (c)

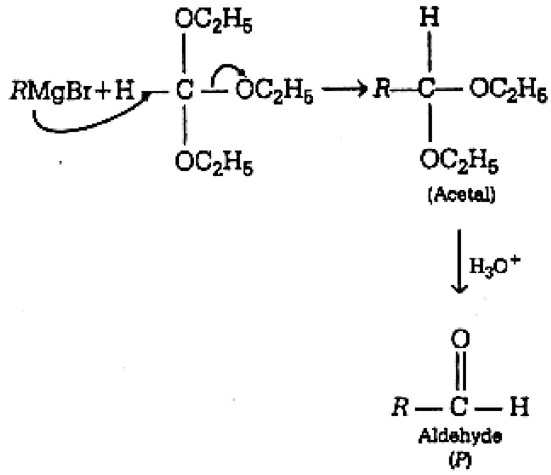
To a part of sodium extract, $FeSO_4$ solution is added and the contents are warmed. A few drops of $FeCl_3$ solution are then added and resulting solution is acidified with conc. HCl . The appearance of bluish green or Prussian blue colouration confirms the presence of nitrogen. The reactions that occur during this test are as

$2NaCN + FeSO_4 \longrightarrow Na_2SO_4 + Fe(CN)_2$

$Fe(CN)_2 + 4NaCN \longrightarrow \underset{sodium\ ferrocyanide}{Na_4[Fe(CN)_6]}$

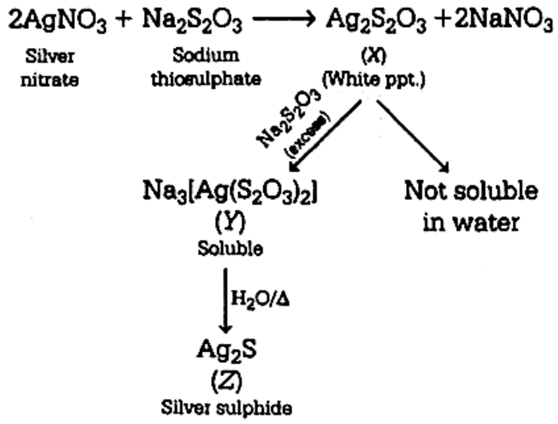
$3Na_4[Fe(CN)_6] + 4FeCl_3 \rightarrow \underset{\substack{Ferric\ ferrocyanide \\ (prussian\ blue)}}{Fee_4[Fe(CN)_6]_3} + 12NaCl$

70. (a)



71. (a)

The sequence of reactions are



Hence, $X=\mathrm{Ag}_2\ \mathrm{S}_2\mathrm{O}_3$

$Y=\mathrm{Na}_3[\mathrm{Ag}(\ \mathrm{S}_2\mathrm{O}_3)_2] Z=\mathrm{Ag}_2\ \mathrm{S}$

72. (a)

From the given cell, the cell reaction is

$2\mathrm{Ag}(s)+\mathrm{Zn}^{2+}(0.1\mathrm{M})\longrightarrow$

$2\mathrm{Ag}^+(0.1\mathrm{M})+\mathrm{Zn}(\mathrm{s})$

The Nernst equation is

$$E_{\text{cell}}=E^\circ{}_{\text{cell}}-\frac{0.0591}{n}\log\frac{[\mathrm{Ag}^+]^2}{[\mathrm{Zn}^{2+}]}$$

$$\text{or, } E_{\text{cell}}=(-1.562)-\frac{(0.0591)}{2}\log\frac{(0.1)^2}{(0.1)}$$

$$\text{or, } E_{\text{cell}}=(-1.562)-\frac{0.0591}{2}\log 10^{-1}$$

$$=-1.562+\frac{0.0591}{2}$$

$$=-1.562+0.02955$$

$$=-1.532\ \mathrm{V}$$

73. (c)

For the reaction $X_2Y_4(l)\longrightarrow 2XY_2(g)$,

$\Delta n_g=$ number of gaseous products

number of gaseous reactants

$\Delta n_g=2-0=2$

Given that, $\Delta U=2\mathrm{kcal}$

$\Delta S=20\mathrm{cal\ K}^{-1}$

From the formula,

$\Delta H=\Delta U+\Delta n_gRT$

Putting the values, we get

$$\Delta H=2+\left(\frac{2\times 2\times 300}{1000}\right)$$

$$=3.2\mathrm{kcal}=3.2\times 10^3\mathrm{cal}$$

Also, $\Delta G=\Delta H-T\Delta S$

$$=3.2\times 10^3-300\times 20$$

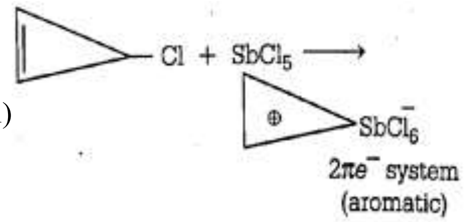
$$=3.2\times 10^3-6\times 10^3$$

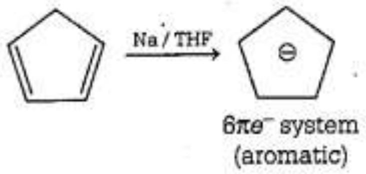
$$=-2800\mathrm{cal}$$

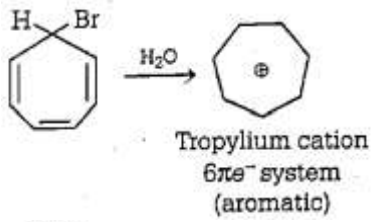
74. (c)

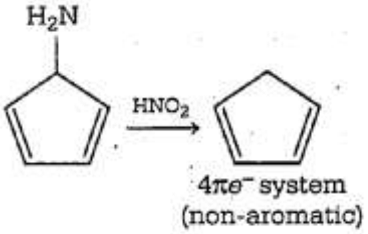
Criteria for aromaticity (Huckel rule)

The cyclic π molecular orbital (electron cloud) formed by overlap of p-orbitals must contain $(4n+2)\pi$-electrons, where $n=$ integer 0,1,2,3, etc. This is known as Huckel rule.

(i) 

(ii) 

(iii) 

(iv) 

75. (a)

Roasted copper pyrite on smelting with sand produces $FeSiO_3$ as fusible slag and Cu_2 S as matte. For removing the gangue, FeS, silica present in the lining of the Bessemer converter, acts as a flux and forms slag (iron silicate) on reaction with FeO .

$$\underset{\text{Gangue}}{FeS} + 3O_2 \longrightarrow 2FeO + 2SO_2$$

$$\underset{\text{Gengue}}{FeO} + SiO_2 \longrightarrow \underset{\text{Slag}}{FeSiO}$$

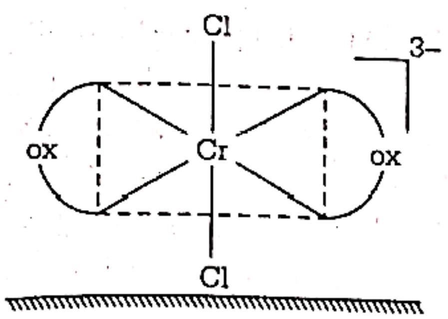
Copper matte mainly contains Cu_2 S and FeS .

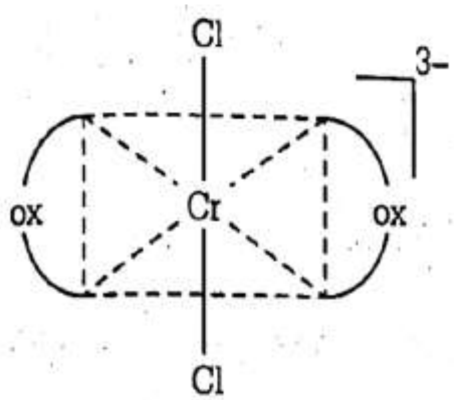
76. (a,c)

Ionisation potential values of noble gases decrease down the group with increase in atomic size. Xenon forms binary fluorides by the direct reaction of elements.

The reason is that only the heavier noble gases form such compounds. Octet of electrons provide the stable arrangements but this statement is not very much valid in cases of higher element which can be ionised easily due to large distance between nucleus and valence electrons.

77. (a,b,d)





trans $[CrCl_2(ox)_2]^{3-}$ isomer-optically inactive (superimposable mirror images and plane of symmetry).
cis- $[CrCl_2(ox)_2]^{3-}$, $[Co(en)_3]^{3+}$ and $[Co(ox)(en)_2]^{+}$ exhibited optical isomerism.

78. (a,d)

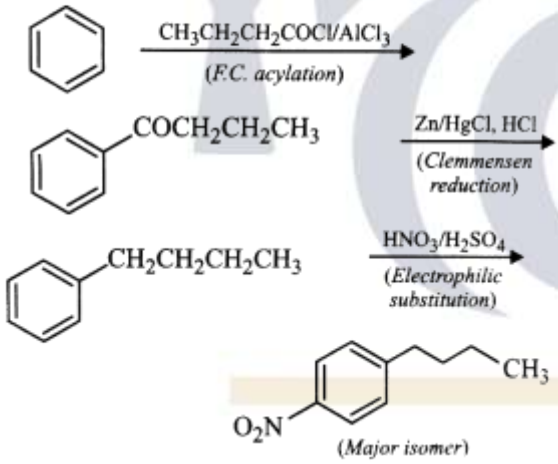
The increase in rate constant of a chemical reaction with increasing temperature are due to the following facts.
The number of collisions among the reactant molecules increases with increasing temperature.
The number of reactant molecules acquiring the activation energy increases with increasing temperature.

79. (a)

Isoelectric point for neutral amino acids ranges from pH 5.5 to 6.3. Acidic amino acids have isoelectric point around 3 while basic amino acids have ranges from pH 7.6 to 10.8 .
At isoelectric point, amino acids exist as Zwitter ions. At low pH , it exists as cation and at high pH , it exists as anion.
Hence, alanine (pH range 2-4) will exist as cation while alanine (pH range 9-11) will exist as anion.
Acidic medium:
$H_3\,N^+ - CH(CH_3)CO_2H$
Basic medium:
$H_2\,N - CH(CH_3)CO_2^-$

80. (b)

The sequence of reaction is as follows:



Mathematics

81. (d)

From option (d), we have
$a-b=c-d \ldots \ldots(i)$
and $a^2-b^2=c^2-d^2 \ldots \ldots(ii)$
Consider, $a^2-b^2=c^2-d^2$
$\Rightarrow (a+b)(a-b)=(c-d)(c+d)$
$\Rightarrow a+b=c+d \ldots \ldots(iii)$
sing Eq. (i)]
On adding Egs. (i) and (iii), we get

$2a = 2c \Rightarrow a = c$
$\Rightarrow b = d$
[using Eq. (iii)]
Thus, $a^n + b^n = c^n + d^n$ for all $n \in N$.

82. (a)

Since, α and β are the roots of $x^2 - px + 1 = 0$.
$\therefore\ \alpha + \beta = p$ and $\alpha\beta = 1$
Also, γ is the root of $x^2 + px + 1 = 0$.
$\therefore \gamma^2 + p\gamma + 1 = 0 \Rightarrow \gamma^2 = -p\gamma - 1$
Now, $(\alpha + \gamma)(\beta + \gamma) = \alpha\beta + \alpha\gamma + \beta\gamma + \gamma^2$
$= 1 + \gamma(\alpha + \beta) - p\gamma - 1 = \gamma(\alpha + \beta - p)$
$= \gamma \times 0 = 0\ [\because \alpha + \beta = p]$

83. (c)

General term of $\left(3^{1/5} + 7^{1/3}\right)^{100}$ is given by $T_{r+1} = {}^{100}C_r\left(3^{1/5}\right)^{100-r}\left(7^{1/3}\right)^r$
$= {}^{100}C_r \cdot 3^{\frac{100-r}{5}} \cdot 7^{\frac{1}{3}}$
For a rational term, $\frac{100-r}{5}$ and $\frac{r}{3}$ must be integer.
Hence, $r = 0,15,30,45,60,75,90$
$\therefore$ There are seven rational terms.
Hence, number of irrational terms
$= 101 - 7 = 94$

84. (c)

Given equation is $(2x + 1)^2 - px + q \neq 0$
$\Rightarrow \quad 4x^2 + 4x + 1 - px + q \neq 0$
$\Rightarrow \quad 4x^2 + (4 - p)x + (1 + q) \neq 0$
Now, $D < 0$
$\Rightarrow (4 - p)^2 - 4(4)(1 + q) < 0$
$\Rightarrow 16 - 8p + p^2 - 16 - 16q < 0$
$\Rightarrow p^2 - 8p - 16q < 0$

85. (a)

Let A be the set of families who own a car and B be the set of families who own a house.
Then, $P(A) = 60\%, P(B) = 30\%$
and $P(A \cap B) = 20\%$
Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 60 + 30 - 20 = 70\%$
Now, families who owns a car or a house but not both $= A\Delta B$
$\Rightarrow P(A \wedge B) = P(A \cup B) - P(A \cap B)$
$= 70 - 20 = 50\% = 0.5$

86. (c)

The number of words starting with CC = 4 !
The number of words starting with CH = 4 !
The number of words starting with CI = 4 !
The number of words starting with CN = 4 ! COCHIN is the first word in the list of words beginning with CO .
$\therefore$ Number of words that appear before the word COCHIN = 96

87. (a)

Given,
$f(x) = \int_0^x f(t)dt$.....(i)
Using Leibnitz theorem, we get
$f'(x) = f(x) \Rightarrow f(x) = ke^x$
On putting $x = 0$ in Eq. (i), we get

$f(0) \quad = \int_0^0 f(t)dt$

$\Rightarrow ke^0 \quad = 0\left[\because \int_a^a f(x)dx = 0\right]$

$k = 0 \quad e^0 = 1$

$\therefore\ f(x) = 0 \Rightarrow f(\log_\theta 5) = 0$

88. (a)

Let $y = \frac{x^2-x+4}{x^2+x+4}$

$x^2y + xy + 4y = x^2 - x + 4$

$\Rightarrow (y-1)x^2 + (y+1)x + 4y - 4 = 0$

For x to be real, discriminant of the above quadratic equation should be greater than or equal to 0 .

$\Rightarrow \quad (y+1)^2 - 4(y-1)(4y-4) \geq 0$

$\Rightarrow \quad (y+1)^2 - 16(y-1)^2 \geq 0$

$\Rightarrow \quad (y+1+4y-4)(y+1-4y+4) \geq 0$

$\Rightarrow \quad (5y-3)(5-3y) \geq 0$

$\therefore\ y \in \left[\frac{3}{5}, \frac{5}{3}\right]$

89. (d)

$2x^2 + y^2 + 2xy + 2x - 3y + 8$

$= \frac{1}{2}[4x^2 + 2y^2 + 4xy + 4x - 6y + 16]$

$= \frac{1}{2}[(y^2 - 8y) + (4x^2 + y^2 + 4xy + 4x + 2y) + 16]$

$= \frac{1}{2}[(y-4)^2 + (2x+y+1)^2 - 1]$

So, least value will be $-\frac{1}{2}$ at $y = 4$ and $x = \frac{-5}{2}$.

90. (d)

If a function $f(x)$ assumes only irrational values which is also continuous, then $f(x)$ must be a constant function.

$\Rightarrow \quad f(x) \quad = \sqrt{2}$

$\therefore \quad f(\sqrt{2}-1) = \sqrt{2} \quad [\because f(\sqrt{2}) = \sqrt{2}]$

91. (a)

Here, $\cos\theta + \sin\theta + \frac{2}{\sin 2\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$

For minimum value, $\sin 2\theta$ must be maximum.

$\therefore\ 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

Hence, $\cos\frac{\pi}{4} + \sin\frac{\pi}{4} + \frac{2}{\sin\frac{\pi}{2}} = \sqrt{2} + 2$

92. (b)

$\lim_{x\to 2}\int_2^x \frac{3t^2}{(x-2)}dt = \frac{\lim_{x\to 2}\int_2^x 3t^2 dt}{\lim_{x\to 2}(x-2)}$

$\frac{\lim\limits_{x\to 2} 3x^2}{1} (using\ L\ hospital; s\ rule$

$=3 \times (2)^2 = 12$

93. (b)

Given, $\cot\frac{2x}{3} + \tan\frac{x}{3} = \mathrm{cosec}\frac{kx}{3}$

Let $\theta = \frac{x}{3}$, then

$\therefore\ \cot 2\theta + \tan\theta = \mathrm{cosec} k\theta$

$\Rightarrow \frac{\cos 2\theta}{\sin 2\theta} + \frac{\sin\theta}{\cos\theta} = \mathrm{cosec} k\theta$

$\Rightarrow \dfrac{2\cos^2\theta - 1}{2\sin\theta\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \operatorname{cosec} k\theta$

$\Rightarrow \dfrac{2\cos^2\theta - 1 + 2\sin^2\theta}{2\sin\theta\cos\theta} = \operatorname{cosec} k\theta$

$\Rightarrow \dfrac{1}{2\sin\theta\cos\theta} = \operatorname{cosec} k\theta$

$\Rightarrow \operatorname{cosec} 2\theta = \operatorname{cosec} k\theta$

$\therefore\ k = 2$

94. (b)

$\sqrt{4\cos^4\theta + \sin^2 2\theta} + 4\cot\theta\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \sqrt{4\cos^4\theta + (2\sin\theta\cos\theta)^2}$

$+2\cot\theta\left[2\cos^2\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)\right]$

$= \sqrt{4\cos^4\theta + 4\sin^2\theta\cos^2\theta}$

$+2\cot\theta\left[1 + \cos\left(\dfrac{\pi}{2} - \theta\right)\right]$

$[\because 2\cos^2\theta = 1 + \cos 2\theta]$

$= \sqrt{4\cos^2\theta(\cos^2\theta + \sin^2\theta)} + 2\cot\theta(1 + \sin\theta)$

$= |2\cos\theta| + 2\cot\theta + 2\cos\theta$

$= -2\cos\theta + 2\cot\theta + 2\cos\theta$

$\left[\text{for } \theta \in \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right), |\cos\theta| = -\cos\theta\right]$

$= 2\cot\theta$

95. (c)

Given, $(\sin x - x)(\cos x - x^2) = 0$

$\Rightarrow\ \sin x = x$ or $\cos x = x^2$

Now, if $\sin x = x$, then only one solution i.e. $x = 0$ is possible. Also, if $\cos x = x^2$, then two solutions are possible. Hence, there are total 3 solutions.

96. (b)

We know that, $\omega = \frac{-1+\sqrt{3}i}{2} 1 - \sqrt{3}i = -2\omega$ and $\omega^2 = \frac{-1-\sqrt{3}i}{2} 1 + \sqrt{3}i = -2\omega^2$

Now, $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{64} + \left(\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^{64}$

$= \left(\dfrac{-2\omega^2}{-2\omega}\right)^{64} + \left(\dfrac{-2\omega}{-2\omega^2}\right)^{64}$

$= \omega^{64} + \dfrac{1}{\omega^{64}} = \omega + \omega^2 \qquad [\because \omega^3 = 1]$

$= -1 \qquad [\because 1 + \omega + \omega^2 = 0]$

97. (b)

We have,

$|z| = \left|z - \dfrac{3}{z} + \dfrac{3}{z}\right|$

$\Rightarrow |z| \leq \left|z - \dfrac{3}{z}\right| + \left|\dfrac{3}{z}\right|$

[using triangle inequality]

$\Rightarrow\ |z| \leq 2 + \left|\dfrac{3}{z}\right| \ \left[\because \left|z - \dfrac{3}{z}\right| = 2\right]$

$\Rightarrow\ |z| \leq 2 + \dfrac{3}{|z|}$

$\Rightarrow\ |z|^2 - 2|z| - 3 \leq 0$

$\Rightarrow\ (|z| - 3)(|z| + 1) \leq 0$

$\Rightarrow\ -1 \leq |z| \leq 3$

Thus, the maximum value of $|z|$ is 3 .

98. (d)

We have, $\frac{5x^2-26x+5}{3x^2-10x+3}<0$

$$\Rightarrow \frac{5x^2-25x-x+5}{3x^2-9x-x+3}<0$$

$$\Rightarrow \frac{5x(x-5)-1(x-5)}{3x(x-3)-1(x-3)}<0 \Rightarrow \frac{(x-5)(5x-1)}{(x-3)(3x-1)}<0$$

$\therefore\ x \in \left(\frac{1}{5},\frac{1}{3}\right) \cup (3,5)$

99. (d)

If the given system of equations has no solution,

then $\begin{vmatrix} 2 & -1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow 2(-2\lambda-1)+1(\lambda-1)-2(1+2)=0$

$\Rightarrow\ -4\lambda-2+\lambda-1-6=0$

$\Rightarrow\ -3\lambda-9=0$

$\Rightarrow\ \lambda=-3$

100. (a)

We have, $f(x)=\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

Taking common $x_1(x-1)$ from R_2 and R_3 respectively, we get

$$f(x)=x(x-1)\begin{vmatrix} 1 & x & x+1 \\ 2 & x-1 & x+1 \\ 3x & x(x-2) & (x+1)x \end{vmatrix}$$

Taking common ($x+1$) from C_3, we get

$$f(x)=x(x-1)(x+1)\begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ 3x & x(x-2) & x \end{vmatrix}$$

Taking common x from R_3, we get

$$f(x)=x^2(x-1)(x+1)\begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ 3 & x-2 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1-R_2, R_2 \to R_2-R_3$, we get

$$f(x)=x^2(x-1)(x+1)\begin{vmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 3 & x-2 & 1 \end{vmatrix}=0$$

[$\because R_1$ is identical with R_2]

$\Rightarrow f(100)=0$

101. (b)

We have, $x_n=\left[\left(1-\frac{1}{3}\right)\left(1-\frac{1}{6}\right)\right.$

$$\Rightarrow x_n = \left[\prod_{n=2}^{n}\left(\frac{n^2+n-2}{n(n+1)}\right)\right]^2$$

$$= \left[\prod_{n=2}^{n}\left(\frac{(n+2)(n-1)}{n(n+1)}\right)\right]^2$$

$$= \left[\prod_{n=2}^{n}\left(\frac{n+2}{n+1}\right)\cdot\prod_{n=2}^{n}\left(\frac{n-1}{n}\right)\right]^2$$

$$= \left[\prod_{n=2}^{n}\left(\frac{n+2}{n+1}\right)\right]^2\left[\prod_{n=2}^{n}\left(\frac{n-1}{n}\right)\right]^2$$

$$\Rightarrow x_n = \left(\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdots\frac{n+2}{n+1}\right)^2\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdots\frac{n-1}{n}\right)^2$$

$$\Rightarrow x_n = \left(\frac{n+2}{3}\right)^2\left(\frac{1}{n}\right)^2 \Rightarrow x_n = \frac{1}{9}\left(\frac{n+2}{n}\right)^2$$

$$\Rightarrow x_n = \frac{1}{9}\left(1+\frac{2}{n}\right)^2$$

$$\Rightarrow \lim_{n\to\infty} x_n = \frac{1}{9}(1+0)^2 = \frac{1}{9}$$

102. (a)

Variance of n natural numbers

$$= \frac{n^2-1}{12} = \frac{(20)^2-1}{12}$$

$$= \frac{400-1}{12} = \frac{399}{12} = \frac{133}{4}$$

103. (b)

Let the coin be tossed n times.

Let getting head is consider to be success.

$\therefore\ p = \frac{1}{2}, q = 1-p = 1-\frac{1}{2} = \frac{1}{2}$

It is given that,

$$P(X=3) = P(X=5)$$

$$\Rightarrow {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3} = {}^nC_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{n-5}$$

$$\Rightarrow {}^nC_3 = {}^nC_5$$

$$\Rightarrow n = 3+5$$

$$\Rightarrow n = 8$$

Now, $P(X=1) = {}^8C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{8-1}$

$= {}^8C_y \Rightarrow x+y=n]$

$${}^8C_1 \times \left(\frac{1}{2}\right)^8 = \frac{1}{32}$$

104. (a)

Total number of ways in which the letters of the word 'PROBABILITY' can be arranged in a row $= \frac{11!}{2!2!}$

Number of ways in which two B's are together $= \frac{10!}{2!}$

$\therefore$ Required probability

$$= \frac{\frac{10!}{2!}}{\frac{11!}{2!\,2!}} = \frac{10!\times 2!}{11!} = \frac{2}{11}$$

105. (d)

(a) If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other, then $\mathbf{a}\cdot\mathbf{b}=0$.
Now consider,
$|\mathbf{a}+\mathbf{b}|^2=(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})$
$=|\mathbf{a}|^2+|\mathbf{b}|^2$
So, option (a) is always true.
(b) If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other, then $\mathbf{a}\cdot\mathbf{b}=0$.
Now consider,

$$|\mathbf{a}+\lambda\mathbf{b}|^2=(\mathbf{a}+\lambda\mathbf{b})\cdot(\mathbf{a}+\lambda\mathbf{b})$$
$$=|\mathbf{a}|^2+\lambda^2|\mathbf{b}|^2$$
$$\Rightarrow |\mathbf{a}+\lambda\mathbf{b}|=\sqrt{|\mathbf{a}|^2+\lambda^2|\mathbf{b}|^2}$$
$$\geq |\mathbf{a}| \text{ for all } \lambda\in R$$

So, option (b) is always true.
(c) Consider,

$$|a+b|^2+|a-b|^2=(a+b)\cdot(a+b)$$
$$=|\mathbf{a}|^2+\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{a}+|\mathbf{b}|^2+|\mathbf{a}-\mathbf{b}|^2-\mathbf{a}\cdot\mathbf{b}$$
$$=2(|\mathbf{a}|^2+|\mathbf{b}|^2)$$

So, option (c) is always true.
(d) Consider,
$\mathbf{a}=-\mathbf{b}$ and $\mathbf{b}\neq 0$
Then, $|\mathbf{a}+\lambda\mathbf{b}|\geq|\mathbf{a}|$
$\Rightarrow |-b+\lambda b|\geq|-b|$
$\Rightarrow |\mathbf{b}||\lambda-1|\geq|\mathbf{b}|\Rightarrow|\lambda-1|\geq 1$
which is not true for all λ, as we consider $\lambda=\frac{1}{2}$, then it is not true.
Hence, option (d) is not always true.

106. (a)

If four points $(x_1,y_1,z_1),(x_2,y_2,z_2),(x_3,y_3,z_3)$ and (x_4,y_4,z_4) are coplanar, then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix}=0$$

Now, $\begin{vmatrix} 3 & 0 & 0 \\ 2 & 0 & -2 \\ 2 & \lambda-1 & 0 \end{vmatrix}=0$

$$\Rightarrow 3(0+2\lambda-2)=0$$
$$\Rightarrow \lambda=1$$

107. (d)

Consider the given equation of lines,
and
$x-(t+\alpha)=0$
$y+16=0$.....(ii)
Since, these lines are concurrent, therefore the system of equations is consistent.

Now, $\begin{vmatrix} 1 & 0 & -(t+\alpha) \\ 0 & 1 & 16 \\ -\alpha & 1 & 0 \end{vmatrix}=0$

$$\Rightarrow 1(0-16)-(t+\alpha)(0+\alpha)=0$$
$$\Rightarrow -16-\alpha(t+\alpha)=0$$
$$\Rightarrow \alpha(t+\alpha)+16=0$$
$$\Rightarrow \alpha^2+t\alpha+16=0$$

Clearly, α should be real.
$\therefore t^2-4\times 16\geq 0\Rightarrow t^2-64\geq 0$
Hence, least positive value of t is 8 .

108. (b)

We have, $a^2\cos^2 A-b^2-c^2=0$
$\Rightarrow a^2\cos^2 A=b^2+c^2$
Also in $\triangle ABC$, we have

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{a^2\cos^2 A - a^2}{2bc}$$
$$= \frac{-a^2(1-\cos^2 A)}{2bc} = \frac{-a^2\sin^2 A}{2bc} < 0$$
$[\because 0 < A < \pi; a, b, c > 0]$

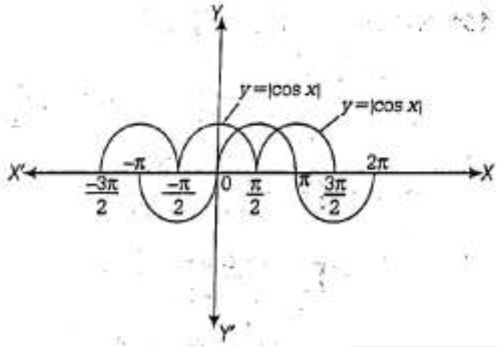
$\Rightarrow \cos A < 0 \Rightarrow A$ lies in IInd quadrant.

$\Rightarrow \frac{\pi}{2} < A < \pi$

109. (a)

Given, $\{x \in R: |\cos x| \geq \sin x\} \cap \left[0, \frac{3\pi}{2}\right]$

If we draw the graphs of $|\cos x|$ and $\sin x$, clearly $|\cos x| \geq \sin x$ when



$$x \in \left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{3\pi}{2}\right]$$
$$\therefore x \in \left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{3\pi}{2}\right] \cap \left[0, \frac{3\pi}{2}\right]$$
$$\Rightarrow x \in \left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{3\pi}{2}\right]$$

110. (a)

We have, $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \cdots\right) = \frac{\pi}{6}$

$$\Rightarrow \sin^{-1}\left(\frac{x}{1-\left(\frac{-x}{2}\right)}\right) = \frac{\pi}{6}\left[\because S_\infty = \frac{a}{1-r}\right]$$
$$\Rightarrow \sin^{-1}\left(\frac{2x}{2+x}\right) = \frac{\pi}{6}$$
$$\Rightarrow \frac{2x}{2+x} = \sin\frac{\pi}{6} \Rightarrow \frac{2x}{2+x} = \frac{1}{2}$$
$$\Rightarrow 4x = 2 + x \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$$

111. (a)

We have, $y = x^3$ and $A(1,1)$

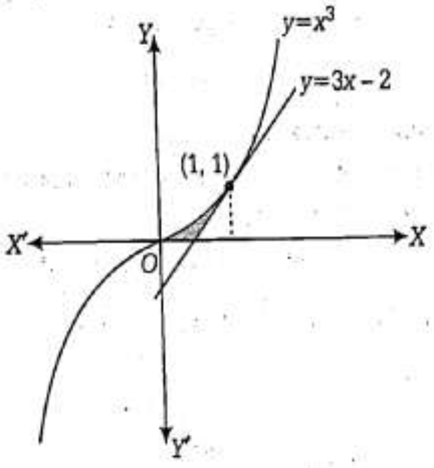
$\therefore \frac{dy}{dx} = 3x^2 (i)$

On putting $x = 1$ in Eq. (i), we get

$\frac{dy}{dx} = 3(1)^2 = 3$

$\therefore$ Equation of tangent at $A(1,1)$ is

$y - 1 = 3(x-1) \Rightarrow y = 3x - 2$



∴ Required area

$= \int_0^1 x^3 dx - \int_{2/3}^1 (3x-2)dx$

$= \left[\frac{x^4}{4}\right]_0^1 - \left[\frac{3x^2}{2} - 2x\right]_{2/3}^1$

$= \frac{1}{4} - \left[\left(\frac{3}{2} - 2\right) - \left(\frac{2}{3} - \frac{4}{3}\right)\right]$

$= \frac{1}{12}$ sq unit

112. (c)

We have, $\log_{0.2}(x-1) > \log_{0.04}(x+5)$

$\Rightarrow \quad \log_{0.2}(x-1) > \log_{(0.2)^2}(x+5)$

$\Rightarrow \quad \log_{0.2}(x-1) > \frac{1}{2}\log_{0.2}(x+5)$

$\Rightarrow \quad 2\log_{0.2}(x-1) > \log_{0.2}(x+5)$

$\Rightarrow \quad \log_{0.2}(x-1)^2 > \log_{0.2}(x+5)$

$\Rightarrow \quad (x-1)^2 < x+5$

$\Rightarrow \quad [\because \log_a x > \log_a y \Rightarrow x < y, \text{if } 0 < a < 1]$

$\Rightarrow \quad x^2 - 2x + 1 < x + 5$

$\Rightarrow \quad x^2 - 3x - 4 < 0$

$\Rightarrow \quad x^2 - 4x + x - 4 < 0$

$\Rightarrow \quad x(x-4) + 1(x-4) < 0$

$\Rightarrow \quad (x-4)(x+1) < 0$

$\Rightarrow \quad x \in (-1,4)$

But $\quad x > 1$

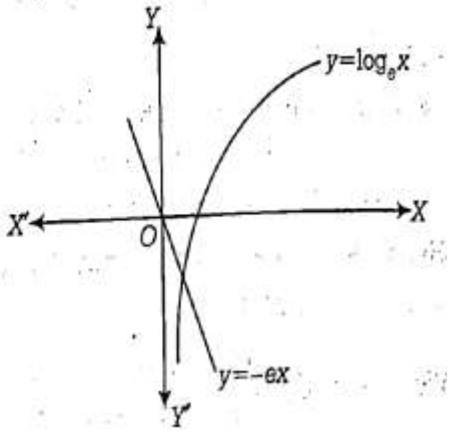
$\Rightarrow \quad x \in (1,4)$

113. (b)

We have, $\log_e x + ex = 0$

$\Rightarrow \log_e x = -ex$

Since, both the graphs intersect at only one point.



Hence, the number of real roots of equation is one.

114. (c)

Given, $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$\Rightarrow \sin x(\sin^2 x - \cos^2 x) - \cos x(\sin x \cos x - \cos^2 x)$

$+\cos x(\cos^2 x - \sin x \cos x) = 0$

$\Rightarrow \sin x(\sin x - \cos x)(\sin x + \cos x)$

$-\cos^2 x(\sin x - \cos x) - \cos^2 x$

$(\sin x - \cos^2 x) = 0$

$\Rightarrow (\sin x - \cos x)(\sin^2 x + \sin x \cos x - 2\cos^2 x) = 0$

$\Rightarrow (\sin x - \cos x)(\sin^2 x + 2\sin x$

$\cos x - \sin x \cos x - 2\cos^2 x) = 0$

$\Rightarrow (\sin x - \cos x)^2(\sin x + 2\cos x) = 0$

$\Rightarrow \sin x - \cos x = 0$ or $\sin x + 2\cos x = 0$

$\Rightarrow \tan x = 1$ or $\tan x = -2$

But $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$\Rightarrow$ Only one solution i.e. $x = \frac{\pi}{4}$

115. (b)

Since, $x_1, x_2, \ldots, x_{15}$ are among $1,2,3, \ldots, 15$. So, x_1 can take any value from $1,2,3, \ldots, 15$. Among these values, one of the number must be 1 , hence product will be 0 .

116. (c)

We have, $f(x) = \begin{cases} \frac{\sin[-x^2]}{[-x^2]}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$

Now, $\lim_{x \to 0} \frac{\sin[-x^2]}{[-x^2]} = \frac{\sin(-1)}{(-1)} = \sin(1)$

Since, $f(x)$ is continuous at $x = 0$.

$\Rightarrow . \lim_{x \to 0} f(x) = f(0) \Rightarrow \sin(1) = \alpha$

117. (a)

Let $f(x) = \frac{a_3 x^4}{4} + \frac{a_2 x^3}{3} + \frac{a_1 x^2}{2} + a_0 x$

$\therefore f(0) = 0, f(1) = \frac{a_3}{4} + \frac{a_2}{3} + \frac{a_1}{2} + a_0 = 0$

$\Rightarrow f(0) = f(1)$

$\Rightarrow f'(x) = 0$ has atleast one real root in $[0,1]$.

[according to Rolle's theorem]

$\therefore f'(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$

Hence, $a_3 x^3 + a_2 x^2 + a_1 x + a_0$ must has a real root in the interval $[0,1]$.

118. (d)

We have, $f(x) = \begin{cases} 0, & x \text{ is irrational} \\ \sin|x|, x & \text{is rational} \end{cases}$

If $f(x)$ is continuous, then $\sin|x|=0$
$\Rightarrow x=k\pi$, where k is an integer.

119. (b)

Given, $s=t^2+at-b+17$
After 5 s, $\left[\frac{ds}{dt}\right]_{t=5}=[2t+a]_{t=5}=0$
$\Rightarrow a=-10$
At $t=5$ s, $s=25$ units
$\therefore 25=5^2+5a-b+17$
$[\because s=25 \text{ and } t=5]$
$\Rightarrow 25=25+5(-10)-b+17$
$\Rightarrow 50-17=-b$
$\Rightarrow b=-33$

120. (c)

$\lim_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n-1}}{n\sqrt{n}}$

$$=\lim_{n\to\infty}\left(\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n-1}+\sqrt{n}}{n\sqrt{n}}-\frac{n}{n\times n}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/2}-\lim_{n\to\infty}\frac{1}{n}\times\frac{n}{n}=\int_0^1\sqrt{x}dx+0=\frac{2}{3}$$

121. (a)

We have,
$\lim_{x\to0}\frac{axe^x-b\log(1+x)}{x^2}=3\left[\frac{0}{0}\text{ form }\right]$
Using L' Hospital's rule, we get
$\lim_{x\to0}\frac{ae^x+axe^x-\frac{b}{1+x}}{2x}=3\left[\frac{0}{0}\text{ form }\right]$
$\Rightarrow a-b=0\Rightarrow a=b$
Using L' Hospital's rule, we get

$$\lim_{x\to0}\frac{ae^x+ae^x+axe^x+\dfrac{b}{(1+x)^2}}{2}=3$$

$$\Rightarrow\quad \lim_{x\to0}2ae^x+axe^x+\frac{b}{(1+x)^2}=6 \quad (i)$$

$\Rightarrow\quad 2a+b=6$
$\Rightarrow\quad 3a=6\Rightarrow a=2$
On putting the value of a in Eq. (i), we get $b=2$

122. (b)

Wè have, $y^2-4y=4x-4a$
$\Rightarrow\quad (y-2)^2-4=4x-4a$
$\Rightarrow\quad (y-2)^2=4x-4a+4$
$\Rightarrow\quad (y-2)^2=4[x-(a-1)]$
Hence, vertex is $(a-1,2)$.
Since, vertex lies between the lines $x+y=3$ and $2x+2y-1=0$, then
$(a-1+2-3)(2a-2+4-1)<0$

$$\Rightarrow (a-2)(2a+1)<0\Rightarrow a\in\left(-\frac{1}{2},2\right)$$

123. (d)

For curve I,

$$4x^2+9y^2=1\Rightarrow\frac{x^2}{\left(\frac{1}{2}\right)^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$$

which is an equation of ellipse with $a=\frac{1}{2}$ and $b=\frac{1}{3}$.
For curve II, $4x^2+y^2=4$

$\Rightarrow \quad \dfrac{x^2}{1}+\dfrac{y^2}{4}=1$

$\Rightarrow \quad \dfrac{x^2}{(1)^2}+\dfrac{y^2}{(2)^2}=1$

which is also an equation of ellipse with $a=1$ and $b=2$.
Hence, there is no intersecting point.

124. (d)

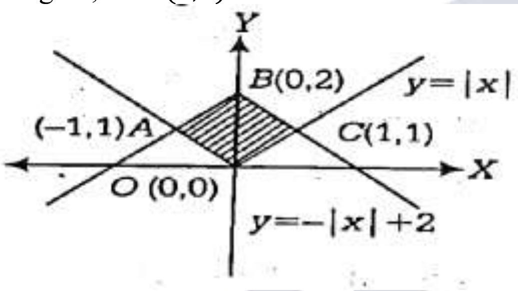
If a line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ lies on a plane $a_2x+b_2y+c_2z=d$, then
(i) $a_1a_2+b_1b_2+c_1c_2=0$
(ii) $a_2x_1+b_2y_1+c_2z_1=d$
According to the question,

$$3(1)+(2+\lambda)\quad(-2)+(-1)(0)=0$$
$$\Rightarrow \quad 3-4-2\lambda=0$$
$$\Rightarrow \quad \lambda=-1/2$$
$$\text{Also,}\quad \lambda-2=0$$
$$\Rightarrow \quad \lambda=2$$

So, there is no such value of λ.

125. (c)

In figure, $B\equiv(0,2)\Rightarrow OB=2$ and $OABC$ is a square.



So, side of length $=\sqrt{2}$ units
$\therefore$ Area of $OABC=2$ sq units

126. (c)

Here, $225=3^2\times5^2$
$\Rightarrow d(225)=3\times3=9$
$1125=3^2\times5^3$
$\Rightarrow\ d(1125)=3\times4=12$
and $640=2^7\times5$
$\Rightarrow\ d(640)=8\times2=16$
Hence, 9, 12 and 16 are in GP.

127. (d)

We know that, $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$

$\Rightarrow \dfrac{-\pi}{2}\le 2\sin^{-1}2a\le\dfrac{\pi}{2}$

$\Rightarrow \dfrac{-\pi}{4}\le \sin^{-1}2a\le\dfrac{\pi}{4}$

$\Rightarrow \sin\left(\dfrac{-\pi}{4}\right)\le 2a\le\sin\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \dfrac{-1}{\sqrt{2}}\le 2a\le\dfrac{1}{\sqrt{2}}$

$\Rightarrow \dfrac{-1}{2\sqrt{2}}\le a\le\dfrac{1}{2\sqrt{2}}\Rightarrow |a|\le\dfrac{1}{2\sqrt{2}}$

128. (c)

If one root of a quadratic equation is of the form $a+ib$, then other root will be $a-ib$.
So, all the roots are $2\pm i,\sqrt{5}\pm 2i$.
$\therefore$ Product of all the roots

$= (2+i)(2-i)(\sqrt{5}+2i)(\sqrt{5}-2i)$
$= (4+1)(5+4) = 5 \times 9 = 45$

129. (a)

We have, $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix} = 1(1+\tan^2\theta) - \tan\theta$
$(-\tan\theta + \tan\theta) + 1(\tan^2\theta + 1)$
$= 2(1+\tan^2\theta) - 0 = 2\sec^2\theta$
$\therefore$ Range of $f = (2, \infty)$

130. (c)

Given, $AB = B$ and $BA = A$
Now, $A^2 + B^2 = A \cdot A + B \cdot B$
$= A(BA) + B(AB) = (AB)A + (BA)B$
$= BA + AB = A + B$

131. (b) $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix}$

$= \begin{vmatrix} 0 & \omega^2 & -\omega \\ 0 & \omega & -\omega^2 \\ -1+\omega & \omega & -\omega^2 \end{vmatrix} [\because C_1 \to C_1 + C_2]$
$= (-1+\omega)(-\omega^4 + \omega^2) = (\omega - 1)(\omega^2 - \omega)$
$= \omega^3 - \omega^2 - \omega^2 + \omega = -3\omega^2$

132. (c) Let $I = \int_0^{\sqrt{3}} f(x^2)dx = \int_0^{\sqrt{3}} \{x^2\}dx$
$= \int_0^{\sqrt{3}} (x^2 - [x^2])dx$
$= \int_0^{\sqrt{3}} x^2 dx - \int_0^{\sqrt{3}} [x^2]dx$
$= \left[\frac{x^3}{3}\right]_0^{\sqrt{3}} - \left[\int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx\right]$
$= \sqrt{3} - \left[\int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx\right]$
$= \sqrt{3} - \left\{0 + [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}}\right\}$
$= \sqrt{3} - \sqrt{2} + 1 - 2\sqrt{3} + 2\sqrt{2}$
$= \sqrt{2} - \sqrt{3} + 1$

133. (b)

We have, $a + b + c = 21$ and $\frac{a+c}{2} = b$
$\Rightarrow a + c + \frac{a+c}{2} = 21$
$\Rightarrow a + c = 14 \Rightarrow \frac{a+c}{2} = 7$
$\Rightarrow b = 7$
So, a can take values from 1 to 6,
c can have values from 8 to 13
or $a = b = c = 7$
$[\because a \le b \le c]$
So, there are 7 such triplets.

134. (a)

For intersecting points, $e^{x^2} = e^{x^2}\sin x$
$\Rightarrow e^{x^2}(\sin x - 1) = 0$
$\Rightarrow e^{x^2} = 0$ or $\sin x = 1$
But $e^{x^2} \ne 0 \Rightarrow \sin x = 1$
$x = \frac{\pi}{2}$
Now,

$y=e^{x^2}$
$\therefore\ \dfrac{dy}{dx}=e^{x^2}\cdot 2x=2xe^{x^2}$
Also, $y=e^{x^2}\sin x$
$\dfrac{dy}{dx}=e^{x^2}\cdot 2x\sin x+e^{x^2}\cos x=2xe^{x^2}$
$[\because \sin x=1, \cos x=0]$
Since, both the curves has equal slope.
Hence, angle between the tangents at intersecting point is 0 .

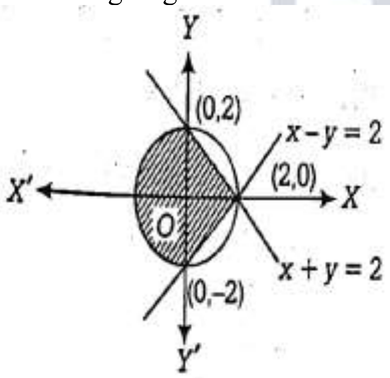
135. (d)

$2\cot^{-1}\frac{1}{2}-\cot^{-1}\frac{4}{3}$

$$=2\tan^{-1}2-\tan^{-1}\frac{3}{4}$$
$$=\pi+\tan^{-1}\frac{4}{-3}-\tan^{-1}\frac{3}{4}$$
$$=\pi-\tan^{-1}\frac{4}{3}-\tan^{-1}\frac{3}{4}$$
$$=\pi-\left\{\tan^{-1}\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\cdot\frac{3}{4}}\right\}$$
$$=\pi-\tan^{-1}\infty=\pi-\frac{\pi}{2}=\frac{\pi}{2}$$

136. (c)

Given that $(2\cos\theta; 2\sin\theta)$ will lie on the circle $x^2+y^2=4$ (from the given figure). Since, point lies on the region containing origin.



So, point will be on the shaded region.
$\therefore\ \theta\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

137. (c)

Let the point be (h,k).
Then, $\left|\dfrac{h-2k+1}{\sqrt{1^2+(-2)^2}}\right|=\sqrt{5}$

$\Rightarrow\ h-2k+1=\pm5$(i)

138. (a) Also, $\left|\frac{2h+3k-1}{\sqrt{2^2+3^2}}\right|=\sqrt{13}$

$\Rightarrow\ 2h+3k-1=\pm13$(ii)
On solving Eqs (i) and (ii), we get four points. So, there are such 4 points.
$\because P(x)=(x-3)(x-5)$
$=Q(x)+(ax+b)$
Given, $P(3)=10$ and $P(5)=6$
$\Rightarrow\ 3a+b=10(ii)$

and $5a+b=6(i)$
On solving Eqs. (i) and (ii), we get

$$a=-2 \text{ and } b=16$$

$\therefore$ Remainder $=-2x+16$

139. (b)

Given, $\frac{dy}{dx}=-(3x^2\tan^{-1}y-x^3)(1+y^2)$

$$\Rightarrow \frac{dy}{dx}=x^3(1+y^2)-3x^2(\tan^{-1}y)(1+y^2)$$

$$\Rightarrow \frac{1}{(1+y^2)}\cdot\frac{dy}{dx}=x^3-3x^2\tan^{-1}y$$

$$\Rightarrow \frac{1}{1+y^2}\cdot\frac{dy}{dx}+3x^2\tan^{-1}y=x^3$$

Put $\tan^{-1}y=t \Rightarrow \frac{1}{1+y^2}\cdot\frac{dy}{dx}=\frac{dt}{dx}$

$$\therefore \frac{dt}{dx}+3tx^2=x^3$$

which is linear differential equation in t.
Now, I $F=e^{\int 3x^2dx}=e^{x^3}$

140. (a)

Given, $y=e^{-x}\cos 2x$.........(i)

$$\therefore \frac{dy}{dx}=e^{-x}(-\sin 2x)2+\cos 2x\cdot e^{-x}(-1)$$

$$\Rightarrow \frac{dy}{dx}=-2\sin 2x\cdot e^{-x}-y$$

$$\Rightarrow \frac{dy}{dx}+y=-2\sin 2x\cdot e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2}+\frac{dy}{dx}=-2[\sin 2x\cdot e^{-x}(-1)$$

$$=2\sin 2x\cdot e^{-x}-4y\ +e^{-x}2\cos 2x]$$

$$\Rightarrow \frac{d^2y}{dx^2}+\frac{dy}{dx}=-\frac{dy}{dx}-y-4y$$

$$\Rightarrow \frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$$

141. (c)

Given, $f(0)=0$ and $f'(0)=2$

$$\therefore \lim_{x\to 0}\frac{1}{x}[f(x)+f(2x)+f(3x)+\cdots+f(2015x)]$$

$$=\lim_{x\to 0}\frac{\left[\begin{array}{l}f'(x)+2f'(2x)+3f'(3x)\\+\cdots+2015f'(2015x)\end{array}\right]}{1}$$ [applying L' Hospital's rule]

$$=\frac{2+2\times 2+3\times 2+\cdots+2015\times 2}{1}$$

$$=2[1+2+3+\cdots+2015]$$

$$=\frac{2(2015)(2015+1)}{2}=\frac{2\times 2015\times 2016}{2}$$

$$=2015\times 2016$$

142. (a)

Given, $17!=3556xy428096000$
Since, 17! is divisible by 9 , so sum of the digits ($48+x+y$) must be divisible by 9 .
So, $x+y$ can be 15 or 6 .
Also, 17 ! is divisible by 11 , so $|10+x-y|$ must be multiple of 11 or 0 . The only possibility is $|x-y|=1$.
$\therefore\ x+y=15$

143. (a)

Let the event of person goes to office by a car, scooter, bus or train be A, B, C and D, respectively.
We have, $P(A)=\frac{1}{7}, P(B)=\frac{3}{7}, P(C)=\frac{2}{7}$

and $P(D)=\frac{1}{7}$

Let $E=$ He reached office in time

We have,

$P\left(\frac{\bar{E}}{A}\right)=\frac{2}{9}, P\left(\frac{\bar{E}}{B}\right)=\frac{1}{9}, P\left(\frac{\bar{E}}{C}\right)=\frac{4}{9}$

and $P\left(\frac{\bar{E}}{D}\right)=\frac{1}{9}$

$$\therefore\ P\left(\frac{A}{E}\right)=\frac{P(A)\cdot \bar{P}\left(\frac{E}{A}\right)}{P(A)\cdot P\left(\frac{E}{A}\right)+P(B)\cdot P\left(\frac{E}{B}\right)+P(C)\cdot P\left(\frac{E}{C}\right)+P(D)\cdot P\left(\frac{E}{D}\right)}$$

$$=\frac{\frac{1}{7}\cdot\frac{7}{9}}{\frac{1}{7}\cdot\frac{7}{9}+\frac{3}{7}\cdot\frac{8}{9}+\frac{2}{7}\cdot\frac{5}{9}+\frac{1}{7}\cdot\frac{8}{9}}$$

$$=\frac{7}{7+24+10+8}=\frac{7}{49}=\frac{1}{7}$$

144. (a)

Let $I=\int \frac{(x-2)}{\{(x-2)^2(x+3)^7\}^{1/3}}dx$

$$=\int \frac{dx}{(x-2)^{-1/3}(x+3)^{7/3}}$$

$$=\int \frac{dx}{(x-2)^2\left(\frac{x+3}{x-2}\right)^{7/3}}$$

Put $\frac{x+3}{x-2}=t \Rightarrow \frac{-5dx}{(x-2)^2}=dt$

$$I=-\frac{1}{5}\int \frac{dt}{t^{7/3}}=-\frac{1}{5}\int t^{-7/3}dt$$

$$=-\frac{1}{5}\left[\frac{t^{-7/3+1}}{-\frac{7}{3}+1}\right]+C=-\frac{1}{5}\left[\frac{t^{-4/3}}{-\frac{4}{3}}\right]+C$$

$$=\frac{3}{20t^{4/3}}+C=\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3}+C$$

145. (d)

We have, $f: N \to R$ such that $f(1)=1$

and $f(1)+2f(2)+3f(3)+\cdots+nf(n)$

$=n(n+1)f(n), \forall n \geq 2$

Clearly, $f(1)+2f(2)=2(2+1)f(2)$

$\Rightarrow\ f(1)=6f(2)-2f(2)$

$\Rightarrow\ f(1)=4f(2)$

$\Rightarrow\ f(2)=\frac{f(1)}{4}=\frac{1}{4}$

Similarly,

$f(1)+2f(2)+3f(3)=3(3+1)f(3)$

$\Rightarrow\ 1+\frac{1}{2}+3f(3)=12f(3)$

$\Rightarrow\ 9f(3)=\frac{3}{2}\Rightarrow f(3)=\frac{1}{6}$

and $f(1)+2f(2)+3f(3)+4f(4)=4(5)f(4)$

$\Rightarrow\ 1+\frac{1}{2}+\frac{1}{2}=16f(4)$

$\Rightarrow\ f(4)=\frac{2}{16}=\frac{1}{8}$

In general, $f(n)=\frac{1}{2n}$

$\therefore\ f(500)=\frac{1}{1000}$

146. (a)

Clearly, 5 distinct balls can be placed into 5 cells in 5^5 ways.

Now, the number of ways of selecting one cell is ${}^5C_1$.

Let the selected cell be empty. Now, we are left with 5 balls and 4 cells.

Now, number of ways of placing 5 distinct balls into 4 cells such that each cell have atleast one ball

$=4^5-{}^4C_1(4-1)^5+{}^4C_2(4-2)^5-{}^4C_3(4-3)^5$

$=1024-4\times 3^5+6\times 2^5-4$

$=1024-972+192-4=1216-976$

$=240$

Thus, the number of ways of placing 5 distinct balls such that exactly one cell remains empty $={}^5C_1\times 240$

$\therefore$ Required probability

$=\frac{{}^5C_1\times 240}{5^5}=\frac{240}{625}=\frac{48}{125}$

147. (c)

Let $S=$ Event that person is smoker

$NS=$ Event that person is non-smoker

$D=$ Event that death is due to lung cancer.

Now, probability of death due to cancer that a person is a smoker,

$P(D)=P(S)\cdot P\left(\frac{D}{S}\right)+P(NS)\cdot P\left(\frac{D}{NS}\right)$

$\Rightarrow 0.006=\frac{20}{100}\times P\left(\frac{D}{S}\right)+\frac{80}{100}\times\frac{1}{10}\times P\left(\frac{D}{S}\right)$

$\Rightarrow \frac{6}{1000}=\frac{2}{10}P\left(\frac{D}{S}\right)+\frac{8}{100}P\left(\frac{D}{S}\right)$

$\Rightarrow\ P\left(\frac{D}{S}\right)\left[\frac{2}{10}+\frac{8}{100}\right]=\frac{6}{1000}$

$\Rightarrow\ P\left(\frac{D}{S}\right)\left[\frac{20+8}{100}\right]=\frac{6}{1000}$

$\therefore\ P\left(\frac{D}{S}\right)=\frac{6}{1000}\times\frac{100}{28}=\frac{3}{140}$

148. (c)

Tangents drawn from external points are of equal length.

$\Rightarrow\ BD=BE=a-r\ [\because CD=r]$

and $AF=AE=b-r\ [\because CF=r]$

$\because\ AE+BE=AB$

$\Rightarrow b-r+a-r=2R$

[$\because$ AB is a diameter of circumcircle]]

$\Rightarrow\ b+a=2R+2r$

$\Rightarrow\ 2(r+R)=a+b$

149. (d)

Given equation is $a\cos\theta + b\sin\theta = c$

$$\Rightarrow a\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) + \frac{2b\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = c$$

$$\left[\because \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \text{ and } \sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right]$$

$$\Rightarrow a\left(1-\tan^2\frac{\theta}{2}\right) + 2b\tan\frac{\theta}{2} = c\left(1+\tan^2\frac{\theta}{2}\right)$$

$$\Rightarrow a - a\tan^2\frac{\theta}{2} + 2b\tan\frac{\theta}{2} - c - c\tan^2\frac{\theta}{2} = 0$$

$$\Rightarrow (c+a)\tan^2\frac{\theta}{2} - 2b\tan\frac{\theta}{2} + (c-a) = 0$$

Let α and β be the roots of the equation.

$\therefore \alpha+\beta = \frac{2b}{c+a}$ and $\alpha\beta = \frac{c-a}{c+a}$

Now, $\tan\frac{\alpha+\beta}{2} = \frac{\frac{2b}{c+a}}{1-\frac{c-a}{c+a}}$

$$= \frac{\frac{2b}{c+a}}{\frac{c+a-c+a}{c+a}} = \frac{b}{a}$$

Since, $\frac{b}{a}$ is a root of the equation.

$$\therefore\ (c+a)\frac{b^2}{a^2} - 2b\left(\frac{b}{a}\right) + c - a = 0$$

$\Rightarrow b^2c + b^2a - 2b^2a + ca^2 - a^3 = 0$

$\Rightarrow -b^2a + b^2c + ca^2 - a^3 = 0$

$\Rightarrow\ b^2c - b^2a + ca^2 - a^3 = 0$

$\Rightarrow\ b^2(c-a) + a^2(c-a) = 0$

$\Rightarrow\ (c-a)(b^2+a^2) = 0$

$\Rightarrow c - a = 0$ or $b^2 + a^2 = 0$

$\Rightarrow\ c = a$ or $b^2 + a^2 = 0$

150. (b)

Let $U_i = \begin{pmatrix} a_i \\ b_i \\ c_i \end{pmatrix}$, where $i = 1,2,3$

$$\therefore\ AU_1 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a_1 \\ 2a_1 + b_1 \\ 3a_1 + 2b_1 + c_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\left[\because AU_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]$$

and $3a_1 + 2b_1 + c_1 = 0$

$\Rightarrow\ 3 + 2(-2) + c_1 = 0$

$\Rightarrow\ 3 - 4 + c_1 = 0$

$\Rightarrow\ c_1 = 1$

Similarly, $AU_2 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a_2 \\ 2a_2 + b_2 \\ 3a_2 + 2b_2 + c_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}\ \because AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}\Bigg]$$

$\therefore a_2 = 2$ and $2a_2 + b_2 = 3$

$\Rightarrow\ 2 \times 2 + b_2 = 3$

$\Rightarrow\ 4 + b_2 = 3$

$\Rightarrow\ b_2 = -1$

and $\ 3a_2 + 2b_2 + c_2 = 0$

$\Rightarrow\ 3\times 2+2(-1)+c_2=0$
$\Rightarrow\ 6-2+c_2=0$
$\Rightarrow\ c_2=-4$

Now, $AU_3=\begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix}\begin{pmatrix}a_3\\b_3\\c_3\end{pmatrix}$

$\Rightarrow \begin{pmatrix}a_3\\2a_3+b_3\\3a_3+2b_3+c_3\end{pmatrix}=\begin{pmatrix}2\\3\\1\end{pmatrix}\ \left[\because AU_3=\begin{pmatrix}2\\3\\1\end{pmatrix}\right]$

$\therefore a_3=2$ and $2a_3+b_3=3$
$\Rightarrow\ 2(2)+b_3=3$
$\Rightarrow\ 4+b_3=3$
$\Rightarrow\ b_3=-1$
and $3a_3+2b_3+c_3=1$
$\Rightarrow\ 3\times 2+2(-1)+c_3=1$
$\Rightarrow\ 6-2+c_3=1$
$\Rightarrow\ c_3=-3$

$\therefore\ U=\begin{pmatrix}1&2&2\\-2&-1&-1\\1&-4&-3\end{pmatrix}$

$\Rightarrow |U|=1(3-4)-2(6+1)+2(8+1)$
$=-1-14+18=3$
Also, adj(U)

$=\begin{bmatrix}-1&-7&9\\-2&-5&6\\0&-3&3\end{bmatrix}^T$

$=\begin{bmatrix}-1&-2&0\\-7&-5&-3\\9&6&3\end{bmatrix}$

$$U^{-1}=\frac{1}{|U|}\operatorname{adj}(U)$$

$$\Rightarrow\quad U^{-1}=\begin{bmatrix}-\frac{1}{3}&-\frac{2}{3}&0\\-\frac{7}{3}&-\frac{5}{3}&-1\\3&2&1\end{bmatrix}$$

Hence, the sum of all elements of U^{-1} is 0 .

151. (b,c)

We have, f is continuous and differentiable function on $[a,b]$.
Also, $f(a)=f(b)=0$.
By Rolle's theorem, there exists $c\in(a,b)$ such that $f'(c)=0$
Thus, there exists $x\in(a,b)$ such that $f'(x)=0$
Let at $x=c\in(a,b), f'(c)=0$
Now, f is continuously differentiable on $[a,b]$.
$\Rightarrow\ f'$ is continuous on $[a,b]$.
Also, f is twice differentiable on (a,b).
$\therefore f'$ is differentiable on (a,b).
and $f'(a)=0=f'(c)$
By Rolle's theorem, there exists $k\in(a,c)$ such that $f''(k)=0$
Thus, there exists $x\in(a,c)$ such that $f''(x)=0$.
So, there exists $x\in(a,b)$ such that $f''(x)=0$.
Let us consider, $f(x)=(x-a)^2(x-b)$,
where $f(a)=f(b)=f'(a)=0$ but $f''(a)\neq 0$ and $f'''(x)\neq 0$ for any $x\in(a,b)$

152. (b,c)

We have, $x\rho y: xy > 0$
(i) Reflexive Suppose $x\rho x \in R$
$\Rightarrow\ x^2 > 0$
which is not true when $x = 0$.
Hence, relation is not reflexive.
(ii) Symmetric $x\rho y \in R$
$\Rightarrow\ xy > 0$
$\Rightarrow\ yx > 0$
$\Rightarrow\ y\rho x \in R$
If $(x, y) \in R$, then, relation is symmetric.
(iii) Transitive $xpy \in R$
$\Rightarrow\ xy > 0$
and $y\rho z \in R$
$\Rightarrow\ yz > 0$
$\Rightarrow\ xy^2z > 0$
$\Rightarrow\ xz > 0$
$\Rightarrow\ (x, z) \in R$
If $(x, y) \in R$, then $(y, z) \in R$
$\Rightarrow\ (x, z) \in R$
Hence, relation is transitive.

153. (a,b,d)

(a) Let $f(x) = \cos x$ and $g(x) = \sin x$
Consider the Wronskian of $f(x)$ and $g(x)$,

$$W = \begin{vmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{vmatrix}$$
$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$
$$= \cos^2 x + \sin^2 x = 1 \neq 0$$

Thus, the functions are linearly independent. So, the general solution of given differential equation is given by $y = A\cos x + B\sin x$, where A and B are real constants.
[$\because$ if y_1 and y_2 are linearly independent solutions of the differential equation $ay'' + by' + c = 0$, then the general solution is $y = c_1y_1 + c_2y_2$, where c_1 and c_2 are constants]
Hence, option (a) is true.

(b) Let $y = A\cos\left(x + \frac{\pi}{4}\right)$

$$= A\left(\cos x \cdot \cos\frac{\pi}{4} - \sin x \cdot \sin\frac{\pi}{4}\right)$$
$$= \frac{A}{\sqrt{2}}(\cos x - \sin x)$$
$$= \frac{A}{\sqrt{2}}\cos x + \left(-\frac{A}{\sqrt{2}}\right)\sin x$$

which is in the form of general solution.
Hence, option (b) is true.
(c) Let $y = A\cos x\sin x$, which cannot be expressed in the form of general solution.
(d) Let $y = A\cos\left(x + \frac{\pi}{4}\right) + B\sin\left(x - \frac{\pi}{4}\right)$

$$= A\left(\cos x \cdot \frac{1}{\sqrt{2}} - \sin x \cdot \frac{1}{\sqrt{2}}\right) + B\left(\sin x \cdot \frac{1}{\sqrt{2}} - \cos x \cdot \frac{1}{\sqrt{2}}\right)$$

$= \cos x\left(\frac{A}{\sqrt{2}} - \frac{B}{\sqrt{2}}\right) + \sin x\left(\frac{B}{\sqrt{2}} - \frac{A}{\sqrt{2}}\right)$ which is in the form of general solution.
Hence, option (d) is true.

154. (a,c)

$0 < \theta < \frac{\pi}{2}$; $\cos\theta$ is strictly decreasing function.

Option (a) When $\theta > \frac{\theta}{2}$, then

$$\cos\theta \leq \cos\frac{\theta}{2}$$

$$\therefore \quad \sqrt{\cos\theta} \leq \cos\frac{\theta}{2}$$

(b) When $\theta > \frac{3\theta}{4}$, then

$$\cos\theta \leq \cos\frac{3\theta}{4}$$

$$\therefore \ (\cos\theta)^{3/4} \leq \cos\frac{3\theta}{4}$$

(c) When $\theta > \frac{5\theta}{6}$, then

$$\cos\theta \leq \cos\frac{5\theta}{6}$$

$$\therefore \ (\cos\theta)^{5/6} \leq \cos\frac{5\theta}{6}$$

(d) When $\frac{7\theta}{8} < \theta$, then

$$\cos\theta \leq \cos\frac{7\theta}{8}$$

But $(\cos\theta)^{7/8} \leq \cos\frac{7\theta}{8}$

155. (a,b,d)

Given equation of hyperbola is

$$16x^2 - 3y^2 - 32x - 12y = 44$$

$$\Rightarrow 16x^2 + 16 - 32x - 3y^2 - 12 - 12y$$

$$= 44 + 4$$

$$\Rightarrow 16(x-1)^2 - 3(y+2)^2 = 48$$

$$\Rightarrow \frac{(x-1)^2}{3} - \frac{(y+2)^2}{16} = 1$$

On comparing the equation with standard equation of hyperbola, we get $a = \sqrt{3}, b = 4$

Now, length of transverse axis

$= 2a = 2\sqrt{3}$

and length of latusrectum

$$= \frac{2b^2}{a} = \frac{2 \times 16}{\sqrt{3}} = \frac{32}{\sqrt{3}}$$

$\therefore$ Eccentricity $(e) = \sqrt{1 + \frac{b^2}{a^2}}$

$$= \sqrt{1 + \frac{16}{3}} = \sqrt{\frac{19}{3}}$$

Equation of directrix is $x = \pm\frac{a}{e}$

$$\Rightarrow x - 1 = \pm\frac{\sqrt{3} \times \sqrt{3}}{\sqrt{19}}$$

$$\Rightarrow \ x = 1 \pm \frac{3}{\sqrt{19}}$$

156. (b,c)

We have, $f(x) = \left[\frac{1}{[x]}\right]$

Domain $= R - \{f(x) = 0\}$

Now, $f(x)$ will be zero, when $[x] = 0$.

$\Rightarrow \ x \in [0,1)$

$\therefore$ Domain of $f(x) = R - (0,1]$

and range of $f(x) = \{-1,0,1\}$

157. (c)

If a quadratic equation has irrational coefficients, then roots are also irrational.
So, option (c) is always false.

158. (a,c)

Given equation of straight line is

$(a-1)x - by + 4 = 0$

Slope $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$

$= \frac{-(a-1)}{-b} = \frac{a-1}{b}$

Equation of hyperbola is

$$xy = 1$$

$$\Rightarrow \quad y = \frac{1}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{x^2}$$

Slope of normal $= x^2 > 0$

$\therefore \frac{a-1}{b} > 0$

$\Rightarrow a - 1 > 0$ and $b > 0$

or $a - 1 < 0$ and $b < 0$

$\Rightarrow a > 1, b > 0$ or $a < 1, b < 0$

159. (c,d)

Given probability of defective part

$= 0.05 = \frac{1}{20}$

Probability of non-defective part

$= 1 - 0.05 = 0.95 = \frac{19}{20}$

We know that, $P(X = r) = {}^nC_r p^r q^{n-r}$

where, $p = \frac{1}{20}, q = \frac{19}{20}$

$r \geq 1$ and $n = ?$

Also, $P(X \geq 1) \geq \frac{1}{2}$

$\Rightarrow 1 - P(X = 0) \geq \frac{1}{2}$

$\Rightarrow 1 - {}^nC_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{n-0} \geq \frac{1}{2}$

$$\Rightarrow \quad 1 - \frac{1}{2} \geq \left(\frac{19}{20}\right)^n$$

$$\Rightarrow \quad \frac{1}{2} \geq \left(\frac{19}{20}\right)^n$$

$$\Rightarrow \quad \frac{1}{2} \geq \left(\frac{95}{100}\right)^n$$

$$\Rightarrow \quad \log 2^{-1} \geq n[\log 95 - \log 100]$$

$$\Rightarrow \quad -\log 2 \geq n[\log 95 - 2]$$

$$\Rightarrow \quad -(0.3) \geq n[1.977 - 2]$$

$$\Rightarrow \quad n \geq \frac{0.3}{0.023}$$

$$\Rightarrow \quad n \geq \frac{300}{23}$$

$$\Rightarrow \quad n \geq 13.04$$

$$\therefore \quad n = 14, 15$$

160. (c)

Given, $f: R \to R$
and $f(2x-1) = f(x), x \in R$
Hence, f is continuous at $x = 1$ and $f(1) = 1$.