Physics

1. Two particles of mass m_1 and m_2; approach each other due to their mutual gravitational attraction only. Then,

(a) accelerations of both the particles are equal
(b) acceleration of the particle of mass m_1 is proportional to m_1
(c) acceleration of the particle of mass m_1 is proportional to m_2
(d) acceleration of the particle of mass m_1 is inversely proportional to m_1

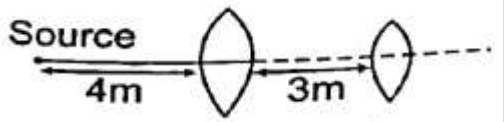
2. Three bodies of the same material and having masses m, m and $3m$ are at temperatures $40°C, 50°C$ and $60°C$, respectively. If the bodies are brought in thermal contact, the final temperature will be

(a) $45°C$ (b) $54°C$
(c) $52°C$ (d) $48°C$

3. A satellite has kinetic energy K, potential energy V and total energy E. Which of the following statements is true?

(a) $K = -V/2$
(b) $K = V/2$
(c) $E = K/2$
(d) $E = -K/2$

4. An object is located 4 m from the first of two thin converging lenses of focal lengths 2 m and 1 m , respectively. The lenses are separated by 3 m . The final image formed by the second lens is located from the source at a distance of



(a) 8.0 m (b) 5.5 m
(c) 6.0 m (d) 6.5 m

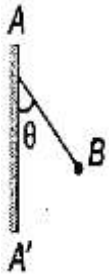
5. A simple pendulum of length L swings in a vertical plane. The tension of the string when it makes an angle θ with the vertical and the bob of mass m moves with a speed v is (g is the gravitational acceleration)

(a) mv^2/L
(b) $mg\cos\theta + mv^2/L$
(c) $mg\cos\theta - mv^2/L$
(d) $mg\cos\theta$

6. The length of a metal wire is L_1 when the tension is T_1 and L_2 when the tension is T_2. The unstretched length of wire is.

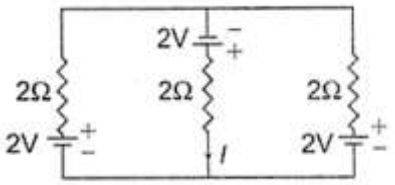
(a) $\frac{L_1+L_2}{2}$ (b) $\sqrt{L_1L_2}$
(c) $\frac{T_2L_1-T_1L_2}{T_2-T_1}$ (d) $\frac{T_2L_1+T_1L_2}{T_2+T_1}$

7. The line AA' is on charged infinite conducting plane which is perpendicular to the plane of the paper. The plane has a surface density of charge σ and B is ball of mass m with a like charge of. magnitude q. B is connected by string from a point on the line AA'. The tangent of angle (θ) formed between the line AA' and the string is



(a) $\frac{q\sigma}{2\varepsilon_0 mg}$ (b) $\frac{q\sigma}{4\pi\varepsilon_0 mg}$
(c) $\frac{q\sigma}{2\pi\varepsilon_0 mg}$ (d) $\frac{q\sigma}{\varepsilon_0 mg}$

8. The current I is in the circuit shown is.



(a) 1.33 A (b) zero
(c) 2.00 A (d) 1.00 A .

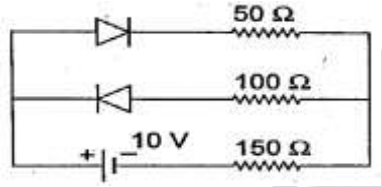
9. A hollow sphere of external radius R and thickness $t(\ll R)$ is made of a metal of density ρ. The sphere will float in water, if.

(a) $t \leq \frac{R}{\rho}$ (b) $t \leq \frac{R}{3\rho}$
(c) $t \leq \frac{R}{2\rho}$ (d) $t \geq \frac{R}{3\rho}$

10. A metal wire of circular cross-section has a resistance R_1. The wire is now stretched without breaking, so that its length is doubled and the density is assumed to remain the same. If the resistance of the wire now becomes R_2, then $R_2 : R_1$ is.

(a) 1:1 (b) 1:2
(c) 4:1 (d) 1:4

11. Assume that each diode as shown in the figure has a forward bias resistance of 50Ω and an infinite reverse bias resistance. The current through the resistance 150Ω is



(a) 0.66 A (b) 0.05 A
(c) zero (d) 0.04 A

12. The rms speed of oxygen is v at a particular temperature. If the temperature is doubled and oxygen molecules dissociate into oxygen atoms, the rms speed becomes.

(a) v (b) $\sqrt{2}v$
(c) $2v$ (d) 4 V

13. Two particles, A and B, having equal charges, after being accelerated through the same potential difference enter into a region of uniform magnetic field and the particles describe circular paths of radii R_1 and R_2, respectively. The ratio of the masses of A and B is.

(a) $\sqrt{R_1/R_2}$ (b) R_1/R_2
(c) $(R_1/R_2)^2$ (d) $(R_2/R_1)^2$

14. A large number of particles are placed around the origin, each at a distance R from the origin. The distance of the centre of mass of the system from the origin is.

(a) equal to R
(b) less than equal to R
(c) greater than R
(d) greater than equal to R

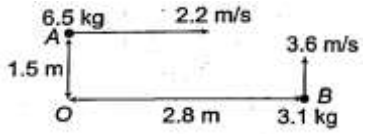
15. A straight conductor 0.1 m long moves in a uniform magnetic field 0.1 The velocity of the conductor is 15 m/s and is directed perpendicular to the field. The emf induced between the two ends of the conductor is.

(a) 0.10 V (b) 0.15 V
(c) 1.50 V (d) 15.00 V

16. A ray of light is incident at an angle i on a glass slab of refractive index μ. The angle between reflected and refracted light is 90°. Then, the relationship between i and μ is

(a) $i = \tan^{-1}\left(\frac{1}{\mu}\right)$
(b) $\tan i = \mu$
(c) $\sin i = \mu$
(d) $\cos i = \mu$

17. Two particles A and B are moving as shown in the figure.



Their total angular momentum about the point O is

(a) 9.8 kg m^2/s (b) zero

(c) 52.7 kg m^2/s (d) 37.9 kg m^2/s

18. A 20 cm long capillary tube is dipped vertically in water and the liquid rises upto 10 cm . If the entire system is kept is a freely falling platform, the length of the water column in the tube will be.

(a) 5 cm (b) 10 cm

(c) 15 cm (d) 20 cm

19. A train is moving with a uniform speed of 33 m/s and an observer is approaching the train with the same speed. If the train blows a whistle of frequency 1000 Hz and the velocity of sound is 333 m/s, then the apparent frequency of the sound that the observer hears is.

(a) 1220 Hz (b) 1099 Hz

(c) 1110 Hz (d) 1200 Hz

20. A photon of wavelength 300 nm interacts with a stationary hydrogen atom in ground state. During the interaction, whole energy of the photon is transferred to the electron of the atom. State which possibility is correct. (Consider, Plank constant $= 4 \times 10^{-15}$eVs, velocity of light $= 3 \times 10^8$ m/s, ionisation energy of hydrogen $= 13.6$eV)

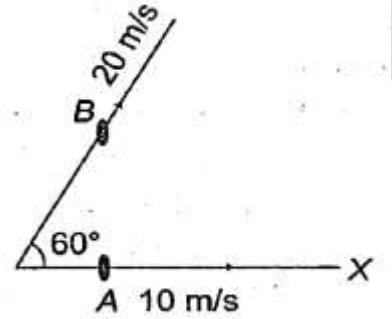
(a) Electron will be knocked out of the atom

(b) Electron will go to any excited state of the atom

(c) Electron will go only to first excited state of the atom

(d) Electron will keep orbiting in the ground state of the atom

21. Particle A moves along X-axis with a uniform velocity of magnitude 10 m/s. Particle B moves with uniform velocity 20 m/s along a direction making an angle of 60° with the positive direction of X-axis as shown in the figure. The relative velocity of B with respect to that of A is



(a) 10 m/s along X-axis

(b) $10\sqrt{3}$ m/s along Y-axis (perpendicular to X-axis)

(c) $10\sqrt{5}$ m/s along the bisection of the velocities of A and B

(d) 30 m/s along negative X-axis

22. When light is refracted from a surface, which of its following physical parameters does not change?

(a) Velocity (b) Amplitude

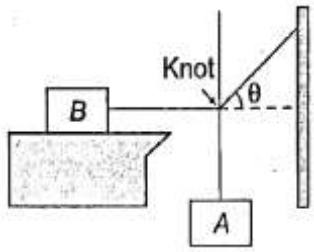
(c) Frequency (d) Wavelength

23. A solid maintained at t_1°C is kept in an evacuated chamber at temperature $t^\circ{}_2$C $(t_2 \gg t_1)$. The rate of heat absorbed by the body is proportional to.

(a) $t_2^4 - t_1^4$ (b) $(t_2^4 + 273) - (t_1^4 + 273)$

(c) $t_2 - t_1$ (d) $t_2^2 - t_1^2$

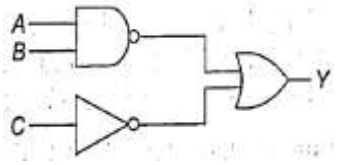
24. Block B lying on a table weighs W. The coefficient of static friction between the block and the table is μ. Assume that the cord between B and the knot is horizontal. The maximum weight of the block A for which the system will be stationary is.



(a) $\frac{W\tan\theta}{\mu}$ (b) $\mu W\tan\theta$

(c) $\mu W\sqrt{1+\tan^2\theta}$ (d) $\mu W\sin\theta$

25. The inputs to the digital circuit are as shown below. The output Y is.



(a) $A+B+\bar{C}$

(b) $(A+B)\bar{C}$

(c) $\bar{A}+\bar{B}+\bar{C}$

(d) $\bar{A}+\bar{B}+C$

26. Two particles A and B having different masses are projected from a tower with same speed. A is projected vertically upward and B vertically downward. On reaching the ground.

(a) velocity of A is greater than that of B

(b) velocity of B is greater than that of A

(c) both A and B attain the same velocity

(d) the particle with the larger mass attains higher velocity

27. The work function of metals is in the range of 2 eV to 5 eV . Find which of the following wavelength of light cannot be used for photoelectric effect? (Consider, Plank constant $= 4\times10^{-15}$eVs, velocity of light $= 3\times10^{8}$ m/s).

(a) 510 nm (b) 650 nm

(c) 400 nm (d) 570 nm

28. A thin plastic sheet of refractive index 1.6 is used to cover one of the slits of a double slit arrangement. The central point on the screen is now occupied by what would have been the 7th bright fringe before the plastic was used. If the wavelength of light is 600 nm , what is the thickness (in μm) of the plastic sheet?

(a) 7 (b) 4

(c) 8 (d) 6

29. The length of an open organ pipe is twice the length of another closed organ pipe. The fundamental frequency of the open pipe is 100 Hz . The frequency of the third harmonic of the closed pipe is.

(a) 100 Hz (b) 200 Hz

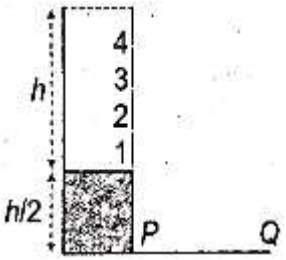
(c) 300 Hz (d) 150 Hz

30. A 5μ F capacitor is connected in series with a 10μ F capacitor. When a 300 V potential difference is applied across this combination, the total energy stored in the capacitors is.

(a) 15 J (b) 1.5 J

(c) 0.15 J (d) 0.10 J

31. A cylinder of height h is filled with water and is kept on a block of height $h/2$. The level of water in the cylinder is kept constant. Four holes numbered 1,2,3 and 4 are at the side of the cylinder and at heights $0, h/4, h/2$ and $3h/4$, respectively. When all four holes are opened together, the hole from which water will reach farthest distance on the plane PQ is the hole number.

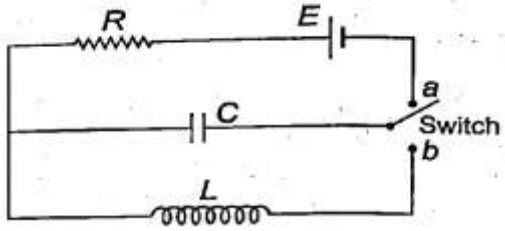


(a) 1 (b) 2
(c) 3 (d) 4

32. The pressure p, volume V and temperature T for a certain gas are related by $p = \frac{AT - BT^2}{V}$, where A and B are constants. The work done by the gas when the temperature changes from T_1 to T_2 while the pressure remains constant, is given by.

(a) $A(I_2 - T_1) + B(T_2^2 - T_1^2)$
(b) $\frac{A(T_2 - T_1)}{V_2 - V_1} - \frac{B(T_2^2 - T_1^2)}{V_2 - V_1}$
(c) $A(T_2 - T_1) - \frac{B}{2}(T_2^2 - T_1^2)$
(d) $\frac{A(T_2 - T_1^2)}{V_2 - V_1}$

33. In the circuit shown below, the switch is kept in position a for a long time and is then thrown to position b. The amplitude of the resulting oscillating current is given by

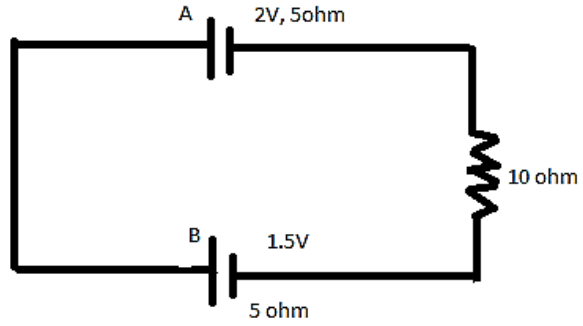


(a) $E\sqrt{L/C}$ (b) E/R
(c) infinity (d) $E\sqrt{C/L}$

34. A charge q is placed at one corner of a cube. The electric flux through any of the three faces adjacent to the charge is zero. The flux through any one of the other three faces is.

(a) $q/3\varepsilon_0$ (b) $q/6\varepsilon_0$
(c) $q/12\varepsilon_0$ (d) $q/24\varepsilon_0$

35. Two cells A and B of emf 2 V and 1.5 V respectively, are connected as shown in figure through an external resistance 10Ω. The internal resistance of each cell is 5Ω. The potential difference E_A and E_B across the terminals of the cells A and B respectively are



(a) $E_A = 2.0$ V, $E_B = 1.5$ V
(b) $E_A = 2.125$ V, $E_B = 1.375$ V
(c) $E_A = 1.875$ V, $E_B = 1.625$ V
(d) $E_A = 1.875$ V, $E_B = 1.375$ V

Directions (Q. Nos. 36-40) Each question has one or more correct option(s),

36. Two charges $+q$ and $-q$ are placed at a distance a in a uniform electric field. The dipole

moment of the combination is $2qa(\cos\theta\hat{i} + \sin\theta\hat{j})$, where, θ is the angle between the direction of the field and the line joining the two charges.
Which of the following statement(s) is/are correct?
(a) The torque exerted by the field on the dipole vanishes
(b) The net force on the dipole vanishes
(c) The torque is independent of the choice of coordinates
(d) The net force is independent of a

37. Find the right condition(s) for Fraunhofer diffraction due to a single slit.
(a) Source is at infinite distance and the incident beam has converged at the slit
(b) Source is near to the slit and the incident beam is parallel
(c) Source is at infinity and the incident beam is parallel.
(d) Source is near to the slit and the incident beam has converged at the slit

38. The conducting loop in the form of a circle is placed in a uniform magnetic field with its plane perpendicular to the direction of the field. An emf will be induced in the loop, if
(a) it is translated parallel to itself.
(b) it is rotated about one of its diameters
(c) it is rotated about its own axis which is parallel to the field
(d) the loop is deformed from the original shape

39. A circular disc rolls on a horizontal floor without slipping and the centre of the disc moves with a uniform velocity v. Which of the following values of the velocity at a point on the rim of the disc can have?
(a) v (b) $-v$
(c) $2v$ (d) Zero

40. Consider two particles of different masses. In which of the following situations the heavier of the two particles will have smaller de-Broglie wavelength?
(a) Both have a free fall through the same height
(b) Both move with the same kinetic energy
(c) Both move with the same linear momentum
(d) Both move with the same speed

Chemistry

41. Match the flame colours of the alkaline earth metal salts in the Bunsen burner.

List I	List II
A. Calcium	**p.** Brick Red
B. Strontium	**q.** Apple Green
C. Barium	**r.** Crimson

	A	B	C
(a)	p	r	q
(b)	r	p	q
(c)	q	r	p
(d)	p	q	r

42. Extraction of gold (Au) involves the formation of complex ions X and Y.

Gold ore $\xrightarrow[CN^-,H_2O,O_2]{\text{Roasting}}$ $HO^- + X \xrightarrow{Zn} Y + Au$

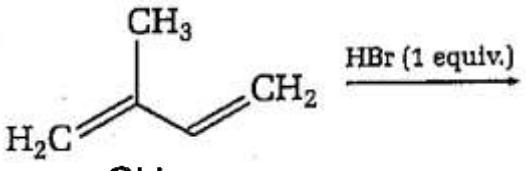
X and Y respectively are
(a) $Au(CN)_2^-$ and $Zn(CN)_4^{2-}$

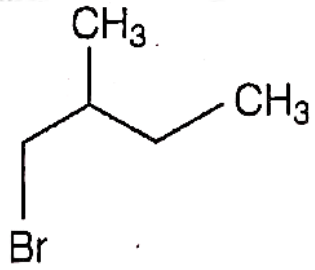
(b) $Au(CN)_4^{3-}$ and $Zn(CN)_4^{2-}$
(c) $Au(CN)_3^-$ and $Zn(CN)_6^{4-}$
(d) $Au(CN)_4^-$ and $Zn(CN)_3^-$

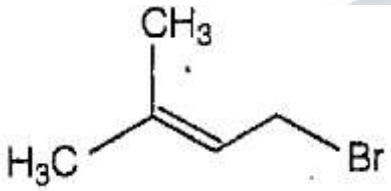
43. The atomic number of cerium (Ce) is 58 . The correct electronic configuration of Ce^{3+} ion Is.

(a) $[Xe]4f^1$ (b) $[Kr]4f^1$
(c) $[Xe]4f^{13}$ (d) $[Kr]4d^1$

44. The major product of the above reaction is.



(a)


(b)


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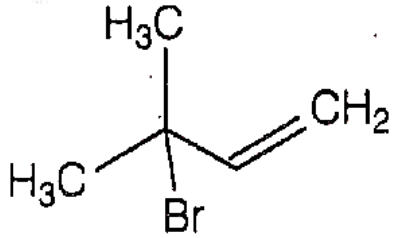
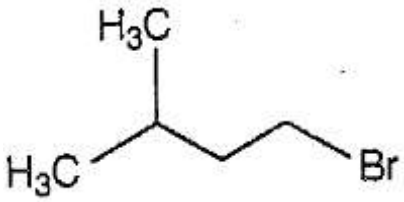
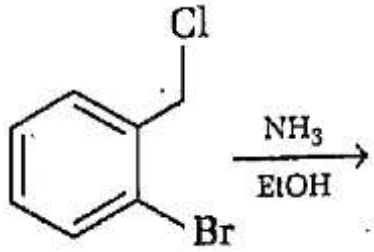
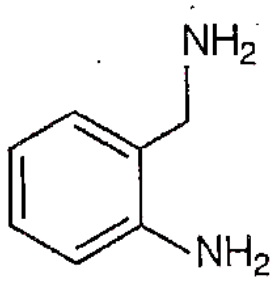
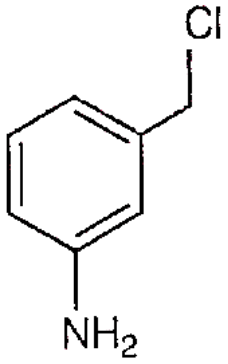


(d)


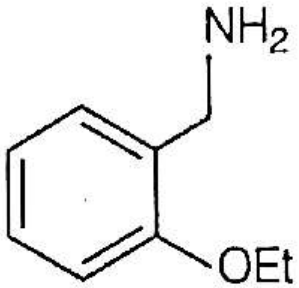
45. The product of the above reaction is.



(a) 

(b) 

(c) 

(d) 

46. Sulphuryl chloride (SO_2Cl_2) reacts with white phosphorus (P_4) to give.

(a) PCl_5, SO_2
(b) $OPCl_3, SOCl_2$
(c) PCl_5, SO_2, S_2Cl_2
(d) $OPCl_3, SO_2, S_2Cl_2$

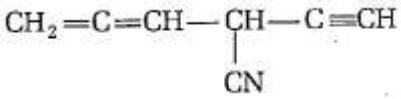
47. The number of lone pairs of electrons on the central atoms of $H_2O, SnCl_2, PCl_3$ an XeF_2 respectively, are.

(a) 2,1,1,3 (b) 2,2,1,3
(c) 3,1,1,2 (d) 2, 1, 2, 3

48. Consider the following salts: $NaCl, HgCl_2, Hg_2Cl_2, CuCl_2, CuCl$ and $AgCl$. Identify the correct set of insoluble salts in water.

(a) $Hg_2Cl_2, CuCl, AgCl$
(b) $HgCl_2, CuCl, AgCl$
(c) $Hg_2Cl_2, CuCl_2, AgCl$
(d) $Hg_2Cl_2, CuCl, NaCl$

49. In the following compound, the number of *sp*-hybridised carbons are.



(a) 2 (b) 3
(c) 4 (d) 5

50. For the reaction $A + 2B \longrightarrow C$, the reaction rate is doubled, if the concentration of A is doubled. The rate is increased by four times when concentrations of both A and B are increased by four times. The order of the reaction is.

(a) 3 (b) 0
(c) 1 (d) 2

51. At a certain temperature, the value of the slope of the plot of osmotic pressure (π) against concentration (C in $\mathrm{molL^{-1}}$) of a certain polymer solution is $291R$. The temperature at which osmotic pressure is measured, is (R is gas constant)

(a) 271°C (b) 18°C
(c) 564 K (d) 18 K

52. The RMS velocity of CO gas molecules at 27°C is approximately 1000 m/s. For N_2 molecules at 600 K , the RMS velocity is approximately.

(a) 2000 m/s
(b) 1414 m/s
(c) 1000 m/s
(d) 1500 m/s

53. A gas can be liquefied at temperature T and pressure p provided

(a) $T = T_c$ and $p < p_c$
(b) $T < T_c$ and $p > p_c$
(c) $T > T_c$ and $p > p_c$
(d) $T > T_c$ and $p < p_c$

54. The dispersed phase and dispersion medium of fog respectively are

(a) solid, liquid
(b) liquid, liquid
(c) liquid, gas
(d) gas, liquid

55. The decreasing order of basic character of K_2O, BaO, CaO and MgO is.

(a) $K_2O > BaO > CaO > MgO$
(b) $K_2O > CaO > BaO > MgO$
(c) $MgO > BaO > CaO > K_2O$
(d) $MgO > CaO > BaO > K_2O$

56. In aqueous alkaline solution, two electrons reduction of HO_2^- gives.

(a) HO^- (b) H_2O
(c) O_2 (d) O_2^-

57. Cold ferrous sulphate solution on absorption of NO develops brown colour due to the formation of

(a) paramagnetic $[Fe(H_2O)_5(NO)]SO_4$
(b) diamagnetic $[Fe(H_2O)_5(N_3)]SO_4$
(c) paramagnetic $[Fe(H_2O)_5(NO_3)][SO_4]_2$
(d) diamagnetic $[Fe(H_2O)_4(SO_4)]NO_3$

58. Among Be, B, Mg and Al , the second ionisation potential is maximum for.

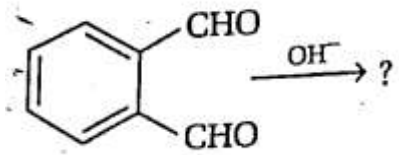
(a) B (b) Be
(c) Mg (d) Al

59. In a mixture, two enantiomers are found to be present in 85% and 15% respectively. The enantiomeric excess (ee) is.

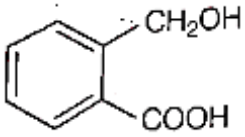
(a) 85% (b) 15%
(c) 70% (d) 60%

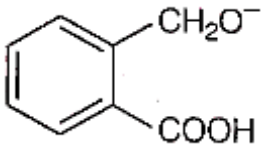
60. 1,4-dimethylbenzene on heating with anhydrous $AlCl_3$ and HCl produces.

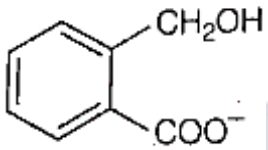
(a) 1, 2-dimethylbenzene
(b) 1,3-dimethylbenzene
(c) 1, 2, 3-trimethylbenzene
(d) ethylbenzene

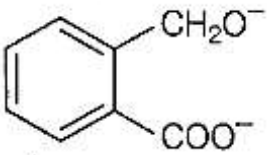
61. 

The product of the above reaction is.

(a) 

(b) 

(c) 

(d) 

62. Suppose the mass of a single Ag-atom is m. Ag metal crystallises in fcc lattice with unit cell of length a. The density of Ag metal in terms of a and m is.

(a) $\frac{4m}{a^3}$ (b) $\frac{2m}{a^3}$
(c) $\frac{m}{a^3}$ (d) $\frac{m}{4a^3}$

63. For the reaction, $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ at 300 K , the value of ΔG° is $-690.9R$. The equilibrium constant value for the reaction at that temperature is (R is gas constant).

(a) 10 atm^{-1} (b) 10 atm
(c) 10 (d) 1

64. At a particular temperature, the ratio of equivalent conductance to specific conductance of a 0.01 N NaCl solution is.

(a) 10^5 cm^3 (b) 10^3 cm^3
(c) 10 cm^3 (d) 10^5 cm^2

65. The units of surface tension and viscosity of liquids respectively are.

(a) $kgm^{-1}\ s^{-1}, Nm^{-1}$
(b) kgs^{-2}, $kg\ m^{-1}\ s^{-1}$
(c) Nm^{-1}, $kg\ m^{-1}\ s^{-2}$
(d) kgs^{-1}, $kg\ m^{-2}\ s^{-1}$

66. The ratio of volumes of $CH_3COOH0.1(N)$ to $CH_3COONa0.1(N)$ required to prepare a buffer solution of pH 5.74 is.
(Given, pK_a of CH_3COOH is 4.74)

(a) 10: 1 (b) 5: 1
(c) 1: 5 (d) 1: 10

67. The reaction of methyl trichloroacetate (Cl_3CCO_2Me) with sodium methoxide (NaOMe)

Generates.
(a) carbocation
(b) carbene
(c) carbanion
(d) carbon radical

68. Best reagent for nuclear iodination of aromatic compounds is.
(a) KI/CH_3COCH_3
(b) $2_2/CH_3CN$
(c) KI/CH_3COOH
(d) b_2/HNO_3

69. In the Lassaigne's test for the detection of nitrogen in an organic compound, the appearance of blue coloured compound is due to.
(a) ferric ferricyanide
(b) ferrous ferricyanide
(c) ferric ferrocyanide
(d) ferrous ferrocyanide

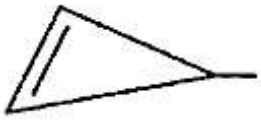
70. In the reaction
$\boldsymbol{RMgBr + HC(OEt)_3 \xrightarrow{\text{Ether, } H_3O^+} P,}$
the product *P* is
(a) RCHO (b) P_2CHOEt
(c) R_3CH (d) $RCH(OEt)_2$

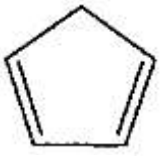
71. Addition of sodium thiosulphate solution to a solution of silver nitrate gives *X* as white precipitate, insoluble in water but soluble in excess thiosulphate solution to give *Y*. On boiling in water, *Y* gives *Z*. *X*, *Y* and *Z* respectively are.
(a) $Ag_2\ S_2O_3, Na_3[Ag(S_2O_3)_2]; Ag_2\ S$
(b) $Ag_2SO_4, Na[Ag(S_2O_3)_2], Ag_2\ S_2$
(c) $Ag_2\ S_2O_3, Na_5[Ag(S_2O_3)_3], AgS$
(d) $Ag_2SO_3, Na_3[Ag(S_2O_3)_2], Ag_2O$

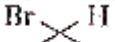
72. At temperature of 298 K , the emf of the following electrochemical cell,
$\boldsymbol{Ag(s)|Ag^+(0.1M)||Zn^{2+}(0.1M)|Zn(s)}$
will be (Given, E° cell $= -1.562$ V)
(a) -1.532 V (b) -1.503 V
(c) 1.532 V (d) -3.06 V

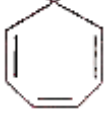
73. For the reaction, $X_2Y_4(l) \longrightarrow 2XY_2(g)$ at 300 K , the values of ΔU and ΔS are 2 kcal and $20 cal K^{-1}$ respectively. The value of ΔG for the reaction is.
(a) -3400 cal (b) 3400 cal
(c) -2800 cal (d) 2000 cal

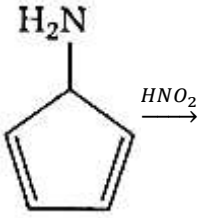
74. The total number of aromatic species generated in the following reactions is.

(i)  $Cl + SbCl_5 \longrightarrow$

(ii)  + Sodium metal $\xrightarrow{THF}$



(iii)  $+H_2O \longrightarrow$

(iv) 

(a) zero (b) 2
(c) 3 (d) 4

75. Roasted copper pyrite on smelting with sand produces

(a) $FeSiO_3$ as fusible slag and Cu_2 S as matte
(b) $CaSiO_3$ as infusible slag and Cu_2O as matte
(c) $Ca_3(PO_4)_2$ as fusible slag and Cu_2 S as matte
(d) $Fe_3(PO_4)_2$ as infusible slag and Cu_2 S as matte

Directions (Q. Nos. 76-80) Each question has one or more correct option(s),

76. Ionisation potential values of noble gases decrease down the group with increase in atomic size. Xenon forms binary fluorides by the direct reaction of elements. Identify the correct statement(s) from below.

(a) Only the heavier noble gases form such compounds
(b) It happens because the noble gases have higher ionisation energies
(c) It happens because the compounds are formed with electronegative ligands
(d) Octet of electrons provide the stable arrangements

77. Optical isomerism is exhibited by (ox= oxalate anion; en = ethylenediamine).

(a) cis $-[CrCl_2(OX)_2]^{3-}$
(b) $[Co(en)_3]^{3+}$
(c) trans $-[CrCl_2(Ox)_2]^{3-}$
(d) $[Co(ox)(en)_2]^{+}$

78. The increase in rate constant of a chemical reaction with increasing temperature is (are) due to the fact(s) that.

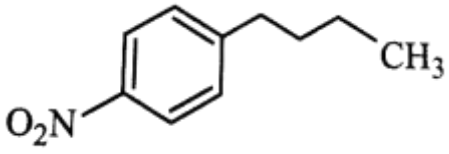
(a) the number of collisions among the reactant molecules increases with increasing temperature
(b) the activation energy of the reaction decreases with increasing temperature
(c) the concentration of the reactant molecules increases with increasing temperature
(d) the number of reactant molecules acquiring the activation energy increases with increasing temperature

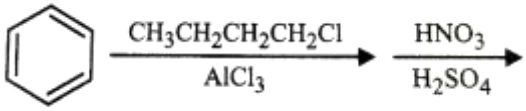
79. Within the list shown below, the correct pair of structures of alanine in pH ranges 2-4 and 9-11 is.

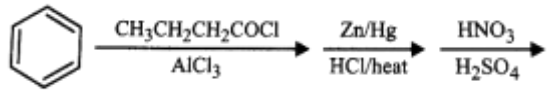
I. $H_3\ N^+ - CH(CH_3)CO_2H$
II. $H_2\ N - CH(CH_3)CO_2^-$
III. $H_3\ N^+ - CH(CH_3)CO_2^-$
IV. $H_2\ N - CH(CH_3)CO_2H$

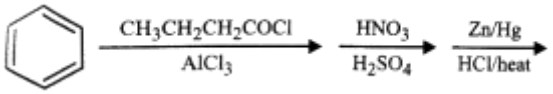
(a) I and II
(b) I and III
(c) II and III
(d) III and IV

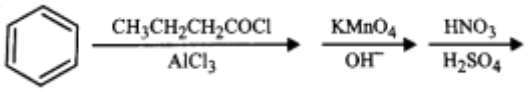
80. Identify the correct method for the synthesis of the compound shown below from the following alternatives.



(a) 

(b) 

(c) 

(d) 

Mathematics

81. Let a, b, c and d be any four real numbers. Then, $a^n + b^n = c^n + d^n$ holds for any natural number n, if

(a) $a + b = c + d$
(b) $a - b = c - d$
(c) $a + b = c + d, a^2 + b^2 = c^2 + d^2$
(d) $a - b = c - d, a^2 - b^2 = c^2 - d^2$

82. If α and β are the roots of $x^2 - px + 1 = 0$ and γ is a root of $x^2 + px + 1 = 0$, then $(\alpha + \gamma)(\beta + \gamma)$ is.

(a) 0 (b) 1
(c) -1 (d) p

83. The number of irrational terms in the binomial expansion of $\left(3^{1/5} + 7^{1/3}\right)^{100}$ is

(a) 90 (b) 88
(c) 94 (d) 95

84. The quadratic expression $(2x + 1)^2 - px + q \neq 0$ for any real x, if

(a) $p^2 - 16p - 8q < 0$
(b) $p^2 - 8p + 16q < 0$
(c) $p^2 - 8p - 16q < 0$
(d) $p^2 - 16p + 8q < 0$

85. In a certain town, 60% of the families own a car, 30% own a house and 20% own both car and house. If a family is randomly chosen, then what is the probability that this family owns a car or a house but not both?

(a) 0.5 (b) 0.7
(c) 0.1 (d) 0.9

86. The letters of the word 'COCHIN' are permuted and all the permutations are arranged in alphabetical order as in English dictionary. The number of words that appear before the word 'COCHIN', is

(a) 360 (b) 192
(c) 96 (d) 48

87. Let $f: R \to R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then, the value of $f(\log_e 5)$ is.

(a) 0 (b) 2
(c) 5 (d) 3

88. Let $f: R \to R$ be defined as $f(x) = \frac{x^2 - x + 4}{x^2 + x + 4}$.

Then, range of the function $f(x)$ is

(a) $\left[\frac{3}{5}, \frac{5}{3}\right]$
(b) $\left(\frac{3}{5}, \frac{5}{3}\right)$
(c) $\left(-\infty, \frac{3}{5}\right) \cup \left(\frac{5}{3}, \infty\right)$
(d) $\left[-\frac{5}{3}, -\frac{3}{5}\right]$

89. The least value of $2x^2 + y^2 + 2xy + 2x - 3y + 8$ for real numbers x and y, is

(a) 2 (b) 8
(c) 3 (d) $-1/2$

90. Let $f: [-2, 2] \to R$ be a continuous function such that $f(x)$ assumes only irrational values.

If $f(\sqrt{2})=\sqrt{2}$, then .

(a) $f(0)=0$

(b) $f(\sqrt{2}-1)=\sqrt{2}-1$

(c) $f(\sqrt{2}-1)=\sqrt{2}+1$

(d) $f(\sqrt{2}-1)=\sqrt{2}$

91. The minimum value of $\cos\theta+\sin\theta+\frac{2}{\sin 2\theta}$ for $\theta\in(0,\pi/2)$, is

(a) $2+\sqrt{2}$ (b) 2

(c) $1+\sqrt{2}$ (d) $2\sqrt{2}$

92. The value of $\lim_{x\to 2}\int_2^x \frac{3t^2}{(x-2)}dt$ is.

(a) 10 (b) 12

(c) 8 (d) 16

93. If $\cot\frac{2x}{3}+\tan\frac{x}{3}=\operatorname{cosec}\frac{kx}{3}$, then the value of k is.

(a) 1 (b) 2

(c) 3 (d) -1

94. If $\theta\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$, then the value of $\sqrt{4\cos^4\theta+\sin^2 2\theta}+4\cot\theta\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ is

(a) $-2\cot\theta$ (b) $2\cot\theta$

(c) $2\cos\theta$ (d) $2\sin\theta$

95. The number of real solutions of the equation $(\sin x-x)(\cos x-x^2)=0$ is.

(a) 1 (b) 2

(c) 3 (d) 4

96. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{64}+\left(\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^{64}$ is.

(a) 0 (b) -1

(c) 1 (d) i

97. Find the maximum value of $-|z|$ when $\left|z-\frac{3}{z}\right|=2$, where z being a complex number.

(a) $1+\sqrt{3}$ (b) 3

(c) $1+\sqrt{2}$ (d) 1

98. Given that, x is a real number satisfying $\frac{5x^2-26x+5}{3x^2-10x+3}<0$, then.

(a) $x<\frac{1}{5}$

(b) $\frac{1}{5}<x<3$

(c) $\dot{x}>5$

(d) $\frac{1}{5}<x<\frac{1}{3}$ or $3<x<5$

99. The value of λ such that the system of equations.
$2x-y-2z=2; x-2y+z=-4;$
$x+y+\lambda z=4$, has no solution, is

(a) 3 (b) 1 .

(c) 0 (d) -3

100. If $f(x)=\begin{vmatrix}1 & x & x+1\\ 2x & x(x-1) & (x+1)x\\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1)\end{vmatrix}$
Then, $f(100)$ is equal to

(a) 0 (b) 1

(c) 100 (d) 10

101. Let $x_n=\left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{6}\right)^2\left(1-\frac{1}{10}\right)^2\ldots\left(1-\frac{1}{\frac{n(n+1)}{2}}\right)^2, n\geq 2$. Then, the value of $\lim_{n\to\infty}x_n$ is

(a) $1/3$ (b) $1/9$
(c) $1/81$ (d) 0

102. The variance of first 20 natural numbers is.

(a) $133/4$ (b) $279/12$
(c) $133/2$ (d) $399/4$

103. A fair coin is tossed at a fixed number of times. If the probability of getting exactly 3 heads equals the probability of getting exactly 5 heads, then the probability of getting exactly one head is.

(a) $1/64$ (b) $1/32$
(c) $1/16$ (d) $1/8$

104. If the letters of the word 'PROBABILITY' are written down at random in a row, then probability that two B's are together, is.

(a) $\frac{2}{11}$ (b) $\frac{10}{11}$
(c) $\frac{3}{11}$ (d) $\frac{6}{11}$

105. Which of the following is not always true?

(a) $|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2$, if $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other
(b) $|\mathbf{a}+\lambda\mathbf{b}| \geq |\mathbf{a}|$ for all $\lambda \in R$, if $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other
(c) $|\mathbf{a}+\mathbf{b}|^2 + |\mathbf{a}-\mathbf{b}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2)$
(d) $|\mathbf{a}+\lambda\mathbf{b}| \geq |\mathbf{a}|$ for all $\lambda \in R$, if $\mathbf{a}$ is parallel to $\mathbf{b}$

106. If the four points with position vectors $-2\hat{\imath}+\hat{\jmath}+\hat{k}, \hat{\imath}+\hat{\jmath}+\hat{k}, \hat{\jmath}-\hat{k}$ and $\lambda\hat{\jmath}+\hat{k}$ are coplanar, then λ is equal to.

(a) 1 (b) 2
(c) -1 (d) 0

107. The least positive value of t, so that the lines $x = t+\alpha, y+16=0$ and $y = \alpha x$ are concurrent, is.

(a) 2 (b) 4
(c) 16 (d) 8

108. In $\triangle ABC$, if $a^2\cos^2 A - b^2 - c^2 = 0$, then.

(a) $\frac{\pi}{4} < A < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < A < \pi$
(c) $A = \frac{\pi}{2}$ (d) $A < \frac{\pi}{4}$

109. $\{x \in R: |\cos x| \geq \sin x\} \cap \left[0, \frac{3\pi}{2}\right]$ is equal to.

(a) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{3\pi}{2}\right]$
(b) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.
(c) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$
(d) $\left[0, \frac{3\pi}{2}\right]$

110. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \cdots\right) = \frac{\pi}{6}$, where $|x| < 2$, then the value of x is.

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) $-\frac{2}{3}$ (d) $-\frac{3}{2}$

111. The area of the region bounded by the curve $y = x^3$, its tangent at $(1, 1)$ and X-axis, is.

(a) $\frac{1}{12}$ sq unit (b) $\frac{1}{6}$ sq unit
(c) $\frac{2}{17}$ sq unit (d) $\frac{2}{15}$ sq unit

112. If $\log_{0.2}(x-1) > \log_{0.04}(x+5)$, then.

(a) $-1 < x < 4$
(b) $2 < x < 3$
(c) $1 < x < 4$
(d) $1 < x < 3$

113. The number of real roots of equation $\log_e x + ex = 0$ is.

(a) 0 (b) 1
(c) 2 (d) 3

114. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ **in the interval** $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ **is**

(a) 0 (b) 2
(c) 1 (d) > 2

115. Let $x_1, x_2, \ldots, x_{15}$ **be 15 distinct numbers chosen from** $1, 2, 3, \ldots, 15$**. Then, the value of** $(x_1 - 1)(x_2 - 1)(x_3 - 1) \ldots (x_{15} - 1)$ **is.**

(a) always ≤ 0
(b) 0
(c) always even
(d) always odd

116. Let $[x]$ **denotes the greatest integer less than or equal to** x**. Then, the value of** α **for which the function** $f(x) = \begin{cases} \frac{\sin[-x^2]}{[-x^2]}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$ **is continuous at** $x = 0$**, is.**

(a) $\alpha = 0$
(b) $\alpha = \sin(-1)$
(c) $\alpha = \sin(1)$
(d) $\alpha = 1$

117. For all real values of a_0, a_1, a_2, a_3 **satisfying** $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$**, the equation** $a_0 + a_1 x + a_2 x^2 + a_3 x^3 = 0$ **has a real root in the interval.**

(a) $[0,1]$ (b) $[-1,0]$
(c) $[1,2]$ (d) $[-2,-1]$

118. Let $f: R \to R$ **be defined as**

$f(x) = \begin{cases} 0, & x \text{ is irrational} \\ \sin|x|, & x \text{ is rational} \end{cases}$

Then, which of the following is true?

(a) f is discontinuous for all x
(b) f is continuous for all x
(c) f is discontinuous at $x = k\pi$, where k is an integer
(d) f is continuous at $x = k\pi$, where k is an integer

119. A particle starts moving from rest from a fixed point in a fixed direction. The distance s **from the fixed point at a time** t **is given by** $s = t^2 + at - b + 17$**, where** a **and** b **are real numbers. If the particle comes to rest after 5 s at a distance of** $s = 25$ **units from the fixed point, then values of** a **and** b **are, respectively**

(a) $10, -33$ (b) $-10, -33$
(c) $-8,33$ (d) $-10,33$

120. $\lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n-1}}{n\sqrt{n}}$ **is equal to.**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) 0

121. If $\lim_{x\to 0} \frac{axe^x - b\log(1+x)}{x^2} = 3$**, then the values of** a **and** b **are, respectively.**

(a) 2,2 (b) 1,2
(c) 2,1 (d) 2,0

122. If the vertex of the conic $y^2 - 4y = 4x - 4a$ **always lies between the straight lines** $x + y = 3$ **and** $2x + 2y - 1 = 0$**, then.**

(a) $2 < a < 4$
(b) $-\frac{1}{2} < a < 2$

(c) $0 < a < 2$
(d) $-\frac{1}{2} < a < \frac{3}{2}$

123. Number of intersecting points of the conics $4x^2 + 9y^2 = 1$ and $4x^2 + y^2 = 4$ is
(a) 1 (b) 2
(c) 3 (d) 0

124. The value of λ for which the straight line $\frac{x-\lambda}{3} = \frac{y-1}{2+\lambda} = \frac{z-3}{-1}$ may lie on the plane $x - 2y = 0$, is.
(a) 2 (b) 0
(c) $-\frac{1}{2}$ (d) there is no such λ

125. Area of the region bounded by $y = |x|$ and $y = -|x| + 2$ is.
(a) 4 sq units
(b) 3 sq units
(c) 2 sq units
(d) 1 sq unit

126. Let $d(n)$ denotes the number of divisors of n including 1 and itself. Then, d (225), d (1125) and $d(640)$ are.
(a) in AP
(b) in HP
(c) in GP
(d) consecutive integers

127. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} 2\alpha$ has a real solution, if.
(a) $|a| > \frac{1}{\sqrt{2}}$
(b) $\frac{1}{2\sqrt{2}} < |a| < \frac{1}{\sqrt{2}}$
(c) $|a| > \frac{1}{2\sqrt{2}}$
(d) $|a| \leq \frac{1}{2\sqrt{2}}$

128. If $(2 + i)$ and $(\sqrt{5} - 2i)$ are the roots of the equation $(x^2 + ax + b)(x^2 + cx + d) = 0$, where a, b, c and d are real constants, then product of all the roots of the equation is.
(a) 40 (b) $9\sqrt{5}$
(c) 45 (d) 35

129. If $f: [0, \pi/2) \to R$ is defined as $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$. Then, the range of f is.
(a) $(2, \infty)$ (b) $(-\infty, -2]$
(c) $[2, \infty)$ (d) $(-\infty, 2]$

130. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ equals.
(a) $2AB$ (b) $2BA$
(c) $A + B$ (d) AB

131. If ω is an imaginary cube root of unity, then the value of the determinant $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix}$ is
(a) -2ω (b) $-3\omega^2$
(c) -1 (d) 0

132. Let $f(x)$ denotes the fractional part of a real number x. Then, the value of $\int_0^{\sqrt{3}} f(x^2)dx$ is.
(a) $2\sqrt{3} - \sqrt{2} - 1$
(b) 0
(c) $\sqrt{2} - \sqrt{3} + 1$
(d) $\sqrt{3} - \sqrt{2} + 1$

133. Let $S = \{(a, b, c) \in N \times N \times N: a + b + c = 21, a \leq b \leq c\}$ and

$T = \{(a, b, c) \in N \times N \times N : a, b, c$ are in AP $\}$, where N is the set of all natural numbers. Then, the number of elements in the set $S \cap T$ is

(a) 6 (b) 7
(c) 13 (d) 14

134. Let $y = e^{x^2}$ and $y = e^{x^2} \sin x$ be two given curves. Then, angle between the tangents to the curves at any point of their intersection is.

(a) 0 (b) π
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

135. The value of $2\cot^{-1}\frac{1}{2} - \cot^{-1}\frac{4}{3}$ is.

(a) $-\frac{\pi}{8}$ (b) $\frac{3\pi}{2}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

136. If the point $(2\cos\theta, 2\sin\theta)$ for $0 \in (0, 2\pi)$ lies in the region between the lines $x + y = 2$ and $x - y = 2$ containing the origin, then θ lies in.

(a) $\left(\theta, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$
(b) $[0, \pi]$
(c) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
(d) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

137. Number of points having distance $\sqrt{5}$ from the straight line $x - 2y + 1 = 0$ and a distance $\sqrt{13}$ from the line $2x + 3y - 1 = 0$, is.

(a) 1 (b) 2
(c) 4 (d) 5

138. Let $P(x)$ be a polynomial, which when divided by $(x - 3)$ and $(x - 5)$ leaves remainders 10 and 6, respectively. If the polynomial is divided by $(x - 3)(x - 5)$, then the remainder is.

(a) $-2x + 16$ (b) 16
(c) $2x - 16$ (d) 60

139. The integrating factor of the differential equation $\frac{dy}{dx} + \left(3x^2\tan^{-1}y - x^3\right)\left(1 + y^2\right) = 0$ is

(a) e^{x^2} (b) e^{x^3}
(c) e^{3x^2} (d) e^{3x^3}

140. If $y = e^{-x}\cos 2x$, then which of the following differential equation is satisfied?

(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$
(b) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 2y = 0$
(c) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 2y = 0$
(d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 5y = 0$

141. Let $f: R \to R$ be differentiable at $x = 0$. If $f(0) = 0$ and $f'(0) = 2$, then the value of $\lim_{x\to 0}\frac{1}{x}[f(x) + f(2x) + f(3x) + \cdots + f(2015x)]$ is.

(a) 2015 (b) 0
(c) 2015×2016 (d) 2015×2014

142. If x and y are digits such that $17! = 3556xy428096000$, then $x + y$ equals

(a) 15 (b) 6
(c) 12 (d) 13

143. A person goes to office by car, scooter, bus and train, probability of which are $1/7, 3/7, 2/7$ and $1/7$, respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $2/9, 1/9, 4/9$, and $1/9$, respectively. Given that he reached office in time, the probability that he travelled by a car, is.

(a) $1/7$ (b) $2/7$
(c) $3/7$ (d) $4/7$

144. The value of $\int \frac{(x-2)}{\{(x-2)^2(x+3)^7\}^{1/3}}dx$ is

(a) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3}+C$

(b) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{3/4}+C$

(c) $\frac{5}{12}\left(\frac{x-2}{x+3}\right)^{4/3}+C$

(d) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{5/3}+C$

145. Let $f: N \to R$ be such that $f(1)=1$ and $f(1)+2f(2)+3f(3)+\cdots+nf(n)=n(n+1)f(n)$, for all $n \in N, n \geq 2$, where N is the set of natural numbers and R is the set of real numbers. Then, the value of $f(500)$ is.

(a) 1000 (b) 500

(c) $1/500$ (d) $1/1000$

146. If 5 distinct balls are placed at random into 5 cells, then the probability that exactly one cell remains empty, is.

(a) $48/125$ (b) $12/125$

(c) $8/125$ (d) $1/125$

147. A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006 . What is the probability of death due to lung cancer given that a person is a smoker?

(a) $1/140$ (b) $1/70$

(c) $3/140$ (d) $1/10$

148. In a $\triangle ABC$, if $\angle C = 90^\circ, r$ and R are the inradius and circumradius of the $\triangle ABC$ respectively, then $2(r+R)$ is equal to.

(a) $b+c$

(b) $c+a$

(c) $a+b$

(d) $a+b+c$

149. Let α and β be two distinct roots of $a\cos\theta + b\sin\theta = c$, where a, b, c are three real constants and $\theta \in [0, 2\pi]$. Then, $\alpha+\beta$ is also a root of the same equation, if.

(a) $a+b=c$

(b) $b+c=a$

(c) $c+a=b$

(d) $c=a$

150. For a matrix $A=\begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix}$, if U_1, U_2 and U_3 are 3×1 column matrices satisfying $AU_1=\begin{pmatrix}1\\0\\0\end{pmatrix}, AU_2=\begin{pmatrix}2\\3\\0\end{pmatrix}, AU_3=\begin{pmatrix}2\\3\\1\end{pmatrix}$ and U is 3×3 matrix whose columns are U_1, U_2 and U_3. Then, sum of the elements of U^{-1} is

(a) 6 (b) 0

(c) 1 (d) $2/3$

Directions (Q. Nos. 151-160) Each question has one or more correct option.

151. A relation ρ on the set of real number R is defined as $\{x\rho y: xy>0\}$. Then, which of the following is/are true?

(a) ρ is reflexive and symmetric

(b) ρ is symmetric but not reflexive

(c) ρ is symmetric and transitive

(d) ρ is an equivalence relation

152. A relation ρ on the set of real number R is defined as $\{x\rho y: xy>0\}$. Then, which of the following is/are true?

(a) ρ is reflexive and symmetric

(b) ρ is symmetric but not reflexive
(c) ρ is symmetric and transitive
(d) ρ is an equivalence relation

153. If $\cos x$ and $\sin x$ are solutions of the differential equation

$$a_0\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=0$$

where a_0, a_1 and a_2 are real constants, then which of the following is/are always true?

(a) $A\cos x + B\sin x$ is a solution, where A and B are real constants
(b) $A\cos\left(x+\frac{\pi}{4}\right)$ is a solution, where A is a real constant
(c) $A\cos x\sin x$ is a solution, where A is a real constant
(d) $A\cos\left(x+\frac{\pi}{4}\right)+B\sin\left(x-\frac{\pi}{4}\right)$ is a solution, where A and B are real constants

154. Which of the following statements is /are correct for $0<\theta<\frac{\pi}{2}$?.

(a) $(\cos\theta)^{1/2}\le\cos\frac{\theta}{2}$
(b) $(\cos\theta)^{3/4}\ge\cos\frac{3\theta}{4}$
(c) $\cos\frac{5\theta}{6}\ge(\cos\theta)^{5/6}$
(d) $\cos\frac{7\theta}{8}\le(\cos\theta)^{7/8}$

155. Let $16x^2-3y^2-32x-12y=44$ represents a hyperbola. Then,

(a) length of the transverse axis is $2\sqrt{3}$
(b) length of each latusrectum is $32/\sqrt{3}$
(c) eccentricity is $\sqrt{19/3}$
(d) equation of a directrix is $x=\frac{\sqrt{19}}{3}$

156. For the function $f(x)=\left[\frac{1}{[x]}\right]$, where $[x]$ denotes the greatest integer less than or equal to x, which of the following statements are true?

(a) The domain is $(-\infty,\infty)$
(b) The range is $\{0\}\cup\{-1\}\cup\{1\}$
(c) The domain is $(-\infty,0)\cup[1,\infty)$
(d) The range is $\{0\}\cup\{1\}$

157. Which of the following is/are always false?

(a) A quadratic equation with rational coefficients has zero or two irrational roots
(b) A quadratic equation with real coefficients has zero or two non-real roots
(c) A quadratic equation with irrational coefficients has zero or two irrational roots
(d) A quadratic equation with integer coefficients has zero or two irrational roots

158. If the straight line $(a-1)x-by+4=0$ is normal to the hyperbola $xy=1$, then which of the following does not hold?

(a) $a>1, b>0$
(b) $a>1, b<0$
(c) $a<1, b<0$
(d) $a<1, b>0$

159. Suppose a machine produces metal parts that contain some defective parts with probability 0.05 . How many parts should be produced in order that the probability of atleast one part being defective is $1/2$ or more? (Given that, $\log_{10}95=1.977$ and $\log_{10}2=0.3$)

(a) 11 (b) 12
(c) 15 (d) 14

160. Let $f: R\to R$ be such that $f(2x-1)=f(x)$ for all $x\in R$. If f is continuous at $x=1$ and $f(1)=1$, then

(a) $f(2)=1$
(b) $f(2)=2$
(c) f is continuous only at $x=1$
(d) f is continuous at all points