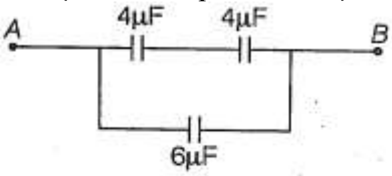
Physics

1. **(b)** $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$

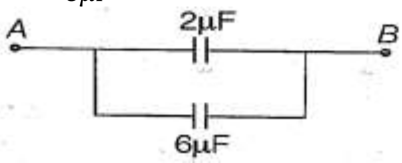
$C = 2\mu$ F

2μ F will be parallel to 4μ F, so the resultant capacitance



$C' = C_1 + C_2 = 2\mu F + 4\mu F$

$= 6\mu F$



$$\frac{1}{C''} = \frac{1}{4\mu F}+\frac{1}{4\mu F}=\frac{2}{4}\mu F$$

$$C'' = 2\mu F$$

$$C_{AB} = 2\mu F + 6\mu F = 8\mu F$$

2. **(b)** $R=\frac{\rho l}{A}$

where, ρ = resistivity

I = length of the resistance wire

A = area of cross-section

When the wires are connected in series, then

$R = R_1 + R_2$

$$R=\frac{\rho_1 l_1}{A_1}+\frac{\rho_1 l_2}{A_2}$$

$$=\frac{\rho_1 l_1}{A}+\frac{\rho_2 l_2}{A} \quad (\because A_1 = A_2)$$

ρ_{eq} = resistivity of combination

$I = I_1 + I_2$

$A = A$

Thus, $\rho_{\text{eq}}:\frac{l_1+l_2}{A}=\frac{\rho_1 l_1+\rho_2 l_2}{A}$

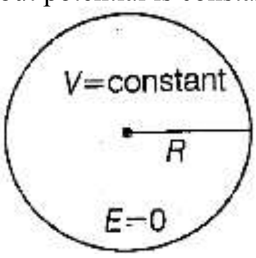
$$\rho_{\text{eq}}=\frac{\rho_1 I_1+\rho_2 I_2}{I_1+I_2}$$

3. **(b)**

Inside a hollow charged sphere, the electric field intensity is always zero.

$\therefore \ E = 0$

but potential is constant.



Potential on the surface of the hollow sphere

$$V=\frac{KQ}{R}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}=\frac{Q}{4\pi\varepsilon_0 R}$$

This is the potential (V) inside the charged hollow metal sphere.

4. **(c)** The de-Broglie wavelength associated with electron is given by,

$$\lambda=\frac{h}{\sqrt{2meV}}$$
where, $h=$ Planck's constant
$m=$ mass of electron
$e=$ charge on electron
$V=$ potential difference
$\lambda=1\text{Å}=10^{-10}$ m

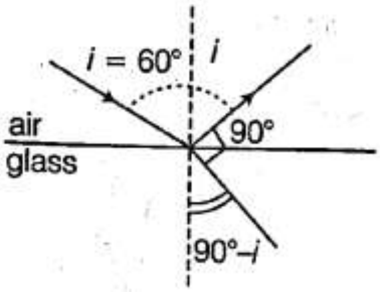
$$\lambda^2=\frac{h^2}{2\text{meV}}\Rightarrow V=\frac{h^2}{2\text{me}\lambda^2}$$
$$=\frac{(6.6\times10^{-34})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times(1\times10^{-10})^2}$$
$$=\frac{6.6\times10^{-34}\times6.6\times10^{-34}}{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times10^{-20}}$$
$$=\frac{6.6\times6.6\times10^2}{2\times9.1\times1.6}=\frac{4356\times10^2}{2912}=150\text{ V}$$

5. (b)
Sure! Here's the same solution without line gaps:
Use the photoelectric equation: $eV_0=\frac{hc}{\lambda}-\phi$
Given $h=6.63\times10^{-34}$ J \ cdotps, $c=3\times10^8$ m/s, $\lambda=600\times10^{-9}$ m, $\phi=2.27$eV Calculate photon energy:
$E=\frac{6.63\times10^{-34}\times3\times10^8}{600\times10^{-9}}=3.315\times10^{-19}$ J
Convert to eV: $E=\frac{3.315\times10^{-19}}{1.6\times10^{-19}}=2.072$eV
Apply photoelectric equation: $eV_0=2.072-2.27=-0.198$eV
Final answer: $V_0\approx-0.2$ V, so the correct option is (b) -0.2 V .

6. (b) The number of de-Broglie wavelength contained in the second Bohr orbit of Hydrogen atom is 2 .

7. (d) Given: Wavelength of 2nd Balmer line $(n=4\rightarrow2)=600$ nm
Using Rydberg formula for both transitions:
$$\frac{1}{\lambda_1}=R\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=R\cdot\frac{3}{16},\Rightarrow R=\frac{1}{600}\cdot\frac{16}{3}$$
Now, for 3rd Lyman line (n = 4 → 1):
$$\frac{1}{\lambda_2}=R\left(1-\frac{1}{4^2}\right)=R\cdot\frac{15}{16}\Rightarrow\frac{1}{\lambda_2}=\frac{1}{600}\cdot\frac{16}{3}\cdot\frac{15}{16}$$
$$=\frac{15}{1800}=\frac{1}{120}\Rightarrow\lambda_2=120\text{ nm}$$
Correct answer: Not matching any option exactly - closest is (d) 200 nm , but actual answer is 120 nm .

8. (d)
Here, angle of incidence is 60°



$$\mu=\frac{\sin i}{\sin r}$$
here,
$\sin i=\mu\sin r$
$r=90-i$
$\sin i=\mu\sin(90°-i)=\mu\cos i$
$$\mu=\frac{\sin i}{\cos i}=\tan i=\tan 60°$$

$\mu = \sqrt{3}$

9. (c)

Thickness of glass plate $= t$

Refractive index $= \mu$

Velocity of light $= c$

$$\text{Speed of light } = \frac{\text{distance}}{\text{time}} = \frac{t}{T}$$

$$T = \frac{t}{\text{speed of light}}$$

Speed of light in glass plate

$$V = \frac{c}{\mu}$$

$$T = \frac{t}{c/\mu} = \frac{\mu t}{c}$$

10. (d)

Given, $x = at + bt^2$

$$[x] = [bt^2]$$

$$\text{Unit of } b = \frac{x}{t^2} = \frac{\text{metre}}{(\mathrm{h})^2} = \frac{\mathrm{m}}{\mathrm{h}^2}$$

11. (c) Given, $|A+B| = |A-B|$

$(A+B)^2 = (A-B)^2$

$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$2AB\cos\theta + 2AB\cos\theta = 0$

$4AB\cos\theta = 0$

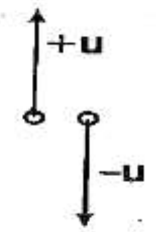
Here

$\mathrm{A} \neq 0, B \neq 0$

$\therefore \quad \cos\theta = 0$

or $\quad \theta = 90^\circ$

12. (b) Given that velocity of ascending body = **u**

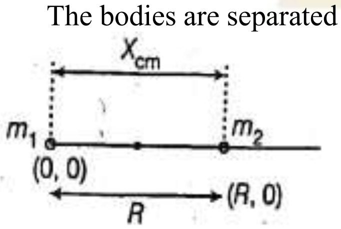


Let there is no air resistance when body falls freely in downward direction, the direction of falling body will opposite to the ascending body.

∴ The velocity at the same height while the body falls freely = −u

13. (a)

The bodies are separated by a distance R, so the coordinates of m_1 and m_2 will be $(0,0)$ and $(R,0)$



From the formula of centre of mass,

$$X_{\mathrm{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{m_1 \times 0 + m_2 \times R}{m_1 + m_2}$$

$$X_{\mathrm{cm}} = \frac{m_2 \cdot R}{m_1 + m_2}$$

14. (b)$T_1 = 273 + 20 = 293\ \mathrm{K}$

$T_2 = 273 + 40 = 313$ K
The velocity of sound wave in gases or air is given by

$$v = \sqrt{\frac{\gamma R T_1}{M}}$$

where, γ = Ratio of C_p / C_V
R = gas constant

$$v_1 = \sqrt{\frac{\gamma R T_1}{M}}$$

$$\therefore \; v_2 = \sqrt{\frac{\gamma R T_2}{M}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{293}{313}}$$

Given, $v_1 = 344.2$ m/s

$$v_2 = v_1 \sqrt{\frac{313}{293}}$$

$$\begin{aligned} \therefore \quad &= 344.2 \times \sqrt{1068} \\ &= 344.2 \times 1.03 \text{ m/s} \\ &= 355.75 \text{ m/s} \\ &\simeq 356 \text{ m/s} \end{aligned}$$

15. (b)
Mass of gas = 4 g
From perfect gas equation
$pV = nRT$
where n = number of moles
here, $\left(n = \frac{4}{2} = 2\right)$
$\therefore \; pV = 2RT$
It will be the perfect gas equation for 4 g of hydrogen.

16. (d)
Suppose temperature of Sun = T
When it is doubled, $T' = 2T$
From law of radiation (Stefan's law)
$E \propto T^4 = \sigma T^4$
σ = Stefan's constant.

$$E_1 \propto T^4$$
$$E_2 \propto (2T)^4$$
$$\frac{E_1}{E_2} = \frac{T^4}{2^4 \cdot T^4}$$
$$E_2 = 16 E_1.$$

17. (c)
In SHM, the acceleration of vibrating particle is proportional to displacement
$|\mathbf{a}| = \omega^2 x.$
Here $a = 16 \text{ cm/s}^2$
$= 16 \times 10^{-2} \text{ m/s}^2$
Displacement, $x = 4 \text{ cm} = 4 \times 10^{-2}$ m

$$16 \times 10^{-2} = \omega^2 (4 \times 10^{-2})$$
$$\omega^2 = 4$$
$$\omega = \pm 2 \text{rad/s}$$
$\therefore$
$$\omega = \frac{2\pi}{T}$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi = 3.142 \text{ s}$$

18. (a) The work done in stretching the spring, $W = \frac{1}{2}Kx^2$

$K =$ force constant of spring

$x =$ extension in length of the spring

$$W_1 = \frac{1}{2}Kx_1^2$$
$$W_2 = \frac{1}{2}Kx_2^2$$
$$x_1 = 1$$
$$x_2 = 1 + 1 = 2$$
$$\frac{W_1}{W_2} = \frac{\frac{1}{2}K(1)^2}{\frac{1}{2}K(2)^2} = \frac{1}{4}$$
$$W_2 = 4W_1$$
$$W_1 = 10 \text{ J}$$
$$W_2 = 40 \text{ J}$$

$\therefore$ More work required $= 40 \text{ J} - 10 \text{ J} = 30 \text{ J}$

19. (c)$v_{\text{ms}} = \sqrt{\frac{3RT}{M}}$

$R =$ gas constant

$T =$ absolute temperature

$M =$ mass of gas

$$v_{\text{rms}} \text{ of } H_2 = \sqrt{\frac{3RT}{M_{H_2}}}$$
$$v_{\text{rms}} \text{ of } O_2 = \sqrt{\frac{3RT}{M_{O_2}}}$$
$$\frac{v_{\text{rms}} \cdot H_2}{v_{\text{rms}} \cdot O_2} = \sqrt{\frac{M_{O_2}}{M_{H_1}}}$$

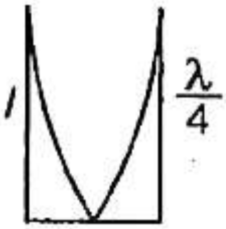
Given

$v_{\text{rms}} H_2 = c$

$$\Rightarrow \frac{c}{v_{\text{rms}} O_2}$$
$$= \sqrt{\frac{M_{O_2}}{M_{H_1}}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$$
$$v_{\text{ms}} = \frac{c}{4}$$

20. (d) The tube is closed at one end. So, it will act like a closed organ pipe.

The lowest frequency produced in the tube



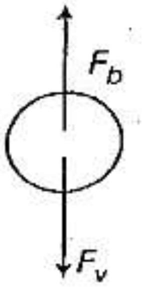
$l = \frac{\lambda}{4}, \lambda = 4$
$l = 1$ m
$l = 84$ Hz
$v = n\lambda = 84 \times 4$
When wave propagate in air, the velocity

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

Here,

$$p = 1.0 \times 10^5 \ \mathrm{N/m^2}$$
$$\rho = 1.2 \ \mathrm{kg/m^3}$$
$$v^2 = \frac{\gamma p}{\rho}$$
$$\gamma = \frac{v^2 \cdot \rho}{p}$$
$$= \frac{(84 \times 4)^2 \times 1.2}{1.0 \times 10^5}$$
$$= \frac{135472.2}{10^5} = 1.354$$
$$\approx 1.4$$

21. (d) The gas bubble is rising through the liquid of density $1.75 \ \mathrm{g/cm^3}$ with constant speed. The acting forces on bubble are



Buoyancy force of liquid = F_b (in upward direction)
Viscous force due to liquid = F_V (in downward direction)
$\therefore \ F_V = F_b$
For free falling in a viscous medium
$F_V = 6\pi\eta N_T$
η = viscosity coefficient of the liquid
r = radius of bubble
$F_b = mg = v \times \rho . g$

$$= \frac{4}{3}\pi r^3 \rho_e . g$$

$$6\pi\eta v_t = \frac{4}{3}\pi r^3 \rho_e \cdot g$$

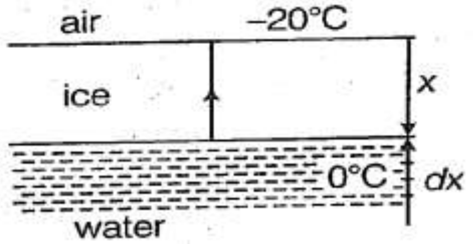
$$\eta = \frac{24\pi \cdot r^3 \rho_e \cdot g}{3 \times 6\pi v_T} = \frac{2}{9} \cdot \frac{r^2 \rho_e \cdot g}{v_T}$$

here,

$D = 2\text{ cm}, r = 1\text{ cm} = 1 \times 10^{-2}\text{ m}$
$\rho_e = 1.75\text{ g/cm}^3 = 1.75 \times 10^{+3}\text{ kg/m}^3$
$g = 10\text{ m/s}^2$
$v_T =$ terminal velocity
$= 0.35 \times 10^{-2}\text{ m/s}$

$$\eta = \frac{2}{9} \times \frac{(10^{-2})^2 \times 1.75 \times 10^{+3} \times 10}{0.35 \times 10^{-2}}$$
$$= \frac{2}{9} \times \frac{10^{-4} \times 1.75 \times 10^3 \times 10}{0.35 \times 10^{-2}}$$
$$= \frac{2 \times 175 \times 10 \times 10}{9 \times 35} \times 10\text{ poise}$$
$= 1089$ poise

22. (d) Temperature of water $= 0°\text{C}$
Temperature of surrounding atmosphere $= -20°\text{C}$
Density of ice $= \rho$



Coefficient of thermal conductivity $= k$
latent heat of melting $= L$
The rate of heat flow $H = \frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{x}$
$$= \frac{kA[(0 - (-20)]}{x}$$
$$= \frac{kA \times 20}{x} \; (\because mL = Q)$$
$$\frac{dQ}{dt} = \frac{dm}{dt} \cdot L$$
$$dm = \rho A \cdot dx$$
$$\frac{dQ}{dt} = \rho A \cdot \frac{dx}{dt} L = \frac{kA \cdot 20}{x}$$
$$\int_0^Z x \cdot dx = \frac{20k}{\rho L} \int_0^t dt \Rightarrow Z^2 = \frac{40k}{\rho L} t$$
So, thickness Z increases as a function of time t according to the above equation.

23. (a)
Let the radius of single drop $= r$
Radius of small drop $= \frac{2\text{ mm}}{2}$
$= 1\text{ mm}$
$= 1 \times 10^{-3}\text{ m}$
Surface tension of water $= 0.072\text{ N/m}$
The volume of large drop must be equal volume of all small drops.
$$\frac{4}{3}\pi R^3 = 1000\left(\frac{4}{3}\pi r^3\right) = (1000)^{1/3} \cdot r = 10.rl$$
$\therefore$ R
The initial potential energy of drops system $U_i = s \times n \times 4\pi r^2$
$= s \times 1000 \times 4\pi r^2$
$= 1000s \cdot 4\pi r^2$
When the droplets coalesce, the final potential energy of the system $U_f = s \cdot 4\pi R^2$
$= s.4\pi \times 100r^2$

Energy loss in the formation of large drop

$\Delta E = U_f - U_i$

$= s \cdot 4\pi 100 r^2 - 1000 s \cdot 4\pi r^2$

$= s \cdot \pi r^2 (400 - 4000)$

$= -3600 s \cdot \pi r^2$

$= -3600 \times 0.72 \times 3.14 \times (1 \times 10^{-3})^2$

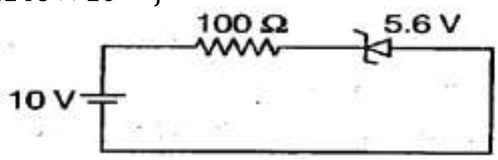
From catenation

$= 813.888 \times 10^{-6}$

$= 8.1388 \times 10^{-4}$ J

$= 8.146 \times 10^{-4}$ J

24. (d)



Breakdown voltage of Zener diode $= 5.6$ V

Voltage of source $= 10$ V

Potential difference $\Delta V = (10 - 5.6)\text{V} = 4.4$ V

Resistance of circuit $= 100\Omega$

The current passing through the Zener diode

$I = \dfrac{\Delta V}{R}$

$= \dfrac{4.4}{100}$

$= 4.4 \times 10^{-2}$ A

$= 44 \times 10^{-3}$ A

$= 44$ mA

25. (c) Current gain $\beta = 45$

We know that, $\beta = \frac{\Delta_c}{\Delta_b}$

$\therefore \frac{\Delta_c}{\Delta_b} = 45$……(i)

Potential drop across $1\text{k}\Omega$ or $1 \times 10^3\Omega$

Collector resistor is 5 V .

$V = I_C R$

$I_C = \dfrac{V}{R} = \dfrac{5}{1 \times 10^3}$

$= 5 \times 10^{-3}$ A

From Eqs. (i) and (ii),

$\Delta_B = \dfrac{\Delta_C}{45}$

$= \dfrac{5 \times 10^{-3}}{45} = \dfrac{1}{9} \times 10^{-3}$

$= 0.111 \times 10^{-3}$ A

$= 111 \times 10^{-6}$ A $= 111\mu$ A

26. (d) Given:

$\vec{E} = 3\hat{\imath} + 6\hat{\jmath} + 2\hat{k}, \vec{B} = 2\hat{\imath} + 3\hat{\jmath}, \vec{v} = 2\hat{\imath} + 3\hat{\jmath}$

Step 1:

$\vec{v} \times \vec{B} = 0$ (same direction $\rightarrow$ cross product zero)

Step 2:

$\vec{F} = e\vec{E} = -1.6 \times 10^{-19}(3\hat{\imath} + 6\hat{\jmath} + 2\hat{k}) \Rightarrow \vec{F} = (-4.8\hat{\imath} - 9.6\hat{\jmath} - 3.2\hat{k}) \times 10^{-19}$

Step 3:

$|\vec{F}| = 10^{-19} \times \sqrt{4.8^2 + 9.6^2 + 3.2^2} = 10^{-19} \times 11.2 = 1.79 \times 10^{-18}$ N

27. (c)

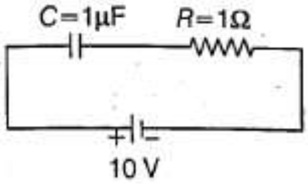
Given, $L_1 = 6\text{mH} = 6 \times 10^{-3}\text{H}$

$L_2 = 8\text{mH} = 8 \times 10^{-3}\text{H}$

If two coils L_1 and L_2 are connected in series, the equivalent self-inductance L for assembly

$$L = L_1 + L_2 + 2\sqrt{L_1 L_2}$$
$$= 6 \times 10^{-3} + 8 \times 10^{-3} + 2\sqrt{6 \times 8 \times 10^{-6}}$$
$$= 14 \times 10^{-3} + 2\sqrt{48} \times 10^{-3}$$
$$= 14 \times 10^{-3} + 2 \times 7 \times 10^{-3}$$
$$= 28 \times 10^{-3}\text{H} = 28\text{mH} \ (\because \sqrt{48} = 6.92 = 7)$$

28. (c)



The time constant of the circuit

$$\tau = C.R$$

Here,

$$= 1\mu\text{ F} = 1 \times 10^{-6}$$
$$R = 1\text{M}\Omega = 1 \times 10^{6}\Omega$$
$$\tau = 1 \times 10^{-6} \times 1 \times 10^{6} = 1\text{ s}$$

We know that in one (1) time constant 63% charging is done.

$$\therefore \quad \frac{63}{100} \times q_{\max}$$
$$= \frac{63}{100} \times 1 \times 10^{-6} \times 10$$
$$= \frac{63}{100} \times 1\mu\text{ F} \times 10 = 6.3\mu\text{C}$$
$$\Rightarrow \quad V = \frac{q}{C} = \frac{6.3 \times 10^{-6}\text{C}}{1 \times 10^{-6}\text{ F}} = 6.3\text{ V}$$

29. (b) In series combination,

$R = R_1 + R_2$

Same current will pass in R_1 and R_2. According to question, R_1 is increased by 0.5%.

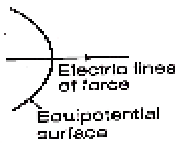
$\therefore$ Increment in the resistance

$$= \frac{0.5}{100} \times 400$$
$$= 2.0\Omega$$

So to keep the potential unchanged in second resistance the change required will be 2.0Ω increment.

30. (b) The electric lines of force are always perpendicular to the equipotential surface

$\theta = 90^\circ$



31. (a)

The current $I = I_0 e^{-\lambda t}$ is of exponential nature.

$$I = \frac{dQ}{dt} = I_0 e^{-\lambda t}$$
$$dQ = I_0 e^{-\lambda t} dt$$
$$\int_0^Q dQ = I_0^t \int_{t=0}^{t=\infty} e^{-\lambda t} \cdot dt$$
$$Q = I_0 \times \frac{1}{\lambda} = \frac{I_0}{\lambda}$$

32. (b)

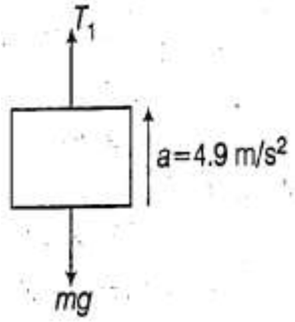
For Fraunhoffer diffraction to occur, both source and screen should be at infinity, it is essential condition for diffraction pattern.

33. (a)

4.79×10^{-4}cal

When temperature is raised from 100°C to 1000°C and volume is 10 liters, this is the required heat. Using $Q = aV(T_2^4 - T_1^4)$, this matches option (a).

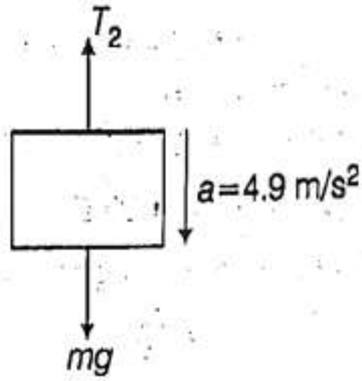
34. (a) (i)When the body of mass m lifted up the forces acting on the body



$$T_1 - mg = \frac{mg}{2}$$

$$T_1 = mg + \frac{mg}{2} = \frac{3mg}{2} \text{(i)}$$

(ii) When the body lowered the acting forces



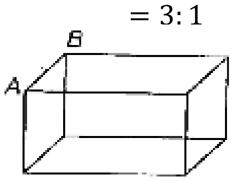
$$(mg - T_2) = \frac{mg}{2}$$

$$T_2 = mg - \frac{mg}{2} = \frac{mg}{2} \text{(i)}$$

On dividing Eq. (i) from Eq. (ii) we get

$$\therefore \frac{T_1}{T_2} = \frac{3mg/2}{mg/2} = \frac{3}{1}$$

$$= 3:1$$

35. (c) 

Let when link AB is removed, x is an equivalent of remaining part of cube without link. AB and rest part will be in parallel, so according to question

$$\frac{7}{12} = \frac{R_1 R_2}{R_1 + R} = \frac{1(x)}{1+x} = \frac{x}{1+x}$$

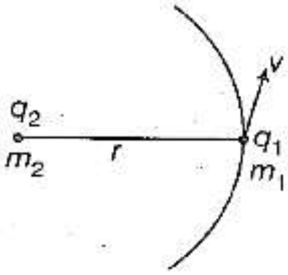
$$\frac{7}{12} = \frac{x}{1+x}$$

$$12x = 7 + 7x$$

$$12x - 7x = 7$$

$$5x = 7$$

$$x = \frac{7}{5}\Omega$$

36. (a,c) 

The necessary centripetal force, to move in a circular path is provided by the Coulomb's force.

$$\frac{m_1 v^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$v = \sqrt{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r m_1}} \quad \text{(i)}$$

We know, $v = r\omega$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi \cdot r}{V} \quad \text{(ii)}$$

Substitution the value of v from, Eq. (i) we get

$$T = \frac{2\pi r}{\sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 r \cdot m_1}}}$$

$$T = \sqrt{\frac{4\pi^2 r^2 \times 4\pi\varepsilon_0 \cdot r \cdot m_1}{q_1 q_2}}$$

$$T = \sqrt{\frac{16\pi^3 \varepsilon_0 r^3 \cdot m_1}{q_1 q_2}}$$

None of the given options is correct.
Both the charges are positive or both negative. (Since, answers have the product $q_1 q_2$ inside square root. Therefore, circular motion is not possible.
The statement of question is wrong. Either q_1 or q_2 should be negative.

37. (a,c) The intensity of emitted electrons is

$$I \propto \frac{1}{r^2}$$

$$I_1 \propto \frac{1}{d^2}$$

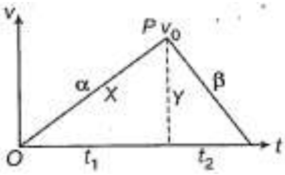
$$I_2 \propto \frac{1}{(d/2)^2} = 4 \cdot \frac{1}{d^2}$$

and $I \propto$ Number of photons per second $\propto N$

$$\therefore N \propto \frac{1}{r^2}$$

$\therefore$ Number of emitted electrons will become 4 times.
From Einstein's photo-electric equation
$KE_{\max} = h\nu - \phi$
where, $\phi =$ work function of surface
$\nu =$ frequency of incident photon
Since, ν remains unchanged, thus $KE_{\max}$ as well as stopping potential remains unchanged.
$\because KE_{\max} = eV_s \propto \nu$
where, $V_s =$ stopping potential.

38. (a,b)



Shape of the graph OP shows the acceleration in train $\therefore \tan\theta =$ acceleration.

$$\alpha = \frac{v_0}{t_1}$$

Shape of the graph pt shows the deceleration in train.

$\therefore$ Similarly,

$$\beta = \frac{v_0}{t_2}$$

$$\therefore \frac{\beta}{\alpha} = \frac{v_0/t_2}{v_0/t_1} = \frac{t_1}{t_2}$$

Displacement = area of graph obtained between v - t

$$x = \frac{1}{2}t_1 \cdot v_0$$

$$\therefore \quad y = \frac{1}{2}t_2 \cdot v_0$$

$$\Rightarrow \quad \frac{x}{y} = \frac{t_1}{t_2}$$

$$\frac{x}{y} = \frac{t_1}{t_2} = \frac{\beta}{\alpha}$$

39. (a) Let mass of the drop $= m$

Weight of the drop $= mg$

It will act downward.

The force due.to surface tension on the drop $= \sigma 2\pi R$

Where, . R =radius of tap

It will act in upward direction.

The drop of water will detach when

$$mg > \sigma 2\pi R$$

But, $$m = v \times \rho = \frac{4}{3}\pi r^3 \rho g$$

$$\therefore \quad \frac{4}{3}\pi r^3 \rho g > \sigma \times 2\pi R$$

$$r > \left[\frac{3}{2}\frac{\sigma \cdot R}{\rho g}\right]^{1/3}$$

Hence, none of the given options is correct.

40. (a,d) Current-carrying rectangular coil is placed in a non-uniform magnetic field. So, the force on each arm of the coil will different giving the resultant force as non-zero.

Thus, option (a) is correct.

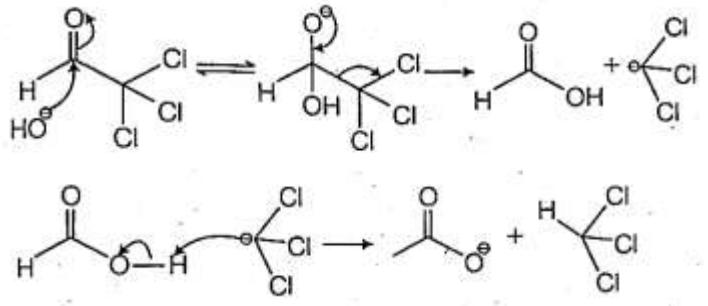
Also, the torque acting on current-carrying coil in non-uniform magnetic field will be non-zero.

Thus, option (d) is also correct.

Chemistry

41. (a) For a carbonyl group to give Cannizzaro reaction, it should not have an α-hydrogen atom.

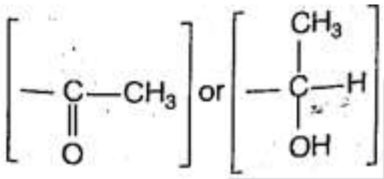
Among the given compound Ne_3CCHO, C_6H_5CHO and HCHO respond to Cannizaro reaction but Cl_3CCHO do not due to the following reason.



When hydroxyl group attacks the carbonyl group a tetrahedral intermediate forms. This tetrahedral intermediate will revert to a carbonyl compound by expelling the best leaving group. The $\overset{\ominus}{C}Cl_3$ carbonium is resonance stabilised therefore it is better leaving group than $\overline{O}H$. These initial products exchange a proton to give ion of carboxylic acid and trichloromethane ($CHCl_3$).

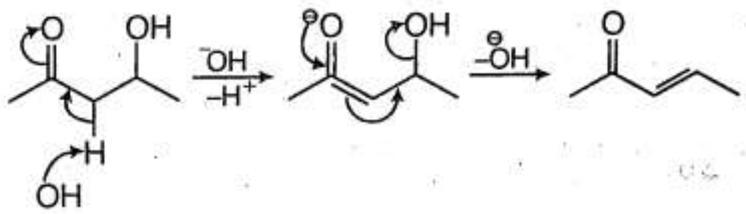
42.(c) Primary (1°)-alcohols do not give Lucas reagent test at room temperature. $CH_3CH_2CH_2OH$ does not undergo S_N1 or S_N2 at room temperature.

43. (c) Iodoform test is used to detect the presence of

 in a molecule.

In the given compounds only CH_3COOH does not fulfill the structural requirement of iodoform test hence, it does not respond to iodoform test.

44. (b)



In alkaline medium the acidic proton present in the molecule is abstracted to give enolate ion. In the next step hydroxide ion if present at β-position leaves giving $\alpha-\beta$-unsaturated carbonyl compound. In (a), (c) and (d), dehydration cannot take place due to improper placement of groups.

45.(c) The correct order is $2<1<3<4$. Since, more the number of $2^\circ-\mathrm{NH}-$ group, more is the basic nature.

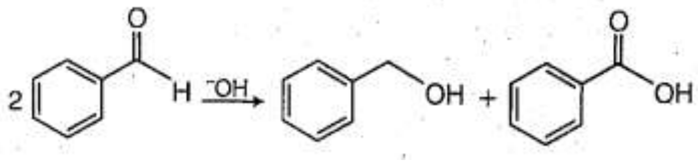
(i) Thus, 3 and 4 are more basic than 1 and 2 .

(ii) Between 3 and 4,4 has one two $-NH_2$ group while 3 has only one - NH_2 group.

$\therefore$ 4 is more basic than 3 .

(iii) Between 1 and 2, 1 has one - NH_2 group with no resonance, thus is more basic than 2.

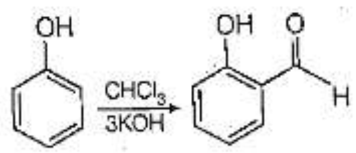
46. (a)$C-C$ bond is not formed in Cannizaro reaction while other reactions result in the formation of $C-C$ bond.Cannizaro reaction



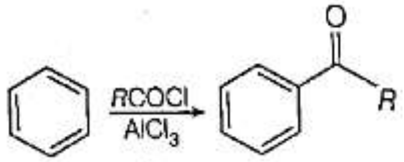
Wurtz reaction

$$2R-X+2\mathrm{Na}\longrightarrow R-R+2\overset{\oplus}{\mathrm{Na}}\overset{\ominus}{\mathrm{X}}$$

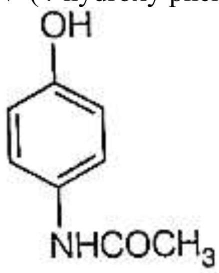
Reimer-Tiemann reaction



(a)
Friedel-Crafts acylation



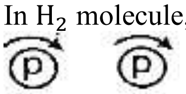
47. (a) Option (a) is false as colloidal sols are not homogeneous.
Colloidal sols are heterogeneous mixture of dispersed phase and dispersion medium.
48. (b)
Paracetamol is 4-acetamidophenol, i.e.
N -(4-hydroxy phenyl) ethanamide (IUPAC Name).



49.(d)
The option (d) is false as Ionic radii of trivalent lanthanides decreases with increase in atomic number due to lanthanoid contraction.
50. (a)
Only KNO_3 on heating do not give NO_2 but all other give
(a) $2KNO_3 \xrightarrow{\Delta} 2KNO_2 + O_2$
(b) $2\,Pb(NO_3)_2 \xrightarrow{\Delta} 2PbO + 4NO_2 + O_2$
(c) $2Cu(NO_3)_2 \xrightarrow{\Delta} 2CuO + 4NO_2 + O_2$
(d) $2AgNO_3 \xrightarrow[\text{Red hot}]{\Delta} 2Ag + O_2 + 2NO_2$
Heavy metal nitrates liberate nitrogen dioxide on heating.
51. (b) Due to hydrogen bonding, HF shows highest boiling point.
Thereafter van der Waals' force decide the boiling point.
Larger the size or molecules mass, greater is the van der Waals' forces. Hence, higher is the boiling point.
$\because$ van der Waals' forces is more for HI than HBr , which is in turn is more than HCl .
Hence, correct order is HF > HI > HBr > HCl.
52. (c) PCl_5 exists as $[PCl_4]^+$and $[PCl_6]$ in solid state.

$2PCl_5 \longrightarrow [PCl_4]^+[PCl_6]^{\top}$

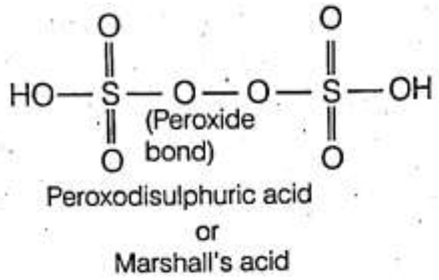
53. (b)(i) Ortho and para hydrogens have different nuclear spin.
In H_2 molecule, two protons in two H atoms with parallel spin are called ortho-hydrogen



Nucleus of hydrogen atom with same spin (ortho)
and with opposite spin are called para-hydrogen.
Nucleus of hydrogen atom with different spin (para)
(ii) Ortho-hydrogen is more stable than para form at and above room temperature only, therefore para form always tends to revert in ortho-form. Thus, not correct.

(iii) The per cent composition of ortho and para changes with the temperature. Thus, their ratio also change.
(iv) They have slightly different boiling point.
Hence, (b) is the answer.

54. **(c)** $H_2S_2O_8$ shows - O - O- (peroxy) bonding as follows:



55. **(d)** Iron (Fe) is used to obtain Cu-metal from $CuSO_4(aq)$ because Fe is more reactive than Cu (and also easily available and is cheaper) thus, replace the copper from $CuSO_4$ to give copper (Cu).
$CuSO_4(aq) + Fe(s) \longrightarrow FeSO_4(aq) + Cu(s)$

56. **(c)**
Radium in isolated form and in its chloride (i.e. Ra^+) only differ in the number of orbital electron.
But, radioactivity is independent of chemical environment of an ion or atom, i.e. radioactivity is the phenomenon which does not depend on the orbital electrons but depends only on the composition of nucleus.

57. **(b)** The correct order of solubility of sulphates in water is
$BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$
Solubility of 2nd group sulphates decreases as we move down the group due to less release of hydration energy.
$Be^{2+} < Mg^{2+} < Ca^{2+} < Sr^{2+} < Ba^{2+}$ (Ionic Size) As hydration energy decreases more rapidly than latice energy, the solubility decreases down the group.

$$\text{Hydration Energy} \propto \frac{\text{Charge}}{\text{Size}}$$

58. **(c)** Given, energy of one mole of H_2 bonds $= 436$ kJ

$$\because E = \frac{n \cdot hc}{\lambda} \text{ (energy for one H-Hbond)}$$

$$\therefore \lambda = \frac{n \cdot hc}{E}$$

$$\lambda = \frac{6.634 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{ ms}^{-1} \times 6.023 \times 10^{23}}{436 \times 10^3 \text{ J}} \text{ m}$$

$$= 0.2736 \times 10^{-34+8+23-3}$$
$$= 0.2736 \times 10^{-6} \text{ m}$$
$$= 273.6 \text{ nm} \approx 274 \text{ nm}$$

59. **(b)** Correct order of $O-O$ bond length is
$H_2O_2 > O_3 > O_2$
(i) Between O_3 and O_2
$\because$ Bond order of $O_2 > O_3$
Thus, bond length of $O_3 > O_2$.

$$\left(\because \text{ Bond order } \propto \frac{1}{\text{Bond length}}\right)$$

(ii) Between O_3 and H_2O_2
$\because$ O_3 has π-bonds in resonance so, has shorter bond length than $O-O$ in H_2O_2.
$\therefore$ Hence, (b) is the answer.

60. **(b)**
Calcium carbide is calcium acetylide, having structure
$Ca^{2+} \cdot \overset{\ominus}{C} \equiv \overset{\ominus}{C}$
Thus, it has one sigma (σ) and two pi (π) bonds.

61. **(c)**
Let $Z =$ atomic number
$M =$ atomic mass

$$ {}_{Z}^{M}E \xrightarrow{-\alpha} {}_{Z-2}E^{M-4} \xrightarrow{-2\beta} {}_{Z}E^{M-4} $$

Hence, ${}_{z}^{M}E$ and ${}_{2}E^{M-4}$ has same atomic number with different atomic mass thus, they are isotopes.

62. (d) The element belonging to 4 th period and 15 th group is arsenic (As). The outer electronic configuration of 15 th group element is ns^2np^3.

Since arsenic belongs to 4th period, therefore it has filled ' d ' orbital too. Thus, the electronic configuration of As is $[\mathrm{Ar}]4s^2 3d^{10} 4p^3$.

So, it has filled ' s ' and ' d ' orbital and half-filled p-orbital.

63. (a)

For exothermic reaction,

$\because\ K_\rho \propto \frac{1}{T}$

Thus, as $\frac{1}{T}$ increases (i.e. T decreases), K_p increases.

$\because \Delta H$ for exothermic reactions is negative.

64. (c) Given,

$p^\circ =$ vapour pressure of pure solvent

$p =$ vapour pressure of solution

$n_1 =$ moles of solute

$n_2 =$ moles of solvent

Thus, according to Raoult's law.

$$\frac{p^\rho - p}{p^\rho} = x_1 = \frac{n_1}{n_1 + n_2}$$

or, $$1 - \frac{p}{p^\rho} = \frac{n_1}{n_1 + n_2}$$

$$\Rightarrow \frac{p}{p^\rho} = 1 - \frac{n_1}{n_1 + n_2}$$

$$\frac{n_1 + n_2 - n_1}{n_1 + n_2}$$

$$\frac{p}{p^\rho} = \frac{n_2}{n_1 + n_2}$$

$$p = p^\circ \left(\frac{n_2}{n_1 + n_2}\right)$$

65. (c)

In Schottky defect, equal number of cations and anions are missing creating equal number of cation and anion vacancies.

66. (c)

For a spontaneous reaction, $\Delta G =$ negative and $\Delta S =$ positive

$\because\ \Delta G = \Delta H - T\Delta S = -$ ve

Hence, $\Delta H - T\Delta S = -ve$ only when $\Delta S = +ve$ and $\Delta H = -ve$.

67. (b)

KCl is more ionic than NaCl while NaCl is more ionic than LiCl .

Also, more the ionic nature, more is the conductivity thus, order of equivalent conductance at infinite dilution is KCl > NaCl > LiCl.

68. (b)

For a compound A_xB,

$K_{sp} = x^x \cdot y^y \cdot S^{x+y}$. (where, $S =$ solubility)

For MX_4

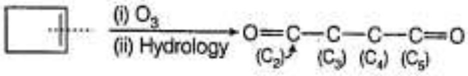
$K_{sp} = 1 \times (4)^4 \cdot S^{1+4}$

$K_{sp} = 256 \times S^5$

$\therefore\ S = \left(\frac{K_{sp}}{256}\right)^{1/5}$

69. (b)

Only  gives one compound on ozonolysis.



All other options give more than one aldehyde/ketone on ozonolysis.

70. (b)

For a compound to be aromatic,

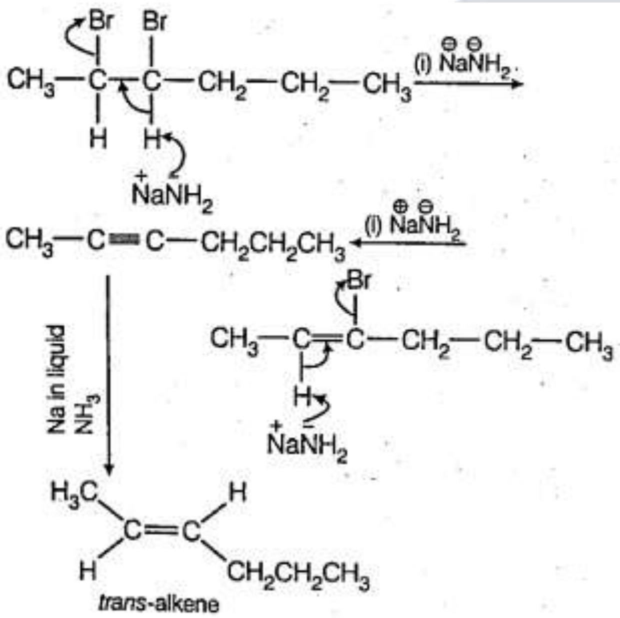
(i) It should be planar

(ii) It should have $(4n + 2)\pi$ electrons

(iii) It should be able to delocalise its π electron density.

Among the given compounds, 1, 3, 5, 7-cyclooctatetraene or cyclooctatetraene is not aromatic as its structure is non-planar

71. (b)



72. (d)

(i) It is a test for ammonia.

$$NH_4Cl + NaOH \xrightarrow{\Delta} \underset{\text{(Basic)}}{NH_3} + NaCl + H_2O$$

$\downarrow$

Turns red litmus blue

(ii) It is a test for SO_4^{2-} ions (Orange)

$K_2Cr_2O_7 + SO_2 + SO_4^{2-} + HCl \longrightarrow$

$K_2Cr_2O_4 + SO_2 + H_2SO_4$
Orange

$\downarrow \Delta$

$Cr_2(SO_4)_3 + K_2SO_4 + H_2O$
(Green)

73. (c)

Let the distance travelled in $\cdot\, T$ time $= 2\pi r$
(Circumference of orbit)

Velocity $(v) = \frac{2\pi r}{T} \ldots \ldots (i)$

Also, $v = \frac{n \cdot h}{2\pi m r} \ldots \ldots (ii)$

$\frac{1}{T(\text{ Time period })} =$ frequency $\nu = v \times \frac{1}{2\pi r}$

From Eq. (ii),

$$\nu = \frac{1}{T} = \frac{nh}{2\pi r \times 2\pi m r}$$

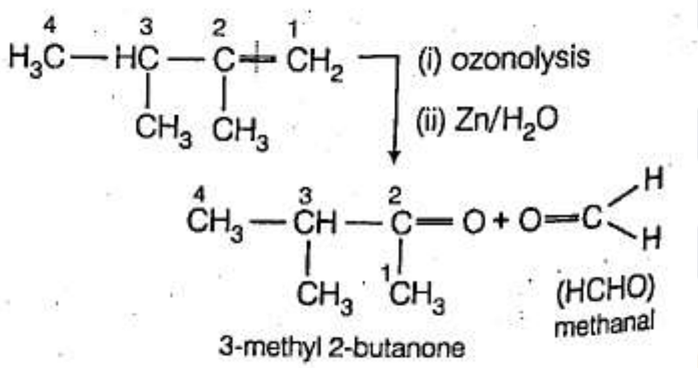
$$\nu = \frac{nh}{4\pi^2 r^2 m}$$

T (time taken in one revolution)

$$= \frac{4\pi^2 r^2 m}{nh}$$

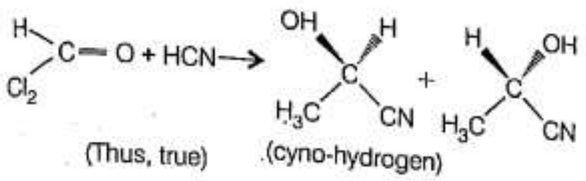
74. (d) $\because$ rms velocity $(v_{\text{rms}}) \propto \sqrt{\frac{1}{M}}$ Thus, smaller the mass, more will be the rms speed. H_2 has minimum molar mass, So it has highest value of rms speed.

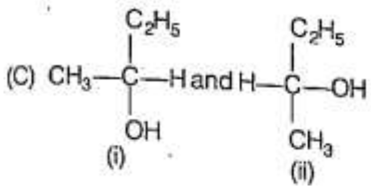
75. (b)



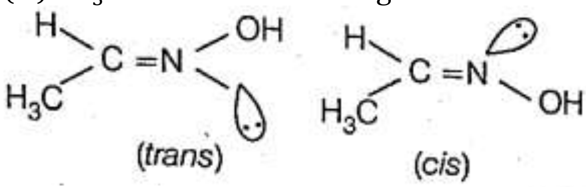
76. (b,d)

(a) These are cis and trans isomers not enantiomers (Thus, false).
(b) CH_3CHO on reaction with HCN gives racemic mixture of cyanohydrin (Thus, true).
(Thus, true)



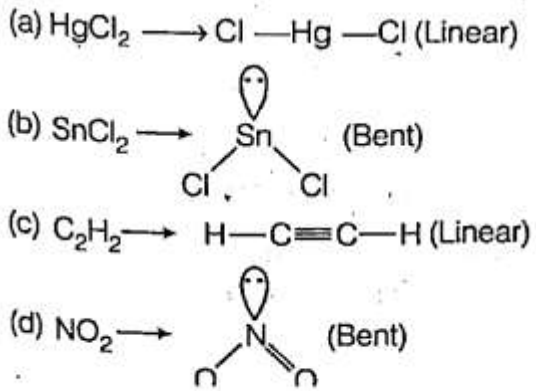


(i) and (ii) both have R-configuration thus, are not enantiomers.
(D) $CH_3 - CH = NOH$ shows geometrical isomers.



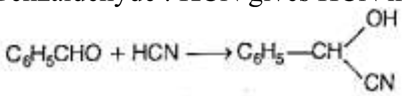
77. (a,b,d)
(A) When NaCl and $K_2Cr_2O_7$ warmed with H_2SO_4 (i.e. in acidic medium), they produce deep red vapours of chromyl chloride (CrO_2Cl_2).
(B) When NaOH is passed, the product CrO_3 formed in step (a) will react with NaOH and gives yellow colour solution.
(C) Chlorine gas is not evolved, thus false.
(D) In the given reaction, chromyl chloride (CrO_2Cl_2) is formed. All reactions are as follows:
(i) $K_2Cr_2O_7 + 2H_2SO_4 \longrightarrow 2KHSO_4 + 2CrO_3 + H_2O$
(ii) $NaCl + H_2SO_4 \longrightarrow NaHSO_4 + HCl$
(iii) $\underset{\text{from (i)}}{CrO_3} + \underset{\text{from (ii)}}{2HCl} \longrightarrow CrO_2Cl_2$
(iv) $CrO_2Cl_2 + 2NaOH \longrightarrow \underset{\text{Yellow solution}}{Na_2CrO_4} + 2HCl$

78. (a,c) $HgCl_2$ and C_2H_2 both show linear shape as of CO_2.
($\because$ both have zero lone pair of electrons)
(a) $HgCl_2 \longrightarrow Cl - Hg - Cl$ (Linear)

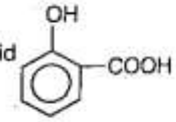


79. (a,b,d)
pH of buffer solutions is given by -
$$pH = pK_a + \log\frac{[\text{ salt }]}{[\text{ acid }]}$$
80. (a,c)
(a) Ethyl chloride gives ethyl cyanide, when it reacts with KCN (ethanolic),
$C_2H_5Cl + KCN \xrightarrow{\text{Ethanolic}} C_2H_5 - CN$
(b) It does not react with KCN .

(c) Benzaldehyde : KCN gives HCN in alcoholic medium and HCN gives cyanohydrin with benzaldehyde.



(d) Salicylic acid



does not react.

Mathematics

81. (b)

Given, $x\frac{dy}{dx}+y=xe^x$

$\Rightarrow \frac{dy}{dx}+\frac{y}{x}=e^x$......(i)

On comparing Eq. (i) by $\frac{dy}{dx}+Py=Q$, we get

$$P=\frac{1}{x} \text{ and } Q=e^x$$

$$\therefore \quad IF=e^{\int \frac{1}{x}dx}=e^{\log x}=x$$

Hence, solution of differential equation,

$$y\cdot x=\int xe^x dx+C$$

$$\Rightarrow \quad xy=xe^x-\int e^x dx+C$$

$$\Rightarrow \quad xy=xe^x-e^x+C$$

$$\Rightarrow \quad xy=e^x(x-1)+C \quad \text{(ii)}$$

$$\because \quad xy=e^x\phi(x)+C \quad \text{(given)}$$

On comparing Eqs. (ii) and (iii), we get

$\phi(x)=(x-1)$

82. (a) The general equation of the parabola is $x=ay^2+b$, where a and b are arbitrary constants.
So, the differential equation is of order 2 .

83. (c) The equation of the line is $y=x+\lambda$

On comparing it with $y=mx+c$, we get

$m=1$ and $c=\lambda$.

The equation of the ellipse is

$2x^2+3y^2=1$

or $\frac{x^2}{\left(\frac{1}{\sqrt{2}}\right)^2}+\frac{y^2}{\left(\frac{1}{\sqrt{3}}\right)^2}=1$

On comparing it with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we get

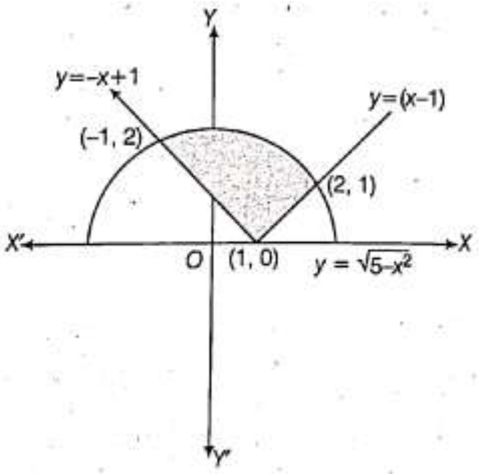
$a^2=\frac{1}{2}$ and $b^2=\frac{1}{3}$.

If the line touches the ellipse, then

$$c^2=a^2m^2+b^2 \Rightarrow \lambda^2=\frac{1}{2}\cdot 1+\frac{1}{3}=\frac{5}{6}$$

$$\Rightarrow \lambda^2=\frac{5}{6} \Rightarrow \lambda=\sqrt{\frac{5}{6}}$$

84. (c) The graphs of $y=|x-1|$ and $y=\sqrt{5-x^2}$ are shown in the figure and the shaded region is the required region bounded by the two curves.



Let A be the area bounded by the given curves. Then,

$$A=\int_{-1}^{1}\left(\sqrt{5-x^2}+x-1\right)dx+\int_{1}^{2}\left(\sqrt{5-x^2}-x+1\right)dx$$

$$=\int_{-1}^{1}\sqrt{5-x^2}dx+\int_{1}^{2}\sqrt{5-x^2}dx$$

$$+\int_{-1}^{1}(x-1)dx+\int_{1}^{2}(-x+1)dx$$

$$=\int_{-1}^{2}\sqrt{5-x^2}dx+\left[\frac{x^2}{2}-x\right]_{-1}^{1}+\left[-\frac{x^2}{2}+x\right]_{1}^{2}$$

$$=\left[\frac{1}{2}x\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2}-\frac{5}{2}$$

$$=1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}+1+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}-\frac{5}{2}$$

$$=-\frac{1}{2}+\frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\times\sqrt{1-\frac{1}{5}}+\frac{1}{\sqrt{5}}\sqrt{1-\frac{4}{5}}\right)$$

$$=-\frac{1}{2}+\frac{5}{2}\sin^{-1}(1)=\frac{5\pi}{4}-\frac{1}{2}\text{ sq units}$$

85. (b) Sum of distances of point P from point (8,0) and (0,12) will be minimum, if points are collinear.

$\therefore$ Equation of line at point (8,0) and (0,12).

$$\frac{x}{8}+\frac{y}{12}=1$$

$$\Rightarrow\quad\frac{y}{12}=1-\frac{x}{8}$$

$\Rightarrow$

$$\Rightarrow\quad y=12-\frac{x}{8}\times 12$$

$\Rightarrow$

$$\Rightarrow\quad y=12-\frac{3}{2}x$$

$\therefore$ Points $(x,y)\equiv(2,9),(4,6)$ and $(6,3)$.

Hence, the number of such points P in S is 3 .

86. (b) Given, $T=2\pi\sqrt{\frac{l}{g}}$

On differentiating w.r.t. I, we get

$$\frac{dT}{dl}=\frac{2\pi}{\sqrt{g}}\cdot\frac{1}{2\sqrt{l}}$$

$$\therefore \quad \Delta T=\frac{dT}{dl}\Delta l=\frac{\pi}{\sqrt{gl}}\cdot\left(\frac{2l}{100}\right)$$

$$=2\pi\sqrt{\frac{l}{g}}\cdot\frac{1}{100}=\frac{T}{100}$$

$$\Rightarrow \quad \frac{\Delta T}{T}=\frac{1}{100}=1\%$$

Hence, approximate change in the time period is 1%.

87. (a) Let the direction ratio's of two diagonals of a cube are $(1,1,1)$ and $(-1,1,1)$.

$$\therefore \cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$=\frac{(1)\times(-1)+(1)\times(1)+(1)\times(1)}{\sqrt{(1)^2+(1)^2+(1)^2}\sqrt{(-1)^2+(1)^2+(1)^2}}$$

$$=\frac{-1+1+1}{\sqrt{3}\sqrt{3}}=\frac{1}{3}$$

88. (c) Given, $\log_a x, \log_b x$ and $\log_c x$ are in AP.

$$\therefore \quad 2\log_b x=\log_a x+\log_c x$$

$$\Rightarrow \quad 2\log_b x=\frac{1}{\log_x a}+\frac{1}{\log_x c}$$

$$\Rightarrow \quad \frac{2}{\log_x b}=\frac{\log_x ac}{\log_x a\log_x c}$$

$$\Rightarrow \quad 2\log_x c=\frac{\log_x b}{\log_x a}(\log_x ac)$$

$$\Rightarrow \quad 2\log_x c=\log_a b\cdot\log_x ac$$

$$\Rightarrow \quad \log_x c^2=\log_x(ac)^{\log_a b}$$

$$\therefore \quad c^2=(ac)^{\log_a b}$$

89. (c) We have,

$$1+(1+a)x+(1+a+a^2)x^2+\cdots\infty$$

$$=\sum_{n=1}^{\infty}(1+a+a^2+\cdots+a^{n-1})x^{n-1}$$

$$=\sum_{n=1}^{\infty}\left(\frac{1-a^n}{1-a}\right)x^{n-1}$$

$$=\sum_{n=1}^{\infty}\frac{x^{n-1}}{1-a}-\sum_{n=1}^{\infty}\frac{a^nx^{n-1}}{1-a}$$

$$=\frac{1}{1-a}\sum_{n=1}^{\infty}x^{n-1}-\frac{a}{1-a}\sum_{n=1}^{\infty}(ax)^{n-1}$$

$$=\frac{1}{1-a}[(1+x+x^2+\cdots\infty)]$$

$$-\frac{a}{1-a}[\{1+ax+(ax)^2+\cdots\infty\}]$$

$$=\frac{1}{(1-a)}\cdot\frac{1}{(1-x)}-\frac{a}{(1-a)(1-ax)}$$

$$=\frac{1}{(1-ax)(1-x)}$$

90. **(a)**Given, $\log_{0.3}(x-1)<\log_{0.09}(x-1)$

$\Rightarrow \quad \log_{0.3}(x-1)<\log_{(0.3)^2}(x-1)$

$\Rightarrow \quad \log_{0.3}(x-1)^2<\log_{0.3}(x-1)$

$\Rightarrow \quad (x-1)^2>x-1$

$\Rightarrow \quad x^2+1-2x-x+1>0$

$\Rightarrow \quad x^2-3x+2>0$

$\Rightarrow \quad (x-1)(x-2)>0$

$\Rightarrow \quad x<1, x>2 \Rightarrow x>2$

$\therefore \quad (2, \infty)$

Hence, x lies in the interval $(2, \infty)$.

91. **(b)**We have,

$$\sum_{n=1}^{13}\left(i^n+i^{n+1}\right)$$

$$=\sum_{n=1}^{13} i^n+\sum_{n=1}^{13} i^{n+1}$$

$$=i\left(\frac{1-i^{13}}{1-i}\right)+i^2\left(\frac{1-i^{13}}{1-i}\right)$$

$$=i\left(\frac{1-i}{1-i}\right)-\left(\frac{1-i}{1-i}\right)$$

$$=i-1$$

92. **(a)**We have,

$$|z_1|=|z_2|=|z_3|=1$$

$\Rightarrow \quad |z_1|^2=|z_2|^2=|z_3|^2=1$

$\Rightarrow \quad z_1\bar{z}_1=z_2\bar{z}_2=z_3\bar{z}_3=1$

$\Rightarrow \quad \frac{1}{z_1}=\bar{z}_1 \frac{1}{z_2}=\bar{z}_2 \frac{1}{z_3}=\bar{z}_3$

Now, $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$

$\Rightarrow \quad |\bar{z}_1+\bar{z}_2+\bar{z}_3|=1$

$\Rightarrow \quad |z_1+z_2+z_3|=1$

$\therefore \quad |z_1+z_2+z_3|=1$

93. **(a)** Given, p, q are roots of equation $x^2+px+q=0$.

$\therefore\ p+q=-p \Rightarrow 2p+q=0$..........(i)

and

$pq=q$

$p=1$

in Eq. (i), we get

$$2\times 1+q=0$$

$\Rightarrow \quad q+2=0$

$\Rightarrow \quad q=-2$

94. **(d)** Given equation is

As,

$x^2-3x+k=0$

$$-\frac{b}{2a}=\frac{3}{2}\notin(0;1)$$

Since, both roots cannot lie between 0 and 1 .

So, no value of k is possible.

95. **(c)**

Given word is 'ARRANGE'.

If R's occur together, number of letters in word $= 6$
To arrange R's together number of ways $= \frac{2!}{2!} = 1$
$\therefore$ Number of ways to permuted 'ARRANGE' $= \frac{6!}{2!}$

96. (b)

Given, $\frac{1}{{}^5C_r} + \frac{1}{{}^6C_r} = \frac{1}{{}^4C_r}$

$$\Rightarrow \frac{r!\,(5-r)!}{5!} + \frac{r!\,(6-r)!}{6!} = \frac{r!\,(4-r)!}{4!}$$
$$\Rightarrow \frac{(5-r)!}{5} + \frac{(6-r)!}{30} = (4-r)!$$
$$\Rightarrow \frac{(5-r)}{5} + \frac{(6-r)(5-r)}{30} = 1$$
$$\Rightarrow 30 - 6r + 30 - 11r + r^2 = 30$$
$$\Rightarrow r^2 - 17r + 30 = 0$$
$$\Rightarrow r^2 - 15r - 2r + 30 = 0$$
$$\Rightarrow r(r-15) - 2(r-15) = 0$$
$$\Rightarrow (r-2)(r-15) = 0$$
$$\therefore r = 15,2$$

Hence, the value of r is 2 .

97. (a) For $n = 1,2,3 \ldots$, we find that $n^3 + 2n$ takes values 3 , 12,33, ..., which are divisible by 3 .

98. (b) Coefficient of x^{17} in $(x-1)(x-2)\ldots(x-18)$

$= -(1+2+3+\cdots+18)$

$= -(171) = -171$

99. (a) Given,

$1 + {}^nC_1\cos\theta + {}^nC_2\cos 2\theta + \cdots + {}^nC_n\cos n\theta$

which is real part of complex number.

$\left({}^nC_0 + {}^nC_1 e^{i\theta} + \cdots\right)$

i.e. $\operatorname{Re}\left({}^nC_0 + {}^nC_1 e^{i\theta} + \cdots\right)$

$= \operatorname{Re}\left(1 + e^{i\theta}\right)^n = \operatorname{Re}(1 + \cos\theta + i\sin\theta)^n$

$= \operatorname{Re}\left(2\cos^2 n\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\right)$

$\left[\begin{array}{c}\because 1 + \cos\theta = 2\cos^2\frac{\theta}{2} \text{ and} \\ \sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\end{array}\right]$

$= \left(2\cos\frac{\theta}{2}\right)^n \operatorname{Re}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)^n$

$= \left(2\cos\frac{\theta}{2}\right)^n \operatorname{Re}\left(\cos\frac{n\theta}{2} + i\sin\frac{\theta}{2}\right)$

[by De moivre's theorem]

$= \left(2\cos\frac{\theta}{2}\right)^n \cdot \cos\frac{n\theta}{2}$

100. **(c)**Let $\Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$= \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$

On taking $\frac{1}{\log x}, \frac{1}{\log y}, \frac{1}{\log z}$ common from R_1, R_2 and R_3, we get

$$=\frac{1}{\log x \cdot \log y \cdot \log z}\left|\begin{array}{lll}\log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z\end{array}\right|$$

$\therefore \Delta=0$ [$\because$ all rows are identical]

101. **(c)** We have, $|B|=64$

$\Rightarrow \quad|\operatorname{adj} A|=64$

$\Rightarrow \quad|A|^2=64$

$\therefore \quad|A|= \pm 8$

102. **(c)** Given,

$$Q=\left[\begin{array}{cc}\cos \frac{\pi}{4} & -\sin \frac{\pi}{4} \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4}\end{array}\right] \text { and } x=\left[\begin{array}{c}\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\end{array}\right]$$

Let $Q(\theta)=\left[\begin{array}{cc}\cos \theta & -\sin \theta \\ \sin \theta & \cos \theta\end{array}\right]$

$\because \; Q^3(\theta)=Q(3 \theta)$

$$\therefore \; Q^3\left(\frac{\pi}{4}\right)=\left[\begin{array}{cc}\cos \frac{3 \pi}{4} & -\sin \frac{3 \pi}{4} \\ \sin \frac{3 \pi}{4} & \cos \frac{3 \pi}{4}\end{array}\right]=\left[\begin{array}{cc}-1 / \sqrt{2} & -1 / \sqrt{2} \\ 1 / \sqrt{2} & -1 / \sqrt{2}\end{array}\right]$$

$$\text{Now, } Q^3\left(\frac{\pi}{4}\right) x=\left[\begin{array}{cc}\frac{-1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{array}\right]\left[\begin{array}{c}\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\end{array}\right]=\left[\begin{array}{c}-\frac{1}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{1}{2}\end{array}\right]$$

$$\therefore \; Q^3\left(\frac{\pi}{4}\right) x=\left[\begin{array}{c}-1 \\ 0\end{array}\right]$$

103. **(c)** Reflexivity For reflexive $(x, x) \in R$.

$x+2 x=3 x$

which is divisible by 3 .

$\Rightarrow \; x R x \in R$

Hence, $x R y$ is reflexive.

Symmetric Let $x+2 y=3 \lambda$

$$\begin{aligned} \Rightarrow x &= 3 \lambda-2 y \\ \text{Now, } y+2 x &= y+2(3 \lambda-2 y) \\ &= y+6 \lambda-4 y \\ &= 6 \lambda-3 y \\ \Rightarrow y+2 x &= 3(2 \lambda-y) \end{aligned}$$

which is divisible by 3 .

$\therefore \; (x, y) \in R \Rightarrow(y, x) \in R$

Hence, x Ry is symmetric.

Transitive $\quad x+2 y=3 \lambda$

and $\quad y+2 z=3 \mu$

On adding Eqs. (i) and (ii), we get

$x+3 y+2 z=3 \lambda+3 \mu$

$\Rightarrow \; x+2 z=3(\lambda+\mu-y)$

which is divisible by 3 .

$\because \; (x, y) \in R$ and $(y, z) \in R$

$\Rightarrow \; (x, z) \in R$

Hence, x Ry is transitive.

$\because R$ is reflexive, symmetric and transitive.

$\therefore R$ is an equivalence relation.

104. **(c)** We have,

$A = 5^n - 4n - 1 = (1+4)^n - 4n - 1$
$= ({}^nC_0 + {}^nC_1 \times 4 + {}^nC_2 \times 4^2 + \cdots + {}^nC_n \times 4^n)$
$= 4^2({}^nC_2 + {}^nC_3 \times 4 + \cdots + {}^nC_n \times 4^{n-2})$
$\therefore$ A contains some multiples of 16 .
Clearly, B contains all multiples of 16 including 0 .
$\therefore\ A \subseteq B$.

105. **(c)**Given, $f(x) = (x^2+1)^{35}, \forall x \in R$.
Since, $f(x)$ is even function.
Hence, the function is not one-one.
And $f(x) > 0, \forall x \in R$.
Hence, the function is not onto.
$\therefore$ Given, the function is neither one-one nor onto.

106. **(c)** Let $x_i = a_1, a_2, a_3, \ldots, a_n$;
and $y_i = \lambda a_1, \lambda a_2, \lambda a_3, \ldots, \lambda a_n$.
$\therefore\ y_i = \lambda x_i$
$\because\ \bar{y} = \dfrac{\Sigma y_i}{N}$
$= \dfrac{\Sigma \lambda x_i}{n}\ [\because y_i = \lambda x_i, N = n]$
$= \lambda \dfrac{\Sigma x_i}{n}$
$= \lambda \bar{x}$

$$\sigma_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{N}}$$
$$= \sqrt{\frac{\Sigma(\lambda x_i - \lambda \bar{x})^2}{n}}$$
$$= \sqrt{\frac{\lambda^2 \Sigma(x_i - \bar{x})^2}{n}}$$
$$= \sqrt{\lambda^2 \sigma_x^2} = |\lambda|\sigma_x = |\lambda|\sigma$$

107. **(a)** Given, $P(A \cap B) = \frac{1}{6}$
$P(A \cup B) = \dfrac{31}{45}$
and $P(\bar{B}) = \frac{7}{10}$
$\therefore\ P(B) = 1 - \dfrac{7}{10} = \dfrac{3}{10}$
Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow\ \dfrac{31}{45} = P(A) + \dfrac{3}{10} - \dfrac{1}{6}$
$\therefore\ P(A) = \dfrac{31}{45} + \dfrac{1}{6} - \dfrac{3}{10}$
$\therefore\ = \dfrac{5}{9}$
Then, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{5}{9}} = \frac{3}{10} > \frac{1}{6}$
and
$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{3}{10}} = \frac{5}{9} > \frac{1}{6}$$

and, $P(A) \times P(B) = \frac{5}{9} \times \frac{3}{10} = \frac{1}{6} = P(A \cap B)$

Hence, A and B are independent.

108. **(b)** We have, $\cos 15^\circ \cdot \cos 7\frac{1}{2}^\circ \cdot \sin 7\frac{1}{2}^\circ$

$$= \frac{1}{2}\cos 15^\circ \left(2\cos 7\frac{1}{2}^\circ \cdot \sin 7\frac{1}{2}^\circ\right)$$

$$= \frac{1}{2}\cos 15^\circ \left[\sin 2\left(7\frac{1}{2}^\circ\right)\right] \left[\because \begin{array}{c} 2\sin\theta \cdot \cos\theta \\ = \sin 2\theta \end{array}\right]$$

$$= \frac{1}{2}\cos 15^\circ \sin 15^\circ$$

$$= \frac{1}{2} \times \frac{1}{2}(2\sin 15^\circ \cos 15^\circ)$$

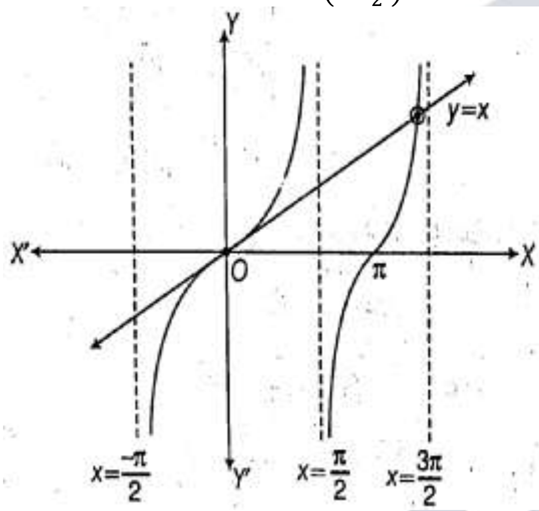
$$= \frac{1}{4}\sin 30^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

109. **(c)** Given equation is

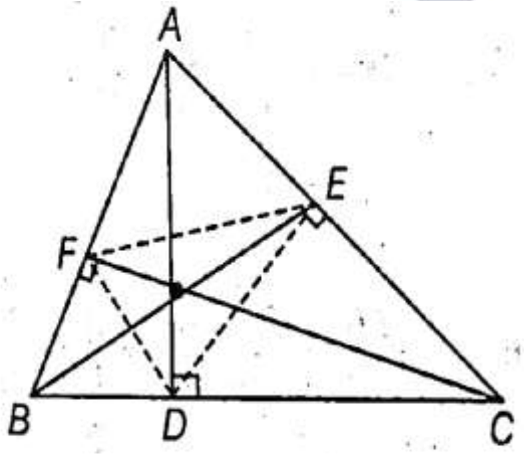
$$\tan x - x = 0$$

$$\Rightarrow \quad \tan x = x$$

Solutions are abscissae of points of intersection of the curves $y = \tan x$ and $y = x$. From the figure, it is clearly visible that solution lies in $\left(\pi, \frac{3\pi}{2}\right)$.



110. **(a)** Let circumradius of $\triangle DEF$ be R'. We know, $\angle FDE = 180^\circ - 2A$ and $FE = R\sin 2A$.



Now, by sine rule in $\triangle DEF$

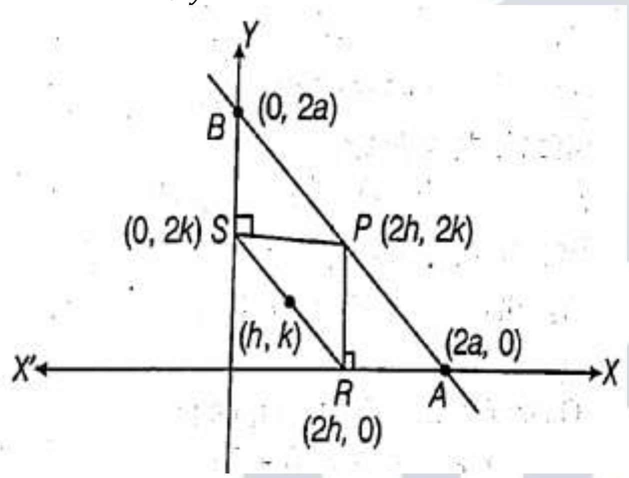
$$2R' = \frac{EF}{\sin \angle FDE} = \frac{R\sin 2A}{\sin(180^\circ - 2A)}$$

$$\therefore \quad R' = \frac{R}{2}$$

111. **(a)** Let the four points be $A(-a, -b), B(a, b), C(0,0)$ and $D(a^2, ab)$.

If A, B and C are collinear.
Then,
$$\begin{vmatrix} -a & -b & 1 \\ a & b & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$$
$\Rightarrow \quad -a(b-0) + b(a-0) + 1(0) = 0$
$\Rightarrow \quad -ab + ab = 0$
Hence, A, B and C ae collinear.
Again, if B, C and D are collinear.
Then, $\begin{vmatrix} a & b & 1 \\ 0 & 0 & 1 \\ a^2 & ab & 1 \end{vmatrix} = 0$
$\Rightarrow \quad a(0-ab) - b(0-a^2) + 1(0) = 0$
$\Rightarrow \quad -a^2b + a^2b = 0$
Hence, B, C and D are collinear.
So, the points A, B, C and D are always collinear.

112. **(b)** Let the equation of line AB is
$$\frac{x}{2a} + \frac{y}{2a} = 1$$
$\Rightarrow \quad x + y = 2a$



Let the coordinates of the mid-point of RS be (h, k).
So, R and S are $(2h, 0)$ and $(0, 2k)$.
Hence, the mid-point of RS is (h, k).
$\because P$ lies on line AB.
Then, from Eq. (i), we have
$2h + 2k = 2a$
$\Rightarrow \quad h + k = a$
So, the locus of (h, k) is $x + y = a$.

113. **(b)** Given three sides of a triangle are
$x + 8y - 22 = 0. \ldots.. \text{(i)}$
$5x + 2y - 34 = 0 \ldots\ldots\text{..(ii)}$
$2x - 3y + 13 = 0 \ldots \ldots. \text{(iii)}$
On solving Eqs. (i) and (ii), we get
$x_1 = 6, y_1 = 2 \ldots.. (say)$

On solving Eqs. (ii) and (iii), we get
$x_2 = 4, y_2 = 7 \ldots\ldots$.(say)
On solving Eqs. (i) and (iii); we get
$x_3 = -2, y_3 = 3 \ldots\ldots$(say)
If the points of the vertices of the triangle are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3), then the area of triangle
$$= \frac{1}{2}\begin{vmatrix} 6 & 2 & 1 \\ 4 & 7 & 1 \\ -2 & 3 & 1 \end{vmatrix}$$

$=\frac{1}{2}[6(7-3)-2(4+2)+1(12+14)]$

$=\frac{1}{2}(24-12+26)=\frac{1}{2}(12+26)$

$=\frac{1}{2}\times 38$

$=19$ sq units

114. **(c)** Let the given points be $A(a,b)$ and $B(-a,-b)$.
The equation of line passing through the points A and B is

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$\Rightarrow \quad y-b=\frac{-b-b}{-a-a}(x-a)$$

$$\Rightarrow \quad y-b=\frac{2b}{2a}(x-a)$$

$$\Rightarrow \quad y-b=\frac{b}{a}(x-a)$$

$$\Rightarrow \quad ay-ab=bx-ab$$

$$\Rightarrow \quad bx=ay \text{(i)}$$

Since, from the given points (a^2,ab) and (a,b) are satisfy the Eq. (i), but (a^2,ab) is the required answer. because (a,b) is already given in question.

115. **(d)** Given equations of straight lines

$\frac{x}{a}+\frac{y}{b}=K$.......(i)

And $\frac{x}{a}-\frac{y}{b}=\frac{1}{K}$.......(ii)

Let the point of intersection be (α,β).
So, from Eqs. (i) and (ii), we get

$\frac{\alpha}{a}+\frac{\beta}{b}=K$

and

$\frac{\alpha}{a}-\frac{\beta}{b}=\frac{1}{K}$.

$\Rightarrow \left(\frac{\alpha}{a}\right)^2-\left(\frac{\beta}{b}\right)^2=1\Rightarrow\frac{\alpha^2}{a^2}-\frac{\beta^2}{b^2}=1$

$\therefore$ Locus $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, which is the equation of a hyperbola.

116. **(a)** Equation of circle is $x^2+y^2=9$
And equation of line is $3x+4y=0$
Equation of line parallel to the line (ii),

$3x+4y=k$

$$y=\frac{-3x}{4}+\frac{k}{4} \text{(iii)}$$

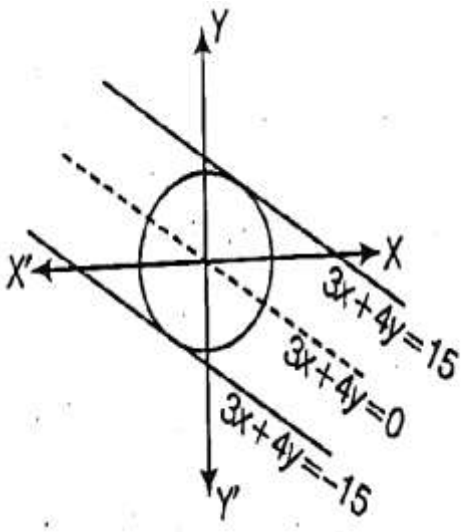
$\because$ Circle touches the line (iii), so by the condition: of tangency,

$$c^2=a^2(1+m^2)$$

$$\Rightarrow \left(\frac{k}{4}\right)^2$$

$$=9\left[1+\left(\frac{-3}{4}\right)^2\right]$$

$$k=\pm 15$$



Hence, $3x + 4y = 15$ touches the circle in the first quadrant.

117. **(b)** Given equations are

X+y=4……..(i)

And x-y=2….(ii)

From Eqs. (i) and (ii), we get

$x = 3$ and $y = 1$

The line through this point making an angle $\tan^{-1}\frac{3}{4}$ with the x- axis is

$$(y-1) = \frac{3}{4}(x-3) \qquad \left[\because m = \frac{3}{4}\right]$$

$$\Rightarrow y = \frac{3x}{4} - \frac{5}{4} = \frac{3x-5}{4} \qquad \ldots \text{(iii)}$$

Since, this line intersects the parabola

$y^2 = 4(x-3)$ at points (x_1, y_1) and (x_2, y_2), respectively.

$\therefore$ Putting $y = \frac{3x-5}{4}$ in equation of parabola, we get

$$\left(\frac{3x-5}{4}\right)^2 = 4(x-3)$$

$$\Rightarrow 9x^2 - 94x + 217 = 0$$

$$\Rightarrow x_1 + x_2$$

$$\therefore = \frac{94}{9} \text{ and } x_1x_2 = \frac{217}{9}$$

$$= \sqrt{\left(\frac{94}{9}\right)^2 - 4 \times \frac{217}{9}}$$

$$= \frac{32}{9}$$

118. **(a)** Given equation of ellipse is

$16x^2 + 25y^2 + 32x - 100y = 284$

Simplifying the given equation, we have ellipse as

$$\frac{(x+1)^2}{25} + \frac{(y-2)^2}{16} = 1$$

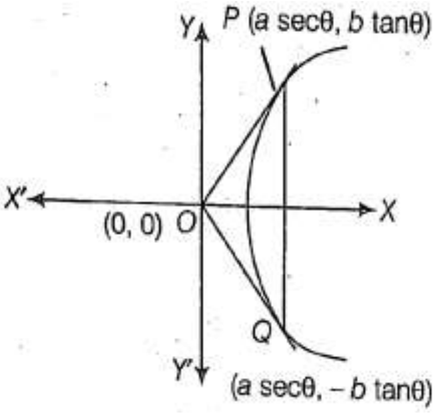
So, the auxiliary equation of circle is

$$(x+1)^2 + (y-2)^2 = 25$$

$$\Rightarrow \quad x^2 + 2x + 1 + y^2 - 4y + 4 = 25$$

$$\therefore \quad x^2 + y^2 + 2x - 4y - 20 = 0$$

119. **(d)** Given equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\triangle OPQ$ is an equilateral triangle. PQ is double ordinate of the hyperbola.



Let the coordinates of P and Q be ($a\sec\theta, b\tan\theta$) and (a $\sec\theta, -b\tan\theta$), respectively.
In $\triangle OPQ$,

$$OP = PQ$$

$$\Rightarrow \quad a^2\sec^2\theta + b^2\tan^2\theta = (2b\tan\theta)^2$$

$$\Rightarrow \quad a^2\sec^2\theta = 3b^2\tan^2\theta$$

$$\Rightarrow \quad \sin^2\theta$$

$$\Rightarrow \quad \frac{a^2}{3b^2}$$

Now, $\sin^2\theta < 1$

$$\Rightarrow \quad \frac{a^2}{3b^2} < 1$$

$$\Rightarrow \quad \frac{b^2}{a^2} > \frac{1}{3}$$

$$\Rightarrow \quad 1 + \frac{b^2}{a^2} > \frac{4}{3}$$

$$\therefore \quad e^2 > \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$$

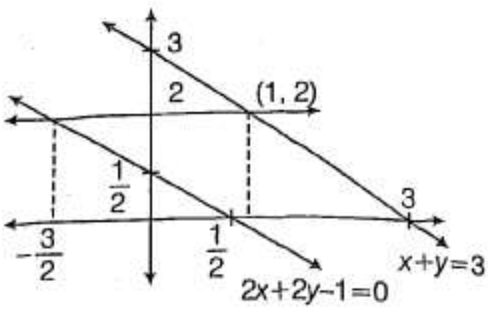
120. **(b)** Equation of the vertex of conic

$$y^2 - 4y = 4x - 4a$$

$$\Rightarrow \quad y^2 - 4y + 4 = 4x - 4a + 4$$

$$\Rightarrow \quad (y-2)^2 = 4[x-(a-1)]$$

$\therefore$ Vertex = (a – 1, 2)



From figure,
clearly,

$$-\frac{3}{2} < a - 1 < 1$$

$$\therefore \; -\frac{1}{2} < a < 2$$

121. **(b)** Equation of line joining the points (1,1,1) and (0,0,0) is

$$\frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0} = \lambda \ldots\ldots (say)$$

$$\Rightarrow \quad x = y = z = \lambda$$

So, the point is $(\lambda, \lambda, \lambda)$.
The point intersects the plane $2x + 2y + z = 10$.

$\therefore \quad 2(\lambda) + 2(\lambda) + \lambda = 10$

$\Rightarrow \quad 5\lambda = 10$

$\Rightarrow \quad \lambda = 2$

Hence, the point is $(2,2,2)$.

122. **(c)** Equations of planes are

$x + y + 2z - 6 = 0$.....(i)

And $2x - y + z - 9 = 0$......(ii)

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$= \frac{1\times2+1\times(-1)+2\times1}{\sqrt{1+1+4}\sqrt{4+1+1}}$.....(ii)

And

$$\frac{2-1+2}{\sqrt{6}\times\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \cos\left(\frac{\pi}{3}\right)$$

$$= \theta = \frac{\pi}{3}$$

123. **(c)**

Given,

$$y = (1+x)(1+x^2)(1+x^4) + \cdots + (1+x^{2n})$$

$$\Rightarrow \log y = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \cdots + \log(1+x^{2n})$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{3x^2}{1+x^3} + \cdots + \frac{2nx^{2n-1}}{1+x^{2n}}$$

$$\Rightarrow \quad \frac{dy}{dx} = y\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{3x^2}{1+x^3} + \cdots + \frac{2nx^{2n-1}}{1+x^{2n}}\right]$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=0} = y\left(\frac{1}{1+0}\right) = 1$$

124. **(c)** Given that $f(x)$ is an odd differentiable function.

Then, $\quad f(-x) = -f(x)$

$\Rightarrow \quad -f'(-x) = -f'(x)$

$\Rightarrow \quad f'(-x) = f'(x)$ ... ... (i)

Put $x = 3$ in Eq. (i), we get

$$f'(-3) = f'(3)$$

$\therefore \quad f'(-3) = 2 \quad [\because f'(3) = 2]$

125. **(c)** We have, $\lim_{x\to1}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$

$$= \lim_{x\to1}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{(1+\sqrt{x})(1-\sqrt{x})}}$$

$$= \lim_{x\to1}\left(\frac{1+x}{2+x}\right)^{\frac{1}{(1+\sqrt{x})}}$$

$$= \left(\frac{1+1}{2+1}\right)^{\frac{1}{(1+1)}} = \left(\frac{2}{3}\right)^{\frac{1}{2}} = \sqrt{\frac{2}{3}}$$

126. **(d)** Given, $f(x) = \tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right] + \tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$

$$= \tan^{-1}\left[\frac{\log e - 2\log x}{\log e + 2\log x}\right] + \tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$$

Let $2\log x = \tan\theta$, we get

$$= \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] + \tan^{-1}\left[\frac{3+\tan\theta}{1-3\tan\theta}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right] + \tan^{-1}[\tan\tan^{-1}(3+\theta)]$$

$$= \frac{\pi}{4} - \theta + \tan^{-1}3 + \theta = \frac{\pi}{4} + \tan^{-1}3$$

$\because f(x) = \text{constant}$

$\therefore f''(x) = 0$

127. **(a)** Let $I = \int \frac{\log\sqrt{x}}{3x}dx$

Again, let $\log\sqrt{x} = z \Rightarrow \frac{1}{2x}dx = dz$

$$\therefore \quad \text{l} = \int \frac{2z}{3}dz = \frac{2}{3}\int zdz$$

$$= \frac{2}{3}\cdot\frac{z^2}{2} + C = \frac{1}{3}(\log\sqrt{x})^2 + C$$

128. **(c)** Let $I = \int 2^x[f'(x) + f(x)\log 2]dx$

Consider $g(x) = 2^x f(x)$

$\Rightarrow \quad g'(x) = 2^x f'(x) + 2^x f(x)\log 2$

$\Rightarrow \quad g'(x) = 2^x[f'(x) + f(x)\log 2]$

$\therefore$

$$\therefore \quad \text{l} = \int g'(x)dx = g(x) + C = 2^x f(x) + C$$

129. **(b)** Let

$$I = \int_0^1 \log\left(\frac{1}{x} - 1\right)dx$$

$$= \int_0^1 \log\left(\frac{1-x}{x}\right)dx$$

Let

$$I = \int_0^1 \log\left(\frac{x}{1-x}\right)dx = -1$$

$$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$$2I = 0 \Rightarrow I = 0$$

130. **(a)** $\because \lim_{n\to\infty}\left[\frac{\sqrt{n+1}+\sqrt{n+2}+\cdots+\sqrt{2n-1}}{n^{\frac{3}{2}}}\right]$

$$= \lim_{n\to\infty}\left[\sqrt{1+\frac{1}{n}} + \sqrt{1+\frac{2}{n}} + \cdots + \sqrt{1+\frac{n-1}{n}}\right]\frac{1}{n}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{n}\sqrt{1+\frac{r}{n}}$$

$$= \int_0^1 \sqrt{1+x}dx = \left[\frac{2}{3}(1+x)^{3/2}\right]_0^1 = \frac{2}{3}(2\sqrt{2}-1)$$

131. **(a)** We have,

$1^3 + 3^3 + 5^3 + 7^3 + \cdots$

$Now\ T_n = (2n-1)^3$

$= 8n^3 - 3(2n)^2(1) + 3(2n)(1)^2 - (1)^3$

$= 8n^3 - 12n^2 + 6n - 1$

$S_n = \Sigma T_n$

$$= \frac{8n^2(n+1)^2}{4} - \frac{12n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2} - n$$

$= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3n(n+1) - n$

$= n[2n(n+1)^2 - 2(n+1)(2n+1) + 3(n+1) - 1]n$

$=[2n(n^2 + 1 + 2n) - 2(2n^2 + 3n + 1)$

$=n[2n^3+2n+4n^2-4n^2-6n-2+3n+2]$
$=n[2n^3-n]=n^2(2n^2-1)$

132. (a)

Given, α and β are roots of $ax^2+bx+c=0$.

$\therefore \quad \alpha+\beta=\frac{-b}{a}$

and

$\alpha\beta=\frac{c}{a}$

Now, if the roots are α^2 and β^2, then

$\alpha^2+\beta^2 \quad =(\alpha+\beta)^2-2\alpha\beta$

$$=\left(\frac{-b}{a}\right)^2-\frac{2c}{a}=\frac{b^2}{a^2}-\frac{2c}{a}\ldots\text{(i)}$$

And $\alpha^2\beta^2=(\alpha\beta)^2=\left(\frac{c}{a}\right)^2=\frac{c^2}{a^2}$

The equation whose roots are α^2 and β^2, is

$$x^2-\left(\frac{b^2}{a^2}-\frac{2c}{a}\right)x+\frac{c^2}{a^2}=0$$

$\therefore\ a^2x^2-(b^2-2ac)x+c^2=0.$

133. (a)

Given,

$(2-\omega)(2-\omega^2)+2(3-\omega)(3-\omega^2)+\cdots+(n-1)(n-\omega)(n-\omega^2)$

Now,

$T_n=(n-1)(n-\omega)(n-\omega^2)$
$=(n-1)(n^2-n\omega-n\omega^2+\omega^3)$
$=(n-1)(n^2+n+1)=n^3-1$

$\therefore\ S_n=\Sigma T_n$

$$=\Sigma n^3-\Sigma 1=\frac{n^2(n+1)^2}{4}-n$$

134. (c)

Given,

${}^nC_{r-1}=36\ldots\text{(i)}$
${}^nC_r=84\ldots\text{(ii)}$
${}^nC_{r+1}=126\ldots\text{(iii)}$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{r}{n-r+1}=\frac{36}{84}$$

$\Rightarrow \quad 84r=36n-36r+36$

$\Rightarrow \quad 120r$
$=36n+36\ldots\text{(iv)}$

Also, on dividing Eq. (ii) by Eq. (iii), we get

$$\frac{r+1}{n-r}=\frac{84}{126}$$

$\Rightarrow \quad 126r+126=84n-84r$

$\Rightarrow \quad 210r$
$=84n-126\ldots\text{(v)}$

On solving Eqs. (iv) and (v), we get

$n=9$ and $r=3$

$\therefore \quad {}^nC_8$

135. (b) Number of males $=14$

and females $=6$

Total $=14+6=20$

Males above 40yr = 8
and females above 40yr = 3
Now, P (selected person is female and above 40 yr) $= \frac{3}{6} = \frac{1}{2}$

136. **(b)** Given equation is

$$x^3 - yx^2 + x - y = 0$$

$\Rightarrow \quad x^2(x-y) + (x-y) = 0$

$\Rightarrow \quad (x^2+1)(x-y) = 0$

Now, $\quad x^2 + 1 \neq 0$

and $\quad x - y = 0$

$\therefore \quad x = y$

So, the equation represents a straight line.

137. **(b)** Let (h,k) be the coordinates of the mid-point of a chord, which subtends a right angle at the origin.
Then, equation of the chord is

$$hx + ky - 1 = h^2 + k^2 - 1 \ldots. (\text{using T} = \text{S})$$

$\Rightarrow \quad hx + ky = h^2 + k^2$

The combined equation of the pair of lines joining the origin to the points of intersection of $x^2 + y^2 = 1$ and $hx + ky = h^2 + k^2$ is

$$x^2 + y^2 - 1\left(\frac{hx+ky}{h^2+k^2}\right)^2 = 0$$

Lines given by the above equation are at right angle.
Therefore, coefficient of x^2 + coefficient of $y^2 = 0$
i.e.

$$h^2 + k^2 = \frac{1}{2}$$

$$x^2 + y^2 = \frac{1}{2}$$

138. **(c)** Let $P(at^2, 2at)$ be a moving point on the parabola $y^2 = 4ax$ and $S(a,0)$ be its focus. Let $Q(h,k)$ be the mid-point of SP.
Then,

$$h = \frac{at^2 + a}{2}$$

and

$$k = \frac{2at + 0}{2}$$

$$\Rightarrow \frac{2h}{a} = t^2 + 1$$

and

$$t = \frac{k}{a}.$$

$$\Rightarrow \frac{2h}{a} = \frac{k^2}{a^2} + 1$$

$$\Rightarrow k^2 = 2ah - a^2$$

[on eliminatingt]
Thus, the locus of (h,k) is $y^2 = 2ax - a^2$
Now,

$$y^2 = 2ax - a^2$$

$$\Rightarrow (y-0)^2 = 2a\left(x - \frac{a}{2}\right)$$

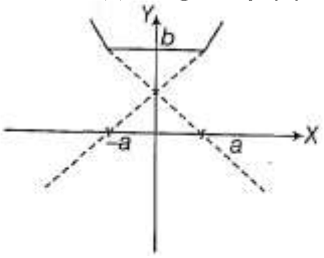
The equation of the directrix of this parabola is

$x-\frac{a}{2}=-\frac{a}{2}$, i.e. $x=0$

139. **(b)** Let,

$$I=\int_0^2 x^2[x]dx$$
$$=\int_0^1 x^2\cdot 0dx+\int_1^2 x^2\times 1dx$$
$$=\left[\frac{x^3}{3}\right]_1^2=\left[\frac{8}{3}-\frac{1}{3}\right]=\frac{7}{3}$$

140. **(c)** Graph of $f(x)$,



There are two sharp turns. Hence, $f(x)$ cannot be differentiable at two points.

141. **(d)** Given,

$|\mathbf{a}+\mathbf{b}|<|\mathbf{a}-\mathbf{b}|$

Now,

$$|\mathbf{a}+\mathbf{b}|^2<|\mathbf{a}-\mathbf{b}|^2$$
$$\Rightarrow (a+b)\cdot(a+b)<(a-b)(a-b)$$
$$\Rightarrow |a|^2+|b|^2+2a\cdot b<|a|^2+|b|^2-2a\cdot b$$
$$\Rightarrow 4a\cdot b<0$$
$$\Rightarrow \mathrm{a}\cdot\mathrm{b}<0$$
$$\Rightarrow |\mathbf{a}||\mathbf{b}|\cos\theta<0$$
$$\Rightarrow \cos\theta<0$$
$\Rightarrow \theta\in$ obtuse angle

$\therefore$ **a** and **b** are inclined at an obtuse angle.

142. **(b)** Given,

$y\frac{dy}{dx}+by^2=a\cos x, 0<x<1$ …. (i)

Let

$y^2=z$

$$\Rightarrow 2y\frac{dy}{dx}=\frac{dz}{dx} \quad \dots\dots(ii)$$
$$\Rightarrow y\frac{dy}{dx}=\frac{1}{2}\frac{dz}{dx}$$
$$\therefore \frac{1}{2}\frac{dz}{dx}+by^2=a\cos x \;\dots\dots(USing\ Eq.ii)$$
$$\Rightarrow \frac{dz}{dx}+2by^2=2a\cos x$$

Now, $\mathrm{F}=e^{2bjdx}=e^{2bx}$

$$\therefore z\cdot e^{2bx}=\int 2a\cos x\cdot e^{2bx}dx$$
$$\Rightarrow y^2e^{2bx}=\frac{2a}{4b^2+1}(\sin x+2b\cos x)e^{2bx}+C$$
$$\Rightarrow (4b^2+1)y^2=2a(\sin x+2b\cos x)+Ce^{-2bx}$$

Here, c is an arbitrary constant.

143. **(a)** Given that, the ordinate decreases at the same rate at which the abscissa increases, therefore

$\frac{dy}{dt}=-\frac{dx}{dt}$…..(i)

Also, equation of ellipse

$16x^2+9y^2=400.....(ii)$
On differentiating w.r.t. t, we get

$\Rightarrow 16\times 2\times\frac{dx}{dt}+9\times 2y\frac{dy}{dt}=0$

$\Rightarrow 16\times\frac{dx}{dt}+9y\frac{dy}{dt}=0$

$\Rightarrow 9y\frac{dy}{dt}=-16\times\frac{dx}{dt}$

$\Rightarrow 9y\frac{dy}{dt}=-16\times\left(-\frac{dy}{dt}\right)$ [using Eq. (i)]

$\Rightarrow 9y=16x\Rightarrow\ y=\frac{16}{9}x$(iii)

Using Eq. (iii) in Eq. (ii), we get

$16x^2+9\left(\frac{16}{9}x\right)^2=400$

$\Rightarrow 16x^2+\frac{(16)^2}{9}x^2=400$

$\Rightarrow 16x\left[x+\frac{16}{9}x\right]=400$

$\Rightarrow 16x\left[\frac{25}{9}x\right]=400\Rightarrow x^2=\frac{400\times 9}{25\times 16}$

$\Rightarrow x^2=9\Rightarrow\ x=\pm 3$

$\therefore$ Required points are $\left(3,\frac{16}{3}\right)$ and $\left(-3,-\frac{16}{3}\right)$.

144. **(a)** In a dictionary, the words at each stage are arranged in alphabetical order.
In the given problem, we must consider the words beginning with C, C, H, I, N, O in order. So,
Number of words starting with $CC=4$!
Number of words starting with CH $=4$!
Number of words starting with CI $=4$!
Number of words starting with CN $=4$!
Next word starting with CO i.e. COCHIN $=1$
$\therefore$ Required number of words $=4\times 4!=96$

145. **(d)** We have,

$$A=\begin{bmatrix}2&0&0\\0&2&0\\2&0&2\end{bmatrix}$$

$$\therefore A^2=\begin{bmatrix}2&0&0\\0&2&0\\2&0&2\end{bmatrix}\begin{bmatrix}2&0&0\\0&2&0\\2&0&2\end{bmatrix}=\begin{bmatrix}4&0&0\\0&4&0\\8&0&4\end{bmatrix}$$

$$=\begin{bmatrix}2^2&0&0\\0&2^2&0\\2\cdot 2^2&0&2^2\end{bmatrix}$$

Now,

$$A^n=\begin{bmatrix}2^n&0&0\\0&2^n&0\\n\cdot 2^n&0&2^n\end{bmatrix}$$

But

$$A^n=\begin{bmatrix}a&0&0\\0&a&0\\b&0&a\end{bmatrix}$$

[given]

$$\therefore\begin{bmatrix}a&0&0\\0&a&0\\b&0&a\end{bmatrix}=\begin{bmatrix}2^n&0&0\\0&2^n&0\\n\cdot 2^n&0&2^n\end{bmatrix}$$

Hence,
$a=2^n$

and
$b = n2^n$.

146. **(b,d)** Given equation of ellipse is
$4x^2 + 9y^2 = 1.......(i)$
On differentiating, we get
$\Rightarrow\ 8x + 18yy' = 0....$(ii)
$\Rightarrow\ y' = \frac{-8x}{18y} = m,$
Also, $8x = 9y$
[equation of line]
On differentiating, we get
$\Rightarrow 8 = 9y'$
$$\Rightarrow\quad y' = \frac{8}{9} = m, \text{(iii)}$$
[∵ tangents are parallel to this line]
So, by Eqs. (ii) and (iii), we get
$$\frac{-8x}{18y} = \frac{8}{9}$$
$$\Rightarrow\quad -x = 2y$$
$$\Rightarrow\quad x = -2y$$
On substituting $x = -2y$ in Eq. (i), we get
$$4(-2y)^2 + 9y^2 = 1$$
$$\Rightarrow\quad 16y^2 + 9y^2 = 1 \Rightarrow 25y^2 = 1$$
$$\Rightarrow\quad y = \pm\frac{1}{5} \therefore x = \mp\frac{2}{5}$$
So, required points are $\left(\frac{2}{5}, -\frac{1}{5}\right)$ and $\left(-\frac{2}{5}, \frac{1}{5}\right)$.

147. **(a,b)** $\int_{-3000}^{3000} \left(\sum_{r'=2014}^{2016} \phi(t-r')\phi(t-2016)\right)dt$
$$= \int_{-3000}^{3000} [\phi(t-2014)\phi(t-2016) + \phi(t-2015)\phi(t-2016)$$
$$+\phi(t-2016)\phi(t-2016)]dt$$
$$= \int_{-3000}^{3000} \phi(t-2016)[\phi(t-2014) + \phi(t-2015)$$
$$+\phi(t-2016)]dt$$
$$= \int_{-3000}^{2016} 0dt + \int_{2016}^{2017} 1\cdot(0+0+1)dt + \int_{2017}^{3000} 0dt$$
$= 0 + 1 + 0 = 1$, which is a real number.

148. **(a,c)** Given,
$$x^2 + y^2 - 10x + 21 = 0$$
$$\Rightarrow\quad x^2 - 10x + y^2 + 21 = 0$$
Now, for real roots of $x, D \geq 0$
$$100 - 4(y^2 + 21) \geq 0$$
$$\Rightarrow\quad y^2 \leq 4 \Rightarrow -2 \leq y \leq 2$$
Also,
$y^2 = -x^2 + 10x - 21$
Now, for real roots of y,
$-4(x^2 - 10x + 21) \geq 0$
$-x^2 + 10x - 21 \geq 0$
$$\Rightarrow\ (x-7)(x-3) \leq 0\ \therefore 3 \leq x \leq 7$$

149. **(a,c)** Given, $z = \sin\theta - i\cos\theta$
$= \cos\left(\theta - \frac{\pi}{2}\right) + i\sin\left(\theta - \frac{\pi}{2}\right) = e^{i\left(\theta - \frac{\pi}{2}\right)}$
Now,

$$z^n = e^{i\left(n\theta - \frac{n\pi}{2}\right)} = \cos\left(n\theta - \frac{n\pi}{2}\right) + i\sin\left(n\theta - \frac{n\pi}{2}\right)$$

$$\Rightarrow \frac{1}{z^n} = e^{i\left(\frac{n\pi}{2} - n\theta\right)} = \cos\left(n\theta - \frac{n\pi}{2}\right) - i\sin\left(n\theta - \frac{n\pi}{2}\right)$$

$$\therefore\ z^n + \frac{1}{z^n} = 2\cos\left(n\theta - \frac{n\pi}{2}\right) = 2\cos\left(\frac{n\pi}{2} - n\theta\right)$$

and $z^n - \frac{1}{z^n} = 2i\sin\left(n\theta - \frac{n\pi}{2}\right)$

150. **(a,b)**Given, $f[f(x)] = x$

Now, $f^{-1}(x) = f(x)$

i.e. $f(x)$ is bijective.

Hence, $f(x)$ has to be one-one and onto.

151. **(a,c)**We know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) = P(A \cup B) + P(A \cap B)$(i)

Given,

$\frac{3}{4} \leq P(A \cup B) \leq 1$

and

$\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$

So,

$\frac{7}{8} \leq P(A \cup B) + P(A \cap B) \leq \frac{11}{8}$

$\therefore P(A) + P(B) \geq \frac{7}{8}$ and $P(A) + P(B) \leq \frac{11}{8}$

152. **(b,d)**$\because$ a, b and c are respectively the AM, GM and HM of first and $(2n-1)$ th terms.

and also, we know

$AM \geq GM \geq HM$

$\therefore$ $a \geq b \geq c$ and $AM \cdot HM = G^2$

$a \cdot c = b^2$

153. **(b,d)**Let (h, k) be the point on line $x + y + 1 = 0$.

So,

$h + k + 1 = 0$

$h = -1 - k$(i)

Given, distance between (h, k) and line $3x + 4y + 2 = 0$ is $\frac{1}{5}$.

$\therefore\ \frac{1}{5} = \left|\frac{(3h) + (4k) + 2}{\sqrt{3^2 + 4^2}}\right|$

$\Rightarrow\ \frac{1}{5} = \left|\frac{3(-1-k) + 4k + 2}{\sqrt{25}}\right|$

$\Rightarrow\ \frac{1}{5} = \pm\left(\frac{-3 - 3k + 4k + 2}{5}\right)$

$\Rightarrow\ \frac{1}{5} = \pm\left(\frac{k-1}{5}\right)$

$\Rightarrow\ \pm(k-1) = 1$

$\Rightarrow\ k = 0$ and 2

$\therefore\ h = -1, -3$

Hence, the required points are $(-1,0)$ and $(-3,2)$.

154. **(a,b)**Given, equation of the parabola

$x^2 = ay$

i.e. . $y = \frac{x^2}{a}$.....(i)

and the equation of line

$y - 2x = 1$

i.e.
$y = 2x + 1$
From Eqs. (i) and (ii), we get
$\Rightarrow \frac{x^2}{a} = 2x + 1$
$x^2 = 2ax + a$
$x^2 - 2ax - a = 0$………(iii)
$= \frac{2a \pm \sqrt{4a^2 + 4a}}{2}$
$= \frac{2\left(a \pm \sqrt{a^2 + a}\right)}{2} = a \pm \sqrt{a^2 + a}$
Points of intersection of the parabola and the line are
$\left(a + \sqrt{a^2 + a}, 2\left(a + \sqrt{a^2 + a}\right) + 1\right)$
and $\left(a - \sqrt{a^2 + a}, 2\left(a - \sqrt{a^2 + a} + 1\right)\right.$
$\because$ Distance between the points $= \sqrt{40}$
$\sqrt{40} = \sqrt{\left(2\sqrt{a^2 + a}\right)^2 + \left(4\sqrt{a^2 + a}\right)^2}$
$40 = 4(a^2 + a) + 16(a^2 + a)$
$2 = a^2 + a \Rightarrow a = 1 - 2$
155. **(b,c)** Given,
$f'(x) = (x - 1)^2(4 - x)$
The sign scheme of $f'(x)$
$\frac{++_1 -}{4}$
Clearly; $f(x)$ is increasing, for $x \in (-\infty, 4)$ and decreasing, for $n \in (4, \infty)$.
Since, $f'(x) = 0$ at $x = 4$.
So, $x = 4$ is a critical point.