## Physics

**1.** **(b)** The speed of a particle executing simple harmonic motion is $v=\omega\sqrt{a^2-x^2}$

where, $a=$ Amplitude

$\omega=$ Angular frequency

$x=$ Displacement

or $v^2=\omega^2(a^2-x^2)$

According to the question,

$$v_1^2=\omega^2(a^2-x_1^2)$$
$$v_2^2=\omega^2(a^2-x_2^2)$$
$$v_1^2-v_2^2=\omega^2(x_2^2-x_1^2)$$
$$\omega=\sqrt{\frac{v_1^2-v_2^2}{x_2^2-x_1^2}}$$

Here

$v_1=13\text{ m/s}$

$v_2=12\text{ m/s}$

$x_1=3\text{ m}$

$x_2=5\text{ m}$

$$\omega=\sqrt{\frac{(13)^2-(12)^2}{(5)^2-(3)^2}}$$
$$=\sqrt{\frac{169-144}{25-9}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$$
$$\omega=2\pi r$$
$$v=\frac{\omega}{2\pi}=\frac{5/4}{2\pi}$$
$$v=\frac{5}{8\pi}$$

**2.** **(a)** The formula of frequency of 3 vibrations produced in tensed wire is

$$v=\frac{n}{2}\sqrt{\frac{T}{m}}$$

$m=$ Mass per unit length of the wire $=\frac{m}{L}$

$M=$ Mass of wire

$L=$ Length of wire

$n=$ Number of loops produced in wire

$$v=\frac{n}{2L}\sqrt{\frac{T}{M/L}}=\frac{n}{2}\sqrt{\frac{T}{ML}}$$

**3.** **(d)** When the sealed capillary tube is submerged vertically into water, the pressure inside the tube Changes

For the inside capillary,

$$p_1V_1=p_2V_2$$
$$p_0(lA)=p'(l-x)A$$

where $p'$ is pressure in capillary after being submerged

$$\therefore\ p'=\frac{p_0l}{l-x}$$

According to question, since level of water inside capillary coincides with outside.

$$\therefore\quad p'-p_0=\frac{2\gamma}{r}$$
$$\therefore\quad \frac{p_0l}{l-x}-p_0=\frac{2\gamma}{r}=\frac{l}{\left(1+\frac{p_0r}{2\gamma}\right)}$$

**4.** **(a)** The Bulk modulus $k$

$k = -\frac{p}{\Delta V/V}$.....(i)

Where, $p =$ Pressure

$\Delta V =$ Change in volume

$V =$ Volume of liquid

From (i)

$$\frac{p}{k} = -\frac{\Delta V}{V} = \frac{\Delta p}{p}$$

$$\Rightarrow p = \frac{k\Delta p}{p}$$

$$\Delta p = 0 \cdot 01\% = 0 \cdot 01/100$$

$$p = \frac{k}{10000}$$

**5.** **(c)** Initial temperature of ideal gas,

$T_{1.} = 273 + 27 = 300 \text{ K}$

when temperature of gas is raised by $6^\circ$C, the final temperature of gas

$T_2 = 273 + 6 + 27$

$= 306 \text{ K}$

Let initial velocities are $v_{\text{rms }1}$, and $v_{\text{rms }2}$

$$v_{\text{rms}} \propto \sqrt{T}$$

$$\frac{v_{\text{rms}_1}}{v_{\text{rms}_2}} = \sqrt{\frac{T_1}{T_2}}$$

$$v_{\text{rms}_2} = \sqrt{\frac{T_2}{T_1}} \times v_{\text{rms}_1}$$

$$= \sqrt{\frac{306}{300}} \times v_{\text{rms}_1}$$

$$= 1.00 \times v_{\text{rms}_1}$$

So, it will increase by 1%

**6.** **(c)** The internal energy

$\Delta U = nC_v\Delta T$

$C_v =$ Specific heat of gas at constant volume

$$\Rightarrow \Delta U = n \cdot \frac{3R}{2}\left(\frac{4p_0V_0}{nR} - \frac{p_0V_0}{nR}\right)$$

$$= n \cdot \frac{3R}{2}\left(\frac{4p_0V_0 - p_0V_0}{nR}\right)$$

$$= n \cdot \frac{3R}{2} \cdot \frac{3p_0V_0}{nR}$$

$$= \frac{9}{2}p_0V_0(\text{i})......(\text{i})$$

Work done by the gas

$W = (2p_0 + p_0)\frac{V_0}{2} = \frac{3p_0V_0}{2}$.........(ii)

From first law of thermodynamics,

$\Delta Q = dW + dU$

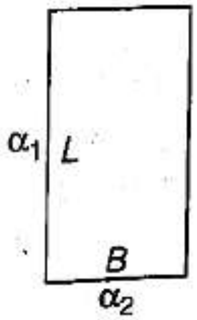
$$= \frac{3p_0V_0}{2} + \frac{9}{2}p_0V_0$$

[ from Eqs. (i) and (ii) ]

$$= \frac{3p_0V_0}{2} + \frac{9}{2}p_0V_0$$

$$= \frac{12p_0V_0}{2} = 6p_0V_0$$

**7. (d)** The coefficient of linear expansion along its length $= \alpha_1$




The coefficient of linear expansion along its breadth $= \alpha_2$
Increase in length,
$L_t = l_0(1 + \alpha_1 \Delta t)$
Increase in breadth,
$B_t = b_0(1 + \alpha_2 \Delta t_2)$
Let coefficient of surface expansion is $\beta$

$$\begin{aligned}
\text{Area} &= \text{length} \times \text{breadth} \\
&= l_0(1 + \alpha_1 \Delta t) \times b_0(1 + \alpha_2 \Delta t) \\
&= l_0 b_0 (1 + \alpha_1 \Delta t)(1 + \alpha_2 \Delta t) \\
&= S_0(1 + \alpha_1 \Delta t + \alpha_2 \Delta t + \cdots)
\end{aligned}$$

re, $S_0 = l_0 \cdot b_0$
$= \text{Initial area of surface}$
In state of expansion,

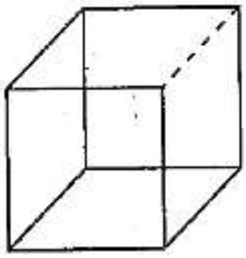
$$\begin{aligned}
S_t &= L_t \times B_t \\
&= L_0 b_0 (1 + \alpha_1 \Delta t)(1 + \alpha_2 \Delta t) \\
&= S_0(1 + \alpha_1 \Delta t + \alpha_2 \Delta t + \cdots) \\
S_t &= S_0(1 + \beta \Delta t)
\end{aligned}$$

$$\therefore S_0(1 + \beta \Delta t) = S_0(1 + \alpha_1 \Delta t + \alpha_2 \Delta t + \cdots)$$

$$\beta \cdot \Delta t = \alpha_1 \Delta t + \alpha_2 \Delta t$$

$$\beta = \alpha_1 + \alpha_2$$

**8. (a)** The positive charge $Q$ is situated at the centre of a cube

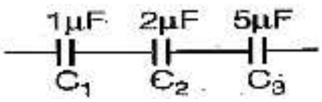


According to Gauss theorem, the flux from any closed surface

$$\phi_E = \frac{Q}{\varepsilon_0}$$

Cube has 6 faces, so electric flux from any face $\phi_E = \frac{Q}{6\varepsilon_0}$

**9. (c)**




When the capacitors are connected in series, the resultant capacitance of combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{5} = \frac{17}{10} \mu \text{ F}$$

$$C = \frac{10}{17} \mu \text{ F}$$

The charge will be same on all the capacitors in series

$Q = CV = \frac{10}{17} \times 10 = \frac{100}{17}$

The potential difference across $2.0\mu$ F capacitor

$V' = \frac{Q}{C} = \frac{100/17}{2} = \frac{50}{17}$ volt

**10. (a)**

$\frac{Q_1}{30 \text{ cm}} Q_2^{Q_2}$

Checking the given options one by one

(i) $Q_1 = Q_2 = 0.4\text{C}$

The force on $Q_1$ due to $Q_2$

$$F = k_p \frac{Q_1 \times Q_2}{30 \times 10^{-2}}$$
$$= k \frac{Q_1 Q_2 \times 100}{30}$$
$$= k \frac{0.4 \times 0.4 \times 100}{30}$$
$$= \frac{k \times 0.16 \times 100}{30}$$

(ii) When $Q_1 = 0.8\text{C} Q_2 \approx 0$

$$F = \frac{k \times Q_1 Q_2}{30 \times 10^{-2}} \approx 0$$

(iii) when $Q_1 \approx 0, Q_2 = 0.8\text{C}$

$F = 0$

(iv) $Q_1 = 0.2C Q_2 = 0.6C$

$$F = \frac{k \cdot 0.2 \times 0.6}{30 \times 10^{-2}}$$
$$= \frac{k \times 0.12 \times 10^2}{30}$$

Thus we find that, in option(a), the force on $Q_1$ will be maximum.

$\therefore$ correct answer is $Q_1 = Q_2 = 0.4\text{C}$

**11. (b)** By Biot-Savart's law, the magnetic fielc due to a current carrying wire is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$$

So,

$$B \propto \frac{1}{r^2}$$

So, the magnetic field due to current decreases as inverse square of distance of the point of observation.

**12. (b)** In galvanometer $I \propto \theta = G\theta$

Where, $G = \frac{k}{NAB}$

When coil is in square of shape,

$$G = \frac{k}{NAB} = \frac{k}{Na^2 B} \ldots\ldots\ldots (i)$$

When coil is in circular shape of radius

a. $/\sqrt{\pi}$

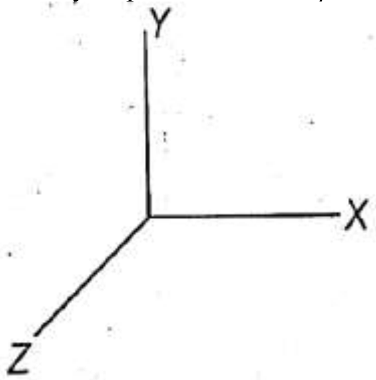
G$= \frac{k}{NAB} = \frac{k}{N\pi r^2 \cdot B}$

$= \frac{k}{N\pi\left(\frac{a}{\sqrt{\pi}}\right)^2 \cdot B} = \frac{k}{Na^2 B}$........(ii)

So, value of $G$ is same for both type of shape of coil. So, the torque $e = NBIA \sin\theta$ will be same in both the case

$e : e = 1 : 1$

**13.** **(a)** Velocity of proton $= 10^6$ m/s along $y$-direction




Electric field $= 2 \times 10^4$ V/m
$\frac{e}{m}$ for proton $= 10^{-8}$C/kg
The magnetic field,

$$B = \frac{E}{v}$$
$$= \frac{2 \times 10^4}{10^6} = 2 \times 10^{-2}\ \mathrm{T}$$

The radius of circular path

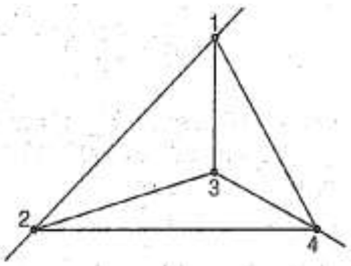
$$\gamma = \frac{mv}{q_p \cdot B} = \frac{10^{-8} \times 10^6}{2 \times 10^{-2}}$$
$$= \frac{1}{2} = 0.5\ \mathrm{m}$$

**14.** **(b)**
The impedance of LCR circuit

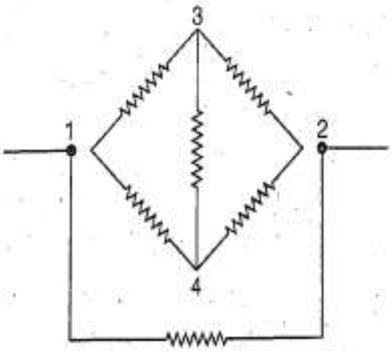
$$Z = \sqrt{R^2 + \left(\omega_c - \frac{1}{\omega_c}\right)^2}$$

Where, $R =$ Resistance
$L =$ Inductance
$C =$ Capacitance
When frequency of AC voltage applied to a series LCR circuit, it will first increase and resonance state $\omega_L = \frac{1}{\omega_C}$, it will decrease. In that state ( $Z = R$ ), it will be minimum. So, it will first increases and then decreases.

**15.** **(d)** Six wires each of resistance $r$ form a tetrahedron as shown in the following figure




The equivalent circuit of this tetrahedron



It is circuit of Wheatstone the equivalent resistance of upper circuit

$$\frac{1}{R}=\frac{1}{2r}+\frac{1}{2r}=\frac{2}{2r}=\frac{1}{r}$$

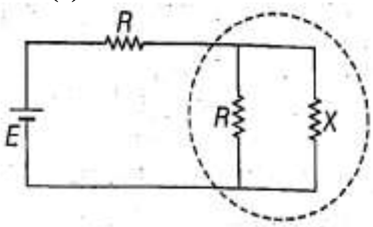
$$R=r$$

It will in parallel with outer resistance

$$\frac{1}{R_{\text{eq}}}=\frac{1}{r}+\frac{1}{r}=\frac{2}{r}$$

$$R_{\text{eq}}=\frac{r}{2}$$

**16. (c)**




For above circuit

$$\frac{1}{R'}=\frac{1}{R}+\frac{1}{X}=\frac{R+X}{RX}$$

$$R'=\frac{RX}{R+X}$$

The current in the circuit

$$i=\frac{E}{R+R'}=\frac{E}{\left(R+\frac{RX}{R+X}\right)}$$

$$V_{RX}=\frac{E\cdot\frac{RX}{R+X}}{\left(R+\frac{RX}{R+X}\right)}=\frac{EX}{R+2X}$$

Power dissipated in the circuit

$$P_X=\frac{V_{RX}^2}{X}=\frac{E^2\cdot X^2}{X(R+2X)^2}$$

$$=\frac{E^2X}{(R+2X)^2}$$

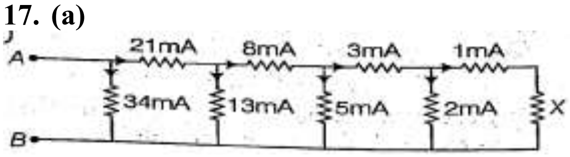
$$\frac{dP_X}{dX}=E^2\frac{(R-2X)}{(R+2X)^3}$$

$$\Rightarrow dP_X=\frac{E^2(R-2X)}{(R+2X)^3}\cdot dX$$

( $dP_X$ ) will be zero for all ( $dX$ ) if

$$X=\frac{R}{2}\left[dP_X=\frac{E^2\left(R-2\times\frac{R}{2}\right)}{\left(R+2\times\frac{R}{2}\right)^3}=0\right.$$

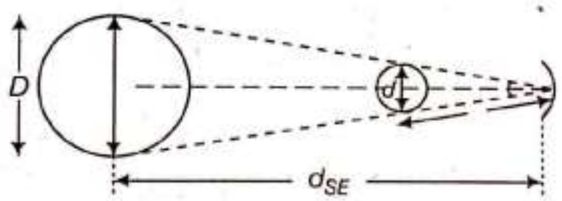
**17. (a)**




The potential between points $A$ and $B$

$V_{AB}=34$ volt

**18. (d)**




The angle subtended by sun on mirror

$$Q=\frac{D}{d_{SE}}=\frac{d}{f}$$

$$\therefore d=\frac{D}{d_{SE}}\times f$$

According to the question,

$$\frac{D}{d_{SE}}=0.009$$

$$f=\frac{r}{2}=\frac{0.4}{2}=0.2\text{ m}$$

$$\therefore\ d=0.009\times 0.2\text{ m}$$
$$=9\times 2\times 10^{-4}\text{ m}$$
$$=18\times 10^{-4}\text{ m}$$
$$=1.8\times 10^{-3}\text{ m}$$

**19. (a)** For maximum intensity,

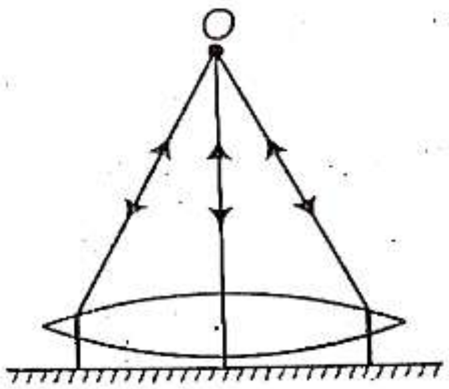
$$I_{\max}=I_1+I_2+2\sqrt{I_1I_2}\cos\delta$$
$$\delta=0$$

$$I_{\max}=L+\frac{L}{4}+2\sqrt{L\times\frac{L}{4}}\cdot\cos 0$$
$$=\frac{5L}{4}+\frac{2L}{2}$$
$$=\frac{5L+4L}{4}$$
$$=\frac{9L}{4}$$

$$I_{\min}=I_1+I_2-2\sqrt{I_1I_2}\cos\theta$$
$$\theta=0^\circ$$
$$=L+\frac{L}{4}-2\sqrt{L\times\frac{L}{4}}\times 1$$
$$=\frac{5L}{4}-L=\frac{L}{4}$$

Thus, maximum and minimum intensities will be $\left(\frac{9L}{4},\frac{L}{4}\right)$.

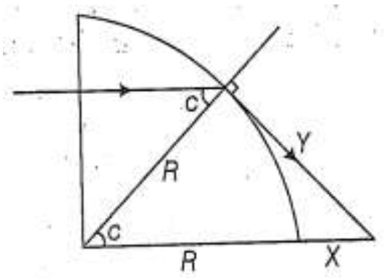
**20. (b)** As the lens is placed on a horizontal thin plane mirror.



So due to role of plane mirror the image will be formed at the same point $O$ above 0: 1 m distance above the center of the lens.

**21. (c)** Given, radius $R = 0.05$ m

Refractive index $n = 1.5$




The critical angle, $\sin c = \frac{1}{n} = \frac{1}{1\cdot 5}$

$$= \frac{10}{15} = \frac{2}{3}$$

From above figure,

$$\cos c = \frac{R}{R+X}$$

$$\sqrt{1-\sin^2 c} = \frac{R}{R+X}$$

$$\sqrt{1-\frac{4}{9}} = \frac{R}{R+X}$$

$$\frac{\sqrt{5}}{3} = \frac{R}{R+X}$$

$$\sqrt{5}(R+X) = 3R$$

$$\therefore\ \sqrt{5}R + \sqrt{5}X = 3R$$

$$\therefore\ X = (3\sqrt{5}-5)\times 10^{-2}\text{ m}$$

**22. (d)** de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore\ \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

$\lambda_1 =$ Wave length of electron

$= 0.4\times 10^{-10}$ m

$E_1 = 1.0\text{keV}$

$\lambda_2 = 1.0\times 10^{-10}$ m

$E_2 = ?$

$$\frac{0.4\times 10^{-10}}{1\times 10^{-10}} = \sqrt{\frac{E_2}{1\cdot 0\text{keV}}}$$

$$\frac{4}{10} = \sqrt{\frac{E_2}{1}}$$

$$\frac{16}{100} = \frac{E_2}{1}$$

$$E_2 = 0.16\text{keV}$$

**23.** (c) From Einstein's photoelectric equation,

$hv = W + eV$

$W =$ Work function

$v =$ Frequency of incident photon

$V =$ Stopping potential

For a photon of frequency $v_1$,
$hv_1 = W + eV_1$.........(i)
For photon of frequency $v_2$,
$hv_2 = W + eV_2$
Divided Eq. (i) by Eq. (ii),

$$\frac{hv_1}{hv_2} = \frac{W + eV_1}{W + eV_2}$$

$$\frac{v_1}{v_2} = \frac{W + eV_1}{W + eV_2}$$

$\Rightarrow Wv_1 + eV_2v_1 = Wv_2 + eV_1v_2$
Solving above equation,

$$\Rightarrow e = \frac{W(v_2 - v_1)}{V_2v_1 - V_1v_2}$$

**24. (a)** Half life of radon $= 3 \cdot 8$ days
Total half-lives in 19 days

$$= \frac{19}{3 \cdot 8}$$

$$= \frac{190}{38}$$

$= 5$ half-lives
Initially mass of radon $= 0.064$ kg

$$0.064 \xrightarrow{T_1} 0.032 \xrightarrow{T_2}$$

$$0.016 \xrightarrow{T_3} 0.008$$

$$\xrightarrow{T_4} 0.004 \xrightarrow{T_5} 0.002$$

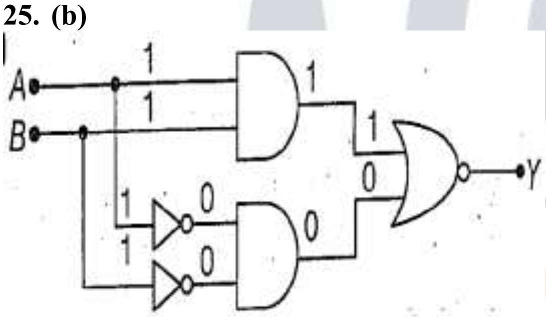
So in five half-life periods, the Radon sample will reduce to 0.002 kg

Or $\frac{N}{N_0} = \left(\frac{1}{2}\right)^5$

$$\Rightarrow N = 0.064 \times \frac{1}{32}$$

$= 0.002$ kg

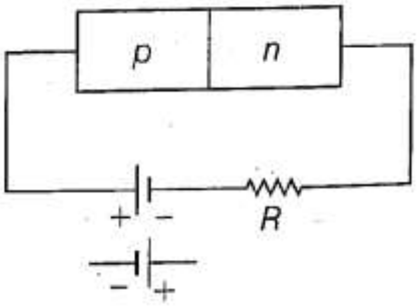
**25. (b)**




The output of logic gate circuit
for

$Y = \overline{A \cdot B + \bar{A}\bar{B}}$
$A = 1$
$B = 1 \Rightarrow Y = 0$
and for
$A = 0, B = 0$
$\Rightarrow Y = 0$

**26. (d)** When the battery and resistance is connected with pn diode,




In case of forward bias that $p$ is connected to $+ve$ terminal of battery, current flows.
If one reverse the polarity i.e., $p$-end is made negative, no current flows in semiconductor diode.

**27. (b)** The dimensions of universal constant $G$, is obtained by Newton's law of gravitation

$F=\frac{G m_1 m_2}{r^2}$

$G=\frac{F \times r^2}{m_1 m_2}$

$F=$ Force

$r=$ Distance

$m_1$ and $m_2$ are masses

$$G=\frac{[\mathrm{M}^1\ \mathrm{L}^1\ \mathrm{T}^{-2}][\mathrm{L}^2]}{[\mathrm{M}][\mathrm{M}]}$$

$$=\frac{[\mathrm{M}^1\ \mathrm{L}^3\ \mathrm{T}^{-2}]}{[\mathrm{M}^2]}=[\mathrm{M}^{-1}\ \mathrm{L}^3\ \mathrm{T}^{-2}].]$$

**28. (a)** In the absence of external force the speed of centre of mass is always zero i.e.

$u_{\mathrm{cm}}=0$

$\because\ \mathrm{F}_{\text{ext}}=0$ then,

$v_{\mathrm{cm.}}=0$ always

**29. (b)** Three vectors are mutually perpendicular, so the scalar or dot product of two vectors will zero. So,

$\mathbf{A} \cdot \mathbf{B}=\mathbf{0}, \mathbf{B} \cdot \mathbf{C}=0, \mathbf{A} \cdot \mathbf{C}=\mathbf{0}$

So, $\mathbf{A} \cdot \mathbf{B}=(a \hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}) \cdot(\hat{\mathbf{i}}+b \hat{\mathbf{j}}+\hat{\mathbf{k}})=0$

$\Rightarrow a+b+1=0$..........(i)

$\mathbf{B} \cdot \mathbf{C}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}} \cdot \hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}=$

$\Rightarrow 1+b+c=0$.......(ii)

$\mathbf{A} \cdot \mathbf{C}=(a \hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}) \cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+c \hat{\mathbf{k}})=0$

$\Rightarrow\ a+1+c=0$.........(iii)

Addition of Eqs. (i), (ii) and (iii),

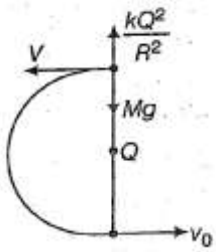
$2(a+b+c)+3=0$

$$\begin{aligned}
\Rightarrow\ a+b+c &= -\frac{3}{2} \\
-1+c &= -\frac{3}{2} \\
c &= -\frac{3}{2}+1 \\
&= -\frac{1}{2}=0.5 \\
\therefore\ a &= -\frac{1}{2} \\
b &= -\frac{1}{2} \\
c &= -\frac{1}{2}
\end{aligned}$$

**30. (a)** From Newton's second law

$F = ma$
$m = 1$ kg
$F = 1 \times a = \frac{dv}{dt}$
According to the question,
$F = kt = \frac{dv}{dt}$
$dv = kt \cdot dt$
Integration gives
$v = \frac{kt^2}{2} = \frac{dx}{dt}$
$x = \frac{kt^3}{6} = \frac{1 \times 6 \times 6 \times 6}{6}$
$= 36$ m

**31. (b)** According to the question, when tension in the string is zero.




As $T = 0$

$$Mg - \frac{KQ^2}{R^2} = \frac{mv^2}{R} \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

$\frac{mv^2}{R}$ = centripetal force required to keep the body in a circular path

$$\Rightarrow v = 0 \left[\because Mg = \frac{kQ^2}{R^2}\right]$$

So the required work done to keep charge in vertical motion

$$W_g = \Delta KE = \text{ change in K.E.}$$

$$\therefore \Rightarrow mg(2R) = \frac{1}{2}mv_0^2$$

$$v_0^2 = 2g \cdot 2R = 4gR$$

$$v_0 = \sqrt{4gR} = 2\sqrt{gR}$$

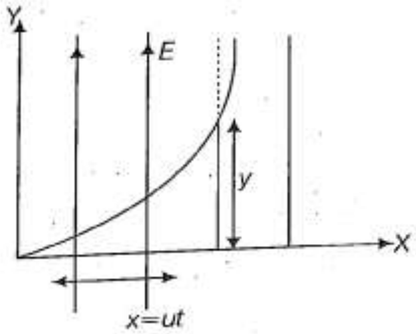
**32. (b)** Let mass of bullet $= m$
Mass of block $= M$
Velocity of bullet $= v = 300$ m/s
Velocity of combined system $M + m = V$
Here, from momentum conservation
$\frac{M+m}{m}V = v$
$\Rightarrow V = \frac{300 \times 42 \times 10^{-2}}{4.2 \times 10^{-2} + 9(4.2 \times 10^{-2})}$
$= 30$ m/s
Now, heat produced = Loss in kinetic energy of bullet
$= \frac{1}{2}mv^2 - \frac{1}{2}(M+m)V^2$

$= \frac{1}{2} \times 4.2 \times 10^{-2}(300)^2 - \frac{1}{2}(4.2 \times 10^{-2}$

$+9 \times 4.2 \times 1)(30)^2$
$= 6.3 \times 270$

$= 1701$ J
$= \frac{1701}{4.2}$ Cal
$= 405$Cal

**33. (d)** From parabola




$y = \frac{1}{2} \times \left[\frac{Ee}{m}\right] \times \frac{X^2}{u^2}$
$= \frac{E \cdot e}{2mu^2} \cdot X^2$

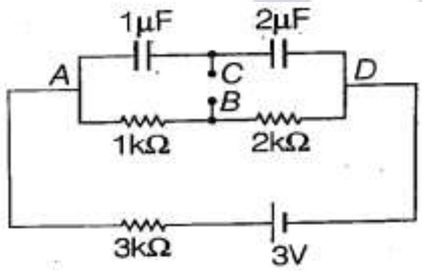
As $X = 4ay$ (for parabola)

$\therefore \quad X^2 = \frac{2mu^2}{E \cdot e} \cdot y$

$a = \frac{2mu^2}{4E \cdot e} = \frac{mu^2}{2E \cdot e}$

**34. (b)**

**35. (b)**




From circuit the current,

$I = \frac{E}{R}$

The equivalent resistance of above circuit

$= 3 + 2 + 1$
$= 6\text{k}\Omega$
$= 6 \times 10^3 \Omega$

$E = 3$volt

$I = \frac{3}{6 \times 10^3}$
$= 0.5 \times 10^{-3}$ A

Potential,

$V_{AD} = iR = 0.5 \times 10^{-3} \times 3 \times 10^3$
$= 1.5$ V

Charge,

$$\frac{1}{C}=\frac{1}{1}+\frac{1}{2}=\frac{3}{2}$$
$$C=\frac{2}{3}$$
$$Q=C\cdot V_{AD}$$
$$=\frac{2}{3}\times 1\cdot 5$$
$$=1\mu\mathrm{C}$$

Applying KVL (Kirchhoff's Voltage Law) from $B$ to $C$

$$V_B-0.5\times 10^{-3}\times 2\times 10^{3}+\frac{1}{2}=V_C$$
$$V_B-V_C=1-\frac{1}{2}=0.5\ \mathrm{V}$$

**36. (b,c)** $v=\sqrt{\frac{\gamma RT}{M}}=\sqrt{\frac{\gamma P}{\rho}}$

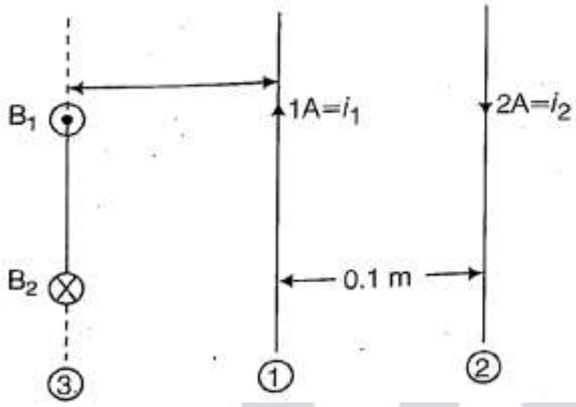
(i) Velocity of sound is proportional to $\sqrt{T}$

$\therefore\ v\propto\sqrt{T}$

(ii) When $\rho$ is constant, the velocity of sound is proportional to root of pressure $P$

$\therefore\ v\propto\sqrt{P}$

**37.** (c)




According to question, the third wire should not feel magnetic force due to wire (1) and (2). It can be only possible in the condition
that fields due to wire (1) and (2) must act opposite to each other and must be equal. The magnetic field due to long straight wire,

$$B=\frac{\mu_0\cdot I}{2\pi x}$$
$$\therefore\ B_1=B_2$$
$$\frac{\mu_{\dot{d}_1}}{2\pi x}=\frac{\mu_{\dot{d}_2}}{2\pi(0\cdot 1+x)}$$

Here $i_1=1$ A

$i_2=2$ A

$$\frac{\mu_0\times 1}{2\pi x}=\frac{\mu_0\times 2}{2\pi(0\cdot 1+x)}$$
$$2x=(0\cdot 1+x)$$
$$x=0\cdot 1\ \mathrm{m}$$

**38. (b, d)**

We know that magnetic susceptibility

$\chi=\mu_{r-1}$

and

$$\mu_r=\frac{\mu}{\mu_0}$$

= Relative permeability

For paramagnetic substance,

$\chi > 0$

$\therefore \quad \mu_r > 1$

$\therefore \quad u > \mu_0$

For diamagnetic substance,

$\chi < 0$

$\therefore \quad \mu_r \quad < 1$

$\therefore \quad \mu < \mu_0$

For ferromagnetic substance,

If $0 < \mu < \mu_0$

$\chi \gg 1$

$\mu \gg M_0$

Then substance will not be paramagnetic.

Hence option (a) is incorrect and option (b) and (d) are correct.

**39. (a, b, c, d)** We know that, $v \propto \frac{1}{n}$ (speed)

$$E_n \propto \frac{1}{n^2}$$

$$\therefore \quad r_n \propto n^2$$

$$\therefore \quad E_n r_n \propto n^0$$

$$E_n r_n \propto E_1 r_1$$

$$\frac{E_n r_n}{E_1 r_1} = \text{ constant } \quad (\because \text{ slope } = 0)$$

$$r_n v_n \propto n^2 \times \frac{1}{n} \propto n$$

$$\therefore \quad \frac{r_n v_n}{r_1 v_1} = n$$

$$(\because \text{ slope } = 1)$$

$$\therefore \quad \frac{r_n \propto n^2}{r_1} = n^2$$

$$\log\left(\frac{r_n}{r_1}\right) = 2\log n$$

$$\frac{r_n}{E_n} \propto n^4$$

$$\frac{r_n}{E_n} \times \frac{E_1}{r_1} = n^4$$

$$\therefore \quad \log\left(\frac{r_n E_1}{E_n r_1}\right) = 4\log(n)(\because \text{ slope } = 2)$$

**40. (d)** Given, steel ball bounces on a steel plate held horizontally, so

$V_{\text{upward}} = eV_{\text{downward}}$

According to the relation,

$h_n = e^{2nh}$

$$\therefore t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2he^2}{g}} + 2\sqrt{\frac{2he^4}{g}} + \cdots$$

$$= \sqrt{\frac{2h}{g}}[1 + 2e + 2e^2 + \cdots]$$

$$= \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]$$

$\therefore t = 10\text{sec}$

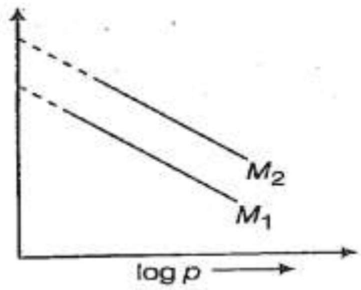
$$\therefore 10 = \sqrt{\frac{2(0\cdot 4)}{10}}\left(\frac{1+e}{1-e}\right)$$

Solving above relation

$$\therefore\ e = \frac{25\sqrt{2}-1}{25\sqrt{2}+1} \approx \frac{17}{18}$$

## Chemistry

**41.** **(a)** For ideal gas, $\log\frac{k}{M_2}, \log\frac{k}{M_1}$




$\therefore\ pV = \frac{W}{M}RT$.........(i)

Let, $WRT = K$

Taking log of both the sides of Eq. (i)

$$\log p + \log V = \log\frac{K}{M}$$

or $\log V = -\log p + \log\frac{K}{M}$

[On compairing Eq. (ii) with $y = mx + C$ ]

$$\log\frac{K}{M_2} > \log\frac{K}{M_1} \text{ or } M_1 > M_2$$

(i) $n + l \to 4 + 1 = 5$

(ii) $n + l \to 4 + 0 = 4$

**42.** **(b)** $\because$ Surface tension $(T)$ = Work done per unit area

$$\therefore \begin{aligned} T &= \frac{dW}{dA} \\ &= \frac{F\cdot dx}{dA}\ (F^{-1} = \mathrm{MT}^{-2}) \end{aligned}$$

$\therefore$ Dimension of surface tension $= \mathrm{ML}^0\ \mathrm{T}^{-2}$

**43.** **(a)** According to ( $n + l$ ) rule;

(i) More be the sum of $(n + l)$ value, more be the energy.

(ii) For same value of sum, more be the value of $n$, more be the energy.

For

(iii) $n+l \rightarrow 3+2=5$
(iv) $n+l \rightarrow 3+1=4$
Hence, order of energy will be
(iv) < (ii) < (iii) < (i)

**44. (b)**

Electronic configuration for $\mathrm{Cr}(24)$
$=[\mathrm{Ar}]_{18} 4s^1 3d^5$
$\therefore$ 19 th electron enters in 4s-orbital and its set of quantum numbers is
$n=4, l=0, m=0, m_s=+\frac{1}{2}$

**45. (d)** $\because$ Number of milli equivalents of NaOH

$(n')=NV=0.1 \times 20=2$

$\therefore$ Number of equivalent $=0.002$

Number of equivalent of base ( NaOH )
= Number of equivalent of acid

$$0.002=\frac{W}{E}=\frac{0.126}{E}$$

$$\therefore E=\frac{0.126}{0.002}=63$$

**46. (c)** Given, weight ratio: $W_{\mathrm{CH}_4}: W_{\mathrm{SO}_2}=1: 2$

$$\because n=\frac{W}{m}\left[\begin{array}{r}n= \text{ no. of moles} \\ w= \text{ mass} \\ M= \text{ molar mass}\end{array}\right.$$

$$\therefore \frac{n_1}{n_2}=\frac{W_1}{M_1} \times \frac{M_2}{W_2}\left\{\begin{array}{l}n_1=n_{\mathrm{SO}_2} \\ n_2=n_{\mathrm{CH}_4}\end{array}\right.$$

$$\frac{n_1}{n_2}=\frac{W_{\mathrm{SO}_2}}{M_{\mathrm{SO}_2}} \times \frac{M_{\mathrm{CH}_4}}{W_{\mathrm{CH}_4}}$$

$$=\frac{2}{64} \times \frac{16}{1}$$

$$\frac{n_1}{n_2}=\frac{1}{2}, \text{ i.e. } 1: 2$$

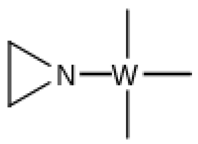
Also $n \propto N$
$\therefore$ Ratio of number of molecules is $1: 2$.

**47. (a)**

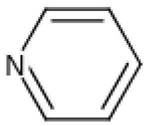
$, {}_{9}\mathrm{F}^{18} \longrightarrow {}_{x}\mathrm{E}^{y}+{}_{+1}e^{0}$
$\therefore \quad x=8, y=18,$
$\therefore \quad x_x \mathrm{E}^y={}_{8}\mathrm{O}^{18}$
$\because$ Position has one unit of the charge and zero mass.
Thus, $\mathrm{F}^{18}$ changes to $\mathrm{O}^{18}$ therefore resulting decay product is $\mathrm{C}_6\mathrm{H}_5\mathrm{O}^{18}$.

**48. (d)** $\because$ On dissolving NaCN in de-ionised water, following reaction takes place.
$\mathrm{CN}^{-}+\mathrm{H}_2\mathrm{O} \rightleftharpoons \mathrm{HCN}+\mathrm{OH}^{-}$
$\because \mathrm{CN}^{-}$ions come from the salt of strong base and weak acid.
$\therefore$ The solution becomes basic and $\mathrm{pH}>7$.

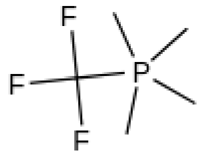
**49. (a)** $\because$ In $\mathrm{Me}_3$ N $\rightarrow$ N is $sp^3$-hybrid



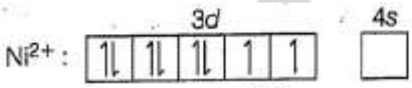

In $\mathrm{C}_5\mathrm{H}_5$ N $\rightarrow$ N is $sp^2$-hybrid

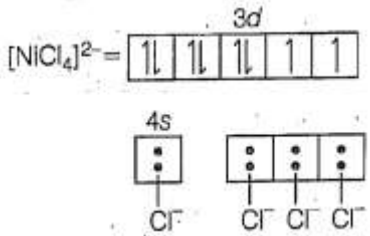



and in MeCN → N is $sp$-hybrid
Also electronegativity of any element $\alpha$%
$s$-character in hybrid orbitals.
Thus, order of electronegativity for nitrogen is
$MeCN > C_5H_5\ N < Me_3\ N$
**50. (c)** Shape of $XeF_5^-$ →

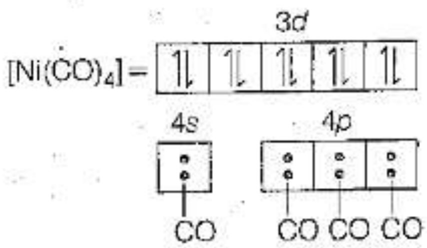



The Xe show $sp^3d^3$-hybridisation in $XeF_5^-$,
Thus, its geometry is
Pentagonal-bipyramidal.
But, number of electron pair $= \frac{8+5+1}{2} = 7$
Number of bond pair $= 5^2$, number of lone pair $= 2$
A lot of electrons are present at axil position and all bonds are in same plane. Hence, the shape is planar.
**51. (a)** According to molecular orbital theory, outer configuration for
$[C_2] = C_{12} \rightarrow \pi p_x^2 = \pi p_y^2$
Thus, is diamagnetic in nature.
$[B_2] = B_{10} -> \pi p_x^1 = \pi p_y^1$
Thus, is paramagnetic in nature.
**52. (b)** Configuration for metal $(M)$ in various complex.

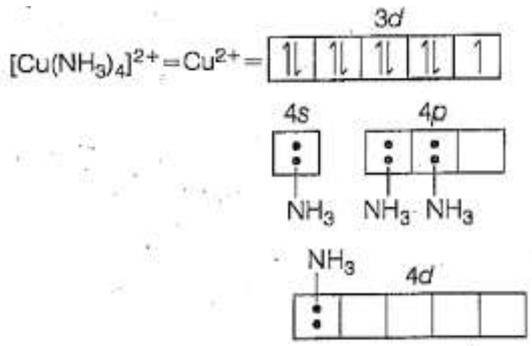



has 2 unpaired electrons




Ni° has zero unpaired electrons with $sp^3$-hybridisation.




$Cu^{2+}$ has one unpaired electron with $dsp^2$-hybridisation.
∴ (b) is the correct answer, i.e.
Number of unpaired electron (2,0,1)

**53. (a)** ∵ F-atom has 2 nd highest electron affinity (Cl has highest) in the periodic table
∴ Among the given options
F -has highest electron gain affinity

**54. (a)** $PbCl_2$ react with HCl as follows

$PbCl_2(s) + Cl^- \xrightarrow{\text{Cold}} [PbCl_3]^-(aq)$

$PbCl_2(s) + 2Cl^- \underset{\text{of HCl}}{\xrightarrow{\text{Excess}}} [PbCl_4]^{2-}(aq)$

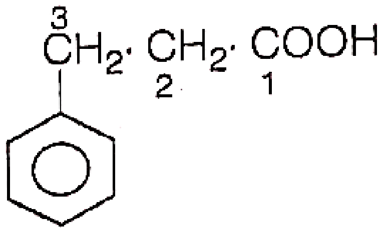
Thus, addition of excess of $Cl^-$ions change the $PbCl_2$ as soluble complex of $[PbCl_4]^{-2}$. Hence, becomes soluble.

**55. (a)**
Higher be the oxidation number of central atom in oxo-acids, more strongly it behave as Bronsted-acid.
Oxidation number for Cl in $HClO_3 \rightarrow +5$
Oxidation number for Cl in $HClO_2 \rightarrow +3$
Oxidation number for Cl in $HClO \rightarrow +1$
Oxidation number for Cl in $HBrO \rightarrow +1$
Hence, $HClO_3$ is the strongest Bronsted acid in aqueous solution.

**56. (d)**
As size of cation decreases, ionic character decreases, thus basicity also decreases
Hence, correct order is
$La^{3+} > Ce^{3+} > Eu^{3+} > Lu^{3+}$

**57. (d)** ∵ If $CO_3^{2-}$, $SO_3^{2-}$ and $SO_4^{2-}$ are present alongwith $BaCl_2$, these can also show white precipitate (as precipitate of all these are also white).

**58. (a)**
IUPAC name of $Ph \cdot CH_2 \cdot CH_2CO_2H$ is, 3-phenylpropanoic acid

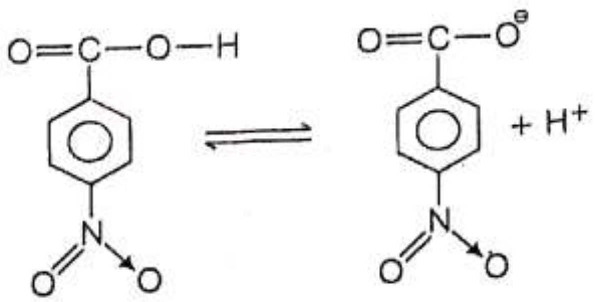



**59. (d)**
Isomerisation of 1-butyne to 2-butyne can be achived by treatment with ethanolic KOH

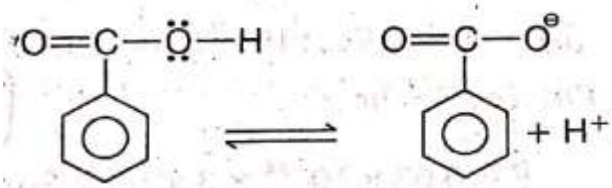
$H_3C - CH_2 - C \equiv CH \underset{KOH(ethanno}{\xrightarrow{isomeris}}$

1-butyne
$CH_3 - C \equiv C - CH_3$
2-butyne

**60. (c)** ∵ In p-nitrobenzoic acid (Z) $- NO_2$ present as electron withdrawing group, thus is most acidic

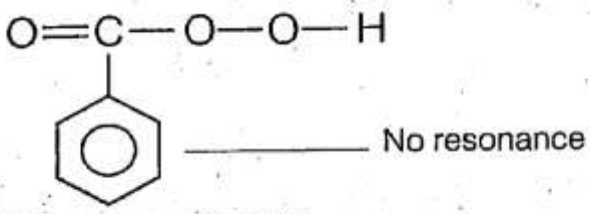


$(X)$ ∵ Benzoic acid is stabilised via resonance, thus is more acidic than per oxybenzoic acid.



Benzoic acid

$(Y)$ peroxy benzoic acid does not show any such resonance.

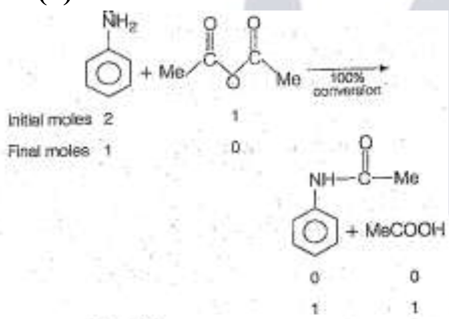


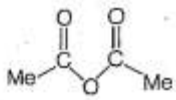

(Peroxy benzoic acid)

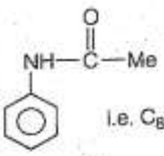
(Peroxy benzoic acid)

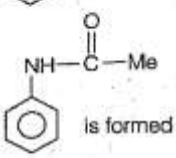
Thus is least acidic.
Hence, correct order is $Z > X > Y$.

**61. (b)** The reaction between aniline and acetic anhydride is as follows




As  is limiting reagent

M. nt of  i.e. $C_8H_9ON$ is 135 g/mol

As 1 mole of  is formed

**62. (c)**

**63. (a)**

ADP is adenosine diphosphate (2-phosphate groups) and ATP is adenosine triphosphate (3 phosphate groups)
Thus, they differ in number of phosphate units.

**64. (c)**

When primary amine react with chloroform and ethanolic potassium hydroxide, they produce -isocyamides as main product.
The reaction is also known as carbyl amine reaction.

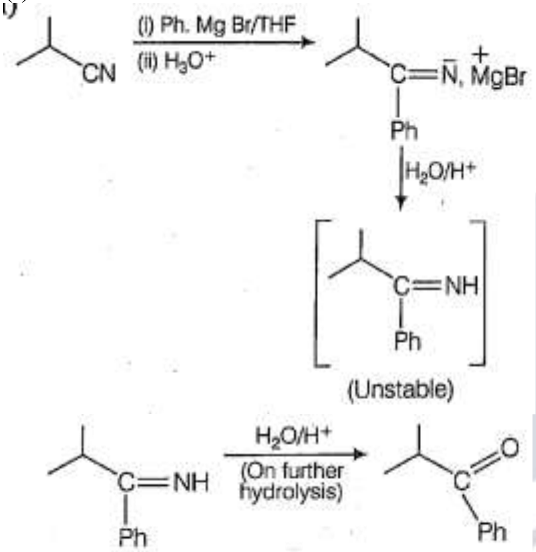
$$Ph - NH_2 \xrightarrow[\text{KOH (ethanolic)}]{CHCl_3} Ph \cdot N \longrightarrow C$$

(Nauseating smell)

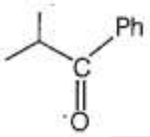
**65. (b)**

The given reaction occurs as follows :

(i)




(ii)




**66. (c)** As per given relation,

$N_1V_1 + N_2V_2 = N(V_1 + V_2)$

$N_1 = 2N_2 = 5$

$V_1 = 500V_2 =$ say- $x$

Thus,

$2 \times 500 + 5x = 3(x + 500)$ or $2x = 500$

i.e. $x = 250$

Hence, maximum volume of

$3\text{MHCl} = 500 + 250 = 750$ mL

**67. (a)**

$\because E = hv = \frac{hc}{\lambda} = hc \cdot \bar{v} \left(\because \frac{1}{\lambda} = \bar{v}\right)$

where $E =$ energy of photon

$c =$ velocity of photon ( $=$ light)

$\lambda =$ wavelength of photon

$h =$ plank's constant.

$\therefore$ For (a)

$E = 6.63 \times 10^{-34} \times 3 \times 10^8 / 300 \times 10^{-9}$

$E = 1.98 \times 10^{-25} \cdot \text{ J}/300 \times 10^{-9}$

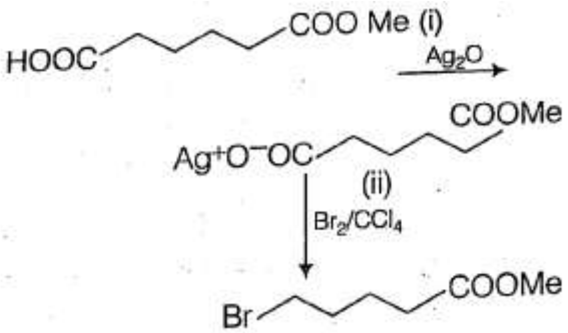
$\therefore E = \dfrac{1.98 \times 10^{-25} \text{ J}}{300 \times 10^{-9} \text{ m}}$

(a) $\rightarrow E = 6.6 \times 10^{-19}$ J
For (b) $E = hv = 6.63 \times 10^{-34} \times 3 \times 10^{8}$
(b) $\rightarrow E = 1.98 \times 10^{-25}$ J
For (c) $E = hc \times \bar{v}$
$\left(\because \frac{1}{\lambda} = \bar{v}\right)$
$E = 6.63 \times 10^{-34} \times 3 \times 10^{8} \times 30 \times 10^{-2}$
(c) $\Rightarrow E = 5.96 \times 10^{-26}$
For (d) d $\rightarrow E = 6.62 \times 10^{-27}$ J
Hence, highest energy for photon is in (a).
**68. (c)** $\because$ For Isotonic solution
$\pi_1 = \pi_2$ i.e. $i_1 C_1 RT = i_2 C_2 RT$
For option (c)
Thus, $i_1 \times C_1 = i_2 \times C_2$
Hence, are isotonic, i.e.
$0.06 = 0.06$
In all other options, $i_1 \times C_1 \neq i_2 \times C_2$
Thus, are not isotonic.
**69. (d)** $Cr_2O_7^{2-} \longrightarrow Cr^{3+}$ Change in +3
Oxi no. $= +6$
Total change in number of electrons
$= 2 \times 3 = 6$ mole $= 6$ F
**70. (d)** For the given reactions,
$2H_2O \rightleftharpoons 2H_2 + O_2, K_1 = 6.4 \times 10^{-8}$
or, $H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2, K_1' = \sqrt{K_1}$
or $H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O, K_1'' = 1/\sqrt{K_1}$ ... (i)
$2CO_2 \rightleftharpoons 2CO + O_2, K_2 = 1.6 \times 10^{-6}$
or $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2, K_2' = \sqrt{K_2}$...(ii)
Form (i) and (ii) [on adding]

$$K = \frac{\sqrt{K_2}}{\sqrt{K_1}} = \frac{\sqrt{1.6 \times 10^{-6}}}{\sqrt{6.4 \times 10^{-8}}}$$

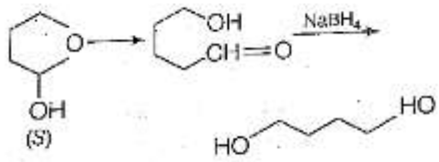
$$K = \sqrt{\frac{10^2}{4}} = \sqrt{25} = 5$$

**71. (d)** $\because$ For be packing,
$\sqrt{3}a = 4r \therefore r = \frac{\sqrt{3}}{4} \cdot a$
where $a =$ edge length
$r =$ radius of lattice sphere
**72. (b)** When drop of line juice is added to mixture of $I^- + IO_3^-$. The following reaction take place and violet colour appears due to formation of $I_2$
$I^- + IO_3^- + H^+ \longrightarrow I_2 + H_2O$
**73. (d)** The reaction occurs as follows :



The above reaction is an example of Borodine Hunsdiecker reaction.

**74.** **(c)** The reaction occurs as follows :



**75.** **(b)** The reaction for given reaction is

$HCl + NaOH \longrightarrow NaCl + H_2O$

Number of moles $xy$ $xy$ 0 0

Added $NV$ $NV$

Number of moles 0 0 $xy$ $xy$

left

$\therefore N(\text{ solution }) = \frac{\text{Number of milliequivalent}}{\text{vol. of solution in mililetre}}$

**76.** **(b,c,d)** During electroysis of molten NaCl , when water is added, the following statements are true

(a) false

(b) $\because$ $Na + H_2O \longrightarrow NaOH + \frac{1}{2}H_2$

and discharge potential of hydrogen is less than of sodium. So, $H_2$ will evolve.

(c) Some amount of caustic soda (NaOH) will be formed

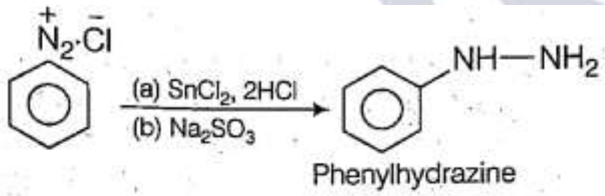
[As shown in (b)]

(d) Fire can likely take place because relations given given in (b) is highly exothermic.

$\therefore$ (b), (c) and (d) are correct.

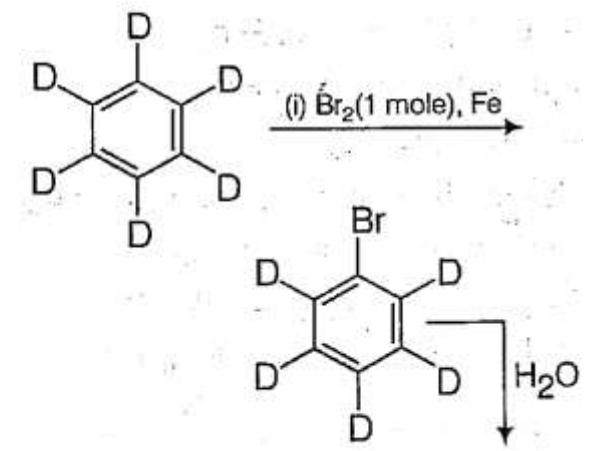
**77.** **(b,c)** Among the given statements (b) and (c) are correct these are the facts.

**78.** **(a, b)** Reduction of benzene diazoniums chloride occurs as follows:



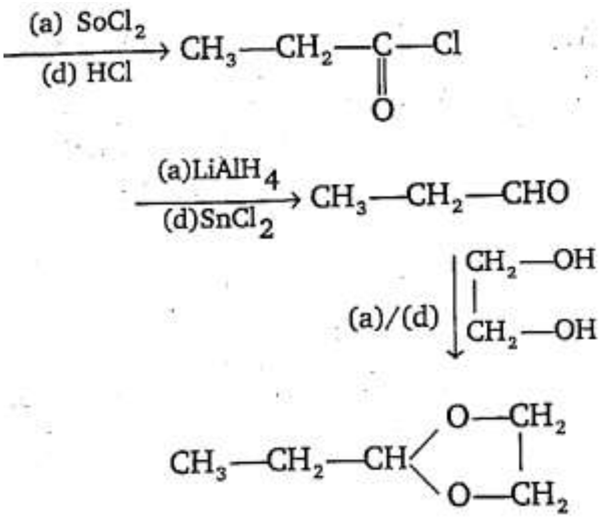
Hence, (a) and (b) are the correct answers.

**79.** **(a)** The reaction for 1 mole of hexadeuterisobenzene occurs as follows:




$H_2O$ is added to remove insoluble bromodueterobenzene as from the solution

**80. (a,d)** $CH_3 - CH_2 - COOH$




Hence, (a) and (d) are correct answers.

## MATHEMATICS

**81. (b)** Total number of 5 -digit numbers having all the digits distinct $= {}^{10}P_5 - {}^{9}P_4$\

$= \frac{10!}{5!} - \frac{9!}{5!} = \frac{10 \times 9!}{5!} - \frac{9!}{5!}$

$= \frac{9!}{5!}(10-1) = \frac{9!}{(9-4)!}(9)$

$= 9 \times {}^{9}P_4$

**82. (b)** $(p+1)(p+2)(p+3)\ldots(p+q)$ is the product of $q$ consecutive natural number $(p, q \in N)$. The product of $q$ consecutive natural number is always divisible by $q!$

**83. (b)** $(1+x+x^2)^9 = a_0 + a_1x + a_2x^2 + \ldots + a_{18}x^{18}$

Put $x = -1$, we get

$(1-1+1)^9 = a_0 - a_1 + a_2 + \cdots + a_{18} \ldots (i)$

$\Rightarrow 1 = a_0 - a_1 + a_2 + \text{v}..+a_{18}$(i)

Put $x = 1$, we get

$(1+1+1)^9 = a_0 + a_1 + a_2 + \cdots + a_{18}$

$\Rightarrow 3^9 = a_0 + a_1 + a_2 + \cdots + a_{18}$(ii)

On adding Eqs. (i) and (ii), we get

$3^9 + 1 = 2(a_0 + a_2 + \cdots + a_{18})$

$\Rightarrow a_0 + a_2 + a_4 + \cdots + a_{18} = \frac{3^9 + 1}{2}$

$= \frac{19683 + 1}{2}$

$= \frac{19684}{2}$

$= 9842$, which is even number.

**84. (d)** We have, $8x - 3y - 5z = 0$,

$5x - 8y + 3z = 0$

$3x + 5y - 8z = 0$

$\therefore D = \begin{vmatrix} 8 & -3 & -5 \\ 5 & -8 & 3 \\ 3 & 5 & -8 \end{vmatrix}$

$= 8(64 - 15) + 3(-40 - 9) - 5(25 + 24)$

$= 8 \times 49 + 3 \times (-49) - 5 \times 49$

$= 0$

$\therefore$ The system has infinitely many non-zero solutions.

**85. (b)** All the orthogonal matrix are non-singular matrix.

$\therefore Q$ is proper subset of $P$.

**86. (b)** We know that,

$\det(AB) = \det(A)\det(B)$

$\therefore \quad \det(AB) = 0$

$\Rightarrow \quad \det(A) \cdot \det(B) = 0$

$\Rightarrow \quad \begin{vmatrix} x+2 & 3x \\ 3 & x+2 \end{vmatrix} \begin{vmatrix} x & 0 \\ 5 & x+2 \end{vmatrix} = 0$

$\Rightarrow \quad \{(x+2)^2 - 9x\}\{x(x+2) - 0\} = 0$

$\Rightarrow \quad (x^2 + 4x + 4 - 9x)x(x+2) = 0$

$\Rightarrow \quad x(x+2)(x^2 - 5x + 4) = 0$

$\Rightarrow \quad x(x+2)(x-1)(x-4) = 0$

$\Rightarrow \quad x = 0, -2, 1, 4$

**87. (a)** We have,

$|A| = \begin{vmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}$

$= 1[1 - (-\cos\theta)(\cos\theta)]$

$-\cos\theta[-\cos\theta + \cos\theta] + 0(\cos^2\theta + 1)$

$= 1 + \cos^2\theta$

Now, we know that

$-1 \leq \cos\theta \leq 1$

$\Rightarrow \quad 0 \leq \cos^2\theta \leq 1$

$\Rightarrow \quad 1 \leq 1 + \cos^2\theta \leq 2$

$\Rightarrow \quad 1 \leq |A| \leq 2$

$\therefore \quad |A| \in [1,2]$

**88. (a)** Since, $f: R \to R$ is injective and $f(x)f(y) = f(x+y), \forall x, y \in R$.

$\therefore f(x) = a^x$

Again, $f(x), f(y), f(z)$ are in GP.

$\Rightarrow \quad (f(y))^2 = f(x) \cdot f(y)$

$\Rightarrow \quad a^{2y} = a^x \cdot a^z$

$\Rightarrow \quad a^{2y} = a^{x+x}$

$\Rightarrow \quad 2y$

$\Rightarrow \quad x + z$

$\therefore x, y, z$ are in AP.

**89. (d)** For every real number $x, x^2 \geq 0$

$\therefore\ (x, x) \in P$
Hence, $P$ is reflexive.
Now, let $(x, y) \in P$
$\Rightarrow\ xy \geq 0$
$\Rightarrow\ yx \geq 0$
$\Rightarrow\ (y, x) \in P$
Hence, $P$ is symmetric.
Again, $(-1,0) \in P$ and $(0,2) \in P$.
But $(-1,2) \notin P$
as $(-1)(2) = -2 < 0$
$\therefore$ $P$ is not transitive.

**90. (b)**

We have, $x\rho y \Rightarrow x - y$ is zero or irrational.
Now, $x - x = 0$
$\Rightarrow\ (x, x) \in \rho$
$\therefore$ $\rho$ is reflexive.
Again, if $x - y$ is either zero or irrational, then $y - x$ will also be either zero or irrational.
$\Rightarrow\ (x, y) \in \rho$
$\Rightarrow\ (y, x) \in \rho$
$\therefore$ $\rho$ is symmetric.
Again, $(2, \sqrt{3}) \in \rho$ and $(\sqrt{3}, 4) \in \rho$
But $(2,4) \notin \rho$
$\therefore$ $\rho$ is not transitive.

**91. (d)** We have,

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_q + \cdots + x_n}{n}$$

$\Rightarrow \qquad \Sigma x \qquad = n\bar{x}$(i)

If $x_q$ is replaced by $x_q'$, then new total will be $\Sigma x' = \Sigma x - x_q - x_q'$
$\therefore$ New mean will be

$$\bar{x}' = \frac{\Sigma x'}{n}$$

$$\bar{x}' = \frac{\Sigma x - x_q + x_q'}{n}$$

$\bar{x}' = \frac{n\bar{x} - x_q + x_q'}{n}$ [from Eq…(i)_

**92. (b)** In a non-leap year, total number of days is 365. Out of them, there are 52 weeks and 1 day extra. Thus, à non-leap year always has 52 Sunday. The remaining 1 day can be
Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday.
Out of these 7 cases, we have Sunday in one case.
$\therefore$ Total number of outcomes = 7
Number of favourable outcomes = 1
Hence, required probability $= \frac{1}{7}$

**93. (d)** Let, $f(x) = \sin x(\sin x + \cos x)$

$$= \sin^2 x + \sin x \cos x$$

$$= \frac{1 - \cos 2x}{2} + \frac{2\sin x\cos x}{2}$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2x + \frac{1}{2}\sin 2x$$

$$= \frac{1}{2} + \frac{1}{2}(\sin 2x - \cos 2x)$$

Now, $-\sqrt{(1)^2 + (1)^2} \leq \sin 2x - \cos 2x$

$$\leq \sqrt{(1)^2+(1)^2}$$

$$\left[\because \sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}\right]$$

$$\Rightarrow -\sqrt{2} \leq \sin 2x - \cos 2x \leq \sqrt{2}$$

$$\Rightarrow -\frac{\sqrt{2}}{2} \leq \frac{\sin 2x - \cos 2x}{2} \leq \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{1}{2} - \frac{\sqrt{2}}{2} \leq \frac{1}{2} + \frac{\sin 2x - \cos 2x}{2} \leq \frac{1}{2} + \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{1-\sqrt{2}}{2} \leq f(x) \leq \frac{1+\sqrt{2}}{2}$$

$$\therefore \frac{1-\sqrt{2}}{2} \leq k \leq \frac{1+\sqrt{2}}{2}$$

**94. (a)** We have;

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}.$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2}\cdot\frac{x+1}{x+2}}\right] = \frac{\pi}{4}$$

$$\Rightarrow \frac{(x-1)(x+2)+(x-2)(x+1)}{(x-2)(x+2)-(x-1)(x+1)} = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1} = 1$$

$$\Rightarrow \frac{2x^2-4}{-3} = 1$$

$$\Rightarrow 2x^2 - 4 = -3$$

$$\Rightarrow 2x^2 = 1$$

$$\Rightarrow x^2 = \frac{1}{2}$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

**95. (b)** Given equations are

$$2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0 \ldots\ldots(\text{i})$$

$$2x^2 + 3xy + 4y^2 + 1 = 0 \ldots\ldots\ldots(\text{ii})$$

Let the origin be transferred to $(p, q)$ axes being parallel to the previous axes; then the equation (i) becomes.

$$2(x'+p)^2 + 3(x'+p)(y'+q)$$

$$+4(y'+q)^2 + (x'+p) + 18(y'+q) + 25 = 0$$

$$\Rightarrow 2x'^2 + 2p^2 + 4x'p + 3x'y' + 3x'q + 3py'$$

$$+3pq + 4y'^2 + 4q^2 + 8y'q + x' + p$$

$$+18y' + 18q + 25 = 0$$

$$\Rightarrow 2x'^2 + 4y'^2 + 3x'y' + (4p+3q+1)x'$$

$$1 + (3p+8q+18)y' + 2p^2 + 3pq$$

$$+4q^2 + p + 25 = 0$$

From Eq. (ii) coefficient of $x'$ and $y'$ must be zero.

$$\therefore\ 4p + 3q + 1 = 0 \ldots(\text{iii})$$

$$3p + 8q + 18 = 0 \ldots(\text{iv})$$

By solving Eqs. (iii) and (iv), we get

$p = 2, q = -3$

**96. (a)** Let the coordinates of $C$ be $(\alpha, \beta)$.

$\therefore$ Coordinates of centroid

$$= \left(\frac{2-2+\alpha}{3}, \frac{-3+1+\beta}{3}\right)$$
$$= \left(\frac{\alpha}{3}, \frac{\beta-2}{3}\right)$$

Since, centroid lie on $2x + 3y = 1$

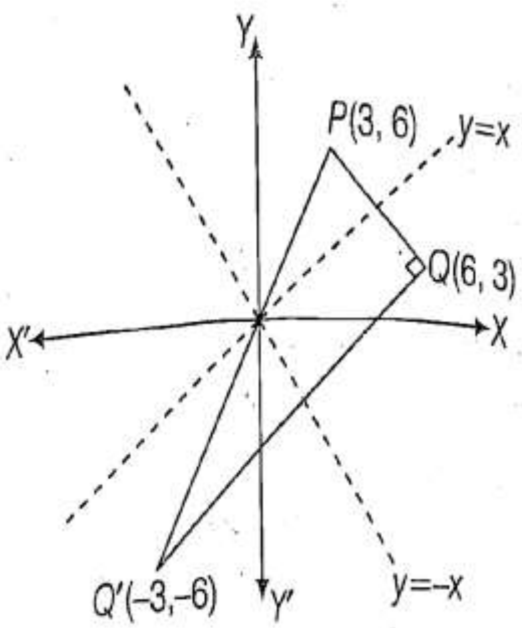
$\therefore \quad \frac{2\alpha}{3} + 3\left(\frac{\beta-2}{3}\right) = 1$

$\Rightarrow \quad \frac{2\alpha}{3} + \frac{3\beta-6}{3} = 1$

$\Rightarrow \quad 2\alpha + 3\beta - 6 = 3 \Rightarrow 2\alpha + 3\beta = 9$

$\therefore$ Locus of point $C$ will be $2x + 3y = 9$.

**97. (d)**




The coordinates of $Q$ and $Q'$ will be $(6,3)$ and $(-3,-6)$, respectively.

Now, Slope of $PQ = \frac{6-3}{3-6} = -1$

and Slope of $QQ' = \frac{-6-3}{-3-6} = 1$

$\therefore$ Slope of $PQ \times$ Slope of $QQ' = -1 \times 1 = -1$

$\therefore \triangle\, PQQ'$ is right angled triangle at $Q$.

$\therefore$ Circumcentre will be the mid-point of

hypotenuse $PQ' = \left(\frac{-3+3}{2}, \frac{-6+6}{2}\right)$

$= (0,0)$

**98. (b)** Let $(h, k)$ be any point on the line $7x - 9y + 10 = 0$, then $7h - 9k + 10 = 0$

$\Rightarrow \; 7h = 9k - 10$

$\Rightarrow \; h = \frac{9k-10}{7} \ldots\ldots\ldots..(i)$

Now, perpendicular distance from point $(h, k)$ to the line $3x + 4y = 5$ is $d_1$

$$d_1 = \frac{3h + 4k - 5}{\sqrt{3^2 + 4^2}}$$

$\Rightarrow \; d_1 = \frac{3h+4k-5}{5}$(ii)

and perpendicular distance from $(h, k)$ to the line $12x + 5y = 7$ is $d_2$

$\therefore \quad d_2 = \frac{12h+5k-7}{\sqrt{12^2+5^2}}$........(iii)

$\Rightarrow \quad d_2 = \frac{12h+5k-7}{13}$

Now, $d_1 - d_2 = \frac{3h+4k-5}{5} - \frac{12h+5k-7}{13}$

$\Rightarrow d_1 - d_2$

$= \frac{13(3h+4k-5) - 5(12h+5k-7)}{65}$

$= \frac{39h+52k-65-60h-25k+35}{65}$

$= \frac{-21h+27k-30}{65}$

$= \frac{-21\left(\frac{9k-10}{7}\right)+27k-30}{65}$

[from Eq.(i)]

$= \frac{-27k+30+27k-30}{65} = 0$

$\Rightarrow d_1 - d_2 = 0 \Rightarrow d_1 = d_2$

**99. (d)** Given, equation of circles are

$x^2 + y^2 - 4x - 4y = 0$ and $2x^2 + 2y^2 = 32$

or $x^2 + y^2 - 4x - 4y = 0$

and

$x^2 + y^2 = 16$

$\therefore$ Equation of common chord is

$(x^2 + y^2 - 4x - 4y) - (x^2 + y^2 - 16) = 0$

$\Rightarrow\ -4x - 4y + 16 = 0$

$\Rightarrow\ x + y = 4$

This common chord passes through (2,2),.

i.e. centre of first circle.

Also, (0,0) is at the circumference of the first circle.

$\therefore$ Common chord will subtent $\frac{\pi}{2}$ angle at (0,0).

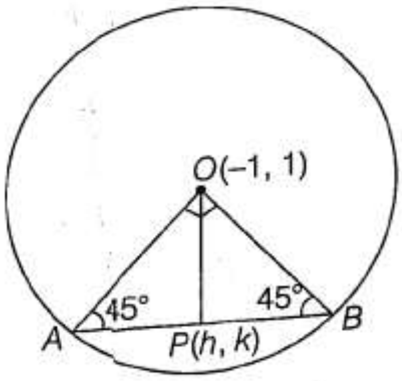
**100.** **(c)** Given, equation of circle is

$x^2 + y^2 + 2x - 2y - 2 = 0$

$\Rightarrow\ (x+1)^2 + (y-1)^2 = 4$

$\therefore$ Centre $(-1,1)$ and radius $= 2$

Let $(h,k)$ be the mid-point of chord.




From figure,

$OP = \sqrt{(h+1)^2 + (k-1)^2}$

In $\triangle\, OAP$,

$\sin 45^\circ = \frac{OP}{OA}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{(h+1)^2 + (k-1)^2}}{2}$

On squaring both sides, we get

$(h+1)^2 + (k-1)^2 = 2$

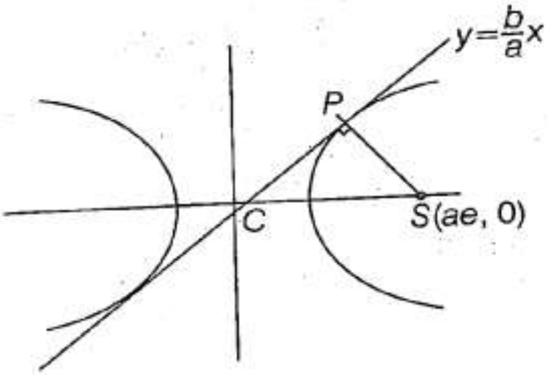
$\Rightarrow h^2 + k^2 + 2h - 2k = 0$

$\therefore$ Locus of $P$ will be

$x^2 + y^2 + 2x - 2y = 0$

**101.** **(b)** Given, equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$




From figure,

$$SP = \left|\frac{abe}{\sqrt{b^2 + a^2}}\right|$$

$$= \left|\frac{abe}{ae}\right| = b$$

and $CS = ae$

Again, $\triangle SPC$ is right angled triangle at $P$.

$$\therefore\ CP = \sqrt{CS^2 - SP^2}$$

$$= \sqrt{a^2e^2 - b^2}$$

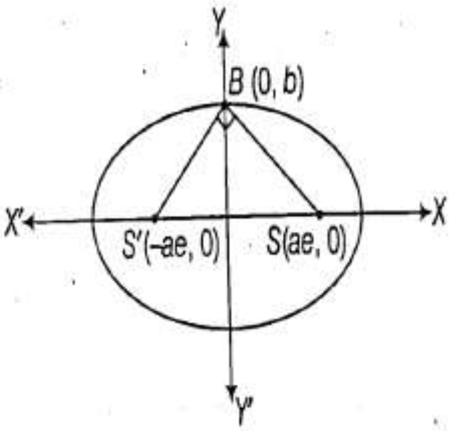
$$= \sqrt{a^2\left(1 + \frac{b^2}{a^2}\right) - b^2}$$

$$= \sqrt{a^2 + b^2 - b^2} = a$$

$\therefore$ Area of rectangle $= CP \times SP$

$= ab$

**102.** **(b)**




Slope of $SB, m_1 = \frac{b-0}{0-ae} = -\frac{b}{ae}$

and slope of $S'B, m_2 = \frac{b-0}{0-(-ae)} = \frac{b}{ae}$

Since, $\angle SBS'$ is a right angle.

$\therefore \quad m_1 m_2 = -1$

$\Rightarrow \quad \frac{-b}{ae} \times \frac{b}{ae} = -1$

$\Rightarrow \quad b^2 = a^2 e^2$

$\Rightarrow \quad \frac{b^2}{a^2} = e^2$

$\Rightarrow \quad 1 - e^2 = e^2$

$\Rightarrow \quad 2e^2 = 1$

$\Rightarrow \quad e^2 = \frac{1}{2}$

$\Rightarrow \quad e = \pm \frac{1}{\sqrt{2}}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$ or $e = -\frac{1}{\sqrt{2}}$

**103.** **(a)** Given, equation

$x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$

$\Rightarrow (x + y)^2 = 5x - 5y + 5$

$\Rightarrow (x + y)^2 = 5(x - y + 1)$

$\therefore$ Axis of the parabola is $x + y = 0$

**104.** **(c)** Given, equation of hyperbola is

$x^2 - y^2 + 1 = 0$

$\Rightarrow y^2 - x^2 = 1$

On comparing it with $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, we get

$a = b = 1$

Now, $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{1}{1}} = \sqrt{2}$

$\therefore$ Foci $= (0, \pm \text{be}) = (0, \pm\sqrt{2})$

Since, line joining foci of hyperbola is diameter of circle.

$\therefore$ Centre of circle $= \left(\frac{0+0}{2}, \frac{\sqrt{2}-\sqrt{2}}{2}\right) = (0,0)$

and radius $= \frac{1}{2}\sqrt{(0-0)^2 + (\sqrt{2} - (-\sqrt{2}))^2}$

$= \frac{1}{2}\sqrt{(2\sqrt{2})^2} = \sqrt{2}$

$\therefore$ Equation of circle will be

$$(x - 0)^2 + (y - 0)^2 = (\sqrt{2})^2$$

$\Rightarrow \quad x^2 + y^2 = 2$

**105.** **(d)**

Equation of the plane will be

$$\begin{vmatrix} x-1 & y-2 & z+3 \\ 2-1 & -2-2 & 1-(-3) \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z+3 \\ 1 & -4 & 4 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow \; 1[4(y - 2) + 4(z + 3)] = 0$

[Expanding along $R_3$ ]

$\Rightarrow \quad 4y - 8 + 4z + 12 = 0$

$\Rightarrow \quad 4y + 4z + 4 = 0$

$\Rightarrow \quad y + z + 1 = 0$

**106.** **(b)** $\Delta = \begin{vmatrix} 1 & -1 & 1 \\ 2 & -3 & 0 \\ 1 & 0 & 3 \end{vmatrix}$

$= 1(-9-0)-(-1)\cdot(6-0)+1(0-(-3))$
$= -9+6+3=0$
Since, $\Delta = 0$
∴ Lines are coplanar.

**107.** **(a)** We have,

$$\frac{f(x)}{f(y)} = f(x-y)$$

$$\Rightarrow \quad f(x) = a^{kx}$$

$$\therefore \quad f'(x) = ka^{kx}\log a$$

Again, $f'(0) = P$

$$\Rightarrow \quad ka^0 \log a = P$$

$$\Rightarrow \quad k\log a = P$$

Also, $f'(5)$

$$\Rightarrow \quad ka^{5k}\log a = q$$

$$\Rightarrow \quad a^{5k}P = q$$

$$\Rightarrow \quad a^{5k} = \frac{q}{P}$$

Now, $f'(-5) = ka^{-5k}\log a$

$$= \frac{k\log a}{a^{5k}}$$

$$= \frac{p}{\left(\frac{q}{p}\right)} = \frac{p^2}{q}$$

**108.** **(c)** We have,

$$f(x) = \frac{\log_5 \log_3 x}{\log 5}$$

$$= \frac{\log\log_3 x}{\log 5\left(\frac{\log x}{\log 3}\right)}$$

$$= \frac{\log 5}{\log x - \log\log 3}$$

$$= \frac{\log 5}{\log 5}\cdot\frac{1}{\log x}$$

$$\therefore \ f'(x) = \frac{1}{\log 5\cdot\log e}$$

$$\therefore \ f'(e) = \frac{1}{e\log 5} \ [\because \log e = 1]$$

$$= \frac{1}{e\log 5}$$

$$= \frac{1}{e\log 5}.$$

**109.** **(c)** We have, $F(x) = e^x, G(x) = e^{-x}$

$\therefore H(x) = G(F(x))$
$= G(e^x)$
$= e^{-e^x}$
$\therefore \frac{dH}{dx} = e^{-e^x} \cdot (-e^x)$
$= -e^x e^{-e^x}$
$\therefore \left.\frac{dH}{dx}\right|_{x=0} = -e^0 \cdot e^{-e^0}$
$= -1 \cdot e^{-1}$
$= -e^{-1}$
$= \frac{-1}{e}$

**110.** **(c)** $\lim_{x\to0} \frac{2f(x)-3f(2x)+f(4x)}{x^2}$
$= \lim_{x\to0} \frac{2f'(x) - 3f'(2x)\cdot 2 + f'(4x)\cdot 4}{2x}$
$= \lim_{x\to0} \frac{f'(x) - 3f'(2x) + 2f'(4x)}{x}$
$= \lim_{x\to0} \frac{f''(x) - 3f''(2x)\cdot 2 + 2f''(4x)\cdot 4}{1}$
$= \lim_{x\to0} f''(x) - 6f''(2x) + 8f''(4x)$
$= f''(0) - 6f''(0) + 8f''(0)( f"(0) = K)$
$= k - 6k + 8k$
$= 3k$

**111.** **(a)** $y = e^{m\sin^{-1}x}$
$\Rightarrow \frac{dy}{dx} = e^{m\sin^{-1}x} \cdot \frac{m}{\sqrt{1-x^2}}$
$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = me^{m\sin^{-1}x}$
$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = my$.......(i)
$\Rightarrow \sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{1}{2\sqrt{1-x^2}}(-2x)\frac{dy}{dx} = m\frac{dy}{dx}$
$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\sqrt{1-x^2}\frac{dy}{dx}$
$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\cdot(my)$
[From Eq. (i)]
$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$
$\therefore k = m^2$

**112.** **(d)** Given curve,
$y = x^2 + 2ax + b$
$At\ x = \alpha,$
$y = \alpha^2 + 2a\alpha + b$
$and\ atx = \beta,$
$y = \beta^2 + 2a\beta + b$
and at
$\therefore$ Slope of line joining $(\alpha, \alpha^2 + 2a\alpha + b)$ and $(\boldsymbol{\beta}, \boldsymbol{\beta}^2 + 2a\boldsymbol{\beta} + b)$ is
$= \frac{(\alpha^2 + 2a\alpha + b) - (\beta^2 + 2a\beta + b)}{\alpha - \beta}$

$$= \frac{\alpha^2 + 2a\alpha + b - \beta^2 - 2a\beta - b}{\alpha - \beta}$$

$$= \frac{(\alpha^2 - \beta^2) + 2a(\alpha - \beta)}{\alpha - \beta}$$

$$= \frac{(\alpha - \beta)(\alpha + \beta) + 2a(\alpha - \beta)}{\alpha - \beta}$$

$$= \alpha + \beta + 2a$$

Slope of given curve $= \frac{dy}{dx}$

$= 2x + 2a$

Now, according to question, tangent is parallel to the chord. Therefore,

$2x + 2a = \alpha + \beta + 2a$

$$\Rightarrow \ 2x = \alpha + \beta \Rightarrow x = \frac{\alpha + \beta}{2}$$

**113.** **(c)** We have,

$f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19$

$\Rightarrow f'(x) = 13x^{12} + 11x^{10} + 9x^8$
$+7x^6 + 5x^4 + 3x^2 + 1$

$\therefore f'(x)$ has no real root.

$\therefore f(x) = 0$ has not more than one real root.

**114.** **(b)**

Since, Lagrange's mean value theorem is applicable on $f(x)$.

$$\therefore \quad \lim_{x \to 0} \frac{x^p}{(\sin x)^q} = f(0)$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{x^p}{(\sin x)^q} = 0$$

Above equation holds only when $p > q$.

**115.** **(b)**

Let $y = \lim_{x \to 0} (\sin x)^{2\tan x}$

$$\Rightarrow \log y = \lim_{x \to 0} \log(\sin x)^{2\tan x}$$

$$= 2\lim_{x \to 0} \tan x \log \sin x$$

$$= 2\lim_{x \to 0} \frac{\log \sin x}{\cot x}$$

$$= 2\lim_{x \to 0} \frac{\frac{1}{\sin x} \cdot \cos x}{-\mathrm{cosec}^2 x}$$

$$= 2\lim_{x \to 0} (-\sin x \cos x)$$

$$= 2 \times 0 = 0$$

$\therefore \ \log y = 0 \Rightarrow y = e^0 = 1$

**116.** **(c)** Let $I = \int \ \cos(\log x) dx$

Put $\log x = t$

$\Rightarrow x = e^t$

$\therefore dx = e^t dt$

$\therefore I = \int e^t \cos t dt$

$= \frac{e^t}{1^2 + 1^2}[\cos t + \sin t] + C$

$\left[\because \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} [a\cos bx + b\sin bx] + C\right]$

$\Rightarrow I = \frac{e^t}{2}[\cos t + \sin t] + C$

$= \frac{x}{2}[\cos(\log x) + \sin(\log x)] + C$

$\therefore f(x) = \frac{x}{2}[\cos(\log x) + \sin(\log x)]$

**117. (a)**

Let $I = \int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx$

$= \int \frac{1 - 1/x^2}{x^2 + 3 + 1/x^2} dx$

$= \int \frac{1 - 1/x^2}{\left(x^2 + \frac{1}{x^2}\right) + 3} dx$

$= \int \frac{1 - 1/x^2}{\left(x + \frac{1}{x}\right)^2 - 2 + 3} dx$

$= \int \frac{1 - 1/x^2}{\left(x + \frac{1}{x}\right)^2 + 1} dx$

Let $x + \frac{1}{x} = t$

$\Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dt$

$\therefore I = \int \frac{dt}{t^2 + 1}$

$= \tan^{-1} t + C$

$= \tan^{-1}\left(x + \frac{1}{x}\right) + C \left[\because t = x + \frac{1}{x}\right]$

**118. (b)** For $x > 10$, we have

$|\sin x| < 1$ and $1 + x^8 > 10^8$

$\Rightarrow \frac{1}{1 + x^8} \leq 10^{-8}$

$\therefore \left|\int_{10}^{19} \frac{\sin x}{1 + x^8} dx\right| \leq \int_{10}^{19} \frac{\sin x \mid}{1 + x^8} dx$

$\leq \int_{10}^{19} 10^{-8} = 9 \times 10^{-8} < 10^{-7}$

**119. (d)** We have,

$I_1 = \int_0^n [x]dx$

$= \int_0^1 [x]dx + \int_1^2 [x]dx$

$+ \int_2^3 [x]dx + \cdots + \int_{n-1}^n [x]dx$

$= \int_0^1 0dx + \int_2^3 1dx + \int_2^3 2dx$

$= 0 + [x]_1^2 + 2[x]_2^3 + \cdots + \int_{n-1}^n (n-1)dx$

$= (2-1) + 2(3-2) + \cdots +$

$= 1 + 2 + 3 + \cdots + (n-1)[x]_{n-1}^n$

$= \frac{(n-1)(n-1+1)}{2}$

$= \frac{n(n-1)}{2}$

Now, $I_2 = \int_0^n \{x\}dx = \int_0^n x - [x]dx$

$= \int_0^n xdx - \int_0^n [x]$

$= \frac{\frac{1}{2}^2}{2}\Bigg]_0^n - \frac{n(n-1)}{2}$

$= \frac{n^2 - n^2 + n}{2}$

$= \frac{n}{2}$

$\therefore \frac{I_1}{I_2} = \frac{\frac{n(n-1)}{2}}{\frac{n}{2}}$

$= (n-1)$

**120.** **(b)** $\lim_{n\to\infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \cdots + \frac{1}{2n}\right]$

$= \lim_{n\to\infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \cdots + \frac{n}{n^2+n^2}\right]$

$= \lim_{n\to\infty} \sum_{r=1}^{\infty} \frac{n}{n^2+r^2}$

$= \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{\infty} \frac{1}{1+\left(\frac{r}{n}\right)^2}$

$= \int_0^1 \frac{1}{1+x^2}dx$

$= [\tan^{-1} x]_0^1$

$= \tan^{-1} 1 - \tan^{-1} 0$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$

**121.** **(d)** We know that, $0 \le x \le 1$

$\Rightarrow \quad x^2 \le 1$

$\Rightarrow \quad e^{x^2} \le e$

$\Rightarrow \quad \int_0^1 e^{x^2} dx \le \int_0^1 e dx$

$\Rightarrow \quad \int_0^1 e^{x^2} dx \le e$

$\therefore \quad \int_0^1 e^{x^2} \in [1, e]$

**122.** **(c)** Let $I = \int_0^{100} e^{x-[x]} dx$

$= 100 \int_0^1 e^{x-[x]} dx$

[$\because x - [x]$ is a periodic function of period 1 and $\int_0^{mT} f(x) dx = m \int_0^T f(x) dx$, where $T$ is period of $f(x)$]

$= 100 \int_0^1 e^x dx [\because x - [x] = x \text{ for } 0 < x < 1]$

$= 100[e^x]_0^1$

$= 100[e^1 - e^0]$

$= 100(e - 1)$

**123.** **(a)** We have,Let

$$(x + y)^2 \frac{dy}{dx} = a^2$$

$$x + y = v$$

$$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\therefore \quad v^2 \left(\frac{dv}{dx} - 1\right) = a^2$$

$$\Rightarrow \quad v^2 \frac{dv}{dx} = v^2 + a^2$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{v^2 + a^2}{v^2}$$

$$\Rightarrow \quad \frac{v^2}{v^2 + a^2} dv = dx$$

On integrating both sides, we get

$$\int \frac{v^2}{v^2 + a^2} dv = \int dx$$

$$\Rightarrow \int \left(1-\frac{a^2}{v^2+a^2}\right)dv = x + C'$$

$$\Rightarrow v - \frac{a^2}{a}\tan^{-1}\frac{v}{a} = x + C'$$

$$\Rightarrow x + y - a\tan^{-1}\frac{x+y}{a} = x + C'$$

$$\Rightarrow y = a\tan^{-1}\frac{x+y}{a} + C'$$

$$= \frac{y-C'}{a} = \tan^{-1}\frac{x+y}{a}$$

$$\Rightarrow \left(\frac{x+y}{a}\right) = \tan\left(\frac{y+C}{a}\right),$$

where $-C' = C$.

**124.** **(b)** We have,

$$x^2(x^2-1)\frac{dy}{dx} + x(x^2+1)y = x^2-1$$

$$\Rightarrow \frac{dy}{dx} + \frac{x^2+1}{x(x^2-1)}y = \frac{1}{x^2}$$

$$\therefore \text{IF} = e^{\int \frac{x^2+1}{x(x^2-1)}dx}$$

$$= e^{\int \frac{x^2+1}{x(x-1)(x+1)}dx}$$

Let $\frac{x^2+1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$\Rightarrow x^2 + 1 = A(x-1)(x+1) + Bx(x+1)$
$+Cx(x-1)$

Put $x = 0$,

$\therefore \quad 1 = -A$

$\Rightarrow \quad A = -1$

Put $\quad x = 1$,

$\therefore \quad 2 = 2B$

$\Rightarrow \quad B = 1$

Put $\quad x = -1$,

$\therefore \quad 2 = 2C$

$\Rightarrow \quad C = 1$

$$\therefore \frac{x^2+1}{x(x-1)(x+1)} = \frac{-1}{x} + \frac{1}{x-1} + \frac{1}{x+1}$$

$$\therefore \text{IF} = e^{\int\left(\frac{-1}{x}+\frac{1}{x-1}+\frac{1}{x+1}\right)dx}$$

$$= e^{[-\log x + \log(x-1) + \log(x+1)]}$$

$$= e^{\log\left(\frac{x^2-1}{x}\right)} = \frac{x^2-1}{x}$$

$$= x - \frac{1}{x}$$

**125.** **(b)** Let $a_n$ be the general term of a GP whose first term is $a$ and common ratio is $r$.
Now according to the question,

$a_n = a_{n+1} + a_{n+2}$

$ar^{n-1} = ar^n + ar^{n+1}$

$r^{n-1} = r^n + r^{n+1}$

$1 = \frac{r^n}{r^{n-1}} + \frac{r^{n+1}}{r^{n-1}}$

$1 = r + r^2$

$r^2 + r - 1 =$

$$r = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

Since, GP consists only positive terms

$$\therefore\ r = \frac{\sqrt{5} - 1}{2}$$

**126.** **(a)** We have,

$\log_5 x \cdot \log_x 3x \cdot \log_{3x} y = \log_x x^3$

$$\Rightarrow \frac{\log x}{\log 5} \times \frac{\log 3x}{\log x} \times \frac{\log y}{\log 3x} = 3\log_x x$$

$$\left[\because \log_b a = \frac{\log a}{\log b} \text{ and } \log a^m = m\log a\right]$$

$$\Rightarrow \quad \frac{\log y}{\log 5} = 3$$

$$\Rightarrow \quad \log y = 3\log 5$$

$$\Rightarrow \quad \log y = \log 5^3$$

$$\Rightarrow \quad y = 5^3 = 125$$

**127.** **(c)** $\frac{(1+i)^n}{(1-i)^{n-2}} = \frac{(1+i)^n}{(1-i)^n(1-i)^{-2}}$

$$= \left(\frac{1+i}{1-i}\right)^n (1-i)^2$$

$$= \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n (1 + i^2 - 2i)$$

$$= \left(\frac{1+2i+i^2}{1-i^2}\right)^n (1 + i^2 - 2i)$$

$$= \left(\frac{1+2i-1}{1-(-1)}\right)^n (1 - 1 - 2i)$$

$$= \left(\frac{2i}{2}\right)^n (-2i)\ [\because i^2 = -1]$$

$$= i^n(-2i) = -2i^{n+1}$$

**128.** **(d)** $\frac{z+i}{z-i} = \frac{(x+iy)+i}{(x+iy)-i}$

$$= \frac{x + (1+y)i}{x + (y-1)i}$$

$$= \frac{x + (y+1)i}{x + (y-1)i} \times \frac{x - (y-1)i}{x - (y-1)i}$$

$$= \frac{x^2 - x(y-1)i + x(y+1)i - (y+1)(y-1)i^2}{x^2 - (y-1)^2 i^2}$$

$$= \frac{x^2 + i[-xy + x + xy + x] + (y^2 - 1)}{x^2 + (y-1)^2}$$

$$= \frac{x^2 + y^2 - 1}{x^2 + (y-1)^2} + \frac{2x}{x^2 + (y-1)^2} i$$

Now, $\frac{z+i}{z-i}$ is purely imaginary.

$$\therefore\ \text{Re}\left(\frac{z+i}{z-i}\right) = 0$$

$$\Rightarrow \quad \frac{x^2+y^2-1}{x^2+(y-1)^2}=0$$
$$\Rightarrow \quad x^2+y^2=1$$

$\therefore (x,y)$ lies on a circle.

**129.** **(a)** Given equation,

$2px^2+(2p+q)x+q=0$

$\therefore D=(2p+q)^2-4(2p)(q)$

$=4p^2+q^2+4pq-8pq$

$=4p^2+q^2-4pq$

$=(2p-q)^2$

$=$ a perfect square

$\therefore$ Given equation has rational roots

**130.** **(b)** Out of 7 consonants, the number of ways of selecting 3 consonants $= {}^7C_3$

Similarly, number of ways of selecting 2 vowels out of 4 vowels $= {}^4C_2$

$\therefore$ Total number of words formed

$= {}^7C_3 \times {}^4C_2 \times {}^5P_5$

$=\frac{7\times 6\times 5}{3\times 2\times 1}\times\frac{4\times 3}{2\times 1}\times 5!$

$=7\times 5\times 2\times 3\times 120=25200$

**131.** **(b)** We have, $A=\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}$

$A^2=A\cdot A$

$=\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}$

$\therefore \quad =\begin{bmatrix}1+0+0&1+1+0&1+1+1\\0+0+0&0+1+0&0+1+1\\0+0+0&0+0+0&0+0+1\end{bmatrix}$

$=\begin{bmatrix}1&2&3\\0&1&2\\0&0&1\end{bmatrix}=\begin{bmatrix}1&2&\frac{2(2+1)}{2}\\0&1&2\\0&0&1\end{bmatrix}$

Again, $A^3=A^2\cdot A$

$=\begin{bmatrix}1&2&3\\0&1&2\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}$

$=\begin{bmatrix}1+0+0&1+2+0&1+2+3\\0+0+0&0+1+0&0+1+2\\0+0+0&0+0+0&0+0+1\end{bmatrix}$

$=\begin{bmatrix}1&3&6\\0&1&3\\0&0&1\end{bmatrix}=\begin{bmatrix}1&3&\frac{3(3+1)}{2}\\0&1&3\\0&0&1\end{bmatrix}$

$\therefore \quad A^n=\begin{bmatrix}1&n&\frac{n(n+1)}{2}\\0&1&n\\0&0&1\end{bmatrix}$

**132.** **(d)** We have,

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

$$+\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2} & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} a & a+1 \\ -b & b+1 \\ c & c-1 \\ & c+1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^{n}c \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} a(1+(-1)^{n+2}) & a+1 & a-1 \\ b(-1+(-1)^{n+1}) & b+1 & b-1 \\ c(1+(-1)^{n}) & c-1 & c+1 \end{vmatrix}=0$$

$\therefore$ $n$ is any odd integer.

**133.** **(c)** We have,

$R=\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$,

$S=\{(1,1),(2,2),(3,3),(1,3),(3,1)\}$.

$\therefore R\cup S=\{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1)\}$.

Since, $(2,1)\in R\cup S,(1,3)\in R\cup S$ but $(2,3)\notin R\cup S$

$\therefore R\cup S$ is reflexive and symmetric but not transitive

**134.** **(a)** Given circle

$x^2+y^2-4x-6y+9=0$

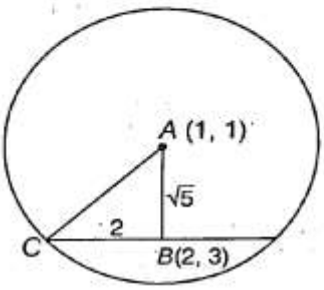
$\therefore$ Centre $=\left(-\frac{1}{2}\times(-4),-\frac{1}{2}\times(-6)\right)$

$=(2,3)$

Radius $=\sqrt{(-2)^2+(-3)^2-9}$

$=\sqrt{4+9-9}$

$=2$



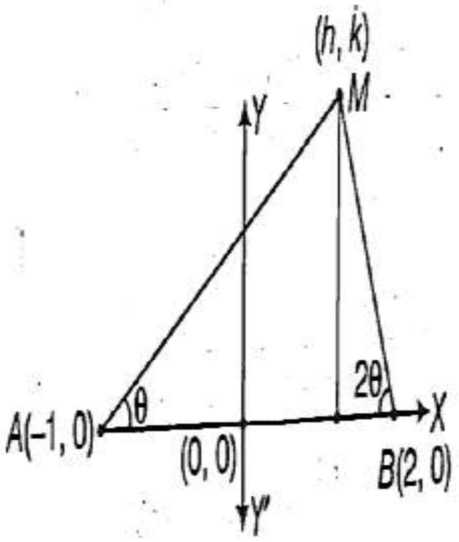

$\therefore\ AB=\sqrt{(2-1)^2+(3-1)^2}=\sqrt{1+4}=\sqrt{5}$

$\therefore AC=\sqrt{AB^2+BC^2}=\sqrt{(\sqrt{5})^2+(2)^2}$

$=\sqrt{5+4}=\sqrt{9}=3$

$\therefore$ Radius of required circle $=3$

**135.** **(d)**




Let $\angle MAB = \theta$, then $\angle MBA = 2\theta$

$\tan\theta = \frac{k}{1+h}$ and $\tan 2\theta = \frac{k}{2-h}$,

then $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{k}{2-h}$

$$\Rightarrow \frac{2(k/1+h)}{1-(k/1+h)^2} = \frac{k}{2-h}$$

$$\Rightarrow \frac{2/(1+h)}{\frac{(1+h)^2-k^2}{(1+h)^2}} = \frac{1}{2-h}$$

$$\Rightarrow \frac{2}{1+h} \times \frac{(1+h)^2}{(1+h)^2-k^2} = \frac{1}{2-h}$$

$$\Rightarrow \frac{2(1+h)}{(1+h)^2-k^2} = \frac{1}{2-h}$$

$\Rightarrow 1 + h^2 + 2h - k^2 = 2(1+h)(2-h)$

$\Rightarrow 1 + h^2 + 2h - k^2 = 2(2 - h + 2h - h^2)$

$\Rightarrow 1 + h^2 + 2h - k^2 = 2(2 + h - h^2)$

$\Rightarrow 1 + h^2 + 2h - k^2 = 4 + 2h - 2h^2$

$\Rightarrow 1 + 3h^2 - k^2 = 4$

$\Rightarrow 3h^2 - k^2 = 3$

which represents hypérbola (d).

**136.** **(c)** We have,

$$f(x) = \int_{-1}^{x} t\,\Big|dt = \int_{-1}^{0}\Big|\,t\,\Big|dt + \int_{0}^{x}\Big|\,t\mid dt$$

$$= -\int_{-1}^{0} tdt + \int_{0}^{x} tdt$$

$$= \left[-\frac{t^2}{2}\right]_{-1}^{0} + \left[\frac{t^2}{2}\right]_{0}^{x}$$

$$= -\frac{1}{2}(0^2 - (-1)^2) + \frac{1}{2}(x^2 - 0^2)$$

$$= -\frac{1}{2}(-1) + \frac{1}{2}(x^2) = \frac{1}{2}(1 + x^2)$$

**137.** **(b)** We have, $f(x) = \lim\limits_{n\to\infty} n\left(x^{1/n} - 1\right)$

$$= \lim_{n\to\infty} \frac{x^{1/n} - 1}{1/n}$$

Let $\frac{1}{n} = y$

$$\therefore\ f(x) = \lim_{y\to 0} \frac{x^y - 1}{y}$$
$$= \log x$$
$$\therefore\ f(xy) = \log(xy)$$
$$= \log x + \log y$$
$$= f(x) + f(y)$$

**138.** **(b)** $I = \int_0^{100\pi} \sqrt{1 - \cos 2x}dx$

$$= \int_0^{100\pi} \sqrt{2\sin^2 x}dx$$
$$= \sqrt{2}\int_0^{100\pi} \sin x \mid dx$$
$$= \sqrt{2} \times 100 \int_0^{\pi} \sin x |dx|$$

[$\sin x$ | has period of $\pi$]

$= 100\sqrt{2}\int_0^{\pi} \sin x dx$
$= 100\sqrt{2}[-\cos x]_0^{\pi}$
$= 100\sqrt{2}[(-\cos\pi) - (-\cos 0)]$
$= 100\sqrt{2}[-(-1) - (-1)]$
$= 100\sqrt{2} \times 2$
$= 200\sqrt{2}$

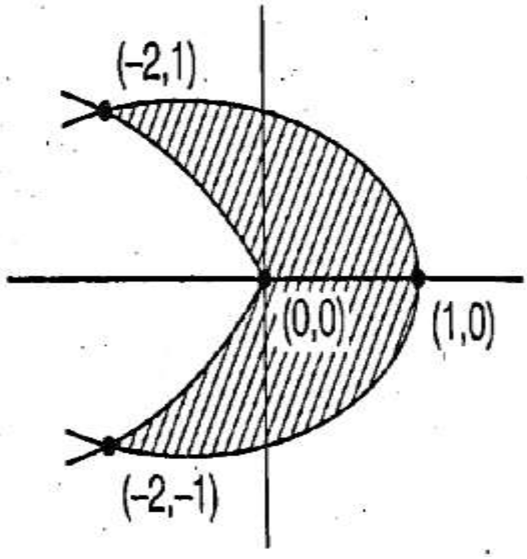
**139.** **(a)** We have,

$$x = -2y^2$$
$$\Rightarrow\quad y^2 = -\frac{1}{2}x$$
and $\quad x = 1 - 3y^2$
$$\Rightarrow\quad y^2 = -\frac{1}{3}(x-1)$$

From Eqs. (i) and (ii), we get

$$-\frac{1}{2}x = -\frac{1}{3}(x-1)$$
$$\Rightarrow\quad 3x = 2(x-1)$$
$$\Rightarrow\quad 3x = 2x - 2$$
$$\Rightarrow\quad x = -2$$
$$\therefore\quad y^2 = -\frac{1}{2}(-1) = 1$$
$$\Rightarrow\quad y = \pm 1$$

$\therefore$ Point of intersection of two curves is $(-2, \pm 1)$




$\therefore$ Required Area $= 2\int_0^1 \sin x dx$

$$= 2\left[y - \frac{y^3}{3}\right]_0^1$$

$$= 2\left[1 - \frac{1}{3}\right]$$

$$= \frac{4}{3} \text{ sq units}$$

**140.** **(a)** We have,

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

$$\Rightarrow a = 3 \text{ and } b = \sqrt{5}$$

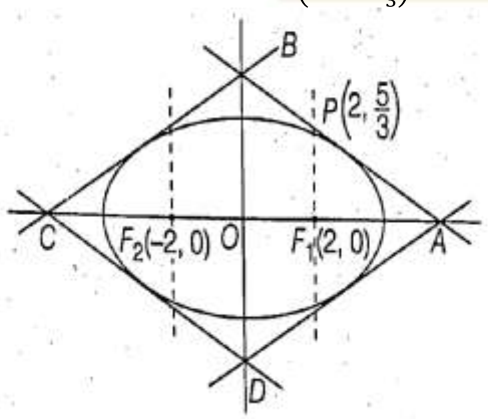
$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$$

$\therefore$ Foci $= (\pm ae, 0)$

$$= \left(\pm 3 \times \frac{2}{3}, 0\right) = (\pm 2, 0)$$

$\therefore$ Ends of latusrectum $= \left(\pm 2, \pm \frac{5}{3}\right)$




Equation of tangent at the end of latusrectum $P$ is

$$\frac{x\times 2}{9}+\frac{y\times 5/3}{5}=1$$

$$\Rightarrow \frac{2}{9}x+\frac{1}{3}y=1$$

$$\Rightarrow 2x+3y=9$$

$$\therefore\ OA=\frac{9}{2} \text{ and } OB=3$$

$\therefore$ Area of quadrilateral $ABCD$

$$= 4\times \text{ Area of } \triangle OAB$$

$$= 4\times\frac{1}{2}\times OA\times OB$$

$$= 4\times\frac{1}{2}\times\frac{9}{2}\times 3$$

$= 27$ sq units.

**141.** **(c)** We have,

$$f(x)=\sin x-\cos x-Kx+5$$

$$\Rightarrow f'(x)=\cos x+\sin x-K$$

For decreasing, $f'(x)<0$

$$\Rightarrow \cos x+\sin x-K<0$$

$$\Rightarrow K>\cos x+\sin x$$

$$\Rightarrow K>\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right]$$

$$\Rightarrow K>\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)$$

$$\therefore K>\sqrt{2}$$

**142.** **(b)** Let $\mathbf{x}=\alpha\hat{\mathbf{i}}+\beta\hat{\mathbf{j}}+\gamma\hat{\mathbf{k}}$

Then, $\mathbf{x}\times\hat{\mathbf{i}}=-\boldsymbol{\beta}\hat{\mathbf{k}}+\boldsymbol{\gamma}\hat{\mathbf{j}}$

$$x\times\hat{\mathbf{j}}=\hat{\mathbf{k}}-\gamma\hat{\mathbf{i}}$$

$$x\times\hat{\mathbf{k}}=-\alpha\hat{\mathbf{j}}+\beta\hat{\mathbf{i}}$$

Now, $(\mathbf{x}\times\hat{\mathbf{i}})^2=(\mathbf{x}\times\hat{\mathbf{i}})\cdot(\mathbf{x}\times\hat{\mathbf{i}})$

$$=(-\beta\hat{\mathbf{k}}+\gamma\hat{\mathbf{j}})\cdot(-\beta\hat{\mathbf{k}}+\gamma\hat{\mathbf{j}})$$

$$=\beta^2+\gamma^2$$

Similarly, $(\mathbf{x}\times\hat{\mathbf{j}})^2=\alpha^2+\gamma^2$.

and $(\mathbf{x}\times\hat{\mathbf{k}})^2=\alpha^2+\beta^2$

$$\therefore (\mathbf{x}\times\hat{\mathbf{i}})^2+(\mathbf{x}\times\hat{\mathbf{j}})^2+(\mathbf{x}\times\mathbf{k})^2$$

$$=\beta^2+\gamma^2+\alpha^2+\gamma^2+\alpha^2+\beta^2$$

$$=2(\alpha^2+\beta^2+\gamma^2)=2|\mathbf{x}|^2$$

**143.** **(c)** Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are two unit vectors.

Then, $|\hat{\mathbf{a}}+\hat{\mathbf{b}}|=1$

$$\Rightarrow |\mathbf{a}+\hat{\mathbf{b}}|^2=1$$

$$\Rightarrow (\hat{\mathbf{a}}+\hat{\mathbf{b}})\cdot(\hat{\mathbf{a}}+\hat{\mathbf{b}})=1$$

$$\Rightarrow |\hat{\mathbf{a}}|^2+|\hat{\mathbf{b}}|^2+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=1$$

$$\Rightarrow 1+1+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=1$$

$$\Rightarrow 1$$

$$\Rightarrow 2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=-1$$

Again, $|\hat{\mathbf{a}}-\hat{\mathbf{b}}|^2=(\hat{\mathbf{a}}-\hat{\mathbf{b}})\cdot(\hat{\mathbf{a}}-\hat{\mathbf{b}})$

$$=|\hat{\mathbf{a}}|^2+|\hat{\mathbf{b}}|^2-2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}$$

$$=1+1-(-1)$$

$$=1+1+1=3$$

$$\therefore |\hat{\mathbf{a}}-\hat{\mathbf{b}}|=\sqrt{3}$$

**144.** **(a)** We have, $x^2+x+1=0$

$\Rightarrow \qquad x = \dfrac{-1 \pm \sqrt{3}i}{2}$

$\Rightarrow \alpha = \dfrac{-1+\sqrt{3}i}{2} \qquad$ and $\beta = \dfrac{-1-\sqrt{3}i}{2}$

or $\qquad \alpha = e^{\frac{i2\pi}{3}}$ and $\beta$

$e^{\frac{-2\pi i}{3}}$

$\therefore \qquad \alpha^n + \beta^n = e^{\frac{2n\pi i}{3}} + e^{\frac{-2n\pi i}{3}}$

$$= 2\left(\frac{e^{\frac{2n\pi i}{3}} + e^{\frac{-2n\pi l}{3}}}{2}\right)$$

$$= 2\cos\left(\frac{2n\pi}{3}\right)$$

**145.** **(c)** Let $y = \frac{x^2+2x+4}{2x^2+4x+9}$

$\Rightarrow 2x^2y + 4xy + 9y = x^2 + 2x + 4$

$\Rightarrow (2y-1)x^2 + (4y-2)x + 9y - 4 = 0$

$\Rightarrow x = \dfrac{-(4y-2) \pm \sqrt{(4y-2)^2 - 4(2y-1)(9y-4)}}{2(2y-1)}$

Since, $x$ is real number.

$\Rightarrow (4y-2)^2 - 4(2y-1)(9y-4 \geq 0$

$\Rightarrow 4(2y-1)^2 - 4(2y-1)(9y-4) \geq 0$

$\Rightarrow 4(2y-1)(2y-1-9y+4) \geq 0$

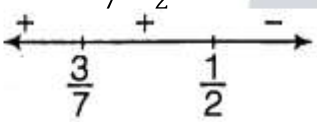
$\Rightarrow \quad 4(2y-1)(3-7y) \geq 0$

$\Rightarrow \quad 4(2y-1)(7y-3) \leq 0$

$\Rightarrow \dfrac{3}{7} \leq y \leq \dfrac{1}{2}$

$+$

$\dfrac{3}{7} + \dfrac{1}{2}$




$\therefore$ Maximum value of $y = \frac{1}{2}$

**146.** **(c, d)** We have,

$ax^2 + bx + 1 = 0$

For real roots, $D \geq 0$

$\therefore \qquad b^2 - 4a \geq 0$

$\Rightarrow \qquad b^2 \geq 4a$

$\therefore \quad (a,b) = (1,2),(1,3),(2,3)$

$\therefore$ Number of ordered pairs $(a,b) = 3$

and $a$ is always less than $b$.

**147.** **(a, c)** Equation of tangent to $y^2 = 4ax$ at $(at^2, 2at)$ will be

$x - yt = -at^2$.........(i)

Also, equation of normal to $x^2 - y^2 = a^2$ at $(a\sec\theta, a\tan\theta)$ will be

$\dfrac{x}{a\sec\theta} + \dfrac{y}{a\tan\theta} = 2$

$\Rightarrow x + y\operatorname{cosec}\theta = 2a\sec\theta$........(ii)

Since, Eqs. (i) and (ii) are identical.

$\therefore t = -\operatorname{cosec}\theta$ or $t = 2\tan\theta$

**148.** **(c)** We have,

$$x^2-6x+4y+1=0$$
$$\Rightarrow \quad (x-3)^2-9+4y+1=0$$
$$\Rightarrow \quad (x-3)^2+4y-8=0$$
$$\Rightarrow \quad (x-3)^2$$
$$\Rightarrow \quad =-4(y-2)$$

It represents parabola whose vertex is (3,2),

$\therefore$ Focus $=(3,-1+2)=(3,1)$

**149.** **(b)** Let function $f(x)=x^2(x-1)$

$\Rightarrow \quad f'(x)=3x^2-2x$

and $\quad f''(x)=6x-2$

Now, $\quad f(0)=f(1)=f'(0)=0$

Then, according to question, at $x=\frac{1}{3}, f''(x)=0$

(i.e. $c=1/3$ for some $C \in R$ )

**150.** **(b, c)** We have,

$f(x) \quad = \quad x^n$

$\Rightarrow \quad f'(x) \quad = nx^{n-1}$

Now, $f'(\alpha+\beta)=f'(\alpha)+f'(\beta)$

$\Rightarrow\ n(\alpha+\beta)^{n-1}=n\alpha^{n-1}+n\beta^{n-1}$

$\Rightarrow\ (\alpha+\beta)^{n-1}=\alpha^{n-1}+\beta^{n-1}$

From options we see that $n=2$, satisfy the above equation.

$\therefore\ n=2$

**151.** **(c)**

$$I=\int_0^a \frac{dx}{1+f(x)}$$
$$=\int_0^a \frac{1}{1+f(a-x)}dx$$
$$=\int_0^a \frac{dx}{1+\frac{1}{f(x)}}\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$
$$I=\int_0^a \frac{f(x)}{f(x)+1}dx$$

On adding Eqs. (i) and (ii), we get

$$2I=\int_0^a \frac{f(x)+1}{f(x)+1}dx$$
$$\Rightarrow \quad 2I=\int_0^a 1dx$$
$$\Rightarrow \quad 2I=[x]_0^a$$
$$\Rightarrow \quad 2I=a$$
$$\Rightarrow \quad I=\frac{a}{2}$$

**152.** **(b, d)** We have,

$$xy=1-2x$$
$$\Rightarrow y=\frac{1-2x}{x}$$
$$\Rightarrow \frac{dy}{dx}=\frac{-2x-(1-2x)\cdot 1}{x^2}$$
$$=\frac{-2x-1+2x}{x^2}=\frac{-1}{x^2}<0$$

Since, $ax+by+c=0$ is tangent to the curve $xy=1-2x$.

$\therefore\ \frac{-a}{b}<0 \Rightarrow \frac{a}{b}>0$

$\Rightarrow$ Either $a>0, b>0$ or $a<0, b<0$.

**153. (b,c)**

(b, c) We know, $S = ut + \frac{1}{2}at^2$.....(i)
Condition for first,
As, velocity is constant $u = u, v = u$ (constant)
$\therefore\ a = 0, S = ut$ ... (ii) [from Eq. (i)]
Condition for second $u = 0, a = f$ (constant)

$$\therefore\ S = \frac{1}{2}at^2$$

$$S = \frac{1}{2}ft^2 \text{.(iii)}$$

[from Eq. (i)]
On differentiating Eqs. (ii) and (iii), we get

$$\frac{dS}{dt} = u$$

and $\frac{dS}{dt} = \frac{1}{2}f(2t)$.........(iv)

$$\frac{dS}{dt} = ft$$

$u = ft$
$t = \frac{u}{f}$........(V)
Thus, they will be at the greatest distance at the end of the $\frac{u}{f}$ from the start.
For greatest distance
Put

$$S = \frac{1}{2}ft^2$$

$$t = \frac{u}{f},$$

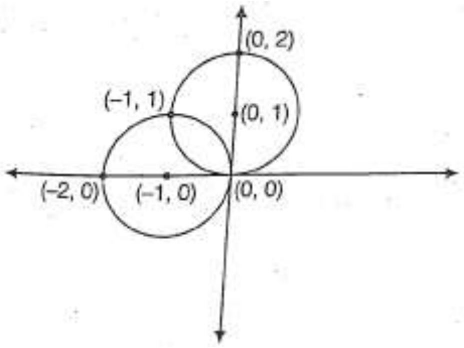
$$S = \frac{1}{2}f\left(\frac{u^2}{f^2}\right)$$

$$S = \frac{u^2}{2f}$$

Hence, option (b) and (c) are correct.

**154. (a, c)** We have, $|z - i| = |z + 1| = 1$
Let, $z = x + iy$
$|z - i| = 1$
$|x + iy - i| = 1$
$x^2 + (y - 1)^2 = 1 \ldots \ldots (i)$
Also $|z + 1| = 1$
$|x + iy + 1| = 1$
$|(x + 1) + iy| = 1$

$\Rightarrow\ (x + 1)^2 + y^2 = 1$....(ii)
From Eqs. (i) and (ii), we get
$x = y = 0$ and $x = -1, y = 1$
$\therefore z = 0, -1 + i$




**155.** **(c)** We know that,

$$1 + a^2 > 0, a \in R$$

$\Rightarrow \quad (a, a) \in \rho$

$\therefore$ $\rho$ is reflexive

Again, let $(a, b) \in \rho$

$\Rightarrow 1 + ab > 0$

$\Rightarrow 1 + ba > 0$

$\Rightarrow (b, a) \in \rho$

$\therefore$ $\rho$ is symmetric

Now, $(1, -0 \cdot 1) \in \rho$

and $(-0 \cdot 1, -9) \in \rho$

but $(1, -9) \notin \rho$

$\therefore$ $\rho$ is not transitive.