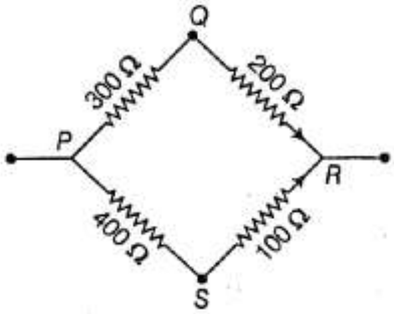
Physics

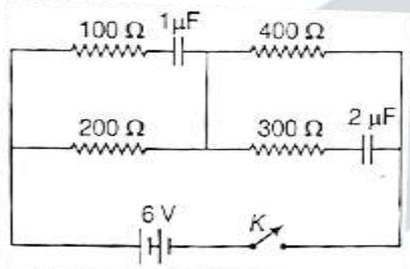
1. **(d)** For maximum equivalent resistance across the diagonal of the square, the given resistors connected as.



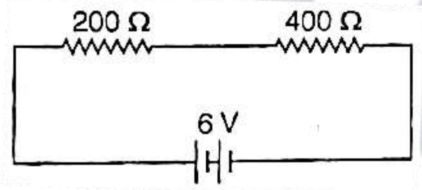
Resistance of PQR arm, $R_1 = 300 + 200 = 500\Omega$
Resistance of PSR arm, $R_2 = 400 + 100\Omega = 500\Omega$ The equivalent resistance between P and R.

$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{500} + \frac{1}{500} = \frac{1+1}{500}$$
$$\therefore\ R_{\mathrm{eq}} = \frac{500}{2} = 250\Omega$$

2. **(c)**



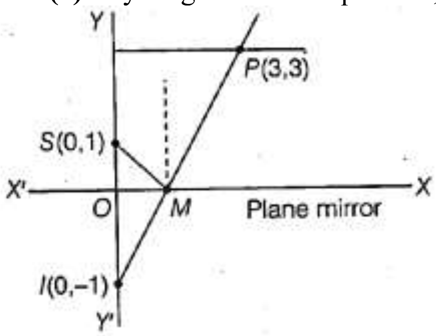
In steady state, arm having capacitors does not flow current. So, we can neglect them.
∴ The given circuit reduces to



$\therefore\ R_{\text{net}} = 200 + 400 = 600\Omega$
∴ Current in circuit,

$$I = \frac{V}{R} = \frac{6}{600} = 0.01\text{ A}$$
$$= 10\text{ mA}$$

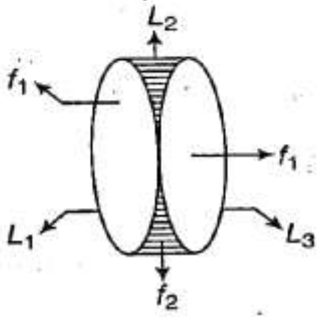
3. **(a)** Ray diagram for the question,



I is image of source S by the plane mirror placed perpendicularly along X-axis.

$\because SM = IM$
$\therefore PM + MS = PM + MI = PI$
$\therefore\ PI = \sqrt{(3-0)^2 + (3+1)^2}$
$= \sqrt{9+16} = \sqrt{25} = 5$

4. **(b)** The given combination of lenses



Here, f_1 = focal length of equiconvex lenses of glass. f_2 = focal length of lens formed by water (concave). The focal length of the combination.

$$\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{f_1} - \frac{1}{f_2} + \frac{1}{f_1} = \frac{2}{f_1} - \frac{1}{f_2}$$

$$\frac{1}{F} = \frac{2f_2 - f_1}{f_1 f_2} \Rightarrow F = \frac{f_1 f_2}{2f_2 - f_1} [\because f_3 = f_1]$$

$$F = \frac{f_1}{2 - \frac{f_1}{f_2}} \dots (\text{i})$$

Here, $\frac{1}{f_1} = (\mu_g - 1)\frac{2}{R}$, for L_1 and L_3

and $\frac{1}{f_2} = (\mu_w - 1)\left(-\frac{2}{R}\right)$, for L_2

Given, $\mu_w < \mu_g$

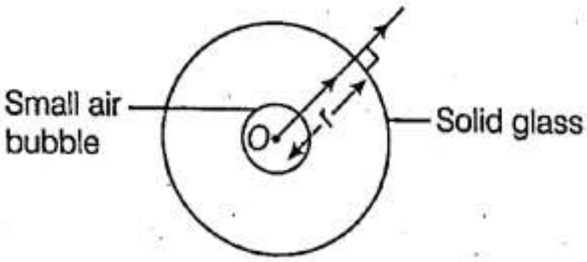
Thus, $\frac{f_1}{f_2} < 1$

So, $F > \frac{f_1}{2}$

From Eqs. (i) and (ii), we get

$\frac{f_1}{2} < F < f_1$ or $\frac{f}{2} < F < f$ $(\because f_1 = f)$

5. **(d)**



As the object is at centre of the sphere.

$\therefore$ All rays will fall normally on surface, hence they do not deflect. Thus, a virtual image is formed at the centre O.

$\therefore$ Apparent depth = Real depth

6. **(c)** Young's double slit experiment with white light which consist wavelength from range 4000Å to 7000Å.

$\because$ At the centre of the screen, the path difference is zero for all wavelengths. The bright fringes of these wavelengths overlap at the centre. Thus, the white fringe at the centre is formed.

7. **(a)** $\because$ Linear velocity of an electron in a orbit of H like atom,

$$v = 2.18 \times 10^6 \times \frac{Z}{n}\ \text{m/s} = \frac{c_0}{137} \cdot \frac{Z}{n}$$

where, Z = number of protons in nucleus,

n = principal quantum number

and c_0 = speed of light in free space,
Thus, we have

$$v \propto \frac{1}{n}$$

8. (d) Given, $t_{1/2} = 3$ days
Number of active nuclei remaining after time t,

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

After $t = 2$ days,

$$N_1 = N_0 \left(\frac{1}{2}\right)^{2/3} = \frac{N_0}{2^{2/3}} = \frac{N_0}{4^{1/3}} = 0.63N_0$$

$$N_1 = 0.63N_0$$

After $t = 3$ days,

$$N_2 = \frac{N_0}{2} = 0.5N_0$$

Number of nuclei that will decay on the 3rd day, $N_3 = N_2 - N_1 = 0.63N_0 - 0.5N_0 = 0.13N_0$
In the term, fraction is 0.13 .

9. (d) $\because$ Kinetic energy of a electron due to accelerated by a potential V, KE = eV

$$\frac{1}{2}m_e v^2 = eV$$

$$\Rightarrow \quad \frac{1 \times p^2}{2m_e} = eV$$

$$\therefore \quad p = \sqrt{2eVm_e}$$

$\because$ de-Broglie wavelength of a particle having momentum p,

$$\lambda = \frac{h}{p}$$

According to question,

$$\frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1} = \frac{\sqrt{2eV_2m_e}}{\sqrt{2eV_1m_e}} = \sqrt{\frac{V_2}{V_1}}$$

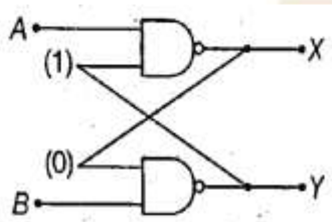
$$\Rightarrow \quad \lambda_2 = 2\lambda_1$$

$$\Rightarrow \quad \frac{\lambda_1}{2\lambda_1} = \sqrt{\frac{V_2}{10000}}$$

$$\therefore \quad \text{Potential } V_2 = \frac{10^4}{4} = 2500 \text{ V}$$

(given)

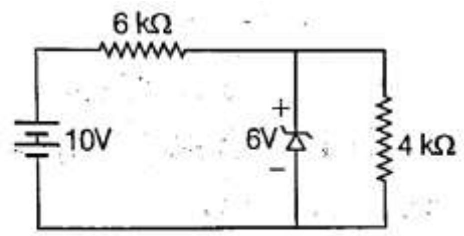
10. (c)



$$X = \overline{AY} = \overline{Y} = 0$$

$$Y = \overline{BX} = 1$$

11. (a)



In the given circuit, the zener diode is used as a voltage regulating device. The voltage across 6kΩ resistance is $(10 - 6)\text{V} = 4\text{ V}$

Current through 6kΩ resistor,

$$I = \frac{4\text{ V}}{6\text{k}\Omega} = \frac{4}{6 \times 10^3} = \frac{2}{3} \times 10^{-3} = \frac{2}{3}\text{ mA}$$

12. (c) In representation of simple harmonic motion in ellipse as velocity along X-axis and displacement along Y-axis.

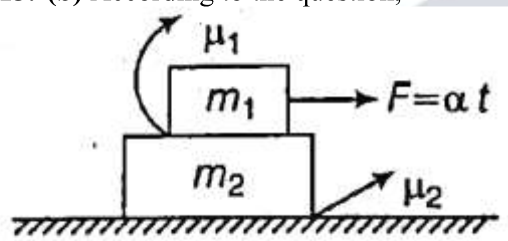
$$\because \frac{\text{Major axis (along } X \text{ - axis)}}{\text{Minor axis (along } X \text{ - axis)}} = 20\pi$$

$$\Rightarrow \frac{\omega A}{A} = 20\pi \Rightarrow \omega = 20\pi$$

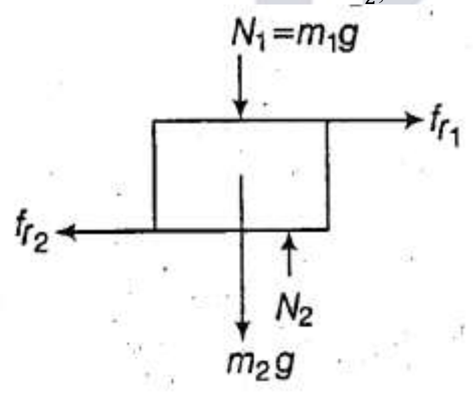
$$\Rightarrow 2\pi n = 20\pi$$

The frequency of the simple harmonic motion $\therefore\ n = 10\text{ Hz}$

13. (b) According to the question,



FBD of lower block of mass m_2,



$\therefore\ N_2 = m_2 g + N_1 = (m_1 + m_2)g$

$\therefore$ Lower block never moves.

$\therefore\ f_{r_2} \geq f_{r_1}$

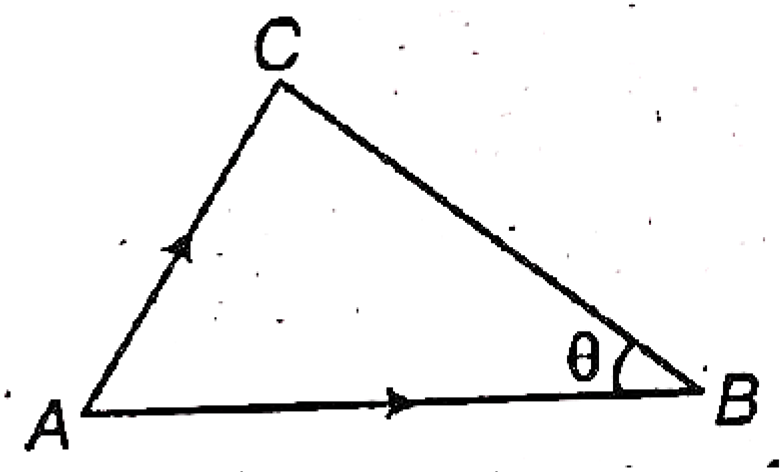
$\mu_2 N_2 \geq \mu_1 N_1 \Rightarrow \mu_2 (m_1 + m_2) g \geq \mu_1 m_1 g$

$$\Rightarrow \frac{m_1 + m_2}{m_1} \geq \frac{\mu_1}{\mu_2}$$

$$\therefore\ \frac{\mu_1}{\mu_2} \leq 1 + \frac{m_2}{m_1}$$

$$\therefore\ \left.\frac{\mu_1}{\mu_2}\right|_{\max} = 1 + \frac{m_2}{m_1}$$

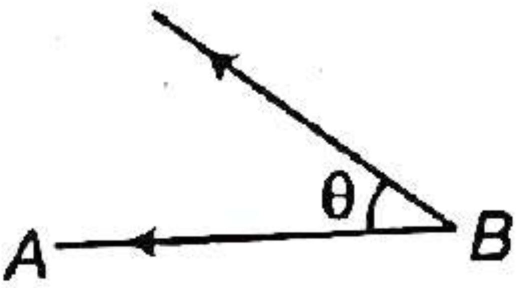
14. (a)



$$\mathbf{AB} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} \text{ and } \mathbf{AC} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$$\therefore \quad \mathbf{BA} = -(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$\angle ABC$ is angle between BA and BC



$$\begin{aligned} \mathbf{BC} &= \mathbf{AC} + \mathbf{BA} \\ &= \mathbf{AC} - \mathbf{AB} \\ &= \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} - (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} \end{aligned}$$

$$\therefore \ \mathbf{BA} \cdot \mathbf{BC} = |\mathbf{BA}||\mathbf{BC}| \cos\theta$$

$$-(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

$$= \left(\sqrt{3^2 + 1^2 + 1^2}\right) \cdot \left(\sqrt{2^2 + 1^2}\right) \cos\theta$$

$$\Rightarrow \quad 6 - 1 \quad = \sqrt{11 \times 5} \cdot \cos\theta$$

$$\therefore \quad \cos\theta \quad = \frac{5}{\sqrt{55}} = \sqrt{\frac{5}{11}} = \theta = \cos^{-1} = \sqrt{\frac{5}{11}}$$

15. (c) According to the question,

$$v \propto \frac{1}{\sqrt{x}}$$

$v = \frac{k}{\sqrt{x}}$..........(i)

$$\frac{dv}{dt} = \frac{d}{dt} \cdot \frac{K}{\sqrt{x}} = k \cdot \frac{-1}{2} \cdot x^{-\frac{1}{2}-1} \cdot \frac{dx}{dt}$$

$$a = -\frac{k}{2} \cdot x^{-\frac{3}{2}} \cdot \frac{k}{\sqrt{x}} \left[\because \frac{dx}{dt} = v \text{ and } v = \frac{k}{\sqrt{x}} \text{ from Eq.(i)}\right]$$

$$= \frac{-k^2}{2} \cdot \frac{1}{x^2}$$

$\therefore$ Force, F = Mass $\times$| Acceleration |

$= m\frac{k^2}{2}\cdot\frac{1}{x^2}$

$F = \frac{k^2}{2}\cdot\frac{m}{x^2}$

$F \propto \frac{1}{x^2}$

(magnitude)

16. (c) ∵ Escape velocity from a planet,

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^2}R}$$

$= \sqrt{2gR}$(i)

According due to gravity

$$g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3\cdot\rho}{R^2} = \frac{4}{3}G\pi R\rho$$

∴ Radius $R = \frac{3g}{4\pi G\rho}$(ii)

From Eqs. (i) and (ii), we get

Thus,

$$v_e \propto \frac{g}{\sqrt{\rho}}$$

$$\therefore \frac{v_{e_1}}{v_{e_2}} = \frac{g_1}{\sqrt{\rho_1}} \times \frac{\sqrt{\rho_2}}{g_2} = \frac{5}{2}\times\frac{1}{\sqrt{2}}$$

$\left(\text{given, } \frac{g_1}{g_2} = \frac{5}{2} \text{ and } \frac{\rho_1}{\rho_2} = 2:1\right)$

$$= \frac{5}{2\sqrt{2}}$$

17. (c) According to the question, time period, $T \propto r^a\rho^b S^c$

$T = kr^a\rho^b S^c$.......(i)

Thus, putting dimension, we get

$[\mathrm{T}] = [\mathrm{L}]^a[\mathrm{ML}^{-3}]^b[\mathrm{MT}^2]^c$

$[\mathrm{T}] = [\mathrm{M}]^{b+c}\cdot[\mathrm{L}]^{a-3b}\cdot[\mathrm{T}]^{-2c}$

Equating the dimensions of both sides, we get

$b + c = 0, a - 3b = 0$

and $-2c = 1$

∴ $c = -\frac{1}{2}$

∴ $b = \frac{1}{2}$

and $a = 3b = \frac{3}{2}$

Putting these value of a, b and c into Eq. (i), we get

$T = k\cdot r^{\frac{3}{2}}\rho^{\frac{1}{2}}\cdot S^{-\frac{1}{2}}$

$= k\sqrt{\frac{\rho}{S}r^3}$

18. (d) ∵ Stress, $\frac{F}{\Delta A} = 1\%$ of $Y = \frac{Y}{100}$

But Young's modulus, $Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{\Delta A}}{\frac{\Delta l}{l}}$

$$\therefore \quad Y = \frac{\frac{Y}{100}}{\frac{\Delta l}{l}} \left(\text{putting } \frac{F}{\Delta A} = \frac{Y}{100}\right)$$

$$\therefore \quad \frac{\Delta l}{l} = \frac{1}{100}$$

Poisson's ratio, $\sigma = \frac{-\frac{\Delta r}{r}}{\frac{\Delta l}{l}}$

$$\therefore \frac{\Delta r}{r} = -\sigma \cdot \frac{\Delta l}{l} = \frac{-0.3}{100}$$

$$\frac{\Delta r}{r} = \frac{-0.3}{100}$$

$\therefore$ Change in volume, $\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta l}{l}$

$$= \frac{2 \times (-0.3)}{100} + \frac{1}{100}$$

$$= \frac{1 - 0.6}{100} = \frac{0.4}{100}$$

$\therefore \ \Delta V\% = 0.4\%$

19. (b) Terminal velocity, $v = \frac{2}{9} r^2 \frac{(\rho - \sigma)}{\eta} g$

Neglecting buoyancy effect of the fluid,

$$v = \frac{2}{9} \cdot \frac{\rho}{\eta} r^2 g$$

Putting the given values, we get

$$v = \frac{2}{9} \times \frac{10^3 \times (0.9 \times 10^{-3})^2}{1.8 \times 10^{-5}} \times 9.8 = 98 \text{ ms}^{-1}$$

20. (c) Let the mass of ice $= m$

Applying calorimetry principle, heat given $=$ heat taken

$$(m_1 + m_2) s_1 (t_1 - t) = \frac{mL}{2} + m s_2 (t - t_2)$$

Putting values, we get

$$(10 + 50) \times 1 \times (15 - 0) = \frac{m}{2} \times 80 + m \times 0.5[0 - (-10)]$$

$$\Rightarrow 60 \times 15 = 40m + \frac{m}{2} \times 10 = 45m$$

$$\therefore \ m = \frac{60 \times 15}{45} = 20 \text{ g}$$

21. (c) Heat capacity of an ideal gas in a thermodynamic process,

$\because$ Ideal gas is monoatomic,

$$C_V = \frac{fR}{2} = \frac{3R}{2}$$

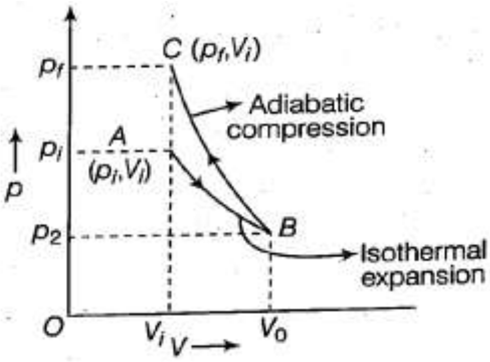
$$C_{\text{process}} = C_V + \frac{p}{n} \cdot \frac{dV}{dT}$$

$$= \frac{3}{2} R + \frac{p}{n} \cdot \frac{V_0}{2T}$$

$$\therefore C_{\text{process}} = \frac{3}{2} R + \frac{nRT}{n2T}$$

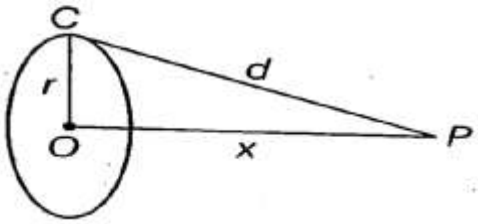
$$= \frac{3}{2} R + \frac{R}{2} = 2R$$

22. (b) In $p - V$ diagram, the slope of an adiabatic curve at any point in steeper than that of isothermalcurve at that point.



Thus, $p_f > p_i$

23. (b) A charged circular ring of radius R is shown in figure.



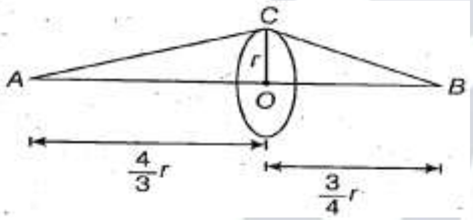
From the figure, $d = \sqrt{x^2 + r^2}$

$\therefore$ Potential due to a uniform ring of positive charge q at point P,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{d}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{x^2 + r^2}} \quad \text{(i)}$$

Now,



$$\therefore CB = \sqrt{OC^2 + OB^2} = \sqrt{r^2 + \left(\frac{3}{4}\right)^2 r^2} = \frac{5}{4}r$$

Similarly, $AC = \frac{5}{3}r$

$\therefore$ Work done in bringing a point charge $-q$ from A to B,

$$W = -q(V_B - V_A)$$

$$= -q\left[\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{CB} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{CA}\right]$$

$$= \frac{-1}{4\pi\varepsilon_0} \cdot q^2 \left[\frac{1}{\frac{5}{4}r} - \frac{1}{\frac{5}{3}r}\right]$$

$$= \frac{-1}{4\pi\varepsilon_0} \cdot \frac{q^2}{5r}[4 - 3] = -\frac{1}{5} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r}$$

24. (a) For surface $ABCD$ electric flux is zero. Because at surface $ABCD$ net electric field is zero.

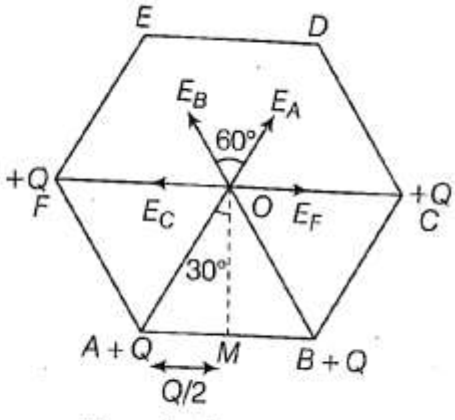
Using Gauss's law,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

Electric flux through the five faces of the cube,

$$\phi_E = \frac{Q}{\varepsilon_0}$$

25. (c)



(Regular hexagon)

In $\triangle AOM, \sin 30^\circ = \frac{AM}{AO}$

$$\therefore\ AO = \frac{\frac{a}{2}}{1/2} = a$$

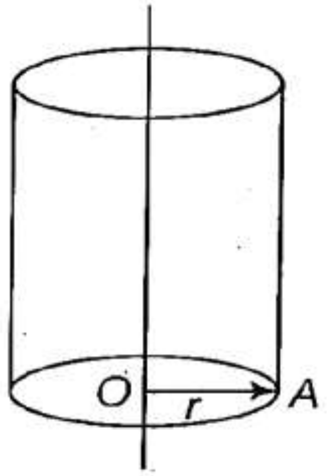
For maximum electric field at center O charges should be placed at F, A, B and C.
$\because$ Electric field due to charges at F and C is equal and opposite at O.
$\therefore$ Net electric field at centre O due to charges at A and B.
Angle between E_A and E_B is 60°.
$\therefore E_{\text{net}}$ at O,

$$E_{\text{net}} = \sqrt{E_A^2 + E_B^2 + 2E_A E_B \cos 60^\circ}\ (\because E_A = E_B = E)$$

$$= \sqrt{E^2 + E^2 + 2E^2 \cdot \frac{1}{2}} \quad \left(\because \cos 60^\circ = \frac{1}{2}\right)$$

$$= E\sqrt{3} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{a^2}\sqrt{3}$$

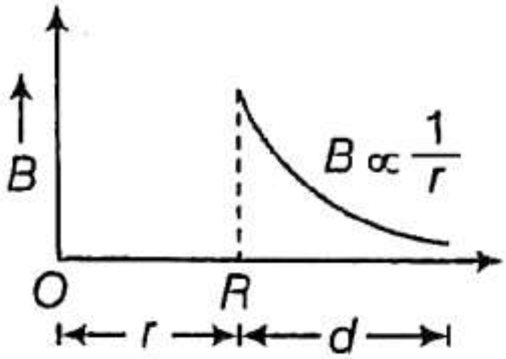
26. (b) $\because$ When a charged particle moves at a circular path in uniform magnetic field, it time period is independent from its speed.
$\therefore$ Time period of proton will not change with change in its speed.
27. (c) $\because$ Magnetic field due to a hollow cylinder of radius r.



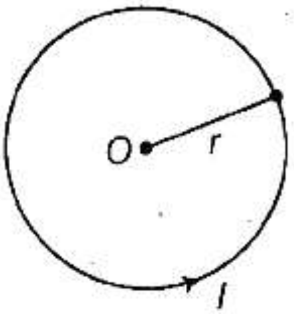
$$B_{\text{inside}} = 0(\text{ for } d < r)$$

$$B_{\text{outside}} = \frac{\mu_0}{2\pi} \cdot \frac{i}{d} (\text{ for } d \geq r)$$

So, graph between B and d.



28. (b)



$\because \quad B = \frac{\mu_0}{2} \cdot \frac{I}{r}$

$I_1 = \frac{V}{R_1}$

Case (I) $l_1 = 2\pi r$ (length of the loop),

$R_1 = \rho \frac{l_1}{A}$ and $B_1 = \frac{\mu_0}{2} \cdot \frac{I_1}{r}$.......(i)

Case (II) $l_2 = 2\pi \cdot 2r = 2l_1$

$R_2 = \rho \cdot \frac{l_2}{A} = \rho \cdot \frac{2l_1}{A} = 2R_1$

$I_2 = \frac{V}{R_2} = \frac{V}{2R_1} = \frac{I_1}{2}$

$B_2 = \frac{\mu_0}{2} \cdot \frac{I_2}{2r} = \frac{\mu_0}{2} \cdot \frac{I_1}{2 \cdot 2r}$(ii)

From Eqs. (i) and (ii),

$$\frac{B_2}{B_1} = \frac{\frac{\mu_0}{2} \cdot \frac{I_1}{2 \cdot 2r}}{\frac{\mu_0}{2} \cdot \frac{I_1}{r}} = \frac{1}{4}$$

Magnetic field $B_2 = \frac{B_1}{4}$

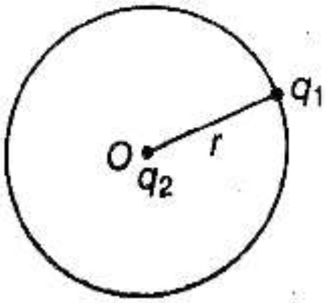
29. (d) $\because$ Resonance frequency, $f_0 = \sqrt{f_1 f_2}$

$= \sqrt{200 \times 800} = 400$ Hz

30. (c)

Initially, there will be no voltage drop across capacitor, so intensity of bulb will rise sharply and gradually voltage drop across capacitor will increase as a result voltage drop across bulb decreases, so intensity of bulb will decreases.

31. (d)



$$F_{\text{centripetal}} = mr\omega^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$$

$$\therefore\ \omega^2 = \frac{1}{4\pi\varepsilon_0 m}\cdot\frac{q_1q_2}{r^3} \Rightarrow \omega = \sqrt{\frac{q_1q_2}{4\pi\varepsilon_0 mr^3}}$$

$$\omega \propto \frac{1}{r^{\frac{3}{2}}}\text{(i)}$$

Current produced due to moving q_1,

$i = \frac{\omega}{2\pi}q_1$.......(ii)

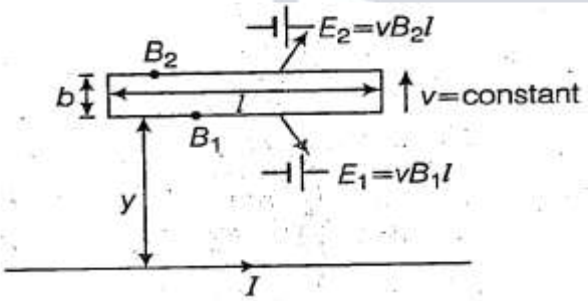
Thus, magnetic field due to motion of q_1 in circular path at point O

$$B = \frac{\mu_0}{2}\cdot\frac{i}{r} = \frac{\mu_0}{2}\cdot\frac{\omega q_1}{2\pi r}\quad \text{[using Eq. (ii)]}$$

Hence, $B \propto \frac{\omega}{r}$

Magnetic field $B \propto \frac{1}{r^{5/2}}$

32. (a)



$\therefore$ Motional emf in the loop,

$$E = E_1 - E_2 = vl(B_1 - B_2)$$

$$= vl\left[\frac{\mu_0}{2\pi}\cdot\frac{I}{y} - \frac{\mu_0 I}{2\pi(y+b)}\right]$$

$$= \frac{\mu_0}{2\pi}\cdot I\cdot vl\left[\frac{y+b-y}{y(y+b)}\right] = \frac{\mu_0}{2\pi}\cdot\frac{vlI}{y(y+b)}b$$

$$\because\ b \ll y$$

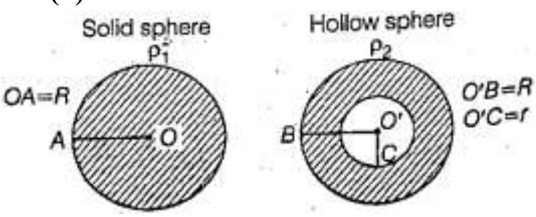
$$\therefore\ E = \frac{\mu_0}{2\pi}\cdot\frac{vlI}{y^2}b$$

$\because v =$ constant $(\therefore y = vt)$

Thus, $E = \frac{\mu_0}{2\pi}\cdot\frac{vlI}{v^2t^2}b$

Hence, $E \propto \frac{1}{t^2}$

33. (d)



$\therefore \quad M_1 = \rho_1 \frac{4}{3}\pi R^3$ and $M_2 = \rho_2 \frac{4}{3}\pi(R^3 - r^3)$

$\therefore \quad \rho_1 \frac{4}{3}\pi R^3 = \rho_2 \frac{4}{3}\pi(R^3 - r^3)$

$\Rightarrow \quad \rho_1 R^3 = \rho_2 (R^3 - r^3)$

$\Rightarrow \quad \frac{\rho_1}{\rho_2} = 1 - \frac{r^3}{R^3}$

$\therefore \quad \frac{r}{R} = \left(1 - \frac{\rho_1}{\rho_2}\right)^{\frac{1}{3}}$

Moment of inertia,

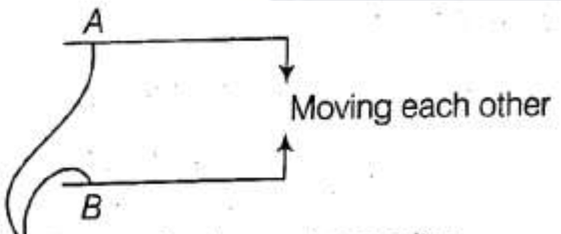
$$\frac{I_H}{I_S} = \frac{\frac{2}{5}M_2\left[\frac{R^5 - r^5}{R^3 - r^3}\right]}{\frac{2}{5}M_1 R^2}$$

$$= \frac{\rho_2 \frac{4}{3}\pi(R^3 - r^3)\left[\frac{R^5 - r^5}{(R^3 - r^3)}\right]}{\rho_1 \frac{4}{3}\pi R^3 R^2}$$

$$= \frac{\rho_2}{\rho_1}\left[\frac{R^5 - r^5}{R^5}\right] = \frac{\rho_2}{\rho_1}\left[1 - \left(\frac{r}{R}\right)^5\right]$$

$$\therefore \frac{I_H}{I_S} = \frac{\rho_2}{\rho_1}\left[1 - \left(1 - \frac{\rho_1}{\rho_2}\right)^{\frac{5}{3}}\right]$$

34. (a)



Plates of a charged capacitor moving toward each other due to electrostatic attraction.

Force $= \frac{1}{2} \cdot \frac{\sigma}{\varepsilon_0} q$

$\therefore$ Relative acceleration of plates,

$a_{\text{rel}} = \frac{2F}{M} \ldots\ldots$(i)

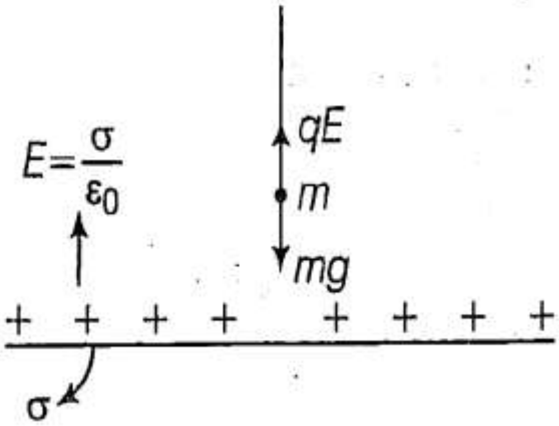
Potential difference, $V = Ed$

where, $d = d_0 - \frac{1}{2}a_{\text{rel}}\, t^2$ ($\because$ plates are moving)

$\therefore \; V = E\left[d_0 - \frac{1}{2}a_{\text{rel}}\, t^2\right]$

This is a equation of a parabola with downward concavity.

35. (d)



∴ Apparent weight of the bob, $w' = mg - qE$

$$mg' = mg - qE$$

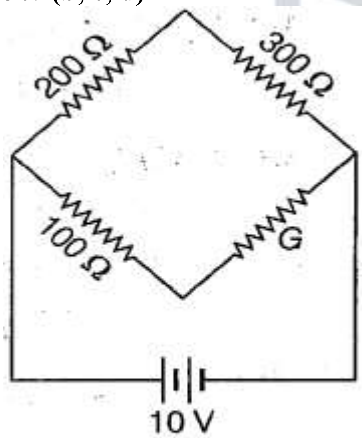
$$\therefore \quad g' = g - \frac{qE}{m}$$

∵ Time period of a pendulum,

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

$$= 2\pi\sqrt{\frac{L}{g - \frac{qE}{m}}} = 2\pi\sqrt{\frac{L}{\left(g - \frac{q\sigma}{\varepsilon_0 m}\right)}}$$

36. (b, c, d)



$$I_G = \frac{10}{100 + G}$$

$$300I_3 = \frac{10}{100 + G}G$$

$$I_3 = \frac{G}{30}\left[\frac{1}{100 + G}\right]$$

$$I_3 + I_G = \frac{1}{(100 + G)}\left[10 + \frac{G}{30}\right] = \frac{(300 + G)}{(100 + G) \times 30}$$

According to the problem,

$$10 = \left(\frac{200}{3} + \frac{300G}{300 + G}\right)\left[\frac{(300 + G)}{(100 + G)30}\right]$$

$[\because V = RI]$

$\Rightarrow 900(100 + G) = 60000 + 1100G$

$\Rightarrow 30000 = 200G$

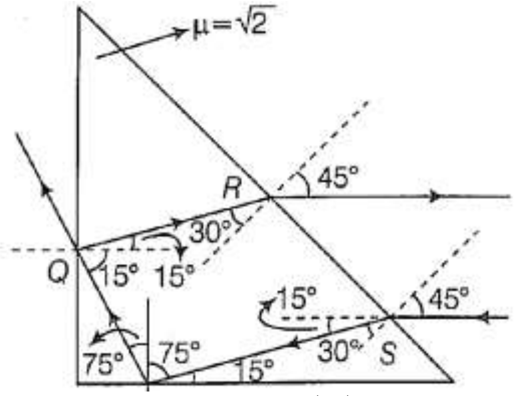
$\therefore G = 150\Omega$

$$I_G = \frac{10}{100 + 150} = \frac{10}{250} = 40 \text{ mA}$$

As $\frac{200}{100}=\frac{300}{150}$

So, $I_{200}=I_{300}$

37. $(\boldsymbol{a}, \boldsymbol{c})$



Critical angle $\theta_C=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=45^{\circ}$

Snell's law at points, $(\because n_1 \sin\theta_1=n_2 \sin\theta_2)$

$1 \sin 45^{\circ}=\sqrt{2} \sin\theta$

$\therefore\ \theta=30^{\circ}$

At P $\because$ incidence angle at $P=75^{\circ}>\theta_C$.

$\therefore$ TIR at point P.

At Q $\because$ incidence angle at $Q=15^{\circ}<\theta_C$.

$\therefore$ Partial reflection and refraction at Q.

As $SP \| QR$, emergent ray at R is parallel to incident ray at S.

$\therefore$ Net deviation $=180^{\circ}$.

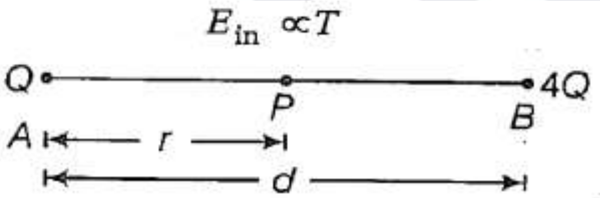
38.($\boldsymbol{a}, \boldsymbol{d}$ **)** The intensity of a sound source appears to be periodic due to

(i) source intensity is periodic.

(ii) source is producing a sound composed of two nearby frequencies.

39.(b) $\because$ Internal energy of an ideal gas depends only on the temperature of gas.

40. (a)



$\because V_P$ is minimum.

$\therefore\ E_P=0 \Rightarrow E_A=E_B$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{4Q}{(d-r)^2}$$

$$\Rightarrow \frac{1}{r}=\frac{2}{d-r} \Rightarrow d-r=2r$$

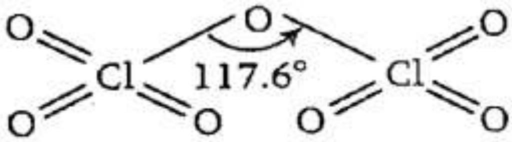
$\therefore$ Distance of P from the $Ar=\frac{d}{3}$ units to the right of A

Chemistry

41. (d) Cl_2O_7 is the anhydride of $HClO_4$. It reacts with water slowly to give perchloric acid.

$Cl_2O_7+H_2O \longrightarrow \underset{\text{Perchloric acid}}{2HClO_4}$

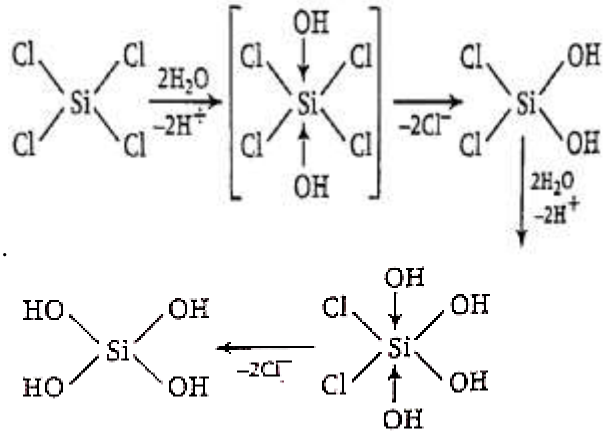
Its structure is as follows:



It is less reactive than other oxides of chlorine and does not react with P, S, coal or paper at room temperature.

42. (d) The main reason that $SiCl_4$ is easily hydrolysed as compared to CCl_4 is that Si can extend its

coordination number beyond four because it has vacant d-orbital. On the other hand, carbon has no d-orbital thus, it cannot extend its coordination number beyond four, so its halides coordinationed (hydrolysed) by water. The are not attacked (hydrolysed) ordinate with water vacant d-orbitals of Si can coordinates are hydrolysed molecules
The hydrolysis of $SiCl_4$ occurs due to coordination of - OH with empty 3 d-orbitals of Si -atom of $SiCl_4$ molecule



43. (d) Silver chloride dissolves in excess of ammonium hydroxide solution. The cation present in the resulting solution is $[Ag(NH_3)_2]^+$
In aqueous solution, silver chloride exist as Ag^+ and Cl^-. Ag^+present in solution reacts with NH_3 to form $[Ag(NH_3)_2]^+$.
$Ag^+Cl^- + 2NH_3 \longrightarrow [Ag(NH_3)_2]^+Cl^-$
44. (a) The ease of hydrolysis of carbonyl compounds depend on the group attached to the carbonyl

carbon. Among the given options, chlorine (-Cl) group is electron withdrawing group which makes the carbonyl group electrophilic. Hence, hydrolysis can occur most easily. The order for electron withdrawing tendency among given options is as follows:

$-Cl > -OCOCH_3 > -OC_2H_5 > -NH_2$
So, the correct order for the ease of hydrolysis in the given compounds is as follows:

$$\underset{\text{I}}{CH_3COCl} > \underset{\text{II}}{CH_3\,COOCOCH_3} > \underset{\text{III}}{CH_3COOC_2H_5} > \underset{\text{IV}}{CH_3CONH_2}$$

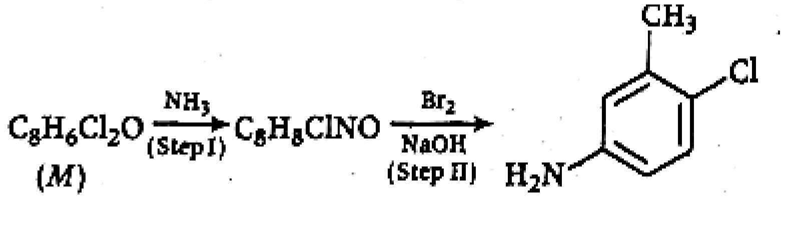
45.(d) $CH_3 - C \equiv CMgBr$ can be prepared by the reaction of $CH_3 - C \equiv CH$ with CH_3MgBr. This method is used in preparation of higher alkynes from lower alkynes. The chemical equation for the formation of $CH_3 - C \equiv CMgBr$ is given below

$CH_3 - C \equiv CH + CH_3\overset{2+}{Mg}Br \longrightarrow$
$CH_3 - C \equiv \overset{-2+\ \ +}{C\ \ Mg}X^-$

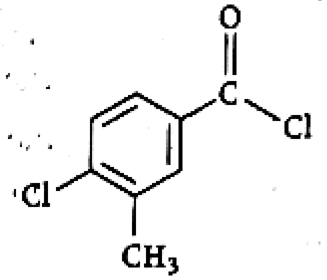
46. (b)
The number of alkene(s) that can produce 2-butanol by the successive treatment of B_2H_6 in tetrahydrofuran solvent and alkaline H_2O_2 solution is 2 . The alkenes are cis-but-2- ene and trans-but-2-ene. The hydration product is obtained in accordance with opposite Markownikoff's rule. In this reaction, addition takes place through the initial formation of π-complex which changes into a cyclic four centre transition state with the addition of boron atom to the less hindered carbon atom.

$$H_3C - HC \underset{\text{But-2-ene}}{=} CH - CH_3 \xrightarrow[\text{(ii) } H_2O_2/OH^-]{\text{(i) } B_2H_6} CH_3 - \underset{\text{Butan-2-ol}}{\underset{OH}{CH}} - CH_2CH_3$$

Butan-2-ol
47. (b) Given sequence of chemical equation is

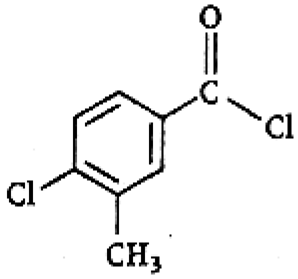


The structural formula of $C_8H_6Cl_2O(M)$ will be



The various step involves in the above road map can be explain as follows

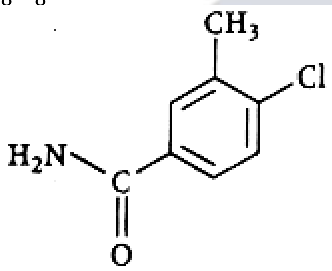
Step-I It involves the formation of an amide on reaction of ammonia and acetyl chloride so, M must be



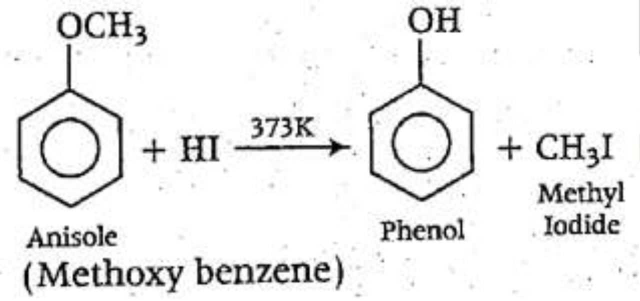
Step-II It involves the Hofmann-bromamide reaction.

$C_8H_8ClNO \xrightarrow[NaOH]{Br_2}$

In this reaction, migration of an alkyl or aryl group takes place from carbonyl carbon of the amide to the N-atom. So, C_8H_8ClNO must be an amide with structural formula.



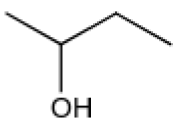
48. **(b)** Methoxybenzene on treatment with HI produces phenol and methyl iodide. This is due the more stable aryl oxygen bond. Here, methyl phenyl oxonium ion is formed by the protonation of ether. The bond between $O - CH_3$ is weaker than the bond between $O - C_6H_5$ because the carbon of phenyl group is sp^2-hybridised and there is a partial double bond character. Thus, the reaction yields phenol and alkyl halide.



49. **(b)** Given sequence of chemical equation is

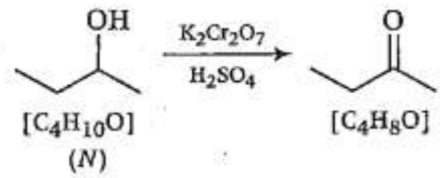
$$\underset{(N)}{C_4H_{10}O} \xrightarrow[\substack{H_2SO_4 \\ (\text{step I})}]{K_2Cr_2O_7} C_4H_8O \xrightarrow[\substack{\text{Warm} \\ (\text{step II})}]{I_2/NaOH} CHI_3$$

Here, N will be

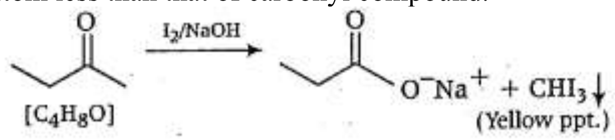


(IUPAC name = Butan-2-ol)

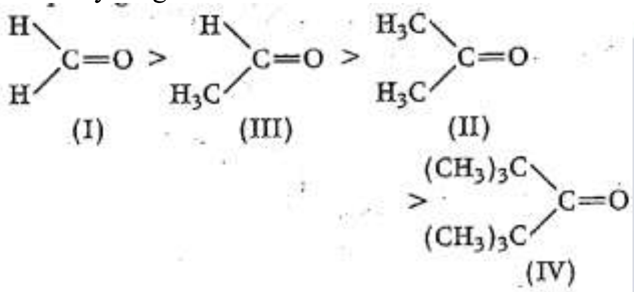
Step 1 It involves the oxidation of secondary alcohol to corresponding alcohol in the presence of $K_2Cr_2O_7$ and H_2SO_4.



Step II It involves the iodoform reaction. Ketone formed in step I have one methyl group linked to the carbonyl carbon atom are oxidised by sodium hypohalite to sodium salts of corresponding carboxylic acids having one carbon atom less than that of carbonyl compound.

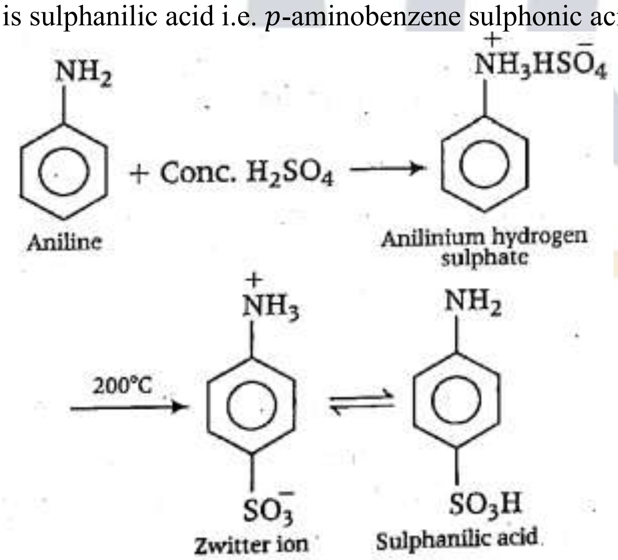


50. (a) The correct order of reactivity for the addition reaction of the given carbonyl compounds with ethylmagnesium iodide is I > III > II > IV.



This order can be explained on the basis of following two factors:
(i) Inductive effect Greater the number of alkyl (electron releasing) groups attached to carbonyl group, greater will be the electron density on carbonyl carbon. Thus, it lowers the attack of nucleophile and hence, reactivity decreases.
(ii) Steric effect As the number of alkyl group attached to carbonyl carbon increases, the attack of nucleophile on carbonyl group becomes more and more difficult due to steric hinderance.

51. (d) When aniline is treated with conc. H_2SO_4 followed by heating at $200^{\circ}C$, the product obtained is sulphanilic acid i.e. p-aminobenzene sulphonic acid.



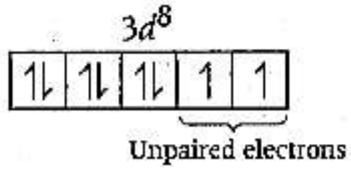
52. (c) $n=2, l=0, m=-1$ is not a possible electronic configuration. With $n=2$ and $l=0$, the orbital is $2s$ having only one m_l value i.e. 0 . As the value of $m_l=-l$ to $+l$

Other options are correct.

	Orbital
$n=3, l=0, m=0$	$3s$

$n=3, l=1, m=-1$	$3p$
$n=2, l=1, m=0$	$2p$

53. (b) The number of unpaired electrons in Ni (atomic number $=28$) are 2 . It can be easily concluded from the electronic configuration of Ni.
Electronic configuration of
$\mathrm{Ni}=1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^8$
Orbitals of Ni in



54. (c) The strongest H -bond is $\mathrm{F}-\mathrm{H}-\cdots--\mathrm{F}$ because in this case, a hydrogen is bonded to a most electronegative atom i.e. fluorine. In all other options, hydrogen is bonded to oxygen and sulphur that are less electronegative atom than fluorine.
The correct electronegativity order of elements are as follows: $\mathrm{F}>\mathrm{O}>\mathrm{S}$

55. (d) Given,
Half-life of $C^{14}, t_{1/2}=5760$ years
Initial concentration of sample of $C^{14}, N_0=200\mathrm{mg}$
Final concentration of sample of $\mathrm{C}^{14}, N_t=25\mathrm{mg}$
The given decay is radioactive and all radioactive decay follows first order kinetics.

$$t_{1/2}=\frac{0.693}{\lambda}$$

$$\therefore \lambda=\frac{0.693}{t_{1/2}}=\frac{0.693}{5760}\mathrm{yr}^{-1}$$

We know that, for first order reaction

$$t=\frac{2.303}{\lambda}\log\frac{N_0}{N_t}=\frac{2303}{\frac{0.693}{5760}}\log\frac{[200]}{[25]}$$

$$=\frac{2303\times 5760}{0.693}\log 8$$

$$=17{,}286.78\mathrm{yrs}\approx 17{,}280\mathrm{yrs}$$

Therefore, the time taken to change 200 mg to 25 mg is 17,286.78yr, which is very close to option (d) i.e. 17280 yrs .

56. (d) Ferric ion forms a prussian blue precipitate due to the formation of $\mathrm{Fe}_4[\mathrm{Fe(CN)}_6]_3$. This complex is formed during the determine action of presence of nitrogen in the given sample. In this method, to portion of sodium fusion extract, freshly prepared ferrous sulphate, FeSO_4 solution is added and warmed. Then about 2 to 3 drops of FeCl_3 solution are added and acidified with conc. HCl. The appearance of a prussian blue colour indicate the presence of nitrogen.

$\mathrm{FeSO}_4+2\mathrm{NaOH}\longrightarrow \mathrm{Fe(OH)}_2+\mathrm{Na}_2\mathrm{SO}_4$

$6\mathrm{NaCN}+\mathrm{Fe(OH)}_2\longrightarrow \underset{sodium Ferrocyanide}{\mathrm{Na}_4[\mathrm{Fe(CN)}_6]}+2\mathrm{NaOH}$

$3\mathrm{Na}_4[\mathrm{Fe(CN)}_6]+4\mathrm{FeCl}_3\longrightarrow$

$\mathrm{Fe\ e}_4[\mathrm{Fe(CN)}_6]_3+12\mathrm{NaCl}$

Ferric ferrocyanide

(prussian blue)

57. (c) The nucleus ${}^{64}_{29}\mathrm{Cu}$ accepts an orbital electron to yield ${}^{64}_{28}\mathrm{Ni}$. The atomic number of Cu is 29 , which is equal to the number of electrons and also equal to the number of protons.
When ${}^{64}_{29}\mathrm{Cu}$ accepts an orbital electron then electrons subtract from the atomic number, i.e. $29-1=28$
${}^{64}_{29}\mathrm{Cu}+{}_{-1}e^0\longrightarrow {}^{64}_{28}\mathrm{Ni}$

58. (d) Mass of an electron $=9.108\times 10^{-31}$ kg

Mass of one mole of electron
$= (9.108 \times 10^{-31} \times 6.023 \times 10^{23})$kg
Then, number of mole of electron in 1 kg

$$= \frac{1}{9.108 \times 6.023 \times 10^{-8}}$$

$$= \frac{1}{9.108 \times 6.023} \times 10^{8} \text{ mole of } e^{-}$$

59. (d) Given, equal weights of ethane and hydrogen are mixed in an empty container at 25°C.

H_2

Initial gram weight $= \text{cog}_2\text{H}_6$ $\quad w$ g $\quad = \frac{w}{30}$ $\frac{w}{2}$

Number of moles

According to Henry's law,

$$\frac{p_{H_2}}{p_{\text{total}}} = \chi_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{C_2H_6}} = \frac{\frac{w}{2}}{\frac{w}{2} + \frac{w}{30}}$$

$$\frac{p_{H_2}}{p_{\text{total}}} = \frac{\frac{w}{2}}{\frac{15w + w}{30}} = \frac{w}{2} \times \frac{30}{16w} = \frac{15}{16}$$

So, the fraction of total pressure exerted by hydrogen is 15: 16.

60. (a) Given, the heat of neutralisation of a strong base and a strong acid is 13.7 kcal .
The reaction of neutralisation is as follows
$HCl + NaOH \longrightarrow NaCl + H_2O; \Delta H = -13.7$kcal
1 mol 1 mol
According to question,
$HCl + NaOH \longrightarrow NaCl + H_2O$.....(i)
0.6 mol 0.25 mol
In equation (i), NaOH acts as a limiting reagent. For 1 mole of NaOH and 1 mole of HCl , heat of neutralisation = 13.7kcal.
∴ For 0.25 mole of NaOH and 0.6 mole of HCl , heat of neutralisation $= 13.7 \times 0.25 \Rightarrow 3.425$kcal

61. (a) Number of X atoms at the corners = 8
Number of X atoms per unit cell $= 8 \times \frac{1}{8} = 1$ atom
Number of Y atoms at the centre of the body = 1 atom Hence, the formula of the compound is XY.

62. (c) The number of electrons involved in the reaction are three as shown below
$Al^{3+} + 3e^{-} \longrightarrow Al$
It means the conversion of every aluminium ion to aluminium atom requires three electrons.
Therefore, the amount of electricity required for one mole of Al^{3+} ions = 3 F.

63. (b) Given,
$Zn \longrightarrow Zn^{2+} + 2e^{-}; E^{\circ} = +0.76$ V
$Fe \longrightarrow Fe^{2+} + 2e^{-}; E^{\circ} = +0.41$ V
On reversing the above equation (i) and (ii), we get
$Zn^{2+} + 2e^{-} \longrightarrow Zn; E^{\circ} = -0.76$ V
$Fe^{2+} + 2e^{-} \longrightarrow Fe; E^{\circ} = -0.41$ V
To find, the standard emf of the cell with the reaction.

Oxidation (anode)

$$Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe$$

Reduction (cathode)

So,

$$E^{\circ}_{\text{cell}} = E^{\circ}_{Fe^{2+}/Fe} - E^{\circ}_{Zn}{}^{2+}/Zn$$

$$= -0.41 \text{ V} + 0.76 \text{ V} = +0.35 \text{ V}$$

64. (b) Given,
$N_2 + 3H_2 \rightleftharpoons 2NH_3; K_1 = \frac{[NH_3]^2}{[N_2][H_2]^3}$.....(i)

$N_2 + O_2 \rightleftharpoons 2NO; K_2 = \frac{[NO]^2}{[N_2][O_2]}$.....(ii)

$H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O; K_3 = \frac{[H_2O]}{[H_2][O_2]^{1/2}}$....(iii)

The chemical equation for the oxidation of 2 mol of NH_3 to give NO is

$2NH_3 + \frac{5}{2}O_2 \rightleftharpoons 2NO + 3H_2O$....(iv)

To get the equation (iv) from equation (i), (ii) and (iii) following steps are followed:

Reversing equation (i), we get

$2NH_3 \rightleftharpoons N_2 + 3H_2$ so $K' = \frac{1}{K_1}$.....(v)

Multiplying equation (iii) by 3, we get

$3H_2 + \frac{3}{2}O_2 \rightleftharpoons 3H_2O$ so $K' = K_3^3$.......(vi)

Adding equation (ii) and (vi)

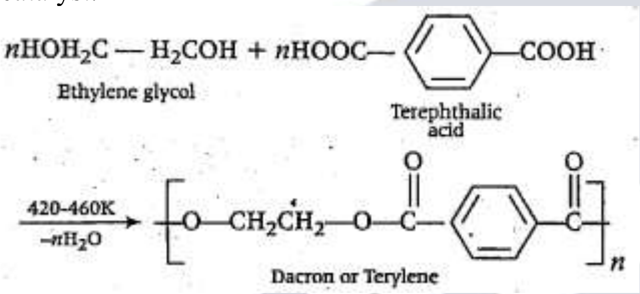
$$N_2 + 3H_2 + \frac{5}{2}O_2 \rightleftharpoons 2NO + 3H_2O$$

so, $K' = K_2 \cdot K_3^3$(vii)

On combining (v) and (vii), we get the required equation having equilibrium constant $K' = K_2 \cdot \frac{K_3^3}{K_1}$.

65. (c)

Dacron is a condensation polymer. It is also known as terylene. It is a polymer obtained by condensation reaction between ethylene glycol and terephthalic acid at $420 - 460$ K in the presence of zinc acetate-antimony trioxide catalyst.



66.(b) H_2SO_4 (sulphuric acid) is present in maximum amount in 'acid rain'. Oxides of nitrogen and sulphur, released into the atmosphere from thermal power plants, industries and automobiles are the main sources of acid rain. These oxides on oxidation followed by hydrolysis give sulphuric acid and nitric acid that alongwith HCl are responsible for the acidity of rain. The oxidation reaction is catalysed by particulate matter present in the polluted atmosphere.

$2SO_2(g) + O_2(g) + 2H_2O(l) \longrightarrow 2H_2SO_4(aq)$

$4NO_2(g) + O_2(g) + 2H_2O(l) \longrightarrow 4HNO_3(aq)$

67.(b) The correct set of oxides arranged in the proper order of basic, amphoteric, acidic are BaO, Al_2O_3, SO_2.

Oxides of non-metallic elements are acidic such as CO_2, NO_2, SO_2 etc. Oxides of less electropositive elements (such as $BeO, Al_2O_3, Bi_2O_3, ZnO$ etc.) are amphoteric i.e. these behaves as acids toward strong bases and as bases towards strong acids. Oxides of electropositive elements (Na_2O, CaO, Tl_2O, BaO etc.) are basic and contain discrete O^{2-} ions.

68. (b) Out of the given outer electronic configuration of atoms, the highest oxidation state is achieved by $(n-1)d^5ns^2$ i.e. 7 .

A large number of oxidation state is due to the fact the $(n-1)d$-electrons may get involved along with ns electrons in bonding as electrons in $(n-1)$ d-orbitals are in an energy state comparable to ns-electrons.

Oxidation state of other options are as follows:

Electronic configuration	Oxidation state
$(n-1)d^8ns^2$	$+2, +3, +4$
$(n-1)d^3ns^2$	$+2, +3, +4, +5$
$(n-1)d^5ns^1$	$+2, +3, +4, +5, +6$

69. (c) At room temperature, the reaction between water and fluorine produces F^-, O_2 and H^+.
Fluorine, being non-polar molecule, readily dissolves with water and forms mixture of oxygen and ozone as shown below

$$2\,F_2 + 2H_2O \longrightarrow 4HF + O_2$$
$$3\,F_2 + 3H_2O \longrightarrow 6HF + O_3$$

HF exist as $H^+$and F^-in a solution.

70. (d) $BeCO_3$ is least thermally stable. The thermal stability of carbonates increases down the group i.e from Be to Ba .

$$\underset{(523\ K)}{BeCO_3} < \underset{(813\ K)}{MgCO_3} < \underset{(1173\ K)}{CaCO_3} < \underset{(1562\ K)}{SrCO_3} < \underset{(1633\ K)}{BaCO_3}$$

$BeCO_3$ is unstable to the extent that it is stable only in atmosphere of CO_2. These carbonates show reversible decomposition in closed container.

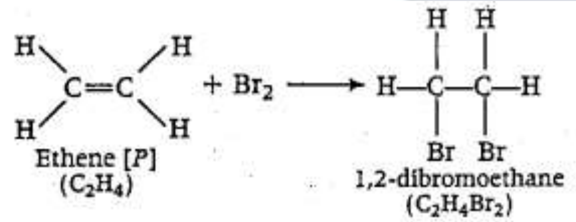
$$MCO_3 \rightleftharpoons MO + CO_2$$

More stable the oxide is formed, lesser will be stability of carbonates.

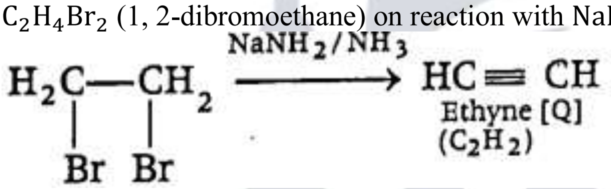
71. (a) Given reactions are

$$[P] \xrightarrow{Br_2} C_2H_4Br_2 \xrightarrow[NH_3]{NaNH_2} [Q]$$
$$[Q] \xrightarrow[Hg^{2+},\Delta]{20\%/H_2SO_4} [R] \xrightarrow{Zn\cdot Hg/HCl} [S]$$

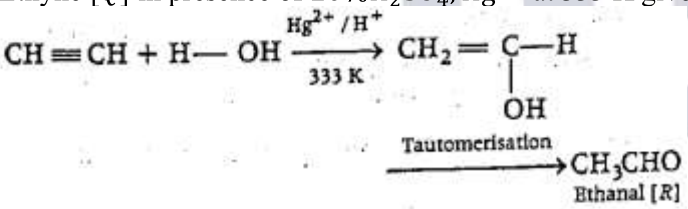
$[P]$ is an alkene with two carbon atoms i.e. ethene. When it reacts with Br_2, addition reaction occurs and $C_2H_4Br_2$ is formed.



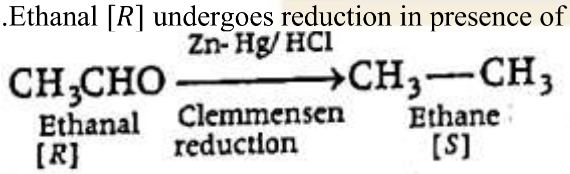
$C_2H_4Br_2$ (1, 2-dibromoethane) on reaction with $NaNH_2/NH_3$ gives ethyne (C_2H_2).



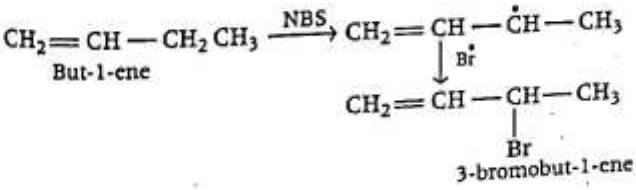
Ethyne $[Q]$ in presence of $20\%H_2SO_4, Hg^{2+}$ at 333 K gives ethanal.



.Ethanal $[R]$ undergoes reduction in presence of $Zn - Hg/HCl$ to give ethane [S]



72. (a) The number of possible organobromine compounds which can be obtained in the allylic bromination of but-l-ene with N -bromosuccinimide is 1 .



73. (b) Given, specific heat = 0.16

Let metal chloride be MCl_x then, $\frac{6.4}{\text{specific heat}}$ = Atomic weight of metal $\frac{6.4}{0.16}$ = Atomic weight of metal

Atomic weight = 40

40 is the atomic weight of calcium. According to question, metal chloride ($M\mathrm{Cl}_x$) have ≈ 65% chlorine present in it.

$$\frac{x \times \text{ Atomic weight of chlorine}}{40 + x \times \text{ Atomic weight of chlorine}} \times 100 = 65$$

$$\frac{x \times 35.5}{40 + x \times 35.5} \times 100 = 65$$

$$x = 209 \approx 2(\text{ approx. })$$

So, the formula of metal chloride will be MCl_2.

74. (a) For a reversible adiabatic process,

$$pT^{\frac{\gamma}{1-\gamma}} = \text{ constant } \ldots\ldots\text{(i)}$$

According to question, during a reversible adiabatic process the pressure of a gas is found to be proportional to the cube of its absolute temperature.

$$p \propto T^3$$

$$pT^{-3} = \text{ constant } \qquad \text{(ii)}$$

Equating equation (i) and (ii), we get

$$\frac{\gamma}{1-\gamma} = -3$$

$$\gamma = -3(1-\gamma)$$

$$\gamma = -3 + 3\gamma$$

$$-2\gamma = -3$$

$$\gamma = \frac{3}{2}$$

As we know, the ratio of molar heat capacities at constant pressure and constant volume is represented by γ. So, the ratio of $\frac{C_P}{C_V}$ for the gas is $\frac{3}{2}$.

75. (a) According to question,

$[X] + \text{Dil. } \mathrm{H_2SO_4} \longrightarrow [Y]$:

Colourless, suffocating gas

$[Y] + \mathrm{K_2Cr_2O_7} + \mathrm{H_2SO_4} \longrightarrow$

Green colouration of solution.

All sulphites when treated with dil. $\mathrm{H_2SO_4}$ gives colourless and suffocating sulphur dioxide gas.

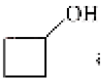
$$\underset{\substack{\text{Sulphite ion}\\[X]}}{\mathrm{SO_3^{2-}}(s)} + \text{Dil.}\mathrm{H_2SO_4}(aq) \longrightarrow \mathrm{SO_4^{2-}}(aq) + \underset{\substack{\text{Sulphur}\\ \text{dioxide}[Y]}}{\mathrm{SO_2}(g)} + \mathrm{H_2O}(l)$$

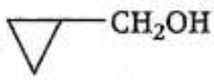
$\mathrm{SO_2}$ gas [Y] turns acidified potassium dichromate paper green due to reduction of Cr (VI) to Cr (III).

$$\mathrm{K_2Cr_2O_7}(aq) + \mathrm{H_2SO_4}(aq) + \underset{[Y]}{3\mathrm{SO_2}(g)} \longrightarrow$$

$$\mathrm{K_2SO_4}(aq) + \underset{\text{Green}}{\mathrm{Cr_2(SO_4)_3}(aq)} + \underset{2}{\mathrm{H_2O}(l)}$$

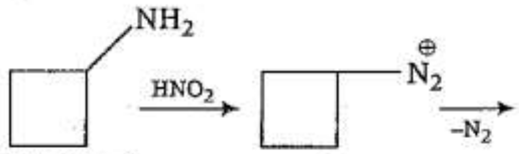
76. (a, c) The possible product(s) to be obtained from the reaction of cyclobutyl amine with $\mathrm{HNO_2}$ is/are

cyclobutanol = 

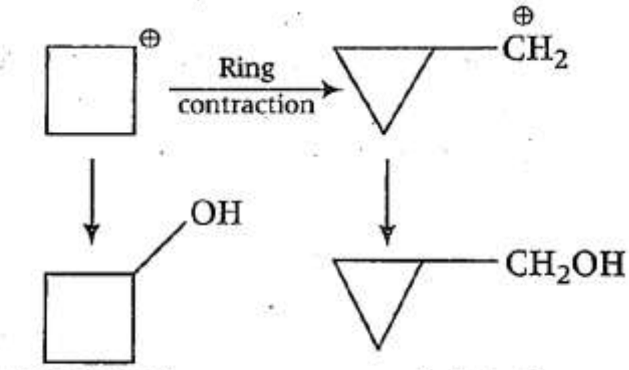
and cyclopropylmethanol = 

:

. The chemical reactions involved are as follows



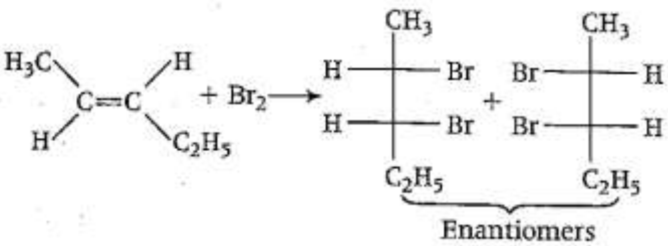
Cyclobutylamine



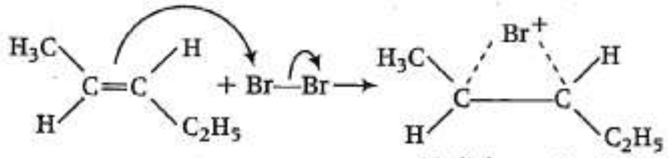
Cyclobutanol

Cyclobutyl methanol

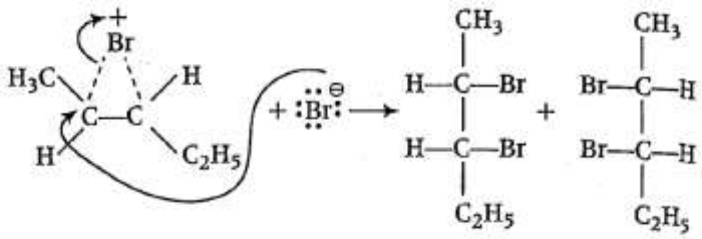
77. ($\boldsymbol{a}, \boldsymbol{d}$) The major product(s) obtained in the given reaction are



The mechanism for the above given reaction is as follows. When the π-electrons of the alkene approach a molecule of Br_2, one of the bromine atom accepts them and releases the shared electrons to the other bromine atom. A cyclic bromonium ion is formed.


(Cyclic bromonium ion)

In next step, Br^{-}attacks a carbon atom of the bromonium ion. This release the strain in the three membered ring and form a vicinal dibromide.



78. **(b, c)** The correct statements for peroxide ion (O_2^{2-}) are that it is diamagnetic and it has bond order one.

Electronic configuration of O_2^{2-} (peroxide ion) is $\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, [\pi 2p_x^2 = \pi 2p_y^2]$
$[\pi^* 2p_x^2 = \pi^* 2p_y^2], \sigma^* 2p_z^0$

$\because$ Bond order =

No. of electrons present in bonding

No. of electrons present in non-bonding

2

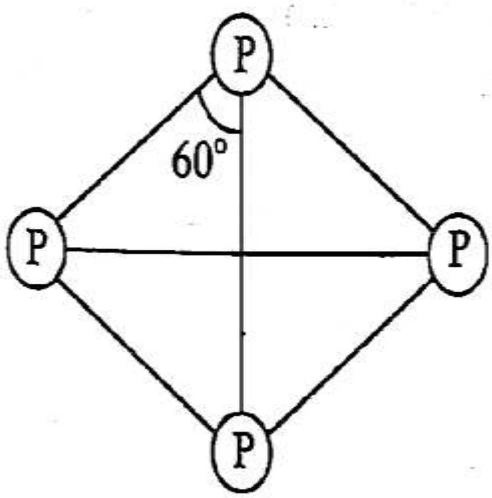
$$\therefore \ \mathrm{BO} = \frac{10-8}{2} = \frac{2}{2} = 1$$

Number of unpaired electrons $= 0$

So, O_2^{2-} is diamagnetic in nature.

79. **($\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}$)** Among the given options, the extensive variables are H (enthalpy), E (internal energy), V (volume).These variables have values that depends upon the quantity or size of matter present in the system. Other examples are heat capacity, entropy, free energy, length and mass.

80. (a, c, d) White phosphorus (P_4) has 6 P — P single bonds, 4 lone pairs of electrons and P − P − P angle of 60°. It is a translucent white waxy solid. It has large atomic size and less electronegativity. So, it forms single bond instead of $p\pi - p\pi$ multiple bond. It consists of discrete tetrahedral P_4 molecule as shown in the figure.



Mathematics

81. (a) We know that, $\sin 30° = \frac{1}{2} = 0.5$

In 1st quadrant $\sin x$ is increasing function.

$\therefore \quad \sin 31° > \sin 30°$

$\Rightarrow \quad \sin 31° > 0.5$

82. (c) We have, $f_1(x) = e^x$

$f_2(x) = e^{f_1(x)}$

Now, $f_n(x) = e^{f_{n-1}(x)}$

On taking log both sides, we get

$\log\{f_n(x)\} = f_{n-1}(x) \log e$

$\frac{d}{dx}\log(f_n(x)) = \frac{d}{dx} f_{n-1}(x) \; (\because \log e = 1)$

$\frac{1}{f_n(x)} \frac{d}{dx} f_n(x) = f'_{n-1}(x)$

$\frac{d}{dx} f_n(x) = f_n(x) f'_{n-1}(x)$ (i)

Now, $f_1'(x) = e^x$

and $f_2(x) = e^{f_1(x)}$

$\Rightarrow \log f_2(x) = f_1(x) \log e = f_1(x)$

$\Rightarrow \frac{1}{f_2(x)} \cdot f_2'(x) = f_1'(x)$

$\Rightarrow f_2'(x) = f_2(x) \cdot f_1'(x)$

$= f_2(x) \cdot e^x \; (\because f_1'(x) = e^x)$

$= f_2(x) \cdot f_1(x), [\because e^x = f_1(x)] \ldots$ (ii)

From Eq. (i),

$\frac{d}{dx} f_n(x) = f_n(x) \cdot f_{n-1}(x) \; \ldots . f_1(x)$ [using Eq. (ii)]

83. (b) We have, $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$

$f(x)$ is defined for all x satisfying

$$\frac{1-|x|}{2-|x|} \geq 0 \Rightarrow \frac{|x|-1}{|x|-2} \geq 0$$

$\Rightarrow \quad |x| \leq$ 1or $|x| > 2$

$\Rightarrow \quad x \in [-1,1]$ or $x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow \quad x \in (-\infty, -2) \cup [-1,1] \cup (2, \infty)$

84. (c) We have, $f: [a, b] \longrightarrow R$ be differentiable on $[a, b]$ and $k \in R$, also $f(a) = 0 = f(b)$

and $J(x)' = f'(x) + kf(x)$
Let $g(x) = kxf(x)$ which is continuous in $[a, b]$ and differentiable in (a, b) such that
$g(a) = 0 = g(b)$
Then, for every $c \in (a, b), g'(c) = 0$
(by Rolle's theorem)
Now, $g'(x) = kf(x) + kxf'(x)$
$\Rightarrow \quad g(c) = kf(c) + kcf'(c)$
$\Rightarrow \quad kf(c) + kcf'(c) = 0$
$\Rightarrow f(x) = 0$,for every $x = . c \in (a, b)$
$\therefore J(x) = 0$ has atleast one root in (a, b).

85. (c) We have,

$f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$
$\therefore f(1-h) = 3(1-h)^{10} - 7(1-h)^8$
$+5(1-h)^6 - 21(1-h)^3 + 3(1-h)^2 - 7$
$= 3(1 - 10h + 45h^2 - 120h^3 + \cdots \ldots + h^{10})$
$-7(1 - 8h + 28h^2 - 56h^3 + \cdots .. + h^8)$
$+5(1 - 6h + 15h^2 - 20h^3 + \cdots .. + h^6)$
$-2\mathrm{i}(1 - 3h + 3h^2 - h^3)$
$+3(1 - 2h + h^2) - 7$
$\Rightarrow f(1-h) = -24 + 53h + h^2(-46) + h^3(-47) + \cdots .$
and
$f(1) = -24$

$$\therefore \lim_{h \to 0} \frac{f(1-h) - f(1)}{h^3 + 3h}$$
$$= \lim_{h \to 0} \frac{-24 + 53h + h^2(-46) + h^3(-47) + \cdots - (-24)}{h(h^2 + 3)}$$
$$= \lim_{h \to 0} \frac{53h + h^2(-46) + h^3(-47) + \cdots}{h(h^2 + 3)}$$
$$= \lim_{h \to 0} \frac{53 + h(-46) + h^2(-47) + \cdots}{h^2 + 3} = \frac{53}{3}$$

86. (a) Let $g(x) = e^{-x} f(x)$
such that $g(a) = 0, g(b) = 0$ and $g(x)$ is continuous and differentiable.
Then, for atleast one value of $c \in (a, b)$ such that $g'(c) = 0$
Now, $g'(x) = e^{-x} f'(x) + (-e^{-x}) f(x)$
$\Rightarrow \ g'(c) = e^{-c} f'(c) + (-e^{-c}) f(c) = 0$
$\Rightarrow \ e^{-c} f'(c) = e^{-c} f(c) \Rightarrow f'(c) = f(c)$

87. (b) We have,
$f: R \to R$ be a twice continuously differentiable function such that $f(0) = f(1) = f'(0) = 0$
Now, for atleast one value of $c_1 \in (0,1)$,

$f'(c_1) = 0$ (by Rolle's theorem)
Again, $f'(0) = 0 = f'(c_1)$
$\Rightarrow$ $f''(c) = 0$ for some $c \in (0, c_1)$
(by Rolle's theorem)

88. (b) We have,

$\int e^{\sin x}\left(\frac{x\cos^3 x-\sin x}{\cos^2 x}\right)dx = e^{\sin x}f(x)+c$

$\Rightarrow \int e^{\sin x}(x\cos x - \sec x\tan x)dx = e^{\sin x}f(x)+c$

$\Rightarrow \int e^{\sin x}(x\cos x - 1 + 1 - \sec x\tan x)dx$

$\Rightarrow = e^{\sin x}f(x)+c$

$\Rightarrow \left[e^{\sin x}\cos x(x-\sec x) + e^{\sin x}(1-\sec x\tan x)\right]dx$

$\Rightarrow = e^{\sin x}f(x)+c$

$\Rightarrow \int \frac{d}{dx}\{e^{\sin x}(x-\sec x)\}dx = e^{\sin x}f(x)+c$

$\Rightarrow e^{\sin x}(x-\sec x) = e^{\sin x}f(x)+c$

$f(x) = x - \sec x$

89. (c) We have,

$$\int f(x)\sin x\cos x dx = \frac{1}{2(b^2-a^2)}\log(f(x))+c$$

$$\Rightarrow f(x)\sin x\cos x = \frac{1}{2(b^2-a^2)}\cdot\frac{1}{f(x)}\cdot f'(x)$$

$$\Rightarrow f(x)\sin 2x = \frac{1}{b^2-a^2}\cdot\frac{f'(x)}{f(x)}$$

$$\Rightarrow \sin 2x = \frac{1}{b^2-a^2}\frac{f'(x)}{(f(x))^2}$$

$$\Rightarrow f\sin 2x dx = \frac{1}{b^2-a^2}\int\frac{f'(x)}{(f(x))^2}dx$$

$$\Rightarrow \frac{-\cos 2x}{2} = \frac{1}{b^2-a^2}\cdot\left(\frac{-1}{f(x)}\right)$$

$$\Rightarrow \frac{\cos 2x(b^2-a^2)}{2} = \frac{1}{f(x)}$$

$$\Rightarrow f(x) = \frac{2}{(b^2-a^2)\cos 2x}$$

90. (d) Given, $M = \int_0^{\pi/2}\frac{\cos x}{(x+2)}dx$

and

$$N = \int_0^{\pi/4}\frac{\sin x\cos x}{(x+1)^2}dx$$

$$= \int_0^{\pi/4}\frac{1}{2}\cdot\frac{\sin 2x}{(x+1)^2}dx$$

Put. $2x = t \Rightarrow dx = \frac{dt}{2}$

$$\therefore N = \int_0^{\pi/2}\frac{\sin t}{4(t/2+1)^2}dt$$

$$= \int_0^{\pi/2}\frac{\sin t}{(t+2)^2}dt$$

$$\therefore M - N = \int_0^{\pi/2}(\cos x)\cdot\frac{1}{(x+2)}dx$$

$$-\int_0^{\pi/2}\frac{\sin x}{(x+2)^2}dx$$

$$= \left(\frac{\sin x}{x+2}\right)_0^{\pi/2} - \int_0^{\pi/2} -\frac{\sin x}{(x+2)^2}dx$$

$$= \frac{\sin\pi/2}{\pi/2+2} = \frac{1}{\frac{\pi+4}{2}} = \frac{2}{\pi+4}$$

91. (b) We have,

Let $\quad I=\int_{1/2014}^{2014}\frac{\tan^{-1}x}{x}dx$

$\Rightarrow \quad x=\frac{1}{t}$

Now, $\quad dx=\frac{-1}{t^2}dt$

$$I=\int_{2014}^{1/2014}\frac{\tan^{-1}(1/t)}{1/t}\left(\frac{-1}{t^2}dt\right)$$

$$=\int_{1/2014}^{2014}\frac{\cot^{-1}t}{t}dt$$

$$=\int_{1/2014}^{2014}\frac{\cot^{-1}x}{x}dx$$

On adding Eqs. (i) and (ii), we get

$$2I=\int_{1/2014}^{2014}\frac{\pi/2}{x}dx=\frac{\pi}{2}(\log x)_{1/2014}^{2014}$$

$$=\frac{\pi}{2}(\log 2014-\log 1/2014)$$

$$\Rightarrow I=\frac{\pi}{4}\left(\log 2014-\log\frac{1}{2014}\right)$$

$$=\frac{\pi}{4}(\log 2014+\log 2014)$$

$$=\frac{\pi}{4}(2\log 2014)=\frac{\pi}{2}\log 2014$$

92. (c) We have,

$$I=\int_{\pi/4}^{\pi/3}\frac{\sin x}{x}dx$$

Since, $\frac{\sin x}{x}$ is a decreasing function.

$$\therefore \quad \frac{\pi}{12}\times\frac{\sin\pi/3}{\pi/3}\le I\le\frac{\pi}{12}\times\frac{\sin\pi/4}{\pi/4}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{8}\le I\le\frac{\sqrt{2}}{6}$$

93. (b) Let

$$I=\int_{\pi/2}^{5\pi/2}\frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)}+e^{\tan^{-1}(\cos x)}}dx$$

$$=\int_{0}^{5\pi/2}\frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)}+e^{\tan^{-1}(\cos x)}}dx$$

$$-\int_{0}^{\frac{\pi}{2}}\frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)}+e^{\tan^{-1}(\cos x)}}dx \ldots.(I)$$

$$I=\int_{0}^{5\pi/2}\frac{e^{\tan-1}(\cos x)}{e^{\tan^{-1}(\cos x)}+e^{\tan^{-1}(\sin x)}}dx$$

$$-\int_{0}^{\frac{\pi}{2}}\frac{e^{\tan^{-1}(\cos x)}}{e^{\tan^{-1}(\cos x)}+e^{\tan^{-1}(\sin x)}}dx \ldots.(II)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{5\pi/2} dx - \int_0^{\pi/2} dx$$

$$\Rightarrow \quad 2I = (x)_0^{5\pi/2} - (x)_0^{\pi/2}$$

$$\Rightarrow \quad 2I = \frac{5\pi}{2} - \frac{\pi}{2} = 2\pi \Rightarrow I = \pi$$

94. **(c)** We have,

$$\lim_{n\to\infty} \frac{1}{n}\left\{\sec^2\frac{\pi}{4n} + \sec^2\frac{2\pi}{4n} + \cdots. + \sec^2\frac{n\pi}{4n}\right\}$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\sec^2\left(\frac{r\pi}{4n}\right) = \int_0^1 \sec^2\left(\frac{\pi x}{4}\right)$$

$$= \frac{4}{\pi}\left[\tan\left(\frac{\pi x}{4}\right)\right]_0^1$$

$$= \frac{4}{\pi} \times 1 = \frac{4}{\pi}$$

95. **(c)** Given, $y^2 = 2d(x + \sqrt{d})$

$\Rightarrow 2yy_1 = 2d \Rightarrow d = yy_1$

From Eq. (i),

$$y^2 = 2yy_1(x + \sqrt{yy_1})$$

$$\Rightarrow y^2 - 2yy_1x = \sqrt{yy_1} \cdot 2yy_1$$

$\Rightarrow (y^2 - 2yy_1x)^2 = 4(yy_1)^3$

So, degree of above equation is 3 .

96. **(c)** We have,

$$(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1 + x^2}\right)y = \frac{4x^2}{1 + x^2}$$

Here, IF $= e^{\int \frac{2x}{1+x^2}} = e^{\log(1+x^2)} = 1 + x^2$

$$\therefore y(1 + x^2) = \int (1 + x^2) \times \frac{4x^2}{(1 + x^2)} dx + C$$

$$\Rightarrow y(1 + x^2) = \int 4x^2 dx + C$$

$$\Rightarrow y(1 + x^2) = \frac{4x^3}{3} + C$$

$$\Rightarrow y(1 + x^2) = \frac{4x^3}{3} - 1 \ (\mathrm{y}(0) = -1)$$

$$\Rightarrow y = \frac{4x^3}{3(1 + x^2)} - \frac{1}{1 + x^2}$$

$$\therefore y(1) = \frac{4}{6} - \frac{1}{2} = \frac{1}{6}$$

97. **(c)** We have, $x = \frac{1}{2}vt$

$\Rightarrow \quad x=\frac{1}{2}\frac{dx}{dt}t\ (v=\frac{dx}{dt})$

$\Rightarrow \quad \frac{2dt}{t}=\frac{dx}{x}$

$\Rightarrow \quad 2\cdot\int\frac{dt}{t}=\int\frac{dx}{x}$

$\Rightarrow \quad 2\log|t|+\log|c|=\log|x|_{\text{a}}$

$\Rightarrow \quad \log(t^2\cdot c)=\log x$

$\Rightarrow \quad x=t^2c$

$\Rightarrow \quad \frac{dx}{dt}=2tc$

$\Rightarrow \quad \frac{d^2x}{dt^2}=2c$

$\Rightarrow$ acceleration f is constant.

98. (b) We have,

equation of parabola $y=x^2$

Let $P(\boldsymbol{\alpha},\boldsymbol{\alpha}^2)$ is a point on the parabola,

$\therefore \quad y-\alpha^2=2\alpha(x-\alpha)$

$\because \quad \left[\because \frac{dy}{dx}=2x\Rightarrow\frac{dy}{dx}=2\alpha\right]$

$\Rightarrow \quad y=2\alpha x-\alpha^2$

Also, given $y=-x^2+4x-4$

$\therefore\ -x^2+4x-4=2\alpha x-\alpha^2$

$\Rightarrow\ x^2+2x(\alpha-2)+(4-\alpha^2)=0$

Discriminant $=0$

$$4(\alpha-2)^2-4(4-\alpha^2)=0$$

$$\Rightarrow \quad (\alpha-2)^2-(4-\alpha^2)=0$$

$$\Rightarrow \quad \alpha^2-4\alpha+4-4+\alpha^2=0$$

$$\Rightarrow \quad \alpha^2-2\alpha=0$$

$$\Rightarrow \quad \alpha=0,\alpha=2$$

99. (d) $2b=(n+2)$ th form

$=a+(n+2-1)d$

$\Rightarrow \quad 2b=a+(n+1)d$

$\Rightarrow \quad d=\frac{2b-a}{n+1}$

$\therefore$

m th mean $=a+m\left(\frac{2b-a}{n+1}\right)$

and also,

$b=(n+2)$ th form

$=2a+(n+2-1)d$

$=2a+(n+1)d$

$d=\frac{b-2a}{n+1}$

$\Rightarrow\ d=\frac{b-2a}{n+1}$

$\therefore\ m$ th mean $=2a+m\left(\frac{b-2a}{n+1}\right)$.

According to the question,

$$a+m\left(\frac{2b-a}{n+1}\right)=2a+m\left(\frac{b-2a}{n+1}\right)$$

$$\Rightarrow \quad \frac{a}{b}=\frac{m}{n+1-m}$$

100. **(c)** We have,

$$x+\log_{10}(1+2^x)=x\log_{10}5+\log_{10}6$$

$$\Rightarrow \log_{10}(1+2^x)=x\log_{10}5+\log_{10}6-x$$

$$=\log_{10}5^x+\log_{10}6-x\log_{10}10$$

$$\log_{10}(1+2^x)=\log_{10}\left(\frac{5^x\cdot 6}{10^x}\right)$$

$$1+2^x=\frac{5^x\cdot 6}{10^x}=\frac{6}{2^x}$$

$$2^x(1+2^x)=6$$

$$(\text{let}2^x=t)$$

$$t(1+t)=6$$

$$t^2+t-6=0$$

$$(t+3)(t-2)=0$$

$$t+3=0$$

$$\text{or } t-2 \quad =0$$

$$\Rightarrow t \quad =2(neglect\ t=-3)$$

$$\Rightarrow 2^x \quad =2\Rightarrow x=1$$

101. **(b)** We have, $Z_r=\sin\frac{2\pi r}{11}-i\cos\frac{2\pi r}{11}$

$$=-i\left(\cos\frac{2\pi r}{11}+i\sin\frac{2\pi r}{11}\right)$$

$$=-ie^{\frac{i2\pi r}{11}}$$

$$\therefore \sum_{r=0}^{10}Z_r=-i\sum_{r=0}^{10}e^{\frac{i2\pi r}{11}}$$

$$=-i\times 0=0$$

102. **(c)** We know that, if z_1, z_2 and z_3 are the vertices of an equilateral triangle. Then,

$z_1^2+z_2^2+z_3^2-z_1z_2-z_2z_3-z_3z_1=0$.....(i)

Now, but we have

$$\frac{z_1}{z_2}+\frac{z_2}{z_1}=1$$

$$\Rightarrow \quad z_1^2+z_2^2=z_1z_2$$

$$\Rightarrow \quad z_1^2+z_2^2-z_1z_2=0$$

Here, $\quad z_3=0$

Hence, given points form an equilateral triangle.

103. **(a)** We have equations

$$x^2+b_1x+c_1=0$$

$$D_1=b_1^2-4c_1$$

and .

$$x^2+b_2x+c_2=0$$

$$D_2=b_2^2-4c_2$$

$$Now D_1+D_2=b_1^2+b_2^2-4(c_1+c_2)$$

$$=b_1^2+b_2^2-2b_1b_2\ [\because b_1b_2=2(c_1+c_2)]$$

$$=(b_1-b_2)^2\geq 0$$

104. **(a)** Number of ways of selection of n objects from $2n$ objects, where as n objects are identical in out of $2n$ objects.

n identical and no different object $=1$ ways

$(t))=\ ^nC_0$

$n-1$ identical and 1 different object $=1\times\ ^nC_1$

$n-2$ identical and 2 different object $=1\times\ ^nC_2$

……………………………………………..
0 identical and n different objects $= 1 \times {}^nC_n$
$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots \ldots + {}^nC_n = 2^n$

105. **(c)** ${}^nC_r + 2 \cdot {}^nC_{r+1} + {}^nC_{r+2}$
$= {}^nC_r + {}^nC_{r+1} + {}^nC_{r+1} + {}^nC_{r+2}$
$= {}^{n+1}C_{r+1} + {}^{n+1}C_{r+2}$
$(\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$
$= {}^{n+2}C_{r+2}$

106. **(a)** $(101)^{100} - 1 = (1+100)^{100} - 1$
$=(1 + {}^{100}C_1 \cdot 100 + {}^{100}C_2 100^2 + \cdots \ldots) - 1$
$= {}^{100}C_1 100 + {}^{100}C_2(100)^2 +$
${}^{100}C_3(100)^3 + \cdots \ldots . + {}^{100}C_{100}(100)^{100}$
$=10^4(1 + {}^{100}C_2 + {}^{100}C_3 10^2 + \cdots .$
$+ {}^{100}C_{100}(100)^{98}$
$=10^4(1 +$ an integer multiple of 10)

107. **(a)** For greatest term of $(1+x)^n$, we have

$$\frac{n}{2} < \frac{n+1}{1+x} < \frac{n}{2} + 1$$

$$\Rightarrow \quad \frac{n}{2} < \frac{n+1}{1+x} \text{ and } \frac{n+1}{1+x} < \frac{n}{2} + 1$$

$$\Rightarrow \quad 1 + x < \frac{n+1}{n/2} \text{ and } \frac{n+1}{\frac{n}{2}+1} < 1 + x$$

$$\Rightarrow \quad x < \frac{n+1-n/2}{n/2}$$

$$\text{and} \quad \frac{n+1-(n/2-1)}{\frac{n+2}{2}} < x$$

$$\Rightarrow \quad x < \frac{n+2}{n} \text{ and } \frac{n}{n+2} < x$$

$$\Rightarrow \quad \frac{n}{n+2} < x < \frac{n+2}{n}$$

108. **(a)** We have,

$$A = \begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix}$$

$= -1(1+12) - 7(2+9)$
$= -13 - 77 = -90 \ldots$(i)

$$\text{And let } B = \begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$$

$$= 3 \times 5 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ -7 & -1 & -1 \end{vmatrix}$$

$$= 15 \begin{vmatrix} 13 & -11 & 1 \\ -7 & -1 & 5 \\ 0 & 0 & -6 \end{vmatrix} (R_3 \to R_3 - R_2)$$

$= 15\{0 - 0 - 6(-13 - 77)\}$
$= 15\{(-6)(-90)\} = 90 \times 90 \ldots .$(ii)
From Eqs. (i) and (ii),
$B = A^2$

109. **(c)** We have,

$$a_r = (\cos 2r\pi + i\sin 2r\pi)^{1/9} = e^{\frac{2r\pi i}{9}}$$

Now, $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} & e^{\frac{6\pi i}{9}} \\ e^{\frac{8\pi i}{9}} & e^{\frac{10\pi i}{9}} & e^{\frac{12\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}$

$= e^{\frac{2\pi i}{9}} \times e^{\frac{8\pi i}{9}} \begin{vmatrix} 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ 1 & e^{\frac{2\pi i}{9}} & e^{\frac{4\pi i}{9}} \\ e^{\frac{14\pi i}{9}} & e^{\frac{16\pi i}{9}} & e^{\frac{18\pi i}{9}} \end{vmatrix}$

$= e^{\frac{2\pi i}{9}} \times e^{\frac{8\pi i}{9}} \times 0$ ($\because R_1$ and R_2 are identical)

$= 0$

110. **(d)** We have, $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2 - 1 & y & n^2(2n+3) \\ 4r^3 - 2nr & z & n^3(n+1) \end{vmatrix}$

$\Rightarrow \sum_{r=1}^{n} S_r = \begin{vmatrix} 2\sum_{r=1}^{n} r & x & n(n+1) \\ \sum_{r=1}^{n} (6r^2 - 1) & y & n^2(2n+3) \\ \sum_{r=1}^{n} (4r^3 - 2nr) & z & n^3(n+1) \end{vmatrix}$

$= \begin{vmatrix} n(n+1) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix}$

$= 0$ ($\because C_1$ and C_3 are identical)

Hence, $\sum_{r=1}^{n} S_r$ is independent of x, y, z and n.

111. **(c)** For non-trivial solution,

We have, $\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$

$\Rightarrow 1(3bc - 2bc) - (4ac - 2ac) + 1(4ab - 3ab) = 0$

$\Rightarrow \quad bc - 2ac + ab = 0$

$\Rightarrow \quad bc + ab = 2ac$

$\Rightarrow \quad b(a+c) = 2ac$

$\Rightarrow \quad b = \dfrac{2ac}{a+c}$

$\Rightarrow a, b, c$ are in HP.

112. **(c)** On the set R,

$x\rho y \Rightarrow x - y$ is zero or irrational number.

Now, $\quad x\rho x$

$\Rightarrow \quad x - x = 0$

$\Rightarrow \rho$ is reflexive.

If $x\rho y \Rightarrow x - y$ is zero or irrational.

$= -(y - x)$ is zero or irrational.

$\Rightarrow y\rho x$ is zero or irrational.

$\Rightarrow \rho$ is symmetric.

And if

$x\rho y \Rightarrow x - y$ is 0 or irrational.

$y\rho z \Rightarrow y - z$ is 0 or irrational.

Then, $(x - y) + (y - z) = x - z$ may be 0 or rational.

$\Rightarrow \rho$ is not transitive.

113. **(d)** On the set R of real numbers

For reflexive,

$x\rho x \Rightarrow (x, x) \in R$
$\Rightarrow x > |x|$ which is not true.
$\Rightarrow \rho$ is not reflexive.
For symmetric,
$(x, y) \in R \Rightarrow x > |y|$
and $(y, x) \in R \Rightarrow y > |x|$
So, $x > |y| \neq y > |x|$
$\Rightarrow \rho$ is not symmetric.
For transitive,
$(x, y) \in R \Rightarrow x > |y| (y, z) \in R \Rightarrow y > |z|$
$\Rightarrow x > |z| \Rightarrow (x, z) \in R$
$\Rightarrow \rho$ is transitive.

114. **(c)** We have, $f: R \to R$, defined by $f(x) = e^x$
and $g: R \to R$ defined by $g(x) = x^2$
Now, We have
$(g \circ f)(x) = g(f(x))$
$= g(e^x)$
$= (e^x)^2$
$= e^{2x}, \forall x \in R$
$\Rightarrow g \circ f$ is injective and g is neither injective nor surjective.
$\Rightarrow g \circ f$ is injective but $g(x)$ is not bijective.

115. **(b)** We have, $P(H) = \frac{1}{2}, P(T) = \frac{1}{2}$
Now, $P(X \geq 1) = 1 - P(X = 0)$
$= 1 - {}^nC_0(p)^0(q)^n = 1 - \left(\frac{1}{2}\right)^n$
$\because \quad 1 - \frac{1}{2^n} \geq 0.9$
$\Rightarrow \quad 01 \geq \frac{1}{2n}$
$\Rightarrow \quad \frac{1}{10} \geq \frac{1}{2^n}$
$\Rightarrow \quad 2^n \geq 10$
$\Rightarrow \quad n \geq 4$
$\therefore$ Minimum number of tossed $= 4$

116. **(a)**Let X be the event that student will be successful.
X_1 be the event that student will be pass in test-I.
X_2 be the event that student will be pass in test-II.
X_3 be the event that student will be pass in test-III.
$\therefore P(X) = P(X_1 \cap X_2 \cap X_3') + P(X_1 \cap X_2' \cap X_3)$
$+ P(X_1 \cap X_2 \cap X_3)$
$\Rightarrow P(X) = P(X_1) \cdot P(X_2) \cdot P(X_3') + P(X_1) \cdot P(X_2') \cdot P(X_3)$
$+ P(X_1) \cdot P(X_2) \cdot P(X_3)$
$\Rightarrow \frac{1}{2} = p \cdot q \cdot \frac{1}{2} + p(1 - q)\frac{1}{2} + pq\frac{1}{2}$
$\Rightarrow \frac{1}{2} = p \cdot q \cdot \frac{1}{2} + p \cdot \frac{1}{2} - p \cdot q \cdot \frac{1}{2} + p \cdot q \cdot \frac{1}{2}$
$\Rightarrow \frac{1}{2} = p \cdot \frac{1}{2} + p \cdot q \cdot \frac{1}{2}$
$\Rightarrow (p + pq) = 1$
$\Rightarrow p(1 + q) = 1$

117. **(a)** We have, $\sin 6\theta + \sin 4\theta + \sin 20 = 0$

$\Rightarrow \sin 6\theta + \sin 2\theta + \sin 4\theta = 0$
$\Rightarrow 2\sin 4\theta \cdot \cos 2\theta + \sin 4\theta = 0$
$\Rightarrow \sin 4\theta(2\cos 2\theta + 1) = 0$
$\Rightarrow \sin 4\theta = 0$ or $2\cos 2\theta + 1 = 0$
$\Rightarrow 4\theta = n\pi$ or $\cos 2\theta = \frac{-1}{2} = \cos\frac{2\pi}{3}$
$\Rightarrow \theta = \frac{n\pi}{4}$ or $2\theta = 2n\pi \pm \frac{2\pi}{3}$
$\Rightarrow \theta = \frac{n\pi}{4}$ or $\theta = n\pi \pm \frac{\pi}{3}$, where $n \in z$

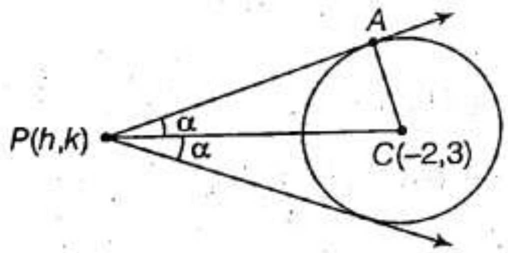
118. **(c)** We have, $0 \le A \le \frac{\pi}{4}$

$$\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$$
$$= \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\frac{\cot A + \cot^3 A}{1 - \cot^4 A}\right)$$
$$= \tan^{-1}\left(\frac{1}{2}\cdot\frac{2\tan A}{1-\tan^2 A}\right) + \tan^{-1}\left(\frac{\tan A}{\tan^2 A - 1}\right)$$
$$= \tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right) - \tan^{-1}\left(\frac{\tan A}{1-\tan^2 A}\right)$$
$$= 0$$

119. **(b)** Put $x = x' + 2$
and
$y = y' + 3$
$\therefore \quad x^2 + y^2 - 4x - 6y + 9 = 0$
$\Rightarrow \quad (x'+2)^2 + (y'+3)^2 - 4(x'+2)$
$\Rightarrow \quad -6(y+3) + 9 = 0$
$\quad x'^2 + 4 + 4x' + y'^2 + 9$
$\Rightarrow \quad +6y' - 4x' - 8 - 6y' - 18 + 9 = 0$
$\Rightarrow \quad x'^2 + y'^2 - 4 = 0$
$\Rightarrow \quad x^2 + y^2 = 4$

120. **(d)** We have equation of circle
$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$
Here, $C \equiv (-2,3)$

Radius $= \sqrt{(-2)^2 + (3)^2 - (9\sin^2\alpha + 13\cos^2\alpha)}$
$= \sqrt{4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha}$
$= \sqrt{13 - 13(1-\sin^2\alpha) - 9\sin^2\alpha}$
$= \sqrt{13\sin^2\alpha - 9\sin^2\alpha}$
$= \sqrt{4\sin^2\alpha} = 2\sin\alpha$

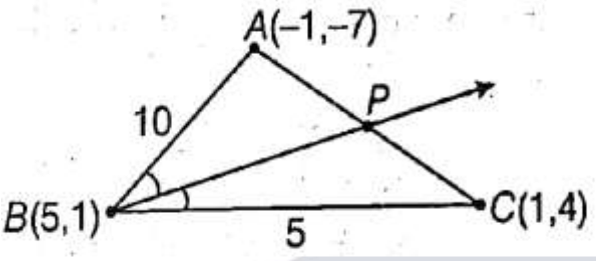


Here, $\sin\alpha = \frac{AC}{PC}$

$\Rightarrow \quad PC\sin\alpha = AC$
$\Rightarrow \quad PC^2\sin^2\alpha = AC^2 = (2\sin\alpha)^2$
$\Rightarrow \quad [(h+2^2+(k-3)^2]\sin^2\alpha = 4\sin^2\alpha$
$\Rightarrow \quad (h+2^2+(k-3)^2 = 4$
$\Rightarrow \quad h^2+4+4h+k^2+9-6k = 4$
$\Rightarrow \quad h^2+k^2+4h-6k+9 = 0$
Hence, locus of a point is
$x^2+y^2+4x-6y+9=0$

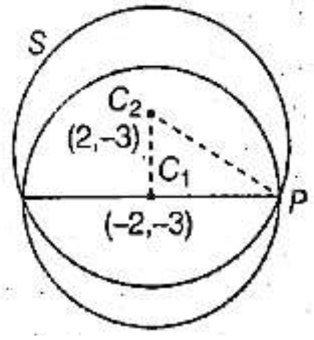
121. **(d)** Given, point $P(1,5)$ image of the point $P(1,5)$ about the line $y=x$ is $Q(5,1)$ and image of the point Q on line $y=-x$ is $R(-1,-5)$
$\therefore$ Required circumcentre = Mid-point of P and R
$=\left(\frac{1-1}{2},\frac{5-5}{2}\right)=(0,0)$

122. **(b)** Here, $AB=\sqrt{(5+1)^2+(1+7)^2}=\sqrt{36+64}=10$



$BC=\sqrt{(1-5)^2+(4-1)^2}=\sqrt{16+9}=5$
By angle bisector theorem,
$$AP:CP=10:5=2:1$$
$\therefore P\left(\frac{2\times1+1\times(-1)}{2+1},\frac{2\times4+1\times(-7)}{2+1}\right)=P\left(\frac{1}{3},\frac{1}{3}\right)$
Required equation of BP is.
$$y-1=\frac{\frac{1}{3}-1}{\frac{1}{3}-5}(x-5)$$
$\Rightarrow \quad y-1=\frac{-2}{-14}(x-5)$
$\Rightarrow \quad 7y-7=x-5$
$\Rightarrow \quad 7y=x+2$

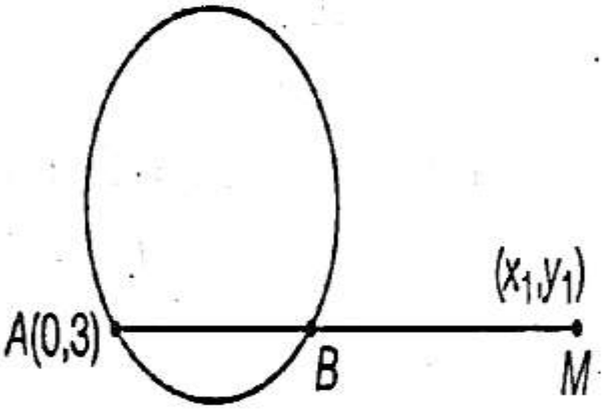
123. **(a)** Given, equation of circle is
$x^2+y^2+4x+6y-12=0$
Centre $C(-2,-3)$
and radius $=\sqrt{(-2)^2+(-3)^2+12}=5$



$x^2+y^2+4x+6y-12=0$
$\therefore \quad C_1C_2=\sqrt{(2+2)^2+(-3+3)^2}$
$\qquad =\sqrt{(4)^2+(0)^2}=4$
$\therefore$ Radius of circle, S

$= \sqrt{(4)^2 + (5)^2} = \sqrt{16+25} = \sqrt{41}$ unit

124. **(c)** Given, $AM = 2AB$



$\Rightarrow B$ is mid-point of AM.

$\therefore$ Coordinate of B is $\left(\frac{0+x_1}{2}, \frac{3+y_1}{2}\right)$

$= \left(\frac{x_1}{2}, \frac{y_1+3}{2}\right)$

Since, B lies on the circle $x^2 + 4x + (y-3)^2 = 0$

$\therefore \left(\frac{x_1}{2}\right)^2 + 4\left(\frac{x_1}{2}\right) + \left(\frac{y_1+3}{2} - 3\right)^2 = 0$

$\Rightarrow \frac{x_1^2}{4} + 2x_1 + \left(\frac{y_1-3}{2}\right)^2 = 0$

$\Rightarrow \frac{x_1^2}{4} + 2x_1 + \frac{y_1^2 + 9 - 6y_1}{4} = 0$

$\Rightarrow x_1^2 + y_1^2 + 8x_1 - 6y_1 + 9 = 0$

Hence, locus of a point is

$x^2 + y^2 + 8x - 6y + 9 = 0$

125. **(a)** Given equation of ellipses is $x^2 + 9y^2 = 9$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{1} = 1$

Here,

$a = 3, b = 1$

$c = \sqrt{(3)^2 - (1)^2} = \sqrt{8}$

$\therefore$ Eccentricity of ellipse, $e = \frac{c}{a}$

$\Rightarrow e = \frac{\sqrt{8}}{3}$

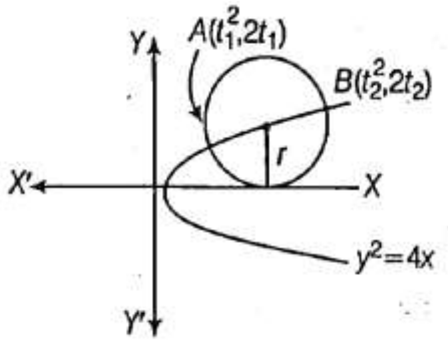
$\therefore$ Eccentricity of hyperbola $= \frac{3}{\sqrt{8}}$

$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{9}{8}$

$\Rightarrow \frac{b^2}{a^2} = \frac{1}{8}$

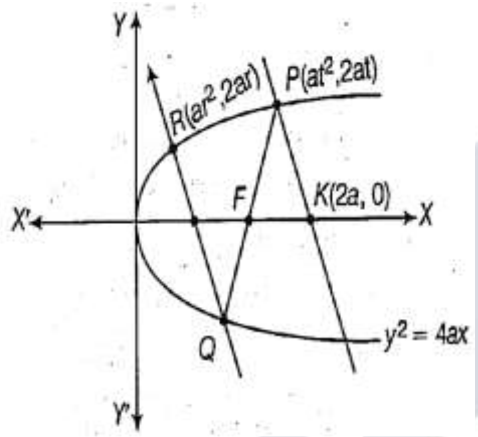
$\Rightarrow a^2 : b^2 = 8 : 1$

126. **(c, d)** Centre of circle $= \left(\frac{t_1^2 + t_2^2}{2}, (t_1 + t_2)\right)$



Since, circle touch the x-axis, so equation of tangent is $y=0$
$\because$ Radius $=$ Perpendicular distance from centre to the tangent
$\Rightarrow$ Radius $=|t_1+t_2|=r$
Slope of $AB=\frac{2}{t_1+t_2}=\frac{2}{\pm r}$

127. (d)



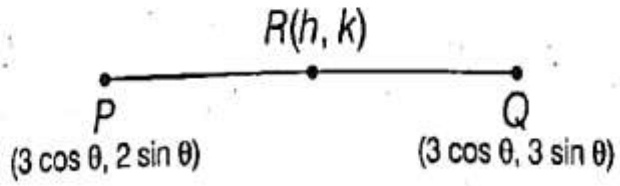
Here, coordinate of Q will be $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$.
Slope of $QR=\frac{2}{r-\frac{1}{t}}$
Slope of $PK=\frac{2at}{at^2-2a}=\frac{2t}{t^2-2}$
Since, Slope of $QR=$ Slope of PK

$$\therefore \quad \frac{2}{r-\frac{1}{t}}=\frac{2t}{t^2-2}$$

$$\Rightarrow \quad r=\frac{t^2-1}{t}$$

128. (a) Since, point P on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$
$\therefore\ P(3\cos\theta, 2\sin\theta)$
Now, equation of line parallel of Y-axis is
$x=3\cos\theta$
and above line meets circle at Q
$\therefore Q(3\cos\theta, 3\sin\theta)$
Given,

$$\frac{PR}{RQ}=\frac{1}{2}$$



$\therefore h=\frac{3\cos\theta+6\cos\theta}{3}, k=\frac{-3\sin\theta+4\sin\theta}{3}$

$\Rightarrow\ h=3\cos\theta, k=\frac{7}{3}\sin\theta$

$\Rightarrow\ \cos\theta=h/3, \sin\theta=\frac{3k}{7}$

Now, $\cos^2\theta+\sin^2\theta=h^2/9+\frac{9k^2}{49}=1$

Hence, locus of a point is $\frac{x^2}{9}+\frac{9y^2}{49}=1$

129. **(a)** Equation of line through $Q(1,-2,3)$ and parallel to the line $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is $\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}=\lambda$

Since, point P lies on above line.

$\therefore\ P(\lambda+1,4\lambda-2,5\lambda+3)$

Since, P lies on the given plane.

$\therefore\ 2(\lambda+1)+3(4\lambda-2)-4(5\lambda+3)+22=0$

$\Rightarrow\ 2\lambda+2+12\lambda-6-20\lambda-12+22=0$

$\Rightarrow\ -6\lambda+6=0$

$\Rightarrow\ \lambda=1$

$\therefore\ P(2,2,8)$

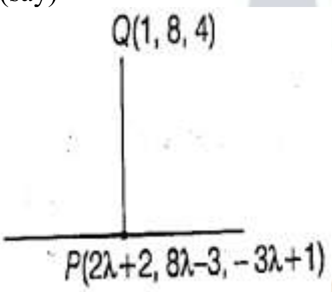
$\therefore\ PQ=\sqrt{(2-1)^2+(2+2)^2+(3-8)^2}$

$\Rightarrow PQ=\sqrt{1+16+25}=\sqrt{42}$

130. **(d)** Equation of line joining points $(0,-11,4)$ and $(2,-3,1)$ is

$\frac{x-2}{2}=\frac{y+3}{8}=\frac{z-1}{-3}=\lambda$

(say)



Let P is any point of the above line then coordinate of P is $(2\lambda+2,8\lambda-3,-3\lambda+1)$.

$\therefore$ DR's of $'PQ$ is $(2\lambda+1,8\lambda-11,-3\lambda-3)$

Now, $(2\lambda+1)(2)+(8\lambda-11)(8)+(-3\lambda-3)$

$(-3)=0$

$\Rightarrow\quad 4\lambda+2+64\lambda-88+9\lambda+9=0$

$\Rightarrow\quad 77\lambda-77=0$

$\Rightarrow\quad \lambda=1$

$\therefore$ Required foot of perpendicular,

$P(4,5,-2)$

131. **(a)**

$x^2+y^2=(20)^2=400$

We have,

$\frac{dy}{dt}=2\text{ft/sec}$

When

$x = 12$
then
$(12)^2 + y^2 = 400$
$\Rightarrow\ 144 + y^2 = 400$
$\Rightarrow\ y^2 = 400 - 144 = 256$
$\Rightarrow\ y = 16$
Now, $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$
$\Rightarrow\ x\frac{dx}{dt} = -y\frac{dy}{dt}$
$\Rightarrow\ 12\left(\frac{dx}{dt}\right) = -16(2)$
$\Rightarrow\ \frac{dx}{dt} = \frac{-8}{3}$

132. **(b)** Here, let $p = \frac{1}{m}$
then $(a^p + b^p)^{1/p} = \left(a^{1/m} + b^{1/m}\right)^m$
$= a + b + k, k \geq 0$
$\therefore\quad a^p + b^p \geq (a+b)^p \geq (a+b)$
$\Rightarrow\quad J(p) \geq I(p)$

133. **(c)** We have,
$\vec{\delta} = \vec{\alpha} + \lambda\vec{\beta}$
$= (\hat{\imath} + \hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} - \hat{k})$
$\Rightarrow\quad \vec{\delta} = (1+\lambda)\hat{\imath} + (1-\lambda)\hat{\jmath} + (1-\lambda)\hat{k}$
Given, $\frac{\vec{\delta}\cdot\vec{\gamma}}{|\vec{\gamma}|} = \frac{1}{\sqrt{3}}$
$\Rightarrow\quad \frac{(1+\lambda)}{\sqrt{3}}(-1) + (1-\lambda)(1) + (1-\lambda)(-1)$
$\Rightarrow\quad \lambda = -2$
$\therefore\quad \vec{\delta} = -\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$

134. **(c)** $\vec{\alpha} = \lambda(\vec{\beta} \times \vec{\gamma}) = \lambda\left(|\vec{\beta}||\vec{\gamma}||\sin 30°|\right.$
$\Rightarrow\ |\vec{\alpha}| = |\lambda|\left(|\beta||\vec{\gamma}|\cdot\frac{1}{2}\right)$
$\Rightarrow\ 1 = |\lambda|\cdot 1\cdot 1\cdot\frac{1}{2}$
$\Rightarrow\ 1\lambda\ | = 2$
$\Rightarrow\ \lambda = \pm 2$
$\therefore\ 1\vec{\alpha} = \pm 2(\vec{\beta} \times \vec{\gamma})$

135. **(d)** Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$
and $\operatorname{Re}(z_1) > 0 \Rightarrow x_1 > 0$
$\operatorname{Im}(z_2) < 0$
$\Rightarrow\ y_2 < 0$
Given, $|z_1| = |z_2|$
$\Rightarrow\ |z_1|^2 = |{z_2}^2|$
$\Rightarrow\ z_1\bar{z}_1 = z_2\bar{z}_2$
Now, $\left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \left(\frac{z_1 + z_2}{z_1 - z_2}\right)$
$= \left(\frac{z_1 + z_2}{z_1 - z_2}\right) + \left(\frac{\bar{z}_1 + \bar{z}_2}{\bar{z}_1 - \bar{z}_2}\right)$
$= \frac{z_1\bar{z}_1 + z_2\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_2 + z_1\bar{z}_1 + z_1\bar{z}_2 - z_2\bar{z}_1 + z_2\bar{z}_2}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)}$
$= \frac{2(|z_1|^2 - |z_2|^2)}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)} = 0(\because |z_1|^2 = |z_2|^2)$

$= \dfrac{z_1 + z_2}{z_1 - z_2}$ is purely imaginary.

136. **(a)** Given, numbers are 1,2,3 … … . .20
Here, number of ways of selecting four consecutive numbers = 17
∴ Required number of selecting 4 non-consecutive numbers = ${}^{20}C_4 - 17$

$$= \frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2 \times 1} - 17$$
$$= 285 \times 17 - 17$$
$$= 284 \times 17$$

137. **(b)** We have, $\begin{pmatrix} \cos \pi/4 & \sin \pi/4 \\ -\sin \frac{\pi}{4} & \cos \frac{\pi}{4} \end{pmatrix}^n$

Let $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$\Rightarrow A = \begin{pmatrix} k & k \\ -k & k \end{pmatrix} \left(\text{where, } k = \frac{1}{\sqrt{2}} \right)$

$\Rightarrow A^2 = \begin{pmatrix} k & k \\ -k & k \end{pmatrix}\begin{pmatrix} k & k \\ -k & k \end{pmatrix} = \begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix}$

$A^4 = \begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2k^2 \\ -2k^2 & 0 \end{pmatrix}$

$A^4 = \begin{pmatrix} -4k^4 & 0 \\ 0 & -4k^4 \end{pmatrix} = \begin{pmatrix} -4 \times \frac{1}{4} & 0 \\ 0 & -4 \times \frac{1}{4} \end{pmatrix}$

$\Rightarrow A^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\Rightarrow A^8 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

138. **(d)** We have, $\rho = \{(x, y) \in N \times N : 2x + y = 41\}$
For reflexive,

$x\rho x \Rightarrow 2x + x = 41$
$\Rightarrow 3x = 41$
$\Rightarrow x = \frac{41}{3} \notin N$

So, ρ is not reflexive.
For symmetric,
$x\rho y \Rightarrow 2x + y = 41$
and
$x\rho y \not\Rightarrow 2y + x = 41$
$x\rho y \neq y\rho x$
So, ρ is not symmetric.
For transitive,
and

$x\rho y \Rightarrow 2x + y = 41$

$y\rho z \Rightarrow 2y + z = 41$

$\Rightarrow x\phi z$
$\Rightarrow \rho$ is not transitive.

139. **(*a*)** Given, $f(x) = \begin{vmatrix} (1+x)^a & (2+x)^b & 1 \\ 1 & (1+x)^a & (2+x)^b \\ (2+x)^b & 1 & (1+x)^a \end{vmatrix}$

For constant term put $x = 0$

$$f(0) = \begin{vmatrix} 1 & 2^b & 1 \\ 1 & 1 & 2^b \\ 2^b & 1 & 1 \end{vmatrix}$$

$$= 1(1 - 2^b) - 2^b(1 - 2^{2b}) + 1(1 - 2^b)$$

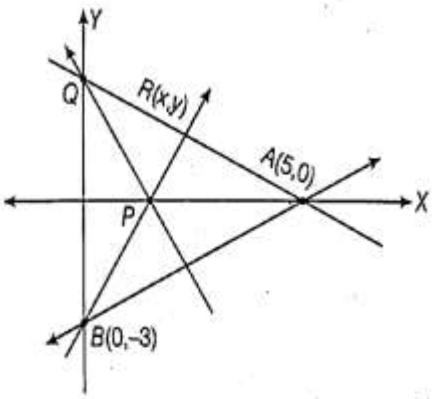
$$= 1 - 2^b - 2^b + 2^{3b} + 1 - 2^b$$

$$= 2 - 3 \cdot 2^b + 2^{3b}$$

140. **(a)** Equation of line AB is

$$\frac{x}{5} + \frac{y}{-3} = 1$$

$$\Rightarrow \quad 3x - 5y = 15$$



Perpendicular line to AB is

$5x + 3y = \lambda$

Coordinate of P is $\left(\frac{\lambda}{5}, 0\right)$

and coordinate of Q is $(0, \lambda/3)$

Now, equation of line AQ is

$$x/5 + \frac{y}{\lambda/3} = 1$$

$$\Rightarrow \frac{x}{5} + \frac{3y}{\lambda} = 1$$

$$\Rightarrow \frac{3y}{\lambda} = 1 - \frac{x}{5}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{3y}\left(1 - \frac{x}{5}\right) \ldots\ldots (i)$$

and equation of line BP is

$$\frac{x}{\lambda/5} + \frac{y}{-3} = 1$$

$$\Rightarrow \frac{5x}{\lambda} - \frac{y}{3} = 1$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{5x}\left(\frac{y}{3} + 1\right) \ldots\ldots (ii)$$

From Eqs. (i) and (ii),

$$\frac{1}{3y}\left(1 - \frac{x}{5}\right) = \frac{1}{5x}\left(\frac{y}{3} + 1\right)$$

$$\Rightarrow \quad 5x\left(1 - \frac{x}{5}\right)$$

$$\Rightarrow \quad 3y\left(\frac{y}{3} + 1\right)$$

$$\Rightarrow \quad 5x - x^2 = y^2 + 3y$$

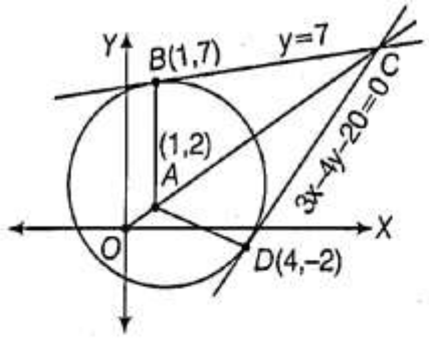
$$x^2 + y^2 - 5x + 3y = 0$$

which is a circle.

141. **(c)** Given, equation of circle is

$x^2 + y^2 - 2x - 4y - 20 = 0$

Center (1,2) and radius $= \sqrt{(1)^2 + (2)^2 + 20} = 5$



Coordinate of intersecting point of tangents at B and D is $C(16,7)$.

$\therefore$ Area of quadrilateral $ABCD$

$= 2 \times \text{ar}(\triangle ABC)$

$= 2 \times \frac{1}{2} \times 15 \times 5 = 75$ sq units

142. **(b)** At $x = -\frac{\pi}{2}$

LHL $= -2$

RHL $= -A + B$

For continuity, LHL = RHL $= f(-\pi/2)$

$\Rightarrow -A + B = 2 \ldots\ldots$(i)

At $x = \pi/2$

LHL $= A + B$

RHL $= 0$

For continuity, LHL = RHL $= f(\pi/2)$

$\Rightarrow A + B = 0 \ldots\ldots$(ii)

On solving Eqs. (i) and (ii), we get

$A = -1$ and $B = 1$

143. **(c)** Given equation of curve,

$y = x^2 - x + 1$

$\Rightarrow \frac{dy}{dx} = 2x - 1$

Slope of normal $= \frac{1}{1-2x}$

Now, at $x_1 = 0, y_1 = 1$

$\therefore$ Slope of normal at $(0,1) = 1$

$\therefore$ Equation of normal, $y - 1 = 1(x - 0)$

$\Rightarrow x - y + 1 = 0 \ldots\ldots$(i)

At $x_2 = -1, y_2 = 3$

Slope of normal at $(-1,3) = \frac{1}{3}$

Equation of normal, $y - 3 = \frac{1}{3}(x + 1)$

$\Rightarrow 3y - 9 = x + 1$

$\Rightarrow x - 3y + 10 = 0 \ldots\ldots$(ii)

At $x_3 = \frac{5}{2}, y_3 = \frac{19}{4}$

Slope of normal at $\left(\frac{2}{5}, \frac{19}{4}\right) = -\frac{1}{4}$

Equation of normal, $y - \frac{19}{4} = -\frac{1}{4}\left(x - \frac{5}{2}\right)$

$\Rightarrow x + 4y = \frac{43}{2}$

$\Rightarrow 2x + 8y = 43 \ldots\ldots\ldots$(iii)

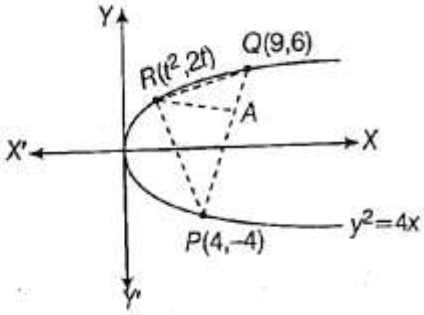
Here, intersecting point of Eqs. (i) and (ii) is $\left(\frac{7}{2}, \frac{9}{2}\right)$ and normal (iii) passes through it. Hence, normals are concurrent.

144. **(b)** Let

$f(x) = x\log x + x - 3$

$\Rightarrow f'(x) = x \cdot \frac{1}{x} + \log x + 1$
$\Rightarrow f'(x) = \log x + 2 > 0$
$\Rightarrow f(1) = -2$ and $f(3) = 3\log 3, f(1) \cdot f(3) < 0$
Hence, exactly one root in $x \in (1,3)$ as $f(x) > 0$

145. **(c)** Equation of PQ is $2x - y = 12$

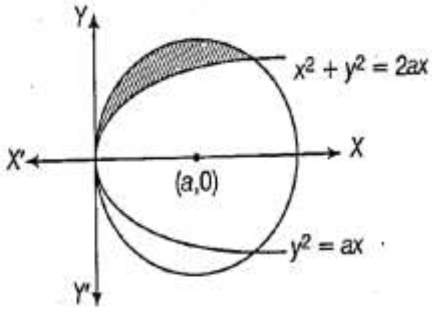


Perpendicular distance
$AR = \left|\frac{2t^2 - 2t - 12}{\sqrt{5}}\right| = \frac{2}{\sqrt{5}}(t-3)(t+2)$
AR is Maximum, at $t = \frac{1}{2}$
$\therefore R$ is $\left(\frac{1}{4}, 1\right)$

146. **(b)** $I = \int_0^1 \frac{x^3 \cos 3x}{2+x^2} dx$
Here, $-1 < \cos 3x < 1$
$\Rightarrow -x^3 < x^3 \cos 3x < x^3$
$\Rightarrow \frac{-x^3}{x^2} < \frac{-x^3}{x} < \frac{-x^3}{2+x^2} < \frac{x^3 \cos 3x}{2+x^2} < \frac{x^3}{2+x^2} < \frac{x^3}{x} < \frac{x^3}{x^2}$
$\Rightarrow \int_0^1 -x^2 dx < I < \int_0^1 x^2 dx$
$\Rightarrow \left(\frac{-x^3}{3}\right)_0^1 < I < \left(\frac{x^3}{3}\right)_0^1$
$\Rightarrow \frac{-1}{3} < I < \frac{1}{3}$

147. **(c, d)** Given, curve, $12y = x^3$
and $\frac{dy}{dt} > \frac{dx}{dt}$........(i)
Now, $12\frac{dy}{dt} = 3x^2\frac{dx}{dt}$....(ii)
From Eqs. (i) and (ii), we get
$3x^2\frac{dx}{dt} > 12\frac{dx}{dt}$
$\Rightarrow \quad 3x^2 > 12$
$\Rightarrow \quad x^2 - 4 > 0$
$\Rightarrow \quad (x-2)(x+2) > 0$
$\Rightarrow \quad x \in (-\infty, -2) \cup (2, \infty)$

148. **(b)** Given, equation of circle
$x^2 + y^2 = 2ax$
$\Rightarrow (x-a)^2 + y^2 = a^2$
and equation of parabola is $y^2 = ax, a > 0$



Intersection points of circle and parabola

$\Rightarrow \quad x^2 + ax = 2ax$

$\Rightarrow \quad x^2 = ax$

$\Rightarrow \quad x^2 - ax = 0$

$\Rightarrow \quad x(x-a) = 0$

$\Rightarrow \quad x = 0, a$

Intersecting points are $(0,0)$ and (a, a).

$$\therefore \text{ Required area } = \frac{\pi a^2}{4} - \int_0^a \sqrt{ax}\,dx$$

$$= \frac{\pi a^2}{4} - \sqrt{a}\left(\frac{x^{3/2}}{3/2}\right)_0^a$$

$$= \frac{\pi a^2}{4} - \frac{2a^2}{3} = a^2\left(\frac{\pi}{4} - \frac{2}{3}\right)$$

149. $(\boldsymbol{a}, \boldsymbol{d})$ Let α and β are the roots of $x^2 + ax + b = 0$ and the roots of $x^2 - cx + d = 0$ are α^4 and β^4.

Now, $\alpha + \beta = -a, \alpha\beta = b$

and $\alpha^4 + \beta^4 = c, \alpha^4\beta^4 = d$

From Eqs. (i) and (ii),

$b^4 = d$ and $\alpha^4 + \beta^4 = c$

$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = c$

$\Rightarrow \quad [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 = c$

$\Rightarrow \quad (a^2 - 2b)^2 - 2b^2 = c$

$\Rightarrow \quad 2b^2 + c = (a^2 - 2b)^2$

$\Rightarrow \quad 2b^2 + c = a^4 + 4b^2 - 4a^2b$

$\Rightarrow \quad 2b^2 - c = 4a^2b - a^4$

$\Rightarrow \quad 2b^2 - c = a^2(4b - a^2)$

and for equation $x^2 - 4bx + 2b^2 - c = 0$

$D = (4b)^2 - 4(1)(2b^2 - c)$

$= 16b^2 - 8b^2 + 4c = 8b^2 + 4c$

$= 4(2b^2 + c) = 4(a^2 - 2b)^2 > 0 =$ real root

Now, $f(0) = 2b^2 - c = a^2(4b - a^2) < 0 \ (\because a^2 > 4b)$

$=$ roots are opposite in sign

150. $(\boldsymbol{b}, \boldsymbol{c})$ Required number of ways $= {}^{20}C_2 \times 2\,!$

$= {}^{20}P_2$

151. **(a,c)** $A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

$$\Rightarrow A' = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = -A$$

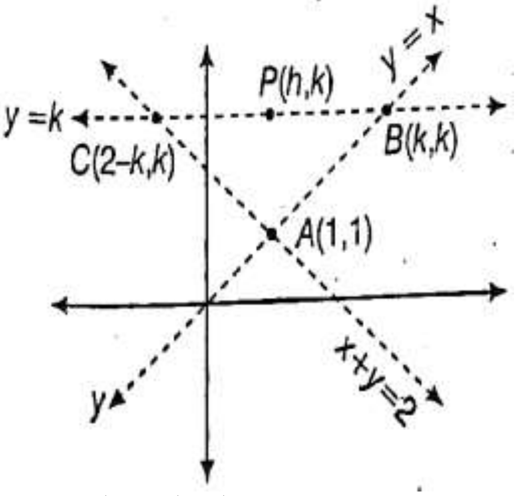
$\Rightarrow A$ is a skew-symmetric matrix.

$|A| = 0 + 1(0+1) - 1(1-0)$

$= 0 + 1 - 1 = 0 =$ Singular

$\Rightarrow A$ is not invertible.

152. **(a,b)**Given, ar$(\triangle ABC) = h^2$



$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ 2-k & k & 1 \end{vmatrix} = \pm h^2$$

$\Rightarrow 1(k-k) - 1(k-2+k) + 1(k^2-2k+k^2) = \pm 2h^2$

$\Rightarrow -(2k-2) + (2k^2-2k) = \pm 2h^2$

$\Rightarrow 2-2k+2k^2-2k = \pm 2h^2$

$\Rightarrow 2k^2-4k+2 = \pm 2h^2$

$\Rightarrow k^2-2k+1 = \pm h^2$

Hence, locus of a point is

$\Rightarrow (k-1)^2 = h^2$

$\Rightarrow y-1 = \pm x$

$\Rightarrow y-1$ or $x = -(y-1)$

153. **(b)** Given, $2a_1 = 2\sin\theta$

$\Rightarrow a_1 = \sin\theta$

and $3x^2+4y^2 = 12$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$

Here, $a^2 = 4$ and $b^2 = 3$

$\therefore \quad b^2 = a^2(1-e^2)$

$\Rightarrow \quad 3 = 4(1-e^2)$

$\Rightarrow \quad e^2 = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \quad e = \frac{1}{2}$

Focus, $F(ae,0) = F\left(2\times\frac{1}{2},0\right)$

$= F(1,0)$

For hyperbola foci are same

$$a_1e_1 = ae = 1$$

$\therefore \quad (\sin\theta)e_1 = 1$

$\Rightarrow \quad e_1 = \text{cosec}\theta$

and $\quad b_1^2 = a_1^2(e_1^2-1) = a_1^2e_1^2 - a_1^2$

$\Rightarrow \quad b_1^2 = 1-\sin^2\theta = \cos^2\theta$

$$\frac{x^2}{a_1^2} - \frac{y^2}{b_1^2} = 1$$

$\Rightarrow \quad \frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

$\Rightarrow \quad x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$

154. **(a,c)** $f(x)=\cos\left(\frac{\pi}{x}\right)$

$\Rightarrow f'(x)=-\sin\left(\frac{\pi}{x}\right)\left(\frac{-\pi}{x^2}\right)=\frac{\pi}{x^2}\sin\frac{\pi}{x}$

For increasing function, $f'(x)>0$

$\Rightarrow \sin\left(\frac{\pi}{x}\right)>0 \Rightarrow 2k\pi<\frac{\pi}{x}<(2k+1)\pi$

$\Rightarrow \frac{1}{2k}>x>\frac{1}{2k+1}$

For decreasing function, $f'(x)<0$

$\Rightarrow \sin\left(\frac{\pi}{x}\right)<0$

$\Rightarrow \frac{\pi}{x}\in[(2k+1)\pi,(2k+2)\pi]$

$\Rightarrow x\in\left(\frac{1}{2k+2},\frac{1}{2k+1}\right)$

155. **(c)** Given, $y=\log_a\left(x+\sqrt{x^2+1}\right), a>0, a\neq 1$

$\Rightarrow a^y=\left(x+\sqrt{x^2+1}\right)$

$\Rightarrow a^{-y}=\frac{1}{x+\sqrt{x^2+1}}$

$=\sqrt{x^2+1}-x$

$\Rightarrow a^y-a^{-y}=2x \Rightarrow x=\frac{a^y-a^{-y}}{2}$

$\Rightarrow f^{-1}(y)=\frac{e^{y\log a}-e^{-y\log a}}{2}$

$\Rightarrow f^{-1}(y)=\sinh(y\log a)\ \left(\because \frac{e^x-e^{-x}}{2}=\sinh(x)\right)$