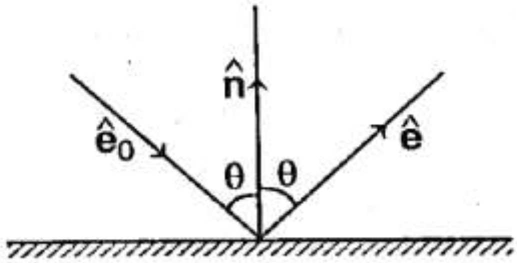
Physics

1. **(b)** According to the question,



We know that incident ray, reflected ray and normal lie in the same plane.
and angle of incidence = angle of reflection
Therefore $\hat{n}$ will be along the angle bisector of $\hat{\mathbf{e}}$ and - $\hat{\mathbf{e}}_0$,
i.e.
$\hat{n}=\frac{\hat{e}+(-\hat{e}_0)}{|\hat{e}-\hat{e}_0|}$.........(i)
[$\because$ Bisector will along a vector dividing in same ratio as the ratio of sides forming that angle]
But $\hat{\mathbf{n}}$ is a unit vector where $|\hat{\mathbf{e}}-\hat{\mathbf{e}}_0|=OC$
$=20P=2|\hat{\mathbf{e}}|$
$\cos\theta=2\cos\theta$
Substituting this value in Eq. (i), we get

$$\hat{n}=\frac{\hat{e}-\hat{e}_0}{2\cos\theta}$$
$$\hat{e}=\hat{e}_0+(2\cos\theta)\hat{n}$$
$$\hat{e}=\hat{e}_0-2(\hat{n}\cdot\hat{e}_0)\hat{n}[\because \hat{n}\cdot\hat{e}_0=-\cos\theta]$$

2. **(b)** According to the question,

$$X\xrightarrow{\alpha\text{-particle}}Y\xrightarrow{\beta\text{-particle}}Z\text{ (say)}$$

According to Rutherford Soddy law of radioactive decay, the rate of decay of radioactive along at any instant is proportional to the number of atoms present at that instant

$$\frac{dN}{dt}=-\lambda N$$

Since, $\lambda=$ constant (decay constant) and
$N=$ number of atoms
Rate of decay of Y particle is given as,

$$\lambda_X N_X-\lambda_Y N_Y=\frac{dN_Y}{dt}=0$$

[$\because$ Decay rate for β-particle become constant after some time]
Given, rate of emission of β-particle $=10^7/\mathrm{h}$
$\therefore \lambda_X N_X=\lambda_Y N_Y=10^7/\mathrm{h}$

3. **(d)** As we know that de-Broglie wavelength is
given as $\lambda=\frac{h}{p}=\frac{h}{mv}$
where, $h=$ Planck constant
$p=$ momentum of particle
$v=$ velocity of particle
and $m=$ mass of the particle.
Eq. (i) can be written as,

$$\lambda=\frac{h}{2m(\mathrm{KE})}=\frac{h}{\sqrt{2mqv}}\ [\because \mathrm{KE}=qv]$$

where, KE = Kinetic energy of particle
Hence, $\lambda\propto\frac{1}{\sqrt{m}}$
Now, $\frac{\lambda_{\text{proton}}}{\lambda_{\text{electron}}}=\sqrt{\frac{m_{\text{electron}}}{m_{\text{proton}}}}$
Given, mass of proton, $m_{\text{proton}}=2000m_{\text{electron}}$

$$\frac{\lambda_{\text{proton}}}{\lambda_{\text{electron}}} = \sqrt{\frac{1}{2000}}$$

$$\frac{\lambda_{\text{proton}}}{\lambda_{\text{electron}}} = \frac{1}{20\sqrt{5}} \Rightarrow \lambda_p = \frac{\lambda_e}{20\sqrt{5}}$$

4. **(d)** According to the Bohr's atomic model,

Angular momentum, $L = mvr = \frac{nh}{2\pi}$.......(i)

Since, angular velocity $\omega = \frac{v}{r}$

$\Rightarrow\ v = r\omega$

From Eq. (i), we get

$$m(r\omega)r = \frac{nh}{2\pi}$$

$$m\omega r^2 = \frac{nh}{2\pi}$$

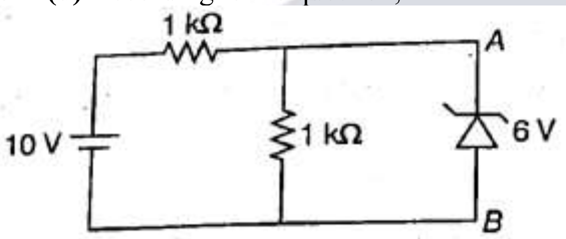
$$\omega = \frac{nh}{2\pi mr^2} \text{(ii)}$$

Since, the radius of the electron in n^{th} orbit of Bohr's atomic model is given as, $r = \frac{n^2h^2\varepsilon_0}{\pi m z^2}$ ...

Squaring the Eq: (iii) and substituting its value in Eq. (ii), we get

$$\omega = \frac{nh(\pi mze)^2}{2\pi m(n^4h^4\varepsilon_0^2)} \Rightarrow \omega \propto \frac{1}{n^3}$$

5. **(d)** According to the question,



Input voltage $V_S = 10$ V

Source resistance $R_S = 1\text{k}\Omega$

Zener diode voltage $V_Z = 6$ V

$\because$ Breakdown voltage of zener diode is $= 6$ V, and the potential difference across the zener diode 5 V .

$\therefore$ Current flow in zener diode $I_Z = 0$

6. **(b)** By using the de-Morgan's law

$\overline{A+B} = \overline{A} \cdot \overline{B}$

Hence, option (b) is the correct.

7. **(b)** As we know that,

Impulse, $I =$ force $(F) \times$ small time interval

$I = ma \times t \qquad [\because F = ma]$

$I = [\text{M}][\text{LT}^{-2}] \times [\text{T}]$

$I = [\text{M L T}^{-1}]$

Hence, the correct dimensional formula for impulse is given by [MLT^{-1}].

8. **(c)** Given,

Maximum error in the measurement of mass $= 0.3\%$ Maximum error in the measurement of length $= 0.2\%$

We know that,

Error in density is given as,

Density, $\rho = \frac{\text{mass } (m)}{\text{volume } (V)} = \frac{m}{L^3}$

where, $L =$ side of cube

Error in density is given as,

$$\left(\frac{\Delta\rho}{\rho}\right) = \frac{\Delta m}{m} + \frac{3\Delta L}{L}$$

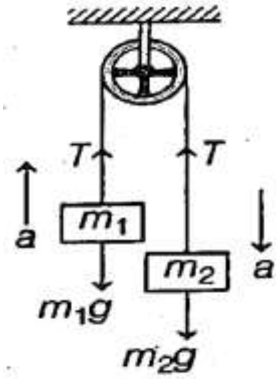
or $\left(\frac{\Delta\rho}{\rho}\right) \times 100 = \left(\frac{\Delta m}{m} + \frac{3\Delta L}{L}\right) \times 100$

Substituting the given values, we get

$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = (0.3\% + 3(0.2)\%) = 0.3\% + 0.6\%$

$\therefore$ Maximum percentage error measurement of density $\left(\frac{\Delta\rho}{\rho}\right)_{\max} = 0.9\%$.

9. **(d)** According to the question, we can draw the following diagram



Here, T is the tension in the string.

In equilibrium condition,

For mass $m_1, T - m_1g = m_1a$.......(i)

For mass $m_2, m_2g - T = m_2a$........(ii)

After adding Eqs. (i) and (ii), we get

$m_2g - m_1g = m_1a + m_2a$

$(m_2 - m_1)g = (m_1 + m_2)a$

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)}g$$

10. **(c)** Given, Power (P) = constant

Kinetic Energy (KE) $= \frac{1}{2}mv^2$

We know that, $P = \frac{\text{KE}}{\Delta t} \Rightarrow P = \frac{mv^2}{2}$

$\because P$ = constant,

Hence, velocity of the body $v \propto \sqrt{t}$

As, Velocity $v = \frac{ds}{dt}$

From Eqs. (i) and (ii), we get

So,

$\frac{ds}{dt} \propto \sqrt{t}$

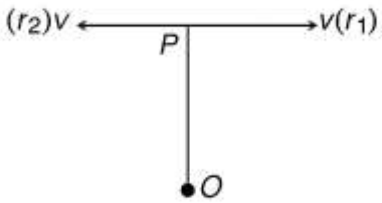
Integrating the above equation w.r.t. time (t),

$\int \frac{ds}{dt} \propto \int \sqrt{t}$

we get, displacement of the body $s \propto t^{3/2}$

$\because$ Displacement $s = at^{3/2}$, where a is constant.

11. **(c)** According to the question,



Representation of position vectors of two particles (drawn from the point P)

In two dimension, the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ represented as

$\mathbf{r}_1 = vt\hat{\mathbf{i}} - \frac{1}{2}gt^2\hat{\mathbf{j}}$......(i)

$\mathbf{r}_2 = vt(-\hat{\mathbf{i}}) - \frac{1}{2}gt^2(\hat{\mathbf{j}})$.....(ii)

$\because$ We know that, when the two vectors are mutually perpendicular, i.e.

So,

$\theta = 90^\circ$

$\mathbf{r}_1 \cdot \mathbf{r}_2 = r_1 r_2 \cos 90^\circ$

$\mathbf{r}_1 \cdot \mathbf{r}_2 = 0$

Substituting the values $\mathbf{r}_1$ and $\mathbf{r}_2$ in the above relation, we get

$$\left((vt)\hat{\mathbf{i}} - \frac{1}{2}gt^2\hat{\mathbf{j}}\right) \cdot \left(vt(-\hat{\mathbf{i}}) - \frac{1}{2}gt^2(\hat{\mathbf{j}})\right) = 0$$

$-v^2t^2 + \frac{1}{4}4g^2t^4 = 0$(where $, \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1)$

$$v^2t^2 = \frac{1}{4}g^2t^4$$

$$\Rightarrow v^2 = \frac{1}{4}g^2t^2$$

$\therefore$ Magnitude of velocity of the particles, $v = \frac{1}{2}gt$

We know that, separation distance between particles at a time t

$$\Delta x = 2vt$$

$$\Delta x = 2 \times v \times \frac{2v}{g} \Rightarrow \Delta x = \frac{4v^2}{g}$$

12. (a) According to the Kepler's third law

$T^2 \propto r^3$

where, $T =$ time period of revolution

$r =$ radius

Now, $\left(\frac{T_E}{T_P}\right)^2 = \left(\frac{r_E}{r_P}\right)^3 = \frac{r_E}{r_P} = \left(\frac{T_E}{T_P}\right)^{2/3}$

$$\frac{r_E}{r_P} = \left(\frac{2\pi/\omega_E}{2\pi/\omega_P}\right)^{2/3} \quad \left[\because T = \frac{2\pi}{\omega}\right]$$

$$\frac{r_E}{r_P} = \left(\frac{\omega_P}{\omega_E}\right)^{2/3}$$

According to the question, $\omega_P = 2\omega_E$ and $r_E = R$

$$\frac{R}{r_P} = \left(\frac{2\omega_E}{\omega_E}\right)^{2/3}$$

$$\Rightarrow \frac{R}{r_P} = (2)^{2/3}$$

$$r_P = \frac{R}{(2)^{2/3}} = R(2)^{-2/3}$$

13. (c) Given, Decrement in the length = 1%.

Poisson's ratio for material of the rod, $\sigma = 0.2$

As we know that,

Volume, $v = \pi r^2 l$, [where, l is the length of the rod]

$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta l}{l}$......(i)

Since, Poisson's ratio, $\sigma = \frac{-\Delta D/D}{\Delta L/L}$

$$= \frac{-\Delta r/r}{\Delta L/L}$$

So, Eq. (i) can be written as,

$$\frac{\Delta V}{V}=\frac{\Delta l}{l}(1-2\sigma)$$

$$\frac{\Delta V}{V}\times 100=\left(\frac{\Delta l}{l}\times 100\right)\times(1-2\sigma)$$
$$=-1\times[1-2\times(0.2)]$$
$$=-1\times[1-0.4]$$
$$=-0.6\%$$

Here, negative sign shows the decrement in the volume.

14. (c) When the spherical body falls with constant velocity, i.e. terminal velocity then the net force becomes zero, i.e the weight of body is equal to the buoyancy force.

Hence, $F_{\text{net}}=0$

15. (c) According to the question,

Area of both bodies A and $B=A$

Temperature of body $A=27^{\circ}\mathrm{C}=27+273\ \mathrm{K}$

Temperature of body $B=177^{\circ}\mathrm{C}=177+273\ \mathrm{K}$

Now, by Stefan-Boltzmann law, thermal energy radiated per second by a body

$$Q=\sigma A T^4$$

where, $A=$ Area

$T=$ temperature

and $\sigma=$ Stefan-Boltzmam's constant

So, the ratio of thermal energy radiated per second by A to that by B is

$$\frac{Q_1}{Q_2}=\frac{\sigma A}{\sigma A}\left(\frac{T_1}{T_2}\right)^4$$

Now, $\frac{Q_1}{Q_2}=\left(\frac{T_1}{T_2}\right)^4$

$$\frac{Q_1}{Q_2}=\left(\frac{273+27}{273+177}\right)^4$$

$$\frac{Q_1}{Q_2}=\left(\frac{300}{450}\right)^4$$

$$\frac{Q_1}{Q_2}=\left(\frac{2}{3}\right)^4=\left(\frac{16}{81}\right)$$

Ratio of thermal energy radiated per second

$Q_1:Q_2=16:81$

16. (b) For a gas at temperature T, the internal energy,

$$U=\frac{f}{2}\mu RT$$

where, $f=$ degree of freedom

$\Rightarrow$ Change in energy, $\Delta U=\frac{f}{2}\mu R\Delta T$

Also, as we know for any gas heat supplied at constant volume

$(\Delta Q_V)=\mu C_V\Delta T=\Delta U$........(ii)

where, $C_V=$ molar specific heat at constant volume From Eqs. (i) and (ii), we get

$$C_V=\frac{1}{2}fR$$

For diatomic gas, degree of freedom, $f=5$

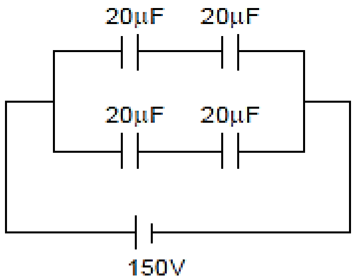
$$C_V=\frac{5}{2}R$$

17. (a)

Since, the whole system is isolated, this means, there is no transfer of heat between the system and the surrounding. Also, the right side container is initially vacuum, so the gas could easily rush there without any resistive force. As the right side container has no temperature. Thus, the temperature of the ideal gas would remain same even if it enters right side chamber.

Therefore, final temperature attained at the equilibrium will be T.

18. (d) The given circuit shows a balanced Wheatstone bridge. Now, the circuit becomes



In the branch AB, both capacitor are arranged in the series combination. Hence, its equivalent capacitance is given by

$$\frac{1}{C_{AB}}=\frac{1}{20}+\frac{1}{20}=\frac{2}{20}=\frac{1}{10}$$

$$C_{AB}=10\mu\text{ F}$$

Similarly, in branch CD,

$C_{CD}=10\mu$ F

Now, C_{AB} and C_{CD} are connected in parallel combination. Hence, the equivalent capacitance of the circuit,

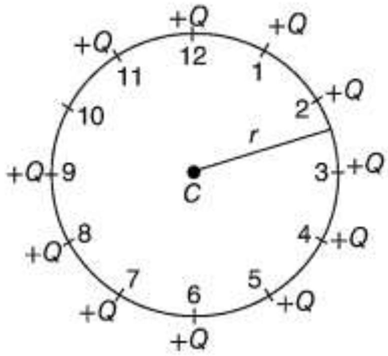
$C_{\text{eq}}=10+10=20\mu$ F

We know that, charge $Q=C_{\text{eq}}V$

$Q=20\times10^{-6}\times150$ V

Amount of charge stored $Q=3\times10^{-3}$C

19. (a) The clock diagram is as shown below



In the above diagram, charge $+Q$ is not placed at 10 h position.

So, net electric field strength at centre C,

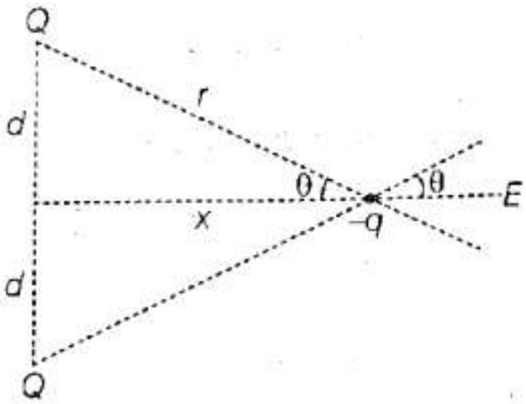
$$\mathrm{E_{net}}=\mathrm{E_1+E_2+E_3+E_4+E_5+E_6+E_7+E_8+E_9+E_{11}+E_{12}}$$

$$\Rightarrow\mathrm{E_{net}}=\mathrm{E_1+E_2+E_3+E_4+E_5+E_6+(-E_1-E_2-E_3-E_5-E_6)}$$

$$\Rightarrow\mathrm{E_{net}}=\mathrm{E_4}=\frac{Q}{4\pi\varepsilon_0r^2}$$, from centre towards the

Mark 10

20. (a) According to the question,



Force experienced by the charge $-q$ due to charge Q

$F=-\frac{2kQq}{r^2}\cos\theta$ ... (i) $\left[\text{where}, k=\frac{1}{4\pi\varepsilon_0}\right]$

From diagram, $\cos\theta=\frac{x}{r}$

By substituting the value of $\cos\theta$ in Eq. (i)

$$F=-\frac{2kQq}{r^2}\cdot\frac{x}{r}$$

or

$$F=-\frac{2kQx}{r^3}$$

or

$$F=-\frac{2kQx}{(x^2+d^2)^{3/2}}\begin{bmatrix}\because r^2=x^2+d^2\\ r=\sqrt{x^2+d^2}\end{bmatrix}$$

For, $x\ll d$, so x^2 can be neglected

$$F=-\frac{2kQx}{d^3}$$

So, the force developed by negative charge ($-q$) due to the system of the charges as shown in the figure is,

$$F=\frac{-4kQqx}{d^3}$$

$\Rightarrow\ F\propto x$

So, the forced developed by negative charge is directly proportional to the distance x.

21. (d) If velocity of particle, v is perpendicular to magnetic field, B i.e. $\theta=90^\circ$, then particle will experience maximum magnetic force, i.e. $F_{\max}=qvB$. This force acts in a direction perpendicular to the motion of charged particle. Therefore the trajectory of the particle is a circle. In this case path of charged particle is circular and magnetic force provides the necessary centripetal force,

i.e.

$$qvB=\frac{mv^2}{r}$$

$\Rightarrow$ Radius of path, $r=\frac{mv}{qB}$

$$r=\frac{p}{qB}$$

$[\because p=mv]$

where, $p=$ momentum of the particle

$\therefore r\propto$ momentum

Hence, option (d) is correct.

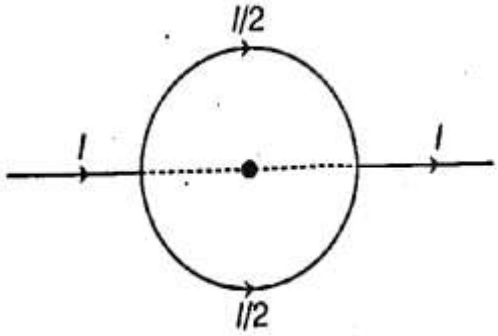
22. (d) Given, electric current in circular wire $=I$

Radius of wire $=r$

Charge on particle $=q$

Speed of the particle $=v$

The given loop can be as shown in the figure below.

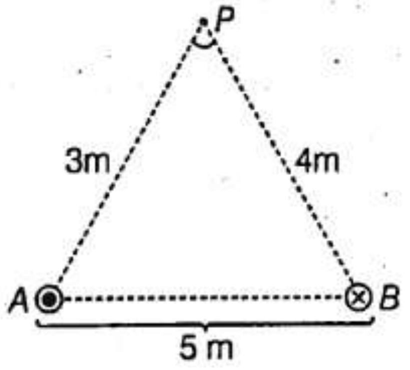


$\because$ Net magnetic field at the centre $=0$

So, magnetic force $\mathbf{F}_{\text{magnetic}}=q(\mathbf{v}\times\mathbf{B})=0$

So, the magnetic force experienced by the particle when it passes through the centre is $F=0$

23. (c) According to the question,

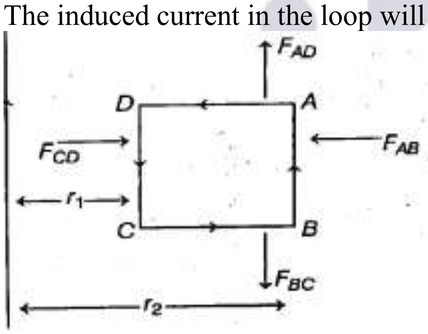


Magnetic field due to first wire (A) is $B_1=\frac{\mu_0 \times I}{2\pi \times 3}$
Magnetic field due to second wire (B) is $B_2=\frac{\mu_0 \times I}{2\pi \times 4}$
Net magnitude of magnetic field $B=\sqrt{B_1^2+B_2^2}$

$$B=\frac{\mu_0 I}{2\pi}\sqrt{\frac{1}{9}+\frac{1}{16}}=\frac{\mu_0 I \times 5}{2\pi \times 3 \times 4}$$

Magnetic field $B=\frac{5}{24} \times \frac{\mu_0 I}{\pi}$

24. (c) As we know that, magnetic field due to a long wire at a distance x from it is given by
$B=\frac{\mu_0 I}{2\pi x}$, where, I is the current flowing through the wire $\beta \propto I$
∴ Magnetic flux associated with the square loop,
$\phi \propto \beta \propto I$
Now, if the current increase, then ϕ also increases.
Direction of long wire will be $\otimes$.
This means, magnetic field due to induced current will be opposite to the existing magnetic field, i.e. according to Lenz's law,
The induced current in the loop will be in the anti-clockwise direction. Now,

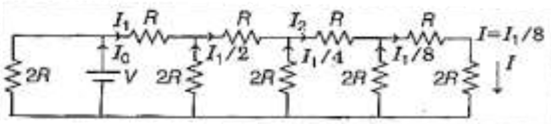


Since, wires attract each other, if current flowing through them is in same direction and repel each other, if currents are in opposite direction.
∴ Part CD of the loop will experience a force of repulsion, whereas part AB will experience attraction. Parts BC and AD will not experience any force. Thus, the overall force will be a force of repulsion because AB is closer to straight.
The force between two current carrying conductors is inversely proportional to the distance between them

$$F \propto \frac{1}{r}$$

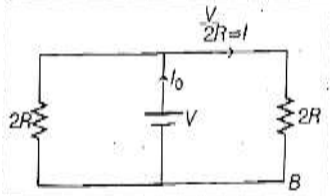
∵ $r_1<r_2$
So, $F_{CD}>F_{AB}$
∴ $F_{\text{net}}=F_{CD}-F_{AB}$
Hence, the loop will moves away from the wire.

25. (c) According to the question,



Total resistance of the given circuit
$R_{eq}=2R$

Now, circuit



$\therefore\ I=\dfrac{V/2R}{8}\ (\because I_1=V/2R)$

So, current I in the circuit $I=\frac{V}{16R}\left(\because I=\frac{I_1}{8}\right)$

26. (d) According to the balanced condition of Wheatstone bridge,

In the first case,

$$\frac{P}{Q}=\frac{R}{S}$$

$$\frac{P}{Q}=\frac{5}{S}\ \ldots\text{(i)}$$

In the second case

$$\frac{P}{Q}=\frac{7}{S+3}\ \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{5}{S}=\frac{7}{S+3}$$

$$5(S+3)=7S$$

$$5S+15=7S$$

$$2S=15$$

$$S=7.5\Omega$$

27. (a) Given, inductance of inductor, $L=60\text{mH}$

$=60\times10^{-3}\text{H}$

Phase difference between voltage and current in $L-R$ circuit, $\theta_1=60^\circ$

Capacitance of capacitor, $C=0.5\mu\text{ F}=0.5\times10^{-6}\text{ F}$

Phase difference between voltage and current in R - C circuit, $\theta_2=30^\circ$

For $L-R$ circuit,

$$\tan\theta_1=\frac{X_L}{R}$$

$$\tan\theta_1=\frac{\omega L}{R}\ \ldots\text{ (i) } [\because X_L=\omega L]$$

Similarly, for R - C circuit,

$$\tan\theta_2=\frac{X_C}{R}$$

$$\tan\theta_2=\frac{\frac{1}{\omega C}}{R}\ \ldots\text{ (ii) }\left[\because X_C=\frac{1}{\omega C}\right]$$

From Eqs. (i) and (ii), we get

$$\frac{\tan\theta_1}{\tan\theta_2}=\frac{\omega L}{R\times\frac{1}{\omega CR}}=\omega^2 LC$$

$$\frac{\tan 60^\circ}{\tan 30^\circ}=\omega^2 LC$$

$$\frac{\sqrt{3}}{1/\sqrt{3}}=\omega^2 LC$$

$$\omega^2 LC=3$$

$$\omega^2=\frac{3}{LC}$$

$$\omega^2=\frac{3}{60\times10^{-3}\times0.5\times10^{-6}}$$

$$\omega^2=\frac{3}{30\times10^{-9}}$$

$$\omega=\sqrt{10^8}$$

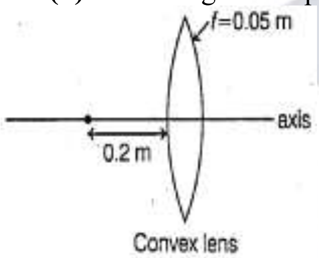
$$\omega=10^4$$

As we know that,

$$\omega=2\pi f$$

$$f=\frac{\omega}{2\pi}=\frac{10^4}{2\pi}\text{ Hz}=\frac{1}{2\pi}\times10^4\text{ Hz}$$

28. (d) According to the question, we can draw the following diagram



Given, $u=-0.2$ m and $f=0.05$ m As we know that,

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}\text{........(i)}$$

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$$

$$\frac{1}{v}=\frac{1}{0.05}-\frac{1}{0.2}$$

$$\frac{1}{v}=\frac{100}{5}-\frac{10}{2}$$

$$\frac{1}{v}=20-5\Rightarrow v=\frac{1}{15}\text{ m}$$

Now, differentiating eq. (i), we get

$$-\frac{dv}{v^2}=-\frac{du}{u^2}\ \therefore\ dv=du\frac{v^2}{u^2}$$

$$A_{\max}=A\times\left(\frac{1}{15}\right)^2\times\left(\frac{1}{(-0.2)}\right)^2$$

$$A_{\max}=A\times\frac{1}{225}\times25$$

$$A_{\max}=\frac{A}{9}$$

Here A is in cm .

Hence, $A_{\max}=\frac{A}{9}\times10^{-2}$ m

29. (c) As we know that,

fringe width, $\beta = \frac{\lambda D}{d}$
where, $\lambda =$ wavelength of light
$D =$ Distance of the screen from the slits
$d =$ separation between the slits
Now, $\frac{\Delta\beta}{\beta} \times 100 = \frac{\Delta D}{D} \times 100 + \frac{\Delta d}{d} \times 100$
According to the question,
$$\frac{\Delta\beta}{\beta} \times 100 = 0.5 + (-0.3) = 0.5 - 0.3 = 0.2\%$$
Hence, the fringe width increases by 0.2%.

30. (c) Given, initial frequency of light,
$f_1 = 4 \times 10^{14}\ \mathrm{s}^{-1}$
Final frequency of light, $f_2 = 5 \times 10^{14}\ \mathrm{s}^{-1}$
Change in wavelength , $\Delta\lambda = \lambda_1 - \lambda_2$
or
$$\Delta\lambda = \frac{c}{f_1} - \frac{c}{f_2}$$
$$= \frac{3 \times 10^8}{4 \times 10^{14}} - \frac{3 \times 10^8}{5 \times 10^{14}}$$
$$= \frac{3 \times 10^8}{10^{14}}\left(\frac{1}{4} - \frac{1}{5}\right)$$
$$= \frac{3 \times 10^8}{10^{14}} \times \frac{1}{20}$$
$$= 1.5 \times 10^{-7}\ \mathrm{m}$$
Now, we know that,
Angular width of central maxima,
$$\theta = \frac{2\lambda}{d} \text{ or } \Delta\theta = \frac{2\Delta\lambda}{d}$$
$$d = \frac{2\Delta\lambda}{\Delta\theta}$$
Here, $\Delta\theta = 0.6$ radian
$$d = \frac{2 \times 1.5 \times 10^{-7}}{0.6} = 5 \times 10^{-7}\ \mathrm{m}$$

31. (a) We know that, energy stored in the capacitor
$$= \frac{1}{2}CE^2$$
and energy supplied by the source of emf
$= CE^2$
$\therefore$ Energy dissipated in resistance R
= Energy supplied by the source of emf E,
Energy stored in the capacitor
$$= CE^2 - \frac{1}{2}CE^2$$
$$= \left(1 - \frac{1}{2}\right)CE^2 = \frac{1}{2}CE^2$$

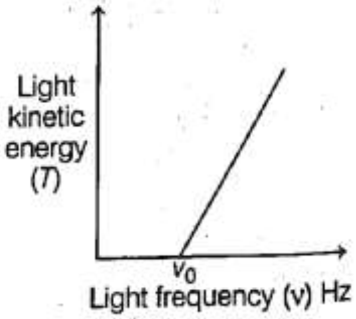
32. (a) Given, cross sectional area of nozzle
$$A = \frac{5}{\sqrt{21}} \times 10^{-3}\ \mathrm{m}^2$$
and rate of heat transfer $Q = \frac{1}{10} = 10^{-1}\ \mathrm{m}^3/\mathrm{s}$
$\because$ Heat transfer $Q = AV$
$$\therefore V = \frac{Q}{A} = \frac{10^{-1}\ \mathrm{m}^3/\mathrm{s}}{\frac{5}{\sqrt{21}} \times 10^{-3}\ \mathrm{m}^2} = 2\sqrt{21} \times 10\ \mathrm{m/s}$$
When it hits a rigid wall then maximum possible increase in temperature of water can be expressed as

$\frac{1}{2}mv^2 = ms\Delta T$.......(i)
where, $m =$ mass, $s =$ specific heat of water
$= 1\text{cal/g} = 4.2 \times 10^3$ J/kg
and $\Delta T =$ increase in temperature
From Eq. (i), $\Delta T = \frac{1}{2}\frac{v^2}{s}$

$$= \frac{\left(2\sqrt{21} \times 10^1\right)^2}{2 \times 4.2 \times 10^3}$$
$$= \frac{84 \times 10^2}{8.4 \times 10^3}$$
$$= 1^\circ\text{C}$$

33. (a) Maximum loss in $K \cdot E =$ Total $K \cdot E$ energy before collision of two ice blocks
$$K \cdot E_{\text{max loss}} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$
This maximum loss in kinetic energy will be equal to loss in heat to melt two ice block.
$\Rightarrow mv^2 = (ms\Delta\theta + mL)2$......(i)
According to question,
$L = 3.36 \times 10^5 \text{ J kg}^{-1}$
$S = 2100 \text{ J kg}^{-1} \text{ K}^{-1}$
$\Delta\theta = 8^\circ\text{C}$
From Eq. (i) $v = \sqrt{2(s\Delta\theta + L)}$
$= \sqrt{2(2100 \times 8 + 336 \times 10^5)}$
$v = 840 \text{ ms}^{-1}$
34. (a) According to the question,
Charge on particle is $= q$
Velocity of particle $= v$
Due to uniform electric field, electric force on particle
$F_{\text{electric}} = qE$.........(i)
Due to a uniform magnetic field, magnetic force on particle is given by,
$F_{\text{magnetic}} = q(v \times B)$
When v is perpendicular to E and B, which are mutually perpendicular to each other,
$F_{\text{electric}} = F_{\text{magnetic}}$
From Eqs. (i) and (ii)
$$qE = qvB$$
$$\therefore \quad v = \frac{E}{B}$$
So, $v = \frac{E}{B}$ is the condition for which the particle moves undeflected in its original trajectory.
35. (c) In given, Series R-L-C circuit
Resistance $R = 100\Omega$
Inductance of Inductor, $L = 20\text{mH} = 20 \times 10^{-3}\text{H}$
Resonance frequency $f = \frac{1250}{\pi}$ Hz
Source voltage $V_{DC} = 25$ V
According to resonant frequency,
$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$
or $$(2\pi f_0)^2 = \frac{1}{LC}$$
or $$4\pi^2 \frac{1250 \times 1250}{\pi \times \pi} = \frac{1}{LC}$$
or $$C = \frac{1000}{1250 \times 1250 \times 4 \times 20}$$
or $$C = 8 \times 10^{-6} \text{ F}$$

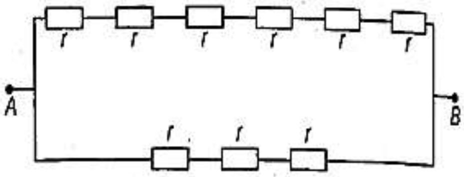
We know that, Charge $Q_S = CV$
$Q_S = 8 \times 10^{-6} \times 25 = 0.2\text{mC}$
So, the amount of charge stored in each plate of capacitor is 0.2 mC .
36. (b, d) In photoelectric effect, if the incident light had a frequency less than a minimum frequency v_0, then no electrons are ejected regardless of the light's amplitude. This minimum frequency is also called the threshold frequency, and the value of v_0 depends on the metal. For frequency greater than v_0, electrons would be ejected from the metal. Furthermore, the kinetic energy of the photoelectrons is proportional to the light frequency (v). The relationship between photoelectron kinetic energy T and light frequency v is shown in graph below



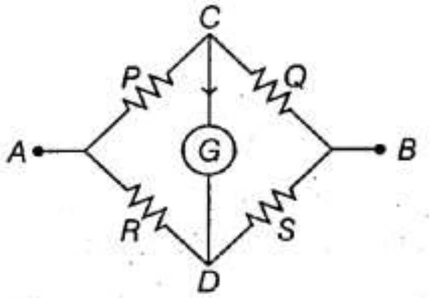
However, for a given photosensitive material and frequency of incident light, number of photoelectrons emitted per second is directly proportional to the intensity of incident light i.e., Number of electrons emitted $\propto J$.'
Also, photoelectric emission is an instantaneous process.
37. (b) The equation of state for an ideal gas undergoing adiabatic process is given as,
$TV^{\gamma-1} =$ constant
Let the temperature after adiabatic compression as given in the question be T then,

$$T_0 V^{\gamma-1} = T\left(\frac{V_0}{2}\right)^{\gamma-1}$$
$$\Rightarrow \quad T = T_0 2^{\gamma-1}$$

Now, heat released at volume $\frac{V_0}{2}$ to achieve temperature T_0.
The net heat released can be determined by the equation.
$$\Delta Q = \mu C_V \Delta T^R$$
$$= \mu \times \frac{R}{\gamma-1}\left(T_0 2^{\gamma-1} - T_0\right)$$
$$= \frac{\mu R T_0}{\gamma-1}\left(2^{\gamma-1}-1\right)(\because \mu T_0 R = p_0 V_0)$$
$\therefore$ Heat released $= \frac{p_0 V_0}{\gamma-1}\left(2^{\gamma-1}-1\right)$
38. (a, d) Given, initial velocity of particle,
$u = 10\ \text{ms}^{-1}$
Range, $R = 5$ m
Gravitational acceleration,
$g = 10\ \text{ms}^{-2}$
As we known that,
Range of projectile, $R = \frac{u^2 \sin 2\theta}{g}$
$$5 = \frac{(10)^2 \sin 2\alpha}{10}$$
$$\sin 2\alpha = \frac{5}{10}$$
$$\sin 2\alpha = \frac{1}{2}$$
$$\alpha = 15^\circ$$
or $(90^\circ - 15^\circ = 75^\circ)$
39. (a,c) The circuit diagram is as shown below,

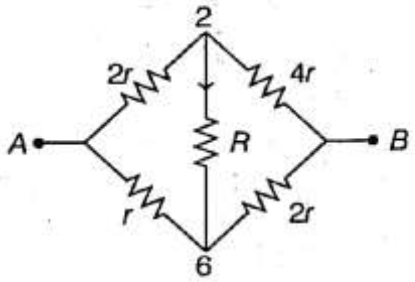


According to question, when given pair of point in options are connected through a resistance R, then equivalent resistance between point A and $B(R_{AB})$ remains unchanged. It is only possible when current does not flows through resistance R and circuit become Wheatstone bridge as shown in the figure below,



At the balanced condition, $\frac{P}{Q} = \frac{R}{S}$

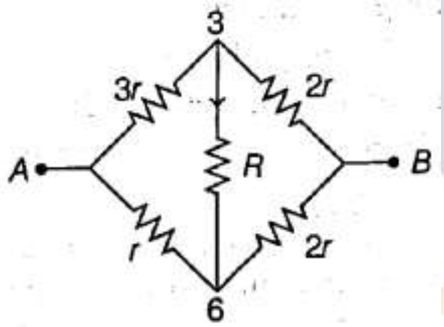
The current flow in CD branch will be zero. Now by checking each option, from option (a), circuit is,



Now, $\because \frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{2r}{4r} = \frac{r}{2r} \Rightarrow \frac{1}{2} = \frac{1}{2}$

Hence, option (a) is correct.

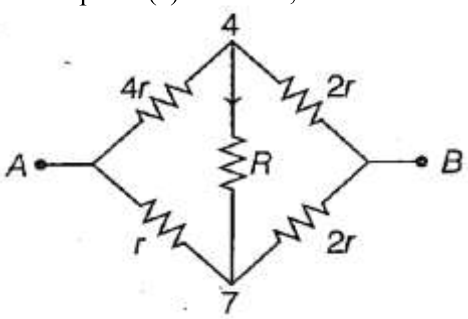
From option (b), circuit is



Now, $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{3r}{3r} \neq \frac{r}{2r}$

So, option (b) is also incorrect.

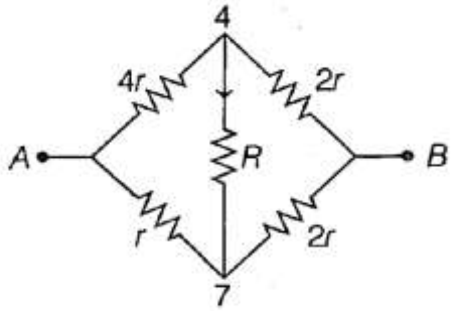
From option (c) circuit is,



Now, $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{4r}{2r} = \frac{2r}{r} \Rightarrow \frac{2}{1} = \frac{2}{1}$

So, option (c) is also correct.

From option (d), circuit is

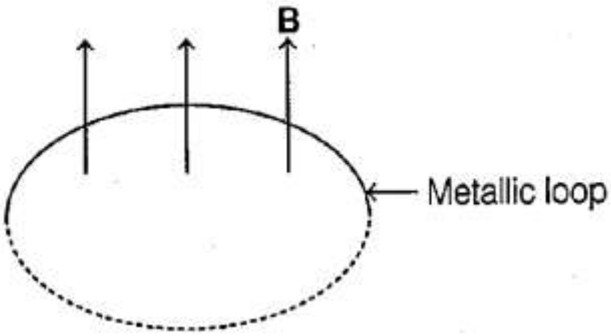


Now, $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{4r}{2r} \neq \frac{r}{2r}$

So, option (d) is also incorrect.

Hence, option (a) and (c) are correct.

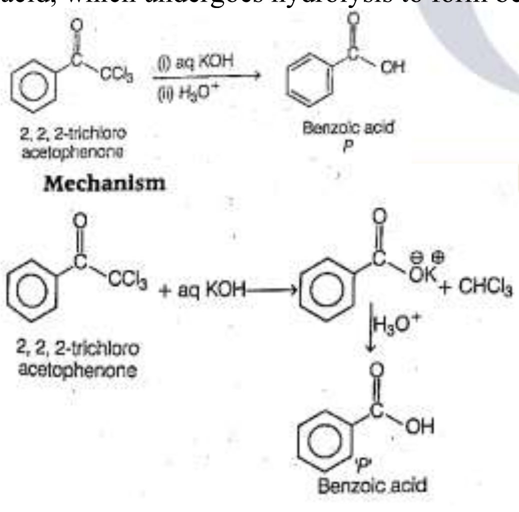
40. (b,d) According to the question,



Given, A metallic loop is placed in uniform magnetic field B. Magnetic field B and metallic loop are perpendicular to each other. Then, if metallic loop is moved along **B** or rotated about its own axis, the net flux associated with it remains constant. Thus, no emf will be induced in these cases. However, when the loop is squeezed to a smaller area or rotated about one of its diameters. Then its flux changes. Thus, emf is induced. So, option (b) and (d) both are correct.

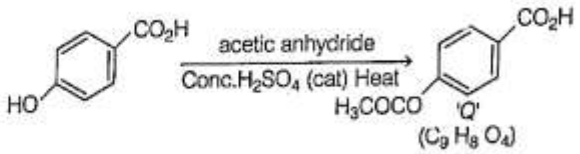
Chemistry

41. (c) 2, 2, 2-trichloroacetophenone reacts with aqueous KOH to form potassium Salt of benzoic acid, which undergoes hydrolysis to form benzoic acid as the final product.

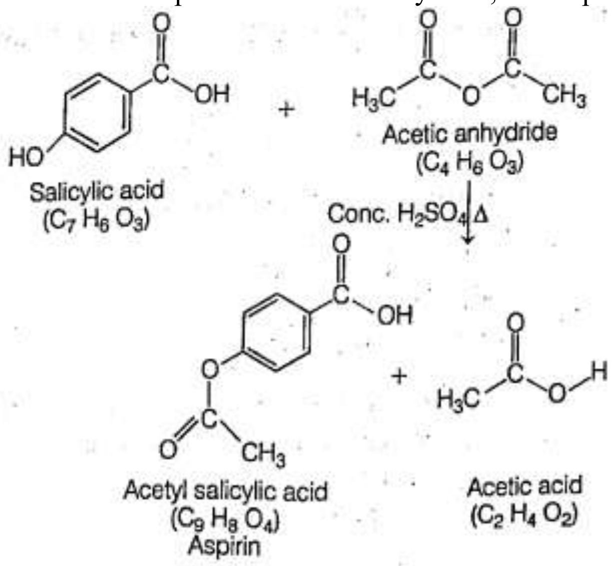


The product of the above reaction P is benzoic acid. Thus, option (c) is correct answer.

42. (a) When salicylic acid reacts with acetic anhydride in the presence of conc. H_2SO_4 and heat, the product will be (Q) acetyl salicylic acid ($C_9H_8O_4$).

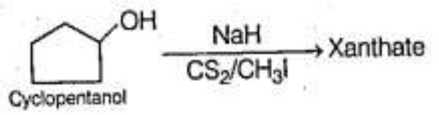


Mechanism In presence of acid anhydride, nucleophilic acyl substitution reaction takes place.

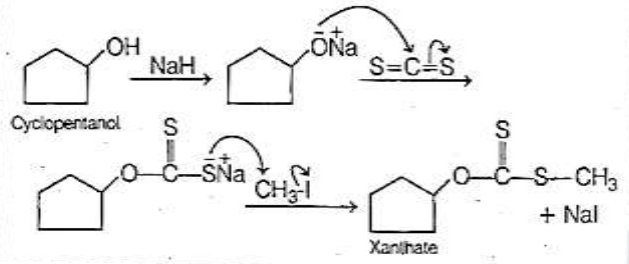


So, the option (a) is correct answer.

43. (d) When cyclopentanol on reaction with NaH followed by CS_2 and CH_3I produces a xanthate.



Mechanism



So, the option (d) is correct answer.

44. (b) $CH_3-\underset{\underset{O}{\|}}{C}-Cl$ hydrolyses to form CH_3COOH even at 25°C, which subsequently reacts with $NaHCO_3$ (sodium bicarbonate) present in the same medium to form carbon dioxide (CO_2). The reaction involved is given below.

$CH_3COCl + NaHCO_3 \longrightarrow CH_3COONa + HCl + CO_2$ Thus, the option (b) is correct answer.

45. (a) In BH_4^--indicated atom is not a nucleophilic site because there is no lone-pair on ' B ' atom.

Hence, the option (a) is correct answr.

46. (b) One millimole $= 0.001$ mol

i.e. 1×10^{-3} mol.

$\because$ 96500C is required for 1 mol of electrons.

$\therefore$ 193C give $\frac{193\times1}{96500} = 0.002$ mol, i.e. 2×10^{-3} mol of electrons.

Thus, value of n in M^{n+} is $= \frac{0.002}{0.001} = 2$

Hence, option (b) is the correct answer.

47. (c) Key Point

For buffer solutions,
$$pH = pK_a + \log\frac{[\text{ Salt }]}{[\text{ Acid }]/[\text{ Base }]}$$
Moles of $CH_3COOH = 5\text{ mL} \times 0.01\text{mmol} = 0.5\text{mmol}$
Moles of $CH_3COONa = 10\text{ mL} \times 0.05\text{mmol}$
$= 0.5\text{ m mol}$
Total volume of solution $= 5 + 10 = 15\text{ mL}$
$$\therefore\ pH = pK_a + \log\frac{0.5/15}{0.5/15} \Rightarrow pH = pK_a$$
The value of pK_a is the lowest for the mixture of CH_3COOH and CH_3COONa (acidic buffer). Therefore, it has lowest pH .
Hence, pH value for option (c) is the lowest.

48. (b) Given, for two first order reactions.
$A \rightarrow B, k = 0.693\text{ min}^{-1} \ldots.(\text{i})$
and $A \rightarrow C, t_{1/2} = 0.693\text{ min}\ldots.(\text{ii})$
For first order reaction
$t_{1/2} = \frac{0.693}{k}\ldots.$
Thus, for the reaction $A \rightarrow B$
$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.693} = 1.0\text{ min}$$
Also, for first order reaction, lower the $t_{1/2}$ value, faster is the reaction.
$$\because\ t_{1/2} \propto \frac{1}{k}$$
and k for $A \rightarrow C$ is $\frac{0.693}{0.693} = 1\text{ min}^{-1}$
Thus, reaction (2) is faster than reaction (1) or reaction (1) is slower than reaction (2).
Hence, option (b) is the correct answer.

49. (d) For the given relation $H_2O(l) \rightleftharpoons H_2O(v)$.
(i) At equilibrium $\Delta G = 0$
(ii) Entropy change, i.e. ΔS is the measurement of randomness. It increases in vapour-state, i.e. $\Delta S = (+)$ ve or $\Delta S > 0$
(iii) When liquid H_2O changes into vapour H_2O, it requires heat energy. Thus, $\Delta H > 0$
Hence, correct set is
$\Delta G = 0, \Delta H > 0$ and $\Delta S > 0$
So, option (d) is the correct answer.

50. (b) Dimensions of $\left(\frac{ab}{V^2}\right)$ are shown below.
$$\frac{a}{V^2} \times b = \text{ pressure } \times \text{ volume}$$
$$= \frac{\text{force}}{\text{area}} \times \text{ volume}$$
$$= \frac{MLT^2}{L^2} \times L^3 = ML^2\ T^2$$
The units of energy (joule) is,
$1\text{ J} = 1\text{ kg m}^2\text{ s}^{-2} = ML^2\ T^{-2}$
Therefore, the dimensions of $\left(\frac{ab}{V^2}\right)$ is same as that of energy. So, the option (b) is correct.

51. (c) In the equilibrium $H_2 + I_2 \rightleftharpoons 2HI$, if at a given temperature, the concentrations of the reactants are increased, the value of the equilibrium constant K_C will remain the same because equilibrium constant does not depend on the molar concentration of reactants.
The option (c) is correct answer.

52. (d) Electrolysis of copper sulphate solution using Cu-electrodes,
Ions present: $Cu^{2+}, H^+, SO_4^{2-}, OH^-$
At cathode $Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$
At anode $Cu(s) - 2e^- \rightarrow Cu^{2+}(aq)$
So, the option (d) is correct answer.

53. (c) The three quantum numbers n, l and m that describe an orbital has integer values of 0,1,2,3 and so on.

Name	Symbol	Range of Values
Principal quantum number	n	$1 \leq n$
Azimuthal quantum number	$/$	$0 \leq 1 \leq n-1$
Magnetic quantum number	m	$-1 \leq m \leq 1$

The electronic arrangement of option (c) is absurd because
$n=2$
$l=0$
$m=0$
To sum up, where $n=2, l=0, m=0$.
Hence, for $l=0$, value of m should be zero.
54. (c) The quantity hv / K_{B} corresponds to temperature. $hv=3 / 2 K_{B} T$
where, $T=$ temperature
$K_{\mathrm{B}}=$ Boltzmann constant
The percentage of $\mathrm{H}_{2} \mathrm{O}$ in the solid is
$=(100-64)=36 \%$
$$\frac{hv}{K_{B}}=\frac{3}{2} T$$
Since, $\frac{3}{2}$ is a constant, the value of $\frac{hv}{K_{\mathrm{B}}}$ corresponds to temperature. Therefore option (c) is correct.
55. (c) Mass of $\mathrm{H}_{2} \mathrm{O}=\frac{36}{100} \times 250=90 \mathrm{~g}$
Moles of $\mathrm{H}_{2} \mathrm{O}=\frac{90}{18}=5 \mathrm{~mol}$
In the crystalline solid $\mathrm{MSO}_{4} \cdot n \mathrm{H}_{2} \mathrm{O}$, the value of n is 5 .
Hence, option (c) is correct answer.
56. (a) Given, mass of gas $(W)=7.5 \mathrm{~g}$
Volume of gas at STP (V) $=56 \mathrm{~L}$
$\because$ Moles of gas at STP $=\frac{V(\mathrm{~L})}{22.4(\mathrm{~L})}=\frac{W}{M}$
Where $M=$ molar mass of gas.
$$\therefore M=\frac{W \times 22.4}{V}$$
$$=\frac{7.5 \times 22.4}{5.6}=30.00 \mathrm{~g} \mathrm{~mol}^{-1}$$
Among the given options
Molar mass(M) of
(a) $\mathrm{NO}=14+16=30.00 \mathrm{~g} \mathrm{~mol}^{-1}$
(b) $\mathrm{N}_{2} \mathrm{O}=28+16=44.00 \mathrm{~g} \mathrm{~mol}^{-1}$
(c) $\mathrm{CO}=12+16=28.00 \mathrm{~g} \mathrm{~mol}^{-1}$
(d) $\mathrm{CO}_{2}=12+32=44.00 \mathrm{~g} \mathrm{~mol}^{-1}$
$\because$ Molar mass of $\mathrm{NO}=30 \mathrm{~g} \mathrm{~mol}^{-1}$.
Thus, the given gas is NO.
Hence, (a) is the correct option.
57. (b) Let, initial concentration $(a)=100$
Given, half-life $\left(t_{1 / 2}\right)=60$ days
To find, radioactivity,
i.e. $(a-x)$ after time T(180 days)
$\because T=n \times t_{1 / 2}$
where, $n=$ no. of half-lives

$180 = n \times 60$

$n = \frac{180}{60} = 3$

and $(a-x) = \frac{a}{2^n} = \frac{100}{2^3} = \frac{100}{8}$

$(a-x) = 12.5\%$

Hence, (b) is the correct answer.

58. (d) Given, ${}_{82}A^{210} \longrightarrow B \longrightarrow C \longrightarrow {}_{82}D^{206}$

i.e. difference in mass no. between A and D is of 4 units i.e., mass no. is decreased by 4 units while atomic no. remains the same, i.e. 82 for A and D. Also,

(i) On emission of each β-particle, mass no. remains the same but atomic no. is increased by one unit.

(ii) On emission of each α-particle, mass no. is decreased by 4 units, while atomic no. is decreased by 2 units.

Thus, on emission of $2-\beta$ and $1-\alpha$ particle, we get ${}_{82}D^{206}$,

i.e.

${}_{82}A^{210} \xrightarrow{-\beta} {}_{83}A^{210} \xrightarrow{-\beta} {}_{84}A^{210} \xrightarrow{-\alpha} {}_{82}A^{206}$

Thus, option (d) is the correct answer.

59. (a) Key Point For similar electronic configuration of outer most shell, size will decide the value of ionisation energy. More the size, lesser is the value of ionisation energy.

Electronic configuration	**IInd I.E**
Zn^+ion ($Z = 30$) = [Ar] $3d^{10}4s^1$,	(1734 kJ/ mol).
Cd^+ion ($Z = 48$) = [Kr]$4d^{10}5s^1$,	(1631 kJ/ mol)
Hg^+ion ($Z = 80$) = [Xe]$4f^{14} \cdot 5d^{10}6s^1$	(1809 kJ/ mol)

$\because$ Size of $Cd^+ > Zn^+$, thus it has lower IInd ionisation energy while due to lanthanoid effect (i.e., poor screening by $4f$ and $5d$ electrons, Hg^+has higher IInd ionisation-energy.

Hence, correct order is,

Zn > Cd < Hg and option (a) is the correct answer.

60. (b) Key Point A compound having more ionic character, has more melting point.

Be and Ca belong to the same group and ionic character increases down the group.

Thus, $BeCl_2$ is less ionic than $CaCl_2$. This means that melting point of $CaCl_2$ is higher than that of $BeCl_2$.

Also, Hg is a transition metal, its compound is less ionic than $BeCl_2$. Therefore, order of melting point will be

$CaCl_2 > BeCl_2 > HgCl_2$

or

(ii) > (i) > (iii)

Hence, option (b) is the correct answer.

61. (d) Key point The species which has one or more unpaired electrons have non-zero magnetic moment.

Electronic configuration of

$Na^+(Z = 11) = 1s^2 2s^2 2p^6$

$Mg(Z = 12) = 1s^2 2s^2 2p^6 3s^2$

$F^-(Z = 9) = 1s^2 2s^2 2p^6$

$Ar^+(Z = 18) = 1s^2 2s^2 2p^6 3s^2 3p^5$

As, Ar^+has an unpaired electron, it has non-zero magnetic moment. Hence, (d) is the correct option.

62. (b) Key Point

(i) Half-filled/fully-filled configuration have $(-)$ ve or low electron affinity i.e., $(+)$ ve or high electron gain enthalpy.

(ii) Smaller the size, more is value of electron affinity, i.e. more $(-)$ ve will be the electron gain enthalpy.

On moving across a period, the size of atoms decreases due to increase in nuclear charge.

(i) Electron configuration of the elements are shown below.

$C(Z=6)=1s^2 2s^2 2p^2$
$N(Z=7)=1s^2 2s^2 2p^3$
$O(Z=8)=1s^2 2s^2 2p^4$

(ii) Due to half-filled electronic configuration of nitrogen (i.e. 3 electrons in $3p$ orbitals), it is highly stable and has $(+)$ ve value of electron gain enthalpy (i.e. about 30.9 kJ/mol).
(iii) As size of ' O ', is smaller than that of ' C ', ' O ' atom has higher $(-)$ ve electron gain enthalpy (about -141.1 kJ/mol) than that of carbon (C-atom) [about- 122.3 kJ/mol].
Hence, correct order is $O > C > N$ and (b) is the correct option.

63. (a) Key point As the percentage of s character in a bond increases, the bond angle also increases.
Therefore, as the bond angle increases, the percentage of p-character decreases.
In the given options,
Ammonia (NH_3) has bond angle $= 107.6°$ and Phosphine (PH_3) has bond angle $= 93.5°$
$\because$ Bond angle $\propto \frac{1}{p\text{-character}}$ and $\propto s$-character
Bond angle of $NH_3 > PH_3$ due to lone-pair lone-pair repulsion.
Thus, the lone pair on NH_3 has less p-character. Hence, option (a) is the correct answer.

64. (b) Chlorine bleach is NaOCl . The hypochlorite ion in chlorine bleach dissociates to give nascent oxygen as shown below.
$OCl^- \longrightarrow [O] + Cl^-$
Therefore, the reactive species in chlorine bleach is OCl^-.
So, the option (b) is correct.

65. (d) Co-ordinate compound of Co (III) dissociates into 3 ions in the solution means it has two ionisable-ions out side the coordination sphere with oxidation state of cobalt (Co) $= +3$

In option (a)
Name : Hexaammine cobalt (III) chloride
Formula : $[Co(NH_3)_6]Cl_3$
$\because$ Oxidation state of Co $= (+)3$

In option (b)
Name : Pentaammine sulphatocobalt (III) chloride
Formula : $[Co(NH_3)_5(SO_4)]Cl$
$\because$ Oxidation state of Co $= +3$

In option (c)
Name : Pentaamminechloridocobalt (III) sulphate
Formula : $[Co(NH_3)_5Cl]SO_4$
$\because$ Oxidation state of Co $= +3$

In option (d)
Name : Pentaamminechloridocobalt (III) chloride
Formula : $[Co(NH_3)_5Cl]Cl_2$
$\because$ Oxidation state of Co $= +3$
Dissociation of this compound is shown below:
$[CO(NH_3)_5Cl]Cl_2 \rightarrow CO\underbrace{(NH_3)_5Cl]^{2+} + 2Cl^-}_{3\text{ ions}}$
This compound can give total 3-ions, as follows. one $[Co(NH_3)_5Cl]^{2+}$ and two Cl^-ions.
Hence, option (d) is the correct answer.

66.(b) In the Bayer's process, the leaching of alumina is done by using NaOH .

$$\underset{\text{Bauxite ore}}{Al_2O_3(s)} + 2NaOH(aq) \xrightarrow{150°C,\ 35\text{ atm}} \underset{\substack{\text{Sodium}\\ \text{aluminate}\\ \text{(soluble in water)}}}{2Na[Al(OH)_4]} + 3H_2O$$

$$2Na[Al(OH)_4] + CO_2 \rightarrow Al_2O_3 \cdot xH_2O + 2NaHCO_3$$

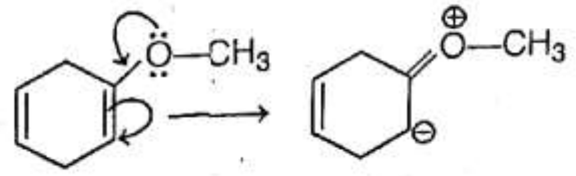
$$Al_2O_3 \cdot xH_2O(s) \xrightarrow{1470\text{ K}} \underset{\text{Pure alumina}}{Al_2O_3(s)} + xH_2O(g)$$

So, the option (b) is correct answer.

67. (d) Among the given options,

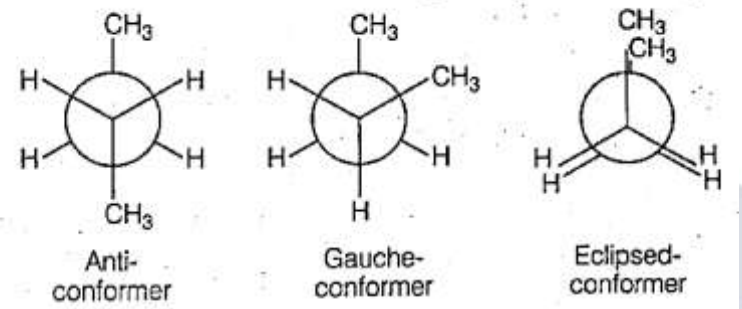
${}_{92}U^{238}$ is an isotope of uranium but cannot be used as a nuclear-fuel. U-238 is non-fissile, i.e., it does not undergo fission by thermal neutrons. The energy released when U − 238 absorbs a neutron is insufficient to carry out nuclear fission.
Hence, (d) is the correct option.

68. (d) The molecule in which a lone pair of electrons undergoes conjugation with an alternate double bond is said to be delocalised. The molecule that has delocalised lone-pair of electrons is,



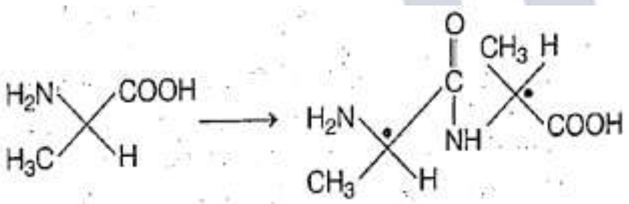
So, option (d) is correct.

69. (d) The conformation of n-butane, commonly known as eclipsed, gauche and anti-conformations can be interconverted by rotation around C2 − C3 linkage.



70.(b) In halogen acids, as the size of halogen atom increases, the bond between halogen and hydrogen atom weakens. Therefore, the case with which the bond can be broken increases.
The correct order of the addition reaction rates of halogen acids with ethylene is hydrogen iodide (HI) > Hydrogen bromide (HBr)
> Hydrogen chloride (HCl)
So, the option (b) is correct answer.

71. (d) Total number of isomeric linear dipeptides which can be synthesised from racemic mixture of alanine are four (4), by (R) type and (S) type.
(∵ Has two chiral centres)
i.e.



The possible isomers are
$(RR), (SR), (RS)$ and (SS).
Hence, total (4) isomeric linear dipeptides for alanine can be synthesised and option (d) is the correct answer.

72. (d) From the given graph

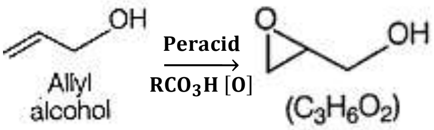
$$\text{Slope } = \frac{(\text{ rate })^{1/2}}{[A]} = 4$$

$\because$ rate $(r) = k[A]^n$
Where, $k =$ rate constant
$[A] =$ concentration at time t(in min).
$n =$ order .
According to the graph ($n = 2$)

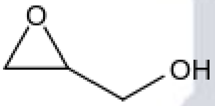
$$\therefore\ k = \frac{\text{rate}}{[A]^n} = [4]^2 = 16$$

Hence, $n = 2$ and
$k = 16.0 \text{dm}^3 \text{ mol}^{-1} \text{ min}^{-1}$

and option (d) is the correct answer.
73. (a) Key Point
According to Kirchhoff's equation
$\Delta H_{(f)} = \Delta H_{(i)} + C_P(\Delta T)$
At constant pressure
$\Delta C_p = 0$
Therefore, heat of formation (ΔH_f) does not depend on temperature and value of ΔC_p,
i.e. when $\Delta C_p = 0$
Hence, option (a) is the correct answer.
74. (d) A copper coin was electroplated with zinc (Zn) and then heated at high temperature until there is a change in colour. The resulting colour will be golden.
The golden colour is due to the zinc migrating through the copper to form the alpha-form of brass alloy (percentage of Cu > 65% and that of Zn < 35%).
$\mathrm{Zn} + \mathrm{Cu} \xrightarrow{\Delta} \text{Brass}$
Thus, the option (d) is correct answer.
75. (a)

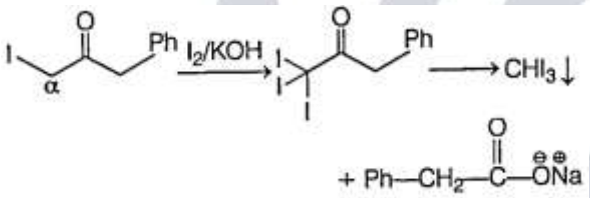


Oxidation of allyl alcohol with a peracid gives a compound of molecular formula $C_3H_6O_2$, which contains an asymmetric carbon atom.
The structure of the compound is

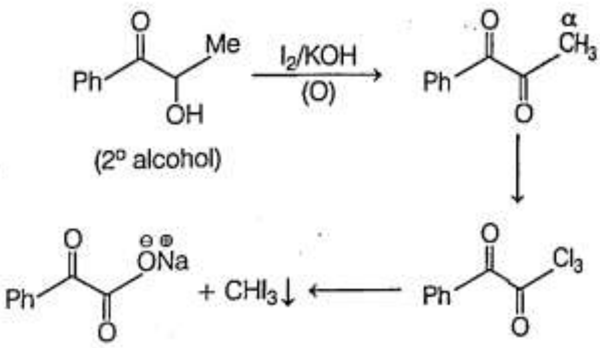


So, the option (a) is correct answer.
76. (a, b) Haloform reaction with I_2 and KOH

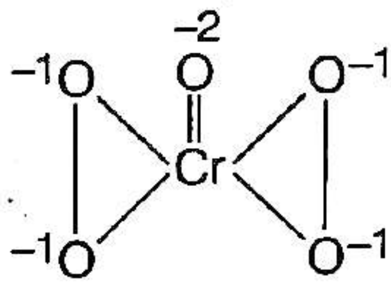
(a)



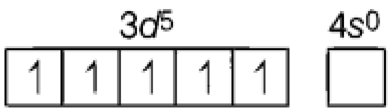
(b)



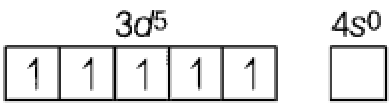
77. (a, b)
(a) In CrO_5 oxidation no. of Cr is +6 .



(b) For the reaction (in ideal case)
$N_2O_4(g) \longrightarrow 2NO_2(g)$
$\Delta H = \Delta U + p \cdot \Delta V$
$\because\ \Delta H = \Delta U + \Delta n_g RT$
$(\because p\Delta V = \Delta n_g RT)$
Where, $\Delta n_g =$ (gaseous moles of product)
(gaseous moles of reactant)
$\Delta n_g = \dot{2} - 1 = 1$
Thus, $\Delta H > \Delta U$
(c) pH of 0.1 NH_2SO_4 is less than of 0.1 HCl at 25° is a wrong statement.
pH of H_2SO_4
$\because pH = -\log[H^+] = -\log[10^{-2}]$
$\because\ (N = C \times Z)$ and ($Z = 2$ for H_2SO_4)
pH $= 2$ and **pH** of **HCl**
$pH = -\log[10^{-1}] = 1$
Thus, pH of $H_2SO_4 >$ pH of HCl .
(d) $\frac{RT}{F} = 0.0591$ at 25°C is a wrong statement.
Where, R = Gas constant = 8.314
T = Temperature = 273 + 25 = 298 K
F = Charge over one mole of electrons
= 96500C
Thus, $\frac{RT}{F} = \frac{8.314 \times 298}{96500} = 0.02567$
The correct value is $\frac{2.303RT}{F} = 0.0591$
Hence, only option (a) and (b) are correct.
78. (b, c, d) Among the given species
(a) H_2O and F are weak field ligands and do not cause pairing of electrons in the central metal atom. As CN is a strong field ligand, it causes pairing of electrons in the central metal atom.
Now,
Oxidation state of :
(b) Fe in $[Fe(H_2O)_6]Cl_3 = (+)3$ and electronic configuration of Fe^{3+} ions is $3d^5 4s^0$
$Fe^{3+} =$

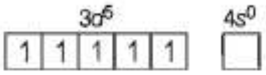


i.e. have 5 unpaired electrons
(c) Fe in $K_3[FeF_6]$
Oxidation no. of Fe = (+)3
electronic configuration of Fe^{3+} is $3d^5 4s^0$



i.e. have 5 unpaired electrons.
(d) Oxidation state of Mn in $K_4[MnF_6] = (+) \cdot 2$ and electronic configuration of Mn^{2+} is $3d^5 4s^0$
$Mn^{2+} =$

$Fe^{3+} = Mn^{2+} =$



i.e., have 5 unpaired electrons.
Hence, options (b) (c) and (d) are the correct options.

79. (a, c)

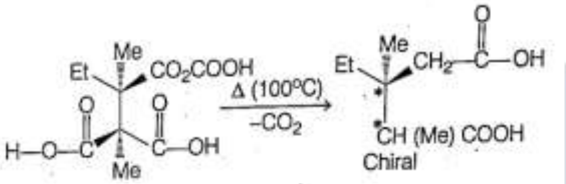
(a) NH_4NO_3 will evolve NH_3 pungent smelling gas with NaOH .
NO_2 brown gas with con. H_2SO_4
No reaction with water

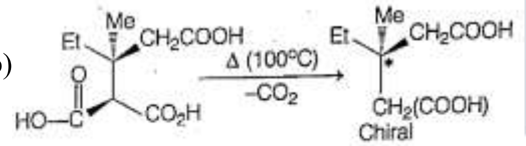
Thus, NH_4NO_3 can be used to label all three beakers.

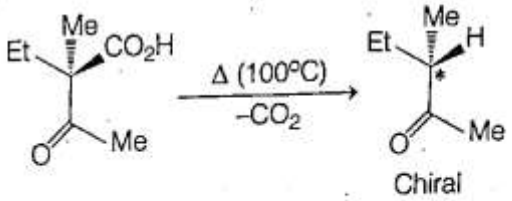
(c) $(NH_4)_2CO_3$ will evolve pungent smelling gas NH_3 with NaOH .
Effervescence of CO_2 gas with con. H_2SO_4
No reaction with water.

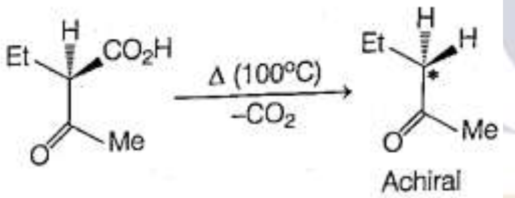
Thus, $(NH_4)_2CO_3$ can be used to label all three beakers.
Thus, options (a, c) are correct.

80. (d)

(a)


(b)


(c)


(d)


Mathematics

81. (b) We have,

$$\lim_{x \to 0^+} x^n \ln x$$

$$= \lim_{x \to 0^+} \frac{\ln x}{x^{-n}}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-nx^{-n-1}} \text{ [using L'Hospital's rule]}$$

$$= \lim_{x \to 0^+} \frac{-1}{nx^{-n}} = 0 \left[\because \frac{1}{x-n} = 0, \text{ when } x = 0\right]$$

82. (b) Let $I = \int \cos x \log\left(\tan\frac{x}{2}\right) dx$

$$= \log\left(\tan\frac{x}{2}\right) \cdot \sin x - \int \sin x \cdot \frac{1}{\tan\frac{x}{2}} \cdot \sec^2\frac{x}{2} \cdot \frac{1}{2} dx$$

$= \sin x \log\left(\tan\frac{x}{2}\right) - \int \sin x \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx$

$= \sin x \cdot \log\left(\tan\frac{x}{2}\right) - \int \frac{\sin x}{\sin x} dx$

$= \sin x \cdot \log\left(\tan\frac{x}{2}\right) - \int 1 dx$

$= \sin x \log\left(\tan\frac{x}{2}\right) - x + c$

$\therefore f(x) = c - x$

83. (d) Let $I = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\} dx$

Put $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta d\theta$

$\therefore I = \int \cos\left\{2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right\}(-2\sin 2\theta)d\theta$

$= -2\int \cos\left\{2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right\}\sin 2\theta d\theta$

$= -2\int \cos\{2\tan^{-1}(\tan\theta)\}\sin 2\theta d\theta$

$= -2\int \cos 2\theta \sin 2\theta d\theta$

$= \int \cos 2\theta d(\cos 2\theta)$

$= \frac{\cos^2 2\theta}{2} + C$

$= \frac{x^2}{2} + C$

Which is an equation of parabola.

84. (b) Let $I = \int_{-\frac{\pi}{4}}^{\pi/4} \left(\lambda|\sin x| + \frac{\mu \sin x}{1+\cos x} + \gamma\right) dx$

$= \int_{-\pi/4}^{\pi/4} (\lambda|\sin x| + \gamma)dx + \mu\int_{-\frac{\pi}{4}}^{\pi/4} \frac{\sin x}{1+\cos x} dx$

Let $f(x) = \frac{\sin x}{1+\cos x}$

$\Rightarrow f(-x) = \frac{\sin(-x)}{1+\cos(-x)} = \frac{-\sin x}{1+\cos x} = -f(x)$

$\therefore f(x)$ is an odd function.

So, $\int_{-\frac{\pi}{4}}^{\pi/4} \left(\frac{\sin x}{1+\cos x}\right) dx = 0$

$\therefore I = \int_{-\frac{\pi}{4}}^{\pi/4} \lambda|\sin x| + \gamma\Big) dx$

$\therefore I$ is independent of μ.

85. (a) We have,

$$\lim_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt - \int_{x+y}^a e^{\sin^2 t}dt\right]$$
$$= \lim_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt + \int_a^{x+y} e^{\sin^2 t}dt\right]$$
$$= \lim_{x\to 0}\frac{1}{x}\int_y^{x+y} e^{\sin^2 t}dt$$
$$= \lim_{x\to 0}\frac{\int_y^{x+y} e^{\sin^2 t}dt}{x}$$
$$= \lim_{x\to 0}\frac{e^{\sin^2(x+y)}(1+0)-0}{1} = e^{\sin^2 y}$$

86. (d) Let $I = \int 2^{2^x}\cdot 2^x dx$

Put $t = 2^x$

$\Rightarrow dt = 2^x \log 2 dx$

$\therefore\ I = \int \frac{2^t}{\log 2}dt$

$= \frac{2^t}{(\log 2)^2} + C = \frac{2^{2^x}}{(\log 2)^2} + C$

$\therefore\ A = \frac{1}{(\log 2)^2}$

87. (d) Let $I = \int_{-1}^{1}\left\{\frac{x^{2015}}{e^{|x|}(x^2+\cos x)} + \frac{1}{e^{|x|}}\right\}dx$

$= \int_{-1}^{1}\frac{x^{2015}}{e^{|x|}(x^2+\cos x)}dx + \int_{-1}^{1}\frac{1}{e^{|x|}}dx$

Let $f(x) = \frac{x^{2015}}{e^{|x|}(x^2+\cos x)}$ and $g(x) = \frac{1}{e^{|x|}}$

Again, $f(-x) = \frac{(-x)^{2015}}{e^{1-x}((-x)^2+\cos(-x))}$

$= \frac{-x^{2015}}{e^{|x|}(x^2+\cos x)} = -f(x)$

and $g(-x) = \frac{1}{e^{|-x|}} = \frac{1}{e^{|x|}} = g(x)$

$\because$ $f(x)$ is odd function and $g(x)$ is even function.

$\therefore \int_{-1}^{1} f(x)dx = 0$ and $\int_{-1}^{1} g(x)dx = 2\int_0^1 g(x)dx$

$$\therefore\ I = 2\int_0^1 \frac{1}{e^{|x|}}dx$$
$$= 2\int_0^1 \frac{1}{e^x}dx = 2\int_0^1 e^{-x}dx$$
$$= -2[e^{-x}]_0^1$$
$$= -2[e^{-1} - e^0]$$
$$= -2(e^{-1} - 1)$$
$$= 2(1 - e^{-1})$$

88. (c) $\lim_{n\to\infty}\frac{3}{n}\left[1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \cdots + \sqrt{\frac{n}{n+3(n-1)}}\right]$

$= \lim_{n\to\infty}\dot{\bar{n}}\left[\sqrt{\frac{n}{n+3\times 0}} + \sqrt{\frac{n}{n+3\times 1}} + \sqrt{\frac{n}{n+3\times 2}} + \sqrt{\frac{n}{n+3\times 3}} + \cdots + \sqrt{\frac{n}{n+3(n-1)}}\right]$

$= \lim_{n\to\infty}\frac{3}{n}\left[\sum_{r=0}^{n-1}\sqrt{\frac{n}{n+3r}}\right]$

$= \lim_{n\to\infty}\frac{3}{n}\left[\sum_{r=0}^{n-1}\sqrt{\frac{1}{1+3\left(\frac{r}{n}\right)}}\right]$

$$= 3\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\sqrt{\frac{1}{1+3\left(\frac{r}{n}\right)}}$$

$$= 3\int_0^1\sqrt{\frac{1}{1+3x}}dx$$

$$= 3\int_0^1 (1+3x)^{-1/2}dx$$

$$= 3\left[\frac{2\sqrt{1+3x}}{3}\right]_0^1$$

$$= [4-2]$$

$$= 2$$

89. (c) $\left(1+e^{\frac{x}{y}}\right)dx+\left(1-\frac{x}{y}\right)e^{x/y}dy=0$

$\Rightarrow \left(1+e^{x/y}\right)dx=-e^{x/y}\left(1-\frac{x}{y}\right)dy$

$\Rightarrow \frac{dx}{dy}=\frac{-e^{x/y}(1-x/y)}{(1+e^{x/y})}\ldots$(i)

This is homogeneous differential equation.

So, put $x=vy$

$\frac{dx}{dy}=v+y\frac{dv}{dy}\ldots$.(ii)

Therefore, $v+y\frac{dv}{dy}=\frac{-e^v(1-v)}{1+e^v}$

[from Eqs. (i) and (ii)]

$$\Rightarrow y\frac{dv}{dy}=\frac{-e^v+ve^v}{1+e^v}-v$$

$$=\frac{-e^v+ve^v-v(1+e^v)}{1+e^v}$$

$= -e^v+ve^v-v-ve^v$

$$\Rightarrow y\frac{dv}{dy}=\frac{-(e^v+v)}{1+e^v}$$

$\Rightarrow \frac{1+e^v}{v+e^v}dv=-\frac{dy}{y}$ [variable separation]

$\Rightarrow \int\frac{1+e^v}{v+e^v}dv=-\int\frac{dy}{y}$ [by integration]…(iii)

Let $v+e^v=t\Rightarrow(1+e^v)dv=dt$

$\therefore \int\frac{1+e^v}{v+e^v}dv=\int\frac{dt}{t}=\log t=\log(v+e^v)$….(iv)

$\Rightarrow \log(v+e^v)=-\log y+\log C$

[from Eqs. (iii) and (iv)]

$\Rightarrow \log(v+e^v)+\log y=\log C$

$\Rightarrow y(v+e^v)=C$

$$\Rightarrow y\left(\frac{x}{y}+e^{x/y}\right)=C$$

$$\Rightarrow y\left(\frac{x+ye^{x/y}}{y}\right)=C$$

$\Rightarrow x+ye^{x/y}=C$

90. (d) $(x+y)^2\frac{dy}{dx}=a^2, a\neq 0$

Let $x+y=t$

$$1+\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{dt}{dx}-1$$

Therefore, $t^2\left(\frac{dt}{dx}-1\right)=a^2$

$\Rightarrow t^2\frac{dt}{dx}-t^2=a^2$

$\Rightarrow t^2\frac{dt}{dx}=a^2+t^2$

$\Rightarrow \frac{t^2}{a^2+t^2}dt=dx$ [variable separation]

$\Rightarrow \int\frac{t^2}{(t^2+a^2)}dt=\int dx$ [integration]

$\Rightarrow \int\frac{t^2+a^2-a^2}{(t^2+a^2)}dt=x+C'$

[doing a^2 adding and subtracting]

$\Rightarrow \int\left(1-\frac{a^2}{t^2+a^2}\right)dt=x+C'$

$\Rightarrow \int dt-\int\frac{a^2}{t^2+a^2}dt=x+C'$

$\Rightarrow t-a^2\int\frac{dt}{t^2+a^2}=x+C'$

$\Rightarrow t-a^2\left[\frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)\right]=x+C'$

$\Rightarrow (x+y)-a\tan^{-1}\left(\frac{x+y}{a}\right)$

$=x+C$[on putting the value of t]

$\Rightarrow \frac{y-C'}{a}=\tan^{-1}\left(\frac{x+y}{a}\right)$

$\Rightarrow \frac{x+y}{a}=\tan\left(\frac{y-C'}{a}\right)$

$\Rightarrow \frac{x+y}{a}=\tan\left(\frac{y+C}{a}\right)$

[where, $C=-C'$ is an arbitrary constant]

$\Rightarrow \tan\left(\frac{y+C}{a}\right)=\frac{x+y}{a}$

91. (b) Given hyperbola equation

$H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$P(4,3)$ lie on H

$\Rightarrow \frac{16}{a^2}-\frac{9}{b^2}=1$

Normal equation at $P(4,3)$ for H

$$a^2y_1(x-x_1)+b^2x_1(y-y_1)=0$$

$$\Rightarrow \quad 3a^2(x-4)+4b^2(y-3)=0$$

Normal cuts the X-axis at $(16,0)$

$\Rightarrow \quad 3a^2(16-4)+4b^2(0-3)=0$

$\Rightarrow \quad 3a^2(12)+4b^2(-3)=0$

$\Rightarrow \quad 36a^2-12b^2=0$

$\Rightarrow \quad 3a^2-b^2=0$

$\Rightarrow \quad 3a^2=b^2$

$\therefore$ Eccentricity of the hyperbola is

$$e=\frac{\sqrt{a^2+b^2}}{a}$$

$$=\frac{\sqrt{a^2+3a^2}}{a}=\frac{\sqrt{4a^2}}{a}=\frac{2a}{a}=2$$

92. (b) Let V be the volume of spherical ballon of radius r.

Then, $V=\frac{4}{3}\pi r^3$

$\Rightarrow \log V=\log\left(\frac{4\pi}{3}\right)+3\log r \Rightarrow \frac{1}{V}\frac{dV}{dr}=0+\frac{3}{r}$

$\Rightarrow \frac{1}{V}\frac{dV}{dr}=\frac{3}{r} \Rightarrow \frac{1}{V}\frac{\Delta V}{\Delta r}=\frac{3}{r}$

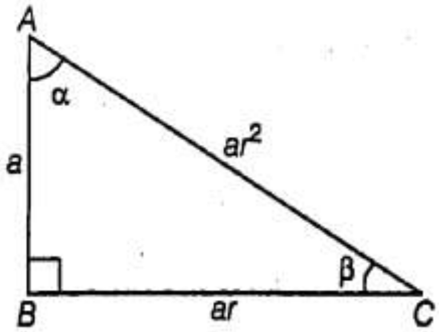
$\Rightarrow \frac{\Delta V}{V}=3\frac{\Delta r}{r}$

$\Rightarrow \frac{\Delta V}{V}\times 100=3\frac{\Delta r}{r}\times 100$

$\Rightarrow \frac{\Delta V}{V}\times 100=3\times 01=0.3$

$\therefore$ Percentage increase in volume is 0.3%

93. (b) Let $\triangle ABC$ be a right angled triangle at B. Let $\angle A$ and $\angle C$ be α and β



Since, sides are in GP so sides are a, ar, ar^2

Now,

$AC^2=AB^2+BC^2$

$\Rightarrow (ar^2)^2=a^2+(a\cdot r)^2$

$\Rightarrow a^2r^4=a^2+a^2r^2$

$\Rightarrow r^4-r^2-1=0$

$\Rightarrow r^2=\frac{1+\sqrt{5}}{2}\ [r^2>0]$

$\Rightarrow r=\sqrt{\frac{\sqrt{5}+1}{2}}$

$\therefore \tan\alpha=\frac{BC}{AB}=\frac{ar}{a}$

$\Rightarrow r=\sqrt{\frac{\sqrt{5}+1}{2}}$

and $\tan\beta=\frac{AB}{BC}=\frac{a}{ar}=\frac{1}{r}$

$=\frac{1}{\sqrt{\frac{\sqrt{5}+1}{2}}}=\sqrt{\frac{\sqrt{5}-1}{2}}$

94. (b) We have,

$$\log_2^6 + \frac{1}{2x} = \log_2\left(2^{\frac{1}{x}} + 8\right)$$

$\Rightarrow \quad 1 + \log_2^3 + \frac{1}{2x} = \log_2\left(2^{\frac{1}{x}} + 8\right)$

$\Rightarrow \quad \dfrac{2^{\frac{1}{x}} + 8}{3} = 2^{1+\frac{1}{2x}}$

$\Rightarrow \quad 2^{\frac{1}{x}} + 8 = 3 \cdot 2^{\frac{1}{2x}}$

$\Rightarrow \quad 2^{\frac{1}{x}} + 8 = 6 \cdot 2^{\frac{1}{2x}}$

Let $\quad y = 2^{\frac{1}{2x}}$

$\Rightarrow \quad y^2 + 8 = 6y$

$\Rightarrow \quad y^2 - 6y + 8 = 0$

$\Rightarrow \quad (y-4)(y-2) = 0$

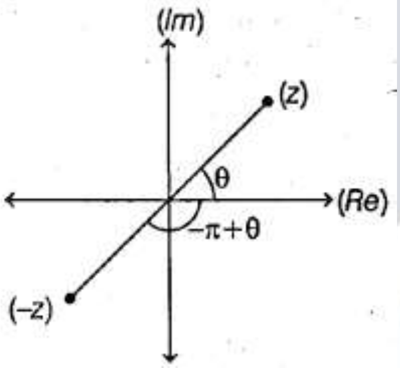
$\Rightarrow \quad y = 4,2$

$\Rightarrow \quad \dfrac{1}{2^{2x}} = 4 \text{ and } 2^{\frac{1}{2x}} = 2$

$\Rightarrow \quad \dfrac{1}{2x} = 2 \text{ and } \dfrac{1}{2x} = 1$

$\Rightarrow \quad x = \dfrac{1}{4}, \dfrac{1}{2}$

95. (c)



$\therefore \arg(z) - \arg(-z) = \theta - (-\pi + \theta)$
$= \pi$

96. (b) We have,

$(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta) \ldots$

$\Rightarrow \quad (\cos n\theta + i\sin n\theta) = 1$

$\Rightarrow \quad e^{i\theta} \cdot e^{i(2\theta)} \cdot e^{i(3\theta)} \cdot \ldots e^{i(n\theta)} = 1$

$\Rightarrow \quad e^{i\theta(1+2+3+\cdots+n)} = 1$

$\Rightarrow \quad \cos\left(\frac{n(n+1)}{2}\theta\right) + i\sin\left(\frac{n(n+1)}{2}\right)\theta = 1 + 0i$

$\Rightarrow \quad \cos\left(\frac{n(n+1)}{2}\theta\right) = 1$

$\Rightarrow \quad \dfrac{4k}{2} = 1$

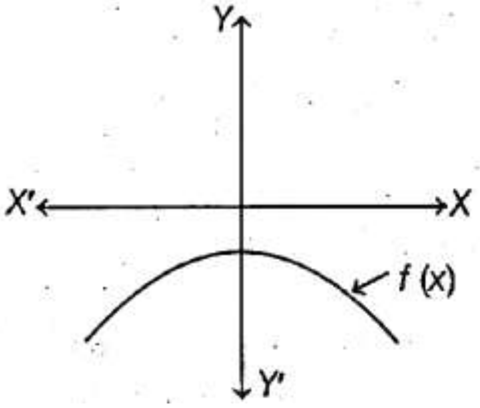
$\Rightarrow \quad \dfrac{n(n+1)}{n(n+1)}\pi$

97. (d) Let $f(x) = ax^2 + bx + c$

$\Rightarrow f(1) = a + b + c < 0$

Again, $f(x)$ has imaginary zeros. So, $a < 0$.

Also, $f(0) = c$. Since $f(x)$ is downward parabola. So, $c < 0$.



98. (a)

A	B
4	2
3	3
2	4

∴ Total number of ways

$= {}^6C_4 \times {}^6C_2 + {}^6C_3 \times {}^6C_3 + {}^6C_2 \times {}^6C_4$

$= \frac{6\times5}{2\times1} \times \frac{6\times5}{2\times1} + \frac{6\times5\times4}{3\times2\times1} \times \frac{6\times5\times4}{3\times2\times1}$

$+ \frac{6\times5}{2\times1} \times \frac{6\times5}{2\times1}$

$= 15 \times 15 + 20 \times 20 + 15 \times 15$

$= 225 + 400 + 225$

$= 850$

99. (b) Required number of ways

$= {}^7C_4 \times D(3)$

$= {}^7C_3 \times \left[3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right)\right]$

$= \left[\begin{array}{l} \because {}^nC_r = {}^nC_{n-r}, \text{ and} \\ D(n) = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots\right] \end{array}\right]$

$= {}^7C_3 \times 2 = 2({}^7C_3)$

100. (d) We have,

$7^{2n} + 16n - 1$

Put $n = 1$, we get

$7^2 + 16 - 1$

$= 49 + 15$

$= 64$

which is divisible by 64.

101. (b) Given, $\left(3^{\frac{1}{8}} + 5^{\frac{1}{4}}\right)^{84}$

here, $T_{r+1} = {}^{84}C_r \left(3^{\frac{1}{8}}\right)^{84-r} \left(5^{\frac{1}{4}}\right)^r$

$= {}^{84}C_r \left(3^{\frac{84-r}{8}}\right) \cdot 5^{\frac{r}{4}}$

Since, T_{r+1} is rational for $r = 4,12,20,28,36,44, 52,60,68,76,84$

∴ no. of rational terms $= 11$

Here, total number of terms $= 85$

∴ Number of irrational terms $= 85 - 11$

$= 74$

102. **(c)** Given, that $A-3I_3$

$$= \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} - 3\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} - \begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$$

$$= \begin{bmatrix}-2&1&1\\1&-2&1\\1&1&-2\end{bmatrix}$$

$$= [-2(4-1)-1(-2-1)+1(1+2)]$$

$$= [-2(3)-1(-3)+1(3)]$$

$$= [-6+3+3] = 0$$

Now, det $= 0$

So, matrix $A-3I_3$ is non-invertible matrix.

103. **(c)**

Since, $\mathrm{adj}(M') = (\mathrm{adj} M'$

$= \mathrm{adj}(M') - (\mathrm{adj} M'$

$[\because \mathrm{adj}(A) = (\mathrm{adj} A)']$

$= 0$, a null matrix

104. **(a)** Given,

$$A = \begin{bmatrix}5&5x&x\\0&x&5x\\0&0&5\end{bmatrix}$$

$\therefore\ A^2 = A\cdot A$

$$= \begin{bmatrix}5&5x&x\\0&x&5x\\0&0&5\end{bmatrix}\begin{bmatrix}5&5x&x\\0&x&5x\\0&0&5\end{bmatrix}$$

$$= \begin{bmatrix}25&25x+5x^2&5x+25x^2+5x\\0&x^2&5x^2+25\\0&&0\end{bmatrix}$$

$$= 25(25x^2-0)$$

$$= 25(25x^2)$$

Since, given that $|A|^2 = 25$

Now, $25(25x^2) = 25$

$\Rightarrow\ 25x^2 = 1$

$\Rightarrow\ x^2 = \dfrac{1}{25}$

$\Rightarrow\ x = \pm\dfrac{1}{5}$

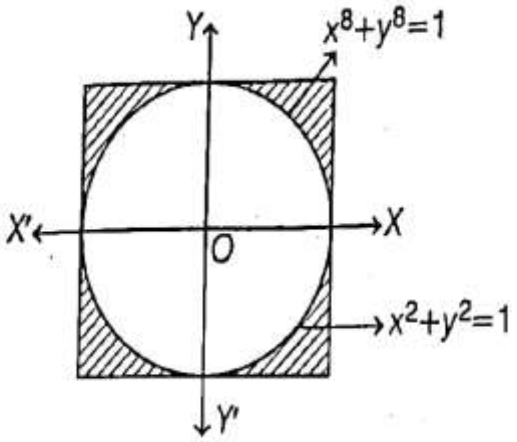
$\Rightarrow\ x = \dfrac{1}{5}$

105. **(d)** Since, product of two non-null matrix can be a null matrix.

Therefore, may be

$A \neq O_3, B \neq O_3$

106. **(a)**



From the graph, it is clear that there is common region which satisfies $x^2 + y^2 = 1$ and $x^8 + y^8 < 1$
$\because\ P \cap T = P$

107. **(d)** We have,

$f(x) = x^2 - \frac{x^2}{1+x^2}$

$\therefore\ f(-x) = (-x)^2 - \frac{(-x)^2}{1+(-x)^2}$

$= x^2 - \frac{x^2}{1+x^2} = f(x)$

$\therefore f(-x) = f(x)$.

So, $f(x)$ is many one

Again, $f(x) = x^2 - \frac{x^2}{1+x^2}$

$= \frac{x^2 + x^4 - x^2}{1 + x^2} = \frac{x^4}{1 + x^2}$

$\therefore$ Range of $f(x)$ will be $[0, \infty)$

But, co-domain of $f(x) = R$

$\therefore$ Range $\neq$ co-domain. So, $f(x)$ is onto.

108. **(d)** We observe the following properties:

Reflexivity : Let a be an arbitrary element of R, Then, $a \in R$

$\Rightarrow 1 + a \cdot a = 1 + a^2 > 0$ $[\because a^2 > 0$ for all $a \in R]$

$\Rightarrow\ (a, a) \in R_1$ [By def. of R_1]

Thus, $(a, a) \in R_1$ for all $a \in R$. So, R_1 is reflexive on R.

Symmetry : Let $(a, b) \in R$.

Then, $(a, b) \in R_1$

$\Rightarrow\ 1 + ab > 0$ $[\because ab = b$ for all $]$

$\Rightarrow\ 1 + ba > 0 \Rightarrow (b, a) \in R_1$

Thus, $(a, b) \in R_1 \neq (b, a) \in R$, for all $a, b \in R$

So, R_1 is symmetric on R.

Transitivity : we observe that $\left(1, \frac{1}{2}\right) \in R_1$ and $\left(\frac{1}{2}, -1\right) \in R_1$ but $(1, -1) \notin R_1$ because

$1 + 1 \times (-1) = 0 \ngtr 0$

So, R_1 is not transitive on R.

109. **(c)** Let $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

$P(C) = \frac{1}{4}, P(D) = \frac{1}{5}$

Now, $P(A \cup B \cup C \cup D)$

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})$

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)$

$= 1 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right) = 1 - \frac{1}{5} = \frac{4}{5}$

110. **(d)** Given that,

$$\sigma(X) = 26 = \sqrt{\text{var}(x)}$$

$$\sigma(1-4x) = \sqrt{\text{var}(1-4x)}$$

$$= \sqrt{16\text{var}(x)}$$

$$= 4 \times \sqrt{\text{var}(x)} = 4 \times 26 = 10.4$$

111. **(a)** Given that, $e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = t$

$\therefore \quad t - t^{-1} - 4 = 0$

$\Rightarrow \quad t - \frac{1}{t} - 4 = 0$

$\Rightarrow \quad t^2 - 1 - 4t = 0$

$$t = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

Now, $e^{\sin x} = 2 \pm \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 + \sqrt{5}$ and $e^{\sin x} = 2 - \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 + \sqrt{5}$

$[\, e^x > 0 \,]$

Also, $-1 \le \sin x \le 1$

$\therefore \frac{1}{e} \le e^{\sin x} \le e$

$\therefore e^{\sin x} \ne 2 + \sqrt{5}$

Hence, there is no value of x.

112. **(d)** Given, that angles of a triangles are $2x, 3x$ and $7x$.

Since, $2x + 3x + 7x = 180$

$\Rightarrow \; 12x = 180^\circ \Rightarrow x = 15^\circ$

So, angles are $30^\circ, 45^\circ$ and 105°

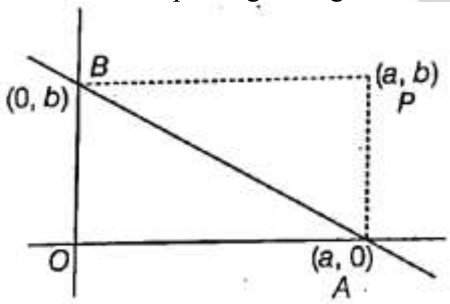
Now, $\frac{a}{\sin 30^\circ} = 2R$

$\Rightarrow \; \frac{a}{\frac{1}{2}} = 2 \times 10 \; [\because R = 10 \text{ cm}]$

$\Rightarrow \; 2a = 20 \Rightarrow a = 10 \text{ cm}$

113. **(b)** Let the equation of line be $\frac{x}{a} + \frac{y}{b} = 1$

Since, the line passing through a fixed point (x_1, y_1).



$\therefore \; \frac{x_1}{a} + \frac{y_1}{b} = 1$

Since, $OAPB$ is a rectangle, therefore the coordinate of P will be (a, b).

Hence, locus of P is

$\frac{x_1}{x} + \frac{y_1}{y} = 1$

114. **(b)** Given line,

$$\sqrt{3}x + y = 1$$

$\Rightarrow \quad y = -\sqrt{3}x + c$

We know that, $m = -\sqrt{3}$

$[\because y = mx + c]$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan 60^\circ = \left|\frac{m_1 - (-\sqrt{3})}{1 + m_1(-\sqrt{3})}\right| \quad [\because \theta = 60^\circ]$$

$$\sqrt{3} = \left|\frac{m_1 + \sqrt{3}}{1 - \sqrt{3}} m_1\right|$$

$$\pm\sqrt{3} = \frac{m_1 + \sqrt{3}}{1 - \sqrt{3}m_1}$$

taking + sign	**taking - sign**
$+\sqrt{3} = \frac{m_1 + \sqrt{3}}{1 - \sqrt{3}m_1}$	$-\sqrt{3} = \frac{m_1 + \sqrt{3}}{1 - \sqrt{3}m_1}$
$\sqrt{3}(1 - \sqrt{3}m_1) = m_1 + \sqrt{3}$	$\Rightarrow -\sqrt{3}(1 - \sqrt{3}m_1) = m_1 + \sqrt{3}$
$\Rightarrow \sqrt{3} - 3m_1 = m_1 + \sqrt{3}$	$\Rightarrow -\sqrt{3} - 3m_1 = m_1 + \sqrt{3}$
	$\Rightarrow 3m_1 - m_1 = 2\sqrt{3}$
$\Rightarrow 4m_1 = 0$	$\Rightarrow 2m_1 = 2\sqrt{3}$
$\Rightarrow m_1 = 0$	$\Rightarrow m_1 = \sqrt{3}$

$\therefore$ given point $(3, -2)$

$\therefore\ y = mx + c$

$-2 = \sqrt{3}(3) + c$

$-2 = 3\sqrt{3} + c$

$c = -2 - 3\sqrt{3}$

Hence, required equation

$$\therefore \quad \begin{aligned} y &= mx + c \\ y &= \sqrt{3}x - 2 - 3\sqrt{3} \\ y - \sqrt{3}x &+ 2 + 3\sqrt{3} = 0 \\ y - x\sqrt{3} &+ 2 + 3\sqrt{3} = 0 \end{aligned}$$

115. **(a)** Let (α, β) be the given point, let $Q(x, y)$ be the foot of the perpendicular, and let O be the origin. The line can have any direction

$\angle PQO = 90^\circ$

Point Q lies on the circle having diameter OP.

The locus of point Q.

$(x - 0)(x - \alpha) + (y - 0)(y - \beta)$

$\Rightarrow \quad x^2 - x\alpha + y^2 - y\beta = 0$

$\Rightarrow \quad x^2 + y^2 - \alpha x - \beta y = 0$

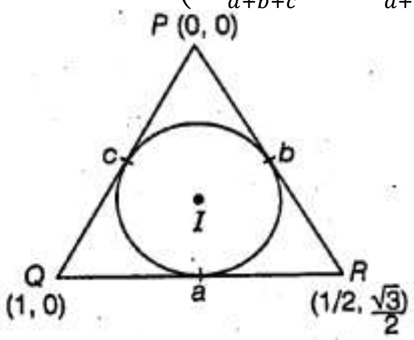
116. **(b)** Let coordinate of the point be $(\alpha, -\alpha)$

Since, $(\alpha, -\alpha)$ lie on $2ax + 4ay + c = 0$

and $7bx + 3by - d = 0$
$\therefore$ $2a\alpha - 4a\alpha + c = 0$
$\Rightarrow$ $-2a\alpha + c = 0$
$\Rightarrow$ $\alpha = \frac{c}{2a} \ldots .(i)$
Also, $7b\alpha - 3b\alpha - d = 0$
$\Rightarrow$ $4b\alpha - d = 0$
$\Rightarrow$ $\alpha = \frac{d}{4b} \ldots .(ii)$
From Eqs. (i) and (ii),
$\frac{c}{2a} = \frac{d}{4b}$
$\Rightarrow$ $\frac{2ad}{4} = 4bc$
$\Rightarrow$ $\frac{ad}{bc} = \frac{4}{2}$
$\Rightarrow$ $\frac{ad}{bc} = \frac{2}{1}$
$\Rightarrow$ $ad : bc = 2 : 1$

117. **(a)** In a circle, AB is a diameter where the coordinate of A is (p, q) and let the coordinate of $B(x_1, y_1)$.
Equation of circle in diameter form is
$(x - p)(x - x_1) + (y - q)(y - y_1) = 0$
$\Rightarrow x^2 - (p + x_1)x + px_1 + y^2 - (y_1 + q)y + qy_1 = 0$
$\Rightarrow x^2 - (p + x_1)x + y^2 - (y_1 + q)y + px_1 + qy_1 = 0$
Since, this circle touches X-axis
$\therefore$ $y = 0$
$\Rightarrow$ $x^2 - (p + x_1)x + px_1 + qy_1 = 0$
Also, the discriminant of above equation will be equal to zero because circle touches X-axis.
$\therefore$ $(p + x_1)^2 = 4(px_1 + qy_1)$
$\Rightarrow p^2 + x_1^2 + 2px_1 = 4px_1 + 4qy_1$
$\Rightarrow x_1^2 - 2px_1 + p^2 = 4qy_1$
$\Rightarrow$ $(x_1 - p)^2 = 4qy_1$
Therefore, the locus of point B is
$(x - p)^2 = 4qy$

118. **(c)** $I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$



$a = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1, b = 1, c = 1$
Centre of the circle.
$$= \left(\frac{1 \times 0 + 1 \times 1 + 1 \times \frac{1}{2}}{1 + 1 + 1}, \frac{1 \times 0 + 1 \times 0 + 1 \times \sqrt{3}/2}{1 + 1 + 1}\right)$$

$$= \left(\frac{0+1+\frac{1}{2}}{3}, \frac{0+0+\frac{\sqrt{3}}{2}}{3} \right)$$
$$= \left(\frac{\frac{3}{2}}{3}, \frac{\frac{\sqrt{3}}{2}}{3} \right)$$
$$= \left(\frac{3}{2} \times \frac{1}{3}, \frac{\sqrt{3}}{2} \times \frac{1}{3} \right)$$
$$= \left(\frac{1}{2}, \frac{1}{2\sqrt{3}} \right)$$

119. **(c)** Given, equation of hyperbola is

$$\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1,$$

Here, $a^2 = \cos^2 \alpha$ and $b^2 = \sin^2 \alpha$

[i.e. comparing with standard equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$]

We know that, foci $= (\pm ae, 0)$,

where, $ae = \sqrt{a^2 + b^2} = \sqrt{\cos^2 \alpha + \sin^2 \alpha} = 1$

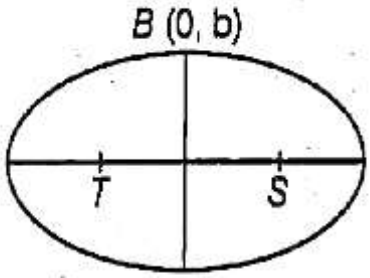
$\Rightarrow$ foci $(\pm 1, 0)$, where vertices are $(\pm \cos \alpha, 0)$

Eccentricity, $ae = 1$ or $e = \frac{1}{\cos \alpha}$.

Hence, foci remains constant with change in α.

120. **(c)** Equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Foci are $S(ae, 0), T(-ae, 0)$.

$B(0, b)$ is the end of the minor axis.

STB is an equilateral triangle.

$$SB = ST$$
$$\Rightarrow \quad SB^2 = ST^2$$
$$\Rightarrow \quad a^2e^2 + b^2 = 4a^2e^2$$
$$\Rightarrow \quad b^2 = 3a^2e^2$$
$$\Rightarrow \quad a^2(1 - e^2) = 3a^2e^2$$
$$\Rightarrow \quad 1 - e^2 = 3e^2$$
$$\Rightarrow \quad 4e^2 = 1$$
$$\Rightarrow \quad e^2 = \frac{1}{4}$$
$$\Rightarrow \quad e = \frac{1}{2}$$

Eccentricity of the ellipse, $e = \frac{1}{2}$

121. **(a)** Given equation is

$$3x^2 - 3y^2 - 18x + 12y + 2 = 0$$

It can be written as

$$\frac{(x-3)^2}{\left(\sqrt{\frac{13}{3}}\right)^2}-\frac{(y-2)^2}{\left(\sqrt{\frac{13}{3}}\right)^2}=1$$

here, $a=b=\sqrt{\frac{13}{3}}$

$$\therefore\ e=\sqrt{1+\frac{b^2}{a^2}}$$

$$=\sqrt{1+1}[\because a=b]$$

$$=\sqrt{2}$$

$\therefore$ Equation of the directrix are

$$x=3\pm\sqrt{\frac{13}{6}}$$

122. **(c)** Given equation of ellipse is

$$3x^2+4y^2=48$$

$$\Rightarrow\frac{3x^2}{48}+\frac{4y^2}{48}=1\Rightarrow\frac{x^2}{16}+\frac{y^2}{12}=1$$

Here, $a=4$ and $b=2\sqrt{3}$

$$\therefore\ e=\sqrt{1-\frac{b^2}{a^2}}$$

$$=\sqrt{1-\frac{12}{16}}=\sqrt{\frac{4}{16}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$$

$\therefore$ Coordinates of P are $=(2,3)$

$\because(4\cos\theta,2\sqrt{3}\sin\theta)=(2,3)$

$$\left[\because p=\left(ae,\frac{b^2}{a}\right)\right]$$

By comparing, we get

$$4\cos\theta=2 \text{ and } 2\sqrt{3}\sin\theta=3$$

$$\Rightarrow\quad\cos\theta=\frac{1}{2}\text{ and }\sin\theta=\frac{\sqrt{3}}{2}$$

$$\Rightarrow\quad\cos\theta=\cos\frac{\pi}{3}\text{ and }\sin\theta=\sin\frac{\pi}{3}$$

$$\Rightarrow\quad\theta=\frac{\pi}{3}\text{ and }\theta=\frac{\pi}{3}$$

$\therefore$ eccentric angle of P is $\frac{\pi}{3}$.

123. **(b)** Direction ratios of line joining the points $(1,2,-3)$ and $(-1,-2,1)$ is $(-1-1,-2-2,1+3)$

i.e. $(-2,-4,4)$

Let the direction ratios of the normal to the plane is (a,b,c), then

$2a+3b+4c=0$..........(i)

and $-2a-4b+4c=0$(ii)

By Eqs. (i) and (ii), we get

$$\frac{a}{3(4)-(4)(-4)} = \frac{b}{(-2)(4)-(2)(4)} = \frac{c}{(2)(-4)-(-2)(3)}$$

$$\Rightarrow \quad \frac{a}{12+16} = \frac{b}{-8-8} = \frac{c}{-8+6}$$

$$\Rightarrow \quad \frac{a}{28} = \frac{b}{-16} = \frac{c}{-2}$$

$$\Rightarrow \quad \frac{a}{14} = \frac{b}{-8} = \frac{c}{-1}$$

Hence, direction ratios are $(14,-8,-1)$.

124. **(c)** The given points are $A(1,2,3)$ and $B(3,4,5)$

The direction ratios of line segment AB is given by $(x_2-x_1),(y_2-y_1),(z_2-z_1)$

i.e. $(3-1),(4-2),(5-3)=(2,2,2)=(1,1,1)$

Since, the plane bisects AB at right angles, AB is the normal to the plane. Which is $\mathbf{n}$ Therefore, $\mathbf{n}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Let c be the mid-point of AB

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$=c\left(\frac{1+3}{2},\frac{2+4}{2},\frac{3+5}{2}\right)$$

$$=c(2,3,4)$$

Let this vector be $\mathbf{a}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

Hence, the required equation of vector

$\{\mathbf{r}-(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})\}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\mathbf{k})=0$

We know that, $\mathbf{r}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\mathbf{k}$

$\Rightarrow\{(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})-(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})\}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=0$

$\Rightarrow(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$

$\quad=(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$

$\Rightarrow x+y+z=2+3+4$

$\Rightarrow x+y+z=9$

125. **(a)** It is clear that, the limit of the interior angle of a regular polygon of n sides as $n\to\infty$ is π or 180°.

126. **(a)** Given,

$h\{f(x)\}=x$

differentiating w.r.t. x, we get

$$h'\{f(x)\}.f'(x)=1$$

$$\Rightarrow \quad f'(x)=\frac{1}{h'\{f(x)\}}$$

$$\Rightarrow f'(x)=\frac{\frac{1}{1}}{1+\log\{f(x)\}}$$

$$\left[\begin{array}{l}\because h'(x)=\dfrac{1}{1+\log x} \\ \text{(given)}\end{array}\right]$$

$$\Rightarrow f'(x)=1+\log(f(x))$$

127. **(c)** We have,

$f(x)=\cos x^2$

Let T be the period of $f(x)$. Then

$$f(x+T)=f(x)$$

$$\Rightarrow \quad \cos(x+T)^2=\cos x^2$$

But there is no value of T for which

$\cos(x+T)^2=\cos x^2$

$\therefore f(x)$ is not periodic

128. **(c)** Let $L=\lim_{x\to 0^+}(e^x+x)^{1/x}$

$\Rightarrow \log L=\lim_{x\to 0^+}\dfrac{\log(e^x+x)}{x}$

$\Rightarrow \log L=\lim_{x\to 0^+}\dfrac{\frac{1}{(e^x+x)}\cdot e^x+1}{1}$

[using L' Hospital rule]

$\Rightarrow \log L=\lim_{x\to 0^+}\dfrac{e^x+1}{e^x+x}$

$\Rightarrow \log L=2$

$\Rightarrow\ L=e^2$

129. **(a)** Let $g(x)=e^{-x}f(x)$

$\therefore\ g'(x)=e^{-x}f'(x)-e^{-x}f(x)$

$\qquad =e^{-x}(f'(x)-f(x))$

Since, $f'(x)>f(x)$, so $f'(x)-f(x)>0$

$\therefore e^{-x}(f'(x)-f(x))>0$

$\Rightarrow\ g'(x)>0$

$\therefore g(x)$ is an increasing function.

Now, $g(x)>g(0)\forall x>0$

$\Rightarrow e^{-x}f(x)>e^0 f(0)$ [$\because f(0)=0$]

$\Rightarrow e^{-x}f(x)>0$

$\Rightarrow\ f(x)>0,\forall x>0$

130. **(b, c)** Given that $f:[1,3]\to R$ be a continuous and differentiable in (1,3).

and $f'(x)=|f(x)|^2+4$

By applying LMVT, there exist at least one point $c\in(1,3)$ such that

$\dfrac{f(3)-f(1)}{3-1}=f'(c)\left[\because f'(c)=\dfrac{f(b)-f(a)}{b-a}\right]$

$\dfrac{f(3)-f(1)}{2}=f'(c)$

$\dfrac{f(3)-f(1)}{2}=[f|c|^2+4]$

$(\because f'(c)=|f(c)|^2+4)$

$f(3)-f(1)=2\cdot|f(c)|^2+8$

$f(3)-f(1)\geq 8$

It is clear from the given options that (B) and (C) the correct.

131. **(c)** Let $f(x)=x^2+2x+3$

$$a=f(x)_{\min}=\frac{-D}{4a}=\frac{-(4-12}{4}=\frac{8}{4}=2$$

$$\text{and } b=\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}=\lim_{\theta\to 0}\frac{1-1+2\sin^2\theta/2}{\theta^2}$$

$$=\lim_{\theta\to 0}\frac{2\sin^2\theta/2}{(\theta/2)^2\cdot 4}=\lim_{\theta\to 0}\frac{1}{2}\cdot\left[\frac{\sin^2\theta/2}{(\theta/2)^2}\right]$$

$$=\frac{1}{2}\cdot\lim_{\theta\to 0}\frac{\sin^2\theta/2}{(\theta/2)^2}$$

$$=\frac{1}{2}\cdot 1=\frac{1}{2}$$

$$b=\frac{1}{2}$$

Now, $\sum_{r=0}^{n} a^r\cdot b^{n-r}$

$$= \sum_{r=0}^{n} \left(2^r \left(\frac{1}{2}\right)^{n-r}\right)$$
$$= \sum_{r=0}^{n} 2^r \cdot 2^{(r-n)}$$
$$= \sum_{r=0}^{n} 2^{2r-n}$$
$$= 2^{-n} \sum_{r=0}^{n} 2^{2r}$$
$$= 2^{-n}[1 + 2^2 + 2^4 + 2^6 + \cdots + 2^{2n}]$$
$$= 2^{-n}\left[\frac{1 \cdot (2^2)^{n+1} - 1}{2^2 - 1}\right]\left\{\because S_n = \frac{a(r^n - 1)}{r - 1}\right\}$$
$$= 2^{-n}\left[\frac{4^{n+1} - 1}{3}\right]$$
$$= \frac{4^{n+1} - 1}{3 \cdot 2^n}$$

132. **(a)** Let $a = 4, b = 1$ and $n = 2$

$\therefore\ I(n) = a^{1/n} - b^{1/n}$

$= 4^{1/2} - 1^{1/2} = 2 - 1 = 1$

and $J(n) = (a - b)^{1/n}$

$= (4 - 1)^{1/2} = \sqrt{3}$

$\therefore\ J(n) > I(n)$

Similarly, we can prove for other values of a, b, n.

133. **(d)** Given,

$$|\hat{\alpha}| = |\hat{\beta}| = |\hat{\gamma}| = 1$$

and $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$

Now, $\hat{\alpha} \times (\hat{\beta} \times \hat{\gamma}) = \frac{1}{2}(\hat{\beta} + \hat{\gamma})$

$\Rightarrow\ (\hat{\alpha} \cdot \hat{\gamma})\hat{\beta} - (\hat{\alpha} \cdot \hat{\beta})\hat{\gamma} = \frac{1}{2}\hat{\beta} + \frac{1}{2}\hat{\gamma}$

On comparing we get,

$-(\hat{\alpha} \cdot \hat{\beta}) = \frac{1}{2}$

$\Rightarrow |\hat{\alpha}||\hat{\beta}|\cos\theta = -\frac{1}{2}$

$\cos\theta = -\frac{1}{2}[\because |\hat{\alpha}| = |\hat{\beta}| = 1]$

$\Rightarrow= 120°$ or $\frac{2\pi}{3}$

134. **(d)** The position vectors of the points A, B, C and D are $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}, 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}, 5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $4\hat{\mathbf{i}} - \hat{\mathbf{j}} - \lambda\hat{\mathbf{k}}$ respectively.

BA $= (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

CA $= (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

DA $= (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (4\hat{\mathbf{i}} - \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}})$

$= -\hat{\mathbf{i}} - \hat{\mathbf{j}} - (1 + \lambda)\hat{\mathbf{k}}$

These are coplanar, so

$$\begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & -3 \\ -1 & -1 & -(1+\lambda) \end{vmatrix} = 0$$

$$= \begin{vmatrix} -1 & -1 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 1+\lambda \end{vmatrix} = 0$$

$1[1+\lambda-3]+1[2+2\lambda-3]+3[2-1]=0$

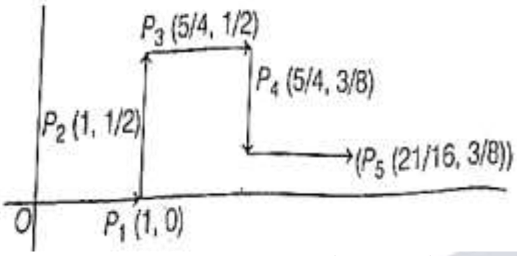
$\Rightarrow -1[\lambda-2]+1[2\lambda-1]+3=0$

$-\lambda+2+2\lambda-1+3$

$\lambda+4=0$

$\lambda=-4$

135. **(b)** According to the given information in question, we can draw the situation of particle at different stages as following



Here, $x_1 = 1, x_2 = 1, x_3 = \frac{5}{4}, x_4 = \frac{5}{4}$ and $x_5 = \frac{21}{16}$

$$y_1 = 0, y_2 = \frac{1}{2}, y_3 = \frac{1}{2}, y_4 = \frac{3}{8} \text{ and } y_5 = \frac{3}{8}$$

$$\therefore\ x_n = 1 + \frac{1}{4} + \frac{1}{4} + \cdots \infty = \frac{1}{1-\frac{1}{4}} = \frac{1}{3} = \frac{4}{3}$$

$$\text{and } y_n = 0 + \frac{1}{2} - \frac{1}{8} + \cdots \infty$$

$$= \frac{\frac{1}{2}}{1-\left(-\frac{1}{4}\right)} = \frac{\frac{1}{2}}{\left(1+\frac{1}{4}\right)} = \frac{\frac{1}{5}}{4} = \frac{2}{5}$$

$$\therefore\ (\alpha, \beta) = \left(\frac{4}{3}, \frac{2}{5}\right)$$

136. **(a)** $|z|+|z-1|a$ is complex number

$|z|+|-(1-z)||a|=|-a|$

$|z|+|1-z|\{\because |a|=|-a|\}$

$|z|+1-z| \geq |z+1-z|\{\because |z_1|+|z_2| \geq |z_1+z_2|\}$

$|z|+|1-z| \geq |1|$

$|z|+|1-z| \geq 1$

$|z|+|-(z-1)| \geq 1$

$|z|+|z-1| \geq 1$

Minimum value of $|z|+|z-1|=1$

137. **(c)** Given, system of equations

$\lambda x + y + 3z = 0$

$2x + \mu y - z = 0$

$5x + 7y + z = 0$

System has infinitely many solutions in R, if

$$\begin{vmatrix} \lambda & 1 & 3 \\ 2 & \mu & -1 \\ 5 & 7 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda(\mu+7)-1(2+5)+3(14-5\mu)=0$

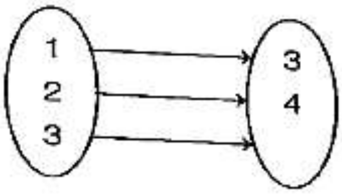
$\Rightarrow \quad \lambda(\mu+7)-7+3(14-5\mu)=0$

$\Rightarrow \quad \lambda\mu+7\lambda-7+42-15\mu=0$

$\Rightarrow\ \lambda\mu+7\lambda-15\mu+35=0$

By checking the options, we get option (c)
$(\lambda = 1, \mu = 3)$ satisfies the given equation.

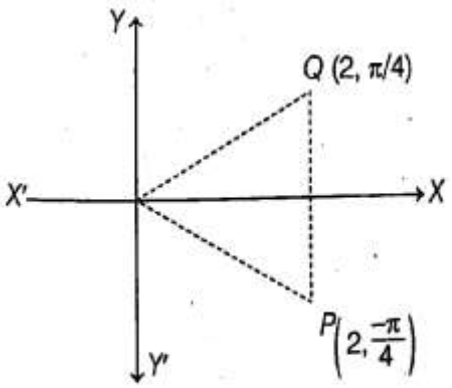
138. **(c)** Let $A = \{1,2,3\}$
and $B = \{2,3,4\}$



here, $f^{-1}(3) = 1$
$f^{-1}(4) = 2$ and $f^{-1}(\lambda) = 3$
It is clear that option (C) $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$ is correct.

139. **(b)** We have,
$g \circ f : S \to U$ is an onto function.
Let Z be any arbitrary element such that $Z \in U$.
Now, gof : $S \to U$ is onto
$\Rightarrow g \circ f(x) = Z$, for $x \in S$
$\Rightarrow g(f(x)) = Z$
$\Rightarrow\ g(y) = Z$, where $y = f(x) \in T$ for all $Z \in U$
$\therefore$ For all $\dot{Z} \in U$, there exists $y = f(x) \in T$ such that $g(y) = Z$
$\therefore\ g : T \to U$ is an onto function.

140. **(a)** Since, Q is the point for which PQ is bisected perpendicularly by the initial line (X-axis).
Therefore, Q will be the image of P in X-axis
$\therefore$ Coordinates of Q is $(2, \pi/4)$



141. **(b)** Let the length of conjugate axis $= b$ and the length of transvers axis $= a$
Given that, $b. > a$
Now, $b^2 > a^2$

$$\therefore \quad \text{Eccentricity } (e) = \sqrt{1 + \frac{b^2}{a^2}}$$

$$e^2 = \left(\sqrt{1 + \frac{b^2}{a^2}}\right)^2 > 2$$

$$e > \sqrt{2}$$

142. **(a)** Given, $\lim\limits_{x \to 0^+} \frac{x}{p}\left[\frac{q}{x}\right]$

$$= \lim_{x\to 0^+} \frac{x}{p}\left(\frac{q}{x} - \left(\frac{q}{x}\right)\right)$$
$$= \frac{[q]}{p} - \frac{x}{p}\left\{\frac{q}{x}\right\}$$
$$= \frac{[q]}{p} - 0 = \frac{[q]}{p}$$

143. **(b)** Given,

$$f(x) = x^4 - 4x^3 + 4x^2 + c$$
$$\because f(1) = 1 + c$$
and $f(2) = 2^4 - 4(2)^3 + 4(2)^2 + c$
$$= 16 - 32 + 16 + c$$
$$= c$$

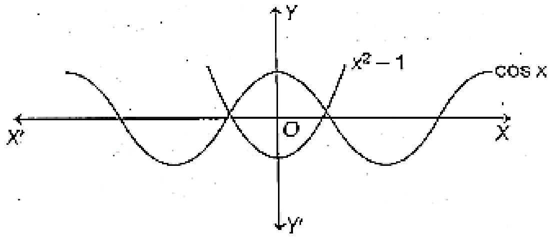
Now, $f(1)\cdot f(2) = c(1+c)$

Let $f(1)\cdot f(2) < 0$

$\Rightarrow c(1+c) < 0 \Rightarrow c \in (-1,0)$

$\therefore$ For $c \in (-1,0)$, $f(x)$ has exactly one zero is $(1,2)$.

144. **(a)**



It is clear from the graph that it intersects at exactly two points.

145. **(c)** We have,

$\frac{dx}{dt} = 1$ and $y = \frac{10}{x}$

Now, $y = \frac{10}{x}$

On differentiating both the sides w.r.t. t, we get

$$\Rightarrow \frac{dy}{dt} = \frac{-10}{x^2} \times \frac{dx}{dt}$$
$$= \frac{-10}{(5)^2} \times 1$$
$$= \frac{-10}{25} = \frac{-2}{5}\left[x = 10, \frac{dx}{dt} = 1\right]$$

$\therefore y$ decreases at the rate of $\frac{2}{5}$ unit per second.

146. **(b,d)**We have,

$$I_n = \int_0^1 x^n \tan^{-1} x\,dx$$
$$= \left[\tan^{-1} x \cdot \frac{x^{n+1}}{n+1}\right]_0^1 - \int_0^1 \frac{x^{n+1}}{n+1} \cdot \frac{1}{1+x^2}\,dx$$
$$= \frac{\pi}{4(n+1)} - \frac{1}{n+1}\int_0^1 \frac{x^2 \cdot x^{n-1}}{1+x^2}\,dx$$
$$= \frac{\pi}{4(n+1)} - \frac{1}{n+1}\int_0^1 \frac{(1+x^2)x^{n-1} - x^{n-1}}{1+x^2}\,dx$$

$$= \frac{\pi}{4(n+1)} - \frac{1}{n+1}\int_0^1 x^{n-1}dx + \frac{1}{n+1}\int_0^1 \frac{x^{n-1}}{1+x^2}dx$$

$$= \frac{\pi}{4(n+1)} - \frac{1}{n+1}\left[\frac{x^n}{n}\right]_0^1 + \frac{1}{n+1}$$

$$[x^{n-1}\cdot\tan^{-1}x]_0^1 - \int_0^1 \tan^{-1}x\cdot(n-1)x^{n-2}dx$$

$$= \frac{\pi}{4(n+1)} - \frac{1}{n(n+1)} + \frac{\pi}{4(n+1)} - \frac{n-1}{n+1}$$

$$\int_0^1 x^{n-2}\tan^{-1}x dx$$

$$\Rightarrow I_n = \frac{\pi}{2(n+1)} - \frac{1}{n(n+1)} - \frac{n-1}{n+2}I_{n-2}$$

$$\Rightarrow I_n + \frac{n-1}{n+2}I_{n-2} = \frac{\pi}{2(n+1)} - \frac{1}{n(n+1)}$$

Put $n = n+2$, we get

$$I_{n+2} + \frac{n+1}{n+3}I_n = \frac{\pi}{2(n+3)} - \frac{1}{(n+2)(n+3)}$$

$$\Rightarrow (n+3)I_{n+2} + (n+1)I_n = \frac{\pi}{2} - \frac{1}{n+2}$$

$$\therefore a_n = n+3, b_n = n+1, c_n = \frac{\pi}{2} - \frac{1}{n+2}$$

$\therefore$ a_n and b_n are in AP.

147. **(c)** Let B travels x units, $v = u + at$

According to problem, $ht = f(t+m)$

$$\Rightarrow \quad ht = ft + fm$$

$$\Rightarrow \quad ht - ft = fm$$

$$\Rightarrow \quad t(h-f) = fm$$

$$\Rightarrow \quad \frac{h-f}{f} = \frac{m}{t}$$

$$\Rightarrow \ t = m\left(\frac{f}{h-f}\right)$$

$$\Rightarrow \ t^2 = m^2\left(\frac{f}{h-f}\right)^2 \text{.....(i)}$$

Again, $n + x = \frac{1}{2}f(t+m)^2$

$$\Rightarrow \ h + \frac{1}{2}ht^2 = \frac{1}{2}f(t+m)^2 \text{....(ii)}$$

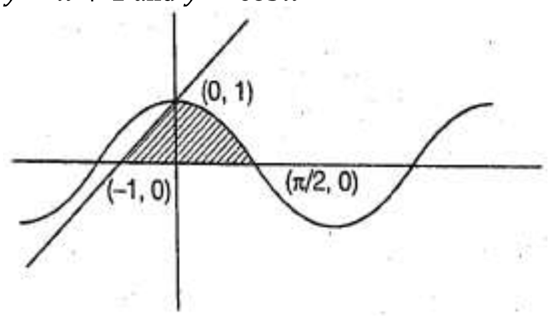
From Eqs. (i) and (ii)

$$\frac{m}{n} = \frac{2(h-f)}{mfh}$$

$$\Rightarrow \quad (h-f)n = \frac{m^2fh}{2}$$

148. **(b)** We have,

$y = x + 1$ and $y = \cos x$



$\therefore$ Require area $= \int_{-1}^{0} (x+1)dx + \int_{0}^{\pi/2} \cos x dx$

$= \left[\frac{x^2}{2} + x\right]_{-1}^{0} + [\sin x]_0^{\pi/2}$

$= \left[(0) - \left(\frac{1}{2} - 1\right)\right] + [1-0]$

$= \frac{1}{2} + 1 = \frac{3}{2}$ sq unit.

149. **(b)** We have,

x_1, x_2 be the roots of equation $x^2 - 3x + a = 0$

$\therefore\ x_1 + x_2 = 3$ and $x_1 x_2 = a$

Also, x_3, x_4 be the roots of equation $x^2 - 12x + b = 0$

$\therefore\ x_3 + x_4 = 12$ and $x_3 x_4 = b$

Again, x_1, x_2, x_3, x_4 are in GP

$\therefore\ x_1 = A, x_2 = AR, x_3 = AR^2, x_4 = AR^3$

Now, $x_1 + x_2 = 3$

$\Rightarrow \quad A(1+R) = 3 \ldots. \text{(i)}$

and $\quad x_3 + x_4 = 12$

$\Rightarrow \quad AR^2(1+R) = 12 \ldots \text{(ii)}$

From Eqs. (i) and (ii), we have

$R^2 = 4 \Rightarrow R = \pm 2$

When, $R = 2, A = 1$ and when, $R = -2A = -3$

$\therefore$ Numbers are either 1,2,4,8 or $-3,6,-12,24$

But $x_1 < x_2 < x_3 < x_4$.

So, $x_1 = 1, x_2 = 2, x_3 = 4, x_4 = 8$

$\therefore ab = x_1 x_2 x_3 x_4$

$= 1 \times 2 \times 4 \times 8 = 64$

150. **(a)** Let $Z = \frac{1 - i\cos\theta}{1 + 2i\cos\theta}$

$\therefore\ \bar{z} = \frac{1 + i\cos\theta}{1 - 2i\cos\theta}$

Since, z is a real number, then $z - \bar{z} = 0$

$\Rightarrow \frac{1 - i\cos\theta}{1 + 2i\cos\theta} = \frac{1 + i\cos\theta}{1 - 2i\cos\theta}$

$\Rightarrow (1 - i\cos\theta)(1 - 2i\cos\theta) = (1 + i\cos\theta)(1 + 2i\cos\theta)$

$\Rightarrow \mathbf{1} - 2i\cos\theta - i\cos\theta - 2\cos^2\theta$

$= 1 + 2i\cos\theta + i\cos\theta - 2\cos^2\theta$

$\Rightarrow\ 6i\cos\theta = 0$

$\Rightarrow\ \cos\theta = 0$

$\Rightarrow\ \theta = (2n+1)\frac{\pi}{2}, n \in I$

151. **(b)** We have,

$A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix}$

Again, we have

$\Rightarrow \quad |A - \lambda I| = 0$

$\Rightarrow \quad \begin{vmatrix} 3-\lambda & 0 & 3 \\ 0 & 3-\lambda & 0 \\ 3 & 0 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad (3-\lambda)[(3-\lambda)^2 - 0] - 0 + 3[0 - 3(3-\lambda)] = 0$

$\Rightarrow \quad (3-\lambda)^3 - 9(3-\lambda) = 0$

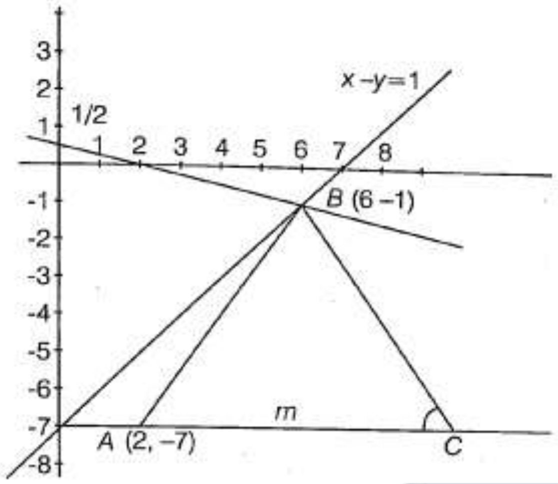
$\Rightarrow \quad (3-\lambda)[(3-\lambda)^2 - 9] = 0$

$\Rightarrow \quad (3-\lambda)(9 + \lambda^2 - 6\lambda - 9) = 0$

$\Rightarrow \quad (3-\lambda)(\lambda^2 - 6\lambda) = 0$

$\Rightarrow \quad (3-\lambda)\lambda(\lambda-6)=0$
$\Rightarrow \quad \lambda = 3, 0, 6$

152. **(a,b)** Given equations of lines are and
$x - y = 7$
$x + 4y = 2$
By solving Eqs. (i) and (ii), we get the point $B(6, -1)$



Let the slope of AC be m then, $AB = AC$

$$\therefore \left|\frac{m+\frac{1}{4}}{1-\frac{m}{4}}\right| = \left|\frac{-\frac{1}{4}-1}{1-\frac{1}{4}}\right|$$

$\Rightarrow m = \dfrac{-23}{7}, 1$

When $m = -\frac{23}{7}$, then equation of line

$$y + 7 = \frac{-23}{7}(x-2)$$

$\Rightarrow 7y + 49 = -23x + 46$
$\Rightarrow 23x + 7y + 3 = 0$
When $m = 1$, then equation of line
$y + 7 = \quad (x-2)$
$\Rightarrow x - y - 9 \quad = 0$

153. **($\boldsymbol{a, c}$)** Given equation of hyperbola can be write as

$$\frac{x^2}{1} - \frac{y^2}{(\sqrt{5})^2} = 1$$

here, $a = 1$ and $b = \sqrt{5}$
Since, $y = mx \pm \sqrt{a^2m^2 - b^2}$

$y = mx \pm \sqrt{m^2 - 5}$

$\Rightarrow \quad (8-2m)^2 = \left(\sqrt{m^2-5}\right)^2$

$\Rightarrow \quad (8-2m)^2 = m^2 - 5$
$\Rightarrow \quad 64 + 4m^2 - 32m = m^2 - 5$
$\Rightarrow \quad 3m^2 - 32m + 69 = 0$
$\Rightarrow \quad m = \dfrac{22}{3}, 3$ [by s

When $m = \frac{23}{3}$, then equation of tangent

$$(y-8)=\frac{23}{3}(x-2)$$

$\Rightarrow \quad 3y-24=23x-46$

$\Rightarrow \quad 23x-3y-46+24=0$

$\Rightarrow \quad 23x-3y-22=0$

When $m=3$, then equation of tangent

$$(y-8)=3(x-2)$$

$\Rightarrow \quad y-8=3x-6$

$\Rightarrow \quad 3x-y-6+8=0$

$\Rightarrow \quad 3x-y+2=0$

Hence, option (a) and (c) are correct.

154. **(a, c)** From option (a) We have,

$= f'(a)\cdot f(b)=0$

$(b)<0$

Again, let $h(x)=f'(x)+f(x)g'(x)$

$\Rightarrow h(a)=f'(a)+f(a)g'(a)=f'(a)$

and $h(b)=f'(b)+f(b)g'(b)=f'(b)$

$\therefore\ h(a)\cdot h(b)=f'(a)\cdot f'(b)<0$

$\therefore\ h(x)=0$ has root between (a,b)

From option (c)

We have, $g(a)=g(b)=0$

$\Rightarrow\ g'(a)\cdot g'(b)<0$

Again, let $m(x)=g'(x)+kg(x)$

$\Rightarrow\ m(a)=g'(a)+kg(a)=g'(a)$

$\Rightarrow\ m(b)=g'(b)+kg(b)=g'(b)$

$\therefore\ m(a)\cdot m(b)=g'(a)\cdot g'(b)<0$

$\therefore\ m(x)=0$ has root between (a,b).

So, option (a) and (c) are correct.

155. **(d)** Given, $f(x)=\frac{x^3}{4}-\sin\pi x+3$

differentiating w.r.t. x, we get

$$f'(x)=\frac{3x^2}{4}-\pi\cos(\pi x)$$

at

$x=-2f(-2)=1$

and at $x=2f(2)=5$