Physics

1. **(b)** Let intensity from one slit $= I_1$
Intensity from other slit $= I_2$
According to question,

$\Rightarrow$ $$I_1 = 1.5 I_2$$

$\Rightarrow$ $$\frac{I_1}{I_2} = 1.5 = \frac{3}{2}$$

$\therefore$ Ratio of intensity, $$\frac{I_{\max}}{I_{\min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$$

$$= \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)^2$$

$$= \left(\frac{1.732+1.414}{1.732-1.414}\right)^2$$

$$= \left(\frac{3.146}{0.318}\right)^2 = (9.89)^2 = 98$$

2. **(d)** Given, slit width, $a = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$
Wavelength, $\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$
Distance of screen, $D = 50 \text{ cm} = 50 \times 10^{-2} \text{ m}$
$\therefore$ Linear separation $= \frac{2D\lambda}{a}$

$$= \frac{2 \times 50 \times 10^{-2} \times 600 \times 10^{-9}}{0.5 \times 10^{-3}}$$

$$= 12 \times 10^{-4} \text{ m}$$

$$= 1.2 \times 10^{-3} \text{ m} = 1.2 \text{ mm}$$

3. **(b)** Wavelength of emitted radiation is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For Lyman series,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \text{ where } n = 2,3,4, \ldots.$$

The range of wavelength in this series is given by

$$\frac{1}{\lambda_{\max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4} \text{ cm}^{-1}$$

and $\frac{1}{\lambda_{\min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R \text{ cm}^{-1}$

Thus for Lyman : $\left[\frac{1}{R} \text{ to } \frac{4}{3R} \text{ cm}\right]$

For Balmer series,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \text{ where } n = 3,4,5, \ldots$$

The range of wavelength in this series is given by
and

$$\frac{1}{\lambda_{\max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \text{ cm}^{-1}$$

$$\frac{1}{\lambda_{\min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4} \text{ cm}^{-1}$$

Thus, for Balmer: $\left[\frac{4}{R} \text{ to } \frac{36}{5R} \text{ cm}\right]$

For Paschen series,

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \text{ where } n = 4,5,6, \ldots$$

The range of wavelength in this series is given by
and

$$\frac{1}{\lambda_{\max}}=R\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=\frac{7R}{144}\ \text{cm}^{-1}$$
$$\lambda_{\min}=R\left(\frac{1}{3^2}-\frac{1}{\infty^2}\right)=\frac{R}{9}\ \text{cm}^{-1}$$
Thus, for Paschen : $\left[\frac{9}{R}\text{ to }\frac{144}{7R}\ \text{cm}\right]$
For Brackett series,
$\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$, where $n=5,6,7,\ldots$
The range of wavelength in this series is given by
$$\frac{1}{\lambda_{\max}}=R\left(\frac{1}{4^2}-\frac{1}{5^2}\right)=\frac{9R}{400}\ \text{cm}^{-1}$$
and. $\frac{1}{\lambda_{\min}}=R\left(\frac{1}{4^2}-\frac{1}{\infty^2}\right)=\frac{R}{16}\ \text{cm}^{-1}$
Thus, for Brackett : $\left[\frac{16}{R}\text{ to }\frac{400}{9R}\ \text{cm}\right]$
For Pfund series,
$\frac{1}{\lambda}=R\left(\frac{1}{5^2}-\frac{1}{n^2}\right)$, where $n=6,7,8,\ldots.$
The range of wavelength in this series is given by
and
$$\frac{1}{\lambda_{\min}}=R\left(\frac{1}{5^2}-\frac{1}{\infty^2}\right)=\frac{R}{25}\ \text{cm}^{-1}$$
Thus, for Pfund: $\left[\frac{25}{R}\text{ to }\frac{900}{11R}\ \text{cm}\right]$
Hence, wavelength range of $\left(\frac{7}{5R}\text{ to }\frac{19}{5R}\ \text{cm}\right)$ does not emit.

4. (a) The process involved is as
$${}_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + v + Q$$
$\therefore$ Mass defect, $m_x=M_x-Zm_e$
and
$m_y=M_{\dot{y}}-(Z+1)m_e$
where, M_x is atomic mass of X, M_y is atomic mass of Y and m_e is mass of an electron.
Total mass defect, $\Delta m=m_x-m_y=\left(M_x-M_y-m_e\right)$
$\therefore$ Binding energy, $E=\Delta mc^2$
So, maximum energy of β-particle emitted
$=\left(M_x-M_y-m_e\right)c^2$

5. (a) Given, binding energy per nucleon of nuclei with mass number $119=7.6\text{MeV}$
Binding energy per nucleon of nuclei with mass number $238=8.6\text{MeV}$
According to question, fission of nucleus $238=119+119$.
Hence, total energy of nucleus before fission,
$E_i=238\times 8.6\text{MeV}$
Total energy of nucleus after fission,
$E_f=238\times 7.6\text{MeV}$
$\therefore$ Energy released in the process of fission
$=E_i-E_f$
$=238\times 8.6-238\times 7.6$
$=238\text{MeV}$
Hence, released energy is closest to 214 MeV .

6. (d) Given, load resistance, $R_L=6\text{k}\Omega=6000\Omega$
Base voltage, $V_B=15\text{mV}=15\times 10^{-3}\ \text{V}$
Base current, $I_B=20\mu\ \text{A}=20\times 10^{-6}\ \text{A}$
Collector current, $I_C=1.8\ \text{mA}=1.8\times 10^{-3}\ \text{A}$
$\therefore$ Voltage gain $=\beta$ (Current gain) $\times$ Resistance gain
$$=\frac{I_C}{I_B}\times\frac{R_L}{R_B}$$

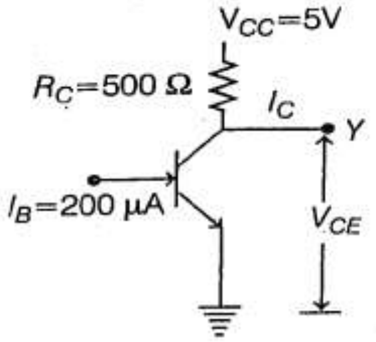
$= \frac{I_C R_L}{I_B R_B} = \frac{I_C R_L}{V_B} [\because V_B = I_B R_B]$

$= \frac{1.8 \times 10^{-3} \times 6000}{15 \times 10^{-3}}$

$= 720$

7. **(a)** Given, β of transistor $= 48$

Base current, $I_B = 200\mu$ A $= 200 \times 10^{-6}$ A



We know that; current gain, $\beta = \frac{I_C}{I_B}$

$\Rightarrow \quad 48 = \frac{I_C}{200 \times 10^{-6}}$

$\Rightarrow \quad I_C = 48 \times 200 \times 10^{-6}$

$= 96 \times 10^{-4}$ A

Now, $V_{CC} = I_C R_C + V_{CE}$

$\Rightarrow V_{CE} = V_{CC} - I_C R_C$

From figure, $V_{CC} = 5$ V and $R_C = 500\Omega$

$\Rightarrow V_{CE} = 5 - 96 \times 10^{-4} \times 500$

$= 5 - 4.8 = 0.2$ V

8. **(d)** Given, frequency, $v = k\delta E$

where, k is constant and δE is change in energy.

$\Rightarrow k = \frac{v}{\delta E} = \frac{v}{hv} \quad [\because \delta E = hv]$

$= \frac{1}{h}$

$\because$ We know that, energy, $E = hv$

$\Rightarrow h = \frac{E}{v} = \frac{ML^2 T^{-2}}{T^{-1}}$

$h = [ML^2 T^{-1}]$

$\therefore$ Dimension of $k = \frac{1}{h} = \frac{1}{[ML^2 T^{-1}]} = [M^{-1} L^{-2} T]$

9. **(b)** Given, $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, \mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$,

$\mathbf{C} = \frac{1}{\sqrt{5}}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$\because \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -1 \\ 2 & -1 & 1 \end{vmatrix} = -3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\therefore \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = \frac{1}{\sqrt{5}}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})(-3\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

$= \frac{6}{\sqrt{5}} - \frac{6}{\sqrt{5}} = 0$

10. **(a)**

$v = 360 \text{ km/h} = 100 \text{ m/s}.$

$h = 500 \text{ m}$

$$R = u\sqrt{\frac{2h}{g}} = 100 \times \sqrt{\frac{2 \times 500}{10}} = 1000 \text{ m}$$

11. (a) A galvanometer can be converted to a voltmeter of full scale deflection V_0 by connecting a series resistance R_1, then

$$R_1 = \frac{V_0}{I_g} - G$$

$\Rightarrow \quad V_0 = I_g(G + R_1) \ldots (i)$

where, I_g = current through the galvanometer and G = resistance of galvanometer.

Similarly, a galvanometer can be converted to an ammeter by connecting a shunt resistance R_2, then

$$R_2 = \left(\frac{I_g}{I_0 - I_g}\right)G$$

$\Rightarrow I_g = I_0\left(\frac{R_2}{G + R_2}\right)$(ii)

where, I_0 = total current.

Now, from Eq. (i), we get

$\frac{V_0}{I_g} - R_1 = G\ldots$(iii)

From Eq. (ii), we get

$$G + R_2 = \frac{I_0}{I_g} \times R_2$$

$\Rightarrow \quad G = \frac{I_0}{I_g} \times R_2 - R_2$...(iv)

From Eqs. (iii) and (iv), we get

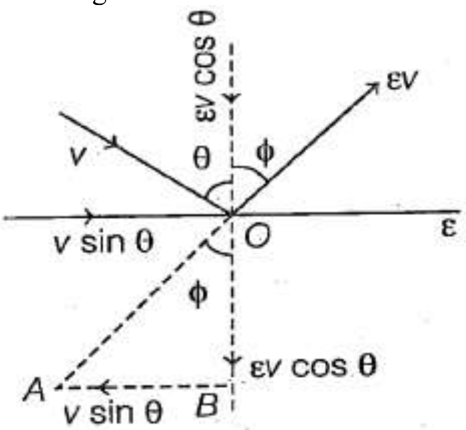
$$\frac{V_0}{I_g} - R_1 = \frac{I_0}{I_g} \times R_2 - R_2$$

$$\Rightarrow \quad \frac{V_0}{I_g} - \frac{I_0}{I_g} \times R_2 = R_1 - R_2$$

$$\Rightarrow \quad \frac{1}{I_g}(V_0 - I_0R_2) = R_1 - R_2$$

$$\Rightarrow \quad I_g = \frac{V_0 - I_0R_2}{R_1 - R_2}$$

12. (a) Given, a tennis ball hits the floor with a speed v making an angle θ with the normal as shown in the figure below.



The coefficient of restitution $= \varepsilon$

Now, from the above diagram,

In $\triangle AOB, \tan\phi = \frac{v\sin\theta}{\varepsilon v\cos\theta} = \frac{\tan\theta}{\varepsilon}$

$\therefore$ Angle of reflection, $\phi = \tan^{-1}\left(\frac{\tan\theta}{\varepsilon}\right)$

13. **(c)** Time period of second pendulum, $T = 2$ s

Displacement equation of simple pendulum,

$$x = A\sin\omega t$$

Given, $t = 2.25 \text{ s} = 2 + 0.25$

$$= 2 + \frac{1}{4}$$

$$\therefore\ t = T + \frac{T}{8}$$

Velocity of second pendulum,

$$v = \frac{dx}{dt}$$

$$= \frac{d}{dt}\cdot(A\sin\omega t)(\text{EQ (i)})$$

$$v = A\omega\cos\omega t$$

Velocity at $t = T$,

$$v = A\omega\cos\left(\frac{2\pi}{T}\cdot T\right) = A\omega = v_0\ldots.(\text{ii})$$

Velocity at $t = T + \frac{T}{8}$,

$$v_1 = A\omega\cos\omega\left(T + \frac{T}{8}\right)$$

$$= A\omega\cos\frac{2\pi}{T}\left(T + \frac{T}{8}\right)$$

$$= A\omega\cos 2\pi\cdot\frac{9}{8}$$

$$= A\omega\cos\left(2\pi + \frac{\pi}{4}\right)$$

$$= A\omega\cos\frac{\pi}{4}$$

$$= \frac{A\omega}{\sqrt{2}}$$

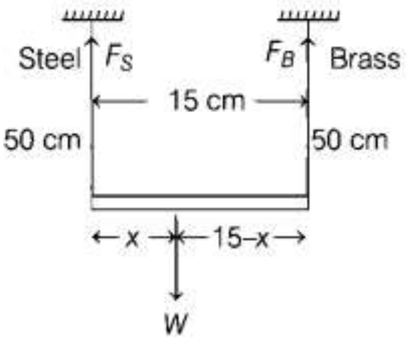
$$v_1 = \frac{v_0}{\sqrt{2}}\ \text{fromEq(ii)}$$

By equation of motion,

$$v_1^2 = v_0^2 - 2gh$$

$$\Rightarrow \left(\frac{v_0}{\sqrt{2}}\right)^2 = v_0^2 - 2gh \Rightarrow h = \frac{v_0^2}{4g}$$

14. **(b)** Let x be the distance from steel wire, where load is applied as shown in the figure below.



Given, Young's modulus of steel,

$Y_{\text{steel}} = 2\times 10^{12}$ dyne /cm^2

Young's modulus of brass, $Y_{\text{brass}} = 1\times 10^{12}$ dyne /cm^2

Cross-sectional area of wires, $A = 0.005$ cm^2

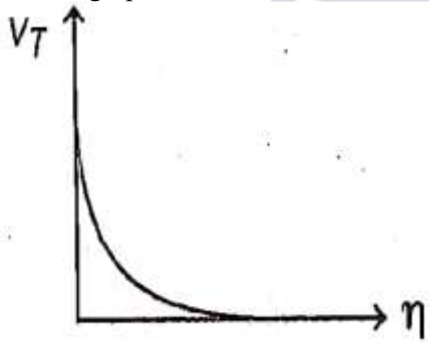
Extension in wires, $\Delta l = 0.1$ cm
We know that,
Young's modulus, $Y_{\text{steel}} = \frac{\text{stress}}{\text{strain}} = \frac{F_S/A}{\frac{\Delta l}{l}} = \frac{F_S \cdot l}{\Delta l A}$

$$\Rightarrow \quad F_S = Y_{\text{steel}} \times \frac{\Delta l}{l} \times A$$
$$= 2 \times 10^{12} \times \frac{0.1}{50} \times 0.005$$
$$\Rightarrow \quad F_S = 2 \times 10^7 \text{ dyne}$$

Similarly,
$$F_B = Y_{\text{brass}} \times \frac{\Delta l}{l} \times A$$
$$= 1 \times 10^{12} \times \frac{01}{50} \times 0.005$$
$$= 10^7 \text{ dyne}$$

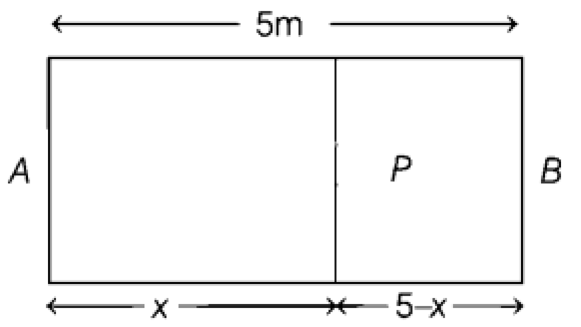
From given figure,
$$F_S \times x = F_B \times (15 - x)$$
$$\Rightarrow \quad 2 \times 10^7 \times x = 10^7 \times (15 - x)$$
$$\Rightarrow \quad 2x = 15 - x$$
$$\Rightarrow \quad 3x = 15 \Rightarrow x = 5 \text{ cm}$$

15. **(c)** We know that,
terminal velocity, $v_T = \frac{2r^2}{9\eta}(\rho_S - \rho_L)g$
$\Rightarrow v_T \propto \frac{1}{\eta}$, where η is viscosity of the liquid.
Hence, graph between terminal velocity v_T and viscosity of liquid η is as shown below.



16. **(a)** Given, during ab, heat rejected, $Q_{ab} = -50$ J
During ca, heat absorbed, $Q_{ca} = 80$ J
During bc, heat transferred, $Q_{bc} = 0$ J
Work done by the gas, $W = 40$ J
$\therefore$ Area of closed curve $abca = Q_{ab} + Q_{bc} + Q_{ca}$
Area $= -50 + 0 + 80$
Area $= 30$ J

17. **(b)** Given, length of rectangular container, $l = 5$ m
The number of moles of two gases are
$\mu_1 = \frac{M}{32}$ and $\mu_2 = \frac{M}{18}$
The given situation is shown in the figure given below.



'where, x is the distance of movable partition P from the left wall A.
At equilibrium, $p_1 = p_2 = p$
and $T_1 = T_2 = T$
By ideal gas equation,

$$\frac{p_1 V_1}{\mu_1} = \frac{p_2 V_2}{\mu_2} = RT$$

$$\Rightarrow \quad \frac{V_1}{\mu_1} = \frac{V_2}{\mu_2}$$

$$\frac{a \cdot x}{\mu_1} = \frac{a \cdot (5-x)}{\mu_2} [\because a = \text{ area of each wall } A \text{ and } B]$$

$$\frac{x}{\mu_1} = \frac{5-x}{\mu_2}$$

$$\Rightarrow \quad \frac{x}{M/32} = \frac{5-x}{M/18}$$

$$\Rightarrow \quad 32x = 18(5-x)$$

$$\Rightarrow \quad 32x = 90 - 18x$$

$$\Rightarrow \quad 50x = 90$$

$$\Rightarrow \quad x = \frac{90}{50} = 1.8 \text{ m}$$

18. (d) Given, mass of boiling water $= 100$ g
Mass of cold water $= 300$ g
Fall in temperature $= 100 - 20 = 80^\circ$C
Let heat capacity of calorimeter $= ms$
Specific heat of water $= 1 \text{calg}^{-1} {}^\circ\text{C}^{-1}$
$\therefore$ Heat lost by boiling water $= ms\Delta T = 100 \times 1 \times 80$
and heat gained by cold water
$= 300 \times 1 \times 10 + ms \times 10$
Now, according to the principle of calorimetry, heat lost = heat gained

$$100 \times 1 \times 80 = 300 \times 1 \times 10 + ms \times 10$$

$$\Rightarrow \quad ms = 500 \text{cal}/{}^\circ\text{C}$$

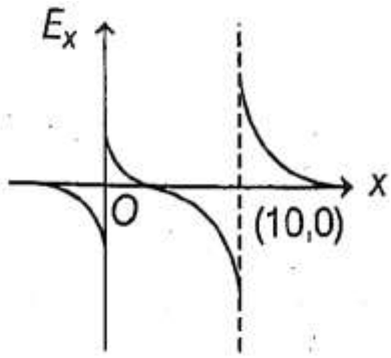
Let specific heat of metal is s_b.
Mass of block $= 1$ kg $= 1000$ g
Rise in temperature of metallic block,
$\Delta t = 19 - 10 = 9^\circ$C
Then, again from principle of calorimetry,

$$m_{\text{mixture}} \times s\Delta T + ms\Delta T = m_{\text{block}} \times s_b \times \Delta t$$

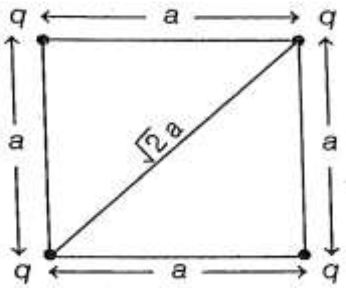
$$(100 + 300) \times 1 \times (20 - 19) + 500 \times (20 - 19) = 1000 \times s_b \times 9$$

$$\Rightarrow s_b = 0.1 \text{cal/g}^\circ\text{C}$$

19. (a) Given, $q_1 = 1 \times 10^{-6}$C, $q_2 = 3 \times 10^{-6}$C
$\therefore \; q_2 = 3q_1; q_2 > q_1$
As we know that, electric field, $E \propto \frac{1}{x^2}$
So, the graph showing variation of E_x in x-direction is as given below.



20. (c) The given system of charges is as shown below,



$\because$ Potential energy, PE $= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{kq_1q_2}{r}$

Here,
$q_1 = q_2 = q$
$r = a$
and

$\Rightarrow$ PE $= \frac{kqq}{a} = \frac{kq^2}{a}$

Due to four identical point masses,

$\text{PE}_i = \frac{4kq^2}{a} + \frac{2kq^2}{\sqrt{2}a}$

According to law of conservation of energy,
$\text{KE}_i + \text{PE}_i = \text{KE}_f + \text{PE}_f$
Initially, $\text{KE}_i = 0$
and at infinity, $\text{PE}_f = 0$

$\therefore$ $\text{KE}_f = \text{PE}_i = \frac{4kq^2}{a} + \frac{2kq^2}{\sqrt{2}a}$

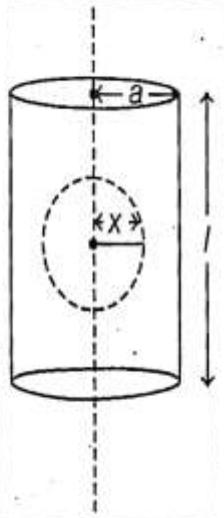
$= \frac{kq^2}{a}(4+\sqrt{2})$

$\because$ In CGS unit, $k = 1$

$\therefore$ KE $= \frac{q^2}{a}(4+\sqrt{2})$

21. (b) Given, radius of solid cylinder $= a$
Charge density of cylinder $= \rho$
Dielectric constant of material of cylinder $= k$

The electric field at a radial distance $x (x < a)$ as shown is calculated as



From Gauss' law,

$\int \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{k\varepsilon_0}$, where k is dielectric constant of material.

$\Rightarrow E(2\pi x l) = \frac{\rho(\pi x^2 l)}{k\varepsilon_0}$ [$\because$ charge, $Q_{\text{in}} = \rho(\pi x^2 l)$]

$\Rightarrow \; E = \dfrac{\rho x}{2k\varepsilon_0}$

22. **(a)** A galvanometer can be converted to a voltmeter of full scale deflection V_0 by connecting a series resistance R_1, then

$$R_1 = \frac{V_0}{I_g} - G$$

$$\Rightarrow \quad V_0 = I_g(G + R_1) \quad \text{.....(i)}$$

where, I_g = current through the galvanometer and G = resistance of galvanometer.

Similarly, a galvanometer can be converted to an ammeter by connecting a shunt resistance R_2, then

$$R_2 = \left(\frac{I_g}{I_0 - I_g}\right) G$$

$$\Rightarrow I_g = I_0\left(\frac{R_2}{G + R_2}\right) \text{(ii)}$$

where, I_0 = total current.

Now, from Eq. (i), we get

$$\frac{V_0}{I_g} - R_1 = G \text{.....(iii)}$$

From Eq. (ii), we get

$$G + R_2 = \frac{I_0}{I_g} \times R_2$$

$$\Rightarrow \quad G = \frac{I_0}{I_g} \times R_2 - R_2 \quad \text{....(iv)}$$

From Eqs. (iii) and (iv), we get

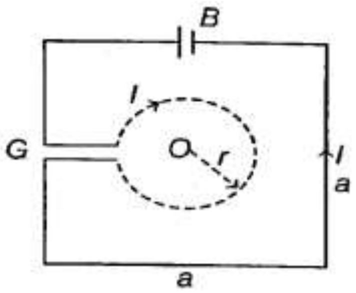
$$\frac{V_0}{I_g} - R_1 = \frac{I_0}{I_g} \times R_2 - R_2$$

$$\Rightarrow \quad \frac{V_0}{I_g} - \frac{I_0}{I_g} \times R_2 = R_1 - R_2$$

$$\Rightarrow \quad \frac{1}{I_g}(V_0 - I_0 R_2) = R_1 - R_2$$

$$\Rightarrow \quad I_g = \frac{V_0 - I_0 R_2}{R_1 - R_2}$$

23. **(d)** According to given figure,



Given, $a : r = 8 : \pi$

$\frac{a}{r} = \frac{8}{\pi} \Rightarrow r = \frac{\pi a}{8}$........(i)

Magnetic field due to square, $\mathbf{B}_1 = \frac{\mu_0 I}{\pi \frac{a}{2}} \times \sqrt{2}$ outward

Magnetic field due to circular loop, $\mathbf{B}_2 = \frac{\mu_0 I}{2r}$,
inward

Strength of magnetic field at the common centre O,

$$B = B_1 + B_2 = \frac{\mu_0 I}{2r} - \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

Substituting value of r from Eq (i), we get

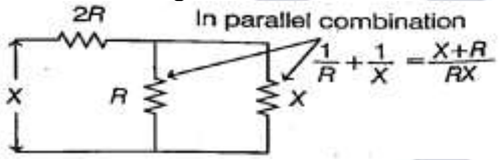
$$B = \frac{\mu_0 I \times 8}{2\pi a} - \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

$$= \frac{\mu_0 I}{\pi a}(4 - 2\sqrt{2})$$

$$= \frac{\mu_0 I}{\pi a} 2\sqrt{2}(\sqrt{2} - 1)$$

24. (a) A wire is bent to form a D-shaped loop carrying current I, where the curved part is semi-circle of radius R. The loop is placed in a uniform magnetic field **B** which is directed into the plane of the paper. As single current by flowing in the loop, so net magnetic force on a closed current loop in a uniform magnetic field **B** is zero.

25. (d) For equivalent resistance, between the terminals A and B of the infinite resistance network, we redraw the given circuit as



Now the $2R$ and $\frac{RX}{X+R}$ is in series, so

$$X = 2R + \frac{RX}{R + X}$$

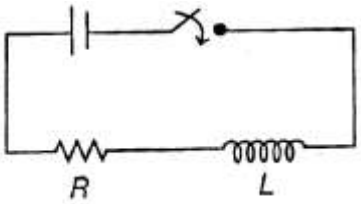
$$X^2 - 2RX - 2R^2 = 0$$

$$\Rightarrow \quad X = \frac{2R \pm \sqrt{4R^2 - 4(1)(-2R^2)}}{2}$$

$$\Rightarrow \quad X = \frac{2R \pm \sqrt{12R^2}}{2} = \frac{2R \pm 2\sqrt{3}R}{2} = (1 \pm \sqrt{3})R$$

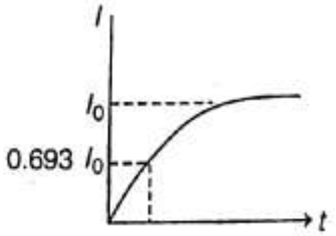
$$\Rightarrow \quad X = (1 + \sqrt{3})R$$

26. (c) When a DC voltage is applied at the two ends of a circuit kept in a closed box, it is observed that the current gradually increases from zero to a certain value and then remains constant. As capacitor blocks DC. So, the circuit must contains a resistor and an inductor in series as shown below.



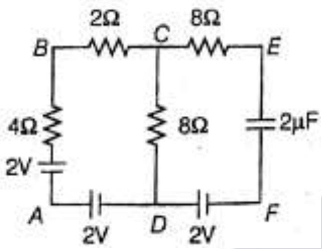
From Graph

Growth of current, $I = I_0\left(1 - e^{-t/\tau_L}\right)$
At
$t = 0, I = 0$
$At\ t = \tau_{L'}$
$$I = I_0\left(1 - \frac{1}{e}\right) = 0.693I_0$$

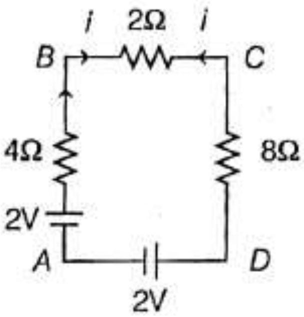


where, $\tau_L = \frac{L}{R}$ = time constant and $I_0 = \frac{E}{R}$ = maximum current.

27. (c) The given circuit is as shown below.

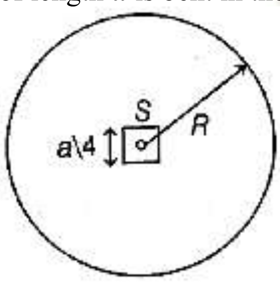


In steady state, the combination of resistance and capacitance in loop CEFD will not work. So, we can eliminate this combination and hence the equivalent circuit is as shown below.



$\because$ In the above circuit, the current i will flow from B to C and same current i will flow from C to B.
Hence, total current through the 2Ω resistor is zero.

28. (b) A conducting wire of length L bent in the form of a circle of radius R and another conductor of length a is bent in the form of a square. The two loops are then placed in same



plane such that the square loop is exactly at the centre of circular loop as shown in the figure
For circular loop,
$2\pi R = L$.........(i)
For square of side S,
$4S = a$......(ii)

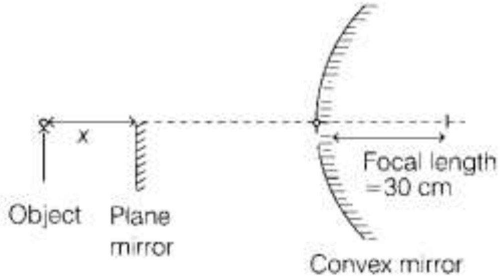
Flux linked, $\phi = \frac{\mu_0 I S^2}{2R}$

Mutual inductance, $M = \frac{\phi}{I} = \frac{\mu_0 S^2}{2R}$

Substituting R from Eq. (i) and S from Eq. (ii), we get

$M = \frac{\mu_0 a^2}{2(16)} \frac{2\pi}{L} = \frac{\mu_0 a^2 \pi}{16L}$

29. (a) The given situation is as shown in figure below.



Object distance from convex mirror, $u = -60$ cm

Focal length of convex mirror, $f = 30$ cm

Using mirror formula,

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{60} = \frac{1}{30}$

$\Rightarrow \frac{1}{v} = \frac{1}{30} + \frac{1}{60} = \frac{2+1}{60} = \frac{3}{60}$

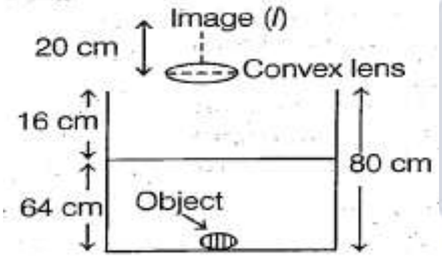
$\Rightarrow v = 20$ cm

Distance between the object and image $= 60 + 20 = 80$ cm

As there is no parallex between the image formed by the two mirrors, i.e. the images are formed at the same place.

So, the plane mirror is at a distance of 40 cm from the object.

30. (a) A thin convex lens is placed just above an empty vessel of depth 80 cm . The image (I) of a coin kept at the bottom of the vessel is thus formed 20 cm above the lens. If now water is poured in the vessel upto a height of 64 cm as shown in the figure below.



Given, object distance, $u = -80$ cm

Image distance, $v = +20$ cm

Using lens formula, we get

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \frac{1}{f} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80}$

$\Rightarrow f = 16$ cm

Now, water is poured in the vessel upto height of 64 cm , then object distance,

$u = 16 + \frac{64}{\mu} = 16 + \frac{64 \times 3}{4} = 64$ cm

(given, refractive index, $\mu = 4/3$)

Again, using lens formula,

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \frac{1}{16} = \frac{1}{v} + \frac{1}{64}$(u=-64cm)

$\Rightarrow \frac{1}{v} = \frac{3}{64}$

$\therefore v = 21.33$ cm, above the lens.

31. (c) Given, resistance of the circular loop $= 20\Omega$

Electric charge flowing due to the induced current, $q = I \cdot dt = \frac{\Delta\phi}{R} = \frac{\phi_2 - \phi_1}{R}$, where ϕ_1, ϕ_2 are magnetic flux at $t_1 = 0$ and $t_2 = 20$ ms.

As the loop is perpendicular to the magnetic field

B, the flux passing through the loop is,

$\phi = \mathbf{B} \cdot \mathbf{A} = BA\cos 90^\circ = 0$

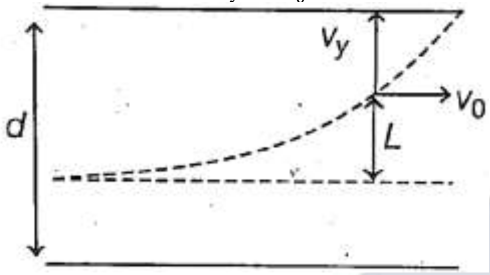
Hence, $q = \frac{\Delta\phi}{R} = \frac{\phi_2 - \phi_1}{R} = 0$

$\therefore\ q = 0\text{C}$

32. (b) Distance between the parallel plates $= d$

The axial distance of beam from the centre of parallel plates $= L$

As, time, $t = \frac{\text{distance}}{\text{velocity}} = \frac{L}{v_0}$



y-component of velocity, $v_y = \left(\frac{qE}{m}\right) t = \frac{e}{m} \cdot \frac{V}{d} \times \frac{L}{v_0}$

$\Rightarrow\ v_y = \dfrac{eVL}{mdv_0}$

$\therefore \tan\theta = \dfrac{v_y}{v_z} = \dfrac{eLV}{mdv_0 \cdot v_0} = \dfrac{eVL}{mdv_0^2}$

$\Rightarrow\ \theta = \tan^{-1}\left(\dfrac{eVL}{mdv_0^2}\right)$

33. (a) Due to dragging movement of the block, work (W_f) will be done against force of friction (f).

Now, force of friction, $f = \mu mg$

where, $\mu =$ coefficient of static friction $= 0.1$,

mass, $m = 20$ kg and $g = 10\ \text{ms}^{-2}$.

$\Rightarrow\ f = 0.1 \times 20 \times 10 = 20$ N

This work done (W_f) will produce heat energy,

$H = mc\Delta T$

Hence, $W_f = H$

$\Rightarrow$ Force $\times$ Displacement $= mc\Delta T$

$\Rightarrow\ f \times vt = mc\Delta T\ [\because s = v \cdot t]$

Here, $v = 0.5\ \text{ms}^{-1}, t = 2.1$ s

and $c = 0.1$ CGS unit $= 0.1 \times 4.2 \times 10^3$ SI unit

$\Rightarrow\ 20 \times 0.5 \times 2.1 = 20 \times 0.1 \times 4.2 \times 10^3 \times \Delta T$

$\Rightarrow\ \Delta T = \dfrac{20 \times 0.5 \times 2.1}{20 \times 0.1 \times 4.2 \times 10^3} = 0.0025^\circ\text{C}$

34. (c) We have, power $= \frac{\text{work done}}{\text{time}}$

Given, time $= 1$ min $= 60$ s

Now, work done per cycle $= (130 - 30)$

$= 100\text{cal} = 100 \times 4.2$ J

As 2 J of energy is consumed due to friction by the engine, hence work done per cycle

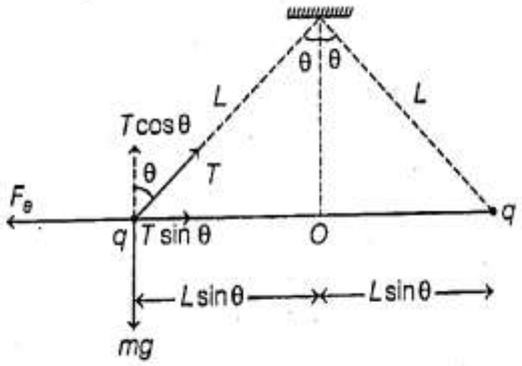
$= 100 \times 4.2 - 2$

$= 420 - 2 = 418$ J

Total work done for 90 cycles = 418×90 J

$\therefore$ Power $= \frac{418\times90}{60} = 627$ W

35. (b) Let force acting due to electrostatic attraction of charges be F_e as shown in the figure below



Now,

$T\sin\theta = F_e$......(i)

$T\cos\theta = mg$....(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{F_e}{mg} = k\frac{q_1 \times q_2}{r^2}$$

Here, $q_1 = q_2 = q$

Distance, $r = 2L\sin\theta$

$\Rightarrow \tan\theta = \frac{kq^2}{(2L\sin\theta)^2 \cdot mg}$....(iii)

Now, in the second case, when the charges are tripled, new charge $q' = 3q$

As the angle between the strings are doubled, θ becomes 2θ, hence from Eq. (iii), we get

$$\tan 2\theta = \frac{k(3q)(3q)}{(2L\sin 2\theta)^2 \cdot mg}$$

$$\tan 2\theta = \frac{9kq^2}{(2L\sin 2\theta)^2 mg} \text{(iv)}$$

On dividing Eq. (iv) by Eq. (iii), we get

$$\frac{\tan 2\theta}{\tan\theta} = \frac{9kq^2}{4L^2\sin^2 2\theta \cdot mg} \times \frac{4L^2\sin^2\theta \cdot mg}{kq^2}$$

$$\frac{\tan 2\theta}{\tan\theta} = \frac{9\sin^2\theta}{\sin^2 2\theta}$$

As, $\tan 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

and $\sin 2\theta = 2\sin\theta\cos\theta$

$$\Rightarrow \frac{2}{1-\tan^2\theta} = \frac{9\sin^2\theta}{4\sin^2\theta\cos^2\theta} = \frac{9}{4\cos^2\theta}$$

$$\Rightarrow \frac{2}{1-\tan^2\theta} = \frac{9}{4}\sec^2\theta$$

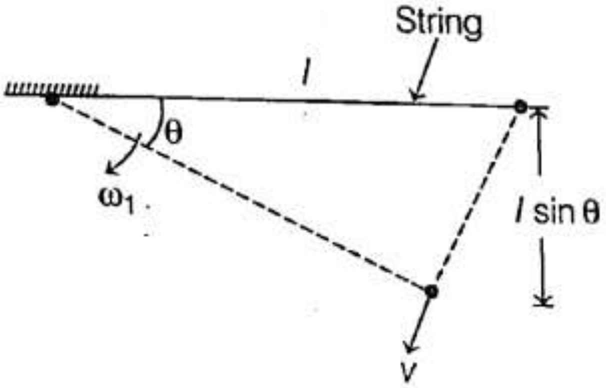
$$\Rightarrow \frac{2}{1-\tan^2\theta} = \frac{9}{4}(1+\tan^2\theta)$$

$[\because 1+\tan^2\theta = \sec^2\theta]$

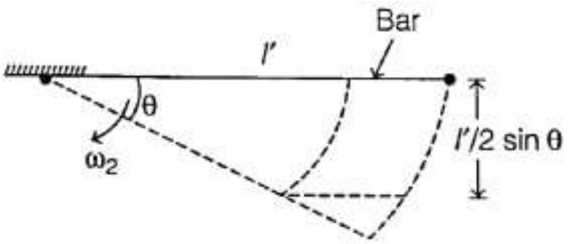
Let $\tan^2\theta = x$

$\Rightarrow \quad 8 = 9 - 9x^2$
$\Rightarrow \quad 9x^2 = 1$
$\Rightarrow \quad x^2 = \frac{1}{9} \Rightarrow x = \frac{1}{3}$
$\Rightarrow \quad \tan^2\theta = \frac{1}{3}$
$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$
$\Rightarrow \quad \theta = 30^\circ$
Hence, the angle between the strings
$= 2\theta = 2 \times 30^\circ = 60^\circ$

36. (b) The photoelectric current depends on the intensity of incident radiation by relation,
$i \propto I$
But the intensity of radiation is inversely proportional to the square of distance between the source and the photoelectric surface,
i.e. $\quad I \propto \frac{1}{d^2}$
$\therefore \quad i \propto \frac{1}{d^2}$
So, when the distance (d) is doubled, the photoelectric current will decrease. The stopping potential and maximum kinetic energy are independent of distance, so they remain same.
37. (d) As, inner radii are not given, so we cannot calculate their masses. Also, it is not given about the nature of material of spheres.
The rotational kinetic energy of sphere,
$(\mathrm{KE})_{\mathrm{rot}} = \frac{1}{2} I\omega^2$
where, $I =$ moment of inertia
and $\omega =$ angular speed.
It is given that, the two metallic spheres have same moment of inertia (I) about their respective diameters. So, their rotational kinetic energies will be equal when rotated with equal uniform angular speed (ω) about their respective diameters.
38. (a) The velocity of the particle at extreme position,
$v = \sqrt{2gh}$ [from $v^2 - u^2 = 2gh$]
Let when released from extreme position, the pendulum traverse an angular distance of θ as shown below.



Here, $h = l\sin\theta$
Then, linear velocity, $v = \sqrt{2gl\sin\theta}$
and angular velocity, $\omega_1 = \frac{v}{l} = \frac{\sqrt{2gl\sin\theta}}{l}$
$= \sqrt{\frac{2g\sin\theta}{l}}$
For a bar moved through the same angular distance θ as shown below.



Here, distance travelled, $h=\frac{l'}{2}\sin\theta$, where l' is the length of bar.
Potential energy = Rotational kinetic energy

$$mgh=\frac{1}{2}I\omega_2^2$$

$$\Rightarrow mg\frac{l'}{2}\sin\theta=\frac{1}{2}\frac{m(l')^2}{3}\times\omega_2^2\left[\because \text{ for rod, } I=\frac{ml^2}{3}\right]$$

$$\Rightarrow gl'\sin\theta=\frac{(l')^2}{3}\omega_2^2$$

$$\Rightarrow \omega_2=\sqrt{\frac{3gl'\sin\theta}{(l')^2}}=\sqrt{\frac{3g\sin\theta}{l'}}$$

As the motions of pendulum and bar are synchronous, so their angular velocities will be equal, i.e.

$$\Rightarrow \quad \omega_1=\omega_2$$

$$\Rightarrow \quad \sqrt{\frac{2g\sin\theta}{l}}=\sqrt{\frac{3g\sin\theta}{l'}}$$

$$\Rightarrow \quad \frac{2}{l}=\frac{3}{l'}\Rightarrow l'=\frac{3}{2}l$$

$\therefore$ The length of the bar is $\frac{3}{2}l$.

39. (d) Given, $R=400\Omega, L=250\text{mH}=250\times10^{-3}\text{H}$,
$C=2.5\mu\text{ F}=2.5\times10^{-6}\text{ F}, V_S=5\text{ V}$
and $\omega=2\text{kHz}=2000\text{rad/s}=2\times10^3\text{rad/s}$
$\therefore$ Inductive reactance,
$X_L=\omega L=2\times10^3\times250\times10^{-3}=500\Omega$
Capacitive reactance,

$$X_C=\frac{1}{\omega C}=\frac{1}{2\times10^3\times2.5\times10^{-6}}=200\Omega$$

Impedance,

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$
$$=\sqrt{(400)^2+(500-200)^2}$$
$$=\sqrt{250000}=500\Omega$$

The peak voltage across capacitor,

$$(V_P)_C=I_PX_C=\frac{V_S}{Z}X_C=\frac{5}{500}\times200=2\text{ V}$$

$\therefore$ Peak value of electrostatic energy of capacitor,

$$(U_P)_C=\frac{1}{2}C(V_P)_C^2$$
$$=\frac{1}{2}\times2.5\times10^{-6}\times(2)^2$$
$$=5\times10^{-6}\text{ J}$$

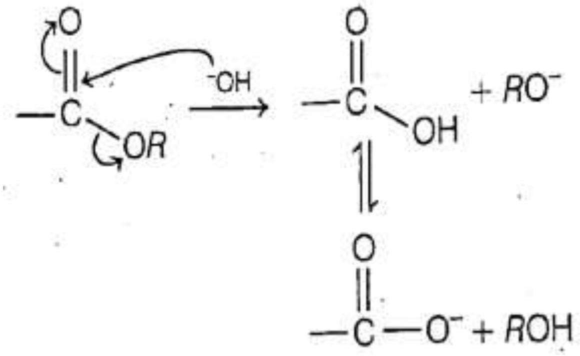
or $(U_P)_C=5\mu\text{ J}$

40. (a, c, d)
It is given that the charged particle is moving with constant velocity in a region of no gravity, so following cases are possible
(i) The particle may move in a straight line in any direction, when E = 0 and B = 0.
(ii) The particle may move in a circle with constant velocity. The centripetal force for circular motion is provided by

magnetic field.
So, $E=0$ and $B \neq 0$.
(iii) The particle may move in a helical path with constant velocity. For helical motion, the condition is that both fields must be present, i.e. $E \neq 0$ and $B \neq 0$.

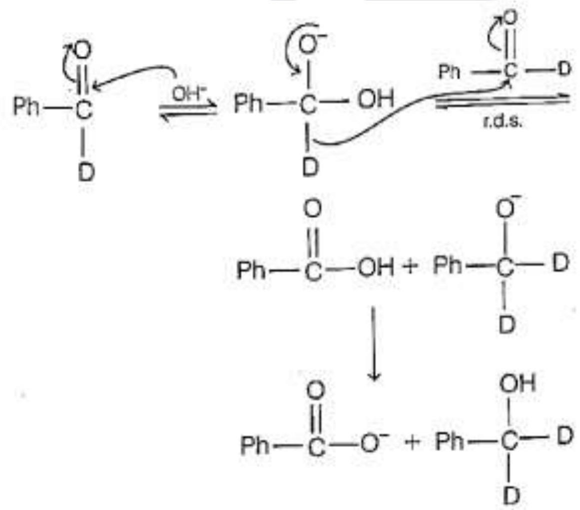
Chemistry

41. (c) Alkaline hydrolysis of an ester (carboxylic acid derivative) follows acyl S_N2 mechanism. It is as follows:



Rate of S_N2 mechanism depends on the polar nature of $>C=O$ group of - COOR group. Electron withdrawing group ($-R>-I$) increases the rate of S_N2 as it promotes the nucleophilic attack whereas electron donating group ($+R>+I$) decreases the rate of S_N 2reaction as it demotes the nucleophilic attack.
Here, the nature and order of functional groups attached para to benzene ring are:
$\underset{(-R)}{-NO_2(I)} > \underset{(+I)}{-Me(III)} > \underset{(+R)}{-OMe(II)}$
So, the order of rate of alkaline hydrolysis is
$I > III > II$

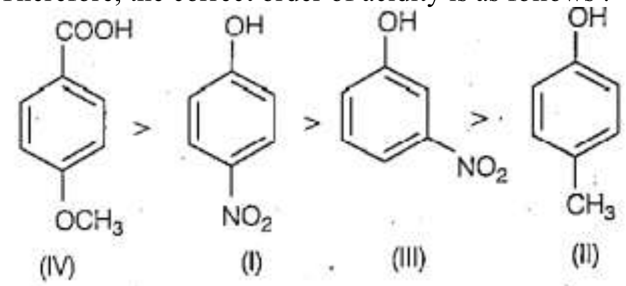
42. (c) Given reaction proceed via shift of D^-.
It is shown as follows:



The alcohol formed in is $Ph-CD_2-OH$.

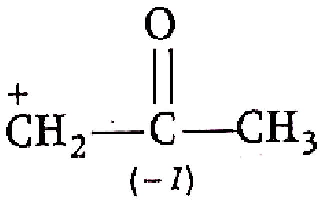
43.(b) Among the given compounds, compound IV is a carboxylic acid whereas rest are phenols.
Carboxylic acids are more acidic than phenol due to more stability of carboxylate ion than phenoxide ion. So, compound IV is the most acidic among the given compounds.
Further, acidicity of phenols increases by the presence of electron withdrawing groups (EWG) such as - NO_2. The effect of EWG is more pronounced at ortho- and para-positions than at m-position due to combined effect of $-I$ and $-R$ effect of $-NO_2$ group at p-position. On the other hand, electron donating group (EDG) such as $-CH_3$ decreases the acidity of phenols.

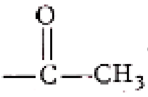
Therefore, the correct order of acidity is as follows :



44. (d) Carbocations are stabilised by the presence of electron donating group ($+R, +I, +H$) and destabilised by presence of electron withdrawing group$(-R, -H, -I)$.

I.



-I-effect of -  destabilises the carbocation. So, it is least stable carbocation.

II.

It is stabilised by $+R$-effect. So, it is the most stable carbocation.

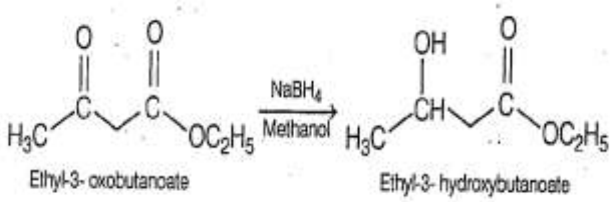
III. $\overset{+}{C}H_2 - CH_3$

Hyperconjugation of $-CH_3$ group stabilises the ion to some extent but not as effective as $-OCH_3$. Therefore, its stability is intermediate between I and II.

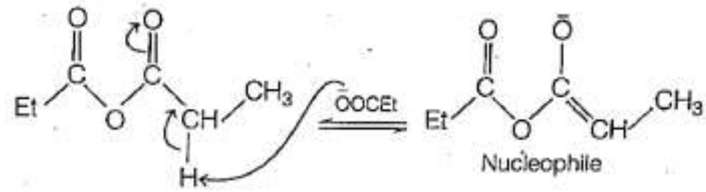
Hence, the correct order of stability of carbocations is II > III > I.

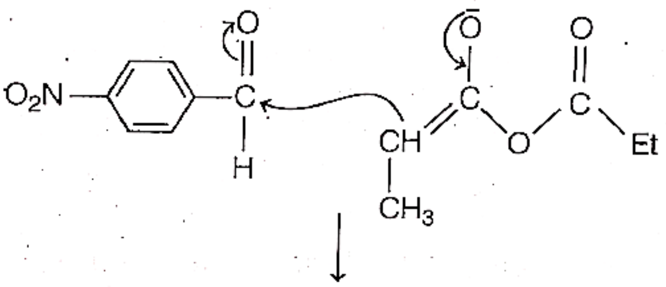
So, the correct option is (d).

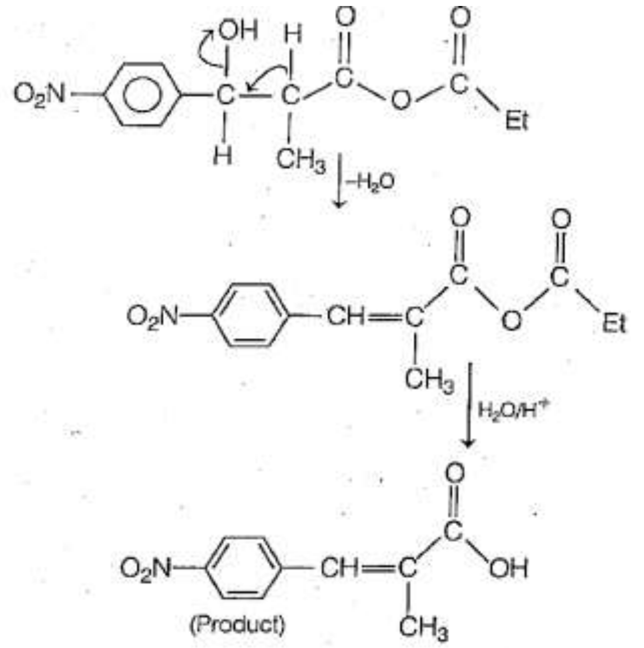
45. (c) $NaBH_4$ reduces aldehydes, ketones and acid chlorides into alcohols. It cannot reduce ester and amides. Therefore, in the given reaction, $NaBH_4$ reduces only the keto group without affecting the ester group. The reaction takes place as:



46. (a)







47. **(d)** Given,

$$n = 3$$
$$l = 2$$

so, subshell $= 3d$

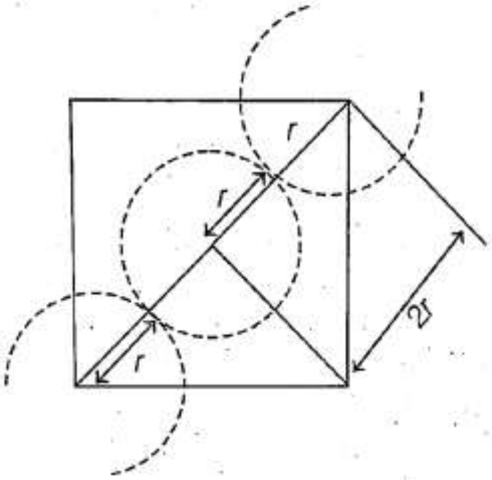
Since, five orientations of d-orbitals are degenerated, therefore $m = -1$ can be assigned to any one of them which means all the $3d$-orbitals are filled. Therefore, the expected maximum number of electrons in an atom of the given quantum numbers are 30 .

So, the correct option is (d).

48. **(b)** The given crystal lattice is face centred cubic lattice, which has $Z_{\text{eff}} = 4$.

Also, for fcc, atomic radius $(r) = \frac{a}{2\sqrt{2}}$.

The arrangement of Au-atom in a fcc lattice is shown below:



From the above figure, it is clear that the closest distance between two Au-atoms $= 2r$

$$= 2 \times \frac{a}{2\sqrt{2}} = \frac{a}{\sqrt{2}}$$

49. **(a)** For $2A \rightleftharpoons B + C, K_1 = 1.0$

$K_1 = \frac{[B][C]}{[A]^2}$.....(i)

For $2B \rightleftharpoons C + D, K_2 = 16$

$K_2 = \frac{[C][D]}{[B]^2}$.....(ii)

For $2C + D \rightleftharpoons 2P, K_3 = 25$

$K_3 = \frac{[P]^2}{[C]^2[D]}$.....(iii)

Add equations (i), (ii) and (iii), we get

$$K_1 \cdot K_2 \cdot K_3 = \frac{[P]^2}{[B][A]^2}$$

$$25 \times 16 \times 1 = \frac{[P]^2}{[B][A]^2} \text{(iv)}$$

On reversing and taking the square root of equation (iv), we get the resultant K for the reaction, $P \rightleftharpoons A + \frac{1}{2}B$ as follows:

$$K = \sqrt{\frac{1}{K_1 \cdot K_2 \cdot K_3}}$$

$$= \frac{[B]^{1/2}[A]}{[P]}$$

$$= \sqrt{\frac{1}{25 \times 16}}$$

$$\therefore \quad K = \frac{1}{20}$$

So, the option (a) is correct answer.

50. (a)

According to Hardy-Schulze rule, "greater the valency of coagulating ion or flocculating ion (oppositively charged ion) added, the greater is its power to cause coagulation. To coagulate a positively charged sol, i.e. $Fe(OH)_3$ the maximum power of coagulating of negative ion is PO_4^{3-}. So, the correct option is (a).

51. (b) Let, the number of moles of ethanol $= n$

The number of moles of water $= N$

Now, mole fraction of ethanol

$$(\chi_B) = \left[\frac{n}{n+N}\right] = 0.08$$

Mole fraction of water $(\chi_A) =$ 1-mole fraction of ethanol $= 0.92$

$$\text{Molality } = \frac{\chi_B \times 1000}{(1-\chi_B) \times M_A}$$

$$\therefore m = \frac{1000 \times \text{ mole fraction of ethanol}}{(1 - \text{ mole fraction of ethanol }) \times \text{ molecular mass of } H_2O}$$

$$= \frac{1000 \times 0.08}{(1-.08) \times 18} = 4.83 \text{ mol/kg}$$

So, the correct option is (b).

52. (b) The balanced chemical reaction is

$Pb(NO_3)_2(aq) + 2KI(aq) \longrightarrow PbI_2 \downarrow + 2KNO_3(aq)$

Initial moles of $Pb(NO_3)_2 = 5 \times 0.1 \times 10^{-3}$

$= 5 \times 10^{-4}$

Initial moles of KI $= 10 \times 0.02 \times 10^{-3}$

$= 2 \times 10^{-4}$

Therefore,

$Pb(NO_3)_2(aq) + 2KI(aq) \longrightarrow PbI_2 \downarrow + 2KNO_3(aq)$

Initially 5×10^{-4} $\quad 2 \times 2 \times 10^{-4}$ $\quad 0 \quad 0$

It is clear that for the formation of 1 mole of PbI_2, we require 4×10^{-4} moles of KI. But the given. moles of KI are 2×10^{-4}. Hence, it is a limiting reagent and the amount of PbI_2 formed will depend upon the moles of KI .

Thus, amount of PbI_2 precipitated will be about $= 2 \times 10^{-4}$ mol

53. (d) Given,

Density $= 1.964$ g/L

Pressure $= 76$ cmHg $= 1$ atm

We know that density,

$$d = \frac{RT}{RM}$$

$$\therefore\ M = \frac{0.0821 \text{ L atm/K mol} \times 273 \text{ K}}{1 \text{ atm} \times 1.964 \text{ g/L}}$$

$$= 44 \text{ g/mol}$$

Among the given gases, CO_2 gas has molecular mass of 44 g/mol.

54. (a) Equal masses, i.e. w kg of ethane and hydrogen are mixed in an empty container.

$\therefore$ Moles of $C_2H_6 = \frac{w}{30}$

Moles of $H_2 = \frac{w}{2}$

From Raoult's law, we have

$p_{H_2} = p_{\text{total}} \times$ mole fraction of H_2

or $p_{H_2} = p_{\text{total}} \times \frac{\frac{w}{2}}{\frac{w}{2}+\frac{w}{30}}$

$$\therefore\ p_{H_2} = p_{\text{total}} \times \frac{15}{16}$$

$$\frac{p_{H_2}}{p_{\text{total}}} = \frac{15}{16}$$

Thus, fraction of total pressure exerted by hydrogen is 15:16.

55. (c) For the work against vacuum, $W = 0$

For an adiabatic process, $q = 0$

From first law of thermodynamics, we have

$$\Delta U = q + W$$

$$\therefore\ \Delta U = 0$$

56. (c) Given,

mass of ethylene glycol $= 10$ g

molecular mass $= 62$ g/mol

K_f (water) $= 1.86$ K kg mol^{-1}

As we know that,

$$\Delta T_f = K_f \times \text{ molality}$$

$$= 1.86 \times \left(\frac{10}{62} \times \frac{1000}{90}\right)$$

$$= 3.3^\circ\text{C}$$

$\therefore$ Temperature at which the ice begins to separate $= -3.3^\circ$C

57. (a) Given,

Radius of orbit of H -atom $= 0.53 \times 10^{-8}$ cm

For first Bohr orbit, $n = 1$

As we know,

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr}$$

$$v = \frac{1 \times 6.626 \times 10^{-27}}{2 \times 3.14 \times 91 \times 10^{-28} \times 0.53 \times 10^{-8}}$$

$$v = 2.188 \times 10^{8} \text{ cm/s}$$

58. (d) Among.the given statements, option (d) is incorrect. It can be corrected as, F is more electronegative than O because it is the most electronegative element in periodic table.

Rest all the given statements are true.

59. (a) The electronic configuration of uranium is ${}_{92}\text{U}: [\text{Rn}]^{86} 5f^3 6d^1 7s^2$.

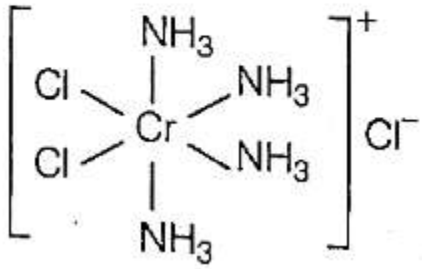
It consist of your electrons, i.e. three from f-orbital and one from d-orbital.

60. **(c)** On prolonged electrolysis, the density of liquid water increases as the proportion of D_2O increases. This is because on prolonged electrolysis, the concentration of heavy water (D_2O) in liquid water increases which has higher density than ordinary water (H_2O). If electrolysis continued, then almost pure D_2O is obtained.

61. (a) Orbital angular momentum $= \sqrt{l(l+1)}\frac{h}{2\pi}$.
For $4f$-orbital, $l = 3$,
$\therefore$ Orbital angular momentum $= \sqrt{3(3+1)}\frac{h}{2\pi}$
$= 2\sqrt{3}\frac{h}{2\pi}$
For $4s$-orbital,
$l = 0$
$\therefore$ Orbital angular momentum $= 0$.
So, the difference between orbital angular momentum $= 2\sqrt{3}\frac{h}{2\pi} - 0 = 2\sqrt{3}\frac{h}{2\pi}$
Therefore, the difference between orbital angular momentum of an electron in a $4f$-orbital and another electron in $4s$-orbital is $2\sqrt{3}$.

62. (d) Number of atoms $= \frac{\text{mass}}{\text{molar mass}} \times N_A \times$ number of atoms in 1 mole.
The calculation of number of atoms in given options are as follow :
(a) Number of atoms in 1 g of Ag
$= \frac{1}{107.87} \times N_A \times 1 = 0.0092 N_A$
(b) Number of atoms in Ig of Fe
$= \frac{1}{55.85} \times N_A \times 1 = 0.0179 N_A$
(c) Number of atoms in 1 g of Cl_2
$= \frac{1}{70} \times N_A \times 2 = 0.0285 N_A$
(d) Number of atoms in 1 g of Mg
$= \frac{1}{24.30} \times N_A \times 1 = 0.0411 N_A$
Thus, 1 g of Mg contains the maximum number of atoms among the given options.

63. (d)



Let the oxidation state of central atom (Cr) be x. Then,
$(x \times 1) + (4 \times 0) + (2 \times -1) = +1$
$x - 2 = +1$
$x = +3$
Thus, the correct IUPAC name of complex is cis-tetraamminedichlorochromium (III) chloride.

64. (b) We know,
mass of N_A atoms $= 12$ g of C
or mass of 6.023×10^{23} atoms $= 12$ g
$\therefore$ Mass of 1 atom $= \frac{12}{6.023 \times 10^{23}}$ g $= 1.9923 \times 10^{-23}$ g

65. (b) We know that,
Bond order
$= \frac{1}{2}$ [number of electrons in bonding orbitals number of electrons in antibonding orbitals] The electronic configuration and bond order (B.O.) of given options is as follows:
(i) $He_2(4e^-) = \sigma(ls)^2\sigma^*(ls)^2$
B.O $= \frac{2-2}{2} = 0$
(ii) $He_2^+(3e^-) = \sigma(1s)^2\sigma^*(1s)^1$

$$B.O = \frac{2-1}{2} = \frac{1}{2}$$

(iii) $He_2^{2+}(2e^-) = \sigma(1s)^2$

$$B.O = \frac{2-0}{2} = 1$$

Therefore, the bond order of He_2, He_2^+ and He_2^{2+} are respectively $0, \frac{1}{2}, 1$.

66. (c) Colourless, efflorescent sodium salt is $Na_2S_2O_3$. The colourless gas evolved in SO_2.

Complete reaction is as follows:

$$\underset{\text{colourless efflorescent}}{Na_2S_2O_3(s)} + 2H^+(\text{dil.}) \longrightarrow$$

$$\underset{\text{White ppt.}}{S(s)} + 2Na^+(aq) + SO_2\uparrow + H_2O(l)$$

Now, when the colourless gas is passed through it acidified dichromate solution turns green.

The reaction taking place is

$$K_2Cr_2O.(aq) + 3SO_2(aq) + 2H^+(aq) \longrightarrow$$

$$k^+(aq) + \underset{(\text{ Green })}{2Cr^{3+}(aq)} + H_2O(l) + 3SO_4^{-2}(aq)$$

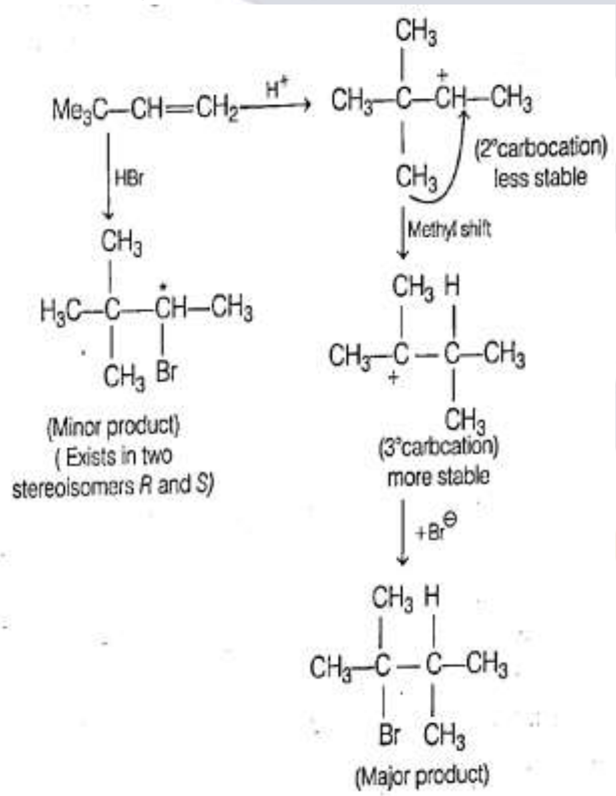
67. (c) Given reaction is an example of thermite process. In thermite reaction, aluminium metal is oxidised by an oxide of another metal most commonly iron oxide.

So, Fe is the metal (M).

68. (a)

For the extraction of Ca by electro-reduction of molten $CaCl_2$ some CaF_2 is added to the electrolyte because CaF_2 decreases the melting point of $CaCl_2$. It keep the electrolyte in liquid state of temperature than the m.p. of $CaCl_2$ So, option (a) is correct.

69. (c) The given reaction proceeds as follows:



Therefore, the total number of alkyl bromides (including stereoisomers) formed in the reaction are 3.

70.(d) The given reaction can be completed as follows:

71. (d) Compounds which do not possess any type of symmetry are referred as asymmetric.

Among the given options, only compound (d) do not possess any type of symmetry, i.e. absence of plane of symmetry and centre of symmetry, so compound (d) is asymmetric. Options (a), (b) and (c) have plane of symmetry.

Hence, correct option is (d).

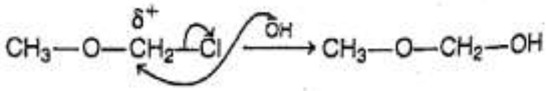
72. (b) Rate law : $r = K[A]^{\alpha}[B]^{\beta}$

As the $t_{1/2}$ does not depend on the concentration of B. Therefore, order w.r.t. B follows first order kinetics.
$\therefore\ B = 1$
Now, according to the question,
$$\frac{r_2}{r_1} = \frac{[2A]^\alpha[2B]^\beta}{[A]^\alpha[B]^\beta}$$
$\Rightarrow 4 = 2^\alpha \cdot 2^\beta$
$2^2 = 2^{\alpha+\beta}$
$\alpha + \beta = 2$
As, $\beta = 1$ because B follows first order kinetic.
$\alpha = 1$
$\therefore$ Overall order $= 2$
Hence, unit of rate constant for second order $= \mathrm{Lmol^{-1}\ s^{-1}}$.
73. (d) Given,
$K_{sp}(SrCO_3) = 7.0 \times 10^{-10}$
$K_{sp}(SrF_2) = 7.9 \times 10^{-10}$
$[CO_3^{2-}] = 1.2 \times 10^{-3}\mathrm{M}$
Now, for the reaction,
$SrCO_3 \rightleftharpoons Sr^{2+} + CO_3^{2-}$
$K_{sp} = [Sr^{2+}][CO_3^{2-}]$
$7 \times 10^{-10} = [Sr^{2+}] \times 1.2 \times 10^{-3}$
$$\therefore\ [Sr^{2+}] = \frac{7 \times 10^{-10}}{1.2 \times 10^{-3}}$$
Also, $SrF_2 \rightleftharpoons Sr^{2+} + 2\,F^-$
$K_{sp} = [Sr^{2+}][F^-]^2$
$$[F^-]^2 = \frac{K_{sp}}{[Sr^{2+}]}$$
$$[F^-] = \sqrt{\frac{K_{sp}}{[Sr^{2+}]}}$$
$$= \sqrt{\frac{7.9 \times 10^{-10}}{7 \times 10^{-10}}}$$
$[F^-] = 3.7 \times 10^{-2}\mathrm{M}$

74. (c) As the gas molecule shows 2-electron magnetic moment which means it contains 2 unpaired electrons.
Among the given gases, O_2 contains 2 unpaired electrons.
$O_2 = 8 + 8 = 16$
Electronic configuration of O_2
$= \sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2$
$< \sigma 2p_z^2 < \pi 2p_x^2 = \pi 2p_y^2 < \pi^* 2p_x^1 = \pi p_y^1 < \sigma^* 2p_z$
$$\therefore\ \text{Bond order of } O_2 = \frac{1}{2}(10 - 5) = \frac{5}{2} = 2.5$$
Now, in oxidised form, O_2 exists as O_2^+,
$O_2 \xrightarrow{-e^-} O_2^+(15e^-)$
So, bond order of $O_2^+ = 2$
As bond ordder $\propto \frac{1}{\text{Bond length}}$
The bond length, in the oxidised form, of molecule gets decreased. Thus, the gas molecule is O_2.
75. (d) For the given reaction statement (d) is correct and applicable. For this reaction, a mixed S_N1 and S_N2 pathway is followed. The given reaction can proceed via S_N1 pathway, So as,
$CH_3 - O - CH_2 - Cl \longrightarrow$

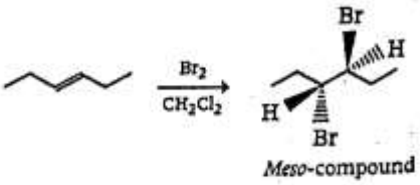
$[CH_3-\ddot{O}-\overset{+}{C}H_2 \longleftrightarrow CH_3-\overset{+}{O}=CH_2]$

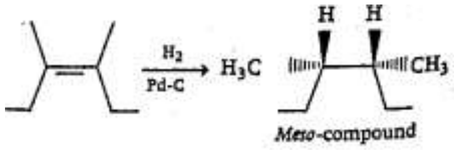
Here, the carbocation is stabilised by resonance. The reaction can also be proceeded via S_N2 path, since the transition state is resonance stabilised.

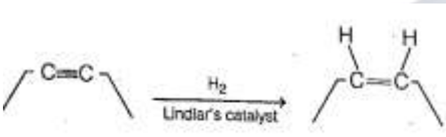


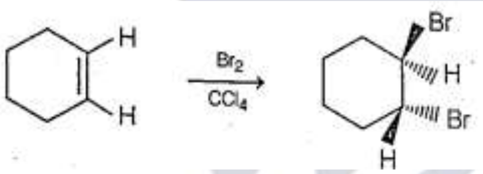
So, the correct option is (d).

76.(a, b) Both reactions (a) and (b) give a mesocompound as the main product the reactions are as follows:

(a) 

(b) 

(c) 

(d) 

77. (a, b, d) For spontaneous polymerisation, $\Delta G =$ negative

Polymerisation process involves the following reaction:

$$nA \longrightarrow \left(A - A - A - A \right)_n$$

So, $\Delta S =$ negative due to association of atoms.

Also, we know that,

$\Delta G = \Delta H - T\Delta S$ or $\Delta H = \Delta G + T\Delta S$

Hence, $\Delta H =$ negative.

78. (a, b, d) Statements (a, b and d) are incorrect whereas statement (c) is correct. The corrected form are as follows :

A sink of SO_2 pollutant is ocean.

FGD (Flue-gas desulfurisation) is a process of removing SO_2 from atmosphere.

The catalyst used to convert CCl_4 to CF_4 by HF is AgF .

79. (a, c, d) SiO_2 is attacked by HF , hot NaOH and fluorine. Reactions involved are as follows :

(a) $SiO_2(s) + 6HF(aq) \longrightarrow H_2SiF_6(aq) + 2H_2O(l)$

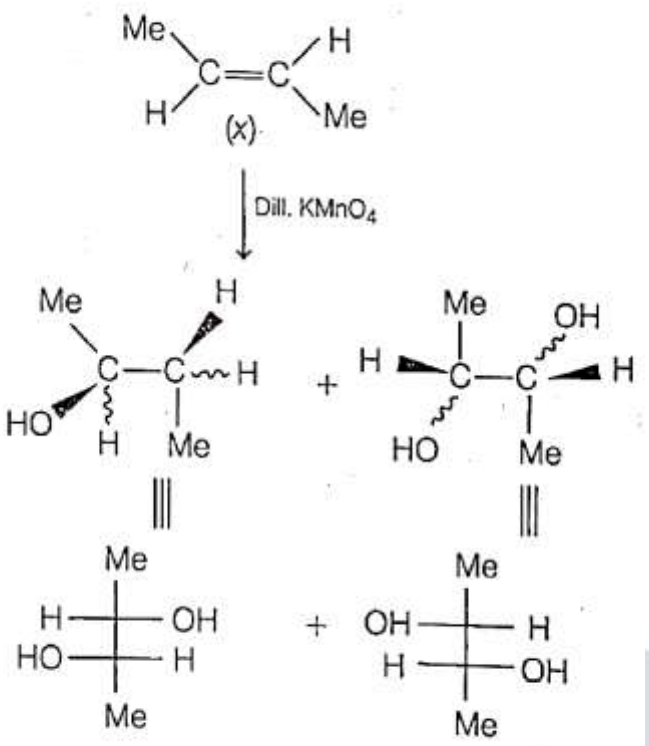
(c) $SiO_2(s) +$ (hot) $2NaOH(aq) \rightarrow Na_2SiO_3(s) + H_2O(l)$

(d) $SiO_2(s) + F_2(g) \longrightarrow SiF_4(aq) + O_2(g)$

So, the correct options are (a) (c) and (d).

80. (a,c) Complete reaction is as follows:

$$Me - C \equiv C - Me \xrightarrow[\text{EtOH, } -33^\circ C]{Na/NH_3 \text{ (liq.)}}$$



Racemic mixture

So, the correct options are (a) and (c).

Mathematics

81. (a) We have,

$$\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$$

$$\Rightarrow\ y = b\cos(n\log x - n\log n)$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = -b\sin(n\log x - n\log n) \times \frac{n}{x}$$

$$\Rightarrow\ x\frac{dy}{dx} = -nb\sin(n\log x - n\log n)$$

Again differentiating both sides w.r.t. x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{n^2 b\cos(n\log x - n\log n)}{x}$$

$$\Rightarrow\ x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -n^2y$$

$$\Rightarrow\ x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + n^2y = 0$$

82. (a) We have,

$\phi(x) = f(x) + f(1-x)$ and $f''(x) < 0$ in $[0,1]$

$\phi'(x) = f'(x) - f'(1-x)$

For monotonic increasing, $f'(x) - f'(1-x) \geq 0$

$\Rightarrow f'(x) \geq f'(1-x)$

Since, $f'(x)$ is decreasing.

$\therefore x \leq 1 - x \Rightarrow x \leq \frac{1}{2}$

$\Rightarrow \phi(x)$ is monotonic increasing in $\left[0,\frac{1}{2}\right]$ and monotonic decreasing in $\left[\frac{1}{2},1\right]$

83. (c) Let

$$I = \int \frac{f(x)\phi'(x) + \phi(x)f'(x)}{(f(x)\phi(x)+1)\sqrt{f(x)\phi(x)-1}}dx$$

Put $f(x)\phi(x)-1=t^2$

$\Rightarrow (f(x)\phi'(x)+\phi(x)f'(x))dx = 2tdt$

$$\therefore \quad I = \int \frac{2tdt}{(t^2+2)t}$$

$$\Rightarrow \quad I = 2\int \frac{dt}{t^2+2}$$

$$= \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)+c$$

$$\Rightarrow \quad I = \sqrt{2}\tan^{-1}\sqrt{\frac{f(x)\phi(x)-1}{2}}+c$$

84. (d) We have,

$$I = \sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}xdx + \sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}xdx$$

$$I = \sum_{n=1}^{10}\left[\int_{-2n-1}^{-2n}\sin^{27}xdx + \int_{2n}^{2n+1}\sin^{27}xdx\right]$$

Let $I_1 = \int_{-2n-1}^{-2n}\sin^{27}xdx$

put $x=-t \Rightarrow dx=-dt$

When $x=-2n-1$

$\Rightarrow t=2n+1$ and $x=-2n$

$\Rightarrow t=2n$

$\therefore I_1 = \int_{2n+1}^{2n}\sin^{27}(-t)(-dt)$

$$= \int_{2n+1}^{2n}\sin^{27}tdt$$

Hence, $I = \sum_{n=1}^{10}\left[\int_{2n+1}^{2n}\sin^{27}xdx + \int_{2n}^{2n+1}\sin^{27}xdx\right]$

$$I = \sum_{n=1}^{10}\left[-\int_{2n}^{2n+1}\sin^{27}xdx + \int_{2n}^{2n+1}\sin^{27}xdx\right] = 0$$

85. (b) Let $I = \int_0^2 [x^2]dx$

$\Rightarrow I = \int_0^1 0.dx + \int_1^{\sqrt{2}}dx + \int_{\sqrt{2}}^{\sqrt{3}}2dx + \int_{\sqrt{3}}^{2}3dx$

$\Rightarrow I = 0+\sqrt{2}-1+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}$

$\Rightarrow I = 5-\sqrt{2}-\sqrt{3}$

86. (d) Given, curve

$y^2=x^3$

On differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{2y}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(m^2,m^3)} = \frac{3(m^4)}{2m^3} = \frac{3}{2}m$$

$\therefore$ Slope of tangent at (m^2,m^3) is $\frac{3}{2}m$

Now slope of normal at (M^2,M^3) to the curve $y^2=x^3$ is

$$-\frac{dx}{dy} = \frac{-2y}{3x^2}$$

$$\Rightarrow \left(-\frac{dx}{dy}\right)_{(M^2,M^3)} = \frac{-2M^3}{3M^4} = \frac{-2}{3M}$$

Given, slope of tangent = slope of normal

$\therefore \quad \frac{3m}{2} = \frac{-2}{3M}$

$\Rightarrow \quad mM = \frac{-4}{9}$

87. (c) Given, $x^2 + y^2 = a^2$

On differentiating both sides w.r.t. x, we get

$= 2x + 2y\frac{dy}{dx} = 0$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{y^2}$$

$$Let\ I = \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$I = \int_0^a \sqrt{1 + \frac{x^2}{y^2}}\, dx$$

$$I = \int_0^a \sqrt{\frac{x^2 + y^2}{y^2}}\, dx$$

$$I = \int_0^a \frac{a}{\sqrt{a^2 - x^2}} dx \ [\because x^2 + y^2 = a^2]$$

$$I = a\left[\sin^{-1}\frac{x}{a}\right]_0^a$$

$$\Rightarrow \quad I = a\left[\sin^{-1}\frac{a}{a} - \sin^{-1} 0\right]$$

$$\Rightarrow \quad I = a[\sin^{-1} 1 - \sin^{-1} 0]$$

$$= a\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2}a$$

88. (c) We have,

$$\lim_{n\to\infty} \sum_{j=0}^{n} \frac{1}{n} f\left(\frac{j}{n}\right)$$

By standard formula, we write

$$\lim_{n\to\infty} \sum_{j=0}^{n} \frac{1}{n} f\left(\frac{j}{n}\right) = \int_0^1 f(x)dx$$

Where, $\lim_{n\to\infty} \left(\frac{j}{n}\right)$ (as $j = 0$) $= 0$

and $\lim_{n\to\infty} \left(\frac{j}{n}\right)$ (as $j = n$) $= 1$

89. (c) We have,

$$y' + yf'(x) - f(x)$$

$$\Rightarrow \quad f'(x) = 0$$

$$\Rightarrow \quad y' + yf'(x) = f(x)f'(x)$$

Which is a linear differentiable equation of the form $\frac{dy}{dx} + Py = Q$

$\therefore$ I.F $= e^{\int f'(x)dx} = e^{f(x)}$

Solution of given differential equation is

$$y \cdot e^{f(x)} = \int f(x)f'(x) \cdot e^{f(x)} dx$$

$y \cdot e^{f(x)} = \int te^t dt[\because \text{ put } f(x) = t \Rightarrow f'(x)dx = dt]$

$$\begin{aligned}
\Rightarrow \quad y \cdot e^{f(x)} &= te^t - e^t + c \\
\Rightarrow \quad y \cdot e^{f(x)} &= f(x)e^{f(x)} - e^{f(x)} + c \\
\Rightarrow \quad y &= f(x) - 1 + ce^{-f(x)} \\
\Rightarrow \quad y + 1 &= f(x) + ce^{-f(x)}
\end{aligned}$$

When $x \to \infty, f(x) = 0$ and $y = 0$

$\therefore \quad 0 + 1 = 0 + c$

$\Rightarrow \quad c = 1$

Hence, $y + 1 = f(x) + e^{-f(x)}$

90. (b) We have,

$x\sin\left(\frac{y}{x}\right) dy = \left(y\sin\left(\frac{y}{x}\right) - x\right) dx$

$\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$

put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = v - \operatorname{cosec} v$

$\Rightarrow \quad -\frac{dv}{\operatorname{cosec} v} = \frac{dx}{x}$

$\Rightarrow \quad \int -\sin v dv = \int \frac{dx}{x}$

$\Rightarrow \quad \cos v = \log xc$

$\Rightarrow \quad \cos\frac{y}{x} = \log cx \ldots\ldots (i)$

Put $x = 1, y = \frac{\pi}{2}$ in Eq. (i), we get

$\cos\frac{\pi}{2} = \log c$

$\Rightarrow \quad \log c = 0$

$\Rightarrow \quad c = 1$

$\cos\frac{y}{x} = \log x$

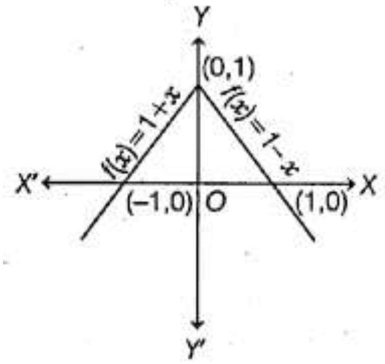
91. (c) We have,

$f(x) = 1 - \sqrt{x^2}$

$f(x) = 1 - |x|$

$f(x) = \begin{cases} 1 + x, & x < 0 \\ 1 - x, & x \geq 0 \end{cases}$

Graph of $f(x)$



Clearly, $f(x)$ has maxima at $x = 0$.

92. (a) Given,

$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$f'(x) = 6x^2 - 18ax + 12a^2$

For maxima or minima $f'(x) = 0$

$6x^2 - 18ax + 12a^2 = 0$

$x^2 - 3ax + 2a^2 = 0$

$(x - 2a)(x - a) = 0$

$x = 2a, a$

$f''(x) = 12x - 18a$

$and\ f''(2a) = 24a - 18a > 0$

$f''(a) = 12a - 18a < 0$

$\therefore f(x)$ is maximum at $x = a$

and minimum at $x = 2a$

$\therefore\ p = a, q = 2a$

Given, $p^2 = q$

$\Rightarrow\ a^2 = 2a$

$\Rightarrow\ a = 2$

93. (a) $\frac{6a}{5b} + \frac{10b}{3a}$

We know that, AM $\geq$ GM

$$\therefore \frac{\frac{6a}{5b} + \frac{10b}{3a}}{2} \geq \sqrt{\frac{6a}{5b} \times \frac{10b}{3a}}$$

$$\Rightarrow \therefore \frac{6a}{5b} + \frac{10b}{3a} \geq 2 \times 2 = 4$$

Hence, least possible value of $\frac{6a}{5b} + \frac{10b}{3a}$ is 4 .

94. (a) We have,

$2\log(x + 1) - \log(x^2 - 1) = \log 2$

$$\Rightarrow\ \log\frac{(x + 1)^2}{x^2 - 1} = \log 2$$

$\Rightarrow\ (x + 1)^2 = 2(x^2 - 1)$

$\Rightarrow\ x^2 + 2x + 1 = 2x^2 - 2$

$\Rightarrow\ x^2 - 2x - 3 = 0$

$\Rightarrow\ (x - 3)(x + 1) = 0$

$\Rightarrow\ x = 3, x \neq -1$

95. (c) Let complex number

$$P = \cos\theta + i\sin\theta$$

$\therefore\quad |P| =$ land $\theta \in [0,2\pi)$

Now, $P^4 = (\cos\theta + i\sin\theta)^4$

$\Rightarrow\ P^4 = \cos 4\theta + i\sin 4\theta$

Imaginary part of $P^4 = 0$

$\therefore\quad \sin 4\theta = 0$

$$\Rightarrow\quad 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$$

$$0 \leq \theta \leq 2\pi$$

$$\therefore\quad 0 \leq \frac{n\pi}{4} < 2\pi$$

$$= 0 \leq x < 8$$

$\Rightarrow\quad n = 8$

Hence, 8 such complex number.

96. (b) Given,

$z\bar{z} + (2 - 3i)z + (2 + 3i)\bar{z} + 4 = 0$

This is the form of

$z\bar{z} + \bar{a}z + a\bar{z} + b$

Here, $a = 2 + 3i, b = 4$

$\therefore$ Radius of circle $= \sqrt{|a|^2 - b}$

$= \sqrt{(4+9) - 4}$

$= \sqrt{13 - 4} = 3$

97. **(c)** The expression $ax^2 + bx + c$ has the same sign as that of a for all x if $D \le 0$

$\therefore\ b^2 - 4ac \le 0$

98. **(d)**

In 12 storied building 3 person leave the lift any 10 floors.

$\therefore$ Total number of ways

$$= {}^{10}P_3 = \frac{10!}{7!} = 10 \times 9 \times 8 = 720$$

99. **(b)** ${}^{n}C_m = k \cdot {}^{n-1}C_{m-1}$

$$\Rightarrow \quad \frac{n!}{m!\,(n-m)!} = k \cdot \frac{(n-1)!}{(m-1)!\,(n-m)!}$$

$$\Rightarrow \quad \frac{n(n-1)!}{m(m-1)!} = \frac{k \cdot (n-1)!}{(m-1)!}$$

$$\Rightarrow \quad n = mk$$

100. **(a)** Since, AM $\ge$ GM

$$\therefore \quad \frac{1+3+5+7+\cdots+(2n-1)}{n} > (J(n))^{\frac{1}{n}}$$

$$\Rightarrow \quad \frac{n^2}{n} > (J(n))^{\frac{1}{n}}$$

$$\Rightarrow \quad n^n > J(n)$$

$$\Rightarrow \quad I(n) > J(n)$$

101. **(b)** $(1+x)^{15} = c_0 + c_1x + c_2x^2 + \cdots + c_{15}x^{15}$

Now, $\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \cdots + 15\frac{c_{15}}{c_{14}}$

$$\frac{15+1-1}{1} + \frac{2(15+1-2)}{2} + \frac{3(15+1-3)}{3}$$

$$+\cdots+ \frac{15(15+1-15)}{15}$$

$$\therefore \quad \left[\because \frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \frac{n+1-r}{r} \text{ Here, } n = 15\right]$$

$$15 + 14 + 13 + \cdots + 1$$

$$\frac{15\ X\ 16}{2} = 120$$

102. **(d)** Given, $A = \begin{bmatrix} 3-t & 1 & 0 \\ -1 & 3-t & 1 \\ 0 & -1 & 0 \end{bmatrix}$

$$\Rightarrow \quad \begin{aligned} |A| &= (3-t)(0+1) - 1(0) + 0 \\ |A| &= 3 - t \end{aligned}$$

Given, $|A| = 5$

$\therefore\ 3 - t = 5$

$\Rightarrow\ t = -2$

103. **(c)** Given, $A = \begin{bmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{bmatrix}$

Since, A is not invertible.

$\therefore |A| = 0$

$|A| = 12(18+4) - 24(3x+2) + 5(-2x+6) = 0$

$\Rightarrow 264 - 72x - 48 - 10x + 30 = 0$

$\Rightarrow 82x = 246 \Rightarrow x = 3$

104. **(a)** Given, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $|A| = 1$ and $|A - \lambda I| = 0$ has imaginary roots.

$|A| = ad - bc = 1$

$A - \lambda I = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$

$|A - \lambda I| = (a-\lambda)(d-\lambda) - bc$

$|A - \lambda I| = ad - (a+d)\lambda + \lambda^2 - bc$

$= \lambda^2 - (a+d)\lambda + 1 (\because ad - bc = 1)$

$|A - \lambda I|$ has imaginary roots.

$\therefore\ (a+d)^2 - 4 < 0 \Rightarrow (a+d)^2 < 4$

105. **(d)** Let $A = \begin{vmatrix} a^2 & bc & c^2+ab \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix}$

Taking common a, b, c from C_1, C_2 and C_3 respectively.

$\therefore\ A = abc \begin{vmatrix} a & c & c+a \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

Apply $R_1 \to R_1 + R_2 + R_3$

$A = abc \begin{vmatrix} 2(a+b) & 2(b+c) & 2(c+a) \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

$A = 2abc \begin{vmatrix} a+b & b+c & c+a \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

Apply $R_1 \to R_1 - R_2$

$A = 2abc \begin{vmatrix} 0 & c & c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

$A = 2abc^2 \begin{vmatrix} 0 & 1 & 1 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

Apply $C_2 \to C_2 - C_3$

$A = 2abc^2 \begin{vmatrix} 0 & 0 & 1 \\ a+b & b-a & a \\ b & b & c \end{vmatrix}$

$A = 2abc^2|(a+b)b - (b-a)b|$

$= 2abc^2|ab + b^2 - b^2 + ab|$

$A = 4a^2b^2c^2$

$k = 4$

106. **(d)** We have,

$f : S \to R$

$f\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = ad - bc$

$f\left[\begin{pmatrix} 4 & 3 \\ 2 & 3 \end{pmatrix}\right] = 12 - 6 = 6$

$f\left[\begin{pmatrix} 2 & 6 \\ 1 & 6 \end{pmatrix}\right] = 12 - 6 = 6$

$\therefore$ Hence, f is not one-one.

Since, $O \in R$ but S does not contain any singul matrix.

$\therefore\ f$ is not onto.

Hence, f is neither one-one nor onto.

107. **(b)** Given,

$a\rho b = a - b$ is zero or irrational.

For reflexive,

$(a, a) = a - a = 0$. Hence, ρ is reflexive.

For symmetric,

$(a, b) \in \rho \Rightarrow (b, a) \in \rho$

$\Rightarrow a - b$ is zero or irrational then $b - a$ is zero or irrational.

$\therefore\ \rho$ is symmetric.

For transitive,
If $a=3, b=\sqrt{3}$ and $c=5$
$(a, b) \in \rho,(b, c) \in \rho \Rightarrow(a, c) \notin \rho$
$\therefore \rho$ is not transitive.

108. **(b)** Let the unit vector in ZOX plane be
$\mathbf{a}=x\hat{\mathbf{i}}+z\hat{\mathbf{k}},|\mathbf{a}|=1$
$\mathbf{a} \cdot \alpha=|\mathbf{a}||\alpha| \cos 45^{\circ}$

$$\Rightarrow (x\hat{\mathbf{i}}+z\hat{\mathbf{k}}) \cdot(2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})=1 \times 3 \times \frac{1}{\sqrt{2}}$$

$[\because \alpha=2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}]$

$$2x-z=\frac{3}{\sqrt{2}}$$

and $\mathbf{a} \cdot \boldsymbol{\beta}=|\mathbf{a}||\boldsymbol{\beta}| \cos 60^{\circ}$

$$\Rightarrow(x\hat{\mathbf{i}}+z\hat{\mathbf{k}}) \cdot(\hat{\mathbf{j}}-\hat{\mathbf{k}})=1 \times \sqrt{2} \times \frac{1}{2}\ [\because \beta=\hat{\mathbf{j}}-\hat{\mathbf{k}}]$$

$$-z=\frac{1}{\sqrt{2}}$$

$$z=-\frac{1}{\sqrt{2}} \text{ and } x=\frac{1}{\sqrt{2}}$$

$$\therefore \quad a=\frac{1}{\sqrt{2}}\hat{\mathbf{i}}-\frac{1}{\sqrt{2}}\hat{\mathbf{k}}$$

109. **(d)** Four person A, B, C, D throw an unbiased die, turn by turn, in succession till one gets an even number and win the game.
Probability of an even number $=\frac{1}{2}$.

$$\therefore \quad P(A)=P(B)=P(C)=P(D)=\frac{1}{2}$$

$$P(\bar{A})=P(\bar{B})=P(\bar{C})=P(\bar{D})=\frac{1}{2}$$

Required probability
$=P(A)+P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})P(A)$
$+P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})P(A)+\cdots$

$$=\frac{1}{2}+\left(\frac{1}{2}\right)^{5}+\left(\frac{1}{2}\right)^{9}+\cdots$$

$$=\frac{1/2}{1-(1/2)^{4}}=\frac{1/2}{15/16}=\frac{8}{15}$$

110. **(b)** Given, $P=\frac{10}{100}=\frac{1}{10}$

$$q=1-P=1-\frac{1}{10}=\frac{9}{10}$$

$$P(X \geq 1) \geq \frac{50}{100}=\frac{1}{2}$$

$$P(X \geq 1)=1-P(X=0)$$

$$P(X \geq 1)=1-{}^{n}C_{0}\left(\frac{1}{10}\right)^{0}\left(\frac{9}{10}\right)^{n}$$

$$1-\left(\frac{9}{10}\right)^{n} \geq \frac{1}{2}$$

$$\left(\frac{9}{10}\right)^{n} \leq \frac{1}{2}$$

For $n=7$ the least number of round to must so fire.

111. **(c)** By using Sandwich theorem.

112. **(b)** We have,

$y = e^x(A\cos x + B\sin x)$

$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$

$\Rightarrow \frac{dy}{dx} = y + e^x(-A\sin x + B\cos x)$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-A\sin x + B\cos x)$

$+e^x(-A\cos x - B\sin$

$\Rightarrow \frac{d^2y}{dx} = \frac{dy}{dx} + \left(\frac{dy}{dx} - y\right) - y$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{2dy}{dx} - 2y$

$\Rightarrow \frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0$

113. **(d)** We have,

$$r\cos\left(\theta - \frac{\pi}{3}\right) = 2$$

$r\left[\cos\theta\cos\frac{\pi}{3} + \sin\theta\sin\frac{\pi}{3}\right] = 2$

$$\frac{r\cos\theta}{2} + \frac{r\sin\theta \cdot \sqrt{3}}{2} = 2$$

$$x + \sqrt{3}y = 4 \; [\because x = r\cos\theta, y = r\sin\theta]$$

$\therefore$ It represents a straight line.

114. **(d)** Let (h, k) be the centre and r be the radius of the variable circle.

Since, variable circle touches the circle $x^2 + y^2 = a^2$

$\therefore \sqrt{h^2 + k^2} = a$......(i)

Also variable circle touches the circle $x^2 + y^2 = 4ax$

$\therefore \sqrt{(h-2a)^2 + k^2} = 2a + r$........(ii)

From Eqs. (i) and (ii), we get

$\sqrt{(h-2a)^2 + k^2} - \sqrt{h^2 + k^2} = a$

$\therefore$ Locus of centre of variable circle is

$\sqrt{(x-2a)^2 + y^2} - \sqrt{x^2 + y^2} = a$

which represents the hyperbola.

115. **(b)** We have, $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ have common line.

$y = x$ satisfies both lines.

$\therefore x^2 + 2x^2 + ax^2 = 0$

$3x^2 + ax^2 = 0 \Rightarrow a = -3$

Equation of lines are

$x^2 + 2xy - 3y^2 = 0$ and $-3x^2 + 2xy + y^2 = 0$

$(x + 3y)(x - y) = 0$ and $(3x + y)(x - y) = 0$

The other lines are $x + 3y = 0$ and $3x + y = 0$.

116. **(b)** Given lines,

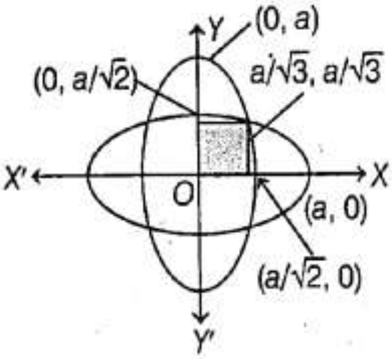
$4x + 2y = 9$ and $2x + y + 6 = 0$

Lines having distance from origin are $\frac{9}{2\sqrt{5}}$ and $\frac{6}{\sqrt{5}}$ respectively.

The point O divide the segment PQ in the ratio $= \frac{9}{2\sqrt{5}} \div \frac{6}{\sqrt{5}} = 3:4$

117. **(a)** Given curve

$x^2 + 2y^2 = a^2$ and $2x^2 + y^2 = a^2$



$\therefore$ Required area $= \left(\frac{a^2}{6} + \int_{a/\sqrt{3}}^{a/\sqrt{2}} \sqrt{a^2 - 2x^2}dx\right) \times 2$

$= \frac{a^2}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

118. **(a)** Given, the radius of circle $= \sqrt{17}$

With centre on positive sides of X-axis

$\therefore$ Equation of circle is $(x-a)^2 + y^2 = 17$

Since, it passes through $(0,1)$

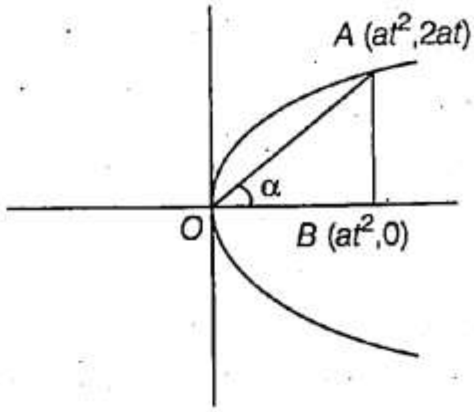
$\therefore \quad a^2 + 1 = 17$

$\Rightarrow \quad a = 4$

$\therefore$ Equation of circle is

$x^2 + y^2 - 8x - 1 = 0$

119. **(b)** Given, parabola $y^2 = 4ax$



Let the point A on parabola $(at^2, 2at)$ in $\triangle OAB$

$\cot\alpha = \frac{at^2}{2at} = \frac{t}{2}$

$t = 2\cot\alpha$

In $\triangle OAB$

$\sin\alpha = \frac{AB}{OA}$

$\Rightarrow \quad OA = AB\text{cosec}\alpha = 2at\text{cosec}\alpha.$

$\therefore \quad OA = 2a(2\cot\alpha)\text{cosec}\alpha$

$OA = 4a\cot\alpha\text{cosec}\alpha$

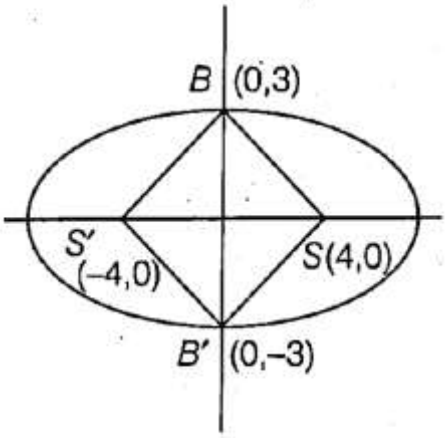
$\therefore$ Length of chord $= 4a\cot\alpha\text{cosec}\alpha$

120. **(d)** A double PQ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Such that $\triangle OPQ$ is an equilateral where O is origin. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\theta, -b\tan\theta)$ $\triangle OPQ$ is an equilateral

$\therefore OP = PQ = OQ$
$a^2\sec^2\theta + b^2\tan^2\theta = 4b^2\tan^2\theta$
$a^2\sec^2\theta = 3b^2\tan^2\theta$
$\frac{b^2}{a^2} = \frac{\sec^2\theta}{3\tan^2\theta} = \frac{1}{3}\text{cosec}^2\theta$
$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{\text{cosec}^2\theta}{3}} = \frac{1}{\sqrt{3}}\sqrt{3+\text{cosec}^2\theta}$
$e > \frac{2}{\sqrt{3}}$

121. **(c)** Equation of ellipse
$\frac{x^2}{25} + \frac{y^2}{9} = 1$



Here, $B = (0,3), B' = (0,-3)$
$S = (4,0), S' = (0,-4)$
Area of rhombus $SBS'B' = \frac{1}{2}SS' \times BB'$
$= \frac{1}{2} \times 8 \times 6 = 24$ sq units

122. **(d)** Given, equation of latusrectum of parabola is
$x + y = 8$
and equation of tangent at vertex is $x + y = 12$ Since both are parallel.
$\therefore$ The distance between latusrectum and equation of tangent at vertex is ' a '
$\therefore\ a = \frac{4}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$
$\therefore$ Length of latusrectum $= 4a = 8\sqrt{2}$ unit

123. **(a)**
Given equation of lines are $\frac{x-1}{3} = \frac{y+2}{2} = \frac{z}{-4}$. and $\frac{x}{2} = \frac{y-1}{-3} = \frac{z-2}{2}$
Equation of plane is
$a(x-2) + b(y+1) + c(z+3) = 0 \ldots$ (i)
Now, given lines are parallel to it.
$\therefore 3a + 2b - 4c = 0$
and $2a - 3b + 2c = 0$
Elimination of a, b and c gives
$\begin{vmatrix} x-2 & y+1 & z+3 \\ 3 & 2 & -4 \\ 2 & -3 & 2 \end{vmatrix} = 0 \Rightarrow [(x-2)(4-12) - (y+1)(6+8) +$
$(z+3)(-9-4)] = 0$
$\Rightarrow -8x + 16 - 14y - 14 - 13z - 39 = 0$
$\Rightarrow 8x + 14y + 13z = -37$
$\Rightarrow 8x + 14y + 13z + 37 = 0$

124. **(b)** Given line,

$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$
and plane $2x-2y+z=5$
Angle between line and plane is

$\sin\theta=\frac{(3)(2)+4(-2)+5(1)}{\sqrt{3^2+4^2+5^2}\sqrt{2^2+(-2)^2+(1)^2}}$

$\sin\theta=\frac{6-8+5}{\sqrt{50}\times 3}$

$\sin\theta=\frac{1}{5\sqrt{2}}=\frac{\sqrt{2}}{10}$

125. **(c)** Given, $f(x)=\sin x+\cos ax$
$\sin x$ is periodic with 2π.
$f(x)$ is periodic only on $\sin x$ and $\cos x$ both are periodic.
$\therefore$ $\cos ax$ is periodic at $\frac{2\pi}{a}$.
LCM of 2π and $\frac{2\pi}{a}$ is possible only a is rational number.

126. **(c)** Given, $f(x)=\sqrt{\left(\frac{1}{\sqrt{x}}-\sqrt{x+1}\right)}$

$f(x)$ is defined $\frac{1}{\sqrt{x}}-\sqrt{x+1}\geq 0$ and $x>0$
$x^2+x-1\leq 0, x>0$

$x=\frac{-1\pm\sqrt{5}}{2}, x>0$

$\therefore$ Domain of $f(x)$ is $\left(0,\frac{\sqrt{5}-1}{2}\right]$.

127. **(c)** Given, $y=2x^2-3x+2$

$\frac{dy}{dx}=4x-3$

x changes from 2 to 1.99
$\therefore$ $\Delta x=1.99-2=-0.01$
The differential of y when x charges 2 to 1.99 is
$dy\ =(4x-3)\Delta x$
$dy\ =(8-3)[-(0.01)]$
$dy\ =-0.05$

128. **(c)** We have,

$$\lim_{x\to 0}\left(\frac{1+cx}{1-cx}\right)^{\frac{1}{x}}=4$$

$\Rightarrow$ $e^{\lim_{x\to 0}\left(\frac{1+cx}{1-cx}-1\right)\times\frac{1}{x}=4}$

$\Rightarrow$ $e^{\lim_{x\to 0}\left(\frac{2cx}{1-cx}\right)\times\frac{1}{x}=4}$

$\Rightarrow$ $e^{2c}=4$

Now, $\lim_{x\to 0}\left(\frac{1+2cx}{1-2cx}\right)^{\frac{1}{x}}=e^{4c}=(e^{2c})^2=(4)^2=16$

129. **(a)** Given,
f is twice continuously and differentiable and $f(0)=f(1)=f'(0)=0$
$f(x)=Kx^2(x-1)$
$f(x)=K(x^3-x^2)$
$\therefore$ $f'(x)=K(3x^2-2x)$

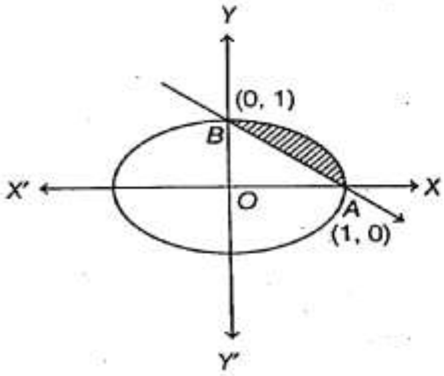
$f'(x)=0\Rightarrow x=$ oand $\frac{2}{3}$

Hence, Rolle's theorem is applicable in $f'(x)$.
$\therefore f''(c)=\mathbf{0}$ for some $c\in R$

130. **(b)** We have,

$f(x)=x^{13}+x^{11}+x^9+x^7+x^5+x^3+x+12$
$f'(x)=13x^{12}+11x^{10}+9x^8+7x^6+5x^4+3x^2+1$
$f'(x)>0$ for all value of $x\in R$.
Hence, $f(x)$ has exactly are real roots.

131. **(c)** Area of region
$\{(x,y)=x^2+y^2\leq 1<x+y\}$
$\therefore\ x^2+y^2=1$ and $x+y=1$



Area of shaded region = Area of quadrant - Area of triangle

$$=\frac{\pi}{4}(1)^2-\frac{1}{2}\times 1\times 1$$
$$=\frac{\pi}{4}-\frac{1}{2}$$

132. **(b)** Let $f(x)=\cos x+x\sin x-1$
$f'(x)=-\sin x+x\cos x+\sin x$
$f'(x)=x\cos x>0; x\in\left(0,\frac{\pi}{2}\right)$
$f(x)$ is increasing on $x\in\left(0,\frac{\pi}{2}\right)$.
$\therefore\ \cos x+x\sin x-1>0$
$\Rightarrow\ \cos x+x\sin x>1$

133. **(c)** Given parabola
put$y=ax^2+bx+C$
$x=1, y=1$, we get
$1=a+b+c$(i)
$y=x$ is tangent of the parabola at $(1,1)$.
$\therefore\ \left(\frac{dy}{dx}\right)_{(1,1)}=2ax+b$ ....(ii)
$\Rightarrow\ 1=2a+b$
Also curve passes through $(-1,0)$
$\therefore\ 0=a-b+c$....(iii)
From Eqs. (i), (ii) and (iii), we get
and

$$a=c=\frac{1}{4}$$
$$b=\frac{1}{2}$$

134. **(c)** Given $\alpha=\hat{\mathbf{i}}+a\hat{\mathbf{j}}+a^2\hat{\mathbf{k}}, \beta=\hat{\mathbf{i}}+b\hat{\mathbf{j}}+b^2\hat{\mathbf{k}}$ and
$\gamma=\hat{\mathbf{i}}+c\hat{\mathbf{j}}+c^2\hat{\mathbf{k}}$

$$\therefore\ \begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\neq 0$$

Now, $\begin{vmatrix}a & a^2 & 1+a^3\\ b & b^2 & 1+b^3\\ c & c^2 & 1+c^3\end{vmatrix}=0$

$$\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} (1 + abc) = 0$$

$$\Rightarrow abc + 1 = 0 \left[\because \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \right]$$

$\Rightarrow abc = -1$

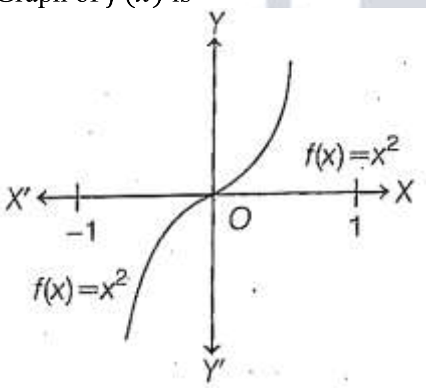
135. **(c)** Given, z_1 and z_2 be two imaginary roots of
$z^2 + pz + q = 0$
$\therefore z_1 + z_2 = -p, z_1 z_2 = q$
z_1, z_2 and origin form an equilateral triangle.
$\therefore z_1^2 + z_2^2 = z_1 z_2$
$\Rightarrow (z_1 + z_2)^2 - 2z_1 z_2 = z_1 z_2$
$\Rightarrow p^2 - 2q = q \Rightarrow p^2 = 3q$

136. **(a)** Given,
$P(x) = ax^2 + bx + c, Q(x) = -ax^2 + dx + c$
Discriminant for $P(x) = b^2 - 4ac$
and Discriminant for $Q(x) = d^2 + 4ac$
If $ac > 0, Q(x)$ has real roots and $ac < 0, P(x)$ has real roots.
$\therefore P(x) \cdot Q(x) = 0$ has atleast two real roots.

137. **(d)** We have,
$f(x) = x|x|, x \in [-1,1]$

$$f(x) = \begin{cases} -x^2, & -1 \leq x < 0 \\ x^2, & 0 \leq x < 1 \end{cases}$$

Graph of $f(x)$ is



Clearly $f(x)$ is bijective function.

138. **(d)** We have,
$f(x) = \sqrt{x^2 - 3x + 2}$
$g(x) = \sqrt{x}$

$$f \circ g = f(g(x)) = \sqrt{x - 3\sqrt{x} + 2}$$

fog is defined $x - 3\sqrt{x} + 2 \geq 0$ and $x \geq 0$
$(\sqrt{x} - 1)(\sqrt{x} - 2) \geq 0$
$x \in (-\infty, 1] \cup [4, \infty)$

$$g \circ f = g(f(x)) = \sqrt{\sqrt{x^2 - 3x + 2}}$$

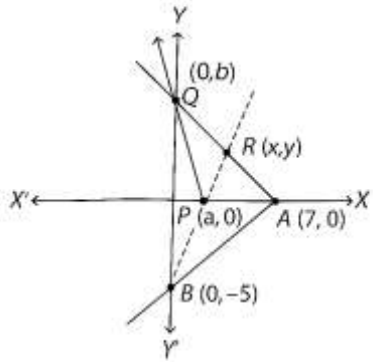
$g \circ f$ is defined $x^2 - 3x + 2 \geq 0$
$(x - 1)(x - 2) \geq 0$
$x \in (-\infty, 1] \cup [2, \infty)$

$\therefore$ Domain of $f \circ g$ is $(-\infty, 1] \cup [4, \infty)$
Domain of $g \circ f$ is $(-\infty, 1] \cup [2, \infty)$
$\therefore$ $S \cap T$ is an interval.

139. **(b)** Given, ρ_1 and ρ_2 be two equivalent relation by theorem,
$\rho_1 \cap \rho_2$ is equivalence relation but $\rho_1 \cup \rho_2$ is not so.

140. **(c)** Since, the length of tangent between the point of contact and the point where it meets the directrix subtends right angle at the corresponding focus.

141. **(c)**



Here, P is the orthocentre of $\triangle ABQ$.
$\therefore$ Slope of $BR \times$ Slope of $AR = -1$

$$\Rightarrow \left(\frac{k+5}{h}\right) \times \left(\frac{k}{h-7}\right) = -1$$

$$\Rightarrow x^2 + y^2 - 7x + 5y = 0$$

142. **(b)** Let

$$I = \lim_{n\to\infty} \sum_{k=1}^{n} \int_{(1/k+\beta)}^{1/(k+\alpha)} \frac{dx}{1+x}$$

$$I = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\log(1+x)\right]_{\frac{k+\alpha}{k+\beta}}^{1}$$

$$I = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\log\left(1+\frac{1}{k+\alpha}\right) - \log\left(1+\frac{1}{k+\beta}\right)\right]$$

$$I = \lim_{n\to\infty} \sum_{k=1}^{n} \log\left(\frac{k+\alpha+1}{k+\alpha}\right)\left(\frac{k+\beta}{k+\beta+1}\right)$$

$$I = \lim_{n\to\infty} \log\left(\frac{\alpha+2}{\alpha+1}\right)\left(\frac{\beta+1}{\beta+2}\right)\left(\frac{\alpha+3}{\alpha+2}\right)\left(\frac{\beta+2}{\beta+3}\right)$$

$$I = \lim_{n\to\infty} \left(\log\left(\frac{\beta+1}{\alpha+1}\right)\left(\frac{\alpha+n+1}{\beta+n+1}\right)\right)\left(\frac{\beta+n}{\beta+n+1}\right)$$

$$I = \log_e\left(\frac{\beta+1}{\alpha+1}\right)$$

143. **(c)** Let $P = \lim_{x\to1}\left(\frac{1}{\log x} - \frac{1}{x-1}\right)$

$$P = \lim_{x\to1}\left(\frac{x-1-\log x}{(x-1)\log x}\right)$$

Apply L-Hospital rules,

$$P = \lim_{x\to1'}\left(\frac{1-\frac{1}{x}}{\frac{x-1}{x}+\log x}\right)$$

$$= \frac{x-1}{(x-1)+x\log x}$$

Again apply L-Hospital rule,

$$P = \lim_{x\to 1} \frac{1}{1 + \frac{x}{x} + \log x}$$

$$= \lim_{x\to 1} \frac{1}{2 + \log x}$$

$$P = \frac{1}{2+0} = \frac{1}{2}$$

144. **(b)** We have,

$$y = \frac{1}{1 + x + \log x}$$

$$(1 + x + \log x)y = 1$$

On differentiating both sides, we get

$$(1 + x + \log x)\frac{dy}{dx} + \left(1 + \frac{1}{x}\right)y = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x+1)y}{x(1 + x + \log x)}$$

$$\Rightarrow x\frac{dy}{dx} = -(x+1)(y^2) \left[\because y = \frac{1}{1 + x + \log x}\right]$$

$$\Rightarrow x\frac{dy}{dx} = -y(x+1)y$$

$$\Rightarrow x\frac{dy}{dx} = y(y\log x - 1)[\because (1+x)y = 1 - y\log x]$$

145. **(b)** Given, curve $y = be^{-\frac{x}{a}}$

$$\Rightarrow \frac{dy}{dx} = -\frac{b}{a}e^{-\frac{x}{a}}$$

$$\therefore \left(\frac{dy}{dx}\right)(0, b) = -\frac{b}{a}$$

$\therefore$ Equation of tangent

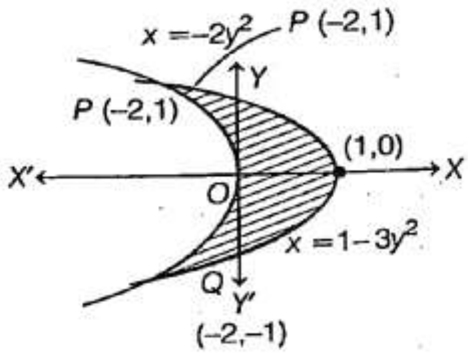
$$y - b = -\frac{b}{a}(x - 0)$$

$$\Rightarrow y - b = -\frac{bx}{a}$$

$$\Rightarrow \frac{y}{b} - 1 = \frac{-x}{a}$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

146. **(b)** Intersecting points of both curves are $P(-2,1)$ and $Q(-2,-1)$.



$\therefore$ Area of the bounded region

$$= \int_{-1}^{1} \left((1 - 3y^2) - (-2y^2)\right)dy$$

$$= 2\int_0^1 (1-y^2)dy$$

$$= 2\left(y-\frac{y^3}{3}\right)_0^1$$

$$= 2\left(1-\frac{1}{3}\right) = \frac{4}{3} \text{ sq unit}$$

147. **(a, d)** We have, $v = u - gt$
at $t = 6, v = 0$

$$= u - gt = 0$$

$\Rightarrow\ u = 6g = 192\text{ft/sec}\,.(\because g = 32\text{ft/sec}^2)$

Now, $h = ut - \frac{1}{2}gt^2$

$$= 192.6 - \frac{1}{2}\times 32\times 6\times 6$$

$$= 576\text{ft}$$

148. **(a, c)** Given, equation

$$x^{(\log_3 x)^2} - \frac{9}{2}\log_3(x+5) = 3\sqrt{3}$$

$$\Rightarrow\left((\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right)\log_3 x = \frac{3}{2}\log_3 3$$

put $\log_3 x = t$

$$\therefore \left(t^2 - \frac{9}{2}t + 5\right)t = \frac{3}{2}$$

$$\Rightarrow 2t^3 - 9t^2 + 10t - 3 = 0$$

$$\Rightarrow (t-3)(t-1)\cdot\left(t-\frac{1}{2}\right) = 0$$

$$\Rightarrow t = 3, 1, \frac{1}{2}$$

$$\Rightarrow \log_3 x = 3, 1, \frac{1}{2}$$

$$\Rightarrow x = 3^3, 3^1, 3^{\frac{1}{2}}$$

149. **(b)** Given, total number of questions are n and total wrong answers are 2047
Also, total wrong answer given

$$= \Sigma k \times \left(2^{n-k} - 2^{n-(k+1)}\right)$$

$\Rightarrow\quad 2^{n-1} + \cdots + 1 = 2047$

$\Rightarrow\quad 2^n = 2048 = 2^{11}$

$\Rightarrow\quad n = 11$

150. **(c, d)** Given, $P(A' \cap B') = \frac{3}{5}$

$$\Rightarrow\quad 1 - P(A\cup B) = \frac{3}{5}$$

$$\Rightarrow\quad P(A\cup B) = \frac{2}{5}$$

$$\Rightarrow P(A) + P(B) - P(A\cap B) = \frac{2}{5}$$

$\Rightarrow P(A) + P(B) - P(A)P(B) = \frac{2}{5}$.........(i)

(since A and B are independent events)

Also, given $P(A\cap B) = \frac{1}{20}$

$\Rightarrow\ P(A)P(B) = \frac{1}{20}$.........(ii)

From Eqs. (i) and (ii), we get

$$P(A) = \frac{1}{4} \text{ or } \frac{1}{5}$$

151. ($\boldsymbol{a, b}$) Equation of straight line having intercepts on the coordinate axes are a and b, are

$\frac{x}{a}+\frac{y}{b}=1$……(i)

Given, $a+b=-1 \Rightarrow b=-(a+1)$

Line (i) passing through the point (4,3).

$$\therefore \quad \frac{4}{a}+\frac{3}{b}=1$$

$$\Rightarrow \quad \frac{4}{a}-\frac{3}{a+1}=1$$

$$\Rightarrow \quad 4(a+1)-3a=a(a+1)$$

$$\Rightarrow \quad a^2+a-a=4$$

$$\Rightarrow \quad a=\pm 2$$

When $a=2$, then $b=-3$ and when $a=-2$, then $b=1$

$\therefore$ Equation of straight line are $\frac{x}{2}-\frac{y}{3}=1, \frac{x}{-2}+\frac{y}{1}=1$

152. **(d)** Equation of tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$ at point (x_1, y_1) is

$\frac{xx_1}{2}+\frac{yy_1}{1}=1$……(i)

Let mid-point of intercept be $P(h,k)$.

$$\therefore \quad h=\frac{1}{x_1}$$

$$\Rightarrow \quad x_1=\frac{1}{h}$$

$$\text{and} \quad k=\frac{1}{2y_1}$$

$$\Rightarrow \quad y_1=\frac{1}{2k}$$

$\therefore$ Required locus of the mid-point

$$=\frac{1}{2x^2}+\frac{1}{4y^2}=1$$

153. **(a)** Given, $y=\frac{x^2}{(x+1)^2(x-2)}$

By using partial fraction,

$$\frac{x^2}{(x+1)^2(x+2)}=\frac{4}{(x+2)}-\frac{3}{x+1}+\frac{1}{(x+1)^2}$$

$$\therefore \; y=\frac{4}{x+2}-\frac{3}{x+1}+\frac{1}{(x+1)^2}$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{-4}{(x+2)^2}+\frac{3}{(x+1)^2}-\frac{2}{(x+1)^3}$$

Again differentiating w.r.t. x

$$\frac{d^2y}{dx^2}=\frac{8}{(x+2)^3}-\frac{6}{(x+1)^3}+\frac{6}{(x+1)^4}$$

$$=2\left[\frac{4}{(x+2)^3}-\frac{3}{(x+1)^3}+\frac{3}{(x+1)^4}\right]$$

154. **(c)** Given, $f(x)=\frac{1}{3}x\sin x-(1-\cos x)$

$\therefore \lim\limits_{x\to 0}\frac{f(x)}{x^k}=\lim\limits_{x\to 0}\frac{x\sin x-3(1-\cos x)}{3x^k}$

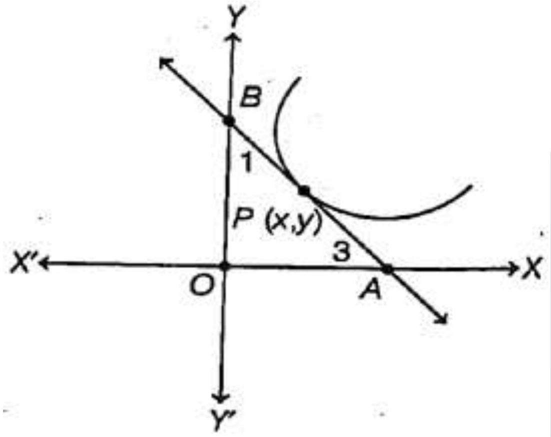
$=\lim\limits_{x\to 0}\frac{2x\sin\frac{x}{2}\cos\frac{x}{2}-6\sin^2\frac{x}{2}}{3x^k}$

$=\frac{1}{3}\lim\limits_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)\cdot\lim\limits_{x\to 0}\left(\frac{2x\cos\frac{x}{2}-6\sin\frac{x}{2}}{2x^{k-1}}\right)$

$=\frac{1}{3}\lim\limits_{x\to 0}\left(\frac{x\cos\frac{x}{2}-3\sin\frac{x}{2}}{x^{k-1}}\right)$

For, $\lim\limits_{x\to 0}\frac{f(x)}{x^k}\neq 0\Rightarrow k-1=1\Rightarrow k=2$

155. **(a, c)** Given, $AP:BP=3:1$



Equation of tangent AB is $Y-y=\frac{dy}{dx}(X-x)$

$\therefore$ Coordinates of A are $\left(\frac{x\frac{dy}{dx}-y}{y'},0\right)$.

and coordinates of B are $\left(0,y-x\frac{dy}{dx}\right)$.

Now, $y=\frac{1\times 0+3\times\left(y-x\frac{dy}{dx}\right)}{4}$

$\Rightarrow 4y=3y-3x\frac{dy}{dx}$

$\Rightarrow 3x\frac{dy}{dx}+y=0$

$\Rightarrow \frac{3dy}{y}+\frac{dx}{x}=0$

$\Rightarrow 3\log y+\log x=c$

$\Rightarrow xy^3=c$

at $x=1,y=1$, we get $c=1$

$\therefore xy^3=1$

$\Rightarrow x(3y^2)\frac{dy}{dx}+y^3=0$

$\Rightarrow 3x\frac{dy}{dx}+y=0$

$\Rightarrow \frac{dy}{dx}=-\frac{y}{3x}$

$\therefore \left(\frac{dy}{dx}\right)_{(1,1)}=-\frac{1}{3}$

$\therefore$ Slope of normal $=3$

$\therefore$ Equation of normal $=y-1=3(x-1)$

$\Rightarrow y-3x+2=0.$