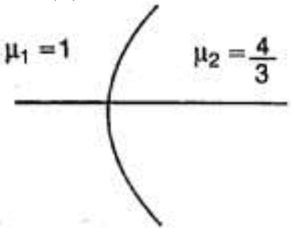
Physics

1. **(d)** The situation is as shown in the figure below



Given,
$P = 5\text{D}$
$\therefore\ f = \frac{1}{P} = \frac{1}{5}\ \text{m} = 20\ \text{cm}$
Let $u = -\infty$ (object distance)
Then, $v = f$ (by sign convention)
Using formula for refraction at spherical surface,
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$
$$\frac{4}{3 \times (20)} - \frac{1}{(-\infty)} = \frac{\frac{4}{3} - 1}{R}\ [\because v = 20\ \text{cm}]$$
$$\Rightarrow \frac{1}{15} = \frac{1}{3R}$$
$$\Rightarrow R = 5\ \text{cm}$$

2. **(c)** Given, $D = 1.2\ \text{m}, d = 0.02\ \text{cm} = 0.02 \times 10^{-2}\ \text{m}$
According to question,
$$\frac{3D\lambda}{d} - \frac{5D\lambda}{2d} = 0.18 \times 10^{-2}$$
$$\lambda\ \left[\frac{3 \times 1.2}{0.02 \times 10^{-2}} - \frac{5 \times 1.2}{2 \times 0.02 \times 10^{-2}}\right] = 0.18 \times 10^{-2}$$
$$\Rightarrow\ \lambda\left[3 - \frac{5}{2}\right] \times 1.2 = 0.18 \times 10^{-2} \times 0.02 \times 10^{-2}$$
$$\Rightarrow\ \lambda = \frac{0.18 \times 10^{-4} \times 0.02 \times 2}{1.2}\ \text{m}$$
$$\Rightarrow\ \lambda = 0.006 \times 10^{-4}\ \text{m}$$
$$\lambda = 600 \times 10^{-9}\ \text{m}$$
$$\Rightarrow\ = 600\ \text{nm}$$

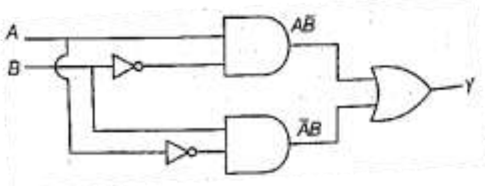
3. **(c)** Let initially the electron in hydrogen atom be in ground state, i.e. $n_1 = 1$ and after bombarding, it jumps to π th energy level i.e. $n_2 = n$, then
$$12.5\text{eV} = \left[\frac{136}{(1)^2} - \frac{136}{n^2}\right]\text{eV}$$
$$\Rightarrow\ 1 - \frac{1}{n^2} = \frac{12.5}{13.6}$$
$$\Rightarrow\ 1 - \frac{125}{13.6} = \frac{1}{n^2}$$
$$\Rightarrow\ \frac{1}{n^2} = 0.080$$
$$\Rightarrow\ n = 3.53 \approx 4$$
So, $n_2 = n = 4$ th excited state.

4. **(a)** Radius (r) of nth orbit in H -atom $= \frac{n^2}{Z} \times 0.529\text{Å}$
Speed (v) of electron in nth orbit in H -atom
$$= 2.19 \times 10^6 \left(\frac{Z}{n}\right) \text{m/s}$$

Energy (E) of electron in nth orbit in H-atom
$=\frac{-13.6}{n^2}$ eV
So, the quantity which is proportional to quantum number n is n.

5. **(a)** The net voltage in the circuit is $(10-9)\mathrm{V}=1$ V This leads to reverse bias of diode in the circuit. Hence, current through the diode will equal to 0 mA .

6. **(c)** The given logic circuit is



Output is written as

$$Y=A\bar{B}+\bar{A}B$$

When then $Y=0, B=1,$

$$\begin{aligned}&=0.\overline{1}+\overline{0}.1\\&=0.0+1.1\\&=0+1\\&=1\end{aligned}$$

When $A=0, B=0,$
then

$$\begin{aligned}Y&=0\cdot\overline{0}+\overline{0}\cdot 0\\&=0\cdot 1+1\cdot 0\\&=0+0=0\end{aligned}$$

7. **(d)**
According to question, from dimensional analysis,
$R \propto e^a m^b h^c$
$\Rightarrow \mathrm{R}=\mathrm{ke}^{\mathrm{a}}\ \mathrm{m}^{\mathrm{b}}\ \mathrm{h}^{\mathrm{c}}$
Using dimensional formula of R, e, m and h ,
$[M^0L^{-1}T^0]=\left[M^{1/2}L^{3/2}T^{-1}\right]^a[M]^b[ML^2T^{-1}]^c$

$\left\{\text{Since, } F=\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2}=\frac{e^2}{r^2}\right.$

$\Rightarrow e^2=Fr^2$

$[e]=[Fr^2]^{1/2}=[MLT^{-2}\cdot L^2]^{1/2}=\left[M^{1/2}L^{3/2}T^{-1}\right]\}$

$\Rightarrow [M^0L^{-1}T^0]=\left[M^{a/2}L^{3a/2}T^{-a}\right][M^b][M^cL^{2c}T^{-c}]$

$\Rightarrow [M^0L^{-1}T^0]=\left[M^{\frac{a}{2}+b+c}L^{\frac{3a}{2}+2c}T^{-a-c}\right]$

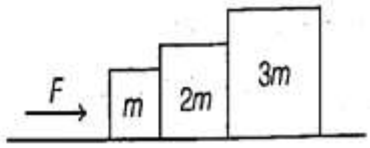
$\therefore \frac{a}{2}+b+c=0 \ldots$ (i)

$\frac{3a}{2}+2c=-1 \ldots$ (ii)

$-\mathrm{a}-\mathrm{c}=0 \ldots$ (iii)

On solving Eqs. (i), (ii) and (iii), we get
$\mathrm{a}=2, \mathrm{b}=1, \mathrm{c}=-2$
$\therefore \mathrm{R}\propto \mathrm{e}^2\mathrm{mh}^{-2}$
$\Rightarrow R\propto\frac{e^2 m}{h^2}$

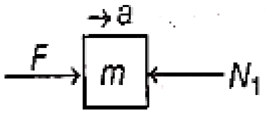
8. **(a)** Given,



Since, the table is frictionless, hence acceleration of the whole system is given as

$$a = \frac{F}{m + 2m + 3m} = \frac{F}{6m}$$

Free body diagram at 1st block (left most)

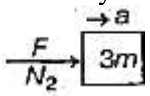


From Newton's equation of motion,

$$F - N_1 = ma = m \times \frac{F}{6m} \quad \left(\because a = \frac{F}{6m}\right)$$

$$\Rightarrow N_1 = \frac{5F}{6}$$

Free body diagram of 3rd block (right most)



From Newton's equation of motion,

$$N_2 = 3ma = 3m \times \frac{F}{6m} \left(\because a = \frac{F}{6m}\right)$$

$$\Rightarrow N_2 = \frac{F}{2}$$

Hence, $F > N_1 > N_2$.

9. **(a)** At maximum compression of spring, total kinetic energy of first block = potential energy of spring

$$\Rightarrow \frac{1}{2}mv^2 \quad = \frac{1}{2}kx^2$$

$$\Rightarrow x = \sqrt{\frac{m}{k}} \cdot v$$

10. **(c)** During first half of the motion (i.e. upto $x = 20$ m) acceleration in increasing linearly is given by

$$a = \frac{5}{20}x + 5 \Rightarrow a = \frac{x}{4} + 5$$

$$\Rightarrow \frac{dv}{dt} = \frac{x}{4} + 5 \, (\because \mathrm{a} = \frac{dv}{dt})$$

$$\Rightarrow \frac{v \cdot dv}{dx} = \frac{x}{4} + 5 \left(\because \frac{dx}{dt} = v \Rightarrow dt = \frac{dx}{v}\right)$$

$$\Rightarrow \int_5^v v.dv = \int_0^{20} \left(\frac{x}{4} + 5\right) \cdot dx$$

$$\Rightarrow \left[\frac{v^2}{2}\right]_5^v = \left[\frac{x^2}{8} + 5x\right]_0^2$$

$$\Rightarrow \frac{v^2}{5} - \frac{25}{2} = \frac{400}{8} + 100$$

$$\Rightarrow \frac{v^2}{2} = 150 + \frac{25}{2} \Rightarrow v^2 = 325$$

Now, in second half of motion, acceleration is constant, i.e. $a = 10 \text{ m/s}^2$

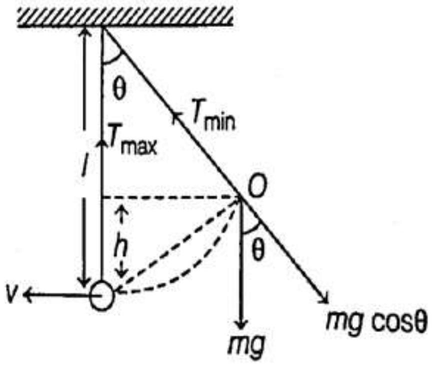
$$\Rightarrow \quad v^2 - u^2 = 2as$$

$$\Rightarrow \quad v'^2 - v^2 = 2as \qquad (\text{take, } v = v', u = v)$$

$$\Rightarrow \quad v'^2 - 325 = 2 \times 10 \times (35 - 20) \quad (\because v^2 = 325)$$

$$\Rightarrow \quad v' = \sqrt{625} = 25 \text{ m/s}$$

11. (a) The given situation is shown below



$T_{min} = mg\cos\theta$.....(i)

$T_{\max} - mg = \frac{mv^2}{l}$......(ii)

By law of conservation of energy,

$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow mg(l - l\cos\theta) = \frac{1}{2}mv^2 \ [\because h = l - l\cos\theta]$$

$$2mg(l - l\cos\theta) = mv^2$$

$$\Rightarrow 2mgl(1 - \cos\theta) = mv^2 \text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$T_{\max} - mg = \frac{2mgl(1-\cos\theta)}{l}$$

$$\Rightarrow T_{\max} - mg = 2mg(1 - \cos\theta)$$

$$\Rightarrow 2T_{\min} - mg = 2mg(1 - \cos\theta) \ [\because T_{\max} = 2T_{\min}]$$

$$\Rightarrow 2mg\cos\theta - mg = 2mg(1 - \cos\theta) \ \text{[from Eq. (i)]}$$

$$\Rightarrow 2\cos\theta - 1 = 2(1 - \cos\theta)$$

$$\Rightarrow 2\cos\theta - 1 = 2 - 2\cos\theta$$

$$\Rightarrow 4\cos\theta = 3$$

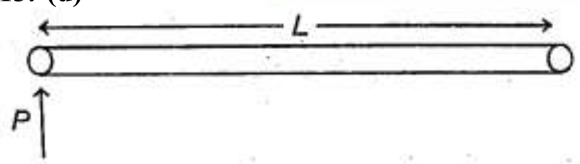
$$\Rightarrow \cos\theta = \frac{3}{4}$$

$$\therefore T_{\max} = 2T_{\min} = 2mg\cos\theta = 2mg \times \frac{3}{4} = \frac{3}{2}mg$$

$$\Rightarrow T_{\max} = \frac{3}{2}mg$$

12. (b) In projectile motion, horizontal component at velocity remains constant with time. Hence, correct representation of graph between t and u_x is shown in option (b).

13. (d)



The given impulse acts as both linear and an angular impulse.

Linear impulse $= P = mv_{\text{CM}}$ (where, v_{CM} = velocity of centre at mass of rod)

Angular impulse $= P \times \frac{L}{2} = I_{\text{CM}}\omega$

where, $I_{\text{CM}} = \frac{mL^2}{12}$

and ω is angular velocity at rod about centre of mass,

i.e

$$\omega = \frac{PL}{2I_{CM}}$$

Kinetic energy

$$=\left[\frac{1}{2}mv_{\mathrm{CM}}^2+\frac{1}{2}I_{\mathrm{CM}}\omega^2\right]$$
$$=\left[\frac{1}{2}\times m\times\left(\frac{P}{m}\right)^2+\frac{1}{2}\times\frac{mL^2}{12}\times\frac{P^2L^2\times 144}{4\times m^2\times L^4}\right]$$
$$=\frac{P^2}{2m}[1+3]$$
$$=\frac{2P^2}{m}$$

14. (a) Centre of mass of three particles (i.e. 1 kg, 2 kg and 3 kg)

$$\frac{1\mathbf{r}_1+2\mathbf{r}_2+3\mathbf{r}_3}{6}=[\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}]$$
$$\left[\because CM=\frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2+m_3\mathbf{r}_3}{m_1+m_2+m_3}\right]$$

where, $\mathbf{r}_1, \mathbf{r}_2$ and $\mathbf{r}_3$ are position vector of mass 1 kg 2 kg and 3 kg , respectively.

Now, centre of mass of two particles (i.e. 3 kg and 2 kg)

$$\frac{3r_4+2r_5}{5}=[-\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}}]$$

where, $\mathbf{r}_4$ and $\mathbf{r}_5$ are position vector of mass 1 kg and 2 kg .

Now, according to question, if 5 kg mass is added to whole particle system, then entire mass lies on CM of first system,

$$\frac{1r_1+2r_2+3r_3+3r_4+2r_5+5r_6}{16}=[1\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}]$$

where, $\mathbf{r}_6$ is position vector of 5 kg mass.

$\Rightarrow 6\hat{\mathbf{i}}+12\hat{\mathbf{j}}+18\hat{\mathbf{k}}-5\hat{\mathbf{i}}+15\hat{\mathbf{j}}-10\hat{\mathbf{k}}+5\mathbf{r}_6=16\hat{\mathbf{i}}+32\hat{\mathbf{j}}+48\hat{\mathbf{k}}$

$\Rightarrow\ 5\mathbf{r}_6=15\hat{\mathbf{i}}+5\hat{\mathbf{j}}+40\hat{\mathbf{k}}$

$\Rightarrow\ \mathbf{r}_6=3\hat{\mathbf{i}}+\mathbf{j}+8\hat{\mathbf{k}}$

Hence, position of 5 kg mass is (3,1,8)

15. (b) Given, density of body $=1.2\times 10^3\ \mathrm{kg/m^3}=\rho_b$

Density of liquid $=24\times 10^3\ \mathrm{kg/m^3}=\rho_l$

Height of fall $=1\ \mathrm{m}=H$

Let the volume of the body be V.

Then, buoyant force acting on the body when it is totally immersed in liquid $=(\rho, Vg)$

Weight at the body $=(\rho_b\ \mathrm{V}\ g)$

Net upward force acting on the body $=(\rho_1-\rho_b)Fg$

Net deceleration produced

$$(a)=\frac{\text{Net upward force}}{\text{Mass of body}}$$
$$(a)=\frac{(\rho_1-\rho_b)Vg}{\rho_bV}$$
$$(a)=\frac{(\rho_1-\rho_b)}{\rho_b}g$$

Let initial velocity of the body be u, then

$$\frac{1}{2}(\rho_bV)u^2=\rho_bVgH$$
$$\Rightarrow\qquad u^2=2gH$$

Final velocity of the body will be zero.

$$v^2-u^2=2as$$
$$(0)^2-2gH=-2\frac{(\rho_1-\rho_b)g}{\rho_b}\times s$$
$$s=\frac{\rho_b}{\rho_1-\rho_b}\cdot H=\left(\frac{1.2\times 10^3}{2.4\times 10^3-1.2\times 10^3}\right)\cdot 1$$

$\frac{1.2}{1.2} = 1 \text{ m}$

16. (b) Given densities of both spheres are same. Let it be ρ unit and density of liquid be σ and η be its viscosity.

Let radius of sphere S_1 be r_1 and that of sphere S_2 be r_2 and volumes are V_1 and V_2, respectively.

Now,

$$\frac{m_1}{V_1} = \frac{m_2}{V_2}$$

$$\frac{m_1}{\frac{4}{3}\pi r_1^3} = \frac{m_2}{\frac{4}{3}\pi r_2^3}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{m_1}{m_2}\right) = \frac{8}{1}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{2}{1}$$

($\because$ density is same)

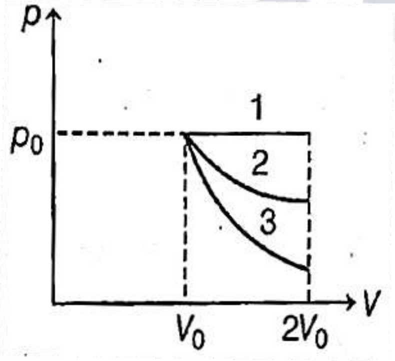
Now, terminal velocity is given by

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{\frac{2r_1^2(\rho-\sigma)g}{9\eta}}{\frac{2r_2^2(\rho-\sigma)g}{9\eta}}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 = 2^2 = 4$$

17. (d) Given figure is



Process 1 is isobaric expansion ($p =$ constant). Hence, temperature of gas will increase, therefore ΔU_1 will be positive.

Process 2 is an isothermal process, $\Delta U_2 = 0$

Process 3 is an adiabatic expansion, hence temperature of gas will fall.

$\therefore \ \Delta U_3 =$ negative

Therefore, $\Delta U_1 > \Delta U_2 > \Delta U_3$

18. (b) The van der Waals' equation for real gas is as follows

$$p + \frac{an^2}{V^2}(V - nb) = nRT$$

where, p is the pressure, a and b are constants, V is the volume, T is the temperature and n is the number of moles.

van der Waals' equation can be rearranged as

$$p = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

Dividing first part with n and the 2nd part with n^2, we get

$$p=\frac{\frac{nRT}{n}}{\frac{V}{n}-\frac{nb}{n}}-\frac{\frac{an^2}{n^2}}{\frac{V^2}{n^2}}$$

$$\Rightarrow p=\frac{RT}{\frac{V}{n}-b}-\frac{a}{\frac{V^2}{n^2}}\text{(i)}$$

The equation given in question is as follows

$p=\frac{RT}{2V-b}-\frac{a}{4b^2}$.....(ii)

By comparing Eqs. (i) and (ii), we get

$$\frac{V}{n}=2V$$

$$\Rightarrow n=\frac{V}{2V}=\frac{1}{2}$$

$\Rightarrow$ (i.e. $n=\frac{1}{2}$ mole)

By using mole formula to determine the amount of O_2 as follows

Mole $=\frac{\text{Mass}}{\text{Molar mass}}$....(iii)

The molar mass of O_2 is $32\ \mathrm{g/mol}$. Substituting in Eq. (iii), we get

$$\Rightarrow \frac{1}{2}=\frac{\text{Mass}}{32 g/\text{mol}}$$

$\Rightarrow$ Mass of $O_2=16$ g

19. **(b)** We know that, latent heat of fusion at ice is

79.7 cal per gram.

Let final temperature be T.

Then,

$m_1 s\Delta T=m_2 L$

$\Rightarrow 300\times 1\times(25-T)=100\times 75$

$=25-T=25$

$=T=0^\circ\mathrm{C}$

After that total energy left, $Q=4.7\times 100=470\mathrm{cal}$

Total mass of water $=400$ g

Amount of water again converted into ice,

$$m=\frac{Q}{L_{\text{ice}}}=\frac{470}{79.7}$$

$\Rightarrow\quad m=5.9$ g

Thus, whole mass is converted into water at 0°C and about 5.9 g water is again converted into ice whose temperature is also 0°C.

After achieving the temperature of 0°C, latent heat of fusion is required firstly for conversion of water into ice, then further lowering of temperature as possible.

So, the final temperature will be 0°C.

20. **(d)** The electric field along Z-axis due to uniformly charged circular ring of radius a in XY-plane

is given by

$$E=\frac{kqz}{(z^2+a^2)^{3/2}}$$

where, q is the net charge on the ring and z is axial distance on Z - axis from the centre of the ring. Now, at point M in the graph shown, electric field is maximum.

So,

$$\frac{dE}{dZ}=\frac{kq(z^2+a^2)^{3/2}-\frac{3}{2}(z^2+a^2)^{3/2}\cdot 2z^2}{(z^2+a^2)}$$

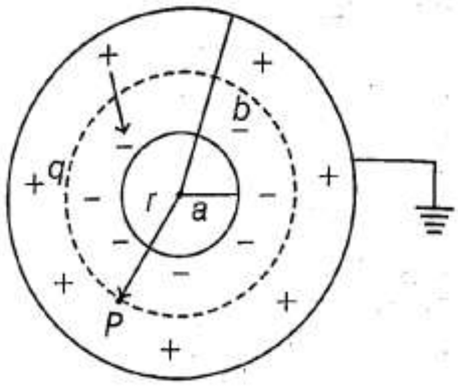
For maxima, $\frac{dE}{dZ}=0$

$\Rightarrow \quad (z^2+a^2)^{3/2} = \frac{3}{2} \times 2z^2(z^2+a^2)^{1/2}$

$\Rightarrow \quad z^2+a^2 = 3z^2 \Rightarrow 2z^2 = a^2$

$\Rightarrow \quad \frac{z}{a} = \frac{1}{\sqrt{2}}$

21. (a) The given situation is shown below



A radial electric field **E** exists in the region between the two shells due to charge on inner shell only.
Electric field at point P is calculated according to Gauss's law,
i.e

$\phi_E = E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$

$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$.....(i)

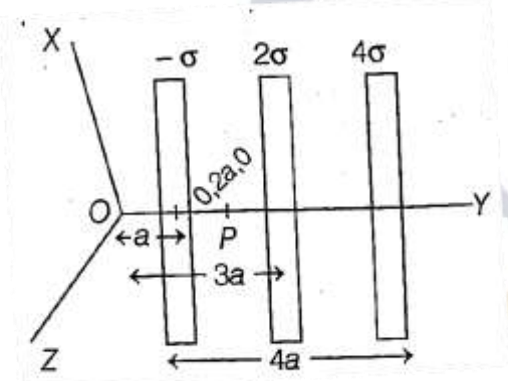
The potential at the centre of sphere,

$V = -\int_a^b \mathbf{E} \cdot d\mathbf{r} = \int_a^b E \cdot dr = \int_a^b \frac{q}{4\pi\varepsilon_0 r^2} dr$

[from Eq. (i)]

$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

22. (b) The given situation is shown in the figure

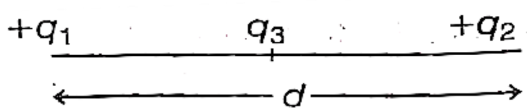


We have to calculate electric field at point $P(0,2a,0)$.
Net electric field at point P,

$E = \frac{-\sigma}{2\varepsilon_0}\hat{\mathbf{j}} - \frac{2\sigma}{2\varepsilon_0}\hat{\mathbf{j}} - \frac{4\sigma}{2\varepsilon_0}\hat{\mathbf{j}}$

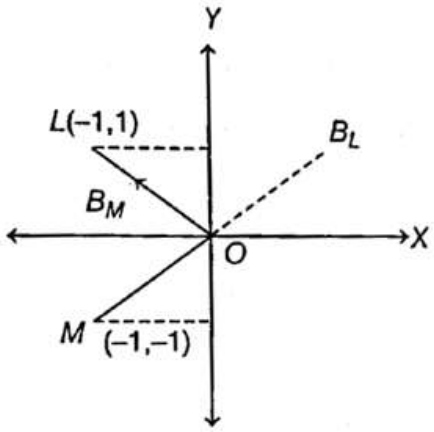
$E = \frac{-7\sigma}{2\varepsilon_0}\hat{\mathbf{j}}$

23. (c) The given situation is shown below



Case 1 If charge q_3 is taken as positives.
In this situation, force on charge q_3 due to charges q_1 and q_2 will be repulsive and in opposite direction. Hence, at a certain location between $+q_1$ and $+q_2$, charge q_3 will be in equilibrium, when both repulsive forces will be equal in magnitude.
Case 2 If charge q_3 is taken as negative.
In this situation, force on charge q_3 due to charges q_1 and q_2 will be attractive and in opposite direction. Hence, at a certain location between $+q_1$ and $+q_2$, charge q_3 will be in equilibrium, when both attractive forces will be equal in magnitude.

24. (b) The given situation is shown below



Length, $OL = \sqrt{1^2 + (-1)^2} = \sqrt{2}$
and
$OM = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$
$\because$ Magnetic field at point O due to wire L,
$$B_L = \frac{\mu_0}{2\pi} \cdot \frac{I}{\sqrt{2}} \left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) \left[\because B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}\right]$$
Magnetic field at point O due to wire M,
$$B_M = \frac{\mu_0}{2\pi} \cdot \frac{I}{\sqrt{2}} \left(\frac{-\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)$$
$\therefore$ Resultant magnetic field at the origin O,
$$B = B_L + B_M$$
$$= \frac{\mu_0 I}{2\pi\sqrt{2}} \left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) + \frac{\mu_0}{2\pi} \cdot \frac{I}{\sqrt{2}} \left(\frac{-\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)$$
$$= \frac{\mu_0 I}{2\sqrt{2}\pi} \left[\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right]$$
$$= \frac{\mu_0 I}{2\sqrt{2}} \cdot (\sqrt{2}\hat{\mathbf{j}}) = \frac{\mu_0 I}{2\pi} \hat{\mathbf{j}}$$

25. (d) Charge per unit length of the thin rod $= \lambda$
If l be the length of the rod, the charge on the rod,
$q = l\lambda \ldots$(i)
According to given situation,
$l = 2\pi R$
$\therefore$ From Eq. (i), we get
$q = 2\pi R\lambda$
Current associated,
$$I = \frac{q}{T}$$
$$I = \frac{2\pi R\lambda}{T} \text{(ii)}$$
Magnetic field on the axis at a distance d away from the centre,
$$B = \frac{\mu_0 I R^2}{2(R^2 + d^2)^{3/2}}$$

For $d \gg R$,

$$B = \frac{\mu_0 I R^2}{2(d^2)^{3/2}}$$

$$= \frac{\mu_0 I R^2}{2d^3}$$

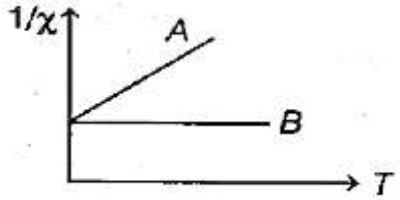
$$= \frac{\mu_0 \left(\frac{2\pi R\lambda}{T}\right) R^2}{2d^3}$$

$$= \frac{\mu_0 \pi \lambda}{T} \cdot \frac{R^3}{d^3}$$

$\Rightarrow B \propto \frac{R^3}{d^3}$ [given, $B \propto \frac{R^m}{d^n}$]

$\therefore m = 3, n = 3$,

26. (d) Since, susceptibility χ of diamagnetic material is independent of temperature.



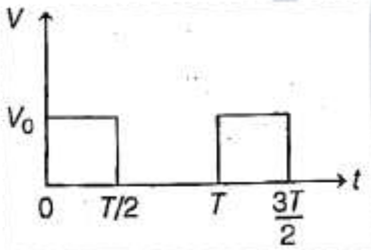
Hence, $\frac{1}{\chi}$ is also independent of temperature.

Therefore, material B is diamagnetic. For paramagnetic substance, susceptibility varies inversely as temperature.

i.e. $\chi \propto \frac{1}{T}$ or $\frac{1}{\chi} \propto T$

Therefore, material A is paramagnetic.

27. (d)

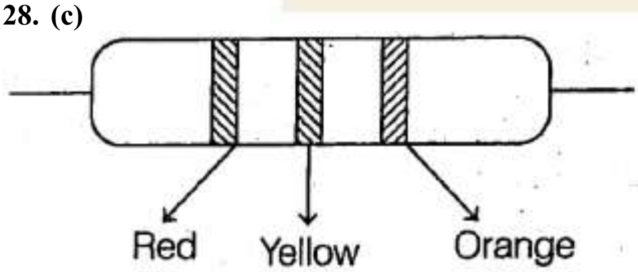


Rms value of potential difference is given as

$$V_{\text{rms}} = \sqrt{\frac{V_0^2 + 0^2}{2}}$$

$$= \frac{V_0}{\sqrt{2}}$$

28. (c)

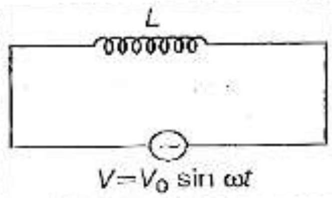


The colours of the four bands of carbon resistors are red, yellow, orange and no fourth band.

$\therefore$	Red	Yellow	Orange	No colour
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
	2	4	3	20%

$\therefore\ R = 24 \times 10^3 \Omega \pm 20\%$
$\quad = 24 k\Omega \pm 20\%$

29. (c)



In pure inductive circuit, alternating voltage leads $\frac{\pi}{2}$ angle from alternating current.

i.e. Phase difference, $\phi = \frac{\pi}{2}$

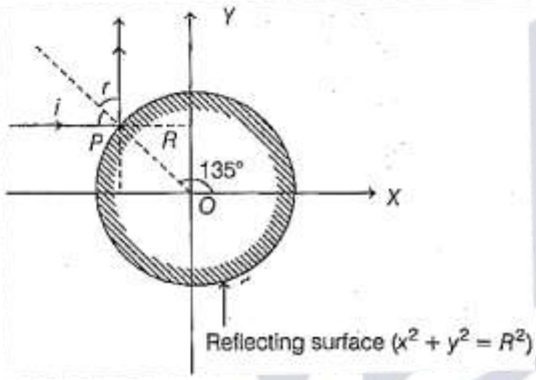
Average power consumed,

$$P = V_{\text{rms}} \times I_{\text{rms}} \times \cos\frac{\pi}{2}$$

$$\Rightarrow \quad P = 0$$

$$\left[\because \cos\frac{\pi}{2} = 0\right]$$

30. (c) The given situation is shown below



Here,
$i = r$
Since,
$i + r = 90°$
$\Rightarrow\ i + i = 90°$
$\Rightarrow\ i = 45°$
$i = r = 45°$
Coordinate at point P is
$(R\cos\theta, R\sin\theta) = (R\cos 135°, R\sin 135°)$

$$= \left[R\left(\frac{-1}{\sqrt{2}}\right), R\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$= \left(\frac{-R}{\sqrt{2}}, \frac{R}{\sqrt{2}}\right)$$

31. (b) For a plane electromagnetic wave, the electric field is given by

$\mathbf{E} = 90\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{\mathbf{k}}\,\text{V/m}$

Here,
$E_0 = 90\ \text{V/m}$

$$\therefore\ B_0 = \frac{E_0}{c}$$

$$= \frac{90}{3 \times 10^8}$$

$$= 30 \times 10^{-8}\ \text{T}$$

$$= 3 \times 10^{-7}\ \text{T}$$

Since, electric field vector and magnetic field vector, both are perpendicular to each other and also perpendicular to direction of propagation of electromagnetic wave. Hence,

$B = B_0 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{\mathbf{j}}\mathrm{T}$

$B = 3 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{\mathbf{j}}\mathrm{T}$

32. (c) Since, both wires are identical, so they have same cross-section A and same length l. Also, in series combination of wires, effective resistance,

$\Rightarrow \quad R = R_1 + R_2$

$$\Rightarrow \quad \frac{(l+l)}{\sigma_{\text{eff}} A} = \frac{1}{\sigma_1 A} + \frac{1}{\sigma_2 A}$$

$$\Rightarrow \quad \frac{2}{\sigma_{\text{eff}}} = \frac{1}{\sigma_1} + \frac{1}{\sigma_2}$$

$$\Rightarrow \quad \sigma_{\text{eff}} = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$$

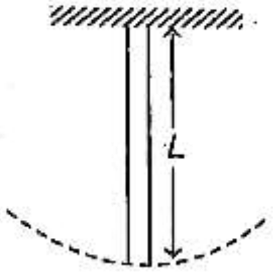
33. (b) Initial length of rod $= L$

Initial angular velocity, $\omega_i = \omega$

When rod pivoted at one end is freely rotated in horizontal plane, then its angular momentum,

$L_i = I_1\omega_1$

$$L_i = \left(\frac{ML^2}{3}\right) \cdot \omega$$



When temperature of the system is increased with temperature ΔT, then change in length of rod ΔL is given as

$\Delta L = \alpha L \Delta T$

Now, new length of the rod,

$L' = \Delta L + L = \alpha L \Delta T + L = L(\alpha \Delta T + 1)$

$\Rightarrow L' = L(1 + \alpha \Delta T)$(i)

Now, new angular momentum,

$$L_f = I_2\omega_2 = \frac{ML'^2}{3} \cdot \frac{\omega}{2}$$

By the law of conservation of angular momentum,

$L_f = L_i$

$$I_2\omega_2 = \frac{ML^2}{3} \cdot \omega$$

$$\frac{ML'^2}{3} \cdot \frac{\omega}{2} = \frac{ML^2}{3} \cdot \omega$$

$$\frac{L'^2}{2} = L^2$$

$\Rightarrow \frac{L^2(1+\alpha\Delta T)^2}{2} = L^2$[from Eq. (i)]

$\Rightarrow \quad (1 + \alpha\Delta T)^2 = 2$

$\Rightarrow \quad 1 + \alpha^2\Delta T^2 + 2\alpha\Delta T = 2$

Since, α is very small, hence the term $\alpha^2\Delta T^2$ may be neglected.

$\therefore \quad 1 + 2\alpha\Delta T = 2 \Rightarrow 2\alpha\Delta T = 1$

$$\Rightarrow \quad \Delta T = \frac{1}{2\alpha}$$

$$L_f = L_i$$

$$I_2\omega_2 = \frac{ML^2}{3} \cdot \omega$$

$$\frac{ML'^2}{3} \cdot \frac{\omega}{2} = \frac{ML^2}{3} \cdot \omega$$

$$\frac{L'^2}{2} = L^2$$

[from Eq. (i)]

$\Rightarrow = L^2$

$$(1 + \alpha\Delta T)^2 = 2$$

$$1 + \alpha^2\Delta T^2 + 2\alpha\Delta T = 2$$

Since, α is very small, hence the term $\alpha^2\Delta T^2$ may be neglected.

$\therefore \quad 1 + 2\alpha\Delta T = 2 \Rightarrow 2\alpha\Delta T = 1$

$\Rightarrow \quad \Delta T = \frac{1}{2\alpha}$

34. (b) As the gravitational field is uniform, therefore center of gravity and the center of mass are at same location.

$\therefore$ The location of centre of mass is

$$h = \frac{\int_0^\infty h dm}{\int_0^\infty dm}$$

$$h = \frac{\int_0^\infty h\rho dh}{\int_0^\infty \rho dh} \quad \ldots(i)$$

But from barometric formula and gas law,

$\rho = \rho_0 e^{-Mgh/RT}$

$\therefore$ Putting the value of ρ in Eq. (i), we have

$$h = \frac{\int_0^\infty h\rho_0 e^{(-Mgh/RT)} dh}{\int_0^\infty \rho_0 e^{-(Mgh/RT)} dh} = \frac{RT}{Mg}$$

35. (b) Under isothermal conditions,

$$p_1V_1 + p_2V_2 = pV$$

$$\Rightarrow \left(p - \frac{4T}{a}\right) \cdot \frac{4}{3}\pi a^3 + \left(p - \frac{4T}{b}\right) \cdot \frac{4}{3}\pi b^3$$

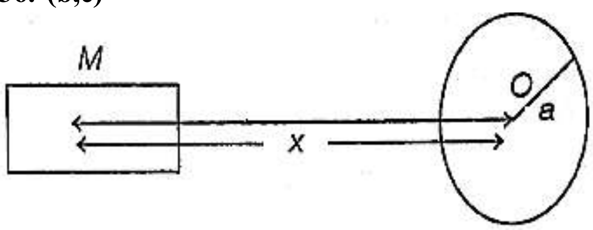
$$= \left(p - \frac{4T}{c}\right) \frac{4}{3}\pi c^3$$

where, T is surface tension of soap bubble.

$\Rightarrow p[c^3 - (a^3 + b^3)] - 4T(a^2 + b^2 - c^2) = 0$

$$\Rightarrow T = \frac{p(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$$

36. (b,c)



Magnetic field at the center O of the circular coil due to bar magnet (axial position)

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}$$

Hence, option (a) is not true.

Induced emf, $e=-\frac{d\phi}{dt}=-\frac{d}{dt}\cdot BA$

$=-\frac{d}{dt}\cdot\frac{\mu_0}{4\pi}\cdot\frac{2M}{x^3}\pi a^2$

$=-\frac{\mu_0 Ma^2}{2}\frac{d}{dt}\cdot\frac{1}{x^3}$

$=-\mu_0 Ma^2\cdot\left(\frac{-3}{x^4}\right)\frac{dx}{dt}$

$\Rightarrow\ e=\frac{3\mu_0 Ma^2}{x^4}\cdot\frac{dx}{dt}$

$\Rightarrow\ e\propto\frac{1}{x^4}$

Hence, option (b) is true.

Induced current, $I=\frac{e}{R}=\frac{3\mu_0 Ma^2}{x^4 R}\frac{dx}{dt}$

$\therefore$ Magnetic moment, $\mu=IA=\frac{3\mu_0 Ma^2}{x^4 R}\frac{dx}{dt}\pi a^2$

$\Rightarrow\mu=\frac{3\pi\mu_0 M}{x^4 R}\frac{dx}{dt}(a^4)$

$\mu\propto a^4$

$\Rightarrow$ Hence, option (c) is true.

Net heat produced,

$H\propto I^2$

$\Rightarrow\quad H\propto\left(\frac{1}{x^4}\right)^2$ ......(i)

$\Rightarrow\quad H\propto\frac{1}{x^8}$

Hence, option (d) is not true.

37. (a,b) Given, $E=a(\cos\omega_0 t+\sin\omega t+\cos\omega_0 t)$

$\omega=10^{15}\ \mathrm{s}^{-1}$

$\omega_0=5\times10^{15}\ \mathrm{s}^{-1}$

Since, $\omega<\omega_0$

$\Rightarrow\ 2\pi v<2\pi v_0$

$\Rightarrow\ v<v_0$ [$v_0=$ threshold frequency]

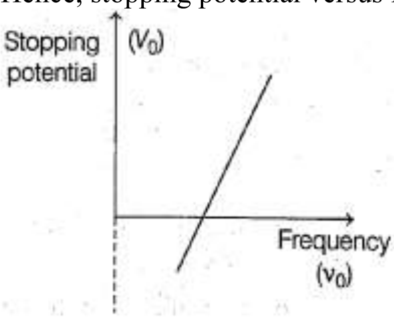
Hence, for light of frequency ω, photoelectric effect is not possible.

According to Einstein's equation,

$eV_0=hv-hv_0$

$eV_0=hv-\phi_0$

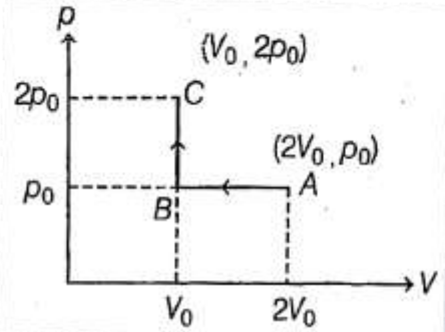
Hence, stopping potential versus frequency graph will be straight line.



Work-function of metal surface,

$\phi_0 = h\nu_0 = h\left(\frac{\omega_0}{2\pi}\right)$
$= 6.62 \times 10^{-34} \times \frac{5 \times 10^{15}}{2\pi}$
$= 5.27 \times 10^{-19}$ J
$= \frac{5.27 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV
$= 3.29$eV
∴ Maximum kinetic energy of photoelectron,
$K_{\max} = -(eV_0) = -[e(-2)] = 2$eV

38. (b,c), Since, given $p - V$ diagram is not a cyclic process, hence the change in internal energy for the whole process is not zero.



Heat rejected by the gas in path AB during isobaric compression,
$dQ_{AB} = nC_p\Delta T$
$= 1 \cdot \frac{5}{2}R(T_B - T_A)$
$= \frac{5R}{2}\left(\frac{p_0V_0}{1 \times R} - \frac{2p_0V_0}{1 \times R}\right) = -5\frac{p_0V_0}{2}$
Heat absorbed by the gas in path BC of isobaric process,
Net heat
$dQ_{BC} = nC_V\Delta T$
$= 1 \times \frac{3R}{2}(T_C - T_B)$
$= \frac{3R}{2}\left(\frac{2p_0V_0}{1 \times R} - \frac{p_0V_0}{1 \times R}\right) = \frac{3p_0V_0}{2}$
$Net\ heat\ = dQ_{AB} + dQ_{BC}$
$= \frac{-5p_0V_0}{2} + \frac{3p_0V_0}{2} = -p_0V_0$
∴ Heat is rejected during the process.
Change in internal energy in process $A \to B$,
$\Delta U = nC_V\Delta T = 1 \times \frac{3}{2}R(T_B - T_A)$

$= \frac{3R}{2}\left(\frac{p_0V_0}{1 \times R} - \frac{2p_0V_0}{1 \times R}\right) \quad = -\frac{3}{2}p_0V_0$

Work done by the gas during entire process is W, then
$W = W_{AB} + W_{BC}$
$= p_0(V_0 - 2V_0) + 0 = -p_0V_0$

39. (b,c) Mass, $m = 0.02$ kg
Potential energy,

$\Rightarrow \quad U = Ax(x-4)$J

$\Rightarrow \quad U = Ax^2 - 4Ax$

Force, $\quad F = -\frac{dV}{dx} = -\frac{d}{dx}(Ax^2 - 4Ax)$

$\Rightarrow \quad F = -2Ax + 4A$

This force is dependent on x, hence particle is not acted upon by a constant force.

From Eq. (i), it is clear that $f \propto -x$

Hence, particle executes simple hormonic motion.

Speed of particle is maximum at equilibrium position, i.e. when $F = 0$

i.e. $\quad -2Ax + 4A = 0$

$\Rightarrow \quad x = 2$ m

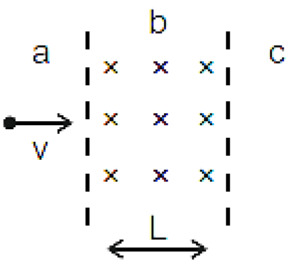
Period of oscillation,

$T = 2\pi\sqrt{\frac{m}{k}}$......(i)

From Eq. (i), $F = -2Ax + 4A$

Since, value of k will be obtained in terms of A. Therefore, the value of time period will be also obtained in terms of A, hence option (d) is not correct.

40. (a,d) The given situation is shown below



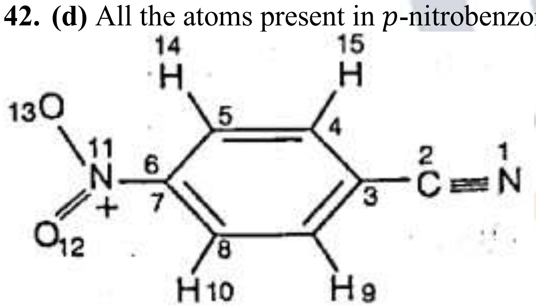
Chemistry

41. (b) The exact order of the boiling points of the given compound is isopentane < n-pentane < butanone < 1-butanol.

The highest boiling will be of butanol because in butanol, molecules are associated due to extensive intermolecular hydrogen bonding.

Now butanone has higher boiling point from n-pentane and iso-pentane because butanone has stronger dipole-dipole interaction and n-pentane has higher boiling point than isopentane because of larger surface area, due to which n-pentane has greater van der Waals' force of attraction.

42. (d) All the atoms present in p-nitrobenzonitrile are in one plane as show in the structure



Since, charge particle enters into magnetic field B perpendicular to it, hence it performs a circular path in magnetic field. Radius of circular path,

$r = \frac{mv}{Bq}$

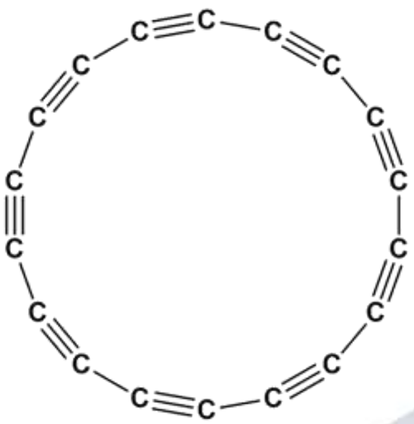
Charge particle will enter in region c, when $L < 2r$

$\Rightarrow \quad L < 2\frac{mv}{Bq}$

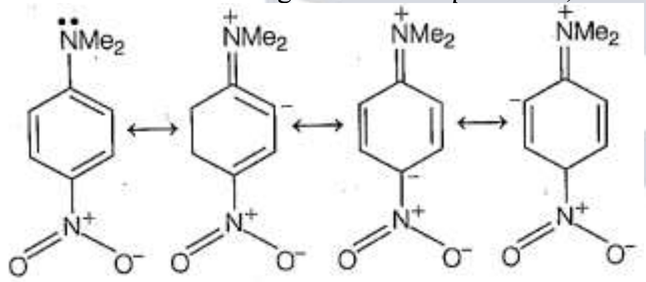
$\Rightarrow \quad \frac{LBq}{2m} < v \Rightarrow v > \frac{qLB}{2m}$

$\Rightarrow \quad v > \left(\frac{1}{2}\right)\frac{qLB}{m}$

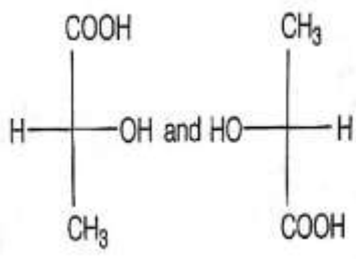
i.e. When $v > \frac{qLB}{m}$, then particle surely will inter into region c, because $\frac{qLB}{m} > \frac{qLB}{2m}$
Hence, option (a) is correct.
Since, charge particle enters into region c, hence path of the particle is a circle not in region b. Hence, option (c) is not correct. Time spent in region b is given as $T = \frac{2\pi m}{Bq}$ which is independent of v.
Hence, option (d) is correct.
Therefore, the maximum number of atom in one plane in this case are 15.
43. **(a)** The total number of triple bonds present in cyclo (18) carbon are 9. Its structure is shown below:



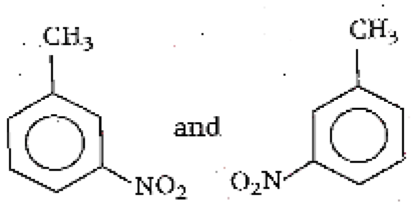
Structure of cyclo (18) carbon
44. **(b)** Resonating structures II and IV of p-nitro-N, N -dimethyl aniline are incorrect. In structure II and IV, valencies of N are not satisfied correctly.
All the correct resonating structures of p-nitro-N, N -dimethyl aniline are as follows



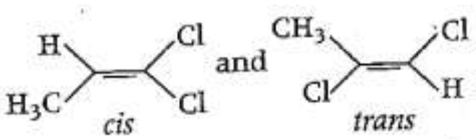
45. **(c)**



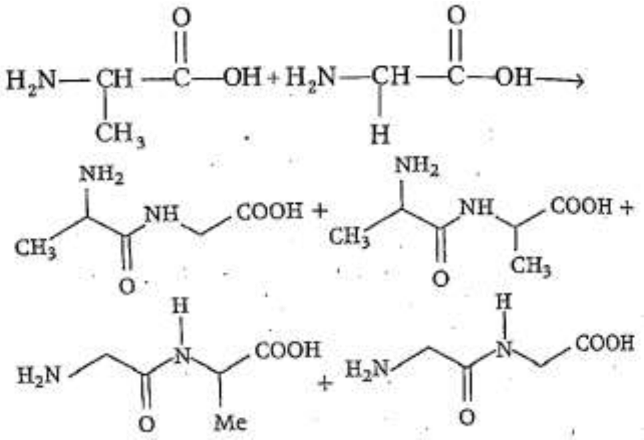
are monomers.
When the same compound is represented in different ways, then all the representations are known as homomers of each other.



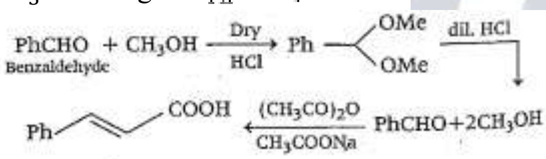
are also homomers.



cis and
trans are constitutional isomers.
Constitutional isomers are the compounds that have the same molecular formula and different connectivity.
46.(d) The correct order of acidity of the given compounds is acetylene < ethanol < p-nitrophenol <
acetic acid.
Ethanol is stronger acid than acetylene. This is because after the loss of acidic proton (H^{+}) the corresponding anion formed by ethanol is stabilised by the electronegative oxygen atom. Now p-nitrophenol is more acidic than ethanol because the phenoxide ion obtained after the deprotonation is stabilised by resonance effect and also by EWG effect (which is caused by - NO_2 group).
Likewise, acetic acid is more acids than all of the given compounds as the formed carboxylate ion is stabilised by resonance which is due to the presence of electronegative oxygen atom.
47. (d) All the given dipeptides can be obtained from the amino acids glycine and alanine.
The general formula of amino acid is $H_2\,N-CH(R)-COOH, R$ is CH_3 for alanine and H for glycine. The structure are as follows



48. (d) Benzaldehyde reacts with 2 moles of methanol in presence of dry HCl to give 
on reaction with dil. HCl
gives respective aldehyde and alcohol. Aldehyde on reaction with $(CH_3CO)_2O$,
CH_3COONa gives 



49. (c) The free energy change of a reaction is
$\Delta G^{\circ}=\Delta H^{\circ}-T\Delta S^{\circ}$
Now, a spontaneous reaction is one that releases free energy and therefore, the sign of ΔG must be negative.
So, when ΔH is negative and ΔS is positive, the sign of ΔG will be negative and the reaction will be spontaneous at all temperature.
50. (b) In acidic medium,
$\overset{+7}{Mn}OO_4^{-}+8H^{+}+5e^{+}\longrightarrow Mn^{2+}+4H_2O$
and $Fe^{2+}\longrightarrow Fe^{3+}+e^{-}$
So, number of moles of $MnO_4^{-}=\frac{1}{5}$ moles…..(i)
Also, in acidic medium,
$\overset{+6}{Cr_2}O_7^{2-}+14H^{+}+6e^{-}\longrightarrow 2Cr^{3+}+7H_2O$
and
$Fe^{2+}\longrightarrow Fe^{3+}+e^{-}$
So, number of moles of $Cr_2O_7^{2-}=\frac{1}{6}$ moles
Now, from Eq. (i)
$\frac{1}{5}$ moles of $MnO_4^{-}=x$ moles of MnO_4^{-}

$\therefore \begin{cases} \frac{1}{5} = x \\ 5x = 1 \end{cases}$ ....(iii)

Put this value in Eq. (ii).

$\frac{1}{6}$ will be $\frac{5x}{6}$

So, the number of moles of $Cr_2O_7^{2-}$ required to oxidise the same amount of Fe^{2+} in acidic medium is $\frac{5}{6}x$ or $0.83x$.

51. (d) Density of the element (d) $= 5.0 \text{ g cm}^{-3}$

Edge length of unit cell $(a) = 200\text{pm}$

$= 200 \times 10^{-10}$ cm

$d = \frac{Z \times M}{a^3 \times N_A}$

[where $M =$ mass of element

$N_A =$ Avogadro number

Z = number of atoms per unit cell]

For body centred cubic lattice, $Z = 2$

$$5 = \frac{2 \times M}{(200 \times 10^{-10})^3 \times 6.022 \times 10^{23}}$$

$$\Rightarrow M = \frac{5 \times (200 \times 10^{-10})^3 \times 6.022 \times 10^{23}}{2}$$

$$= \frac{5 \times (200)^3 \times 10^{30} \times 6.022 \times 10^{23}}{2}$$

$$= 12.044 \text{ g}$$

Now, 12.044 g of element have $\longrightarrow 6.022 \times 10^{23}$ atoms

1 g will have $\longrightarrow \frac{6.022 \times 10^{23}}{12.044}$ atoms

100 g will have $\longrightarrow \frac{6.022 \times 10^{23}}{12.044} \times 100$ atoms

$= 0.5 \times 10^{25}$ atoms or

$= 5 \times 10^{24}$ atoms

52. (d) Let two gases are A and B.

$u_1 =$ molecular velocity of gas A

$u_2 =$ molecular velocity of gas B

$m_1 =$ molecular mass of gas A

$m_2 =$ molecular mass of gas B

The root mean square velocity, $u_{\text{ms}} = \sqrt{\frac{3RT}{M}}$.

(where, $M =$ molecular mass, $R =$ gas constant)

For gas A,

$$u_1 = \sqrt{\frac{3RT}{m_1}}$$

$$u_1^2 = \frac{3RT}{m_1}$$

$$T = \frac{u_1^2 m_1}{3R} \text{(i)}$$

For gas B,

$$u_2 = \sqrt{\frac{3RT}{m_2}}$$

$$u_2^2 = \frac{3RT}{m_2} \quad T = \frac{u_2^2 m_2}{3R} \text{(ii)}$$

From Eq. (i) and (ii),

$$\frac{u_1^2 m_1}{3R} = \frac{u_2^2 m_2}{3R}$$

$\therefore \quad m_1 u_1^2 = m_2 u_2^2$

53. (a) $\Delta T_f = i \times k_f \times m$

$\Delta T_f =$ depression in freezing point of solvent

$i =$ van't Hoff factor

$k_f =$ freezing point depression constant

$m =$ molality

$$i = \frac{\Delta T_f}{m \times k_f} = \frac{2}{\left(\frac{w_{\text{solute}} \times 100}{M_{\text{solute}} \times W_{\text{solvent}}}\right) \times 1.72}$$

$$= \frac{2}{\left(\frac{20 \times 1000}{172 \times 50}\right) \times 1.72}$$

$$\begin{bmatrix} M_{\text{solute}} = M_{(C_{11}H_8O_2)} \\ = 12 \times 11 + 8 \times 1 + 2 \times 16 \\ = 172 \end{bmatrix}$$

$i = \frac{2}{4} = 0.5$

54. (a) The presence of catalyst does not change the value of equilibrium constant. It only increases the rate of forward and backward reaction to equal extent. So, the equilibrium constant in presence of catalyst at 2000 K is same as equilibrium constant in the absence of catalyst at 2000 K , i.e. 4×10^{-4}.

55. (a) For 1st order kinetic, $t_{1/2} = \frac{\ln 2}{k_1} = 40$.

For 2nd order kinetics, $t_{1/2} = \frac{[A_0]}{2k_0} = 20$

From Eq. (i),

$$40 = \frac{\ln 2}{k_1}$$

$\Rightarrow \quad k_1 = \frac{\ln 2}{40}$(iii)

From Eq. (ii),

$20 = \frac{[A_0]}{2k_0}$.........(iv)

$k_0 = \frac{[A_0]}{40}$

Now, ratio $\left(\frac{k_1}{k_0}\right) = \frac{\ln 2}{40} \times \frac{40}{[A_0]}$

$= \frac{\ln 2}{[A_0]} = \frac{0.693}{1.386 \text{ moldm}^{-3}}$

$= 0.5 \text{ mol}^{-1}\text{dm}^3$

56. (d) Among the given solutions, $NaCl, KNO_3$ and HCl are strong electrolytes but conductivity also depends upon size of ions. Smaller the size of ions, greater its conductance, hence greater is the conductivity than larger ions. So, out of NaCl , KNO_3 and HCl ions, $H^+$is the smallest and therefore, 0.1 M HCl will have highest conductivity.

57. (b) $Na_2S + nS \longrightarrow Na_2S_{(n+1)}$

$Na_2SO_3 + S \longrightarrow Na_2S_2O_3$

$X = Na_2S_{(n+1)}$

$Y = Na_2S_2O_3$

58. (d) Let a weak monoacidic base is BOH. BOH is neutralised as follow

$BOH + HCl \longrightarrow BCl + H_2O$

At point of equivalence, all BOH get converted into salt and the concentration of H^+ions is due to the hydrolysis of resultant salt, BCl.

$\underset{C(1-h)}{B^+} + H_2O \rightleftharpoons \underset{Ch}{BOH} + \underset{Ch}{H^+}$

Volume of HCl used

$V_a = \dfrac{N_b V_b}{N_a} = \dfrac{2.5 \times 2 \times 15}{2 \times 5} = 7.5 \text{ mL}$

Now, the concentration of salt, $[BCl]$

$= \dfrac{\text{Conc. of base}}{\text{Total volume}} = \dfrac{2 \times 2.5}{5(7.5 + 2.5)} = \dfrac{1}{10} = 0.1$

So, $K_h = \dfrac{Ch^2}{1-h} = \dfrac{K_w}{K_b} = \dfrac{10^{-14}}{10^{-12}} = 10^{-2}$

or $\dfrac{Ch^2}{1-h} = 10^{-2}$

$\dfrac{0.1 \times h^2}{1-h} = 10^{-2}$

On calculating, $h = 0.27$ which is significant, not negligible.

$\therefore [\mathrm{H}^+] = \mathrm{Ch} = 0.1 \times 0.27 = 2.7 \times 10^{-2} \mathrm{M}$

59. (d) For salt $MX, K_{\text{sp}} = [S][S]$

$K_{\text{sp}} = 4 \times 10^{-8}$ (given)

$\therefore \quad 4 \times 10^{-8} = [S]^2$

$\therefore \quad S = 2 \times 10^{-4}$

For salt $MX_2, K_{\text{sp}} = [S][2S]^2$

$K_{\text{sp}} = 3.2 \times 10^{-14}$

$[S] = \left(\dfrac{3.2 \times 10^{-14}}{4}\right)^{1/3}$

$= (8 \times 10^{-15})^{1/3} = 2 \times 10^{-5}$

For salt $MX_3, K_{\text{sp}} = [3S]^{-3}[S]$

$K_{\text{sp}} = 2.7 \times 10^{-15}$

$[S] = \left(\dfrac{2.7 \times 10^{-15}}{27}\right)^{1/4}$

$= (10^{-16})^{1/4} = 10^{-4}$

$\therefore$ Solubility of MX order is

$MX > MX_3 > MX_2$

60. (c) Reduction hydrogen half-cell is

$\mathrm{H}^+|(x\mathrm{M})|\mathrm{Pt}(\mathrm{H}_2)$; Pressure p_{H_2}

Half-cell reaction is

$2\mathrm{H}^+(aq) + 2e^- \longrightarrow \mathrm{H}_2(g)$

$n = 2$

Reaction quotient $\Rightarrow Q = \frac{p_{\mathrm{H}_2}}{[\mathrm{H}^+]^2}$

$E_{\text{red}} = E^\circ_{\text{red}} - \dfrac{0.0591}{n} \log Q$

$E_{\text{red}} = -\dfrac{0.0591}{2} \log Q$

Case (a), $\log Q = \log \dfrac{1}{1} = 0 \quad \therefore E_{\text{red}} = 0$

Case (b), $\log Q = \log \dfrac{1}{4} = -\text{ve} \quad \therefore E_{\text{red}} = +\text{ve}$

Case (c), $\log Q = \log \dfrac{2}{1} = +\text{ve} \quad \therefore E_{\text{red}} = -\text{ve}$

Case (d), $\log Q = \log \dfrac{2}{4} = -\text{ve} \quad \therefore E_{\text{red}} = +\text{ve}$

$\therefore E_{\text{red}}$ is negative for option (c).

61. (d) Solubility of saturated solution = Solubility product

$\therefore \ K_{\text{sp}} = 4 \times 10^{-5}$(for BaSO_4)

Let the solubility of $BaSO_4$ in $0.1M Na_2SO_4$ be x.

$BaSO_4 \rightleftharpoons \underset{x}{Ba^{2+}} + \underset{x}{SO_4^{2-}}$

$Na_2SO_4 \rightleftharpoons \underset{\substack{2\times0.1 \\ (0.2)}}{2Na^{+}} SO_4^{2-}$

$\therefore$ Total $(SO_4^{2-}) = x + 0.1$

$\approx 0.1M$

Thus, $K_{sp} = [Ba^{2+}][SO_4^{2-}]$

$4 \times 10^{-5} = [x](0.1)$

$\therefore x = 4 \times 10^{-4}$

$\therefore$ Solubility $= 4 \times 10^{-4} M$

62. (c) The given reaction is as follows:

$Co(NO_3)_2 + 7Na_2NO_2 + 4CH_3COOH \longrightarrow$

$Na_3[Co(NO_2)_6] + 3NO_2 + 2H_2O + 4CH_3COONa$

Sodium cobalt nitrile

With the salt of potassium, sodium cobalt nitrile forms insoluble double salt $K_2Na[Co(NO_2)_6] \cdot H_2O$. This salt is yellow coloured ppt.

63. (b) The metal (M) is copper and the reactions are as follows:

$2Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + SO_2 \uparrow$

$Cu_2S + 2Cu_2O \xrightarrow{\Delta} 6Cu + SO_2 \uparrow$

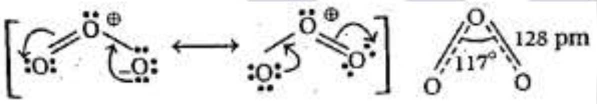
64. (d) When CO_2 is passed through ammoniacal solution of NaCl , sodium bicarbonate $(NaHCO_3)$ and (NH_4Cl) are formed.

$NH_4C\underset{(X)}{aCl} + CO_2 \longrightarrow \underset{\substack{\text{Sodium} \\ \text{bicarbonate} \\ (Y)}}{NaHCO_3} + \underset{\text{chloride}}{NH_4Cl}$

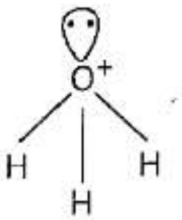
On further heating $NaHCO_3$, we get Na_2CO_3.

$2NaHCO_3 \longrightarrow \underset{\substack{\text{Sodium} \\ \text{carbonate} \\ \text{(white ppt.)} \\ (Z)}}{Na_2CO_3} + H_2O + CO_2$

65. (a) Structure of ozone (i.e. O_3) has delocalised π-electrons. Its resonance structure are as follows:



66. (b) The structure of the hydronium ion is pyramidal because it has three bonding pairs of electrons and one lone pair of electrons. Its structure is as follows:



Hybridisation $= sp^3$ and shape $=$ pyramidal

67. (b) Mass on the reactant side $= 18.00567$amu Mass of the proton $= 1.00782$amu Energy absorbed in the reaction $= 1.16$MeV

$^{14}N(\alpha, p)\, ^{17}O = 0.00124$amu

This reaction can be simplified as

$^{14}_{7}N + {}^{4}_{2}\alpha \xrightarrow{\Delta E} {}^{17}_{8}O + {}^{1}_{1}H$

where, ΔE is the energy absorbed.

In this reaction, highly energetic alpha particle with kinetic energy $1.16 MeV^{are}$ absorbed by $^{14}_{7}N$ nucleus to form $^{17}_{8}O$ isotope and a proton.

Hence,

$$\underbrace{{}^{14}_{7}N + {}^{4}_{2}\alpha}_{18.00567\text{ amu}} \xrightarrow{1.16\text{MeV}} {}^{17}_{8}O + \underbrace{{}^{1}_{1}H}_{1.00782\text{ amu}}$$

According to conversation of mass-energy.

$18.00567\text{amu} = ({}^{17}_{8}O + 1.0078 - 0.00124)\text{amu}$

${}^{17}_{8}O = 18.00567 - 1.0078 + 0.00124$

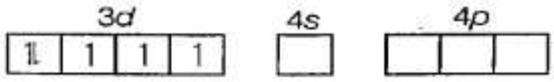
$= 16.99914\text{amu}$

68. (a) A solution of $NaNO_3$, when treated with a mixture of Zn dust and NaOH (caustic soda) yields ammonia gas.

Complete reaction is as follows

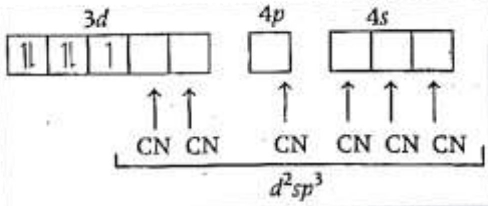
$$\underset{\text{Zinc metal}}{Zn} + \underset{\text{Caustic soda}}{NaOH} + \underset{\text{Sodium nitrate}}{NaNO_3} \longrightarrow \underset{\text{Sodium zincate}}{Na_2ZnO_2} + \underset{\text{Ammonia}}{H_2O} + \underset{NH_3}{NH_3} \uparrow$$

69. (a) In $K_3[Fe(CN)_6]$Fe present in +3 oxidation state outer electronic configuration of Fe^{3+} =



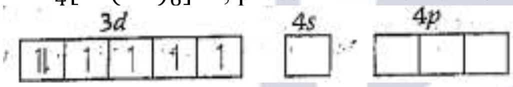
As CN is a strong field ligand. So, it causes the pairing of electrons.

Therefore, $K_3[Fe(CN)_6]$ =



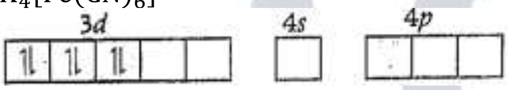
Thus, in this complex, number of unpaired electron is one.

In $K_4[Fe(CN)_6]$Fe, present in +2 oxidation state outer, electronic configuration of Fe^{2+}



As in this case also the ligand is CN (which is a strong field). So, it again causes the pairing of the electron. This can be shown in following way.

$K_4[Fe(CN)_6]$ =



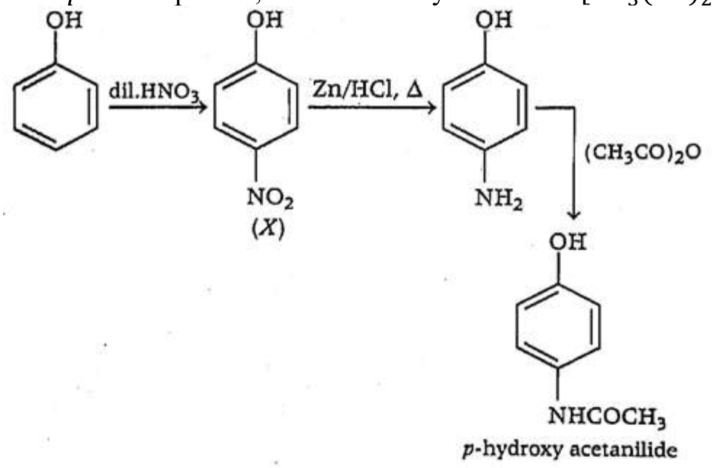
Thus, in this case the number of unpaired electron is zero.

70.(d) When two complexes have same number of unpaired electrons, they have same value of magnetic moment. The number of unpaired electrons present in $[Cu(H_2O)_6]^{2+}$, $[Mn(H_2O)_6]^{4+}$, $[Mn(H_2O)_6]^{3+}$, $[Fe(H_2O)_6]^{3+}$ and $[Cr(H_2O)_6]^{3+}$ are 0,3,4,5 and 3 respectively.

Thus, the magnetic moment of $[Mn(H_2O)_6]^{3+}$ and $[Cr(H_2O)_6]^{3+}$ are identical as they have same number of unpaired electrons (i.e., 3).

71. (a) Methoxymethyl chloride (CH_3OCH_2Cl) hydrolysed most easily and neopentyl chloride $[(CH_3)_3C - Cl]$ hydrolysed most slowly. Primary halide readily undergoes S_N2 meqhanism and forms intermediate whereas tertiary halides due to steric hinderance reacts slowly with water.

72. (c) Phenol on hitration with dil. HNO_3 gives p-nitro phenol (X) which on heating with Zn/HCl, form p-amino phenol, which on acetylation with $[CH_3(CO)_2O]$ gives p-hydroxy acetanilide: Reaction is as follows:



73. (a) de-Broglie wavelength, $\lambda = \frac{h}{mv}$

$\therefore \quad \frac{\lambda_{He}}{\lambda_{Ne}} = \frac{M_{Ne}}{M_{Ne}} = \frac{20}{4}$

$\therefore \quad \lambda_{He} = 5\lambda_{Ne}$

74. (a) Given,

$x_{solute} = 0.1$

Molarity $=$ Molality

Density of solution $= 2 \text{ g cm}^{-3}$

$= \frac{2 \times 10^{-3}}{10^{-6}}$ kg

$= 2 \times 10^{3}$ kg/m^3

$= 2$ kg/L

$\frac{m_{solute}}{M_{solute}} = \frac{m_{solute}}{M_{solute}}$

Volume Mass of solvent (kg)

Volume of solution $=$ Mass of solvent (kg)

$\frac{\text{Mass of solution}}{d} = \text{Mass of solvent}$

$\therefore \frac{\text{Mass of solution}}{\text{Mass of solvent}} = 2$

$\therefore \frac{\text{Mass of solute} + \text{Mass of solvent}}{\text{Mass of solvent}} = 2$

$\therefore \frac{\text{Mass of solute}}{\text{Mass of solvent}} = 1 \ldots\ldots \text{(i)}$

$\frac{\eta_{solute}}{\eta_{solute} + \eta_{solvent}} = 0.1$

$\therefore \frac{m_{solute}}{M_{solute}} = 0.1$

$\frac{m_{solute}}{M_{solute}} + \frac{m_{solvent}}{M_{solvent}}$

$\frac{1}{\frac{1}{M_{solute}} + \frac{1}{M_{solvent}}} = 01 \quad \{\because M_{solute} = m_{solvent}$

$\therefore \frac{M_{solute}}{M_{solvent}} = 9 \qquad$ using (i) }

$x_{\text{solute}} = 0.1$

Molarity $=$ Molality

Density of solution $= 2\text{ g cm}^{-3}$

$= \dfrac{2 \times 10^{-3}}{10^{-6}}$ kg

$= 2 \times 10^{3}\text{ kg/m}^{3}$

$= 2\text{ kg/L}$

$$\frac{m_{\text{solute}}}{M_{\text{solute}}} = \frac{m_{\text{solute}}}{M_{\text{solute}}}$$

Volume Mass of solvent (kg)

Volume of solution $=$ Mass of solvent (kg)

$$\frac{\text{Mass of solution}}{d} = \text{Mass of solvent}$$

$$\therefore \frac{\text{Mass of solution}}{\text{Mass of solvent}} = 2$$

$$\therefore \frac{\text{Mass of solute} + \text{Mass of solvent}}{\text{Mass of solvent}} = 2$$

$$\therefore \frac{\text{Mass of solute}}{\text{Mass of solvent}} = 1$$

$$\frac{\eta_{\text{solute}}}{\eta_{\text{solute}} + \eta_{\text{solvent}}} = 0.1$$

$$\therefore \frac{m_{\text{solute}}}{M_{\text{solute}}} = 0.1$$

$$\frac{m_{\text{solute}}}{M_{\text{solute}}} + \frac{m_{\text{solvent}}}{M_{\text{solvent}}}$$

$$\frac{1}{M_{\text{solute}}} + \frac{1}{M_{\text{solvent}}} = 0.1 \text{(i)}$$

$$\therefore \frac{M_{\text{solute}}}{M_{\text{solvent}}} = 9 \qquad \text{(i)}$$

using (i) (i)

$$\frac{1}{M_{\text{solute}}} + \frac{1}{M_{\text{solvent}}} = 0.1$$

$$\therefore \frac{M_{\text{solute}}}{M_{\text{solvent}}} = 9$$

using (i)

75. (d) $IE_{\text{Na}} = 490\text{ kJ/mol}$

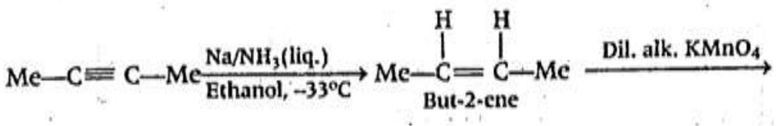
$$\eta_{\text{Na}} = \frac{m_{\text{Na}}(g)}{M_{\text{Na}}}$$

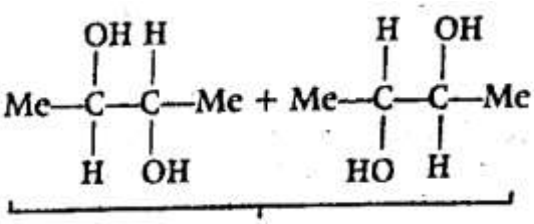
$$= \frac{5.75 \times 10^{-3}}{23}$$

$\therefore$ $\text{IE}_{\text{Na}} = 490 \times 0.25 \times 10^{-3}$

$= 0.1225\text{ kJ}$

76. (b,c) Butyne (Me-C = C-Me) on reaction with Na/NH_3 (liq.) undergoes reduction to give but-2-ene which on reaction with dil. alkaline $KMnO_4$ gives diol.





Butan-2, 3-diol

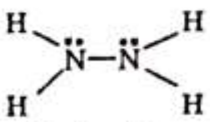
77. (d)

78. (c,d) Buffer solutions are made by mixing weak acid with its salt of strong base or mixing weak base with its salt of strong acid. It is never formed by mixing strong acid and strong base. The pair(s) of solutions which' form a buffer upon mixing are (c) HNO_3 and $CH_3CO\overset{-}{O}\overset{+}{N}$, (d) i.e. CH_3COOH and $CH_3CO\overset{-}{N}\overset{+}{Na}$

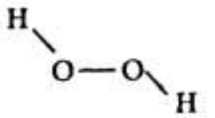
79. (b, c) Reaction of silver nitrate solution with phosphorus acid produces phosphoric acid and metallic silver. Complete reaction is as follows:

$$\underset{\text{Phosphorus acid}}{H_3PO_3} + AgNO_3 + H_2O \longrightarrow \underset{\text{Phosphtone acid}}{H_3PO_4} + Ag\downarrow + 2HNO_3$$

80. (b, c, d) N_2H_4 and H_2O_2 show similarity in reducing and oxidising nature. They have same hybridisation of central atoms i.e. sp^3. The structure of hydrazine and hydrogen peroxide as follows.



Hydrazine (N_2H_4)



Hydrogen peroxide (H_2O_2)

Mathematics

81. (c) Given, $I = \lim_{x\to 0}\left(\frac{e^x - x - 1 - \frac{x^2}{2}}{x^2}\right)$

$$= \lim_{x\to 0} \frac{\sin\left(\frac{e^x - x - 1 - \frac{x^2}{2}}{x^2}\right)}{\frac{e^x - x - 1 - \frac{x^2}{2}}{x^2}} \times \frac{e^x - x - 1 - \frac{x^2}{2}}{x^2}$$

$$= \lim_{x\to 0} \frac{e^x - x - 1 - \frac{x^2}{2}}{x^2} \quad \frac{0}{0}\ from$$

$$= \lim_{x\to 0} \frac{e^x - 1 - 0 - x}{2x} \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

$$= \lim_{x\to 0} \frac{e^x - 1}{2}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{e^x - 1}{x} \times x$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{e^x - 1}{x} \times \lim_{x\to 0} x$$

$$= 0$$

82. (b) Given, $|f'(x)| \leq 5$ for all x

$$\Rightarrow \int_0^1 -5dx \leq \int_0^1 f'(x)dx \leq \int_0^1 5dx$$

$$\Rightarrow \quad -5 \leq f(1) - 0 \leq 5$$

$$\Rightarrow \quad -5 \leq f(1) \leq 5$$

$$\Rightarrow \quad f(1) \in [-5,5]$$

83. **(c)** Let $I=\int \frac{\sin 2x}{(a+b\cos x)^2}dx$

$$\Rightarrow I=\int \frac{2\sin x\cos x}{(a+b\cos x)^2}dx$$

Put $a+b\cos x=t$

$$\Rightarrow \cos x=\frac{t-a}{b} \Rightarrow -b\sin x dx=dt$$

$$I=-\frac{2}{b}\int \left(\frac{t-a}{b}\right)\frac{1}{t^2}dt$$

$$=-\frac{2}{b^2}\int \frac{t-a}{t^2}dt$$

$$=-\frac{2}{b^2}\left[\int \frac{1}{t}dt-\int \frac{a}{t^2}dt\right]$$

$$=-\frac{2}{b^2}\int \left[\log(t)-\frac{t^{2+1}}{-2+1}\right]+C$$

$$=-\frac{2}{b^2}\left[\log|t|+\frac{a}{t}\right]+C$$

$$\therefore I=-\frac{2}{b^2}\left[\log|a+b\cos x|+\frac{a}{a+b\cos x}\right]+C$$

$$\Rightarrow \alpha=-\frac{2}{b^2}$$

84. **(c)** Given, $g(x)=\int_x^{2x}\frac{f(t)}{t}dt, x>0$

$$\Rightarrow g'(x)=\frac{f(2x)}{2x}\frac{d}{dx}(2x)-\frac{f(x)}{x}\frac{d}{dx}(x)$$

(Using Leibnitz Rule for Differentiation)

$$=\frac{f(2x)}{2x}(2)-\frac{f(x)}{x}$$

$$=\frac{f(2x)-f(x)}{x}$$

$$=\frac{f(x)-f(x)}{x} \quad (\because f(2x)=f(x))$$

$\Rightarrow g'(x)=0$

$\Rightarrow g(x)$ is constant function

85. **(b)** We have,

$$=\int_1^3 \frac{|x+1|}{|x-2|+|x-3|}dx$$

$$=\int_1^2 \frac{x-1}{x+2-x+3}dx+\int_2^3 \frac{x-1}{x-2-x+3}dx$$

$$=\int_1^2 \frac{x-1}{-2x+5}dx+\int_2^3 (x-1)dx$$

$$=-\frac{1}{2}\int_1^{2-2x+2} \frac{x}{-2x+5}dx+\left(\frac{x^2}{2}-x\right)_2^3$$

$$= -\frac{1}{2}\int_1^2 \frac{-2x+2}{-2x+5}dx + \left(\frac{x^2}{2} - x\right)_2^3$$

$$= -\frac{1}{2}\int_1^2 \frac{-2x+5-3}{-2x+5}dx + \left(\frac{9}{2} - 3\right) - (2-2)$$

$$= -\frac{1}{2}\left[\int_1^2 \frac{-2x+5}{-2x+5}dx - \int_1^2 \frac{3}{-2x+5}dx + \frac{3}{2}\right.$$

$$= -\frac{1}{2}\left[\int_1^2 dx + \frac{3}{2}\log[-2x+5]_1^2\right] + \frac{3}{2}$$

$$= \frac{3}{4}\log 3 + 1$$

86. (c) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left\{\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right\}^{\frac{1}{2}} dx$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[\left(\frac{x+1}{x-1} - \frac{x-1}{x+1}\right)^2\right]^{\frac{1}{2}} dx$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[\frac{4x}{x^2-1}\right] dx$$

$$= \int_{-\frac{1}{2}}^{0} \left[\frac{4x}{x^2-1}\right] dx - \int_0^{\frac{1}{2}} \left[\frac{4x}{x^2-1}\right] dx$$

$$= -4\int_{-\frac{1}{2}}^{0} \frac{x}{1-x^2}dx + 4\int_0^{\frac{1}{2}} \frac{x}{1-x^2}dx$$

$$= 2[\log(1-x^2)]_{-\frac{1}{2}}^{0} - 2[\log(1-x^2)]_0^{\frac{1}{2}}$$

$$= -2\log\left(1-\frac{1}{4}\right) - 2\log\left(1-\frac{1}{4}\right)$$

$$= -4\log\frac{3}{4} = 4\log\frac{4}{3}$$

87. (c) Let $I = \int_{\log_e 2}^{x} \frac{1}{e^x-1}dx$

Put $e^x - 1 = t \Rightarrow e^x = t+1$

$e^x dx = dt$

$\Rightarrow dx = \frac{dt}{e^x} = \frac{dt}{t+1}$

When $x = x, t = e^x - 1$

and when $x = \log_e 2, t = e^{\log_e 2} - 1 = 2 - 1 = 1$

$$I = \int_1^t \frac{1}{t(t+1)}dt$$

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$$

$1 = A(t+1) + B(t)$

$t = 0, A = 1$

$t + 1 = 0 \Rightarrow t = -1, B = -1$

$$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\Rightarrow I = \int_1^t \left(\frac{1}{t} - \frac{1}{t+1}\right)dt$$

$$= [\log_e t - \log_e(t+1)]_1^t$$
$$= \left[\log_e \frac{t}{t+1}\right]_1^t$$
$$= \left[\log_e \frac{t}{t+1} - \log_e \frac{1}{2}\right]$$
$$= \log_e \frac{2t}{t+1}$$
$$\frac{2t}{2t+1} = \frac{3}{2}$$

$$\Rightarrow 4t = 3t + 3$$
$\Rightarrow$ t = 3

$$\Rightarrow e^x - 1 = t$$
$$\Rightarrow e^x - 1 = 3$$
$$\Rightarrow e^x$$
$$\Rightarrow e^x = 4$$
$$x = \log 4$$

88. (c) Equation of normal
$$Y - y = \frac{-1}{dy/dx}(X - x)$$
When $y = 0$.
Then,
$$G = \left(x + y\frac{dy}{dx}, 0\right)$$
According to the question,
$$x + y\frac{dy}{dx} = 2x \Rightarrow y\frac{dy}{dx} = x$$
$$\Rightarrow \int ydy = \int xdx$$
$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C \Rightarrow \frac{y^2 - x^2}{2} = C$$
$\Rightarrow$ which is hyperbola.

89. (b) We know that the equation of an ellipse whose center is at origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Differentiate both sides w.r.t x, we get
$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx} = -\frac{2x}{a^2}$$
$$\Rightarrow \frac{y}{x}\frac{dy}{dx} = -\frac{b^2}{a^2}$$
Again differentiate both sides w.r.t χ, we get
$$\frac{y}{x}\frac{d}{dx}\left(\frac{dy}{dx}\right) + y'\frac{d}{dx}\left(\frac{y}{x}\right) = 0$$
$$\Rightarrow \frac{y}{x}y'' + y'\left(\frac{xy' - y \times 1}{x^2}\right) = 0$$
$$\Rightarrow \frac{y}{x}y'' + y'\left(\frac{xy' - y}{x^2}\right) = 0$$
$$\Rightarrow xyy'' + x(y')^2 - yy' = 0$$
Which is the required differential equation.

90. (a) Given, $x\frac{dy}{dx} + y = x\frac{f(xy)}{f'(xy)}$

$$\Rightarrow \quad \frac{d}{dx}(xy) = x\frac{f(xy)}{f'(xy)}$$

$$\Rightarrow \quad \frac{f'(xy)}{f(xy)}d(xy) = xdx$$

$$\Rightarrow \quad \int \frac{f'(xy)}{f(xy)}d(xy) = \int xdx$$

$$\Rightarrow \quad \log|f(xy)| = \frac{x^2}{2} + C$$

$$\Rightarrow \quad |f(xy)| = e^{\frac{x^2}{2}} + C$$

$$= ke^{\frac{x^2}{2}}$$

91. (d)

$-y = (x-1)^2 - 1$

$y = 1 - (x-1)^2$

Hence the required area is

$$\int_0^1 [1-(x-1)^2]dx = \left[x - \frac{(x-1)^3}{3}\right]_0^1$$

$$= 1 - \left[-\frac{(-1)^3}{3}\right]$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, $y = \frac{2}{3}x$ will divide the entire area in 2 equal parts.

92. (c)

$$\int_0^5 \max\{x^2, 6x-8\}dx$$

$$= \int_0^2 x^2dx + \int_2^4 (6x-8)dx + \int_4^5 x^2dx$$

$$= \left[\frac{x^3}{3}\right]_0^2 + \left[\frac{6x^2}{2} - 8x\right]_2^4 + \left[\frac{x^3}{3}\right]_4^5$$

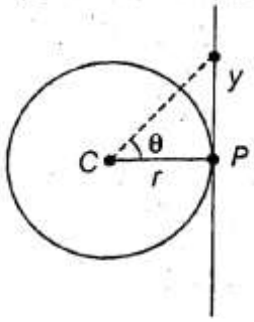
$$= \frac{8}{3} + [3x^2 - 8x]_2^4 + \frac{125}{3} - \frac{64}{3}$$

$$= (48 - 32 - 12 + 16) + \frac{69}{3}$$

$$= 20 + 23$$

$$= 43$$

93. (b) Given, radius of circular track is 10 m .



and speed of man $= 10$ m/sec. i.e. $v = r\frac{d\theta}{dt} = 10$

Now, $\tan\theta = \frac{y}{r}$

$\Rightarrow \; y = r\tan\theta$

$\Rightarrow \dfrac{dy}{dt} = r\sec^2\theta \cdot \dfrac{d\theta}{dt}$

$= \sec^2\theta \cdot r\dfrac{d\theta}{dt}$

$= \sec^2 60° \cdot (10)$

$= (2)^2 \times 10$

$= 40\ \text{m/sec.}$

94. (c) Let B travels x units,

$v = u + at$

According to the problem,

$f't = f(t+m)$

$\Rightarrow \dfrac{f'-f}{f}$

$\Rightarrow \dfrac{m}{t}$ (i)

$t^2 = m^2\left(\dfrac{f}{f'-f}\right)^2$ (i)

Again, $n + x = \frac{1}{2}f(t+m)^2$

$\Rightarrow n + \frac{1}{2}f't^2 = \frac{1}{2}f(t+m)^2$.....(ii)

Now from Eqs. (i) and (ii), we get

$\dfrac{m}{n} = \dfrac{2(f'-f)}{mff'}$

$\Rightarrow (f'-f)n = \dfrac{1}{2}ff'm^2$

95. (b) Given, $\alpha + 3\beta$ is collinear with γ

$\therefore \alpha + 3\beta = \lambda_1\gamma$

$\Rightarrow \beta = \dfrac{\lambda_1}{3}\gamma - \dfrac{\alpha}{3}$(i)

and $\beta + 2\gamma$ is collinear with α

$\therefore \beta + 2\gamma = \lambda_2\alpha$

$\Rightarrow \beta = \lambda_2\alpha - 2\gamma$......(ii)

From Eqs. (i) and (ii), we get

$\dfrac{\lambda_1}{3}\gamma - \dfrac{\alpha}{3} = \lambda_2\alpha - 2\gamma$

$\Rightarrow \alpha\left(\lambda_2 + \dfrac{1}{3}\right) = \gamma\left(\dfrac{\lambda_1}{3} + 2\right)$

$\Rightarrow \lambda_2 + \dfrac{1}{3} = 0$ and $\dfrac{\lambda_1}{3} + 2 = 0$

$\Rightarrow \lambda_2 = -\dfrac{1}{3}$ and $\dfrac{\lambda_1}{3} = -2$

$\Rightarrow \lambda_1 = -6$ and $\lambda_2 = -\dfrac{1}{3}$

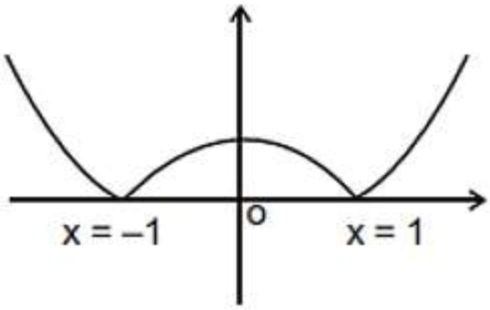
From Eqs. (i) and (ii), $\beta = -2\gamma - \frac{\alpha}{3}$

$\therefore \alpha + 3\beta + 6\gamma = \alpha + 3\left(-2\gamma - \dfrac{\alpha}{3}\right) + 6\gamma$

$= 0$

96. (c) Given, $f(x) = |x^2 - 1|, x \in R$.

The graph of $f(x)$ is clearly, from graph.



$f(x)$ has a local minima at $x = \pm 1$ and has local maxima at $x = 0$

97. (d) Given,

$$\frac{2}{3}\log_b a + \frac{3}{5}\log_c b + \frac{5}{2}\log_a c = 3$$

$$\Rightarrow \frac{2\log a}{3\log b} + \frac{3\log b}{5\log c} + \frac{5\log c}{2\log a} = 3$$

Since, A.M $= \dfrac{\frac{2}{3}\log_b a + \frac{3}{5}\log_c b + \frac{5}{2}\log_a c}{3}$

$$= \frac{3}{3} = 1$$

and G.M $= \left(\frac{2}{3}\log_b a \cdot \frac{3}{5}\log_c b \cdot \frac{5}{2}\log_a c\right)^{\frac{1}{3}}$

$$= \frac{2\log a}{3\log b} \times \frac{3\log b}{5\log c} \times \frac{5\log c}{2\log a} = 1$$

$\therefore$ A.M $=$ G.M

$$\therefore \frac{2\log a}{3\log b} = 1 \Rightarrow a^2 = b^3$$

$$\Rightarrow a = (3^6)^{\frac{1}{2}} = 3^3 = 27$$

98. (a) Given, $h(p) = \frac{1}{2}\{h(p+1) + h(p-1)\}$

for every $p = 1,2,\dots,99$

$2h(p) = h(p+1) + h(p-1)$

$\Rightarrow h(p-1), h(p), h(p+1)$ are in AP.

Now, $h(100) = 20$

$\Rightarrow \quad h(0) + 99d = 20$

$\Rightarrow \quad 5 + 99d = 20$ (h(0) = 5)

$\Rightarrow \quad d = \dfrac{15}{99} = \dfrac{5}{33}$

$\therefore \quad h(1) = h(0) + d$

$$= 5 + \frac{5}{33} = 5 + 0.15$$

$$= 5.15$$

99. (d) Let $z = e^{i\theta}$

Also, let $w = \dfrac{e^{i\theta}}{1 - e^{2i\theta}} = \dfrac{1}{e^{-i\theta} - e^{i\theta}}$

$$= \frac{1}{-2 \cdot i\sin\theta} = \frac{i}{2\sin\theta}$$

$\Rightarrow W$ is purely imaginary

$\Rightarrow$ Locus of point is Y-axis.

100. (c) We have $z^2 = w^2$

$\Rightarrow z^2 - w^2 = 0$

$\Rightarrow\ (z-w)(z+w)=0$
⇒z=w,z=-w
$\Rightarrow |z|=|w|$
$\therefore B\subset A$

101. **(c)** Given, $x^2-6x-2=0$
$\Rightarrow x^{n-2}(x^2-6x-2)=0$
$\Rightarrow x^n-6x^{n-1}-2x^{n-2}=0$
When, $n=10$,
Then, $x^{10}-6x^9-2x^8=0$
$\Rightarrow\ -1x^{10}-2x^8=6x^9$
Since, α and β are roots of (1).
$\therefore \alpha^{10}-2\alpha^8=6\alpha^9$
and $\beta^{10}-2\beta^8=6\beta^9$
$\therefore (\alpha^{10}-\beta^{10})-2\alpha^8-\beta^8)\quad=6(\alpha^9-\beta^9)$
$\Rightarrow\quad a_{10}-2a_8=6a_9\ (\because a_n=\alpha^n-\beta^n)$
$$\Rightarrow\quad \frac{a_{10}-2a_8}{2a_9}=3$$

102. **(d)** For $x\in R, x\neq -1$
$(1+x)^{2016}+x(1+x)^{2015}+x^2(1+x)^{2014}+\cdots$
$$+x^{2016}=\sum_{i=0}^{2016} a_i\cdot x^i$$
Here, coefficient of x^{17}
$={}^{2016}C_{17}+{}^{2015}C_{16}+{}^{2014}C_{15}+\cdots+{}^{1999}C_0$
$={}^{2016}C_{1999}+{}^{2015}C_{1999}+{}^{2014}C_{1999}+\cdots+{}^{1999}C_{1999}$
$$={}^{2017}C_{2000}=\frac{(2017)!}{(2000)!\,(2017-2000)!}$$
$$=\frac{2017!}{(2000)!\,(17)!}$$

103. **(c)** Given, word "EQUATION"
Here, consonants-Q, T, N
and Vowel- E,U,A,I,O
$\therefore$ Number of word $={}^5C_3\times 3!\times {}^3C_2\times 3!$
$=10\times 6\times 3\times 6$
$=1080$

104. **(c)** According to the question,
Consider, $x_1+x_2+x_3+x_4=10$, where $x_i\geq 2$
$\Rightarrow (x_1-2)+(x_2-2)+(x_3-2)+(x_4-2)=2$
$\Rightarrow y_1+y_2+y_3+y_4=2$ where $y_i\geq 2$,
(where, $x_1-2=y_1, x_2-2=y_2, x_3-2=y_3$ and $x_4-2=y_4$)
$\therefore$ Number of ways $={}^{2+4-1}C_{4-1}={}^5C_3=10$

105. **(a)** We have,
$1!+2!+3!+4!+5!+\cdots+99!$
Since unit digit of 5! ,6! ,7!, ... 99 are zero.
$\therefore 1!+2!+3!+4!+5!+\cdots+99!$
$=1+2+6+24=33$
$\therefore$ Unit digit $=3$

106. **(c)** Given,
$(01\quad 2)\mathrm{M}=(1\quad 0\quad 0)$
and $(3\quad 4\quad 5)\mathrm{M}=(0\quad 1\quad 0)$
$\Rightarrow\quad (6\quad 8\quad 10)\mathrm{M}=(0\quad 2\quad 0)$
On subtracting Eq. (i) from Eq. (ii), we get $(6810)\mathrm{m}-(012)\mathrm{M}=(020)-(1\quad 0\quad 0)\Rightarrow (678)\mathrm{M}=(-120)$

107. **(c)** Given,

$A=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos t & \sin t\\ 0 & -\sin t & \cos t\end{bmatrix}$

Since, $\det(A-\lambda I_3)=0$

$\Rightarrow \begin{vmatrix}1-\lambda & 0 & 0\\ 0 & \cos t-\lambda & \sin t\\ 0 & -\sin t & \cos t-\lambda\end{vmatrix}=0$

$\Rightarrow \quad (1-\lambda)\{(\cos t-\lambda)^2+(\sin t)^2\}=0$

$\Rightarrow \quad (1-\lambda)(\cos^2 t+\lambda^2-2\lambda\cos t+\sin^2 t)=0$

$\Rightarrow \quad (1-\lambda)(\lambda^2-2\lambda\cos t+1)=0$

$\Rightarrow \quad \lambda^2-2\lambda\cos t+1-\lambda^3+2\lambda^2\cos t-\lambda=0$

$\Rightarrow \quad -\lambda^3+\lambda^2(1+2\cos t)-\lambda(1+2\cos t)+1=0$

$\Rightarrow \quad \lambda^3-\lambda^2(1+2\cos t)+\lambda(1+2\cos t)-1=0$

$\therefore \quad \lambda_1+\lambda_2+\lambda_3=(1+2\cos t)$

$\Rightarrow \quad 1+2\cos t=\sqrt{2}+1$ (Given)

$\Rightarrow \cos t=\frac{1}{\sqrt{2}} \Rightarrow t=-\frac{\pi}{4},\frac{\pi}{4}$

108. **(c)** Given A and B be two non-singular skew symmetric matrices.

Such that $AB=BA$

$\Rightarrow A^T=-A$ and $B^T=-B$ ... ... (i)

Since matrices are non-singular

$\therefore$ order of these matrices are even.

Now, $A^2B^2(A^TB)^{-1}(AB^{-1})^T$

$=A^2B^2(-AB)^{-1}(AB^{-1})^T$

$=-A^2B^2(B^{-1}A^{-1})(B^T)^{-1}A^T$

$=-A^2(B^2B^{-1})A^{-1}(-B)^{-1}(-A)$

$=-A^2B\cdot A^{-1}B^{-1}A$

$=-A^2B(BA)^{-1}A \qquad (\because AB=BA$

$=-A^2B(AB)^{-1}A$

$=-A^2BB^{-1}A^{-1}A$

$=-A^2(BB^{-1})(A^{-1}A)$

$=-A^2$

109. **(d)** We have, $a_n=ar^{n-1}$

$\Rightarrow \log a_n=\log(ar^{n-1})$

$\Rightarrow \log a_n=\log a+(n-1)\log r$

Now, $\begin{vmatrix}\log a_n & \log a_{n+1} & \log a_{n+2}\\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5}\\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix}$

$=\begin{vmatrix}\log a+(n-1)\log r & \log a+n\log r\\ \log a+(n+2)\log r & \log a+(n+3)\log r\\ \log a+(n+5)+\log r & \log a+(n+6)\log r\end{vmatrix}$

$\log a+(n+1)\log r$

$\log a+(n+4)\log r$

$\log a+(n+7)\log r$

On applying $R_2\to R_2-R_1, R_3\to R_3-R_1$

$\begin{vmatrix}\log a+(n-1)\log r & \log a+n\log r & \log a+(n+1)\log r\\ 3\log r & 3\log r & 3\log r\\ 6\log r & 6\log r & 6\log r\end{vmatrix}$

$=0$ (Since R_2 and R_3 are proportional)

110. **(a)** Given, $(A\cap C)\cup(B\cap C')=\phi$

$\Rightarrow A\cap C=\phi$......(i)

and

$B\cap C'=\phi$........(ii)

From Eqs. (i) and (ii), we get

$A \cap B = \phi$

111. **(b)** Since,

and

$n(A) \neq n(T)$

$n(A) \neq n(U)$

$\therefore$ There does not exist bijective mapping between A and T, U.

112. **(d)** Even numbers on die are 2,4,6.

and odd numbers on die are 1,3,5.

$$\therefore \quad P(\text{ even }) = \frac{3}{6} = \frac{1}{2}$$

$$\text{and} \quad P(\text{ odd }) = \frac{3}{6} = \frac{1}{2}$$

According to the question,

$$P\left(A_{(\text{wins})}\right) = \frac{1}{2} + \left(\frac{1}{2}\right)^4 \times \frac{1}{2} + \left(\frac{1}{2}\right)^8 \times \frac{1}{2} + \left(\frac{1}{2}\right)^{12} \times \frac{1}{2} + \cdots \infty$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^9 + \left(\frac{1}{2}\right)^3 + \cdots \infty$$

$$= \frac{\frac{1}{2}}{1 - \left(\frac{1}{2}\right)^4} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{1}{2} \times \frac{16}{15} = \frac{8}{15}$$

113. **(a)** We have,

Mean, $np = 4$

and variance, $npq = 2$

$$\therefore \quad \frac{npq}{np} = \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow \quad q = \frac{1}{2} \therefore p = \frac{1}{2} \text{ and } n = 8$$

$$\therefore P(x = 2) = {}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6$$

$$= {}^8C_2 \left(\frac{1}{2}\right)^8$$

$$= \frac{8 \times 7}{2 \times 1} \times \frac{1}{256} = \frac{7}{64}$$

114. **(b)** Given,

$S_n = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18$

$+\cot^{-1} 32 + \cdots$ to n^{th} term .

$= \cot^{-1} 2 \times 1^2 + \cot^{-1} 2 \times 2^2 + \cot^{-1} 2 \times 3^2$

$+\cot^{-1} 2 \times 4^2 + \cdots$

$\therefore n^{\text{th}}$ term $= \cot^{-1} 2n^2$

$$\Rightarrow t_n = \tan^{-1}\left(\frac{1}{2n^2}\right)$$

$$= \tan^{-1}\left(\frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)}\right)$$

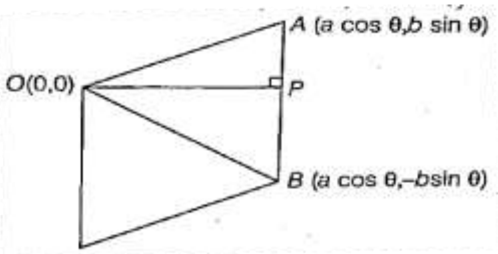
$$= \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$$\therefore S_n = \tan^{-1}(2n+1) - \tan^{-1}(1)$$

$$\therefore \lim_{n \to \infty} S_n = \tan^{-1}(\infty) - \tan^{-1}(1)$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

115. **(a)** We have vertices of parallelogram $O(0,0)$, $A(a\cos\theta, b\sin\theta)$ and $B(a\cos\theta, -b\sin\theta)$



Here, $OA = OB$

$\therefore$ Area of parallelogram $= 2 \times$ area of $\triangle OAB$

$$= 2 \times \left(\frac{1}{2} \times OP \times AB\right)$$

$$= 2 \times \frac{1}{2} \times a\cos\theta \times 2b\sin\theta$$

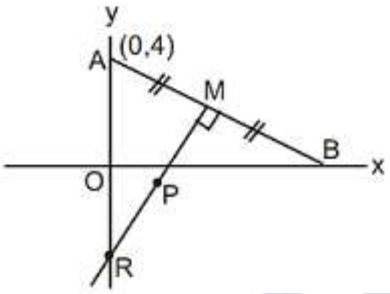
$$= 2 \times ab(\sin\theta\cos\theta)$$

$$= ab\sin 2\theta$$

$\therefore$ Area will be maximum when $\theta = \frac{\pi}{4}$.

$\Rightarrow$ Maximum Area $= ab$ when $\theta = \frac{\pi}{4}$

116. **(a)** Let point $B = (2p, 0)$



$\therefore$ Coordinates of M are $(p, 2)$

Now, Slope of $AB = \frac{4-0}{0-2p} = -\frac{2}{p}$

$\therefore$ Slope $MR = \frac{p}{2}$

$\therefore$ Equations of $MR = y - 2 = \frac{p}{2}(x - p)$

On putting $x = 0$, we get

$$y - 2 = -\frac{p^2}{2} \Rightarrow y = 2 - \frac{p^2}{2}$$

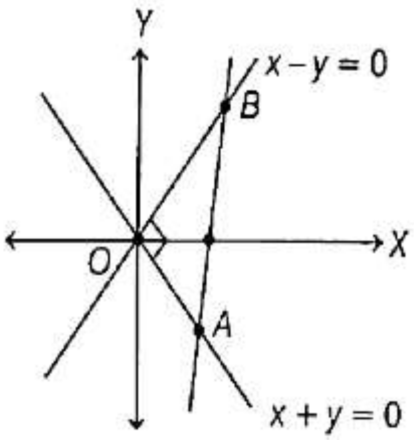
$\therefore$ Coordinates of R is $\left(0, 2 - \frac{p^2}{2}\right)$

$\therefore$ Coordinates of P is $\left(\frac{p}{2}, \frac{2-\frac{p^2}{2}+2}{2}\right)$

i.e. $\left(\frac{p}{2}, 2 - \frac{p^2}{4}\right)$

$\therefore y = 2 - x^2 \Rightarrow y + x^2 = 2$

117. **(b)**



Let coordinates of A are $(\alpha, -\alpha)$ and coordinates of B are (β, β)

$\therefore \beta+\alpha=2h$ and $\beta-\alpha=2k$

Now, Area of $\triangle AOB=\frac{1}{2}\times OA\times OB$

$$=\frac{1}{2}\sqrt{2\alpha^2}\sqrt{2\beta^2}=|\alpha\beta|=c$$

$$\Rightarrow \quad \alpha^2\beta^2=c^2$$

$$\Rightarrow \quad 16\alpha^2\beta^2=16c^2$$

$$\Rightarrow \quad \{(\beta+\alpha)^2-(\beta-\alpha)^2\}^2=16c^2$$

$$\Rightarrow \quad (4h^2-4k^2)^2=16c^2$$

$$\Rightarrow \quad h^2-k^2=c^2$$

$\therefore$ Locus of the mid point P of MR is $(x^2-y^2)^2=c^2$

118. **(a)** Given equation of parabola

$6y=2a^3x^2+3a^2x-12a$

$\Rightarrow 6y+12a$

$$\Rightarrow =2a\left(a^2x^2+\frac{3}{2}ax\right)$$

$$=2a\left(a^2x^2+2\cdot\frac{3}{4}ax+\frac{9}{16}\right)-\frac{9}{8}a$$

$$\Rightarrow 6y+\frac{105}{8}a$$

$$\Rightarrow =2a\left(ax+\frac{3}{4}\right)^2$$

Let vertices are (h,k)

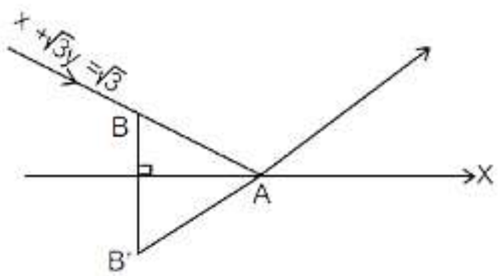
$$\therefore \quad h=\frac{-3}{4a} \text{ and } k=\frac{-35a}{16}$$

$$\therefore \quad hk=\frac{105}{64}$$

$\therefore$ locus of the vertices is

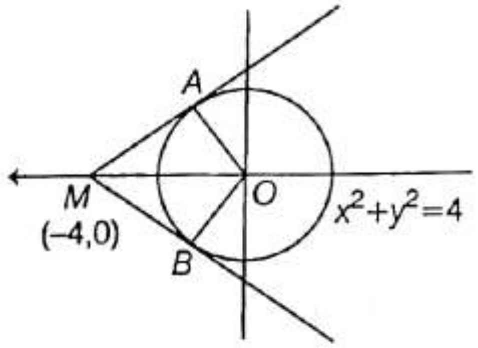
$$xy=\frac{105}{64}$$

119. **(b)** Given, equation of ray of light $x+\sqrt{3}y=\sqrt{3}$



On putting $y = 0$, we get $x = \sqrt{3}$
$\therefore$ Coordinates of A are $(\sqrt{3}, 0)$
Now, let $B(0,1)$ then $B' = (0, -1)$
$\therefore$ Equation of AB' is $\frac{y-0}{0+1} = \frac{x-\sqrt{3}}{\sqrt{3}-0}$
$\Rightarrow \sqrt{3}y = x - \sqrt{3}$

120. (a)



Given, equation of circle $x^2 + y^2 = 4$
Centre $= (0,0)$ and radius $= 2$
$\therefore\ MA = \sqrt{(4)^2 - (2)^2} = \sqrt{12} = 2\sqrt{3}$
$\therefore$ Area of quadrilateral $MAOB$

$= 2 \times$ area of $\triangle MAO$

$= 2 \times \frac{1}{2} \times MA \times OA$

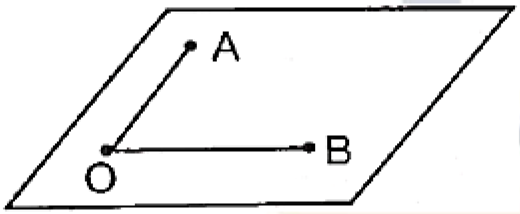
$= 2\sqrt{3} \times 2$

$= 4\sqrt{3}$ sq. units.

121. (b) Given, equation of parabola $y^2 = x$

We know that for $y^2 = 4ax$
if we draw a normal from $(h, 0)$ on it then condition for three normal is $h > 2a$
$\therefore d > \frac{1}{2}$

122. (b)



According to question the coordinates of A and B are $(0, b, c)$ and $(a, 0, c)$ respectively.
Now, perpendicular vector $\mathbf{OA} \times \mathbf{OB}$

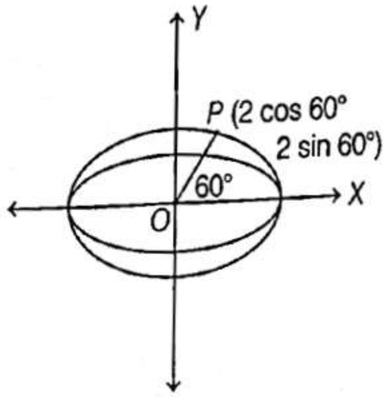
$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & b & c \\ a & \mathbf{0} & c \end{vmatrix}$

$= \hat{\mathbf{i}}(bc - 0) - \hat{\mathbf{j}}(0 - ac) + \hat{\mathbf{k}}(0 - ab)$

$= bc\hat{\mathbf{i}} + ac\hat{\mathbf{j}} - ab\hat{\mathbf{k}}$

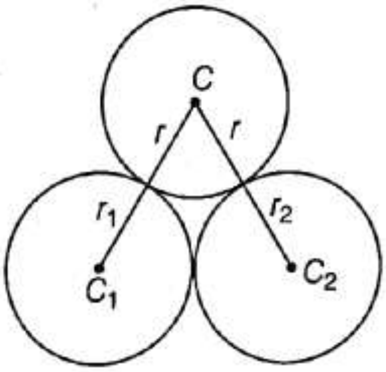
$\therefore$ Required equation of plane is $bcx + acy - abz = 0$

123. (b) Given, equation of ellipse $x^2 + 2y^2 = 4$
$\Rightarrow \frac{x^2}{4} + \frac{y^2}{2} = 1$........(i)



Now, equation of auxiliary circle is $x^2+y^2=4$.
$\therefore$ Point on the auxiliary circle with eccentric angle 60° is $P(2\cos 60^\circ, 2\sin 60^\circ)$
i.e., $P(1,\sqrt{3})$

124. **(b)** According to the question, we have.



and
$CC_1 = r + r_1$
$CC_2 = r + r_2$
$\therefore\ C_2 - CC_1 = (r+r_2)-(r+r_1) = r_2 - r_1$
$\Rightarrow$ Locus of centre is Hyperbola.

125. **(c)** Let direction cosines of line is (l,l,l).
$\therefore\quad l^2+l^2+l^2=1$
$\Rightarrow\quad 3l^2 = 1 \Rightarrow l = \dfrac{1}{\sqrt{3}}$
$\therefore$ Direction cosines of line is $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$
Now, equation of a line passing through $P(2,-1,2)$
$\dfrac{x-2}{1/\sqrt{3}} = \dfrac{y+1}{1/\sqrt{3}} = \dfrac{z-2}{1/\sqrt{3}} = k$
$\Rightarrow x = k+2 y = k-1$ and $z = k+2$
Since line (i) meets the plane
$2x+y+z=9$ at Q
$\therefore\ 2(k+2)+(k-1)+(k+2)=9$
$\Rightarrow\ 2k+4+k-1+k+2=9$
$\Rightarrow\ 4k+5=9$
$\Rightarrow\ 4k=4\ \Rightarrow\ k=1$
$\therefore$ Coordinates of Q are $(3,0,3)$
$\therefore\ PQ = \sqrt{(3-2)^2+(0+1)^2+(3-2)^2}$
$= \sqrt{1+1+1} = \sqrt{3}$ units.

126. **(b)** Given, $y = \sin^{-1}\left\{\frac{5x+12\sqrt{1-x^2}}{13}\right\}$

$= \sin^{-1}\left\{\frac{5x}{13}+\frac{12\sqrt{1-x^2}}{13}\right\}$

Let $\sin\theta_1 = \frac{5}{13}$ and $\cos\theta_2 = x$

$\therefore y = \sin^{-1}(\sin\theta_1 \cdot \cos\theta_2 + \cos\theta_1 \cdot \sin\theta_2)$

$= \sin^{-1}\{\sin(\theta_1+\theta_2)\}$

$= \theta_1 + \theta_2 = \sin^{-1}\frac{5}{13} + \cos^{-1} x$

$\Rightarrow \quad y_1 = -\frac{1}{\sqrt{1-x^2}}$

and $\quad y_2 = \frac{-x}{(1-x^2)\sqrt{1-x^2}}$

$\Rightarrow \quad y_2(1-x^2) = xy_1$

$\Rightarrow \quad y_2(1-x^2) - xy_1 = 0$

$\therefore \quad a = 1, b = -1$

$\therefore (a,b) = (1,-1)$

127. **(c)** Given, $2f(x) + 3f(-x) = 15 - 4x$……..(i)

On replacing x by - x in Eq. (i), we get

$2f(-x) + 3f(x) = 15 + 4x$…(ii)

On solving Eqs. (i) and (ii), we get

$f(x) = 4x + 3$

$\therefore \quad f(2) = 4 \times 2 + 3 = 11$

128. **(c)** Given, $f_1(x) = x, f_2(x) = 2 + \log_e x$

Now, Let $h(x) = f_2(x) - f_1(x)$

$= (2 + \log_e x) - x$

$= 2 + \log_e x - x$

Here, $h(0^+) < 0, h(l) > 0, h(e) > 0$ and $h(e^2) < 0$ and value of $h(x)$ for all $x \geq e^2$ is negative.

$\therefore h(x) = 0$ has two roots is $(0,1)$ and (e, e^2)

129. **(c)** Given equation, $6^x + 8^x = 10^x$

$\Rightarrow \left(\frac{6}{10}\right)^x + \left(\frac{8}{10}\right)^x = 1$

$\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x = 1$

Let $\frac{3}{5} = \sin\theta \Rightarrow \frac{4}{5} = \cos\theta$

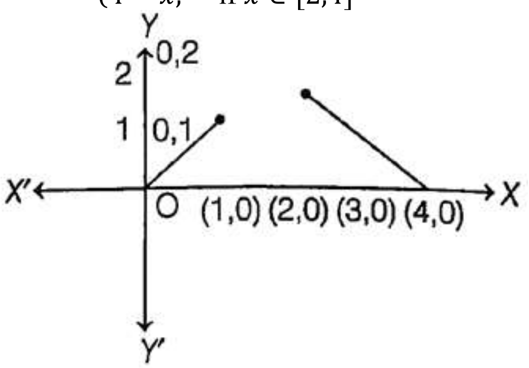
$\therefore (\sin\theta)^x + (\cos\theta)^x = 1$

it is possible only $x = 2$

$\therefore$ given equation has exactly one real root.

130. **(b)** We have, $f: D \to R$ where $D = [0,1] \cup [2,4]$

$f(x) = \begin{cases} x, & \text{if } x \in [0,1] \\ 4-x, & \text{if } x \in [2,4] \end{cases}$



Clearly $f(x)$ is increasing in $[0,1]$ and decreasing in $[2,4]$

$\therefore f(x)$ is neither continuous and nor differentiable in D.

$\therefore$ Rolles theorem is not applicable to f in D

131. **(b)** $f(x)$ be a continuous periodic function with period T

Given,

$$I=\int_a^{a+T} f(x)dx$$

$$I=\int_0^T f(x)dx$$

Hence, I does not depend on ' a '.

132. **(c)** We have, $b=\int_0^1 \frac{e^t}{t+1}dt$

Let

$$I=\int_{a-1}^{a}\frac{e^{-t}}{t-a-1}dt$$

$$I=\int_{a-1}^{a}\frac{e^{-(a+a-1-t)}}{a+a-1-t-a-1}dt$$

$$I=\int_{a-1}^{a}\frac{e^{t-2a+1}}{a-2-t}dt$$

put $t-(a-1)=x \Rightarrow dt=dx$

when $t=a, x=1$ and $t=a-1 \Rightarrow x=0$ Type equation here.

$$\therefore \quad I=\int_0^1 \frac{e^{x+a-1-2a+1}}{a-2-x-a+1}dx$$

$$\Rightarrow \quad I=\int_0^1 \frac{e^x \cdot e^{-a}}{-(x+1)}dx$$

$$\Rightarrow \quad I=-e^{-a}\int_0^1 \frac{e^x}{x+1}dx$$

$$\Rightarrow I=-e^{-a}(b)\left[\because \int_0^1 \frac{e^t}{t+1}dt=b\right]$$

$\Rightarrow I=-be^{-a}$

133. **(a)** We have,

$f(x)=\log_e(1+e^{10x})-\tan^{-1}(e^{5x})$

$$\Rightarrow f'(x)=\frac{10e^{10x}}{1+e^{10x}}-\frac{5e^{5x}}{1+e^{10x}}$$

$$\Rightarrow f'(x)=\frac{10e^{10x}-5e^{5x}}{1+e^{10x}}$$

$$\Rightarrow \ f'(0)=\frac{10-5}{1+1}=\frac{5}{2}$$

We know that,

$f(x+\Delta x)-f(x)=f'(x)dx$

$$\Rightarrow f(x+\Delta x)-f(x)=\frac{5}{2}\times 0.2[\because dx=0.2]$$

$\Rightarrow \Delta f(x)=0.5$

134. **(b)** We have,

$[\because f(c)=c]$

$\Rightarrow \ (c-1)^2=(\sqrt{c})^2$

$\Rightarrow \qquad c^2-2c+1=c$

$\Rightarrow \qquad c^2-3c+1=0$

$$\Rightarrow \qquad c=\frac{3\pm\sqrt{5}}{2}$$

$\therefore f$ has unique fixed point in $[1,\infty]$

135. **(c)** Let $L=\lim\limits_{x\to\infty}\left(\frac{3x-1}{3x+1}\right)^{4x}$

$L = e^{\lim_{x\to\infty}\left(\frac{3x-1}{3x+1}-1\right)4x}$

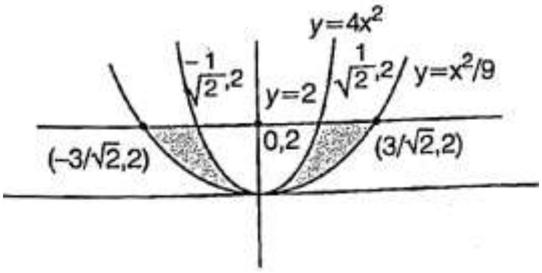
$L = e^{\lim_{x\to\infty}\left(\frac{3x-1-3x-1}{3x+1}\right)4x}$

$L = e^{\lim_{x\to\infty}\frac{-2\times 4}{3+\frac{1}{x}}}$

$L = e^{-8/3}$

136. **(a)** Given curve, $y = 4x^2, y = \frac{x^2}{9}$ and $y = 2$

Graph of given curve



Area of shaded region

$$= 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy$$

$$= 2\int_2^0 \frac{\sqrt{y}}{2} dy$$

$$= 5 \times \frac{2}{3}\left[y^{3/2}\right]_0^2$$

$$= \frac{10}{3}\left[2^{3/2} - 0\right]$$

$$= \frac{10}{3} \times 2\sqrt{2}$$

$$= \frac{20}{3}\sqrt{2} \text{ sq unit.}$$

137. **(c)** We have,

$a(\alpha \times \beta) + b(\beta \times \gamma) + c(\gamma + \alpha) = 0$

Clearly $a(\alpha \times \beta), (\beta \times \gamma)$ and $(\gamma + \alpha)$ are coplanar vector $\therefore \alpha, \beta$ and γ are also coplanar

138. **(b)** Given equation

$y^2 = 2x^3$

$\Rightarrow 2y\frac{dy}{dx} = 6x^2$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{4}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(h,k)} = \frac{3h^2}{k}$

Slope of tangent is perpendicular to line $4x = 3y$

$\therefore \left(\frac{3h^2}{k}\right)\left(\frac{4}{3}\right) = -1$

$\Rightarrow 4h^2 = -k \quad \ldots.(i)$

and

$k^2 = 2h^3$

From Eqs. (i) and (ii)

$(4h^2)^2 = 2h^3$

$\Rightarrow 16h^4 = 2h^3$

$\Rightarrow h = \frac{1}{8}$

$\therefore k = -4\left(\frac{1}{8}\right)^2$

$\Rightarrow k = -4 \times \frac{1}{64} = \frac{-1}{64}$

$\therefore (h,k) = \left(\frac{1}{8}, \frac{-1}{16}\right)$ only

139. **(d)** Given, $(bc + ca + ab)^6$

General term $= \frac{6!}{p!\,q!\,r!}(bc)^p(ca)^q(ab)^r$

$= \frac{6!}{p!\,q!\,r!}(a)^{q+r}(b)^{p+r}(c)^{p+q}$

Coefficient of $a^3b^4c^5$ in the expansion $(bc + ca + ab)^6$

$\therefore\ q + r = 3, p + r = 4, p + q = 5$

$= 2(p + q + r) = 12$

$= p + q + r = 6$

$\therefore p = 3, q = 2, r = 1$

$\therefore$ Coefficient of $a^3b^4c^5$ is $\frac{6!}{3!\,2!\,1!} = 3\left(\frac{6!}{3!\,3!}\right)$

140. **(c)** Given a, b, c are in G.P and $\log\left(\frac{5c}{2a}\right), \log\left(\frac{7b}{5c}\right). \log\left(\frac{2a}{7b}\right)$ and A.P

$\therefore b^2 = ac$

and $2\log\left(\frac{7b}{5c}\right) = \log\left(\frac{5c}{2a}\right) + \log\left(\frac{2a}{7b}\right)$

$\Rightarrow \frac{49b^2}{25c^2} = \frac{5c}{7b}$

$\Rightarrow (7b)^3 = (5c)^3$

$\Rightarrow 7b = 5 \Rightarrow c = \frac{7}{5}b$

and $b^2 = ac$

$\Rightarrow a\left(\frac{7}{5}b\right)$

$\Rightarrow b = \frac{7}{5}a$

Sides are $\frac{5b}{7}, b, \frac{7}{5}b$

$\therefore a, b, c$ are the length of the sides of scalene triangle.

141. **(b)** We have,

$$\begin{vmatrix} a^2+10 & ab & ac \\ ab & b^2+10 & bc \\ ac & bc & c^2+10 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a(a^2+10) & ab^2 & ac^2 \\ a^2b & b(b^2+0) & bc^2 \\ a^2c & b^2c & c(c^2+10) \end{vmatrix}$$

Taking common a, b, c from $\vec{R}_1, R_2$ and $\vec{R}_3$ respectively

$$= \frac{abc}{abc}\begin{vmatrix} a^2+10 & b^2 & c^2 \\ a^2 & b^2+10 & c^2 \\ a^2 & b^2 & c^2+10 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$= \begin{vmatrix} a^2+b^2+c^2+10 & b^2 & c^2 \\ a^2+b^2+c^2+10 & b^2+10 & c^2 \\ a^2+b^2+c^2+0 & b^2 & c^2+10 \end{vmatrix}$$

$$= (a^2+b^2+c^2+10)\begin{vmatrix} \mathrm{I} & b^2 & c^2 \\ \mathrm{I} & b^2+10 & c^2 \\ \mathrm{I} & b^2 & c^2+10 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (a^2+b^2+c^2+10)\begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{vmatrix}$$

$$= (a^2+b^2+c^2+10)(100)$$

$\because$ If is divisible by 100.

142. **(b)** We have,

$S = \{(x, y): y = x + 1, 0 < x < 2\}$

For reflexive $(x, x): \in S$

$x = x + 1 \notin S$

$\therefore$ S is not reflexive

$\therefore$ S is not equivalence relation

$T = \{(x, y): x - y$ is an integer $\}$

For reflexive $(x, x) \in T$

$x = x - x = 0 \in \mathbb{E}R$

T is reflexive

For symmetric

$(x, y) = x - y$ is an integer

$(y, x) = y - x$ is also integer

$\therefore$ T is symmetric

For Transitive

$(x, y) \in T, (y, z) \in Ta \Rightarrow (x, z) \in T$

$(x, y) = x - y$ is an integer

$(y, z) = y - z$ is also integer

$\therefore (x - y) + (y - z) = x - z$ is an integer

$\therefore (x, y) \in T$

Hence, T is an equivalence relation.

143. **(d)** Given plane,

$P_1 = lx + my = 0$ and $P_2 = z = 0$

Plane through common line of P_1 and P_2 :

$\therefore\ P_3 = lx + my + nz = 0$

angle between P_1 and $P_3 = \alpha$

$$\therefore\ \cos\alpha = \frac{l^2+m^2}{\sqrt{l^2+m^2+n^2}\sqrt{l^2+m^2}}$$

$$\Rightarrow \cos\alpha = \sqrt{\frac{l^2+m^2}{l^2+m^2+n^2}}$$

$$\Rightarrow \cos^2\alpha = \frac{l^2+m^2}{l^2+m^2+n^2}$$

$$\Rightarrow \sec^2\alpha = \frac{l^2+m^2+n^2}{l^2+m^2} = 1 + \frac{n^2}{l^2+m^2}$$

$$\Rightarrow \quad (\sec^2\alpha - 1)(l^2+m^2) = n^2$$

$$\Rightarrow \quad n^2 = \tan^2\alpha(l^2+m^2)$$

$$= n = \pm\sqrt{(l^2+m^2)}\tan\alpha$$

$\therefore$ Equation becomes

$lx + my \pm z\tan\alpha\sqrt{l^2+m^2} = 0$

144. **(c)** Given ellipse

and

$x^2 + 2y^2 - 6x - 12y + 20 = 0$

$2x^2 + y^2 - 10x - 6y + 15 = 0$

The point of intersection of the given ellipse is

$$(x^2+2y^2-6x-12y+20)+\lambda(2x^2+y^2-10x-6y+15)=0$$

$$(1+2\lambda)x^2+(2+\lambda)y^2-(6+10\lambda)x-(12+6\lambda)y+20+5\lambda=0$$

The points lie on circle

$\therefore$ Equation becomes circle

$\therefore\ 1+2\lambda=2+\lambda \Rightarrow \lambda=1$

$\therefore$ Equation of circle is

$3x^2+3y^2-16x-18y+35=0$

$\therefore$ Centre of circle is $\left(\frac{8}{3},3\right)$

145. (a)

We have, $I=\int_{\pi/4}^{\pi/3}\frac{\sin x}{x}dx$

Let $f(x)=\frac{\sin x}{x}$

$f(x)$ is decreasing functions

$$\Rightarrow f(\pi/3)<f(x)<f(\pi/4)$$

$$\Rightarrow \frac{3\sqrt{3}}{2\pi}<f(x)<\frac{2\sqrt{2}}{\pi}$$

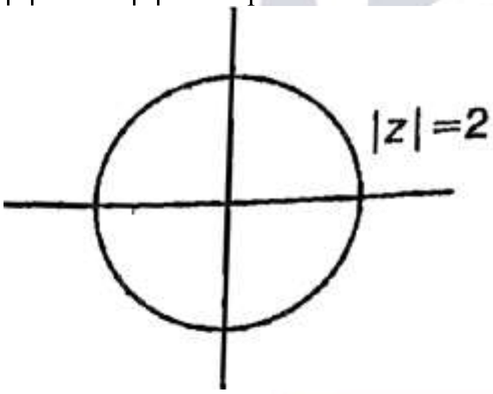
$$\Rightarrow \frac{3\sqrt{3}}{2\pi}\int_{\pi/4}^{\pi/3}dx<I<\frac{2\sqrt{2}}{\pi}\int_{\pi/4}^{\pi/3}dx$$

$$\Rightarrow \frac{3\sqrt{3}}{2\pi}\left(\frac{\pi}{3}-\frac{\pi}{4}\right)<I<\frac{2\sqrt{2}}{\pi}(\pi/3-\pi/4)$$

$$\Rightarrow \frac{\sqrt{3}}{8}<I<\frac{\sqrt{2}}{6}$$

146. (b, c) have $|z+i|-|z-1|=|z|-2$

$|z|-2 \Rightarrow |z|=2$ represent a circle.



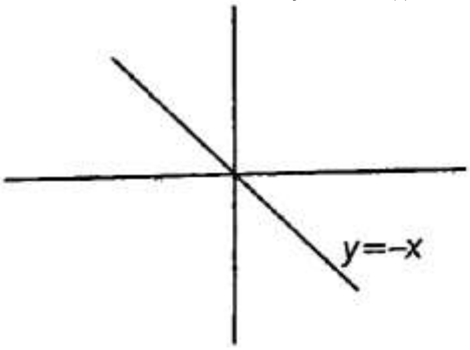
$$|z+i|-|z-1|=0$$

$$\Rightarrow |z+i|=|z-1|$$

$$\Rightarrow x^2+(y+1)^2=(x-1)^2+y^2$$

$$\Rightarrow x^2+y^2+2y+1=x^2-2x+1+y^2$$

$$=y=-x\text{(i)}$$



$\therefore\ |z| = 2 \Rightarrow x^2 + y^2 = 4 \ldots\ldots$(ii)

From Eqs. (i) and (ii)

$$x^2 + x^2 = 4$$

$$\Rightarrow \quad 2x^2 = 4x = \pm\sqrt{2}$$

and $y == \mp\sqrt{2}$

$$\therefore \quad z = x + iy$$

$$z = \pm\sqrt{2} \mp \sqrt{2i}$$

$$z = \pm\sqrt{2}(1 \mp i)$$

and $$z = \sqrt{2}(1 - i)$$

$$z = \sqrt{2}(-1 + i)$$

147. **(d)** We have,

$$\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 7x-2 & 17x+6 & 12x-1 \end{vmatrix} = 0$$

$R_3 \to R_3 - 3R_1 - 2R_2$, we get

$$\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\therefore$ infinite value of x is possible.

148. **(b)** We have

$7^{7^{7^{7}}}$ (22 time 7)

$\Rightarrow 7^{7^{7^{7}}}$ is an odd number

$$\therefore 7^{(2m+1)} = 7^{2m} \times 7$$

$$= (7^2)^m \times 7$$

$$= (49)^m \times 7$$

$$= (48+1)^m \times 7$$

$$= 7(1 + 48k)$$

$$= 7 + (48 \times 7k$$

$\therefore$ remainder $= 7$

149. **(c, d)** (a) $I_1 = \int_{-2}^{2} \frac{dx}{4+x^2}$

It is possible to put $x = \tan t$

Option (a) is incorrect.

(b) $I_2 = \int_0^1 \sqrt{x^2+1}dx$

It is possible to put $x = \tan t$

$\therefore$ Option (b) is also incorrect

Hence, option (c) and (d) are correct

150. **(a, c)** Let equation of plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\text{Centroid } = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

$$\therefore \quad \frac{a}{3} = 1, \frac{b}{3} = r, \frac{c}{3} = r^2$$

$$\Rightarrow \quad a = 3, b = 3rc = 3r^2$$

Equation of plane be

$$\frac{x}{3} + \frac{y}{3r} + \frac{z}{3r^2} = 1$$

Now plane is passes through $(5, 5, -12)$

$\therefore \quad \frac{5}{3}+\frac{5}{3r}-\frac{12}{3r^2}=1$

$\Rightarrow \quad 5r^2+5r-12=3r^2$

$\Rightarrow \quad 2r^2+5r-12=0$

$\Rightarrow \quad 2r^2+8r-3r-12=0$

$\Rightarrow \quad =(2r-3)(r+4)=0$

$\Rightarrow \quad r=3/2,-4$

151. **(a)** Let the equation of circle is

$(x-\alpha)^2+(\gamma-\beta)^2=r^2$

$\therefore \quad x=\alpha+r\cos\theta$

$\therefore \quad y=\beta+r\sin\theta$

$P\equiv(\alpha+r\cos\theta,\beta+r\sin\theta)$

Let $\theta=(a,b),R=(h,k)$

$\therefore \quad h=\frac{\alpha+r\cos\theta}{2},k=\frac{\beta+r\sin\theta}{2}$

$r\cos\theta=2h-\alpha \text{and} r\sin\theta=2k-\beta$

$\therefore \quad r^2(\cos^2\theta+\sin^2\theta)=(2h-\alpha)^2+(2k-\beta)^2$

$$=\left(h-\frac{\alpha}{2}\right)^2+\left(k-\frac{\beta}{2}\right)^2=\left(\frac{r}{2}\right)^2$$

$\Rightarrow$ Locus of R is $\left(x-\frac{\alpha}{2}\right)^2+\left(y-\frac{\beta}{2}\right)^2=\frac{r^2}{4}$ Which represents a circle the locus of R is circle.

152. **(a)** Let $I=\lim_{n\to\infty}\left\{\frac{\sqrt{n}}{\sqrt{n^3}}+\frac{\sqrt{n}}{\sqrt{(n+4)^3}}+\frac{\sqrt{n}}{\sqrt{(n+8)^3}}\right.$

$+\frac{\sqrt{n}}{\sqrt{[n+4(n-1)]^3}}$

$\Rightarrow \quad I=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{\sqrt{n}}{\sqrt{(n+4r)^3}}$

$\Rightarrow \quad I=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{n\sqrt{n}}{\sqrt{(n+4r)^3}}$

$\Rightarrow \quad I=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{\sqrt{\left(1+\frac{4r}{n}\right)^{3/2}}}$

$\Rightarrow \quad I=\int_0^1\frac{dx}{(1+4x)^{3/2}}$

$\Rightarrow \quad I=\left[\frac{(1+4x)^{-3/2+1}}{4(-3/2+1)}\right]_0^1$

$\Rightarrow \quad I=\frac{-1}{2}[(1+4x)-1/2]_0^1$

$\Rightarrow \quad I=\frac{-1}{2}[5^{-1/2}-1]$

$\Rightarrow \quad I=\frac{-1}{2}\left[\frac{1}{\sqrt{5}}-1\right]$

$\Rightarrow \quad I=\frac{-1}{2}\left[\frac{1-\sqrt{5}}{\sqrt{5}}\right)$

$\Rightarrow \quad I=\frac{\sqrt{5}-1}{2\sqrt{5}}=\frac{\sqrt{5}(\sqrt{5}-1)}{10}=\frac{5-\sqrt{5}}{10}$

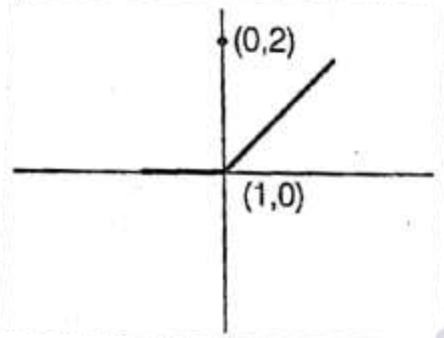
153. **(a, d)** We have, $f(x) = \begin{cases} 0, & \text{if} \quad -1 \le x < 0 \\ 1, & \text{if} \quad x = 0 \\ 2, & \text{if} \quad 0 < x \le 1 \end{cases}$

$$F(x) = \int_{-1}^{x} f(t)dt$$

$$F(x) = \int_{-1}^{0} 0dt + \int_{0}^{1} 2dt$$

$$F(x) = \begin{cases} 0, & -1 \le x \le 0 \\ 2x, & 0 < x \le 1 \end{cases}$$

Graph of $i(x)$ is



Clearly $F(x)$ is continuous in $[-1,1]$

$F'(x)$ does not exist at $x = 0$

154. **(b, c)** We have,

$$f(x) = \tan^{-1} - \frac{1}{2}\log x \text{ on } \left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$$

$$f'(x) = \frac{1}{1+x^2} - \frac{1}{2x}$$

$$f'(x) = \frac{2x - 1 - x^2}{2x(1+x^2)}$$

$$f'(x) = \frac{-(x^2 - 2x + 1)}{2x(1+x^2)}$$

$$f'(x) = \frac{-(x-1)^2}{2x(1+x^2)}$$

$f'(x)$ is decreasing function $\forall x > 0$

$f_{\max}$ at $x = \frac{1}{\sqrt{3}}$ and $f_{\min}$ at $x = \sqrt{3}$

$$\therefore f\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\frac{1}{\sqrt{3}} - \frac{1}{2}\log\frac{1}{\sqrt{3}}$$

$$= \frac{\pi}{6} + \frac{1}{4}\log 3$$

$$f(\sqrt{3}) = \tan^{-1}\sqrt{3} - \frac{1}{2}\log\sqrt{3}$$

$$= \frac{\pi}{3} - \frac{1}{4}\log 3$$

$$\therefore f_{\max} = \frac{\pi}{6} + \frac{1}{4}\log 3$$

$$f_{\min} = \frac{\pi}{3} - \frac{1}{4}\log 3$$

155. **(d)** We have, f and g be periodic function with periods T_1 and T_2 respectively $f + g$ is periodic if $T_1 = T_2$