Physics

1. **(c)** Electric field due to line charge is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 d}$$

where, $\lambda =$ linear charge density.

$$\lambda = \frac{q}{l} \Rightarrow q = \lambda l$$

Electric force due to one wire on another wire,

$$F_E = q \cdot E$$

$$= \lambda l \cdot \frac{\lambda}{2\pi\varepsilon_0 d}$$

$$\Rightarrow F_E = \frac{\lambda^2 l}{2\pi\varepsilon_0 d}$$

Now, current due to moving charged wire is given as

$$I = \frac{dq}{dt} = \frac{d}{dt}(\lambda dl) \left[\because v = \frac{dl}{dt}\right]$$

$$= \lambda v$$

Magnetic force on one wire due to another wire,

$$F_B = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Since, both forces balance each other, thus

$$F_E = F_B$$

$$\Rightarrow \frac{\lambda^2 l}{2\pi\varepsilon_0 d} = \frac{\mu_0(\lambda v)(\lambda v)l}{2\pi d} \quad [\because I_1 = I_2 = I]$$

$$\Rightarrow \frac{1}{\varepsilon_0\mu_0} = v^2$$

$$v = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = c$$

2. **(b)** We know that, $E = -\frac{dV}{dr}$

Thus, electric field from (-4 to -2),

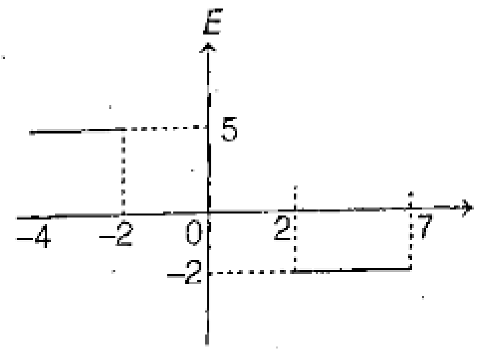
$$E = -\left(\frac{10-0}{4-(-2)}\right) = \frac{10}{2} = 5\ \mathrm{N/C}$$

Electric field from (-2 to 2),

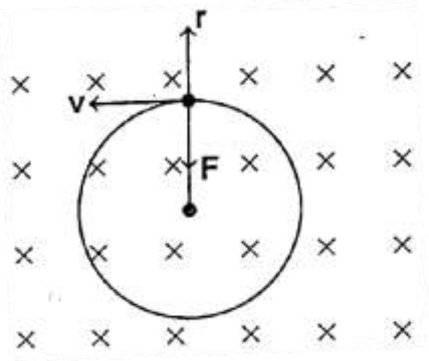
$$E = -\left(\frac{10-10}{2-(-2)}\right) = 0$$

Electric field from (2 to 7),

$$E = -\left(\frac{10-0}{2-7}\right) = -2\ \mathrm{N/C}$$



3. **(a)** According to the question, given situation can be represented by the figure below.



From the Lorentz force, we have
$F = q(v \times B)$
Hence, force is acting directly towards the centre.
Thus, both angular momentum (L) and torque (T) will have the same direction. i.e. **T||L**.

4. (a) Given, $\mathbf{B} = B_0\left(2 - \frac{x}{a}\right)\hat{\mathbf{k}}$
Force on a current carrying wire is given as
$$\Rightarrow \quad dF = IB \cdot dl$$
$$\Rightarrow \quad \int dF = \int IB_0\left(2 - \frac{x}{a}\right)dx$$
$$F = IB_0 \int_a^{2a} \left(2 - \frac{x}{a}\right)dx$$
$$= IB_0 \left|2x - \frac{x^2}{2a}\right|_a^{2a} = IB_0\left(\frac{a}{2}\right)$$
Therefore, the value of k is 1 .

5. (b) Given, change in flux, $d\phi = 5$ Wb
Emf due to change in flux is given as
$E = \frac{d\phi}{dt} \Rightarrow E \cdot dt = 5$
Now, it is given that, $R = 100\Omega$
Thus, current, $I = \frac{V}{R} = \frac{E}{R}$....(i)
Also,
$I = \frac{dq}{dt}$.........(ii)
where, dq is the charge flowing per unit time. Thus, from Eqs. (i) and (ii), we have
$$\frac{E}{R} = \frac{dq}{dt} \Rightarrow \frac{E \cdot dt}{R} = dq$$
$$\Rightarrow \frac{5}{100} = dq \Rightarrow dq = 0.05\text{C}$$

6. (b) At any moment, the phase difference between current and voltage is given by
$\tan\phi = \frac{X_L}{R}$
Since, voltage leads current by $\frac{\pi}{4}$, thus given circuit will be RL series circuit
$\therefore \tan\frac{\pi}{4} = \frac{X_L}{R} \Rightarrow R = X_L = \omega L$
From option (b), $R = 1\text{k}\Omega = 1000\Omega$
$$\therefore X_L = \omega L \qquad [\because \omega = 100\text{rad/s}]$$
$$= 100 \times 10$$
$$= 1000\Omega$$
Thus, the condition is satisfied. Hence option (b) is correct.

7. (a) The constant voltage source provides a constant voltage to the load resistance regardless of variations or changes, in the load resistance. For this to happen, the source must have an internal resistance which is very low as compared to the resistance of the load.

8. (a) de-Broglie wavelength of a charged particle having energy ε is given by

$\lambda = \frac{h}{\sqrt{2m\varepsilon}}$
Thus,
$\lambda_e = \frac{h}{\sqrt{2m_e\varepsilon_1}}$
$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha\varepsilon_2}}$
$\lambda_p = \frac{h}{\sqrt{2m_p\varepsilon_3}}$
Since, $\lambda_e = \lambda_\alpha = \lambda_p$ and $m_\alpha > m_p > m_e$
Thus, we can say
$\frac{1}{\sqrt{m_\alpha\varepsilon_2}} = \frac{1}{\sqrt{m_p\varepsilon_3}} = \frac{1}{\sqrt{m_e\varepsilon_1}}$
$\therefore\ \varepsilon_1 > \varepsilon_3 > \varepsilon_2$

9. (c) Intensity at any point on the screen is given by
$I = 4I_0\cos^2\frac{\phi}{2}$
where, I_0 is the intensity of either wave and ϕ is the phase difference between two waves.
Phase difference $(\phi) = \frac{2\pi}{\lambda} \times$ path difference
$= \frac{2\pi}{\lambda} \times \lambda = 2\pi$
$\therefore I = 4I_0\cos^2\left(\frac{2\pi}{2}\right)$
$\Rightarrow I_0 = \frac{1}{4}\cos^2(\pi) = 4I_0 = I$
$I_0 = \frac{1}{4}$......(i)
When path difference is $\frac{\lambda}{4}$, then
$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$
$$I = 4I_0\cos^2\left(\frac{\pi}{4}\right) = 2I_0 = 2 \times \frac{I}{4} = \frac{I}{2}$$

10. (a) Given, $\frac{I_{\max}}{I_{\min}} = \frac{4}{1}$

Let I_1 and I_2 be the intensities of interfering waves, then $\frac{\left(\sqrt{I_2}+\sqrt{I_1}\right)^2}{\left(\sqrt{I_2}-\sqrt{I_1}\right)^2} = \frac{I_{\max}}{I_{\min}}$

$$\Rightarrow \left(\frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1}\right)^2 = \frac{4}{1}$$
$$\Rightarrow \frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1} = \frac{2}{1} \Rightarrow \sqrt{\frac{I_2}{I_1}} = \frac{3}{1}$$
or
$\frac{I_1}{I_2} = \frac{1}{9}$

11. (c) Given, angular resolution of human eye,
$\theta = 5.8 \times 10^{-4}$rad
The linear distance between two successive dots in a printer,
$l = \frac{254}{300} = 0.846 \times 10^{-2}$ cm
At a distance d cm, the gap l will subtend an angle which is given by

$d=\frac{l}{\theta}=\frac{0.846\times10^{-2}}{5.8\times10^{-4}}=14.59\text{ cm}$

12. **(d)** The angular momentum is quantized to even multiple of $\frac{h}{2\pi}$. Hence, the quantum numbers that are allowed are $n=2$ and $n=4$ ... . So,

$$E=13.6\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$
$$=13.6\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$$
$$=13.6\left(\frac{1}{4}-\frac{1}{16}\right)$$
$$=255\text{eV}$$

Now, given that, $hc=1242\text{MeV}-\text{nm}$

Hence, $\lambda=\frac{hc}{255\text{eV}}=\frac{1242\text{eV}-\text{nm}}{2.55\text{eV}}$

$=487$ nm

13. **(b)** Given, $V_s=10$ V

$V_z=6$ V

Maximum current $(I_{z\max})=40$ mA

$=40\times10^{-3}$ A

Minimum current $(I_{z\min})=10\text{ mA}=10\times10^{-3}$ A

Maximum value of series resistance R_S in the voltage regulator circuit is given as

$$R_{\max}=\frac{V_s-V_z}{I_{z\min}}$$
$$=\frac{10-6}{10\times10^{-3}}$$
$$=400\Omega$$

14. **(a)** Given expression,

$\bar{A}(A+B)+(B+AA)(A+\bar{B})$

$=\bar{A}(A+B)+(B+A)(A+\bar{B})$

$=A\bar{A}+\bar{A}B+AB+B\bar{B}+AA+A\bar{B}$

$=A+A\bar{B}+B(\bar{A}+A)[\because A\bar{A}=B\bar{B}=0\text{ and }A\cdot A=A]$

$=A+A\bar{B}+B(\bar{A}+A)$

$=A(1+\bar{B})+B(1)$

$=A+B$

$[\because A+\bar{A}=1]$

$[\because 1+\bar{B}=1]$

15. **(c)** The percentage error in Z is given as

$$\frac{\Delta Z}{Z}\%=4\frac{\Delta A}{A}+\frac{1}{3}\frac{\Delta B}{B}+\frac{\Delta C}{C}+\frac{3}{2}\frac{\Delta D}{D}$$
$$=4\times4+\frac{1}{3}\times2+3+\frac{3}{2}\times1$$
$$=16+\frac{2}{3}+3+\frac{3}{2}$$
$$=\frac{127}{6}\%$$

16. **(c)** Given, $S=\beta k_B A$

where, dimensional formula of $S=[\text{M}^1\text{ L}^2\text{ T}^{-2}\text{ K}^{-1}]$

Dimensional formula of $k_B=[\text{M}^1\text{ L}^2\text{ T}^{-2}\text{ K}^{-1}]$

Dimensional formula of $A=[\text{L}^2]$

Thus, $[\text{M}^1\text{ L}^2\text{ T}^{-2}\text{ K}^{-1}]=\beta[\text{M}^1\text{ L}^2\text{ T}^{-2}\text{ K}^{-1}][\text{L}^2]$

$\Rightarrow\ \beta=[L^{-2}]$

17. **(d)** According to work-energy theorem, change in kinetic energy = work done = force × distance

Thus, force $= \frac{\text{change in energy}}{\text{change in distance}}$

At $x = 10$ m, we have

$$\mathbf{F} = \frac{20-40}{12-8}\hat{\mathbf{i}} = -5\hat{\mathbf{i}}\mathbf{N}$$

18. (d) At position A, direction of $\mathbf{p}$ is along $+\mathbf{ve}$ Y-axis.

At position A, direction of $\mathbf{L}$ is along + ve Z-axis because $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and $\mathbf{r}$ is along $+$ve$X$-axis.

Thus, $\alpha = \mathbf{p} \times \mathbf{L}$

$= \hat{\mathbf{i}} \times \hat{\mathbf{k}}$

$= -\hat{\mathbf{j}}$ or $-$ ve$Y$-axis

19. (b) According to the question,

$$A = \sqrt{A^2 + A^2 + 2AA\cos\delta}$$

$$\Rightarrow \quad \cos\delta = -\frac{1}{2}$$

or $\quad \delta = 120°$

or $\quad = \frac{2\pi}{3}$

20. (c) Maximum height of projectile,

$h = \frac{u^2\sin^2\theta}{2g}$......(i)

At maximum height velocity of projectile,

$v = u\cos\theta$

$\therefore$ Momentum at highest point $= mu\cos\theta$

$\therefore$ Angular momentum about origin,

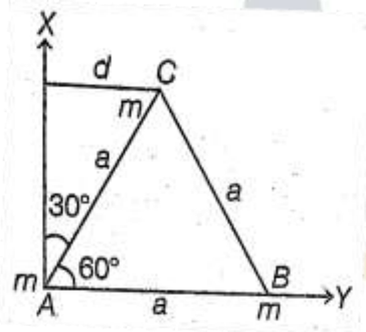
$$p = mu\cos\theta \times h$$

$$= mu\cos\theta \times \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{mu^3\cos\theta\sin^2\theta}{2g}$$

$\Rightarrow p \propto u^3$

21. (d) Moment of inertia about line $AX = ma^2 + md^2$



We know that,

$$\Rightarrow \sin\theta = \frac{d}{a}$$

$$\Rightarrow \frac{1}{2} = \frac{d}{a} \; (\because \theta = 30°)$$

$$\therefore \quad d = \frac{a}{2}$$

$$I_{Ax}$$

$$\Rightarrow = ma^2 + m\left(\frac{a}{2}\right)^2$$

$$= ma^2 + \frac{ma^2}{4} = \frac{5}{4}ma^2$$

22. (d) Given, initial speed, $v=\sqrt{3}v_e$

We know that, $v_e=\sqrt{\frac{2GM}{R}}$

At the surface of earth,

Total energy = PE + KE

$$=-\frac{GMm}{R}+\frac{1}{2}mv^2$$

$$=-\frac{GMm}{R}+\frac{1}{2}m\left(\sqrt{3}\sqrt{\frac{2GM}{R}}\right)^2$$

$$=-\frac{GMm}{R}+\frac{1}{2}3m\times\frac{2GM}{R}$$

$$=-\frac{GMm}{R}+\frac{3GMm}{R}=\frac{2GMm}{R}$$

At infinity, total energy = PE + KE

$$=0+\frac{1}{2}mv^2$$

From the principle of conservation of energy,

$$\frac{2GMm}{R}=\frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{4GM}{R}=v^2$$

$$\Rightarrow \quad v=\sqrt{\frac{4GM}{R}}$$

$$=\sqrt{2\times\frac{2GM}{R}}=\sqrt{2}v_e$$

23. (d) According to the Hooke's law, for deformation of string of length l,

$\frac{F_1}{A}\propto\frac{\Delta l_1}{l}$......(i)

and in second case,

$\frac{F_2}{A}\propto\frac{\Delta l_2}{l}$.........(ii)

From Eqs. (i) and (ii), we have

$$\frac{F_1}{F_2}=\frac{\Delta l_1}{\Delta l_2}=\frac{l_1-l}{l_2-l}$$

$$\Rightarrow \quad (F_2-F_1)l=F_2l_1-F_1l_2$$

$$\Rightarrow \quad l=\frac{F_2l_1-F_1l_2}{F_2-F_1}$$

24. (c) Let r be the radius of small mercury drop.

Thus, total volume of 27 drops $=27\times\frac{4}{3}\pi r^3$

Total volume of bigger drop $=\frac{4}{3}\pi R^3$

∴ According to given situation,

$$\Rightarrow\frac{4}{3}\pi R^3 \quad =27\times\frac{4}{3}\pi r^3$$

$R=3r$

Now, surface energy = surface tension × area

$=T\times A$

Surface energy of 27 drops $=T\times 27\times(4\pi r^2)$

$=27\times 4\pi r^2T$

Surface energy of bigger drop $=T\times 4\pi R^2$

$=T\times 4\pi(3r)^2$

$=9\times 4\pi r^2T$

Increase in surface energy $= 9 \times 4\pi r^2 T - 27 \times 4\pi r^2 T$
$= -18 \times 4\pi r^2 T$
$\therefore$ Relative increase in surface energy
$= \dfrac{-18 \times 4\pi r^2 T}{27 \times 4\pi r^2 T} = -\dfrac{2}{3}.$

25. (a) For the given isotherm, the temperature of the curve EF is greater than the curve CD and AB.
As we know that, most probable speed of the molecule is given as

$$v_p = \sqrt{\frac{2RT}{M}}$$

Thus, the v_p at 3 and 4 will be same. Hence, v_p at $3 = v_p$ at $4 > v_p$ at $2 > v_p$ at 1 .

26. (b) $U = Ap^2V, A =$ constant
For adiabatic process, $dQ = 0$
$\therefore$ From first law of thermodynamics,

$$dQ = dU + dW$$

$$\Rightarrow \quad dU + dW = 0 \ (\because dQ = 0)$$

$$\Rightarrow \quad d(Ap^2V) + pdV = 0$$

$$\Rightarrow \quad (Ap^2 + p)dV + 2ApVdp = 0$$

$$\Rightarrow \quad (Ap^2 + p)dV = -2ApVdp$$

$$\Rightarrow \quad \frac{dV}{-2AV} = \frac{pdp}{p(Ap+1)}$$

$$\Rightarrow \quad \frac{dp}{(Ap+1)} + \frac{1}{2A} \cdot \frac{dv}{v} = 0$$

Integrating on both sides,

$$\Rightarrow \quad \int \frac{dp}{(Ap+1)} + \frac{1}{2A}\int \frac{dV}{V} = \int 0$$

$$\Rightarrow \quad \frac{\ln(Ap+1)}{A} + \frac{1}{2A}\ln V = \ln C$$

$$\Rightarrow \quad \ln(Ap+1) + \ln V^{1/2} = \ln C$$

$$\Rightarrow \quad \ln\left[V^{1/2}(Ap+1)\right] = \ln C$$

$$\Rightarrow \quad V^{1/2}(Ap+1) = C$$

$$\Rightarrow \quad V(Ap+1)^2 = C \text{ (constant)}$$

27. (b) The given process BC is isothermal and AB is isochoric.
From A to B, we have as
$p_0 \longrightarrow 2p_0$
$p \propto T$
Hence, $T_B = 2T_A = 2T_0$

$$Q_{AB} = \Delta U_{AB} + W_{AB} = C_V \Delta T + 0$$

$$\Rightarrow Q_{AB} = \frac{5}{2}R(2T_0 - T_0) = \frac{5}{2}p_0V_0$$

$\left(\because C_V = \dfrac{5}{2}R \text{ for diatomic gas and } pV = RT\right)$

From B to C, we have

$$Q_{BC} = \Delta U_{BC} + W_{BC} = 0 + R(2T_0)\ln\frac{2V_0}{V_0}$$

$$= 2RT_0 \ln 2$$

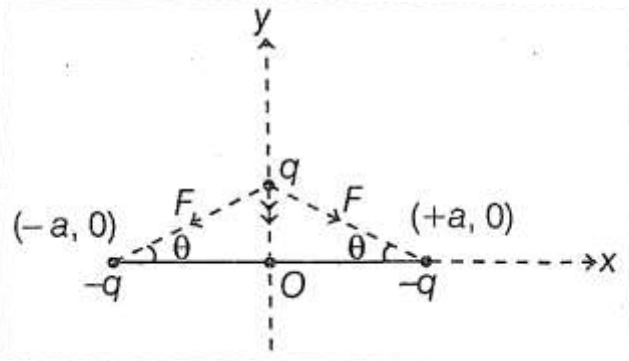
$$= 2\ln 2 p_0V_0$$

Hence, $Q_{\text{otal}} = Q_{AB} + Q_{BC}$

$$= (25 + 2\ln 2p_0V_0$$

$$= 3.9p_0V_0$$

28. (b) Consider the diagram given below.



When the charge q is displaced momentarily, it starts oscillating. The net force acting on q is given by $F_{\text{net}} = 2F\sin\theta$

$$= -\frac{2kq \times q}{\left(\sqrt{y^2+a^2}\right)^2} \cdot \frac{y}{\sqrt{y^2+a^2}} = -\frac{2kq^2 y}{\left(\sqrt{y^2+a^2}\right)^{\frac{3}{2}}}$$

For $a \gg y$, we have

$F_{\text{net}} = -\frac{2kq^2 y}{a^3}$ or $F_{\text{net}} \propto -y$

29. (a) We can find the force between q_a and q_b by calculating electric field first.
From Gauss's law,

$$\oint E \cdot ds = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \Rightarrow E \cdot 4\pi c^2 = \frac{q_a}{\varepsilon_0}$$

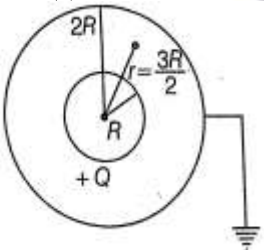
($\because$ For cavity a electric field is to be calculated upto q_b)

$$\Rightarrow E_a = \frac{q_a}{4\pi\varepsilon_0 c^2}$$

Now, force on q_b due to q_a is given by

$$F = E_a \cdot q_b = \frac{q_a q_b}{4\pi\varepsilon_0 c^2}$$

30. (c) Consider the figure given below.

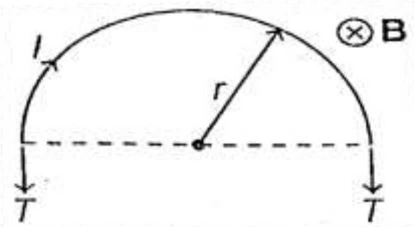


Due to grounding, charge on outer sphere $= 0$ Now, potential due to inner sphere,

$$V = \frac{kQ}{r} = \frac{kQ}{\frac{3R}{2}}$$

$$= \frac{2kQ}{3R} \text{ or } \frac{1}{4\pi\varepsilon_0}\frac{2Q}{3R}$$

31. (b) Considering the semi-circular wire of radius r. Force acting on it is shown below



Let T be the force exerted by each spring.

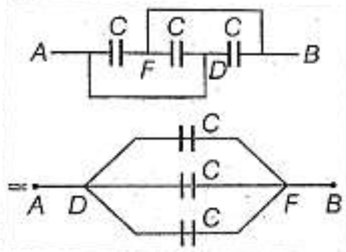
$\therefore \quad T + T = IBl$

$2T = IBl$

$2T = IB \cdot \pi r \; (\because l = \pi r)$

$T = \dfrac{IB\pi r}{2}$

32. (b)



Here, all the three capacitors are connected in parallel. So, equivalent capacitance will be

$C_{\text{eq}} = C + C + C = 3C$

33. (c) Given, initial velocity, $u = 0$ m/s

Final velocity, $v = 100$ m/s

Mass of ball, $m = 50 \times 10^{-3}$ kg

Time of contact $= 0.02$ s

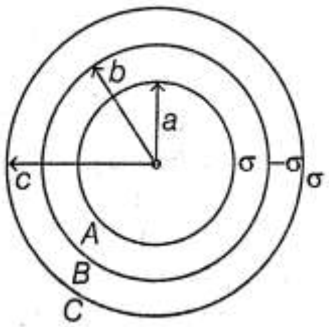
Impulse, $F = \frac{m(v-u)}{t}$

$= \dfrac{50 \times 10^{-3}(100 - 0)}{0.02}$

$= \dfrac{5 \times 100}{2}$

$= 250$ N

34. (d) Net potential at any point $(r = b)$ of shell B



$=$ potential due to shell A + potential due to shell B + potential due to shell C.

$V = V_{A,\text{ out}} + V_{B,\text{ surface}} + V_{C,\text{ in}}$

$= V_{A,\text{ out}} + V_{B,\text{ surface}} + V_{C,\text{ surface}}$

$V = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{q_a}{b} + \dfrac{q_b}{b} + \dfrac{q_c}{c}\right]$

$= \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{\sigma 4\pi a^2}{b} + \dfrac{(-\sigma)4\pi b^2}{b} + \dfrac{(\sigma)4\pi c^2}{c}\right]$

$= \dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2}{b} - b + c\right]$

35. (b) Given $pV^3 = C, C_V = \frac{3R}{2}, n = 1$ mol

For polytropic process,

$pV^x = C$

$\therefore \; x = 3$

$C = C_V + \dfrac{R}{1-x} = \dfrac{3R}{2} + \dfrac{R}{1-3} = R$

Heat supplied $(Q) = nC\Delta T$

$= 1 \times R \times (T_2 - T_1) = R(T_2 - T_1)$

Now, As
$pV = nRT$……..(i)
$\therefore\ pV = RT (\because n = 1)$
$\therefore\ R = \frac{pV}{T} = \frac{p_1 V_1}{T_1}$
Also,
$pV^3 = C$
$\Rightarrow \left(\frac{RT}{V}\right) V^3 = C$
[From Eq. (i)]
$\Rightarrow TV^2 = C \Rightarrow T_1 V_1^2 = T_2 V_2^2$
$\therefore\ T_2 = \frac{T_1 V_1^2}{V_2^2}$
Heat supplied $(Q) = \frac{p_1 V_1}{T_1}\left[\frac{T_1 V_1^2}{V_2^2} - T_1\right] = p_1 V_1 \left[\frac{V_1^2}{V_2^2} - 1\right]$
36. (a, c, d) Given, resistance per unit length of loop $= \lambda \Omega/\text{m}$
(a) Current in the loop, $I = \frac{E}{R}$
$I = \frac{Blv}{R} (\because \text{emf} E = Blv)$……(i)
We know that,
$\frac{R}{2(a+b)} = \lambda \Rightarrow R = \lambda(2a + 2b)$#(ii)
Using Eqs. (i) and (ii), we get
$I = \frac{Bbv}{\lambda(2a + 2b)}$
So, option (a) is correct.
(b) Using right hand thumb rule, direction of current in the loop will be clockwise, looking from the top. But as the flux is changing, so according to Lenz's law, direction of current will be anti-clockwise.
So, option (b) is incorrect.
(c) As the arm PS and QR has same potential, then $V_P - V_S = V_Q - V_R$ is also same.
So, option (c) is correct.
(d) As the arm SR is parallel to the velocity. So, there will not be any current or induction.
So, option (d) is correct.
37. (a, b,)
(a) Given, energy of photons $= 10.2\text{eV}$
Energy for nth state,
$$E_n = -13.6 Z^2 \times \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$
$$= -13.6 \times \left(2^2 \left(\frac{1}{2^2} - \frac{1}{4^2}\right) (\because Z = 2)\right.$$
$$= -13.6 \times 4 \times \left(\frac{4-1}{16}\right)$$
$$= -3.4 \times \frac{3}{4}$$
$$= -10.2\text{eV}$$
$E = 10.2\text{eV}$
So, option (a) is correct.
(b) Number of lines in emission spectra
$= \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6\ [\because n = 4]$
So, option (b) is correct.
(c) For smallest wavelength of He^+spectrum the final state should be $n_f = \infty$
So, option (c) is incorrect.
(d) He^+electron cannot jump from $n = 2$ to $n = 3$ as the energy of photon is less than required.
So, option (d) is incorrect.

38. (a, b) Given, trajectory of the particle,

$\mathbf{r} = b\cos\omega t\hat{\mathbf{i}} + b\sin\omega t\hat{\mathbf{j}}$

It's component can be written as

$x = b\cos\omega t$............(i)

$y = b\sin\omega t$........(ii)

Both Eqs. (i) and (ii) together represent the circle in $x-y$ plane. Thus, the trajectory of the particle in $x-y$ plane is circle.

Now, $\frac{d\mathbf{r}}{dt} = \omega b(-\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}})$

$$\frac{d^2\mathbf{r}}{dt^2} = -b\omega^2(\cos\omega t\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}})$$

$$= -\omega^2(\mathbf{r})$$

Therefore, $\mathbf{a} = -\omega^2\mathbf{r}$

Hence, option (c) and (d) are incorrect.

Energy of the particle in circular motion is given as $\frac{1}{2}m\omega^2 A^2$ (where, A is the maximum displacement from the centre), which is constant.

Hence, $\frac{E}{\omega}$ is also constant.

So, option (a) and (b) are correct.

39. (a,d) Load F can be given as

$$F = \left(\frac{AY}{L}\right)\Delta x$$

where, $A =$ cross-sectional area

$Y =$ Young's modulus

$L =$ length

$\Delta x =$ elongation in length.

i.e., F versus Δx graph is a straight line of slope $\frac{YA}{L}$. $(\text{ slope })_A > (\text{ slope })_B$

$$\left(\frac{YA}{L}\right)_A > \left(\frac{YA}{L}\right)_B$$

or

$(A)_A > (A)_B$

As we know that, they are of same material. Hence, $Y_B = Y_A$

So, option (a) and (d) are correct.

40. (a, c) According to Gauss' theorem

$$\text{Net flux through the surface } = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Here, $q_{\text{enclosed}} = 0$

So, $\phi_{\substack{\text{curved}\\\text{surface}}} + \phi_{\substack{\text{circular}\\\text{surface}}} = 0$

$$\phi_{\substack{\text{curved}\\\text{surface}}} = -\phi_{\substack{\text{circular}\\\text{surface}}}$$

$$= -\mathrm{E}\cdot\mathrm{S} = -ES\cos 180^\circ$$

$$= ES \quad (Cos 180^\circ = -1)$$

$$= E\cdot\pi R^2 = \pi R^2 E$$

As, electrostatic field is conservative in nature. So, work done is path independent.

So, option (a) and (c) are correct.

Chemistry

41. (b) The given reaction take place as follows:

$$\underset{\text{Magnesium carbonate}}{MgCO_3} \xrightarrow{\text{dil. } 2HCl} \underset{\text{Magnesium chloride}}{MgCl_2} + H_2O + CO_2$$

$$MgCl_2 + 2NH_3 + 2H_2O \longrightarrow Mg(OH)_2 + 2NH_4Cl$$

Magnesium chloride

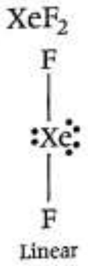
$Mg(OH)_2$ (Magnesium hydroxide) $+ Na_2HPO_4$ (Disodium hydrogen phosphate) $+ NH_4Cl \longrightarrow \underset{white.ppt}{Mg(NH_4)PO_4}$

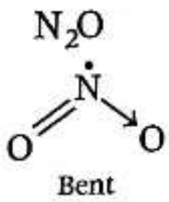
$NH_3 + H_2O + NaCl$

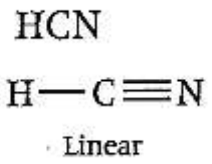
∴ The formula of white ppt. is $Mg(NH_4)PO_4$.

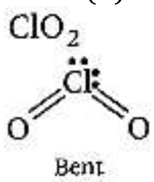
42. (d) The non-linear molecule pair is ClO_2 and NO_2.

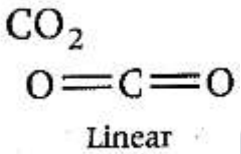
The structure of the given molecules are as follows :

(i) 

(ii) 

(iii) 

(iv) 

(v) 

43. (a) Number of atoms in body centred unit cell

$= 8$ atoms are present at 8 corners and 1 atom is present at body centre.

So, total contribution $= 8 \times \frac{1}{8} + 1 \times \frac{1}{1}$

$= 1 + 1 = 2$

Number of atoms in face centred cubic unit cell

$= 8$ atoms are present at 8 corners each and 6 atoms are present at 6 faces each atom contributes one half to unit cell.

$= 8 \times \frac{1}{8} + 6 \times \frac{1}{2}$

Hence, total number of atoms in one face unit cell $= 1 + 3 = 4$

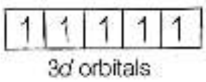
44. (c) Atomic number of Mn is 25.

Electronic configuration of Mn $= [Ar]3d^5 4s^2$

Electronic configuration of $Mn^{2+} = [Ar]3d^5$

There are five $3d$ orbitals, each orbital occupied by single electron. Thus, number of unpaired electrons is 5 .

$Mn^{2+} = [Ar]3d^5$



45. (b)

$$(V_{avg})_{O_2} = \sqrt{\frac{8RT_2}{\pi M_{O_2}}}$$

$$(V_{avg})_{H_2} = \sqrt{\frac{8RT_1}{\pi M_{H_2}}}$$

$$\therefore \sqrt{\frac{8RT_2}{\pi M_{O_2}}} = \sqrt{\frac{8RT_1}{\pi M_{H_2}}}$$

$$\therefore \sqrt{\frac{T_1}{2}} = \sqrt{\frac{T_2}{32}}$$

$$\therefore \frac{T_1}{T_2} = \frac{1}{16}$$

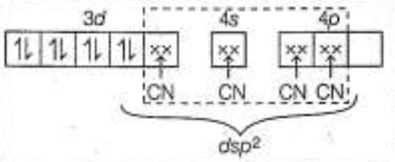
$$\frac{T_1}{T_2} = \frac{1}{16} \Rightarrow T_1 : T_2 = 1:16 \Rightarrow T_2 : T_1 = 16:1$$

Hence no option is matched with the answer.

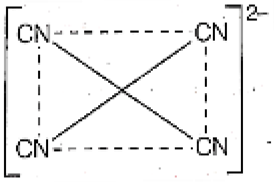
46.(b) The formula of sodium nitroprusside in $Na_2[Fe(CN)_5NO]$.

47. (c) Ni^{2+} ion has $3d^8$ outer electronic configuration. $CN^-$is a strong field ligand. So, pairing of electron takes place.

Electronic configuration of $[Ni(CN)_4]^{2-}$



Therefore, $[Ni(CN)_4]^{2-}$ is dsp^2 hybridisation. As all the electrons are paired, hence it is diamagnetic. Geometry of $[Ni(CN)_4]^{2-}$ is square planar.



48. **(d)** Extensive hydrogen bonding is present in water molecules as compare to HF molecules. It is due to the presence of twice the hydrogen bonding than HF. Number of hydrogen bonds, despite these being individually weaker, thus $\mathbf{H_2O}$ has high boiling point than HF.

49. (a) Only Zn and Al produce nascent hydrogen with alkaline medium.

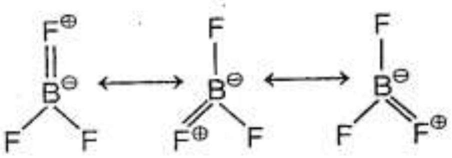
$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + 2[H]$

$Al + 2NaOH \longrightarrow Na_2AlO_2 + 2[H]$

50. (d) Bond order

$$= \frac{\text{Number of bonds present between atoms}}{\text{Total number of resonance structures}}$$

$\because$ BF_3 has three resonance structures.



So, bond order $= \frac{4}{3} = 1\frac{1}{3}$

51. (c) Tritium is the heaviest and only radioactive isotope of hydrogen.

52. (b) The stability of these halides decreases down in the group due to decrease in bond dissociation enthalpy.

Thus, the correct order of acidity is

HF < HCl < HBr < HI.

53. (a) The given reaction take place as follows

$$\underset{\text{Sodium salt}}{2NaCl} + \underset{\text{Lead nitrate}}{Pb(NO_3)_2} \longrightarrow \underset{\substack{\text{(White ppt.)}\\\text{lead chloride}}}{PbCl_2 \downarrow} + \underset{\text{Sodium nitrate}}{2NaNO_3}$$

$PbCl_2$ is soluble in hot water and forms white ppt. in cold condition.

This show that Cl^-acid radical is present in the salt.

54. (c) Oxidation state of Cr in $K_2Cr_2O_7$.

Let the 0.5 of Cr be x.

So, $2(+1) + 2(x) + 7(-2) = 0$

$2 + 2x - 14 = 0$

$2x = 14 - 2$

$x = \frac{12}{2} \Rightarrow x = +6$

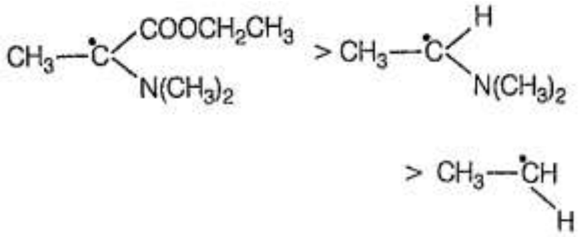
Oxidation of Cr is +6 .

Oxidation state of Cr in CrO_5.

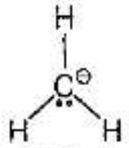
In this molecule four oxygen atom present as peroxide and one oxygen as oxide.

Let the oxidation state of Cr be x in CrO_5.
So,
$$x + 4(-1) + 1 \times (-2) = 0$$
$$x - 4 - 2 = 0$$
$$x = +6$$
∴ Oxidation state of Cr is +6 .

55. (d) The stability of free radical increases with increases is order a



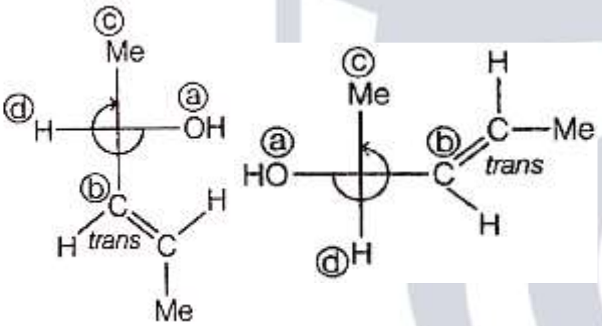
56. (d) $\overset{\ominus}{\ddot{C}}H_3$ carbanion has 3 bonding pairs and 1 lone pair. Its structure is as follow



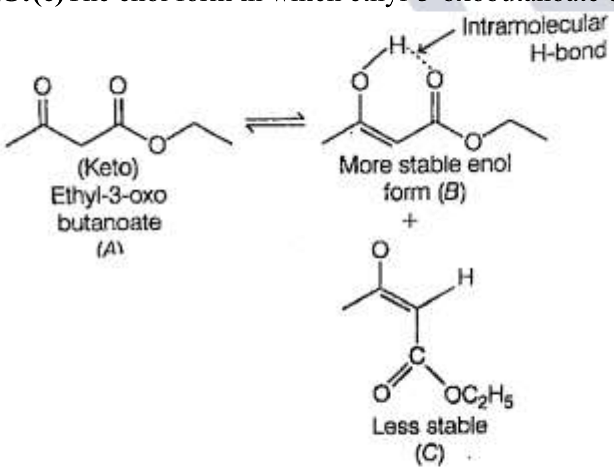
Hybridisation in sp^3

In $H_2\overset{\ominus}{C} - CHOCH_3$ molecule C is also sp^3
hybridisation, because it contain 3 bond pairs and 1 lone pair.

57. (b) They both have S-configuration. So, they are homomers.

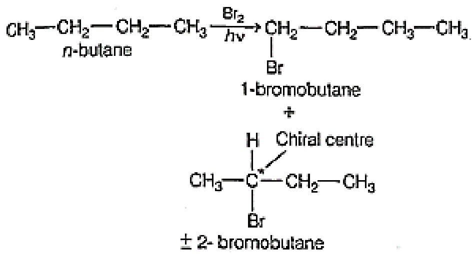


58. (c) The enol form in which ethyl-3-oxobutanoate exists is



(B) form is more stable than (C) form due the presence of intramolecular hydrogen bonding.

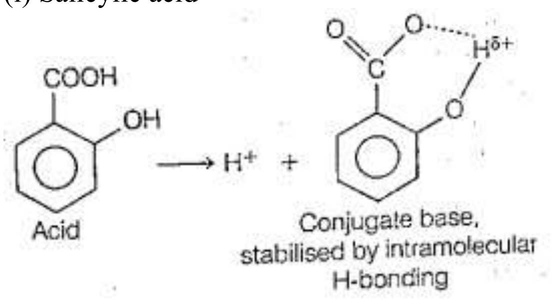
59. (c) Monobromination of n-butane in form of free radical take place as follows:



∴ Total product = 3

60. (a) The acidic strength of an acid increases with the increase in stability of its conjugate base by intramolecular hydrogen bonding.

(i) Salicylic acid



(ii) 2, 6-dihydroxybenzoic acid

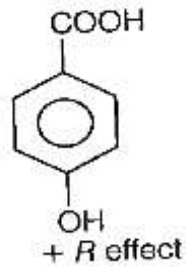
Acid → H^+ +

Also, pK a value of 2,6 -dihydroxybenzoic acid is greater than salicylic acid.

∴ It is a stronger acid.

(iii) In 4-hydroxybenzoic acid - OH group show $+R$ effect.

Thus, it decrease the acidic character 4-hydroxybenzoic acid.



∴ The acidity order is 2,6 -dihydroxybenzoic acid > salicylic acid > 4-hydroxybenzoic acid.

61. (c) Given, molecular weight of oxalic acid = 126

Volume = 100 mL

Normality = 0.1 N

Mass =?

We know,

$$\text{Normality} = \frac{\text{Equivalent weight}}{\text{Volume (Litre)}}$$

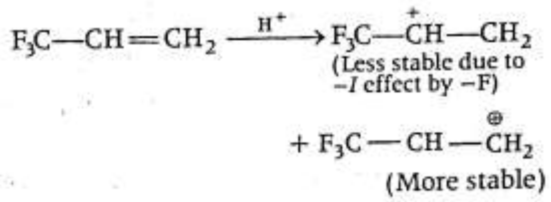
$$\therefore \text{ Equivalent weight } = \frac{\text{Molecular weight}}{\eta \text{ - factor}}$$

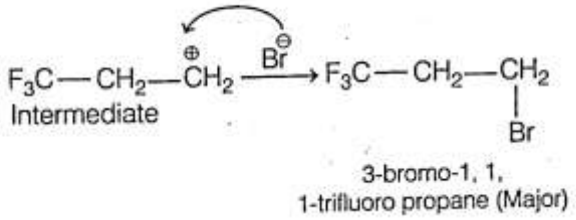
$$= \frac{126}{2} [\eta \text{ - factor } = 2 \text{ for oxalic acid)}$$

$$\therefore\ 0.1 = \frac{\text{Mass}}{63 \times \frac{100}{1000}} \Rightarrow \text{ Mass } = \frac{0.1 \times 63 \times 100}{1000}$$

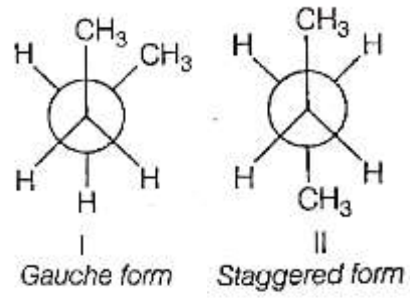
$$\therefore \text{ Mass } = 0.63 \text{ g}$$

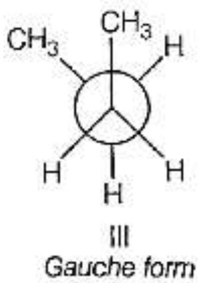
62. (a) The given reaction take place as follows:





63. (a) The given conformers of n-butane are





64. (c)

$C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$

78 g of C_6H_6 required $= \frac{15}{2}$ mole of O_2

39 g $\left(\frac{1}{2}\text{ mole }\right)C_6H_6$ requires $= \frac{15}{2} \times \frac{1}{2}$ mole of $O_2 = \frac{15}{4}$ mole of O_2

$\because$ Moles $= \frac{\text{Volume (L)}}{22.4}$

$\frac{15}{4} = \frac{\text{Volume}}{22.4}$

$\therefore$ Volume $= 84$ L(O_2)

65. (b) Avagadro's law is valid for ideal gas.

According to Avagadro's law, volume of ideal gas is directly proportional to number of moles of gas at constant temperature and pressure.

66. (b) Let the two oxides be M_2O_x and M_2O_y.

Let atomic weight of metal, $M = a$

$\therefore$ For MO_x,

ratio of weight

$2a : 16x = 25 : 4$.....(i)

For MO_y,

ratio of weight

$2a : 16y = 25 : 6$........(ii)

From (i) and (ii), we get

$\therefore 16x : 16y \quad = 4 : 6$

$\therefore x : y \quad = 2 : 3$

$\therefore$ The two oxides may be M_2O_2 or M_2O_3.
Now, $2a:16x^2 = 25:4$

$\therefore\ a = \frac{25}{4} \times 16 = 100$

67. (b) de-Broglie equation,

$\lambda = \frac{h}{mv}$

where, $\lambda =$ wavelength
$m =$ mass
$v =$ velocity
$h =$ Plank's constant

$\therefore\ \lambda \propto \frac{1}{m}$

We know,
$m_{He^{2+}} > m_p > m_e$
So,
$\lambda_e > \lambda_p > \lambda_{He^{2+}}$
or
$e > p > \alpha$

68. (a) Number of drops in 1 mL water = 25

$\therefore$ Volume of 1 drop $= \frac{1}{25}$ mL

$\therefore$ Mass of water = Density $\times$ Volume

$= 1 \times \frac{1}{25}$

$= \frac{1}{25}$ g

$\therefore$ Moles of water $= \frac{\text{Mass}}{\text{Molecular mass}}$

$= \frac{1}{25 \times 18}$

$\therefore$ Molecules of water $= N_0 \times$ Moles of water

$= \frac{N_0}{25 \times 18} = \frac{0.02}{9} N_0$

69. (d) Radius of ' H ' atom in ground state $= \gamma_1$
Radius of He^+ion in ground state $= r_2$

$\because\ r \propto \frac{n^2}{Z}$

where, $n =$ principal quantum number
$Z =$ atomic number

$\therefore\ \frac{r_1}{r_2} = \frac{Z_2}{Z_1} = \frac{2}{1} \quad \{\because n_1 = n_2 = 1\}$

$\Rightarrow\ \frac{r_2}{r_1} = \frac{1}{2}$

70. (b) Let principal quantum number $= n$
The value of azimuthal quantum number $l = 0$ to $(n-1)$.
The value of magnetic quantum number $m = -l$ to $+l$.
Spin quantum number $(s) = +\frac{1}{2}, -\frac{1}{2}$

$\therefore$ The correct set of quantum number (n, l, m, s) is $\left(4, 3, -2, -\frac{1}{2}\right)$

For $n = 4$,
and
$l = 0,1,2,3; l = 3$
$m = -2, -1, 0, 1, +2$

71. (b) $(C_{rms})_{H_2} = \sqrt{\frac{3RT}{2}} = \sqrt{\frac{3 \times R \times 150}{2}}$

$$= \sqrt{225R}$$

$$(C_{mp})_{He} = \frac{(C_{rms})_{H_2}}{2}$$

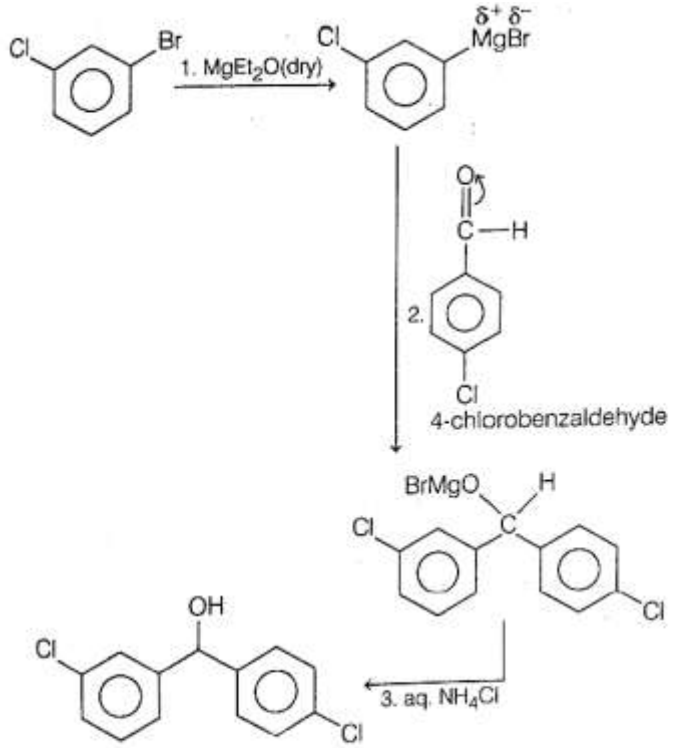
$$(C_{mp})_{He} = \sqrt{\frac{2RT}{4}}$$

$$\sqrt{\frac{2RT}{4}} = \frac{\sqrt{225R}}{2}$$

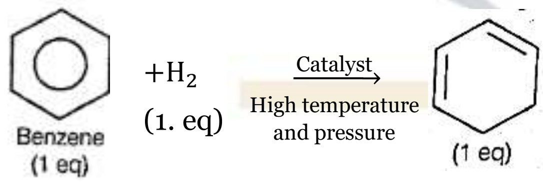
$$T = 112.5 \text{ K}$$

72. **(b)** Electron affinity of $S > O$ and $Cl > F$. This is so because of the small size of oxygen and fluorine (in respective case), due to which they got higher charge density and thus electronic repulsion increases when new electron is added.

73. **(a)** The given reaction take place as follows



74. **(c)**The hydrogenation reaction of benzene take place as follows



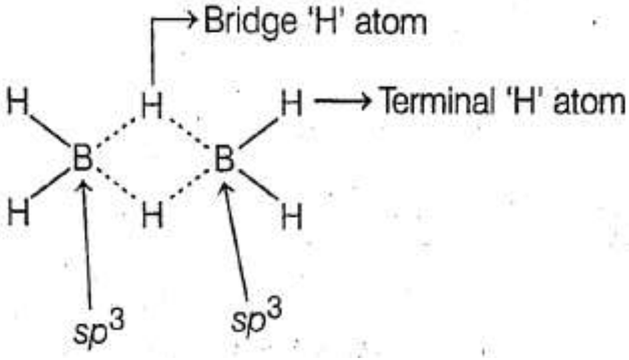
75. **(a)** (a) Relative lowering of vapour pressure

$$\frac{p^0 - p}{p^0} = x_\beta$$

∴ It is independent of temperature.
∴ It is a correct statement.
(b) Osmotic pressure does not depends on nature of solute.
(c) Boiling point is a property of solvent.
∴ ΔT_p is also dependent on nature of solvent.
∴ b, c and d are incorrect statements.

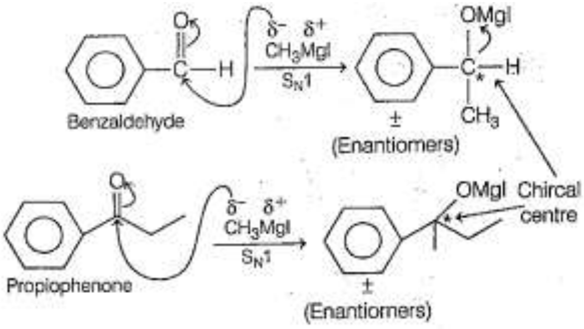
76. **(b, c)** During preparation of NH_3 in Haber's process Mo as well as mixture of Al_2O_3 and K_2O are used as promoters.

77. (a, c, d) The structure of B_2H_6 is as follows:

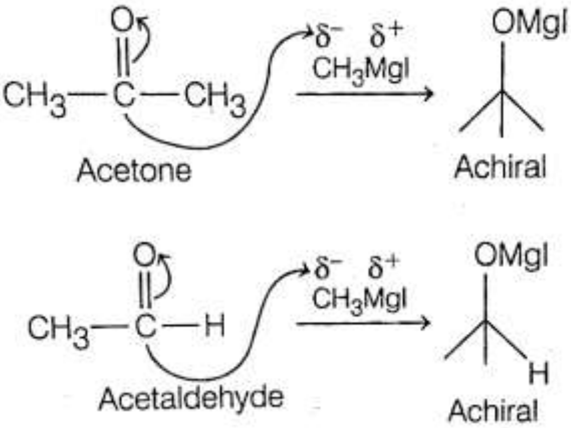


∴ All B atoms are sp^3 hybridised.
It contains two $3C-2e^-$ bonds and four $2C-2e^-$ bonds.
In B_2H_6, two types of H atoms are present i.e. bridge and terminal H atom.
It is diamagnetic as all the electrons are paired.
∴ Statement a, c and d are correct statements.

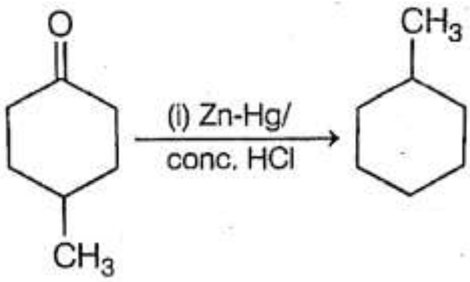
78. (a, b) The given compound reacts with methyl magnesium iodide as follows

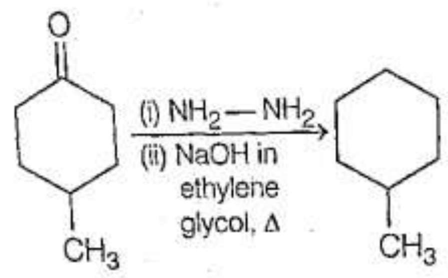


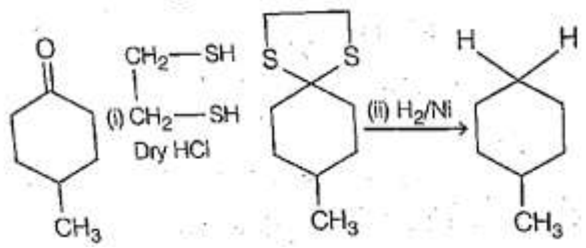
As products in a and b reactions have chiral center, therefore they produce enantiomeric products.



79. (a, b, c) The given reaction is a reduction reaction. It take place in following ways:







80. (b, c) For a weak acid and weak base salt solution. If $K_a > K_b$ i.e. $\mathrm{p}KK_b > \mathrm{p}KK_a$ than $\mathrm{pH} = 1/2$ $\left(\mathrm{p}K_w + \mathrm{p}KK_a - \mathrm{p}K_p\right)$ value is less than 7.
∴ (b) is an incorrect statement.
The pH of 10^{-8}MHCl is less than 7.
∴ Statement (c) is also incorrect.

Mathematics

81. (d) LHL

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin(a+1)x + \sin x}{x}$$

$$= \lim_{h\to 0} \frac{\sin(a+1)(0-h) + \sin(0-h)}{(0-h)}$$

[Put $x = 0 - h$; when $x \to 0^-$, then $h \to 0$]

$$= \lim_{h\to 0} \frac{\sin(a+1)h + \sin h}{h}$$

$$= \lim_{h\to 0} (a+1) \frac{\sin(a+1)h}{(a+1)h} + \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (a+1)\cdot 1 + 1$$

$$= a + 2$$

$$= \lim_{x\to 0^+} f(x)$$

$$= \lim_{x\to 0^+} \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{1/2}}$$

$$= \lim_{h\to 0} \frac{(h+bh^2)^{1/2} - h^{1/2}}{bh^{1/2}}$$

$$= \lim_{h\to 0} \frac{h^{1/2}\left[\left(1+\frac{1}{2}bh+\cdots\right)-1\right]}{bh^{1/2}}$$

$$= 0$$

and $f(0) = C$ for continuity
LHL $= \lim_{x\to 0^-} f(x) =$ RHL
$x \to 0^-$
$\therefore\ a + 2 = 0 = c^{\lim_{x\to 0^+} f(x) = f(0)}$
$a = -2, c = 0, b$ is arbitrary non-zero real.

82. (c) Given, $y = \sqrt{\log_{10} \frac{3x - x^2}{2}}$

To find-Domain

$$\frac{3x - x^2}{2} > 0$$
$$x(3 - x) > 0$$
$$x \in (0,3)$$

Now,

$$\log_{10}\left[\frac{3x - x^2}{2}\right] \geq 0$$
$$\Rightarrow \quad \frac{3x - x^2}{2} \geq 1$$
$$\Rightarrow \quad 3x - x^2 \geq 2$$
$$\Rightarrow \quad x^2 - 3x + 2 \leq 0$$
$$\Rightarrow \quad (x - 1)(x - 2) \leq 0$$
$$\Rightarrow \quad x \in [1,2]$$

83. (c) We know
$|x|$ is not differentiable at $x = 0$ where as,
$|x|^3$ is differentiable at $x = 0$
Also $|x|^2 = x^2$ is differentiable for all real x.
If $a_1 = 0$, then
$f(x) = a_3|x|^3 + a_2|x|^2 + a_0$
is differentiable at $x = 0$ which is possible when $a_1 = 0$.

84. (a) $y = e^{\tan^{-1} x}$

$$\frac{dy}{dx} = \frac{e^{\tan^{-1} x}}{1 + x^2}$$
$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1 + x^2) \cdot \frac{e^{\tan^{-1} x}}{1 + x^2} - e^{\tan^{-1} x}(2x)}{.\,(1 + x^2)^2}$$
$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(\mathrm{k} - 2x)e^{\tan^{-1} x}}{(1 + x^2)^2}$$
$$\Rightarrow \frac{d^2y}{dx^2}(1 + x^2) = (1 - 2x)\frac{dy}{dx}$$
$$\therefore \ (1 + x^2)y_2 + (2x - 1)y_1 = 0$$

85. (c) $\lim_{x \to 0} \frac{\ln\sqrt{\frac{1+x}{1-x}}}{x}$
which is indeterminant form.
By substitution method,
Put $x = \cos 2\theta$

$$2\theta = \cos^{-1} x$$
$$\theta = \frac{1}{2}(\cos^{-1} x)$$

When $x \to 0$, then $\left(\theta \to \frac{\pi}{4}\right)$

$$\Rightarrow \lim_{\theta \to \pi/4} \frac{\ln(\cot^2\theta)^{1/2}}{\cos 2\theta}$$
$$\Rightarrow \lim_{\theta \to \pi/4} \frac{\ln(\cot\theta)}{\cos 2\theta}$$
$$\Rightarrow \lim_{\theta \to \pi/4} \frac{\ln|\cot\theta|}{\cos 2\theta}$$
$$\lim_{\theta \to \pi/4} \frac{\frac{1}{\cot\theta} \times -\mathrm{cosec}^2\theta}{-(\sin 2\theta)(2)} = \frac{2}{2} = 1$$

86. (a) Let us assume $g(x) = e^{-x}f(x)$

$\because f'(c)-f(c)$ or
$f'(x)-f(x)$ expression gives us following
$f'(x)-f(x)=0$
$e^{-x}f'(x)+e^{-x}(-1)f(x)=0$
$\frac{d}{dx}[e^{-x}f(x)]=0$
From the condition and nature of e^{-x}, we can say that
(i) $g(x)$ is continuous in $[a,b]$
(ii) $g(x)$ is differentiable in (a,b)
(iii) $g(a)=e^{-a}f(a)=0$ and $g(b)=e^{-b}f(b)=0$
$[\because f(a)=0=f(b)]$
From Rolle's theorem,
$g'(c)=0$ for at least one $c\in(a,b)$
$\Rightarrow e^{-c}f'(c)-e^{-c}f(c)=0$
$\Rightarrow f'(c)-f(c)=0[\because e^{-c}\neq 0]$
$\Rightarrow f'(c)=f(c)$ for at least one $c\in(a,b)$

87. (d) Let $I=\int\cos(\ln x)dx$

$$I=x\cos\ln x+\int x\sin(\ln x)\frac{1}{x}dx$$
$$\Rightarrow\quad I=x\cos\ln(x)+\int\sin(\ln x)dx$$
$$\Rightarrow\quad x\cos(\ln x)+\left[x\sin(\ln x)-\int x\cos(\ln x)\cdot\frac{1}{x}dx\right.$$
$$\Rightarrow\quad 2I=x[\cos(\ln x)+\sin(\ln x)]$$
$$\Rightarrow\quad I=\frac{x}{2}[(\cos\ln x)+\sin\ln x]+c$$

88. (a) Let $f(x)=x$
Solve by options,
Alternate (A),
$$\int_0^c f(x)dx=(1-c)f(c)$$
$$\int_0^c xdx=(1-c)c$$
$$\left[\frac{x^2}{2}\right]_0^c=(1-c)c$$
$$\frac{c^2}{2}-0=(1-c)c\text{ or }\frac{c}{2}=(1-c)$$
$$c=2-2c$$
$$3c=2\Rightarrow c=\frac{2}{3}$$
$\therefore$ Which is true.
If alternate A is true.
$\because$ B is false vice-versa.
For alternate (D), $\int_0^c xdx=\frac{c^2}{2}$
which dependent on c.

89. (b) $\int\sqrt{x}(1-x^3)^{-1/2}dx$
$$\int\sqrt{x}\left[1-\left(x^{3/2}\right)^2\right]^{-1/2}dx$$
Let $x^{3/2}=t$
$\frac{3}{2}x^{1/2}dx=dt$

$$I = \frac{2}{3}\int (1-t^2)^{-1/2}dt$$

$\Rightarrow$ $$= \frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}}$$

$\Rightarrow$ $$I = \frac{2}{3}\sin^{-1} t + c$$

$\Rightarrow$ $$I = \frac{2}{3}\sin^{-1}(x^{3/2}) + c$$

$\Rightarrow$ $$= \frac{2}{3}g[f(x)] + c$$

$\therefore$ $g(x) = \sin^{-1} x$ and $f(x) = x^{3/2}$

90. (a)

$I_1 = \int_0^{\pi/2} \frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x}+(\sin x)^{\cos x}} dx$......(i)

Put $x = \left(\frac{\pi}{2} - x\right)$

$I_2 = \int_0^{\pi/2} \frac{(\sin x)^{\cos x}}{(\sin x)^{\cos x}+(\cos x)^{\sin x}}$.....(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1dx$$

$\Rightarrow$ $$2I = [x]_0^{\pi/2} \Rightarrow 2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

91. (b) Given,

$\int_0^x \frac{bt\cos 4t - a\sin 4t}{t^2} dt = \frac{a\sin 4x}{x} - 1$

Differentiating both side w.r.t. x, $\frac{bx\cos 4x - a\sin 4x}{x^2} = \frac{a\{4x\cos 4x - \sin 4x\}}{x^2}$

On comparing, we get

$b = 4a$

$\therefore a = \frac{1}{4}$ and $b = 1$

92. (b) Given, $\int_{\sin x}^{\cos x} e^{-t^2}dt$

$\Rightarrow$ $$g(\cos x)\frac{d}{dx}(\cos x) - g(\sin x)\frac{d}{dx}(\sin x)$$

$\because$ $$f'(x)\int_{a(x)}^{b(x)} g(t)dx$$

$$f'(x) = g(b)\frac{d}{dx}(b) - g(a)\frac{d(a)}{dx}$$

$\Rightarrow$ $$e^{-\cos^2 x}(-\sin x) - e^{-\sin^2 x}(\cos x)$$

At $x = \frac{\pi}{4}$

$\Rightarrow$ $$-e^{-1/2}\left(\frac{1}{\sqrt{2}}\right) - e^{-1/2}\left(\frac{1}{\sqrt{2}}\right)$$

$\Rightarrow$ $$-\sqrt{2}e^{-1/2}$$

$\Rightarrow$ $$-\sqrt{\frac{2}{e}}$$

93. (a) $x\frac{dy}{dx} + y = x\frac{f(xy)}{f'(xy)}$

i.e. $\frac{d}{dx}(xy) = x\frac{f(x,y)}{f'(x,y)} \Rightarrow \frac{f'(xy)}{f(xy)}d(xy) = xdx$

$$\Rightarrow \int \frac{f'(xy)}{f(xy)} d(xy) = \int xdx$$

$$\Rightarrow \log[f(xy)] = \frac{x^2}{2} + c$$

$$\Rightarrow f(xy) = e^{\left(\frac{x^2}{2}+c\right)}$$

$$= e^{\frac{x^2}{2}} \cdot e^c = ce^{\frac{x^2}{2}}$$

94. (c) Let the slope of tangent at point $P(x, y)$ be m.
Equation of tangent will be $(Y - y) = m(X - x)$
If $Y = 0$, then $X = x - \frac{y}{m}$ and if $X = 0$, then $Y = y - mx$. Since, $P(x, y)$ is mid-point of $A\left(x - \frac{y}{m}, 0\right)$ and $B(0, y - mx)$.

$$\therefore x - \frac{y}{m} = 2x \text{ and } y = -mx$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \int \frac{dy}{y} = -\int \frac{dx}{x} \Rightarrow \log y = -\log x + \log c$$

$$\log xy = \log c$$

$$xy = c$$

Since, the line passing through the point (3,2).
$\therefore c = 6$
The equation of curve is $xy = 6$

95. (d) $\cos y \frac{dy}{dx} = e^{x+\sin y} + x^2 e^{\sin y}$
On separating the variables, we get
$\cos y e^{-\sin y} dy = (e^x + x^2)dx$
On integrating both side,

$$\int \cos y e^{-\sin y} dy = \int (e^x + x^2)dx$$

Let $e^{-\sin y} = t$
$\therefore e^{-\sin y}(-\cos y)dy = dt$

$$\Rightarrow -\int dt = e^x + \frac{x^3}{3}$$

$$\Rightarrow -t = e^x + \frac{x^3}{3} + c$$

$$\Rightarrow -e^{-\sin y} = e^x + \frac{x^3}{3} + c$$

$$-e^x - \frac{x^3}{3} - e^{-\sin y} = c$$

$$e^x + \frac{x^3}{3} + e^{-\sin y} = c$$

Hence, $f(x) = \left(e^x + \frac{x^3}{3}\right)$

96. (a) Equation of the tangent at any point $\left[\frac{9t^2}{4}, \frac{9t}{2}\right]$
On the parabola is
$ty = x + \frac{9t^2}{4}$ and it passes through (4,10)

$y^2 = 9x$

$\therefore a = \dfrac{9}{4}$

$y = mx + \dfrac{a}{m}$

$\Rightarrow y = mx + \dfrac{9}{4m} \because \left[\because a = \dfrac{9}{4}\right]$

Since, it passes through (4,10)

$\Rightarrow \quad 10 = 4m + \dfrac{9}{4m}$

$\Rightarrow \quad 16m^2 - 40m + 9 = 0$

$\Rightarrow \quad m = \dfrac{1}{4}, \dfrac{9}{4}$

Equation the tangents are

$Y = \dfrac{1}{4}x + 9$ and $y = \dfrac{9}{4}x + 1$

$x - 4y + 36 = 0$

$9x - 4y + 4 = 0$

$y = mx + \dfrac{a}{m}$

touches at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

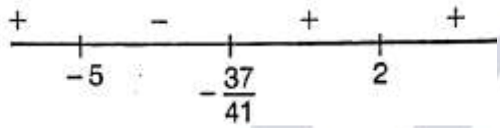
$\therefore$ Point of contact are (36,18) and $\left[\frac{4}{9}, 2\right]$.

97. (c) $f(x) = (x-2)^{17}(x+5)^{24}$

$f'(x) = 17(x-2)^{16}(x+5)^{24} + (x-2)^{17}(24)(x+5)^{23}$

$= (x-2)^{16}(x+5)^{23}\{17x + 85 + 24x - 48\}$

$= (x-2)^{16}(x+5)^{23}(41x+37) = 0$



$\therefore f(x)$ has neither a maximum nor a minimum d: $x = 2$

98. (b) Since **c** is a unit vector coplanar with **a** and h

Let $\mathbf{c} = x\mathbf{a} + y\mathbf{b}$

$x(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + y(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$(x+y)\hat{\mathbf{i}} + (x-y)\hat{\mathbf{j}} - (x-y)\hat{\mathbf{k}}$

Since, **c** is perpendicular to **a**.

$\therefore \quad x + y + x - y + x - y = 0$

$3x = y$

Since $|\mathbf{c}| = 1$

$\therefore \quad (x+y)^2 + (x-y)^2 + (x-y)^2 = 1$

$\Rightarrow \quad (4x)^2 + (-2x)^2 + (-2x)^2 = 1$

$24x^2 = 1 \Rightarrow x = \dfrac{1}{2\sqrt{6}}$

$\mathbf{c} = 4x\hat{\mathbf{i}} - 2x\hat{\mathbf{j}} + 2x\hat{\mathbf{k}}$

$= \dfrac{2}{\sqrt{6}}\hat{\mathbf{i}} - \dfrac{1}{\sqrt{6}}\hat{\mathbf{j}} + \dfrac{1}{\sqrt{6}}\hat{\mathbf{k}}$

i.e. $\quad \mathbf{c} = \dfrac{1}{\sqrt{6}}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$\mathbf{a} = d_1\hat{\mathbf{i}} + d_2\hat{\mathbf{j}} + d_3\hat{\mathbf{k}}$

Since, **d** is perpendicular to **a** and **c**.

$\mathbf{d} \cdot \mathbf{a} = \mathbf{0}$

$\therefore \quad d_1 + d_2 - d_3 = \mathbf{0}$

and $\mathbf{d} \cdot \mathbf{c} = \mathbf{0}$

$$2d_1 - d_2 + d_3 = 0 \Rightarrow 3d_1 = 0$$
$$d_1 = 0$$
$$\therefore \quad d_2 = d_3$$

Since $|\mathbf{d}| = 1$

$$d_1^2 + d_2^2 + d_3^2 = 1$$
$$\therefore \quad d_2 = d_3 = \frac{1}{\sqrt{2}}$$

Hence, $d = \frac{1}{\sqrt{2}}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$

99. (c) Length of perpendicular from the center of circle on tangent = radius of circle.

$$r = \left|\frac{3x_1 + y_1}{\sqrt{(3)^2 + (1)^2}}\right| \Rightarrow r = \frac{5}{\sqrt{10}} = \sqrt{\frac{5}{2}}$$

Equation of circle $(x-b)^2 + (y-k)^2 = r^2 (h, b)$

Centre points

$$(x-2)^2 + (y+1)^2 = \left(\sqrt{\frac{5}{2}}\right)^2$$
$$x^2 + 4 - 4x + y^2 + 1 + 2y = \frac{5}{2}$$

Let the equation of other tangent which is passing through origin is $y = mx$

$$x^2 + 4 - 4x + m^2x^2 + 1 + 2mx = \frac{5}{2} \; [\because y = mx]$$
$$\Rightarrow x^2(1+m^2) + (2m-4)x + \frac{10-5}{2} = 0$$
$$\Rightarrow (1+m^2)x^2 + (2m-4)x + \frac{5}{2} = 0$$

Compare with $ax^2 + bx + c = 0$, where

$a = (1+m^2), b = 2m - 4$

and $c = \frac{5}{2}$

$$\Rightarrow (2m-4)^2 - 4(1+m^2) \times \frac{5}{2} = 0$$
$$\Rightarrow 4m^2 + 16 - 16m - 4 \times \frac{5}{2} - 4m^2 \times \frac{5}{2} = 0$$
$$\Rightarrow 4m^2 - 16m + 16 - 10 - 10m^2 = 0$$
$$\Rightarrow -6m^2 - 16m + 6 = 0$$
$$\Rightarrow -2(3m^2 + 8m - 3) = 0$$
$$\Rightarrow (3m-1)(m+3) = 0$$
$$\Rightarrow 3m - 1 = 0, m + 3 = 0$$
$$m = \frac{1}{3}; m = -3$$

(i) $m = -3, y = -3x, 3x + y = 0$

(ii) $m = \frac{1}{3}, y = \frac{1}{3}x, 3y - x = 0$

Other tangent $3y - x = 0$

$x - 3y = 0$

100. (b) The given curves are

$y^2 + 8x = 16$ and $y^2 - 24x = 48$

On solving both equations, we get $x = -1$ and $y = \pm\sqrt{24}$

$\therefore$ Required area

$$= \int_{-\sqrt{24}}^{\sqrt{24}} (x_1 - x_2) dy$$

$= \int_{-\sqrt{24}}^{\sqrt{24}} \left\{\left(2-\frac{y^2}{8}\right)-\left(\frac{y^2}{14}-2\right)\right\} dy$

$= \int_{-\sqrt{24}}^{\sqrt{24}} \left(4-\frac{4y^2}{24}\right) dy$

$= \int_{-\sqrt{24}}^{\sqrt{24}} \left(4-\frac{y^2}{6}\right) dy$

$= \left(4y-\frac{y^3}{18}\right)_{\sqrt{-24}}^{\sqrt{24}}$

$= \left(4\sqrt{24}-\frac{24\sqrt{24}}{18}\right)-\left(-4\sqrt{24}+\frac{24\sqrt{24}}{18}\right)$

$= 8\sqrt{24}-\frac{48\sqrt{24}}{18}$

$= 8\sqrt{24}-\frac{8\sqrt{24}}{3}$

$= \frac{16\sqrt{24}}{3}$

$= \frac{16\times 2\sqrt{6}}{3}$

$= \frac{32\sqrt{6}}{3}$ sq units

101. **(a)** $V = a - kt^2$

$\frac{dV}{dt} = a - kt^2$

$\int_0^V dV = \int_0^t (a-kt^2)dt$

$V =$ at $-\frac{kt^3}{3}$(i)

When V is max$a = 0$

$\therefore a - Kt^2 = 0$

$\Rightarrow t = \sqrt{\frac{a}{k}}$

Using this in Eq. (i)

$V_{\max} = a\sqrt{\frac{a}{k}} - \frac{k}{3}\cdot\frac{a}{k}\sqrt{\frac{a}{k}} = \frac{2}{3}a\sqrt{\frac{a}{k}}$

102. **(b)** By given condition $2[\log 2b - \log 3c]$

$= \log a - \log 2b + \log 3c - \log a$

$\log 3c - \log 2b$

$\Rightarrow \quad 3[\log 2b - \log 3c] = 0$

$2b = 3c$

Also, $b^2 = ac$

$\therefore \quad \frac{9c^2}{4} = ac$

$\Rightarrow \quad c = \frac{4a}{9}$

$b = \frac{3c}{2} = \frac{3}{2}\cdot\frac{4}{9}a = \frac{2a}{3}$

Since, $a + b = a + \frac{2a}{3} = \frac{5a}{3} > c$

$$b+c=\frac{2a}{3}+\frac{4a}{9}=\frac{10a}{9}>a$$
$$c+a=\frac{13a}{9}>b$$

$\therefore a, b, c$ are the sides of triangle. Also, as a is the greatest side,
$\angle A$ of $\triangle ABC$,

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{\frac{4a^2}{9}+\frac{16a^2}{81}-a^2}{2\cdot\frac{2}{3}a\cdot\frac{4}{9}a}$$
$$=\frac{-29}{48}<0$$

$\therefore \triangle ABC$ is an obtuse angle.

103. **(b)** We have

$$a_n=(1^2+2^2+3^2+\cdots+n^2)^n$$
$$and b_n=n^n(n!)$$

Also, written as

$$a_n=\left[\frac{n(n+1)(2n+1)}{6}\right]^n$$

and $b_n=n^n(n!)$

By putting the values of $n=1,2,3,4$ ..., we get

$a_n>b_n, \forall n$

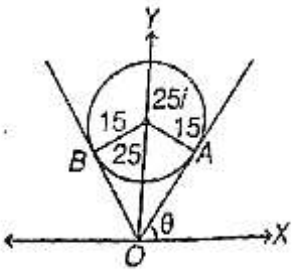
104. **(d)** In terms of prime factors, 100 ! can be written as $2^a\cdot 3^b\cdot 5^c 7^d$....

Now,

$$E_5(100!)=\left[\frac{100}{5}\right]+\left[\frac{100}{5^2}\right]$$
$$=20+4=24$$
$$100!=2^{97}\cdot 3^6\cdot 5^{24}\cdot 7^d \ldots$$
$$=2^{73}\cdot 3^6\cdot\left(2\times 5^{24}\cdot 7^d\ldots\right.$$
$$=2^{73}\cdot 3^6\cdot 10^{24}\cdot 7^d$$

Hence, number of zeroes at the end of 100! is 24 .

105. **(b)** If $|z-25i|\leq 15$, then z lies either in the interior and on the boundary of the circle with center at $c(25i)$ and radius is equal to 15.



From figure, it is clear that argument at least for point A and argument is greatest for point B.

From right $\triangle OAC$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{OA}{OC}=\frac{20}{25}$$
$$\frac{\pi}{2}-\theta=\cos^{-1}\left(\frac{4}{5}\right)$$

Now, for $|z-25i|\leq 15$

| Maximum ($\arg z$) - Minimum $\arg z$ |

$$=\arg(B)-\arg(A)\Big|=\angle BOA=2\left(\frac{\pi}{2}-\theta\right)$$
$$=2\cos^{-1}\left(\frac{4}{5}\right)$$

106. **(d)** $z=x-iy$

$(z)^{1/3} = p + iq$.......(i)

Value of $\left(\frac{x}{p}+\frac{y}{q}\right)/(p^2+q^2)$

Taking cube on both sides on equation,

$[(z)^{1/3}] = (p+iq)^3$

$= p^3 + (iq)^3 + 3pqi(p+iq)$

$= p^3 - iq^3 + 3p^2qi - 3pq^2$

$z = p^3 - 3pq^2 - i(q^3 - 3p^2q)$

$z = p(p^2-3q^2) - iq(q^2-3p^2)$....(iii)

On comparing Eqs. (ii) and (iii),

we get $x = p(p^2-3q^2), y = q(q^2-3p^2)$

Now,

$$\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{p^2+q^2} = \frac{\frac{p(p^2-3q^2)}{p}+\frac{q(q^2-3p^2)}{q}}{p^2+q^2}$$

$$= \frac{p^2-3q^2+q^2-3p^2}{p^2+q^2}$$

$$= \frac{-2(p^2+q^2)}{p^2+q^2} = -2$$

107. **(a)** Discriminant $= (D) = \sqrt{b^2-4ac}$

$(2a-b)^2 > 0$, as 2 times of a is greater than b, then value of D is positive ($D > 0$).

So, roots are rational.

108. **(c)** The possible arrangements of n white and n black balls alternately in the following ways.

(i) WBWB (ii) BWBW .....

Thus, the required number of ways

$= n! \times n! + n! \times n! = 2(n!)^2$

109. **(b)** $f(n) = 2^{n+1}$ ......(i)

$g(n) = 1 + (n+1)2^n$ ....(ii)

$g(n) = 1 + (n+1)\frac{f(n)}{2}$ $[f_n = 2^{n+1}]$

$2^n = \frac{f(n)}{2}$

$g(1) = f(1) + 1$

$n > 1, \frac{n+1}{2} > 1$

$g(n) > \left(\frac{n+1}{2}\right) f(n) > f(n)$

$f(n) < g(n)$

110. **(d)** Let $A = \{a_1, a_2, a_3, \dots, a_n\}$

$1 \le i \le n$

(i) $a_i \in P$ and $a_i \in Q$

(ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$

(iv) $a_i \notin P$ and $a_i \notin Q$

$P \cap Q = \phi$

$[f_n = 2^{n+1}]$

Since, case in favour 3 i.e. (ii), (iii) and (iv).

Total element in $A = n$

$\therefore$ Required number of ways $= 3^n$

111. **(d)** $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & (a-4) \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ a \end{bmatrix}$

$x + 2y + 4z = 6$

$2x + y + 2z = 4$

$x + 2y + (a-4)z = a$

For unique solution, $|A| \neq 0$, then the system is consistant independent and have unique solution.

$\therefore\ a \neq 8$

112. **(b)** $\begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ (x-1) & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2^3 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, then

$$\Delta x = \begin{vmatrix} (x-2) & (x-1)^2 & x^3 \\ 1 & (2x-1) & (3x^2+3x+1) \\ 2 & 4x & 6x^2+12x+8 \end{vmatrix}$$

Applying $R_3 \to R_3 - 2R_2$, then

$$\begin{vmatrix} (x-2) & (x-1)^2 & \vdots & x^3 \\ 1 & (2x-1) & & 3x^2+3x+1 \\ 0 & 2 & & 6x+6 \end{vmatrix}$$

Expand w.r.t. C, then

$\Delta(x) = (x-2)\{(2x-1)(6x-6) - 2(3x^2+3x+1)\}$

$-1\{(x-1)^2(6x+6) - 2x^3\}$

$\therefore$ Coefficient of x in $\Delta(x) = -8 + 6 = -2$

113. **(b)** $P \begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix}$

For matrix 3×3 matrix,

$|\mathrm{Adj}A| = |A|^2$

$|\mathrm{Adj}A| = 16$

$\Rightarrow 1(12 - 12 - \alpha(4-6) + 3(4-6) = 16$

$\Rightarrow 2\alpha - 6 = 16$

$\Rightarrow 2\alpha = 16 + 6$

$\Rightarrow \alpha = \dfrac{22}{2}$

$\Rightarrow \alpha = 11$

114. **(b)** $A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$

$A^{2018} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 1+i \\ 0 & i^2 \end{bmatrix}$$

Then,

$$A^3 = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 & 1+i \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} 1 & i \\ 0 & i^3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & i \\ 0 & i^3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+i \\ 0 & i^4 \end{bmatrix}$$

So, $A^{2018} = \begin{bmatrix} 1 & 1+i \\ 0 & i^{2018} \end{bmatrix}$

On comparing with $A^{2018} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

So, $a = 1$

$d = i^{2018} = (i^2)^{1009} = (-1)^{1009} = -1$

Hence, $a + d = 1 - 1 = 0$

115. **(d)** $g \circ f : S \to U$

gof is injective or (one-one) function:

So, $g(fx_1) = g(fx_2)$

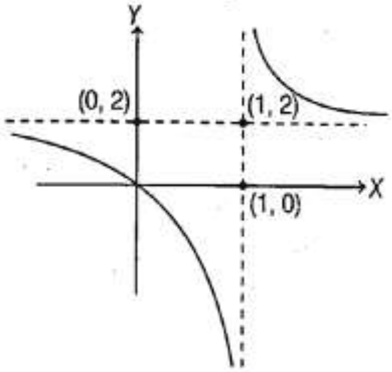
$f(x_1) = f(x_2)$

$\therefore$ ***g*** is obviously injective.

116. **(d)** $f(x) = \frac{2x}{x-1}$

$$y = \frac{2x}{x-1} = \frac{2x-2+2}{x-1} = \frac{2(x-1)+2}{(x-1)}$$
$$= \frac{2(x-1)}{(x-1)} + \frac{2}{(x-1)}$$
$\therefore\ (y-2)(x-1) = 2$

onto $\rightarrow$ co-domain $=$ range



Thus, f is both one-one and onto.

117. **(b)** Given that the probability of three events are

$P(A) = \dfrac{3x+1}{3} \Rightarrow P(B) = \dfrac{1-x}{4} \Rightarrow P(C) = \dfrac{1-2x}{2}$

As we know that probability of event lies between 0 and 1.

This can be represented as $0 \leq P(x) \leq 1$. So let us use this theorem for each probability one by one.

For given event A, we have $0 \leq P(A) \leq 1$

Let us now substitute the value given

$0 \leq \left[P(A) = \frac{3x+1}{3}\right] < 1$.....(i)

We get the interval of x from Eq. (i),

$$0 \leq \frac{3x+1}{3} \leq 1$$

$\Rightarrow\ \frac{-1}{3} \leq x \leq \frac{2}{3}$....(ii)

Similarly, for the given event B, we have

$0 \leq P(B) \leq 1$

Let us now substitute the value given.

Let us simplify further to find the interval of x

$$0 \leq \frac{1-x}{4} \leq 1$$

$-3 \leq x \leq 1$(iii)

For event c

$0 \leq p(c) \leq 1$

$$0 \leq \frac{1-2x}{2} \leq 1$$

$\Rightarrow\ 1 - 2x \leq 2$

$\Rightarrow\ -\frac{1}{2} \leq x \leq \frac{1}{2}$...(iv)

Now, three mutually exclusive events are

$0 \leq P(A \cup B \cup C) \leq 1$

$\therefore$ [Probability must be 0 to 1]

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
$-P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

So, probability of intersection any of these event is 0

$P(A \cap B) = P(B \cap C) = P(C \cap A)$
$= P(A \cap B \cap C) = 0$

So, the union of events become

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - 0 - 0 - 0$

$\Rightarrow P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$0 < \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \leq 1$

$0 \leq 13 - 3x \leq 12$

$\frac{1}{3} \leq x \leq \frac{13}{3} \ldots\ldots (v)$

Finally, we have four inequalities

$-\frac{1}{3} \leq x \leq \frac{2}{3}$ ... [from Eq. (ii)]

$-3 \leq x \leq 1$ ... [from Eq. (iii)]

$-\frac{1}{2} \leq x \leq \frac{1}{2}$ ... from Eq. (iv)

$\frac{1}{3} \leq x \leq \frac{13}{3}$ ... from Eq. (v)

Hence, the common of each inequality is $\frac{1}{3} \leq x \leq \frac{1}{2}$.

118. **(b)** Determinant of order 2 will be of the form

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\Delta = ad - bc$

The total number of ways of choosing a, b, c and d is

$2^4 = 16$

Now, $\Delta \neq 0$ if and only if either $ad = 1$

$bc = 0$ or $ad = 0, bc = 1$ for $ad = 1 (a, d) = (1,1)$

For $bc = 0 (b, c) = (0,1), (1,0), (0,0)$

So, three cases are there.

Similarly, for $ad = 0$ and $bc = 1$

There will be three cases.

$\therefore$ Number of favourable cases $= 3 + 3 = 6$

So, required probability $= \frac{6}{16} = \frac{3}{8}$

119. **(a)** We know,

$(\cot\alpha_1)(\cot\alpha_2)(\cot\alpha_3) \ldots (\cot\alpha_n) = 1$

$\Rightarrow$

$\cos\alpha_1 \cos\alpha_2 \ldots \cos\alpha_n = \sin\alpha_1 \sin\alpha_2 \sin\alpha_3 \ldots \sin\alpha_n$

Multiply ($\cos\alpha_1 \cos\alpha_2 \cos\alpha_3 \ldots . \cos\alpha_n$) both side

$(\cos\alpha_1 \cos\alpha_2 \ldots . \cos\alpha_n)^2$

$= (\cos\alpha_1 \sin\alpha_1)(\cos\alpha_2 \sin\alpha_2) \ldots (\cos\alpha_n \sin\alpha_n)$

$\Rightarrow (\cos\alpha_1 \cos\alpha_2 \ldots \cos\alpha_n)^2 = \frac{1}{2^n}$

$(\sin 2\alpha_1)(\sin 2\alpha_2) \ldots (\sin 2\alpha_n)$

$\Rightarrow (\cos\alpha_1 \cos\alpha_2 \ldots . \cos\alpha_n)^2 \leq \frac{1}{2^n}$

$\Rightarrow \cos\alpha_1 \cos\alpha_2 \ldots . \cos\alpha_n \leq \frac{1}{2^{n/2}}$

Thus, the maximum value of

$(\cos\alpha_1, \cos\alpha_2 \ldots . \cos\alpha_n)$ is $\frac{1}{2^{n/2}}$

120. **(d)** Let the equation of the line be $ax + by + c = 0$

According to the given condition, we have

$\left|\frac{2a+c}{\sqrt{a^2+b^2}}\right| + \left|\frac{2b+c}{\sqrt{a^2+b^2}}\right| + \left|\frac{a+b+c}{\sqrt{a^2+b^2}}\right| = 0$

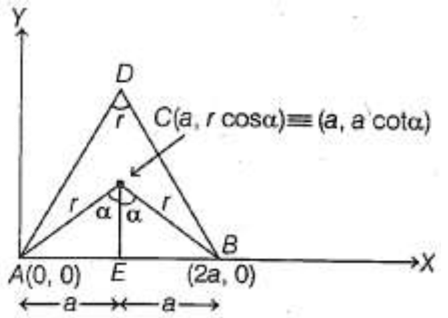
$\Rightarrow \pm\frac{2a+c}{\sqrt{a^2+b^2}} \pm \left(\frac{2b+c}{\sqrt{a^2+b^2}}\right) \pm \left(\frac{a+b+c}{\sqrt{a^2+b^2}}\right) = 0$

$\Rightarrow 2a + c + 2b + c + a + b + c = 0$

$\Rightarrow 3a + 3b + 3c = 0$

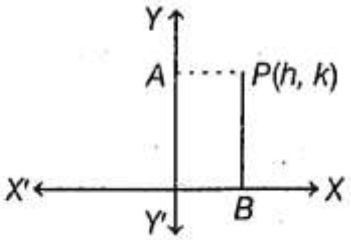
$\Rightarrow a+b+c=$
Comparing the line $ax+by+c=0$ and $a+b+c=0$ and get the fixed point.
$\therefore$ The required fixed point is (1,1).

121. **(b)** Let $AC=BC=r$



Now, from $\triangle ACE$
$r\sin\alpha=a$
$r=a\text{cosec}\alpha$
$\Rightarrow$ Now, again in $\triangle ACE$
$CE=r\cos\alpha=(a\text{cosec}\alpha)\cos\alpha=a\cot\alpha$
Now locus of the vertex will be a circle with radius $r=a\text{cosec}\alpha$ and cetnre $C(a,a\cot\alpha)$
$\therefore$ Required locus is
$(x-a)^2+(y-a\cot\alpha)^2=a^2\text{cosec}^2\alpha$
$x^2+a^2-2ax+y^2+a^2\cot^2\alpha-2ay\cot\alpha$
$=a^2\text{cosec}^2\alpha$
$\Rightarrow x^2+y^2-2ax-2ay\cot\alpha=0$

122. **(c)** Let the locus of a point in a plane be $P(h,k)$.



According to the question,
$|PA|+|PB|=1$
$\Rightarrow$ $\Rightarrow$
$|h|+|k|=1$
Hence, locus of a point is $|x|+|y|=1$
which represents the straight line.

123. **(b)** Let $\theta=\sin^{-1}\frac{3}{5}$

$\Rightarrow\ \sin\theta=\frac{3}{5}\Rightarrow\tan\theta=\frac{3}{4}$

So, equation of the line passes through $(-1,1)$ with slope $\frac{3}{4}$ is $y-1=\left(\frac{3}{4}\right)(x+1)$
$3x-4y+7=0$
Which meets the curve $x^2=4y-9$ at points for which $3x-(x^2+9)+7=0$ or $x^2-3x+2=0$
$\Rightarrow x=1,2$ and the coordinates of A are $\left(1,\frac{10}{4}\right)$ and of B are $\left(2,\frac{13}{4}\right)$.

So, that $|AB|=\sqrt{(2-1)^2+\left(\frac{13}{4}-\frac{10}{4}\right)^2}=\frac{5}{4}$ units

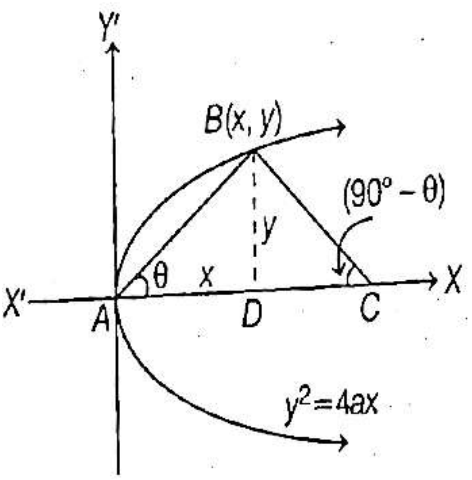
124. **(c)** We know that,
The equation of common chord of two circles $S=x^2+y^2+2gx+2fy+c=0$
and $S'=x^2+y^2+2g'x+2f'y+c'=0$ is
$2x(g-g')+2y(f-f')+c-c'=0$
i.e. $S-S'=0$

125. **(b)** An equation of the normal at P is $3x\cos\theta+2y\cot\theta=9+4$

$\Rightarrow\ 3x + xy\text{cosec}\theta = 13\sec\theta \ldots$(i)
An equation of normal at Q is
$3x + 2y\text{cosec}\phi = 13\sec\phi \ldots\ldots$(ii)
On subtracting Eq. (ii) from Eq. (i), we get
$\therefore\ 2y[\text{cosec}\theta - \text{cosec}\phi] = 13(\sec\theta - \sec\phi)]$
$\Rightarrow 2y[\text{cosec}\theta - \sec\theta] = 13(\sec\theta - \text{cosec}\phi)]$
$\left[\because \phi = \frac{\pi}{2} - \theta\right]$
$y = -\frac{13}{2}$

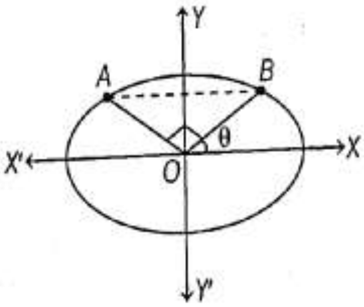
126. **(d)** Let the coordinates of $Qbe(h,k)$ and α and β be the mid-point of PQ.
Then,
$$\alpha = \frac{2+h}{2}$$
$$h = 2\alpha - 2$$
and
$$\beta = \frac{0+k}{2}.$$
$$k = 2\beta$$
Since, $Q(h,k)$ lies on $(y-6)^2 = 2(x-4)$
$\Rightarrow\ (k-6)^2 = 2(h-4) \ldots\ldots$(i)
Put the value of h and k in Eq. (i)
$$(2\beta - 6)^2 = 2(2\alpha - 2 - 4)$$
$$\Rightarrow\ 4(\beta - 3)^2 = 2(2\alpha - 6)$$
$$\Rightarrow\ 2(\beta - 3)^2 = 2(\alpha - 3)$$
$$\Rightarrow\ \beta^2 + 9 - 6\beta = \alpha - 3$$
$$\alpha - \beta^2 + 6\beta - 12 = 0$$
$\therefore$ The locus of (h,k) is $x - y^2 + 6y - 12 = 0$

127. **(d)** We know that,



In $\triangle ABD$, we have
$\tan\theta = \frac{y}{x} \ldots\ldots\ldots$(i)
In $\triangle BCD$, we have $\tan(90° - \theta) = \frac{y}{CD}$
$$CD = y\tan\theta = \frac{y^2}{x} \quad \text{[by using Eq. (i)]}$$
$$CD = \frac{4ax}{x} = 4a\ \ (y^2 = 4ax)$$

128. **(a)** Suppose OB makes an angle θ with the positive direction of the X-axis, then OA makes an angle of $\theta + \frac{\pi}{2}$ with the positive direction of the X-axis.



Let $OA = r_1$ and $OB = r_2$ coordinates of A and B are $(-r_1 \sin\theta, r_1 \cos\theta)$ and $(r_2 \cos\theta, r_2 \sin\theta)$ respectively. As B lies on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{r_2^2 \cos^2\theta}{a^2} + \frac{r_2^2 \sin^2\theta}{b^2} = 1$$

$$\frac{1}{OB^2} = \frac{1}{r_2^2} = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} \ldots.(i)$$

Similarly, as A lies on the ellipse

$$\frac{1}{OA^2} = \frac{1}{r_1^2} = \frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2} \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{OA^2} + \frac{1}{OB^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

129. **(c)** The equation of the given first plane is

$x + y + z - 1 = 0 \ldots\ldots(i)$

where $(1,1,1)$ are direction ratio of normal to the plane.

The equation of second plane is

$2x + 3y - z + 4 = 0 \ldots.(ii)$

where $(2,3,-1)$ are direction ratio of normal to the second plane.

We know that, the equation of any plane passes through the intersection of two given planes

$ax + by + cz + d = 0$ and $a_1 x + b_1 y + c_1 z + d_1 = 0$ is

$ax + by + cz + d + \lambda(a_1 x + b_1 y + c_1 z + d_1) = 0$ ...(iii)

where λ is arbitrary constant.

Here, $a = 1, b = 1, c = 1, d = -1$

$a_1 = 2, b_1 = 3, c_1 = -1, d_1 = 4$

Putting these values in Eq. (iii), we get

$x + y + z - 1 + \lambda(2x + 3y - z + 4) = 0$

$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 - \lambda) - 1 + 4\lambda = 0 \ldots(iv)$

It is given that the plane (iv) Is parallel to the X-axis, then the plane (iv) is perpendicular to the YZ, plane. Therefore, the normal of the plane (iv) and the YZ-plane are also perpendicular to each other.

Equation of YZ plane is $x = 0$ i.e.

$$1 \cdot x + 0 \cdot y + 0 \cdot z = 0$$

$$\Rightarrow 1(1 + 2\lambda) + 0(1 + 3\lambda) + 0(1 - \lambda) = 0$$

$$1 + 2\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$$

Put the value of $\lambda = -\frac{1}{2}$ in Eq. (iv), we get

$$x\left(1 - \frac{2}{2}\right) + y\left(1 - \frac{3}{2}\right) + z\left(1 + \frac{1}{2}\right) - 1 + 4\left(-\frac{1}{2}\right) = 0$$

$$x \cdot 0 + y\left(-\frac{1}{2}\right) + \frac{3z}{2} - 1 - 2 = 0$$

$$-y + 3z - 6 = 0$$

$\therefore$ Required equation of plane $= y - 3z + 6 = 0$

130. **(d)** Given, the equations of the line,

$x - 2y + 4z + 4 = 0 \ldots\ldots\ldots$ (i)
$x + y + z - 8 = 0 \ldots\ldots\ldots$ (ii)
and equation of the plane
$x - y + 2z + 1 = 0$.......(iii)
Solve Eqs. (i), (ii) and (iii), we get
$x = 2, y = 5$ and $z = 1$
Hence, the required point is (2,5,1).

131. **(d)** For $0 < x < 1$, we have $\frac{1}{2}x^2 < x^2 < x$

i.e. $-x^2 > -x$. So that, $e^{-x^2} > e^{-x}$
Hence, $\int_0^1 e^{-x^2}\cos^2 x dx > \int_0^1 e^{-x}\cos^2 x dx$.
Also $\cos^2 x \leq 1$, therefore
$\int_0^1 e^{-x^2}\cos^2 x dx \leq \int_0^1 e^{-x^2} dx < \int_0^1 e^{-x^2/2} dx = I_4$
Hence, I_4 is the greatest integral.

132. **(b)** $\lim_{x\to\infty}\left(\frac{x^2+1+2x-2x}{x+1} - ax - b\right) = 0$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{(x+1)^2 - 2x}{x+1} - ax - b\right) = 0$$

$$\Rightarrow \lim_{x\to\infty}\left((x+1) - \frac{2x}{x+1} - ax - b\right) = 0$$

$$\Rightarrow \lim_{x\to\infty}\left[(x+1) - \left(\frac{2x+2-2}{x+1}\right)\right] - \lim_{x\to\infty}(ax+b) = 0$$

$$\Rightarrow \lim_{x\to\infty}\left[x+1 - \left(\frac{2x+2-2}{x+1}\right)\right] = \lim_{x\to\infty}(ax+b)$$

$$\Rightarrow \lim_{x\to\infty}\left[x+1-2+\frac{2}{x+1}\right] = \lim_{x\to\infty}(ax+b)$$

$\Rightarrow \lim_{x\to\infty}(x-1) + 0 = a\lim_{x\to\infty} x + b$
$\Rightarrow \lim_{x\to\infty} x - 1 = a\lim_{x\to\infty} x + b$
On comparing the above equation, we get $a = 1, b = -1$

133. **(b)** We have, $z = \log\tan\left(\frac{x}{2}\right)$

Now, $\frac{dz}{dx} = \frac{1}{2}\frac{\sec^2(x/2)}{\tan(x/2)} = \frac{1}{\sin x} = \text{cosec} x$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} = \text{cosec} x\frac{dy}{dz}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dz}\left(\frac{dz}{dx}\right)\frac{dy}{dx}$$

$$= \frac{d}{dz}\left(\text{cosec} x\frac{dy}{dz}\right)\text{cosec} x$$

$$= \left[\text{cosec} x\frac{d^2y}{dz^2} - \text{cosec}\, x\cot x\frac{dx}{dz}\frac{dy}{dz}\right]\text{cosec} x$$

Putting these values in the given differential equation, we have

$$\frac{d^2y}{dx^2} + \cot x\frac{dy}{dx} + 4y\text{cosec}^2 x = 0$$

$$\Rightarrow \text{cosec}^2 x\frac{d^2y}{dz^2} - \text{cosec}\, x\cot x\frac{dy}{dz} + \cot x$$

$$\text{cosec} x\frac{dy}{dz} + 4y\text{cosec}^2 x = 0$$

$$\Rightarrow \text{cosec}^2 x\left(\frac{d^2y}{dz^2} + 4y\right) = 0 \Rightarrow \frac{d^2y}{dz^2} + 4y = 0$$

On comparing $\frac{d^2}{dx^2} + ky = 0$, then
$k = 4$

134. **(d)** Equation of tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$
Since, tangent passes through $(-1, -6)$

$$-6 = -m + \frac{1}{m}$$

$\Rightarrow \quad m^2 - 6m - 1 = 0$

On solving above equation, we get

$m_1 = 6.16$

$m_2 = -0.16$

$m_1 m_2 = 616 \times (-0.16)$

$m_1 m_2 \cong -1$

$\therefore$ The angle between them is 90° (approx).

135. **(d)** Given, $\boldsymbol{\beta} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\gamma = \hat{\mathbf{i}} + \hat{\mathbf{k}}$

So, $[\alpha \quad \beta \quad \gamma] = \alpha \cdot [\beta \times \gamma]$

$$= \alpha \cdot \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix} = \alpha \cdot [\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}]$$

$= |\alpha|\sqrt{1+4+1}\cos\theta$

$= |\alpha|\sqrt{6}\cos\theta$

$= \sqrt{6}\cos\theta$

$\text{Max}[\alpha \quad \beta \quad \gamma] = \sqrt{6} \times 1 = \sqrt{6}$

Hence, max value $[\alpha \cdot \beta \cdot \gamma] = \sqrt{6}$

136. **(c)** We have $f(x) = e^{\sin x} + e^{\cos x}$

$$\therefore \frac{e^{\sin x} + e^{\cos x}}{2} \geq e^{\sqrt{\sin x + \cos x}}$$

$$\frac{e^{\sin x} + e^{\cos x}}{2} \geq e^{-\frac{\sqrt{2}}{2}}$$

$$e^{\sin x} + e^{\cos x} \geq 2e^{-\frac{\sqrt{2}}{2}}$$

$$\geq 2e^{-\frac{1}{\sqrt{2}}}$$

Thus, maximum value of $f(x)$ is $2e^{1/\sqrt{2}}$.

137. **(b)** Let the line be $\frac{x}{a} + \frac{y}{b} = 1$

$\therefore A$ is $(a, 0)$, B is $(0, b)$.

Also, O is $(0,0)$.

$\therefore$ Equation of circumcircle to $\triangle OAB$ is

$x^2 + y^2 - ax - by = 0$

Tangent at $(0,0)$ to the circle is

$$x \cdot 0 + y \cdot 0 + \frac{a}{2}(x+0) + \frac{b}{2}(y+0) = 0$$

$$= ax + by = 0$$

$$m = \frac{a \cdot a + b \cdot 0}{\sqrt{a^2+b^2}} = \frac{a^2}{\sqrt{a^2+b^2}}$$

$$n = \frac{a \cdot 0 + b \cdot b}{\sqrt{a^2+b^2}} = \frac{b^2}{\sqrt{a^2+b^2}}$$

$$m + n = \frac{a^2+b^2}{\sqrt{a^2+b^2}} = \sqrt{a^2+b^2}$$

Radius of circle $= \frac{\sqrt{a^2+b^2}}{2}$

Hence, diameter of circle is $m + n$.

138. **(a)** Tangent at P is $ty = x + at^2$, which meets axis at $T(-at^2, 0)$. Normal at P is $tx + y = 2at + at^3$, which meets axis at $G(2a + at^2, 0)$

$\therefore \angle TPG = \frac{\pi}{2}$, so TG is diameter of the circle.

Equation of circle, is

$(x + at^2)(x - 2a - at^2) + (y - 0)(y - 0) = 0$
$x^2 + y^2 - 2ax - at^2(2a + at^2) = 0$
Here,
$g = -a$
$c = at^2(2a + at^2)$
$f = 0$
So, radius $(r) = \sqrt{g^2 + f^2 - c}$
$\Rightarrow r = \sqrt{(-a)^2 + 0 + at^2(2a + at^2)}$
$r = a\sqrt{1 + 2t^2 + t^4}$
$= a(1 + t^2)$
$r = a(1 + t^2)$

139. **(b)** Suppose α, β be the roots of the given equation Then,
$\alpha + \beta = a - 2$ and $\alpha\beta = -(a + 1)$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a - 2^2 + 2(a + 1)$
$\alpha^2 + \beta^2 = a^2 - 2a + 6$
$= (a - 1)^2 + 5$
$\alpha^2 + \beta^2 \geq 5.$
So, the minimum value of $\alpha^2 + \beta^2$ is 5,
which attains at $a = 1$

140. **(b)** $\frac{\log x^2}{\log 25} + \left[\frac{\log x}{\log 5}\right]^2 < 2$

$\Rightarrow \frac{2\log x}{2\log 5} + \left[\frac{\log x}{\log 5}\right]^2 - 2 < 0 \left[\text{Here, } y = \frac{\log x}{\log 5}\right]$
$\Rightarrow y^2 + y - 2 < 0$
$\Rightarrow -2 < y < 1$
$\Rightarrow -2 < \frac{\log x}{\log 5} < 1$
$\Rightarrow -2\log 5 < \log x < \log 5$
$\Rightarrow \frac{1}{25} < x < 5$, i.e. $x \in \left(\frac{1}{25}, 5\right)$

141. **(d)** Given, $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$ and $|A - \lambda I_2| = 0$
Now, $A - \lambda I_2 = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$= \begin{bmatrix} 2-\lambda & 3 \\ x & y-\lambda \end{bmatrix}$
$\therefore |A - \lambda I_2| = 0$
$\Rightarrow \begin{vmatrix} 2-\lambda & 3 \\ x & y-\lambda \end{vmatrix} = 0$
$\Rightarrow (2 - \lambda)(y - \lambda) - 3x = 0$
$\Rightarrow 2y - 2\lambda - \lambda y + \lambda^2 - 3x = 0$
When $\lambda = 4$, then $2y - 8 - 4y + 16 - 3x = 0$
$\Rightarrow 3x + 2y - 8 = 0$……(i)
and when $\lambda = 8$ then $2y - 16 - 8y + 64 - 3x = 0$
$\Rightarrow 3x + 6y - 48 = 0$……..(ii)
On solving Eqs. (i) and (ii), we get
$x = -4, y = 10$

142. **(a)** Let coordinates of P_1 be $(at_1^2, 2at_1)$, then coordinates of P_2 are $\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$.
Similarly, if coordinates of P_3 are $(at_2^2, 2at_2)$.
Then, coordinates of P_4 are $\left(\frac{a}{t_2^2}, \frac{-2a}{t_2}\right)$.
An equation of the chord P_1P_3 is

$$\frac{x-at_1^2}{at_2^2-at_1^2}=\frac{y-2at_1}{2at_2-2at_1}$$

or $2(x+at_1t_2)=(t_1+t_2)y\ldots$(i)

Similarly, the equation of the chord

$$2\left(x+\frac{a}{t_1t_2}\right)=-\left(\frac{1}{t_1}+\frac{1}{t_2}\right)y$$

$$2(t_1t_2x+a)=-(t_1+t_2)y\text{(ii)}$$

Similarly, the equation the chord P_2P_4 is

Adding Eqs. (i) and (ii), we get

$$2(1+t_1t_2)(x+a)=0$$

$$x+a=0$$

Therefore, intersection of point P_1P_3 and P_2P_4 lies on directrix.

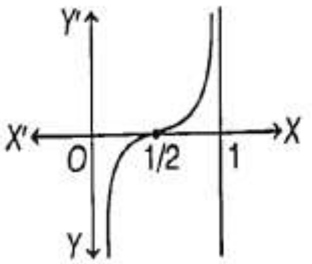
143. **(c)** Given

$$,f(x)=\frac{2x-1}{1-|2x-1|}$$

$$\therefore f(x)=\begin{cases}\dfrac{2x-1}{1+2x-1}, & 0<x<\dfrac{1}{2}\\ \dfrac{2x}{1-2x+1}, & \dfrac{1}{2}\le x<1\end{cases}$$

$$=\begin{cases}\dfrac{2x-1}{2x}, & 0<x<\dfrac{1}{2}\\ \dfrac{2x-1}{2-2x}, & \dfrac{1}{2}\le x<1\end{cases}$$

Now, let us draw the graph of above function.



From the above graph it is clear that $f(x)$ is one-one and since the range of $f(x)$ is real number, which implies that codomain is equal to range.

$\therefore f(x)$ is one-one and onto both and hence $f(x)$ is bijective.

144. **(c)** Given, $\int_0^x \sqrt{1-(f'(t))^2}dt=\int_0^x f(t)dt, 0\le x\le\frac{\pi}{2}$

On differentiating both sides w.r.t. x by using Leibnitz rule, we get

$$\Rightarrow \sqrt{1-[f'(x)]^2}=f(x)$$

$$\Rightarrow f'(x)=\pm\sqrt{1-[f(x)]^2}$$

$$\Rightarrow \int\frac{f'(x)}{\sqrt{1-[f(x)]^2}}dx=\pm\int dx$$

$$\sin^{-1}[f(x)]=\pm x+c$$

Put $x=0\Rightarrow\sin^{-1}[f(0)]=\pm 0+c$

$\Rightarrow c=\sin^{-1}(0)=0\quad[\because f(0)=0]$

$\therefore\quad f(x)=\pm\sin x$

$\because f$ is a non-negative function defined in $\left[0,\frac{\pi}{2}\right]$

$\therefore f(x)=\sin x$

And we know that, $\sin x<x,\forall x>0$

$\therefore \sin\left(\frac{1}{2}\right)<\frac{1}{2}$ and $\sin\left(\frac{1}{3}\right)<\frac{1}{3}\Rightarrow f\left(\frac{1}{2}\right)<\frac{1}{2}$

and $f\left(\frac{1}{3}\right)<\frac{1}{3}$

Similarity $f\left(\frac{4}{3}\right)<\frac{4}{3}$ and $f\left(\frac{2}{3}\right)<\frac{2}{3}$

145. **(d)** Let the coordinates of P be (h, k).

$\therefore \quad PQ=2k$ and $OP=OQ=PQ$

$\Rightarrow \quad \sqrt{h^2+k^2}=2k \Rightarrow h^2+k^2=4k^2$

$\Rightarrow \quad \therefore h^2=3k^2$

$\therefore \quad (h, k)$ lies on hyperbola $\Rightarrow \dfrac{h^2}{a^2}-\dfrac{k^2}{b^2}=1$

$\Rightarrow \quad \dfrac{3k^2}{a^2}-\dfrac{k^2}{b^2}=1$

$\Rightarrow \quad \dfrac{3b^2-a^2}{a^2b^2}=\dfrac{1}{k^2}>0$

$\Rightarrow \quad \dfrac{3}{a^2}-\dfrac{1}{b^2}>0 \Rightarrow \dfrac{3}{a^2}>\dfrac{1}{b^2}$

$\Rightarrow \quad \dfrac{b^2}{a^2}>\dfrac{1}{3} \Rightarrow e^2-1>\dfrac{1}{3}$

$\Rightarrow \quad e^2>\dfrac{4}{3} \Rightarrow e>2/\sqrt{3}$

146. **(c)** If h is the height of the balloon when stone is dropped. Then, using equation

$h=-ut+\frac{1}{2}gt^2$

Here $u=v\text{ft/sec}, t=4\text{sec}$

$h=-vt+\frac{1}{2}32t^2=-4v+256$

$h=256-4v \quad$ (i)

If balloon rises $h$ft after dropping the balloon and height attained by balloon is 4 sec .

$h'=$ velocity of balloon $\times$ time taken

$\quad =v\times 4$

$h'=4v$

Total height of the balloon from the ground

$h+h'=256+4v-4v$

$\quad\quad =256\text{ft}$

147. **(a, c, d)** Given $f(x)=x^2+x\sin x-\cos x$

Now $f'(x)=\dfrac{d}{dx}(x^2)+\dfrac{d}{dx}(x\sin x)-\dfrac{d}{dx}(\cos x)$

$\Rightarrow \; f'(x)=2x+x\cos x+\sin x+\sin x$

$f'(x)=2x+x\cos x+2\sin x$

$\Rightarrow f(x)$ is an increasing function.

Now, $f(0)=-1$

$f(\infty)=+\infty$

$f(-\infty)=+\infty$

$\because \; f(-\infty)=+\text{ve}$ and $f(0)=-\text{ve}$

$\therefore$ There must be atleast one negative root in $(-\infty, 0)$.

$\because \; f(0)=-$ ve and $f(\infty)=+$ ve

$\therefore$ Then, must be atleast one positive root in $(0, \infty)$.

148. **(a,d)** We use properties of argument of complex numbers.

(i) $\arg\left(\frac{z_1}{z_2}\right)=\arg(z_1)-\arg(z_2)+2k\pi(K=0$ or

lor -1)

(ii) $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)+2k\pi(K=0$ or lor -1)

149. **(b,d)** Let

$$\Delta=\begin{vmatrix}\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta\\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta\\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0\end{vmatrix}$$

Apply $C_1\rightarrow C_1-\cot\phi C_2$, we get

$$\Delta=\begin{vmatrix}0 & \sin\theta\sin\phi & \cos\theta\\ 0 & \cos\theta\sin\phi & -\sin\theta\\ -\sin\theta/\sin\phi & \sin\theta\cos\phi & 0\end{vmatrix}$$

On expanding along C_1, we get

$$\Delta=-\frac{\sin\theta}{\sin\phi}[-\sin\phi\sin^2\theta-\cos^2\theta\sin\phi]$$

$=\sin\theta$, this is independent of ϕ

Also, $\frac{d\Delta}{d\theta}=\cos\theta$

$$\Rightarrow \left.\frac{d\Delta}{d\theta}\right|_{\theta=\frac{\pi}{2}}=\cos\left(\frac{\pi}{2}\right)=0$$

150. **(b)** Reflexivity Let a be an arbitrary element of A.

Then, $a\in A$

$(a,a)\in R$ and $(a,a)\in S$ [$\because$ R and S are reflexive]

$(a,a)\in R\cap S$

Thus, $(a,a)\in R\cap S$ for all $a\in A$. So, $R\cap S$ is a reflexive relation on A.

Symmetry Let $a,b\in A$ such that $(a,b)\in R\cap S$

Then $(a,b)\in R\cap S$

$(a,b)\in R$ and $(a,b)\in Sb(b,a)\in R$

and $(b,a)\in S$ [$\because$ S and R are symmetric]

$(b,a)\in R\cap S$ for all $(a,b)\in R\cap S$

So, $R\cap S$ in symmetric on A.

Transitive Let $a,b,c\in A$ such that $(a,b)\in R\cap S$ and $(b,c)\in R\cap S$

Then, $(a,b)\in R\cap S$ and $(b,c)\in R\cap S$

$\Rightarrow\ \{(a,b)\in R$ and $(b,c)\in S\}$

$\Rightarrow\ \{(a,b)\in R$ and $(b,c)\in R\}$

and $\{(a,b)\in S,(b,c)\in S\}$

$\Rightarrow\ (a,c)\in R(a,c)\in S$

$\therefore$ R and S are transitive so

$(a,b)\in R(b,c)\in R\Rightarrow(a,c)\in R$

$(a,b)\in S$ and $(b,c)\in S\Rightarrow(a,c)\in S$

$(a,c)\in R\cap S$

Thus, $(a,b)\in R\cap S$

$(a,c)\in R\cap S$

So, $R\cap S$ is transitive on A.

Hence, $R\cap S$ is an equivalence relation on A.

151. **(c)** Let (h,k) be the mid-point of a chord passing through the positive end of the minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Then, the equation of the chord is

$$\frac{hx}{a^2}+\frac{ky}{b^2}-1=\frac{h^2}{a^2}+\frac{k^2}{b^2}-1$$

$$\Rightarrow\ \frac{hx}{a^2}+\frac{ky}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

Thus, curve passes through the minor axis $(0,b)$.

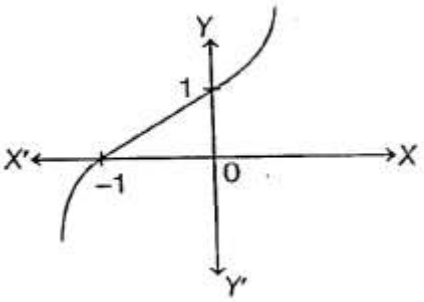
$\therefore\ \frac{k}{b}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$.

Hence, the locus of (h,k) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{y}{b'}$ which represent an ellipse.

152. **(b, c)** The above equation $(y-y_1)=m(x-x_1)$ is given. $y-y_1-m(x-x_1)=0$ is a family of lines

$y - y_1 = 0 \Rightarrow y = y_1$
$x - x_1 = 0 \Rightarrow x = x_1$
(b) There will be a set of parallel lines.
(c) all lines intersect the line $x = x_1$.

153. **(c)** $P(0) = 1$
$P'(x) > 0; \forall x \in R$
Let $P(x) = x^3 + 1$
$P'(x) = 3x^2$



(px)may have always be increasing as shown in above.
$P(x)$ may have negative real root.

154. **(c)** Perimeter of sector = 20 m

$$\frac{\theta}{360°}2\pi r + 2r = 20$$
$$\Rightarrow \frac{\theta}{360°}\pi r + r = 10$$
$$\Rightarrow \frac{\theta}{360°} = \frac{10 - r}{\pi r}$$

We know that , area of sector $A = \frac{\theta}{360°}\pi r^2$

$$A = \frac{10 - r}{\pi r} \cdot \pi r^2$$
$$A = (10 - r)r$$
$$\frac{dA}{dr} = \frac{d}{dr}(10r - r^2)$$
$$= 10 - 2r$$

For maximum area

$$\frac{dA}{dr} = 0$$
$$\Rightarrow 10 - 2r = 0$$
$$\Rightarrow r = 5$$
$$\Rightarrow \frac{d^2A}{dr^2} = \frac{d}{dr}(10 - 2r) = -2$$
$$\Rightarrow \frac{d^2A}{dr^2} < 0$$

Maximum value of $r = 5$ m

155. **(a,b,c)** Given $y = x + 5$
Comparing with $y = mx + c$
$m = 1$ and $c = 5$
Solve by option
Option (a) Condition of tangency
$$c = \frac{a}{m} \Rightarrow s = \frac{5}{1}$$
which is true.
Option (b) $9x^2 + 16y^2 = 144$
$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Condition of tangency $c^2 = a^2m^2 + b^2$
$25 = 16 \times 1 + 9 = 25$
which is true.
Option (c) $\frac{x^2}{29} - \frac{y^2}{4} = 1$
$\because$ Condition of tengency
$c^2 = a^2m^2 - b^2$
$25 = 29 \times 1 - 4 = 25$
which is true.
Option (d) Now length of perpendicular from center $(0,0)$ to the line $y = x + 5$ is $\frac{|5|}{\sqrt{2}}$.
i.e. $\frac{5}{\sqrt{2}} \neq$ radius (5)