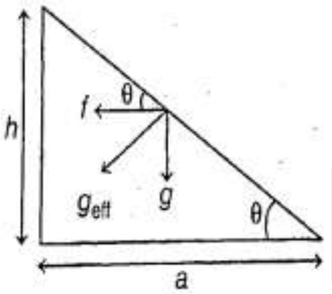
Physics

1. **(b)** In a SHM,
Acceleration, $f=-\omega^2 x$
Time period, $T=\frac{2\pi}{\omega}$
$\therefore\ |fT|=\left|-\omega^2 x\times\frac{2\pi}{\omega}\right|=2\pi\omega x$
$\therefore\ |fT|\propto x$
Hence, $|fT| vs x$ graph is a straight line.

2. **(a)** Given, $v=4\ \mathrm{m/s}$
At time, $t=0, y=3\sin 2\pi\left(-\frac{x}{3}\right)$
$\therefore$ At time $t, y=3\sin\frac{2\pi}{3}(vt-x)$
$\therefore$ At time $4\ \mathrm{s}, y=3\sin\frac{2\pi}{3}(4\times 4-x)$
$\therefore\ y=3\sin 2\pi\left(-x\frac{-16}{3}\right)$

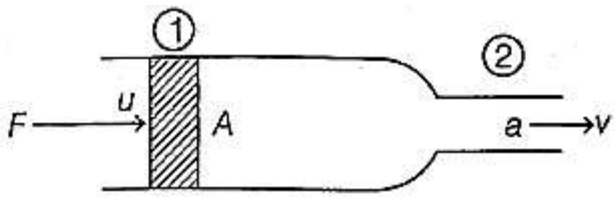
3. **(a)** The FBD of the liquid above the level is



$\therefore\ \tan\theta=\frac{f}{g}=\frac{h}{a}\ \therefore\ h=\frac{f}{g}a$

4. **(b)** The velocities of the molecules of ideal gas are 1,3,5,5,6 and 5 m/s.
$\therefore$ Average speed,
$\bar{v}=\frac{\sum v}{n}=\frac{1+3+5+5+6+5}{6}\Rightarrow\frac{25}{6}=4.1\ \mathrm{m/s}$
Root mean square speed, $v_{\text{rms}}=\sqrt{\frac{\Sigma v^2}{n}}$
$\therefore v_{\text{rms}}=\sqrt{\frac{1^2+3^2+5^2+5^2+6^2+5^2}{6}}$
$=\sqrt{\frac{121}{6}}=\frac{11\sqrt{6}}{6}=4.5\ \mathrm{m/s}$
$\therefore\ v_{\text{rms}}>\bar{v}$

5. **(c)**



Let u, and v are the speed at inlet and outlet of the pump.
$\therefore$ Applying Bernoulli's theorem at points 1 and 2 .
$p_0+\frac{F}{A}+\frac{1}{2}\rho u^2=p_0+\frac{1}{2}\rho v^2$
$\therefore\frac{F}{A}=\frac{1}{2}\rho(v^2-u^2)\ldots(i)$
By continuity equation, $Au=av$

$\Rightarrow u = \frac{av}{A}$....(ii)

From Eqs. (i) and (ii), we get

$$\frac{F}{A} = \frac{1}{2}\rho\left[v^2 - \frac{a^2v^2}{A^2}\right] = \frac{1}{2}\rho v^2\left(\frac{A^2 - a^2}{A^2}\right)$$

As, $A \gg a \Rightarrow \frac{A^2 - a^2}{A^2} = 1$,

$$v = \sqrt{2F/\rho A}$$

6. **(a)** Given, $\frac{dH}{dt} = \text{constant}(K)$.......(i)

Where, $H = ms\theta$

$\therefore$ From Eq. (i),

$$ms\frac{d\theta}{dt} = K$$

$$\therefore \quad \frac{d\theta}{dt} \propto \frac{1}{s}$$

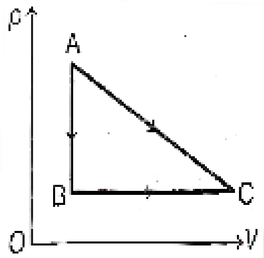
i.e., slope of θ vs t graph $\propto \frac{1}{s}$

As $(\text{Slope})_B > (\text{Slope})_A$

$\therefore$ $(\text{Specific heat})_A > (\text{Specific heat})_B$

7. **(c)** Given, $W_{AC} = 200$ J

$Q_{AC} = 280$ J



By first law of thermodynamics

$$Q_{AC} = \Delta U_{AC} + W_{AC}$$

$$\therefore \quad \Delta U_{AC} = Q_{AC} - W_{AC} = 280 - 200 = 80 \text{ J}$$

$$\therefore \quad \Delta U_{A\to B\to C} = 80 \text{ J}$$

From process $A \to B \to C$,

$W_{ABC} = 80$ J

$\therefore$ By first law of thermodynamics

$$Q_{ABC} = \Delta U_{ABC} + W_{ABC} = 80 + 80 = 160 \text{ J}$$

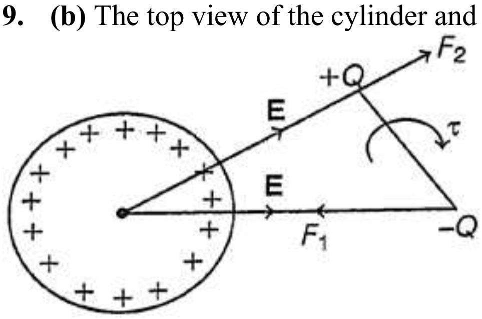
8. **(b)**

The electric field at the center of the semicircle is:

$$\vec{E} = \frac{\lambda_0}{8\pi\varepsilon_0 R}\hat{j}$$

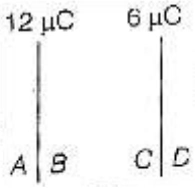
The direction of the electric field is upward along the positive y-axis due to the symmetric sine distribution of charge.

9. **(b)** The top view of the cylinder and the dipole is



Hence, the dipole will experience a net force and a net torque in clockwise direction.

10. **(a)** Given, $Q_1 = 12\mu$C, $Q_2 = 6\mu$C



The charges on the outer surfaces A and D of the plates are

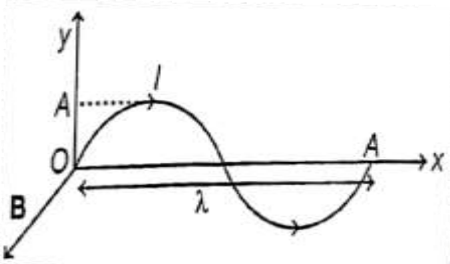
$$Q_A = Q_D = \frac{Q_1 + Q_2}{2} = \frac{12 + 6}{2} = 9\mu\text{C}$$

$$\therefore\ Q_B = Q_1 - Q_A = 12 - 9 = 3\mu\text{C}$$

$$Q_C = -Q_B = -3\mu\text{C}$$

Hence, the charges on the surfaces A, B, C and D are $9\mu\text{C}, 3\mu\text{C}, -3\mu\text{C}, 9\mu\text{C}$, respectively.

11. (c)

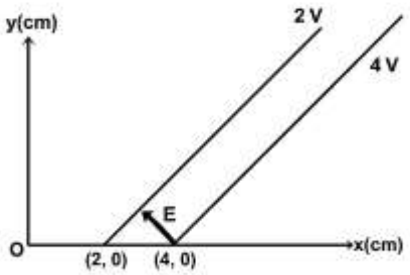


The magnetic force on a current carrying wire is

$$\mathbf{F} = \mathbf{I l} \times \mathbf{B} = \mathbf{I}\lambda \times \mathbf{B}$$

$$F = I\lambda B \sin 90^\circ = I.\lambda B$$

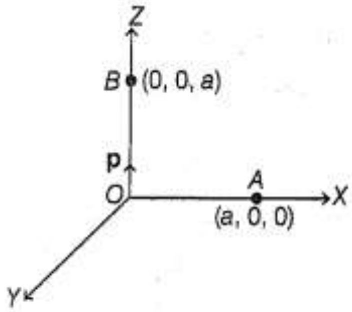
12. (b)



The x-component of $\mathbf{E}$ is given by

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{2-4}{2-4} = -1\ \text{V/cm} = -100\ \text{V/m}$$

13. (d)



The work required to move a charge (q) from point A to B is

$$W_{\text{ext}} = q(V_B - V_A) = q\left(\frac{kp}{a^2} - 0\right)$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Pq}{a^2} \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$$

14. (b) The electric field of EMW is

$\mathbf{E} = E_0(\hat{\mathbf{i}} + \hat{\mathbf{j}})\sin(kz - \omega t)$

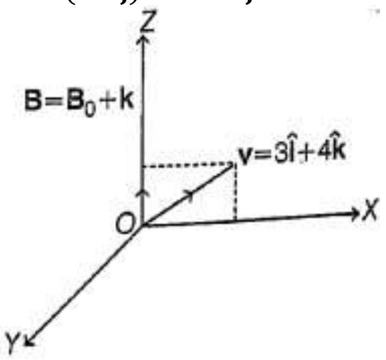
Here, the EM wave is propagating in $+Z$-axis direction.

$\therefore\ \hat{\mathbf{v}} = \hat{\mathbf{k}}$

∴ The direction of magnetic field **B** is

$\hat{\mathbf{v}} \times \mathbf{E} = \hat{\mathbf{k}} \times (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = -\hat{\mathbf{i}} + \hat{\mathbf{j}}$

15. (b)



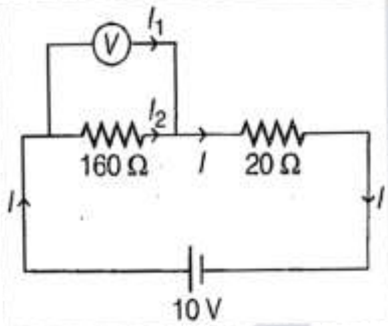
As the component of velocity is along **B** as well as perpendicular to $\mathbb{E}$. Hence, the trajectory of the particle is helical path.

Also, $v_z = 4\text{ m/s}, z = 2\text{ m}$

$\therefore \quad z = v_z \cdot t$

$\therefore \quad t = \frac{z}{v_z} = \frac{2}{4} = \frac{1}{2}\text{ s}$

16. (c) The given circuit is



Reading of voltmeter, $V = 8\text{ V}$

∴ Potential difference across resistance 20Ω

$V' = 10 - 8 = 2\text{ V}$

∴ Current through 20Ω resistor,

$I = \frac{V'}{R_1} = \frac{2}{20} = \frac{1}{10}\text{ A}$

Current through 160Ω resistor,

$I_2 = \frac{V}{R_2} = \frac{8}{160} = \frac{1}{20}\text{ A}$

∴ Current through voltmeter,

$$I_1 = \left(\frac{160}{160 + R_V}\right) I$$

$$\Rightarrow \quad I - I_2 = \left(\frac{160}{160 + R_V}\right) I$$

$$\Rightarrow \quad \frac{1}{20} = \left(\frac{160}{160 + R_V}\right) \times \frac{1}{10}$$

$$\therefore \quad R_V = 160\Omega$$

17. (a) $I_1 : I_2 = n : 1$

$\therefore \quad I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2,$

$\quad I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\therefore \quad \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2 - \left(\sqrt{I_1} - \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2 + \left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$= \frac{4\sqrt{I_1}\sqrt{I_2}}{2(I_1 + I_2)} = \frac{\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{\sqrt{n I_2^2}}{(n+1)I_2} = \frac{\sqrt{n}}{n+1}$$

For $\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$ to be maximum, $\frac{d}{dx}\left(\frac{\sqrt{n}}{n+1}\right) = 0$

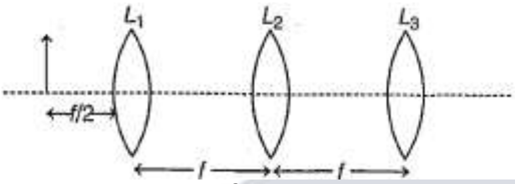
$$\Rightarrow \frac{(n+1)\cdot\frac{1}{2\sqrt{n}} - \sqrt{n}(1)}{(n+1)^2} = 0$$

$\Rightarrow n + 1 - 2n = 0$

$\therefore n = 1$

18. (c)

19. (a)



For Lens $L_1 u = -\frac{f}{2}, f = +f$

$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-\frac{f}{2}} = \frac{1}{f}$

$\therefore v = -f$ and $m = v/u = 2$

For Lens $L_2 u = -2f, v = 2f$

$\therefore m = \frac{v}{u} = -1$

For Lens $L_3 u = +f, f = +f$

$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} - \frac{1}{f} = \frac{1}{f} \Rightarrow v = \frac{f}{2}$

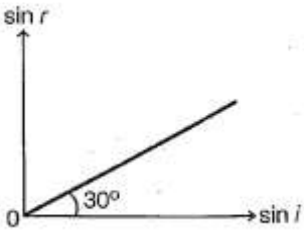
and

$m = \frac{v}{u} = \frac{1}{2}$

$\therefore m = m_1 \cdot m_2 \cdot m_3 = 2 \times (-1) \times \frac{1}{2} = -1$

Hence, final image will be $\frac{f}{2}$ behind the lens L_3 and the magnification is -1 .

20. (b)



$$\tan 30^\circ = \frac{\sin r}{\sin i}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sin r}{\sin i} \ldots (i)$$

By Snell's law,

$$\mu_x \sin i = \mu_y \sin r \Rightarrow \frac{c}{v_x} \sin i = \frac{c}{v_y} \sin r$$

$$\frac{v_y}{v_x} = \frac{\sin r}{\sin i} \ldots.(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{v_y}{v_x} = \frac{1}{\sqrt{3}} \Rightarrow v_x = \sqrt{3} v_y$$

21. (b) Given, for hydrogen atom, potential energy in first excited state,

$(\mathrm{PE})_2 = 0$ (Assumption)

But, $(\mathrm{TE})_2 = (\mathrm{PE})_2 + (\mathrm{KE})_2$

$$\frac{-13.6}{4} = (\mathrm{PE})_2 + \frac{13.6}{4}$$

$$\therefore \ (\mathrm{PE})_2 = -2\left(\frac{13.6}{4}\right) = -6.8 \mathrm{eV}$$

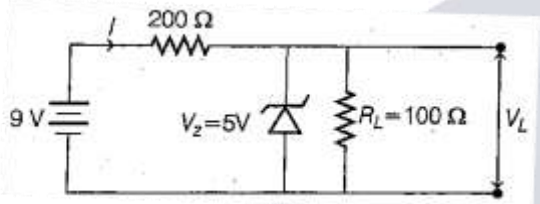
$\therefore$ According to the question,

$$-6.8 + x = 0$$

$$\therefore \ x = 6.8 \mathrm{eV}$$

$\therefore$ Total energy of $n = \infty$ state is $E_\infty = 0 + x = 6.8 \mathrm{eV}$

22. (b)



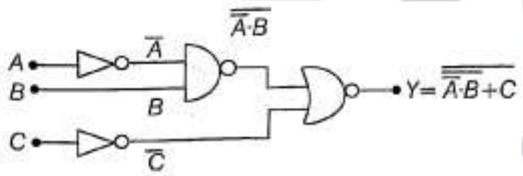
The current through the circuit is

$$I = \frac{9}{200 + 100} = \frac{3}{100} \mathrm{~A}$$

$$\therefore V_L = I \cdot R_L = \frac{3}{100} \times 100 = 3 \mathrm{~V}$$

As $V_L < V_Z$. So, the Zener diode will not activate and $V_L = 3$ V.

23. (c)



The output, $Y = \overline{\overline{\bar{A} \cdot B} + \bar{C}}$

Let $D = \bar{A} \cdot B$

$$\therefore \ Y = \overline{\bar{D} + \bar{C}} = \overline{\overline{D \cdot C}} = D \cdot C = \bar{A} \cdot B \cdot C$$

$$\therefore \ Y = \bar{A} \cdot B \cdot C$$

This is AND gate with input $\bar{A}, B$ and C

For $Y = 1; \bar{A} = 1, B = 1, C = 1 \Rightarrow$ one combination.

$\therefore$ The combination of A, B and C which gives $Y = 0$ is

$2^n - 1 = 2^3 - 1 = 7$

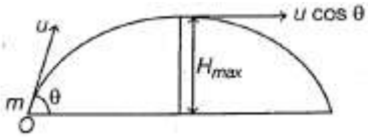
24. (d) By Bragg's law,

For maxima of X-ray diffracted from the crystal,

$2 d \sin \theta = n \lambda$

Where, $n = 1,2,3, \ldots$

25. (d)



$r_{\perp}=H_{\max}=\dfrac{u^2\sin^2\theta}{2g}, v=u\cos\theta$

Angular momentum at the maximum height about point O is

$$L=mvr_{\perp}=m(u\cos\theta)\left(\frac{u^2\sin^2\theta}{2g}\right)$$

$$\therefore L=\left(\frac{mu^3}{2g}\right)\cdot\sin^2\theta\cos\theta$$

As, $\sin^2\theta$ varies from 0 to 1 between 0 to $\pi/2$.
$\cos\theta$ varies from 1 to 0 between 0 to $\pi/2$.
$\therefore \sin^2\theta\cdot\cos\theta$ varies from 0 to 0 between 0 to $\pi/2$.
At $\theta=\frac{\pi}{4}, \sin^2\theta\cdot\cos\theta=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

26. (d) Given, $m=2$ kg, $r=5$ m, $v=2\sqrt{5}$ m/s

$$a_t=\frac{dv}{dt}=3\ \mathrm{m/s^2}$$

$\therefore$ Centripetal acceleration or radial acceleration is

$$a_r=\frac{v^2}{r}=\frac{(2\sqrt{5})^2}{5}=4\ \mathrm{m/s^2}$$

$\therefore$ Net acceleration, $a=\sqrt{a_t^2+a_r^2}$
$\therefore\ a=\sqrt{(3)^2+(4)^2}=5\ \mathrm{m/s^2}$
$\therefore$ The force action on the body is
$F=ma=2\times5=10$ N

27. (a) Measurement of the object $=6.50$ cm
$\therefore$ Number of significant figures $=3$
Now, when the measured value is converted from one unit to another unit, then the number of significant figures remains same in both cases.
$\therefore$ The number of significant figures in 0.0650 m is 3 .

28. (a) By conservation of angular momentum,

$$I\omega=(I+MR^2)\cdot\omega'$$

$$\therefore\quad \frac{\omega'}{\omega}=\frac{I}{I+mR^2}$$

$\therefore$ The fractional loss of angular velocity is

$$\frac{\omega-\omega'}{\omega}=1-\frac{\omega'}{\omega}=1-\frac{I}{I+mR^2}$$

$$=\frac{mR^2}{I+mR^2}$$

29. (c) The acceleration due to gravity at height H from the surface of planet is
$g_H=\frac{GM}{(R+H)^2}$...(i)
The acceleration due to gravity at depth H below the surface of planet is
$g_H=\frac{GM}{R^3}(R-H)$.....(ii)
From Eq. (i) and (ii), we get

$$\frac{GM}{(R+H)^2}=\frac{GM}{R^3}(R-H)$$

$$\Rightarrow \quad H^2+HR-R^2=0$$

$$\Rightarrow \quad H=\frac{-R\pm\sqrt{R^2-4\times1\times(-R^2)}}{2\times1}$$

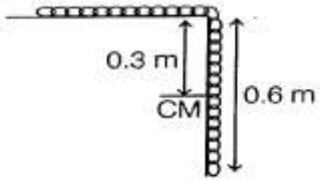
$$=\frac{-R\pm\sqrt{5}R}{2}$$

$$\Rightarrow \quad H=\left(\frac{-1\pm\sqrt{5}}{2}\right)R$$

$$\Rightarrow \quad H\propto R$$

Hence, H vs R graph is straight line.

30. **(d)** $M=0.4$ kg, $L=4$ m



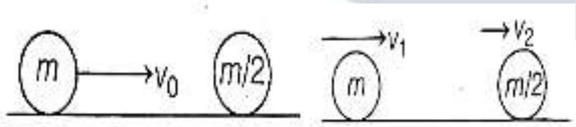
$$W_{\text{ext}}=U_f-U_i=0-(-mgh)$$

$$=mgh=\left(\frac{M}{L}\times0.6\right)\times10\times0.3$$

$$=\frac{0.4}{4}\times0.6\times3$$

$$=0.18\text{ J}$$

31. **(c)** Let the collision of two balls



$\therefore$ By conservation of momentum,

$$mv_0=mv_1+\frac{m}{2}v_2$$

$$\therefore v_0=v_1+\frac{v_2}{2} \quad (i)$$

For elastic collision, $e=1=\frac{v_2-v_1}{v_0-0}$

$\therefore\ v_0=v_2-v_1$....(ii)

From Eqs. (i) and (ii), we get

$$\frac{3v_2}{2}=2v_0$$

$$\Rightarrow \quad v_2=\frac{4v_0}{3}$$

Similarly, the velocity of 3rd ball, $v_3=\left(\frac{4}{3}\right)^2v_0$

$\therefore$ Velocity of nth ball, $v_n=\left(\frac{4}{3}\right)^{n-1}v_0$

32. **(c)** The time period of the satellite near surface of planet is

$$T^2=\frac{4\pi^2}{GM}R^3=\frac{4\pi^2}{G\cdot\frac{4}{3}\pi R^3\cdot\rho}R^3$$

$$\therefore \quad T^2=\frac{3\pi}{\rho G}$$

$\therefore$ For the earth, $T_e^2=\frac{3\pi}{\rho_eG}$...(i)

For the moon, $T_m^2=\frac{3\pi}{\rho_mG}$....(ii)

As $T_e = T_m = 90\text{rev/min}$

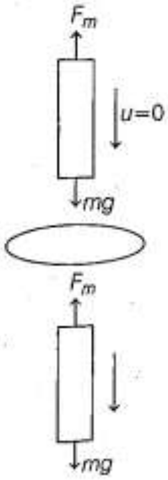
$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{3\pi}{\rho_e G} = \frac{3\pi}{\rho_m G}$$

$\therefore \rho_e = \rho_m$

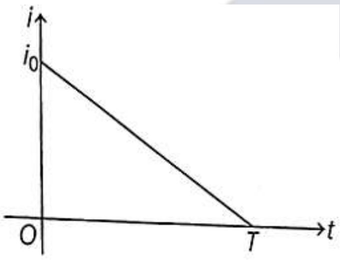
33. (a) Initially only gravity force acts on the magnet.

$\therefore$ The speed of magnet increases from zero to certain value. When the magnet reaches near the ring, then opposing magnetic force F_m acts on the magnet. So, its speed decreases.



Similarly when magnet crosses the ring and goes away, then only gravity force acts and the speed of magnet increases.

34. (a)



$\therefore\ i = -\frac{i_0}{T}t + i_0 \ldots.(i)$

The charge flowing through the coil is

$Q =$ Area of $i - t$ graph

$$Q = \frac{1}{2} \times i_0 \times T$$

$\therefore\ i_0 = 2Q/T$

$\therefore$ From Eq. (i),

$$i = -\frac{1}{T}\cdot\frac{2Q}{T}\cdot t + \frac{2Q}{T} = \frac{2Q}{T}\left(1 - \frac{t}{T}\right)$$

$\therefore$ Amount of heat generated in the coil is,

$$H = \int_0^T i^2 R dt = \int_0^T \left(\frac{2Q}{T}\right)^2 \left(1 - \frac{t}{T}\right)^2 R \cdot dt$$

$$= \frac{4Q^2 R}{T^2} \cdot \int_0^T \left(1 - \frac{2}{T}t + \frac{t^2}{T^2}\right) dt$$

$$= \frac{4Q^2 R}{T^2}\left[t - \frac{2}{T}\cdot\frac{t^2}{2} + \frac{1}{T^2}\cdot\frac{t^3}{3}\right]_0^T = \frac{4Q^2 R}{3T}$$

35. (a) Given, gravitational potential,

$$V = -\frac{GM}{r} + \frac{A}{r^2}$$

By principle of homogeneity,

$$\left[\frac{GM}{r}\right] = \left[\frac{A}{r^2}\right]$$

Now, $[G]=\left[\frac{Fr^2}{m_1m_2}\right]$

$=\frac{[MLT^{-2}][\mathrm{L}^2]}{[\mathrm{M}^2]}=[\mathrm{M}^{-1}\,\mathrm{L}^3\,\mathrm{T}^{-2}]$

$\therefore\ [A]=[\mathrm{M}^{-1}\,\mathrm{L}^3\,\mathrm{T}^{-2}]\mathrm{M}][\mathrm{L}]=[\mathrm{L}^4\,\mathrm{T}^{-2}]$

According to the question,

$[A]=[G]^x[M]^y[c]^z$

$\Rightarrow [\,\mathrm{L}^4\,\mathrm{T}^{-2}]=[\mathrm{M}^{-1}\,\mathrm{L}^3\,\mathrm{T}^{-2}]^x[M]^y[\mathrm{LT}^{-1}]^z$

$\therefore -x+y=0\Rightarrow x=y$ (i)

$3x+z=4$ (ii)

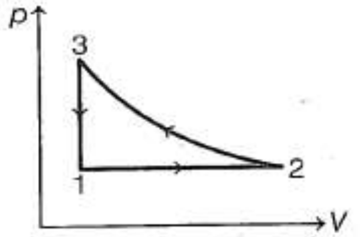
$-2x-z=-2$ (iii)

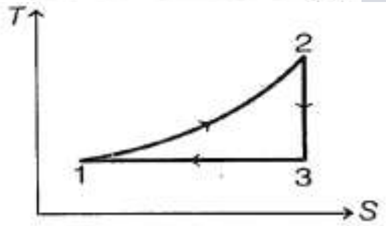
From Eqs. (i), (ii) and (iii), we get

$x=y=2, z=-2$

$\therefore A=G^2M^2c^{-2}=\frac{G^2M^2}{c^2}$

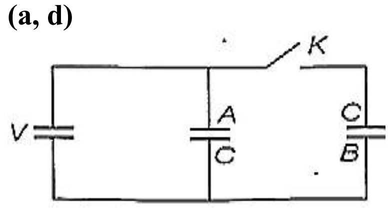
36. (b, c, d)





From $p-V$ diagram,
Process $1-2$ is isobaric process.
Process 3-1 is isochoric process
From T-S diagram,
Process 2-3 is constant entropy process
i.e., $dS=0$
$\therefore\ Q=TdS=0$
$\therefore$ Process 2-3 is adiabatic process.
Also, in cyclic process
$W_{\text{net}}=Q_{\text{net}}\neq 0$

37. (a, d)



Initially switch K is closed,

Energy stored in capacitor $A, E_A=\frac{1}{2}CV^2$

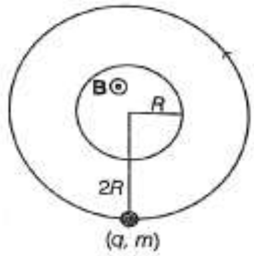
Energy stored in capacitor $B, E_B=\frac{1}{2}CV^2$

$\therefore$ Total energy stored in two capacitors,

$E_i=E_A+E_B=\frac{1}{2}CV^2+\frac{1}{2}CV^2=CV^2$

When switch K is open, and a dielectric ($K=3$) is inserted in both capacitors, then
Energy stored in capacitor A,
$E_A=\frac{1}{2}(3C)V^2=\frac{3}{2}CV^2$
Energy stored in capacitor $B, E_p=\frac{(CV)^2}{2\times3C}=\frac{1}{6}CV^2$
$\therefore$ Total energy stored in two capacitors,
$E_f=E_A+E_B=\left(\frac{3}{2}+\frac{1}{6}\right)CV^2=\frac{5}{3}CV^2$

38. (a)



$\mathbf{B}=(4t^2-2t+6)\hat{\mathbf{k}} \Rightarrow \frac{dB}{dt}=(8t-2)$
By Maxwell's equation,
For $r<R$,

$$\oint E_{\text{ind}}\cdot dl=\frac{d\phi_B}{dt}=-\left(\frac{dB}{dt}\right)A$$

$\Rightarrow \quad E_{\text{ind}}(2\pi r)=-(8t-2)\cdot\pi R^2$

$\therefore \quad E_{\text{ind}}=-(4t-1)\cdot\frac{R^2}{r}$

$\Rightarrow \quad E_{\text{ind}} \propto \frac{1}{r} \Rightarrow E_{\text{ind}}$ vs r does not vary linearly.
As E_{ind} is a non-conservative field. So, it will form a closed loop.
Now, $\left(\frac{dB}{dt}\right)_{t=0}=8\times0-2=-2$
$\therefore B$ will decrease. So, by Lenz's law the charge particle will move in anti-clockwise direction to oppose the decreasing magnetic field.
Now, $|E_{\text{ind}}|_{t=2\text{ s}}=(4t-1)\cdot\left.\frac{R^2}{2R}\right|_{t=2}=\frac{7R}{2}$
$\therefore$ Acceleration of charge particle is
$a=\frac{qE_{\text{ind}}}{m}=\frac{q}{m}\cdot\frac{7R}{2}=\frac{7qR}{2m}$

39. (a, b, c) $\times\times\times\times\times_m$
$\times\times\times\times\times$
$(q,m)\rightarrow v$
$\times\times\times\times\times$
$\times\times\times\times\times$

The radius of the circular path of moving electron is
$r_n=mv_n/Bq\ldots\ldots$(i)
By Bohr's quantization,
$mv_nr_n=\frac{nh}{2\pi}=n\hbar\ldots$(ii)
From Eqs (i) and (ii),

$$Bqr_n\cdot r_n=n\hbar \Rightarrow r_n^2=n\hbar/Bq \Rightarrow r_n\propto\sqrt{n}$$

Again, $\frac{m^2v_n^2}{Bq}=n\hbar \Rightarrow v_n^2=\frac{Bqn\hbar}{m^2}$

$$\therefore v_n=\frac{\sqrt{Bq\hbar\cdot n}}{m}$$

$\therefore$ For minimum velocity of electron, $n = 1$

$\therefore \; v_{\min} = \sqrt{qB\hbar/m}$

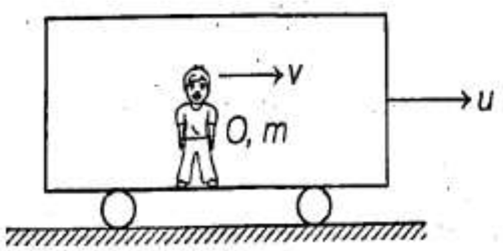
Now, $E_n = \mathrm{KE} + \mathrm{PE} = \mathrm{KE} - 2(\mathrm{KE})$

$= -\mathrm{KE} = -\frac{1}{2}mv_n^2 = -\frac{1}{2}m \cdot \frac{Bq\hbar n}{m^2}$

$\therefore \; E_n \times n$

Also, $hv \propto n \Rightarrow v \propto n$

40. (a, c)



The kinetic energy of ball measured by girl,

$K = \frac{1}{2}mv^2$

By work-energy theorem,

$W_{\text{girl}} = K_f - K_i = \frac{1}{2}m(v+u)^2 - \frac{1}{2}mu^2$

$=$ Gain in KE

Now, force on the girl by the train for small time dt is, $F = mv/dt$

Displacement of girl, $s = udt$

$\therefore$ Work done by train on the girl $= F \cdot S$

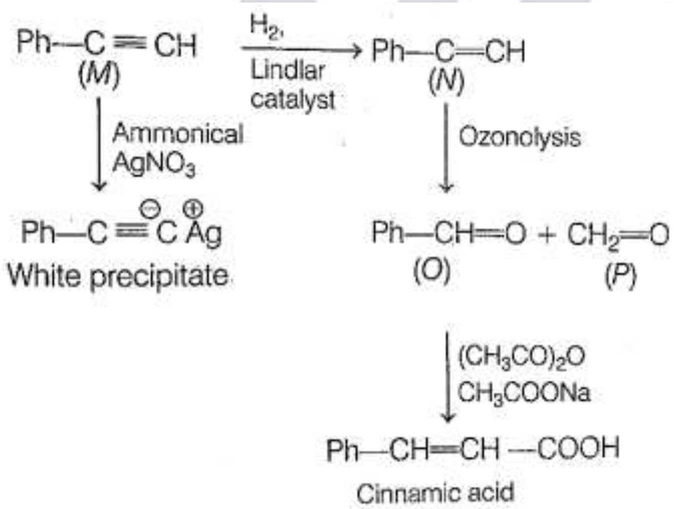
$= \left(\frac{mv}{dt}\right)(udt) = mvu$

Chemistry

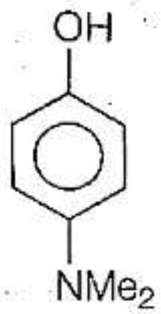
41. (c) The correct order of boiling points of given compounds is II < I < III.

Butan-2-ol shows stronger hydrogen bonding than N -ethylethanamine. While ethoxyethane have dipole association which is weaker than hydrogen bonding. Thus, the order followed is ethoxyethane < N-ethylethanamine < butan-2-ol.

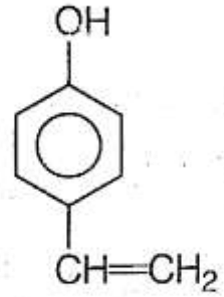
42. (a)



43.(d)



Hyperconjugation is Shown by Ch₃ | **+R effect is shown by NMe_2**

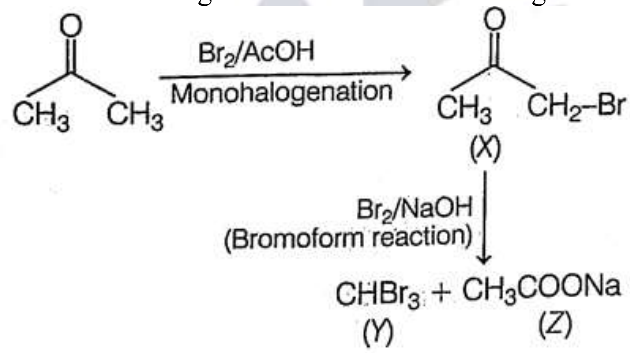


$-R$ effect is shown by $CH=CH_2$

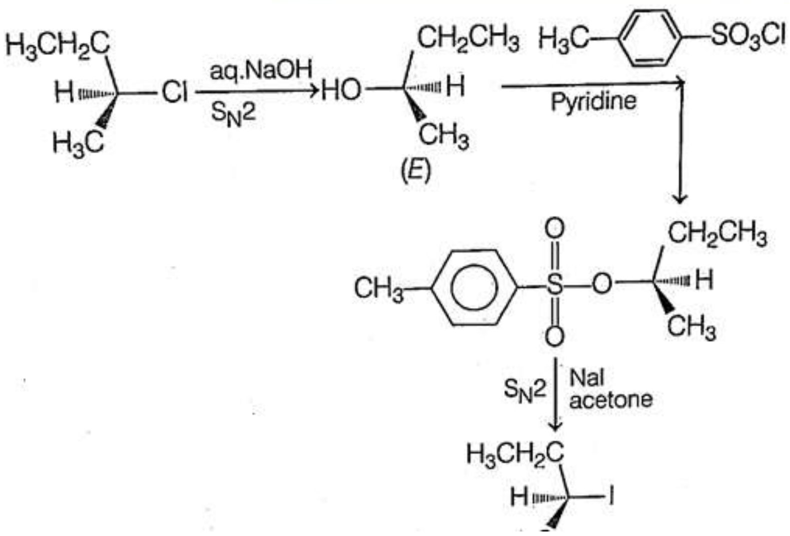
$-R$ effect is shown by NO_2 group

Electron donating group decreases the acidity while electron withdrawing group increases the acidity: Thus, the correct order is IV > III > I > II.

44. (a) In acidic medium, during the formation of X, monohalogenation will takes place. After that ' X ' formed undergoes bromoform reaction to give Y and Z. The reaction will takes place as follows:



45. (d)



Thus, E and F are given in option d .

46. (b) Given, $m_1 = 100$ g
$V_1 = V_1$ m/s
$m_2 = 50$ g
$V_2 = 1.5V_1$ m/s

$\frac{\lambda(m_1)}{\lambda(m_2)} = ?$

Using the formula of de-Broglie's wavelength, we will get

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{m_2 V_2}{m_1 V_1} = \frac{50 \times 1.5V_1}{100V_1} = \frac{3}{4}$$

So, the ratio is 3: 4.

47. (d) For body centred cubic system, edge length is given by

$$a = \frac{4r}{\sqrt{3}}$$

On substituting the given value of $r = 75$pm we will get

$$a = \frac{4 \times 75}{\sqrt{3}} = 173.2\text{pm}$$

48. (c) Given,

$T_1 = T$ $\qquad T_2 = 2T$

$M_1 = M$ $\qquad M_2 = \frac{M}{2}$

$v_{\text{rms}(1)} = x$ m/s $\qquad v_{\text{rms}(2)} = ?$

On substituting the above values, we will get

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{M_1} \times \frac{M_2}{T_2}} = \sqrt{\frac{T}{M} \times \frac{M/2}{2T}} = \frac{1}{2}$$

$$\therefore \frac{x}{v_2} = \frac{1}{2} \Rightarrow v_2 = 2x \text{ m/s}$$

49. (a) (I) Mass of mole of N_2 is given by

Mass $=$ Number of moles $\times$ molar mass
$= 1 \times 28$ g
$= 28$ g

(II) Mass of 0.5 mole of O_3
$= 0.5 \times 48 = 24$ g

(III) Mass of 3.011×10^{23} molecules of O_2
$= \frac{1}{2}$ mole of O_2
$= \frac{1}{2} \times 32 = 16$ g

(IV) Mass of 0.5 g atom of O_2
$= \frac{1}{2}$ mole of atoms of O
$= \frac{1}{2} \times 16 = 8$ g

Thus, the order of increasing mass is
IV < III < II < I

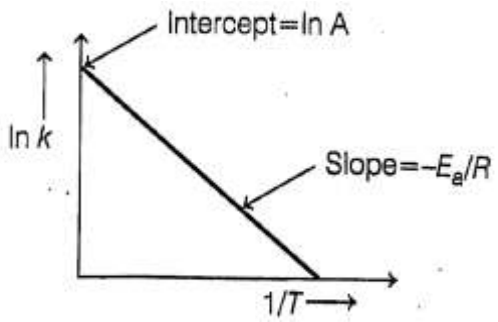
50. (d) From Arrhenius equation, we can say that

$k = Ae^{-E_a/RT}$

Taking natural logarithm on both side of equation

$$\ln k = \frac{-E_a}{RT} + \ln A$$

$$\text{or } \ln k = \ln A - \frac{E_a}{RT} \text{ (i)}$$



On comparing Eq. (i) with straight line equation, we will get the linear plot in graph of $\ln k \vee s\frac{1}{T}$

51. **(b)** Given,

$\Lambda^{\circ}{}_{H_3COONa} = 91\Omega^{-1}\ cm^2 eq^{-1}$

$\Lambda^{\circ}_{HCl} = 426.16\Omega^{-1}\ cm^2 eq^{-1}$

$\Lambda^{\circ}_{NaCl} = 126.45\Omega^{-1}\ cm^2 eq^{-1}$

Using Kohlrausch's law,

$\Lambda^{\circ} CH_3COOH = \Lambda^{\circ} CH_3COONa + \Lambda^{\circ} HCl - \Lambda^{\circ}{}_{NaCl}$

$\Lambda^{\circ} CH_3COOH = 91 + 426.16 - 126.45$

$\Lambda^{\circ}{}_{CH_3COOH} = 390.71 ohm^{-1}\ cm^2 eq^{-1}$

52. **(c)** For the given reaction, $A + B \rightarrow C$, rate equation is given by

$R = k[A]^x[B]^y$

where, x and y are the orders w.r.t. A and B respectively. So, from the given data we can write

$100 = k[1]^x[10]^y$....(i)

$1 = k[1]^x[1]^y$......(ii)

$10 = k[10]^x[1]^y$.....(iii)

On dividng Eq. (ii) by Eq. (i), we will get

$$\frac{1}{100} = \frac{k[1]^x[1]^y}{k[1]^x[10]^y}$$

$$\Rightarrow \frac{1}{100} = \left(\frac{1}{10}\right)^y$$

$y = 2$

Now, dividing Eq. (iii) by Eq. (ii), we will get

$$\frac{10}{1} = \frac{k[10]^x[1]^y}{k[1]^x[1]^y}$$

$\Rightarrow \quad 10 = (10)^x$

$\Rightarrow \quad x = 1$

So, the reaction is 2 nd order w.r.t. B and 1st order w.r.t. A.

53. **(c)**

54. **(b)** Given,

pH = 7.0 $R = 1.987 cal/mol\ K$

$H_2O \rightleftharpoons H^+(aq) + OH^-(aq)$

$K_w = [H^+][OH^-] = 10^{-7} \times 10^{-7}$

$= 10^{-14}$ as pH = 7

Now, using the equation,

$\Delta G^{\circ} = -2.303 RT \log K_w$

$= -2.303 \times 1.987 \times 298 \times 14 cal/mol$

$= 19091.3 cal/mol \approx 19091 cal/mol$

55. **(c)** $KHCO_3$ is appreciably soluble in water, thus cannot be separated from reaction medium easily. So, it does not precipitates out like sodium bicarbonate. Hence, it is not prepared by solvay process.

$(NH_4)HCO_3 + KCl \longrightarrow \underset{\text{Appreciably soluble}}{KHCO_3(aq)} + NH_4Cl(aq)$

56. **(b)** For radioactive decay, half-life is given by

$T_{1/2} = \frac{0.693}{\lambda}$........(i)

Also, $T_{1/2} = \lambda$ (Given)
Let both of them are equal to x, then Eq. (1) will become

$$x = \frac{0.693}{x} \Rightarrow x^2 = 0.693$$

or $\quad x = (0.693)^{1/2}$

$\Rightarrow \quad T_{1/2} = \lambda = (0.693)^{1/2}$

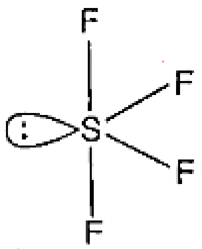
57. (b) The order of ionisation energy of given elements is He > Ne > Ar > Kr. Since, the energy of photon is sufficient to ionise both Ar and Kr. Thus, the mixture contains $Ar^+ + Kr^+$.

58. (b) Given, $\pi = 3 \times 10^{-4}$ atm
$T = 27^\circ C = 300$ K
$R = 0.0821 \text{Latm mol}^{-1} \text{K}^{-1}; V = 4$ L
Mass $= 4$ g
Using the formula, $\pi = i \times CRT$
where, C is the concentration i.e. molarity which is

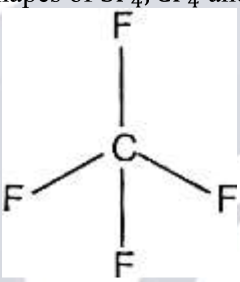
given by molarity $= \dfrac{\text{Number of moles}}{\text{Volume of solution (L)}}$

On substituting the values,
$3 \times 10^{-4} = 1 \times C \times 0.0821 \times 300$
$C = 1.22 \times 10^{-5}$
or $\frac{4/M}{4} = 1.22 \times 10^{-5}$
$\Rightarrow M = 81967 \approx 82000$ g/mol

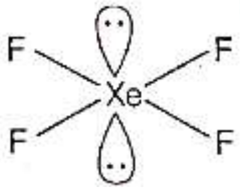
59. (d) The molecular shapes of SF_4, CF_4 and XeF_4 are as follows



1 lone pair see-saw Shape



2 lone pairs square planar shape



2 lone pairs Square planar shape

Thus, the given molecules have different molecular shapes with 1,0 and 2 lone pairs of electrons on the central atom respectively.

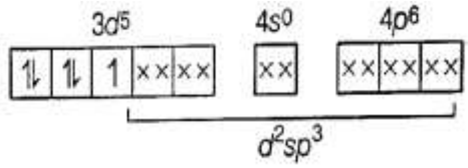
60. (c) Among the given options NO_2^+ has nitrogen in sp-hybridisation.

Number of electronic lobes $= 2 + \frac{1}{2}[5 - 4 - 1]$

$$= \underset{\textit{Bond Paris}}{\underset{\uparrow}{2}} + \underset{\textit{Lone paris}}{\underset{\uparrow}{0}}$$

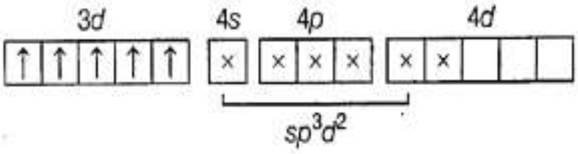
Thus NO_2^+ has sp-hybridisation.

61. (a) In both the compounds Fe is in +3 oxidation state. Thus,
$Fe^{3+} = [Ar]3d^5 4s^0$
Now, for $[Fe(CN)_6]^{3-}$



Pairing of electron will takes place due to presence of strong field ligand i.e CN^- but has one unpaired electron thus paramagnetic.

Now, for $[FeF_6]^{3-}$,



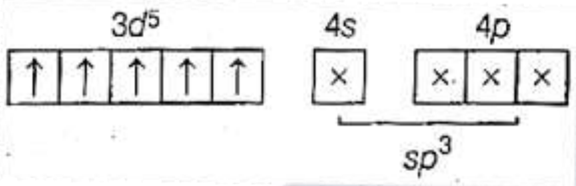
F^- is a weak field ligand. So, no pairing of electron, thus has five unpaired electrons. Therefore, paramagnetic.

62. (a) For $[FeCl_4]^-$

Fe is in +3 oxidation state. Thus,

$Fe^{3+} = [Ar]3d^5 4s^0$

In this complex,

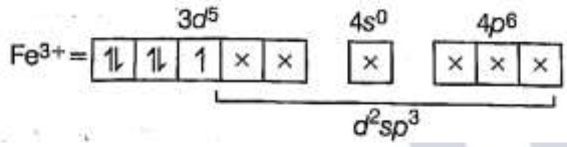


Cl^- is a weak field ligand. So, no pairing of electron will take place. Thus, it has 5 unpaired electrons ($n = 5$)

$$\therefore \quad \mu = \sqrt{n(n+2)}\ \text{BM} = \sqrt{5(5+2)}\text{BM}$$
$$= 5.9\text{BM}$$

Now, for $[Fe(CN)_6]^{3-}$

Fe is in +3 oxidation state and CN^- is a strong field ligand. So, pairing of electron will takes place.



Thus, $n = 1$

$\therefore \ \mu = \sqrt{1(1+2)} = 1.732\text{BM}$

63. (b) BrF_3 self ionises as BrF_2^+ and BrF_4^-.

$2BrF_3 \rightleftharpoons BrF_2^+ + BrF_4^-$.

It is because of their relatively more stable structure.

64. (b) Among the given options Pr^{3+} has $4f^2$ configuration.

$Pr(59) \quad = [Xe]4f^3 6s^2$

$\therefore Pr^{3+} \quad = [Xe]4f^2$

65. (d) Incorrect statement is given in option (d).

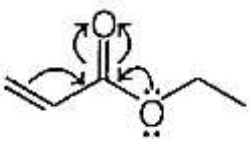
In $[CoF_6]^{3-}$ complex, Co is in +3 oxidation state.

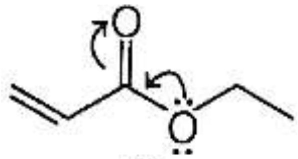
$Co^{3+} = [Ar]3d^6 4s^0$

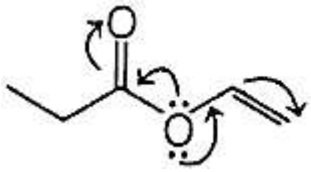
Also, F^- is a weak field ligand. So, no pairing of electron will takes place. Thus, the compound is paramagnetic with 4 unpaired electrons instead of 2 .

66. (d) $\rangle C{=}O$ bond has the most single bond

character in compound II followed by I and II respectively.

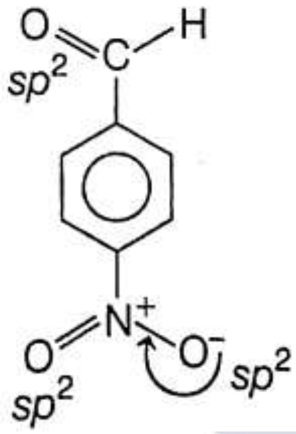
(II) 

(I) 

(III) 

Thus, the order is II > I > III.

67. (a) Among the given options 4-nitrobenzaldehyde has all the atoms in a single plane.



Here, all atoms are sp^2 hybridised, other than hydrogen.

68. (b) The IUPAC name of the given compound is

$$\overset{4}{C}H_3\overset{3}{C}H{=}\overset{2}{\underset{\underset{1}{C}HO}{C}}{-}CH_2{-}CH_3$$

2-ethylbut-2-enal

69. (c) The correct order of carbocation is III > IV > I > II.

$$>H_2\overset{\oplus}{C}-CH{=}CH-NMe_2$$

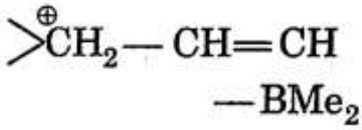
(NMe_2 group shows more $+M$ effect have most stable.)

$$>H_2\overset{\oplus}{C}-CH{=}CH-OMe$$

(OMe group shows $+M$ effect but it is less than NMe_2 thus less stable than III but more than IV.)

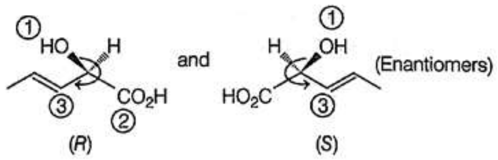
$$>H_2\overset{+}{C}-CH{=}CH-CH_3$$

(Contains no electron withdrawing or donating group but is stabilised by $+R$ effect.)

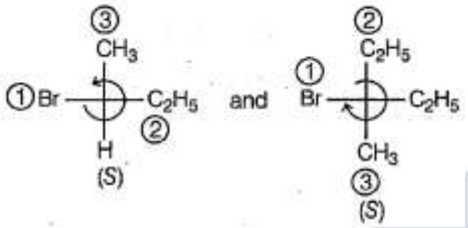


(Since it contains boron which
is electron deficient. Hence,
is least stable.)

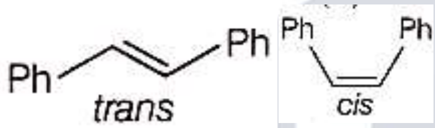
70. (c) Compounds given in (1) are enantiomers of each other.



Compounds given in (2) are identical or are homomers of each other.

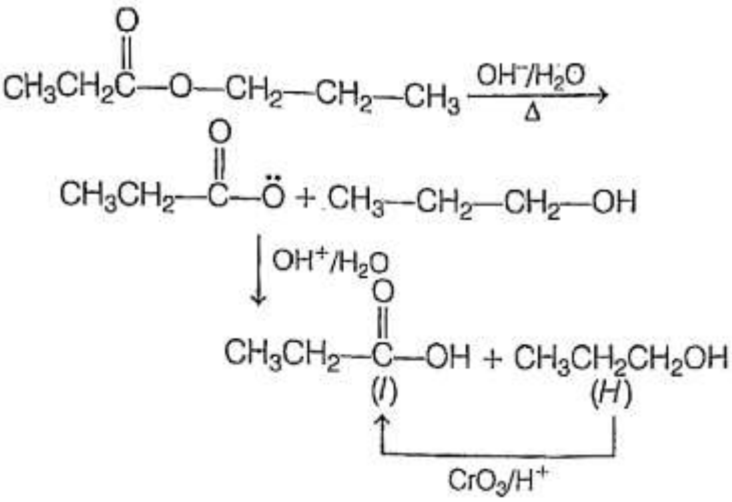


Comounds given in (3) are diastereomers



(Diastrereomers)

71. (c) The compound (G) is given in option (c).



72. (b) As temperature is directly proportional to kinetic energy, thus when temperature is halved the average kinetic energy will also be halved. Also, we know that average speed is given by
or

$$V_{av} = \sqrt{\frac{2RT}{M}}$$

$$V_{av} \propto \sqrt{\frac{T}{M}}$$

Case I

$$V_{av} \propto \sqrt{\frac{T}{M}}$$

Case II

$$V_{av} \propto \sqrt{\frac{T}{2 \times 2M}} = \frac{1}{2}\sqrt{\frac{T}{M}}$$

So, average speed is also halved.

73. (b) Given

Mass of compound (Solute) = 63 g

Mass of solvent $= 500$ g

Molecular weight $= 126$ g

Density $= 1.126$ g/mL

Molarity = ?

Mass of solution = Mass of solute + Mass of solvent

$= 63 + 500 = 563$ g

Number of moles $= \frac{63}{126} = 0.5$ moles

Volume $= \frac{\text{Mass}}{\text{Density}} = \frac{563}{1.126}$ mL

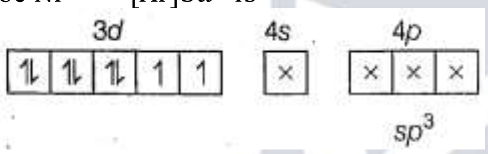
Now, using the formula of molarity,

$$M = \frac{\text{Number of moles of solute}}{\text{Volume of solution (in L)}}$$

$$1 = \frac{0.5 \times 1000}{563/1.126}$$

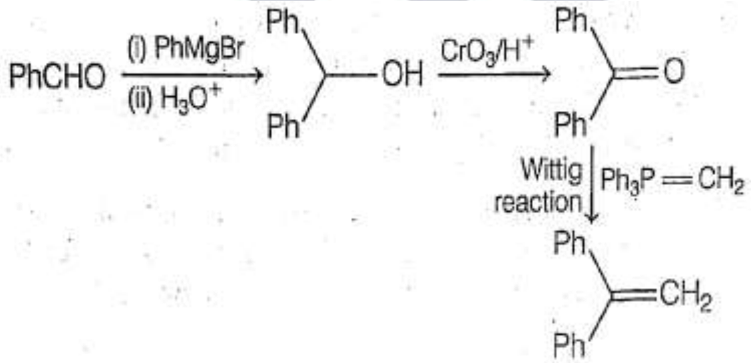
$$1 = \frac{1.126 \times 1000 \times 0.5}{563} = 1 \text{ Molar or } 1.0\text{M.}$$

74. (a) In the given complex $[NiX_4]^{2-}$, Ni is present in +2 oxidation state. Thus, it configuration will be $Ni^{2+} = [Ar]3d^8 4s^0$



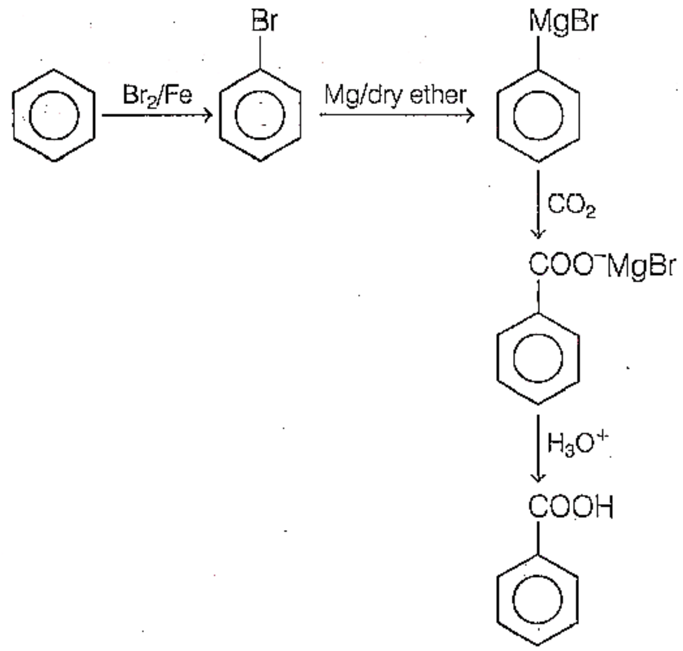
Thus, the given compound is paramagnetic with two unpaired electrons and sp^3 hybridisation.

75. (a) ' L ' is the given sequence of reaction is benzaldehyde.

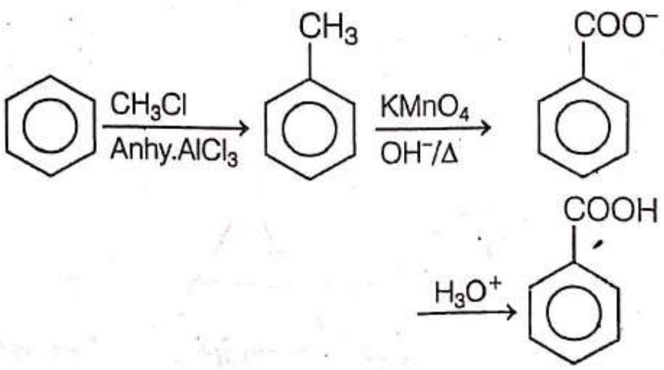


76. (a, c, d) The correct set of reaction(s) to synthesise benzoic acid starting from benzene is given in option a, c and d.

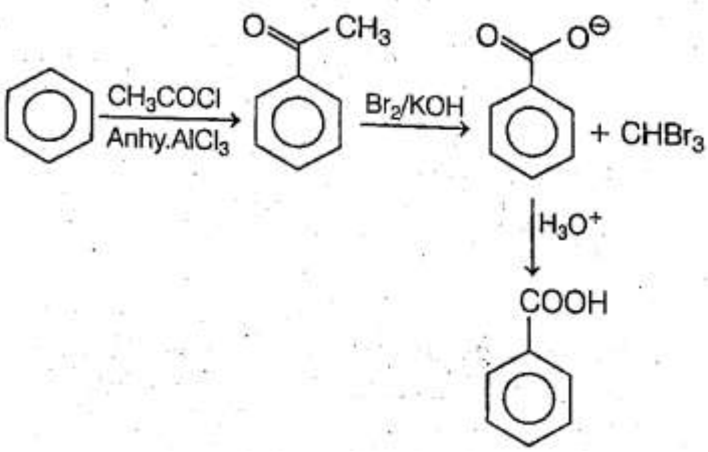
(a)



(c)



(d)



77. (a,c,d) Statement given in option (a) is applicable above critical temperature. That is a gas cannot be liquefied above critical temperature, a liquid phase cannot be distinguished from a gas phase and density changes with p or V.

78. (b,c) Acidic buffer is formed by a salt of weak acid and strong base + weak acid, while basic buffer is formed by salt of weak base and a strong acid + weak base.

(b) $NH_4OH + HCl$ (2:1 mole ratio)

$$\longrightarrow \underset{\text{Basic buffer}}{\underset{1\quad:\quad 1}{NH_4Cl + NH_4OH}}$$

(c) $CH_3COOH + NaOH$ (2:1 mole ratio)

$$\longrightarrow CH_3COONa + CH_3COOH$$
$$1 \quad : \quad 1$$
Acidic buffer

Thus, among the given options buffer solutions are formed in b and c.

79. (a,d)

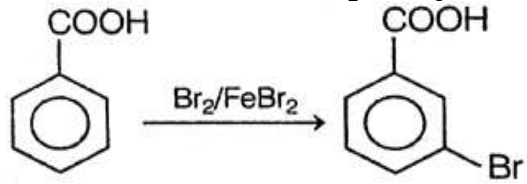
Since, the orbital given is $5d$. Hence, $n = 5$ and $l = 2$.

So, the value of m can vary from +1 to -1 . Thus, the value of m can be $+2, +1, 0, -1, -2$.

80. (a,d)

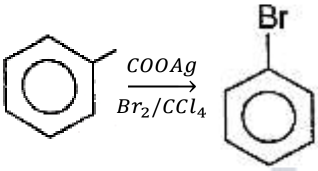
$PhCH = CHCH_3 \underset{CCl_4}{\overset{Br_2}{\rightarrow}} PhCHBrCHBrCH_3$

In this reaction addition of Br_2 takes place in presence of CCl_4 solvent.



$CH_3CH_2COOH \underset{Br_2}{\overset{\text{Red P}}{\rightarrow}} CH_3CHBrCOOH$

The above reaction is a Hell-Volhard Zelinsky reaction and does not use Br_2/CCl_4.



The above reaction is a borodine Hunsdiecker reaction, which uses Br_2/CCl_4.

Thus, options given in (a, d) uses Br_2 in CCl_4.

Mathematics

81. (d) If $x + 2y - 9 = 0, 3x + 5y - 5 = 0$, $ax + by - 1 = 0$ are concurrent, then

$$\begin{vmatrix} 1 & 2 & -9 \\ 3 & 5 & -5 \\ a & b & -1 \end{vmatrix} = 0$$

$\Rightarrow 1(-5 + 5b) - 2(-3 + 5a) - 9(3b - 5a) = 0$

$\Rightarrow -5 + 5b + 6 - 10a - 27b + 45a = 0$

$\Rightarrow 35a - 22b + 1 = 0$

Comparing with $35x - 22y + 1 = 0$

$(x, y) = (a, b)$

$\therefore 35x - 22y + 1$ passes through (a, b).

82. (b) We have, $A(0,4), B(2t, 0)$

Since, L is mid-point of AB,

$\therefore\ L\left(\frac{0+2t}{2}, \frac{4+0}{2}\right)$ i.e. $L(t, 2)$.....(i)

$m =$ Slope of $AB = \frac{0-4}{2t-0} = \frac{-2}{t}$....(i)

$\therefore$ Slope of line perpendicular to $AB = \frac{-1}{m_1} = \frac{t}{2}$

So, equation of perpendicular bisector of AB is

$$y - 2 = \left(\frac{t}{2}\right)(x - t)$$

Putting, $x = 0, y - 2 = \left(\frac{t}{2}\right)(-t)$

$$\Rightarrow\ y = 2 - \frac{t^2}{2}$$

So, $M\left(0, 2 - \frac{t^2}{2}\right)$

As, N is mid-point of LM,

So, $N\left(\frac{t+0}{2}, \frac{2+2-\frac{t^2}{2}}{2}\right)$

i.e., $N\left(\frac{t}{2}, 2-\frac{t^2}{4}\right)$

Let $\frac{t}{2}=h, 2-\frac{t^2}{4}=k$

$\Rightarrow\ k=2-\left(\frac{t}{2}\right)^2=2-h^2$

$\Rightarrow h^2+k-2=0 \Rightarrow h^2=2-k$

i.e., Locus of N is $x^2=2-y$, which is a parabola.

83. (a) $4a^2+9b^2-c^2+12ab=0$

$\Rightarrow\ (2a)^2+(3b)^2+2(2a)(3b)-c^2=0$

$\Rightarrow\ (2a+3b)^2-c^2=0$

$\Rightarrow\ (2a+3b-c)(2a+3b+c)=0$

$\Rightarrow\ (-2a-3b+c)(2a+3b+c)=0$

$\Rightarrow\ -2a-3b+c=0$ or $2a+3b+c=0$

Comparing with $ax+by+c=0$

$(x,y)=(-2,-3)$ or $(2,3)$

So, given family of lines are concurrent at $(2,3)$ or $(-2,-3)$.

84. (b) Tangent at $(a\cos\theta, b\sin\theta)$ to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$ (parametric form)

Putting, $y=0 \Rightarrow x=a\sec\theta$.....(i)

Putting, $x=0 \Rightarrow y=b\cdot\operatorname{cosec}\theta$......(ii)

So, $T(a\sec\theta, 0)$ and $T_1(0, b\operatorname{cosec}\theta)$....(iii)

Now, $(0,T)(0,T_1)=\frac{a}{|\cos\theta|}\times\frac{b}{|\sin\theta|}$

$\because\ \theta\in\left(0,\frac{\pi}{2}\right)$,

$\therefore |\cos\theta|=\cos\theta, |\sin\theta|=\sin\theta$

So,

$(0,T)(0,T_1)=\frac{ab}{\sin\theta\cos\theta}=\frac{2ab}{2\sin\theta\cos\theta}=\frac{2ab}{\sin 2\theta}$

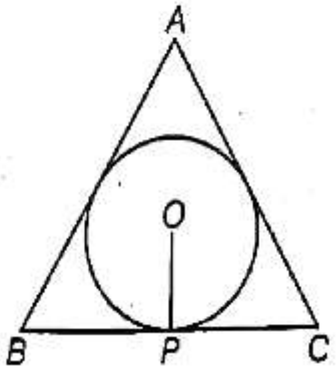
$\theta\in\left(0,\frac{\pi}{2}\right)\Rightarrow 2\theta\in(0,\pi)\Rightarrow \sin 2\theta\in(0,1]$

i.e., $\max.(\sin 2\theta)=1$

$\therefore \min\{(0,T)(0,T_1)\}=2ab$

$0<\theta<\frac{\pi}{2}$

85. (b) $AB=AC=15$ units, $BC=10$ units



OP = radius of incircle.

Now, inradius $r=\frac{\Delta}{s}$

where, Δ = area of $\triangle ABC$

s = semi-perimeter of $\triangle ABC$

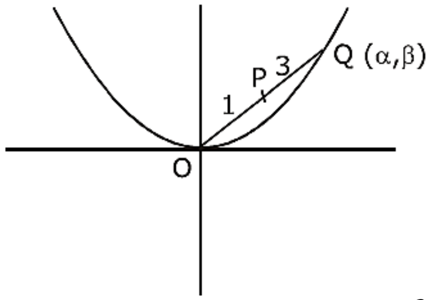
$\Delta=\sqrt{(S)(S-AB)(S-BC)(S-CA)}$ sq units

$=\sqrt{20\times5\times10\times5}$ sq units $=50\sqrt{2}$ sq units

$\therefore OP = r = \frac{50\sqrt{2}}{20} = \frac{5}{\sqrt{2}}$ units

86. (d) Vertex O, of parabola $x^2 = 8y$ is $O(0,0)$

Let Q be a point on parabola i.e., $Q(4t, 2t^2), P(h, k)$ divides OQ internally in ratio $1:3$.



$$h = \frac{1 \times 4t + 3 \times 0}{1+3} = t, k = \frac{1 \times 2t^2 + 3 \times 0}{1+3} = \frac{t^2}{2}$$

$\Rightarrow k = \frac{h^2}{2}$

i.e., Locus of P is $x^2 = 2y$.

87. (c) Given lines are $L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2: \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

$\Rightarrow$ Direction ratios of $L_1: \langle 2,3,4 \rangle$

$\Rightarrow$ Direction ratios of $L_2: \langle 3,4,5 \rangle$

Since, plane contains these two lines, so normal to plane $\| \overrightarrow{L_1} \times \overrightarrow{L_2}$

$$\mathbf{L}_1 \times \mathbf{L}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(15-16) - \hat{\mathbf{j}}(10-12) + \hat{\mathbf{k}}(8-9)$$

$$= -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

$\therefore$ Direction ratios of normal to plane are $\langle -1,2,-1 \rangle$.

L, passes through (1,2,3).

So, plane also passes through (1,2,3)

$\therefore$ Its equation is

$(-1)(x-1) + 2(y-2) + (-1)(z-3) = 0$

i.e., $x - 2y + z = 0$

Other parallel plane is $\alpha x - 2y + z - k = 0$

Since, they are parallel

$\therefore \frac{\alpha}{1} = \frac{-2}{-2} = \frac{1}{1} \Rightarrow \alpha = 1$

Distance between these planes $= \sqrt{6}$ units

$$\Rightarrow \frac{|0-(-k)|}{\sqrt{(1^2) + (-2)^2 + (1)^2}} = \sqrt{6} \Rightarrow |k| = 6$$

88. (d) We have, hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$

$A(2\sec\theta, 3\tan\theta), B(2\sec\phi, 3\tan\phi)$

Equation of normal in parametric form gives equation of normal at A and B as:

$2x\cos\theta + 3y\cot\theta = 13$....(i)

$2x\cos\phi + 3y\cot\phi = 13$......(ii)

Also, $\theta + \phi = \frac{\pi}{2} \Rightarrow \phi = \frac{\pi}{2} - \theta$

$\therefore$ Eq. (ii) becomes,

$$2x\cos\left(\frac{\pi}{2} - \theta\right) + 3y\cot\left(\frac{\pi}{2} - \theta\right) = 13$$

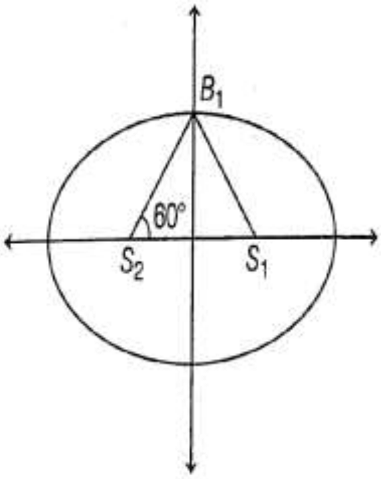
$\Rightarrow 2x\sin\theta + 3y\tan\theta = 13$......(iii)

$\sin\theta \times (1) - \cos\theta \times (3)$,

$3y(\cos\theta - \sin\theta) = 13(\sin\theta - \cos\theta)$

$\Rightarrow y=\dfrac{-13}{3} \Rightarrow \beta=\dfrac{-13}{3}$

89. (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



Let eccentricity be e.

$\therefore\ S_1(ae,0), S_2(-ae,0), B_1(0,b)$

Given, $\angle B_1S_2S_1=60^\circ$

$\therefore$ Slope of $B_1S_2=\tan 60^\circ$

$\Rightarrow\ \dfrac{b-0}{0-(-ae)}=\sqrt{3}$

$\Rightarrow\ b=ae\sqrt{3}$.........(i)

Also, $b^2=a^2(1-e^2)$

$\Rightarrow (ae)^2(3)=a^2(1-e^2)$

$\Rightarrow 3e^2=1-e^2 \Rightarrow e^2=\dfrac{1}{4}$

$\Rightarrow\ e=\dfrac{1}{2}$

90. (d) $y=\underbrace{\log_e \log_e \log_e \ldots \log_e x}_{n\text{-times}}$

Using chain rule of differentiation,

$$\frac{dy}{dx}=\frac{1}{\underbrace{\log_e \log_e \ldots \log_e x}_{(n-1)\text{ times}}}\times\frac{1}{\underbrace{\log_e \ldots \log_e x}_{(n-2)\text{ times}}}\times\ldots$$
$$\times\frac{1}{\log_e \log_e x}\times\frac{1}{\log_e x}\times\frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{(\log^{n-1} x)(\log^{n-2} x)\ldots(\log^2 x)(\log x)(x)}$$

$$\Rightarrow x\log x\log^2 x \ldots \log^{n-1} x\frac{dy}{dx}=1$$

$$\Rightarrow\ x\log x\log^2 x \ldots \log^{n-1} x\log^n x\frac{dy}{dx}=\log^n x$$

91. (a) Direction ratios of normal to plane

$2x-y+2z-1=0$ are $\langle 2,-1,2\rangle$

Direction ratios of X-axis are $\langle 1,0,0\rangle$

Let θ be angle between the normal and X-axis,

$$\therefore \cos\theta=\frac{|(2)(1)+(-1)(0)+(2)(0)|}{\sqrt{(2)^2+(-1)^2+(2^2)}\cdot\sqrt{1^2+0^2+0^2}}=\frac{2}{3}$$

$\therefore\ \theta=\cos^{-1}(2/3)$ is the required angle.

92. (c) $[x^2] = \begin{cases} 0, & 0 \le x^2 < 1 \\ 1, & 1 \le x^2 < 2 \\ 2, & 2 \le x^2 < 3 \\ 3, & 3 \le x^2 < 4 \\ 4, & 4 \le x^2 < 5 \\ \vdots & \end{cases} (\because x^2 \ge 0)$

$$\Rightarrow [x^2] = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < \sqrt{2} \\ 2, & \sqrt{2} \le x < \sqrt{3} \; (\because x > 0) \\ 3, & \sqrt{3} \le x < 2 \\ 4, & 2 \le x < \sqrt{5} \end{cases}$$

$$f(x) = [x^2]\sin \pi x = \begin{cases} 0, & 0 \le x < 1 \\ \sin \pi x, & 1 \le x < \sqrt{2} \\ 2\sin \pi x, & \sqrt{2} \le x < \sqrt{3} \\ 3\sin \pi x, & \sqrt{3} \le x < 2 \\ 4\sin \pi x, & 2 \le x < \sqrt{5} \\ \vdots & \end{cases}$$

at $x = 1$, LHL $= 0$,
RHL $= \sin \pi = 0 = f(l)$
$\therefore f(x)$ is continuous at $x = 1$
at $x = 2$, LHL $= 3\sin(2\pi) = 0$
RHL $= 4\sin(2\pi) = 0 = f(2)$
$\therefore f(x)$ is continuous at $x = 2$ and so on.
$\therefore f(x)$ is continuous at only those points which are perfect square.

93.(d) $f(x) = \begin{cases} 1, & x = 1 \\ e^{(x^{10}-1)} + (x-1)^2 \sin\left(\frac{1}{x-1}\right), & x \ne 1 \end{cases}$

$\lim_{x\to1} f(x) = e^{1^{10}-1} + (1-1)^2 \times$ oscillating number between (-1 and 1) = $1 + 0 = 1 = f(1)$
$\therefore f(x)$ is continuous at $f(1)$

$$f'(x) = e^{(x^{10}-1)} \times 10x^9 + (x-1)^2 \cos\left(\frac{1}{x-1}\right) \times \frac{-1}{(x-1)^2} + \sin\left(\frac{1}{x-1}\right) \times 2(x-1)$$

$\lim_{x\to1} f'(x) = \left(e^{1^{10}-1}\right) \times 10(1)^9 + 0 \times$ oscillating number between (-1 and 1) $+0 = 10$
$\therefore f'(l)$ exist and is 10 .

94. (b) Let

$$L = \lim_{x\to\infty} x - [(x-a_1)(x-a_2)\dots(x-a_n)]^{\frac{1}{n}}$$

Let $x = \frac{1}{t'}$ as $x \to \infty \Rightarrow t \to 0^+$

$$\therefore L = \lim_{t\to0^+} \frac{1}{t} - \left[\left(\frac{1-a_1t}{t}\right)\left(\frac{1-a_2t}{t}\right)\cdots\left(\frac{1-a_nt}{t}\right)\right]^{\frac{1}{n}}$$

$$= \lim_{t\to0^+} \frac{1 - [(1-a_1t)(1-a_2t)\dots(1-a_nt)]^{\frac{1}{n}}}{t}$$

$$= \lim_{t\to0^+} \frac{1 - [1 - S_1t + S_2t^2 + \dots + (-1)^n S_n t^n]^{\frac{1}{n}}}{t}$$

This is $\frac{0}{0}$ form,
Using L-hospital rule,

$$L=\lim_{t\to 0^+}-\frac{1}{n}(1-S_1t+S_2t^2\ldots+(-1)^nS_nt^n)^{\frac{1}{n}-1}$$
$$\times(-S_1+2S_2t+\cdots+n(-1)^nS_nt^{n-1})$$
$$=\frac{-1}{n}\times 1\times(-S_1)=\frac{S_1}{n}$$
$$=\frac{a_1+a_2+\cdots+a_n}{n}$$

95. (b) $\cos^{-1}\left(\frac{y}{b}\right)=\log_e\left(\frac{x}{n}\right)^n=n\ln\left(\frac{x}{n}\right)$
$=n\ln(x)-n\ln(n)$
$=\cos\left[\cos^{-1}\left(\frac{y}{b}\right)\right]=\cos(n\ln x-n\ln n)^{\text{are}}$
$\Rightarrow \frac{y}{b}=\cos(n\ln x-n\ln n)$
$\Rightarrow y=b\cos(n\ln x-n\ln n)$
$\Rightarrow y_1=-b\sin(n\ln x-n\ln n)\times\frac{n}{x}$
$\Rightarrow y_2=-bn\left[x\times\cos(n\ln x-n\ln n)\times\frac{n}{x}\right.$
$=\left.\frac{-bn[n\cos(n\ln x-n\ln n)-\sin(n\ln x-n\ln n)]}{x^2}\right]$
Now, $Ay_2+By_1+Cy=0$
$\Rightarrow \frac{-Abn[n\cos(n\ln x-n\ln n)-\sin(n\ln x-n\ln n)]}{x^2}$
$-\frac{Bbn\sin(n\ln x-n\ln n)}{x}+Cb\cos(n\ln x-n\ln n)=0$
$\Rightarrow -An^2\cos(n\ln x-n\ln n)+An\sin(n\ln x$
$-n\ln n)-Bnx\sin(n\ln x-n\ln n)$
$+Cx^2\cos(n\ln x-n\ln n)=\mathbf{0}$
$\Rightarrow (-An^2+Cx^2)\cos(n\ln x-n\ln n)$
$+(An-Bnx)\sin(n\ln x-n\ln n)=\mathbf{0}$
$\Rightarrow -An^2+Cx^2=0\Rightarrow Cx^2=An^2$
and $An-Bnx=0\Rightarrow A=Bx$ (i)
$\therefore A=x^2, B=x, C=n^2$

96. (b) $f:[1,3]\to R$
$f'(x)=[f(x)]^2+4\forall x\in(1,3)$
Here, $[f(x)]^2\geq 0,\forall x\in(1,3)$
$\Rightarrow [f(x)]^2+4\geq 4$ ......(i)
$\Rightarrow f'(x)\geq 4$
By Lagrange's mean value theorem, $\exists C\in(1,3)$
$$f'(x)=\frac{f(3)-f(1)}{2}$$
$\Rightarrow f(3)-f(1)=2f'(x)$
From Eq. (i), $2f'(x)\geq 8$
$\therefore f(3)-f(1)\geq 8$

97. (c) $f'(2)=6, f'(1)=4$
$$L=\lim_{h\to 0}\frac{f(2+2h+h^2)-f(2)}{f(1+h-h^2)-f(1)}$$
$$=\lim_{h\to 0}\frac{\frac{f(2+2h+h^2)-f(2)}{2+2h+h^2-2}\times(2+2h+h^2-2)}{\frac{f(1+h-h^2)-f(1)}{1+h-h^2-1}\times(1+h-h^2-1)}$$
$$=\frac{f'(2)}{f'(1)}\times\lim_{h\to 0}\frac{h(2+h)}{h(1-h)}$$

$= \frac{6}{4} \times 2 = 3$

98. **(d)** Let $I_1 = \int_0^n [x]dx$

$$= \int_0^1 0 \cdot dx + \int_1^2 1 \cdot dx + \int_2^3 2dx + \cdots$$

$$+ \int_{n-2}^{n-1} (n-2)dx + \int_{n-1}^{n} (n-1)dx$$

$$= 1(x)_0^1 + 2(x)_2^3 + 3(x)_3^4 + \cdots + (n-2)(x)_{n-2}^{n-1} = +(n-1)(x)_{n-1}^{n}$$

$$= 1 + 2 + 3 + \cdots + (n-1) = \frac{n(n-1)}{2}$$

Let $I_2 = \int_0^n \{x\}dx = \int_0^n (x - [x])dx$

$$= \int_0^n xdx - \int_0^n [x]dx$$

$$= \left(\frac{x^2}{2}\right)_0^n - I_1$$

$$= \frac{n^2}{2} - \frac{n(n-1)}{2} = \frac{n^2 - n^2 + n}{2} = \frac{n}{2}$$

$$\therefore \frac{\int_0^n [x]dx}{\int_0^n \{x\}dx} = \frac{\frac{n(n-1)}{2}}{\frac{n}{2}} = n - 1$$

99. **(c)** Given, $I = \int \frac{x^2 dx}{(x\sin x + \cos x)^2}$

$$= \int \frac{x}{(x\sin x + \cos x)^2} \times \frac{x\cos x}{\cos x} dx$$

$$\frac{d}{dx}(x\sin x + \cos x)$$

$$= x\cos x + \sin x - \sin x$$

$$= x\cos x$$

$$\Rightarrow I = \frac{x}{\cos x} \int \frac{x\cos x}{(x\sin x + \cos x)^2} dx$$

$$- \int \left(\frac{d}{dx}\left(\frac{x}{\cos x}\right) \int \frac{x\cos dx}{(x\sin x + \cos x)^2}\right) dx$$

$$\Rightarrow \text{I} = \frac{x}{\cos x}\left(\frac{-1}{x\sin x + \cos x}\right)$$

$$- \int \frac{(\cos x + x\sin x)}{\cos^2 x}\left(\frac{-1}{x\sin x + \cos x}\right) dx$$

$$\Rightarrow I = \frac{-x}{\cos x(x\sin x + \cos x)} + \int \sec^2 x$$

$$\Rightarrow I = \frac{-x}{\cos x(x\sin x + \cos x)} + \tan x + C$$

$$\therefore f(x) = \frac{-x}{\cos x(x\sin x + \cos x)}$$

100. **(d)** $\int \frac{dx}{(x+1)(x-2)(x-3)}$

Let $\frac{1}{(x+1)(x-2)(x-3)} \equiv \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow 1 \equiv A(x-2)(x-3)+B(x+1)(x-3)$
$+C(x+1)(x-2)$
$1 \equiv A(x^2-5x+6)+B(x^2-2x-3)$
$+C(x^2-x-2)$
$\Rightarrow\ 1 \equiv x^2(A+B+C)+x(-5A-2B-C)$
$+(6A-3B-2C)$
$\Rightarrow\ A+B+C=0, -5A-2B-C=0,$
$\Rightarrow\ A=\frac{1}{12}, B=\frac{-1}{3}, C=\frac{1}{4}$
$\therefore\ \int \frac{dx}{(x+1)(x-2)(x-3)} = \frac{1}{12}\int \frac{dx}{x+1}$
$=\frac{1}{12}\log_e|x+1| - \frac{1}{3}\log_e|x-2|$
$=\frac{1}{12}\log_e\left(\frac{|x+1||x-3|^3}{|x-2|^4}\right)+C$
$\therefore\ R=12$

101. **(b)** $y=\frac{x}{\log_e|cx|}$

$\Rightarrow\ \ y\log_e|cx| = x$
$\Rightarrow\ \ y\times\frac{1}{cx}\times c+\log_e|cx|\frac{dy}{dx}=1$
$\Rightarrow\ \ \frac{y}{x}+\frac{x}{y}\left(\frac{y}{x}+\phi\left(\frac{x}{y}\right)\right)=1$
$\Rightarrow\ \ \frac{y}{x}+1+\frac{x}{y}\phi\left(\frac{x}{y}\right)=1 \Rightarrow \phi\left(\frac{x}{y}\right)=\frac{-y^2}{x^2}$

102. **(a)** For $n\in N$ and $x\in\left(0,\frac{1}{2}\right)$

$\therefore \frac{x^{2n}=(x^2)^n \le x^2}{\sqrt{1-x^{2n}}} \le \frac{1}{\sqrt{1-x^2}}$
$\Rightarrow \int_0^{1/2}\frac{dx}{\sqrt{1-x^{2n}}} \le \int_0^{1/2}\frac{dx}{\sqrt{1-x^2}} \le \sin^{-1}(x)|_0^{1/2}$
$\le \sin^{-1}\left(\frac{1}{2}\right)-\sin^{-1}(0) \le \frac{\pi}{6}$

103. **(b)** $I_n=\int_0^{\frac{\pi}{2}}\cos^n x\cos nx dx$

$I_1\ =\int_0^{\frac{\pi}{2}}\cos^2 xdx$ ... (i)
$\Rightarrow\ I_1\ =\int_0^{\frac{\pi}{2}}\cos^2\left(\frac{\pi}{2}-x\right)dx=\int_0^{\frac{\pi}{2}}\sin^2 xdx3$….(ii)
On adding Eqs. (i) and (ii), we get
$2I_1\ =\int_0^{\frac{\pi}{2}}dx=\frac{\pi}{2}\Rightarrow I_1=\frac{\pi}{4}$….(iii)
$I_2\ =\int_0^{\frac{\pi}{2}}\cos^2 x\cos 2xdx$….(iv)
$\Rightarrow I_2\ =\int_0^{\frac{\pi}{2}}\cos^2\left(\frac{\pi}{2}-x\right)-\cos 2\left(\frac{\pi}{2}-x\right)dx$
$=-\int_0^{\frac{\pi}{2}}\sin^2 x\cos 2xdx$
On adding Eqs. (iv) and (v), we get
$2I_2=\int_0^{\frac{\pi}{2}}\cos^2 2xdx$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1+\cos 4x}{2}\right) dx = \frac{x + \frac{\sin 4x}{4}}{2}\Bigg|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$

$\therefore\ I_2 = \frac{\pi}{8} \ldots\ldots (\mathrm{vI})$

$I_3 = \int_0^{\frac{\pi}{2}} \cos 3x \cos 3x dx$

$$= \int_0^{\frac{\pi}{2}} \frac{3\cos x + \cos 3x}{4} \cdot \cos 3x dx$$

$$= \frac{3}{8}\int_0^{\frac{\pi}{2}} 2\cos x \cos 3x dx + \frac{1}{8}\int_0^{\frac{\pi}{2}} 2\cos^2 3x dx$$

$$= \frac{3}{8}\int_0^{\frac{\pi}{2}} (\cos 4x + \cos 2x) dx + \frac{1}{8}\int_0^{\frac{\pi}{2}} (1+\cos 6x) dx$$

$$= \frac{3}{8}\left(\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right)_0^{\frac{\pi}{2}} + \frac{1}{8}\left(x + \frac{\sin 6x}{6}\right)_0^{\frac{\pi}{2}}$$

$$= \frac{3}{8} \times 0 + \frac{1}{8} \times \frac{\pi}{2} = \frac{\pi}{2^4}$$

$$\Rightarrow I_2 = \frac{I_1}{2}, I_3 = \frac{I_1}{2^2}$$

$\therefore I_1, I_2, I_3$ are inGP

104. **(d)** $z = \log\left(\tan\left(\frac{x}{2}\right)\right)$

$$\Rightarrow \frac{dz}{dx} = \frac{1}{\tan\left(\frac{x}{2}\right)} \times \sec^2\left(\frac{x}{2}\right) \times \frac{1}{2}$$

$$= \frac{1}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} = \frac{1}{\sin x} = \operatorname{cosec} x \ldots (\mathrm{i})$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx} \ldots\ldots (ii) \ldots$$

$$= \operatorname{cosec} x \frac{dy}{dz}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\operatorname{cosec} x \frac{dy}{dz}\right)$$

$$= \frac{d}{dz}\left(\operatorname{cosec} x \frac{dy}{dz}\right) \times \frac{dz}{dx}$$

$$\left[\operatorname{cosec} x \frac{d^2y}{dz^2} + \frac{dy}{dz} \times (-\operatorname{cosec} x \cot x)\left(\frac{dx}{dz}\right)\right]\left(\frac{dz}{dx}\right)$$

$$= \left[\operatorname{cosec} x \frac{d^2y}{dz^2} - \frac{dy}{dz} \times \operatorname{cosec} x \cot x \times \frac{1}{\operatorname{cosec} x}\right] \operatorname{cosec} x$$

$$= \operatorname{cosec}^2 x \frac{d^2y}{dz^2} - \operatorname{cosec} x \cot x \frac{dy}{dz}$$

$$\therefore \frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0$$

$$\Rightarrow \operatorname{cosec}^2 x \frac{d^2y}{dz^2} - \operatorname{cosec} x \cot x \frac{dy}{dz}$$

$$+ \operatorname{cosec}^2 x \cot x \frac{dy}{dz} + 4y \operatorname{cosec}^2 x = 0$$

$$\Rightarrow \frac{d^2y}{dz^2} + 4y = 0$$

105. **(c)** $y = e^{kx} \Rightarrow \frac{dy}{dx} = ke^{kx} = ky$

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = k^2e^{kx} = k^2y$

$\Rightarrow \quad \left(\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx}\right)\left(\dfrac{dy}{dx} - y\right) = y\dfrac{dy}{dx}$

$\Rightarrow \quad (k^2y + ky)(ky - y) = y \times ky$

$\Rightarrow \quad (k^2 + k)(k - 1)y^2 = ky^2$

$\Rightarrow \quad k(k + 1)(k - 1) - k = 0$

$\Rightarrow \quad k[k^2 - 1 - 1] = 0$

$\Rightarrow \quad k(k^2 - 2) = 0$

$\Rightarrow \quad k = 0, \pm\sqrt{2}$

106. **(a)** Focii of hyperbola $(\pm ae, 0) = (\pm 2, 0)$

$\Rightarrow ae = 2$

Let equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

It passes through $P(\sqrt{2}, \sqrt{3})$

$\Rightarrow \dfrac{2}{a^2} - \dfrac{3}{b^2} = 1 \Rightarrow \dfrac{2}{a^2} - \dfrac{3}{a^2(e^2 - 1)} = 1$

$\Rightarrow \dfrac{2}{a^2} - \dfrac{3}{a^2e^2 - a^2} = 1 \Rightarrow \dfrac{2}{a^2} - \dfrac{3}{4 - a^2} = 1$

Let $a^2 = x$

$\Rightarrow \dfrac{2}{x} - \dfrac{3}{4 - x} = 1$

$\Rightarrow 8 - 2x - 3x = 4x - x^2$

$\Rightarrow x^2 - 9x + 8 = 0$

$\Rightarrow (x - 1)(x - 8) = 0$

$\Rightarrow x = 1,8$

$\Rightarrow a^2 = 1,8$

$\Rightarrow b^2 = 4 - a^2 = 3, -4$

But, $b^2 \neq -4$

$\therefore \; a^2 = 1, b^2 = 3$

So, equation of hyperbola is $\frac{x^2}{1} - \frac{y^2}{3} = 1$

Equation of tangent at $P(\sqrt{2}, \sqrt{3})$ is

$\dfrac{x \cdot \sqrt{2}}{1} - \dfrac{y \cdot \sqrt{3}}{3} = 1$

$\Rightarrow y = x\sqrt{6} - \sqrt{3}$

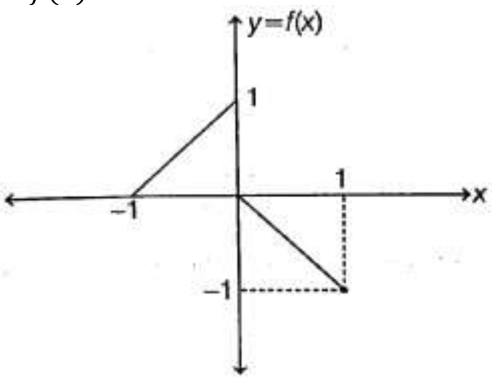
107. **(c)** $f(x) = \begin{cases} x + 1, & -1 \leq x \leq 0 \\ -x, & 0 < x \leq 1 \end{cases}$

At $x = 0$, LHL $= 0 + 1 = 1$

RHL $= -0 = 0$

Since, LHL $\neq$ RHL

$\therefore \; f(x)$ is discontinuous at $x = 0$



108. **(c)** $x = 100t - \frac{25t^2}{2}$, x is the height.

For maximum height, $\frac{dx}{dt} = 0$

$$\Rightarrow \quad 100 \div \frac{25 \times 2t}{2} = 0$$

$$\Rightarrow \quad t = 4 \text{ s}$$

$\therefore$ Maximum height $= 100(4) - \frac{25(4)^2}{2}$ m $= 200$ m

109. **(c)** $\alpha = \hat{\mathrm{i}} + a\hat{\mathrm{j}} + \hat{\mathrm{k}}, \beta = 0\hat{\mathrm{i}} + \hat{\mathrm{j}} + a\hat{\mathrm{k}}$,

$\gamma = a\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$[\alpha\beta\gamma] = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$$

$= 1(1-0) - 0 + a(a^2 - 1)$

$= a^3 - a + 1 = f(a)$(Let)

For maximum scalar triple product,

$f'(a) = 0 \Rightarrow 3a^2 - 1 = 0$

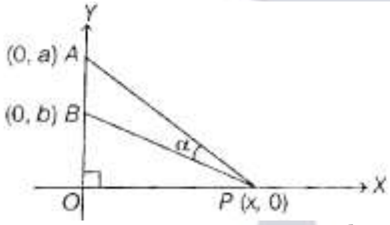
$$\Rightarrow \ a = \pm\frac{1}{\sqrt{3}}$$

Now, $f''(a) = 6a$

$$\Rightarrow f''\left(\frac{1}{\sqrt{3}}\right) > 0, f''\left(\frac{-1}{\sqrt{3}}\right) < 0$$

$\therefore a = \frac{-1}{\sqrt{3}}$ is the point of maxima.

110. **(a)** $\angle APB = \alpha = \angle APO - \angle BPO$



$$\Rightarrow \ \alpha = \tan^{-1}\left(\frac{a}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$$

For maximum value of α,

$$\frac{d\alpha}{dx} = 0 \Rightarrow \frac{1}{1 + \frac{a^2}{x^2}} \times \frac{-a}{x^2} - \frac{1}{1 + \frac{b^2}{x^2}} \times \frac{-b}{x^2} = 0$$

$$\Rightarrow \frac{-a}{x^2 + a^2} + \frac{b}{x^2 + b^2} = 0$$

$$\Rightarrow b(x^2 + a^2) = a(x^2 + b^2)$$

$$\Rightarrow x^2 = ab$$

111. **(a)** We have, hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Since, vertical chord is symmetric about X-axis.

$\therefore$ Average length of vertical chord from a to $2a$,

$$= \frac{2\int_a^{2a} y dx}{\int_a^{2a} dx} = \frac{2\int_a^{2a} \frac{b}{a}\sqrt{x^2-a^2}dx}{(x)_a^{2a}}$$

$$= \frac{\frac{2b}{a}\int_a^{2a}\sqrt{x^2-a^2}dx}{a} = \frac{2b}{a^2}\int_0^{2a}\sqrt{x^2-a^2}dx$$

$$= \frac{2b}{a^2}\left(\frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|\right)_a^{2a}$$

$$= \frac{2b}{a^2}\left[\frac{(2a)\sqrt{4a^2-a^2}}{2} - \frac{a^2\ln\left|2a+\sqrt{4a^2-a^2}\right|}{2} - \frac{a\sqrt{a^2-a^2}}{2} + \frac{a^2\ln\left|a+\sqrt{a^2-a^2}\right|}{2}\right]$$

$$= \frac{2b}{a^2}\left[\sqrt{3}a^2 - \frac{a^2\ln|2+\sqrt{3}|a|}{2} + \frac{a^2\ln|a|}{2}\right]$$

$$= 2b\left(\sqrt{3} + \frac{\ln\left|\frac{a}{(2+\sqrt{3})a}\right|}{2}\right)$$

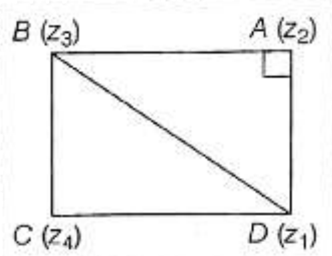
$$= b(2\sqrt{3} - \ln|2+\sqrt{3})$$

112. **(a)**Reflection of $\bar{a}z + a\bar{z} = 0$ in real axis is obtained by transforming Z to $\bar{z}$

So, required line is $\bar{a}Z + a(Z) = 0$

$\Rightarrow \bar{a}z + az = 0$

113. **(d)** At A, using rotation theorem,



$$\frac{z_3 - z_2}{z_1 - z_2} = \frac{BA}{DA}e^{-i\left(\frac{\pi}{2}\right)}$$

Since, it is a square, $AB = AD$

$\Rightarrow \frac{z_3 - z_2}{z_1 - z_2} = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right) = -i$

$\Rightarrow z_3 - z_2 = -i(z_1 - z_2)$

$\Rightarrow z_3 = -iz_1 + (1+i)z_2$

114. **(c)** $a_1, a_2, \ldots, a_n$ are in AP with common difference $r > 0$

i.e., $a_i = a_1 + (i-1)r \ldots .(\text{i})$

Mean of square $= \frac{\sum_{i=1}^{n} a_i^2}{n}$

$$= \frac{\sum_{i=1}^{n}[a_1^2 + 2a_1 r(i-1) + r^2(i-1)^2]}{n}$$

$$= \frac{na_1^2 + 2a_1 r(0+1+2+\cdots+(n-1)) + r^2(0^2+1^2+2^2+..+(n-1)^2)}{n}$$

$$= \frac{na_1^2 + 2a_1 r \times \frac{n(n-1)}{2} + r^2 \times \frac{n(n-1)(2n-1)}{6}}{n}$$

$= \frac{6a_1^2+6a_1r(n-1)+r^2(n-1)(2n-1)}{6} \ldots$(i)

Square of mean $= \left(\frac{\sum_{i=1}^{n} a_i}{n}\right)^2$

$= \left[\frac{\frac{n}{2}(2a_1+(n-1)r)}{n}\right]^2$

$= \left(a_1 + \frac{(n-1)r}{2}\right)^2$

$= a_1^2 + ra_1(n-1) + \frac{r^2(n-1)^2}{4}$

Required difference

$$= a_1^2 + a_1r(n-1) + \frac{r^2(n-1)(2n-1)}{6}\Bigg] - \left[a_1^2 + ra_1(n-1) + \frac{r^2(n-1)^2}{4}\right]$$

$$= \frac{r^2(n-1)}{2}\left[\frac{2n-1}{3} - \frac{n-1}{2}\right]$$

$$= \frac{r^2(n-1)}{2}\left(\frac{4n-2-3n+3}{6}\right)$$

$$= \frac{r^2(n^2-1)}{12}$$

115. **(b)** $1, \log_9(3^{1-x}+2), \log_3(4\cdot 3^x - 1)$ are in AP

$\Rightarrow 2\log_9(3^{1-x}+2) = 1 + \log_3(4\cdot 3^x - 1)$

$\Rightarrow \frac{2}{2}\log_3(3^{1-x}+2) = \log_3 3 + \log_3(4\cdot 3^x - 1)$

$\Rightarrow 3^{1-x} + 2 = 3(4\cdot 3^x - 1)$

Let $3^x = t$,

$\Rightarrow \frac{3}{t} + 2 = 12t - 3$

$\Rightarrow 3 + 2t = 12t^2 - 3t$

$\Rightarrow 12t^2 - 5t - 3 = 0$

$\Rightarrow t = \frac{5 \pm \sqrt{25+4\cdot 12\cdot 3}}{2\cdot 12} = \frac{5\pm\sqrt{169}}{24}$

$= \frac{5\pm 13}{24} = \frac{-1}{3}, \frac{3}{4}$ (but $3^x > 0$)

$\Rightarrow 3^x = \frac{-1}{3}, \frac{3}{4}$

$\Rightarrow 3^x = \frac{3}{4}$

$\Rightarrow x = \log_3\left(\frac{3}{4}\right) = \log_3 3 - \log_3 4$

$\Rightarrow x = 1 - \log_3 4$

116. **(d)** The number of ways in which n different objects can be distributed into different person $= n^n$

And number of ways in which each person gets exactly one objects $= n!$

$\therefore$ Required number of ways $= n^n - n!$

117. **(d)** Let $p(x) = x^2 + px - q^2$

Since, one of root $p(x)$ be less than 2 and other be greater than 2

$\therefore p(2) < 0$

$\Rightarrow (2)^2 + p(2) - q^2 < 0$

$\Rightarrow 4 + 2p - q^2 < 0$

118. **(d)** Given, word in VERTICAL.

Here, vowel are E, I, A
Number of ways that out of 8 alphabets
3 vowels (EIA) can be chosen are ${}^8C_3$
and remaining 5 letters can be arranged in 5! ways.
∴ Required number of ways
$= {}^8C_3 \times 5! = \frac{8!}{3!}$

119. **(a)** Since, A and B are orthogonal matrices.
∴ $AA' = I$ and $BB' = I \ldots.(i)$
⇒ $|AA'| = 1$ and $|BB'| = 1$
⇒ $|A||A'| = 1$ and $|B||B'| = 1$
⇒ $|A|^2 = 1$ and $|B|^2 = 1$….(ii)
Also, given, $|A| + |B| = 0$
⇒ $|A| = -|B|$
⇒ $|A||B| = -1$ $\ldots.(iii)$
Now, $|A+B| = |A \cdot I + B \cdot I| = |ABB' + AA'B|$
[$\because B \cdot I = I \cdot B$], [Using Eq. (i)]
$= |A(B' + A')B|$
$= |A||B' + A'||B|$
$= -|B' + A'|$ [Using Eq. (iii)]
$= -|(A+B)'|$
$= -|A+B|$
⇒ $2|A+B| = 0$
⇒ $|A+B| = 0$
⇒ $A+B$ is singular.

120. **(b)** Given, $P(n) = 3^{2n+1} + 2^{n+2}, n \in N$
At $n = 1, P(1) = 3^3 + 2^3 = 35$
Which is divisible by $(3+2) = 5$
i.e. a prime number.

121. **(a)** Given, $n(A) = n$
Since, set Q contains just one element more than set P. and sets P and Q are subsets of A.
∴ Required number of ways
$= {}^nC_0 \times {}^nC_1 + {}^nC_1 \times {}^nC_2 + {}^nC_2 \times {}^nC_3$
$+ \cdots + {}^nC_{n-1} \times {}^nC_n$
$= {}^{2n}C_{n-1}$

122. **(a)** We have, $S_n = \alpha^n + \beta^n$
∴ $S_1 = \alpha + \beta$
$S_2 = \alpha^2 + \beta^2$
$S_3 = \alpha^3 + \beta^3$
$S_4 = \alpha^4 + \beta^4$

Now, $\begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$

$= \begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$
$$= \{(1-\alpha)(1-\beta)(\alpha-\beta)\}^2$$
$$= \{1-(\alpha+\beta)+\alpha\beta\}^2(\alpha-\beta)^2$$
$$= \left(1+\frac{b}{a}+\frac{c}{a}\right)^2\left(\frac{b^2}{a^2}-\frac{4c}{a}\right)$$
$$= \frac{(a+b+c)^2}{a^2} - \frac{(b^2-4ac)}{a^2}$$
$$= (b^2-4ac)\frac{(a+b+c)^2}{a^4}$$
$\therefore k = b^2 - 4ac$

123. **(a)** We have, $A = \begin{bmatrix} 2 & 0 & 3 \\ 4 & 7 & 11 \\ 5 & 4 & 8 \end{bmatrix}$

$\Rightarrow |A| = 2(56-44) + 3(16-35) = 2(12) + 3(-19)$
$= 24 - 57 = -33$
Which is divisible by 11 .

124. **(c)** We have, $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}, r = 1,2,3, \ldots$

$\Rightarrow |M_r| = r^2 - (r-1)^2$
$= r^2 - r^2 - 1 + 2r = 2r - 1$
$\therefore |M_1| + |M_2| + |M_3| + \cdots + |M_{2008}|$
$$= \sum_{r=1}^{2008} |M_r| = \sum_{r=1}^{2008} (2r-1)$$
$= 1 + 3 + 5 + \cdots + 4015$
$= 1 + 3 + 5 + \cdots + (2 \times 2008 - 1)$
$= (2008)^2$

125. **(a)** We have, $P(A \cap B) = \frac{1}{12}$

Since, A and B are two independent events.
$\therefore\ P(A \cap B) = P(A)P(B) = \frac{1}{12}$….(i)
Also, $P(A' \cap B') = \frac{1}{2}$
$$\Rightarrow\ P(\overline{A \cup B}) = \frac{1}{2}$$
$$\Rightarrow\ 1 - P(A \cup B) = \frac{1}{2}$$
$$\Rightarrow P(A \cup B) = \frac{1}{2}$$
$$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{1}{2}$$
$\Rightarrow P(A) + P(B) - \frac{1}{12} = \frac{1}{2}$ [Using Eq. (i)]
$\Rightarrow P(A) + P(B) = \frac{7}{12}$…(ii)
Now, from Eqs. (i) and (ii), we get

$$P(A)\left\{\frac{7}{12}-P(A)\right\}=\frac{1}{12}$$

$\Rightarrow \quad \frac{7}{12}P(A)-P(A)^2=\frac{1}{12}$

$\Rightarrow \quad P(A)^2-\frac{7}{12}P(A)+\frac{1}{12}=0$

$\Rightarrow \quad 12P(A)^2-7P(A)+1=0$

$\Rightarrow \quad (3P(A)-1)(4P(A)-1)=0$

$\Rightarrow \quad P(A)=\frac{1}{3}$ or $P(A)=\frac{1}{4}$

$\therefore$ When $P(A)=\frac{1}{3}$, then $P(B)=\frac{1}{4}$

and when $P(A)=\frac{1}{4}$, then $P(B)=\frac{1}{3}$

126. **(b)** Given, A, B, C are subsets of X

Option (a), $A\cup(B \mid C)$

$\Rightarrow \quad A\cup(B\cap C')$

$\Rightarrow \quad (A\cup B)\cap(A\cup C')$

which is wrong.

Option (b), $|A|B) \mid C$

$= (A\cap B'\cap C'$

$= A\cap(B'\cap C')$

$= A\cap(B\cup C)'$

$= A \mid (B\cup C)$

Which is true.

127. **(b)** We know that product of two transitive relation is not transitive.

128. **(d)** Given, $r^2\cos^2\left(\theta-\frac{\pi}{3}\right)=2$

$\Rightarrow \quad r^2\left(\cos\left(\theta-\frac{\pi}{3}\right)\right)^2=2$

$\Rightarrow \quad r^2\left(\cos\theta\cos\frac{\pi}{3}+\sin\theta\sin\frac{\pi}{3}\right)^2=2$

$\Rightarrow \quad r^2\left(\frac{\cos\theta}{2}+\frac{\sqrt{3}}{2}\sin\theta\right)^2=2$

$\Rightarrow \quad \frac{1}{4}(r\cos\theta+\sqrt{3}r\sin\theta)^2=2$

$\Rightarrow \quad (x+\sqrt{3}y)^2=8$

$\Rightarrow \quad (x+\sqrt{3}y)^2-(2\sqrt{2})^2=0$

$\Rightarrow \quad (x+\sqrt{3}y+2\sqrt{2})(x+\sqrt{3}y-2\sqrt{2})=0$

Which represents a pair of lines.

129. **(a)** We have, $E_k=\{(a,b)\in S: ab=k\}$

and $p_k=P(E_k)$

Here, $E_1 = \{(1,1)\}$

$E_2 = \{(1,2),(2,1)\}$

$E_4 = \{(2,2),(1,4),(4,1)\}$

$E_5 = \{(1,5),(5,1)\}$

$E_8 = \{(2,4),(4,2)\}$

$E_{10} = \{(2,5),(5,2)\}$

E_{14}, E_{16}, E_{17} are not possible

$\therefore\ p_1 = \frac{1}{36}, p_2 = \frac{2}{36},$

$p_4 = \frac{3}{36}, p_5 = \frac{2}{36}, p_8 = \frac{2}{36}$

and $p_{10} = \frac{2}{36}$

$\therefore p_1 < p_{10} < p_4$ which is correct.

130. **(b)** Given, $\frac{1}{6}\sin\theta, \cos\theta, \tan\theta$ are in GP.

$\therefore\ \cos^2\theta = \frac{1}{6}\sin\theta\tan\theta$

$\Rightarrow\ 6\cos^2\theta = \frac{\sin^2\theta}{\cos\theta}$

$\Rightarrow\ 6\cos^3\theta - \sin^2\theta - 1 = 0$

$\Rightarrow\ (2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 2) = 0$

$\Rightarrow\ 2\cos\theta - 1 = 0$

$\Rightarrow\ \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

131. **(a)** Given, equation of curve $y = e^{a\sin x}$

Taking log on both sides, we get

$\log y = a\sin x \log e$

$\Rightarrow\ \log y = a\sin x$....(i)

On differentiating both sides with respect to x, we get

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = a\cos x$

$\frac{dy}{dx} = y(a\cos x)$

$= y\left(\frac{\log y}{\sin x}\cdot\cos x\right)$

[Using Eq. (i)]

$\Rightarrow\ \frac{dy}{dx} = y(\log y\cdot\cot x)$

$\Rightarrow \tan x\frac{dy}{dx} = y\log y$

132. **(c)** Given, $\sin^2 x + \sin^2 y = 1$

$\Rightarrow\ \sin^2 y = 1 - \sin^2 x$

$\Rightarrow\ \sin^2 y = \cos^2 x$

$\Rightarrow\ \sin y = \pm\cos x$

If $\sin y = \cos x = \sin\left(\frac{\pi}{2} - x\right)$

$\Rightarrow\ y = n\pi + (-1)^n\left(\frac{\pi}{2} - x\right), n \in Z$

Which represents equation of line with slope ±1 and if $\sin y = -\cos x = -\sin\left(\frac{\pi}{2} - x\right)$

$= \sin\left(x - \frac{\pi}{2}\right)$

$\Rightarrow\ y = n\pi + (-1)^n\left(x - \frac{\pi}{2}\right), n \in Z$

which represents equation of line with slope ±1 So, there are infinitely many lines with slope ±1.

133. **(b)** We have

$$\lim_{n\to\infty}\left[\left(\frac{1}{2\cdot 3}+\frac{1}{2^2\cdot 3}\right)+\left(\frac{1}{2^2\cdot 3^2}+\frac{1}{2^3\cdot 3^2}\right)+\cdots+\left(\frac{1}{2^n\cdot 3^n}+\frac{1}{2^{n+1}\cdot 3^n}\right)\right]$$

$$=\lim_{n\to\infty}\sum_{x=1}^{n}\left(\frac{1}{2^x\times 3^x}+\frac{1}{2^{x+1}\times 3^x}\right)$$

$$=\lim_{x\to\infty}\sum_{x=1}^{n}\left(\frac{1}{6^x}+\frac{1}{6^x\cdot 2}\right)$$

$$=\lim_{x\to\infty}\sum_{x=1}^{n}\frac{1}{6^x}\left(1+\frac{1}{2}\right)$$

$$=\frac{3}{2}\lim_{x\to\infty}\sum_{x=1}^{n}\frac{1}{6^x}$$

$$=\frac{3}{2}\lim_{x\to\infty}\left(\frac{1}{6}+\frac{1}{6^2}+\frac{1}{6^3}+..+\frac{1}{6^n}\right)$$

$$=\frac{3}{2}\times\frac{1/6}{1-1/6}=\frac{3}{2}\times\frac{1}{6}\times\frac{6}{5}=\frac{3}{10}$$

134. **(d)** Given, $f(x)=\sin\left(\frac{1}{x^3}\right)$

and $-2\pi<x<0$

$\Rightarrow -8\pi^3<x^3<0$

$\Rightarrow -\frac{1}{8\pi^3}>\frac{1}{x^3}>-\infty$

Clearly, $f(x)$ will takes values between -1 and 1 so changes sign infinitely many times.

135. **(d)** Let I = $=\int_0^{2\pi}\theta\sin^6\theta\cos\theta d\theta\ldots$(i)

$$=\int_0^{2\pi}(2\pi-\theta)\sin^6(2\pi-\theta)\cos(2\pi-\theta)d\theta$$

$$=\int_0^{2\pi}(2\pi-\theta)\sin^6\theta\cos\theta d\theta\ \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I=\int_0^{2\pi}2\pi\sin^6\theta\cos\theta d\theta$$

$$\Rightarrow 2I=2\pi\int_0^{2\pi}\sin^6\theta\cos\theta d\theta$$

$$=2\pi\times 2\int_0^{\pi}\sin^6\theta\cos\theta d\theta[\because f(2a-x)=f(x)]$$

$$\Rightarrow \mathrm{I}=2\pi\int_0^{\pi}\sin^6\theta\cos\theta d\theta$$

$\Rightarrow I=2\pi\times 0$

[since, $f(2a-x)=-f(x)$

$\left.\int_0^{2a}f(x)dx=0\right]$

$I=0$

136. **(c)** We have, $x=\sin\theta$ and $y=\sin k\theta$

Differentiate x and y w.r.t. θ,

$$\frac{dx}{d\theta}=\cos\theta \text{ and } \frac{dy}{d\theta}=k\cos k\theta$$

$$y_1=\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}=\frac{k\cos k\theta}{\cos\theta}$$

Again, differentiate $\frac{dy}{dx}$ w.r.t. x,

$$y_2 = \frac{d^2y}{dx^2}$$

$$= \frac{\cos\theta[-k^2\sin k\theta] - k\cos k\theta(-\sin\theta)}{\cos^2\theta}\frac{d\theta}{dx}$$

$$= \frac{k\cos k\theta\sin\theta - k^2\cos\theta\sin k\theta}{\cos^3\theta}$$

Now, $(1-x^2)y_2 - xy_1 - \alpha y = 0$

$$(1-\sin^2\theta)\left[\frac{k\cos k\theta\sin\theta - k^2\cos\theta\sin k\theta}{\cos^3\theta}\right]$$

$$-\sin\theta\times\frac{k\cos k\theta}{\cos\theta} - \alpha\sin k\theta = 0$$

$\Rightarrow k\cos k\theta\sin\theta - k^2\cos\theta\sin k\theta$

$-k\cos k\theta\sin\theta - \alpha\sin k\theta\cos\theta = 0$

$\Rightarrow \alpha = -k^2$

137. **(c)** We have, $x^{2/3} + y^{2/3} = a^{2/3}$

Let

$x = a\cos^3\theta$ and $y = a\sin^3\theta$

$$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$$

$$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$$

$$= -\tan\theta$$

Equation of tangent is

$$(y - a\sin^3\theta) = -\tan\theta(x - a\cos^3\theta)$$

$$y\cos\theta - a\sin^3\theta\cos\theta = -x\sin\theta + a\cos^3\theta\sin\theta$$

$$x\sin\theta + y\cos\theta = a\sin\theta\cos\theta(\cos^2\theta + \sin^2\theta)$$

$$\frac{x}{a\cos\theta} + \frac{y}{a\sin\theta} = 1$$

$\Rightarrow a\cos\theta$ is x-intercept and $a\sin\theta$ is y-intercept

Let $A(a\cos\theta, 0)$ and B is $(0, a\sin\theta)$ are the points, where the tangents cuts X-axis and Y-axis, respectively.

$AB = \sqrt{a^2\cos^2\theta + a^2\sin^2\theta} = a$

$\therefore$ The required position of tangent is constant.

138. **(a)** We have, $y = \sin x$

As, $\sin x$ is continuous function in $[0,\pi]$, average

$$\text{ordinate} = \frac{1}{\pi - 0}\left|\int_0^\pi y\,dx\right|$$

$$= \frac{1}{\pi}\left|\int_0^\pi \sin x\,dx\right| = \frac{1}{\pi}[\cos x]_0^\pi$$

$$= \frac{1}{\pi}[1+1] = \frac{2}{\pi}$$

$\therefore$ The average ordinate of $y = \sin x$ over $[0,\pi]$ is $\frac{2}{\pi}$

139. **(c)** We have, $f(x) = e^{\sin x} + e^{\cos x}$

$f'(x) = e^{\sin x}(\cos x) - e^{\cos x}(\sin x)$

$\Rightarrow\ f'(x) = 0$

$\Rightarrow\ e^{\sin x}\cos x = e^{\cos x}\sin x$

$\tan x = e^{\sin x - \cos x}$

when $x = \frac{\pi}{4}, \frac{9\pi}{4}, \frac{17\pi}{4}, \ldots$

$f''(x) = e^{\sin x}\cos^2 x + e^{\sin x}(-\sin x)$

$+e^{\cos x}(\sin^2 x) + e^{\cos x}(-\cos x)$

$f''\left(\frac{\pi}{4}\right) = \frac{1}{2}e^{\frac{1}{\sqrt{2}}} + e^{\frac{1}{\sqrt{2}}}\left(-\frac{1}{\sqrt{2}}\right) + e^{\frac{1}{\sqrt{2}}}\left(\frac{1}{2}\right) - \frac{1}{\sqrt{2}}e^{\frac{1}{\sqrt{2}}}$

$= e^{\frac{1}{\sqrt{2}}} - \sqrt{2}e^{\frac{1}{\sqrt{2}}}$

$f''(x) < 0$

Hence f(x) is maximum at infinitely many points.

140. **(c)** We have, volume of parallelepiped with a × b, b × c and c × a is 9 cu . units.

$\Rightarrow [\mathrm{a\times b\ b\times c\ c\times a}] = 9$

$\Rightarrow (\mathrm{a\times b})\cdot\{(\mathrm{b\times c})\times(\mathrm{c\times a})\} = 9$

$\Rightarrow (\mathrm{a\times b})\cdot\{[\mathrm{bca}]\mathrm{c} - [\mathrm{bcc}]\mathrm{a}\} = 9$

$\Rightarrow \{\because (\mathrm{a\times b})\times(\mathrm{c\times d}) = [\mathrm{abd}]\mathrm{c} - [\mathrm{abc}]\mathrm{d}\}$

$\Rightarrow (\mathrm{a\times b})\{[\mathrm{bca}]\mathrm{c}\} = 9\{\because [\mathrm{bcc}] = 0\}$

$\Rightarrow (\mathrm{a\ \times b}).\ \ [\mathrm{b\,c\,a}] = 9$

$\Rightarrow [\mathrm{abc}][\mathrm{abc}] = 9$

$\Rightarrow \{\because [\mathrm{bca}] = [\mathrm{a\,b\,c}]\}\ = [\mathrm{abc}]^2 = 9$

$\because\ (\mathrm{a\times b})\times(\mathrm{c\times d}) = [\mathrm{abd}]\mathrm{c} - [\mathrm{abc}]\mathrm{d}$

$\because (\mathrm{a\times b})\times(\mathrm{b\times c}) = [\mathrm{abc}]\mathrm{b} - [\mathrm{abb}]\mathrm{c} = [\mathrm{abc}]\mathrm{b}$

$\{\because [\mathrm{a\,b\,c}] = 0$

Similarly,

$(\mathrm{b\times c})\times(\mathrm{c\times a}) = [\mathrm{bca}]\mathrm{c}$

and $(\mathrm{c\times a})\times(\mathrm{a\times b}) = [\mathrm{c\,a\,b}]\mathrm{a}$

Volume of required parallel θ piped

$= [(\mathrm{a\times b})\times(\mathrm{b\times c})\quad(\mathrm{b\times c})\times(\mathrm{c\times a})\quad(\mathrm{c\times a})\times(\mathrm{a\times b})]$

$= [[\mathrm{a\quad b\quad c}]b[\mathrm{b\,c\,a}]\mathrm{c}[\mathrm{cab}]\mathrm{a}]$

$= [\mathrm{a\quad b\quad c}]^3[\mathrm{b\quad c\quad a}]$

$= [\mathrm{abc}]^4[\because [\mathrm{bca}] = [\mathrm{abc}]]$

$= 9^2$

$= 81$ cu units

141. **(b)** Given, numbers are $a_1, a_2, a_3, \ldots, a_n$

We know that $AM \geq GM$

$\therefore \frac{\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_n}{a_1}}{n} \geq \left(\frac{a_1}{a_2} \times \frac{a_2}{a_3} \times \ldots \times \frac{a_n}{a_1}\right)^{1/n}$

$\Rightarrow \frac{\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_n}{a_1}}{n} = (1)^{1/n}$

$\Rightarrow \frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_n}{a_1} \geq n$

$\therefore$ Minimum value of $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_n}{a_1}$ is n.

142. **(c)** We have, a quadratic equation, $ax^2 + 2bx + c = 0$ has no real root.

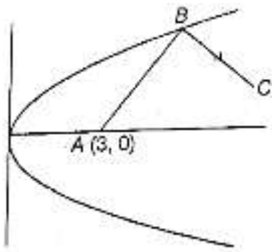
$\Rightarrow\quad (2b)^2 - 4 \times a \times c < 0 \Rightarrow 4b^2 - 4ac < 0$

$\Rightarrow\quad 4(b^2 - ac) < 0 \Rightarrow b^2 < ac$

$b < (ac)^{1/2}$

$b <$ GM of a, b, c
$\therefore a, b, c$ are not in GP.
As, we know that AM $\geq$ GM
$\frac{a+c}{2} \geq (ac)^{1/2}$ and $(ac)^{1/2} > b$
So, $\frac{a+c}{2} > b$
$\therefore a, b$ and c are not in AP.
HM $= \frac{2ac}{a+c} \leq \frac{2}{a+c}\left(\frac{a+c}{2}\right)^2 \Rightarrow \frac{2ac}{a+c} \leq \frac{a+c}{2}$
But, $\frac{a+c}{2} > b$
So, $\frac{2ac}{a+c}$ can be equal to b.
Hence, a, b and c cann't be in AP or GP, but can be in HP.

143. **(b)** We have, equation of parabola i.e.. $y^2 = 12x$



Here, $a = 3$
So, co-ordinate of focus is (3,0)
Let A be the focus of parabola and AB is the incident ray and BC be the reflected ray.
Slope of $AB = \tan\theta = \frac{3}{4}$
Equation of AB is $y - 0 = \frac{3}{4}(x-3)$
$4y = 3x - 9$
Let co-ordinate of B be $\left(\frac{\alpha^2}{12}, \alpha\right)$
Now, $4\alpha = \frac{3\alpha^2}{12} - 9$
$16\alpha = \alpha^2 - 36$
$\alpha^2 - 16\alpha - 36 = 0$
$\alpha = 18$
[Neglecting $\alpha = -2$]
The reflected ray leaves the parabola at $y = 18$

144. **(a)** As, P be an orthogonal matrix,
$PP^T = P^TP = I$
$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{bmatrix}\begin{bmatrix} 0 & x & 0 \\ 1 & 0 & 0 \\ 0 & 0 & y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & x^2 & 0 \\ 0 & 0 & y^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$x^2 = 1$ and $y^2 = 1$
As, $B = PAP^{-1}$, $Bp = PA$
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} x & 0 & 0 \\ 0 & 0 & y \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix}$$
$\Rightarrow x = y = 1$

145. **(a)** We have, $\{(x, y) \in N \times N : 2x + y = 41\}$

The relation R can be written as
$R=\{(1,39),(2,37),(3,35),(4,33),\ldots(20,1)\}$
Domain $A=\{1,2,3,\ldots,20\}$
Range $B=\{1,3,5,7,\ldots,39\}$
$A\subset\{x\in N:1\leq x\leq 20\}$ and $B\subset\{y\in N:1\leq y\leq 39\}$

146. **(b,c)** Let $f(x)=\int_a^{a+T} f(x)dx$

$$=\int_a^0 f(x)dx+\int_0^T f(x)dx+\int_T^{a+T} f(x)dx$$

Let $x-T=y$ in $\int_T^{a+T} f(x)dx$
$\Rightarrow dx=dy$

$$\int_T^{a+T} f(x)dx=\int_0^a f(y+T)dy=\int_0^a f(y)dy$$

$$=\int_0^a f(x)dx$$

Now, substitute it in Eq. (i),

$$f(x)=\int_a^0 f(x)dx+\int_0^T f(x)dx+\int_0^a f(x)dx$$

$$=\int_a^0 f(x)dx+\int_0^T f(x)dx-\int_a^0 f(x)dx$$

$=\int_0^T f(x)dx$, which is independent on a

$f(T)=0$ and $f(T+(-T)=f(0)=1\neq f(T)$
So, T cannot be an irrational number
Let T is a rational numer then
$f(x+T)=f(x)=1$, if x is rational
$f(x+T)=f(x)=0$, if x is irrational
So, T must be rational for $f(x)$ a periodic function.

147. **(a,c)** We have, $f(x)=x^m$
$f(a+b)=(a+b)^m$
$f'(a+b)=m(a+b)^{m-1}$
$f'(a)=m(a)^{m-1}$ and $f'(b)=mb^{m-1}$
Also,
$f'(a+b)=f'(a)+f'(b)$
$m(a+b)^{m-1}=ma^{m-1}+mb^{m-1}$
$(a+b)^{m-1}=a^{m-1}+b^{m-1}$
From the binomial expansion, we can say that
$m-1=1\Rightarrow m=2$
and d false for $n=1$ and $n=3$
Also, for $n=0, f(x)=1$
So, $f'(x)=0; f'(a+b)=0$
So, $f'(a+b)=f'(a)+f'(b)$
Hence, value of m are 0 and 2 .

148. **(b,c)** We have, $f(x)=3\sqrt[3]{x^2}-x^2$

$f'(x) = 3 \times \frac{2}{3} x^{-1/3} - 2x = 2\left[\frac{1}{\sqrt[3]{x}} - x\right]$

$f'(x) = 0$

$\frac{1}{\sqrt[3]{x}} - x = 0 \Rightarrow x = \frac{1}{\sqrt[3]{x}}$

$\Rightarrow x^4 = 1 \Rightarrow x = \pm 1$

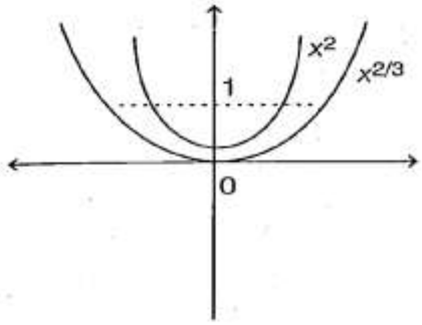
$f''(x) = 2\left[-\frac{1}{3}x^{-4/3} - 1\right] = -2\left[\frac{1}{3(x^4)^{1/3}} - 1\right]$

$f''(x) > 0$, when $x = \pm 1$

$\therefore f(x)$ is maximum at two points $x = 1$ and $x = -1$

So, $f(0) = 0$

From the graph



$\therefore f$ is minimum at $x = 0$

149. **(b)** We have, $\int_0^x (f'(t) - \sin 2t)dt$

$= \int_x^0 f(t) \tan t dt$

Differentiate both sides, we get

$f'(x) - \sin 2x = 0 - f(x) \tan x$

$f'(x) + f(x) \tan x = \sin 2x$

Which is linear differential equation whose

$\text{IF} = e^{\int \tan x dx} = \sec x$

$\therefore$ Equation is $f(x) \times \sec x = \int \sin 2x \times \sec x dx$

$f(x) \times \sec x = 2 \int \sin x dx$

$f(x) \times \sec x = -2\cos x + c$

$f(x) = -2\cos^2 x + c \times \cos x$

$AS\ f(0) = 1$

$1 = -2 + c$

$c = 3$

$\Rightarrow \quad f(x) = -2\cos^2 x + 3\cos x$

$\int_0^{\frac{\pi}{2}} f(x)dx = \int_0^{\frac{\pi}{2}} (-2\cos^2 x + 3\cos x)dx$

$= \int_0^{\frac{\pi}{2}} (1 + \cos 2x + 3\cos x)dx$

$= \left[-\left(x + \frac{\sin 2x}{2}\right) + 3\sin x\right]_0^{\frac{\pi}{2}}$

$= \left(-\frac{\pi}{2} + 0 + 3\right) - (0 + 0 + 0) = 3 - \frac{\pi}{2}$

150. **(b,c)** When the stone is dropped from the balloon, its velocity is the same as the velocity of the balloon at that instant.

The balloon starts from rest, so $u = 0 \text{ft/s}$.

The upward acceleration of the balloon (a) is 4ft/sec^2. The balloon rises for 5 sec before the stone is dropped.

$v = u + at = 0 + 4 \times 5$
$= 20\text{ft/sec}$ in the vertically upward direction.
20ft/sec is the initial velocity of the stone after being dropped.
The height of balloon in 5 sec is

$$v^2 = u^2 + 2ah$$
$$v^2 = 2ah$$
$$(20)^2 = 2 \times 4 \times h$$
$$h = \frac{400}{8} = 50\text{ft}$$

Now, the acceleration on the stone after being dropped is $g = 9.8\text{ft/s}^2$
$u = -20\text{ft/sec}, s = 50\text{ft}, a = 32\text{ft/s}^2, t = 5\text{sec}$
Now,

$$s = ut + \frac{1}{2}at^2$$
$$50 = -20 \times T + \frac{1}{2} \times 32 \times T^2$$
$$8T^2 - 10T - 25 = 0$$
$$T = \frac{5}{2}\ \text{s}$$

Total height of the ballon when store reaches the ground

$$(H) = 50\text{ft} + \left(ut + \frac{1}{2}at^2\right)$$
$$H = 50\text{ft} + \left(20 \times \frac{5}{2} + \frac{1}{2} \times 4 \times \left(\frac{5}{2}\right)^2\right)$$
$$H = 50\text{ft} + 50 + \frac{25}{2}$$
$$H = \frac{225}{2} = 112.5\text{ft}$$

151. **(a,c)** We have, three rings with 15 different letters.
The total number of possible permutation $= 15 \times 15 \times 15 = 3375$
Out of all the possible permutations, only one pattern will open the lock.
$\therefore$ Number of unsuccessful attempts to open the lock $= 3375 - 1 = 3374 = 2 \times 7 \times 241$
$\therefore N$ is the product of three distinct prime numbers.
Clearly 482 divides N because $\frac{N}{482}$

$$= \frac{2 \times 7 \times 2r}{482} = 7$$

152. **(c,d)** We have, $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$
Let $z_1 = a + ib$ and $z_2 = c + id$

$$\left|\frac{(a+c)+i(b+d)}{(a-c)+i(b-d)}\right| = 1$$
$$\Rightarrow |(a+c)+i(b+d)| = |(a-c)+i(b-d)|$$
$$\Rightarrow (a+c)^2 + (b+d)^2 = (a-c)^2 + (b-d)^2$$
$$\Rightarrow a^2 + c^2 + 2ac + b^2 + d^2 + 2bd$$
$$= a^2 + c^2 - 2ac + b^2 + d^2 - 2bd$$
$$\Rightarrow 4ac + 4bd = 0$$
$$ac = -bd$$

Now, $\frac{z_1}{z_2} = \frac{a+ib}{c+id}$

$$= \frac{(a+ib)(c-id)}{(c+id)(c-id)}$$
$$= \frac{(ac+bd)}{c^2+d^2} + i\frac{(bc-ad)}{c^2+d^2}$$
$$= i\left[\frac{bc-ad}{c^2+d^2}\right], [\because ac = -bd]$$

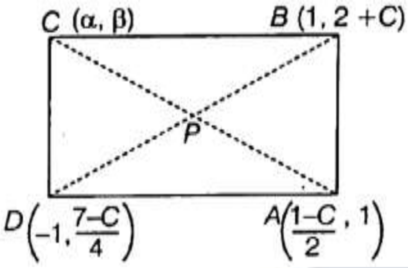
$\therefore \frac{z_1}{z_2}$ is purely imaginary.

Take $z_1 = 0$ and $z_2 \neq 0$

Now, $\left|\frac{z_1+z_2}{z_1-z_2}\right| = \left|\frac{z_2}{-z_2}\right| = |-1| = 1$

So, $\frac{z_1}{z_2} = 0$

153. **(a, c)**



Since, side of rectangle $ABCD$ is parallel to line $y = 2x$

$\therefore$ Let equation of AB is $y = 2x + c$.....(i)

Since, A is on line $y = 1$

$\Rightarrow \quad 1 = 2x + c$

$\therefore \quad x = \frac{1-c}{2}$

$A \equiv \left(\frac{1-c}{2}, 1\right)$

Since, B is on line $x = 1$

$\therefore \quad y = 2 + c$

$\therefore \quad B = (1,2+c)$

Now, equation of DA,

$$y - 1 = \left(-\frac{1}{2}\right)\left(x - \frac{1-c}{2}\right)$$

$\Rightarrow \quad 2y - 2 = -x + \frac{1-c}{2}$

$\Rightarrow \quad x + 2y = 2 + \frac{1-c}{2}$

Since, D is on $x = -1$

$\therefore \quad -1 + 2y = 2 + \frac{1-c}{2}$

$\Rightarrow \quad 2y = 3 + \frac{1-c}{2} = \frac{7-c}{2}$

$\Rightarrow \quad y = \frac{7-c}{4}$

$\therefore \; D \equiv \left(-1, \frac{7-c}{4}\right)$

Let co-ordinates of c are (α, β)

Here, mid-point of AC = mid-point of BD

$$\Rightarrow \quad \left(\frac{\alpha + \frac{1-c}{2}}{2}, \frac{\beta+1}{2}\right) = \left(\frac{1-1}{2}, \frac{2+c+\frac{7-c}{4}}{2}\right)$$

$$\Rightarrow \quad \left(\frac{2\alpha+1-c}{4}, \frac{\beta+1}{2}\right) = \left(0, \frac{3c+15}{8}\right)$$

$$\Rightarrow \quad \frac{2\alpha+1-c}{4} = 0$$

$$\Rightarrow \quad 2\alpha+1-c = 0$$

$$\Rightarrow \quad 2\alpha+1 = c \;....(ii)$$

and $\quad \dfrac{\beta+1}{2} = \dfrac{3c+15}{8}$

$$\Rightarrow \quad 4\beta+4 = 3c+15$$

$$\Rightarrow \quad 4\beta-11 = 3c$$

$$\Rightarrow \quad \frac{4\beta-11}{3} = c \;........(iii)$$

From Eqs. (ii) and (iii), we get

$$2\alpha+1 = \frac{4\beta-11}{3}$$

$$\Rightarrow \quad 6\alpha+3 = 4\beta-11$$

$$\Rightarrow \quad 6\alpha-4\beta = -14$$

$$\Rightarrow \quad 3\alpha-2\beta = -7$$

$\therefore$ Locus of c is $3x-2y=-7$….(iv)
From given options (a) and (c), $(3,8)$ and $(-3,-1)$ satisfied (iv).

154. **(a,c)**We have, R be a relation on A i.e. $R \subseteq A \times A$
$R = \{(a,b) \mid a,b \in A\}$
$R^{-1} = \{(b,a) \mid a,b \in A\}$
Since, R is reflexive; $(a,a) \in R$
$\Rightarrow (a,a) \in R^{-1} \Rightarrow R^{-1}$ is reflexive
Since, R is symmetric; $(a,b),(b,a) \in R$
$\Rightarrow (b,a),(a,b) \in R^{-1} \Rightarrow R^{-1}$ is symmetric
For transitive,
Let $(b,a),(a,c) \in R^{-1}$
$(a,b)(c,a) \in R$
Since, R is transitive $(c,b) \in R$
$\Rightarrow (b,c) \in R^{-1}$
$\therefore R^{-1}$ is transitive
Hence, R^{-1} is an equivalence relation.
For $R \cap R'$ and $R \cup R'$
Since, R and R' are reflexive
$(a,a) \in R$ and $(a,a) \in R' \forall a \in A$,
$(a,a) \in R \cap R'$ and $(a,a) \in R \cup R'$
$\Rightarrow R \cap R'$ and $R \cup R'$ are reflexive
Since, R and R' are symmetric.
If $(a,b) \in R$, then $(b,a) \in R$
Similarly, if $(a,b) \in R'$, then $(b,a) \in R'$
If $(a,b) \in R \cup R'$ and $(a,b) \in R \cap R'$, then $(b,a) \in R \cup R'$ and $(b,a) \in R \cap R'$
$\therefore R \cup R'$ and $R \cap R'$ are symmetric.
Since, R and R' are transitive
If $(a,b),(b,c) \in R$, then $(a,c) \in R$
Similarly,

If $(a, b), (b, c) \in R'$, then $(a, c) \in R'$
If $(a, b), (b, c) \in R \cap R'$, then $(a, c) \in R \cap R'$
$\therefore R \cap R'$ is transitive
If $(a, b) \in R, (b, c) \in R'$, then $(a, b), (b, c) \in R \cup R'$.
But (a, c) may or may not be in $R \cup R'$
$\therefore R \cup R'$ may or may nor be transitive.
Hence, if R and R' are equivalence relation, then R^{-1}, $R \cap R'$ and $R \cup R'$ are also equivalence relation.

155. **(a,c)** If $f(x)$ is an decreasing function, then
for $x > y \Rightarrow f(x) < f(y)$
$f(a^2 + 5a + 3) < f(a + 15)$
$\Rightarrow a^2 + 5a + 3 > a + 15$
$\Rightarrow a^2 + 4a - 12 > 0$
$\Rightarrow (a + 6)(a - 2) > 0$
$\Rightarrow a \in (-\infty, -6) \cup (2, \infty)$