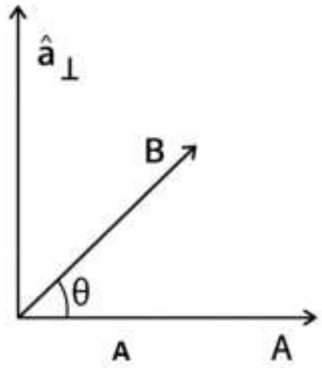
1. (c) We can write, $\mathbf{B}=B\cos\theta\hat{\mathbf{A}}+B\sin\theta\hat{\mathbf{a}}_{\perp}$
Thus, $\mathbf{B}-B\sin\theta\mathbf{a}_{\perp}=B\cos\theta\hat{\mathbf{A}}$



Hence, $\mathbf{B}-B\sin\theta\mathbf{a}_{\perp}$ has direction along $\mathbf{A}$.

2. (b) Given, $P\propto\frac{(qa)^m}{c^n}$
(i) The RHS term does not include dimensions of charge.
(ii) The formula should be consistent in all system of units.
(b) Given, $P\propto\frac{(qa)^m}{c^n}$

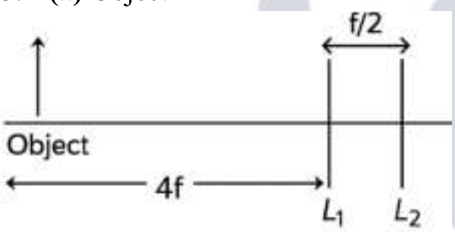
$$\text{Power}=\frac{\text{Electrostatic energy}}{\text{Time}}=\frac{Kq_0q_0}{r\times t}$$

$K=1$ in CGS system
Thus, $\frac{q^2}{rt}=\frac{q^m a^m}{c^n}$
Comparing both sides, we get $m=2$
Thus,

$$\frac{1}{[\mathrm{L}][\mathrm{T}]}=\frac{[\mathrm{LT}^{-2}]^2}{[\mathrm{LT}^{-1}]^{\mathrm{n}}}\Rightarrow[\mathrm{L}^{-1}\mathrm{T}^{-1}]=[\mathrm{L}^{2-\mathrm{n}}\mathrm{T}^{-4+\mathrm{n}}]$$

Thus, we get $-1=-4+n\Rightarrow n=3$

3. (a) Object



For lens L_1,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow\frac{1}{v}-\frac{1}{-4f}=\frac{1}{f}\Rightarrow v=\frac{4f}{3}$$

This image will act as an object for lens L_2.
Thus, $u_2=\frac{4f}{3}-\frac{f}{2}=\frac{5f}{6}$
For lens L_2,

$$\frac{1}{v_2}-\frac{1}{u_2}=\frac{1}{f}\Rightarrow\frac{1}{v_2}-\frac{6}{5f}=\frac{1}{f}\Rightarrow v_2=\frac{5f}{11}$$

Since v_2 is positive, thus final image will be $5f/11$ unit right of L_2.

4. (d) We know that,
$[h]=[\mathrm{ML}^2\mathrm{T}^{-1}]\Rightarrow[C]=[\mathrm{LT}^{-1}]$
Dimensions of length,
$[L]=h^{\alpha}C^{\beta}m^{\gamma}\Rightarrow[L^1]=[\mathrm{ML}^2\mathrm{T}^{-1}]^{\alpha}[\mathrm{LT}^{-1}]^{\beta}[\mathrm{M}]^{\gamma}$
Upon equating, we get

$\alpha + \gamma = 0 \Rightarrow \Rightarrow -\alpha - \beta = 0 \Rightarrow \beta = -\alpha$ $\quad 2\alpha + \beta = 1$

Thus, $2\alpha - \alpha = 1$

$\Rightarrow \alpha = 1 \Rightarrow \beta = -1 \Rightarrow \gamma = -1$

Hence, final formula is $\frac{h}{mc}$.

5. (b) Number of particles = Area under the curve

$= \frac{3}{2} v \cdot a = N \ldots$(i)

Now, the number of particles between speed $1.2 v_0$ to $1.8 v_0$ will be

$(1.8 - 1.2) v_0^a = 0.6 v_0^a$

From Eq. (i), $v_0^a = \frac{N \times 2}{3}$

Thus, number of particle will be

$0.6 \times \frac{N \times 2}{3} = 0.4$ N

6. (d) The internal energy U is an extensive property, meaning it scales linearly with the system size. Entropy S and volume V are also extensive properties. For an extensive quantity, if we scale all extensive variables by a factor λ, the quantity itself should scale by the same factor λ.

Given $U = aS^{4/3} V^{\alpha}$.

If we replace S with λS and V with λV :

$U' = a(\lambda S)^{4/3} (\lambda V)^{\alpha} = a\lambda^{4/3} S^{4/3} \lambda^{\alpha} V^{\alpha} = \lambda^{4/3+\alpha} (aS^{4/3} V^{\alpha}) = \lambda^{4/3+\alpha} U$

For U to be extensive, U' must be equal to λU. Therefore, the exponent of λ must be 1 :

$$\frac{4}{3} + \alpha = 1$$

$$\alpha = 1 - \frac{4}{3}$$

$$\alpha = -\frac{1}{3}$$

The final answer is $-1/3$.

7. (c) Given, potential energy, $U(x) = \frac{-\alpha x}{x^2 + \beta^2}$

Here, $[\beta] = [\mathrm{L}] \Rightarrow [\alpha] = [\mathrm{ML^3\ T^{-2}}]$

We know that $\frac{2\pi}{T} = \omega$ or $\omega = [T^{-1}]$

(a) $\sqrt{\frac{\alpha^3}{m\beta^4}} = \sqrt{\frac{[ML^3T^{-2}]^3}{M[L]^4}} = \sqrt{\frac{M^3L^9T^6}{ML^4}} \neq [T^{-1}]$

(b) $\sqrt{\frac{\alpha}{m\beta^4}} = \sqrt{\frac{[ML^3T^{-2}]}{M[L^4]}} \neq [T^{-1}]$

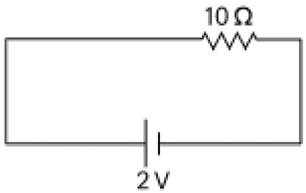
(c) $\sqrt{\frac{\alpha}{m\beta^3}} = \sqrt{\frac{[\mathrm{ML^3\ T^{-2}}]}{[\mathrm{M}][\mathrm{L}]^3}} = [\mathrm{T^{-1}}]$

(d) $\sqrt{\frac{\alpha}{m\beta^6}} = \sqrt{\frac{[ML^3\ T^{-2}]}{[M][L]^6}} \neq [\mathrm{T^{-1}}]$

Hence, option (c) is the correct answer.

8. (d) Longitudinal waves do not have perpendicular components of oscillation w.r.t. their propagation. So, these waves can not be polarised.

9. (b) The given circuit can be reduced to



As, the diode in the 20Ω branch is reversed biased.

Thus, current, $i = \frac{V}{R} = \frac{2}{10} = 0.2$ A

10. (c) According to Gauss's law

$$\phi_E = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

11. (d) The elastic potential energy of a strained body is given by

$$E = \frac{1}{2} \times \text{ stress } \times \text{ strain } \times \text{ volume}$$

12. (b) The mass energy of nucleus is lesser than the total mass energy of its individual protons and neutrons. This mass difference appears as binding energy of the nucleus.

13. (b) We know that,

Kinetic energy, $\text{KE} = \frac{J^2}{2I} \Rightarrow K = \frac{J^2}{2mR^2}$ and total energy of the satellite, $E = -\text{KE} = \frac{-J^2}{2mR^2}$

14. (d) Given:

$m = 10$ kg,

$\mu_s = 0.6$,

$\theta = 60^\circ$

Formula:

$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

Substitute values:

$$F = \frac{0.6 \cdot 10 \cdot 9.8}{\cos 60^\circ + 0.6 \cdot \sin 60^\circ} = \frac{58.8}{0.5 + 0.5196}$$

$$= \frac{58.8}{1.0196} \approx 57.7 \text{ N}$$

15. (d) Given, wavelength, $\lambda = 6000\text{Å}$

refractive index $(\mu) = 1.5$

refraction angle $(r) = 60^\circ$

For dark fringe, $2\mu t \cos r = n\lambda$

Here, $n = 1$ for smallest thickness as at $n = 1$, we will receive first minima.

$\therefore\ 2 \times (1.5) \times t \times \cos(60^\circ) = 6000 \times 10^{-10}$

$$\Rightarrow\ t = \frac{6000 \times 10^{-10} \times 2}{2 \times 1.5} = 4 \times 10^{-7} \text{ m}$$

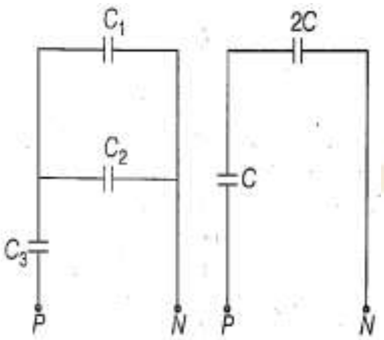
16. (c) All resistances are in parallel combination,

$R_{\text{net}} = R/3$

For maximum power transfer, cells, internal resistance must be equal to circuit resistance.

Thus, $r = \frac{R}{3} \Rightarrow R = 3r$

17. (b) Lets redraw the circuit,



Thus, equivalent capacitance,

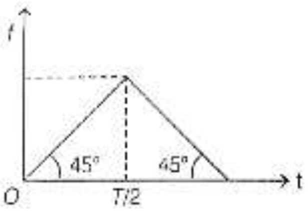
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C} + \frac{1}{2C} \Rightarrow \frac{1}{C_{\text{eq}}} = \frac{2+1}{2C} \Rightarrow C_{\text{eq}} = \frac{2C}{3}$$

18. (c) Condition of minima in diffraction is given as $d\sin\theta = n\lambda$

According to question,

$$n_1\lambda_1 = n_2\lambda_2 \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = \frac{3}{2}$$

19. (d) For $0 \le t < T/2$, the force is increasing thus the velocity will keep on increasing.



We know that,

$$F = m\frac{dv}{dt} \Rightarrow F \cdot dt = mdv$$

Thus, the velocity will keep on increasing.

From graph, $\frac{df}{dt} = 1 \Rightarrow f = t$

Thus, $t \cdot dt = mdv$

$$\int_0^{T/2} t \cdot dt = m\int \ dv \Rightarrow mv = \left(\frac{t^2}{2}\right)_0^{\frac{T}{2}} = \frac{T^2}{8}$$

Hence, the v versus t graph will be a parabola.

For, $\frac{T}{2} < t \leq T \Rightarrow \frac{df}{dt} = -1 \Rightarrow f = -t$

Hence, $-\int_{T/2}^{T} dt = m\int \ dv \Rightarrow \left[\frac{-t^2}{2}\right]_{\frac{T}{2}}^{T} = mv$

$$\Rightarrow mv = -\frac{1}{2}\left[T^2 - \frac{T^2}{4}\right] = \frac{-1}{2}\left[\frac{3T^2}{4}\right] = \frac{-3T^2}{8}$$

Hence, the graph will be parabolic but less curved outward.

Hence, correct option is (d).

20. **(a)** Given, r $= x\hat{\mathbf{i}} + \dot{b}\hat{\mathbf{j}}$ i.e.,

$$v = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} = v_x\hat{\mathbf{i}}$$

Then, $\mathbf{r} \times \mathbf{v} = bv(-\hat{\mathbf{k}})$

Angular momentum, $\mathbf{r} \times m\mathbf{v} = bmv(-\hat{\mathbf{k}})$

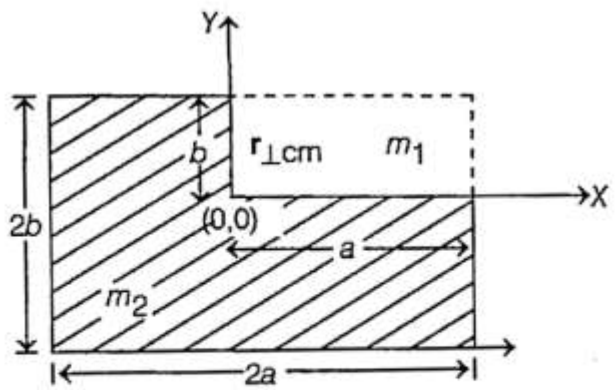
i.e.,

$[\mathbf{L}] = bmv$

$(\because v = \text{constant})$

Hence, the graph will be a straight line.

21. **(d)** Let us consider that the plate has mass be 4 m



Then, $m_2 = 3m$ and $m_1 = m$

Therefore, $m_2x_2 + m_1x_1 = 4m(0)$

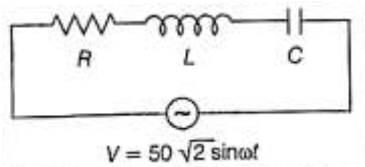
$$3mx + m\frac{a}{2} = 0 \Rightarrow x = -\frac{a}{6}$$

Similarly, $y = \frac{-b}{6}$

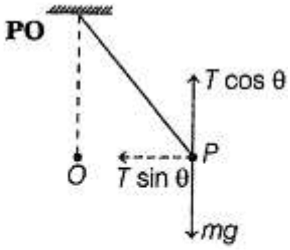
Hence, position of centre of mass will be $\left(\frac{-a}{6}, \frac{-b}{6}\right)$.

22. **(d)** Given, V_{rms} across $V_R = 30$ V V_{rms} across $V_C = 90$ V

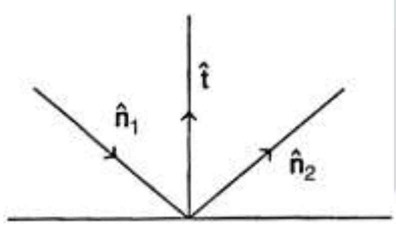
$V = 50\sqrt{2}\sin\omega t$



As we know that,
$V_R^2 + (V_C - V_L)^2 = V^2$
$\Rightarrow 30^2 + (90 - V_L)^2 = 50^2$
$\Rightarrow (90 - V_L)^2 = 40^2 \Rightarrow V_L = 130 \text{ or } 50$
$(V_L)_{\text{peak}} = 130\sqrt{2} \text{ or } 50\sqrt{2}$
Since, $130\sqrt{2}$ is not possible, the correct optio is (d).
23. (c) Net force $= T \sin\theta$ along
$\Rightarrow$ Net torque about O is zero.



$\because \ \tau = \mathbf{r} \times \mathbf{F}$
If both **r** and **F** has same direction, then $\tau = 0$.
24. (a) Let angle of incidence be θ.



Thus, $\hat{n}_1 = \cos\theta(-\hat{j}) + \sin\theta(\hat{i})$
$\hat{n}_2 = \cos\theta(\hat{j}) + \sin\theta(\hat{i})$
$\hat{n}_1 \cdot t = \cos(180^\circ - \theta) = -\cos\theta$
$\hat{n}_1 - \hat{n}_2 = -2\cos\theta(\hat{j})$
$\hat{n}_1 - \hat{n}_2 = -2\cos\theta\hat{t} = +2(\hat{n} \cdot \hat{t})\hat{t}$
$\Rightarrow \ \hat{n}_2 = \hat{n}_1 - 2(\hat{n}_1 \cdot \hat{t})\hat{t}$
25. (c) We know that, $r = \frac{mv}{qB} = \text{constant}$
Einstein's photoelectric equation is given as
$$\frac{hc}{\lambda} = \omega_0 + \frac{1}{2}mv^2 = \omega_0 + \frac{1}{2m}(mv)^2 = \omega_0 + \frac{q^2B^2r^2}{2m}$$
Let $\frac{1}{\lambda} = x$ and $B^2 = y$
Then, $hcx = \omega_0 + \frac{q^2R^2}{2m}y$
Eq. (i) represents $y = mx - c$
Hence, option (c) is correct graph.
26. (b) Given, velocity $\mathbf{v} = v_1\hat{\mathbf{i}} + v_2\hat{\mathbf{j}}$
Force, $\mathbf{F} = F_1\hat{\mathbf{i}} + F_2\hat{\mathbf{j}}$
Let $\mathbf{B} = B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}}$
We know that,
$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = q\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_1 & v_2 & \mathbf{0} \\ B_1 & B_2 & B_3 \end{vmatrix}$$
$= q\left[\hat{\mathbf{i}}(v_2B_3 - 0) - \hat{\mathbf{j}}(v_1B_3 - 0) + \hat{\mathbf{k}}(v_1B_2 - v_2B_1)\right]$

$= q[v_2B_3\hat{\mathbf{i}} - v_1B_3\hat{\mathbf{j}} + (v_1B_2 - v_2B_1)\hat{\mathbf{k}}]$

Thus, $F_1\hat{\mathbf{i}} + F_2\hat{\mathbf{j}} = q[v_2B_3\hat{\mathbf{i}} - v_1B_3\hat{\mathbf{j}} + (v_1B_2 - v_2B_1)\hat{\mathbf{k}}]$

Upon comparison, we get

and $F_1 = qv_2B_3$

and $F_2 = -qv_1B_3$

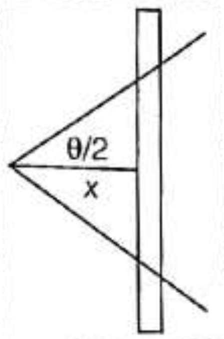
$\Rightarrow$ $F_3 = 0 = q(v_1B_2 - v_2B_1)$

$\Rightarrow$ $v_1B_2 = v_2B_1$

$\frac{B_1}{B_2} = \frac{v_1}{v_2}$

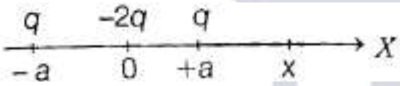
(Here $B_3 \neq 0$)

27. (b) From the diagram, $l = 2vf\tan\theta/2$



Induced emf, $\varepsilon = Blv = 2Bv^2\tan\frac{\theta}{2}$.

28. (d)



From the diagram, electric field at x due to all charges is

$$E = \left[\frac{kq}{(x-a)^2} + \frac{kq}{(x+a)^2} - \frac{2kq}{x^2}\right]\hat{\mathbf{i}}$$

$$= kq\left[\frac{(x+a)^2x^2 + (x-a)^2x^2 - 2(x+a)^2(x-a)^2}{(x-a)^2(x+a)^2x^2}\right]\hat{\mathbf{i}}$$

$$= kq\left[\frac{x^2(2x^2+2a^2) - 2(x^2-a^2)^2}{x^2(x^2-a^2)^2}\right]\hat{\mathbf{i}}$$

$$= kq\left[\frac{2x^4 + 2a^2x^2 - 2x^4 - 2a^4 + 4a^2x^2}{x^2(x^2-a^2)^2}\right]\hat{\mathbf{i}}$$

$$= kq\left[\frac{6x^2a^2 - 2a^4}{x^2(x^2-a^2)^2}\right]\hat{\mathbf{i}}$$

$$\approx kq\left(\frac{6a^2}{x^4}\right)\hat{\mathbf{i}} = 6\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^4}\{\because \text{ Given } qa^2 = Q\}$$

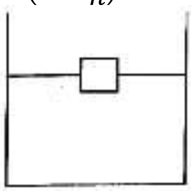
Hence, $\beta = 4$ and $\alpha = 6$

i.e., $2\alpha = 3\beta$

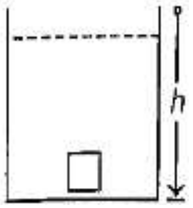
29. (d) Buoyant force $= mg$

$\Rightarrow F_B = mg$

$V\left(1 - \frac{1}{n}\right)\sigma g = Vdg$



$\Rightarrow d = \left(\frac{n-1}{n}\right)\sigma$
Here, d is density of the body and σ is density of water.
Now,
For the body to come back up



$a = \frac{F_b - mg}{m}$

$a = \frac{V\sigma g - Vdg}{Vd} = \left(\frac{\sigma}{d} - 1\right) g =$

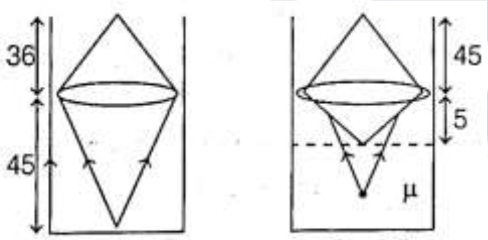
$\left(\frac{n}{n-1} - 1\right) g = \frac{g}{n-1}$

We know that, $h = \frac{1}{2}at^2$

$\Rightarrow t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2h}{g/(n-1)}} = \sqrt{\frac{2h(n-1)}{g}} \Rightarrow t \propto \sqrt{n-1}$

30. **(a)** In case of no friction, applying work energy theorem, change in potential energy = gain in kinetic energy
i.e., $mgR - mgr = \text{KE} \Rightarrow \text{KE} = mg(R-r)$

31. **(d)** Case I Without water



Case I Case II

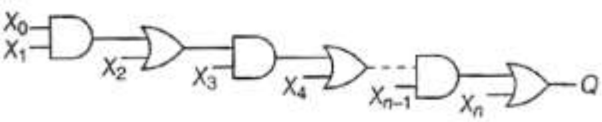
$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{36} - \frac{1}{-45} \Rightarrow f = 20 \text{ cm}$

Case II After pouring water upto 40 cm .

Apparent object distance, $u = \frac{40}{\mu} + 5$

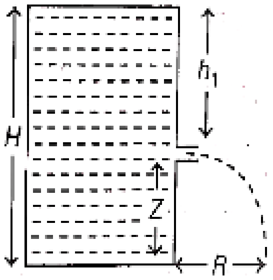
Again, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{1}{48} - \frac{1}{\frac{40}{\mu}+5} \Rightarrow 1.366$

32. **(d)** In the given network of AND and OR gates, output Q can be written as (assuming n as even)



For 1: X_0X_1
For 2: $X_0X_1 + X_2$
For 3: $X_0X_1X_3 + X_2X_3$
For 4: $X_0X_1X_3 + X_2X_3 + X_4$
Thus, $Q = X_0X_1 \dots X_{n-1} + X_2X_3X_5 \dots X_{n-1} = X_{n-2}X_{n-1} + X_n$

33. **(b)** Velocity of efflux $= \sqrt{2gh_1}$



Time to reach the water from height Z to ground,
$t=\sqrt{2Z/g}$
Thus, Range = Velocity of efflux × Time

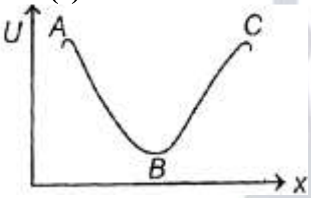
$$=\sqrt{2gh_1}\times\sqrt{\frac{2Z}{g}}=\sqrt{4h_1Z}=\sqrt{(h_1+Z)^2-(h_1-Z)^2}$$

For range to be maximum, $h_1=Z$
We also can see that, $h_1+Z=H$
Thus, $2Z=H$ or $Z=H/2$
34. (b) Given, area of metal plate, $A=10^{-2}\ \mathrm{m}^2$
Thickness of oil, $t=2\times10^{-3}$ m
Viscosity of oil, $\eta=1.55\mathrm{Nsm}^{-2}$
Speed, $v=3\times10^{-2}\ \mathrm{ms}^{-1}$
Force required, to more the plate will be,

$$F=\eta\times\text{ Area }\times\frac{\text{Velocity}}{\text{Thickness}}\left(\because\eta=\frac{F\cdot dx}{A\cdot dv}\right)$$

$$=1.55\times10^{-2}\times\frac{3\times10^{-2}}{2\times10^{-3}}=0.2325\ \mathrm{N}$$

35. (c) The double derivative of potential energy (V) gives the equilibrium point of curve.



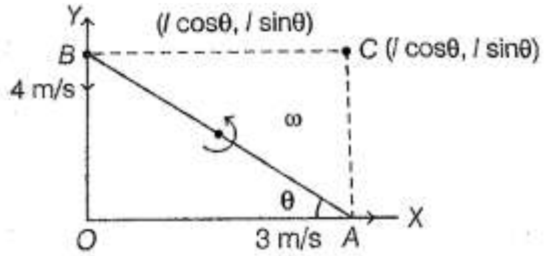
If it's positive, then the particle is in stable equilibrium and if it's negative, then particle is in unstable equilibrium.
Here, A and C represent unstable equilibrium.
B represents stable equilibrium.
36. (d) Given, wavelength, $\lambda=4770\text{Å}$
Energy of incident light, $E=\frac{hc}{\lambda}$

$$=\frac{6.626\times10^{-34}\times3\times10^{8}}{4770\times10^{-10}}=0.00416\times10^{-16}\ \mathrm{J}$$

$$=\frac{0.00416\times10^{-16}}{1.6\times10^{-19}}=0.0026\times10^{3}\mathrm{eV}=2.46\mathrm{eV}$$

For emission of photoelectrons, work function (ω_0) should be less than or equal to energy of incident light.
Thus, from metal D electrons will be emitted.
37. (a,b,c) $\oiint \mathbf{E}\cdot\mathbf{dS}=\frac{Q}{\varepsilon_0}$

$\oiint$ symbol represents surface integral over a closed surface. The surface integral can be calculated with the help of a polar vector.
Here dS represents a polar vector.
dS = rdrdQ
It con:ains Coulomb's law and superposition principle. Q represents all the charges emerged within the stored surface.
38. (a,b,d) Given, length of rod, $l=1$ m
mass of rod, $m=4$ kg
velocity along Y-axis $=4$ m/s
velocity along Y-axis $=3$ m/s



Angular velocity of rod, $\omega = \frac{v_A}{AC} = \frac{v_B}{BC}$

$\Rightarrow \omega^2 = \frac{v_A^2}{AC^2} = \frac{v_B^2}{BC^2} = \frac{v_A^2 + v_B^2}{l^2} = \frac{3^2+4^2}{1^2} = 5^2$

$\Rightarrow \omega = 5\text{rod/s}$

Velocity of centre of mass, $v_{CM} = \omega \times l/2$

$= 5 \times \frac{1}{2} = 2.5 \text{ m/s}$

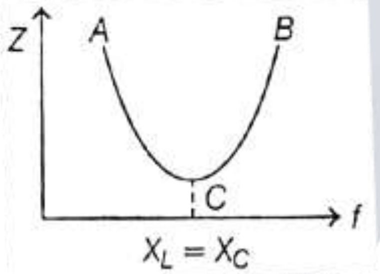
We know that rotational kinetic energy,

$\text{KE} = \frac{1}{2} I\omega^2 = \frac{1}{2}\left[\frac{ml^2}{12}\right]\omega^2 \left(\because I_{\text{rod}} = \frac{ml^2}{12}\right)$

$= \frac{1}{2}\left(\frac{4}{12}\right) \times (5)^2 = \frac{25}{6} \text{ J}$

39. (c, d) At resonance, ($X_L = X_C$)

The impedance Z is minimum.



As the frequency increases, capacitive reactance decreases and inductive reactance increases i.e., for $AC, X_L < X_C \Rightarrow$ Capacitive nature

$BC; X_L > X_C \Rightarrow$ Inductive nature

40. (b,c) Given,

$\mathbf{E}(x,y,z,t) = E_0 \hat{\mathbf{n}} e^{ik_0[(x+y+z)-ct]}$.....(i)

Polarisation is in XZ-plane.

The general equation of EM wave is given by

$\mathbf{E} = E_0 \hat{\mathbf{n}} e^{ik_0[\mathbf{k}\cdot\mathbf{r}-\omega t]}$...(ii)

Comparing both Eqs. (i) and (ii), we get

$k = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

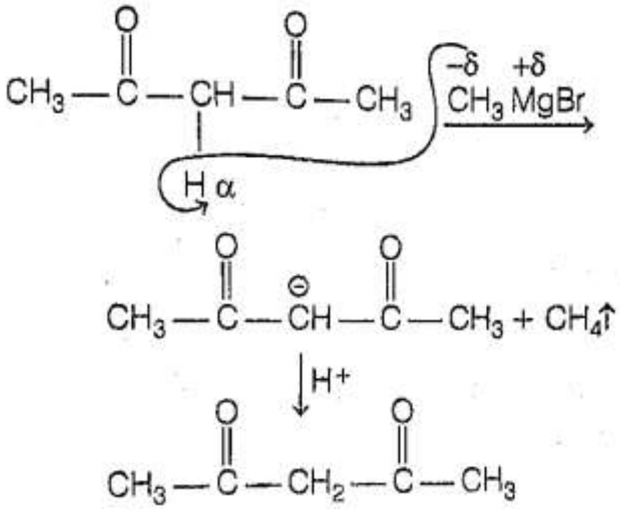
Velocity, $v = \frac{\omega}{k} = \frac{c}{|\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}|} = \frac{c}{\sqrt{3}}$

Hence, refractive index, $\eta = \sqrt{3}$

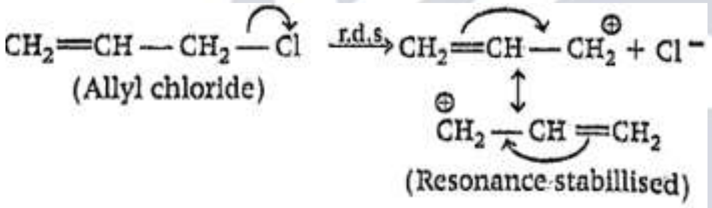
Eis in XZ-plane. Thus, $\mathbf{E} \cdot \mathbf{v} = \mathbf{0} \Rightarrow \hat{\mathbf{n}} = \frac{\hat{\mathbf{i}}-\hat{\mathbf{k}}}{\sqrt{2}}$

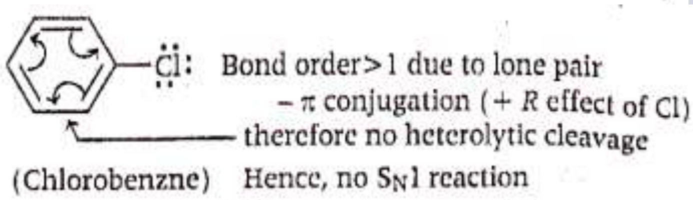
Chemistry

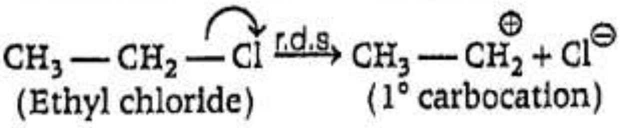
41. (a) The complete reaction is as follows



42. (c) The electronic configuration of Sm is
${}_{62}\mathrm{Sm} = [\mathrm{Xe}]4f^6 6s^2 \Rightarrow \mathrm{Sm}^{2+} = [\mathrm{Xe}]4f^6 6s^0$
Hence, Sm^{2+} ion have $4f^6$ configuration.
43. (d) For metallic conductor, resistance is given by
$R = R_0(1 + \alpha t)$ or $R \propto T$
Hence, conductivity decreases as temperature increases. For semi-conductors, conductivity increases with rise in temperature due to increase in number of electron-hole pairs.
44. (b) The given reaction is,
$I_2 + 2Na_2\,S_2O_3 = 2NaI + Na_2\,S_4O_6$
$I_2 + 2Na_2^{+2}\,S_2O_3 = 2NaI + Na_2^{+2.5}\,S_4O_6$
$2 \times 0.5 = 1$
∴ Number of moles off electron lost per mole of $Na_2\,S_2O_3 = 1$
Hence, equivalent weight of $Na_2\,S_2O_3 = \frac{M}{1} = M$
45. (b) Reactivity of S_N1 reaction ∝ Stability of carbocation formed

(I)


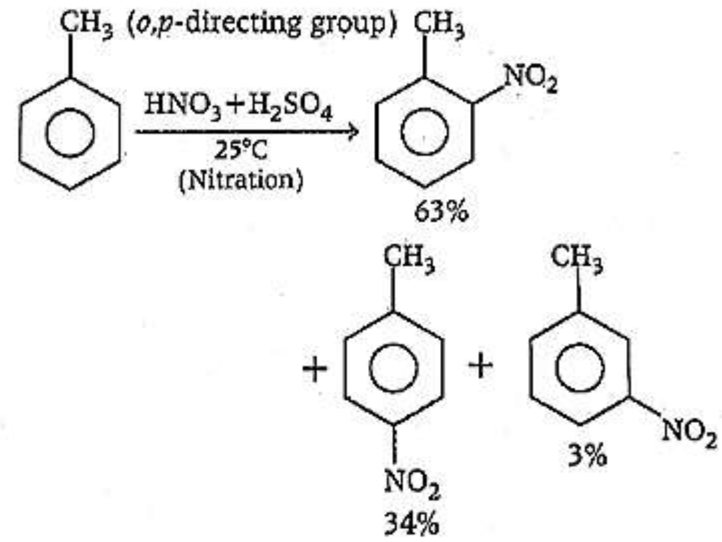
(II)


(III)


∴ I > III > II

46. (c) When toluene react with mixed acid at 25°C it will produce predominantly *o*-nitrotoluene and

p-nitrotoluene. The complete reaction proceeds as follows,



47. (b)

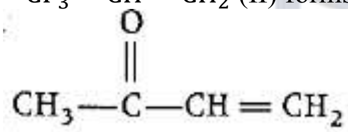
48. (d)

The reactivity of alkenes towards HBr addition depends on the stability of the carbocation intermediate formed during the reaction:

$-\mathrm{MeOCH}=\mathrm{CH_2}$ (III) forms a resonance-stabilized carbocation due to the electron-donating methoxy group, making it the most reactive.

$-\mathrm{CH_3}-\mathrm{CH}=\mathrm{CH_2}$ (I) forms a secondary carbocation, which is moderately stable.

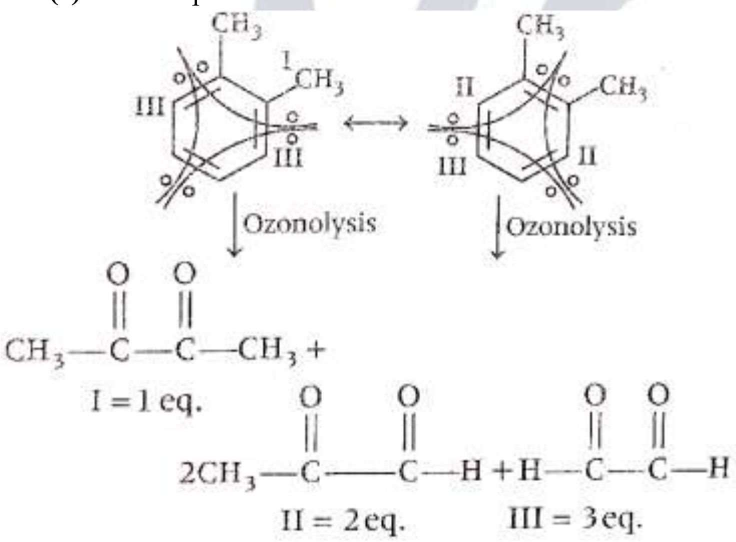
$-\mathrm{CF_3}-\mathrm{CH}=\mathrm{CH_2}$ (II) forms a destabilized carbocation due to the electron-withdrawing $\mathrm{CF_3}$ group.



(IV) forms a sterically hindered carbocation, making it the least reactive.

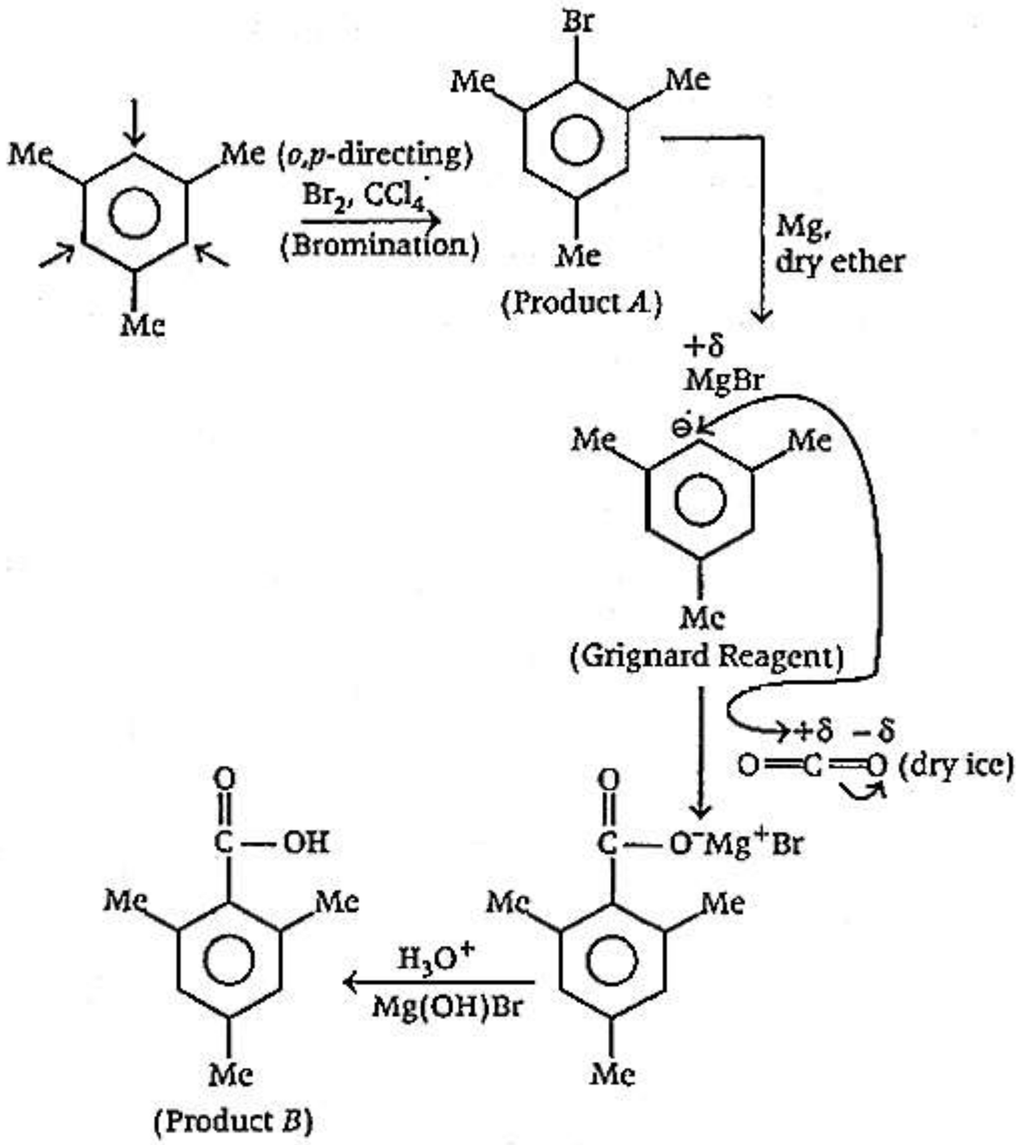
Thus, the order of reactivity is III > I > II > I

49. (c) The complete reaction is as follows.



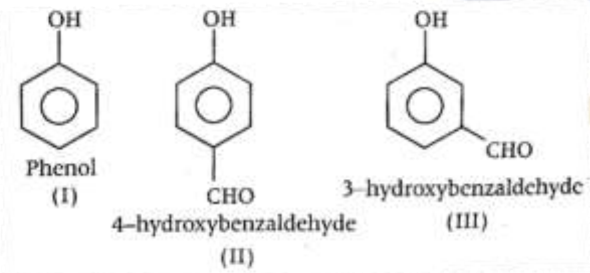
$\therefore$ I: II: III $= 1:2:3$

50. (c) The complete reactions of giving compounds of A and B are,



51. (b) On heating benzene diazonium chloride it produces N_2 gas which escapes from the mixture and NaCN is not formed hence this compound does not give positive Lassaigne's test.

52. (b)

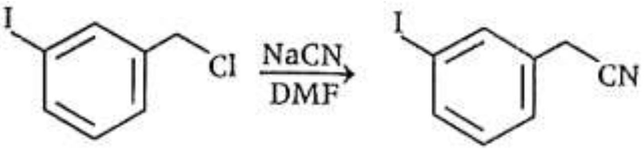


Acidity strength = II > III > I

-CHO is an electron withdrawing group which increases acid strength of phenolic compound.

Also electron withdrawing effect of - CHO is more through para-position ($-R$) as compared to meta-position (-1 only).

53. (c) The major product in given reaction



Halogen attached to benzene ring does not participate in S_N2 reaction.
Here, Cl attached to benzylic carbon can only be substituted. (S_{N^2}).

54. (a) $nA \rightarrow A_n$
Being spontaneous ΔG of the reaction is less than zero.
$\Delta G = \Delta H - T\Delta S$
In polymerisation reaction lot of heat is generated, so ΔH must be negative and the final product is a polymer, which has well defined structure (orderly manner), so ΔS is also negative.

55. (c) $H_2O(l) \rightleftharpoons H^+(aq) + OH^-(aq)$
$K = 10^{-14}$
$\Delta G = -2.303RT\log(K)$
$= -2.303 \times 1.987 \times (273 + 25) \times \log(10^{-14})$
$= 2.303 \times 1.987 \times 298 \times 14 = 19091\text{cal}$
$\approx 19.1\text{kcal mol}^{-1}$

56. (c) Velocity of an electron according to Bohr's model is given as, $V_1 = V_0\frac{Z}{n}$
For 1st orbit of $He^+, V_1 = V_0\frac{2}{1}$ $(Z = 2, n = 1)$
For 3rd orbit of $He^+, V_3 = V_0\frac{2}{3}$ $(z = 2, n = 3)$
Ratio of V_3 and V_1
$V_3 : V_1 = 1 : 3$

57. (b) van der Waals' equation for 1 mole of gas is given by
$\left(p + \frac{a}{V^2}\right)(V - b) = RT$
At high pressure,
$p + \frac{a}{V^2} \approx p$
$\therefore\ p(V - b) = RT$
$pV - pb = RT \Rightarrow pV = RT + pb$
$$Z = \frac{pV}{RT} = \frac{RT}{RT} + \frac{pb}{RT} \Rightarrow Z = 1 + \frac{pb}{RT}$$

58. (a) Statement given in option (a) is incorrect, while (b), (c), (d) are correct statement.
The correct form of statement (a) is,
$(\Delta G_{\text{system}})_{p,T} < 0$

59. (c, d) Statement given in (c), (d) are incorrect, while statements given in (a), (b) correct.
The correct form of incorrect statements is, viscosity and surface tension may increase or decrease upon addition of impurities.

60. (a) From van't Hoff equation,
$$\log\left[\frac{K_2}{K_1}\right] = \frac{\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$
where, K_1 and K_2 are equilibrium constants at T_1 and T_2 temperature respectively.
When $\Delta H > 0$ (endothermic process) then equilibrium constant increases with increase in temperature.
When $\Delta H < 0$ (exothermic process) the equilibrium constant
temperature.
According to Arrhenius equation
$$\log\left(\frac{K_2}{K_1}\right) = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$
E_a (Activation energy) > 0
When temperature increases rate constant will increase.

61. (b) The nuclear reaction is as follows,
${}_{83}Bi^{214} \longrightarrow {}_{84}Po^{214} + \beta$
${}_{84}Po^{214} \longrightarrow {}_{82}Pb^{210} + \alpha$
Number of neutron = Mass number (A)
$210 - 82 = 128$-Atomic number (Z)

62. (b) Bohr's atomic radius is given by

Radius of nth orbit $= 0.529 n^2 / Z$Å
Radius of 1st Bohr orbit of H -atom

$= 0.529 \times \frac{1^2}{1}$Å $= 0.529$Å

Radius of 2nd Bohr orbit of Be^{3+}

$= 0.529\left(\frac{2^2}{4}\right) = 0.529$Å

63. (c) According to the integrated ist order rate law,

$\log[A]_t = \left[\frac{-k}{2.303}\right] t + \log[A]_0$

Compare it with $y = mx + c$

$\log[A] + \frac{l}{t}$

Slope $= \frac{-k}{2.303}$

64. (d) Concentration of H^+in molarity

$= \frac{\text{Total no. of mole of } H^+}{\text{Total volume of solution (L)}}$

$= \frac{\Sigma M \times V \text{ (in L) } \times \text{ Basicity}}{\Sigma V}$

$= \frac{V \times 0.1 \times 1 + V \times 0.2 \times 2}{V + V} = \frac{0.5}{2} = 0.25\mathrm{M}$

65. (b) Elevation in boiling point of aqueous solution $\propto$ number of solute particles

$\Rightarrow KNO_3$ (Strong electrolyte);

$KNO_{3(aq)} \longrightarrow K^+_{(aq)} + NO^-_{3(aq)}$

$\alpha = 1, i = 2, c = 1, \mathrm{N} = 1\mathrm{M}$

$\Rightarrow$ NaCl(Strong electrolyte);

$NaCl(aq) \longrightarrow Na^+(aq) + Cl^-(aq)$

$\alpha = 1, i = 2, c = 1, \mathrm{N} = 1\mathrm{M}$

$\Rightarrow CH_3COOH$ (Weak electrolyte);

$CH_3COOH(aq) \rightleftharpoons CH_3COO^-(aq) + H^+(aq)$

$\alpha < 1, i < 2\mathrm{c} = 1, \mathrm{N} = 1\mathrm{M}$

$\Rightarrow$ Sucrose (Non electrolyte).

$i = 1, c = 1, \mathrm{N} = 1\mathrm{M}$

66. (b) AgF is soluble in water. The other are sparingly soluble and solubility decreases from chloride to iodide.

Hence, the correct solubility order in water is,

$AgI < AgBr < AgCl < AgF$

67. (a) The mentioned reactions are as follows

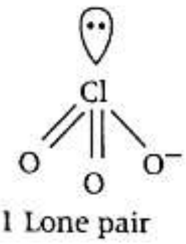
(i) $Cu^{+2}(aq) + 4NH_4^+(aq) \longrightarrow [Cu(NH_3)_4]^{+2} + 4H^+(aq)$

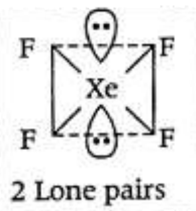
Formation of stable $[Cu(NH_3)_4]^{+2}$ complex increases H^+ion concentration, thus acidity increases.

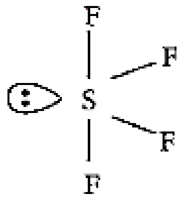
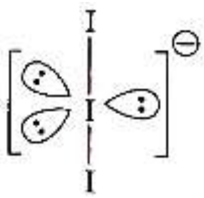
(ii) $HF \rightleftharpoons H^+ + F^-$

SbF_5 removes F^-as $SbF_6^-$and shifts equilibrium forward to increase H^+, thus acidity increases.

68. (d)

(a) 

(b) 

(c)  (d) 

1 Lone pair 3 Lone pairs

I_3^- has highest, i.e., 3 lone pairs on central atom.

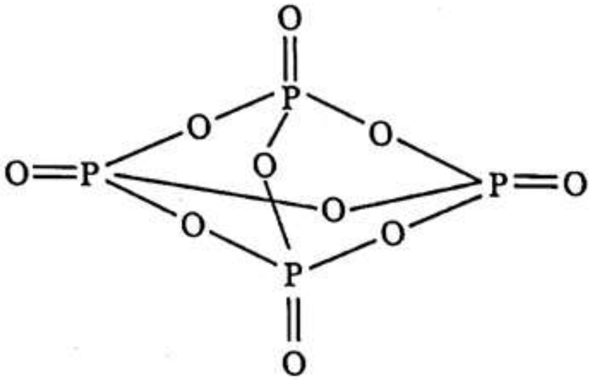
69. (c) Mohr's salt $= FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$ Dissociation of Mohr's salt in water is given as,

$FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O \rightleftharpoons Fe^{+2} + 2NH_4^+ + 2SO_4^{2-}$

5 ions are produced on dissociation.

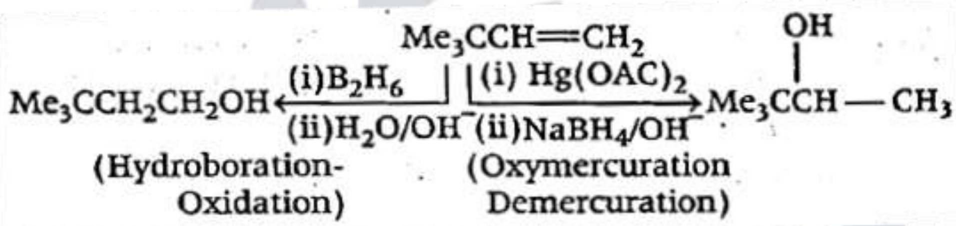
70. (a) MnO_4^{2-} ion exhibits colour due to charge transfer from O^{2-} ion to Mn^{6+} (LMCT) and also exhibits paramagnetism as it has $3d^1$ configuration (one unpaired electron).

71. (a) The structure of P_4O_{10} is,



Hence, total $6P-O-P$ linkage are present in P_4O_{10}.

72. (d) The complete reaction is as follows



73. (d) For 10^{-8}(M)HCl solution, the solution being very dilute, H^+ ion concentration is not significant as compared to H^+ ion furnished from H_2O.

Thus,

$$H^+_{(total)} = H^+_{HCl} + H^+_{H_2O} = 10^{-8} + 10^{-7}$$

$$= 1.1 \times 10^{-7} \text{ (approx)}$$

$$pH = -\log(H^+_{total}) = -\log(1.1 \times 10^{-7})$$

$$= 7 - \log(1.1) = 6.96 \text{ approx}$$

74. (a) First we need to calculate λ_m of aq. CH_3COOH

$$\lambda_m = \frac{\kappa \times 1000}{C(M)} = \frac{1.65 \times 10^{-5} \times 1000}{0.02}$$

$$\lambda_m = 8.25 Scm^2/mol$$

λ_m(CH_3COOH at infinite dilution)

$= \lambda_m^\infty(H^+) + \lambda_m^\infty(CH_3COO^-)$

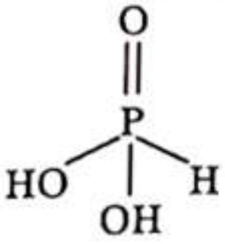
Now, degree of dissociation

$$\alpha = \frac{\lambda_m[CH_3COOH \text{ at } 0.02(M)]}{\lambda_m[CH_3COOH \text{ at infinite dilution }]}$$

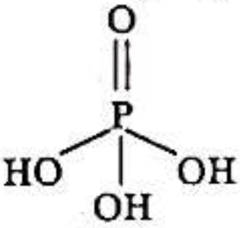
$$\alpha = \frac{8.25}{349.1 + 40.9} \Rightarrow \alpha = 0.021$$

75. (c)

H_3PO_3



H_3PO_4



Phosphorus Acid **Orthophosphoric Acid**

2OH group **3OH group**

76. (a, b, c) Statement given in (a), (b) and (c) are correct, while statement given in (d) is incorrect.

⇒ S_N2 reaction rate depends upon nucleophilicity of the attacking nucleophile. Here, the 2 in the notation stand for biomolecularity and 2nd order kinetics as well.

⇒ tert-butyl substrate fails in S_N2 mechanism due to steric hinderence.

⇒ Optical rotation does not change necessarily from (+) to (-) or vice-versa as product is not necessarily an isomer of reactant.

77. (**b, c**) The configuration of stereocentre in Y is ' R ' while that of option (a), (b) and (c) are R, S and S respectively.

In option (d) the configuration around the double bond is cis, so it cannot be an enantiomer of Y. Hence, only (b) and (c) are the the enantiomers of y.

78. (a, b) Statement given in (a) and (b) are correct, while statements given in (c) and (d) are incorrect.

(a)  Oxidation number (Cr) $= +6$

(b) $N_2O_4(g) \longrightarrow 2NO_2(g)$

$$\therefore \quad \Delta n_g = 2 - 1 = 1$$
$$\Delta H = \Delta U + \Delta n_g RT$$

Hence, $\Delta H > \Delta U$

(c) For 0.1 NHCl, pH is 1

$N_{H_2SO_4} = 0.1$

$\therefore \quad (H^+) = 0.1\ N \Rightarrow N_{HCl} = 0.1$

$\therefore \quad (H^+) = 0.1\ N$

Since, both contain same concentration of H^+ ion, hence, have same pH .

(d) $\frac{2.303RT}{F} = 0.0591$ at $25^\circ C$

79. (b, d) Co^{3+} is a d^6 ion. With strong field ligand like NH_3, $[Co(NH_3)_6]^{3+}$ has no unpaired electron. For Co^{3+}, $F^-$is a weak field ligand. Fe^{2+} is a d^6 ion. With strong field ligand CN^-, $[Fe(CN)_6]^{4-}$ has no unpaired electron. For Fe^{2+}, H_2O is a weak field ligand.

80. (b, c) Statements given in (b) and (c) are correct, while statements given in (a) and (d) are incorrect. I_2 dissolves in aqueous KI to form $KI_3 \cdot I_2$ also dissolves in CCl_4 as both are non-polar compounds.

Mathematics

81. (c) Given that

$$\frac{1}{\sqrt{a}}\int_1^a\left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx < 4$$

Let $I = \int_1^a\left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx = I$

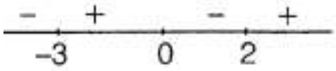
$\Rightarrow I = [x\sqrt{x}+x-2\sqrt{x}]_1^a$

$\quad = (a\sqrt{a}+a-2\sqrt{a}) - 0$

Then, $\frac{1}{\sqrt{a}}\left((\sqrt{a})^3+(\sqrt{a})^2-2(\sqrt{a})\right) < 4$

$\Rightarrow (\sqrt{a})^3+(\sqrt{a})^2-6\sqrt{a} < 0$

Let $\sqrt{a}=t \Rightarrow t(t^2+t-6)<0$
$\Rightarrow\ t(t^2+3t-2t-6)<0$
$\Rightarrow\ t[t(t+3)-2(t+3)]<0$
$t(t+3)(t-2)<0$



Therefore,
$\sqrt{a}\in(-\infty,-3)$ or $\sqrt{a}\in(0,2)$

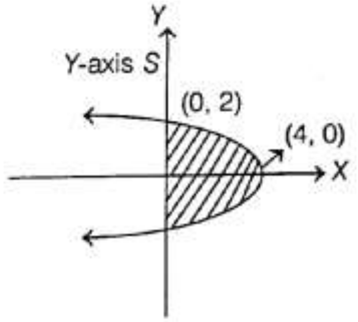
$\Rightarrow a\in(0,4)$

82. (c) $I=\int_0^{\pi} e^{\cos^2 x}\cos^3(2n+1)x dx$..(i)
Using integral property, $x\to\pi-x$
$\Rightarrow I=\int_0^{\pi} e^{\cos^2(\pi-x)}[\cos(2n+1)(\pi-x)]^3 dx$
$\Rightarrow I=\int_0^{\pi} e^{\cos^2(\pi-x)}[\cos\{(2n+1)\pi-(2n+1)x\}]^3 dx$
$\Rightarrow I=\int_0^{\pi} e^{\cos^2(\pi-x)}(-\cos(2n+1)x)^3 dx$
$\Rightarrow I=-\int_0^{\pi} e^{\cos^2(\pi-x)}\cos^3(2n+1)x dx$…(ii)
By adding Eqs. (i) and (ii), we get
$I+I=0\Rightarrow I=0$

83. (b) $\because \frac{dy}{dx}+yf'(x)=f(x)f'(x)$
$\text{IF}=e^{\int f'(x)dx}=e^{f(x)}$
So, $y\cdot e^{f(x)}=\int e^{f(x)}\cdot f(x)f'(x)dx$
Let $f(x)=t\Rightarrow f'(x)dx=dt$
$\Rightarrow\ y\cdot e^t=\int e^t\cdot t\cdot dt\Rightarrow ye^t=e^t(t-1)+C$
$\Rightarrow ye^{f(x)}=e^{f(x)}[f(x)-1]+C$
$\because\ \lim_{x\to\infty} f(x)=0, \lim_{x\to\infty} y(x)=0$
$\Rightarrow\ 0\cdot e^0=e^0(0-1)+C\Rightarrow C=1$
Therefore, $y+1=f(x)+e^{-f(x)}$

84. (d) $xy'+y-e^x=0\Rightarrow \frac{dy}{dx}+\frac{y}{x}=\frac{e^x}{x}$
$\therefore\ \text{IF}=e^{\int\frac{1}{x}dx}=e^{\ln(x)}=x$
Therefore, $yx=\int x\cdot\frac{e^x}{x}\cdot dx\Rightarrow xy=e^x+C$
$\because\ y(a)=b$
$\Rightarrow\ ab=e^a+C\Rightarrow C=ab-e^a$
Therefore,
$xy=e^x+ab-e^a$
Now, $\lim\limits_{x\to 1} y(x)=\lim\limits_{x\to 1}\left[\frac{e^x}{x}+\frac{(ab-e^a)}{x}\right]=e+ab-e^a$

85. (b) $x=4-y^2\Rightarrow y=\sqrt{4-x}$



$\therefore$ Shaded area $(A)=2\int_0^4\sqrt{4-x}\cdot dx$
$=\left|\left[-2(4-x)^{3/2}\times\frac{2}{3}\right]_0^4\right|=\frac{32}{3}$ sq. units

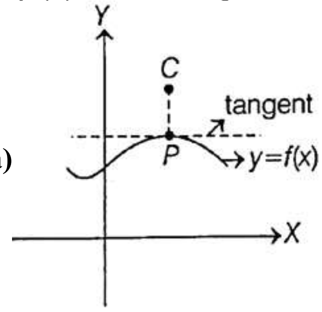
86. (c) $f(x) = \cos x - 1 + \frac{x^2}{2!}$

$f'(x) = -\sin x - 0 + \frac{2x}{2} = x - \sin x$

$\because x - \sin x > 0, \forall x > 0$ and $x - \sin x < 0, \forall x < 0$

Hence, $f(x)$ is increasing in $x \in (0, \infty)$ and $f(x)$ is decreasing in $x \in (-\infty, 0)$.

87. (a)



When the length of PC will be minimum then, P and C will overlap.

Therefore, $PC = 0$

and the tangent will be perpendicular.

Hence, PC is perpendicular to the tangent at P.

88. (c) $x = a\sin(\sqrt{\lambda}t + b)$

When the particle will come to rest.

Then, $\frac{dx}{dt} = 0$

$\Rightarrow \frac{d}{dt}(a\sin\sqrt{\lambda}t + b) = a\sqrt{\lambda}\cos(\sqrt{\lambda}t + b)$

Put, $\frac{dx}{dt} = 0 \Rightarrow \cos(\sqrt{\lambda}t + b) = 0$

$\Rightarrow \sqrt{\lambda}t + b = -\frac{\pi}{2}$ or $\sqrt{\lambda}t + b = \frac{\pi}{2}$

$\Rightarrow t = \frac{-\pi-2b}{2\sqrt{\lambda}}$ or $t = \frac{\pi-2b}{2\sqrt{\lambda}}$

So, $x_1 = a\sin\left(\frac{-\pi-2b}{2\sqrt{\lambda}} \times \sqrt{\lambda} + b\right) = -a$

and $x_2 = a\sin\left(\frac{\pi-2b}{2\sqrt{\lambda}} \times \sqrt{\lambda} + b\right) = a$

Hence, the required distance $= x_2 - x_1 = 2a$

89. (a) $\frac{13}{14}\hat{\imath} + \frac{1}{14}\hat{\jmath}$

90. (c) $f(x) = |x^2 - 1|$

$\Rightarrow f(x) = -(x^2 - 1)$, when $x \in (-1,1)$

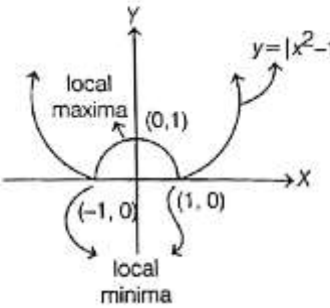
and $f(x) = x^2 - 1$ otherwise

So, $f(x) = \begin{cases} (1 - x^2), & -1 < x < 1 \\ (x^2 - 1), & \text{otherwise} \end{cases}$

At $x = 1, -1 \Rightarrow f(x) = 0$

at $x = 0 \Rightarrow f(x) = 1$

Therefore, graph of $f(x)$ will be



Hence, f has local minima at $x = \pm 1$ and a local maxima at $x = 0$

91. (b) $a_1 = b_1$ and $a_2 = b_2$ for $x > 2$

Let 3rd term of AP$(a_3) = a_1 + 2d = a_1 + 2(a_2 - a_1)$

$\Rightarrow\ a_3 = 2a_2 - a_1 \ldots\ldots$(i)
and 3rd term of GP $(b_3) = b_1 r^2$
$= b_1\left(\frac{b_2}{b_1}\right)^2 = \frac{b_2^2}{b_1} = \frac{a_2^2}{a_1}$
Let GP > AP
$\Rightarrow \frac{a_2^2}{a_1} > 2a_2 - a_1 \Rightarrow a_2^2 > 2a_1a_2 - a_1^2$
$\Rightarrow a_1^2 - 2a_1a_2 + a_2^2 > 0$
$\Rightarrow (a_1 - a_2)^2 > 0$ which is true
Hence, $b_n > a_n, \forall n > 2$.

92. (c) According to the question,
$a_1, a_2, a_3, \ldots$ etc. $a_r - a_{r+1}$ bears constant ratio with $a_r \cdot a_{r+1}$.
i.e. $\frac{a_2-a_1}{a_1 \cdot a_2} = \frac{a_3-a_2}{a_2 \cdot a_3}$
$\Rightarrow \frac{a_2}{a_1 \cdot a_2} - \frac{a_1}{a_1 \cdot a_2} = \frac{a_3}{a_2 \cdot a_3} - \frac{a_2}{a_2 \cdot a_3}$
$\Rightarrow \frac{1}{a_1} - \frac{1}{a_2} = \frac{1}{a_2} - \frac{1}{a_3} \Rightarrow \frac{1}{a_1} + \frac{1}{a_3} = \frac{2}{a_2}$.
Hence, a_1, a_2, a_3 are in HP.

93. (b) z_1 and z_2 are roots of the equation
$z^2 + az + b = 0$.
So, $z_1 + z_2 = -a$ and $z_1 \cdot z_2 = b$
and z_1, z_2 form an equilateral triangle.
So, $z_2 = z_1 e^{i\pi/3}$
$= z_1\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = z_1\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$
$\Rightarrow\ 2z_2 - z_1 = \sqrt{3}iz_1$
$\Rightarrow\ (2z_2 - z_1)^2 = -3z_1^2$
$\Rightarrow\ 4z_2^2 + z_1^2 - 4z_1z_2 = -3z_1^2$
$\Rightarrow\ 4(z_1^2 + z_2^2) = 4z_1z_2$
$\Rightarrow\ (z_1 + z_2)^2 - 2z_1z_2 = z_1z_2$
$\Rightarrow\ (-a)^2 - 3(b)$
$\Rightarrow\ a^2 = 3b$

94. (c) $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$
$\Rightarrow a_0 + a_1 \cdot x^{-1} + \cdots + a_{n-1}x^{-n+1} + a_nx^{-n} = 0$
$\because \cos\theta + i\sin\theta$ is the root.
So, $a_0 + a_1(\cos\theta - i\sin\theta) + a_2(\cos 2\theta - i\sin 2\theta) + \cdots$
By comparing imagina $+a_n(\cos n\theta - i\sin n\theta) = 0$
$-(a_1\sin\theta + a_2\sin 2\theta + \cdots + a_n\sin n\theta) = 0$
$a_1\sin\theta + a_2\sin 2\theta + \cdots + a_n\sin n\theta = 0$

95. (a) $(x^2\log_x 27) \cdot \log_9 x = x + 4$
$\Rightarrow\ x^2 \cdot 3\log_x 3 \cdot \frac{1}{2}\log_3 x = x + 4$
$\Rightarrow\ \frac{3x^2}{2}\log_x 3 \cdot \log_3 x = x + 4$
$\Rightarrow\ \frac{3x^2}{2} = x + 4$
$\Rightarrow\ 3x^2 - 2x - 8 = 0$
$\Rightarrow\ 3x^2 - 6x + 4x - 8 = 0$
$\Rightarrow\ 3x(x - 2) + 4(x - 2) = 0$
$\Rightarrow\ (x - 2)(3x + 4) = 0$
Hence, $x = 2$
$\left[\because x \neq -\frac{4}{3}\right]$

96. (b) $P(x)=ax^2+bx+c$
$Q(x)=-ax^2+dx+c$ and $ac \neq 0$
For real roots,
$b^2-4ac \geq 0$ and $d^2+4ac \geq 0$
Now, if $ac<0$
Then, $b^2-4ac>0$
Then, $P(x) \cdot Q(x)$ has atleast two real roots.
If $ac>0$, then $d^2+4ac>0$
Therefore, $P(x) \cdot Q(x)$ has atleast two real roots.
97. (c) Since, 0 is a solution, then equation must be of the form $ax^2+bx=0$
Where, $a \in \{1,2,\ldots,9\}$ and $b \in \{0,1,2,\ldots,9\}$
$\therefore$ a has 9 choices and b has 10 choices.
Hence, $N=9 \times 10=90$
98. (a)
99. (d) $1,2,3,\ldots,m$ are arranged in random order.
Total number of ways $=m!$
We know that arranging the digits $1,2,3,\ldots,r$ as group and the $(m-r)$ will be remaining digits.
This can be done in $(m-r+1)!$ ways.
Therefore, total number of ways this can be done $=(m-r+1)!\,r!$
100. (a) $A=\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$
$A^2=A \times A$
$$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & -\sin^2\theta+\cos^2\theta\end{bmatrix}$$
$$=\begin{bmatrix}\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$
Now,
$A^{100}=A^{98} \times A^2$
$$=\begin{bmatrix}\cos 98\theta & -\sin 98\theta \\ \sin 98\theta & \cos 98\theta\end{bmatrix}\begin{bmatrix}\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$
Put $\theta=\frac{2\pi}{7}$ in A^{98}
$$=\begin{bmatrix}\cos\left(98 \times \frac{2\pi}{7}\right) & -\sin\left(98 \times \frac{2\pi}{7}\right) \\ \sin\left(98 \times \frac{2\pi}{7}\right) & \cos\left(98 \times \frac{2\pi}{7}\right)\end{bmatrix}$$
$$\begin{bmatrix}\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$
$$=\left[\cos^2(28\pi)+\sin^2(28\pi)\right)\begin{bmatrix}\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$
Hence, $A^{100}=\begin{bmatrix}\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{bmatrix}$
101. (c) $\sum_{k=0}^{7}(-1)^k a_{2k}=a_0-a_2+a_4-a_6+a_8 \ldots \ldots a_{14}$
and $(1+x+x^2+x^3)^5=a_0+a_1k+\cdots a_{15}x^{15}$
Now, put $x=i$
$(1+i+i^2+i^3)^5=a_0+a_1i+a_2i^2+\cdots a_{15}i^{15}$
Now, on equating real part only
$a_0-a_2+a_4-a_6+a_8 \ldots+a_{14}=0$
Hence, $\sum_{k=0}^{7}(-1)^k a_{2k}=0$
102. (c) The general term in the expansion of
$(bc+ca+ab)^{10}$
$$=\frac{10!}{r!\,s!\,t!}(bc)^r(ca)^s(ab)^t$$
$$=\frac{10!}{r!\,s!\,t!}a^{s+t}b^{r+t}c^{r+s}$$

Now, $r+s+t=10$
For the coefficient of $a^{10}b^7c^3$
$s+t=10, r+t=7$ and $r+s=3$, we get
$r=0, s=3$
And $t=7$
Hence, the coefficient of $a^{10}b^7c^3$
$=\frac{10!}{0!\,3!\,7!}=\frac{10\times9\times8}{3\times2}=120$

103. **(d)** $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix}$
$=(x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$
$\because \begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = x^k y^k z^k \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$
Applying, $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$
$= x^k y^k z^k \begin{vmatrix} 0 & (x-y)(x+y) & (x-y)(x^2+xy+y^2) \\ 0 & (y-z)(y+z) & (y-z)(y^2+yz+z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$
$=(xyz)^k(x-y)(y-z)\begin{vmatrix} 0 & x+y & x^2+xy+y^2 \\ 0 & y+z & y^2+yz+z^2 \\ 1 & z^2 & z^3 \end{vmatrix}$
Applying $R_1 \to R_1 - R_2$
$=(xyz)^k(x-y)(y-z)\begin{vmatrix} 0 & x-z & (x-z)(x+y+z) \\ 0 & y+z & y^2+yz+z^2 \\ 1 & z^2 & z^3 \end{vmatrix}$
$=(xyz)^k(x-y)(y-z)(z-x)\begin{vmatrix} 0 & -1 & -(x+y+z) \\ 0 & y+z & y^2+yz+z^2 \\ 1 & z^2 & z^3 \end{vmatrix}$
$=(xyz)^k(x-y)(y-z)(z-x)(xy+yz+xz)$
$=(xyz)^{k+1}(x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$
On comparing with Eq. (i), we get
$k+1=0 \Rightarrow\ k=-1$

104. **(a)** $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\cdot A\cdot\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Let, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}=B$ and $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}=C$
Then, $B\cdot A\cdot C=I$
Multiply Eq. (i) by B^{-1}
$B^{-1}\cdot B\cdot A\cdot C=I\cdot B^{-1}$.....(i)
Multiply Eq. (ii) by C^{-1}
$\Rightarrow\ B^{-1}B\cdot A\cdot C\cdot C^{-1}=I\cdot B^{-1}C^{-1}$....(ii)
$\Rightarrow\ I\cdot A\cdot I=B^{-1}\cdot I\cdot C^{-1}\Rightarrow A=B^{-1}\cdot C^{-1}$
Now, $B^{-1}=\frac{1}{(4-3)}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}=\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$
and $C^{-1}=\frac{1}{(9-10)}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}=\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$
Therefore, $A=\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$
$=\begin{bmatrix} 6-5 & 4-3 \\ -9+10 & -6+6 \end{bmatrix}=\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

105. **(b)** $f(x)=\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^3 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$=\cos x(x^3-2x^2)-x(2\sin x-2x\tan x)$
$=+1(2x\sin x-x^3\tan x)$
$=x^3\cos x-2x^2\cos x-2x\sin x+2x^2\tan x$
$+2x\sin x-x^3\tan x$
$=(x^3-2x^2)\cos x+(2x^2-x^3)\tan x$
$\therefore \dfrac{f(x)}{x^2}=(x-2)\cos x-(x-2)\tan x$
Hence, $\lim_{x\to 0}\frac{f(x)}{x^2}=-2-0=-2$

106. **(c)** Reflexivity Let a be an arbitrary element of p.
Then, $a\in p$
$\Rightarrow\ a^2-4a\times a+3a^2=0$
So, p is reflexive.
Symmetry Let $(a,b)\in p$
$\Rightarrow\ a^2-4ab+3b^2=0$
But $b^2-4ab+3a^2\neq 0,\forall a,b\in R$
So, p is not symmetric.
Transitive Let $(1,2)\in p$ and $(2,3)\in p$
$\Rightarrow\ \ 1-8+6=0$ and $4-24+27=0$
$\Rightarrow\ \ -1\neq 0$ and $7\neq 0$
So, p is not transitive.

107. **(d)** $f(x)=\frac{e^{|x|}-e^{-x}}{e^x+e^{-x}}$
If $x\leq 0, f(x)=\frac{e^{-x}-e^{-x}}{e^x+e^{-x}}=0$
$\Rightarrow\ f(-2)=f(-4)=0$
$\therefore f$ is not a one-one function.
If $x>0, f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{e^{2x}-1}{e^{2x}+1}$
$\because e^{2x}+1>e^{2x}-1,\forall x\in R$
So, $f(x)<1,\forall x\in R$
Then, pre-image of 1 does not exist.
$\therefore f$ is also not an onto function.

108. **(d)** $\because A=\{2,4,6\}$ and $B=\{2,3,5\}$
$n(A\times B)=3\times 3=9$
Hence, the number of relations from A to $B=2^9$

109. **(c)** Total square $=64$
Out of 64 squares, two squares are selected.
Total ways $=\ {}^{64}C_2$ ways $=32\times 63$ ways
Number of ways of selecting the pairs which have the common side $=1/2$
$\therefore$ Favourable cases
$$=\frac{1}{2}\times(4\times 2+24\times 3+36\times 4)$$
$$=\frac{1}{2}\times 224=112$$
[$\because$ each corner has two neighbours, every 24 squares on the border have 3 neighbours and remaining 36 squares have four neighbours]
Hence, required probability
$$=\frac{112}{32\times 63}=\frac{1}{18}$$

110. **(d)** We have, $P(r\leq k$ and $s\leq k)$
$$=\frac{P[(r\leq k)\text{ and }(s\leq k)]}{P(s\leq k)}$$

and $P(s \leq k)=\frac{k}{n}$

and $P[(r \leq k)$ and $(s \leq k)]=\frac{{}^{k}C_{2}}{{}^{n}C_{2}}=\frac{k(k-1)}{n(n-1)}$

Therefore, $P_{k}=\frac{\left(\frac{k(k-1)}{n(n-1)}\right)}{\left(\frac{k}{n}\right)}=\left(\frac{k-1}{n-1}\right)$

111. **(b)** Let, p be the probability of getting a head and q be the probability of not getting head.

The head appears the first time on the even number of throws are 2 or 4 or 6 .

Thus, $\frac{2}{5}=q p+q^{3} p+q^{5} p+\cdots$

$\Rightarrow \quad \frac{2}{5}=\frac{q p}{1-q^{2}}$

$\because \quad q=1-p$

$\Rightarrow \quad \frac{2}{5}=\frac{p(1-p)}{1-(1-p)^{2}} \Rightarrow$

$\Rightarrow \quad 4-2 p=5-5 p \Rightarrow 3 p=1$

$\therefore \quad p=1/3$

112. **(b)** The given expression is

$\cos^{2} \phi+\cos^{2}(\theta+\phi)$

$-[\cos(\theta+\phi)+\cos(\theta-\phi)] \cos(\theta+\phi)$

$=\cos^{2} \phi-\cos(\theta-\phi) \cos(\theta+\phi)$

$=\cos^{2} \phi-\left(\cos^{2} \phi-\sin^{2} \theta\right)=\sin^{2} \theta$

which is independent of ϕ.

113. **(b)** $\because \tan(3 \theta)=\tan(2 \theta+\theta)$

$\Rightarrow \tan(3 \theta)=\frac{\tan 2 \theta+\tan \theta}{1-\tan \theta \cdot \tan 2 \theta}$

$\Rightarrow \tan 3 \theta(1-\tan \theta \cdot \tan 2 \theta)=\tan \theta+\tan 2 \theta \tan \theta+\tan 2 \theta+\tan 3 \theta=0$

$\Rightarrow \tan 3 \theta(1-\tan \theta \tan 2 \theta)+\tan 3 \theta=0$

$\Rightarrow \tan 3 \theta(2-\tan \theta \tan 2 \theta)=0$

$\because \tan 3 \theta \neq 0$

$\therefore$ $\boldsymbol{\tan\theta\tan 2\theta}=2$

Hence, $k=2$

114. **(c)** $r \cos \theta=2 a \sin^{2} \theta$

$\because r \cos \theta=x$ and $r \sin \theta=y \Rightarrow \sin \theta=\frac{y}{r}$

$\Rightarrow \quad x=2 a\left(\frac{y}{r}\right)^{2} \Rightarrow x \cdot r^{2}=2 a y^{2}$

$\because \quad r^{2}=x^{2}+y^{2}$

$\Rightarrow \quad x^{3}+x y^{2}=2 a y^{2} \Rightarrow x^{3}=y^{2}(2 a-x)$

115. **(b)** We have,

$5 x-y-4=0 \ldots .(\mathrm{i})$

And $3 x+4 y-4=0 \ldots$ (ii)

Let the required line intersects the given lines in Eq. (i) and in Eq. (ii) at points (α_{1}, β_{1}) and (α_{2}, β_{2}) respectively. Then, we get

$5 \alpha_{1}-\beta_{1}=4 \Rightarrow \beta_{1}=5 \alpha_{1}-4 \ldots .(\mathrm{iii})$

and $3 \alpha_{2}+4 \beta_{2}-4=0$

$\Rightarrow \beta_{2}=\frac{4-3 \alpha_{2}}{4} \ldots \ldots(\mathrm{iv})$

It is given that mid-point of the segment of the required line between (α_{1}, β_{1}) and (α_{2}, β_{2}) is (1,5)

$\frac{\alpha_{1}+\alpha_{2}}{2}=1 \Rightarrow \alpha_{1}+\alpha_{2}=2 \ldots(\mathrm{v})$

and

$\frac{\beta_{1}+\beta_{2}}{2}=5 \Rightarrow \beta_{1}+\beta_{2}=10$

By Eqs. (iii) and (iv)

$$5\alpha_1 - 4 + 1 - \frac{3}{4}\alpha_2 = 10$$
$$\Rightarrow \quad 20\alpha_1 - 12 - 3\alpha_2 = 40$$
$$20\alpha_1 - 3\alpha_2 \quad = 52 \text{(vi)}$$

By using Eq. (v)

$20\alpha_1 - 3(2 - \alpha_1) = 52$

$\Rightarrow\ 23\alpha_1 = 58 \Rightarrow \alpha_1 = \dfrac{58}{23}$

and $\alpha_2 = 2 - \frac{58}{23} = \frac{-12}{23}$

Similarly, $\beta_1 = \frac{198}{23}$ and $\beta_2 = \frac{32}{23}$

$$y - \frac{198}{23} = \frac{\left(\frac{32}{23} - \frac{198}{23}\right)}{\left(\frac{-12}{23} - \frac{58}{23}\right)}\left(k - \frac{58}{23}\right)$$

$$\Rightarrow \quad y - \frac{198}{23} = \frac{83}{35}\left(x - \frac{58}{23}\right)$$

$$\Rightarrow \quad 35y = 83x + 92 \Rightarrow 83x - 35y + 92 = 0$$

116. **(d)** $\triangle ABC$ has vertex $A(1,2)$

Let the mid-point of AC is D and mid-point of AB is E.

Now, equation of BD is $\Rightarrow x + y = 5$

Equation of CE is $\Rightarrow x = 4$

Let coordinate of $B = (p, q)$ and $C = (r, s)$

$\because$ E lies on the line segment CE.

$\therefore\ \dfrac{p+1}{2} = 4 \Rightarrow p = 7$

and $B(p, q)$ lies on the line BD.

$\therefore\ p + q = 5 \Rightarrow q = -2$

Then, $B \equiv (7, -2)$

$C(r, s)$ lies on the line $CE[x = 4]$

$\therefore\ r = 4$

Coordinate of $D \equiv \left(\frac{r+1}{2}, \frac{s+2}{2}\right)$

$\therefore \dfrac{r+1}{2} + \dfrac{s+2}{2} \quad = 5 \Rightarrow r + s = 7 \Rightarrow s = 3$

$\therefore \qquad C \ = (4,3)$

Therefore, mid-point of

$BC = \left(\dfrac{7+4}{2}, \dfrac{-2+3}{2}\right) = \left(\dfrac{11}{2}, \dfrac{1}{2}\right)$

117. **(c)** Let perpendicular lines are

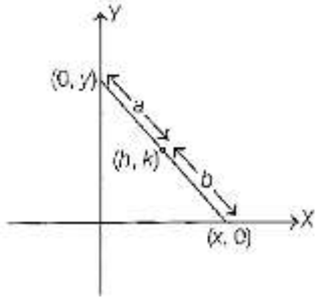
$x = 0$ and $y = 0$

Let, the coordinate of point $= (h, k)$

So, $h = \frac{ax + b(0)}{a+b} = \frac{ax}{a+b}$

and

$k = \frac{a(0) + by}{a+b} = \frac{by}{a+b}$.........(i)



By Pythagoras theorem, $(a+b)^2 = x^2 + y^2$

On substitute the values of x and y from Eqs. (i) and (ii), we get

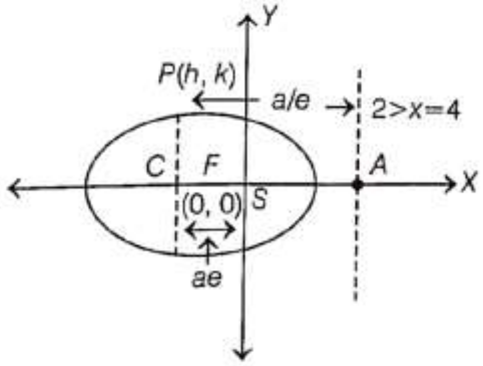
$$\frac{(a+b)^2h^2}{a^2} + \frac{(a+b)^2k^2}{b^2} = (a+b)^2$$

$$\Rightarrow \frac{h^2}{a^2} + \frac{k^2}{b^2} = (a+b)^2$$

which is standard ellipse.

118. **(b)** $x = 4 \Rightarrow$ directrix

Foci = Origin



$$SA = \frac{a}{e} - ae = 4 \Rightarrow a\left(\frac{1-e^2}{e}\right) = 4$$

$$\Rightarrow \frac{a}{e}(1-e^2) = 4\left[\because 1 - e^2 = \frac{b^2}{a^2}\right]$$

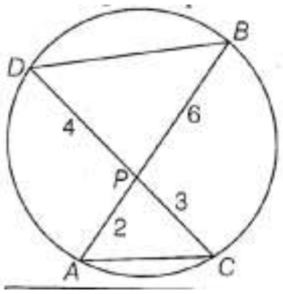
$$\Rightarrow \frac{a}{e} \times \frac{b^2}{a^2} = 4 \Rightarrow \frac{b^2}{ae} = 4$$

$$\Rightarrow p(h,k) = (ae, b)$$

i.e. $ae = h$ and $b = k$

By Eq. (i), $\frac{k^2}{h} = 4 \Rightarrow k^2 = 4h \Rightarrow y^2 = 4x$ which is standard parabola.

119. **(b)** By properties of circle, if two chords AB and CD of circle intersect at $90°$ with each other, then diameter d of circle is given by



$$\Rightarrow \quad d = \sqrt{AP^2 + PB^2 + CP^2 + PD^2}$$

$$\Rightarrow \quad d = \sqrt{2^2 + 6^2 + 3^2 + 4^2} = \sqrt{4 + 36 + 9 + 16}$$

$$d = \sqrt{65}$$

$\therefore$ Radius $= \frac{d}{2} = \frac{\sqrt{65}}{2}$ units

120. **(b)** Let $P_1 \equiv 2x - y + 3z + 5 = 0$

and $P_2 \equiv x + y + z = 1$

So, the equation of plane, say P_3, passing through the line of intersection of both P_1 and P_2 is given by

$$\Rightarrow P_3 \equiv P_1 + \lambda P_2 = 0$$

$$\Rightarrow P_3 \equiv (2x - y + 3z + 5) + \lambda(x + y + z - 1) = 0$$

$$\Rightarrow P_3 \equiv (2 + \lambda)x + (-1 + \lambda)y + (3 + \lambda)z + 5 - \lambda = 0$$

Now, according to question, P_1 is related through $90°$ about line of intersection $\Rightarrow$ Dot product of direction ratios of P_1 and P_3 is equal to 0 .

$\Rightarrow\quad 2(2+\lambda)-1(-1+\lambda)+3(3+\lambda)=0$

$\Rightarrow\quad 4+2\lambda+1-\lambda+9+3\lambda=0$

$$\Rightarrow\quad P_3 \equiv \left(2-\frac{7}{2}\right)x+\left(-1-\frac{7}{2}\right)y+\left(3-\frac{7}{2}\right)z$$
$$+5-\left(\frac{-7}{2}\right)=0$$

$$\Rightarrow\quad P_3 \equiv \frac{-3}{2}x-\frac{9}{2}y-\frac{z}{2}+\frac{17}{2}=0$$

$\therefore$ Required equation of plane is

$3x+9y+z=17$

121. **(b)** We know that, $\cos\theta=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$....(i)

where, $\theta=$ angle between both vectors **a** and **b**.

Now, given $l+m+n=0$

$\Rightarrow\ l=-(m+n)$

and substituting value of l in

$\Rightarrow\quad 2lm+2mn-ln=0$

$\Rightarrow\quad 2(-m-n)m+2mn+(m+n)n=0$

$\Rightarrow\quad -2m^2-2mn+2mn+nm+n^2=0$

$\Rightarrow\quad 2m^2-nm-n^2=0$

$(2m+n)(m-n)=0$

$\Rightarrow$ Either $2m+n=0$ or $m-n=0$

$\Rightarrow\ n=-2m$ or $n=m$

When $n=-2m$	and when $n=m$
$\Rightarrow l=-(m+n)$	$\Rightarrow\ l=-(m+n)$
$\Rightarrow l=-(m-2m)$	$\Rightarrow\ l=-(m+m)$
$\Rightarrow l=m$	$\Rightarrow\ l=-2m$

$\therefore$ Direction ratio of

Line $1\equiv(m,m,-2m)$ and Line $2\equiv(-2m,m,m)$

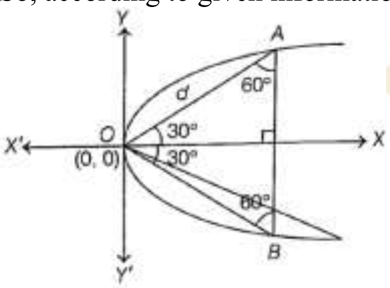
Now by Eq. (i),

$$\cos\theta=\frac{m(-2m)+m(m)+(-2m)(m)}{\sqrt{m^2+m^2+4m^2}\sqrt{4m^2+m^2+m^2}}$$

$$\Rightarrow\ \cos\theta=\frac{-3m^2}{6m^2}=\frac{-1}{2}\Rightarrow\theta=\cos^{-1}\left(\frac{-1}{2}\right)=\frac{2\pi}{3}$$

122. **(a)** Given, equilateral triangle OAB, of side d, is inscribed in a parabola $y^2=4ax$, where $a>0$

So, according to given information



Now, equation of line OA

$\Rightarrow\ y=\frac{1}{\sqrt{3}}x$....(ii)

Now, since OA intersects with parabola, so by solving Eqs. (i) and (ii) both

$$\Rightarrow\quad 4ax=\frac{x^2}{3}$$

$\Rightarrow\quad x^2-12ax=0$

$\Rightarrow\quad x(x-12a)=0$

$\Rightarrow\quad x=0$ or $x=12a$

and

$$y=\frac{1}{\sqrt{3}}(12a)=4\sqrt{3}a$$

$\therefore$ Coordinates of $A=(12a,4\sqrt{3}a)$

So,

$$d=\sqrt{(12a)^2+(4\sqrt{3}a)^2}=\sqrt{144a^2+48a^2}$$
$$=\sqrt{192a^2}=8a\sqrt{3}$$

123. **(a)** Given, $f(x)=\frac{x}{x+1}, f_1(x)=f(x)$

and $f_n(x)=f(f_{n-1}(x))$ for $n\geq 2$

$$\Rightarrow\ f_2(x)=f(f_1(x))=f(f(x))=f\left(\frac{x}{x+1}\right)$$
$$=\frac{\frac{x}{x+1}}{\frac{x}{x+1}+1}=\frac{x}{2x+1}$$
$$\Rightarrow\ f_3(x)=f(f_2(x))=\frac{\frac{x}{2x+1}}{\frac{x}{2x+1}+1}=\frac{x}{3x+1}$$

$\therefore$ By following the patterns,

$$f_n(x)=\frac{x}{nx+1}$$

Now, $f_1(-2)\cdot f_2(-2)\cdot f_3(-2)\ldots f_n(-2)$ will be

$$=\left(\frac{-2}{-2+1}\right)\left(\frac{-2}{-4+1}\right)\left(\frac{-2}{-6+1}\right)\cdots\left(\frac{-2}{-2n+1}\right)$$
$$=\left(\frac{2}{1}\right)\left(\frac{2}{3}\right)\left(\frac{2}{5}\right)\cdots\left(\frac{2}{2n-1}\right)=\frac{2^n}{1\cdot 3\cdot 5\cdots(2n-1)}$$

124. **(b)** Given, $U(x)=\frac{Lx+M}{x^2-2Bx+C}$

$\Rightarrow U(x)\times(x^2-2Bx+C)=Lx+M$

Differentiating w.r.t. x, we get

$U(x)[2x-2B]+U_1(x)[x^2-2Bx+C]=L$

Again, differentiating w.r.t. x, we get

$U_1(x)[2x-2B]+U(x)[2]$

$+U_2(x)[x^2-2Bx+C]+U_1(x)[2x-2B]=0$

On rearranging,

$U_2(x)[x^2-2Bx+C]+U_1(x)[4x-4B]+U(x)\times 2=0$

On comparing with $PU_2+QU_1+RU=0$

$\Rightarrow\ P=x^2-2Bx+C$

$Q=4(x-B)$ and $R=2$

125. **(b)** Given, $2^x+5^x=3^x+4^x$

Here, LHS = RHS for $x=0$ and

At $x=0$

LHS $=2^0+5^0=2$ and RHS $=3^0+4^0=2$

$\therefore$ LHS = RHS and at $x=1$

LHS $=2+5=7$ and RHS $=3+4=7$

$\therefore$ LHS = RHS

$\therefore 2^x+5^x=3^x+4^x$ has only one non-zero real solution.

126. **(a)** Given, $x\log_e x=2-x(x>0)$

$$\Rightarrow\ \log_e x=\frac{2-x}{x}$$

Let $g(x)=\frac{x-2}{x}+\log_e x$

Now, let differentiate $f(x)$ w.r.t. x, we get

$f'(x)=\dfrac{x-2}{x}+\log_e x \Rightarrow f'(x)=g(x)$
$\Rightarrow$ $f(x)$ is primitive of $g(x)$
$\Rightarrow$ Roots of $f(x)=1,2$
So, by Rolle's theorem, if a function has same value at two points, then its derivative must have atleast one root between both the points.
$\therefore$ $g(x)$ has atleast one root in $(1,2)$.

127. **(d)** Given, α and β are roots of the equation $ax^2+bx+c=0$, then it can be written as
$ax^2+bx+c\equiv a(x-\alpha)(x-\beta)\ldots$(i)
Now, we have to find value of

$$=\lim_{x\to\beta}\frac{1-\cos(ax^2+bx+c)}{(x-\beta)^2}$$
$$=\lim_{x\to\beta}\frac{1-\cos(a(x-\alpha)(x-\beta))}{(x-\beta)^2}$$
$$=\lim_{x\to\beta}\frac{2\sin^2(a(x-\alpha)(x-\beta)/2}{a^2(x-\alpha)^2(x-\beta)^2}\times a^2(x-\alpha)^2$$
$$=\frac{1}{2}\lim_{x\to\beta}\frac{\sin^2(a(x-\alpha)(x-\beta)/2}{[a^2(x-\alpha)^2(x-\beta)^2/4]}\times a^2(x-\alpha)^2$$
$$=\frac{1}{2}\times a^2(\beta-\alpha)^2\left[\because\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$
$$=\frac{a^2}{2}(\alpha-\beta)^2$$

128. **(c)** Given, $f(x)=\frac{e^x}{1+e^x}$
Now, $f(a)=\frac{e^a}{1+e^a}$ and $f(-a)=\frac{e^{-a}}{1+e^{-a}}=\frac{1}{e^a+1}$
$$\Rightarrow f(a)+f(-a)=\frac{e^a}{1+e^a}+\frac{1}{1+e^a}=\frac{1+e^a}{1+e^a}=1$$
Now, $I_1=\int_{f(-a)}^{f(a)}xg(x(1-x))dx$
$$\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$$
$$\Rightarrow I_1=\int_{f(-a)}^{f(a)}(1-x)g((1-x)(1-(1-x))dx$$
$$\Rightarrow I_1=\int_{f(-a)}^{f(a)}(1-x)g((1-x)x)dx$$
$$\Rightarrow I_1=\int_{f(-a)}^{f(a)}g(x(1-x))dx-\int_{f(-a)}^{f(a)}xg(x(1-x))dx$$
$$\Rightarrow I_1=I_2-I_1\Rightarrow 2I_1=I_2\Rightarrow\frac{I_2}{I_1}=2$$

129. **(a)** Let $L=\lim_{x\to 1}\int_4^{f(x)}\frac{2t}{x-1}dt=\lim_{x\to 1}\frac{\int_4^{f(x)}2tdt}{x-1}$
By applying L'Hospital rule
$$L=\lim_{x\to 1}\frac{2f(x)\cdot f'(x)-0}{1}$$
$$L=2f(1)\cdot f'(1)=2\times 4\times 2=16$$

130. **(a)** Let $I=\int\frac{\log_e(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx$
Let $\log\left(x+\sqrt{1+x^2}\right)=t$
Differentiating w.r.t. x,
$$\Rightarrow\left(\frac{1}{x+\sqrt{1+x^2}}\right)\times\left(1+\frac{2x}{2\sqrt{1+x^2}}\right)dx=dt$$

$\Rightarrow \left(\frac{1}{x+\sqrt{1+x^2}}\right) \times \left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right) dx = dt$

$\frac{dx}{\sqrt{1+x^2}} = dt$

$I = \int t dt = \frac{t^2}{2} + C = \frac{1}{2}\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2 + C$

Given, $I = f(g(x)) + C \Rightarrow f(x) = \frac{x^2}{2}$

and $g(x) = \log_e\left(x+\sqrt{1+x^2}\right)$

131. **(d)** Let $I(R) = \int_0^R e^{-R\sin x} dx; R > 0$

Now, we know, $x > \sin x$

$\Rightarrow -x < -\sin x \Rightarrow e^{-Rx} < e^{-R\sin x}$

$\Rightarrow \int_0^R e^{-Rx} dx < \int_0^R e^{-R\sin x} dx$

$\Rightarrow \left[\frac{e^{-Rx}}{-R}\right]_0^R < \int_0^R e^{-R\sin x} dx$

$\Rightarrow \left[\frac{e^{-R^2}}{(-R)} - \frac{1}{(-R)}\right] < I(R) \Rightarrow \frac{\left(1-e^{-R^2}\right)}{R} < I(R)$

132. **(b)** Given, $f(x) = x(x-1)(x-2) \ldots (x-100)$

Let $f'(x) = (x-1)(x-2) \ldots (x-100)$

$+x(x-2)(x-3) \ldots (x-100) + \cdots$

$+x(x-1)(x-2) \ldots (x-99)$

$= x(x-1)(x-2 \ldots (x-100)$

$\left[\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-2} + \cdots \frac{1}{x-100}\right]$

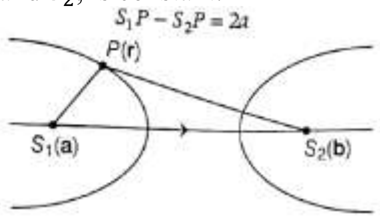
$= \frac{x(x-1)(x-2) \ldots (x-49)(x-50)(x-51) \ldots (x-100)}{101(x-50)}$

$f'(x) = \frac{x(x-1)(x-2) \ldots (x-49)(x-51) \ldots (x-100)}{101}$

$\Rightarrow f'(x)$ have 100 roots

$\Rightarrow$ 50 will be maximas and 50 will be minimas.

133. **(a)** We know that in hyperbola difference of distances of a point, say P, from two fixed points S_1 and S_2, is constant.



Given, $|\mathbf{r} - \mathbf{a}| - |\mathbf{r} - \mathbf{b}| = c$

$\Rightarrow c = 2a$.(i)

We also know, $S_1S_2 = 2ae$ (where, e is eccentricity)

$\Rightarrow S_1S_2 = 2ae = |\mathbf{a} - \mathbf{b}|$

$\Rightarrow e = \frac{|\mathbf{a}-\mathbf{b}|}{2a} \Rightarrow e = \frac{|\mathbf{a}-\mathbf{b}|}{c}$ [from Eq. (i)]

134. **(d)** Let 5 balls be B_1, B_2, B_3, B_4, B_5 and 3 boxes be C_1, C_2, C_3.

Now, we have to place atleast one ball in each box.

So distribution of ball can be done in 2 ways.

(i) 3,1,1 → 3 balls in one box, 1 in second and 1 in last box.

(ii) 2,2,1 → 2 balls in one box, 2 in second and 1 in last box.

Now, first take 3, 1, 1 arrangement.

Total combinations will be

$= 3 \times {}^5C_3 \times {}^2C_1 \times {}^1C_1 = 3 \times 10 \times 2 \times 1 = 60$

and second take 2,2,1 arrangement.

Total combination will be $= 3 \times {}^5C_2 \times {}^3C_2 \times {}^1C_1$
$= 3 \times 10 \times 3 = 90$

$\therefore$ Total ways $= 60 + 90 = 150$

135. **(d)** Here, $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}, B = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}_{3\times1}$

So, to get result $AX = B$ order of X will be 3×1

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\Rightarrow AX = B$

$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}$

By multiplication of matrices

$x - y = 2$...(i)

$\Rightarrow y - z = 1$....(ii)

$\Rightarrow x + y + z = 7$...(iii)

By solving Eqs. (ii) and (iii), we get

$x + y + (y - 1) = 7$

$\Rightarrow x + 2y = 8$(iv)

Soloving Eqs.(i) and (iv)

$x + 2(x - 2) = 8$

$3x = 12$

X=4

and $y = 4 - 2 = 2 \Rightarrow z = 2 - 1 = 1$

$\therefore X = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$

136. **(d)** Given, $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are in AP with common difference θ.

$\alpha_n - \alpha_{n-1} = d = \alpha_2 - \alpha_1 = \alpha_3 - \alpha_2 \ldots$

Now we know,

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\sin(A - B) = \cos A \cos B(\tan A - \tan B)$

$= \sec A \sec B = \dfrac{\tan A - \tan B}{\sin(A - B)}$

$\therefore \sec\alpha_1 \sec\alpha_2 + \sec\alpha_2 \sec\alpha_3 + \cdots + \sec\alpha_{n-1} \sec\alpha_n$

$= \dfrac{(\tan\alpha_2 - \tan\alpha_1)}{\sin(\alpha_2 - \alpha_1)} + \dfrac{(\tan\alpha_3 - \tan\alpha_2)}{\sin(\alpha_3 - \alpha_2)} + \cdots$

$+ \dfrac{(\tan\alpha_n - \tan\alpha_{n-1})}{\sin(\alpha_n - \alpha_{n-1})}$

$\dfrac{\tan\alpha_n - \tan\alpha_1}{\sin\theta} \Rightarrow \operatorname{cosec}\theta(\tan\alpha_n - \tan\alpha_1)$

$= k(\tan\alpha_n - \tan\alpha_1)$

$\therefore k = \operatorname{cosec}\theta$

137. **(a)** Given, x and $y \in R$ and relation on (x, y) is given by $x - y + \sqrt{2}$ is an irrational number

Lets check for

(i) Reflexive

If $(x, x) \in$ Relation

$\Rightarrow x - x + \sqrt{2}$

$\Rightarrow \sqrt{2} \rightarrow$ always an irrational number

$\therefore$ Relation is reflexive.

(ii) Symmetric

Let $(x, y) \in$ Relation, where $x = \sqrt{2}$ and $y = 2$
$\Rightarrow \sqrt{2} - 2 + \sqrt{2} \rightarrow$ an irrational number
But for (y, x)
$\Rightarrow 2 - \sqrt{2} + \sqrt{2} \rightarrow$ rational number
$\therefore (y, x) \notin$ Relation
$\therefore$ Relation is not symmetric.
(iii) Similarly relation is also not transitive.
Hence, relation is not equivalence.

138. **(d)** Given, $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

Clearly, A is not a null matrix and skew-symmetric matrix.
$\therefore$ Option (a) and (b) are incorrect.
Now, $|A| = -1(-1) = 1$
$\Rightarrow |A| \neq 0$, therefore inverse of A exists
$\therefore$ Option (c) is also incorrect.
$\therefore$ Option (d) is correct, $A^2 = I$

139. **(a)** Given, $1000! = 3^n \times m$
Here, we have to find exponent of 3 in 1000 !.

$$\therefore \left[\frac{1000}{3}\right] + \left[\frac{1000}{3^2}\right] + \left[\frac{1000}{3^3}\right] + \left[\frac{1000}{3^4}\right] + \left[\frac{1000}{3^5}\right] + \left[\frac{1000}{3^6}\right] + \left[\frac{1000}{3^7}\right]$$

$= 333 + 111 + 37 + 12 + 4 + 1 = 498$
$\therefore\ 1000! = 3^{498} \cdot m \Rightarrow n = 498$

140. **(a)** Given, A and B are acute angle, also
$\sin A = \sin^2 B$ and $2\cos^2 A = 3\cos^2 B$
$\Rightarrow 2(1 - \sin^2 A) = 3(1 - \sin^2 B) \Rightarrow 2 - 2\sin^2 A = 3 - 3\sin^2 B$
$\Rightarrow 2 - 2\sin^2 A = 3 - 3\sin A \Rightarrow 2\sin^2 A - 3\sin A + 1 = 0$
$\Rightarrow (2\sin A - 1)(\sin A - 1) = 0$
$\Rightarrow \sin A = \frac{1}{2}$ or $1 \Rightarrow A = \frac{\pi}{6}$ or $\frac{\pi}{2}$
But A is acute angle.
$\therefore\ A = \pi/6$
Now, $\sin^2 B = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$
$\sin B = \pm\frac{1}{\sqrt{2}} \Rightarrow B = -\frac{\pi}{4}$ or $\frac{\pi}{4}$
But B is also acute angle.
$\therefore\ B = \pi/4$

141. **(b)** Let the equation of the circle be
$x^2 + y^2 + 2gx + 2fy + c_1 = 0$
Since, it passes through $(0, a)$ and $(0, -a)$.
$\therefore\ 0 + a^2 + 0 + 2fa + c_1 = 0$ ... (i)
$0 + a^2 + 0 - 2fa + c_1 = 0$ …(ii)
Solving Eqs. (i) and (ii),
$4fa = 0 \Rightarrow f = 0$ and $c_1 = -a^2$
Hence, the circle becomes
$x^2 + y^2 + 2gx - a^2 = 0$
Centre $\equiv (-g, 0)$
Radius $\equiv \sqrt{g^2 + a^2}$
Now, line $y = mx + c$ touches the circle, hence perpendicular from centre should be equal to radius.
$\Rightarrow \frac{-mg + c}{\sqrt{m^2 + 1}} = \sqrt{g^2 + a^2}$
$\Rightarrow\ (c - mg)^2 = (m^2 + 1)(g^2 + a^2)$

$\Rightarrow c^2 + m^2g^2 - 2cmg = m^2g^2 + m^2a^2 + g^2 + a^2$
$\Rightarrow g^2 + 2gmc + [a^2(1+m^2) - c^2] = 0$
Roots of above equation give us value of g for both the circles. Let g will give g_1 and g_2, then
$g_1g_2 = a^2(1+m^2) - c^2$
Now, circles will be
and
$x^2 + y^2 + 2g_1x - a^2 = 0$
$x^2 + y^2 + 2g_2x - a^2 = 0$
Now, conditions for two circles to intersect orthogonally is
$2g_1g_2 + 2f_1f_2 = c_1 + c_2$
$\Rightarrow \quad 2(a^2(1+m^2) - c^2) + 0 = -a^2 - a^2 = -2a^2$
$\Rightarrow \quad a^2(1+m^2) - c^2 = -a^2 \Rightarrow a^2(2+m^2) = c^2$

142. **(b)** Given equations of hyperbola is
$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$
Now, let mid-points of chord is (h, k).
Equations of chord when mid-point is known to is us given by $T = S_1$.
$$\Rightarrow \quad \frac{xh}{4} - \frac{yk}{9} - 1 = \frac{h^2}{4} - \frac{k^2}{9} - 1$$
$$\Rightarrow \quad \frac{xh}{4} - \frac{yk}{9} = c \left(\text{where}, c = \frac{h^2}{4} - \frac{k^2}{9}\right)$$
Slope of above equation, $m = 9h/4k$
Now, according to question above chord is parallel to $y = 2x$.
$\therefore \quad m = 2 \Rightarrow 9h/4k = 2$
$\Rightarrow \quad 9h - 8k = 0$
Replacing (h, k) with (x, y)
$\therefore 9x - 8y = 0$

143. **(a)**

144. **(c)** Given, $y = \tan^{-1}\left[\frac{1-2\ln x}{1+2\ln x}\right] + \tan^{-1}\left[\frac{3+2\ln x}{1-3\cdot 2\ln x}\right]$
$y = \tan^{-1}(1) - \tan^{-1}(2\ln x) + \tan^{-1}(3) + \tan^{-1}(2\ln x)$
$y = \tan^{-1}(1) + \tan^{-1}(3) \Rightarrow y$ is a constant.
$\therefore \frac{dy}{dx}$ and $\frac{d^2y}{dx^2} = 0$

145. **(c)** Let $L = \lim_{n\to\infty} \frac{1}{n^{k+1}}[2^k + 4^k + 6^k + \cdots + (2n)^k]$
$$L = \lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{2}{n}\right)^k + \left(\frac{4}{n}\right)^k + \left(\frac{6}{n}\right)^k + \cdots \left(\frac{2r}{n}\right)^k + \cdots \left(\frac{2n}{n}\right)^k\right]$$
$$L = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} 2^k \left(\frac{r}{n}\right)^k$$
Replacing $\frac{1}{n} \to dx, \lim_{n\to\infty}\sum_{r=1}^{n} \to \int_0^1$ and $\frac{r}{n} \to x$
$$L = \int_0^1 2^k x^k dx = 2^k \left[\frac{x^{k+1}}{k+1}\right]_0^1 = \frac{2^k}{k+1}$$

146. **(b,c)** Given, $f = 6 - \sqrt{1.2t}$, where $f = dv/dt$
Now, for max velocity, $dv/dt = 0 \Rightarrow f = 0$
$$\Rightarrow 6 - \sqrt{1.2t} = 0 \Rightarrow 6 = \sqrt{1.2t} \Rightarrow 36 = 1.2t \Rightarrow t = \frac{36}{1.2} = 30 \text{ s}$$
Now, $v = \int_0^{30} f dt = \int_0^{30} 6 - \sqrt{1.2t} dt$

$= \left[6t - \sqrt{1.2}t^{3/2} \times \frac{2}{3}\right]_0^{30}$

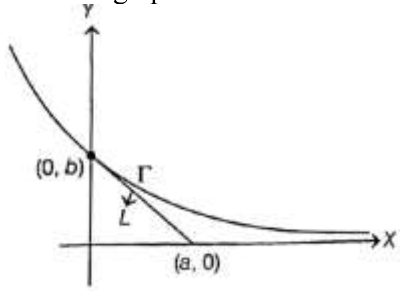
$= 6 \times 30 - \sqrt{1.2} \times (30)^{3/2} \times \frac{2}{3}$

$= 180 - 30\sqrt{36} \times \frac{2}{3} = 180 - 30 \times 6 \times \frac{2}{3}$

$= 180 - 120 = 60\text{ft/s}$

147. **(a,d)**Given, $\Gamma \equiv y = be^{-x/a}$ and $L \equiv \frac{x}{a} + \frac{y}{b} = 1$

Lets draw graph of both



Here line L touches the curve at point on Y-axis and since Γ curve is an exponential functions, graph of its will never touches the X-axis.

148. **(a,c)**Given,

$S = (2n+1)\,{}^nC_0 + (2n-1)\,{}^nC_1 + (2n-3)\,{}^nC_2 \ldots 1 \cdot {}^nC_n$ $S = 1^nC_0 + 3^nC_1 + 5^nC_2 \ldots (2n+1)^nC_n$

Adding both

$2S = (2n+2)^nC_0 + (2n+2)^nC_1$
$+(2n+2)^nC_2 \ldots (2n+2)^nC_n$

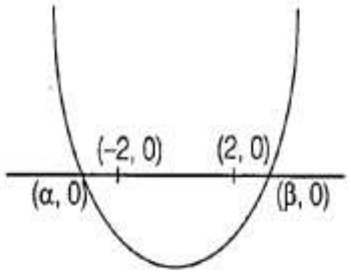
$\Rightarrow 2S = (2n+2)[{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n]$

$\Rightarrow S = (n+1)2^n$

Now, let $f(x) = x^{n+1}$

$\Rightarrow \begin{array}{l} f'(x) = (n+1)x^n \\ f'(2) = (n+1)2^n \end{array}$

149. **(a,c)**According to given information, graph of quadratic is



Given, $a > 0 \Rightarrow$ Graph will open upward

Now $\alpha \rightarrow$ negative and $\beta \rightarrow$ positive

$\therefore\ \alpha\beta =$ negative $= \frac{c}{a} \Rightarrow c < 0$

Also value of quadratic (y) between α and β is negative.

$\Rightarrow f(-1) < 0 \Rightarrow a(-1)^2 + b(-1) + c < 0 \Rightarrow a - b + c < 0$

150. **(a,b,c)**Let

$$\Delta = \begin{vmatrix} a_1 + b_1x & a_1x + b_1 & c_1 \\ a_2 + b_2x & a_2x + b_2 & c_2 \\ a_3 + b_3x & a_3x + b_3 & c_3 \end{vmatrix}$$

Now, at $x = 1$

$$\Delta = \begin{vmatrix} a_1 + b_1 & a_1 + b_1 & c_1 \\ a_2 + b_2 & a_2 + b_2 & c_2 \\ a_3 + b_3 & a_3 + b_3 & c_3 \end{vmatrix} = 0$$

[$\because C_1$ and C_2 are identical]

and at $x = -1$

$$A=\begin{vmatrix}a_1-b_1 & -a_1+b_1 & c_1\\ a_2-b_2 & -a_2+b_2 & c_2\\ a_3-b_3 & -a_3+b_3 & c_3\end{vmatrix}=0$$

[$\because C_1$ and C_2 are identical]

Also, $\Delta=\begin{vmatrix}a_1 & a_1x & c_1\\ a_2 & a_2x & c_2\\ a_3 & a_3x & c_3\end{vmatrix}+\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$

$+\begin{vmatrix}b_1x & a_1x & c_1\\ b_2x & a_2x & c_2\\ b_3x & a_3x & c_3\end{vmatrix}+\begin{vmatrix}b_1x & b_1 & c_1\\ b_2x & b_2 & c_2\\ b_3x & b_3 & c_3\end{vmatrix}=0$

$\Rightarrow\quad 0+(1-x^2)\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}+0=0$

$\Rightarrow\quad \begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$ and $x=\pm1$

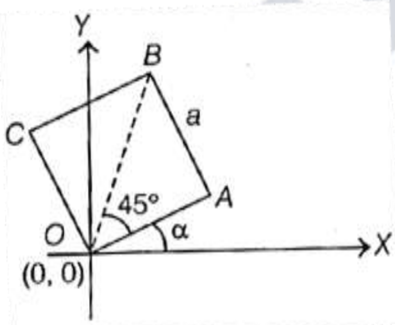
151. **(d)** Given, $f(x)=e^x+e^{-x}$

Let put $x=-x\Rightarrow f(-x)=e^{-x}+e^x$
$\Rightarrow f(x)=f(-x)\Rightarrow f(x)=$ even function
$\therefore\ f(x)$ is a many one function.
Now, e^x and e^{-x} both are always positive.
$\therefore$ Codomain $\neq$ Range $\Rightarrow f(x)$ is not onto.

152. **(a,c)**According to the question, Equation of $OB=y=\tan(45+\alpha)x$

$\Rightarrow y=\left(\dfrac{1+\tan\alpha}{1-\tan\alpha}\right)x$

$\Rightarrow y=\left(\dfrac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}\right)x$



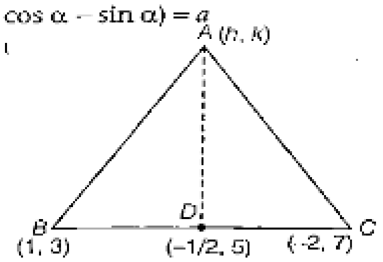
$\Rightarrow\ y(\cos\alpha-\sin\alpha)=(\cos\alpha+\sin\alpha)x$
Now coordinate of $A\equiv(a\cos\alpha, a\sin\alpha)$
Equation of line $AC\equiv(y-a\sin\alpha)$
$=\left(\dfrac{\sin\alpha-\cos\alpha}{\cos\alpha+\sin\alpha}\right)(x-a\cos\alpha)$
$\Rightarrow y(\cos\alpha+\sin\alpha)-a\sin\alpha\, a\cos\alpha - a\sin^2\alpha=x(\sin\alpha-\cos\alpha)$
$-a\cos\alpha\, a\sin\alpha+a\cos^2\alpha$
$\Rightarrow y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$

153. **(c,d)**Given ABC is an isoscceles triangle Now, slope of BC,

$M_{BC}=\dfrac{7-3}{-2-1}=\dfrac{-4}{3}$

Slope of
$AD, M_{AD}=3/4$

$\cos\alpha - \sin\alpha) = a$



Equation of

$AD \equiv (y-5) = \frac{3}{4}\left(x+\frac{1}{2}\right)$

$\equiv 8y - 40 = 6x + 3 \equiv 8y - 6x = 43$

Putting (h, k) in above equation of line.

$\Rightarrow 8k - 6h = 43$ which is required equation.

$\therefore$ Option (c) and (d) will satisfies the above equation.

154. **(a,b)**Let $y = \int_0^{x^2} \frac{t^2-5t+4}{2+e^t} dt$

By applying Leibnitz's rule,

$\frac{dy}{dx} = \left[\frac{x^4-5x^2+4}{2+e^{x^2}}\right] \times 2x$

To find extremum (maximas and minimas), we put $\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{(x^4-5x^2+4)}{2+e^{x^2}} 2x = 0$

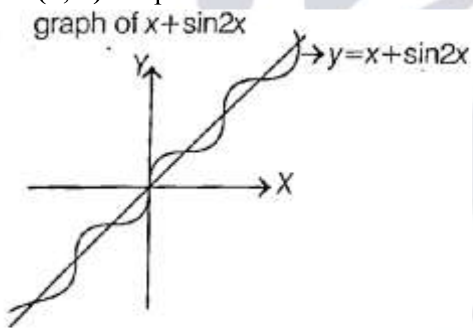
$\Rightarrow \quad x^4 - 5x^2 + 4 = 0$ or $x = 0$

$(\because 2 + e^{x^2}$ is always greater than 0).

$\Rightarrow (x^2-4)(x^2-1) = 0 \Rightarrow x^2 - 4 = 0$ or $x^2 - 1 = 0$

$\Rightarrow \quad x = \pm 2$ or $x = \pm 1$

155. **(b, d)** Graph of $x + \sin 2x$



Clearly it is not periodic.

Now, in $\cos(\sqrt{x}+1), x \geq 0$.

Lets consider $\cos(\sqrt{x}+1)$ be a periodic function, with period T, where $T > 0$

$\therefore f(x+T) = f(x) \Rightarrow \cos\sqrt{x+T} = \cos\sqrt{x}$

In particular, let $x = 0$, then we have

$\cos\sqrt{T} = \cos 0 = 1 \ldots.(i)$

For $x = T$, we have

$\cos\sqrt{2T} = \cos\sqrt{2T} = 1 \ldots.(ii)$

By Eq. (i), $\sqrt{T} = 2n\pi$ and

By Eq. (ii), $\sqrt{2T} = 2m\pi$

or

$\sqrt{2} = \frac{2m\pi}{2n\pi} = \frac{m}{n} =$ Rational number

But $\sqrt{2}$ is an irrational number.

$\therefore$ Due to contradiction $\cos(\sqrt{x}+1)$ is not a periodic function.