1. **(c)** $\frac{v-u}{t} =$
$\frac{(0-50)}{(10-0)} = -5 \text{ m/s}^2$
$u = 50 \text{ m/s}$
$v = u + at = 50 - 5t$
Veocity in first two seconds $t = 2$
$v_{(att=2)} = 40 \text{ m/s}$
From work-energy theorem,
$\Delta K.E. = W = \frac{1}{2}(40^2 - 50^2) \times 10 = -4500 \text{ J}$

2. **(a)** The frequency of the AC supply is 50 Hz . This means the angular frequency, ω, is calculated as:
$\omega = 2\pi f = 2\pi \times 50 = 100\pi$ radians per secor
The given 220 V is the Root Mean Square (RMS) value of the voltage. The peak (maximum) voltage, $V_{\max}$, is related to the RMS value by:
$V_{\max} = 220\sqrt{2}$.
The general expression for an AC voltage is:
$V(t) = V_{\max}\cos(\omega t)$
Substituting the values we calculated:
$V(t) = 220\sqrt{2}\cos(100\pi t)$

3. **(a)** Let's analyze the decay chain step-by-step. We start with a nucleus X having atomic number Z and mass number A. The decay chain is as follows:
$X \xrightarrow{\alpha} X_1$
In alpha decay, an alpha particle (helium nucleus) is emitted, which has a mass number of 4 and an atomic number of 2 .
Therefore:
X_1: atomic number $= Z - 2$, mass number A - 4
$X_1 \xrightarrow{\beta} X_2$
In beta decay, a neutron is converted into a proton, increasing the atomic number by 1 while the mass number remains unchanged.
Therefore:
$X_2 \xrightarrow{\alpha} X_3$
Again, alpha decay reduces the atomic number by 2 and the mass number by 4 .
Therefore:
X_3: atomic number $= (Z-1) - 2 = Z - 3$
$X_3 \xrightarrow{\gamma} X_4$
Gamma decay does not change either the atomic number or the mass number.
Therefore:
X_4: atomic number $= Z - 3,$ mass number = A - 8
We are given that X_4 has a mass number of 172 and an atomic number of 69. This gives us two equations:
$Z - 3 = 69$
$A - 8 = 172$
Now, solve these equations:
From the first equation:
$Z = 69 + 3 = 72$
From the second equation:
$A = 172 + 8 = 180$
Thus, the original nucleus X has an atomic number of 72 and a mass number of 180 .

4. **(b)** Fringe positions are given by the single-slit minima condition
$a\sin\theta = m\lambda \; (m = \pm 1, \pm 2, \dots)$

so the angular fringe spacing scales as $\Delta\theta \sim \lambda/a$. Blue light has a smaller wavelength than red, hence its diffraction fringes are closer together (narrower angular width).

5. **(d)** Given:
Rod ka length $= L$
Cross-sectional area $= \alpha$
Density varies as: $\rho(x) = \rho_0 \frac{x^2}{L^2}$
Distance from one end $= x$
Humein centre of mass x_{cm} chahiye.
Step 1: Mass element dm
Rod ka ek chhota element dx ka mass:

$$dm = \rho(x) \cdot \text{ volume element } = \rho(x) \cdot \alpha dx = \rho_0 \frac{x^2}{L^2} \alpha dx$$

Step 2: Total mass M

$$M = \int_0^L dm = \int_0^L \rho_0 \frac{x^2}{L^2} \alpha dx = \rho_0 \frac{\alpha}{L^2} \int_0^L x^2 dx = \rho_0 \frac{\alpha}{L^2} \cdot \frac{L^3}{3} = \frac{\rho_0 \alpha L}{3}$$

Step 3: Centre of mass

$$x_{cm} = \frac{1}{M} \int_0^L x dm = \frac{1}{M} \int_0^L x \left(\rho_0 \frac{x^2}{L^2} \alpha dx \right) = \frac{\rho_0 \alpha}{M L^2} \int_0^L x^3 dx$$

$$\int_0^L x^3 dx = \frac{L^4}{4}$$

$$x_{cm} = \frac{\rho_0 \alpha}{M L^2} \cdot \frac{L^4}{4} = \frac{\rho_0 \alpha L^2}{4M}$$

Substitute $M = \frac{\rho_0 \alpha L}{3}$:

$$x_{cm} = \frac{\rho_0 \alpha L^2/4}{\rho_0 \alpha L/3} = \frac{3L}{4}$$

6. **(c)** At height $h = R$ above the Earth's surface, the distance from Earth's centre is $2R$.
Gravitational acceleration there is

$$g' = g \frac{R^2}{(2R)^2} = \frac{g}{4}$$

The time period of a simple pendulum of length L in gravitational field g' is

$$T = 2\pi \sqrt{\frac{L}{g'}} = 2\pi \sqrt{\frac{L}{g/4}} = 2\pi \sqrt{\frac{4L}{g}} = 4\pi \sqrt{\frac{L}{g}}$$

Substituting $L = 4.0$ m and $g = \pi^2 \mathrm{m/s^2}$:

$$\sqrt{\frac{L}{g}} = \sqrt{\frac{4}{\pi^2}} = \frac{2}{\pi}, T = 4\pi \times \frac{2}{\pi} = 8 \text{ s.}$$

7. **(b)** We are given:
Mass: $m = 1\text{gm} = 0.001$ kg
Charge: $q = 1\text{C}$
Electric field: $E(t) = E_0 \sin \omega t$, with $E_0 = 2$ N/C and $\omega = 1000\text{rad/s}$
Step 1: Equation of motion
Force on the particle:

$$F = qE(t) = m \frac{dv}{dt}$$

$$\Rightarrow m \frac{dv}{dt} = qE_0 \sin \omega t$$

$$\frac{dv}{dt} = \frac{qE_0}{m} \sin \omega t$$

Step 2: Integrate to find velocity

$$v(t) = \int \frac{qE_0}{m} \sin \omega t dt$$

$$v(t) = \frac{qE_0}{m} \int \sin \omega t dt$$

$$v(t) = \frac{qE_0}{m} \left(-\frac{\cos \omega t}{\omega} \right) + C$$

Since particle is at rest at $t = 0, v(0) = 0$:

$$0 = -\frac{qE_0}{m\omega}\cos 0 + C \Rightarrow C = \frac{qE_0}{m\omega}$$

So, velocity:

$$v(t) = \frac{qE_0}{m\omega}(1 - \cos\omega t)$$

Step 3: Maximum speed

Maximum speed occurs when $1 - \cos\omega t$ is maximum, i.e., $\cos\omega t = -1$:

$$v_{\max} = \frac{qE_0}{m\omega}(1 - (-1)) = \frac{2qE_0}{m\omega}$$

Step 4: Substitute values

$$v_{\max} = \frac{2 \cdot 1 \cdot 2}{0.001 \cdot 1000} = \frac{4}{1} = 4 \text{ m/s}$$

8. **(d)** Given:

Inclined plane with angle $\theta, \tan\theta = 2\mu$

Force to start pushing up: F_1

Force to prevent sliding down: F_2

Step 1: Forces on the body

Let the mass of the body be m.

Weight components along the incline:

Down the incline: $mg\sin\theta$

Normal: $mg\cos\theta$

Friction force: $f = \mu N = \mu mg\cos\theta$

Step 2: Force to push up (overcome friction and gravity)

Minimum force to push up:

$$F_1 = mg\sin\theta + \mu mg\cos\theta$$

Step 3: Force to prevent sliding (act downwards)

Minimum force to prevent sliding down:

$$F_2 = mg\sin\theta - \mu mg\cos\theta$$

(kyunki friction opposes motion, yaha friction act up the slope)

Step 4: Ratio F_1/F_2

$$\frac{F_1}{F_2} = \frac{mg\sin\theta + \mu mg\cos\theta}{mg\sin\theta - \mu mg\cos\theta} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}$$

Step 5: Substitute $\tan\theta = 2\mu$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = 2\mu \Longrightarrow \sin\theta = 2\mu\cos\theta$$

$$\frac{F_1}{F_2} = \frac{2\mu\cos\theta + \mu\cos\theta}{2\mu\cos\theta - \mu\cos\theta} = \frac{3\mu\cos\theta}{\mu\cos\theta} = 3$$

9. **(c)** From acceleration-time graph. Acceleration is constant for first part of motion so, for this part velocity of body increases uniformly with time and as $a = 0$ then the velocity becomes constant. Then again increased because of constant acceleration.

10. **(b)** 8×10^{-6}

11. **(b)** Let the escalator have "length" L (in steps).

Walking alone (escalator off):

$$v_1 t_1 = L$$

Standing alone (escalator on):

$$v_2 t_2 = L$$

Both walking and moving: net speed $v_1 + v_2$, so time

$$t = \frac{L}{v_1 + v_2} = \frac{v_1 t_1}{v_1 + v_2} = \frac{\frac{L}{t_1} t_1}{\frac{L}{t_1} + \frac{L}{t_2}} = \frac{L}{L\left(\frac{1}{t_1} + \frac{1}{t_2}\right)} = \frac{1}{\frac{1}{t_1} + \frac{1}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$$

12. **(d)** Let's find the dimension of X :

$$X = \frac{\varepsilon_0 L \Delta V}{\Delta t}$$

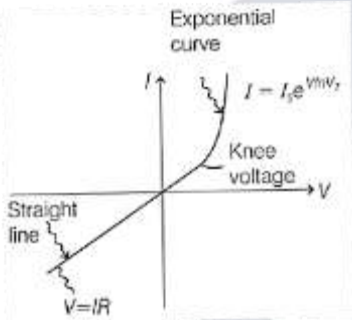
Where:

ε_0 has dimension $[M^{-1}L^{-3}T^4A^2]$
$L \rightarrow [L]$
$\Delta V \rightarrow [ML^2T^{-3}A^{-1}]$
$\Delta t \rightarrow [T]$
So,$X = \frac{[M^{-1}L^{-3}T^4A^2]\cdot[L]\cdot[ML^2T^{-3}A^{-1}]}{[T]}$
Multiply numerator:
$= [M^{-1}L^{-3}T^4A^2]\cdot[L] = [M^{-1}L^{-2}T^4A^2]$
$= [M^{-1}L^{-2}T^4A^2]\cdot[ML^2T^{-3}A^{-1}] = [M^0L^0T^1A^1]$
Now divide by $[T]$:
$= [M^0L^0T^1A^1]/[T] = [M^0L^0T^0A^1] = [A]$
Conclusion: The dimension is that of current

13. (a) When V is negative, diode is in reverse bias and it does not conducts.
So, current-voltage relationship followed from Ohm's law is, $V = IR$
$V - I$ graph is straight line.
As V grows in positive region, diode becomes conductive after knee voltage and after that current is approximately,

$I = I_s e^{V/nV_T}$

where, I_s = saturation current in reverse bias, V = voltage across diode n and V_T are diode constants. So, correct graph



14. (a) where:
R is the Rydberg constant,
n is the principal quantum number of the higher level.
Minimum wavelength
The minimum wavelength occurs when n is as large as possible, i.e., $n \rightarrow \infty$:
$$\frac{1}{\lambda_{\min}} = R\left(1 - \frac{1}{\infty}\right) = R$$
So,
$$\lambda_{\min} = \frac{1}{R} = P$$
Maximum wavelength
The maximum wavelength occurs when $n = 2$, because that's the smallest possible transition in the Lyman series:
$$\frac{1}{\lambda_{\max}} = R\left(1 - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right) = R\frac{3}{4}$$
Thus,
$$\lambda_{\max} = \frac{1}{R\frac{3}{4}} = \frac{4}{3R}$$
Since $P = \frac{1}{R}$, this becomes:
$$\lambda_{\max} = \frac{4P}{3}$$

15. (c) Given:

Energy of electron in n-th orbit:

$E_n = -\frac{13.6}{n^2}\mathrm{eV}$

Electron jumps from $n = 2$ to $n = 1$, so the energy difference is radiated as a photon.

Work function $\phi = 4.2\mathrm{eV}$.

We need to find the stopping potential.

Step 1 - Find the energies of the two states:

For $n = 1$:

$E_1 = -\frac{13.6}{1^2} = -13.6\mathrm{eV}$

For $n = 2$:

$E_2 = -\frac{13.6}{2^2} = -\frac{13.6}{4} = -3.4\mathrm{eV}$

Step 2 - Find the energy of the photon emitted:

$E_{\text{photon}} = E_2 - E_1 = (-3.4) - (-13.6) = 10.2\mathrm{eV}$

Step 3 - Use the photoelectric equation:

The maximum kinetic energy of the emitted electron is:

$K_{\max} = E_{\text{photon}} - \phi$

$K_{\max} = 10.2 - 4.2 = 6\mathrm{eV}$

The stopping potential V_s is related to the maximum kinetic energy by:

$K_{\max} = eV_s$

Since $e = 1$ in eV units,

$V_s = 6\ \mathrm{V}$

16. (b) Given:

Initial height $= h$

Coefficient of restitution $= e$

Neglect air resistance.

Step 1 - Ball falls first time:

It falls from height h, so the distance covered is h.

Step 2 - Ball bounces up:

After hitting the floor, it rises to a height e^2h, because:

$v_{\text{after}} = e \cdot v_{\text{before}}$

and since $h \propto v^2$, so new height $= e^2h$.

Step 3 - Falls again:

It falls from e^2h, so it covers e^2h distance on the way down.

Step 4 - Bounces again:

It rises to e^4h, and so on.

Total distance covered:

$D = h + 2(e^2h + e^4h + e^6h + \cdots)$

The factor 2 appears because after the first fall, each bounce has both an upward and a downward journey.

Step 5 - Summing the series:

$D = h + 2h(e^2 + e^4 + e^6 + \cdots)$

This is a geometric series where first term $a = e^2h$ and common ratio $r = e^2$.

The sum of infinite geometric series is:

$S = \frac{a}{1-r} = \frac{e^2h}{1-e^2}$

So,

$D = h + 2 \cdot \frac{e^2h}{1-e^2}$

$D = h + \frac{2e^2h}{1-e^2}$

Take common denominator:

$D = \frac{(1-e^2)h + 2e^2h}{1-e^2}$
$D = \frac{h(1-e^2+2e^2)}{1-e^2}$
$D = \frac{h(1+e^2)}{1-e^2}$

17. (a) Given:
Zener voltage: $V_z = 5.6$ V
Maximum power: $P_{z,\max} = \frac{1}{4}$ W $= 0.25$ W
Input voltage: $V_{\text{in}} = 10$ V
Series resistor: R_s (to be calculated)
Step 1: Maximum zener current
Zener diode ka maximum current $I_{z,\max}$ tab hoga jab diode full power dissipate kare:

$$P_{z,\max} = V_z \cdot I_{z,\max} \Longrightarrow I_{z,\max} = \frac{P_{z,\max}}{V_z}$$

$$I_{z,\max} = \frac{0.25}{5.6} \approx 0.04464 \text{ A} \approx 44.64 \text{ mA}$$

Step 2: Series resistor R_s calculation
Series resistor ka kaam hai extra voltage drop karna:

$$R_s = \frac{V_{\text{in}} - V_z}{I_{z,\max}}$$

$$R_s = \frac{10-5.6}{0.04464} = \frac{4.4}{0.04464} \approx 98.56\Omega$$

18. (a) Using Newton's second law,

$$\vec{F} = m\vec{a} \Longrightarrow \vec{a} = \frac{\vec{F}}{m} = \left(\frac{a}{m}, \frac{b}{m}, \frac{c}{m}\right)$$

Starting from rest at the origin, the displacement after time t is

$$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 = \frac{1}{2}\left(\frac{a}{m}, \frac{b}{m}, \frac{c}{m}\right)t^2 = \left(\frac{at^2}{2m}, \frac{bt^2}{2m}, \frac{ct^2}{2m}\right)$$

19. (b) For a prism, the angle of minimum deviation $\delta_{\min}$ occurs when the light passes symmetrically through the prism. At this point, the angle of incidence i and angle of emergence e are equal, and the relation is:

$A = i + e - \delta_{\min}$ or simplified as $A = \frac{\delta_{\min} + A}{2}$?

Actually, the correct relation at minimum deviation is:
$\delta_{\min} = 2i - A$ and at minimum deviation, $i = e$
From the δ vs i graph, the minimum deviation corresponds to the lowest point of δ. By symmetry, the prism angle A equals the angle of incidence at minimum deviation. From the graph, this gives $A = 45°$.
So the prism angle is indeed 45°.

20. (a) Given:
Charges: $+q$ at A, $-q$ at B
Distance between A and B: $2L$
C is midpoint of AB
Move charge $+Q$ from C along semicircle CSD or along straight line CBD
Step 1: Electric potential at midpoint C
Electric potential due to a point charge is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

At midpoint C, distance to A and B is L. So, potential at C due to both charges:

$$V_C = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{L} + \frac{-q}{L}\right) = 0$$

Step 2: Work done along any path from C to D
Work done in moving a charge Q in an electric field is:

$W=Q\left(V_{\text{final}}-V_{\text{initial}}\right)$
For semicircle CSD: the potential difference between start and end is
$V_{\mathrm{D}}-V_{C}=\frac{q}{4 \pi \epsilon_{0} L}-0=\frac{q}{4 \pi \epsilon_{0} L}$
$\Longrightarrow W_{1}=\frac{q Q}{4 \pi \epsilon_{0} L}$
For straight line CBD: potential difference is same because electric force is conservative. Path doesn't matter.
$\Longrightarrow W_{2}=\frac{q Q}{4 \pi \epsilon_{0} L}$

21. (b) That means the combination of logic gates in your question behaves like an AND gate, i.e., it outputs 1 only when both inputs are 1 , otherwise it outputs 0 .
If you want, I can also explain why that particular combination reduces to an AND gate. Do you want me to do that?

22. (a) Step 1: Recall the definition of activity
Activity A is the rate of decay of nuclei:
$A=-\frac{d N}{d t}$
For a radioactive substance with decay constant λ :
$\frac{d N}{d t}=-\lambda N \Longrightarrow A=\lambda N$
Step 2: Relationship between N and A
From $A=\lambda N$, we see that activity is directly proportional to the number of undecayed nuclei.
If N decreases exponentially with time (like a decaying curve),
Then A will follow the same shape of exponential decay, just scaled by λ.
Step 3: Graphical interpretation
If the given $N(t)$ curve is an exponential decay,
Then the correct A vs N graph is a straight line passing through the origin (because $A \propto N$).

23. (c) Given:
Cyclic process ABCA in P-T diagram.
Gas is ideal, so $P V=n R T$.
Step 1: Express V in terms of P and T
For an ideal gas:
$V=\frac{n R T}{P}$
So, if we know how P and T change, we can infer V.
If T increases while P is constant $\rightarrow V$ increases linearly.
If P increases while T is constant $\rightarrow V$ decreases.
If both change linearly $\rightarrow$ curve in P-V plane.
Step 2: Analyze the cycle
AB: T increases, P increases (typical slope in $\mathrm{P}-\mathrm{T}$ plot) $\rightarrow$ in $\mathrm{P}-\mathrm{V}$ plot, V may increase or decrease depending on relative rates.
BC: T decreases, P constant $\rightarrow$ V decreases (from $V=n R T / P$).
CA: Returns to initial point $\rightarrow$ forms a closed loop in $\mathrm{P}-\mathrm{V}$ diagram.
Step 3: Shape in P-V diagram
Because it is cyclic and involves both T and P changes, the $\mathrm{P}-\mathrm{V}$ diagram is a closed loop that is slanted, not purely rectangular or triangular.
Among given options, option (c) shows a closed, slanted loop - consistent with the reasoning.

24. (c) The fractional (percentage) error in a quotient $R=V / I$ is the sum of the fractional errors of V and I :
$\frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta I}{I}$.
Here
$\frac{\Delta V}{V}=\frac{0.4}{25}=0.016=1.6 \%$,

$$\frac{\Delta I}{I}=\frac{3}{200}=0.015=1.5\%.$$
So
$$\frac{\Delta R}{R}=0.016+0.015=0.031=3.1\%$$

25.(c) The magnetic moment is
$$\mu=IA=\frac{q\omega}{2\pi}\times(\pi r^2)=\frac{q\omega r^2}{2},$$
and the angular momentum is
$$L=mr^2\omega$$
Thus
$$\frac{\mu}{L}=\frac{\frac{q\omega r^2}{2}}{mr^2\omega}=\frac{q}{2m},$$
which depends only on q and m.

26.(a) Let the initial mass of the bus be m_1.
Initial speed $v \Rightarrow$
$$K_1=\frac{1}{2}m_1v^2, \lambda_1=\frac{h}{m_1v}\equiv\lambda$$
After some passengers leave, the mass becomes m_2.
New speed $2v \Rightarrow$
$$K_2=\frac{1}{2}m_2(2v)^2=2m_2v^2$$
Given $K_2=2K_1=2\left(\frac{1}{2}m_1v^2\right)=m_1v^2$, so
$$2m_2v^2=m_1v^2 \implies m_2=\frac{m_1}{2}$$
New de Broglie wavelength
$$\lambda_2=\frac{h}{m_2(2v)}=\frac{h}{\left(\frac{m_1}{2}\right)2v}=\frac{h}{m_1v}=\lambda$$

27.(d) Displacement:
$$y(t)=2\sin\left(\frac{\pi t}{2}+\phi\right)\text{ cm}$$
so the angular frequency is
$$\omega=\frac{\pi}{2}\text{ s}^{-1}.$$
Acceleration amplitude is
$$a_{\max}=\omega^2A=\left(\frac{\pi}{2}\right)^2\cdot 2=\frac{\pi^2}{2}\text{ cm/s}^2$$

28.(b) If the 1μ F and 2μ F capacitors are in series across a voltage V :

Charge on series capacitors is the same: $Q=C_{\text{eq}}V$

Equivalent capacitance: $C_{\text{eq}}=\frac{C_1C_2}{C_1+C_2}=\frac{1\cdot 2}{1+2}=\frac{2}{3}\mu$ F

Voltage across each:
$$V_1=\frac{Q}{C_1}, V_2=\frac{Q}{C_2}$$
Let total charge be $Q=\frac{2}{3}V$. Then:
$$Q_1=C_1V_1=1\cdot\frac{2}{3}\cdot 2=4\mu\text{C}, Q_2=2\cdot\frac{2}{3}=8\mu\text{C}$$

29.(c) Step 1: Hydrostatic pressure condition

At the interface where the liquids meet, pressures on both sides must be equal.

Suppose the heights of the liquids in the U-tube are such that:

Left arm: liquid with density ρ_1 and height h_1, and ρ_3 with height h_3

Right arm: liquid with density ρ_2 and height h_2, and ρ_3 with height h_3

At the same horizontal level (interface of ρ_3 with ρ_1 and ρ_2), pressure is equal:

$\rho_1 g h_1 + \rho_3 g h_3 = \rho_2 g h_2 + \rho_3 g h_3$

Here, h_3 cancels out:

$\rho_1 h_1 = \rho_2 h_2$

Step 2: Relation for ρ_3

From the figure (typical U-tube problem), the height of ρ_3 is half of the difference of the other two liquids:

$$\rho_3 = \frac{\rho_2 - \rho_1}{1/2} = 2(\rho_2 - \rho_1)$$

30. **(a)** Let:

Total volume of granite $= V$

Volume in water $= V_1$

Volume in mercury $= V_2$

Step 1: Weight of granite

$W = \rho g V$

Step 2: Upthrusts from water and mercury

Upthrust from water $= \rho_1 g V_1$

Upthrust from mercury $= \rho_2 g V_2$

Step 3: Equilibrium condition

Weight of granite $=$ Total upthrust $\Rightarrow \rho g V = \rho_1 g V_1 + \rho_2 g V_2$

Also, $V = V_1 + V_2$. So:

$\rho(V_1 + V_2) = \rho_1 V_1 + \rho_2 V_2$

$\rho V_1 + \rho V_2 = \rho_1 V_1 + \rho_2 V_2$

$(\rho - \rho_1)V_1 = (\rho_2 - \rho)V_2$

Step 4: Ratio of volumes

$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$

31. **(a)** Energy per photon:

$$E = \frac{hc}{\lambda} = \frac{(6.6 \times 10^{-34}\ \mathrm{J\ s})(3.0 \times 10^{8}\ \mathrm{m/s})}{660 \times 10^{-9}\ \mathrm{m}} = 3.0 \times 10^{-19}\ \mathrm{J}$$

Photons emitted per second: $N = 10^{20}$.

Total energy per second (power):

$P = NE = 10^{20} \times 3.0 \times 10^{-19}\ \mathrm{J/s} = 30\ \mathrm{W}$.

32. **(b)** Let β be the true volume-expansion coefficient of the liquid, A the linear coefficient of copper, and let B be the linear coefficient of silver. Then the vessel's volume-expansion coefficients are 3A (copper) and 3B (silver), and the apparent expansions are

$C = \beta - 3A, S = \beta - 3B$.

Eliminating β,

$$\beta = C + 3A = S + 3B \Longrightarrow 3B = C + 3A - S \Longrightarrow B = \frac{C + 3A - S}{3}$$

So the linear coefficient of expansion of silver is $\frac{C+3A-S}{3}$.

33. **(b)** We are given the stationary wave equation:

$$y = 5\sin\left(\frac{\pi x}{3}\right)\cos(40\pi t)$$

where x and y are in cm, t in seconds.

We are asked to find the separation between two adjacent nodes.

Step 1: Recall node condition
For a stationary wave on a string, nodes occur where the amplitude is zero, i.e.,
$\sin(kx) = 0 \Rightarrow kx = n\pi, n = 0,1,2, \ldots$
Here, the wave number k is the coefficient of x in the sine term:
$k = \frac{\pi}{3}\ \mathrm{cm}^{-1}$
Step 2: Node positions
Nodes are at:
$x_n = \frac{n\pi}{k} = \frac{n\pi}{\pi/3} = 3n\ \mathrm{cm}, n = 0,1,2, \ldots$
Step 3: Node separation
The distance between two adjacent nodes is:
$x_{n+1} - x_n = 3(n+1) - 3n = 3\ \mathrm{cm}$

34. **(b)** Given:
Temperature of the body: θ
Temperature of the surroundings: θ_0
Rate of cooling: $R = -\frac{d\theta}{dt}$
Newton's Law of Cooling:
$R = k(\theta - \theta_0)$
where $k > 0$ is a constant.
Analysis:
R is proportional to the excess temperature of the body over the surroundings.
This means if we plot R (y-axis) vs θ (x-axis), it will be a straight line.
Slope $= k$
Line passes through $(\theta_0, 0)$, because when $\theta = \theta_0, R = 0$.

35. **(c,d)** Step 1: Recall the formulas
For a monoatomic ideal gas at temperature T :
1.Most probable speed:
$V_p = \sqrt{\frac{2k_BT}{m}}$
2.Mean speed:
$\bar{V} = \sqrt{\frac{8k_BT}{\pi m}}$
3.Root mean square speed:
$V_{rms} = \sqrt{\frac{3k_BT}{m}}$
Also, we know the relationship:
$V_p < \bar{V} < V_{rms}$
since numerically:
$\sqrt{2} \approx 1.414, \sqrt{8/\pi} \approx 1.596, \sqrt{3} \approx 1.732.$
Step 4: Check option (d)
Average kinetic energy of a molecule is $\frac{3}{4}mV_p^2$
Average kinetic energy:
$\langle KE \rangle = \frac{3}{2}k_BT$
$V_p^2 = \frac{2k_BT}{m} \Longrightarrow mV_p^2 = 2k_BT$
So $\frac{3}{4}mV_p^2 = \frac{3}{4} \cdot 2k_BT = \frac{3}{2}k_BT$

36. **(a,b)**

37. **(a,b,d)** Step 1: Identify type of wave
The wave is in the form:

$y(x,t) = A\cos(\omega t + \phi)\cos(kx)$
This is a stationary wave (or standing wave) because it's a product of a time-dependent part and a spacedependent part.
Amplitude at position x is $A_x = A\cos(kx)$
Nodes: positions where amplitude $= 0 \Rightarrow \cos(kx) = 0$
Antinodes: positions where amplitude = maximum $\Rightarrow |\cos(kx)| = 1$
Step 2: Find nodes
$\cos(kx) = 0 \Longrightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$
Here, $k = 10\pi\ \mathrm{m}^{-1}$ (from $\cos(10\pi x)$).
First node: $10\pi x = \frac{\pi}{2} \Longrightarrow x = \frac{1}{20} = 0.05$ m
Next nodes: $x = 0.15$ m, 0.25 m, 0.35 m, ...
So, x = 0.15 m is a node → statement (a) correct.
Step 3: Find antinodes
Antinodes occur at $|\cos(kx)| = 1 \Longrightarrow kx = 0, \pi, 2\pi, \ldots$
$x = 0, \frac{\pi}{10\pi} = 0.1\ \mathrm{m}, 0.2\ \mathrm{m}, 0.3\ \mathrm{m}, \ldots$
So, x = 0.3 m is an antinode → statement (b) correct.
Step 4: Wavelength
For a standing wave:
$k = \frac{2\pi}{\lambda} \Longrightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2$ m
Statement (d) correct.

38.(c) Step 1: Write dimensions of each constant
1.Gravitational constant G :
$[G] = \frac{[M]^{-1}[L]^3}{[T]^2} = M^{-1}L^3T^{-2}$
2.Speed of light C :
$[C] = LT^{-1}$
3.Planck's constant h :
$[h] = ML^2T^{-1}$
Step 2: Set up the dimensional equation
$[L] = [G]^x[C]^y[h]^z$
Substitute dimensions:
$L^1 = (M^{-1}L^3T^{-2})^x \cdot (LT^{-1})^y \cdot (ML^2T^{-1})^z$
Combine powers:
Mass: M^{-x+z}
Length: $L^{3x+y+2z}$
Time: $T^{-2x-y-z}$
So dimensional equation:
$M^0L^1T^0 = M^{-x+z}L^{3x+y+2z}T^{-2x-y-z}$
Step 3: Equate exponents
1.Mass: $-x + z = 0 \Longrightarrow z = x$
2.Length: $3x + y + 2z = 1$
Substitute $z = x$: $3x + y + 2x = 1 \Longrightarrow 5x + y = 1$
3. Time: $-2x - y - z = 0$
Substitute $z = x$: $-2x - y - x = 0 \Longrightarrow -3x - y = 0 \Longrightarrow y = -3x$
Step 4: Solve for x, y, z
From Length eq: $5x + y = 1 \Longrightarrow 5x - 3x = 1 \Longrightarrow 2x = 1 \Longrightarrow x = 1/2$
Then:
$y = -3x = -3/2, z = x = 1/2$
So one solution is:
$x = 1/2, y = -3/2, z = 1/2$
The negative exponent in C is fine; it just means C is in the denominator.
Step 5: Express L in words

$L \sim \frac{h^{1/2} G^{1/2}}{C^{3/2}}$

This is the Planck length.
So the correct answer in options corresponds to:
Option (c) or (d) depending on notation, i.e., $y = -3/2, z = 1/2$

39.(a,c) Given:
Sphere S_1 initially at rest.
Sphere S_2 initially moving along x-axis with velocity v.
After collision:
$\vec{v}_2' = \frac{v}{2}\hat{y}$ (perpendicular to original direction)
Masses: m_1, m_2
Hume chahiye $\vec{v}_1'$ ki magnitude aur direction.
Step 1: Momentum conservation
x-direction:
$m_2 v = m_1 v_{1x}' + m_2 v_{2x}'$
Lekin $v_{2x}' = 0$ (kyunki S_2 y-direction mein move kar rahi hai)
$m_2 v = m_1 v_{1x}' \Longrightarrow v_{1x}' = \frac{m_2}{m_1} v$
y-direction:
$0 = m_1 v_{1y}' + m_2 v_{2y}' \Longrightarrow v_{1y}' = -\frac{m_2}{m_1} \cdot \frac{v}{2} = -\frac{m_2 v}{2m_1}$
Step 2: Velocity magnitude of S_1

$$v_1' = \sqrt{(v_{1x}')^2 + \left(v_{1y}'\right)^2} = \sqrt{\left(\frac{m_2}{m_1} v\right)^2 + \left(-\frac{m_2 v}{2m_1}\right)^2}$$

$$v_1' = \frac{m_2 v}{m_1}\sqrt{1 + \frac{1}{4}} = \frac{m_2 v}{m_1} \cdot \frac{\sqrt{5}}{2}$$

Matches option (a)
Step 3: Direction of S_1

$$\theta = \tan^{-1}\left(\frac{v_{1y}'}{v_{1x}'}\right) = \tan^{-1}\left(\frac{-\frac{m_2 v}{2m_1}}{\frac{m_2}{m_1} v}\right) = \tan^{-1}\left(-\frac{1}{2}\right)$$

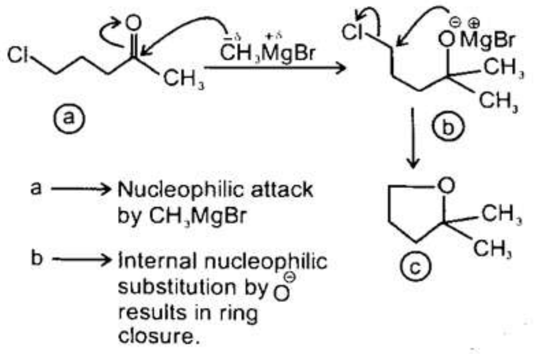
Matches option (c)

40. **(d)** If you look at the vector diagram:
Vector $\vec{f}$ is the resultant of vectors $\vec{d}$ and $\vec{e}$.
Adding $\vec{d}$ and $\vec{e}$ using the tip-to-tail method gives $\vec{f}$.
The other combinations ($\vec{b} + \vec{e}, \vec{b} + \vec{c}, \vec{d} + \vec{c}$) do not result in $\vec{f}$.

41. **(d)** 1.Step 1: Count valence electrons of Xe
Xe is in group 18 → 8 valence electrons.
2.Step 2: Determine bonding and lone pairs
Xe is bonded to O and 2 F atoms → total 3 bonds.
Total electrons used in bonding: 3 bonds × 2 electrons = 6 electrons.
Remaining electrons = 8 − 6 = 2 electrons → 1 lone pair? Wait, let's check geometry carefully.
3.Step 3: Consider formal charges / expanded octet
Xe can expand octet → it can accommodate more than 8 electrons.
Structure: O is double bonded to Xe (2 electrons shared), each F is single bonded. So Xe has:
O double bond → 2 pairs shared
2 F single bonds → 2 pairs shared
Total bonding pairs = 4 pairs → 8 electrons.
Xe originally has 8 valence electrons; 4 pairs are used in bonds → no electrons left? But XeOF 2 is known to have 2 lone pairs on Xe .
So, Xe has 2 lone pairs +3 sigma bonds → 5 regions of electron density.
4.Step 4: Hybridization

5 electron regions → sp^3 d hybridization (commonly written as sp^3 d).
Number of lone pairs = 2

42.(c)



43.(b) General concept:
In nucleophilic substitution reactions, the rate depends on factors such as:
The nature of the leaving group ($I > Br > Cl_r$ etc.).
The structure of the carbon attached (primary < secondary < tertiary).
The stability of the intermediate or transition state.
Since the question doesn't show the actual structures of I, II, III, IV, it seems you've already been provided them in a figure or context and you've mentioned that the correct answer is (b) II < I < III < IV.

44.(b) Concept involved:
MeMgBr (methyl magnesium bromide) is a Grignard reagent.
Grignard reagents are strong bases and nucleophiles.
They react easily with compounds that have acidic hydrogen atoms (like terminal alkynes, alcohols, etc.).
The reaction produces methane (CH_4) when MeMgBr abstracts a proton from an acidic hydrogen atom.
Now, what's option b ?
Option b refers to ethyne (acetylene, $HC \equiv CH$) - a terminal alkyne.
Why?
The hydrogen in the terminal alkyne ($\equiv C - H$) is acidic because the sp-hybridized carbon holds the electron density closer, making the proton easier to abstract.
Reaction:
$HC \equiv CH + MeMgBr \rightarrow HC \equiv C^{-}MgBr + CH_4$
This shows that one hydrogen from the terminal alkyne is removed by MeMgBr , forming methane.

45.(c) In an adiabatic process, there is no heat exchange, so $Q = 0$.
In free expansion, the gas expands into a vacuum, meaning there is no external pressure doing work, so $W = 0$.
From the first law of thermodynamics, we have:
$\Delta U = Q - W$
Substituting the values, we get:
$\Delta U = 0 - 0 = 0$
For an ideal gas, the internal energy U depends only on the temperature. Since $\Delta U = 0$, the temperature does not change during the process. This characteristic is described as an isothermal process.
Thus, the adiabatic free expansion of an ideal gas is isothermal, which corresponds to:

46.(b) Given:
Molecular formula: C_8H_{16}, which means it's an alkene (since it has 2 n hydrogens less than the saturated alkane C_8H_{18}).
It is optically active → It must have a chiral center or be such that its geometry leads to chirality.
On ozonolysis, it gives acetone ($CH_3 - CO - CH_3$) as one of the products.
Step 1- Ozonolysis:
Ozonolysis cleaves the double bond and forms carbonyl groups (aldehydes or ketones).
If one of the products is acetone $(CH_3 - CO - CH_3)_r$, that means one side of the double bond had $C(CH_3)_2$.
So the double bond must have been something like:

$(CH_3)_2C = -$
Step 2 - Build the alkene structure:
The other side of the double bond must account for the rest of the carbon atoms:
Total $C = 8$
Acetone part $= 3$ carbons $\rightarrow C_3$
So remaining $= 8 - 3 = 5 \rightarrow$ must form the other fragment.
Hence, the other fragment must be a C_5 compound $\rightarrow$ it must produce a 5 -carbon fragment after cleavage.
That fragment can be something like pentanal, or other ketone/aldehyde, depending on how the molecule is structured.
Step 3 - Optical activity:
The alkene must be such that the molecule is chiral and optically active $\rightarrow$ the structure must lack symmetry or have a chiral center.
Final Structure:
One correct structure is:
$(CH_3)_2C = CH - CH_2 - CH_2 - CH_3$
This is 2,3-dimethyl-2-hexene.
It is optically active because of the asymmetry, and on ozonolysis, the double bond between C2 and C3 breaks:
The $(CH_3)_2C =$ part becomes acetone.
The rest becomes pentanal.

47.(a) According to Graham's law of diffusion:

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

Where: $r_1/r_2 =$ ratio of diffusion rates
$M_1, M_2 =$ molar masses
Given:
Hydrocarbon diffuses $360\ cm^3$ in 30 min $\rightarrow$ rate $= \frac{360}{30} = 12\ cm^3/min$
SO2 diffuses $360\ cm^3$ in 60 min $\rightarrow$ rate $= \frac{360}{60} = 6\ cm^3/min$
So, $\frac{r_1}{r_2} = \frac{12}{6} = 2$

$$2 = \sqrt{\frac{M_{SO_2}}{M_{HC}}}$$

Square both sides:

$$4 = \frac{M_{SO_2}}{M_{HC}}$$

$$M_{HC} = \frac{M_{SO_2}}{4}$$

Molar mass of $SO_2 = 32 + 2 \times 16 = 64$ g/mol

$$M_{HC} = \frac{64}{4} = 16 \text{ g/mol}$$

Hydrocarbon with molar mass 16 g/mol $\rightarrow CH_4$

48. **(a)** Structure of B_2H_6 (diborane):
Diborane has 6 hydrogen atoms.
Out of these, 4 hydrogens are terminal, each bonded to a boron atom in a conventional way.
The remaining 2 hydrogens are bridging, connecting the two boron atoms in a 3 -center-2-electron bond.

49.(b) The reaction you are referring to is the reaction of an aromatic halide with $NaNH_2$ in liquid NH_3, which is a classic Bucherer reaction / amination via nucleophilic aromatic substitution.
Here's the reasoning:
1.The starting compound is probably chlorobenzene derivative with a methoxy $(-OCH_3)$ group, since the products involve anisidines (methoxyanilines).

2.$NaNH_2$ in liquid NH_3 is a strong nucleophile/base. It tends to substitute halogens on an aromatic ring, preferably at positions ortho and para to electron-withdrawing groups. However, the $-OCH_3$ group is an electron-donating group, which activates the meta position for nucleophilic substitution via the Meisenheimer complex formation.

3.Therefore, the major product will have the $-NH_2$ group at the meta position relative to $-OCH_3$, giving m-anisidine.

50.(a) Given:

Initial amount $[A]_0 = 100$ moles

Half-life $t_{1/2} = 10$ min

Time $t = 20$ min

(i) Zero-order reaction:

$[A] = [A]_0 - kt$

Half-life for zero-order:

$$t_{1/2} = \frac{[A]_0}{2k} \Longrightarrow k = \frac{[A]_0}{2t_{1/2}} = \frac{100}{20} = 5 \text{ mol/min}$$

After 20 min :

$[A] = 100 - 5 \cdot 20 = 0$ moles

(ii) First-order reaction:

$$[A] = [A]_0 e^{-kt}, t_{1/2} = \frac{\ln 2}{k} \Longrightarrow k = \frac{\ln 2}{10} \approx 0.0693 \text{ min}^{-1}$$

After 20 min :

$[A] = 100e^{-0.0693 \cdot 20} = 100e^{-1.386} \approx 100 \cdot 0.25 = 25$moles

(iii) Second-order reaction:

$$\frac{1}{[A]} = \frac{1}{[A]_0} + kt, t_{1/2} = \frac{1}{k[A]_0} \Longrightarrow k = \frac{1}{100 \cdot 10} = 0.001 \text{ min}^{-1} \text{ mol}^{-1}$$

After 20 min :

$$\frac{1}{[A]} = \frac{1}{100} + 0.001 \cdot 20 = 0.01 + 0.02 = 0.03 \Longrightarrow [A] = \frac{1}{0.03} \approx 33.33\text{moles}$$

51. (c) Step 1: Convert pH to $[H^+]$

$[H^+] = 10^{-pH}$

For A:

$[H^+]_A = 10^{-6}$M

For B:

$[H^+]_B = 10^{-4}$M

Step 2: Find $[H^+]$after mixing equal volumes

When equal volumes are mixed, the final $[H^+]$is the average:

$$[H^+]_{\text{final}} = \frac{[H^+]_A + [H^+]_B}{2} = \frac{10^{-6} + 10^{-4}}{2}$$

$$[H^+]_{\text{final}} = \frac{0.000001 + 0.0001}{2} = \frac{0.000101}{2} = 5.05 \times 10^{-5}\text{M}$$

Step 3: Convert back to pH

$\text{pH} = -\log[H^+] = -\log(5.05 \times 10^{-5})$

$\text{pH} \approx 4.3$

Step 4: Conclusion

The pH of the new solution lies between 4 and 5 .

52.(d) Given:

Compounds: PQ_2 aur PQ_3

Mass of solute $m = 1$ g, mass of solvent $M_s = 51$ g $= 0.051$ kg

Freezing point depression $\Delta T_f = 0.8^\circ$C for PQ_2 and 0.625°C for PQ_3

K_f of benzene $= 5.1$ K $\cdot$ kg/mol

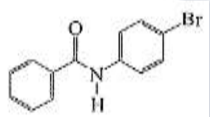
Step 1: Molality from freezing point depression

$\Delta T_f = K_f \cdot m_{\text{molal}}$

$$m_{\text{molal}} = \frac{\Delta T_f}{K_f}$$

For PQ_2 :

$m_1 = \frac{0.8}{5.1} \approx 0.15686 \text{ mol/kg}$
For PQ_3 :
$m_2 = \frac{0.625}{5.1} \approx 0.12255 \text{ mol/kg}$
Step 2: Moles of solute
Molality $m = \frac{\text{moles of solute}}{\text{kg of solvent}}$
For PQ_2 :
$n_1 = m_1 \cdot 0.051 \approx 0.15686 \cdot 0.051 \approx 0.007999 \approx 0.008 \text{ mol}$
For PQ_3 :
$n_2 = m_2 \cdot 0.051 \approx 0.12255 \cdot 0.051 \approx 0.00625 \text{ mol}$
Step 3: Molar masses
$M = \frac{\text{mass}}{\text{moles}}$
For PQ_2 :
$M_1 = \frac{1}{0.008} \approx 125 \text{ g/mol}$
For PQ_3 :
$M_2 = \frac{1}{0.00625} \approx 160 \text{ g/mol}$
Step 4: Atomic masses
Let atomic masses of $P = x, Q = y$
$PQ_2: x + 2y = 125$
$PQ_3: x + 3y = 160$
Subtract first from second:
$(x + 3y) - (x + 2y) = 160 - 125 \Longrightarrow y = 35$
$x + 2(35) = 125 \Longrightarrow x + 70 = 125 \Longrightarrow x = 55$
Atomic masses: $\mathrm{P} = 55, \mathrm{Q} = 35$

53. **(c)** 

54. **(d)** This question is about the common ion effect on the solubility of AgCl . Let's reason carefully:
1.AgCl solubility in pure water (H_2O) - No common ions, so solubility is maximum among solutions with common ions.
2.AgCl in 1MNaCl – NaCl provides Cl^-ions, so due to the common ion effect, solubility of AgCl decreases.
3.AgCl in 1M$CaCl_2$ – $CaCl_2$ gives 2Cl^-ions per formula unit, so the common ion effect is stronger than in NaCl . Therefore, solubility decreases even more.
4.AgCl in 1M$NaNO_3$ – $NaNO_3$ does not provide Cl^-, so there is no common ion effect, solubility is almost like in water (slightly less due to ionic strength, but higher than in NaCl or $CaCl_2$).
So the increasing order of solubility:
$CaCl_2 < NaCl < H_2O \approx NaNO_3$

55. **(c)** The compounds are:
1. I: $HO_2C - (CH_2)_4 - COOH$ (adipic acid)
2. II: $CO_2H - (CH_2)_2 - CO_2H$ (succinic acid)
3. III: Not explicitly given here, but by context seems like shorter-chain dicarboxylic acid.
4. IV: Likely the next shorter-chain one (malonic acid: $HOOC - CH_2 - COOH$)
Reasoning:
The acid strength of dicarboxylic acids increases as the distance between the two -COOH groups decreases because of inductive electron-withdrawing effect.
Longer chains → weaker interaction → weaker acid.
Shorter chains → stronger acid.
So, in increasing acid strength:
$HO_2C - (CH_2)_4 - COOH < HOOC - CH_2 - CH_2 - COOH$
$< HOOC - CH_2 - COOH < HOOC - COOH$
This corresponds to I < IV < II < III.

56.(a) Hydrogen bond strength depends on the electronegativity of the atoms involved and the polarity of the bond donating the H . Stronger H -bonds occur when H is attached to highly electronegative atoms (like O or N) and the acceptor is also highly electronegative.

Let's analyze the options:

1.$C-H \cdots O \rightarrow$ H is attached to carbon (less electronegative), so the bond is weak.

2.$N-H \cdots O \rightarrow$ H attached to nitrogen; stronger than $C-H$ bond.

3.$O-H \cdots O \rightarrow$ H attached to oxygen; strong H -bond.

4.$O-H \cdots F \rightarrow$ H attached to oxygen, F is highly electronegative; very strong H -bond.

Weakest hydrogen bond: $C-H \cdots O \rightarrow$ option (a).

57. (b) We are asked: Number of oxygen atoms in 0.36 g of water (H_2O).

Step 1: Molar mass of water

$M_{\mathrm{HzO}}=2(1)+16=18 \mathrm{~g/mol}$

Step 2: Number of moles of water

$n=\frac{\text{mass}}{\text{molar mass}}=\frac{0.36}{18}=0.02 \mathrm{~mol}$

Step 3: Number of water molecules

Number of molecules $=n \times N_A=0.02 \times 6.022 \times 10^{23} \approx 1.204 \times 10^{22}$

Step 4: Number of oxygen atoms

Each water molecule has 1 oxygen atom, so:

Number of oxygen atoms $=1.204 \times 10^{22} \approx 1.205 \times 10^{22}$

58.(b) Given:

$\Lambda_m^{\circ}(\mathrm{Ba(OH)_2})=523.28$

$\Lambda_m^{\circ}(\mathrm{BaCl_2})=280.0$

$\Lambda_m^{\circ}(\mathrm{NH_4Cl})=129.8$

We need $\Lambda_m^{\circ}(\mathrm{NH_4OH})$.

Step 1: Find $\Lambda_m^{\circ}(\mathrm{OH^-})$

$\mathrm{Ba(OH)_2} \rightarrow \mathrm{Ba^{2+}}+2\mathrm{OH^-}$

$\Lambda_m^{\circ}(\mathrm{Ba(OH)_2})=\Lambda_m^{\circ}(\mathrm{Ba^{2+}})+2\Lambda_m^{\circ}(\mathrm{OH^-})$

$\mathrm{BaCl_2} \rightarrow \mathrm{Ba^{2+}}+2\mathrm{Cl^-}$

$\Lambda_m^{\circ}(\mathrm{BaCl_2})=\Lambda_m^{\circ}(\mathrm{Ba^{2+}})+2\Lambda_m^{\circ}(\mathrm{Cl^-})$

Subtracting gives:

$\Lambda_m^{\circ}(\mathrm{Ba(OH)_2})-\Lambda_m^{\circ}(\mathrm{BaCl_2})=2\Lambda_m^{\circ}(\mathrm{OH^-})-2\Lambda_m^{\circ}(\mathrm{Cl^-})$

$523.28-280=2\left(\Lambda_m^{\circ}(\mathrm{OH^-})-\Lambda_m^{\circ}(\mathrm{Cl^-})\right)$

$243.28=2\left(\Lambda_m^{\circ}(\mathrm{OH^-})-\Lambda_m^{\circ}(\mathrm{Cl^-})\right)$

$\Lambda_m^{\circ}(\mathrm{OH^-})-\Lambda_m^{\circ}(\mathrm{Cl^-})=121.64$

$\Lambda_m^{\circ}(\mathrm{OH^-})=121.64+\Lambda_m^{\circ}(\mathrm{Cl^-})=121.64+?$

From $\mathrm{NH_4Cl}$: $\mathrm{NH_4}^{+}+\mathrm{Cl^-}$

$\Lambda_m^{\circ}(\mathrm{NH_4Cl})=\Lambda_m^{\circ}(\mathrm{NH_4^+})+\Lambda_m^{\circ}(\mathrm{Cl^-})$

$129.8=\Lambda_m^{\circ}(\mathrm{NH_4^+})+\Lambda_m^{\circ}(\mathrm{Cl^-})$

Step 2: Express $\Lambda_m^{\circ}(\mathrm{NH_4OH})$

$\mathrm{NH_4OH} \rightarrow \mathrm{NH_4^+}+\mathrm{OH^-}$

$\Lambda_m^{\circ}(\mathrm{NH_4OH})=\Lambda_m^{\circ}(\mathrm{NH_4^+})+\Lambda_m^{\circ}(\mathrm{OH^-})$

$=\left(\Lambda_m^{\circ}(\mathrm{NH_4^+})+\Lambda_m^{\circ}(\mathrm{Cl^-})\right)+\left(\Lambda_m^{\circ}(\mathrm{OH^-})-\Lambda_m^{\circ}(\mathrm{Cl^-})\right)$

$=\Lambda_m^{\circ}(\mathrm{NH_4Cl})+\left(\Lambda_m^{\circ}(\mathrm{OH^-})-\Lambda_m^{\circ}(\mathrm{Cl^-})\right)$

$=129.8+121.64=251.44$

59.(d) +2 and +2

60. **(d)** The question is about acid-catalyzed dehydration of alcohols. Let's analyze why option (d) is correct.

Key principle: Under acidic conditions (like H_2SO_4), alcohols undergo dehydration to form alkenes. The rate of dehydration depends on carbocation stability:

Tertiary alcohols > Secondary alcohols > Primary alcohols.

Resonance-stabilized carbocations (like allylic or benzylic) also form more readily.

So the alcohol that can form the most stable carbocation under acidic conditions will dehydrate most easily.

Option (d) corresponds to a tertiary alcohol, which forms a stable tertiary carbocation, hence it dehydrates most readily compared to primary or secondary alcohols in the other options.
If you want, I can draw the carbocations for all four options to show exactly why (d) is fastest. This makes it visually clear. Do you want me to do that?

61. (b) Reaction:

$_5B^{10} + {}_2He^4 \rightarrow X + {}_0n^1$

Step 1: Mass number aur atomic number balance karna
1.Mass number (A):
Left side: $10 + 4 = 14$
Right side: $A_X + 1 \rightarrow A_X = 14 - 1 = 13$
2. Atomic number (Z) :
Left side: $5 + 2 = 7$
Right side: $Z_X + 0 \rightarrow Z_X = 7$
Step 2: Identify X
Atomic number $Z = 7 \rightarrow$ element Nitrogen (N)
Mass number $A = 13 \rightarrow$ isotope N -13
Isliye, X = $_7N^{13}$

62.(d) Given:

Empty cylinder = 15.0 kg
Full cylinder = 30.0 kg → mass of LPG = 30 − 15 = 15 kg
After use, cylinder mass = 24.2 kg → mass of LPG used = 30 − 24.2 = 5.8 kg
Gas = normal butane C_4H_{10}, behaves ideally
Temperature $T = 27^\circ C = 300$ K
We are to find the volume of used gas at 1 atm and 27°C.
Step 1: Moles of gas used
Molar mass of butane C_4H_{10} :
$M = 4 \times 12 + 10 \times 1 = 58$ g/mol $= 0.058$ kg/mol
Mass of gas used: 5.8 kg

$$n = \frac{\text{mass}}{\text{molar mass}} = \frac{5.8}{0.058} = 100\text{moles}$$

Step 2: Ideal gas law to find volume
$PV = nRT$
$P = 1$ atm $= 1.013 \times 10^5$ Pa
$n = 100$ moles
$R = 8.314$ J/mol · K
$T = 300K$

$$V = \frac{nRT}{P} = \frac{100 \times 8.314 \times 300}{1.013 \times 10^5}$$

Calculate step by step:
1.$100 \times 8.314 = 831.4$
2.$831.4 \times 300 = 249420$ J
3.Divide by $1.013 \times 10^5 \approx 101300$

$$V \approx \frac{249420}{101300} \approx 2.46 \text{ m}^3$$

63.(c) Let's break down how molecular oxygen (O_2) is reduced step-by-step:

Adding one electron to O_2 gives the superoxide ion:
$O_2 + e^- \rightarrow O_2^{\cdot -}$
(This is a one-electron reduction.)
Adding a second electron gives the peroxide ion:
$O_2 + 2e^- \rightarrow O_2^{2-}$
(This is a two-electron reduction.)
When O_2 undergoes a four-electron reduction, the process is:
$O_2 + 4e^- \rightarrow 2O^{2-}$
Here, each oxygen atom ends up as an oxide ion (O^2).
So, the four-electron reduced form of O_2 is oxide

64.(c) Kjeldahl's method works by digesting the sample in strong acid, converting nitrogen into ammonium ions, which are then quantified. However, it is not suitable for nitrogen compounds where nitrogen is present in forms that do not break down easily under digestion conditions, such as:

Aromatic azo compounds (like azobenzene),

Nitro compounds,

Nitrogen in heterocyclic rings that are resistant to digestion.

Azobenzene ($C_6H_5 - N = N - C_6H_5$) contains nitrogen in an azo linkage ($-N = N -$) which is stable and does not easily convert into ammonium during digestion. Hence, it cannot be estimated by the Kjeldahl method.

For the other compounds (which are not fully written here but assumed to be typical organic or inorganic nitrogen-containing substances), the nitrogen is usually present in forms that decompose under acidic digestion.

65.(b) The SN1 reaction depends mainly on two factors:

1.Carbocation stability - the more stable the carbocation intermediate, the faster the reaction.

2.Leaving group ability - a better leaving group helps the process.

Among typical options, compound (b) must be the one that forms the most stable carbocation, often a tertiary carbon, allylic, benzylic, or resonance-stabilized system.

So, compound (b) is the most reactive because:

It likely forms a tertiary or resonance-stabilized carbocation, which is more stable.

The carbocation forms quickly, allowing the SN1 mechanism to proceed faster.

If you want, you can send me the exact structures of (a), (b), (c), and (d), and I'll explain in detail why (b) is the correct choice!

66.(b) The coagulating power of electrolytes depends on the charge of their ions. The higher the charge, the greater the coagulating power because multivalent ions neutralize the charges on the colloidal particles more effectively.

Here the order of charges is:

$Na^+ \rightarrow +1$

$Ba^{2+} \rightarrow +2$

$Al^{3+} \rightarrow +3$

So the coagulating power increases as:

$Na^+ < Ba^{2+} < Al^{3+}$

67.(c) Let's determine the effective number of atoms for each element in one unit cell of the cubic lattice:

A atoms :

Atoms are placed at the 8 corners of the cube.

Each corner atom is shared by 8 neighboring unit cells.

Effective atoms per unit cell: $8 \times \frac{1}{8} = 1$.

B atoms :

B atoms are located at the center of the cube, which is entirely within the unit cell.

Effective atoms per unit cell: 1 .

C atoms :

Atoms are positioned at the center of each of the 12 edges.

Each edge-centered atom is shared by 4 unit cells.

Effective atoms per unit cell: $12 \times \frac{1}{4} = 3$.

With these contributions, the total number of atoms per unit cell is:

$A = 1$

$B = 1$

$C = 3$

Thus, the formula of the compound is: ABC_3.

68. **(b)** The key to answering this question is understanding that a molecule is paramagnetic when it has one or more unpaired electrons. Let's examine each option briefly:

SO_2 (Sulfur dioxide): In this molecule, all electrons are paired, making it diamagnetic.

NO_2 (Nitrogen dioxide): This molecule has an odd number of electrons, which results in an unpaired electron. Hence, it is paramagnetic.

SiO_2 (Silicon dioxide): This compound has all electrons paired, so it is diamagnetic.

CO_2 (Carbon dioxide): This molecule also has all electrons paired, rendering it diamagnetic.
Thus, the paramagnetic oxide among the given options is:

69.(d) The reduction of nitrogen (N_2) to ammonia (NH_3) can be written as:
$N_2 + 6H^+ + 6e^- \rightarrow 2NH_3$
This shows that 6 electrons are required to reduce one molecule of N_2 into 2 molecules of NH_3.
But the question asks: how many electrons are needed to reduce N_2 to NH_3 ?
So for 1 molecule of N_2, the total electrons required are 6 .

70.(a) 1. Total number of electrons:
He has 2 electrons.
H has 1 electron.
The molecule has a +1 charge, so it loses 1 electron.
→ Total electrons $= 2 + 1 - 1 = 2$ electrons.
2.Molecular orbital filling:
The order for diatomic molecules up to 2nd period is:
$\sigma(1s), \sigma^*(1s)$
→ The first 2 electrons will fill the bonding orbital $\sigma(1s)$.
3.Bond order formula:
$$\text{Bond order } = \frac{(\text{bonding electrons } - \text{ antibonding electrons})}{2}$$
Here:
Bonding electrons $= 2$ (in $\sigma 1$ s)
Antibonding electrons $= 0$
$$\text{Bond order } = \frac{2-0}{2} = 1$$

71. (d) Given:
Boiling time at sea level $= 4.0$ min, at temperature T_1
Boiling time at mountain top $= 8.0$ min, at temperature T_2
We are to find the expression for activation energy E_a.
Step 1 - Use Arrhenius equation:
$$k = Ae^{-\frac{E_n}{RT}}$$
Where:
$k =$ rate constant
$E_a =$ activation energy
$R =$ gas constant
$T =$ temperature in Kelvin
Since boiling time is inversely proportional to rate constant:
$$k \propto \frac{1}{t}$$
Thus,
$$\frac{k_1}{k_2} = \frac{t_2}{t_1}$$
Given $t_1 = 4, t_2 = 8$:
$$\frac{k_1}{k_2} = \frac{8}{4} = 2$$
Step 2 - Take ratio of Arrhenius expressions:
$$\frac{k_1}{k_2} = \frac{Ae^{-\frac{E_0}{Rn_1}}}{Ae^{-\frac{E_0}{Rn_2}}} = e^{\frac{E_n}{R}\left(\frac{1}{t_2}-\frac{1}{T_1}\right)}$$
So,
$$2 = e^{\frac{E_n}{R}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}$$
Take natural logarithm on both sides:
$$\ln 2 = \frac{E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$\ln 2=\frac{E_a}{R} \frac{T_1-T_2}{T_1 T_2}$

Step 3 - Solve for E_a :

$E_a=\ln 2 \cdot R \cdot \frac{T_1 T_2}{T_1-T_2}$

Since $\ln 2 \approx 0.693$, the final expression becomes:

$E_a=0.693 R \frac{T_1 T_2}{T_1-T_2}$

72. **(b)** First, calculate the moles of HgO :

$n(\mathrm{HgO})=\frac{0.217 \mathrm{~g}}{217 \mathrm{~g} / \mathrm{mol}}=0.001 \mathrm{~mol}$

According to the reaction:

$\mathrm{HgO}+4 \mathrm{I}^{-}+\mathrm{H}_2 \mathrm{O} \longrightarrow \mathrm{HgI}_4^{2-}+2 \mathrm{OH}^{-}$

Each mole of HgO produces 2 moles of hydroxide ions (OH^{-}). So, the moles of OH formed are:

$n(\mathrm{OH}^{-})=2 \times 0.001=0.002 \mathrm{~mol}$

Next, these hydroxide ions are titrated with HCl . The reaction between HCl and OHD is:

$\mathrm{HCl}+\mathrm{OH}^{-} \longrightarrow \mathrm{H}_2 \mathrm{O}$

This reaction occurs in a 1:1 mole ratio. Therefore, 0.002 moles of HCl are needed to neutralize the hydroxide produced.

Given that the concentration of HCl is 0.01 M , the volume of HCl required is calculated by:

Given that the concentration of HCl is 0.01 M , the volume of HCl required is calculated by:

$\text{Volume }=\frac{\text{Moles of HCl}}{\text{Concentration of HCl}}=\frac{0.002 \mathrm{~mol}}{0.01 \mathrm{M}}=0.2 \mathrm{~L}$

Converting liters to milliliters:

$0.2 \mathrm{~L}=200 \mathrm{~mL}$

Thus, 200 mL of 0.01 M HCl is required at the equivalence point.

73. **(c)** Since I can't see the actual reactions or diagrams, confirm whether:

1.You want an explanation for why option (c) is correct based on the reaction type.

2.You can upload or describe the reaction steps so I can fully analyze it.

Please share the missing details - the reaction scheme, reagents, or the compounds involved - and I'll explain thoroughly.

74. **(b)** $\underset{1-\alpha}{A}(g) \rightleftharpoons \underset{\alpha}{B}(g)+\underset{\alpha}{C}(g)$

$P=P_A+P_B+P_C$

$k_P=\frac{(P_b)(O_C)}{P_A}$

Total moles $=1-\alpha+\alpha+\alpha=1+\alpha$

$P_A=\frac{1-\alpha}{1+\alpha} \cdot P, P_B=P_C=\frac{\alpha}{1+\alpha} \cdot P$

$k_p=\frac{\left(\frac{\alpha P}{1+\alpha}\right)^2}{\left(\frac{1-\alpha}{1+\alpha}\right)}=\frac{\alpha^2}{1-\alpha^2} \Rightarrow \alpha^2=\frac{k_p}{P+k_p}$

(a) $P \gg k_P \Rightarrow \alpha^2 \simeq \frac{k_P}{P} \Rightarrow \alpha \ll 1 \alpha$

(b) $P \uparrow, \alpha \downarrow$

(c) $k_P \gg P \Rightarrow \alpha^2 \simeq \alpha \simeq \alpha$

(d) $P \uparrow, \alpha \uparrow, \alpha$

75. **(d)** Effervescence (bubbling) occurs when a compound reacts with sodium bicarbonate (NaHCO_3) to release carbon dioxide (CO_2) gas.

For this to happen, the compound must be acidic enough to react with the bicarbonate ion. Typically, carboxylic acids (-COOH group) react with NaHCO_3 as follows:

$\mathrm{R}-\mathrm{COOH}+\mathrm{NaHCO}_3 \rightarrow \mathrm{R}-\mathrm{COONa}+\mathrm{H}_2 \mathrm{O}+\mathrm{CO}_2$ (gas)

Now let's analyze the options:

(a) Some non-acidic compound → No reaction

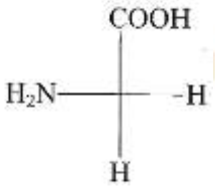
(b) $\mathrm{CH}_3 \mathrm{COCH}_3$ (Acetone): It's a ketone, not acidic → No reaction

(c) Another non-acidic compound → No reaction
(d) A carboxylic acid → Reacts with $NaHCO_3$, producing CO_2 → Effervescence!
So, only option (d) contains a carboxylic acid group, which is acidic enough to react with sodium bicarbonate and give off bubbles of carbon dioxide.

76. (a,c,d) Since the actual compound structure isn't provided, but from the correct answers you've given - (a), (c), and (d) — it hints that the compound is likely an aldehyde or a ketone with an -OH group nearby, such as acetylacetone (2,4-pentanedione) or something similar. Here's why the statements are correct:
(a) It exhibits tautomerism
This is true because compounds like β-diketones (e.g. acetylacetone) undergo keto-enol tautomerism. The proton shifts between carbonyl carbon and the adjacent carbon, forming an equilibrium between keto and enol forms.
(c) It gives reddish-violet coloration with $FeCl_3$ solution
Enolic forms have active hydrogen that can complex with Fe^{3+} ions, giving characteristic colorations like reddish-violet.
(d) It gives precipitate with 2,4-dinitrophenylhydrazine solution
Compounds with carbonyl groups ($C=O$) react with 2,4-DNP to form a hydrazone precipitate, which confirms the presence of aldehyde or ketone functionality.

77. (a,c) We are given:
X is extensive → depends on the size/amount of the system (e.g., volume V, internal energy U, mass m)
x is intensive → independent of system size (e.g., temperature T, pressure P, density ρ)
We need to check which of the statements about combinations of X and x are correct:
xX is extensive:
x is intensive (independent of size), X is extensive (proportional to size).
When you multiply an intensive quantity with an extensive quantity, the result scales with system size, so it is extensive.
X/x is extensive:
X extensive, x intensive → dividing an extensive by an intensive quantity still scales with system size, so X/x is extensive.

78. (a,c) The question is about separating ions by precipitation, which depends on the solubility differences of their salts. Let's analyze:
Precipitation method works if one ion forms an insoluble salt while the other stays soluble under given conditions.
Now, looking at the options:
Eu (II) and Dy (III) → They have different charges and different solubility properties, so their salts (like sulfates or hydroxides) have different solubilities.
Eu (II) and Yb(II) → Both are +2 , but their salts' solubility differs (for example, carbonates or hydroxides). Can be separated.



79. (a)

80. **(a)** To identify **P** and **Q**, we need to consider the type of reaction. Although the exact reaction isn't shown, this looks like a classic double displacement and complex formation reaction involving cadmium salts.
From the options:
1.Option (a): $P=K_2[Cd(CN)_4], Q=CdS$
Cadmium can react with KCN to form potassium tetracyanocadmate(II).
Cadmium salts (like $CdSO_4$ or $Cd(NO_3)_2$) react with H_2S to give CdS precipitate.
2.Option (b): $P=CdS, Q=K_2[Cd(CN)_4]$
This reverses the order; less likely.
3.Option (c): $P=Cd(NO_3)_2, Q=CdSO_4$
These are both soluble salts; reaction would not yield a precipitate.

4.Option (d): $P = [Cd(H_2O)_4](NO_3)_2, Q = [Cd(NO_3)_4]^{2-}$
This is more complex and less likely for a standard lab reaction.

81. (a) Step 1: Continuity at $x = 1$
For $f(x)$ to be differentiable everywhere, it must be continuous at $x = 1$. So:
$\lim_{x\to1} f(x) = \lim_{x\to1} f(x)$
$(1^2 + 3 \cdot 1 + a) = b \cdot 1 + 2$
$1 + 3 + a = b + 2$
$4 + a = b + 2$
$a - b = -2 \Rightarrow a = b - 2$ ......(1)
Step 2: Differentiability at $x = 1$
The derivatives from left and right must be equal:
$f'_-(1) = f'_+(1)$
$\frac{d}{dx}(x^2 + 3x + a) = 2x + 3 \Rightarrow f'_-(1) = 2 \cdot 1 + 3 = 5$
$\frac{d}{dx}(bx + 2) = b \Rightarrow f'_+(1) = b$
Equating:
$b = 5$ ......(2)
Step 3: Solve for a
From (1): $a = b - 2 = 5 - 2 = 3$

82.(b) Step 1: Determine the degree and form of $p(x)$
A local maximum or minimum occurs where $p'(x) = 0$.
So $p'(x)$ must vanish at $x = 1$ and $x = 3$.
Least degree polynomial means $p'(x)$ has exactly these roots and no more.
$p'(x) = k(x - 1)(x - 3)$
where k is a constant.
Step 2: Integrate to find $p(x)$
$p(x) = \int k(x - 1)(x - 3)dx$
Expand $(x - 1)(x - 3) = x^2 - 4x + 3$:
$p(x) = \int k(x^2 - 4x + 3)dx = k\left(\frac{x^3}{3} - 2x^2 + 3x\right) + C$
So the polynomial is:
$p(x) = k\left(\frac{x^3}{3} - 2x^2 + 3x\right) + C$
Step 3: Use given points to find k and C
1.$p(1) = 6$:
$p(1) = k\left(\frac{1}{3} - 2 + 3\right) + C = k\left(\frac{4}{3}\right) + C = 6 \Longrightarrow \frac{4}{3}k + C = 6$
2.$p(3) = 2$:
$p(3) = k\left(\frac{27}{3} - 18 + 9\right) + C = k(0) + C = C = 2$
So $C = 2$.
Plug back into first equation:
$\frac{4}{3}k + 2 = 6 \Longrightarrow \frac{4}{3}k = 4 \Longrightarrow k = 3$
Step 4: Find $p'(x)$ and $p'(0)$
$p'(x) = k(x - 1)(x - 3) = 3(x - 1)(x - 3)$
$p'(0) = 3(0 - 1)(0 - 3) = 3 \cdot (-1) \cdot (-3) = 9$

83.(c) Step 1: Find derivative
$f'(x) = \frac{d}{dx}(2x^3 - 3x^2 - 12x + 4) = 6x^2 - 6x - 12$
Step 2: Set derivative = 0
$6x^2 - 6x - 12 = 0 \Longrightarrow x^2 - x - 2 = 0$

Factorize:
$x^2 - x - 2 = (x-2)(x+1) = 0$
So the critical points are:
$x = 2, x = -1$
Step 3: Use second derivative to classify
$f''(x) = \frac{d}{dx}(6x^2 - 6x - 12) = 12x - 6$
At $x = 2$:
$f''(2) = 12(2) - 6 = 24 - 6 = 18 > 0 \Longrightarrow$ local minimum at $x = 2$
At $x = -1$:
$f''(-1) = 12(-1) - 6 = -12 - 6 = -18 < 0 \Longrightarrow$ local maximum at $x = -1$
Step 4: Conclusion
The function has one local maximum at $x = -1$ and one local minimum at $x = 2$.

84. **(b)** Step 1: Differentiate $\phi(x)$
$\phi'(x) = f'(x) + \frac{d}{dx}[f(2a-x)] = f'(x) - f'(2a-x)$
because by chain rule, $\frac{d}{dx}f(2a-x) = f'(2a-x)\cdot(-1)$.
Step 2: Analyze $\phi'(x)$ on $[0, \mathrm{a}]$
For $x \in [0, a]$, notice that $2a - x \in [a, 2a]$.
We are given $f''(x) > 0$ for $x \in [0, a]$, which means $f'(x)$ is increasing on $[0, \mathrm{a}]$.
Now consider:
$\phi'(x) = f'(x) - f'(2a-x)$
At $x = 0$: $\phi'(0) = f'(0) - f'(2a)$ (we don't know $f'(2a)$, but let's think about monotonicity).
At $x = a$: $\phi'(a) = f'(a) - f'(a) = 0$.
Also, for $x \in [0, a]$, $f'(2a-x) > f'(x)$ because $2a - x \geq a$ and f' is increasing (though strictly speaking $f'' > 0$ is only given for $[0, \mathrm{a}]$; beyond a we don't know, but the standard assumption is symmetry gives decreasing).
Hence:
$\phi'(x) = f'(x) - f'(2a-x) \leq 0$ on $[0, a]$
So $\phi(x)$ is decreasing on $[0, \mathrm{a}]$.

85. Given:
1. $g(f(x)) = |\sin x|$
2. $f(g(x)) = (\sin\sqrt{x})^2$
Check each option:
(a) $f(x) = \sin^2 x, g(x) = \sqrt{x}$
$g(f(x)) = \sqrt{\sin^2 x} = |\sin x| \boldsymbol{V}$
$f(g(x)) = f(\sqrt{x}) = \sin^2(\sqrt{x}) = (\sin\sqrt{x})^2$

86. **(b)** where $n \in \mathbb{N}$.
We are to find which among the options this expression is divisible by.
Step 1 - Check each option by plugging in values of n
Let's try $n = 1$ first.
For $n = 1$:
$2^{4(1)} - 15(1) - 1 = 2^4 - 15 - 1 = 16 - 15 - 1 = 0$
So for $n = 1$, the expression is 0 , hence divisible by any number.
For $n = 2$:
$2^{4(2)} - 15(2) - 1 = 2^8 - 30 - 1 = 256 - 31 = 225$
So for $n = 2$, the expression is 225 .
Check divisibility:
125: $225/125 = 1.8 \rightarrow$ not divisible
225: $225/225 = 1 \rightarrow$ divisible
325: $225/325 < 1 \rightarrow$ not divisible
425: $225/425 < 1 \rightarrow$ not divisible
So for $n = 2$, the expression is divisible only by 225 .
Check $n = 3$:

$2^{4(3)} - 15(3) - 1 = 2^{12} - 45 - 1 = 4096 - 46 = 4050$
125: $4050/125 = 32.4 \rightarrow$ not divisible
225: $4050/225 = 18 \rightarrow$ divisible
325: $4050/325 = 12.46 \rightarrow$ not divisible
425: $4050/425 = 9.53 \rightarrow$ not divisible
Again divisible only by 225 .

87. **(a)** Given:
$\frac{2z_1}{3z_2}$ is purely imaginary
This means its real part is zero.
Let's write z_1 and z_2 in general form:
$z_1 = a + bi, z_2 = c + di$
So,
$$\frac{2z_1}{3z_2} = \frac{2(a+bi)}{3(c+di)}$$
Multiply numerator and denominator by the conjugate of the denominator:
$$= \frac{2(a+bi)(c-di)}{3(c+di)(c-di)} = \frac{2(a+bi)(c-di)}{3(c^2+d^2)}$$
Now expand the numerator:
$$= \frac{2[(ac+bd) + i(bc-ad)]}{3(c^2+d^2)}$$
For this expression to be purely imaginary, the real part must be zero, i.e.:
$ac + bd = 0$
So this is our key equation.
Step 2: Find $|(z_1 - z_2)/(z_1 + z_2)|$
Now compute $(z_1 - z_2)$ and $(z_1 + z_2)$:
$z_1 - z_2 = (a-c) + i(b-d)$
$z_1 + z_2 = (a+c) + i(b+d)$
The modulus is:
$$\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = \frac{\sqrt{(a-c)^2 + (b-d)^2}}{\sqrt{(a+c)^2 + (b+d)^2}}$$
Step 3: Simplify the expression using $ac + bd = 0$
Now square both sides to make simplification easier:
$$\left|\frac{z_1 - z_2}{z_1 + z_2}\right|^2 = \frac{(a-c)^2 + (b-d)^2}{(a+c)^2 + (b+d)^2}$$
Expand the numerator:
$$= \frac{a^2 - 2ac + c^2 + b^2 - 2bd + d^2}{(a+c)^2 + (b+d)^2} = \frac{a^2 + b^2 + c^2 + d^2 - 2(ac+bd)}{(a+c)^2 + (b+d)^2}$$
Since $ac + bd = 0$, this becomes:
$$= \frac{a^2 + b^2 + c^2 + d^2}{(a+c)^2 + (b+d)^2}$$
Expand the denominator:
$$= \frac{a^2 + b^2 + c^2 + d^2}{a^2 + 2ac + c^2 + b^2 + 2bd + d^2} = \frac{a^2 + b^2 + c^2 + d^2}{a^2 + b^2 + c^2 + d^2 + 2(ac+bd)}$$
Since $ac + bd = 0$, this simplifies to:
$$= \frac{a^2 + b^2 + c^2 + d^2}{a^2 + b^2 + c^2 + d^2} = 1$$
Thus,
$$\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = \sqrt{1} = 1$$

88. **(b)** Let's carefully solve the integral:
$$I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$$

Step 1 - Symmetry approach
This integral looks like one where using the substitution $x=a+b-t$ or $x=9-t$ can simplify it.
Let's define:
$$I=\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$$
and another integral by substituting $x=9-t$:
$$I'=\int_3^6 \frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}} dx$$
Step 2 - Add both integrals
Add I and I' :
$$I+I'=\int_3^6\left(\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}+\frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}}\right) dx$$
Combine the terms inside:
$$=\int_3^6 \frac{\sqrt{x}+\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}} dx$$
$$=\int_3^6 1 dx$$
$=6-3=3$
So, $I+I'=3$
Step 3 - Observe that $I=I'$
Because the substitution $x=9-t$ keeps the limits same (3 to 6), and the form of the integral remains unchanged.
Thus, $I=I'$
So,
$2I=3$
$$I=\frac{3}{2}$$

89. **(a)** Step 1: Find the coordinates of the intersection points P and Q.
The equation of the line is $y=\sqrt{3}x-3$.
The equation of the parabola is $y^2=x+2$.
Substitute the expression for y from the line equation into the parabola equation:
$(\sqrt{3}x-3)^2=x+2$
$3x^2-6\sqrt{3}x+9=x+2$
$3x^2-(6\sqrt{3}+1)x+7=0$
This is a quadratic equation in x. Let the roots of this equation be x_1 and x_2, which are the xcoordinates of the points P and Q. Using Vieta's formulas, we have:
$$x_1+x_2=\frac{6\sqrt{3}+1}{3}$$
$$x_1x_2=\frac{7}{3}$$
The corresponding y-coordinates are $y_1=\sqrt{3}x_1-3$ and $y_2=\sqrt{3}x_2-3$.
So, the points P and Q are $(x_1,\sqrt{3}x_1-3)$ and $(x_2,\sqrt{3}x_2-3)$.
Step 2: Calculate the distances XP and XQ.
The coordinates of point X are $(\sqrt{3},0)$.
$$XP^2=\left(x_1-\sqrt{3}\right)^2+\left(\sqrt{3}x_1-3-0\right)^2=\left(x_1-\sqrt{3}\right)^2+\left(\sqrt{3}\left(x_1-\sqrt{3}\right)\right)^2$$
$$XP^2=\left(x_1-\sqrt{3}\right)^2+3\left(x_1-\sqrt{3}\right)^2=4\left(x_1-\sqrt{3}\right)^2$$
$$XP=2\left|x_1-\sqrt{3}\right|$$
Similarly,
$$XQ^2=\left(x_2-\sqrt{3}\right)^2+\left(\sqrt{3}x_2-3-0\right)^2=\left(x_2-\sqrt{3}\right)^2+\left(\sqrt{3}\left(x_2-\sqrt{3}\right)\right)^2$$
$$XQ^2=\left(x_2-\sqrt{3}\right)^2+3\left(x_2-\sqrt{3}\right)^2=4\left(x_2-\sqrt{3}\right)^2$$

$XQ = 2|x_2 - \sqrt{3}|$

Step 3: Calculate the product $XP \cdot XQ$.

$XP \cdot XQ = 4|(x_1 - \sqrt{3})(x_2 - \sqrt{3})| = 4|x_1 x_2 - \sqrt{3}(x_1 + x_2) + 3|$

Substitute the values of $x_1 + x_2$ and $x_1 x_2$ from Step 1:

$$XP \cdot XQ = 4\left|\frac{7}{3} - \sqrt{3}\left(\frac{6\sqrt{3}+1}{3}\right) + 3\right|$$

$$XP \cdot XQ = 4\left|\frac{7}{3} - \frac{18+\sqrt{3}}{3} + \frac{9}{3}\right|$$

$$XP \cdot XQ = 4\left|\frac{7 - 18 - \sqrt{3} + 9}{3}\right|$$

$$XP \cdot XQ = 4\left|\frac{-2-\sqrt{3}}{3}\right| = 4\left(\frac{2+\sqrt{3}}{3}\right) = \frac{4(2+\sqrt{3})}{3}$$

90. **(a)** We are given:

$f(x) = |1 - 2x|$

This is an absolute value function, which can potentially be non-differentiable at points where the expression inside the absolute value is zero.

1. Find where it might not be differentiable:

Set $1 - 2x = 0$,

$$2x = 1 \Rightarrow x = \frac{1}{2}$$

So, the critical point is at $x = \frac{1}{2}$.

2. Check continuity at $x = \frac{1}{2}$:

For continuity, the left-hand limit, right-hand limit, and the value of the function at that point must be equal.

Left-hand limit $x \to \frac{1}{2}^-$:

For $x < \frac{1}{2}, 1 - 2x > 0$, so

$f(x) = 1 - 2x$

$$\lim_{x \to \frac{1}{2}^-} f(x) = 1 - 2\left(\frac{1}{2}\right) = 0$$

Right-hand limit $x \to \frac{1}{2}^+$:

For $x > \frac{1}{2}, 1 - 2x < 0$, so

$f(x) = -(1 - 2x) = 2x - 1$

$$\lim_{x \to \frac{1}{2}^+} f(x) = 2\left(\frac{1}{2}\right) - 1 = 0$$

Function value at $x = \frac{1}{2}$:

$$f\left(\frac{1}{2}\right) = \left|1 - 2 \cdot \frac{1}{2}\right| = |1 - 1| = 0$$

Since all are equal, $f(x)$ is continuous at $x = \frac{1}{2}$.

3. Check differentiability at $x = \frac{1}{2}$:

Find the left-hand derivative and right-hand derivative.

Left-hand derivative:

$f(x) = 1 - 2x, f'(x) = -2$

So,

$$\lim_{x \to \frac{1}{2}^-} f'(x) = -2$$

Right-hand derivative:

$f(x) = 2x - 1, f'(x) = 2$

So,

$\lim_{x\to\frac{1}{2}^+} f'(x) = 2$

Since the left-hand derivative -2 and right-hand derivative 2 are not equal, the function is not differentiable at $x = \frac{1}{2}$.

91. (a) Given:

f' is the inverse of g, meaning $f = g^{-1}$.

$g'(x) = \frac{1}{1+x^n}$.

We are asked to find $f'(x)$, i.e., the derivative of the inverse function.

Key formula:

If f and g are inverses, then:

$$f'(x) = \frac{1}{g'(f(x))}$$

Applying this:

$$f'(x) = \frac{1}{g'(f(x))}$$

Substitute $g'(x)$:

$$f'(x) = \frac{1}{\frac{1}{1+(f(x))^n}}$$

$$f'(x) = 1 + (f(x))^n$$

92.(b) Given matrix:

$$A = \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix}$$

It is orthogonal, meaning $A^T A = I$, where I is the identity matrix.

Step 1 - Find A^T

$$A^T = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$

Step 2 - Compute $A^T A$

We calculate $A^T A$ and equate it to I.

Let's compute each entry.

(1,1) entry:

$(0)(0) + (2b)(2b) + (c)(c) = 4b^2 + c^2$

Set equal to 1 :

$4b^2 + c^2 = 1$

(2,2) entry:

$(a)(a) + (b)(b) + (-c)(-c) = a^2 + b^2 + c^2$

Set equal to 1 :

$a^2 + b^2 + c^2 = 1$

(3,3) entry:

$(a)(a) + (-b)(-b) + (c)(c) = a^2 + b^2 + c^2$

Same as above, so:

$a^2 + b^2 + c^2 = 1$

(1,2) entry:

$(0)(a) + (2b)(b) + (c)(-c) = 2b^2 - c^2$

Set equal to 0 :

$2b^2 - c^2 = 0$

(1,3) entry:

$(0)(a) + (2b)(-b) + (c)(c) = -2b^2 + c^2$

Set equal to 0 :

$-2b^2 + c^2 = 0$

Same as above.

(2,3) entry:

$(a)(a) + (b)(-b) + (-c)(c) = a^2 - b^2 - c^2$

Set equal to 0 :
$a^2 - b^2 - c^2 = 0$
Step 3 - Solve the equations
From $2b^2 - c^2 = 0$:
$c^2 = 2b^2$
From $4b^2 + c^2 = 1$:
Substitute $c^2 = 2b^2$:
$4b^2 + 2b^2 = 1$
$6b^2 = 1$
$b^2 = \frac{1}{6}$
$b = \pm\frac{1}{\sqrt{6}}$
Find c :
$c^2 = 2b^2 = 2 \times \frac{1}{6} = \frac{1}{3}$
$c = \pm\frac{1}{\sqrt{3}}$
From $a^2 + b^2 + c^2 = 1$:
$a^2 + \frac{1}{6} + \frac{1}{3} = 1$
Convert $\frac{1}{3}$ to $\frac{2}{6}$:
$a^2 + \frac{1}{6} + \frac{2}{6} = 1$
$a^2 + \frac{3}{6} = 1$
$a^2 + \frac{1}{2} = 1$
$a^2 = 1 - \frac{1}{2} = \frac{1}{2}$

93.(c) Given:
$A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$
And $|A|^2 = 25$, which means $|A| = \pm 5$, but determinant is usually taken as positive, so $|A| = 5$.
Step 1 - Find $|A|$:
Since this is an upper triangular matrix, the determinant is the product of the diagonal entries.
So,
$|A| = 5 \times \alpha \times 5 = 25\alpha$
We are given that:
$|A|^2 = 25$
That is:
$(25\alpha)^2 = 25$
Step 2 – Solve for α :
$625\alpha^2 = 25$
Divide both sides by 25:
$25\alpha^2 = 1$
Divide both sides by 25:
$\alpha^2 = \frac{1}{25}$
Take square root:
$|\alpha| = \frac{1}{5}$

94.(a) Step 1: Simplify by substituting $y = 0$
Take $y = 0$:

$f\left(\frac{x}{3}\right) = \frac{f(x)+f(0)+f(0)}{3} = \frac{f(x)+2f(0)}{3}$.

Multiply both sides by 3:

$3f\left(\frac{x}{3}\right) = f(x) + 2f(0) \Rightarrow f(x) = 3f\left(\frac{x}{3}\right) - 2f(0)$

This is a functional equation of linear type.

Step 2: Try linear solution

Assume $f(x) = ax + b$. Then $f(0) = b$. Check if it satisfies the original equation:

LHS:

$f\left(\frac{x+y}{3}\right) = a\frac{x+y}{3} + b = \frac{a(x+y)}{3} + b$

RHS:

$\frac{f(x)+f(y)+f(0)}{3} = \frac{(ax+b)+(ay+b)+b}{3} = \frac{a(x+y)+3b}{3} = \frac{a(x+y)}{3} + b$

Both sides match exactly.

So $f(x) = ax + b$ works.

Step 3: Differentiability at $x = 0$

We are told f is differentiable at $x = 0$. Our solution $f(x) = ax + b$ is differentiable everywhere, so no contradiction arises.

Step 4: Conclude

Hence, the function must be linear.

95.(c) We are asked:

Let f be differentiable for all real x. If $f(2) = -4$ and $f'(x) \geq 6$ for all $x \in [2,4]$, then find information about $f(4)$.

Let's solve step by step.

Step 1: Use the Mean Value Theorem (MVT)

Since f is differentiable on [2,4] and hence continuous on [2,4], by the MVT, there exists $c \in (2,4)$ such that

$f'(c) = \frac{f(4)-f(2)}{4-2} = \frac{f(4)-(-4)}{2} = \frac{f(4)+4}{2}$.

Step 2: Use the derivative bound

We are given $f'(x) \geq 6$ for all $x \in [2,4]$. So in particular, $f'(c) \geq 6$. Therefore,

$\frac{f(4)+4}{2} \geq 6 \Rightarrow f(4) + 4 \geq 12 \Rightarrow f(4) \geq 8$

96.(c) We are asked to find $P(E \mid F) - P(F \mid E)$ given:

$P(E) = 0.3$

$P(E \cup F) = 0.5$

E and F are independent.

Let's solve step by step.

Step 1: Find $P(F)$

For any two events:

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Since E and F are independent:

$P(E \cap F) = P(E)P(F) = 0.3 \cdot P(F)$

Plug into union formula:

$0.5 = 0.3 + P(F) - (0.3 \cdot P(F))$

$0.5 = 0.3 + P(F)(1 - 0.3)$

$0.5 = 0.3 + 0.7P(F)$

$0.7P(F) = 0.2 \Rightarrow P(F) = \frac{0.2}{0.7} = \frac{2}{7}$

Step 2: Find $P(E \mid F)$ and $P(F \mid E)$

$P(E \mid F) = \frac{P(E\cap F)}{P(F)} = \frac{0.3 \cdot P(F)}{P(F)} = 0.3$ (independence!)

$P(F \mid E) = \frac{P(E \cap F)}{P(E)} = \frac{0.3 \cdot P(F)}{0.3} = P(F) = \frac{2}{7}$

Step 3: Compute the difference

$P(E \mid F) - P(F \mid E) = 0.3 - \frac{2}{7} = \frac{21-20}{70} = \frac{1}{70}$

97. **(d)** The function
$f(x) = x - [x]$
is just the fractional-part function, usually denoted $\{x\} = x - \lfloor x \rfloor$.
On any open interval $(n, n+1)$ (where $n \in \mathbb{Z}$), $\lfloor x \rfloor = n$ is constant, so
$f(x) = x - n$
is continuous there.
At an integer $x = n$, the left- and right-hand limits disagree:
As $x \to n^-$,
$f(x) = x - (n-1) \to n - (n-1) = 1$
As $x \to n^+$,
$f(x) = x - n \to 0$
(and $f(n) = 0$).
Hence each integer n is a jump discontinuity.
the points of discontinuity are exactly the integers, $\mathbb{Z}$.

98. **(d)** Step 1: Recall formula
For a quadratic $x^2 + bx + c = 0$ with roots α, β :
$\alpha + \beta = -b, \alpha\beta = c$
Also, the sum of squares of roots is:
$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
Step 2: Identify coefficients
Here, the equation is:
$x^2 - (a-2)x - (a-1) = 0$
So:
$\alpha + \beta = a - 2, \alpha\beta = -(a-1)$
Step 3: Express sum of squares
$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (a-2)^2 - 2[-(a-1)] = (a-2)^2 + 2(a-1)$
Simplify:
$(a-2)^2 + 2(a-1) = a^2 - 4a + 4 + 2a - 2 = a^2 - 2a + 2$
So the sum of squares is:
$S(a) = a^2 - 2a + 2$
Step 4: Minimize $S(a)$
$S(a) = a^2 - 2a + 2$ is a quadratic in a. Minimum occurs at:
$a = -\frac{-2}{2 \cdot 1} = 1$

99. **(b)** Step 1: Split the integral at 0
Because of the absolute value, it's natural to split the integral:
$$I = \int_{-1}^{0} \frac{x^3 + (-x) + 1}{x^2 + 2(-x) + 1} dx + \int_0^1 \frac{x^3 + x + 1}{x^2 + 2x + 1} dx$$
For $x < 0, |x| = -x$
For $x > 0, |x| = x$
So:
$$I = \int_{-1}^{0} \frac{x^3 - x + 1}{x^2 - 2x + 1} dx + \int_0^1 \frac{x^3 + x + 1}{x^2 + 2x + 1} dx$$
Step 2: Factor denominators
Notice that:
$x^2 - 2x + 1 = (x-1)^2, x^2 + 2x + 1 = (x+1)^2$
So:
$$I = \int_{-1}^{0} \frac{x^3 - x + 1}{(x-1)^2} dx + \int_0^1 \frac{x^3 + x + 1}{(x+1)^2} dx$$
Step 3: Use substitution for symmetry
For the first integral, let $x = -t$, then $dx = -dt$ and when $x = -1 \to t = 1, x = 0 \to t = 0$.

$$\int_{-1}^{0}\frac{x^3-x+1}{(x-1)^2}dx=\int_{1}^{0}\frac{(-t)^3-(-t)+1}{(-t-1)^2}(-dt)$$
Simplify numerator:
$(-t)^3-(-t)+1=-t^3+t+1$
Denominator:
$(-t-1)^2=(-(t+1))^2=(t+1)^2$
Also, $dx=-dt$, so flipping limits and taking care of negative:
$$\int_{-1}^{0}\frac{x^3-x+1}{(x-1)^2}dx=\int_{0}^{1}\frac{-t^3+t+1}{(t+1)^2}dt$$
Step 4: Combine integrals
Now both integrals are from 0 to 1:
$$I=\int_{0}^{1}\frac{-x^3+x+1}{(x+1)^2}dx+\int_{0}^{1}\frac{x^3+x+1}{(x+1)^2}dx$$
Add the fractions:
$(-x^3+x+1)+(x^3+x+1)=2x+2=2(x+1)$
Denominator: $(x+1)^2$
So the integral becomes:
$$I=\int_{0}^{1}\frac{2(x+1)}{(x+1)^2}dx=\int_{0}^{1}\frac{2}{x+1}dx$$
Step 5: Integrate
$$I=2\int_{0}^{1}\frac{1}{x+1}dx=2[\ln|x+1|]_0^1=2(\ln 2-\ln 1)=2\ln 2$$

100. **(c)** Step 1: Condition for vectors to be coplanar
Vectors $\vec{u},\vec{v},\vec{w}$ are coplanar if their scalar triple product is zero:
$[\vec{u},\vec{v},\vec{w}]=\vec{u}\cdot(\vec{v}\times\vec{w})=0$
Step 2: Compute $\vec{v}\times\vec{w}$
$\vec{v}\times\vec{w}=(\lambda\vec{b}+4\vec{c})\times((2\lambda-1)\vec{c})$
Use distributive property:
$\vec{v}\times\vec{w}=\lambda(2\lambda-1)(\vec{b}\times\vec{c})+4(2\lambda-1)(\vec{c}\times\vec{c})$
But $\vec{c}\times\vec{c}=0$, so:
$\vec{v}\times\vec{w}=\lambda(2\lambda-1)(\vec{b}\times\vec{c})$
Step 3: Compute scalar triple product
$[\vec{u},\vec{v},\vec{w}]=\vec{u}\cdot(\vec{v}\times\vec{w})=(\vec{a}+2\vec{b}+3\vec{c})\cdot(\lambda(2\lambda-1)(\vec{b}\times\vec{c}))$
Factor out $\lambda(2\lambda-1)$:
$[\vec{u},\vec{v},\vec{w}]=\lambda(2\lambda-1)(\vec{a}+2\vec{b}+3\vec{c})\cdot(\vec{b}\times\vec{c})$
But $\vec{b}\cdot(\vec{b}\times\vec{c})=0$ and $\vec{c}\cdot(\vec{b}\times\vec{c})=0$. So only $\vec{a}$ term contributes:
$(\vec{a}+2\vec{b}+3\vec{c})\cdot(\vec{b}\times\vec{c})=\vec{a}\cdot(\vec{b}\times\vec{c})\neq 0$
(since $\vec{a},\vec{b},\vec{c}$ are non-coplanar).
Hence:
$[\vec{u},\vec{v},\vec{w}]=\lambda(2\lambda-1)(\vec{a}\cdot(\vec{b}\times\vec{c}))$
Step 4: Find values of λ making it zero
$[\vec{u},\vec{v},\vec{w}]=0 \implies \lambda(2\lambda-1)=0$
$\lambda=0$ or $2\lambda-1=0\Rightarrow\lambda=\dfrac{1}{2}$
Step 5: Conclusion
The vectors are coplanar only for $\lambda=0$ or $\lambda=1/2$.
Therefore, they are non-coplanar for all other values.

101. **(c)** Step 1: Properties of ω
If ω is a non-trivial cubic root of unity, then:
$\omega^3=1,\omega\neq 1$
Also, we know the useful relation:
$1+\omega+\omega^2=0\Rightarrow\omega^2=-1-\omega$
And the modulus squared property:

$|\omega| = 1, |\omega^2| = 1$
Step 2: Express $|a + b\omega + c\omega^2|^2$
Let:
$z = a + b\omega + c\omega^2$
We can write $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then $\omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$.
So:
$$a + b\omega + c\omega^2 = a + b\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + c\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$
Separate real and imaginary parts:
Real part:
$$\mathrm{Re}(z) = a - \frac{1}{2}b - \frac{1}{2}c = a - \frac{b+c}{2}$$
Imaginary part:
$$\mathrm{Im}(z) = \frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}c = \frac{\sqrt{3}}{2}(b-c)$$
So:
$$|z|^2 = \left(a - \frac{b+c}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}(b-c)\right)^2$$
$$|z|^2 = \left(a - \frac{b+c}{2}\right)^2 + \frac{3}{4}(b-c)^2$$
Step 3: Minimize $|z|^2$ for distinct nonzero integers a, b, c
We want a, b, c distinct nonzero integers.
It is usually enough to try small integers 1,2,3 (positive or negative) since minimum occurs for small numbers.
$$|z|^2 = \left(a - \frac{b+c}{2}\right)^2 + \frac{3}{4}(b-c)^2$$
Try $a = 1, b = 2, c = 3$:
Re part $= 1 - (2+3)/2 = 1 - 5/2 = -3/2$
Impart $= \frac{\sqrt{3}}{2}(2-3) = -\frac{\sqrt{3}}{2}$
$|z|^2 = (-3/2)^2 + (-\sqrt{3}/2)^2 = 9/4 + 3/4 = 12/4 = 3$

102. **(b)** We are asked to evaluate:
$$\int_0^{1.5} [x^2]dx$$
Here, $[x^2]$ is the floor function (greatest integer $\leq x^2$).
Step 1: Find the intervals where $[x^2]$ is constant
For $x \in [0,1), x^2 \in [0,1) \Rightarrow [x^2] = 0$
For $x \in [1,\sqrt{2}), x^2 \in [1,2) \Rightarrow [x^2] = 1$
For $x \in [\sqrt{2},\sqrt{3}), x^2 \in [2,3) \Rightarrow [x^2] = 2$
Here, the upper limit is 1.5 . Since $\sqrt{2} \approx 1.414, 1.5 > \sqrt{2}$ but $< \sqrt{3} \approx 1.732$. So, split the integral:
$$\int_0^{1.5} [x^2]dx = \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{1.5} 2dx$$
Step 2: Evaluate each integral
1. $\int_0^1 0dx = 0$
2. $\int_1^{\sqrt{2}} 1dx = (\sqrt{2} - 1)$
3. $\int_{\sqrt{2}}^{1.5} 2dx = 2 \cdot (1.5 - \sqrt{2}) = 3 - 2\sqrt{2}$
Step 3: Add them up
$0 + (\sqrt{2} - 1) + (3 - 2\sqrt{2}) = (\sqrt{2} - 1 + 3 - 2\sqrt{2}) = 2 - \sqrt{2}$

103. **(b)** Given: Sum of first n terms of an A.P. is
$S_n = 3n^2 + 5n$

The m-th term $t_m = 164$. Find m.
Step 1: Formula for n-th term
We know the n-th term of an A.P. is:
$t_n = S_n - S_{n-1}$
Here:
$S_n = 3n^2 + 5n$
$S_{n-1} = 3(n-1)^2 + 5(n-1) = 3(n^2 - 2n + 1) + 5n - 5$
$= 3n^2 - 6n + 3 + 5n - 5 = 3n^2 - n - 2$
So, the n-th term is:
$t_n = S_n - S_{n-1} = (3n^2 + 5n) - (3n^2 - n - 2) = 6n + 2$
Step 2: Solve for m
We are given $t_m = 164$, so:
$6m + 2 = 164$
$6m = 162$
$m = 27$

104. **(c)** Given:

$$x = \int_0^y \frac{1}{\sqrt{1+9t^2}} dt$$

and $\frac{d^2y}{dx^2} = ay$
We are to find a.
Step 1 - Find $\frac{dy}{dx}$:
Differentiate both sides of the first equation with respect to x :

$$\frac{dx}{dx} = \frac{d}{dx}\left(\int_0^y \frac{1}{\sqrt{1+9t^2}} dt\right)$$

Using the Fundamental Theorem of Calculus and chain rule:

$$1 = \frac{1}{\sqrt{1+9y^2}} \cdot \frac{dy}{dx}$$

Thus,

$$\frac{dy}{dx} = \sqrt{1+9y^2}$$

Step 2 - Find $\frac{d^2y}{dx^2}$:
Differentiate $\frac{dy}{dx}$ with respect to x :

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\sqrt{1+9y^2}\right)$$

$$= \frac{1}{2\sqrt{1+9y^2}} \cdot 18y \cdot \frac{dy}{dx}$$

$$= \frac{9y}{\sqrt{1+9y^2}} \cdot \sqrt{1+9y^2}$$

$= 9y$
Step 3 - Equating $\frac{d^2y}{dx^2} = ay$:
$9y = ay$
So,$a = 9$

105. **(c)** Step 1- Expand permutations

$$^9P_5 = \frac{9!}{(9-5)!} = \frac{9!}{4!}$$

$$^9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!}$$

So, the left-hand side becomes:

$$\frac{9!}{4!} + 5 \cdot \frac{9!}{5!}$$

Now, factor $\frac{9!}{5!}$:

$= \frac{9!}{4!} + 5 \cdot \frac{9!}{5!} = \frac{9!}{5!}\left(\frac{5!}{4!} + 5\right)$

$= \frac{9!}{5!}(5+5)$

$= \frac{9!}{5!} \cdot 10$

$= 10 \cdot \frac{9!}{5!}$

Step 2 – Expand ${}^{10}P_r$

${}^{10}P_r = \frac{10!}{(10-r)!}$

We want to find r such that:

$10 \cdot \frac{9!}{5!} = \frac{10!}{(10-r)!}$

But $10! = 10 \cdot 9!$, so:

$10 \cdot \frac{9!}{5!} = \frac{10 \cdot 9!}{(10-r)!}$

Cancel $10 \cdot 9$! on both sides:

$\frac{1}{5!} = \frac{1}{(10-r)!}$

So:

$(10-r)! = 5!$

Thus:

$10 - r = 5$

$r = 10 - 5 = 5$

106. **(d)** Step 1 - Recall the vector identities

1.The magnitude of the cross product:

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

2.The dot product:

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

The problem gives:

$|\vec{a} \times \vec{b}|^2 = k^2 - (\vec{a} \cdot \vec{b})^2$

Substitute the above identities:

$(|\vec{a}||\vec{b}|\sin\theta)^2 = k^2 - (|\vec{a}||\vec{b}|\cos\theta)^2$

$(7 \cdot 1 \cdot \sin\theta)^2 = k^2 - (7 \cdot 1 \cdot \cos\theta)^2$

$49\sin^2\theta = k^2 - 49\cos^2\theta$

Bring $49\cos^2\theta$ to the left-hand side:

$49\sin^2\theta + 49\cos^2\theta = k^2$

Factor 49 :

$49(\sin^2\theta + \cos^2\theta) = k^2$

Since $\sin^2\theta + \cos^2\theta = 1$:

$49 \cdot 1 = k^2$

So,$k^2 = 49$

Thus,$k = 7$

Step 2 - Finding θ

We found that $k = 7$, and this holds for any value of θ, because the identity $\sin^2\theta + \cos^2\theta = 1$ is always true.

So, θ can be arbitrary!

Final Conclusion

$k = 7$

θ is arbitrary (can be any angle)

107. **(d)** Given:

$P(\cos\alpha, \sin\beta)$

$Q(\sin\alpha, \cos\beta)$
$R(0,0)$
$0 < \alpha, \beta < \frac{\pi}{4}$
We are to check whether these points are collinear or one lies on the line segment between the others.
Step 1 - Check collinearity
Three points are collinear if the area of the triangle formed by them is zero.
Area formula:
Area $= \frac{1}{2}\left|x_P(y_Q - y_R) + x_Q(y_R - y_P) + x_R(y_P - y_Q)\right|$
Substituting the coordinates:
$= \frac{1}{2}[\cos\alpha(\cos\beta - 0) + \sin\alpha(0 - \sin\beta) + 0(\sin\beta - \cos\beta)]$
$= \frac{1}{2}(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$
$= \frac{1}{2}\cos(\alpha + \beta)$
For collinearity, this area must be zero, so $\cos(\alpha + \beta) = 0$.
But $0 < \alpha, \beta < \frac{\pi}{4}$, hence $0 < \alpha + \beta < \frac{\pi}{2}$, so $\cos(\alpha + \beta)$ cannot be zero!
Conclusion: The area is non-zero $\Rightarrow$ the points are non-collinear.

108. **(d)** A real skew-symmetric matrix A must satisfy $A^T = -A$. Hence
All diagonal entries a_{ii} obey $a_{ii} = -a_{ii}$, so $a_{ii} = 0$.
For each off-diagonal pair $i
There are $\frac{n(n-1)}{2}$ such independent pairs, so the total number of skew-symmetric $n \times n$ matrices with entries in $\{-1,0,1\}$ is
$3^{\frac{n(n-1)}{2}}$.

109. **(b)** Given:
The cubic equation is $x^3 + qx + r = 0$, roots are α, β, γ, in A.P.
So $\beta = \alpha + d, \gamma = \alpha + 2d$.
Step 1-Sum of roots:
$\alpha + \beta + \gamma = 3\alpha + 3d = 0 \Rightarrow \alpha + d = 0 \Rightarrow \beta = 0$
So $\beta = 0, \alpha = -dr\gamma = 2d$.
Step 2 - The matrix:
$$A = \begin{bmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{bmatrix} = \begin{bmatrix} -d & 0 & d \\ 0 & d & -d \\ d & -d & 0 \end{bmatrix}$$
Step 3 - Find rank:
Adding all three rows gives zero row $\rightarrow$ rows are linearly dependent.
But not all rows are zero $\rightarrow$ rank is 2 .

110. **(d)** Telescoping product $\rightarrow$ a double-angle
One shows by induction (or by repeated half-angle/angle-doubling algebra) that
$f_n(x) = \tan\frac{x}{2}(1 + \sec x)(1 + \sec 2x) \cdots \cdots (1 + \sec 2^n x) = \tan(2^n x)$
Plug in $x = \frac{\pi}{16}$:
$f_n\left(\frac{\pi}{16}\right) = \tan\left(2^n \cdot \frac{\pi}{16}\right) = \tan\left(\frac{2^n\pi}{16}\right)$
Solve
We want $\tan\left(\frac{2^n\pi}{16}\right) = 1$. That means
$\frac{2^n\pi}{16} = \frac{\pi}{4} + k\pi, k \in \mathbb{Z} \implies 2^n = 16\left(k + \frac{1}{4}\right) = 4(4k + 1)$.
The only power of two of the form $4m + 1$ is 1 itself, giving
$2^n = 4 \implies n = 2, k = 0$.
Hence

$f_2\left(\frac{\pi}{16}\right) = \tan\left(\frac{2^2\pi}{16}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

111. **(c)** The expression becomes: ${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$.
Apply the identity ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$ repeatedly:
Combine ${}^{47}C_4 + {}^{47}C_3$:
${}^{47}C_4 + {}^{47}C_3 = {}^{48}C_4$.
The expression now is:
${}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$.
Combine ${}^{48}C_4 + {}^{48}C_3$:
${}^{48}C_4 + {}^{48}C_3 = {}^{49}C_4$.
The expression now is:
${}^{49}C_4 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$.
Combine ${}^{49}C_4 + {}^{49}C_3$:
${}^{49}C_4 + {}^{49}C_3 = {}^{50}C_4$
The expression now is:
${}^{50}C_4 + {}^{50}C_3 + {}^{51}C_3$.
Combine ${}^{50}C_4 + {}^{50}C_3$:
${}^{50}C_4 + {}^{50}C_3 = {}^{51}C_4$.
The expression now is:
${}^{51}C_4 + {}^{51}C_3$.
Combine ${}^{51}C_4 + {}^{51}C_3$:
${}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$.

112. **(c)** Given:
$\text{adj}(B) = A$
$|P| = |Q| = 1$
We are asked to find $\text{adj}(Q^{-1}BP^{-1})$.
Step 1- Property of adjoint
For any invertible matrix M :
$\text{adj}(M) = |M| \cdot M^{-1}$
Also, if $M = XYZ$, then:
$|M| = |X| \cdot |Y| \cdot |Z|$
$M^{-1} = Z^{-1}Y^{-1}X^{-1}$
Step 2 - Determinant of $Q^{-1}BP^{-1}$
$|Q^{-1}BP^{-1}| = |Q^{-1}| \cdot |B| \cdot |P^{-1}| = \frac{1}{|Q|} \cdot |B| \cdot \frac{1}{|P|}$
Since $|P| = |Q| = 1$, this simplifies to:
$|Q^{-1}BP^{-1}| = |B|$
Step 3 - Inverse of $Q^{-1}BP^{-1}$
$(Q^{-1}BP^{-1})^{-1} = PB^{-1}Q$
Step 4 - Apply adjoint formula
$\text{adj}(Q^{-1}BP^{-1}) = |Q^{-1}BP^{-1}| \cdot (Q^{-1}BP^{-1})^{-1}$
Substituting:
$= |B| \cdot PB^{-1}Q$
But from the given $\text{adj}(B) = A$, we know:
$A = |B| \cdot B^{-1}$
So,
$|B| \cdot B^{-1} = A$
Thus,
$= PAQ$

113. **(d)** Given:
Vectors $\vec{a}, \vec{b}, \vec{c}$ all have the same magnitude, say $|\vec{a}| = |\vec{b}| = |\vec{c}| = k$.
Angles between them:
Between $\vec{a}$ and $\vec{b}$: α
Between $\vec{b}$ and $\vec{c}$; β

Between $\vec{c}$ and $\vec{a}$: γ
We are to find the minimum of:
$\cos\alpha+\cos\beta+\cos\gamma$
Step 1 - Express cosine using dot product
For vectors of equal magnitude k, the cosine between any two is:
$$\cos\alpha=\frac{\vec{a}\cdot\vec{b}}{k^2},\cos\beta=\frac{\vec{b}\cdot\vec{c}}{k^2},\cos\gamma=\frac{\vec{c}\cdot\vec{a}}{k^2}$$
So,
$$\cos\alpha+\cos\beta+\cos\gamma=\frac{1}{k^2}(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$
Thus, minimizing the sum of cosines is equivalent to minimizing $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.
Step 2 - Use an identity
Consider the expression:
$|\vec{a}+\vec{b}+\vec{c}|^2\geq 0$
Expanding it:
$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\geq 0$
Since $|\vec{a}|^2=|\vec{b}|^2=|\vec{c}|^2=k^2$, this becomes:
$3k^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\geq 0$
Therefore,
$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\geq-\frac{3k^2}{2}$$
Dividing by k^2,
$$\cos\alpha+\cos\beta+\cos\gamma\geq-\frac{3}{2}$$
Final Conclusion
The minimum value of $\cos\alpha+\cos\beta+\cos\gamma$ is:
$$-\frac{3}{2}$$

114. **(d)** Let $f(x)=ax^2+bx+c$ be a second degree polynomial.
Step 1: Use $f(1)=f(-1)$
$f(1)=a(1)^2+b(1)+c=a+b+c$
$f(-1)=a(-1)^2+b(-1)+c=a-b+c$
Given $f(1)=f(-1)$:
$a+b+c=a-b+c\Longrightarrow b=0$
So $f(x)=ax^2+c$ and $f'(x)=2ax$.
Step 2: Evaluate $f'(p),f'(q),f'(r)$
$f'(p)=2ap,f'(q)=2aq,f'(r)=2ar$
Given p,q,r are in A.P.:
$q-p=r-q\Longrightarrow 2q=p+r$
Multiply all by $2a$:
$$2aq=2a\cdot\frac{p+r}{2}\Longrightarrow f'(q)=\frac{f'(p)+f'(r)}{2}$$
This shows $f'(p),f'(q),f'(r)$ are also in A.P.

115. **(c)** Step 1: Find intersection point
The intersection point (x_0,y_0) satisfies both equations:
1.$ax+2by+3b=0\Longrightarrow ax+2by=-3b$
2.$bx-2ay-3a=0\Longrightarrow bx-2ay=3a$
We have the system:
$$\begin{cases}ax+2by=-3b\\bx-2ay=3a\end{cases}$$
Step 2: Solve for y
Add the two equations cleverly. To eliminate x, multiply first by b and second by a :
1. $abx+2b^2y=-3b^2$
2.$abx-2a^2y=3a^2$

Now subtract (1) - (2):
$(abx+2b^2y)-(abx-2a^2y)=-3b^2-3a^2$
$2b^2y+2a^2y=-3(a^2+b^2) \Longrightarrow 2(a^2+b^2)y=-3(a^2+b^2)$
$y=-\frac{3}{2}$
So the intersection point has y-coordinate $y_0=-3/2$.
Step 3: Line parallel to x -axis
A line parallel to the x -axis has equation:
$y=$ constant
Since it passes through the intersection, the line is:
$$y=-\frac{3}{2}$$
This is below the x-axis at a distance $3/2$.

116. **(d)** Step 1: Check continuity on [0,2]
The function $f(x)=2+(x-1)^{2/3}$ is a sum of a constant and a power function $(x-1)^{2/3}$.
The function $g(x)=(x-1)^{2/3}$ is continuous everywhere.
Therefore, $f(x)$ is continuous on [0,2].
Step 2: Check differentiability on (0,2)
$$f'(x)=\frac{2}{3}(x-1)^{-1/3}$$
This derivative does not exist at $x=1$ because of the negative exponent.
So f is not differentiable at $x=1$.
Thus, the statement " f is not derivable in (0,2)" is partially correct, but it's better phrased as " f is not derivable at x = 1."
Step 3: Check $f(0)$ and $f(2)$
$f(0)=2+(0-1)^{2/3}=2+1=3$
$f(2)=2+(2-1)^{2/3}=2+1=3$
So $f(0)=f(2)$.
Step 4: Check Rolle's theorem applicability
Rolle's theorem requires:
1.f continuous on [0,2]
2.f differentiable on (0,2) × (fails at $x=1$)
3.$f(0)=f(2)$
Since f is not differentiable at $x=1$, Rolle's theorem cannot be applied.

117. **(c)** Reflexive relation means: for every element $a \in A$, the pair (a,a) must be in the relation.
A set A has n elements.
Total possible pairs in $A \times A=n^2$.
In a reflexive relation, all n diagonal pairs (a,a) are fixed.
The remaining n^2-n pairs can either be in or out of the relation freely.
So, the total number of reflexive relations:
2^{n^2-n}

118. **(a)** We are asked:
Let $f(x)$ be continuous on [0,5] and differentiable on (0,5). If $f(0)=0$ and $|f'(x)| \leq \frac{1}{5}$ for all $x \in (0,5)$, then find a bound on $f(x)$.

This is a classic application of the Mean Value Theorem (MVT).

Step 1: Apply MVT
MVT says: for any $x \in [0,5]$, there exists some $c \in (0,x)$ such that

$$f(x)-f(0)=f'(c)(x-0) \Longrightarrow f(x)=f'(c)\cdot x$$

Step 2: Use the derivative bound
Given $|f'(x)| \leq \frac{1}{5}$, we get

$$|f(x)| = |f'(c) \cdot x| \le \frac{1}{5} \cdot x$$

Since $x \in [0,5]$, the maximum value occurs at $x = 5$:

$$|f(x)| \le \frac{1}{5} \cdot 5 = 1$$

119. **(b)** We are asked to evaluate:
$$\lim_{x\to 0} \frac{\tan([-\pi^2]x^2) - x^2\tan([-\pi^2])}{\sin^2 x}$$
Step 1: Use small-angle approximations
For $x \to 0$, we know:
$\sin x \sim x$ and $\tan y \sim y$ when $y \to 0$.
So $\sin^2 x \sim x^2$.
Step 2: Rewrite the numerator
$\tan([-\pi^2]x^2) - x^2 \tan([-\pi^2]) \sim [-\pi^2 x^2] - x^2 \tan([-\pi^2])$
$= x^2([-\pi^2] - \tan([-\pi^2]))$.
Step 3: Divide by $\sin^2 x \sim x^2$
$$\frac{\tan([-\pi^2]x^2) - x^2\tan([-\pi^2])}{\sin^2 x} \sim \frac{x^2([-\pi^2] - \tan([-\pi^2]))}{x^2} = [-\pi^2] - \tan([-\pi^2]).$$
Step 4: Evaluate the constants
$[-\pi^2]$ denotes the floor of $-\pi^2$, i.e., -10 .
$\tan([-\pi^2]) = \tan(-10)$ (in radians).
So the limit becomes:
$-10 - \tan(-10) = -10 + \tan 10 = \tan 10 - 10$

120. **(c)** Step 1: Transform the equation
We know that $\cos^{-1} x \in [0, \pi]$. The sum of three such terms is 3π. This implies each term is π :
$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi \Rightarrow \cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi$
So,
$\alpha = \beta = \gamma = \cos\pi = -1$
Step 2: Substitute into the expression
$\alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta)$
Substitute $\alpha = \beta = \gamma = -1$:
$(-1)((-1)+(-1)) + (-1)((-1)+(-1)) + (-1)((-1)+(-1))$
$= (-1)(-2) + (-1)(-2) + (-1)(-2)$
$= 2 + 2 + 2 = 6$

121. **(d)** The area of the parallelogram spanned by $\vec{\alpha}$ and $\vec{\beta}$ is
$$\|\vec{\alpha} \times \vec{\beta}\| = \sqrt{\|\vec{\alpha}\|^2\|\vec{\beta}\|^2 - (\vec{\alpha}\cdot\vec{\beta})^2}$$
Compute $\|\vec{\alpha}\|$:
$\vec{\alpha} = (3,0,-1) \implies \|\vec{\alpha}\|^2 = 3^2 + 0^2 + ((-1)^2 = 10$

Given $\|\vec{\beta}\|^2 = (\sqrt{5})^2 = 5$.
Given $\vec{\alpha}\cdot\vec{\beta} = 3$, so $(\vec{\alpha}\cdot\vec{\beta})^2 = 9$.
Plug into the formula:
$\|\vec{\alpha}\times\vec{\beta}\| = \sqrt{10\cdot 5 - 9} = \sqrt{50-9} = \sqrt{41}$

122. **(c)** Step 1: Find derivative
$$f'(x) = \frac{d}{dx}(\alpha\log|x| + \beta x^2 + x) = \frac{\alpha}{x} + 2\beta x + 1.$$
For extreme points, $f'(x) = 0$.
$\frac{\alpha}{x} + 2\beta x + 1 = 0 \Rightarrow \alpha + 2\beta x^2 + x = 0$ (multiply by x)
So the condition for extremum at $x = -1$ and $x = 2$ is:
1.

$x = -1: \alpha + 2\beta(-1)^2 + (-1) = 0$
$\Rightarrow \alpha + 2\beta - 1 = 0$
$\Rightarrow \alpha + 2\beta = 1 \ldots$ (i)
2.
$x = 2: \alpha + 2\beta(2)^2 + 2 = 0$
$\Rightarrow \alpha + 8\beta + 2 = 0$
$\Rightarrow \alpha + 8\beta = -2 \ldots$ (ii)
Step 2: Solve the system of equations
From (i) and (ii):
$\begin{cases} \alpha + 2\beta = 1 \\ \alpha + 8\beta = -2 \end{cases}$
Subtract first from second:
$(\alpha + 8\beta) - (\alpha + 2\beta) = -2 - 1 \Longrightarrow 6\beta = -3 \Longrightarrow \beta = -\frac{1}{2}$
Substitute $\beta = -1/2$ into (i):
$\alpha + 2(-1/2) = 1 \Longrightarrow \alpha - 1 = 1 \Longrightarrow \alpha = 2.$
Step 3: Conclusion
$\alpha = 2, \beta = -\frac{1}{2}.$

123. **(c)** Step 1: Recall the property of adjugate
$A \cdot \operatorname{adj}(A) = |A|I$
Here, $|A| = 6$, so
$A \cdot \operatorname{adj}(A) = 6I_3$
Step 2: Use the determinant property
Also, for any 3×3 matrix:
$|\operatorname{adj}(A)| = |A|^{n-1} = |A|^2$ (since n = 3)
So $|\operatorname{adj}(A)| = 6^2 = 36.$
Step 3: Use the formula for last row to find k
The last row of $\operatorname{adj}(A)$ is $[-1, k, 0]$.
Let's check a simpler approach: In a 3×3 matrix, the sum of products of elements of any row of A with corresponding elements of a row of adj(A) gives $|A|$ for the same row, and 0 for different rows.
Let's denote the rows of A as R_1, R_2, R_3.
The rows of adj(A) are C_1, C_2, C_3(adj(A) is transposed cofactor matrix).
Specifically, $R_3 \cdot \operatorname{adj}(A)_3 = |A| = 6.$
Last row of adj(A) $= [-1, k, 0]$.
Suppose last row of $A = [a, b, c]$. Then
$a(-1) + b(k) + c(0) = 6 \Longrightarrow -a + bk = 6$
We don't know a, b but maybe there's a simpler way: check the determinant of adj(**A**).
Step 4: Compute determinant of $\operatorname{adj}(A)$
$\operatorname{adj}(A) = \begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & 1 \\ -1 & k & 0 \end{bmatrix}$
$|\operatorname{adj}(A)| = 1\begin{vmatrix} 1 & 1 \\ k & 0 \end{vmatrix} - (-2)\begin{vmatrix} 4 & 1 \\ -1 & 0 \end{vmatrix} + 4\begin{vmatrix} 4 & 1 \\ -1 & k \end{vmatrix}$
Compute each minor:
1.$\begin{vmatrix} 1 & 1 \\ k & 0 \end{vmatrix} = 1 * 0 - 1 * k = -k$
2.$\begin{vmatrix} 4 & 1 \\ -1 & 0 \\ 4 & 1 \end{vmatrix} = 4 * 0 - 1 * (-1) = 1$
3.$\begin{vmatrix} 4 & 1 \\ -1 & k \end{vmatrix} = 4 * k - 1 * (-1) = 4k + 1$
Now determinant:
$|\operatorname{adj}(A)| = 1 * (-k) - (-2) * (1) + 4 * (4k + 1) = -k + 2 + 16k + 4 = 15k + 6$
Step 5: Set equal to $|A|^2 = 36$
$15k + 6 = 36 \Longrightarrow 15k = 30 \Longrightarrow k = 2$

124. **(a)** Step 1: Use the logarithm change-of-base formula

Recall:

$\log_a b = \frac{\ln b}{\ln a}, \log_b a = \frac{\ln a}{\ln b},$ etc.

So the determinant becomes:

$$D = \begin{vmatrix} 1 & \frac{\ln b}{\ln a} & \frac{\ln c}{\ln a} \\ \frac{\ln a}{\ln b} & 1 & \frac{\ln c}{\ln b} \\ \frac{\ln a}{\ln c} & \frac{\ln b}{\ln c} & 1 \end{vmatrix}$$

Step 2: Factor out $\frac{1}{\ln a}, \frac{1}{\ln b}, \frac{1}{\ln c}$ from columns

Let's multiply each column by its denominator:

Multiply 1st column by $\ln a$

Multiply 2nd column by $\ln b$

Multiply 3rd column by $\ln c$

Then the determinant becomes:

$$D = \frac{1}{\ln a \ln b \ln c} \begin{vmatrix} \ln a & \ln b & \ln c \\ \ln a & \ln b & \ln c \\ \ln a & \ln b & \ln c \end{vmatrix}$$

Step 3: Observe identical rows

All rows are proportional! So determinant = 0.

125. **(d)** Symmetric form

$$\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0} = t$$

Equivalently,

$(x, y, z) = (3,2,1) + t(3,1,0)$

So its direction vector is

$\mathrm{d} = (3,1,0)$

Compare with the z-axis direction vector

$\mathrm{k} = (0,0,1)$:

$\mathrm{d} \cdot \mathrm{k} = 3 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 = 0.$

A zero dot product means the two lines are perpendicular.

A zero dot product means the two lines are perpendicular.

Since it's neither purely along x nor y, but its direction is orthogonal to the z-axis, the correct choice is

126. **(b)** We are given:

First term $a = 11$

Sum of first 4 terms $S_4 = 56$

Sum of last 4 terms $S_{\text{last } 4} = 112$

Number of terms $= n$ (to be found)

Step 1: Express sum of first 4 terms

For an AP, the sum of first n terms:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

So, sum of first 4 terms:

$$S_4 = \frac{4}{2}[2 \cdot 11 + 3d] = 2[22 + 3d] = 44 + 6d$$

We are given $S_4 = 56$, so

$44 + 6d = 56 \Longrightarrow 6d = 12 \Longrightarrow d = 2$

Common difference $d = 2$

Step 2: Express sum of last 4 terms

Let the number of terms be n. Then the last 4 terms are:

$a + (n-4)d, a + (n-3)d, a + (n-2)d, a + (n-1)d$

Sum of last 4 terms:

$S_{\text{last } 4} = 4a + [(n-4) + (n-3) + (n-2) + (n-1)]d$

Sum of indices:

$(n-4)+(n-3)+(n-2)+(n-1)=4n-10$
So:
$S_{\text{last }4}=4a+(4n-10)d$
Substitute $a=11, d=2, S_{\text{last }4}=112$:
$112=4\cdot 11+(4n-10)\cdot 2$
$112=44+8n-20$
$112=24+8n \Longrightarrow 8n=88 \Longrightarrow n=11$

127. **(c)** Step 1: Split the logarithm

$$\log\frac{4+3\sin x}{4+3\cos x}=\log(4+3\sin x)-\log(4+3\cos x)$$

So the integral becomes:

$$I=\int_0^{\pi/2}\log(4+3\sin x)dx-\int_0^{\pi/2}\log(4+3\cos x)dx$$

Step 2: Use the property of definite integrals
We know that for $f(x)$:

$$\int_0^{\pi/2}f(x)dx=\int_0^{\pi/2}f\left(\frac{\pi}{2}-x\right)dx$$

Apply this to the second integral:

$$\int_0^{\pi/2}\log(4+3\cos x)dx=\int_0^{\pi/2}\log(4+3\sin x)dx$$

Step 3: Subtract the integrals

$$I=\int_0^{\pi/2}\log(4+3\sin x)dx-\int_0^{\pi/2}\log(4+3\sin x)dx=0$$

Step 3: Subtract the integrals

$$I=\int_0^{\pi/2}\log(4+3\sin x)dx-\int_0^{\pi/2}\log(4+3\sin x)dx=0$$

128. **(c)** Step 1: Recall formula for sum of squares of roots

For a quadratic $x^2+px+q=0$ with roots α,β :
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
Here, $\alpha+\beta=-($ coefficient of $x)=a-2$ (careful with the sign!)
The given equation:
$x^2-(a-2)x-(a+1)=0$
Sum of roots: $\alpha+\beta=a-2$
Product of roots: $\alpha\beta=-(a+1)$ (because it's the constant term with negative sign)
Check: Standard quadratic $x^2+px+q=0 \Rightarrow$ sum $=-p$, product $=q$. Here $p=-(a-2)$, so sum $=-(-(a-2))=a-2$. Correct. Product = q = −(a + 1).
Step 2: Express sum of squares
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(a-2)^2-2(-(a+1))$
$\alpha^2+\beta^2=(a-2)^2+2(a+1)$
$\alpha^2+\beta^2=a^2-4a+4+2a+2=a^2-2a+6$
Step 3: Minimize the quadratic
We have $S(a)=a^2-2a+6$.
This is a quadratic in a, opening upwards (coefficient of $a^2>0$)
Minimum occurs at $a=-\frac{-2}{2\cdot 1}=1$
Minimum sum of squares occurs at $a=1$

129. **(b)** Step 1: Use the trick for sum of even coefficients

For a polynomial $f(x)=\sum_{k=0}^{n}a_kx^k$, the sum of even coefficients is:

$$S_{\text{even}}=\frac{f(1)+f(-1)}{2}$$

and the sum of odd coefficients is:

$$S_{\text{odd}}=\frac{f(1)-f(-1)}{2}$$

Here, "even coefficients" means coefficients of x^0, x^2, x^4, ... But the problem asks for $a_2 + a_4 + \cdots + a_{12}$, i.e., even powers except x^0. We'll adjust later.
Step 2: Compute $f(1)$ and $f(-1)$
$f(x) = (1 + x - 2x^2)^6$
$f(1) = (1 + 1 - 2 \cdot 1^2)^6 = (1 + 1 - 2)^6 = 0^6 = 0$
$f(-1) = (1 - 1 - 2 \cdot 1)^6 = (-2)^6 = 64$
Step 3: Sum of coefficients at even powers

$$\text{Sum of all even coefficents } = \frac{f(1) + f(-1)}{2} = \frac{0 + 64}{2} = 32$$

The even coefficients include $a_0 + a_2 + \cdots + a_{12}$.
Since $a_0 = 1$ (constant term), we get:
$a_2 + a_4 + \cdots + a_{12} = 32 - 1 = 31$

130. **(d)** Step 1: Understand the conditions
1.$\vec{a} \cdot \vec{b} = 0 \Longrightarrow \vec{a}$ is perpendicular to $\vec{b}$.
2.$\vec{a} \cdot \vec{c} = 0 \Longrightarrow \vec{a}$ is perpendicular to $\vec{c}$.
So, $\vec{a}$ is perpendicular to the plane formed by $\vec{b}$ and $\vec{c}$.
Step 2: Use cross product property
The cross product $\vec{b} \times \vec{c}$ is perpendicular to both $\vec{b}$ and $\vec{c}$.
So, $\vec{a}$ must be parallel to $\vec{b} \times \vec{c}$.
Hence, $\vec{a} = \pm k(\vec{b} \times \vec{c})$, for some scalar $k > 0$.
Step 3: Check unit vector
Given that $\vec{a}$ is a unit vector: $|\vec{a}| = 1$
Also, $|\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}|\sin\theta = 1 \cdot 1 \cdot \sin(\pi/6) = 1/2$
So, to make $|\vec{a}| = 1$:

$$|\vec{a}| = k|\vec{b} \times \vec{c}| = k \cdot \frac{1}{2} = 1 \Longrightarrow k = 2$$

Thus:
$\vec{a} = \pm 2(\vec{b} \times \vec{c})$

131. **(b)** A non-leap year has 365 days, i.e., $52 \times 7 + 1 = 52$ full weeks plus 1 extra day.
52 weeks → exactly 52 Sundays and 52 Saturdays.
The extra 1 day determines whether the year has 53 Sundays or 53 Saturdays.
1.The extra day can be any day of the week (Sunday, Monday, ..., Saturday).
2.So there are 7 possibilities for the extra day.
If the extra day is Sunday → 53 Sundays.
If the extra day is Saturday → 53 Saturdays.
Hence, favorable outcomes = 2 (Saturday or Sunday).
Total outcomes = 7(all days of the week).

$$\text{Probability } = \frac{2}{7}$$

132. **(a)** Step 1: Interpret the problem
Z_1, Z_2, Z_3 are complex numbers with modulus 1 → they lie on the unit circle.
Their sum is zero → geometrically, the centroid of the triangle formed by these points is at the origin.
This is a special case: three points on the unit circle that sum to zero are the vertices of an equilateral triangle.
Step 2: Find the side length
For an equilateral triangle inscribed in the unit circle:
Let $Z_1 = 1$, then $Z_2 = e^{i2\pi/3}, Z_3 = e^{i4\pi/3}$.
Distance between Z_1 and Z_2 :
$|Z_1 - Z_2| = \left|1 - e^{i2\pi/3}\right| = \sqrt{(1 - \cos 120°)^2 + (\sin 120°)^2}$
Compute carefully:
$\cos 120° = -1/2, \sin 120° = \sqrt{3}/2$

$$|Z_1 - Z_2| = \sqrt{(1 - (-1/2))^2 + (\sqrt{3}/2)^2} = \sqrt{(3/2)^2 + (\sqrt{3}/2)^2}$$

$$= \sqrt{9/4 + 3/4} = \sqrt{12/4} = \sqrt{3}$$

So side length $a = \sqrt{3}$.
Step 3: Area of equilateral triangle
For side a, area is:

$$\text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(\sqrt{3})^2 = \frac{\sqrt{3}}{4}\cdot 3 = \frac{3\sqrt{3}}{4}$$

133. **(b)** Step 1: Expand the determinant
Use the first row for expansion:

$$f(\theta) = 1\cdot\begin{vmatrix} 1 & -\cos\theta \\ \sin\theta & 1\end{vmatrix} - \cos\theta\cdot\begin{vmatrix} -\sin\theta & -\cos\theta \\ -1 & 1\end{vmatrix} + (-1)\cdot\begin{vmatrix} -\sin\theta & 1 \\ -1 & \sin\theta\end{vmatrix}.$$

Step 2: Compute each 2×2 determinant

1.$\begin{vmatrix} 1 & -\cos\theta \\ \sin\theta & 1\end{vmatrix} = (1)(1) - (-\cos\theta)(\sin\theta) = 1 + \sin\theta\cos\theta$

2.$\begin{vmatrix} -\sin\theta & -\cos\theta \\ -1 & 1\end{vmatrix} = (-\sin\theta)(1) - (-\cos\theta)(-1) = -\sin\theta - \cos\theta$

3.$\begin{vmatrix} -\sin\theta & 1 \\ -1 & \sin\theta\end{vmatrix} = (-\sin\theta)(\sin\theta) - (1)(-1)$

$= -\sin^2\theta + 1 = 1 - \sin^2\theta = \cos^2\theta$

Step 3: Substitute back

$f(\theta) = 1\cdot(1 + \sin\theta\cos\theta) - \cos\theta\cdot(-\sin\theta - \cos\theta) - 1\cdot\cos^2\theta$
$= 1 + \sin\theta\cos\theta + \cos\theta(\sin\theta + \cos\theta) - \cos^2\theta$
$= 1 + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta - \cos^2\theta$
$= 1 + 2\sin\theta\cos\theta$
$= 1 + \sin 2\theta$
(since $2\sin\theta\cos\theta = \sin 2\theta$).

Step 4: Find maximum and minimum
$f(\theta) = 1 + \sin 2\theta$
Maximum: $\sin 2\theta = 1 \Longrightarrow f_{\max} = 1 + 1 = 2$
Minimum: $\sin 2\theta = -1 \Longrightarrow f_{\min} = 1 - 1 = 0$

134. **(d)** Step 1: Find $f(f(x))$

$$f(x) = \frac{3x-4}{2x-3}$$

Let $y = f(x) = \frac{3x-4}{2x-3}$. Then

$$f(f(x)) = f(y) = \frac{3y-4}{2y-3} = \frac{3\left(\frac{3x-4}{2x-3}\right) - 4}{2\left(\frac{3x-4}{2x-3}\right) - 3}$$

Simplify numerator:

$$3\cdot\frac{3x-4}{2x-3} - 4 = \frac{9x - 12 - 4(2x-3)}{2x-3} = \frac{9x - 12 - 8x + 12}{2x-3} = \frac{x}{2x-3}$$

Simplify denominator:

$$2\cdot\frac{3x-4}{2x-3} - 3 = \frac{6x - 8 - 3(2x-3)}{2x-3} = \frac{6x - 8 - 6x + 9}{2x-3} = \frac{1}{2x-3}$$

So

$$f(f(x)) = \frac{x/(2x-3)}{1/(2x-3)} = x$$

Step 2: Find $f(f(f(x)))$
$f(f(f(x))) = f(f(f(x))) = f(f(x))\circ f = f(f(x))$ applied to f
Since $f(f(x)) = x$, we have:
$f(f(f(x))) = f(x)$

$$\Rightarrow f(f(f(x))) = \frac{3x-4}{2x-3}$$

135. **(b)** Step 1: Analyze $f(x)$
1.Term 1: $x + |x|$

$$x + |x| = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0\end{cases}$$

2.Term 2: $x - [x]$
This is the fractional part of x: $x - [x] \in [0,1)$.

Step 2: Compare the two terms
We define $f(x) = \max(x + |x|, x - [x])$.
Case 1: $x \geq 0$
$x + |x| = 2x, x - [x] \in [0,1)$
For $x \geq 1/2, 2x \geq 1 > x - [x]$.
For $0 \leq x < 1/2, 2x < 1$. Then $x - [x] = x$, so $2x \geq x$, so still $f(x) = 2x$.
So for $x \geq 0, f(x) = 2x$.
Case 2: $x < 0$
$x + |x| = x + (-x) = 0, x - [x] \in [0,1)$?
For negative $x, [x]$ is the greatest integer $\leq x$.
Example: $x = -1.3 \Longrightarrow [x] = -2 \Longrightarrow x - [x] = -1.3 - (-2) = 0.7$
Fractional part is still positive.
So $f(x) = \max(0, x - [x]) = x - [x]$ for negative x.
Step 3: Split the integral

$$\int_{-3}^{3} f(x)dx = \int_{-3}^{0} f(x)dx + \int_{0}^{3} f(x)dx$$

1.For $x \in [-3,0)$: $f(x) = x - [x]$
Break into integer intervals:
$[-3,-2], [-2,-1], [-1,0]$
On $[-3,-2]$: $[x] = -3 \Longrightarrow f(x) = x - (-3) = x + 3$

$$\int_{-3}^{-2} (x+3)dx = \left[\frac{x^2}{2} + 3x\right]_{-3}^{-2} = (2-6) - (4.5-9) = -4 - (-4.5) = 0.5$$

On $[-2,-1]$: $[x] = -2 \Longrightarrow f(x) = x + 2$
$\int_{-2}^{-1} (x+2)dx = [0.5 + (-?)]$ let's compute carefully:

$$\int_{-2}^{-1} (x+2)dx = \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} = (0.5-2) - (2-4) = (-1.5) - (-2) = 0.5$$

On $[-1,0]$: $[x] = -1 \Longrightarrow f(x) = x + 1$
$\int_{-1}^{0} (x+1)dx = [0+0] - [-0.5 + -1]$?

$$\int_{-1}^{0} (x+1)dx = \left[\frac{x^2}{2} + x\right]_{-1}^{0} = (0+0) - (0.5-1) = 0 - (-0.5) = 0.5$$

So total for negative side: $0.5 + 0.5 + 0.5 = 1.5 = 3/2$
2.For $x \in [0,3]$: $f(x) = 2x$

$$\int_{0}^{3} 2x dx = [x^2]_0^3 = 9 - 0 = 9$$

Step 4: Add contributions

$$\int_{-3}^{3} f(x)dx = 3/2 + 9 = 21/2$$

136. (b) a^2, c^2, b^2 are in A.P.

137. **(b)** Step 1: Find the center of the circle
Perpendicular diameters intersect at the center of the circle. So, solve the system:
$x - y = 0$ and $x + y = 1$
From $x - y = 0 \Rightarrow x = y$.
Substitute into $x + y = 1$:

$$x + x = 1 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}, y = \frac{1}{2}$$

So the center is:

$$C\left(\frac{1}{2}, \frac{1}{2}\right)$$

Step 2: Use the distance formula to find R
The circle passes through the origin $(0,0)$, so the radius is the distance from the center to the origin:

$$R=\sqrt{\left(0-\frac{1}{2}\right)^2+\left(0-\frac{1}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{1}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

138. **(b)** Step 1: Find α and β

$x^2-3x+2=0 \Longrightarrow (x-1)(x-2)=0$

So $\alpha=1$ and $\beta=2$.

Step 2: Consider differentiability of $f(x)$

A function of the form $g(x)=|x-a|$ is not differentiable at $x=a$.

Here, $f(x)=|x-1|+|x-2|$. Normally, it is non-differentiable at $x=1$ and $x=2$.

Step 3: Check the interval $[\alpha,\beta]=[1,2]$

For $x\in[1,2]$, notice $f(x)$ can be rewritten as:

$f(x)=|x-1|+|x-2|=(x-1)+(2-x)=1$

This is constant on $[1,2]$.

Step 4: Differentiability

A constant function is differentiable everywhere, so $f(x)$ is differentiable at every point in $[1,2]$.

Hence, the number of points in $[\alpha,\beta]$ where f is not differentiable $=0$.

139. **(c)** Step 1: Equation of normal to each parabola

1.For $y^2=4ax$, the slope form of the normal is:

$y+mx=2am+am^3$

Or more standard: parametric form of normal using parameter t (for parabola $y^2=4ax$):

$y=tx-at^3+2at$

Similarly, for $x^2=4by$, let the parameter be s. Then normal is:

$x=sy-bs^3+2bs$

Step 2: Set up condition for common normal

A common normal will satisfy both parabola equations. Using the parametric approach:

For $y^2=4ax$, the normal at point $(at^2,2at)$ is:

$y=tx-at^3+2at$

For $x^2=4by$, the normal at $(2bs,bs^2)$ is:

$x=sy-bs^3+2bs$

Step 3: Eliminate x and y

From the first normal: $y=tx-at^3+2at$

From the second normal: $x=sy-bs^3+2bs$

Substitute y from the first into second:

$x=s(tx-at^3+2at)-bs^3+2bs$

$x=stx-sat^3+2ast-bs^3+2bs$

Bring x terms together:

$x-stx=x(1-st)=-sat^3+2ast-bs^3+2bs$

$$x=\frac{-sat^3+2ast-bs^3+2bs}{1-st}$$

Similarly, eliminate y if needed.

Step 4: Maximum number of common normals

The final equation reduces to a quintic in one parameter (say t or s) - degree **5**, which comes from the fact that:

Equating slopes of normals leads to the equation:

$$\left.\frac{dy}{dx}\right|_{\text{first parabola}}=\left.\frac{dy}{dx}\right|_{\text{second parabola}}$$

For $y^2=4ax$, slope of normal: $-1/t$

For $x^2=4by$, slope of normal: $-s$

Set $-1/t=-s \Longrightarrow s=1/t$

Then substitute into the positions → leads to a fifth-degree equation in t → maximum 5 real solutions.

Hence, the maximum number of common normals $=5$.

140. **(b)** Step 1: Find the centers and radii

Circle 1: $x^2+y^2-4x-6y-12=0$

Complete the squares:

$x^2 - 4x + y^2 - 6y = 12$
$x^2 - 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9$
$(x-2)^2 + (y-3)^2 = 25$

So, center $C_1 = (2,3)$, radius $r_1 = 5$.

Circle 2: $x^2 + y^2 + 6x + 18y + 26 = 0$
Complete the squares:

$x^2 + 6x + y^2 + 18y = -26$

$x^2 + 6x + 9 + y^2 + 18y + 81 = -26 + 9 + 81$

$(x+3)^2 + (y+9)^2 = 64$

So, center $C_2 = (-3,-9)$, radius $r_2 = 8$.

Step 2: Find distance between centers

$d = \sqrt{(2-(-3))^2 + (3-(-9))^2} = \sqrt{5^2 + 12^2}$

$= \sqrt{25+144} = \sqrt{169} = 13$

Step 3: Determine number of common tangents
For two circles with radii r_1, r_2 and distance d between centers:

If $d > r_1 + r_2 \rightarrow 4$ tangents (2 direct,2 transverse)

If $d = r_1 + r_2 \rightarrow 3$ tangents (externally tangent)

If $|r_1 - r_2| < d < r_1 + r_2 \rightarrow 2$ tangents (intersecting circles)

If $d = |r_1 - r_2| \rightarrow 1$ tangent (internally tangent)

If $d < |r_1 - r_2| \rightarrow 0$ tangents (one inside another)

Here, $r_1 + r_2 = 5 + 8 = 13$, and $d = 13$.
So the circles are externally tangent, giving 3 common tangents.

141. **(c)** Given:
$\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$
Use $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$, so the equation becomes:
$\sin^{-1} x + \sin^{-1}(1-x) = \frac{\pi}{2} - \sin^{-1} x$
Bring $\sin^{-1} x$ terms together:
$\sin^{-1} x + \sin^{-1}(1-x) + \sin^{-1} x = \frac{\pi}{2}$
$2\sin^{-1} x + \sin^{-1}(1-x) = \frac{\pi}{2}$
Now, the domain of $\sin^{-1} x$ is $[-1,1]$, so $x \in [-1,1]$ and $1 - x \in [-1,1]$, meaning $x \in [0,2]$. The intersection gives $x \in [0,1]$.
Check possible solutions by substituting:
For $x = 0$:
$\sin^{-1}(0) + \sin^{-1}(1) = \cos^{-1}(0)$
$0 + \frac{\pi}{2} = \frac{\pi}{2}$
For $x = 1$:
$\sin^{-1}(1) + \sin^{-1}(0) = \cos^{-1}(1)$
$\frac{\pi}{2} + 0 = 0 \times$
For $x = \frac{1}{2}$:
$\sin^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\frac{1}{2}\right)$
$\frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

$\frac{\pi}{3} = \frac{\pi}{3}$

Thus, solutions are $x = 0$ and $x = \frac{1}{2}$.

142. **(d)** Given:

1.$u + v + w = 3$, where $u, v, w \in \mathbb{R}$

2.$f(x) = ux^2 + vx + w$

3.Functional equation:

$f(x+y) = f(x) + f(y) + xy\ \forall x, y \in \mathbb{R}$

Step 1 - Expand $f(x+y)$

$f(x+y) = u(x+y)^2 + v(x+y) + w$

$= u(x^2 + 2xy + y^2) + v(x+y) + w$

$= ux^2 + 2uxy + uy^2 + vx + vy + w$

Step 2 - Expand $f(x) + f(y) + xy$

$f(x) = ux^2 + vx + w$

$f(y) = uy^2 + vy + w$

So,

$f(x) + f(y) + xy = ux^2 + vx + w + uy^2 + vy + w + xy$

$= ux^2 + uy^2 + vx + vy + 2w + xy$

Step 3 - Equate both sides

Comparing $f(x+y)$ and $f(x) + f(y) + xy$:

$ux^2 + 2uxy + uy^2 + vx + vy + w = ux^2 + uy^2 + vx + vy + 2w + xy$

Now, subtract common terms:

$2uxy + w = 2w + xy$

Bring like terms together:

$2uxy - xy = 2w - w$

$xy(2u - 1) = w$

Since this equation must hold for all x, y, the coefficient of xy must be constant and w must be constant.

Thus,

$2u - 1 = 0 \Rightarrow u = \frac{1}{2}$

and

$w = 0$

Step 4 - Use $u + v + w = 3$

$u + v + w = 3$

$\frac{1}{2} + v + 0 = 3$

$v = 3 - \frac{1}{2} = \frac{5}{2}$

Step 5 - Find $f(1)$

$f(1) = u(1)^2 + v(1) + w$

$= \frac{1}{2} \times 1 + \frac{5}{2} \times 1 + 0$

$= \frac{1}{2} + \frac{5}{2}$

$= \frac{6}{2} = 3$

143. **(a)** Step 1 - Expansion

We are expanding:

$$\left[x + \frac{\sin(1/n)}{x^2}\right]^{3n}$$

We are to find the term independent of x, i.e., the constant term.

In the binomial expansion, the general term is:

$$T_k = \binom{3n}{k} \cdot x^{3n-k} \cdot \left(\frac{\sin\left(\frac{1}{n}\right)}{x^2}\right)^k = \binom{3n}{k} (\sin(1/n))^k x^{3n-k-2k}$$

$$= \binom{3n}{k}(\sin(1/n))^k x^{3n-3k}$$

For this term to be independent of x, the exponent must be zero:

$3n - 3k = 0 \Rightarrow k = n$

So only the term with $k = n$ is independent of x.

Step 2 - Find a_n

So the constant term is:

$$a_n = \binom{3n}{n}(\sin(1/n))^n$$

Step 3 - Evaluate the limit

We are to find:

$$\lim_{n\to\infty} \frac{a_n \cdot n!}{{}^{3n}P_n}$$

Where ${}^{3n}P_n = \frac{(3n)!}{(3n-n)!} = \frac{(3n)!}{2n!}$.

So the expression becomes:

$$\frac{\binom{3n}{n}(\sin(1/n))^n \cdot n!}{\frac{(3n)!}{2n!}} = \frac{\frac{(3n)!}{n!\,(2n)!} \cdot (\sin(1/n))^n \cdot n!}{\frac{(3n)!}{2n!}}$$

$$= \frac{(3n)!}{n!\,(2n)!} \cdot (\sin(1/n))^n \cdot n! \cdot \frac{2n!}{(3n)!}$$

Canceling (3n)!:

$$= \frac{1}{n!\,(2n)!} \cdot (\sin(1/n))^n \cdot n! \cdot 2n! = \frac{2n!}{(2n)!} \cdot (\sin(1/n))^n$$

So:

$$= 2 \cdot \frac{n!}{(2n)!} \cdot (\sin(1/n))^n$$

Step 4 - Approximate as $n \to \infty$

We know that for large n,

$$\sin(1/n) \approx \frac{1}{n}$$

So

$$(\sin(1/n))^n \approx \left(\frac{1}{n}\right)^n$$

Now,

$$\frac{n!}{(2n)!} = \frac{1}{(n+1)(n+2)\ldots(2n)} = \frac{1}{n^n\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{n}{n}\right)}$$

As $n \to \infty$, this product grows much faster than $n!$, and the whole expression tends to 0 .

Thus,

$$\lim_{n\to\infty} 2 \cdot \frac{n!}{(2n)!} \cdot (\sin(1/n))^n = 0$$

144. (a) Given:

$$\cos(\theta+\phi) = \frac{3}{5}, \sin(\theta-\phi) = \frac{5}{13}, 0 < \theta, \phi < \frac{\pi}{4}$$

We are to find $\cot(2\theta)$.

Step 1 - Find $\sin(\theta+\phi)$

We know:

$$\cos^2(\theta+\phi) + \sin^2(\theta+\phi) = 1$$

So,

$$\sin(\theta+\phi) = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Step 2 - Find $\cos(\theta-\phi)$

Similarly,

$\sin^2(\theta-\phi)+\cos^2(\theta-\phi)=1$
So,
$$\cos(\theta-\phi)=\sqrt{1-\left(\frac{5}{13}\right)^2}=\sqrt{1-\frac{25}{169}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$$
Step 3 - Find $\sin(2\theta)$ and $\cos(2\theta)$
Use the identities:
$\sin(2\theta)=\sin\big((\theta+\phi)+(\theta-\phi)\big)$
$=\sin(\theta+\phi)\cos(\theta-\phi)+\cos(\theta+\phi)\sin(\theta-\phi)$
Substitute the known values:
$$\sin(2\theta)=\frac{4}{5}\cdot\frac{12}{13}+\frac{3}{5}\cdot\frac{5}{13}=\frac{48}{65}+\frac{15}{65}=\frac{63}{65}$$
Next,
$\cos(2\theta)=\cos\big((\theta+\phi)+(\theta-\phi)\big)$
$=\cos(\theta+\phi)\cos(\theta-\phi)-\sin(\theta+\phi)\sin(\theta-\phi)$
Substitute:
$$\cos(2\theta)=\frac{3}{5}\cdot\frac{12}{13}-\frac{4}{5}\cdot\frac{5}{13}=\frac{36}{65}-\frac{20}{65}=\frac{16}{65}$$
Step 4 - Find $\cot(2\theta)$
$$\cot(2\theta)=\frac{\cos(2\theta)}{\sin(2\theta)}=\frac{\frac{16}{65}}{\frac{63}{65}}=\frac{16}{63}$$

145. **(a)** Step 1 - Roots of x^2+x+1 :
The polynomial x^2+x+1 has complex roots:
$$x=\frac{-1\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm i\sqrt{3}}{2}$$
Let's denote a root by ω, where ω is a primitive cube root of unity:
$$\omega=\frac{-1+i\sqrt{3}}{2}\ \text{ and }\ \omega^3=1, 1+\omega+\omega^2=0$$
Step $2-\phi(\omega)=0$:
Since x^2+x+1 divides $\phi(x)$, we must have $\phi(\omega)=0$ and $\phi(\omega^2)=0$.
Let's calculate $\phi(\omega)$:
$\phi(\omega)=f(\omega^3)+\omega g(\omega^3)$
But $\omega^3=1$, so:
$\phi(\omega)=f(1)+\omega g(1)=0$
Step 3 - Similarly for ω^2 :
$\phi(\omega^2)=f((\omega^2)^3)+\omega^2 g((\omega^2)^3)$
Since $(\omega^2)^3=\omega^6=1$, we get:
$\phi(\omega^2)=f(1)+\omega^2 g(1)=0$
Step 4 - Solving the system:
We have:
$f(1)+\omega g(1)=0$
$f(1)+\omega^2 g(1)=0$
Subtracting the second from the first:
$(\omega-\omega^2)g(1)=0$
Since $\omega\neq\omega^2$, this implies $g(1)=0$, so $(x-1)$ divides $g(x)$.
Now, substituting $g(1)=0$ into the first equation:
$f(1)=0$
So $(x-1)$ also divides $f(x)$.
Step 5 - Check which option matches:
(x-1) divides both $f(x)$ and $g(x)$, so statement (a) $\phi(x)$ is divisible by $x-1$ makes sense because:
Let's explicitly check:
$\phi(1)=f(1^3)+1\cdot g(1^3)=f(1)+g(1)=0+0=0$
So $x-1$ divides $\phi(x)$.

146. **(a,b)** Step 1: Substitution

Let $y=\sin^2 x$. Then the equation becomes:

$y^2-(p+2)y-(p+3)=0$

Step 2: Quadratic in y

For the quadratic $y^2-(p+2)y-(p+3)=0$ to have a real solution y, the discriminant must be nonnegative:

$\Delta=(-(p+2))^2-4(1)(-(p+3))\geq 0$

$\Delta=(p+2)^2+4(p+3)\geq 0$

$\Delta=p^2+4p+4+4p+12\geq 0$

$p^2+8p+16\geq 0$

$(p+4)^2\geq 0$

Always true, so a real y always exists.

Step 3: Restriction for $\sin^2 x$

Since $y=\sin^2 x$, we must have:

$0\leq y\leq 1$

Step 4: Solve for y

Quadratic formula gives:

$$y=\frac{(p+2)\pm\sqrt{(p+2)^2+4(p+3)}}{2}=\frac{(p+2)\pm\sqrt{p^2+8p+16}}{2}$$

$$=\frac{(p+2)\pm|p+4|}{2}$$

So we have two possibilities:

1.$y_1=\frac{p+2+|p+4|}{2}$

2.$y_2=\frac{p+2-|p+4|}{2}$

Step 5: Consider cases for absolute value

Case 1: $p+4\geq 0\Rightarrow p\geq -4$

$$y_1=\frac{p+2+(p+4)}{2}=\frac{2p+6}{2}=p+3$$

$$y_2=\frac{p+2-(p+4)}{2}=\frac{-2}{2}=-1$$

$y_2=-1$ is invalid $(y\geq 0)$

So valid solution is $y_1=p+3\leq 1\Rightarrow p\leq -2$

Also $p\geq -4$ (from case)

So interval: $-4\leq p\leq -2$

Case 2: $p+4<0\Rightarrow p<-4$

$$y_1=\frac{p+2+(-(p+4))}{2}=\frac{p+2-p-4}{2}=\frac{-2}{2}=-1$$

$$y_2=\frac{p+2-(-(p+4))}{2}=\frac{p+2+p+4}{2}=\frac{2p+6}{2}=p+3$$

$y_1=-1$ invalid

$y_2=p+3\geq 0\Rightarrow p\geq -3$

But this contradicts $p<-4$, so no solution in this case.

Step 6: Combine

Valid interval:

$-3\leq p\leq -2$

So closed interval $[-3,-2]$ if endpoints allowed

Open interval $(-3,-2)$ if endpoints excluded

147. **(c)** Step 1: Rearrange the equation

$x^2-2x+4+3\cos(ax+b)=0$

Think of it as a quadratic in x :

$x^2-2x+[4+3\cos(ax+b)]=0$

For real solutions in x, the discriminant $D\geq 0$:

$D=(-2)^2-4[4+3\cos(ax+b)]\geq 0$

Step 2: Compute discriminant

$4-16-12\cos(ax+b) \geq 0$
$-12-12\cos(ax+b) \geq 0$
$-12(1+\cos(ax+b)) \geq 0 \Rightarrow 1+\cos(ax+b) \leq 0$
$\cos(ax+b) \leq -1$
But $\cos\theta \leq -1 \Rightarrow \cos\theta = -1$.
So equality must hold.
$\cos(ax+b) = -1$
Step 3: Solve $\cos(ax+b) = -1$
$ax+b = \pi + 2n\pi, n \in \mathbb{Z}$
Now, for some real x, this must hold. To make it simple, choose $x=0$:
$a \cdot 0 + b = \pi \Rightarrow b = \pi$
But given $0 \leq b \leq 3$, the only possible value is $\pi/2$ or less? Wait, let's check: $\pi \approx 3.14$, slightly above 3 , so maybe x is not 0 .
Step 4: Choose simplest solution
Take the smallest positive solution of $\cos(ax+b) = -1$:
$ax+b = \pi \Rightarrow a+b = \pi$ (for some $x=1$)
Since $0 \leq a, b \leq 3$, this is feasible.
Step 5: Conclude
$a+b=\pi$

148. **(b,c,d)** 1. Check for minimum and maximum:
The derivative: $f'(x) = 3x^2$.
$f'(x) = 0$ at $x=0 \rightarrow$ critical point.
Evaluate $f(x)$ at endpoints and critical point:
$f(-1) = (-1)^3 = -1, f(0) = 0^3 = 0, f(1) = 1^3 = 1$
Maximum value $= 1$ at $x=1$
Minimum value $= -1$ at $x=-1$
So:
(a) False (minimum is at $x=-1$, not 0)
(b) True (maximum is at $x=1$)
2.Continuity:
$f(x) = x^3$ is a polynomial $\rightarrow$ continuous everywhere, so also continuous on $[-1,1]$
(c) True
3.Boundedness:
On a closed interval $[-1,1]$, $f(x) = x^3$ is continuous $\rightarrow$ bounded (Extreme Value Theorem)
(d) True

149. **(c)** Step 1: Total number of ways
Total ways to choose 3 numbers from 10:
$\binom{10}{3} = 120$
Step 2: Cases
Case A: Minimum is 3
If the minimum is 3 , the other two numbers must be from $\{4,5,\ldots,10\}$ (because all numbers $\geq$ minimum).
Number of ways to choose 2 numbers from $\{4,5,\ldots,10\} \rightarrow$ there are 7 numbers (4 to 10):
$\binom{7}{2} = 21$
Case B: Maximum is 7
If the maximum is 7 , the other two numbers must be from $\{1,2,\ldots,6\}$ (because all numbers $\leq$ maximum).
Number of ways to choose 2 numbers from $\{1,2,\ldots,6\}$:
$\binom{6}{2} = 15$
Step 3: Intersection (Minimum is 3 and Maximum is 7)
Now, minimum $= 3$ and maximum $= 7$. So the three numbers are $3 \leq x \leq 7$, with 3 being the minimum and 7 being the maximum.
The only remaining number must come from $\{4,5,6\} \rightarrow 3$ choices

So intersection $= 3$
Step 4: Apply inclusion-exclusion
Favorable ways $= 21 + 15 - 3 = 33$
Step 5: Probability
$P = \frac{33}{120} = \frac{11}{40}$

150. **(d)** Step 1: Write in standard linear form
$\frac{dp}{dt} - 0.5p = -450$
This is a first-order linear differential equation.
Step 2: Find the integrating factor (IF)
$\text{IF} = e^{\int -0.5dt} = e^{-0.5t}$
Multiply both sides by IF:
$e^{-0.5t}\frac{dp}{dt} - 0.5e^{-0.5t}p = -450e^{-0.5t} \Longrightarrow \frac{d}{dt}(pe^{-0.5t}) = -450e^{-0.5t}$
Step 3: Integrate both sides
$\int \frac{d}{dt}(pe^{-0.5t})dt = \int -450e^{-0.5t}dt$
$pe^{-0.5t} = -450\int e^{-0.5t}dt + C$
$\int e^{-0.5t}dt = \frac{e^{-0.5t}}{-0.5} = -2e^{-0.5t}$
$pe^{-0.5t} = -450(-2e^{-0.5t}) + C = 900e^{-0.5t} + C$
$pe^{-0.5t} = 900e^{-0.5t} + C \Longrightarrow p(t) = 900 + Ce^{0.5t}$
Step 4: Use initial condition
$p(0) = 850 \Longrightarrow 850 = 900 + Ce^0 \Longrightarrow C = -50$
So solution:
$p(t) = 900 - 50e^{0.5t}$
Step 5: Find t when $p(t) = 0$
$0 = 900 - 50e^{0.5t} \Longrightarrow 50e^{0.5t} = 900 \Longrightarrow e^{0.5t} = 18$
$0.5t = \ln 18 \Longrightarrow t = 2\ln 18$

151. **(b)** Step 1: Represent the sum of row elements using a vector
Let
$\mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \in \mathbb{R}^5$
be a column vector of ones.
The sum of the elements of the i-th row of P is the i-th component of $P\mathbf{e}$.
Given that the sum of each row is 1 , we have:
$P\mathbf{e} = \mathbf{e}$
So $\mathbf{e}$ is an eigenvector of P with eigenvalue 1 .
Step 2: Relate this to P^{-1}
If $P\mathbf{e} = \mathbf{e}$, then we can multiply both sides by P^{-1} :
$P^{-1}(P\mathbf{e}) = P^{-1}\mathbf{e} \Longrightarrow \mathbf{e} = P^{-1}\mathbf{e}$.
Step 3: Interpret the result
The vector $P^{-1}\mathbf{e} = \mathbf{e}$ means that the sum of elements in each row of P^{-1} is also 1 .

152. **(c)** Step 1: Use the identity $\cot\theta = \tan(\pi/2 - \theta)$
$\tan(\pi\tan x) = \tan\left(\frac{\pi}{2} - \pi\cot x\right)$
So the equation becomes:
$\tan(\pi\tan x) = \tan\left(\frac{\pi}{2} - \pi\cot x\right)$
Step 2: Equate the arguments (up to $n\pi$)

$\pi \tan x = \frac{\pi}{2} - \pi \cot x + n\pi, n \in \mathbb{Z}$
Divide by π :
$\tan x + \cot x = \frac{1}{2} + n$
Step 3: Simplify in the interval $x \in (0, \pi/2)$
$\tan x + \cot x \geq 2$ (AM-GM inequality). But $\frac{1}{2} + n$ should be ≥ 2, so $n \geq 2$.
For integer n, the RHS cannot equal the LHS exactly in $(0, \pi/2)$.
Step 4: Conclusion
There is no solution in $(0, \pi/2)$, so the solution set is empty $(\emptyset)$.

153. **(b)** Step 1: Substitution in $f(x)$
Let $t = \sin^2\theta \Longrightarrow dt = 2\sin\theta\cos\theta d\theta = \sin 2\theta d\theta$.
Then when $t = 0, \theta = 0$, and when $t = \sin^2 x, \theta = x$.
So,
$$f(x) = \int_0^x \arcsin\left(\sqrt{\sin^2\theta}\right) \cdot \sin 2\theta d\theta = \int_0^x \arcsin(\sin\theta) \cdot \sin 2\theta d\theta$$
Since $0 \leq \theta \leq \pi/2, \arcsin(\sin\theta) = \theta$.
So,
$$f(x) = \int_0^x \theta \cdot \sin 2\theta d\theta$$
Step 2: Substitution in $g(x)$
Similarly, let $t = \cos^2\phi \Longrightarrow dt = -2\cos\phi\sin\phi d\phi = -\sin 2\phi d\phi$.
When $t = 0, \phi = \pi/2$; when $t = \cos^2 x, \phi = x$.
$$g(x) = \int_0^{\cos^2 x} \arccos(\sqrt{t})dt = \int_{\pi/2}^x \arccos(\cos\phi)(-\sin 2\phi d\phi) = \int_x^{\pi/2} \phi\sin 2\phi d\phi$$
because $\arccos(\cos\phi) = \phi$ for $0 \leq \phi \leq \pi/2$.
Step 3: Add $f(x) + g(x)$
$$f(x) + g(x) = \int_0^x \theta\sin 2\theta d\theta + \int_x^{\pi/2} \theta\sin 2\theta d\theta = \int_0^{\pi/2} \theta\sin 2\theta d\theta$$
Notice that the sum does not depend on x.
Step 4: Evaluate $\int_0^{\pi/2} \theta\sin 2\theta d\theta$
Use integration by parts:
$$\int \theta\sin 2\theta d\theta = -\frac{\theta\cos 2\theta}{2} + \frac{1}{2}\int \cos 2\theta d\theta = -\frac{\theta\cos 2\theta}{2} + \frac{\sin 2\theta}{4} + C$$
Evaluate from 0 to $\pi/2$:
$$\left[-\frac{\theta\cos 2\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi/2} = \left[-\frac{\pi/2 \cdot \cos\pi}{2} + \frac{\sin\pi}{4}\right] - [0+0] = -\frac{\pi/2 \cdot (-1)}{2} = \frac{\pi}{4}$$
Wait, check carefully:
$$-\frac{\pi/2 \cdot \cos\pi}{2} = -\frac{\pi/2 \cdot (-1)}{2} = \frac{\pi}{4}$$
$\frac{\sin\pi}{4} = 0$, good.
At $\theta = 0$, both terms $= 0$.
So final answer:
$$f(x) + g(x) = \frac{\pi}{4}$$

154. **(c)** Step 1: Check symmetry
The numerator: $x + x^3 + x^5 \rightarrow$ all terms are odd powers $\rightarrow$ odd function.
The denominator: $1 + x^2 + x^4 + x^6 \rightarrow$ all terms are even powers $\rightarrow$ even function.
Thus, the integrand is odd / even = odd function (odd divided by even = odd).
Step 2: Property of odd function
For any odd function $f(x)$:

$$\int_{-a}^{a} f(x)dx = 0$$

Step 3: Conclusion

Here the integrand is odd, and limits are symmetric about 0 . So:

$$\int_{-100}^{100} \frac{x + x^3 + x^5}{1 + x^2 + x^4 + x^6} dx = 0$$

155. **(b)** We are given:

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}, g(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q} \\ 1 & \text{if } x \notin \mathbb{Q} \end{cases}.$$

These are the classic Dirichlet-type functions, where f is 1 on rationals and 0 on irrationals, and g is the opposite.

Step 1: Continuity of f and g

Both f and g take different values on rationals and irrationals, which are dense in every interval. Therefore, both f and g are discontinuous at every point in $[0,1]$, including $x = 1/2$ and $x = 2/3$.

Step 2: Sum $f + g$

$$(f + g)(x) = f(x) + g(x) = \begin{cases} 1 + 0 = 1 & x \in \mathbb{Q} \\ 0 + 1 = 1 & x \notin \mathbb{Q} \end{cases} = 1 \text{ for all } x.$$

So $f + g$ is constant, hence continuous everywhere, including $x = 2/3$ and $x = 3/4$.