SECTION A

1. p = 3

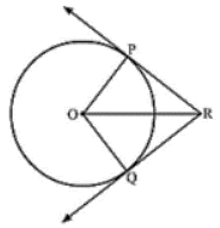
2. 30°

3. $\frac{1}{9}$

4. 120°

SECTION B

5.



Construction Join PO and OQ

In $\triangle POR$ and $\triangle QOR$

$OP = OQ$(Radii)

$RP = RQ$

(Tangents from the external point are congruent)

$OR = OR$(Common)

By SSS congruency, $\triangle POR \cong \triangle QOR$

$\angle PRO = \angle QRO$(C.P.C.T)

Now, $\angle PRO + \angle QRO = \angle PRQ$

$\Rightarrow 2\angle PRO = 120^\circ$

$\Rightarrow \angle PRO = 60^\circ$

Now. In $\triangle POR$

$\cos 60^\circ = \frac{PR}{OR}$

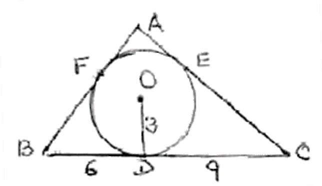
$\Rightarrow \frac{1}{2} = \frac{PR}{OR}$

$\Rightarrow OR = 2PR$

$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + RQ$

6.



Let $AF = AE = x$

$AB = 6 + x, AC = 9 + x, BC = 15$

$\frac{1}{2}[15 + 6 + x + 9 + x] \cdot 3 = 54$

$\Rightarrow x = 3 \therefore AB = 9$ cm, $AC = 12$ cm

and $BC = 15$ cm

7. $4x^2 + 4bx + b^2 - a^2 = 0 \Rightarrow (2x + b)^2 - (a)^2 = 0$

$\Rightarrow (2x + b + a)(2x + b - a) = 0$

$\Rightarrow x = -\frac{a + b}{2}, x = \frac{a - b}{2}$

8. $S_5 + S_7 = 167 \Rightarrow \frac{5}{2}[2a + 4d] + \frac{7}{2}[2a + 6d] = 167$

$24a + 62d = 334$ or $12a + 31d = 167$ ……………………(i)

$S_{10} = 235 \Rightarrow 5[2a + 9d] = 235$ or $2a + 9d = 47$ …………(ii)

Solving (i) and (ii) to get a = 1, d = 5. Hence AP is 1,6,11, …..

9. Here, $AB^2 + BC^2 = AC^2$

$\Rightarrow (4)^2 + (p-4)^2 + (7-p)^2 = (3)^2 + (-4)^2$

$\Rightarrow p = 7$ or 4

since $p \neq 7 \therefore p = 4$

10. Using ar (△ ABC) = 0

$\Rightarrow x(7-5) - 5(5-y) - 4(y-7) = 0$

$2x - 25 + 5y - 4y + 28 = 0$

$2x + y + 3 = 0$

SECTION C

11. $a_{14} = 2a_8 \Rightarrow a + 13d = 2(a + 7d) \Rightarrow a = -d$

$a_6 = -8 \Rightarrow a + 5d = -8$

solving to get $a = 2, d = -2$

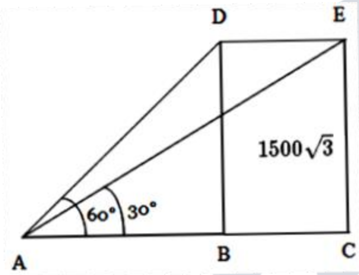
$S_{20} = 10(2a + 19d) = 10(4 - 38) = -340$

12. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0 \Rightarrow (x - \sqrt{6})(\sqrt{3}x + \sqrt{2}) = 0$

$\Rightarrow x = \sqrt{6}, x = -\sqrt{\frac{2}{3}}$

13.



Let D and E be the initial and final positions of the plane respectively.

It is given that $BD = 1500\sqrt{3}$ m

In right △ ABD, we have

$\tan 60° = \frac{BD}{AB}$

$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{AB}$

$\Rightarrow AB = 1500$

In right △ ACE, we have

$\tan 30° = \frac{CE}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AC}$

$\Rightarrow AC = 4500$

Now, Distance $= BC = AC - AB = 4500 - 1500 = 3000$ m

DE = BC = 3000 m

i.e. the plane travels a distance of 3000 m in 15 seconds

the speed of the plane $= \frac{\text{distance}}{\text{time}} = \frac{3000}{15} = 200$ m/s

$= 200 \times \frac{18}{5}$ km/hr

$= 720$ km/hr

14. $AP = \frac{3}{7}AB \Rightarrow AP : PB = 3 : 4$

A	P(x, y)	B

$(-2,-2)$	$3:4$	$(2,-4)$

$$x=\frac{6-8}{7}=-\frac{2}{7}$$
$$y=\frac{-12-8}{7}=-\frac{20}{7}$$
$$P\left(-\frac{2}{7},-\frac{20}{7}\right)$$

15. $\mathrm{P}(\text{ Red })=\frac{1}{4}, \mathrm{P}(\text{ blue })=\frac{1}{3}$

$$\Rightarrow \mathrm{P}(\text{ orange })=1-\frac{1}{4}-\frac{1}{3}=\frac{5}{12}$$
$$\Rightarrow \frac{5}{12}(\text{ Total no. of balls })=10$$
$$\Rightarrow \text{ Total no. of balls }=\frac{10\times 12}{5}=24$$

16. $r=14$ cm. $\theta=60^{\circ}$

$$\text{Area of minor segment }=\pi r^{2}\frac{\theta}{360}-\frac{1}{2}r^{2}\sin\theta$$
$$=\frac{22}{7}\times 14\times 14\times\frac{60}{360}-\frac{1}{2}\times 14\times 14\times\frac{\sqrt{3}}{2}$$
$$=\left(\frac{308}{3}-49\sqrt{3}\right)\mathrm{cm}^{2}\text{ or }17.89\ \mathrm{cm}^{2}\text{ or }17.9\ \mathrm{cm}^{2}\text{ Approx.}$$

Area of Major segment
$$=\pi\mathrm{r}^{2}-\left(\frac{308}{3}-49\sqrt{3}\right)$$
$$=\left(\frac{1540}{3}+49\sqrt{3}\right)\mathrm{cm}^{2}\text{ or }598.10\ \mathrm{cm}^{2}$$
or $598\ \mathrm{cm}^{2}$ Approx.

17. Slant height $(\ell)=\sqrt{(2.8)^{2}+(2.1)^{2}}=3.5$ cm

$$\text{Area of canvas }=2\times\frac{22}{7}\times(2.1)\times 4+\frac{22}{7}\times 2.1\times 3.5$$
$$\text{for one tent }=6.6(8+3.5)=6.6\times 11.5\ \mathrm{m}^{2}$$
$$\text{Area for }100\text{ tents }=66\times 115\ \mathrm{m}^{2}$$
$$\text{Cost of }100\text{ tents }=\text{ Rs. }66\times 115\times 100$$
$$50\%\text{ Cost }=33\times 11500=\text{ Rs. }379500$$

Values: Helping the flood victims

18. Volume of liquid in the bowl $=\frac{2}{3}\cdot\pi\cdot(18)^{3}\ \mathrm{cm}^{3}$

Volume, after wastage $=\frac{2\pi}{3}\cdot(18)^{3}\cdot\frac{90}{100}\ \mathrm{cm}^{3}$

Volume of liquid in 72 bottles $=\pi(3)^{2}\cdot h\cdot 72\ \mathrm{cm}^{3}$

$$\Rightarrow \mathrm{h}=\frac{\frac{2}{3}\pi(18)^{3}\cdot\frac{9}{10}}{\pi(3)^{2}\cdot 72}=5.4\text{ cm}$$

19. Laergest possible diameter $=10$ cm.
of hemisphere

radius $=5$ cm.

Total surface area $=6(10)^{2}+3.14\times(5)^{2}$

$$\text{Cost of painting }=\frac{678.5\times 5}{100}=\frac{\text{Rs. }3392.50}{100}=₹33.9250$$
$$=₹33.93$$

20. Volume of metal in 504 cones $=504\times\frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times 3$ cm.

$$\frac{4}{3}\times\frac{22}{7}\times r^{3}=504\times\frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times 3$$

r = 10.5 cm. ∴ diameter = 21 cm.

Surface area $= 4 \times \frac{22}{7} \times \frac{21}{7} \times \frac{21}{2} \times \frac{21}{2} = 1386\ \text{cm}^2$

SECTION D

21. Let the length of shorter side be x m.

length of diagonal $= (x + 16)m$
and, lenght of longer side $= (x + 14)m$
$x^2 + (x + 14)^2 = (x + 16)^2$
$\Rightarrow x^2 - 4x - 6 = 0 \Rightarrow x = 10$ m
length of sides are 10 m and 24 m.

22. $t_{60} = 8 + 59(2) = 126$

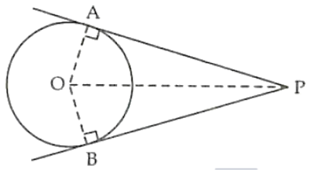
sum of last 10 terms $= (t_{51} + t_{52} + \cdots \ldots \ldots + t_{60})$
$t_{51} = 8 + 50(2) = 108$
Sum of last 10 terms $= 5[108 + 126]$

$= 1170$

23. Let the original average speed of (first) train be x km./h.

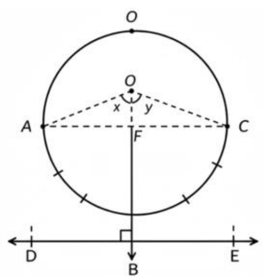
$\frac{54}{x} + \frac{63}{x+6} = 3$
$\Rightarrow 54x + 324 + 63x = 3x(x + 6)$
$\Rightarrow x^2 - 33x - 108 = 0$
Solving to get x = 36
First speed of train = 36 km/h.

24. Given: PA and PB are tangents drawn to circle with centre O from external point P .



To Prove: PA = PB
Construction: Join OA, OB and OP.
Proof: ∠OAP = ∠OBP = 90°.
(Radius ⊥ Tangent)
In △ OAP and △ OBP,
OA = OB (Radii of same circle)
OP = OP (Common side)
∠OAP = ∠OBP (ach 90°)
△ OAP ≅△ OBP (RHS congruency)
PA = PB (c.p.c.t.c.)
Thus, length of tangents drawn from an external point are equal.

25.



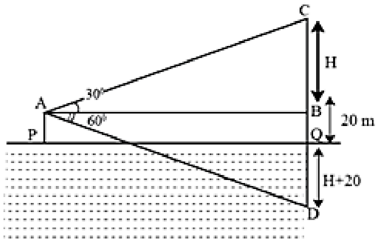
B is mid point of arc (ABC) Correct Fig.
∠1 = ∠2
△ OAF ≅△ OCF SAS.
∠AFO = ∠CFO = 90°
⇒ ∠AFO = ∠DBO = 90°
But these are corresponding angles

AC||DE

26. Constructing △ ABC

Constructing △ AB′C′

27.



correct figure

$\frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow x = \sqrt{3}h.$

$\frac{40 + h}{x} = \tan 60° = \sqrt{3} \Rightarrow x = \frac{40 + h}{\sqrt{3}}$

$\sqrt{3}h = \frac{40 + b}{\sqrt{3}} \Rightarrow h = 20 \text{ m}$

$x = 20\sqrt{3} \text{ m}$

$AC = \sqrt{(20)^2 + (20\sqrt{3})^2} = 40 \text{ m}$

28. **(i)** P(spade or an ace) $= \frac{13+3}{52} = \frac{4}{13}$

(ii) P (a black king) $= \frac{2}{52} = \frac{1}{26}$

(iii) P (neither a jack nor a king) $= \frac{52-8}{52} = \frac{44}{52} = \frac{11}{13}$

(iv) P(either a king or a queen) $= \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

29. $\frac{1}{2}[1(2k + 5) - 4(-5 + 1) - k(-1 - 2k)] = 24$

$\Rightarrow 2k^2 + 3k - 27 = 0$

Solving to get $k = 3, k = -\frac{9}{2}$

30. Radius of circle with centre O is OR

let $OR = x \therefore x^2 + x^2 = (42)^2 \Rightarrow x = 21\sqrt{2}$ m

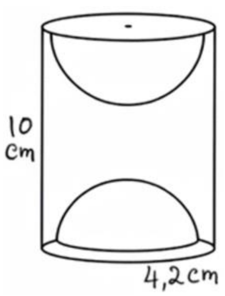
Area of one flower bed = Area of segment of circle with center angle 90^0

$= \frac{22}{7} \times 21\sqrt{2} \times 21\sqrt{2} \times \frac{90}{360} - \frac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2}$

$= 693 - 441 = 252 \text{ m}^2$

Area of two flower beds $= 2 \times 252 = 504 \text{ m}^2$

31.



Total Volume of cylinder $= \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times 10 \text{ cm}^3$

$= 554.40$ cm

$\Rightarrow$ Volume of metal scooped out $= \frac{4}{3} \times \frac{42}{7} \times \left(\frac{42}{10}\right)^3$

$= 310.46 \text{ cm}^3$

Volume of rest of cylinder $= 554.40 - 310.46$

$= 243.94 \text{ cm}^3$

If ℓ is the length of were, then

$$\frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} \times \ell = \frac{24394}{100}$$

$\Rightarrow \ \ell = 158.4$ cm