SECTION A

1. For $\angle ACB = 90^\circ$

$\angle PCA = 60^\circ$

2. $2(2k-1) = k + 9 + 2k + 7$

$k = 18$

3. $\frac{l}{2.5} = 2$

$l = 5$ m

4. No. of red cards and queens: 28

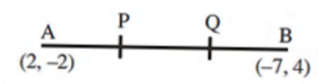
Required Probability: $\frac{24}{52}$ or $\frac{6}{13}$

SECTION B

5. $2(-5)^2 + p(-5) - 15 = 0 \Rightarrow p = 7$

$7x^2 + 7x + k = 0$ gives $49 - 28k = 0 \Rightarrow k = \frac{7}{4}$

6.



P divides AB in $1:2$

Coords of P are: $(-1,0)$

Q is mid-point of PB

Coords of Q are: $(-4,2)$

7. $AP = AS, BP = BQ, CR = CQ$ and $DR = DS$

$AP + BP + CR + DR = AS + BQ + CQ + DS \Rightarrow AB + CD = AD + BC$

8. Let the point be $A(3,0), B(6,4), C(-1,3)$

$AB = \sqrt{9+16} = 5, BC = \sqrt{49+1} = 5\sqrt{2}, AC = \sqrt{16+9} = 5$

$AB = AC$ and $AB^2 + AC^2 = BC^2$: $\triangle$ ABC isosceles, right Δ

9. $a + 3d = 0 \Rightarrow a = -3d$

$a_{25} = a + 24d = 21d$

$3a_{11} = 3(a + 10d) = 3(7d) = 21d$

10. Let $\angle TOP = \theta$ $\therefore \cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$ $\therefore \theta = 60^\circ$ Hence $\angle TOS = 120^\circ$ In $\triangle OTS, OT = OS \Rightarrow \angle OTS = \angle OST = 30^\circ$

SECTION C

11. $BC^2 = AB^2 - AC^2 = 169 - 144 = 25 \therefore BC = 5$ cm

Area of the shaded region = Area of semicircle - area of rt. $\triangle$ ABC

$= \frac{1}{2}(3.14)\left(\frac{13}{2}\right)^2 - \frac{1}{2} \cdot 12 \times 5$

$= 66.33 - 30 = 36.33\ \text{cm}^2$

12. Area of canvas needed $= 2 \times \frac{22}{7} \times (1.5) \times 2.1 + \frac{22}{7} \times 1.5 \times 2.8$

$= \frac{22}{7}[6.3 + 4.2] = \frac{22}{7} \times 10.5 = 33\ \text{m}^2$

cost $= 33 \times 500 =$ ₹16500

13. $PA = PB$ or $(PA)^2 = (PB)^2$

$(a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$

$(a+b)^2 + x^2 - 2ax - 2bx + (b-a)^2 + y^2 - 2by + 2ay$

$= (a-b)^2 + x^2 - 2ax + 2bx + (a+b)^2 + y^2 - 2ay - 2by$

$\Rightarrow 4ay = 4bx$ or $bx = ay$

14. Shaded area $= \pi(14^2 - 7^2) \times \frac{320}{360}$

$= \frac{22}{7} \times 147 \times \frac{8}{9}$

$= \frac{1232}{3} = 410.67 \text{ cm}^2$

15. $\frac{Sn}{S'_n} = \frac{n/2(2a+(n-1)d)}{n/2(2a'+(n-1)d')} = \frac{7n+1}{4n+27}$

$= \frac{a+\frac{n-1}{2}d}{a'+\frac{n-1}{2}d'} = \frac{7n+1}{An+27}$ ......(i)

Since $\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a+(m-1)d'}$, So replacing $\frac{n-1}{2}$ by $m-1$ i.e. $n = 2m-1$ in (i)

$\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a'+(m-1)d'} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$

16. Here $3(x-3+x-1) = 2(x-1)(x-2)(x-3)$

$\Rightarrow 3(2x-4) = 2(x-1)(x-2)(x-3)$

$\Rightarrow 3 = (x-1)(x-3)$ i.e. $x^2 - 4x = 0$

$x = 0, x = 4$

17. Volume of water in conical vessel $= \frac{1}{3} \times \frac{22}{7} \times 25 \times 24 \text{ cm}^2$

$\frac{1}{3} \times \frac{22}{7} \times 25 \times 24 = \frac{22}{7} \times 10 \times 10 \times \text{h}$

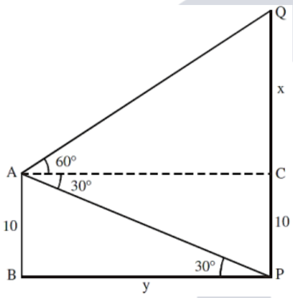
$\Rightarrow \text{h} = 2 \text{ cm}$

18. Volume of sphere $= \frac{4}{3}\pi \cdot (6)^3 \cdot \text{cm}^3$

$\pi r^2 \frac{32}{9} = \frac{4}{3}\pi(6)^3$

$\Rightarrow r = 9$ cm.

19.



Correct Figure

In $\triangle ABP, \frac{y}{10} = \cot 30° = \sqrt{3}$

$y = 10\sqrt{3}$ m

In $\triangle ACQ, \frac{x}{y} = \tan 60° = \sqrt{3}$

$x = \sqrt{3}(10\sqrt{3}) = 30$ m

Height of hill $= 30 + 10 = 40$ m

20. Set of possible outcomes is

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

(i) P(exactly 2 heads) $= 3/8$

(ii) P(at least 2 heads) $= 4/8$ or $1/2$

(iii) P (at least 2 tails) $= 4/8$ or $1/2$

SECTION D

21. Slant height of conical part $= \sqrt{(2.8)^2 + (2.1)^2} = 3.5$ m

Area of canvas/tent $= 2 \times \frac{22}{7} \times 2.8 \times 3.5 + \frac{22}{7} \times 2.8 \times 3.5 \text{ m}^2$

$= 92.4 \text{ m}^2$

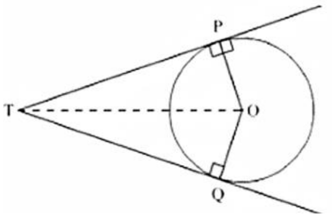
Cost of 1500 tents $= 1500 \times 92.4 \times 120 = ₹16632000$

Share of each school $= \frac{1}{50} \times 1663200$

$= ₹332640/$

"Helping the needy"

22.



Given:
TP and TQ are two tangents drawn from an external point T to the circle $C(O, r)$.
To prove: $TP = TQ$
Construction: Join OT.
Proof:
We know that a tangent to the circle is perpendicular to the radius through the point of contact.
$\angle OPT = \angle OQT = 90^{\circ}$
In $\triangle$ OPT and $\triangle$ OQT,
$OT = OT$ ... (Common)
$OP = OQ$ ... (Radius of the circle)
$\angle OPT = \angle OQT$ ... (90°)
$\triangle OPT \cong \triangle OQT$ ... (RHS congruence criterion)
$\Rightarrow TP = TQ$ ... (CPCT)
Hence, the lengths of the tangents drawn from an external point to a circle are equal.

23. Steps of construction:

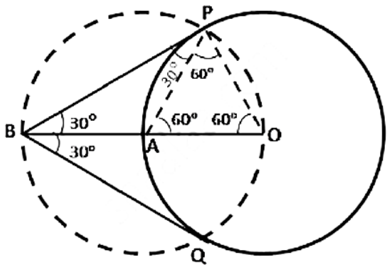
(i) Take a point O on the plane of the paper and draw a circle of radius $OA = 4$ cm.

(ii) Produce OA to B such that $OA = AB = 4$ cm.

(iii) Draw a circle with centre at A and radius AB.

(iv) Suppose it cuts the circle drawn in step (i) at P and Q .

b Join BP and BQ to get the desired tangents.



Justification:
In OAP, $OA = OP = 4$ cm ... (radii of the same circle)
Also, $AP = 4$ cm .... (Radius of the circle with centre A)
$\triangle$ OAP is equilateral.
$\angle PAO = 60^{\circ}$
$\angle BAP = 120^{\circ}$
In BAP, we have $BA = AP$ and $\angle BAP = 120^{\circ}$
$\angle ABP = \angle APB = 30^{\circ}$
Similarly we can get $\angle ABQ = 30^{\circ}$
$\angle PBQ = 60^{\circ}$

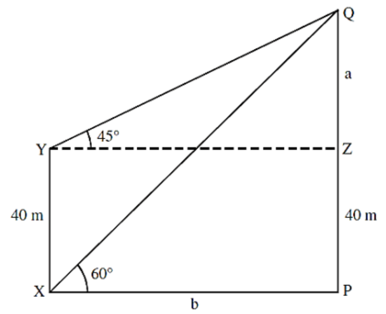
24. AC is tangent to circle with centre 0,

Thus $\angle ACO = 90^{\circ}$
$\triangle AO'D \sim \triangle AOC$
$\Rightarrow \frac{AO'}{AO} = \frac{DO'}{CO}$
$\frac{DO'}{CO} = \frac{r}{3r} = \frac{1}{3}$

25. $(x+4)(x+2+2x+2) = 4(x+1)(x+2)$

$(x+4)(3x+4)=4(x^2+3x+2)$
$\Rightarrow\ x^2-4x-8=0$
$\Rightarrow\ x=\dfrac{4\pm\sqrt{16+32}}{2}=2\pm 2\sqrt{3}$

26.



Correct Figure

In $\triangle YZQ, \dfrac{a}{YZ}=\tan 45^\circ=1$

$\Rightarrow\ YZ=a$ i.e. $a=b$

In $\Delta QPX, \dfrac{a+40}{b}=\dfrac{a+40}{a}=\tan 60^\circ=\sqrt{3}$

$(\sqrt{3}-1)a=40$ or $a=\dfrac{40}{\sqrt{3}-1}=20(\sqrt{3}+1)$

$=20(2.73)=54.60$ m

$PX=54.6$ m

$PQ=54.6+40=94.6$ m

27. Sum of numbers preceeding X

$=\dfrac{(\mathrm{X}-1)\mathrm{X}}{2}$

Sum of numbers following $X=\frac{(49)(50)}{2}-\frac{(X-1)}{2}-X$

$=\dfrac{2450-\mathrm{X}^2-\mathrm{X}}{2}$

$\dfrac{(\mathrm{X}-1)\mathrm{X}}{2}=\dfrac{2450-\mathrm{X}^2-\mathrm{X}}{2}$

$\Rightarrow 2\mathrm{X}^2=2450$

$\mathrm{X}^2=1225$

$=35$

[Since there is a typographic error in the question, which makes it unsolvab hence 4 marks be given to each student]

28. Coords of D are: $\left(\frac{1(1)+2(4)}{3}\right),\left(\frac{1(5)+2(6)}{3}\right)$ i.e. $\left(3,\frac{17}{3}\right)$

Coords of E are: $\left(\frac{1(7)+2(4)}{3},\frac{1(2)+2(6)}{3}\right)$ i.e. $\left(5,\frac{14}{3}\right)$

ar. $\triangle$ ADE $=\frac{1}{2}\left[4(1)+3\left(\frac{14}{3}-6\right)+5\left(6-\frac{17}{3}\right)\right]=\frac{5}{6}$

ar. $\triangle$ ABC $=\frac{1}{2}[4(3)+1(-4)+7(1)]=\frac{15}{2}$

ar. $\triangle$ ADE : ar. $\triangle$ ABC $=\frac{5}{6}:\frac{15}{2}$ or $1:9$

29. x can be any one of 1,2,3 or 4.

y can be any one of 1,4,9 of 16

Total number of cases of xy $=16$

Number of cases, where product is less than $16=8$

{1,4,9,2,8,3,12,4}

Required Probability $=\dfrac{8}{16}$ or $\dfrac{1}{2}$

30. Length of are $\overset{\frown}{\mathrm{AP}}=2\pi\mathrm{r}\frac{\theta}{360}$ or $\frac{\pi \mathrm{r}\theta}{180}$ ….. (i)

$\frac{AB}{r} = \tan\theta \Rightarrow AB = r\tan\theta$ .... (ii)

$\frac{OB}{r} = \sec\theta \Rightarrow OB = r\sec\theta$

$PB = OB - r = r\sec\theta - r$ ...... (iii)

Perimeter $= AB + PB + \overset{\frown}{AP}$

$= r\tan\theta + r\sec\theta - r + \frac{\pi r\theta}{180}$

or $r\left[\tan\theta + \sec\theta - 1 + \frac{\pi\theta}{180}\right]$

31. let x km/h be the speed of the stream

$\frac{32}{24-x} - \frac{32}{24+x} = 1$

$\Rightarrow 32(2x) = (24-x)(24+x)$

$x^2 + 64x - 576 = 0$

$(x+72)(x-8) = 0 \Rightarrow x = 8$

Speed of stream $= 8$ km/h