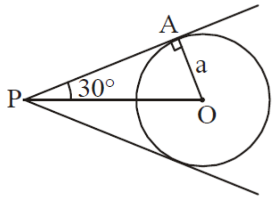
SECTION A

1. $a_{21} - a_7 = 84 \Rightarrow (a + 20d) - (a + 6d) = 84$

$\Rightarrow 14d = 84$

$\Rightarrow d = 6$

2.

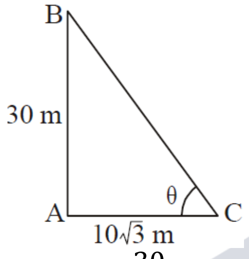


$\angle OPA = 30^\circ$

$\sin 30^\circ = \dfrac{a}{OP}$

$\Rightarrow OP = 2a$

3.



$\tan\theta = \dfrac{30}{10\sqrt{3}} = \sqrt{3}$

$\Rightarrow \theta = 60^\circ$

4. Let the number of rotten apples in the heap be n.

$\dfrac{n}{900} = 0.18$

$\Rightarrow n = 162$

SECTION B

5. Let the roots of the given equation be α and 6α.

Thus, the quadratic equation is $(x - \alpha)(x - 6\alpha) = 0$

$\Rightarrow x^2 - 7\alpha x + 6\alpha^2 = 0$ .... (i)

Given equation can be written as $x^2 - \frac{14}{p}x + \frac{8}{p} = 0$ .... (ii)

Comparing the co-efficients in (i) & (ii) $7\alpha = \frac{14}{p}$ and $6\alpha^2 = \frac{8}{p}$

Solving to get $p = 3$

6. Here $d = \frac{-3}{4}$

Let the nth term be first negative term

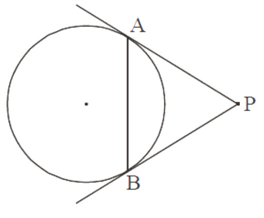
$20 + (n - 1)\left(\dfrac{-3}{4}\right) < 0$

$\Rightarrow 3n > 83$

$\Rightarrow n > 27\dfrac{2}{3}$

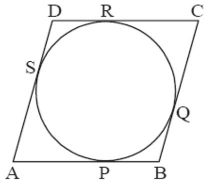
Hence 28th term is first negative term.

7.



Case I:
Correct Figure
Since PA = PB
Therefore in $\triangle$ PAB
$\angle PAB = \angle PBA$
Case II: If the tangents at A and B are parallel then each angle between chord and tangent = 90°

8.



Here AP = AS
BP = BQ
CR = CQ
DR = DS
Adding $(AP + PB) + (CR + RD) = (AS + SD) + (BQ + QC)$
$\Rightarrow AB + CD = AD + BC$

9. Let the coordinates of points P and Q be (0, b) and (a, 0) resp.
$\frac{a}{2} = 2 \Rightarrow a = 4$
$\frac{b}{2} = -5 \Rightarrow b = -10$
$P(0, -10)$ and $Q(4,0)$

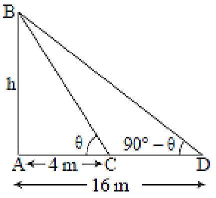
10. $PA^2 = PB^2$
$\Rightarrow (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$
$\Rightarrow 12x = 8y$
$\Rightarrow 3x = 2y$

SECTION C

11. $D = 4(ac+bd)^2 - 4(a^2+b^2)(c^2+d^2)$
$= -4(a^2d^2 + b^2c^2 - 2abcd)$
$= -4(ad-bc)^2$
Since ad ≠ bc
Therefore D < 0
The equation has no real roots

12. Here $a = 5, l = 45$ and $S_n = 400$
$\frac{n}{2}(a + l) = 400$ or $\frac{n}{2}(5 + 45) = 400$
$\Rightarrow n = 16$
Also 5 + 15 d = 45
$\Rightarrow d = \frac{8}{3}$

13.



Correct Figure
$\tan\theta = \frac{h}{4}$ ……… (i)
$\tan(90-\theta) = \frac{h}{16}$

$\Rightarrow \cot\theta = \frac{h}{16}$ .... (ii)

Solving (i) and (ii) to get

$h^2 = 64$

$\Rightarrow h = 8$ m

14. Let the number of black balls in the bag be n.

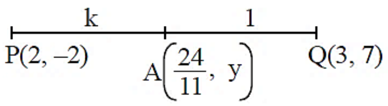
Total number of balls are $15 + n$

Prob(Black ball) = 3 × Prob(White ball)

$$\Rightarrow \frac{n}{15+n} = 3 \times \frac{15}{15+n}$$

$\Rightarrow n = 45$

15.



Let PA: AQ = k: 1

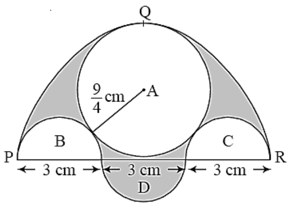
$$\frac{2+3k}{k+1} = \frac{24}{11}$$

$$\Rightarrow k = \frac{2}{9}$$

Hence the ratio is 2: 9.

Therefore $y = \frac{-18+14}{11} = \frac{-4}{11}$

16.



Area of semi-circle PQR $= \frac{\pi}{2}\left(\frac{9}{2}\right)^2 = \frac{81}{8}\pi\ \text{cm}^2$

Area of region A $= \pi\left(\frac{9}{4}\right)^2 = \frac{81}{16}\pi\ \text{cm}^2$

Area of region (B + C) $= \pi\left(\frac{3}{2}\right)^2 = \frac{9}{4}\pi\ \text{cm}^2$

Area of region $D = \frac{\pi}{2}\left(\frac{3}{2}\right)^2 = \frac{9}{8}\pi\ \text{cm}^2$

Area of shaded region $= \left(\frac{81}{8}\pi - \frac{81}{16}\pi - \frac{9}{4}\pi + \frac{9}{8}\pi\right)\text{cm}^2$

$= \frac{63}{16}\pi\ \text{cm}^2$ or $\frac{99}{8}\ \text{cm}^2$

17. Area of region ABDC $= \pi\frac{60}{360} \times (42^2 - 21^2)$

$= \frac{22}{7} \times \frac{1}{6} \times 63 \times 21$

$= 693\ \text{cm}^2$

Area of shaded region $= \pi(42^2 - 21^2) -$ region ABDC

$= \frac{22}{7} \times 63 \times 21 - 693$

$= 4158 - 693$

$= 3465\ \text{cm}^2$

18. Volume of water flowing in 40 min $= 5.4 \times 1.8 \times 25000 \times \frac{40}{60}\ \text{m}^3$

$= 162000\ \text{m}^3$

Height of standing water = 10 cm = 0.10 m

Area to be irrigated $= \frac{162000}{0.10}$

$= 1620000 \text{ m}^2$

19. Here $l = 4$ cm, $2\pi r_1 = 18$ cm and $2\pi r_2 = 6$ cm

$\Rightarrow \pi r_1 = 9, \pi r_2 = 3$

Curved surface area of frustum $= \pi(r_1 + r_2) \times l$ or $(\pi r_1 + \pi r_2) \times l$

$= (9 + 3) \times 4$

$= 48 \text{ cm}^2$

20. Volume of cuboid $= 4.4 \times 2.6 \times 1 \text{ m}^3$

Inner and outer radii of cylindrical pipe $= 30$ cm, 35 cm

Volume of material used $= \frac{\pi}{100^2}(35^2 - 30^2) \times \text{hm}^3$

$= \frac{\pi}{100^2} \times 65 \times 5 \text{ h}$

SECTION D

21. Here $[(5x + 1) + (x + 1)3](x + 4) = 5(x + 1)(5x + 1)$

$\Rightarrow (8x + 4)(x + 4) = 5(5x^2 + 6x + 1)$

$\Rightarrow 17x^2 - 6x - 11 = 0$

$\Rightarrow (17x + 11)(x - 1) = 0$

$\Rightarrow x = \frac{-11}{17}, x = 1$

22. Let one tap fill the tank in x hrs.

Therefore, other tap fills the tank in $(x + 3)$hrs.

Work done by both the taps in one hour is

$\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

$\Rightarrow (2x + 3)40 = 13(x^2 + 3x)$

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow (13x + 24)(x - 5) = 0$

$\Rightarrow x = 5$

(rejecting the negative value)

Hence one tap takes 5 hrs and another 8 hrs separately to fill the tank.

23. Let the first terms be a and a' and d and d' be their respective common differences.

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}(2a' + (n-1)d')} = \frac{7n + 1}{4n + 27}$$

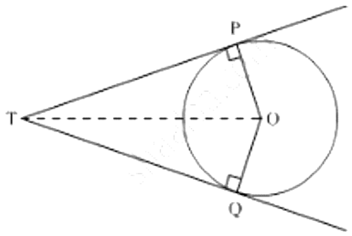
$$\Rightarrow \frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{7n + 1}{4n + 27}$$

To get ratio of 9th terms, replacing $\frac{n-1}{2} = 8$

$\Rightarrow \text{n} = 17$

Hence $\frac{t_9}{t_9} = \frac{a+8d}{a'+8d'} = \frac{120}{95}$ or $\frac{24}{19}$

24.



Given:

TP and TQ are two tangents drawn from an external point T to the circle $C(O, r)$.

To prove: $TP = TQ$

Construction: Join OT.

Proof:

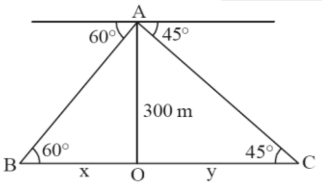
We know that a tangent to the circle is perpendicular to the radius through the point of contact.

$\angle OPT = \angle OQT = 90^\circ$
In $\triangle$ OPT and $\triangle$ OQT,
$OT = OT$ ... (Common)
$OP = OQ$ ... (Radius of the circle)
$\angle OPT = \angle OQT$ ... (90°)
$\Delta OPT \cong \Delta OQT$ ... (RHS congruence criterion)
$\Rightarrow TP = TQ$ ... (CPCT)
Hence, the lengths of the tangents drawn from an external point to a circle are equal.

25. In right angled $\triangle$ POA and $\triangle$ OCA
$\triangle OPA \cong \triangle OCA$
$\angle POA = \angle AOC$ .... (i)
Also $\triangle OQB \cong \triangle OCB$
$\angle QOB = \angle BOC$ ......(ii)
Therefore $\angle AOB = \angle AOC + \angle COB$
$= \frac{1}{2}\angle POC + \frac{1}{2}\angle COQ$
$= \frac{1}{2}(\angle POC + \angle COQ)$
$= \frac{1}{2} \times 180^\circ$
$= 90^\circ$

26. Correct construction of $\triangle$ ABC and corresponding similar triangle

27.



Correct Figure
$\tan 45^\circ = \frac{300}{y}$
$\Rightarrow\ 1 = \frac{300}{y}$ or $y = 300$
$\tan 60^\circ = \frac{300}{x}$
$\Rightarrow\ \sqrt{3} = \frac{300}{x}$ or $x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$
Width of river $= 300 + 100\sqrt{3} = 300 + 173.2$
$= 473.2$ m

28. Points A, B and C are collinear
Therefore $\frac{1}{2}[(k+1)(2k+3-5k) + 3k(5k-2k) + (5k-1)(2k-2k-3)] = 0$
$= (k+1)(3-3k) + 9k^2 - 3(5k-1) = 0$
$= 2k^2 - 5k + 2 = 0$
$= (k-2)(2k-1) = 0$
$\Rightarrow\ k = 2, \frac{1}{2}$

29. Total number of outcomes $= 36$
(i) P(even sum) $= \frac{18}{36} = \frac{1}{2}$
(ii) P(even product) $= \frac{27}{36} = \frac{3}{4}$

30. Area of shaded region $= (21 \times 14) - \frac{1}{2} \times \pi \times 7 \times 7$
$= 294 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 294 - 77$
$= 217\ \text{cm}^2$

Perimeter of shaded region $= 21 + 14 + 21 + \frac{22}{7} \times 7$

$= 56 + 22$
$= 78\ \text{cm}$

31. Volume of rain water on the roof $=$ Volume of cylindrical tank

ie., $22 \times 20 \times h = \frac{22}{7} \times 1 \times 1 \times 3.5$

$\Rightarrow\ \text{h} = \frac{1}{40}\ \text{m}$

$= 2.5\ \text{cm}$

Water conservation must be encouraged or views relevant to it.