SECTION- A

1. $x = 3$ is one root of the equation

$9 - 6k - 6 = 0$

$\Rightarrow\ k = \dfrac{1}{2}$

2. The required numbers are 2 and 4.

HCF of 2 and 4 is 2.

3. $OP = \sqrt{x^2 + y^2}$

4. $a + 6(-4) = 4$

$\Rightarrow\ a = 28$

5. $\cos 67^\circ = \sin 23^\circ$

$\cos^2 67^\circ - \sin^2 23 = 0$

6. $\dfrac{ar\ \triangle ABC}{ar\ \triangle PQR} = \dfrac{AB^2}{PQ^2}$

$= \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$

SECTION - B

7. Let us assume $5 + 3\sqrt{2}$ is a rational number.

$5 + 3\sqrt{2} = \frac{p}{q}$ where $q \neq 0$ and p and q are integers.

$\Rightarrow\ \sqrt{2} = \dfrac{p - 5q}{3q}$

$\Rightarrow\ \sqrt{2}$ is a rational number as RHS is rational

This contradicts the given fact that $\sqrt{2}$ is irrational.

Hence $5 + 3\sqrt{2}$ is an irrational number.

8. $AB = DC$ and $BC = AD$

$\Rightarrow \quad \left.\begin{aligned} x + y &= 30 \\ \text{and} \quad x - y &= 14 \end{aligned}\right\}$

Solving to get $x = 22$ and $y = 8$.

9. $S = 3 + 6 + 9 + 12 + \cdots + 24$

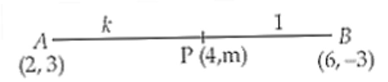
$= 3(1 + 2 + 3 + \cdots + 8)$

$= 3 \times \dfrac{8 \times 9}{2}$

$= 108$

10. Let $AP : PB = k : 1$

$\dfrac{6k + 2}{k + 1} = 4$



$\Rightarrow k = 1$, ratio is $1:1$

Hence $m = \dfrac{-3 + 3}{2} = 0$

11. Total number of possible outcomes $= 36$

(i) Doublets are $(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)$

Total number of doublets $= 6$

Prob(getting a doublet) $= \frac{6}{36}$ or $\frac{1}{6}$

(ii) Favourable outcomes are $(4,6)(5,5)(6,4)$ i.e., 3

Prob (getting a sum 10) $= \frac{3}{36}$ or $\frac{1}{12}$

12. Total number of outcomes $= 98$

(i) Favourable outcomes are $8, 16, 24, \ldots, 96$ i.e., 12

Prob (integer is divisible by 8) $= \frac{12}{98}$ or $\frac{6}{49}$

(ii) Prob(integer is not divisible by 8) $= 1 - \frac{6}{49}$

$= \dfrac{43}{49}$

SECTION- C

13. $404 = 2 \times 2 \times 101 = 2^2 \times 101$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$\therefore$ HCF of 404 and 96 $= 2^2 = 4$

LCM of 404 and 96 $= 101 \times 2^5 \times 3 = 9696$

HCF $\times$ LCM $= 4 \times 9696 = 38784$

Also $404 \times 96 = 38784$

Hence HCF $\times$ LCM $=$ Product of 404 and 96.

14. $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

$2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of $p(x)$

$\mathrm{p(x) = (x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) \times g(x)}$

$= \mathrm{(x^2 - 4x + 1)g(x)}$

$\mathrm{(2x^4 - 9x^3 + 5x^2 + 3x - 1) \div (x^2 - 4x + 1) = 2x^2 - x - 1}$

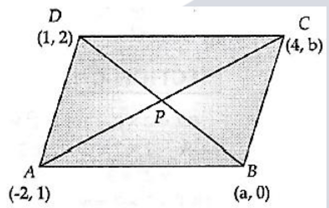
$\mathrm{g(x) = 2x^2 - x - 1}$

$= \mathrm{(2x + 1)(x - 1)}$

Therefore other zeroes are $x = -\frac{1}{2}$ and $x = 1$

Therefore all zcrocs are $2 + \sqrt{3}, 2 - \sqrt{3}, -\frac{1}{2}$ and 1

15.



ABCD is a parallelogram

$\therefore$ diagonals AC and BD bisect each other

Therefore

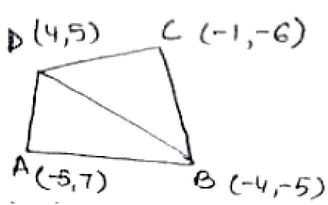
Mid point of BD is same as mid point of AC

$\Rightarrow \left(\dfrac{a+1}{2}, \dfrac{2}{2}\right) = \left(\dfrac{-2+4}{2}, \dfrac{b+1}{2}\right)$

$\Rightarrow \dfrac{a+1}{2} = 1$ and $\dfrac{b+1}{2} = 1$

$\Rightarrow a = 1, b = 1$. Therefore length of sides are $\sqrt{10}$ units each.

OR



Area of quad $ABCD = Ar \triangle ABD + Ar \triangle BCD$

Area of $\triangle$ ABD $= \dfrac{1}{2}|(-5)(-5-5) + (-4)(5-7) + (4)(7+5)$

$= 53$ sq units

Area of $\triangle BCD = \dfrac{1}{2}|(-4)(-6-5) + (-1)(5+5) + 4(-5+6)$

$= 19$ sq units

Hence area of quad. ABCD $= 53 + 19 = 72$ sq units

16. Let the usual speed of the plane be x km/hr.

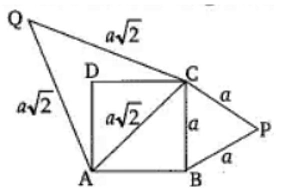
$$\frac{1500}{x}-\frac{1500}{x+100}=\frac{30}{60}$$
$\Rightarrow x^2+100x-300000=0$
$\Rightarrow x^2+600x-500x-300000=0$
$\Rightarrow (x+600)(x-500)=0$
$x\neq -600, \therefore x=500$
Speed of plane $=500$ km/hr

17. Let ABCD be a square with side a. Then diagonal AC $=a\sqrt{2}$.

Since $\triangle$ BCP and $\triangle$ ACQ are equilateral triangles, so they are similar.

$$\frac{\operatorname{ar}(\triangle BCP)}{\operatorname{ar}(\triangle ACQ)}=\frac{BC^2}{AC^2}=\frac{a^2}{(a\sqrt{2})^2}=\frac{1}{2}\quad \ldots\ldots(\mathrm{i})$$

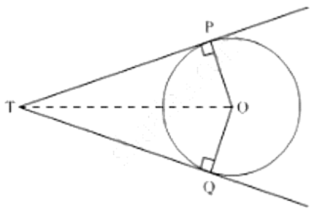
$$\Rightarrow \operatorname{ar}(\triangle BCP)=\frac{1}{2}\operatorname{ar}(\triangle ACQ)$$



OR

Let $\triangle ABC \sim \triangle PQR$.
$$\frac{\operatorname{ar}\triangle ABC}{\operatorname{ar}\triangle PQR}=\frac{AB^2}{PQ^2}=\frac{BC^2}{QR^2}=\frac{AC^2}{PR^2}$$
Given ar $\triangle$ ABC $=$ ar $\triangle$ PQR
$$\Rightarrow \frac{AB^2}{PQ^2}=1=\frac{BC^2}{QR^2}=\frac{AC^2}{PR^2}$$
$\Rightarrow$ AB $=$ PQ, BC $=$ QR, AC $=$ PR
$\Rightarrow$ Therefore $\triangle ABC \cong \triangle PQR$. (sss congruence rule)

18.



Given:
TP and TQ are two tangents drawn from an external point T to the circle C(O, r).
To prove: TP $=$ TQ
Construction: Join OT.
Proof:
We know that a tangent to the circle is perpendicular to the radius through the point of contact.
$\angle OPT=\angle OQT=90^\circ$
In $\triangle$ OPT and $\triangle$ OQT,
OT $=$ OT ... (Common)
OP $=$ OQ ... (Radius of the circle)
$\angle OPT=\angle OQT$ ... (90°)
$\Delta OPT \cong \Delta OQT$ ... (RHS congruence criterion)
$\Rightarrow$ TP $=$ TQ ... (CPCT)
Hence, the lengths of the tangents drawn from an external point to a circle are equal.

19. $4\tan\theta=3$

$\Rightarrow \tan\theta = \frac{3}{4}$

$\Rightarrow \sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$

$$\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1}$$

$= \frac{13}{11}$

OR

$\tan 2A = \cot(A - 18^\circ)$

$\Rightarrow 90^\circ - 2A = A - 18^\circ$

$\Rightarrow 3A = 108^\circ$

$A = 36^\circ$

20. Radius of each arc drawn $= 6$ cm

Area of one quadrant $= (3.14) \times \frac{36}{4}$

Area of four quadrants $= 3.14 \times 36 = 113.04 \text{ cm}^2$

Area of square ABCD $= 12 \times 12 = 144 \text{ cm}^2$

Hence Area of shaded region $= 144 - 113.04$

$= 30.96 \text{ cm}^2$

21. Total surface Area of article = CSA of cylinder + CSA of 2 hemispheres

CSA of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 3.5 \times 10$

$= 220 \text{ cm}^2$

Surface Area of two hemispherical scoops $= 4 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 154 \text{ cm}^2$

Total surface Area of article $= 220 + 154$

$= 374 \text{ cm}^2$

OR

Radius of conical heap $= 12$ m

Volume of rice $= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 \text{ m}^3$

$= 528 \text{ m}^3$

Area of canvas cloth required $= \pi r!$

$l = \sqrt{12^2 + (3.5)^2} = 12.5$ m

Area of canvas required $= \frac{22}{7} \times 12 \times 12.5$

$= 471.4 \text{ m}^2$

22.

Salary (in thousand Rs)	No. of persons (f)	cf
5-10	49	49
10-15	133	182
15-20	63	245
20-25	15	260
25-30	6	266
30-35	7	273

35-40	4	277
40-45	2	279
45-50	1	280

$\frac{N}{2} = \frac{280}{2} = 140$

Median class is 10-15

Median $= l + \frac{h}{f}\left(\frac{N}{2} - C\right)$

$= 10 + \frac{5}{133}(140 - 49)$

$= 10 + \frac{5 \times 91}{133}$

$= 13.42$

Median salary is Rs 13,42 thousand or Rs 13420 (approx)

SECTION - D

23. Let the speed of stream be x km/hr.

The speed of the boat upstream $= (18 - x)$km/hr

and Speed of the boat downstream $= (18 + x)$km/hr

As given in the question,

$\frac{24}{18-x} - \frac{24}{18+x} = 1$

$\Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow (x + 54)(x - 6) = 0$

$x \neq -54, \therefore x = 6$

Speed of the stream $= 6$ km/hr.

OR

Let the original average speed of train be x km/hr.

Therefore $\frac{63}{x} + \frac{72}{x+6} = 3$

$\Rightarrow x^2 - 39x - 126 = 0$

$\Rightarrow (x - 42)(x + 3) = 0$

$x \neq -3 \ \therefore x = 42$

Original speed of train is 42 km/hr.

24. Let the four consecutive terms of the A.P. be

$a - 3d, a - d, a + d, a + 3d.$

By given conditions

$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow 4a = 32$

$\Rightarrow a = 8$

and $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

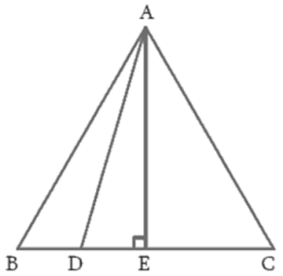
$\Rightarrow 8a^2 = 128d^2$

$\Rightarrow d^2 = 4$

$\Rightarrow d = \pm 2$

Numbers are 2,6,10,14 or 14,10,6,2.

25.



Draw $AE \perp BC$

$\triangle AEB \cong \triangle AEC$ (RHS congruence rule)

$BE = EC = \frac{1}{2}BC = \frac{1}{2}AB$

Let $AB = BC = AC = x$

Now $BE = \frac{x}{2}$ and $DE = BE - BD$

$= \frac{x}{2} - \frac{x}{3}$

$= \frac{x}{6}$

Now $AB^2 = AE^2 + BE^2$ .... (1)

and $AD^2 = AE^2 + DE^2$ .... (2)

From (1) and (2) $AB^2 - AD^2 = BE^2 - DE^2$

$\Rightarrow x^2 - AD^2 = \left(\frac{x}{2}\right)^2 - \left(\frac{x}{6}\right)^2$

$\Rightarrow AD^2 = x^2 - \frac{x^2}{4} + \frac{x^2}{36}$

$\Rightarrow AD^2 = \frac{28}{36}x^2$

$\Rightarrow 9AD^2 = 7AB^2$

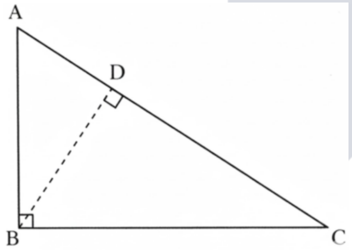
OR

"In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides."

Proof: Let ABC be a right triangle where $\angle B = 90^\circ$.

It has to be proved that $AC^2 = AB^2 + BC^2$

Construction: Draw $BD \perp AC$



In $\triangle ADB$ and $\triangle ABC$,

$\angle ADB = \angle ABC$[Each is right angle]

$\angle BAD = \angle BAC$[Common angle]

Therefore, by AA similarity criterion, $\triangle ADB \sim \triangle ABC$

$\frac{AD}{AB} = \frac{AB}{AC}$ ... ... [Sides are proportional in similar triangles]

$\Rightarrow AD \times AC = AB^2$ ... (1)

Similarly, it can be proved that $\Delta BDC \sim \Delta ABC$

$\frac{CD}{BC} = \frac{BC}{AC}$

$\Rightarrow AC \times CD = BC^2$ ....(2)

Adding equations (1) and (2), we obtain

$AB^2 + BC^2 = AD \times AC + AC \times CD$

$\Rightarrow AB^2 + BC^2 = AC(AD + CD)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AB^2 + BC^2 = AC^2$

26. A $\triangle A'BC'$ whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$ can be drawn as follows:

Step 1

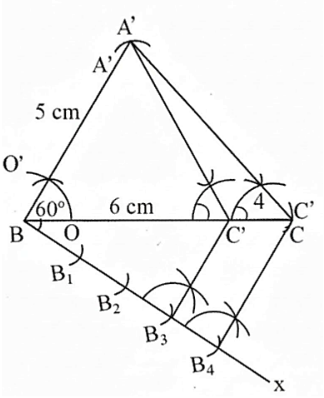
Draw a $\triangle ABC$ with side $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60^\circ$

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3

Locate 4 points (as 4 is greater in 3 and 4), B_1, B_2, B_3, B_4, on line segment BX .
Step 4
Join B_4C and draw a line through B_3, parallel to B_4C intersecting BC at C'
Step 5
Draw a line through C' parallel to AC intersecting AB at A'.$\triangle A'B'$ is the required triangle.



27. LHS $= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$

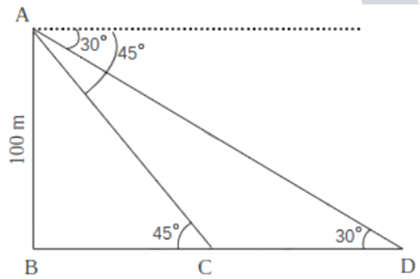
$= \frac{\sin A\left(1-2(1-\cos^2 A)\right)}{\cos A(2\cos^2 A - 1)}$

$= \tan A \frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$

$= \tan A = RHS$

28. Here $r_1 = 15$ cm, $r_2 = 5$ cm and $h = 24$ cm
(i) Area of metal sheet = CSA of the bucket + area of lower end
$= \pi l(r_1 + r_2) + \pi r_2^2$
where $l = \sqrt{24^2 + (15-5)^2} = 26$ cm
Surface area of metal sheet $= 3.14(26 \times 20 + 25)\text{cm}^2$
$= 1711.3\ \text{cm}^2$
(ii) We should avoid use of plastic because it is non-degradable or similar value.

29.



Figure

Let AB be the tower and ships are at points C and D.

$\tan 45^\circ = \frac{AB}{BC}$

$\Rightarrow \frac{AB}{BC} = 1$

$\Rightarrow AB = BC$

Also $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$

$\Rightarrow AB + CD = \sqrt{3}AB$

$\Rightarrow CD = AB(\sqrt{3} - 1)$
$= 100 \times (1.732 - 1)$
$= 73.2$ m

30.

Class	x	f	fx	
11-13	12	3	36	
13-15	14	6	84	
15-17	16	9	144	
17-19	18	13	234	
19-21	20	f	20f	
21-23	22	5	110	For x
23-25	24	4	96	Ef
		$40 + f$	$704 + 20f$	Efx

$$\text{Mean } = 18 = \frac{704 + 20f}{40 + f}$$

$\Rightarrow 720 + 18f = 704 + 20f$
$\Rightarrow f - 8$

OR

Cumulative frequency distribution table of less than type is

Daily income	Cumulative frequency
Less than 100	0
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50