SECTION - A

1. Let the point A be (x, y)

$\frac{1+x}{2} = 2$ and $\frac{4+y}{2} = -3$

$\Rightarrow\ x = 3$ and $y = -10$

Point A is $(3, -10)$

2. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow\ 16 - 4k \geq 0$

$\Rightarrow\ k \leq 4$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots $= 1$

$\Rightarrow \frac{k}{3} = 1 \Rightarrow k = 3$

3. $\tan 2A = \cot(90^\circ - 2A)$

$90^\circ - 2A = A - 24^\circ$

$\Rightarrow\ A = 38^\circ$

OR

$\sin 33^\circ = \cos 57^\circ$

$\sin^2 33^\circ + \sin^2 57^\circ = \cos^2 57^\circ + \sin^2 57^\circ = 1$

4. Numbers are $12, 15, 18, \ldots, 99$

$99 = 12 + (n-1) \times 3$

$\Rightarrow\ n = 30$

5. $AB = 1 + 2 = 3$ cm

$\Delta ABC \sim \Delta ADE$

$\frac{ar(ABC)}{ar(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$

$ar(\triangle ABC) : ar(\triangle ADE) = 9:1$

6. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.) e.g., 1.5, 1.6, 1.63 etc.

SECTION - B

7. Using Euclid's Algorithm

$7344 = 1260 \times 5 + 1044$

$1260 = 1044 \times 1 + 216$

$1044 = 216 \times 4 + 180$

$216 = 180 \times 1 + 36$

$180 = 36 \times 5 + 0$

HCF of 1260 and 7344 is 36.

OR

Using Euclid's Algorithm

$a = 4q + r, 0 \leq r < 4$

$\Rightarrow a = 4q, a = 4q + 1, a = 4q + 2$ and $a = 4q + 3$.

Now $a = 4q$ and $a = 4q + 2$ are even numbers.

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q.

8. $a_n = a_{21} + 120$

$= (3 + 20 \times 12) + 120$

$= 363$

$363 = 3 + (n-1) \times 12$

$\Rightarrow n = 31$ or 31 st term is 120 more than a_{21}.

OR

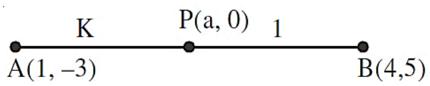
$a_1 = S_1 = 3 - 4 = -1$

$a_2 = S_2 - S_1 = [3(2)^2 - 4(2)] - (-1) = 5$

$d = a_2 - a_1 = 6$

Hence $a_n = -1 + (n-1) \times 6 = 6n - 7$

9.



Let the required point be $(a, 0)$ and required ratio $AP: PB = k: 1$

$a = \dfrac{4k+1}{k+1}$

$0 = \dfrac{5k-3}{k+1}$

$\Rightarrow k = \dfrac{3}{5}$ or required ratio is $3:5$

Point P is $\left(\frac{17}{8}, 0\right)$

10. Total number of outcomes $= 8$

Favourable number of outcomes $(HHH, TTT) = 2$

Prob. (getting success) $= \frac{2}{8}$ or $\frac{1}{4}$

Prob. (losing the game) $= 1 - \frac{1}{4} = \frac{3}{4}$.

11. Total number of outcomes $= 6$.

(i) Prob. (getting a prime number (2,3,5)) $= \frac{3}{6}$ or $\frac{1}{2}$

(ii) Prob. (getting a number between 2 and 6(3,4,5)) $= \frac{3}{6}$ or $\frac{1}{2}$.

12. System of equations has infinitely many solutions

$\dfrac{c}{12} = \dfrac{3}{c} = \dfrac{3-c}{-c}$

$\Rightarrow c^2 = 36 \Rightarrow c = 6$ or $c = -6$ …. (1)

Also $-3c = 3c - c^2 \Rightarrow c = 6$ or $c = 0$ …. (2)

From equations (1) and (2)

$c = 6$

SECTION- C

13. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and $b \neq 0$.

So $\sqrt{2} = \frac{a}{b}$

$\Rightarrow a^2 = 2b^2$

Thus a^2 is a multiple of 2

$\Rightarrow$ a is a multiple of 2.

Let $a = 2m$ for some integer m

$b^2 = 2m^2$

Thus b^2 is a multiple of 2

$\Rightarrow$ b is a multiple of 2

Hence 2 is a common factor of a and b .

This contradicts the fact that a and b are coprimes

Hence $\sqrt{2}$ is an irrational number.

14. Sum of zeroes $= k + 6$

Product of zeroes $= 2(2k-1)$

Hence $k + 6 = \frac{1}{2} \times 2(2k-1)$

$\Rightarrow k = 7$

15. Let sum of the ages of two children be $x yrs$ and father's age be $y yrs$.

$y = 3x$ ……. (1)

and $y + 5 = 2(x + 10)$ ………. (2)

Solving equations (1) and (2)

$x = 15$
and $y = 45$
Father's present age is 45 years.

OR

Let the fraction be $\frac{x}{y}$

$\frac{x-2}{y} = \frac{1}{3}$ .... (1)

and $\frac{x}{y-1} = \frac{1}{2}$......... (2)

Solving (1) and (2) to get $x = 7, y = 15$.

Required fraction is $\frac{7}{15}$

16. Let the required point on y-axis be $(0, b)$

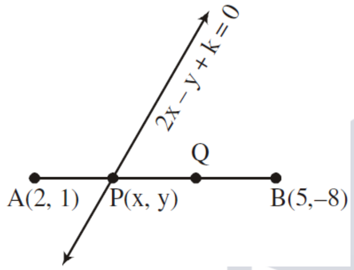
$(5-0)^2 + (-2-b)^2 = (-3-0)^2 + (2-b)^2$

$\Rightarrow\ 29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow\ b = -2$

Required point is $(0, -2)$

OR



AP: PB $= 1:2$

$\mathrm{x} = \frac{4+5}{3} = 3$ and $\mathrm{y} = \frac{2-8}{3} = -2$

Thus point P is $(3, -2)$.

Point (3, -2) lies on $2x - y + k = 0$

$\Rightarrow 6 + 2 + \mathrm{k} = 0$

$\Rightarrow \mathrm{k} = -8$.

17. LHS $= \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$

$= (\sin^2\theta + \cos^2\theta) + \operatorname{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$

$= 7 + \cot^2\theta + \tan^2\theta =$ RHS

OR

LHS $= \left(1 + \frac{1}{\tan A} - \operatorname{cosec} A\right)(1 + \tan A + \sec A)$

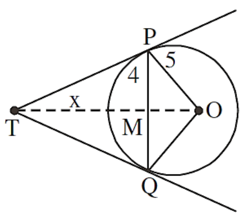
$= \frac{1}{\tan A}(\tan A + 1 - \sec A)(1 + \tan A + \sec A)$

$= \frac{1}{\tan A}[(1 + \tan A)^2 - \sec^2 A]$

$= \frac{1}{\tan A}[1 + \tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 =$ RHS

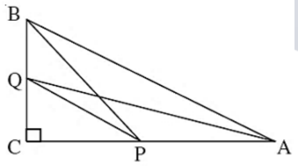
18.



Join OT and OQ.
$TP = TQ$
$TM \perp PQ$ and bisects PQ
Hence $PM = 4$ cm
Therefore $OM = \sqrt{25 - 16} = \sqrt{9} = 3$ cm.
Let $TM = x$
From $\triangle PMT, PT^2 = x^2 + 16$
From $\triangle POT, PT^2 = (x + 3)^2 - 25$
Hence $x^2 + 16 = x^2 + 9 + 6x - 25$
$\Rightarrow 6x = 32 \Rightarrow x = \dfrac{16}{3}$
Hence $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$
$PT = \frac{20}{3}$ cm.

19. $\Delta ACB \sim \triangle ADC$ (AA similarity)
$\Rightarrow \frac{AC}{BC} = \frac{AD}{CD}$ .....(1)
Also $\triangle ACB \sim \triangle CDB$ (AA similarity)
$\Rightarrow \frac{AC}{BC} = \frac{CD}{BD}$ .....(2)
Using equations (1) and (2)
$$\frac{AD}{CD} = \frac{CD}{BD}$$
$\Rightarrow C^2 = AD \times BD$

OR



Correct Figure
$AQ^2 = CQ^2 + AC^2$
$BP^2 = CP^2 + BC^2$
$AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$
$= PQ^2 + AB^2$

20. $AC = \sqrt{64 + 36} = 10$ cm.
Radius of the circle $(r) = 5$ cm.
Area of shaded region = Area of circle $-$ Ar(ABCD)
$= 3.14 \times 25 - 6 \times 8$
$= 78.5 - 48$
$= 30.5\ cm^2$

21. Length of canal covered in 30 min = 5000 m.
Volume of water flown in 30 min $= 6 \times 1.5 \times 5000\ m^3$
If 8 cm standing water is needed
then area irrigated $= \dfrac{6 \times 1.5 \times 5000}{.08} = 562500\ m^2$

22. Modal class is $30 - 40$
$$\text{Mode } = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$
$$= 30 + \left(\frac{16 - 10}{32 - 10 - 12}\right) \times 10$$
$= 36$

SECTION - D

23. Let the smaller tap fills the tank in x hrs
the larger tap fills the tank in $(x - 2)$ hrs.

Time taken by both the taps together $= \frac{15}{8}$ hrs.

Therefore $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$

$\Rightarrow 4x^2 - 23x + 15 = 0$

$\Rightarrow (4x-3)(x-5) = 0$

$x \neq \frac{3}{4} \therefore x = 5$

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp.

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\frac{30}{x-y} + \frac{44}{x+y} = 10$ ....(i)

and $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ....(ii)

Solving (i) and (ii) to get

$x + y = 11$

and $x - y = 5$

Solving (iii) and (iv) to get $x = 8, y = 3$.

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

24. $S_4 = 40 \Rightarrow 2(2a + 3d) = 40 \Rightarrow 2a + 3d = 20$

$S_{14} = 280 \Rightarrow 7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$

Solving to get d = 2

and a = 7

$\text{Sn} = \frac{n}{2}[14 + (n-1) \times 2]$

$= n(n+6)$ or $(n^2 + 6n)$

25. LHS $= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$

Dividing num. & deno. by cos A

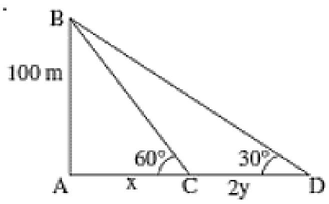
$= \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$

$= \frac{\tan A - 1 + \sec A}{(\tan A - \sec A) + (\sec^2 A - \tan^2 A)}$

$= \frac{\tan A - 1 + \sec A}{(\tan A - \sec A)(1 - \sec A - \tan A)}$

$= \frac{-1}{\tan A - \sec A} = \frac{1}{\sec A - \tan A} =$ RHS

26.



Correct Figure

Let the speed of the boat be ym/min

$\therefore CD = 2y$

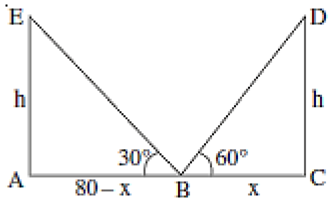
$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x + 2y} \Rightarrow x + 2y = 100\sqrt{3}$

$y = \frac{100\sqrt{3}}{3} = 57.73$

or speed of boat = 57.73 m/min.

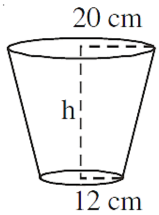
OR



Correct Figure
Let $BC = x$ so $AB = 80 - x$
where AC is the road.
$\tan 60^\circ = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$
and $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{80 - x} \Rightarrow h\sqrt{3} = 80 - x$
Solving equation to get
$x = 20, h = 20\sqrt{3}$
$AB = 60$ m, $BC = 20$ m and $h = 20\sqrt{3}$ m.

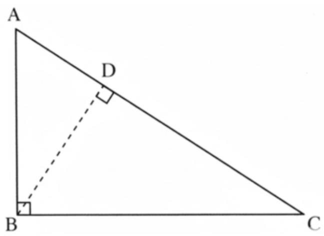
27. Correct construction of $\triangle$ ABC.
Correct construction of triangle similar to triangle ABC.

28.



Volume of the bucket $= 12308.8 \text{ cm}^3$
Let $r_1 = 20$ cm, $r_2 = 12$ cm
$V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$
$12308.8 = \frac{3.14 \times h}{3}(400 + 144 + 240)$
$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784} = 15$ cm
Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$
$\Rightarrow\ l = 17$ cm
Surface area of metal sheet used $= \pi r_2^2 + \pi l(r_1 + r_2)$
$= 3.14(144 + 17 \times 32)$
$= 2160.32 \text{ cm}^2$

29. "In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides."
Proof: Let ABC be a right triangle where $\angle B = 90^\circ$.
It has to be proved that $AC^2 = AB^2 + BC^2$
Construction: Draw $BD \perp AC$



In $\triangle$ ADB and $\triangle$ ABC,
$\angle ADB = \angle ABC$ [Each is right angle]
$\angle BAD = \angle BAC$ [Common angle]
Therefore, by AA similarity criterion, $\triangle ADB \sim \triangle ABC$
$\frac{AD}{AB} = \frac{AB}{AC}$ ..... [Sides are proportional in similar triangles]

$\Rightarrow AD \times AC = AB^2$ ……(1)

Similarly, it can be proved that $\Delta BDC \sim \Delta ABC$

$\frac{CD}{BC} = \frac{BC}{AC}$

$\Rightarrow AC \times CD = BC^2$ …….(2)

Adding equations (1) and (2), we obtain

$AB^2 + BC^2 = AD \times AC + AC \times CD$

$\Rightarrow AB^2 + BC^2 = AC(AD + CD)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AB^2 + BC^2 = AC^2$

This proves the Pythagoras Theorem.

30.

Class	**Frequency**	**Cumulative freq.**
$0-10$	f_1	f_1
$10-20$	5	$5 + f_1$
$20-30$	9	$14 + f_1$
$30-40$	12	$26 + f_1$
$40-50$	f_2	$26 + f_1 + f_2$
$50-60$	3	$29 + f_1 + f_2$
$60-70$	2	$31 + f_1 + f_2$
	$\underline{40}$	

Median $= 32.5 \Rightarrow$ median class is $30-40$.

Now $32.5 = 30 + \frac{10}{12}(20 - 14 - f_1)$

$\Rightarrow f_1 = 3$

Also $31 + f_1 + f_2 = 40$

$\Rightarrow f_2 = 6$