1. **(b)** Given
$A^2 - 3A + I = O$
Multiplying by A^{-1}.
$A - 3I + A^{-1} = O$
$A^{-1} = 3I - A$
So, $x = -1, y = 3$
$x + y = 2$
2. **(c)** $|A| = 2, A$ is 2×2
$|4A^{-1}| = 4^2|A^{-1}|$
$= 16 \cdot \frac{1}{|A|} = 16 \cdot \frac{1}{2} = 8$
3. **(d)** For a 3×3 matrix,
$|adjA| = |A|^{n-1} =| A^2$
$|A|^2 = 64$
$|A| = \pm 8$
4. **(b)** $2A + B = O$
$B = -2A$
$B = -2\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} -6 & -8 \\ -10 & 4 \end{bmatrix}$
5. **(b)** $f'(x) = \log x$
$f(x) = \int \log x dx$
Using integration by parts,
$f(x) = x\log x - x + C$
$= x(\log x - 1) + C$
6. **(a)** $I = \int_0^{\pi/6} \sec^2\left(x - \frac{\pi}{6}\right) dx$
$= \left[\tan\left(x - \frac{\pi}{6}\right)\right]_0^{\pi/6}$
$= \tan 0 - \tan\left(-\frac{\pi}{6}\right)$
$= 0 + \frac{1}{\sqrt{3}}$
$= \frac{1}{\sqrt{3}}$
7. **(c)** $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$
Order $= 2$, Degree $= 1$
$2 + 1 = 3$
8. **(a)** Perpendicular vectors $\Rightarrow$ dot product $= 0$
$(2, p, 1) \cdot (-4, -6, 26) = 0$
$-8 - 6p + 26 = 0$
$18 - 6p = 0$
$p = 3$
9. **(d)** $(i \times j) \cdot j + (j \times i) \cdot k$
$i \times j = k, j \times i = -k$
$k \cdot j + (-k) \cdot k = 0 - 1 = -1$
10. **(b)** $\vec{a} + \vec{b} = i$
$\vec{b} = i - \vec{a}$
$= (1,0,0) - (2, -2, 2) = (-1, 2, -2)$
$|\vec{b}| = \sqrt{(-1)^2 + 2^2 + (-2)^2} = \sqrt{9} = 3$
11. **(d)** $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$
Let each $= \lambda$.

$x = 1 + 2\lambda, y = 1 - 3\lambda, z = \dfrac{1 + 12\lambda}{2}$

Direction ratios:

$(2, -3, 6)$

Magnitude:

$\sqrt{4 + 9 + 36} = 7$

Direction cosines:

$\left(\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$

12. (a) $P\left(\frac{A}{B}\right) = 0.3, P(A) = 0.4$

$P\left(\frac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)}$

$0.3 = \dfrac{P(A \cap B)}{0.8}$

$P(A \cap B) = 0.24$

$P\left(\frac{B}{A}\right) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.24}{0.4} = 0.6$

13. (d) For continuity at $x = 2$,

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$k(2)^2 = 3(2) + 5$

$4k = 11$

$k = \dfrac{11}{4}$

14. (c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

Now,

$(3I + 4A)(3I - 4A) = 9I - 16A^2$

$= 9I + 16I = 25I$

Given

$(3I + 4A)(3I - 4A) = x^2 I$

$x^2 = 25$

$x = \pm 5$

15. (d) $xdy - (1 + x^2)dx = dx$

$xdy = (x^2 + 2)dx$

$\dfrac{dy}{dx} = x + \dfrac{2}{x}$

Integrating.

$y = \int \left(x + \frac{2}{x}\right) dx = \dfrac{x^2}{2} + 2\log x + C$

16. (c) $f(x) = a(x - \cos x)$

$f'(x) = a(1 + \sin x)$

Since

$1 + \sin x \geq 0 \ \forall x$

For $f(x)$ to be strictly decreasing.

$a < 0$

$a \in (-\infty, 0)$

17. (c) $Z = 18x + 9y$

At (2, 72):

$Z = 18(2) + 9(72) = 684$

At (15, 20):

$Z = 18(15) + 9(20) = 450$

At (40, 15):

$Z = 18(40) + 9(15) = 855$
Maximum $= 855$ at $(40,15)$
Minimum $= 450$ at $(15,20)$

18. **(a)** Constraints:
$x - y \geq 0 \Rightarrow y \leq x$
$2y \leq x + 2 \Rightarrow y \leq \frac{x+2}{2}$
$x \geq 0, y \geq 0$
Intersection of
$y = x$ and $y = \frac{x+2}{2}$
gives
$x = \frac{x+2}{2} \Rightarrow x = 2, y = 2$
Corner points are:
$(0,0), (2,2)$
Hence number of corner points $= 2$.

19. **(c)** • Assertion (A):
$f(x) = 2\sin^{-1}(x) + \frac{3\pi}{2}, x \in [-1,1]$
Since
$\sin^{-1}(x) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$2\sin^{-1}(x) \in [-\pi, \pi]$
Adding $\frac{3\pi}{2}$.
$f(x) \in \left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$
So Assertion is True.
• Reason (R):
The principal value range of $\sin^{-1}x$ is
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
not $[0, \pi]$.
So Reason is False.
• Assertion is True, Reason is False.

20. **(d)** • Assertion
Points:
$(1,2,3), (3,-1,3)$
Direction ratios:
$(3-1, -1-2, 3-3) = (2,-3,0)$
Equation of line:
$\frac{x-3}{2} = \frac{y+1}{-3} = \frac{z-3}{0}$
The given assertion has $\frac{y+1}{3}$, which is incorrect.
Hence Assertion is False.
• Reason (R):
The formula
$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$
is correct.
Hence Reason is True.
• So, Assertion is False, Reason is True.

21. **(A)** Given
$f: A \to B, f(x) = 2x$
and

$A = \{1,2,3,4\}$
Then
$f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8$
Since f is onto,
$B = \{2,4,6,8\}$

OR

(B) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\cos^{-1}\left(\cos\frac{3\pi}{4}\right) = \frac{3\pi}{4}$$

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Hence

$$\frac{\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$$

22. Vector collinear with
$\hat{\imath} + \hat{\jmath} + \hat{k}$
has form
$\lambda(\hat{\imath} + \hat{\jmath} + \hat{k})$
Magnitude:
$|\lambda|\sqrt{1+1+1} = |\lambda|\sqrt{3}$
Given magnitude $= 3\sqrt{3}$.
$|\lambda| = 3$
$3(\hat{\imath} + \hat{\jmath} + \hat{k})$ and $-3(\hat{\imath} + \hat{\jmath} + \hat{k})$

23. (A) Given

$$\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$$

$$\vec{c} - \vec{a} = \frac{5}{4}(\vec{b} - \vec{a})$$

$$\vec{c} = \vec{a} + \frac{5}{4}\vec{b} - \frac{5}{4}\vec{a}$$

$$\vec{c} = \frac{5}{4}\vec{b} - \frac{1}{4}\vec{a}$$

OR

(B) Line 1:
$x = 2\lambda + 2, y = 7\lambda + 1, z = -3\lambda - 3$
Direction ratios:
$(2,7,-3)$
Line 2:
$x = -\mu - 2, y = 2\mu + 8, z = 4\mu + 5$
Direction ratios:
$(-1,2,4)$
Dot product:
$(2)(-1) + (7)(2) + (-3)(4) = -2 + 14 - 12 = 0$
Therefore the lines are perpendicular

24. Given

$$y = \left(x + \sqrt{x^2 - 1}\right)^4$$

Taking log.

$$\ln y = 2\ln\left(x + \sqrt{x^2 - 1}\right)$$

Differentiating.

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{\sqrt{x^2 - 1}}$$

$\frac{dy}{dx} = \frac{2y}{\sqrt{x^2 - 1}}$

Squaring.

$\left(\frac{dy}{dx}\right)^2 = \frac{4y^2}{x^2 - 1}$

Hence

$(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$

25. $f(x) = \frac{16\sin x}{4+\cos x} - x$

$f'(x) = \frac{16(4\cos x + 1)}{(4 + \cos x)^2} - 1$

$= \frac{64\cos x + 16 - (4 + \cos x)^2}{(4 + \cos x)^2}$

$= \frac{-(\cos x - 8)^2}{(4 + \cos x)^2}$

Since denominator > 0,

$f'(x) < 0$

for $x \in \left(\frac{\pi}{2}, \pi\right)$.

Therefore,

- $f(x)$ is strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$

26. Evaluate

$I = \int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2\cos x)]dx$

$I = \int_0^{\pi/2} \log(\sin x)dx - \int_0^{\pi/2} \log(2\cos x)dx$

Using the standard results:

$\int_0^{\pi/2} \log(\sin x)dx = \int_0^{\pi/2} \log(\cos x)dx = -\frac{\pi}{2}\log 2$

Hence,

$\int_0^{\pi/2} \log(2\cos x)dx = \frac{\pi}{2}\log 2 + \int_0^{\pi/2} \log(\cos x)dx = 0$

Therefore,

$I = -\frac{\pi}{2}\log 2$

$\int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2\cos x)]dx = -\frac{\pi}{2}\log 2$

27. Find

$\int \frac{1}{\sqrt{x}(\sqrt{x} + 1)(\sqrt{x} + 2)}dx$

Let

$t = \sqrt{x} \ \Rightarrow \ x = t^2, dx = 2tdt$

Then

$I = \int \frac{2}{(t + 1)(t + 2)}dt$

Using partial fractions,

$\frac{2}{(t + 1)(t + 2)} = \frac{2}{t + 1} - \frac{2}{t + 2}$

Hence,

$I = 2\int \left(\frac{1}{t + 1} - \frac{1}{t + 2}\right)dt$

$I = 2\ln|t+1| - 2\ln|t+2| + C$
Substituting $t = \sqrt{x}$,
$$\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)} dx = 2\ln\left|\frac{\sqrt{x}+1}{\sqrt{x}+2}\right| + C$$

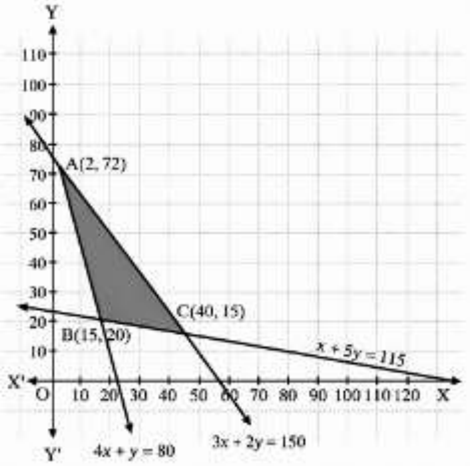
28. **(A)** Particular Solution
$$\frac{dy}{dx} + \sec^2 xy = \tan x\sec^2 x, y(0) = 0$$
This is a linear differential equation.
$P(x) = \sec^2 x$
Integrating factor:
$IF = e^{\int \sec^2 xdx} = e^{\tan x}$
Thus,
$$\frac{d}{dx}(ye^{\tan x}) = \tan x\sec^2 xe^{\tan x}$$
Let $u = \tan x, du = \sec^2 xdx.$
$$ye^u = \int ue^u du = e^u(u-1) + C$$
Hence,
$y = \tan x - 1 + Ce^{-\tan x}$
Using $y(0) = 0$,
$0 = 0 - 1 + C$
$C = 1$
Therefore,
$y = \tan x - 1 + e^{-\tan x}$

OR

(B) Solve
$$xdy - ydx - \sqrt{x^2+y^2}dx = 0$$
$$x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$$
$$\frac{dy}{dx} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}$$
Let
$$v = \frac{y}{x} \Rightarrow y = vx, \frac{dy}{dx} = v + x\frac{dv}{dx}$$
Substituting.
$$v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$$
$$x\frac{dv}{dx} = \sqrt{1+v^2}$$
$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$
Integrating.
$\sinh^{-1} v = \ln|x| + C$
Substituting $v = \frac{y}{z}$,
$$\sinh^{-1}\left(\frac{y}{x}\right) = \ln|x| + C$$
or equivalently.
$$\frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2} = Cx$$
which is the required solution.

29. The given constraints are
$4x + y \geq 80$
$x + 5y \geq 115$

$3x+2y\geq 150$
$x,y\geq 0$
Converting the given inequations into equations, we get
$4x+y=80, x+5y=115, 3x+2y=150, x=0$ and $y=0$
These lines are drawn on the graph and the shaded region ABC represents the feasible region of the given LPP.



It can be observed that the feasible region is bounded. The coordinates of the corner points of the feasible region are $A(2,72), B(15,20)$ and $C(40,15)$.
The values of the objective function, Z at these corner points are given in the following table:

Corner Point	Value of the Objective Function $Z=6x+3y$
$A(2,72)$	$Z=6\times 2+3\times 72 = 228$
$B(15,20)$	$Z=6\times 15+3\times 20 = 150$
$C(40,15)$	$Z=6\times 40+3\times 15 = 285$

From the table, Z is minimum at $x=15$ and $y=20$ and the minimum value of Z is 150 .
Thus, the minimum value of Z is 150 .

30. **(A)** Given

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find k

$\frac{k}{2}+\frac{k}{3}+\frac{k}{6}=1$

$k\left(\frac{3+2+1}{6}\right)=1$

$k=1$

$k=1$

(ii) Find $P(1\leq X<3)$

$P(X=1)+P(X=2)$

$=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

(iii) Mean $E(X)$

$E(X)=\sum xP(x)$

$=1\left(\frac{1}{2}\right)+2\left(\frac{1}{3}\right)+3\left(\frac{1}{6}\right)$

$$= \frac{1}{2} + \frac{2}{3} + \frac{1}{2}$$
$$= 1 + \frac{2}{3} = \frac{5}{3}$$
$$E(X) = \frac{5}{3}$$

OR

(B) Given

$$P(A \cap \bar{B}) = \frac{1}{4}$$
$$P(A \cap B) = \frac{1}{6}$$

Since

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$
$$P(A) = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$$

Because A and B are independent,

$$P(A \cap B) = P(A)P(B)$$
$$\frac{1}{6} = \frac{5}{12}P(B)$$
$$P(B) = \frac{1}{6} \cdot \frac{12}{5} = \frac{2}{5}$$
$$P(A) = \frac{5}{12}, P(B) = \frac{2}{5}$$

31. (A) Evaluate

$$I = \int_0^{x/2} e^x \sin x dx$$

Using

$$\int e^x \sin x dx = \frac{e^x}{2}(\sin x - \cos x) + C$$

Therefore,

$$I = \left[\frac{e^x}{2}(\sin x - \cos x)\right]_0^{\pi/2}$$
$$= \frac{e^{\pi/2}}{2}(1 - 0) - \frac{1}{2}(0 - 1)$$
$$= \frac{e^{\pi/2} + 1}{2}$$
$$\int_0^{\pi/2} e^{x^2} \sin x dx = \frac{e^{\pi/2} + 1}{2}$$

OR

(B) Find

$$\int \frac{dx}{\cos(x - a)\cos(x - b)}$$

Using

$$\tan A - \tan B = \frac{\sin(A - B)}{\cos A \cos B}$$

Hence,

$$\frac{1}{\cos(x - a)\cos(x - b)} = \frac{\tan(x - a) - \tan(x - b)}{\sin[(x - a) - (x - b)]}$$
$$= \frac{\tan(x - a) - \tan(x - b)}{\sin(b - a)}$$

Therefore,

$$I = \frac{1}{\sin(b - a)} \int [\tan(x - a) - \tan(x - b)]dx$$

$$= \frac{1}{\sin(b-a)}\left[-\ln|\cos(x-a)| + \ln|\cos(x-b)|\right] + C$$

$$\int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(b-a)}\ln\left|\frac{\cos(x-b)}{\cos(x-a)}\right| + C$$

32. Relation $R = \{(x, y): xy \text{ is an irrational number }\}$

Reflexive

For reflexive, $(x, x) \in R$ for every $x \in \mathbb{R}$.

Take $x = 1$,

$1 \cdot 1 = 1$

which is rational.

Therefore, R is not reflexive.

Symmetric

If $(x, y) \in R$, then xy is irrational.

Since

$yx = xy$

yx is also irrational.

Therefore, R is symmetric.

Transitive

Take

$x = \sqrt{2}, y = 1, z = \sqrt{2}$.

Then

$xy = \sqrt{2}(\text{ irrational }), yz = \sqrt{2}(\text{ irrational }),$

but

$xz = 2$ (rational).

Hence R is not transitive.

R is neither reflexive nor transitive, but symmetric.

33. **(A)** Find $(AB)^{-1}$

Given

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Using

$(AB)^{-1} = B^{-1}A^{-1}$.

First,

$$A^{-1} = \begin{bmatrix} 3 & -2 & 6 \\ 1 & -1 & 2 \\ 2 & -2 & 5 \end{bmatrix}$$

Therefore,

$$(AB)^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & -2 & 6 \\ 1 & -1 & 2 \\ 2 & -2 & 5 \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} 10 & -7 & 21 \\ 49 & 34 & 103 \\ 17 & -12 & 36 \end{bmatrix}$$

OR

(B) Solve by Matrix Method

$$\begin{cases} x + 2y + 3z = 6 \\ 2x - y + z = 2 \\ 3x + 2y - 2z = 3 \end{cases}$$

Solving.

$x = 1, y = 1, z = 1$

34. **(A)** Points:

$A(3,3,-5), B(1,0,-11)$

Direction vector of AB :

$\overrightarrow{AB} = (-2, -3, -6)$

Line through $(1,2,-4)$ parallel to AB :
Vector equation
$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(-2\hat{i} - 3\hat{j} - 6\hat{k})$
Cartesian equation
$$\frac{x-1}{-2} = \frac{y-2}{-3} = \frac{z+4}{-6}$$
Distance between the lines
Take
$P(1,2,-4), Q(3,3,-5)$
$\overrightarrow{PQ} = (2,1,-1)$
$$d = \frac{|\overrightarrow{PQ} \times \overrightarrow{AB}|}{|\overrightarrow{AB}|}$$
$\overrightarrow{PQ} \times \overrightarrow{AB} = (-9,14,-4)$
$|\overrightarrow{PQ} \times \overrightarrow{AB}| = \sqrt{293}$
$|\overrightarrow{AB}| = 7$
Hence
$$d = \frac{\sqrt{293}}{7}$$
OR

(B) Given
$A(1,2,3), B(3,5,9)$
Direction vector
$\overrightarrow{AB} = (2,3,6)$
Equation of line
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6}$$
Points at distance 14 units from B
$|\overrightarrow{AB}| = 7$
Unit vector along line:
$$\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$$
Required points:
$$B \pm 14\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$$
$= (3,5,9) \pm (4,6,12)$
Hence
$(7,11,21)$
and $(-1,-1,-3)$

35. Area bounded by $y = x^2, y = x + 2$ and x-axis
Intersections:
(1) $y = x^2$ and $y = x + 2$
$x^2 = x + 2$
$(x-2)(x+1) = 0$
$x = 2, -1$
(2) Line meets x-axis at $x = -2$.
(3) Parabola meets x-axis at $x = 0$.
Area enclosed:
$$A = \int_{-2}^{-1} (x+2)dx + \int_{-1}^{0} x^2 dx$$
$$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$$
$$= \frac{1}{2} + \frac{1}{3}$$

$= \frac{5}{6}$

Area $= \frac{5}{6}$ square units

36. From the Venn diagram:

- A only $= 0.32$
- $A \cap B = 0.09$
- B only $= y$
- $B \cap C = x$
- C only $= 0.21$
- Outside all three $= 0.11$

(i) Given:

$P(C) = 0.44$

So,

$x + 0.21 = 0.44$

$x = 0.23$

(ii) Find y

Total probability = 1

$0.32 + 0.09 + y + 0.23 + 0.21 + 0.11 = 1$

$0.96 + y = 1$

$y = 0.04$

(iii) (A) Find $P(\bar{B})$

$P(B) = 0.09 + 0.04 + 0.23 = 0.36$

Therefore,

$P(\bar{B}) = 1 - P(B)$

$= 1 - 0.36$

$P(\bar{B}) = 0.64$

OR

(B) Probability that a person does Yoga of type A or B but not C

Required region:

- A only $= 0.32$
- $A \cap B = 0.09$
- B only $= 0.04$

$P((A \cup B) \cap \bar{C}) = 0.32 + 0.09 + 0.04$

$P((A \cup B) \cap \bar{C}) = 0.45$

37. Given:

- Water flows out at the rate $2\ cm^3/s$

$\frac{dV}{dt} = -2\ cm^3/s$

- Semi-vertical angle of cone $= 45°$

$\tan 45° = \frac{r}{h} = 1$

$r = h$

(i) Volume of water in the conical tank in terms of r

Volume of cone:

$V = \frac{1}{3}\pi r^2 h$

Since $h = r$.

$V = \frac{1}{3}\pi r^3$

$V = \frac{\pi r^3}{3}$

(ii) Rate of change of radius when $r = 2\sqrt{2}$ cm

Differentiate

$V = \frac{\pi r^3}{3}$

with respect to t :

$\frac{dV}{dt} = \pi r^2 \frac{dr}{dt}$

Substitute $\frac{dV}{dt} = -2$:

$-2 = \pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-2}{\pi r^2}$

At

$r = 2\sqrt{2}$

$r^2 = 8$

$\frac{dr}{dt} = -\frac{1}{4\pi}$ cm/s

(iii) (A) Rate at which wet surface area decreases when $r = 2\sqrt{2}$ cm Since $h = r$,

$l = \sqrt{r^2 + h^2} = \sqrt{2r^2} = r\sqrt{2}$

Curved (wet) surface area:

$S = \pi r l = \pi r(r\sqrt{2}) = \pi\sqrt{2}r^2$

Differentiate:

$\frac{dS}{dt} = 2\pi\sqrt{2}r\frac{dr}{dt}$

Substitute $r = 2\sqrt{2}$ and $\frac{dr}{dt} = -\frac{1}{4\pi}$:

$\frac{dS}{dt} = 2\pi\sqrt{2}(2\sqrt{2})\left(-\frac{1}{4\pi}\right)$

$= 8\pi\left(-\frac{1}{4\pi}\right)$

$\frac{dS}{dt} = -2\ \text{cm}^2/\text{s}$

Hence the wet surface area is decreasing at the rate

$2\ \text{cm}^2/\text{s}$

OR

(B) Rate of change of height when slant height $l = 4$ cm Since

$l = r\sqrt{2}, r = h$

$h = \frac{l}{\sqrt{2}}$

If $l = 4$,

$h = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

Thus $r = 2\sqrt{2}$.

Since $r = h_v$

$\frac{dh}{dt} = \frac{dr}{dt}$

From part (ii).

$\frac{dh}{dt} = -\frac{1}{4\pi}$ cm/s

So the height is decreasing at the rate

$\frac{1}{4\pi}$ cm/s

(negative sign indicates decrease).

38. Given

$f(x) = a(x+9)(x+1)(x-3)$

and the curve crosses the y-axis at $(0, -1)$.

(i) Find the value of a

Since $f(0) = -1$.
$a(0 + 9)(0 + 1)(0 - 3) = -1$
$a(9)(1)(-3) = -1$
$-27a = -1$
$a = \frac{1}{27}$
(ii) Find $f''(1)$
Substitute $a = \frac{1}{27}$:
$f(x) = \frac{1}{27}(x + 9)(x + 1)(x - 3)$
First expand:
$(x + 1)(x - 3) = x^2 - 2x - 3$
$(x + 9)(x^2 - 2x - 3) = x^3 + 7x^2 - 21x - 27$
Hence
$f(x) = \frac{1}{27}(x^3 + 7x^2 - 21x - 27)$
Differentiate:
$f'(x) = \frac{1}{27}(3x^2 + 14x - 21)$
$f''(x) = \frac{1}{27}(6x + 14)$
At $x = 1$,
$f''(1) = \frac{1}{27}(6 + 14) = \frac{20}{27}$
$f''(1) = \frac{20}{27}$

39. **(c)** Let's calculate A^2.
$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
Since $A^2 = 0$, any higher power $A^n = 0$ for $n \geq 2$. Thus, $A^{2023} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

40. **(b)** Any square matrix A can be uniquely expressed as $P + Q$, where
$P = \frac{A + A^T}{2}$ (symmetric) and $Q = \frac{A - A^T}{2}$ (skew-symmetric).
$A^T = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}$
$Q = \frac{1}{2}\left(\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

41. **(d)** A non-singular matrix has a determinant not equal to zero.
$\det(A) = 1(3 - a) - 2(2 - 3) + 1(2a - 9) = a - 4$
For the matrix to be non-singular, $a - 4 \neq 0 \Rightarrow a \neq 4$. Thus, $a \in \mathbb{R} - \{4\}$.

42. **(d)** The determinant property states that $|kA| = k^n|A|$ where n is the order of the matrix. Given $n = 2|kA| = k^2|A|$.
$|A| = k^2|A| \Rightarrow |A|(k^2 - 1) = 0$
For a non-trivial matrix where $|A| \neq 0$, we have $k^2 - 1 = 0 \Rightarrow k = 1, -1$. The sum of these values is $1 + (-1) = 0$.

43. **(b)** Integrating the derivative with respect to x :
$f(x) = \int (ax + b)dx = \frac{ax^2}{2} + bx + c$
Using the initial condition $f(0) = 0$, we get $C = 0$. Thus, $f(x) = \frac{ax^2}{2} + bx$.

44. **(c)** The degree of a differential equation is the highest power of the highest-order derivative when the equation is a polynomial in its derivatives. Because this equation contains $\cos\left(\frac{dy}{dx}\right)$, which cannot be expressed as a polynomial in $\frac{dy}{dx}$, its degree is not defined.

45. (d) $(1-y^2)\frac{dx}{dy}+yx=ay$

$$\frac{dx}{dy}+\frac{y}{1-y^2}x=\frac{ay}{1-y^2}$$

$$\text{I.F.}=e^{\int\frac{y}{1-y^2}dy}=e^{-\frac{1}{2}\ln(1-y^2)}=\frac{1}{\sqrt{1-y^2}}$$

46. (d) $\overrightarrow{PQ}=(4-2)\hat{\imath}+(4-1)\hat{\jmath}+(-7+1)\hat{k}=2\hat{\imath}+3\hat{\jmath}-6\hat{k}$

$|\overrightarrow{PQ}|=\sqrt{4+9+36}=7$

Unit vector:

$$\frac{2\hat{\imath}+3\hat{\jmath}-6\hat{k}}{7}$$

47. (b) $\vec{M}=\frac{\vec{A}+\vec{B}}{2}$

$\vec{B}=2\vec{M}-\vec{A}$

$=2(3\hat{\imath}+2\hat{\jmath}-3\hat{k})-(2\hat{\imath}+3\hat{\jmath}-4\hat{k})$

$=4\hat{\imath}+\hat{\jmath}-2\hat{k}$

48. (a) Projection $=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$(2\hat{\imath}+3\hat{\jmath})\cdot(3\hat{\imath}-2\hat{\jmath})=6-6=0$

$\therefore$ Projection $=0$

49. (d) Line through $(1,1,1)$ parallel to z-axis has direction ratios $(0,0,1)$.

$$\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{1}$$

50. (d) Given sum $=9$

Possible outcomes:

$(3,6),(4,5),(5,4),(6,3)$

Total $=4$

Favourable $=2(4,5),(5,4)$

$$P=\frac{2}{4}=\frac{1}{2}$$

51. (c) $\frac{\tan x-1}{\tan x+1}=\frac{\tan x-\tan\frac{\pi}{4}}{1+\tan x\tan\frac{x}{4}}=\tan\left(x-\frac{\pi}{4}\right)$

$$\int\tan\left(x-\frac{\pi}{4}\right)dx=-\ln\left|\cos\left(x-\frac{\pi}{4}\right)\right|+C$$

$$=\ln\left|\sec\left(\frac{\pi}{4}-x\right)\right|+C$$

52. (b) $\Delta=\frac{1}{2}\begin{vmatrix}a&c&e\\b&d&f\\1&1&1\end{vmatrix}$

Therefore,

$$\begin{vmatrix}a&c&e\\b&d&f\\1&1&1\end{vmatrix}=2\Delta$$

Squaring.

$$\begin{vmatrix}a&c&e\\b&d&f\\1&1&1\end{vmatrix}^2=4\Delta^2$$

53. (a) $f(x)=x|x|=\begin{cases}x^2, & x\geq 0\\ -x^2, & x<0\end{cases}$

$f(0)=0$

Continuous at $x=0$.

$$f'(0)=\lim_{h\to0^-}\frac{-h^2}{h}=0,\ f'_+(0)=\lim_{h\to0^+}\frac{h^2}{h}=0$$

Hence differentiable at $x=0$.

54. **(b)** $\tan\left(\frac{x+y}{x-y}\right) = k$

Since k is constant,

$$\frac{x+y}{x-y} = C$$

Differentiate:

$$\frac{d}{dx}\left(\frac{x+y}{x-y}\right) = 0$$

Using quotient rule:

$$(1+y')(x-y) - (x+y)(1-y') = 0$$

$$2xy' - 2y = 0$$

$$\frac{dy}{dx} = \frac{y}{x}$$

55. **(c)** $Z = ax + by$

At (4, 6).

$4a + 6b = 42$

At (3, 2).

$3a + 2b = 19$

Multiply second by 3:

$9a + 6b = 57$

Subtract:

$5a = 15 \Rightarrow a = 3$

$3(3) + 2b = 19$

$2b = 10 \Rightarrow b = 5$

56. **(d)**

Corner Points	$Z = 30x + 24y$
(0,4)	$Z = 30 \times 0 + 24 \times 4 = 96 \rightarrow$ Min.
(8,0)	$Z = 30 \times 8 + 24 \times 0 = 240 \rightarrow$ Max.
$\left(\frac{20}{3}, \frac{4}{3}\right)$	$Z = 30 \times \frac{20}{3} + 24 \times \frac{4}{3} = 200 + 32 = 232$

Then Max. Z − Min. Z = 240 − 96 = 144.

57. **(c)** The maximum value of $\cos^{-1}x$ is π at 1 .

∴ Maximum value of $(\cos^{-1}x)^2$ is π^2.

Range of principal value branch of $\cos^{-1}x$ is $[0, \pi]$.

58. **(a)** $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\Rightarrow 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

59. **(A)** $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$

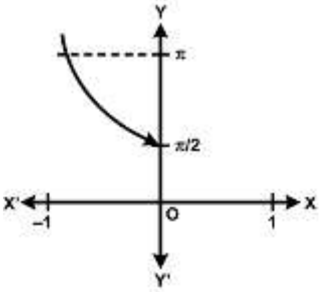
$$= \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{4}\right)\right] + \pi + \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4} + \pi + \frac{\pi}{4}$$

$$= \frac{\pi}{2} + \pi$$

$$= \frac{3\pi}{2}$$

(B)



Range of $\cos^{-1} x$ is $[0, \pi]$.

60. Given:

$3y = ax^3 + 1$

Differentiate w.r.t. t

$3\frac{dy}{dt} = a \times 3x^2 \frac{dx}{dt}$

$\Rightarrow \frac{dy}{dt} = ax^2 \frac{dx}{dt}$

Use given condition at $x = 1$

$\Rightarrow \frac{dy}{dt} = 2\frac{dx}{dt}$

$\frac{dy}{dt} = a(1^2)\frac{dx}{dt} a \frac{dx}{dt}$

$a\frac{dx}{dt} = 2\frac{dx}{dt}$

$\frac{dx}{dt} \neq 0$

$a = 2$

61. Given, $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$

$\therefore \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$

$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

Either $\vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b} - \vec{c})$

But $\vec{b} \neq \vec{c}$

[$\because \vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors]

$\therefore$ Angle between $\vec{a}$ and $\vec{b} - \vec{c}$ is $90°$.

62. Given line is

$\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2} = k(\text{ assume })$

$x = k, y = 2k + 1, z = 2k - 1$

So, let point on the given line is

$P(k, 2k + 1, 2k - 1)$

Distance of point (P) from the origin is

$\sqrt{(k-0)^2 + (2k+1-0)^2 + (2k-1-0)^2}$

Now $\sqrt{k^2 + (2k+1)^2 + (2k-1)^2} = \sqrt{11}$ ... (Given)

$\Rightarrow k^2 + 4k^2 + 1 + 4k + 4k^2 + 1 - 4k = 11$

$\Rightarrow 9k^2 + 2 = 11$

$\Rightarrow 9k^2 = 9$

$\Rightarrow k = \pm 1$

Therefore, point on the line is $(1,3,1)$ or $(-1, -1 - 3)$.

63. **(A)** Given, $y = \sqrt{ax + b}$

Then, $\frac{dy}{dx} = \frac{a}{2\sqrt{ax+b}}$

$\Rightarrow \frac{dy}{dx} = \frac{a}{2y}$

$\Rightarrow y\frac{dy}{dx} = \frac{a}{2}$

Again, differentiating with respect to x , we get

$\Rightarrow y\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{dy}{dx} = 0$

$\Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

Hence Proved.

OR

(B) We have,

$f(x) = \begin{cases} ax + b: 0 < x \leq 1 \\ 2x^2 - x: 1 < x < 2 \end{cases}$

(LHD at $x = 1$)

$= \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1}$

$= \lim_{h \to 0} \frac{f(1-h) - f(1)}{1 - h - 1}$

$= \lim_{h \to 0} \frac{[a(1-h) + b] - [a + b]}{-h}$

$= \lim_{h \to 0} \frac{[a - ah + b - a - b]}{-h}$

$= \lim_{h \to 0} \frac{ah}{a}$

$= a$

(RHD at $x = 1$)

$= \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$

$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{(1+h) - 1}$

$= \lim_{h \to 0} \frac{[2(1+h)^2 - (1+h)] - [2-1]}{h}$

$= \lim_{h \to 0} \frac{[2(1 + h^2 + 2h) - 1 - h] - 1}{h}$

$= \lim_{h \to 0} \frac{[2 + 2h^2 + 4h - 1 - h - 1]}{h}$

$= \lim_{h \to 0} \frac{(2h^2 + 3h)}{h}$

$= \lim_{h \to 0} (2h + 3)$

$= 3$

Since, f(x) is differentiable, so

(LHD at $x = 1$) =(RHD at $x = 1$)

$\therefore a = 3$

Now, LHL $= \lim_{x,1^-} f(x)$

$= \lim_{h \to 0} f(1-h)$

$= \lim_{h \to 0} a(1-h) + b$

$= a + b$

Now, RHL $= \lim_{x \to 1^+} f(x)$

$= \lim_{h \to 0} f(1+h)$

$= \lim_{h \to 0} 2(1+h)^2 - (1+h)$

$= 2 - 1$

$= 1$

$\because$ LHL = RHS

$\therefore a + b = 1$

$\Rightarrow 3 + b = 1$
$b = -2$
Hence, $a = 3$ and $b = -2$.

64. **(A)** Let $I = \int_0^{\pi/4} \log(1 + \tan x)dx$

By using property

$$\int_0^a f(x) = \int_0^a f(a - x)$$

$$I = \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx$$

$$= \int_0^{\pi/4} \log\left[1 + \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right] dx$$

$$= \int_0^{\pi/4} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right] dx$$

$$= \int_0^{\pi/4} \log\left[\frac{2}{1 + \tan x}\right] dx$$

$$= \int_0^{\pi/4} \log 2 - \int_0^{\pi/4} \log(1 + \tan x)dx$$

On adding equations (i) and (ii),

$$2I = \int_0^{\pi/4} \log(1 + \tan x)dx + \int_0^{\pi/4} \log 2dx - \int_0^{\pi/4} \log(1 + \tan x)dx$$

$$\Rightarrow 2I = \int_0^{\pi/4} \log 2dx$$

$$\Rightarrow 2I = \log 2 \int_0^{\pi/4} 1 \cdot dx$$

$$\Rightarrow 2I = \log 2[x]_0^{\pi/4}$$

$$\Rightarrow I = \frac{\log 2}{2} \times \frac{\pi}{4} = \frac{\pi}{8}\log 2$$

OR

(B) Let $I = \int \frac{dx}{\sqrt{\sin^3 x \cos(x-\alpha)}}$

$$= \int \frac{dx}{\sqrt{\frac{\sin^4 x}{\sin x}[\cos x\cos\alpha + \sin x\sin\alpha]}}$$

$$= \int \frac{dx}{\sin^2 x\sqrt{\cot x\cos\alpha + \sin\alpha}}$$

$$= \int \frac{\mathrm{cosec}^2 x dx}{\sqrt{\cot x\cos\alpha + \sin\alpha}}$$

Let $\cot x\cos\alpha + \sin\alpha = t$

Then, $dt = -\mathrm{cosec}^2 x\cos\alpha dx$

$$\therefore I = \int \frac{-dt}{\cos\alpha\sqrt{t}}$$

$$= \frac{-2\sqrt{t}}{\cos\alpha} + C$$

$$= \frac{-2\sqrt{\cot x\cos\alpha + \sin\alpha}}{\cos\alpha} + C$$

$$= -2\sec\alpha\sqrt{\cot x\cos\alpha + \sin\alpha} + C$$

65. Let $I = \int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right) dx$

Take $\cot^{-1}x = t$

$\therefore x = \cot t$

$\therefore dx = -\text{cosec}^2 t dt$

$\therefore I = \int e^t \left(\frac{1+\cot^2 t - \cot t}{1+\cot^2 t}\right)(-\text{cosec}^2 t)dt$

$= \int -e^t \frac{(\text{cosec}^2 t - \cot t)}{\text{cosec}^2 t} \times \text{cosec}^2 t dt$

$= \int -e^t(\text{cosec}^2 t - \cot t)dt$

$= \int e^t(\cot t - \text{cosec}^2 t)dt$

$\therefore$ Now, taking $f(t) = \cot t$

Then $f'(t) = -\text{cosec}^2 t$

$\therefore I = \int e^t[f(t) + f'(t)]dt$

$= e^t f(t) + C$

$= e^{\cot^{-1} x}|\cot(\cot^{-1} x)| + C$

$= e^{\cot^{-1} x} \times x + C$

$= xe^{\cot^{-1} x} + C$

66. Let $I = \int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})} dx$

$= \int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{\frac{(e^{2x}+1)}{e^x} \times \frac{(e^{2x}-1)}{e^x}} dx$

$= \int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{e^{2x}}{(e^{4x}-1)} dx$

Let $e^{2x} = t$

Then, $2e^{2x}dx = dt$

$= \int_2^3 \frac{dt}{2(t^2-1)}$

$= \frac{1}{2}\int_2^3 \frac{dt}{t^2-1^2}$

$= \left[\frac{1}{2} \times \frac{1}{2 \times 1} \log\left|\frac{t-1}{t+1}\right|\right]_2^3$

$= \frac{1}{4}\left[\log\left(\frac{3-1}{3+1}\right) - \log\left(\frac{2-1}{2+1}\right)\right]$

$= \frac{1}{4}\left[\log\frac{2}{4} - \log\frac{1}{3}\right]$

$= \frac{1}{4}\left[\log\frac{1}{2} + \log 3\right]$

$= \frac{1}{4}\left[\log\frac{1}{2} \times 3\right]$

$= \frac{1}{4}\log\frac{3}{2}$

67. (A) $(xy - x^2)dy = y^2 dx$

Divide throughout by y^2 (assuming $y \neq 0$) to simplify:

$\frac{xy - x^2}{y^2} dy = dx$

$\left(\frac{x}{y} - \frac{x^2}{y^2}\right) dy = dx$

To simplify, let us introduce a substitution. Divide the entire equation by x^2 (assuming $x \neq 0$):

$\left(\frac{1}{x} \cdot \frac{y}{x} - \frac{1}{y}\right) dy = \frac{1}{x^2} dx$

$v = \frac{y}{x}$(so that $y = vx$ and $dy = vdx + xdv$)

Substitute into the equation:

$$\left(\frac{1}{x}v-\frac{1}{v}\right)(vdx+xdv)=\frac{1}{x^2}dx$$

$$\left(\frac{v}{x}-\frac{1}{vx}(vdx+xdv)=\frac{1}{x^2}dx\right)$$

Simplify step by step:

Multiply and separate:

$$\frac{v^2}{x}dx+\frac{v}{x}xdv-\frac{1}{vx}vdx-\frac{1}{vx}xdv=\frac{1}{x^2}dx$$

$$\frac{v^2}{x}dx-\frac{1}{x}dx+vdv-\frac{1}{v}dv=\frac{1}{x^2}dx$$

Group terms involving v and x . The equation becomes:

$$\frac{v^2-1}{v}dv=\left(\frac{1}{x^2}-\frac{v^2-z}{x}\right)dx$$

OR

(B) Given equation is

$$(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$$

$$\Rightarrow \frac{dy}{dx}+\frac{2x}{(x^2+1)}\times y=\frac{\sqrt{x^2+4}}{(x^2+1)}$$

It is of the form,

$$\frac{dy}{dx}+Py=Q$$

$$\therefore \text{IF}=e^{\int Pdx}$$

$$=e^{\int\left(\frac{2x}{x^2+1}\right)dx}$$

Let $x^2+l=t$

$2xdx=dt$

$$\therefore I\cdot F=e^{\int\frac{a}{t}}$$

$$=e^{\log t}$$

$$=t$$

$$=x^2+1$$

General solution is given as

$$(\text{I.F.})y=\int(\text{I.F.})Qdx+C$$

$$\Rightarrow (x^2+1)y=\int(x^2+1)\frac{\sqrt{x^2+4}}{(x^2+1)}dx+C$$

$$\Rightarrow (x^2+1)y=\int\sqrt{x^2+1}dx+C$$

$$\Rightarrow (x^2+1)y=\int\sqrt{(x)^2+2^2}dx+C$$

$$\Rightarrow (x^2+1)y=\frac{x}{2}\left(\sqrt{x^2+4}+\frac{4}{2}\log\left|x+\sqrt{x^2+4}\right|\right)+C$$

$$\Rightarrow (x^2+1)y=\frac{x}{2}\sqrt{x^2+4}+2\log\left|x+\sqrt{x^2+4}\right|+C$$

is the required solution of given differential equation.

68. **(A)** When we draw 2 balls, we get no red balls, 1 red ball or 2 red balls.

Then, possible values of X are 0,1,2.

Let, number of red balls be y and number of green balls be y.

The, total number of balls $=2y$

Now, $P(X=0)=P(\text{ no red ball })$

$=(PGG)$

$$=\frac{y}{2y}\times\frac{y}{2y}$$

$= \frac{1}{4}$

$P(X = 1) = P(\text{ one red ball })$

$= P(\text{RG or GR })$

$= \frac{y}{2y} \times \frac{y}{2y} + \frac{y}{2y} \times \frac{y}{2y}$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

$P(X = 2) = P(\text{ two red balls })$

$= P(RR)$

$= \frac{y}{2y} \times \frac{y}{2y}$

$= \frac{1}{4}$

$\therefore$ Probability distribution is

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Now, mean i.e.,

$E(X) = \sum XP(X)$

$= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

OR

(B) Let S denote the success (getting a '6') and F denote the failure (not getting a '6').

Thus, $P(S) = \frac{1}{6} = p, P(F) = \frac{5}{6} = q$

$P(\text{A wins in first throw }) = P(S) = p$

$P(\text{ A wins in third throw }) = P(FFS) = qqp$

$P(\text{ A wins in fifth throw }) = P(\text{ FFFFS }) = qqqqp$

So, $P(\text{A wins }) = p + q^2p + q^4p + \cdots .. = p(1 + q^2 + q^4 + \cdots .)$

$= \frac{p}{1 - q^2}$

$= \frac{\frac{1}{6}}{1 - \frac{25}{36}}$

$= \frac{6}{11}$

$P(\text{B wins }) = 1 - P(\text{A wins })$

$= 1 - \frac{6}{11}$

$= \frac{5}{11}$

So, $P(\text{A wins }) = \frac{6}{11}$ and $P(\text{B wins }) = \frac{5}{11}$.

69. Here, objective function

Min. $Z = 5x + 10y$

Subject to constraints

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x \geq 0, y \geq 0$

Changing inequations to equations, we get

$x + 2y = 120$ ...(i)

x	0	120
y	60	0

$x + y = 60$ ....(ii)

x	0	60
y	60	0

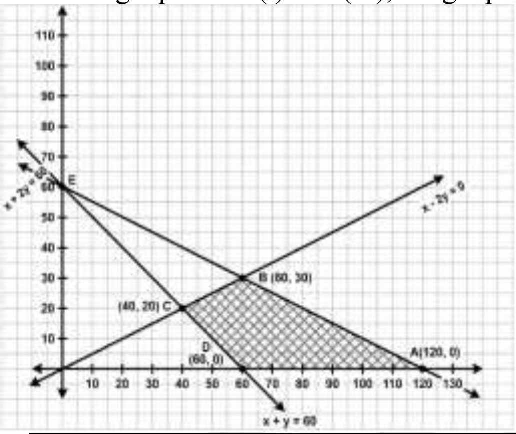
$x - 2y = 0$ ....(iii)

x	0	120
y	0	60

On solving equations (i) and (ii), we get point of intersection $E(0,60)$.

On solving equations (ii) and (iii), we get point of intersection $C(40,20)$.

On solving equations (i) and (iii), we get point of intersection $B(60,30)$.



Corner Points	$Z = 5x + 10y$
$A(120,0)$	$Z = 5 \times 120 + 10 \times 0 = 600$
$B(60,30)$	$Z = 5 \times 60 + 10 \times 30$ $= 300 + 300$ $= 600$
$C(40,20)$	$Z = 5 \times 40 + 10 \times 20$ $= 200 + 200$ $= 400$
$D(60,0)$	$Z = 5 \times 60 + 0 \times 0 = 300 \rightarrow$ Min.

Hence, min. value Z is 300 , when $x = 60$ and $y = 0$.

70. (A) Given, $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

and $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

Now, $AB = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -3+4+0 & -6+2+4 & 0+4-4 \\ 2-2+0 & 4-1-2 & 0-2+2 \\ 2-2+0 & 4-1-3 & 0-2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore AB = I$

$(AB)B^{-1} = IB^{-1}$

$A = B^{-1}$ ....(i)

Now, equations are

$x - 2y = 3$

$2x - y - z = 2$

$-2y + z = 3$

$$\therefore \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

$CX = D$

Here, $C = B^T$

$\therefore C^{-1}(CX) = C^{-1}D$ ...(ii)

$\Rightarrow IX = C^{-1}D$

$\Rightarrow X = C^{-1}D$

$= [B^T]^{-1}D$ ....[from (ii)]

$= [B^{-1}]^T D$

$= [A]^T D$ ...[from (i)]

$$X = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}^T \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -9+4+6 \\ -6+2+3 \\ -12+4+9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Hence $x = 1, y = -1$ and $z = 1$.

OR

(B) Given, $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$f(-\beta) = \begin{bmatrix} \cos(\beta) & \sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(\alpha)\cdot f(-\beta) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta + \sin\alpha\sin\beta & \cos\alpha\sin\beta - \sin\alpha\sin\beta & 0 \\ \sin\alpha\cos\beta - \sin\beta\cos\alpha & \sin\alpha\sin\beta + \cos\alpha\cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0 \\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= f(\alpha - \beta)$

Hence Proved.

71. **(A)** Equation of diagonal PR is i.e., $P(4,2,-6)$ and $R(12,4,5)$ is

$$\frac{x-4}{12-4}=\frac{y-2}{4-2}=\frac{z+6}{5+6}$$

$\Rightarrow \frac{x-4}{8}=\frac{y-2}{2}=\frac{z+6}{11}$ ....(i)

Equation of diagonal QS is i.e., $Q(5,-3,1)$ and $S(11,9,-2)$ is

$$\frac{x-5}{11-5}=\frac{y+3}{9+3}=\frac{z-1}{-2-1}$$

$\Rightarrow \frac{x-5}{6}=\frac{y+3}{12}=\frac{z-1}{-3}$ ....(ii)

For finding point of intersection.

Let $\frac{x-4}{8}=\frac{y-2}{2}=\frac{z+6}{11}=\lambda$

$x=8\lambda+4, y=2\lambda+2, z=11\lambda-6$

Since, the lines (i) and (ii) are intersecting, the point (x, y, z) must satisfy the second line.

$$\therefore \frac{8\lambda+4-5}{6}=\frac{2\lambda+2+3}{12}=\frac{11\lambda-6-1}{-3}$$

$$\Rightarrow \frac{8\lambda-1}{6}=\frac{2\lambda+5}{12}=\frac{11\lambda-7}{-3}$$

$$\Rightarrow \frac{8\lambda-1}{6}=\frac{2\lambda+5}{12}$$

$\Rightarrow 16\lambda-2=2\lambda+5$

$\Rightarrow 14\lambda=7$

$$\lambda=\frac{1}{2}$$

$\therefore$ Point of intersection is

$$x=8\times\frac{1}{2}+4=8$$

$$y=2\times\frac{1}{2}+2=3$$

$$z=11\times\frac{1}{2}-6=\frac{11}{2}-6=\frac{-1}{2}$$

Thus, the diagonals intersect at $\left(8,3,\frac{-1}{2}\right)$

OR

(B) Line i is passing through the point $(-1,3,-2)$ and the lines are

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3} \text{ and } \frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$$

Direction cosines of first line is

$\vec{a}=(\hat{\imath}+2\hat{\jmath}+3\hat{k})$

Direction cosines of second line is

$\vec{b}=(-3\hat{\imath}+2\hat{\jmath}+5\hat{k})$

$\therefore \vec{a}\times\vec{b}=(\hat{\imath}+2\hat{\jmath}+3\hat{k})\times(-3\hat{\imath}+2\hat{\jmath}+5\hat{k})$

$$=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 2 & 3\\ -3 & 2 & 5\end{vmatrix}$$

$=\hat{\imath}(10-6)-\hat{\jmath}(5+9)+\hat{k}(2+6)$

$\dot{A}=4\hat{\imath}-14\hat{\jmath}+8\hat{k}$

Position vector of the given line is

$\dot{B}=-\hat{\imath}+3\hat{\jmath}-2\hat{k}$

Vector equation of line is,

$\dot{r}=\dot{B}+\lambda\dot{A}$

$=(-\hat{\imath}+3\hat{\jmath}+2\hat{k})+\lambda(4\hat{\imath}-14\hat{\jmath}+0\hat{k})$

or $\dot{r}=(-\hat{\imath}+3\hat{\jmath}-2\hat{k})+\lambda(2\hat{\imath}-7\hat{\jmath}+4\hat{k})$

Distance of the line ' l ' passing through the point $(-1,3,-2)$ from the origin is

$=\sqrt{(-1-0)^2+(3-0)^2+(-2-0)^2}$

$=\sqrt{(-1)^2+3^2+2^2}$

$= \sqrt{1+9+4}$
$= \sqrt{14}$ units.

72. Given, $y = \sqrt{4-x^2}$
$\Rightarrow x^2 + y^2 = 4$
For finding point of intersection put $y = \sqrt{3}x$ in $y = \sqrt{4-x^2}$, we get
$\sqrt{3}x = \sqrt{4-x^2}$
$\Rightarrow 3x^2 = 4 - x^2$
$\Rightarrow 4x^2 = 4$
$\Rightarrow x^2 = 1$
$\Rightarrow x = \pm 1$
$\therefore y = \sqrt{3}$
$\therefore$ Coordinates of A is $(1, \sqrt{3})$

$$\therefore \text{ Required Area } = \int_0^{\sqrt{3}} \frac{y}{\sqrt{3}} dy + \int_{\sqrt{3}}^{2} \sqrt{4-y^2} dy$$
$$= \frac{1}{\sqrt{3}}\left[\frac{y^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{y}{2}\sqrt{4-y^2} + \frac{4}{2}\sin^{-1}\left(\frac{y}{2}\right)\right]_{\sqrt{3}}^{2}$$
$$= \frac{1}{\sqrt{3}}\left[\frac{3}{2} - 0\right] + \left[2\sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$
$$= \frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{3}$$
$$= \pi - \frac{2\pi}{3}$$
$$= \frac{\pi}{3} \text{ sq. units}$$

73. We have, $f: [-4,4] \to [0,4]$ defined as $f(x) = \sqrt{16-x^2}$
(i) One-One
$f(x_1) = f(x_2)$
$\Rightarrow \sqrt{16-x_1^2} = \sqrt{16-x_2^2}$
$\Rightarrow 16 - x_1^2 = 16 - x_2^2$
$\Rightarrow x_1^2 = x_2^2$
$\Rightarrow x_1^2 - x_2^2 = 0$
$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$
Here, $x_1 + x_2 = 0$ is also possible.
As if $x_1 = 4$ and $x_2 = -4$.
Then, $x_1 + x_2 = 0$ is also possible.
$\therefore x_1 = -x_2$
But for one-one,
$x_1 = -x_2$
so, f(x) is not one-one.
(ii) Onto
Let, $y = \sqrt{16-x^2}$
$\Rightarrow y^2 = 16 - x^2$
$\Rightarrow x^2 = 16 - y^2$
$\Rightarrow x = \sqrt{16-y^2} \in [0,4]$
So, f(x) is onto.
For $f(a) = \sqrt{7}$, we have
$f(a) = \sqrt{16-a^2}$
$\Rightarrow \sqrt{7} = \sqrt{16-a^2}$
On equating both sides
$\Rightarrow 7 = 16 - a^2$

$\Rightarrow a^2 = 16 - 7$
$\Rightarrow a^2 = 9$
$\Rightarrow a = \pm 3$
Hence, possible values of a are 3 and -3 .

74. **(i)** Area of metal sheet required to made a cylinder open from top $= 75\pi\ \text{cm}^2$.
Given, ' r ' is the radius of the cylinder ' h ' is the height of the cylinder.
$\therefore 2\pi rh + \pi r^2 = 75\pi$
$\Rightarrow 2rh + r^2 = 75$
$\Rightarrow 2rh = 75 - r^2$
$\Rightarrow\ h = \frac{75-r^2}{2r}$ ...(i)
Then, volume of cylinder,
(ii) From (i),
$V = \pi r^2 h$
$= \pi r^2 \times \left(\frac{75-r^2}{2r}\right)$ ...[From (i)]
$= \frac{\pi r}{2}(75 - r^2)\text{cm}^3$
$V = \frac{\pi}{2}(75r - r^3)$
$\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2)$ ...(ii)
(iii) (A) From (ii),
$\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2)$
For maximum volume, put $\frac{dV}{dr} = 0$
$\Rightarrow \frac{\pi}{2}(75 - 3r^2) = 0$
$\Rightarrow \frac{3\pi}{2} \neq 0, 25 - r^2 = 0$
$\Rightarrow 25 = r^2$ [after taking square roots]
$\Rightarrow r = 5$ cm
Now, $\frac{d^2V}{dr^2} = \frac{\pi}{2}(3r^2) < 0$
Hence, the volume is maximum, when $r = 5$ cm.

OR

(B) From
Then, $h = \frac{75-r^2}{2r}$
$= \frac{75 - 5^2}{2(5)}$
$= \frac{75 - 25}{10}$
$= \frac{50}{10}$
$= 5$
For max. volume,
$h = r = 5$ cm
Hence, the given statement is false.

75. Given, $P(L) = \frac{12}{100}$
$P(L)P(\bar{L}) = 1 - \frac{12}{100} = \frac{88}{100}$
and $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$
$P\left(\frac{L}{A}\right) = \frac{24}{100}$

$P\left(\frac{L}{B}\right) = \frac{22}{100}$

$P\left(\frac{L}{C}\right) = \frac{17}{100}$

$P\left(\frac{L}{D}\right) = \frac{9}{100}$

(i) $P\left(\frac{L}{C}\right) = \frac{17}{100}$, from the given data.

(ii) $P\left(\frac{\bar{L}}{A}\right) = \frac{P(\bar{L} \cap A)}{P(A)}$

$= \frac{P(A) - P(L \cap A)}{P(A)}$

$= 1 - \frac{P(L \cap A)}{P(A)}$

$= 1 - P\left(\frac{L}{A}\right)$

$= 1 - \frac{24}{100}$

$= \frac{100 - 24}{100}$

$= \frac{76}{100}$

$= \frac{38}{50}$

$= \frac{19}{25}$

(iii) (A) $P\left(\frac{A}{L}\right) = \frac{P(A \cap L)}{P(L)}$

But $P\left(\frac{L}{A}\right) = \frac{P(A \cap L)}{P(A)}$

$\frac{24}{100} = \frac{P(A \cap L)}{\frac{1}{4}}$

$\Rightarrow P(A \cap L) = \frac{24}{100} \times \frac{1}{4} = \frac{6}{100} = \frac{3}{50}$

$\therefore P\left(\frac{A}{L}\right) = \frac{\frac{3}{50}}{\frac{12}{100}} = \frac{3 \times 100}{12 \times 50} = \frac{1}{2}$

OR

(B) $P\left(\frac{L}{B \cup C}\right) = \frac{P[(L) \cap (B \cup C)]}{P(B \cup C)}$

$= \frac{P[(L \cap B) \cup (L \cap C)]}{P(B \cup C)}$

$= \frac{P(L \cap B) + P(L \cap C) - P(L \cap B)P(L \cap C)}{P(B) + P(C) - P(B)P(C)}$ ... (As they are independent)

$= \frac{\frac{22}{100} \times \frac{1}{4} + \frac{17}{100} \times \frac{1}{4} - \frac{22}{400} \times \frac{17}{400}}{\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \times \frac{1}{4}}$

$= \frac{\frac{22}{400} + \frac{17}{400} - \frac{22 \times 17}{400 \times 400}}{\frac{1}{2} - \frac{1}{16}}$

$= \frac{\left(\frac{39}{400} - \frac{374}{160000}\right)}{\frac{1}{2} - \frac{1}{16}}$

$= \frac{16}{7}\left(\frac{39 \times 400 - 374}{160000}\right)$

$= \dfrac{16 \times 15226}{7 \times 160000}$
$= 0.217$
$= 0.22$

76. Given, the estimated of electric vehicles in use at any time t is given by

$V(t) = \dfrac{1}{5}t^3 - \dfrac{5}{2}t^2 + 25t - 2$

(i) No, the function cannot be used to calculate the number of vehicles in 2000.
As, $t = 1,2,3,\ldots$ where starting year is 2001, 2002, 2003 ...
Therefore, it could not be used to calculate the year before 2001.

(ii) Here, $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

$\dfrac{dV(t)}{dt} = \dfrac{1}{5} \times 3t^2 - \dfrac{5}{2} \times 2t + 25$

$V'(t) = \dfrac{3}{5}t^2 - 5t + 25$

For the function to be increasing $V'(t) > 0$

Here, $\frac{3}{2}t^2 - 5t + 25 > 0$

Hence, function $V(t) > 0$
So, it is an increasing function.

77. (d) Matrix A is symmetric
For a symmetric matrix, $A = A'$ (the transpose of A).
Comparing the non-diagonal elements:
- $a_{12} = a_{21} \Rightarrow 4 = z$
- $a_{13} = a_{31} \Rightarrow x = -3$
- $a_{23} = a_{32} \Rightarrow y = -1$

Thus, the value of $x + y + z = -3 + (-1) + 4 = 0$.

78. (a) Value of $|A| + |adjA|$
Using the standard matrix property $A \cdot (adjA) = |A|I$, we can see that:

$|A|I = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow |A| = 3$

We also know that the determinant of the adjoint matrix is $|adjA| = |A|^{n-1}$. For a 3×3 matrix ($n = 3$):
$|adjA| = 3^{3-1} = 3^2 = 9$
Therefore, $|A| + |adjA| = 3 + 9 = 12$.

79. (c) Skew-symmetric matrices A and B
Since A and B are skew-symmetric, $A' = -A$ and $B' = -B$.
For AB to be symmetric, we must have $(AB)' = AB$.
Using the transpose property $(AB)' = B'A'$:
$B'A' = AB$
$(-B)(-A) = AB$
$BA = AB$

80. (b) Value of $x \in \left[0, \frac{\pi}{2}\right]$

Given $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, the transpose is $A' = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$.
Adding these together:

$A + A' = \begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix}$

We are given that $A + A' = \sqrt{3}I = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{bmatrix}$.

Equating the elements yields $2\cos x = \sqrt{3}$, or $\cos x = \frac{\sqrt{3}}{2}$.

For $x \in \left[0, \frac{\pi}{2}\right]$, the value is $x = \frac{\pi}{6}$.

81. (b) Area of a triangle
The area of a triangle (A) with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) using determinants is given by:

$$A = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Multiplying both sides by 2 gives:

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$$

(Your written question had a typo y_8 in the vertices, but standard formulas require y_3.)

82. (c) Integration of $2^{x+2}dx$

We can rewrite the integral using exponent rules: $\int 2^{x+2}dx = \int 2^x \cdot 2^2 dx = 4\int 2^x dx$.

Recall the standard integration formula $\int a^x dx = \frac{a^x}{\ln a} + C$.

Applying this, we get:

$$4\left(\frac{2^x}{\ln 2}\right) + C = \frac{2^2 - 2^x}{\ln 2} + C = \frac{2^{x+2}}{\ln 2} + C$$

83. (b) $\int \frac{2\cos 2x - 1}{1 + 2\sin x}dx$

$2\cos 2x - 1 = 2(1 - 2\sin^2 x) - 1 = 1 - 4\sin^2 x$

$1 - 4\sin^2 x = (1 - 2\sin x)(1 + 2\sin x)$

Hence,

$$\int \frac{(1 - 2\sin x)(1 + 2\sin x)}{1 + 2\sin x}dx = \int (1 - 2\sin x)dx$$

$= x + 2\cos x + C$

84. (c) $\frac{dx}{x} + \frac{dy}{y} = 0$

Integrating.

$$\int \frac{dx}{x} + \int \frac{dy}{y} = 0$$

$\log x + \log y = C$

$\log(xy) = C$

$xy = C_1$

85. (b) • $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$

Highest derivative $= \frac{d^2y}{dx^2}$

• Order $= 2$

• Power of highest derivative $= 1$

So,

Degree $= 1$

Order × Degree $= 2 \times 1 = 2$

86. (c) Let direction cosines be l, m, n.

Given,

$$l = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Using

$l^2 + m^2 + n^2 = 1$

$\frac{1}{2} + \frac{1}{2} + n^2 = 1$

$n^2 = 0$

$n = 0$

Hence

$\cos\gamma = 0 \Rightarrow \gamma = \frac{\pi}{2}$

87. (a) Projection of $\vec{a}$ on $\vec{b}$ is

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

Given projection $= 0$,
$\vec{a} \cdot \vec{b} = 0$
Since both vectors are non-zero, they are perpendicular.
$\theta = \frac{\pi}{2}$

88. (d) Given
$\overrightarrow{AB} = i + j + 2k, \overrightarrow{AC} = 3i - j + 4k$
If D is midpoint of BC, then
$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$
$$= \frac{(i + j + 2k) + (3i - j + 4k)}{2}$$
$$= \frac{4i + 6k}{2}$$
$$= 2i + 3k$$

89. (a) The value of λ for which the angle is $\pi/2$:
- Direction vector of line 1: $\overrightarrow{b_1} = (2,1,2)$
- Direction vector of line 2: $\overrightarrow{b_2} = (1, \lambda, 1)$
- For the angle to be $\pi/2$, the dot product $\overrightarrow{b_1} \cdot \overrightarrow{b_2}$ must be 0 :

$2(1) + 1(\lambda) + 2(1) = 0 \Longrightarrow 2 + \lambda + 2 = 0 \Longrightarrow \lambda = -4$

90. (a) Find $P(B/A)$:
- $P(A) = 1 - P(\bar{A}) = 1 - \frac{3}{4} = \frac{1}{4}$
- $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/8}{1/4} = \frac{4}{8} = \frac{1}{2}$

91. (d) Value of k for differentiability at $x = 0$:
- Right-hand derivative (at $x \geq 0$): $\frac{d}{dx}(x^2) = 2x$. At $x = 0$, RHD $= 0$.
- Left-hand derivative (at $x < 0$) : $\frac{d}{dx}(kx) = k$. At $x = 0$, LHD $= k$.
- For differentiability, LHD = RHD, so $k = 0$.

92. (a) Find dy/dx for $y = \frac{\cos x - \sin x}{\cos x + \sin x}$:
- Divide numerator and denominator by $\cos x$: $y = \frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)$
- Differentiating with respect to x: $\frac{dy}{dx} = \sec^2\left(\frac{\pi}{4} - x\right) \cdot (-1) = -\sec^2\left(\frac{\pi}{4} - x\right)$

93. (d) Number of feasible solutions:
- The constraints $3x + y \leq 12, x + 2y \leq 10, x \geq 0$, and $y \geq 0$ define a bounded region (a polygon) in the first quadrant.
- Every point within or on the boundary of this region is a "feasible solution." Since there are infinitely many points in any such area, the number of solutions is infinite.

94. (c) The two boundary lines are:
$x + y = 3$
and
$x + 2y = 4$
Their intersection point is:
$x + y = 3, \quad x + 2y = 4$
$y = 1, \quad x = 2$
So the point of intersection is (2,1).
The shaded feasible region lies above both lines and is in the first quadrant.
Check a point in the shaded region, say (4,2) :
- $x + y = 4 + 2 = 6 \geq 3$
- $x + 2y = 4 + 4 = 8 \geq 4$

Thus the inequalities are:
$x + y \geq 3$
$x + 2y \geq 4$

and since the region is in the first quadrant,
$x \geq 0, y \geq 0$
Therefore the possible constraints are:
$x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$

95. (d)• Assertion (A): Range of $[\sin^{-1}x + 2\cos^{-1}x]$ is $[0, \pi]$.
Analysis: Let $f(x) = \sin^{-1}x + 2\cos^{-1}x$.
Since $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, we can rewrite it as:
$$f(x) = (\sin^{-1}x + \cos^{-1}x) + \cos^{-1}x = \frac{\pi}{2} + \cos^{-1}x$$
The range of $\cos^{-1}x$ is $[0, \pi]$.
Adding $\frac{\pi}{2}$ to this range gives: $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.
• Conclusion: The assertion is False.
• Reason (R): Principal value branch of $\sin^{-1}x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
Analysis: This is a standard definition in inverse trigonometry.
• Conclusion: The reason is True.

96. (c) • Assertion (A): A line through (4,7,8) and (2,3,4) is parallel to a line through (-1 ,−2,1) and (1,2,5).
Analysis: Direction ratios (DRs) of the first line: $(4-2, 7-3, 8-4) = (2,4,4)$.
DRs of the second line: $(1-(-1), 2-(-2), 5-1) = (2,4,4)$.
Since the DRs are identical (proportional), the lines are parallel.
• Conclusion: The assertion is True.
• Reason (R): Lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are parallel if $\vec{b}_1 \cdot \vec{b}_2 = 0$.
Analysis: $\overrightarrow{b_1} \cdot \overrightarrow{b_2} = 0$ is the condition for lines to be perpendicular, not parallel. For lines to be parallel, $\vec{b}_1$ must be a scalar multiple of $\vec{b}_2$ (i.e., $\vec{b}_1 = k\vec{b}_2$ or $\vec{b}_1 \times \vec{b}_2 = \vec{0}$).
• Conclusion: The reason is False.

97. We can use the vector identity $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$.Setting $\vec{a} = \vec{r}, \vec{b} = \hat{j}, \vec{c} = \vec{r}$, and $\vec{d} = \hat{k}$:
$(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) = (\vec{r} \cdot \vec{r})(\hat{j} \cdot \hat{k}) - (\vec{r} \cdot \hat{k})(\vec{r} \cdot \hat{j})$
Since $\hat{j} \cdot \hat{k} = 0$, the first term disappears:
$= 0 - (\vec{r} \cdot \hat{k})(\vec{r} \cdot \hat{j})$
From $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, we have $\vec{r} \cdot \hat{k} = 6$ and $\vec{r} \cdot \hat{j} = -2$.
$= -(6)(-2) = 12$
Substituting this back into the original expression:
$12 - 12 = 0$

98. The direction vectors of the lines are $\vec{b}_1 = \alpha\hat{i} - 5\hat{j} + \beta\hat{k}$ and $\vec{b}_2 = 1\hat{i} + 0\hat{j} + 1\hat{k}$.
The angle between them is $\theta = \frac{\pi}{4}$. Using the angle formula $\cos\theta = \frac{|\overrightarrow{b_1} \cdot \overrightarrow{b_2}|}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$:
$$\cos\left(\frac{\pi}{4}\right) = \frac{|\alpha(1) + (-5)(0) + \beta(1)|}{\sqrt{\alpha^2 + (-5)^2 + \beta^2}\sqrt{1^2 + 0^2 + 1^2}}$$
$$\frac{1}{\sqrt{2}} = \frac{|\alpha + \beta|}{\sqrt{\alpha^2 + \beta^2 + 25}\sqrt{2}}$$
Canceling $\sqrt{2}$ from both denominators and squaring both sides gives:
$\alpha^2 + \beta^2 + 25 = (\alpha + \beta)^2$
$\alpha^2 + \beta^2 + 25 = \alpha^2 + \beta^2 + 2\alpha\beta \Longrightarrow 2\alpha\beta = 25$

99. Strictly Increasing Function
Differentiating $f(x) = a(\tan x - \cot x)$ with respect to x :
$f'(x) = a\left(\sec^2 x - (-\csc^2 x)\right) = a(\sec^2 x + \csc^2 x)$
Since squares of real numbers are positive, $\sec^2 x > 0$ and $\csc^2 x > 0$ in their domain. Given that $a > 0$, the product $f'(x) > 0$ for all valid values of x. Since the first derivative is strictly positive, the function is increasing.

100. (A) Substitute the principal values of each inverse trigonometric term:
• $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

• $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

• $\cos^{-1}(0) = \frac{\pi}{2}$

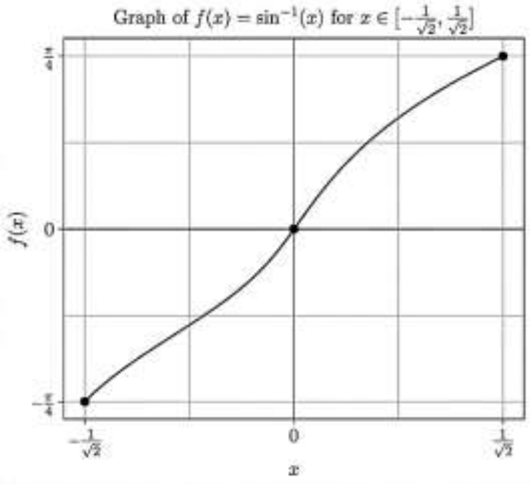
Now, compute the complete sum:

$$3\left(\frac{\pi}{4}\right) + 2\left(\frac{\pi}{6}\right) + \frac{\pi}{2} = \frac{3\pi}{4} + \frac{\pi}{3} + \frac{\pi}{2} = \frac{9\pi + 4\pi + 6\pi}{12} = \frac{19\pi}{12}$$

(B)• Range: $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

• Graph:

Graph of $f(x) = \sin^{-1}x$ for $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$



101. **(A)** If $y = x^{1/x}$, find $\frac{dy}{dx}$ at $x = 1$

Given

$y = x^{1/x}$

Taking logarithm:

$$\log y = \frac{\log x}{x}$$

Differentiate both sides:

$$\frac{1}{y}\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

Hence

$$\frac{dy}{dx} = x^{1/x} \cdot \frac{1 - \log x}{x^2}$$

At $x = 1$.

$y = 1^1 = 1, \log 1 = 0$

Therefore

$$\left.\frac{dy}{dx}\right|_{x=1} = 1 \cdot \frac{1-0}{1} = 1$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 1$$

OR

(B) If

$x = a\sin 2t, y = a(\cos 2t + \log\tan t)$

find $\frac{dy}{dx}$.

Using parametric differentiation,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dz}{dt}}$$

Now,

$$\frac{dx}{dt} = a(2\cos 2t) = 2a\cos 2t$$
and
$$\frac{dy}{dt} = a\left(-2\sin 2t + \frac{d}{dt}(\log\tan t)\right)$$
Since
$$\frac{d}{dt}(\log\tan t) = \frac{\sec^2 t}{\tan t} = \frac{1}{\sin t\cos t} = \frac{2}{\sin 2t}$$
$$\frac{dy}{dt} = a\left(-2\sin 2t + \frac{2}{\sin 2t}\right)$$
$$= 2a\left(\frac{1-\sin^2 2t}{\sin 2t}\right)$$
$$= 2a\frac{\cos^2 2t}{\sin 2t}$$
Therefore
$$\frac{dy}{dx} = \frac{2a\frac{\cos^2 2t}{\sin 2t}}{2a\cos 2t} = \frac{\cos 2t}{\sin 2t}$$
$$\frac{dy}{dx} = \cot 2t$$

102. **(A)** General Solution of $\frac{d}{dx}(xy^2) = 2y(1+x^2)$

The general solution is $y = \frac{2}{5}x^2 + 2 + \frac{C}{\sqrt{x}}$.

(1) Differentiate left side: Use the product rule on $\frac{d}{dx}(xy^2)$ to get $y^2 + x\left(2y\frac{dy}{dx}\right) = 2y(1+x^2)$.

(2) Simplify to linear form: Divide by $2xy$ (assuming $y \neq 0$) to get $\frac{dy}{dx} + \frac{1}{2x}y = \frac{1+x^2}{x}$.

(3) Find Integrating Factor (IF): $IF = e^{\int \frac{1}{2x}dx} = e^{\frac{1}{2}\ln x} = \sqrt{x}$.

(4) Solve the equation: Multiply by IF and integrate:
$$\sqrt{x}y = \int \left(\frac{1}{x} + x\right)\sqrt{x}dx = \int (x^{-1/2} + x^{3/2})dx.$$

(5) Final Integration: $\sqrt{x}y = 2x^{1/2} + \frac{2}{5}x^{5/2} + C$. Dividing by $\sqrt{x}$ gives $y = 2 + \frac{2}{5}x^2 + \frac{C}{\sqrt{x}}$.

OR

(B) Solution of $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$

The solution is $e^{-\frac{y}{x}} = \ln|x| + C$.

(1) Identify type: This is a homogeneous differential equation. Rearrange to
$$\frac{dy}{dx} = \frac{y - xe^{y/x}}{x} = \frac{y}{x} - e^{y/x}.$$

(2) Substitute $v = \frac{y}{x}$: Let $y = vx$, so $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

(3) Variable separation: $v + x\frac{dv}{dx} = v - e^v \Rightarrow x\frac{dv}{dx} = -e^v \Rightarrow e^{-v}dv = -\frac{1}{x}dx$.

(4) Integrate: $\int e^{-v}dv = \int -\frac{1}{x}dx \Rightarrow -e^{-v} = -\ln|x| + C'$.

(5) Back-substitute: $e^{-y/x} = \ln|x| + C$.

103. Evaluate $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$

The value of the integral is 1 .

(1) Apply property: Use $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. Here $a+b=4$.

(2) New integral: Let $I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$. Then
$$I = \int_1^3 \frac{\sqrt{4-(4-x)}}{\sqrt{4-x}+\sqrt{4-(4-x)}}dx = \int_1^3 \frac{\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}dx$$

(3) Add integrals: $2I = \int_1^3 \frac{\sqrt{4-x}+\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}dx = \int_1^3 1dx$.

(4) Solve: $2I = [x]_1^3 = 3 - 1 = 2 \Rightarrow I = 1$.

104. Evaluate $\int_1^e \frac{1}{\sqrt{4x^2-(x\log x)^2}}dx$

The value of the integral is $\frac{\pi}{6}$.

(1) Simplify denominator: Factor out x^2 from the square root $\int_1^e \frac{1}{x\sqrt{4-(\log x)^2}}dx$.

(2) Substitution: Let $u = \log x$, then $du = \frac{1}{x}dx$.

(3) Change limits: When $x = 1, u = 0$; when $x = e, u = 1$.

(4) Integrate: The integral becomes $\int_0^1 \frac{1}{\sqrt{2^2-u^2}}du = \left[\sin^{-1}\left(\frac{u}{2}\right)\right]_0^1$.

(5) Final Value: $\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0) = \frac{\pi}{6} - 0 = \frac{\pi}{6}$.

105. **(A)** Evaluate

$$I = \int \frac{\cos x}{\sin 3x}dx$$

Using

$$\sin 3x = \sin x(3 - 4\sin^2 x)$$

$$I = \int \frac{\cos x}{\sin x(3 - 4\sin^2 x)}dx$$

Let

$$t = \sin x, dt = \cos x dx$$

Then

$$I = \int \frac{dt}{t(3 - 4t^2)}$$

Using partial fractions,

$$\frac{1}{t(3 - 4t^2)} = \frac{1}{3t} + \frac{4t}{3(3 - 4t^2)}$$

Hence

$$I = \frac{1}{3}\int \frac{dt}{t} + \frac{1}{3}\int \frac{4tdt}{3 - 4t^2}$$

$$= \frac{1}{3}\ln|t| - \frac{1}{6}\ln|3 - 4t^2| + C$$

Substituting $t = \sin x$,

$$\int \frac{\cos x}{\sin 3x}dx = \frac{1}{3}\ln|\sin x| - \frac{1}{6}\ln|3 - 4\sin^2 x| + C$$

OR

(B) Evaluate

$$I = \int x^2\log(x^2 + 1)dx$$

Take

$$u = \log(x^2 + 1), dv = x^2dx$$

Then

$$du = \frac{2x}{x^2 + 1}dx, v = \frac{x^3}{3}$$

By integration by parts,

$$I = \frac{x^3}{3}\log(x^2 + 1) - \frac{2}{3}\int \frac{x^4}{x^2 + 1}dx$$

Now

$$\frac{x^4}{x^2 + 1} = x^2 - 1 + \frac{1}{x^2 + 1}$$

Therefore

$$I = \frac{x^3}{3}\log(x^2 + 1) - \frac{2}{3}\int \left(x^2 - 1 + \frac{1}{x^2 + 1}\right)dx$$

$$I = \frac{x^3}{3}\log(x^2 + 1) - \frac{2}{3}\left(\frac{x^3}{3} - x + \tan^{-1}x\right) + C$$

$$I = \frac{x^3}{3}\log(x^2+1) - \frac{2x^3}{9} + \frac{2x}{3} - \frac{2}{3}\tan^{-1}x + C$$

106. Determine graphically the minimum value of $Z = 500x + 400y$

The minimum value is 42,000 , which occurs at the point (20,80).

(1) Identify the constraints:

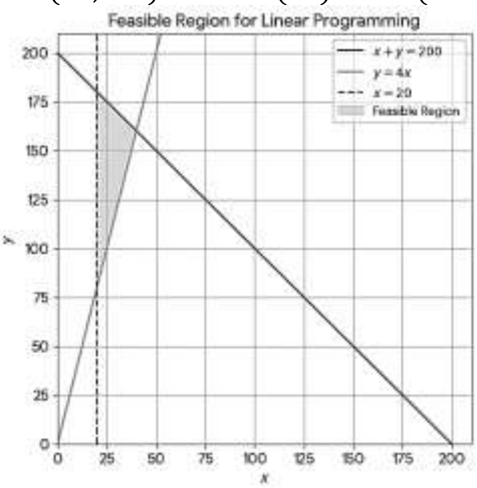
• $x + y \leq 200$
• $x \geq 20$
• $y \geq 4x$
• $y \geq 0$

(2) Find intersection points (vertices):

• Point A: Intersection of $x = 20$ and $y = 4x \rightarrow (20,80)$.
• Point B: Intersection of $y = 4x$ and $x + y = 200 \rightarrow 5x = 200 \Rightarrow (40,160)$.
• Point C: Intersection of $x = 20$ and $x + y = 200 \rightarrow (20,180)$.

(3) Evaluate the objective function at vertices:

• At $(20,80)$: $Z = 500(20) + 400(80) = 42{,}000$.
• At $(40,160)$: $Z = 500(40) + 400(160) = 84{,}000$.
• At $(20,180)$: $Z = 500(20) + 400(180) = 82{,}000$.



107. **(A)** Probability Distribution of X

X = absolute difference of the numbers on two dice.

Total outcomes = 36.

X	Outcomes	Probability
0	6	$\frac{6}{36} = \frac{1}{6}$
1	10	$\frac{10}{36} = \frac{5}{18}$
2	8	$\frac{8}{36} = \frac{2}{9}$
3	6	$\frac{6}{36} = \frac{1}{6}$
4	4	$\frac{4}{36} = \frac{1}{9}$
5	2	$\frac{2}{36} = \frac{1}{18}$

Hence

X	0	1	2	3	4	5

P(X)	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

OR

(B) Bayes' Theorem

Let

- B : biased coin selected
- F : fair coin selected
- H : head occurs

Given

$$P(B) = P(F) = \frac{1}{2}$$

For the biased coin,

$P(H) : P(T) = 1 : 3$

$$P(H \mid B) = \frac{1}{4}$$

For the fair coin,

$$P(H \mid F) = \frac{1}{2}$$

Now

$$P(H) = P(B)P(H \mid B) + P(F)P(H \mid F)$$

$$= \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$

By Bayes' theorem,

$$P(B \mid H) = \frac{P(B)P(H \mid B)}{P(H)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{4}}{\frac{3}{8}} = \frac{1}{8} \cdot \frac{8}{3} = \frac{1}{3}$$

$$P(\text{ biased coin } \mid \text{ head }) = \frac{1}{3}$$

108. Show that $f: \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \frac{5x - 3}{4}$$

is one-one and onto.

(i) One-One

Let

$f(x_1) = f(x_2)$

Then

$$\frac{5x_1 - 3}{4} = \frac{5x_2 - 3}{4}$$

$5x_1 - 3 = 5x_2 - 3$

$x_1 = x_2$

Hence f is one-one (injective).

(ii) Onto

Let $y \in \mathbb{R}$.

We need an $x \in \mathbb{R}$ such that

$f(x) = y$

$$\frac{5x - 3}{4} = y$$

$5x = 4y + 3$

$$x = \frac{4y + 3}{5}$$

Since $\frac{4y+3}{5} \in \mathbb{R}$, there exists an x corresponding to every $y \in \mathbb{R}$.

Hence f is onto (surjective).

Therefore,

$f(x) = \frac{5x - 3}{4}$ is both one-one and onto.

109. Find m

Given:
$x^2 + y^2 = 4$
is a circle of radius 2 .
The line is
$y = mx, m > 0.$
Let the line meet the circle at P.
If the angle made by the line with the positive x-axis is θ, then
$m = \tan\theta.$
The required region in the first quadrant is the sector bounded by:
• x-axis,
• line $y = mx$,
• arc of the circle.
Area of sector:
$\frac{1}{2}r^2\theta = \frac{1}{2}(2)^2\theta = 2\theta$
Given area
$2\theta = \frac{\pi}{2}$
Thus
$\theta = \frac{\pi}{4}.$
Hence
$m = \tan\theta = \tan\frac{\pi}{4} = 1$
$m = 1$

110. **(A)** $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

Show that
$A^3 - 6A^2 + 7A + 2I = O$
Step 1: Characteristic polynomial
$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix}$
Expanding along second column,
$= (2 - \lambda)\begin{vmatrix} 1-\lambda & 2 \\ 2 & 3-\lambda \end{vmatrix}$
$= (2 - \lambda)[(1 - \lambda)(3 - \lambda) - 4]$
$= (2 - \lambda)(\lambda^2 - 4\lambda - 1)$
$= -\lambda^3 + 6\lambda^2 - 7\lambda - 2$
Therefore characteristic equation is
$\lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0$
By Cayley-Hamilton Theorem, replacing λ by A,
$A^3 - 6A^2 + 7A + 2I = O$
Hence proved.

OR

(B) $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$
Step 1: Find A^{-1}
$|A| = 3(-7) - 2(5) = -21 - 10 = -31$
$A^{-1} = \frac{1}{-31}\begin{bmatrix} -7 & -2 \\ -5 & 3 \end{bmatrix} = \frac{1}{31}\begin{bmatrix} 7 & 2 \\ 5 & -3 \end{bmatrix}$
Thus
$A^{-1} = \frac{1}{31}\begin{bmatrix} 7 & 2 \\ 5 & -3 \end{bmatrix}$

Step 2: Solve

$3x + 2y = -3$

$5x - 7y = 11$

or $AX = \begin{bmatrix} -3 \\ 11 \end{bmatrix}$

Hence

$X = A^{-1}\begin{bmatrix} -3 \\ 11 \end{bmatrix}$

$= \frac{1}{31}\begin{bmatrix} 7 & 2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} -3 \\ 11 \end{bmatrix}$

$= \frac{1}{31}\begin{bmatrix} 1 \\ 48 \end{bmatrix}$

Therefore

$x = \frac{1}{31}, y = -\frac{48}{31}$

(Checking: $3x + 5y = 11, 2x - 7y = -3$ gives $x = 2, y = 1$. The coefficient matrix for those equations is different from the given matrix A.)

111. **(A)** (1) Express lines in parametric form

Let the first line be L_1 and the second be L_2. We set each equal to a parameter s and t

• $L_1: \frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4} = s \Rightarrow x = 2s + 1, y = 3s + b, z = 4s + 3$

• $L_2: \frac{x-4}{5} = \frac{y-1}{2} = z = t \Rightarrow x = 5t + 4, y = 2t + 1, z = t$

(2) Solve for parameters s and t

Equate the x and z coordinates of both lines:

• $2s + 1 = 5t + 4 \Rightarrow 2s - 5t = 3$ ….(i)

• $4s + 3 = t$ ….(ii)

Substitute $t = 4s + 3$ from (Eq. 2) into (Eq. 1):

$2s - 5(4s + 3) = 3$

$2s - 20s - 15 = 3 \Rightarrow -18s = 18 \Rightarrow s = -1$

Then, $t = 4(-1) + 3 = -1$.

(3) Determine the value of b

Equate the y coordinates using the found values of s and t :

$3s + b = 2t + 1$

$3(-1) + b = 2(-1) + 1$

$-3 + b = -1 \Rightarrow b = 2$

(4) Find the point of intersection

Substitute $t = -1$ into the parametric form of L_2 :

$x = 5(-1) + 4 = -1$

$y = 2(-1) + 1 = -1$

$z = -1$

The point of intersection is $(-1, -1, -1)$.

OR

(B) Part 1: Equations of all the sides of parallelogram ABCD

The vector equation or Cartesian equation of a line passing through (x_1, y_1, z_1) and (x_2, y_2, z_2) uses direction ratios $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$.

• Side AB (passing through A(4,7,8) and B(2,3,4)):

Direction ratios: $(2 - 4, 3 - 7, 4 - 8) = (-2, -4, -4)$ or $(1,2,2)$

$$\frac{x-4}{1} = \frac{y-7}{2} = \frac{z-8}{2}$$

• Side BC (passing through B(2,3,4) and C(−1,−2,1)):

Direction ratios: $(-1 - 2, -2 - 3, 1 - 4) = (-3, -5, -3)$ or $(3,5,3)$

$$\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{3}$$

• Side CD (passing through C(−1,−2,1) and D(1,2,5)):

Direction ratios: $(1 - (-1), 2 - (-2), 5 - 1) = (2,4,4)$ or $(1,2,2)$

$$\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$$

• Side DA (passing through D(1,2,5) and A(4,7,8)):
Direction ratios: $(4-1,7-2,8-5)=(3,5,3)$
$\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-5}{3}$
Part 2: Coordinates of the foot of the perpendicular from A to CD
Let P be the foot of the perpendicular on line CD.
The general coordinates of any point P on line CD can be written using parameter λ :
$\frac{x+1}{1}=\frac{y+2}{2}=\frac{z-1}{2}=\lambda \Longrightarrow P(\lambda-1,2\lambda-2,2\lambda+1)$
The direction ratios of vector $\overrightarrow{AP}$ are given by $(P-A)$:
$\overrightarrow{AP}=((\lambda-1)-4,(2\lambda-2)-7,(2\lambda+1)-8)=(\lambda-5,2\lambda-9,2\lambda-7)$
Since $\overrightarrow{AP}$ is perpendicular to side CD (which has direction ratios 1,2,2), their dot product is zero:
$1(\lambda-5)+2(2\lambda-9)+2(2\lambda-7)=0$
$\lambda-5+4\lambda-18+4\lambda-14=0$
$9\lambda-37=0 \Longrightarrow \lambda=\frac{37}{9}$
Substitute $\lambda=\frac{37}{9}$ back into the coordinates of point P.
• $x=\frac{37}{9}-1=\frac{28}{9}$
• $y=2\left(\frac{37}{9}\right)-2=\frac{56}{9}$
• $z=2\left(\frac{37}{9}\right)+1=\frac{83}{9}$
• The foot of the perpendicular is $\left(\frac{28}{9},\frac{56}{9},\frac{83}{9}\right)$.

112. **(i)** Find p
Since total probability = 1,
$p+2p+2p+p+2p+p^2+2p^2+(7p^2+p)=1$
$9p+10p^2=1$
$10p^2+9p-1=0$
$(10p-1)(p+1)=0$
$p=\frac{1}{10}$ or $p=-1$
Since probability cannot be negative,
$p=\frac{1}{10}$
(ii) Find $P(X>6)$
$P(X>6)=P(7)+P(8)$
$=2p^2+(7p^2+p)$
$=9p^2+p$
Substituting $p=\frac{1}{10}$.
$P(X>6)=9\left(\frac{1}{100}\right)+\frac{1}{10}$
$=\frac{9}{100}+\frac{10}{100}$
$=\frac{19}{100}$
$P(X>6)=\frac{19}{100}$
(iii) (A) Find $P(X=3m)$, where m is a natural number
Possible values of X are 1,2,3,4,5,6,7,8.
Values of X that are multiples of 3:
$X=3,6$
Therefore
$P(X=3m)=P(X=3)+P(X=6)$
$=2p+p^2$

Substituting $p = \frac{1}{10}$.

$= 2\left(\frac{1}{10}\right) + \frac{1}{100}$

$= \frac{20+1}{100}$

$= \frac{21}{100}$

$P(X = 3m) = \frac{21}{100}$

OR

(B) Find the mean E(X)

$E(X) = \sum xP(X)$

$= 1(p) + 2(2p) + 3(2p) + 4(p) + 5(2p) + 6(p^2) + 7(2p^2) + 8(7p^2 + p)$

Substitute $p = \frac{1}{10}$:

$E(X) = \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{4}{10} + \frac{10}{10} + 6\left(\frac{1}{100}\right) + 14\left(\frac{1}{100}\right) + 8\left(\frac{17}{100}\right)$

$= \frac{25}{10} + \frac{6+14+136}{100}$

$= 2.5 + \frac{156}{100}$

$= 2.5 + 1.56$

$= 4.06$

$E(X) = \frac{203}{50} = 4.06$

113. **(i)** Find the total cost C in terms of x

Since volume $= x^2h_r$

$x^2h = 250$

$h = \frac{250}{x^2}$

Cost of land

$= 5000x^2$

Cost of digging

$= 40000h^2$

$= 40000\left(\frac{250}{x^2}\right)^2$

$= \frac{2500000000}{x^4}$

Therefore,

$C(x) = 5000x^2 + \frac{2500000000}{x^4}$

(ii) Find $\frac{dC}{dx}$

$C(x) = 5000x^2 + 2500000000x^{-4}$

Differentiating,

$\frac{dC}{dx} = 10000x - 10000000000x^{-5}$

$\frac{dC}{dx} = 10000x - \frac{10000000000}{x^5}$

(iii) (A) Find the value of x for which C is minimum

For minimum cost,

$\frac{dC}{dx} = 0$

$10000x - \frac{10000000000}{x^5} = 0$

$10000x^6 = 10000000000$

$x^6 = 1000000 = 10^6$
$x = 10$
Since $x > 0$,
$x = 10$ m
Verification
$$\frac{d^2C}{dx^2} = 10000 + \frac{50000000000}{x^6}$$
which is positive for all $x > 0$.
Hence C is minimum at
$x = 10$ m
and
$$h = \frac{250}{10^2} = \frac{250}{100} = 2.5 \text{ m.}$$

OR

(B) Check whether $C(x)$ is increasing or not for $x > 0$
$$\frac{dC}{dx} = 10000x - \frac{10000000000}{x^5}$$
$$= \frac{10000(x^6 - 10^6)}{x^5}$$
Since $x^5 > 0$ for $x > 0$,
- $\frac{dC}{dx} < 0$ when $0 < x < 10$
- $\frac{dC}{dx} = 0$ when $x = 10$
- $\frac{dC}{dx} > 0$ when $x > 10$

Therefore,
$C(x)$ decreases on $(0,10)$ and increases on $(10, \infty)$
Hence $C(x)$ is not increasing for all $x > 0$; it starts increasing only after $x = 10$.

114. **(i)** Is $h(t)$ a continuous function? Justify.
Yes, $h(t)$ is a continuous function for all $t \geq 0$.
Justification: The path is modeled by the function $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, which is a polynomial function (specifically, a quadratic polynomial). In calculus, all polynomial functions are continuous everywhere over their defined domain.
(ii) Find the time at which the height of the ball is maximum.
To find the time t when the height is at its maximum, we calculate the first derivative of $h(t)$ with respect to t and set it equal to zero:
$$h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$$
Differentiating with respect to t :
$$h'(t) = \frac{d}{dt}\left(-\frac{7}{2}t^2 + \frac{13}{2}t + 1\right)$$
$$h'(t) = -\frac{7}{2}(2t) + \frac{13}{2}$$
$$h'(t) = -7t + \frac{13}{2}$$
Set $h'(t) = 0$ to find the critical value of time:
$$-7t + \frac{13}{2} = 0$$
$$7t = \frac{13}{2}$$
$$t = \frac{13}{14} \text{ seconds } \approx 0.93 \text{ seconds}$$

115. **(b)** Matrix Equation
Given: $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$
This gives us two linear equations:

(1) $x + 2y = 4$
(2) $2x + 5y = 9$
Solving for x in the first equation: $x = 4 - 2y$.
Substituting into the second: $2(4 - 2y) + 5y = 9 \Rightarrow 8 - 4y + 5y = 9 \Rightarrow y = 1$.
Then $x = 4 - 2(1) = 2$.

116. **(a)** Matrix Product

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} (a^2 + b^2) & (-ab + ab) \\ (-ba + ab) & (b^2 + a^2) \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

117. **(a)** Square Matrix Identity

Given $A^2 = A$, we expand $(I + A)^2 - 3A$:
$(I + A)^2 = I^2 + IA + AI + A^2 = I + A + A + A = I + 3A$(since $I^2 = I$ and $A^2 = A$).
Then: $(I + 3A) - 3A = I$.

118. **(d)** Matrix AA'

$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$(a 1×3 matrix) and $A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ (a 3×1 matrix).

$AA' = [1(1) + 2(2) + 3(3)] = [1 + 4 + 9] = [14]$

119. **(a)** Determinant Value

Let $\Delta = \begin{vmatrix} x + y & y + z & z + x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$.

Perform the operation $R_1 \to R_1 + R_2$:

$$\Delta = \begin{vmatrix} x + y + z & x + y + z & x + y + z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Taking $(x + y + z)$ common from R_1 :

$$\Delta = (x + y + z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Since R_I and R_3 are identical, the determinant is **0**.

120. **(c)** Function Differentiability

The function $f(x) = |x|$ forms a "V" shape on a graph. It is continuous everywhere because there are no breaks. However, at $x = 0$, there is a "sharp corner" where the slope changes instantly, making it non-differentiable at that specific point.

121. **(c)** Work: Use the chain rule. $y = \left(\sin(x^3)\right)^2$.

$$\frac{dy}{dx} = 2\sin(x^3) \cdot \frac{d}{dx}\left(\sin(x^3)\right) = 2\sin(x^3) \cdot \cos(x^3) \cdot 3x^2 = 6x^2\sin^3\cos x^3$$

122. **(b)** Work: Since $e^{5\log x} = e^{\log(x^5)} = x^5$, the integral becomes $\int x^5 dx = \frac{x^6}{6} + C$.

123. **(a)** Work: $\int_0^a 3x^2 dx = [x^3]_0^a = a^3$.

Given $a^3 = 8$, so $a = \sqrt[3]{8} = 2$.

124. **(d)** Work: Rewrite the equation in standard form $\frac{dy}{dx} + P(x)y = Q(x)$ by dividing by x :

$$\frac{dy}{dx} - \frac{1}{x}y = 2x$$

$P(x) = -\frac{1}{x}$, so I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

125. **(d)** Work: The highest order derivative is $\frac{d^2y}{dx^2}$, so the order is 2 . Because of the term $\sin\left(\frac{dy}{dx}\right)$, the equation is not a polynomial in its derivatives, so the degree is not defined.

126. **(b)** Work: The magnitude of the vector is $\sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = 5$.

The unit vector is $\frac{\text{Vector}}{\text{Magnitude}} = \frac{1}{5}(4i - 3k)$.

127. **(b)** Reason: The dot product is defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$. For the product to be ≥ 0, $\cos\theta$ must be non-negative. In the standard range of angles between vectors (0 to π), $\cos\theta \geq 0$ when θ is in the first quadrant, including 0 and $\pi/2$.

128. **(d)** Reason: The distance of a point (x, y, z) from the y-axis is given by the formula $\sqrt{x^2 + z^2}$. Substituting (p, q, r), we get $\sqrt{p^2 + r^2}$.

129. **(c)** Reason: Because the inequality is strict ($<$), the boundary line is not included (making it "open"). Testing the origin (0,0): $3(0) + 5(0) = 0$, and $0 < 7$ is true, so the solution plane contains the origin.

130. **(d)** Reason: Let's test the point $(4,2)$:
(1) $2(4) + 2 = 10 \leq 10$ (True)
(2) $4 + 2(2) = 8 \geq 8$ (True)
None of the other points satisfy both conditions.

131. **(d)** Reason: For direction cosines (l, m, n), the property $l^2 + m^2 + n^2 = 1$ must hold.
$\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 = 1 \Rightarrow \frac{3}{a^2} = 1 \Rightarrow a^2 = 3 \Rightarrow a = \pm\sqrt{3}$.

132. **(a)** Reason: Contradiction happens if A speaks truth and B lies, OR A lies and B speaks truth.
$P(\text{ Contradict }) = P(A)P(\text{ not } B) + P(\text{ not } A)P(B)$
$= \left(\frac{4}{5} \times \frac{1}{4}\right) + \left(\frac{1}{5} \times \frac{3}{4}\right) = \frac{4}{20} + \frac{3}{20} = \frac{7}{20}$.

133. **(d)** • Inverse Trigonometric Functions
• Assertion (A) is False. Trigonometric functions are periodic and many-to-one over their natural domains. To make them invertible, their domains must be restricted to specific intervals (called principal value branches). They do not have inverses over their entire natural domains.
• Reason (R) is True. The function $\tan^{-1} x$ (the inverse tangent function) exists and is defined for all real numbers $x \in R$.
• Conclusion: Assertion (A) is false, but Reason (R) is true.

134. **(a)** • Three-Dimensional Lines
• Assertion (A) is True. The direction of the lines is given by the vectors $\vec{b_1}$ and $\vec{b_2}$. Two lines are perpendicular if their direction vectors are perpendicular, which means their dot product must be zero ($\vec{b_1} \cdot \vec{b_2} = 0$).
• Reason (R) is True. The standard formula to find the angle θ between two vector lines is indeed $\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$.
• Explanation: If we substitute $\theta = 90°$ (perpendicular lines) into the formula from the Reason, $\cos(90°) = 0$, which directly leads to $\vec{b_1} \cdot \vec{b_2} = 0$ as stated in the Assertion.
• Conclusion: Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

135. **(A)** Find the domain of
$y = \sin^{-1}(x^2 - 4)$
For $\sin^{-1}(t)$ to be defined.
$-1 \leq t \leq 1$
So,
$-1 \leq x^2 - 4 \leq 1$
Adding 4:
$3 \leq x^2 \leq 5$
Therefore,
$\sqrt{3} \leq |x| \leq \sqrt{5}$
Hence the domain is
$[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

OR

(B) Evaluate
$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$
Since cosine is even,
$\cos\left(\frac{7\pi}{3}\right) = \cos\left(\frac{7\pi}{3}\right)$

$$\frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$$
Therefore,
$$\cos\left(\frac{7\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$$
Hence,
$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

136. If $(x^2 + y^2)^2 = xy$

find $\frac{dy}{dx}$.

Differentiate implicitly:
$$2(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y$$
$$4(x^2 + y^2)\left(x + y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y$$
Collecting $\frac{dy}{dx}$ terms:
$$4y(x^2 + y^2)\frac{dy}{dx} - x\frac{dy}{dx} = y - 4x(x^2 + y^2)$$
$$\frac{dy}{dx} = \frac{y - 4x(x^2 + y^2)}{4y(x^2 + y^2) - x}$$
$$\frac{dy}{dx} = \frac{y - 4x(x^2 + y^2)}{4y(x^2 + y^2) - x}$$

137. Find maximum and minimum values of

$f(x) = 5 + \sin 2x$

Since

$-1 \le \sin 2x \le 1$

adding 5 gives

$4 \le f(x) \le 6$

Therefore,

Maximum value $= 6$

Minimum value $= 4$

138. If projection of $i + j + k$ on $pi + j - 2k$ is $\frac{1}{3}$, find p.

Let $\vec{a} = i + j + k = (1,1,1)$

$\vec{b} = pi + j - 2k = (p, 1, -2)$

Scalar projection of $\vec{a}$ on $\vec{b}$.
$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{1}{3}$$
Now,

$\vec{a}\cdot\vec{b} = p + 1 - 2 = p - 1$

and

$|\vec{b}| = \sqrt{p^2 + 1 + 4} = \sqrt{p^2 + 5}$

Thus,
$$\frac{p - 1}{\sqrt{p^2 + 5}} = \frac{1}{3}$$
Squaring:

$9(p - 1)^2 = p^2 + 5$

$9p^2 - 18p + 9 = p^2 + 5$

$8p^2 - 18p + 4 = 0$

$4p^2 - 9p + 2 = 0$

$(4p - 1)(p - 2) = 0$

$p = \frac{1}{4}, 2$

Checking in the original equation:

For $p = \frac{1}{4}$.

$$\frac{p-1}{\sqrt{p^2+5}} < 0$$

not equal to $\frac{1}{3}$.

Hence rejected.

For $p = 2$,

$$\frac{2-1}{\sqrt{4+5}} = \frac{1}{3}$$

satisfies.

$p = 2$

139. **(A)** Vector equation of the required line

Given lines:

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

Direction ratios:

$\vec{d}_1 = (1,2,3)$

and

$$\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$$

Direction ratios:

$\vec{d}_2 = (-3,2,5)$

A line perpendicular to both will have direction vector

$\vec{d} = \vec{d}_1 \times \vec{d}_2$

$$\vec{d} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -3 & 2 & 5 \end{vmatrix}$$

$= (10-6)\hat{\imath} - (5+9)\hat{\jmath} + (2+6)\hat{k}$

$= 4\hat{\imath} - 14\hat{\jmath} + 8\hat{k}$

$= (2,-7,4)$

Since it passes through $(2,1,3)$,

$\vec{r} = (2\hat{\imath} + \hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} - 7\hat{\jmath} + 4\hat{k})$

OR

(B) Given:

$5x - 3 = 15y + 7 = 3 - 10z = t$

Then

$$x = \frac{t+3}{5}, y = \frac{t-7}{15}, z = \frac{3-t}{10}$$

Direction ratios:

$$\left(\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}\right)$$

Multiplying by 30 ,

$(6,2,-3)$

Magnitude:

$\sqrt{6^2 + 2^2 + (-3)^2} = \sqrt{49} = 7$

Direction cosines:

$$\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$$

For $t = 0$,

$$x = \frac{3}{5}, y = -\frac{7}{15}, z = \frac{3}{10}$$

Hence a point on the line is

$$\left(\frac{3}{5}, -\frac{7}{15}, \frac{3}{10}\right)$$

140. $I = \int \frac{x^2+x+1}{(x+1)^2(x+2)} dx$

Using partial fractions,

$$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{1}{x+2} - \frac{1}{(x+1)^2}$$

Therefore,

$$I = \int \left(\frac{1}{x+2} - \frac{1}{(x+1)^2}\right) dx$$

$$= \ln|x+2| + \frac{1}{x+1} + C$$

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)} dx = \ln|x+2| + \frac{1}{x+1} + C$$

141. **(A)** $I = \int_{x/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right) dx$

Using identities,

$1 - \sin 2x = (\sin x - \cos)^2$

$1 - \cos 2x = 2\sin^2 x$

Hence

$$\frac{1-\sin 2x}{1-\cos 2x} = \frac{(\sin x - \cos x)^2}{2\sin^2 x} = \frac{(\tan x - 1)^2}{2\tan^2 x}$$

Let $u = x - \frac{\pi}{4}$. Then

$$\frac{1-\sin 2x}{1-\cos 2x} = \tan^2 u$$

So $I = e^{\pi/2} \int_0^{\pi/4} e^{2u}\tan^2 u du$

After integration,

$$I = \frac{1}{2}\left(e^{\pi} - e^{\pi/2}\right)$$

OR

(B) $I = \int_{-2}^{2} \frac{x^2}{1+5^x} dx$

Let

$$I = \int_{-2}^{2} \frac{x^2}{1+5^{-x}} dx$$

Adding.

$$2I = \int_{-2}^{2} x^2 \left[\frac{1}{1+5^x} + \frac{1}{1+5^{-x}}\right] dx$$

Since

$$\frac{1}{1+5^x} + \frac{1}{1+5^{-x}} = 1$$

$$2I = \int_{-2}^{2} x^2 dx$$

$$= 2\int_0^2 x^2 dx = 2\left[\frac{x^3}{3}\right]_0^2 = \frac{16}{3}$$

$$I = \frac{8}{3}$$

142. **(A)** $I = \int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$

Let

$t = e^x, dt = e^x dx$

Then

$$I = \int \frac{dt}{\sqrt{5-4t-t^2}}$$

$$= \int \frac{dt}{\sqrt{9-(t+2)^2}}$$

Using
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$$
$$I = \sin^{-1}\left(\frac{e^x + 2}{3}\right) + C$$

OR

(B) $I = \int_0^{\pi/2} \sqrt{\sin x}\cos^5 x dx$

Write
$$\cos^5 x = \cos^4 x\cos x = (1 - \sin^2 x)^2\cos x$$
Let
$$t = \sin x, dt = \cos x dx$$
Limits:
$$x = 0 \Rightarrow t = 0, x = \frac{\pi}{2} \Rightarrow t = 1$$
Thus
$$I = \int_0^1 t^{1/2}(1 - t^2)^2 dt$$
$$= \int_0^1 (t^{1/2} - 2t^{5/2} + t^{9/2})dt$$
$$= \frac{2}{3} - \frac{4}{7} + \frac{2}{11}$$
$$= \frac{52}{231}$$
$$\int_0^{\pi/2} \sqrt{\sin x}\cos^5 x dx = \frac{52}{231}$$

143. **(A)** Differential Equation

Given: $\frac{dy}{dx} = \frac{x+y}{x}$ with $y(1) = 0$.

(1) Simplify: $\frac{dy}{dx} = 1 + \frac{y}{x}$.

(2) Substitute: Let $y = vx$, so $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

(3) Solve:
$$v + x\frac{dv}{dx} = 1 + v \Longrightarrow x\frac{dv}{dx} = 1 \Longrightarrow dv = \frac{dx}{x}$$
Integrating both sides: $v = \ln|x| + C$.

(4) Back-substitute: $\frac{y}{x} = \ln|x| + C \Longrightarrow y = x\ln|x| + Cx$.

(5) Initial Condition: $y(1) = 0 \Longrightarrow 0 = 1\ln(1) + C(1) \Longrightarrow C = 0$.

Particular Solution: $y = x\ln|x|$.

OR

(B) General Solution

Given: $e^x\tan y dx + (1 - e^x)\sec^2 y dy = 0$.

(1) Separate Variables: $(1 - e^x)\sec^2 y dy = -e^x\tan y dx$.
$$\frac{\sec^2 y}{\tan y}dy = \frac{-e^x}{1 - e^x}dx = \frac{e^x}{e^x - 1}dx$$
(2) Integrate: $\int \frac{\sec^2 y}{\tan y}dy = \int \frac{e^x}{e^x - 1}dx$.
$$\ln|\tan y| = \ln|e^x - 1| + \ln C$$
General Solution: $\tan y = C(e^x - 1)$.

144. Linear Programming Problem

Minimise: $z = -3x + 4y$

Constraints: $x + 2y \leq 8, 3x + 2y \leq 12, x, y \geq 0$.

(1) Feasible Region: The region is bounded by the axes and the lines.

- Intercepts for $x + 2y = 8$: (8,0) and (0,4).
- Intercepts for $3x + 2y = 12$: (4,0) and (0,6).
- Intersection Point: Solving the two equations gives (2, 3).

(2) Corner Points and Value of z :

- $(0,0): z = 0$
- $(4,0): z = -3(4) + 4(0) = -12$
- $(2,3): z = -3(2) + 4(3) = 6$
- $(0,4): z = -3(0) + 4(4) = 16$

Minimum Value: -12 at point (4,0).

145. Probability Distribution

Total bulbs = 30. Defective (D) = 6, Non-defective (G) = 24.

With replacement: $P(D) = \frac{6}{30} = \frac{1}{5}, P(G) = \frac{24}{30} = \frac{4}{5}$.

Let X be the number of defective bulbs in 2 draws.

- $P(X = 0) = P(G)P(G) = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25} = 0.64$
- $P(X = 1) = P(DG) + P(GD) = 2 \times \left(\frac{1}{5} \times \frac{4}{5}\right) = \frac{8}{25} = 0.32$
- $P(X = 2) = P(DD) = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25} = 0.04$

Probability Distribution Table:

X	0	1	2
P(X)	0.64	0.32	0.04

Mean (μ): $E(X) = \sum X \cdot P(X) = 0(0.64) + 1(0.32) + 2(0.04) = 0.4$.

146. Find A^{-1} and solve the system

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

Step 1: Determinant

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$$

$= 1(8 - 6) - (-1)(0 + 9) + 2(0 - 6)$

$= 2 + 9 - 12 = -1$

Hence A^{-1} exists.

Step 2: Adjoint

Cofactor matrix:

$$C = \begin{bmatrix} 2 & 9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$$

Adjoint

$$\text{adj}(A) = \begin{bmatrix} 2 & 0 & -1 \\ 9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

Therefore

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ 9 & -2 & 3 \\ 6 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Step 2: solve

Given

$$AX = B, B = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$X = A^{-1}B$

$$= \begin{bmatrix} -2 & 0 & 1 \\ -9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -16 \\ 1 \end{bmatrix}$$

Hence

$x = 1, y = -16, z = 1$

147. Area bounded by $y^2 = 4ax$ and its latus rectum

For $y^2 = 4ax$

Latus rectum is

$x = a$

It intersects the parabola at

$y = \pm 2a$

Required area:

$$A = \int_{-2a}^{2a} \left(a - \frac{y^2}{4a}\right) dy$$

$$= 2\int_{0}^{2a} \left(a - \frac{y^2}{4a}\right) dy$$

$$= 2\left[ay - \frac{y^3}{12a}\right]_0^{2a}$$

$$= 2\left(2a^2 - \frac{8a^3}{12a}\right)$$

$$= 2\left(2a^2 - \frac{2}{3}a^2\right)$$

$$= \frac{8}{3}a^2$$

$$\text{Area } = \frac{8a^2}{3}$$

148. **(A)** Given relation R on $N \times N$:

$(a, b)R(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$

To show R is an equivalence relation:

Reflexive:

$ab(a + b) = ab(a + b)$

Hence $(a, b)R(a, b)$.

Symmetric:

If

$ad(b + c) = bc(a + d)$,

then

$bc(a + d) = ad(b + c)$,

so $(c, d)R(a, b)$.

Transitive:

Given

$(a, b)R(c, d), (c, d)R(e, f)$

we get

$$\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$$

and

$$\frac{1}{c} + \frac{1}{f} = \frac{1}{d} + \frac{1}{e}.$$

Adding and simplifying.

$$\frac{1}{a} + \frac{1}{f} = \frac{1}{b} + \frac{1}{e}$$

which gives

$af(b + e) = be(a + f)$.

Hence $(a, b)R(e, f)$.

Therefore R is reflexive, symmetric and transitive.

R is an equivalence relation.

OR

(B) $f(x) = \frac{4x}{3x+4}, x \neq -\frac{4}{3}$

One-One:

Let

$f(x_1) = f(x_2)$

$$\frac{4x_1}{3x_1 + 4} = \frac{4x_2}{3x_2 + 4}$$

$x_1(3x_2 + 4) = x_2(3x_1 + 4)$

$x_1 = x_2$

Hence f is one-one.

Onto:

Let $y = \frac{4x}{3x+4}$

Then

$$x = \frac{4y}{4 - 3y}$$

This exists for all $y \in R$ except $y = \frac{4}{3}$.

Thus the range is

$$R = \left\{\frac{4}{3}\right\}$$

Since codomain is R, f is not onto.

Hence f is one-one but not onto.

149. **(A)** Lines:

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda$$

$x = 1 + 3\lambda, y = -1 + 2\lambda, z = 1 + 5\lambda$

And

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu$$

$x = -2 + 4\mu, y = 1 + 3\mu, z = -1 - 2\mu$

For intersection:

$1 + 3\lambda = -2 + 4\mu$

$-1 + 2\lambda = 1 + 3\mu$

Solving gives

$\lambda = -14, \mu = -10$

Checking third equation:

$1 + 5(-14) = -69$

$-1 - 2(-10) = 19$

Not equal.

Hence no common point exists.

The lines do not intersect (they are skew lines).

OR

(B) Angle between the lines

$2x = 3y = -z$

Direction ratios:

$$\left(\frac{1}{2}, \frac{1}{3}, -1\right)$$

or

$(3, 2, -6)$

$6x = -y = -4z$

Direction ratios:

$$\left(\frac{1}{6}, -1, -\frac{1}{4}\right)$$

or

$(2, -12, -3)$

Now

$$\cos\theta = \frac{|3(2) + 2(-12) + (-6)(-3)|}{\sqrt{9+4+36}\sqrt{4+144+9}}$$

$$= \frac{|6 - 24 + 18|}{7\sqrt{157}}$$

$$= 0$$

Hence $\theta = 90^\circ$

$$\theta = \frac{\pi}{2}(= 90^\circ)$$

150. To find the derivatives and check for differentiability at $x = 1$, we first rewrite the function $f(x)$ clearly around that point.

Given:

$$f(x) = \begin{cases} |x-3| & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$$

Since we are looking at $x = 1$, for the part $x \geq 1, |x-3|$ behaves like $-(x-3) = 3 - x$ (because $x - 3$ is negative when x is near 1).

So, for $x \in [1,3), f(x) = 3 - x$.

(i) Right Hand Derivative (R.H.D.) at $x = 1$

Using the formula $Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$:

- $f(1) = |1-3| = 2$
- $f(1+h) = 3 - (1+h) = 2 - h$(for small $h > 0$)

$$Rf'(1) = \lim_{h\to 0} \frac{(2-h)-2}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$$

(ii) Left Hand Derivative (L.H.D.) at $x = 1$

Using the formula $Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$:

- $f(1) = 2$
- $f(1-h) = \frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4}$

$$f(1-h) = \frac{1 - 2h + h^2}{4} - \frac{3-3h}{2} + \frac{13}{4} = \frac{1}{4} - \frac{h}{2} + \frac{h^2}{4} - \frac{3}{2} + \frac{3h}{2} + \frac{13}{4}$$

$$f(1-h) = \left(\frac{1}{4} - \frac{6}{4} + \frac{13}{4}\right) + h = \frac{8}{4} + h = 2 + h$$

$$Lf'(1) = \lim_{h\to 0} \frac{(2+h)-2}{-h} = \lim_{h\to 0} \frac{h}{-h} = -1$$

(iii) (A) Is $f(x)$ differentiable at $x = 1$?

A function is differentiable if L.H.D. = R.H.D.

Since $Lf'(1) = -1$ and $Rf'(1) = -1$, the function is differentiable at $x = 1$.

OR

(B) Find $f'(2)$ and $f'(-1)$

- For $x = 2$: This falls in the range $x \geq 1$. Near $x = 2, f(x) = 3 - x$.

$$f'(x) = \frac{d}{dx}(3-x) = -1 \Rightarrow f'(2) = -1$$

- For $x = -1$: This falls in the range $x < 1$.

$$f(x) = \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}$$

$$f'(x) = \frac{2x}{4} - \frac{3}{2} = \frac{x-3}{2}$$

$$f'(-1) = \frac{-1-3}{2} = \frac{-4}{2} = -2$$

151. Given:

$P(E_1) = 0.65$

where E_1 : Many workers are not present.

$P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

where E_2 : All workers are present.
Also,
$P(E \mid E_1) = 0.35$
$P(E \mid E_2) = 0.80$
where E : Construction work completed on time.
(i) Probability that all workers are present
$P(E_2) = 0.35$
$P(E_2) = 0.35$
(ii) Probability that construction is completed on time
Using the law of total probability:
$P(E) = P(E_1)P(E \mid E_1) + P(E_2)P(E \mid E_2)$
$= (0.65)(0.35) + (0.35)(0.80)$
$= 0.2275 + 0.28$
$= 0.5075$
$P(E) = 0.5075$
(iii) (A) Probability that many workers are not present given that work is completed on time
Using Bayes" theorem:
$$P(E_1 \mid E) = \frac{P(E_1)P(E \mid E_1)}{P(E)}$$
$$= \frac{(0.65)(0.35)}{0.5075}$$
$$= \frac{0.2275}{0.5075} = \frac{91}{203}$$
$$P(E_1 \mid E) = \frac{91}{203} \approx 0.448$$
OR
(B) Probability that all workers were present given that work was completed on time
$$P(E_2 \mid E) = \frac{P(E_2)P(E \mid E_2)}{P(E)}$$
$$= \frac{(0.35)(0.80)}{0.5075}$$
$$= \frac{0.28}{0.5075} = \frac{112}{203}$$
$$P(E_2 \mid E) = \frac{112}{203} \approx 0.552$$
(Notice that $\frac{91}{203} + \frac{112}{203} = 1$).

152. • x : Length of the two sides perpendicular to the brick wall (in metres).
• y : Length of the side parallel to the brick wall (in metres).
(i) Relation and Area Function
Total Length of Fencing Wire:
The wire fencing is used for only three sides (two sides of length x and one side of length y). Given that the total length of wire is 200 metres:
$2x + y = 200 \Rightarrow y = 200 - 2x$
Area of the Garden $A(x)$:
The area A is the product of the two adjacent sides x and y.
$A = x \cdot y$
Substituting the expression for y :
$A(x) = x(200 - 2x)$
$A(x) = 200x - 2x^2$
(ii) Maximum Value of $A(x)$
To find the maximum area, we find the derivative of $A(x)$ with respect to x and set it to zero:
$A'(x) = 200 - 4x$
$200 - 4x = 0 \Rightarrow x = 50$
To confirm it's a maximum, we check the second derivative: $A''(x) = -4$. Since $A''(x) < 0$, the function reaches a maximum at $x = 50$.

Maximum Area calculation:
Substitute $x = 50$ back into the area function:
$A(50) = 200(50) - 2(50)^2$
$A(50) = 10{,}000 - 5{,}000$
Maximum $A(x) = 5{,}000\ m^2$

153. **(b)** Reflexive relations on $A = \{3,5\}$:
The number of reflexive relations on a set with n elements is 2^{n^2-n}. Here $n = 2$, so $2^{4-2} = 2^2 = 4$.

154. **(a)** Value of $\sin[\pi/3 + \sin^{-1}(1/2)]$: $\sin^{-1}(1/2) = \pi/6$. Thus, $\sin(\pi/3 + \pi/6) = \sin(\pi/2) = 1$.

155. **(c)** Inverse of square matrix A where $A^2 - A + I = O$:
Multiply the equation by A^{-1}: $A^{-1}(A^2 - A + I) = 0 \Longrightarrow A - I + A^{-1} = 0$. Solving for A^{-1} gives $I - A$.

156. **(c)** Value of x for $A = B^2$:
$B^2 = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$.
Comparing with $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, we get $x^2 = 1$ (so $x = \pm 1$) and $x + 1 = 2$ (so $x = 1$). The common value is $x = 1$.

157. **(d)** Value of α from determinant:
Expanding the determinant: $\alpha(2-4) - 3(1-1) + 4(4-2) = 0 \Longrightarrow -2\alpha + 0 + 8 = 0 \Longrightarrow 2\alpha = 8 \Longrightarrow \alpha = 4$.

158. **(c)** Derivative of x^{2x} :
Let $y = x^{2x}$. Using logarithmic differentiation, $\ln y = 2x\ln x$.
Differentiating both sides: $\frac{1}{y}\frac{dy}{dx} = 2(1 + \ln x)$, so $\frac{dy}{dx} = 2x^{2x}(1 + \ln x)$.

159. **(b)**The function $f(x) = [x]$ is continuous at:
• Reason: The greatest integer function is discontinuous at all integer points because the limit from the left and right differ. It is continuous at all non-integer points like 1.5.

160. **(d)** If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is:
• Reason:
$\cdot \dfrac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$
$\dfrac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t = -16(A\cos 4t + B\sin 4t) = -16x$

161. **(b)** The interval in which $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is:
• Reason: $f'(x) = 6x^2 + 18x + 12$. Setting $f'(x) < 0$ gives $6(x^2 + 3x + 2) < 0$, which factors to $(x+2)(x+1) < 0$. This inequality holds true between the roots -2 and -1 .

162. **(b)** $\int \frac{\sec x}{\sec x - \tan x}dx$ equals:
• Reason: Multiply numerator and denominator by $(\sec x + \tan x)$. Since $\sec^2 x - \tan^2 x = 1$, the integral simplifies to $\int (\sec^2 x + \sec x\tan x)dx$, which is $\tan x + \sec x + C$.

163. **(d)** $\int_{-1}^{1} \frac{|x-2|}{x-2}dx$ is equal to:
• Reason: In the interval $[-1,1]$, $(x-2)$ is always negative. Therefore, $|x-2| = -(x-2)$. The integrand becomes $\frac{-(x-2)}{x-2} = -1$. The integral of -1 from -1 to 1 is $1 - x|_{-1}^{1} = -1(1) = -2$.

164. **(b)** The sum of the order and the degree of $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right) = 0$ is:
• Reason: Differentiating gives $3\left(\frac{dy}{dx}\right)^2\left(\frac{d^2y}{dx^2}\right) = 0$. The order is 2 (highest derivative is y'') and the degree is 1 (power of the highest derivative). $2 + 1 = 3$

165. **(b)** Two vectors are collinear if their corresponding components are proportional:
$\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$

166. **(c)** Magnitude of vector $6\hat{i} - 2\hat{j} + 3\hat{k}$:
$|\vec{a}| = \sqrt{6^2 + (-2)^2 + 3^2}$
$= \sqrt{36 + 4 + 9} = \sqrt{49} = 7$

167. **(a)** Direction cosines are:
$l = \cos 90° = 0$

$m = \cos 135^\circ = -\frac{1}{\sqrt{2}}$

$n = \cos 45^\circ = \frac{1}{\sqrt{2}}$

168. **(d)** For lines:

$2x = 3y = -z$

Direction ratios:

$\left(\frac{1}{2}, \frac{1}{3}, -1\right) \propto (3,2,-6)$

Second line:

$6x = -y = -4z$

Direction ratios:

$\left(\frac{1}{6}, -1, -\frac{1}{4}\right) \propto (2,-12,-3)$

Angle formula:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{3(2) + 2(-12) + (-6)(-3)}{\sqrt{9+4+36}\sqrt{4+144+9}}$$

$$= \frac{6 - 24 + 18}{7\sqrt{157}} = 0$$

$\theta = 90^\circ$

169. **(c)** $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

$$= \frac{7/10}{4/5} = \frac{7}{10} \times \frac{5}{4} = \frac{7}{8}$$

170. **(c)** Probability of at least one head:

$P(\text{ at least one head }) = 1 - P(\text{ no head })$

$$= 1 - \left(\frac{1}{2}\right)^3$$

$$= 1 - \frac{1}{32} = \frac{31}{32}$$

171. **(a)** Two coins are tossed simultaneously.

Sample space i.e., possible outcomes are $\{HT, TH, HH, TT\}$

$E =$ event of getting two heads

F = event of getting at least one head

$P(E) = \frac{1}{4}, P(F) = \frac{3}{4}, P(E \cap F) = \frac{1}{4}$

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{\left(\frac{1}{4}\right)}{\left(\frac{3}{4}\right)}$$

$$= \frac{1}{3}$$

So, Both (A) and (R) are true and (R) is the correct explanation of (A).

172. **(a)** $I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$ ....(i)

Using property of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$I = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}} dx$ ....(ii)

Adding equations (i) and (ii)

$2I = \int_2^8 \frac{\sqrt{10-x}+\sqrt{x}}{\sqrt{10-x}+\sqrt{x}} dx$

$= \int_2^8 dx$

$= [x]_2^8$

$= 8 - 2$

$= 6$

$\Rightarrow I = 3$

R is also true as the property P_4 is

$\int_a^b f(x)dx = \int_a^b f(a+b-x)$

So, Both (A) and (R) are true and (R) is the correct explanation of (A).

173. $f(x) = \tan^{-1}x$

Domain $(-\infty, \infty)$

Range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

174. **(A)** $f(x) = \begin{cases} x^2, \text{ if } x \geq 1 \\ x, \text{ if } x < 1 \end{cases}$

then $f'(x) = \begin{cases} 2x, \text{ if } x \geq 1 \\ 1, \text{ if } x < 1 \end{cases}$

LHD of $f(x) = 1$

RHD of $f(x) = 2x = 2$

Since LHD $\neq$ RHD

It is not differentiable at $x = 1$.

OR

(B) $f(x) = \begin{cases} \frac{\sin^2\lambda x}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

For continuity at $x = 0$

$\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = f(x)$

$\lim_{x\to0^-} \frac{\sin^2\lambda x}{\lambda^2x^2} \times \lambda^2 = \lim_{x\to0^-} 1 \times \lambda^2 = \lambda^2$

$\lim_{x\to0^+} \frac{\sin^2\lambda x}{\lambda^2x^2} \times \lambda^2 = \lim_{x\to0^+} \lambda^2 = \lambda^2$

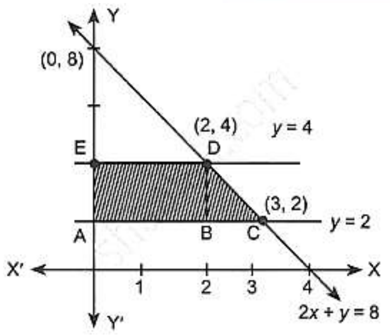
$f(0) = 1$

Since $f(x)$ is continuous.

$\lambda^2 = 1$

$\Rightarrow \lambda = \pm1$

175. $2x + y = 8, y = 2, y = 4$



Required Area $=$ Area of ABDE $+$ Area of BCD

$= \int_0^2 (4-2)dx + \int_2^3 \{(8-2x)-2\}dx$

$= [2x]_0^2 + [6x - x^2]_2^3$

$= 4 + [18 - 9 - 12 + 4]$

$= 4 + 1$

$= 5$ sq. units.

176. **(A)** Let the angle between $\vec{a}$ and $\vec{b}$ be θ It is given that $\vec{a} \times \vec{b}$ is a unit vector.

$\therefore |\vec{a} \times \vec{b}| = 1$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 1$

$\Rightarrow 3 \times \frac{2}{3} \times \sin\theta = 1$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Thus, the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$

OR

(B) $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$

$\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$

Area of parallelogram $= |\vec{a} \times \vec{b}|$

Now $(\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$

$(\vec{a} \times \vec{b}) = \hat{\imath}(-1 + 21) - \hat{\jmath}(1 - 6) + \hat{k}(-7 + 2)$

$(\vec{a} \times \vec{b}) = 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + (-5)^2}$

$= \sqrt{450}$

$= 15\sqrt{2}$ sq. units.

177. The equation of the line $5x - 25 = 14 - 7y = 35z$ can be re-written as

$$\frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}}$$

$$\Rightarrow \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$$

Since the required line is parallel to the given line, so the direction ratios of the required line are proportional to $7, -5, 1$.

The vector equation of the required line passing through the point $(1, 2, -1)$ and having direction ratios proportional to $7, -5, 1$ is

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$

178. $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$

$A^3 - 23A - 40I = 0$

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 19+12+32 & 38-8+16 & 57+4+8 \\ 1+36+32 & 2-24+16 & 3+12+8 \\ 14+18+60 & 28-12+30 & 42+6+15 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$23A = 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix}$$

Now A^3 - 23A-40I

$$\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

179. **(A)**Let $u = \sec^{-1}\frac{1}{\sqrt{1-x^2}}$

$x = \sin\theta$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1}{\cos\theta}$$

$= \sec\theta$

So $u = \sec^{-1}\sec\theta$

$= \theta$

$= \sin^{-1}x$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ ….(i)

Suppose that

$x = \sin\alpha$

$$v = \sin^{-1}\left[2\sin\alpha\sqrt{1-\sin^2\alpha}\right]$$

$= \sin^{-1}[2\sin\alpha \cdot \cos\alpha]$

$= \sin^{-1}[\sin 2\alpha]$

$= 2\alpha$

$= 2\sin^{-1}x$

$\frac{dv}{dx} = \frac{2}{\sqrt{1-x^2}}$ ….(ii)

From (i) and (ii)

$$\therefore \frac{du}{dv} = \frac{1}{2}$$

OR

(B)

$y = \tan x + \sec x$

$$\frac{dy}{dx} = \frac{d}{dx}(\tan x) + \frac{d}{dx}(\sec x)$$

$= \sec^2 x + \sec x\tan x$

$= \sec x(\sec x + \tan x)$

$$= \frac{1}{\cos x}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$$

$$= \frac{1+\sin x}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{1+\sin x}{1-\sin^2 x}$$

$$= \frac{1}{1-\sin x}$$

$$\frac{d^2y}{dx^2} = \frac{(1-\sin x)0 - 1(-\cos x)}{(1-\sin x)^2}$$

$$= \frac{\cos x}{(1 - \sin x)^2}$$

180. **(A)** Let $I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$ ....(i)

Applying property,

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx, \text{ we get}$$

$$I = \int_0^{2\pi} \frac{dx}{1 + e^{\sin(2\pi - x)}}$$

$$= \int_0^{2\pi} \frac{dx}{1 + e^{-\sin x}}$$

$$= \int_0^{2\pi} \frac{dx}{1 + \frac{1}{e^{\sin x}}}$$

$$= \int_0^{2\pi} \frac{e^{\sin x} dx}{e^{\sin x} + 1} \quad ....(ii)$$

On adding equations (i) and (ii), we get

$$2I = \int_0^{2\pi} \frac{dx}{1 + e^{\sin x}} + \int_0^{2\pi} \frac{e^{\sin x} dx}{1 + e^{\sin x}}$$

$$= \int_0^{2\pi} \left(\frac{1 + e^{\sin x}}{1 + e^{\sin x}}\right) dx$$

$$= \int_0^{2\pi} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{2\pi}$$

$$\Rightarrow 2I = [2\pi]$$

$$\Rightarrow I = \pi$$

OR

(B) $\int \frac{x^4}{(x-1)(x^2+1)} dx = \int \frac{(x^4 - 1 + 1)}{(x-1)(x^2+1)} dx$

$$= \int \frac{(x^4 - 1)}{(x-1)(x^2+1)} dx + \int \frac{1}{(x-1)(x^2+1)} dx$$

$$= \int (x+1)dx + \int \frac{1}{(x-1)(x^2+1)} dx$$

$$= \frac{x^2}{2} + x + \int \frac{dx}{(x-1)(x^2+1)}$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\int \left(\frac{1}{x-1} - \frac{x+1}{x^2+1}\right) dx \quad ....\{\because \text{ Partial factorisation }\}$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\left[\int \frac{1}{x-1} dx - \int \frac{xdx}{x^2+1} - \int \frac{dx}{1+x^2}\right]$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log(x-1) - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$$

181. $y^2 \le 2x$

$y \ge x - 4$

$y^2 = 2x$ ...(i)

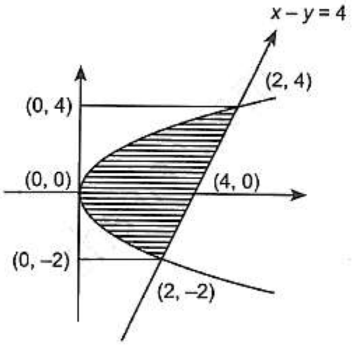
$y - x + 4 = 0$ or $x - y = 4$ ....(ii)

Put the value of x from (ii) in (i), we have

$y^2 = 2(y + 4)$

$y^2 - 2y - 8 = 0$

$\Rightarrow y = 4, -2$



When $y = 4, x = 4 + 4 = 8$
When $y = -2, x = 4 - 2 = 2$
Required area $= \int_{-2}^{4} (y + 4)dy - \int_{-2}^{4} \frac{y^2}{2} dy$
$= \left[\frac{(y + 4)^2}{2}\right]_{-2}^{4} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-2}^{4}$
$= \frac{1}{2}[64 - 4] - \frac{1}{6}[64 + 8]$
$= 30 - 12$
$= 18$ sq. units.

182. **(A)** Let $P = (0,2,3)$
Let M be the foot of the perpendicular drawn from P to the line
$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$ ....(Say)
The coordinates of any point on the line are given by
$x = 5\lambda - 3, y = 2\lambda + 1, z = 3\lambda - 4$
Let $M = (5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$ ....(i)
The direction ratios of PM are
$5\lambda - 3 - 0, 2\lambda + 1 - 2, 3\lambda - 4 - 3$
i.e. $5\lambda - 3\lambda, 2\lambda - 1, 3\lambda - 7$
Since, PM is perpendicular to the line whose direction ratios atr 5,2,3,
$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$
$\therefore 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$
$\therefore 38\lambda - 38 = 0$
$\therefore \lambda = 1$
Substituting $\lambda = 1$ in (1), we get
$M = (5 - 3, 2 + 1, 3 - 4) = (2,3,-1)$
Hence, the coordinates of the foot of perpendicular are $(2,3,-1)$.

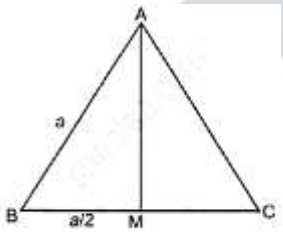
OR

(B) Since $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, we have
$\vec{a} + \vec{b} + \vec{c} = \vec{0} = 0$
or $\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$
Therefore $\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = -|\vec{a}|^2 = -9$ ...(i)
Again, $\vec{b} \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$
or $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} = -|\vec{b}|^2 = -16$ ....(ii)
Similarly $\vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} = -4.$ ....(iii)
Adding (1), (2) and (3), we have
$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{c}) = -29$
or $2\mu = -29$, i.e., $\mu = \frac{-29}{2}$

183. Lines are parallel
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{2}{4}=\frac{3}{6}=\frac{6}{12}$$
$$\frac{1}{2}=\frac{1}{2}=\frac{1}{2}$$
$$\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$$
$$|\vec{b}|=\sqrt{4+9+36}$$
$$=\sqrt{49}$$
$$=7$$
$$\text{S.D.} =\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$
$$\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}$$
$$\vec{a}_2=3\hat{i}+3\hat{j}-5\hat{k}$$
$$\vec{a}_2-\vec{a}_1=2\hat{i}+\hat{j}-\hat{k}$$
$$\vec{b}\times(\vec{a}_2-\vec{a}_1)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ 2 & 1 & -1\end{vmatrix}$$
$$=-9\hat{i}+14\hat{j}-4\hat{k}$$
$$|\vec{b}\times(\vec{a}_2-\vec{a}_1)|=\sqrt{81+196+16}$$
$$=\sqrt{293}$$
$$\text{S.D.} =\frac{\sqrt{293}}{7}\text{ units}$$

184. **(A)** Let ' a ' be the side of an equilateral triangle.



$$\text{Medain AM } =\frac{\sqrt{3}}{2}a$$
$$\text{Given } \frac{d(AM)}{dt}=2\sqrt{3}\text{ cm/s}$$
$$\frac{d(AM)}{dt}=\frac{d\left(\frac{\sqrt{3}}{2}a\right)}{da}\times\frac{da}{dt}$$
$$2\sqrt{3}=\frac{\sqrt{3}}{2}\cdot\frac{da}{dt}$$
$$\Rightarrow\frac{da}{dt}=4\text{ cm/s}$$

OR

(B) Let two numbers be x and y then

$x+y=5$ …(i)

Let $S=x^3+y^3$ …(ii)

$=x^3+(5-x)^3$ ….[From (i)]

$$\frac{dS}{dx}=3x^2+3(5-x)^2(-1)$$
$$\frac{dS}{dx}=3x^2-3(25+x^2-10x)$$
$$=3x^2-75-3x^2+30x$$
$$=30x-75$$

For maximum or minimum

$\frac{dS}{dx} = 0$

$\Rightarrow 30x - 75 = 0$

$\Rightarrow x = \frac{75}{35} = \frac{5}{2}$

When $x = \frac{5}{2}, y = 5 - \frac{5}{2} = \frac{5}{2} \ldots$ [From (i)] $\frac{d^2S}{dx^2} = 30$ which is +ve .

So the sum is least when $x = \frac{5}{2}$ and $y = \frac{5}{2}$

$S = x^2 + y^2$

$= \frac{25}{4} + \frac{25}{4}$

$= \frac{50}{4}$

$= \frac{25}{2}$

185. $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

$= \int_0^{\frac{\pi}{2}} 2\sin x \cos x \tan^{-1}(\sin x) dx$

Let $\sin x = t$

$\cos x dx = dt$

When $x = 0, t = 0$ and $x = \frac{\pi}{2}$

$\Rightarrow t = 1$

$2\int_0^1 t\tan^{-1} t dt$

$= 2\left[\tan^{-1} t \cdot \frac{t^2}{2}\right]_0^1 - 2\int_0^1 \frac{1}{1+t^2} \cdot \frac{t^2}{2} dt$

$= \left(\frac{\pi}{4} \cdot 1 - 0\right) - \int_0^1 \frac{t^2+1-1}{1+t^2} dt$

$= \frac{\pi}{4} - \int_0^1 \left(1 - \frac{1}{1+t^2}\right) dt$

$= \frac{\pi}{4} - [t - \tan^{-1} t]_0^1$

$= \frac{\pi}{4} - \left[1 - \frac{\pi}{4} - 0\right]$

$= \frac{\pi}{4} - 1 + \frac{\pi}{4}$

$= \frac{\pi}{2} - 1$

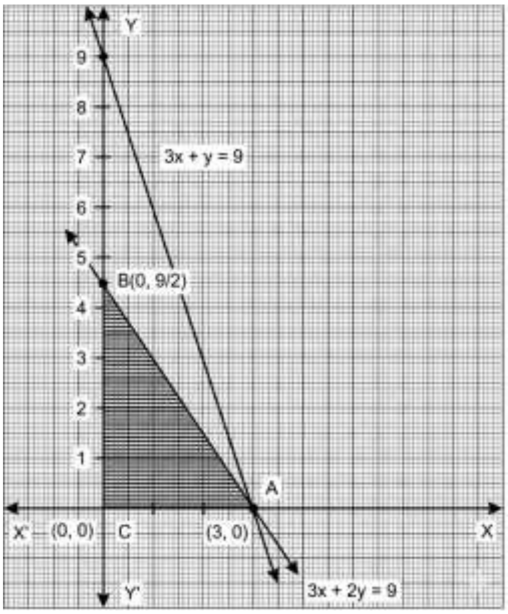
186. $Z_{max} = 70x + 40y$

$3x + 2y = 9$

x	3	0
y	0	9/2

$3x + y = 9$

x	3	0
y	0	9



$z_A = 70 \times 3 + 40 \times 0 = 210$

$z_B = 0 + 40 \times \frac{9}{2} = 180$

$z_C = 0$

So, $Z_{max} = 210$ at $A(3,0)$.

187. **(A)** Let B_1 be the event student knows the answer

B_2 be the event student guesses the answer

$E =$ Event answer is correct

$P(B_1) = \frac{3}{5}$

$P(B_2) = \frac{2}{5}$

$P\left(\frac{E}{B_1}\right) = 1$

$P\left(\frac{E}{B_2}\right) = \frac{1}{3}$

$$P\left(\frac{B_1}{E}\right) = \frac{P(B_1)P\left(\frac{E}{B_1}\right)}{P(B_1)P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)}$$

$$= \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{3}}$$

$$= \frac{\frac{3}{5}}{\frac{9+2}{15}}$$

$$= \frac{9}{11}.$$

OR

(B)

X:	P(X):
8	$\frac{1}{10}$
8	$\frac{1}{10}$
4	$\frac{1}{10}$

4	$\frac{1}{10}$
4	$\frac{1}{10}$
4	$\frac{1}{10}$
4	$\frac{1}{10}$
2	$\frac{1}{10}$
2	$\frac{1}{10}$
2	$\frac{1}{10}$

Probability of having one ticket out of 10 ticket $= \frac{1}{10}$

$\mu = \sum x_i p$

$= 8 \times \frac{1}{10} + 8 \times \frac{1}{10} + 4 \times \frac{1}{10} + 4 \times \frac{1}{10} + 4 \times \frac{1}{10} + 4 \times \frac{1}{10} + 4 \times \frac{1}{10} + 2 \times \frac{1}{10} + 2 \times \frac{1}{10} + 2 \times \frac{1}{10}$

$= \frac{16 + 20 + 6}{10}$

$= \frac{42}{10}$

$= 4.2$

188. **(I)** Number of possible relations from $B \to G$

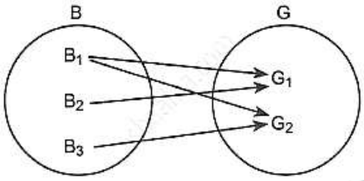
$= 2^{n(B) \times n(G)}$

$= 2^{3 \times 2}$

$= 2^6$

$= 64$

(II)



Every element of set B has two options to map in set G i.e., B_1 can go to G_1 and G_2.

So, 2 ways (i.e., two functions).

$\therefore$ Total function $= 2 \times 2 \times 2 = 8$

(III) $R: B \to B$

$R = \{(x, y): \text{x and y are students of the same sex}\}$

$(b_1, b_1) \in \mathbb{R}$ ... (Reflexive)

$(b_1, b_2) \in R \Rightarrow (b_2, b_1) \in R$ ... (Symmetric)

If $(b_1, b_2) \in R \wedge (b_2, b_3) \in R$

$\Rightarrow (b_1, b_3) \in R$ ...(Transitive)

$\Rightarrow$ It is an equivalence relation.

OR

(III) Given, $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$

$f = \{(b_1, s_1), (b_2, s_2), (b_3, s_1)\}$

Since b_1 and b_2 both are related to same element g_1.

So f is not bijective (one-one).

189. $5x + 3y + z = 160$

$2x + y + 3z = 190$

$x + 2y + 4z = 250$

(I) $AX = B$ form

$$\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}160\\190\\250\end{bmatrix}$$

(II)

$$|A|=\begin{vmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{vmatrix}$$
$$=5(4-6)-3(8-3)+1(4-1)$$
$$=5(-2)-3(5)+3$$
$$=-10-15+3$$
$$=-22$$

(III) $A^{-1}=\frac{1}{|A|}(\text{adj}A)$

$C_{11}=-2, C_{12}=-5, C_{13}=3$
$C_{21}=-10, C_{22}=19, C_{23}=-7$
$C_{31}=8, C_{32}=-13, C_{33}=-1$

$$\text{Adj } A=\begin{bmatrix}-2 & -10 & 8\\-5 & 19 & -13\\3 & -7 & -1\end{bmatrix}$$
$$A^{-1}=\frac{1}{|A|}(\text{adj}A)$$
$$=\frac{1}{-22}\begin{bmatrix}-2 & -10 & 8\\-5 & 19 & -13\\3 & -7 & -1\end{bmatrix}$$
$$A^{-1}=\frac{1}{22}\begin{bmatrix}2 & 10 & -8\\5 & -19 & 13\\-3 & 7 & 1\end{bmatrix}$$

OR

(III) $P=A^2-5A$

$$=\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}-5\begin{bmatrix}5 & 3 & 1\\2 & 1 & 3\\1 & 2 & 4\end{bmatrix}$$
$$=\begin{bmatrix}32 & 20 & 18\\15 & 13 & 17\\13 & 13 & 23\end{bmatrix}-\begin{bmatrix}25 & 15 & 5\\10 & 5 & 15\\5 & 10 & 20\end{bmatrix}$$
$$=\begin{bmatrix}7 & 5 & 13\\5 & 8 & 2\\8 & 3 & 3\end{bmatrix}$$

190. **(I)** $(x^2-y^2)dx+2xydy=0$

$$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$
$$=\frac{x^2\left(\frac{y^2}{x^2}-1\right)}{x^2\left(2\frac{y}{x}\right)}$$
$$=g\left(\frac{y}{x}\right)$$

(II) Let $y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$
$$v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2vx^2}$$
$$=\frac{v^2-1}{2v}$$
$$x\frac{dv}{dx}=\frac{v^2-1}{2v}-v$$
$$=\frac{v^2-1-2v^2}{2v}$$

$= -\frac{(1+v^2)}{2v}$

$-\int \frac{2v}{1+v^2} dv = \int \frac{dx}{x}$

$-\log|1+v^2| - \log|x| + C = 0$

$-\log\left|1+\frac{y^2}{x^2}\right| - \log|x| + C = 0$

$-\log\left|\frac{x^2+y^2}{x^2}\right| - \log|x| + C = 0$

$-\log\left|\frac{x^2+y^2}{x}\right| + C = 0$

191. **(c)** • Domain of $\cos^{-1}x$

The domain of the inverse cosine function, $y = \cos^{-1}x$, consists of all values x for which $\cos y = x$. Since the range of the cosine function is $[-1,1]$, the domain of its inverse is:

192. **(c)** • Element a_{31} in Matrix A

Given the formula $a_{ij} = \frac{1}{2}|-3i+j|$, we substitute $i = 3$ and $j = 1$:

$a_{31} = \frac{1}{2}|-3(3)+1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$

193. **(a)** Value of $2x - y$

Perform the matrix addition:

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

• Solve for y: $2 + y = 5 \Longrightarrow y = 3$

• Solve for x : $2x + 2 = 8 \Longrightarrow 2x = 6 \Longrightarrow x = 3$

• Calculate $2x - y$: $2(3) - 3 = 6 - 3 = 3$

194. **(b)** • Inverse of Matrix A

For $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$:

(1) Determinant $|A|$: $(1 \times 5) - (2 \times 3) = 5 - 6 = -1$

(2) Adjoint: Swap diagonal elements and change signs of off-diagonal elements:

$\text{adj}(A) = \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$

(3) Inverse: $A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

195. **(c)** • Minor M_{23}

The minor M_{23} is the determinant of the submatrix formed by removing the 2 nd row and 3rd column:

$M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = (2 \times 5) - (-3 \times 1) = 10 + 3 = 13$

196. **(b)** Derivative $\frac{dy}{dx}$ at $x = 1$

Given $y = \sec(\tan^{-1}x)$. Let $\theta = \tan^{-1}x \Longrightarrow \tan\theta = x$.

Using the identity $\sec\theta = \sqrt{1+\tan^2\theta}$, we simplify:

$y = \sqrt{1+x^2}$

Now, differentiate with respect to x :

$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$

Substitute $x = 1$:

$\frac{dy}{dx} = \frac{1}{\sqrt{1+1^2}} = \frac{1}{\sqrt{2}}$

197. **(d)** Continuity of $f(x)$ at $x = \frac{\pi}{2}$

For $f(x)$ to be continuous at $x = \frac{\pi}{2}$, the limit must equal the functional value:

$$\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}=5$$

Using L'Hôpital's Rule:

$$\lim_{x\to\frac{\pi}{2}}\frac{-k\sin x}{-2}=\frac{k}{2}=5\Longrightarrow k=10$$

198. **(c)** Integration of $\int\frac{dx}{\sin^2 2x\cdot\cos^2 2x}$

Using the identity $1=\sin^2 2x+\cos^2 2x$:

$$\int\frac{\sin^2 2x+\cos^2 2x}{\sin^2 2x\cos^2 2x}dx=\int(\sec^2 2x+\csc^2 2x)dx=\frac{1}{2}\tan 2x-\frac{1}{2}\cot 2x+C$$

199. **(b)** Integral of an Odd Function

The function $f(x)=\sin^3 x$ is an odd function because $f(-x)=-f(x)$. For any odd function:

$$\int_{-a}^{a}f(x)dx=0$$

200. **(d)** General Solution of $\frac{dy}{dx}=e^{x-y}$

Separate the variables: $e^y dy=e^x dx$. Integrating both sides:

$e^y=e^x+C\Longrightarrow e^x-e^y=-C(\text{ or } e^x-e^y=C')$

201. **(a)** Order and Degree of Differential Equation

For $\left(\frac{d^3y}{dx^3}\right)^2+3x\left(\frac{d^2y}{dx^2}\right)^4=\log x$:

- Order (highest derivative): 3
- Degree (power of highest derivative): 2
- Sum: $3+2=5$

202. **(d)** Condition for $\vec{a}+\vec{b}$ to be a Unit Vector

Given $|\vec{a}|=1,|\vec{b}|=1$, and $|\vec{a}+\vec{b}|=1$:

$|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta=1$

$1+1+2(1)(1)\cos\theta=1\Longrightarrow 2\cos\theta=-1\Longrightarrow\cos\theta=-\frac{1}{2}$

$\theta=\frac{2\pi}{3}$

203. **(c)** If $(2i+6j-22k)\times(i+\lambda j+\mu k)=0$

For cross product to be zero, vectors must be parallel.

$(1,\lambda,\mu)=t(2,6,-22)$

Since first component gives

$1=2t\Rightarrow t=\frac{1}{2}$

Thus

$\lambda=6\left(\frac{1}{2}\right)=3,\mu=-22\left(\frac{1}{2}\right)=-11$

$\lambda-\mu=3-(-11)=14$

204. **(b)** Direction cosines of $\overrightarrow{BA}$

$\vec{A}=(1,2,-3),\vec{B}=(-1,-2,1)$

$\overrightarrow{BA}=\vec{A}-\vec{B}=(2,4,-4)$

Magnitude:

$|\overrightarrow{BA}|=\sqrt{2^2+4^2+(-4)^2}=\sqrt{36}=6$

Direction cosines:

$\left(\frac{2}{6},\frac{4}{6},\frac{-4}{6}\right)=\left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right)$

205. **(b)** Value of λ

First line:

$\frac{x-5}{7}=\frac{2-y}{5}=\frac{z}{1}=t$

Direction ratios:

$(7,-5,1)$
Second line:
$\frac{x}{1}=\frac{2y-1}{\lambda}=\frac{z}{3}=s$
Direction ratios:
$\left(1,\frac{\lambda}{2},3\right)$
For perpendicular lines:
$7(1)+(-5)\left(\frac{\lambda}{2}\right)+1(3)=0$
$10-\frac{5\lambda}{2}=0$
$20=5\lambda$
$\lambda=4$

206. **(d)** Solution set of $2x+y\geq 5$
Line:
$2x+y=5$
Check origin:
$2(0)+0=0\geq 5$
So the solution is the half-plane not containing the origin, and since inequality is $\geq$, the boundary line is included.

207. **(a)** Minimum value of
$z=3x+8y$
Subject to
$0\leq x\leq 20, y\geq 10$
Since coefficients of x and y are positive, minimum occurs at smallest feasible values:
$x=0, y=10$
$z=3(0)+8(10)=80$

208. **(c)** Two events A and B are independent if
For independent events,
$P(A'B')=P(A')P(B')$
$=(1-P(A))(1-P(B))$

209. **(a)** • Assertion (A) & Reason (R)
• Assertion (A): Matrix $A=\begin{bmatrix}0 & -3 & 5\\ 3 & 0 & -2\\ -5 & 2 & 0\end{bmatrix}$
• To check if it's skew-symmetric, we find A^T: $A^T=\begin{bmatrix}0 & 3 & -5\\ -3 & 0 & 2\\ 5 & -2 & 0\end{bmatrix}$.
• Since $A^T=-A$, the assertion is True.
• Reason (R): This is the definition of a skew-symmetric matrix. True.
• Conclusion: Both (A) and (R) are true and (R) is the correct explanation of (A).

210. **(a)**• Assertion (A) & Reason (R)
• Assertion (A): Line passing through $A(-1,0,2)$ and $B(3,4,6)$.
• Direction vector $\vec{b}=\vec{B}-\vec{A}=(3-(-1))\hat{\imath}+(4-0)\hat{\jmath}+(6-2)\hat{k}=4\hat{\imath}+4\hat{\jmath}+4\hat{k}$.
• The given equation uses $\vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$, which is parallel to $4\hat{\imath}+4\hat{\jmath}+4\hat{k}$.
• Vector equation: $\vec{r}=(-\hat{\imath}+0\hat{\jmath}+2\hat{k})+\lambda(\hat{\imath}+\hat{\jmath}+\hat{k})$. This matches. True.
• Reason (R): This is the standard formula for a line. True.
• Conclusion: Both (A) and (R) are true and (R) is the correct explanation of (A).

211. **(A)** Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$
(1) $\tan^{-1}\sqrt{3}=\frac{\pi}{3}$
(2) $\sec^{-1}(-2)=\pi-\sec^{-1}(2)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$
(3) Result: $\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$

OR

(B) Check injectivity and surjectivity of $f: N\rightarrow N, f(x)=x^3$

• Injectivity: For $x_1, x_2 \in \mathbb{N}$. if $x_1^3 = x_2^3$, then $x_1 = x_2$. It is Injective (one-to-one).
• Surjectivity: For $y = 2 \in \mathbb{N}$ (codomain), there is no $x \in \mathbb{N}$ such that $x^3 = 2$. It is not Surjective (into).

212. Differentiation

Given: $\cos y = x\cos(a+y)$

Rewrite as: $x = \frac{\cos y}{\cos(a+y)}$

Differentiate with respect to y :

$$\frac{dx}{dy} = \frac{-\sin y\cos(a+y) - \cos y[-\sin(a+y)]}{\cos^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \cos(a+y)\sin y}{\cos^2(a+y)}$$

Using the identity $\sin(A-B) = \sin A\cos B - \cos A\sin B$:

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\cos^2(a+y)} = \frac{\sin a}{\cos^2(a+y)}$$

Taking the reciprocal:

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

213. Projection of $7\hat{i} - \hat{j} + 8\hat{k}$ on $\hat{i} + 2\hat{j} + 2\hat{k}$

Let $\vec{a} = 7\hat{i} - \hat{j} + 8\hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$

Scalar projection:

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$\vec{a}\cdot\vec{b} = 7(1) + (-1)(2) + 8(2) = 21$

$|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$

Projection $= \frac{21}{3} = 7$

214. **(A)** Unit vector parallel to diagonal AC

Given

$\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\overrightarrow{AD} = \hat{i} - 2\hat{j} - 3\hat{k}$

In a parallelogram,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$

$= (3\hat{i} - 6\hat{j} + 2\hat{k})$

Magnitude:

$|\overrightarrow{AC}| = \sqrt{3^2 + (-6)^2 + 2^2} = \sqrt{49} = 7$

Hence the required unit vector is

$\frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$

OR

(B) Angle between the lines

Direction vectors are

$\vec{d}_1 = (1, -2, -2)$

$\vec{d}_2 = (3, 2, -6)$

Using

$$\cos\theta = \frac{\vec{d}_1\cdot\vec{d}_2}{|\vec{d}_1||\vec{d}_2|}$$

$\vec{d}_1\cdot\vec{d}_2 = 1(3) + (-2)(2) + (-2)(-6) = 11$

$|\vec{d}_1| = \sqrt{1+4+4} = 3$

$|\vec{d}_2| = \sqrt{9+4+36} = 7$

$\cos\theta = \frac{11}{21}$

$\theta = \cos^{-1}\left(\frac{11}{21}\right)$

215. Rate of increase of surface area of an air bubble

Given

$\frac{dr}{dt} = 0.5 \text{ cm/s}$

Surface area of sphere:

$S = 4\pi r^2$

Differentiate w.r.t. t

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

At r = 1.5 cm,

$\frac{dS}{dt} = 8\pi(1.5)(0.5)$

$= 6\pi$

$\frac{dS}{dt} = 6\pi \text{ cm}^2/\text{s}$

216. Evaluate

$I = \int \frac{dx}{1 + \cot x}$

$= \int \frac{\sin x}{\sin x + \cos x} dx$

Write

$\sin x = \frac{(\sin x + \cos x) - (\cos x - \sin x)}{2}$

Hence

$I = \frac{1}{2}\int dx - \frac{1}{2}\int \frac{\cos x - \sin x}{\sin x + \cos x} dx$

Let

$t = \sin x + \cos x$

$dt = (\cos x - \sin x)dx$

Therefore

$I = \frac{x}{2} - \frac{1}{2}\ln|\sin x + \cos x| + C$

$\int \frac{dx}{1 + \cot x} = \frac{x}{2} - \frac{1}{2}\ln|\sin x + \cos x| + C$

217. **(A)** Evaluate

$I = \int_{x/2}^{\pi} e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$

Use identity:

$\frac{1 - \sin x}{1 - \cos x} = \frac{(1 - \sin x)(1 + \sin x)}{(1 - \cos x)(1 + \sin x)} = \frac{\cos^2 x}{(1 - \cos x)(1 + \sin x)} = \frac{1 + \cos x}{1 + \sin x}$

Also,

$\frac{1 + \cos x}{1 + \sin x} = \frac{(\sin x + \cos x + 1)'}{\sin x + \cos x + 1}$

Hence

$I = \int_{\pi/2}^{\pi} e^{\alpha} \frac{d}{dx} \ln(\sin x + \cos x + 1) dx$

Observing.

$\frac{1 + \cos x}{1 + \sin x} = \frac{d}{dx}\left(\frac{e^x}{1 + \sin x}\right) e^{-x}$

Therefore,

$I = \left[\frac{e^x}{1 + \sin x}\right]_{\pi/2}^{\pi} = e^{\pi} - \frac{e^{\pi/2}}{2}$

$I = e^{\pi} - \frac{e^{\pi/2}}{2}$

OR

(B) Evaluate

$I = \int_0^{\pi/4} \log(1 + \tan x)dx$

Let

$I = \int_0^{\pi/4} \log(1 + \tan x)dx$

Put $x = \frac{\pi}{4} - t$,

$I = \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan t}{1 + \tan t}\right) dt$

$= \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan t}\right) dt$

Adding both expressions:

$2I = \int_0^{\pi/4} \log 2dt = \frac{\pi}{4}\log 2$

Hence

$\int_0^{\pi/4} \log(1 + \tan x)dx = \frac{\pi}{8}\log 2$

218. Find

$\int \frac{x}{(x^2 + 1)(x - 1)}dx$

Partial fractions:

$\frac{x}{(x^2 + 1)(x - 1)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 1}$

Comparing coefficients:

$A = \frac{1}{2}, B = -\frac{1}{2}, C = \frac{1}{2}$

Thus

$I = \frac{1}{2}\int \frac{dx}{x - 1} - \frac{1}{2}\int \frac{xdx}{x^2 + 1} + \frac{1}{2}\int \frac{dx}{x^2 + 1}$

$I = \frac{1}{2}\ln|x - 1| - \frac{1}{4}\ln(x^2 + 1) + \frac{1}{2}\tan^{-1}x + C$

219. **(A)** $\frac{dy}{dx} + y\cot x = 4x\operatorname{cosec}x$

Given $y = 0$ when $x = \frac{1}{2}$.

Integrating factor:

$IF = e^{\int \cot xdx} = e^{\ln(\sin x)} = \sin x$

Multiplying by IF:

$\sin x\frac{dy}{dx} + y\sin x\cot x = 4x$

$\frac{d}{dx}(y\sin x) = 4x$

Integrating:

$y\sin x = 2x^2 + C$

Using $y = 0$ at $x = \frac{\pi}{2}$,

$0 = 2\left(\frac{\pi}{2}\right)^2 + C$

$C = -\frac{\pi^2}{2}$

Hence

$$y = \frac{2x^2 - \frac{\pi^2}{2}}{\sin x}$$

OR

(B) $xdy - ydx = \sqrt{x^2 + y^2}dx$

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

Put

$$y = vx$$

Then

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting:

$$x^2\frac{dv}{dx} = x\sqrt{1 + v^2}$$

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

Integrating:

$$\sinh^{-1} v = \ln|x| + C$$

Replacing $v = \frac{H}{x}$.

$$\sinh^{-1}\left(\frac{y}{x}\right) = \ln|x| + C$$

220. $z = 4x + 3y$

Corner points:

$A(0,700), B(100,700), C(200,600), D(400,200)$

Evaluate:

Point	$z = 4x + 3y$
A(0,700)	2100
B(100,700)	2500
C(200,600)	2600
D(400,200)	2200

Maximum:

$$z_{max} = 2600 \text{ at } (200,600)$$

Minimum:

$$z_{min} = 2100 \text{ at } (0,700)$$

221. **(A)** Let

- H : pair of heads actually occurs.
- R : man reports pair of heads.

$$P(H) = \frac{1}{4}, P(H^c) = \frac{3}{4}$$

Truth probability:

$$P(T) = \frac{3}{5}, P(L) = \frac{2}{5}$$

$$P(R \mid H) = \frac{3}{5}$$

$$P(R \mid H^c) = \frac{2}{5}$$

By Bayes' theorem,

$$P(H \mid R) = \frac{P(H)P(R \mid H)}{P(H)P(R \mid H) + P(H^c)P(R \mid H^c)}$$

$$= \frac{\frac{1}{4}\cdot\frac{3}{5}}{\frac{1}{4}\cdot\frac{3}{5}+\frac{3}{4}\cdot\frac{2}{5}}$$
$$= \frac{3}{9} = \frac{1}{3}$$

OR

(B) Let X = number of defective bulbs.
Possible values:
$X = 0,1,2$
Total ways:
$$\binom{10}{2} = 45$$
$P(X = 0)$
$$\frac{\binom{8}{2}}{45} = \frac{28}{45}$$
$P(X = 1)$
$$\frac{\binom{2}{1}\binom{8}{1}}{45} = \frac{16}{45}$$
$P(X = 2)$
$$\frac{\binom{2}{2}}{45} = \frac{1}{45}$$
Distribution:
Mean:
$$E(X) = 0\cdot\frac{28}{45} + 1\cdot\frac{16}{45} + 2\cdot\frac{1}{45}$$
$$= \frac{18}{45} = \frac{2}{5}$$
$$E(X) = \frac{2}{5}$$

222. **(A)** Relation and Equivalence Proof

Given set $A = \{5,6,7,8,9\}$ and relation $R = \{(x,y): |x-y| \text{ is divisible by } 2\}$.
(1) Roster Form
Two numbers have a difference divisible by 2 if they have the same parity (both odd or both even).
• Odds: $\{5,7,9\}$
• Evens: $\{6,8\}$
$R = \{(5,5),(6,6),(7,7),(8,8),(9,9),(5,7),(7,5),(5,9),$
$(9,5),(7,9),(9,7),(6,8),(8,6)\}$
(2) Proof of Equivalence Relation
• Reflexive: For all $x \in A$, $|x-x| = 0$, which is divisible by 2 . Thus, $(x,x) \in R$.
• Symmetric: If $(x,y) \in R$, then $|x-y|$ is divisible by 2 . Since $|y-x| = |x-y|$, $|y-x|$ is also divisible by 2 . Hence, $(y,x) \in R$.
• Transitive: If $(x,y) \in R$ and $(y,z) \in R$, then $x-y = \pm 2k_1$ and $y-z = \pm 2k_2$ for integers k_1, k_2.
Adding the equations: $x-z = 2(\pm k_1 \pm k_2)$, which means $|x-z|$ is divisible by 2 . Hence, $(x,z) \in R$.
Since R is reflexive, symmetric, and transitive, it is an equivalence relation.
(3) Elements related to 7
The elements related to 7 are all odd numbers in set A: $\{5,7,9\}$.

OR

(B) One-One and Onto Proof
Given $f: A \to B$ where $A = R - \{3\}$, $B = R - \{1\}$, and $f(x) = \frac{x-2}{x-3}$.
(1) Testing One-One (Injective)
Let $f(x_1) = f(x_2)$:
$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$
$$(x_1-2)(x_2-3) = (x_2-2)(x_1-3)$$

$x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$
$-3x_1 - 2x_2 = -3x_2 - 2x_1 \Longrightarrow -x_1 = -x_2 \Longrightarrow x_1 = x_2$
Hence, f is one-one.
(2) Testing Onto (Surjective)
Let $y \in B$. Set $y = f(x)$ and solve for x :
$$y = \frac{x-2}{x-3} \Longrightarrow y(x-3) = x-2$$
$$xy - 3y = x - 2 \Longrightarrow x(y-1) = 3y - 2 \Longrightarrow x = \frac{3y-2}{y-1}$$
Since $y \in B(R - \{1\}), y \neq 1$, so x is defined for all values of y. Moreover, x can never equal 3. Thus, every element in codomain B has a unique pre-image in domain A.
Hence, f is onto.

223. Solve using matrices
$x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$
Subtract first from third:
$x + z = 5$
$x = 5 - z$
Substitute in first:
$5 - z - y + 2z = 7$
$y = z - 2$
Substitute in second:
$3(5 - z) + 4(z - 2) - 5z = -5$
$15 - 3z + 4z - 8 - 5z = -5$
$7 - 4z = -5$
$z = 3$
$x = 2, y = 1$
$x = 2, y = 1, z = 3$

224. Area Bounded by Curves
Given curve $y = \sqrt{x} \Longrightarrow x = y^2$, line $x = 2y + 3$, and the x-axis $(y = 0)$.
Step 1: Point of Intersection
Equate x values of the curve and line:
$y^2 = 2y + 3 \Longrightarrow y^2 - 2y - 3 = 0 \Longrightarrow (y-3)(y+1) = 0$
Since $y = \sqrt{x} \geq 0$, we select $y = 3$. The intersection point is (9,3).
Step 2: Set up and Compute Area
Integrating with respect to the y-axis from $y = 0$ to $y = 3$:
$$\text{Area} = \int_0^3 (x_{\text{linc}} - x_{\text{curvo}})dy$$
$$\text{Area} = \int_0^3 (2y + 3 - y^2)dy$$
$$\text{Area} = \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3$$
$$\text{Area} = \left(3^2 + 3(3) - \frac{3^3}{3}\right) - 0 = 9 + 9 - 9 = 9 \text{ sq. units}$$

225. **(A)** Lines:
$\vec{r} = (1,2,-4) + \lambda(2,3,6)$
$\vec{r} = (3,-3,-5) + \mu(-2,3,8)$
Let
$\vec{a} = (1,2,-4), \vec{b} = (3,-3,-5)$
$\vec{d}_1 = (2,3,6), \vec{d}_2 = (-2,3,8)$
Shortest distance:

$$D = \frac{|(\vec{b} - \vec{a}) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}$$

$\vec{d}_1 \times \vec{d}_2 = (6, -28, 12)$

$|\vec{d}_1 \times \vec{d}_2| = \sqrt{964} = 2\sqrt{241}$

$\vec{b} - \vec{a} = (2, -5, -1)$

$(\vec{b} - \vec{a}) \cdot (\vec{d}_1 \times \vec{d}_2) = 140$

Hence

$$D = \frac{140}{2\sqrt{241}}$$

$$\frac{70}{\sqrt{241}}$$

OR

(B) Line:

$$\frac{x-4}{2} = \frac{y}{6} = \frac{z-1}{3} = t$$

So

$x = 4 + 2t, y = -6t, z = 1 + 3t$

Foot of perpendicular from

$P(2,3,-8)$

to line is $Q(4 + 2t, -6t, 1 + 3t)$.

For perpendicular,

$(PQ) \cdot (2, -6, 3) = 0$

$(-2t - 2)(2) + (6t + 3)(-6) + (-3t - 9)(3) = 0$

$-49t - 49 = 0$

$t = -1$

Therefore

$Q = (2, 6, -2)$

Hence foot of perpendicular:

$(2, 6, -2)$

Distance:.

$PQ = \sqrt{(2-2)^2 + (3-6)^2 + (-8+2)^2}$

$= \sqrt{45} = 3\sqrt{5}$

226. Case Study – 1: Spherical Balloon Analysis

This case study focuses on the rates of change for a spherical balloon as it is inflated.

(i) Rates of Change

The quantities (radius, surface area, and volume) change at different rates.

While they all increase as the balloon expands, their rates of change are related nonlinearly. For instance, the surface area depends on the square of the radius (r^2), while volume depends on the cube of the radius (r^3).

(ii) Expressions for Surface Area and Volume

For a sphere with radius r :

- Surface Area (S): $S = 4\pi r^2$
- Volume (V): $V = \frac{4}{3}\pi r^3$

(iii) (A) Rate of Change of Surface Area

Given: $r = 6$ cm and $\frac{dr}{dr} = 2$ cm/s.

To find the rate of change of surface area $\left(\frac{dS}{dr}\right)$, we differentiate $S = 4\pi r^2$ with respect to time t :

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

Substituting the given values:

$$\frac{dS}{dt} = 8\pi(6)(2) = 96\pi \text{ cm}^2/\text{s}$$

OR

(B) Rate of Change of Volume

Given: $r = 6$ cm and $\frac{dr}{dr} = 2$ cm/s.

To find the rate of change of volume $\left(\frac{dV}{dr}\right)$, we differentiate $V = \frac{4}{3}\pi r^3$ with respect to time t :

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Substituting the given values:

$$\frac{dV}{dt} = 4\pi(6)^2(2) = 4\pi(36)(2) = 288\pi \text{ cm}^3/\text{s}$$

227. Case Study - 2

Based on the information provided, here are the step-by-step solutions:

Given:

- Path of the jet: $4y = x^2 \Rightarrow y = \frac{x^2}{4}$
- Position of the soldier: $(0,5)$
- Position of the jet: (x, y)

(i) Express the distance $f(x)$

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

$f(x) = \sqrt{(x - 0)^2 + (y - 5)^2}$

Substitute $y = \frac{x^2}{4}$:

$$f(x) = \sqrt{x^2 + \left(\frac{x^2}{4} - 5\right)^2}$$

(ii) Find $\frac{dS}{dx}$

Given $S = [f(x)]^2$:

$$S = x^2 + \left(\frac{x^2}{4} - 5\right)^2$$

Differentiate with respect to x :

$$\frac{dS}{dx} = 2x + 2\left(\frac{x^2}{4} - 5\right) \cdot \frac{2x}{4}$$

$$\frac{dS}{dx} = 2x + x\left(\frac{x^2}{4} - 5\right) = 2x + \frac{x^3}{4} - 5x$$

$$\frac{dS}{dx} = \frac{x^3}{4} - 3x$$

(iii) (A) Position of the jet when shot down

The soldier shoots when the jet is nearest, which occurs when $\frac{dS}{dx} = 0$.

$$\frac{x^3}{4} - 3x = 0 \Rightarrow x\left(\frac{x^2}{4} - 3\right) = 0$$

Possible values: $x = 0$ or $x^2 = 12 \Rightarrow x = \pm 2\sqrt{3}$.

- If $x = 0, y = 0, S = 25$.
- If $x = \pm 2\sqrt{3}, y = \frac{12}{4} = 3. S = 12 + (3 - 5)^2 = 16$.

Since $16 < 25$, the nearest positions are $(2\sqrt{3}, 3)$ and $(-2\sqrt{3}, 3)$.

OR

(B) Minimum distance

The minimum distance is the value of $f(x)$ at the nearest point:

$f(x) = \sqrt{S} = \sqrt{16}$

Distance $= 4$ units

228. Case Study - 3

There are 10 cards numbered 1 to 10 .

Total number of possible outcomes:

$n(S) = 10$

(i) Probability that the number on the drawn card is greater than 4

Numbers greater than 4 are:
$\{5,6,7,8,9,10\}$
Number of favorable outcomes:
$n(E) = 6$
Therefore,
$P(E) = \frac{6}{10} = \frac{3}{5}$

(ii) If it is known that the number drawn is greater than 4 , find the probability that it is even This is a conditional probability.
Given numbers greater than 4:
$\{5,6,7,8,9,10\}$
Total outcomes $= 6$.
Even numbers among them:
$\{6,8,10\}$
Favorable outcomes $= 3$.
Hence,
$P(\text{ Even} \mid \text{Greater than } 4) = \frac{3}{6} = \frac{1}{2}$