General Instructions :

Read the following instructions very carefully and follow them :

(i) This question paper contains 38 questions. All questions are compulsory.
(ii) Question paper is divided into FIVE Sections-Section A, B, C, D and E.
(iii) In Section A - Question Number 1 to 18 are Multiple Choice Questions (MCQ) type and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B - Question Number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.
(v) In Section C-Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In Section D - Question Number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
(vii) In Section E - Question Number 36 to 38 are case study based questions carrying 4 marks each where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section −D and 2 questions in Section - E.
(ix) Use of calculators is NOT allowed.

SECTION A

1. If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is :

(a) -2 (b) 2
(c) 3 (d) -3

2. If $|A| = 2$, where A is a 2×2 matrix, then $|4A^{-1}|$ equals:

(a) 4 (b) 2
(c) 8 (d) $\frac{1}{32}$

3. Let A be a 3×3 matrix such that $|\text{adj}A| = 64$. Then $|A|$ is equal to :

(a) 8 only (b) -8 only
(c) 64 (d) 8 or -8

4. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and $2A + B$ is a null matrix, then B is equal to :

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$
(c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

5. If $\frac{d}{dx}(f(x)) = \log x$, then $f(x)$ equals :

(a) $-\frac{1}{x} + C$ (b) $x(\log x - 1) + C$
(c) $x(\log x + x) + C$ (d) $\frac{1}{x} + C$

6. $\int_0^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to :

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) $-\sqrt{3}$

7. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is :

(a) 5 (b) 2
(c) 3 (d) 4

8. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is :

(a) 3 (b) -3
(c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

9. The value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ is:

(a) 2 (b) 0
(c) 1 (d) -1

10. If $\vec{a}+\vec{b}=\hat{i}$ and $\vec{a}=2\hat{i}-2\hat{j}+2\hat{k}$, then $|\vec{b}|$ equals:

(a) $\sqrt{14}$ (b) 3

(c) $\sqrt{12}$ (d) $\sqrt{17}$

11. Direction cosines of the line $\frac{x-1}{2}=\frac{1-y}{3}=\frac{2z-1}{12}$ are :

(a) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ (b) $\frac{2}{\sqrt{157}},-\frac{3}{\sqrt{157}},\frac{12}{\sqrt{157}}$

(c) $\frac{2}{7},-\frac{3}{7},-\frac{6}{7}$ (d) $\frac{2}{7},-\frac{3}{7},\frac{6}{7}$

12. If $P\left(\frac{A}{B}\right)=0\cdot 3, P(A)=0\cdot 4$ and $P(B)=0\cdot 8$, then $P\left(\frac{B}{A}\right)$ is equal to :

(a) 0.6 (b) 0.3

(c) 0.06 (d) 0.4

13. The value of k for which $f(x)=\begin{cases}3x+5, & x\geq 2\\ kx^2, & x<2\end{cases}$ is a continuous function, is:

(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$

(c) 11 (d) $\frac{11}{4}$

14. If $A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ and $(3I+4A)(3I-4A)=x^2I$, then the value $(s)x$ is/are :

(a) $\pm\sqrt{7}$ (b) 0

(c) ± 5 (d) 25

15. The general solution of the differential equation $xdy-(1+x^2)dx=dx$ is :

(a) $y=2x+\frac{x^3}{3}+C$ (b) $y=2\log x+\frac{x^3}{3}+C$

(c) $y=\frac{x^2}{2}+C$ (d) $y=2\log x+\frac{x^2}{2}+C$

16. If $f(x)=a(x-\cos x)$ is strictly decreasing in $\mathbb{R}$, then ' a ' belongs to

(a) $\{0\}$ (b) $(0,\infty)$

(c) $(-\infty,0)$ (d) $(-\infty,\infty)$

17. The corner points of the feasible region in the graphical representation of a linear programming problem are $(2,72),(15,20)$ and $(40,15)$. If $z=18x+9y$ be the objective function, then :

(a) z is maximum at (2,72), minimum at (15,20)

(b) z is maximum at (15,20), minimum at (40,15)

(c) z is maximum at (40,15), minimum at (15,20)

(d) z is maximum at (40,15), minimum at (2,72)

18. The number of corner points of the feasible region determined by the constraints $x-y\geq 0, 2y\leq x+2, x\geq 0, y\geq 0$ is:

(a) 2 (b) 3

(c) 4 (d) 5

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. Assertion (A) : The range of the function $f(x)=2\sin^{-1}x+\frac{3\pi}{2}$, where $x\in[-1,1]$, is $\left[\frac{\pi}{2},\frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0,\pi]$.

20. Assertion (A) : Equation of a line passing through the points (1, 2, 3) and (3, −1,3) is $\frac{x-3}{2}=\frac{y+1}{3}=\frac{z-3}{0}$.

Reason (R): Equation of a line passing through points (x_1,y_1,z_1), (x_2,y_2,z_2) is given by $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$.

SECTION B

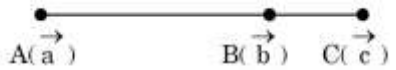
21. **(A)** A function $f: A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1,2,3,4\}$, then find the set B.

OR

(B) Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$

22. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.

23. **(A)** Position vectors of the points A, B and C as shown in the figure below are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.



If $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

OR

(B) Check whether the lines given by equations $x = 2\lambda + 2, y = 7\lambda + 1, z = -3\lambda - 3$ and $x = -\mu - 2, y = 2\mu + 8, z = 4\mu + 5$ are perpendicular to each other or not.

24. If $y = \left(x + \sqrt{x^2 - 1}\right)^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

25. Show that the function $f(x) = \frac{16\sin x}{4+\cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

SECTION C

26. Evaluate : $\int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2\cos x)]dx$.

27. Find : $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}dx$

28. **(A)** Find the particular solution of the differential equation

$$\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x, \text{ given that } y(0) = 0.$$

OR

(B) Solve the differential equation given by

$$xdy - ydx - \sqrt{x^2 + y^2}dx = 0.$$

29. Solve graphically the following linear programming problem :

Maximise $z = 6x + 3y$, subject to the constraints

$4x + y \geq 80$

$x + 5y \geq 115$

$3x + 2y \leq 150$

$x \geq 0, y \geq 0$

30. **(A)** The probability distribution of a random variable X is given below :

X	1	2	3
$P(X)$	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k .

(ii) Find $P(1 \leq X < 3)$.

(iii) Find $E(X)$, the mean of X .

OR

(B) A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find $P(A)$ and $P(B)$.

31. **(A)** Evaluate :

$$\int_0^{\frac{\pi}{2}} e^x \sin x dx$$

OR

(B) Find :

$$\int \frac{1}{\cos(x-a)\cos(x-b)}dx$$

SECTION D

32. A relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y): x \cdot y \text{ is an irrational number }\}$.

Check whether R is reflexive, symmetric and transitive or not.

33. **(A)** If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

OR

(B) Solve the following system of equations by matrix method :

$x + 2y + 3z = 6$

$2x - y + z = 2$

$3x + 2y - 2z = 3$

34. **(A)** Find the vector and the Cartesian equations of a line passing through the point $(1,2,-4)$ and parallel to the line joining the points $A(3,3,-5)$ and $B(1,0,-11)$. Hence, find the distance between the two lines.

OR

(B) Find the equations of the line passing through the points $A(1,2,3)$ and $B(3,5,9)$. Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

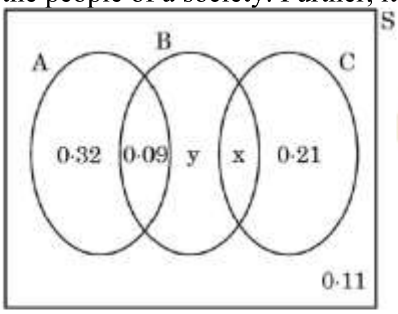
35. Find the area of the region bounded by the curves $x^2 = y, y = x + 2$ and x -axis, using integration.

SECTION E

Case Study – 1

36. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is $0 \cdot 44$.



On the basis of the above information, answer the following questions :

(i) Find the value of x .

(ii) Find the value of y.

(iii) **(A)** Find $P\left(\frac{C}{B}\right)$.

OR

(B) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

Case Study – 2

37. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.

A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is $45°$.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm.

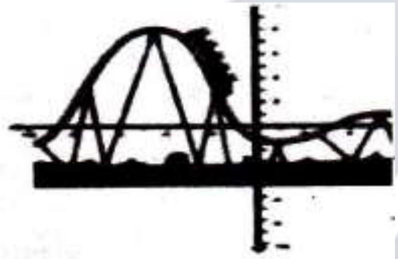
(iii) (A) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $\mathrm{r} = 2\sqrt{2}$ cm.

OR

(B) Find the rate of change of height ' h ' at an instant when slant height is 4 cm .

Case Study – 3

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x+9)(x+1)(x-3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :



(i) Find the value of ' a '.

(ii) Find $\mathrm{f}''(\mathrm{x})$ at $\mathrm{x} = 1$.

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(v) In Section C-Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D - Question Number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

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(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section $-$D and 2 questions in Section - E.

(ix) Use of calculators is NOT allowed.

SECTION A

39. If $\mathrm{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

40. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to

(a) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

41. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the setA is

(a) $\mathbb{R}$ (b) $\{0\}$

(c) $\{4\}$ (d) $\mathbb{R} - \{4\}$

42. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is

(a) 1 (b) -1

(c) 2 (d) 0

43. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to

(a) $a + b$ (b) $\frac{ax^2}{2} + bx$

(c) $\frac{ax^2}{2} + bx + c$ (d) b

44. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is

(a) 2 (b) 1

(c) not defined (d) 0

45. The integrating factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is

(a) $\frac{1}{y^2-1}$ (b) $\frac{1}{\sqrt{y^2-1}}$

(c) $\frac{1}{1-y^2}$ (d) $\frac{1}{\sqrt{1-y^2}}$

46. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are $(2, 1, -1)$ and $(4, 4, -7)$, is

(a) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (b) $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(c) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (d) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

47. Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is

(a) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (b) $4\hat{i} + \hat{j} - 2\hat{k}$

(c) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (d) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

48. Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is

(a) 0 (b) 12

(c) $\frac{12}{\sqrt{13}}$ (d) $\frac{-12}{\sqrt{13}}$

49. Equation of a line passing through point $(1, 1, 1)$ and parallel to z -axis is

(a) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (b) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$

(c) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ (d) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

50. If the sum of numbers obtained on throwing a pair of dice is 9 , then the probability that number obtained on one of the dice is 4 , is :

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$

(c) $\frac{1}{18}$ (d) $\frac{1}{2}$

51. Anti-derivative of $\frac{\tan x-1}{\tan x+1}$ with respect to x is

(a) $\sec^2\left(\frac{\pi}{4}-x\right)+c$

(b) $-\sec^2\left(\frac{\pi}{4}-x\right)+c$

(c) $\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|+c$

(d) $-\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|+c$

52. If (a, b), (c, d) and (e, f) are the vertices of $\triangle$ ABC and $\triangle$ denotes the area of $\triangle$ ABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to

(a) $2\Delta^2$ (b) $4\Delta^2$

(c) 2Δ (d) 4Δ

53. The function $f(x)=x|x|$ is

(a) continuous and differentiable at $x=0$.

(b) continuous but not differentiable at $x=0$.

(c) differentiable but not continuous at $x=0$.

(d) neither differentiable nor continuous at $x=0$.

54. If $\tan\left(\frac{x+y}{x-y}\right)=k$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{-y}{x}$ (b) $\frac{y}{x}$

(c) $\sec^2\left(\frac{y}{x}\right)$ (d) $-\sec^2\left(\frac{y}{x}\right)$

55. The objective function $Z=ax+by$ of an LPP has maximum value 42 at $(4,6)$ and minimum value 19 at $(3,2)$. Which of the following is true ?

(a) $a=9, b=1$ (b) $a=5, b=2$

(c) $a=3, b=5$ (d) $a=5, b=3$

56. The corner points of the feasible region of a linear programming problem are $(0,4)$, $(8,0)$ and $\left(\frac{20}{3},\frac{4}{3}\right)$. If $Z=30x+24y$ is the objective function, then (maximum value of Z - minimum value of Z) is equal to

(a) 40 (b) 96

(c) 120 (d) 144

In the following questions 19 & 20 , a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true, but (R) is false.

(d) (A) is false, but (R) is true.

57. Assertion (A) : Maximum value of $(\cos^{-1}x)^2$ is π^2.

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$.

58. Assertion (A) : If a line makes angles α,β,γ with positive direction of the coordinate axes, then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$.

Reason (R) : The sum of squares of the direction cosines of a line is 1 .

SECTION – B

59. (A) Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)$.

OR

(B) Draw the graph of $\cos^{-1}x$, where $x\in[-1,0]$. Also, write its range.

60. A particle moves along the curve $3y=ax^3+1$ such that at a point with x-coordinate 1, y-coordinate is changing twice as fast at x-coordinate. Find the value of a.

61. If $\vec{a},\vec{b},\vec{c}$ are three non-zero unequal vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, then find the angle between $\vec{a}$ and $\vec{b}-\vec{c}$.

62. Find the coordinates of points on line $\frac{x}{1}=\frac{y-1}{2}=\frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.

63. (A) If $y=\sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2=0$.

OR

(B) If $f(x)=\begin{cases}ax+b & ;0<x\leq 1\\ 2x^2-x & ;1<x<2\end{cases}$ is a differentiable function in (0, 2), then find the values of a and b.

SECTION-C

64. (A) Evaluate $\int_0^{\pi/4}\log(1+\tan x)dx$.

OR

(B) Find $\int \frac{dx}{\sqrt{\sin^3 x\cos(x-\alpha)}}$.

65. Find $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$.

66. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}}\frac{1}{(e^x+e^{-x})(e^x-e^{-x})}dx$

67. (A) Find the general solution of the differential equation :
$(xy-x^2)dy=y^2\,dx$.

OR

(B) Find the general solution of the differential equation :
$(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$

68. (A) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

OR

(B) A and B throw a die alternately till one of them gets a ' 6 ' and wins the game. Find their respective probabilities of wining, if A starts the game first.

69. Solve the following linear programming problem graphically :
Minimize: $Z=5x+10y$
subject to constraints : $x+2y\leq 120, x+y\geq 60, x-2y\geq 0, x\geq 0, y\geq 0$

SECTION - D

70. (A) If $A=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, then find AB and use it to solve the following system of equations:
$x-2y=3$
$2x-y-z=2$
$-2y+z=3$

OR

(B) If $f(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, prove that $f(\alpha)\cdot f(-\beta)=f(\alpha-\beta)$

71. (A) Find the equations of the diagonals of the parallelogram PQRS whose vertices are $P(4,2,-6), Q(5,-3,1), R(12,4,5)$ and $S(11,9,-2)$. Use these equations to find the point of intersection of diagonals.

OR

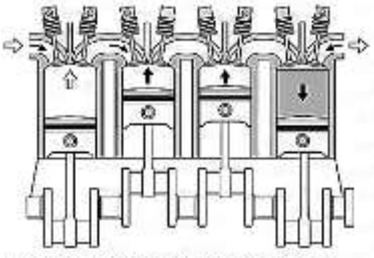
(B) A line l passes through point $(-1,3,-2)$ and is perpendicular to both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.
Find the vector equation of the line l. Hence, obtain its distance from origin.

72. Using integration, find the area of region bounded by line $y=\sqrt{3}x$, the curve $y=\sqrt{4-x^2}$ and y -axis in first quadrant.

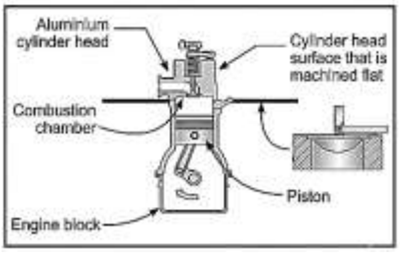
73. A function $f:[-4,4]\rightarrow[0,4]$ is given by $f(x)=\sqrt{16-x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of ' a ' for which $f(a)=\sqrt{7}$.

Section-E

74. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



One complete cycle of a four-cylinder four-stroke engine. The volume displaced is marked



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi\ cm^2$.

Based on the above information, answer the following questions:

(i) If the radius of cylinder is r cm and height is h cm , then write the volume V of cylinder in terms of radius r .

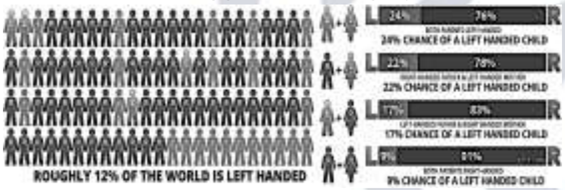
(ii) Find $\frac{dV}{dr}$.

(iii) (A) Find the radius of cylinder when its volume is maximum.

OR

(B) For maximum volume, $h > r$. State true or false and justify.

75. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A : When both father and mother are left handed :

Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :

Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :

Chances of left handed child is 17%.

D : When both father and mother are right handed:

Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find $P(L/C)$

(ii) Find $P(\overline{L}/A)$

(iii) (A) Find $P(A/L)$

OR

(B) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

76. The use of electric vehicles will curb air pollution in the long run.

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

where t represents the time and $t = 1,2,3$ ... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(ii) Prove that the function $V(t)$ is an increasing function.

General Instructions :

Read the following instructions very carefully and follow them :

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE Sections-Section A, B, C, D and E.

(iii) In Section A - Question Number 1 to 18 are Multiple Choice Questions (MCQ) type and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B - Question Number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.

(v) In Section C-Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D - Question Number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

(vii) In Section E - Question Number 36 to 38 are case study based questions carrying 4 marks each where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section −D and 2 questions in Section - E.

(ix) Use of calculators is NOT allowed.

SECTION A

77. If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $x + y + z$ is :

(a) 10 (b) 6

(c) 8 (d) 0

78. If $A \cdot (\text{adj}A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj}A|$ is equal to :

(a) 12 (b) 9

(c) 3 (d) 27

79. A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) $AB = O$ (b) $AB = -BA$

(c) $AB = BA$ (d) $BA = O$

80. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}I$, where $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) 0 (d) $\frac{\pi}{2}$

81. Let A be the area of a triangle having vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3). Which of the following is correct?

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$

(b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2\,A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$

(d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

82. $\int 2^{x+2} dx$ is equal to :

(a) $2^{x+2} + C$ (b) $2^{x+2}\log 2 + C$

(c) $\frac{2^{x+2}}{\log 2} + C$ (d) $2 \cdot \frac{2^x}{\log 2} + C$

83. $\int \frac{2\cos 2x - 1}{1 + 2\sin x} dx$ is equal to :

(a) $x - 2\cos x + C$ (b) $x + 2\cos x + C$

(c) $-x - 2\cos x + C$ (d) $-x + 2\cos x + C$

84. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is :

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $\log x - \log y = C$

(c) $xy = C$ (d) $x + y = C$

85. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(a) 3 (b) 2

(c) 6 (d) not defined

86. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is :

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 0

87. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0 . The angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 0

88. In $\triangle ABC, \overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC , then vector $\overrightarrow{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

89. The value of λ for which the angle between the lines $\vec{r} = \hat{i} + \hat{j} + \hat{k} + p(2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = (1 + q)\hat{i} + (1 + q\lambda)\hat{j} + (1 + q)\hat{k}$ is $\frac{\pi}{2}$ is :

(a) -4 (b) 4

(c) 2 (d) -2

90. If $P(A \cap B) = \frac{1}{8}$ and $P(\bar{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

91. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at $x = 0$ is :

(a) 1 (b) 2

(c) any real number (d) 0

92. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$

(b) $\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

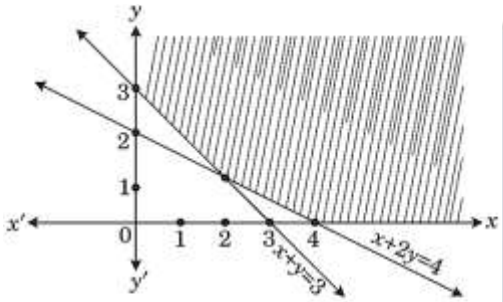
(d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

93. The number of feasible solutions of the linear programming problem given as
Maximize $z = 15x + 30y$ subject to constraints : $3x + y \leq 12, x + 2y \leq 10, x \geq 0, y \geq 0$ is

(a) 1 (b) 2

(c) 3 (d) infinite

94. The feasible region of a linear programming problem is shown in the figure below :



Which of the following are the possible constraints?

(a) $x + 2y \geq 4, x + y \leq 3, x \geq 0, y \geq 0$

(b) $x + 2y \leq 4, x + y \leq 3, x \geq 0, y \geq 0$

(c) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$

(d) $x + 2y \geq 4, x + y \geq 3, x \leq 0, y \leq 0$

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

95. Assertion (A) : Range of $[\sin^{-1}x + 2\cos^{-1}x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1}x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

96. Assertion (A) : A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points $(-1, -2, 1)$ and (1,2,5).

Reason (R) : Lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$.

SECTION B

97. If $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{\jmath}) \cdot (\vec{r} \times \hat{k}) - 12$.

98. If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, find the relation between α and β.

99. If $f(x) = a(\tan x - \cot x)$, where $a > 0$, then find whether f(x) is increasing or decreasing function in its domain.

100. **(A)** Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

OR

(B) Draw the graph of $f(x) = \sin^{-1} x, x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of $f(x)$.

101. **(A)** If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.

OR

(B) If $x = a\sin 2t, y = a(\cos 2t + \log\tan t)$, then find $\frac{dy}{dx}$.

SECTION C

102. **(A)** Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$$

OR

(B) Solve the following differential equation :

$$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

103. Evaluate :

$$\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x} + \sqrt{4-x}} dx$$

104. Evaluate :

$$\int_1^e \frac{1}{\sqrt{4x^2 - (x\log x)^2}} dx$$

105. **(A)** Find :

$$\int \frac{\cos x}{\sin 3x} dx$$

OR

(B) Find :

$$\int x^2 \log(x^2 + 1) dx$$

106. Determine graphically the minimum value of the following objective function:

$z = 500\,x + 400y$

Subject to constraints

$x + y \leq 200$

$x \geq 20$

$y \geq 4x$

$y \geq 0$

107. **(A)** A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X .

OR

(B) There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1: 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

SECTION D

108. Show that a function $f: \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \frac{5x-3}{4}$ is both one-one and onto.

109. The area of the region bounded by the line $y = mx (m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

110. **(A)** If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = O$.

OR

(B) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11, 2x - 7y = -3.$

111. **(A)** Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.

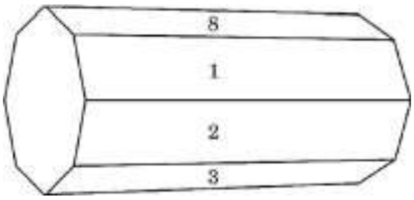
OR

(B) Find the equations of all the sides of the parallelogram ABCD whose vertices are $A(4,7,8), B(2,3,4), C(-1,-2,1)$ and $D(1,2,5)$. Also, find the coordinates of the foot of the perpendicular from A to CD.

SECTION E

Case Study - 1

112. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X:	P(X):
1	P
2	2p
3	2p
4	P
5	2p
6	p^2
7	$2p^2$
8	$7p^2+p$

Based on the above information, answer the following questions :

(i) Find the value of p.

(ii) Find $P(X>6)$.

(iii) **(A)** Find $P(X=3\,m)$, where m is a natural number.

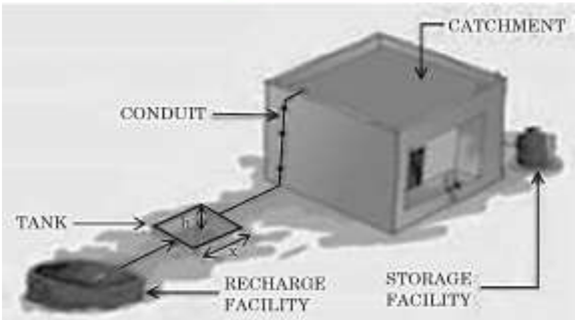
OR

(B) Find the mean $E(X)$.

Case Study - 2

113. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of $250\ m^3$. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ $40{,}000\ h^2$, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions:

(i) Find the total cost C of digging the tank in terms of x.

(ii) Find $\frac{dC}{dx}$.

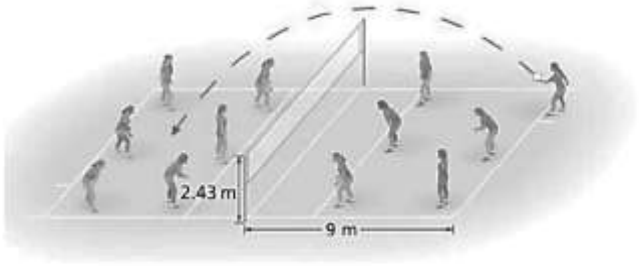
(iii) **(A)** Find the value of x for which cost C is minimum.

OR

(B) Check whether the cost function $C(x)$ expressed in terms of x is increasing or not, where $x > 0$.

Case Study - 3

114. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where $h(t)$ is the height of ball at any time t (in seconds), $(t \geq 0)$.



Based on the above information, answer the following questions :

(i) Is $h(t)$ a continuous function? Justify.

(ii) Find the time at which the height of the ball is maximum.

General Instructions :

Read the following instructions very carefully and follow them :

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE Sections-Section A, B, C, D and E.

(iii) In Section A - Question Number 1 to 18 are Multiple Choice Questions (MCQ) type and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B - Question Number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.

(v) In Section C-Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D - Question Number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

(vii) In Section E - Question Number 36 to 38 are case study based questions carrying 4 marks each where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section −D and 2 questions in Section - E.

(ix) Use of calculators is NOT allowed.

SECTION A

115. **If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then :**

(a) $x = 1, y = 2$ (b) $x = 2, y = 1$

(c) $x = 1, y = -1$ (d) $x = 3, y = 2$

116. **The product $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$ is equal to :**

(a) $\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$ (b) $\begin{bmatrix}(a+b)^2 & 0\\(a+b)^2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}a^2+b^2 & 0\\a^2+b^2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}a & 0\\0 & b\end{bmatrix}$

117. **If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to :**

(a) I (b) A

(c) 2 A (d) 3 I

118. **If a matrix $A = \begin{bmatrix}1 & 2 & 3\end{bmatrix}$, then the matrix AA' (where A' is the transpose of A) is :**

(a) 14 (b) $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}$ (d) $[14]$

119. **The value of** $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ **is**

(a) 0 (b) 1

(c) $x+y+z$ (d) $2(x+y+z)$

120. **The function $f(x) = |x|$ is**

(a) continuous and differentiable everywhere.

(b) continuous and differentiable nowhere.

(c) continuous everywhere, but differentiable everywhere except at $x = 0$.

(d) continuous everywhere, but differentiable nowhere.

121. **If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to :**

(a) $2\sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2(x^3)$

122. **$\int e^{5\log x} dx$ is equal to:**

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$

(c) $5x^4 + C$ (d) $6x^5 + C$

123. **If $\int_0^a 3x^2 dx = 8$, then the value of ' a ' is :**

(a) 2 (b) 4

(c) 8 (d) 10

124. **The integrating factor for solving the differential equation $x\frac{dy}{dx} - y = 2x^2$ is :**

(a) e^{-y} (b) e^{-x}

(c) x (d) $\frac{1}{x}$

125. **The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x\sin\left(\frac{dy}{dx}\right)$ respectively are :**

(a) 2,2 (b) 1,3

(c) 2,3 (d) 2 , degree not defined

126. **A unit vector along the vector $4\hat{i} - 3\hat{k}$ is :**

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$ (b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$ (d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

127. **If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when :**

(a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

128. **Distance of the point (p, q, r) from y-axis is :**

(a) q (b) $|q|$

(c) $|q| + |r|$ (d) $\sqrt{p^2 + r^2}$

129. **The solution set of the inequation $3x + 5y < 7$ is :**

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

130. **Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?**

(a) $(-2,4)$ (b) $(3,2)$

(c) $(-5,6)$ (d) $(4,2)$

131. If the direction cosines of a line are $\left(\frac{1}{a},\frac{1}{a},\frac{1}{a}\right)$, then :

(a) $0 < a < 1$ (b) $a > 2$

(c) $a > 0$ (d) $a = \pm\sqrt{3}$

132. The probability that A speaks the truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is :

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$

(c) $\frac{3}{20}$ (d) $\frac{4}{5}$

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

133. Assertion (A) : All trigonometric functions have their inverses over their respective domains.

Reason (R) : The inverse of $\tan^{-1}x$ exists for some $x \in R$.

134. Assertion (A) : The lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ are perpendicular, when $\overrightarrow{b_1} \cdot \overrightarrow{b_2} = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by $\cos\theta = \frac{\overrightarrow{b_1}\cdot\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$

SECTION B

135. (A) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(B) Evaluate :

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

136. If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.

137. Find the maximum and minimum values of the function given by $f(x) = 5 + \sin 2x$.

138. If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

139. (A) Find the vector equation of the line passing through the point (2,1,3) and perpendicular to both the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$; $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

OR

(B) The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes.

SECTION C

140. Find :

$$\int \frac{x^2 + x + 1}{(x+1)^2(x+2)}dx$$

141. (A) Evaluate :

$$\int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1 - \sin 2x}{1 - \cos 2x}\right)dx$$

OR

(B) Evaluate :

$$\int_{-2}^{2} \frac{x^2}{1 + 5^x}dx$$

142. (A) Find:

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}dx$$

OR

(B) Evaluate :

$$\int_0^{\pi/2} \sqrt{\sin x}\cos^5 x dx$$

143. **(A)** Find the particular solution of the differential equation

$\frac{dy}{dx} = \frac{x+y}{x}, y(1) = 0.$

OR

(B) Find the general solution of the differential equation $e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0.$

144. Solve the following linear programming problem graphically :

Minimise : $z = -3x + 4y$ subject to the constraints

$x + 2y \le 8, 3x + 2y \le 12, x, y \ge 0$

145. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

SECTION D

146. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations $x - y + 2z = 1; 2y - 3z = 1; 3x - 2y + 4z = 3.$

147. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

148. **(A)** If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b)R(c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(B) Let $f: R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

149. **(A)** Show that the following lines do not intersect each other:

$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}; \frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$

OR

(B) Find the angle between the lines

$2x = 3y = -z$ and $6x = -y = -4z.$

SECTION E

Case Study - 1

150. Let f(x) be a real valued function. Then its

• Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$

• Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at $x = a$ if its L.H.D. and R.H.D. at $x = a$ exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, x \ge 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at $x = 1$?

(ii) What is L.H.D. of f(x) at $x = 1$?

(iii) (A) Check if the function f(x) is differentiable at $x = 1$.

OR

(B) Find $f'(2)$ and $f'(-1)$.

Case Study - 2

151. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65 . The probability that many are not present and still the work gets completed on time is 0.35 . The probability that work will be completed on time when all workers are present is $0-80$.

Let : E_1 : represent the event when many workers were not present for the job;
E_2 : represent the event when all workers were present; and
E : represent completing the construction work on time.

Based on the above information, answer the following questions:

(i) What is the probability that all the workers are present for the job?

(ii) What is the probability that construction will be completed on time?

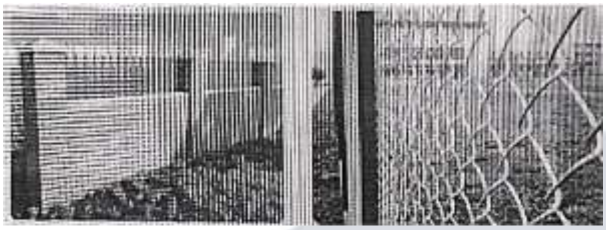
(iii) (A) What is the probability that many workers are not present given that the construction work is completed on time ?

OR

(B) What is the probability that all workers were present given that the construction job was completed on time?

Case Study - 3

152. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

(i) Let ' x ' metres denote the length of the side of the garden perpendicular to the brick wall and ' y ' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden.

(ii) Determine the maximum value of A(x).

General Instructions :

Read the following instructions very carefully and follow them :

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE Sections-Section A, B, C, D and E.

(iii) In Section A - Question Number 1 to 18 are Multiple Choice Questions (MCQ) type and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B - Question Number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.

(v) In Section C-Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D - Question Number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

(vii) In Section E - Question Number 36 to 38 are case study based questions carrying 4 marks each where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section $-D$ and 2 questions in Section - E.

(ix) Use of calculators is NOT allowed.

SECTION - A

153. Let $A=\{3,5\}$. Then number of reflexive relations on A is

(a) 2 (b) 4
(c) 0 (d) 8

154. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

155. If for a square matrix A, $A^2-A+I=O$, then A^{-1} equals

(a) A (b) $A+I$
(c) $I-A$ (d) $A-I$

156. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals

(a) ± 1 (b) -1

(c) 1 (d) 2

157. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is

(a) 1 (b) 2

(c) 3 (d) 4

158. The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x}\log x$

(c) $2x^{2x}(1+\log x)$ (d) $2x^{2x}(1-\log x)$

159. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at

(a) $x = 1$ (b) $x = 1.5$

(c) $x = -2$ (d) $x = 4$

160. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

(a) x (b) $-x$

(c) $16x$ (d) $-16x$

161. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is

(a) $(-1, \infty)$ (b) $(-2, -1)$

(c) $(-\infty, -2)$ (d) $[-1, 1]$

162. $\int \frac{\sec x}{\sec x - \tan x} dx$ equals

(a) $\sec x - \tan x + c$

(b) $\sec x + \tan x + c$

(c) $\tan x - \sec x + c$

(d) $-(\sec x + \tan x) + c$

163. $\int_{-1}^{1} \frac{|x-2|}{x-2} dx, x \neq 2$ is equal to

(a) 1 (b) -1

(c) 2 (d) -2

164. The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is

(a) 2 (b) 3

(c) 5 (d) 0

165. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if

(a) $a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$

(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$

(d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

166. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is

(a) 1 (b) 5

(c) 7 (d) 12

167. If a line makes angles of $90^\circ, 135^\circ$ and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

168. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) $0°$ (b) $30°$

(c) $45°$ (d) $90°$

169. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

170. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is

(a) $\frac{27}{32}$ (b) $\frac{5}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{32}$

Assertion - Reason Based Questions

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(c) (A) is true and (R) is false.
(d) (A) is false, but (R) is true.

171. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

172. **Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx = 3$

Reason (R) : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

SECTION-B

173. Write the domain and range (principle value branch) of the following functions:
$f(x) = \tan^{-1} x$

174. **(A)** If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

OR

(B) Find the value(s) of 'λ', if the function $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.

175. Sketch the region bounded by the lines $2x + y = 8, y = 2, y = 4$ and the y-axis. Hence, obtain its area using integration.

176. **(A)** If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(B) Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.

177. Find the vector and the cartesian equations of a line that passes through the point A(1,2,−1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

SECTION – C

178. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.

179. **(A)** Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$.

OR

(B) If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

180. **(A)** Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$

OR

(B) Find : $\int \frac{x^4}{(x-1)(x^2+1)} dx$

181. Find the area of the following region using integration :

$\{(x, y): y^2 \leq 2x \text{ and } y \geq x-4\}$

182. **(A)** Find the coordinates of the foot of the perpendicular drawn from the point P(0,2,3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

OR

(B) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

183. Find the distance between the lines:

$\vec{r} = (\hat{\imath}+2\hat{\jmath}-4\hat{k})+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$

$\vec{r} = (3\hat{\imath}+3\hat{\jmath}-5\hat{k})+\mu(4\hat{\imath}+6\hat{\jmath}+12\hat{k})$

SECTION - D

184. **(A)** The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

(B) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

185. Evaluate: $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

186. Solve the following Linear Programming Problem graphically :

Maximize : $P = 70x + 40y$

subject to :$3x + 2y \leq 9$

$3x + y \leq 9$

$x \geq 0, y \geq 0$

187. **(A)** In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly?

OR

(B) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

SECTION-E

Case Study-I

188. An organization conducted bike race under two different categories - Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?
(III) Let $R: B \rightarrow B$ be defined by $R = \{(x, y): x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function $f: B \rightarrow G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Case Study-II

189. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160 . From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190 . Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250 .

Based on the above information, answer the following questions:

(I) Convert the given above situation into a matrix equation of the form $AX = B$.

(II) Find $|A|$.

(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$.

Case Study-III

190. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions:

(I) Show that $(x^2 - y^2)dx + 2xydy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

General Instructions :

Read the following instructions very carefully and follow them :

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(ii) Question paper is divided into FIVE Sections-Section A, B, C, D and E.
(iii) In Section A - Question Number 1 to 18 are Multiple Choice Questions (MCQ) type and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B - Question Number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.
(v) In Section C-Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In Section D - Question Number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
(vii) In Section E - Question Number 36 to 38 are case study based questions carrying 4 marks each where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section −D and 2 questions in Section - E.
(ix) Use of calculators is NOT allowed.

SECTION A

191. The domain of the function $\cos^{-1}x$ is :

(a) $[0, \pi]$ (b) $(-1,1)$
(c) $[-1,1]$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

192. In a 3×3 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{1}{2}|-3i + j|$, the element a_{31} is :

(a) -4 (b) 5
(c) 4 (d) 8

193. **If $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$, then $2x-y$ is equal to :**

(a) 3 (b) 13

(c) -3 (d) 0

194. **If $A=\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}$, then A^{-1} is given by :**

(a) $\begin{bmatrix}5 & -2\\-3 & 1\end{bmatrix}$ (b) $\begin{bmatrix}-5 & 2\\3 & -1\end{bmatrix}$

(c) $\begin{bmatrix}-5 & 3\\2 & -1\end{bmatrix}$ (d) $\begin{bmatrix}5 & 3\\1 & 2\end{bmatrix}$

195. **In the determinant $\begin{vmatrix}2 & -3 & 5\\6 & 0 & 4\\1 & 5 & -7\end{vmatrix}$, M_{23} is : (where M_{ij} denotes the minor of element a_{ij})**

(a) 7 (b) -13

(c) 13 (d) -7

196. **If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x=1$ is equal to :**

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 1 (d) $\frac{1}{2}$

197. **The value of k for which the function f given by $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & \text{if } x\neq\frac{\pi}{2}\\ 5, & \text{if } x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, is :**

(a) 6 (b) 5

(c) $\frac{5}{2}$ (d) 10

198. **$\int\frac{dx}{\sin^2 2x\cdot\cos^2 2x}$ equals :**

(a) $\frac{1}{2}[\tan 2x+\cot 2x]+C$

(b) $\tan 2x-\cot 2x+C$

(c) $\frac{1}{2}[\tan 2x-\cot 2x]+C$

(d) $\frac{1}{2}[\cot 2x-\tan 2x]+C$

199. **$\int_{-\pi/4}^{\pi/4}\sin^3 x dx$ equals :**

(a) $2\int_0^{\pi/4}\sin^3 x dx$ (b) 0

(c) 1 (d) $\int_0^{\pi/4}\sin^3 x dx$

200. **The general solution of the differential equation $\frac{dy}{dx}=e^{x-y}$ is:**

(a) $e^x+e^y=C$ (b) $e^x-e^{-y}=C$

(c) $-e^x-e^y=C$ (d) $e^x-e^y=C$

201. **The sum of the order and the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2+3x\left(\frac{d^2y}{dx^2}\right)^4=\log x$, is :**

(a) 5 (b) 6

(c) 7 (d) 4

202. **Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. $\vec{a}+\vec{b}$ is a unit vector, if :**

(a) $\theta=\frac{\pi}{3}$ (b) $\theta=\frac{\pi}{4}$

(c) $\theta=\frac{\pi}{2}$ (d) $\theta=\frac{2\pi}{3}$

203. **If $(2\hat{i}+6\hat{j}-22\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$, then $\lambda-\mu$ is equal to :**

(a) -8 (b) -14

(c) 14 (d) 8

204. If the position vectors of two points A and B are $\hat{i}+2\hat{j}-3\hat{k}$ and $-\hat{i}-2\hat{j}+\hat{k}$ respectively, then the direction cosines of the vector $\overrightarrow{BA}$ are :

(a) $\frac{2}{6}, -\frac{4}{6}, -\frac{4}{6}$ (b) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$

(c) $\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

205. The value of λ for which the lines $\frac{x-5}{7}=\frac{2-y}{5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{2y-1}{\lambda}=\frac{z}{3}$ are at right angles, is :

(a) 2 (b) 4

(c) -4 (d) -2

206. The solution set of the inequation $2x+y\geq 5$ is :

(a) half plane that contains the origin

(b) open half plane not containing the origin and not containing the points on the line $2x+y=5$.

(c) whole xy-plane except the points lying on the line $2x+y=5$.

(d) open half plane not containing the origin, but containing the points on the line $2x+y=5$.

207. The minimum value of $z=3x+8y$ subject to the constraints $x\leq 20, y\geq 10$ and $x\geq 0, y\geq 0$ is :

(a) 80 (b) 140

(c) 0 (d) 60

208. Two events A and B will be independent, if :

(a) A and B are mutually exclusive

(b) $P(A)=P(B)$

(c) $P(A'B')=[1-P(A)][1-P(B)]$

(d) $P(A)+P(B)=1$

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

209. Assertion (A) : Matrix $A=\begin{bmatrix}0 & -3 & 5\\ 3 & 0 & -2\\ -5 & 2 & 0\end{bmatrix}$ is a skew-symmetric matrix.

Reason (R) : If $A'=-A$, then A is a skew-symmetric matrix.

210. Assertion (A) : The vector equation of a line passing through the points $A(-1,0,2)$ and $B(3,4,6)$ is $\vec{r}=-\hat{i}+2\hat{k}+\lambda(\hat{i}+\hat{j}+\hat{k})$.

Reason (R) : The equation of a line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$, is $\vec{r}=\vec{a}+\lambda\vec{b}$.

SECTION B

211. (A) Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

OR

(B) Check the injectivity and surjectivity of the function $f: N\to N$, given by $f(x)=x^3$.

212. If $\cos y = x\cos(a+y)$, and $\cos a \neq \pm 1$, prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$.

213. Find the projection of the vector $7\hat{i}-\hat{j}+8\hat{k}$ on the vector $\hat{i}+2\hat{j}+2\hat{k}$.

214. (A) In a parallelogram ABCD , the sides AB and AD are represented by the vectors $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$ respectively. Find the unit vector parallel to its diagonal $\overrightarrow{AC}$.

OR

(B) Find the angle between the pair of lines given by $\vec{r}=\hat{i}+2\hat{j}-2\hat{k}+\lambda(\hat{i}-2\hat{j}-2\hat{k})$ and $\vec{r}=3\hat{i}-5\hat{j}+\hat{k}+\mu(3\hat{i}+2\hat{j}-6\hat{k})$

215. The radius of an air bubble is increasing at the rate of 0.5 cm/s. At what rate is the surface area of the bubble increasing when the radius is 1.5 cm ?

SECTION C

216. Find :

$$\int \frac{dx}{1+\cot x}$$

217. **(A)** Evaluate :

$$\int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$$

OR

(B) Evaluate :

$$\int_0^{\pi/4} \log(1+\tan x)dx$$

218. Find :

$$\int \frac{x}{(x^2+1)(x-1)}dx$$

219. **(A)** Find a particular solution of the differential equation $\frac{dy}{dx}+y\cot x=4x\operatorname{cosec}x$, $(x\neq 0)$, given that $y=0$ when $x=\frac{\pi}{2}$.

OR

(B) Solve the differential equation $xdy-ydx=\sqrt{x^2+y^2}dx$.

220. The objective function $z=4x+3y$ of a linear programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are $A(0,700)$, $B(100,700)$, $C(200,600)$ and $D(400,200)$. Find the point at which z is maximum and the point at which z is minimum. Also, find the corresponding maximum and minimum values of z.

221. **(A)** A man is known to speak the truth 3 out of 5 times. He throws a pair of different coins and reports that he got a pair of heads. Find the probability that a pair of heads actually occurs.

OR

(B) From a lot of 10 bulbs which includes 2 defectives, a sample of 2 bulbs is drawn at random without replacement. Find the probability distribution of the number of defective bulbs. Hence, find the mean.

SECTION D

222. **(A)** A relation R in the set $A=\{5,6,7,8,9\}$ is given by $R=\{(x,y):|x-y| \text{ is divisible by } 2\}$. Write R in roster form and prove that R is an equivalence relation. Also, find the elements related to element 7.

OR

(B) Let $A=R-\{3\}$ and $B=R-\{1\}$ be two sets. Prove that the function $f:A\rightarrow B$ given by $f(x)=\left(\frac{x-2}{x-3}\right)$ is onto. Is the function f one-one? Justify your answer.

223. Using matrices, solve the following system of linear equations:

$x-y+2z=7;\ 3x+4y-5z=-5;\ 2x-y+3z=12$

224. Find the area of the region bounded by the curve $y=\sqrt{x}$, the line $x=2y+3$ and the x -axis, using integration.

225. **(A)** Find the shortest distance between the lines whose vector equations are :

$\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=3\hat{i}-3\hat{j}-5\hat{k}+\mu(-2\hat{i}+3\hat{j}+8\hat{k})$

OR

(B) Find the foot of the perpendicular drawn from the point $(2,3,-8)$ to the line $\frac{x-4}{2}=\frac{y}{-6}=\frac{z-1}{3}$. Also, find the perpendicular distance of the given line from the given point.

SECTION E

Case Study – 1

226. A balloon is being inflated with the help of an air pump, and it remains spherical. Its radius, the surface area and the volume of air in it are all increasing.

Based on the above, answer the following questions:

(i) Are the quantities : radius, surface area and volume of the spherical balloon changing at the same rate or different rates, when air is filled in it?

(ii) Write the expressions for the surface area (S) and the volume (V) of the balloon at any time ' t ' in terms of radius ' r ' at that instant.

(iii) (A) At the instant when the radius of the balloon is 6 cm and the radius (r) is increasing at the rate of 2 cm/s, find at what rate the surface area (S) of the balloon is increasing.

OR

(B) At the instant when the radius of the balloon is 6 cm and the radius (r) is increasing at the rate of 2 cm/s, find at what rate the volume (V) of the spherical balloon is increasing.

Case Study – 2

227. A fighter-jet of the enemy is flying along the parabolic path $4y = x^2$. A soldier is located at the point $(0,5)$ and is aiming to shoot down the jet when it is nearest to him.

Based on the above, answer the following questions:

(i) Let (x, y) be the position of the jet at any instant. Express the distance between the soldier and the jet as the function $f(x)$.

(ii) Taking $S = [f(x)]^2$, find $\frac{dS}{dx}$.

(iii) (A) What will be the position of the jet when the soldier shoots it down?

OR

(B) What will be the distance between the soldier and the jet at the instant when he shoots it down?

Case Study - 3

228. Read the following passage and answer the questions given below:

There are ten cards numbered 1 to 10 and they are placed in a box and then mixed up thoroughly. Then one card is drawn at random from the box.

Based on the above, answer the following questions:

(i) What is the probability that the number on the drawn card is greater than 4 ?

(ii) If it is known that the number on the drawn card is greater than 4 , then what is the probability that it is an even number?