General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) Question paper is divided into FIVE Sections - Section A, B, C, D and E.
(iii) In Section A - Questions Number 1 to 18 are Multiple Choice Questions ($MCQs$) type and Questions Number 19 & 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B - Questions Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
(v) In Section C-Questions Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In Section D - Questions Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
(vii) In Section E-Questions Number 36 to 38 are case study based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section -D and 2 questions in Section - E.
(ix) Use of calculators is NOT allowed.

SECTION-A

This section has 20 multiple choice questions of 1 mark each.

1. **A function $f: \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :**
   (a) injective but not surjective.
   (b) surjective but not injective.
   (c) both injective and surjective.
   (d) neither injective nor surjective.

2. **If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b = c)$ is :**
   (a) 0 (b) 1
   (c) -10 (d) 10

3. **If A is a square matrix of order 3 such that the value of $|\text{adj} \cdot A| = 8$, then the value of** $|A^T|$ is :
   (a) $\sqrt{2}$ (b) $-\sqrt{2}$
   (c) 8 (d) $2\sqrt{2}$

4. **If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is:**
   (a) -4 (b) 1
   (c) 3 (d) 4

5. **If $[x \quad 2 \quad 0] \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \quad 1] \begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is :**
   (a) -1 (b) 0
   (c) 1 (d) 2

6. **Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j)- minimum (i, j) :**
   (a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
   (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. **If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :**
   (a) -1 (b) 1
   (c) -e (d) $-\frac{1}{e}$

8. **Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :**
   (a) $\sin x e^{\sin^2 x}$ (b) $\cos x e^{\sin^2 x}$
   (c) $-2\cos x e^{\sin^2 x}$ (d) $-2\sin^2 x \cos x e^{\sin^2 x}$

9. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :

(a) 2 (b) 1

(c) 0 (d) -2

10. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :

(a) -60 units/sec (b) 60 units/sec

(c) -70 units/sec (d) -140 units/sec

11. $\int \frac{1}{x(\log x)^2} dx$ is equal to :

(a) $2\log(\log x) + c$ (b) $-\frac{1}{\log x} + c$

(c) $\frac{(\log x)^3}{3} + c$ (d) $\frac{3}{(\log x)^3} + c$

12. The value of $\int_{-1}^{1} x|x| dx$ is :

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $-\frac{1}{6}$ (d) 0

13. Area of the region bounded by curve $y^2 = 4x$ and the X -axis between $x = 0$ and $x = 1$ is :

(a) $\frac{2}{3}$ (b) $\frac{8}{3}$

(c) 3 (d) $\frac{4}{3}$

14. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is :

(a) 4 (b) 3

(c) 2 (d) not defined

15. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3: 1 and S is the mid-point of line segment PR. The position vector of S is :

(a) $\frac{\vec{p}+3\vec{q}}{4}$ (b) $\frac{\vec{p}+3\vec{q}}{8}$

(c) $\frac{5\vec{p}+3\vec{q}}{4}$ (d) $\frac{5\vec{p}+3\vec{q}}{8}$

16. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(a) $\frac{5\pi}{6}$ (b) $\frac{3\pi}{4}$

(c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{4}$

17. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line : $\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(a) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ (b) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ (d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

18. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :

(a) $A \subset B$, but $A \neq B$ (b) $A = B$

(c) $A \cap B = \phi$ (d) $P(A) = P(B)$

Assertion - Reason Based Questions

Direction : In questions numbers 19 and 20, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : Domain of $y = \cos^{-1}(x)$ is $[-1,1]$.

Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

20. **Assertion (A) :** The vectors

$\vec{a} = 6\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$

$\vec{b} = 10\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$\vec{c} = 4\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

represent the sides of a right angled triangle.

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

SECTION – B

This section has 5 Very Short Answer questions of 2 marks each.

21. Find value of k if $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$.

22. **(A)** Verify whether the function f defined by $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ or not.

OR

(B) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

23. The area of the circle is increasing at a uniform rate of $2\ \text{cm}^2/\text{sec}$. How fast is the circumference of the circle increasing when the radius $r = 5$ cm ?

24. **(A)** Find: $\int \cos^3 x e^{\log \sin x}\, dx$

OR

(B) Find: $\int \frac{1}{5+4x-x^2}\, dx$

25. Find the vector equation of the line passing through the point $(2,3,-5)$ and making equal angles with the co-ordinate axes.

SECTION – C

There are 6 short answer questions in this section. Each is of 3 marks.

26. **(A)** Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

OR

(B) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

27. If $x = a\sin^3\theta, y = b\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

28. **(A)** Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}\, dx$

OR

(B) Find : $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

29. **(A)** Find the particular solution of the differential equation

$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}; y(0) = 5.$

OR

(B) Solve the following differential equation :

$x^2 dy + y(x + y)dx = 0$

30. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k}$ and hence verify your answer.

31. The random variable X has the following probability distribution where a and b are some constants :

X	1	2	3	4	5
P(X)	0.2	a	a	0.2	b

If the mean $E(X) = 3$, then find values of a and b and hence determine $P(X \geq 3)$.

SECTION – D

There are 4 long answer questions in this section. Each question is of 5 marks.

32. **(A)** If A $= \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$x + 2y - 3z = 1$
$2x - 3z = 2$
$x + 2y = 3$

OR

(B) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations:

$x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$

33. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

34. **(A)** Find the co-ordinates of the foot of the perpendicular drawn from the point $(2,3,-8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

OR

(B) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through $(2,-1,1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$L_2 : \vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

35. Solve the following L.P.P. graphically :

Maximise $Z = 60x + 40y$
Subject to
$x + 2y \leq 12$
$2x + y \leq 12$
$4x + 5y \geq 20$
$x, y \geq 0$

SECTION-E

In this section there are 3 case study questions of 4 marks each.

36. **(A)** Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

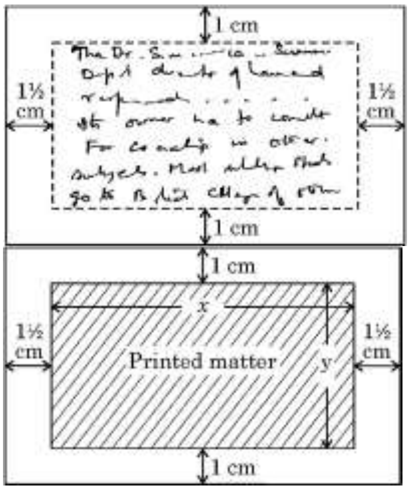
(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(B) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

37. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $11/2$ cm as shown below :



On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

38. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1), P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A \mid E_1)$ and $P(A \mid E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections - A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculators is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

39. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then the value of x, for which A is an identity matrix, is

(a) $\frac{\pi}{2}$ (b) π

(c) 0 (d) $\frac{3\pi}{2}$

40. If the matrix $A = \begin{bmatrix} 0 & 5 & -7 \\ a & 0 & 3 \\ b & -3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the values of ' a ' and ' b ' are :

(a) $a = 5, b = 3$ (b) $a = 5, b = -7$

(c) $a = -5, b = -7$ (d) $a = -5, b = 7$

41. If $\begin{vmatrix} x+2 & x-4 \\ x-2 & x+3 \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 1 & 3 \end{vmatrix}$, then the value of x is :

(a) 1 (b) 2

(c) -2 (d) -1

42. If $\begin{bmatrix} 8 & 14 \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} X$, then matrix X is :

(a) $\begin{bmatrix} 3 & 7 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 7 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 \\ -3 & 7 \end{bmatrix}$

43. The value of k, for which $f(x) = \begin{cases} \frac{\sqrt{3}\cos x + \sin x}{3x + \frac{\pi}{2}}, & x \neq -\frac{\pi}{3} \\ k, & x = -\frac{\pi}{3} \end{cases}$ is continuous at $x = -\frac{\pi}{3}$, is :

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$

(c) $\frac{3}{2}$ (d) 6

44. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = \frac{2}{\sqrt{3}}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

45. If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, then the projection of $(\vec{c} - \vec{b})$ along $\vec{a}$ is:

(a) 15 (b) 5

(c) $\frac{2}{3}$ (d) 1

46. The angle between the lines $\frac{x+1}{2} = \frac{2-y}{-5} = \frac{z}{4}$ and $\frac{x-3}{1} = \frac{y-7}{2} = \frac{5-z}{3}$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

47. The Cartesian equations of a line are given as $6x - 2 = 3y + 1 = 2z - 2$
The direction ratios of the line are :

(a) $2, -1, 3$ (b) $1, -2, -3$

(c) $1, 2, 3$ (d) $3, 1, 2$

48. The solution set of the inequation $2x + 3y < 6$ is :

(a) open half-plane not containing origin

(b) whole xy-plane except the points lying on the line $2x + 3y = 6$

(c) open half-plane containing origin

(d) half-plane containing the origin and the points lying on the line $2x + 3y = 6$

49. The maximum value of the objective function $z = 3x + 5y$ subject to the constraints $x \geq 0, y \geq 0$ and $4x + 3y \leq 12$ is:

(a) 15 (b) 29

(c) 9 (d) 20

50. If the points $A(3, -2), B(k, 2)$ and $C(8, 8)$ are collinear, then the value of k is :

(a) 2 (b) -3

(c) 5 (d) -4

51. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ is equal to :

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) $-\frac{3}{2}$

52. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$ **is equal to :**

(a) $\cot x + \tan x + c$ (b) $-\cot x + \tan x + c$
(c) $\cot x - \tan x + c$ (d) $-\cot x - \tan x + c$

53. The solution of the differential equation $\frac{dy}{dx} = 1 - x + y - xy$ **is :**

(a) $\log|1+y| = x - \frac{x^2}{2} + c$

(b) $\log|1+y| = -x + \frac{x^2}{2} + c$

(c) $e^y = x - \frac{x^2}{2} + c$

(d) $e^{(1+y)} = -x + \frac{x^2}{2} + c$

54. The degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^5 = 0$ **is :**

(a) 2 (b) 3
(c) 4 (d) 5

55. The integrating factor of the differential equation $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$ **is :**

(a) $e^{\sec x}$ (b) $\sec x + \tan x$
(c) $\sec x$ (d) $\cos x$

56. The probabilities of A, B and C solving a problem are $\frac{1}{3}, \frac{1}{5}$ **and** $\frac{1}{6}$ **respectively. The probability that the problem is solved, is :**

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$
(c) $\frac{1}{90}$ (d) $\frac{1}{3}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

57. Assertion (A) : $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

Reason (R): $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

58. Assertion (A) : If the side of a square is increasing at the rate of 0.2 cm/s, then the rate of increase of its perimeter is 0.8 cm/s.

Reason (R): Perimeter of a square = 4 (side).

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

59. (A) Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

OR

(B) Find the domain of the function $y = \cos^{-1}(x^2 - 4)$.

60. (A) Differentiate $\cot^{-1}\left(\sqrt{1+x^2} + x\right)$ w.r.t. x .

OR

(B) If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

61. Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ **is**

(A) strictly increasing
(B) strictly decreasing.

62. Show that of all rectangles inscribed in a given circle, the square has the maximum area.

63. Find : $\int \operatorname{cosec}^3(3x+1)\cot(3x+1)dx$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

64. If $x = a\cos\theta$ and $y = b\sin\theta$, then prove that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

65. Find: $\int \frac{2x}{(x^2+1)(x^2+3)}dx$

66. **(A)** Evaluate : $\int_{-6}^{6} |x+2|dx$

OR

(B) Find: $\int \left(\frac{4-x}{x^5}\right) e^x dx$

67. **(A)** Find the particular solution of the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$, given that $y(1) = 0$.

OR

(B) Solve the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{3}$.

68. The corner points of the feasible region determined by the system of linear constraints are $A(0,40), B(20,40), C(60,20)$ and $D(60,0)$. The objective function of the L.P.P. is $z = 4x + 3y$. Find the point of the feasible region at which the value of objective function is maximum and the point at which the value is minimum. Hence, find the maximum and the minimum values.

69. **(A)** A card is randomly drawn from a well-shuffled pack of 52 playing cards. Events A and B are defined as under :

A : Getting a card of diamond

B : Getting a queen

Determine whether the events A and B are independent or not.

OR

(B) Find the probability distribution of the number of doublets in three throws of a pair of dice.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

70. **(A)** Let $A = \{x \mid x \in Z, 0 \le x \le 12\}$. Show that the relation $R = \{(a,b): a,b \in A, (a-b) \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of elements related to 2.

OR

(B) Let $A = \mathbb{R} - \{4\}$ and $B = \mathbb{R} - \{1\}$ and let function $f: A \to B$ be defined as $f(x) = \frac{x-3}{x-4}$ for $\forall x \in A$. Show that f is one-one and onto.

71. Using matrices, solve the following system of linear equations:

$3x + 4y + 2z = 8; 2y - 3z = 3; x - 2y + 6z = -2$

72. Using integration, find the area of the region bounded by the curve $y = x^2, x = -1, x = 1$ and the x -axis.

73. **(A)** Write the vector equations of the following lines and hence find the shortest distance between them:

$\frac{x+1}{2} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

OR

(B) Find the length and the coordinates of the foot of the perpendicular drawn from the point P(5,9,3) to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the coordinates of the image of the point P in the given line.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

74. The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the relation $y = 4x - \frac{1}{2}x^2$, where x is the number of days it is exposed to sunlight.

Based on the above, answer the following questions:

(i) Find the rate of growth of the plant with respect to sunlight.

(ii) What is the number of days it will take for the plant to grow to the maximum height?

(iii) What is the maximum height of the plant?

Case Study – 2

75. A cricket match is organised between two clubs P and Q for which a team from each club is chosen. Remaining players of club P and club Q are respectively sitting along the lines AB and CD , where the points are $A(3,4,0), B(5,3,3), C(6,-4,1)$ and $D(13,-5,-4)$.

Based on the above, answer the following questions:

(i) Write the direction ratios of vector $\overrightarrow{AB}$.

(ii) Write a unit vector in the direction of $\overrightarrow{CD}$.

(iii) (a) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

OR

(b) Write a vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This Question Paper contains 38 questions. All questions are compulsory.
(ii) Question Paper is divided into five Sections - Section A, B, C, D and E.
(iii) In Section A - Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and Questions no. 19 & 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B - Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
(v) In Section C - Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In Section D - Questions no. 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
(vii) In Section E - Questions no. 36 to 38 are case study based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 3 questions in Section D and 2 questions in Section E.
(ix) Use of calculators is not allowed.

SECTION - A

This section consists of 20 multiple choice questions of 1 mark each.

76. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is :

(a) 0 (b) 5
(c) 10 (d) 25

77. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrixA is :

(a) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$
(c) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (d) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

78. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \cdots$ is :

(a) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

79. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of $|A\,(\text{adj. } A)|$ is :

(a) 100 I (b) 10 I
(c) 10 (d) 1000

80. Given that $[1\ x]\begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$, the value of x is :

(a) -4 (b) -2
(c) 2 (d) 4

81. Derivative of e^{2x} with respect to e^x, is :

(a) e^x (b) $2e^x$
(c) $2e^{2x}$ (d) $2e^{3x}$

82. For what value of k , the function given below is continuous at $x = 0$?

$$\mathbf{f(x)} = \begin{cases} \dfrac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

(a) 0 (b) $\frac{1}{4}$

(c) 1 (d) 4

83. The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{18}$

84. The general solution of the differential equation $xdy + ydx = 0$ is :

(a) $xy = c$ (b) $x + y = c$

(c) $x^2 + y^2 = c^2$ (d) $\log y = \log x + c$

85. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y (y > 0)$ is :

(a) $\frac{1}{x}$ (b) x

(c) y (d) $\frac{1}{y}$

86. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$

87. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(a) an equilateral triangle

(b) an obtuse-angled triangle

(c) an isosceles triangle

(d) a right-angled triangle

88. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is:

(a) a^2 (b) $2a^2$

(c) $3a^2$ (d) 0

89. The vector equation of a line passing through the point $(1, -1, 0)$ and parallel to Y -axis is :

(a) $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} - \hat{j})$ (b) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$

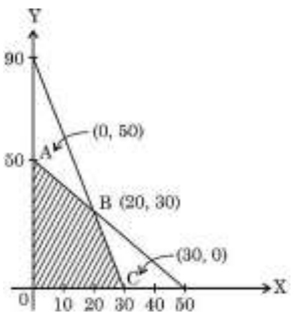
(c) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{k}$ (d) $\vec{r} = \lambda\hat{j}$

90. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to:

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 2 (d) 3

91. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is :



(a) 50 (b) 110

(c) 120 (d) 170

92. The probability distribution of a random variable X is :

X	0	1	2	3	4

P(X)	0.1	k	2 k	k	0.1

where k is some unknown constant.

The probability that the random variable X takes the value 2 is :

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{4}{5}$ (d) 1

93. The function $f(x) = kx - \sin x$ is strictly increasing for

(a) $k > 1$ (b) $k < 1$

(c) $k > -1$ (d) $k < -1$

Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (a), (b), (c) and (d) as given below:

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

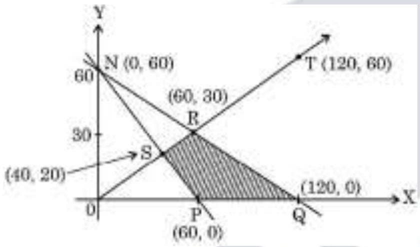
(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

94. Assertion (A) : The relation $R = \{(x, y): (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

95. Assertion (A) : The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.



Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

SECTION – B

In this section there are 5 very short answer type questions of 2 marks each.

96. (A) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

OR

(B) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

97. (A) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(B) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

98. Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

99. The volume of a cube is increasing at the rate of $6\ cm^3/s$. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

100. Find : $\int \frac{1}{x(x^2-1)} dx$.

SECTION – C

In this section there are 6 short answer type questions of 3 marks each.

101. Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, find $\frac{dy}{dx}$.

102. **(A)** Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{1+\cos 2x+\sin 2x}$

OR

(B) Find : $\int e^x\left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right]dx$

103. Find : $\int \frac{3x+5}{\sqrt{x^2+2x+4}} dx$

104. **(A)** Find the particular solution of the differential equation $\frac{dy}{dx} = y\cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

OR

(B) Find the particular solution of the differential equation $\left(xe^{\frac{y}{x}} + y\right) dx = xdy$, given that $y = 1$ when $x = 1$.

105. Solve the following linear programming problem graphically :

Maximise $Z = 2x + 3y$ subject to the constraints:

$x + y \leq 6$
$x \geq 2$
$y \leq 3$
$x, y \geq 0$

106. **(A)** A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(B) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

SECTION – D

In the section there are 4 long answer type questions of 5 marks each.

107. **(A)** Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X -axis and the ordinates $x = -2$ and $x = 2$, using integration.

OR

(B) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2, x = 2$, and the X -axis.

108. **(A)** Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function $f: A \rightarrow B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

OR

(B) Check whether the relation S in the set of real numbers R defined by $S = \{(a, b)$: where $a - b + \sqrt{2}$ is an irrational number $\}$ is reflexive, symmetric or transitive.

109. If $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

$2x + y - 3z = 13$
$3x + 2y + z = 4$
$x + 2y - z = 8$

110. **(A)** Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

OR

(B) If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3, -4, 7)$.

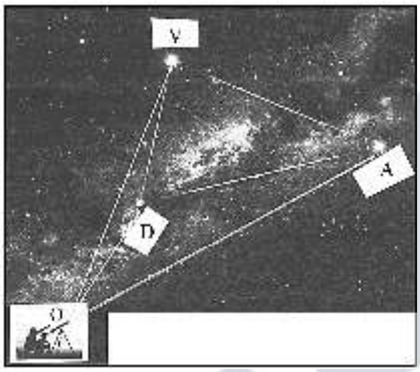
SECTION-E

In this section, there are 3 case study based questions of 4 marks each.

111. A store has been selling calculators at ₹350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.

Based on the above information, answer the following questions:
(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result.
(ii) What rebate in price of calculator should the store give to maximise the revenue?

112. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at $O(0,0,0)$ and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}, 7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions:
(i) How far is the star V from star A ?
(ii) Find a unit vector in the direction of $\overrightarrow{DA}$.
(iii) Find the measure of $\angle VDA$.
OR
(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$?

113. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.
Based on the above information, answer the following questions:
(i) What is the probability that at least one of them is selected?
(ii) Find $P(G \mid \bar{H})$ where G is the event of Jaspreet's selection and $\bar{H}$ denotes the event that Rohit is not selected.
(iii) Find the probability that exactly one of them is selected.
OR
(iii) Find the probability that exactly two of them are selected.

General Instructions :

Read the following instructions very carefully and strictly follow them:
(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This question paper is divided into five Sections - A, B, C, D and **E**.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3

questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculators is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

114. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(a) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (b) $a_{ij} = 1, \forall i, j$

(c) $a_{ij} = 0, \forall i, j$ (d) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

115. Let R_+ denote the set of all non-negative real numbers. Then the function $f: R_+ \to R_+$ defined as $f(x) = x^2 + 1$ is :

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

116. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, $(a + b + c + d)$ is equal to :

(a) 2a (b) 2 b
(c) 2 c (d) 0

117. A function $f(x) = |1 - x + |x||$ is :

(a) discontinuous at $x = 1$ only
(b) discontinuous at $x = 0$ only
(c) discontinuous at $x = 0,1$
(d) continuous everywhere

118. If the sides of a square are decreasing at the rate of 1.5 cm/s, the rate of decrease of its perimeter is :

(a) 1.5 cm/s (b) 6 cm/s
(c) 3 cm/s (d) 2.25 cm/s

119. $\int_{-a}^{a} f(x)dx = 0$, if :

(a) $f(-x) = f(x)$ (b) $f(-x) = -f(x)$
(c) $f(a - x) = f(x)$ (d) $f(a - x) = -f(x)$

120. $x\log x \frac{dy}{dx} + y = 2\log x$ is an example of a :

(a) variable separable differential equation.
(b) homogeneous differential equation.
(c) first order linear differential equation.
(d) differential equation whose degree is not defined.

121. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are:

(a) collinear vectors which are not parallel
(b) parallel vectors
(c) perpendicular vectors
(d) unit vectors

122. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is not true?

(a) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
(b) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
(c) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$
(d) $\cos\alpha + \cos\beta + \cos\gamma = 1$

123. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :

(a) feasible solutions (b) constraints
(c) optimal solutions (d) infeasible solutions

124. Let E and F be two events such that $P(E) = 0 \cdot 1, P(F) = 0 \cdot 3, P(E \cup F) = 0 \cdot 4$, then $P(F \mid E)$ is :

(a) 0.6 (b) 0.4
(c) 0.5 (d) 0

125. If A and B are two skew symmetric matrices, then $(AB + BA)$ is :

(a) a skew symmetric matrix
(b) a symmetric matrix
(c) a null matrix
(d) an identity matrix

126. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is:

(a) 2 (b) -2
(c) ± 2 (d) ∓ 2

127. The derivative of 2^x w.r.t. 3^x is :

(a) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$ (b) $\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$
(c) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$ (d) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$

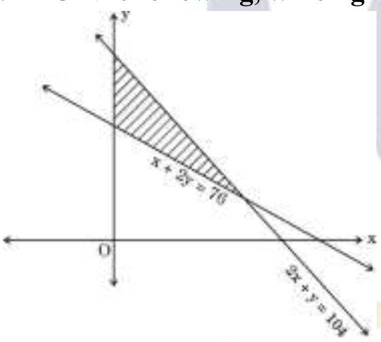
128. If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

(a) $[-6,4]$ (b) $[0,4]$
(c) $[4,6]$ (d) $[0,6]$

129. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π

130. Of the following, which group of constraints represents the feasible region given below ?



(a) $x + 2y \le 76, 2x + y \ge 104, x, y \ge 0$
(b) $x + 2y \le 76, 2x + y \le 104, x, y \ge 0$
(c) $x + 2y \ge 76, 2x + y \le 104, x, y \ge 0$
(d) $x + 2y \ge 76, 2x + y \ge 104, x, y \ge 0$

131. **If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :**

(a) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (b) $30\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(c) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (d) $\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

132. **Assertion (A) :** Every scalar matrix is a diagonal matrix.

Reason (R): In a diagonal matrix, all the diagonal elements are **0**.

133. **Assertion (A) :** Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

Reason (R) : Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

134. Evaluate: $\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$

135. **(A)** If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

OR

(B) Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.

136. **(A)** Evaluate : $\int_0^{\pi/2} \sin 2x \cos 3x dx$

OR

(B) Given $\frac{d}{dx}F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$, find $F(x)$.

137. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4: 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

138. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors.

Prove that $|\vec{a} \times \vec{b}| \le |\vec{a}||\vec{b}|$.

State the condition under which equality holds, i.e., $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

139. **(A)** If $x\cos(p + y) + \cos p \sin(p + y) = 0$, prove that $\cos p \frac{dy}{dx} = -\cos^2(p + y)$, where p is a constant.

OR

(B) Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \frac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$ is a continuous function.

140. **(A)** Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(B) Find the absolute maximum and absolute minimum values of the function f given by $f(x)=\frac{x}{2}+\frac{2}{x}$, on the interval [1,2].

141. Find : $\int \frac{x^2+1}{(x^2+2)(x^2+4)}dx$

142. **(A)** Find: $\int \frac{2+\sin 2x}{1+\cos 2x}e^x dx$

OR

(B) Evaluate : $\int_0^{\pi/4} \frac{1}{\sin x+\cos x}dx$

143. Solve the following linear programming problem graphically :

Maximise $z=4x+3y$,
subject to the constraints
$x+y\leq 800$
$2x+y\leq 1000$
$x\leq 400$
$x,y\geq 0$.

144. The chances of P,Q and R getting selected as CEO of a company are in the ratio 4: 1: 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0.3,0.8 and 0.5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO .

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

145. A relation R on set $A=\{-4,-3,-2,-1,0,1,2,3,4\}$ be defined as $R=\{(x,y): x+y$ is an integer divisible by $2\}$. Show that R is an equivalence relation. Also, write the equivalence class [2].

146. **(A)** It is given that function $f(x)=x^4-62x^2+ax+9$ attains local maximum value at $x=1$. Find the value of ' a ', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(B) The perimeter of a rectangular metallic sheet is 300 cm . It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

147. Using integration, find the area of the region enclosed between the circle $x^2+y^2=16$ and the lines $x=-2$ and $x=2$.

148. **(A)** Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and $\frac{x-1}{0}=\frac{y}{-3}=\frac{z-7}{2}$ and perpendicular to these given lines.

OR

(B) Two vertices of the parallelogram ABCD are given as $A(-1,2,1)$ and $B(1,-2,5)$. If the equation of the line passing through C and D is $\frac{x-4}{1}=\frac{y+7}{-2}=\frac{z-8}{2}$, then find the distance between sides AB and CD . Hence, find the area of parallelogram ABCD .

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study -1

149. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.

A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X=x)=\begin{cases} kx^2, & \text{for } x=1,2,3 \\ 2kx, & \text{for } x=4,5,6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions:

(i) Express the probability distribution given above in the form of a probability distribution table.

(ii) Find the value of k .

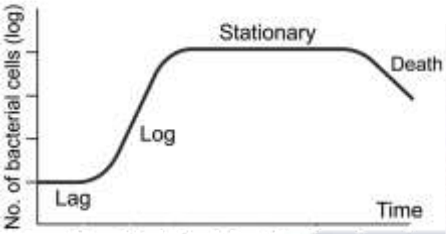
(iii) (A) Find the mean number of hours spent by the student.

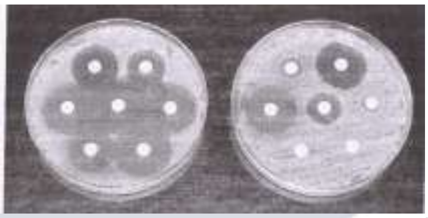
OR

(B) Find $P(1<X<6)$.

Case Study - 2

150. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as : $\frac{dP}{dt}=kP$, where P is the population of bacteria at any time ' t '.

Based on the above information, answer the following questions:

(i) Obtain the general solution of the given differential equation and express it as an exponential function of ' t '.

(ii) If population of bacteria is 1000 at $t=0$, and 2000 at $t=1$, find the value of k.

Case Study - 3

151. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.


Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022-23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices.

(ii) Check whether the system of matrix equations so obtained is consistent or not.

(iii) (A) Find the number of scholarships of each kind given by the school, using matrices.

OR

(B) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school?

General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections —A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculators is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

152. If the sum of all the elements of a 3×3 scalar matrix is 9 , then the product of all its elements is :

(a) 0 (b) 9
(c) 27 (d) 729

153. Let $f: R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :

(a) one-one (b) onto
(c) bijective (d) neither one-one nor onto

154. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(a) 0 (b) 1
(c) 2 (d) 4

155. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$:

(a) 0 (b) 1
(c) 2 (d) infinite

156. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

(a) strictly decreasing on R
(b) strictly increasing on R
(c) neither strictly increasing nor strictly decreasing on R
(d) strictly decreasing on $(-\infty, 0)$

157. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to :

(a) π (b) Zero (0)
(c) $\int_0^{\pi/2} \frac{2\sin x}{1 + \sin x \cos x} dx$ (d) $\frac{\pi^2}{4}$

158. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is :

(a) $\cos x - \sin\left(\frac{y}{x}\right)$ (b) $\frac{y}{x}$
(c) $\frac{x^2 + y^2}{xy}$ (d) $\cos^2\left(\frac{x}{y}\right)$

159. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?

(a) $\vec{a} \cdot \vec{b} \geq |\vec{a}||\vec{b}|$ (b) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$
(c) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$ (d) $\vec{a} \cdot \vec{b} < |\vec{a}||\vec{b}|$

160. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on the x -axis are given by :

(a) (1,0,0) (b) (2,0,0)
(c) $(\sqrt{5}, 0, 0)$ (d) (0,0,0)

161. **The common region determined by all the constraints of a linear programming problem is called :**
(a) an unbounded region
(b) an optimal region
(c) a bounded region
(d) a feasible region

162. **Let E be an event of a sample space S of an experiment, then $P(S \mid E) =$**
(a) $P(S \cap E)$ (b) $P(E)$
(c) 1 (d) 0

163. **If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is false?**
(a) $a_{11} < 0$ (b) $a_{12} + a_{21} = -6$
(c) $a_{13} > a_{31}$ (d) $a_{31} = 0$

164. **The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :**
(a) $\frac{x}{1+x^4}$ (b) $\frac{2x}{1+x^4}$
(c) $-\frac{2x}{1+x^4}$ (d) $\frac{1}{1+x^4}$

165. **The degree of the differential equation $(y'')^2 + (y')^3 = x\sin(y')$ is :**
(a) 1 (b) 2
(c) 3 (d) not defined

166. **The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :**
(a) $2\hat{j}$ (b) $\hat{j}$
(c) $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{k}}{\sqrt{2}}$

167. **Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are:**
(a) $2,-1,6$ (b) $2,1,6$
(c) $2,1,3$ (d) $2,-1,3$

168. **If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :**
(a) 1 (b) 2
(c) 0 (d) -2

169. **If a line makes an angle of 30° with the positive direction of x -axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :**
(a) 90° (b) 120°
(c) 60° (d) 0°

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

170. **Assertion (A) :** For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.
Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

171. **Assertion (A) :** For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.
Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

172. **(A)** Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

(B) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

173. **(A)** If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

OR

(B) If $y = \operatorname{cosec}(\cot^{-1}x)$, then prove that $\sqrt{1+x^2}\frac{dy}{dx} - x = 0$.

174. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}(x \neq 0)$ respectively, find the value of $(M - m)$.

175. Find: $\int \frac{e^{4x}-1}{e^{4x}+1}dx$

176. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1}x$ is strictly increasing in its domain.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

177. **(A)** If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$.

OR

(B) Show that : $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$

178. **(A)** Evaluate: $\int_{-2}^{2}\sqrt{\frac{2-x}{2+x}}dx$

OR

(B) Find: $\int \frac{1}{x[(\log x)^2 - 3\log x - 4]}dx$

179. **(A)** Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0; y = 2$, when $x = 1$.

OR

(B) Find the general solution of the differential equation :
$ydx = (x + 2y^2)dy$

180. The position vectors of vertices of ΔABC are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of ΔABC.

181. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X .

182. Find : $\int x^2 \cdot \sin^{-1}(x^{3/2})dx$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

183. **(A)** Show that a function $f: R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f: R \to A$ becomes an onto function.

OR

(B) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as :
$(a,b)R(c,d) \Leftrightarrow a - c = b - d$
Show that R is an equivalence relation.

184. Find the equation of the line which bisects the line segment joining points A(2,3,4) and B(4,5,8) and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

185. **(A)** Solve the following system of equations, using matrices :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

where $x, y, z \neq 0$

OR

(B) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

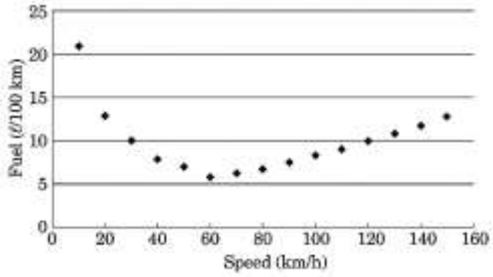
186. If A_1 denotes the area of region bounded by $y^2 = 4x, x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x, x = 4$, find $A_1 : A_2$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study -1

187. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption $F(l/100\text{ km})$ and speed $V(\text{km/h})$ under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

(i) Find F , when $V = 40$ km/h.

(ii) Find $\frac{dF}{dV}$.

(iii) (A) Find the speed V for which fuel consumption F is minimum.

OR

(B) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

Case Study - 2

188. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

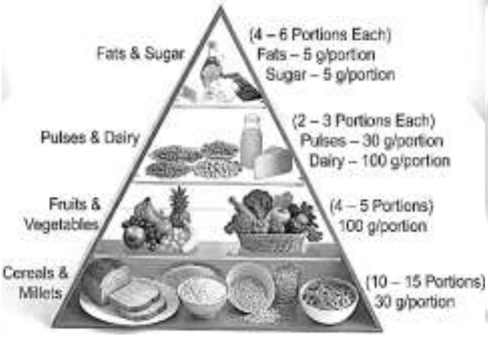


Figure-1

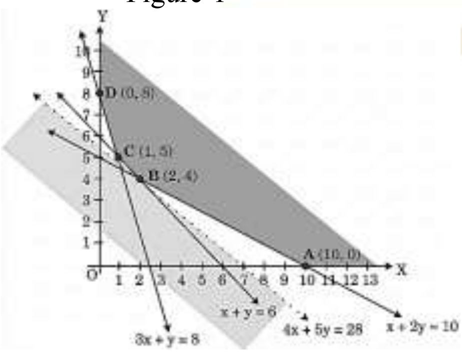


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X(xkg) and food Y(ykg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions:

(i) Identify and write all the constraints which determine the given feasible region in Figure-2.

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.

Case Study - 3

189. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.

Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.
Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.On the basis of the above information, answer the following questions :

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A \mid E_1) + P(A \mid E_2)$.

(iii) (A) Find $P(A)$.

OR

(B) Find $P(E_2 \mid A)$.

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(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculators is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

190. A function $f: R_+ \to R$ (where $R_+$is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

191. If a matrix has 36 elements, the number of possible orders it can have, is:

(a) 13 (b) 3
(c) 5 (d) 9

192. Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$?

(a) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$
(b) f(x) is continuous $\forall x \in \mathbb{R}$
(c) f(x) is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$
(d) f(x) is discontinuous at infinitely many points

193. Let $f(x)$ be a continuous function on $[a, b]$ and differentiable on (a, b). Then, this function $f(x)$ is strictly increasing in (a, b) if

(a) $f'(x) < 0, \forall x \in (a, b)$
(b) $f'(x) > 0, \forall x \in (a, b)$
(c) $f'(x) = 0, \forall x \in (a, b)$
(d) $f(x) > 0, \forall x \in (a, b)$

194. **If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :**

(a) 7 (b) 6
(c) 8 (d) 18

195. **$\int_a^b f(x)dx$ is equal to :**

(a) $\int_a^b f(a-x)dx$
(b) $\int_a^b f(a+b-x)dx$
(c) $\int_a^b f(x-(a+b))dx$
(d) $\int_a^b f((a-x)+(b-x))dx$

196. **Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$. Then, $\hat{a} \cdot \hat{b}$ is equal to :**

(a) $\pm\frac{3}{5}$ (b) $\pm\frac{3}{4}$
(c) $\pm\frac{4}{5}$ (d) $\pm\frac{4}{3}$

197. **The integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} + xy = ax, -1 < x < 1$, is :**

(a) $\frac{1}{x^2-1}$ (b) $\frac{1}{\sqrt{x^2-1}}$
(c) $\frac{1}{1-x^2}$ (d) $\frac{1}{\sqrt{1-x^2}}$

198. **If the direction cosines of a line are $\sqrt{3}k, \sqrt{3}k, \sqrt{3}k$, then the value of k is:**

(a) ± 1 (b) $\pm\sqrt{3}$
(c) ± 3 (d) $\pm\frac{1}{3}$

199. **A linear programming problem deals with the optimization of a/an :**

(a) logarithmic function (b) linear function
(c) quadratic function (d) exponential function

200. **If $P(A \mid B) = P(A' \mid B)$, then which of the following statements is true ?**

(a) $P(A) = P(A')$ (b) $P(A) = 2P(B)$
(c) $P(A \cap B) = \frac{1}{2}P(B)$ (d) $P(A \cap B) = 2P(B)$

201. **$\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to :**

(a) $2x^3$ (b) 2
(c) 0 (d) $2x^3 - 2$

202. **The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is :**

(a) 1 (b) -1
(c) $-2\sqrt{\pi}$ (d) $2\sqrt{\pi}$

203. **The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are:**

(a) 1,2 (b) 2,3
(c) 2,1 (d) 2,6

204. **The vector with terminal point $A(2, -3, 5)$ and initial point $B(3, -4, 7)$ is:**

(a) $\hat{\imath} - \hat{\jmath} + 2\hat{k}$ (b) $\hat{\imath} + \hat{\jmath} + 2\hat{k}$
(c) $-\hat{\imath} - \hat{\jmath} - 2\hat{k}$ (d) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$

205. The distance of point $P(a, b, c)$ from y-axis is :

(a) b (b) b^2

(c) $\sqrt{a^2 + c^2}$ (d) $a^2 + c^2$

206. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is :

(a) 0 (b) 1

(c) 2 (d) 3

207. If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then :

(a) $AB = O$ (b) $AB = -BA$

(c) $BA = O$ (d) $AB = BA$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason(R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

208. Assertion (A) : For matrix$A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi], |A| \in [2, 4]$.

Reason (R): $\cos\theta \in [-1, 1], \forall\theta \in [0, 2\pi]$.

209. Assertion (A) : A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

210. (A) Check whether the function $f(x) = x^2|x|$ is differentiable at $x = 0$ or not.

OR

(B) If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\frac{dy}{dx} = \frac{1+y^4}{4y}$.

211. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

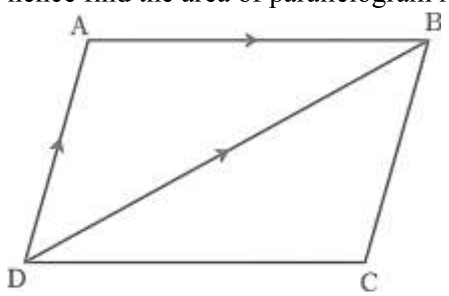
212. (A) Find : $\int x\sqrt{1 + 2x}dx$

OR

(B) Evaluate: $\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

213. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}|\vec{a}|$.

214. In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD .



SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

215. (A) A relation R on set $A = \{1,2,3,4,5\}$ is defined as $R = \{(x, y): |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(B) A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not.a

216. **(A)** If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(B) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.

217. **(A)** Find: $\int \frac{x^2}{(x^2+4)(x^2+9)}dx$

OR

(B) Evaluate : $\int_1^3 (|x-1| + |x-2| + |x-3|)dx$

218. Find the particular solution of the differential equation given by $x^2\frac{dy}{dx} - xy = x^2\cos^2\left(\frac{y}{2x}\right)$, given that when $x = 1, y = \frac{\pi}{2}$.

219. Solve the following linear programming problem graphically :
Maximise $z = 500x + 300y$, subject to constraints
$x + 2y \le 12$
$2x + y \le 12$
$4x + 5y \ge 20$
$x \ge 0, y \ge 0$

220. E and F are two independent events such that $P(\bar{E}) = 0.6$ and $P(E \cup F) = 0.6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$.

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

221. **(A)** If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations:
$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$

OR

(B) If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,find the value of $(a + x) - (b + y)$.

222. **(A)** Evaluate : $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x}dx$

OR

(B) Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx$

223. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.

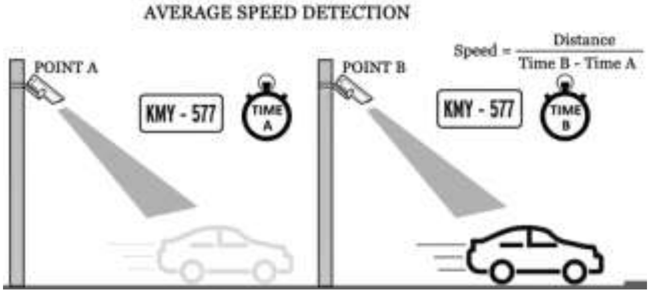
224. The image of point $P(x, y, z)$ with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $P'(1,0,7)$. Find the coordinates of point P.

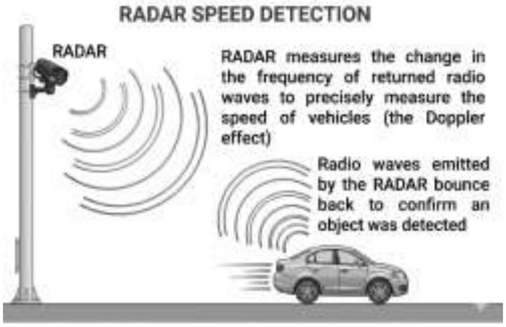
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study -1

225. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.





A camera is installed on a pole at the height of 5 m . It detects a car travelling away from the pole at the speed of 20 m/s. At any point, × m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions:

(i) Expressθ in terms of height of the camera installed on the pole and x .

(ii) Find $\frac{d\theta}{dx}$.

(iii) (A) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

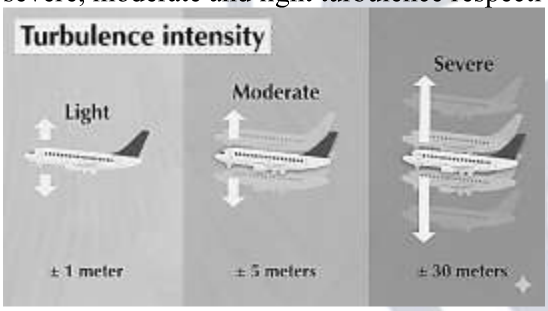
OR

(B) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car.

Case Study - 2

226. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions:

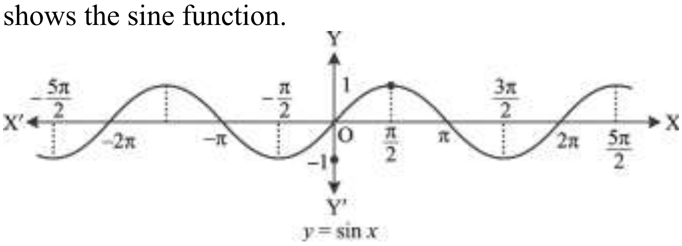
(i) Find the probability that an airplane reached its destination late.

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence.

Case Study - 3

227. If a function $f: X \rightarrow Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g: Y \rightarrow X$ such that $g(y) = x$, where $x \in X$ and $y = f(x), y \in Y$. Function g is called the inverse of function f.

The domain of sine function is R and function sine : $R \rightarrow R$ is neither one-one nor onto. The following graph shows the sine function.



Let sine function be defined from set A to $[-1,1]$ such that inverse of sine function exists, i.e., $\sin^{-1}x$ is defined from $[-1,1]$ to A.

On the basis of the above information, answer the following questions:

(i) If A is the interval other than principal value branch, give an example of one such interval.

(ii) If $\sin^{-1}(x)$ is defined from $[-1,1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$.

(iii) (A) Draw the graph of $\sin^{-1}x$ from $[-1,1]$ to its principal value branch.

OR

(B) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$.